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Abstract

The first chapter studies academic specialization and misallocation of skills in the labor market. I develop a general equilibrium version of the Roy model to study occupations where occupation-specific human capital is obtained through university education and people incur considerable upfront costs to work in a particular occupation. The model embeds a market failure: risk-averse individuals face an incomplete markets problem because they are not able to purchase insurance against adverse occupation-specific shocks. I compare production efficiency and utilitarian welfare in competitive equilibrium to the outcomes of two social planning problems: (i) unconstrained planning problem (ii) 'constrained efficient' planning problem. To get quantitative estimates of the importance of academic specialization, I calibrate the model using data on petroleum, chemical and mechanical engineers. The output loss caused by the lack of insurance depends on model parameters and can potentially be very large.

The objective of the second chapter is to study career concerns in teams and

the possibility of multiple equilibria. I use a information structure where only the joint output of the team, rather than signals for each team member, is observed by the principal. As opposed to the previous literature on the topic, I show the existence of multiple equilibria if either (i) the labor market exhibits increasing returns to perceived talent (ii) there is complementarity in hidden effort. In one equilibrium, both workers exert little effort and have bad career prospects. In the other equilibrium, both workers exert high effort and have good career prospects. I show that linear wage contracts will eliminate the bad equilibrium in case (i) but not in case (ii).

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INCENTIVES

Vesa-Heikki Soini

A DISSERTATION

in

Economics

Presented to the Faculties of the University of Pennsylvania in Partial
Fulfillment of the Requirements for the Degree of Doctor of Philosophy

2016

Supervisor of Dissertation

Guillermo Ordoñez, Associate Professor of Economics

Graduate Group Chairperson

Jesús Fernández-Villaverde, Professor of Economics

Dissertation Committee:

Guillermo Ordoñez, Associate Professor of Economics

Harold Cole, Professor of Economics

Dirk Krueger, Professor of Economics

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ABSTRACT

ESSAYS IN ACADEMIC SPECIALIZATION AND CAREER INCENTIVES

Vesa-Heikki Soini

Guillermo Ordoñez

The first chapter studies academic specialization and misallocation of skills in the labor market. I develop a general equilibrium version of the Roy model to study occupations where occupation-specific human capital is obtained through university education and people incur considerable upfront costs to work in a particular occupation. The model embeds a market failure: risk-averse individuals face an incomplete markets problem because they are not able to purchase insurance against adverse occupation-specific shocks. I compare production efficiency and utilitarian welfare in competitive equilibrium to the outcomes of two social planning problems: (i) unconstrained planning problem (ii) ‘constrained efficient’ planning problem. To get quantitative estimates of the importance of academic specialization, I calibrate the model using data on petroleum, chemical and mechanical engineers. The output loss caused by the lack of insurance depends on model parameters and can potentially be very large.

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Chapter 1

Academic Specialization and Misallocation of Skills in the Labor Market

Abstract

This paper studies academic specialization and misallocation of skills in the labor market. I develop a general equilibrium version of the Roy model to study occupations where occupation-specific human capital is obtained through university education and people incur considerable upfront costs to work in a particular occupation. The model embeds a market failure: risk-averse individuals face an incomplete markets problem because they are not able to purchase insurance against adverse occupation-specific shocks. I compare production efficiency and utilitarian welfare in competitive equilibrium to the outcomes of two social planning problems: (i) unconstrained planning problem (ii) ‘constrained efficient’ planning problem. To get quantitative estimates of the importance of academic specialization, I calibrate the model using data on petroleum, chemical and mechanical engineers. The output loss caused by the lack of insurance depends on model parameters and can potentially be very large.

1.1 Introduction

Academic specialization is increasingly important in the labor market, and for example many technical, medical and legal occupations require knowledge in a narrowly defined field. It is not possible to work in such occupations without first obtaining occupation-specific skills and credentials through specialized university training. At the same time, the demand for the services provided by these occupations is unpredictable and it is difficult to find substitutes for these workers. Does the education system provide an optimal amount of occupation-specific skills?

I develop a general equilibrium version of the Roy (1951) model in which individuals self-select into occupations based on their heterogeneous productivities. These occupational choices determine both output and wage distribution in the economy. The choice of occupation is based on their productivity in that occupation and expected wages of the occupation. Furthermore, people are risk-averse and skills learned at university are occupation-specific. The model embeds a market failure: risk-averse individuals face an incomplete markets problem because they are not able to purchase insurance against adverse occupation-specific shocks. Therefore the social planner's outcome differs from the competitive equilibrium. I study two planning problems: (i) the unconstrained planning problem (ii) the 'constrained efficient' planning problem. The unconstrained problem yields maximal output but assumes that the planner can transfer income *ex post* after occupation-specific

shocks are realized. The second problem does not allow such transfers.

This paper makes two contributions. First, the theoretical model is used to study how the competitive equilibrium allocation could be improved in a world where there is no insurance against adverse occupation-specific shocks. Second, a dynamic version of the model is used to quantify how much more output could be produced if the insurance was available. I use data on petroleum, chemical and mechanical engineers for this purpose. The potential increase in output is estimated to be about 3.7% for these occupations. Given the magnitude of this number, it is puzzling that financial markets do not provide such insurance. Potential reasons for this are discussed in the text.

In a world without insurance, the main theoretical result is that the direction of market failure is the opposite of what one might expect. To illustrate the main forces of the model, imagine that there are two occupations. Both occupations have the same expected productivity but one occupation is risky whereas the other one is safe. An unconstrained planner is going to allocate an equal number of workers to both occupations, because this equalizes expected marginal product of labor across occupations. Due to risk aversion, less people go to the risky occupation in competitive equilibrium. However, the highlight of this section is the constrained efficient allocation which will depend on the wage distribution in the economy. Other things

equal, utilitarian welfare is increasing in the mean but decreasing in the variance of the wage distribution. Putting more people to the risky occupation will increase both the mean and the variance of the wage distribution. I will show that the constrained efficient allocation requires that, compared to the equilibrium, *less* people are allocated to the risky occupation. This feature is only present in a model with heterogeneous productivities.

In general equilibrium, people's choices cause 'pecuniary externalities', i.e. externalities transferred through equilibrium wages. Because markets are incomplete, the first welfare theorem does not apply and pecuniary externalities do not offset each other in equilibrium. As a consequence, it is possible to change labor allocation so that the change in wages improves the outcome. Because of heterogeneous productivities, individuals are always sorted to occupations based on their comparative advantage. This leads inevitably to a wage distribution where people's marginal utilities are different both within and across occupations. The planner is not able to reduce consumption inequality *within* occupations but consumption inequality *across* occupations can be manipulated. Doing so will lead to an increase in utilitarian welfare. Effectively the planner wants to increase wages in the risky occupation to compensate individuals for the risk.

In the beginning of the paper, I study a one-period model. I derive closed-

form solutions for labor allocations, output, utilitarian welfare and optimal taxes for competitive equilibrium and the two social planning problems. The incomplete markets problem affects worker's choices through the risk of wage fluctuations. The fundamental risk of each occupation is related to the variance of occupation-specific TFP (Total Factor Productivity) shocks because these are passed on to occupation-specific wages. The wage risk is mitigated by a low elasticity of substitution between occupation goods and a low capital share of the occupation.

The remainder of the paper studies the unconstrained planning problem in a dynamic version of the model. The purpose is to get a quantitative estimate of the output loss caused by the lack of insurance. In the dynamic model, the inefficiencies that are present in the one-period model will accumulate over time. The unconstrained planner should put more workers to the risky occupations. But in the calibrated version of the model the talent distributions differ across occupations which amplify or mitigate output losses. The move from equilibrium to the unconstrained planner's allocation requires that people are moved from occupations where they are more productive to occupations where they are less productive, or vice versa. The calibrated model is used to run counter-factual analysis of these forces.

The dynamic model also exhibits a decrease in total units of labor for another

reason. In the calibration of my model, the competitive equilibrium exhibits too high variation in the occupational choices compared to the unconstrained planner's problem. The dynamic model exhibits booms and busts for each occupation and people 'over-react' to these changes. After a bust, mean wage of the occupation is low and wage uncertainty tends to be high. This is the time when people's occupational choices tend to be distorted downwards the most since people especially avoid the most uncertain occupations. During a boom period, wages increase and the wage uncertainty decreases. In stationary equilibrium the occupational choices are made by comparing the attractiveness of each occupation with respect to all other occupations. Moreover, all new entrants have to go to some occupation. As sub-optimally few people go to occupations with the highest level of uncertainty, sub-optimally many people have to go to occupations with a lower level of uncertainty. I show that such over-reaction results in a decrease of total efficiency units of labor in the economy, as comparative advantage lines fluctuate too much over time.

The exact magnitude of the effect on production efficiency depends on the elasticity of substitution between occupation goods and to a lesser extent on the capital share of each occupation. The importance of the elasticity of substitution is caused by the fact that the general equilibrium wage mechanism will provide partial insurance against productivity shocks in my model. This result has similarities to

the model of Cole and Obstfeld (1991). The elasticity of substitution governs both the elasticity of wage with respect to existing labor force and the elasticity of wage with respect to the TFP realizations. If the elasticity of substitution is low, people's choices are affected mostly by the existing labor force in each occupation. On the other hand, when the elasticity of substitution increases, people's choices are affected mostly by the expected TFP realizations.

To analyze the impact of various policies and compute some quantitative results, I calibrate the model using data on petroleum, chemical and mechanical engineers. In my model there is a continuum of occupations and therefore I construct a stationary equilibrium with heterogeneous occupations which has similarities to Aiyagari (1994). The structural parameters of the model are estimated by constructing model predictions for the fraction of individuals choosing each discipline in each year and matching these to the data. Since there are no reliable micro-data estimates for the elasticity of substitution for the occupations considered in this paper, I experiment with different parameter values and conclude that the elasticity of substitution is indeed a crucial parameter for production efficiency. The numerical examples show that in a static model the output loss in these occupations is 3.7%. The output loss caused by over-reaction in the dynamic model is approximately 1.2%. The effect of the output losses in these occupations on the aggregate economy is discussed in the quantitative section.

The rest of this chapter is organized as follows. Related literature is discussed below. Section 1.3 introduces a one-period model, defines and characterizes the equilibrium and solves the social planning problems. There is also an example to illustrate the model. Section 1.4 introduces a dynamic model and discusses the solution method. The empirics are in Section 1.5. Section 1.6 shows the quantitative results and section 1.7 discusses policy implications. describes the data and the calibration of model parameters. Section 1.8 concludes. The proofs and derivations are in the Appendix.

1.2 Related Literature

This paper is related to many strands of research. The Roy (1951) model has been previously extended by numerous authors (see Heckman and Honore (1990), Heckman et al. (1998), Heckman and Scheinkman (1987), Heckman and Sedlacek (1985), Rothschild and Scheuer (2013), Borjas (1987) and Gould (2002)). A key difference between this literature and my paper is the assumed distribution for ability. In my model ability for each occupation is distributed according to a Fréchet distribution which makes it possible to obtain closed form solutions for the labor allocation and general equilibrium wages. This makes a closed form analysis of the one period model feasible. Secondly, I add uncertainty about future labor demand to the dynamic model. This creates a market distortion, as occupational choices

are permanent but the state of each occupation changes stochastically over time.

A key concept in my dynamic model is uncertainty about future labor demand for each occupation. This is modeled as occupation-specific business cycles where agents learn sluggishly about the state of each occupation. Starting from Lucas (1987), there is a large literature on the welfare costs of business cycles (Imrohoroglu (1989), Atkeson and Phelan (1994), Krusell and Smith (1999), Krusell et al. (2009), Storesletten et al. (2001)).

Some papers in this literature also consider missing insurance markets and in general estimate the corresponding welfare costs to be very small. My paper does not have aggregate uncertainty and only considers a subset of occupations and hence a generalization to the aggregate economy seems inappropriate. That being said, depending on the value of elasticity of substitution, a calibrated version of my model may potentially give a much higher estimate for the welfare cost of missing insurance markets than what is estimated in the literature.

As sectoral and occupational shocks are often caused by advances in technology, the paper contributes to the literature on technology adoption. Previously Parente and Prescott (1994), Chari and Hopenhayn (1991), as well as Atkeson and Kehoe (2007), among others, have studied the transition of economy after a shock and

technology adoption. Despite the importance of these papers, they do not take into account the role of evolving occupation-specific human capital in the way that this paper does. Krueger and Kumar (2004) study the role of general vs. specific education and relate these to the growth differences between the U.S. and Europe. My paper is complementary but focuses on occupations where specialized education is a prerequisite for being able to work in that occupation. Comin and Hobijn (2004) construct a cross-country dataset containing information about the adoption of several important technologies. Their finding is that human capital is consistently an important determinant of the speed of adoption. Sectoral adjustment has been analyzed theoretically by Matsuyama (1992). However, the analysis in that paper is purely theoretical and does not include a similar general equilibrium framework as this paper.

The discrete choice model was pioneered by McFadden (1974) and has been applied in various contexts by Eaton and Kortum (2002), Fielser (2011), Fuller et al. (1982), and more recently by Hsieh et al. (2013). Additionally there is a large related literature in empirical micro (e.g. Lee and Wolpin (2006) and related literature) but that literature has a different focus. Overall, while there is a large literature on occupation choice, not much work has been done to analyze potential skill misallocation in the labor market using a general equilibrium model with an emphasis on macro issues. This paper aims to fill this gap.

This paper makes the extreme assumption that people choose their occupation once and for all in the beginning of their career. Hence the analysis abstracts from occupation switches which are undoubtedly important in reality. Occupational switches are studied in Kambourov and Manovskii (2008), Kambourov and Manovskii (2009) and subsequent literature. That literature has documented the stylized fact that probability of occupational switches is decreasing in the level of education. A natural follow-up question is what determines the occupation of those highly educated individuals in the first place? This paper sheds light on that question.

1.3 A One-Period Model

This section describes a one-period model. There are three types of agents: workers, a continuum of occupation good firms and final good producers that aggregate the occupation goods. The following subsections describe the model in more detail.

1.3.1 Workers

In the beginning there is no existing labor force in the economy. A continuum of new workers enters the economy and chooses their occupation optimally. After choosing the occupation, the workers will work in the chosen occupations. Labor is

supplied inelastically based on each worker's productivity realization.

Preferences

Agents obtain utility $u(\cdot)$ from consumption and

$$u(c) = \frac{1}{1-\rho} c^{1-\rho}, \quad \rho \in [0, \infty)$$

The only nontrivial decision that agents face is the occupation choice.

Occupation Choice

Individuals choose their occupation $i \in [0, 1]$ in the beginning of the period. Each worker gets a productivity draw ξ_i for each occupation i . If a worker chooses occupation i his labor endowment will be ξ_i . The ξ_i 's are drawn from distribution F which is a Type II extreme value distribution (also known as the Fréchet distribution). Therefore, the cumulative distribution function is:

$$F(z) = \exp(-z^{-\psi})$$

Here $\psi > 1$ is a shape parameter. The mean of the productivity distribution is $\Gamma\left(1 - \frac{1}{\psi}\right)$ where $\Gamma(t) = \int_0^\infty x^{t-1} \exp(-x) dx$ is the gamma function.

1.3.2 Technology

Occupation good producers convert capital and occupation-specific labor into an occupation good. Final good producers convert the the occupation goods to an aggregate final good.

Occupation Good Producer

The firm of occupation i uses capital k and occupation-specific labor l to produce the occupation good. There is a competitive fringe of firms and therefore the firm has no bargaining power. I assume that capital is chosen after the realization of that period's productivity shock. The production function is

$$f(k) = k^\theta l^{1-\theta}$$

where $\theta \in (0, 1)$. The price of capital r is assumed to be exogenous. Wages are determined by general equilibrium conditions.

Final Good Producer

Competitive final good producers aggregate occupation goods according to

$$Y = \left(\int_0^1 \gamma_i Y_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \text{ such that } \int_0^1 \gamma_i di = 1 \quad (1.1)$$

Occupations may face idiosyncratic risk about γ_i but behavior of the aggregate economy is predictable. In most of the paper I assume that γ_i can take two possible

values: γ_g and γ_b where $\gamma_g > \gamma_b$.

1.3.3 Timing

The timing of the one-period model is as follows:

1. In the beginning, there is no existing labor force. The new cohort makes their occupation choice.
2. Realizations of γ_i are observed. These together with the labor endowments determine wages. Individuals consume their earnings.

1.3.4 Definition of Equilibrium

For occupation $i \in [0, 1]$, w_i denotes the wage per labor unit, p_i is the price of the occupation good, L_i is the labor units, K_i is capital and Y_i is the production of the occupation. An equilibrium is a vector $(w_i, p_i, L_i, K_i, Y_i)$ such that

- Given expected utility in occupation i and realized ξ_{ij} , worker j chooses the optimal occupation.
- Worker j in occupation i chooses consumption $c_{ij} = \xi_{ij}w_i$.
- Final good market clears: $\int_j \int_0^1 c_{ij} di dF(j) = Y$.
- Occupation good producer chooses k_i and l_i optimally.
- Market for labor clears: $l_i = L_i$.

- Market for capital clears: $k_i = K_i$.

1.3.5 Preview of Equilibrium

Figure 1.1 shows an example of the equilibrium and the two planning allocations. Moreover, as a benchmark I have also included the constrained efficient allocation for a model where all individuals have the same productivity in all occupations. The key point to note is that, compared to the equilibrium, the direction of the market failure is the opposite for my model and a model with homogeneous productivities. Figure 1.1 is drawn for a model with one risky and one safe occupation.

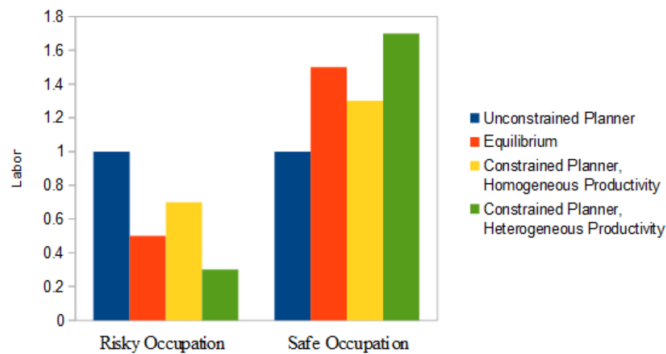


Figure 1.1: Labor Allocations with Two Occupations

1.3.6 Characterization of Equilibrium

Workers choose their occupation based on their expected utility and their labor endowment in that occupation. The equilibrium is characterized by solving the worker's problem, the final good producer's problem and each occupation good

producer's problem.

Final Good Producer

The competitive final good producer chooses Y_i optimally and prices are adjusted so that markets clear. Standard derivations show that the inverse demand is given by

$$p_i = \gamma_i Y_i^{-\frac{1}{\sigma}} Y^{\frac{1}{\sigma}} \quad (1.2)$$

Demand is isoelastic. Since γ_i is stochastic for risky occupations, each occupation faces idiosyncratic risk. Total production in the economy Y affects p_i which can be thought as a pecuniary externality. Since the aggregate behavior of the economy is predictable, the only source of uncertainty is γ_i .

Occupation Good Producer

Each occupation good producer faces an inverse demand (1.2). However, due to the competitive fringe, the firm has to produce occupation goods using a cost-minimizing combination of inputs. Hence the optimization problem is

$$\begin{aligned} \min_{k_j, l_j} \quad & rk_j + w_j l_j \\ \text{s.t.} \quad & k_j^\theta l_j^{1-\theta} \geq \underline{Y} \end{aligned} \quad (1.3)$$

In addition to being the cost-minimizing combination of inputs, the production

has to be such that profits are zero. This requires $p_i Y_i - r K_i - w_i L_i = 0$, where p_i is given in (1.2).

Solving this optimization problem leads to an expression for wages:

$$w_i = c_1 \gamma_i^{\frac{\sigma}{\kappa}} L_i^{-\frac{1}{\kappa}} Y^{\frac{1}{\kappa}} \quad (1.4)$$

where I have used important short-hand notation

$$\kappa = (1 - \theta)\sigma + \theta$$

and collected some parameters to an uninteresting constant

$$c_1 = \left(\left(\frac{\theta}{1 - \theta} \frac{1}{r} \right)^{\theta \frac{\sigma - 1}{\sigma}} (1 - \theta) \right)^{\frac{\sigma}{\kappa}}$$

Equation (1.4) summarizes the response of wages to various realizations of γ_i . A positive productivity shock in occupation i increases wage. Hence the risky occupations will have stochastic wages. The wage rate is decreasing in L_i but increasing in Y . The latter term reflects the externalities that come from other occupations.

Total production in the economy is

$$Y = c_2 \left(\int_0^1 \gamma_i^{\frac{\sigma}{\kappa}} L_i^{\frac{\kappa - 1}{\kappa}} di \right)^{\frac{\kappa}{\kappa - 1}} \quad (1.5)$$

where c_2 is an uninteresting constant given by

$$c_2 = \left(\frac{\theta}{r}\right)^{\frac{\theta}{1-\theta}}$$

Worker's Choice Let v_i denote the value function of an individual who has one efficiency unit of labor in occupation i . In the one-period model, this is the expected utility. In the dynamic model, this will be a value function. Since the utility function is homogeneous of degree $1 - \rho$, the individual's choice rule is to choose discipline i if

$$\xi_i^{1-\rho} v_i \geq \xi_j^{1-\rho} v_j, \quad \forall j \neq i$$

If the individual has ξ_i efficiency units of labor and chooses to go to occupation i , the homogeneity assumption implies that the expected utility for that person is $\xi_i^{1-\rho} v_i$.

It is important to note that when $\rho > 1$, the expected utility is such that $v_i < 0$. The choice rule can be expressed in a convenient form as

$$\xi_i |v_i|^{\frac{1}{1-\rho}} \geq \xi_j |v_j|^{\frac{1}{1-\rho}} \tag{1.6}$$

This rule works for any $\rho \in (0, \infty)$.

Total efficiency units of labor going to occupation i is obtained by integrating the Fréchet distribution and using the cut-off rule in (1.6). Total efficiency units

are

$$L_i = \frac{|v_i|^{\frac{\psi-1}{1-\rho}}}{\left(\int_0^1 |v_j|^{\frac{\psi-1}{1-\rho}} dj\right)^{\frac{\psi-1}{\psi}}} \Gamma\left(1 - \frac{1}{\psi}\right)$$

Substituting in the expected utility of the one-period model and simplifying gives the competitive equilibrium labor allocation:

$$L_{i,eq} = \frac{E\left(\gamma_i^{(1-\rho)\frac{\sigma}{\kappa}}\right)^{\frac{1}{1-\rho} \frac{\kappa(\psi-1)}{\kappa+\psi-1}}}{\left(\int_0^1 E\left(\gamma_j^{(1-\rho)\frac{\sigma}{\kappa}}\right)^{\frac{1}{1-\rho} \frac{\kappa\psi}{\kappa+\psi-1}} dj\right)^{\frac{\psi-1}{\psi}}} \Gamma\left(1 - \frac{1}{\psi}\right) \quad (1.7)$$

Labor in occupation i is therefore determined by an occupation i specific term divided by an integral that summarizes all occupations. This ratio is multiplied by the mean of the Fréchet distribution.

1.3.7 Unconstrained Planner's Problem

Next I will solve for the unconstrained planner's allocation which is also the allocation that maximizes output in the economy. The planner chooses numbers $h_i \in \mathbb{R}_+$ for occupation i such that a worker goes to occupation i if

$$h_i \xi_i \geq h_j \xi_j \quad \forall j \neq i \quad (1.8)$$

The labor units going to sector i can then be solved by integrating the Fréchet distribution and using the cut-off rule in (1.8):

$$L_i = \frac{h_i^{\psi-1}}{\left(\int_0^1 h_j^\psi dj\right)^{\frac{\psi-1}{\psi}}} \Gamma\left(1 - \frac{1}{\psi}\right) \quad (1.9)$$

The unconstrained planner's maximization problem is:

$$\begin{aligned} \max_{h_i, K_i} & \left(\int_0^1 \gamma_i Y_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \\ \text{s.t.} & \\ Y_i &= K_i^\theta L_i^{1-\theta} \\ L_i &= h_i^{\psi-1} \left(\int_0^1 h_j^\psi dj \right)^{\frac{1-\psi}{\psi}} \Gamma\left(1 - \frac{1}{\psi}\right) \end{aligned} \quad (1.10)$$

In words, the planner maximizes output subject to the constraint that occupation good is produced according to the same production function as before and the labor units are consistent with the choice rule.

Planner's Problem: Solution

The planner's problem is differentiable in h_i . Hence the planner's problem is solved by taking the first-order condition with respect to h_i and K_i . As shown by the derivations in the Appendix, the unconstrained planner's allocation is

$$L_{i,om} = \frac{E\left(\gamma_i^{\frac{\sigma}{\kappa}}\right)^{\frac{\kappa(\psi-1)}{\kappa+\psi-1}}}{\left(\int_0^1 E\left(\gamma_j^{\frac{\sigma}{\kappa}}\right)^{\frac{\kappa\psi}{\kappa+\psi-1}} dj\right)^{\frac{\psi-1}{\psi}}} \Gamma\left(1 - \frac{1}{\psi}\right) \quad (1.11)$$

Taxes that Implement the Unconstrained Planner's Allocation

Suppose the unconstrained planner would like to achieve the optimal allocation by imposing occupation-specific taxes. What would those tax rates be and how would they depend on the model parameters? This problem is approached using a tax scheme in which a tax rate τ_i is imposed on workers in occupation i . Therefore, in competitive equilibrium one efficiency unit of labor earns an income of $(1 - \tau_i)w_i$. The purpose is to choose tax rates for each occupation so that the output maximizing allocation is achieved.

Carrying out the same derivations with taxation gives a competitive equilibrium allocation which is a function of the occupation-specific tax rates. Comparing that allocation to the unconstrained planner's allocation shows that the necessary tax rates are given by

$$\tau_{i,eq} = 1 - \frac{E\left(\gamma_i^{\frac{\sigma}{\kappa}}\right)}{E\left(\gamma_i^{(1-\rho)\frac{\sigma}{\kappa}}\right)^{\frac{1}{1-\rho}}} \quad (1.12)$$

This closed-form solution is useful for building intuition for the underlying mechanics of the model. The source of inefficiency is the risk-aversion of workers. Not surprisingly, the inefficiency disappears if $\rho \rightarrow 0$, i.e. the workers become risk-neutral. However, given that $\rho > 0$, the effect of risk-aversion is amplified through the labor share of each occupation θ . This happens because a high capital share amplifies the movements in the stochastic productivity γ_i , making the wages rel-

actively more volatile in the risky occupations. The risky occupations become less attractive which has to be offset by a higher subsidy.

The required subsidies are increasing in the elasticity of substitution across occupation goods. The reason for this is interesting: a low elasticity of substitution implies that a negative productivity shock has a relatively small impact on wages. That is, as output is reduced in that occupation, the price of the occupation good goes up. As a consequence, the revenues and also wages of the occupation change only by a small amount. This amounts to a lower wage risk which mitigates the incomplete markets problem. The extreme case corresponds to $\sigma = 0$ (Leontieff preferences) where a change in γ_i induces an exactly off-setting change in p_i and the wages remain the same after any productivity realization (see equation (1.4)). At the other extreme, if $\sigma \rightarrow \infty$, the occupation goods become perfect substitutes. After any productivity realization γ_i , the price of the occupation good remains constant and the productivity realization has a high impact on wages.

Let us denote the probability of a good productivity realization by π_i . This variable enters the optimal tax rate through the expectations in (1.12). The required subsidy is highest for occupations in which $\pi_i = 0.5$. That is exactly the value which maximizes the variance of the Bernoulli distribution from which the binary productivity draws are taken. The result that subsidy depends on the variance of

productivity realizations could be extended to other probability distributions. On the other hand, the mean of the probability distribution does not matter. The required subsidy is zero if $\pi = 0$ or $\pi = 1$. In other words, workers internalize the mean but not the variance of wages in the model.

Figure 1.2 displays graphically the tax rate that implements the unconstrained planner's allocation.

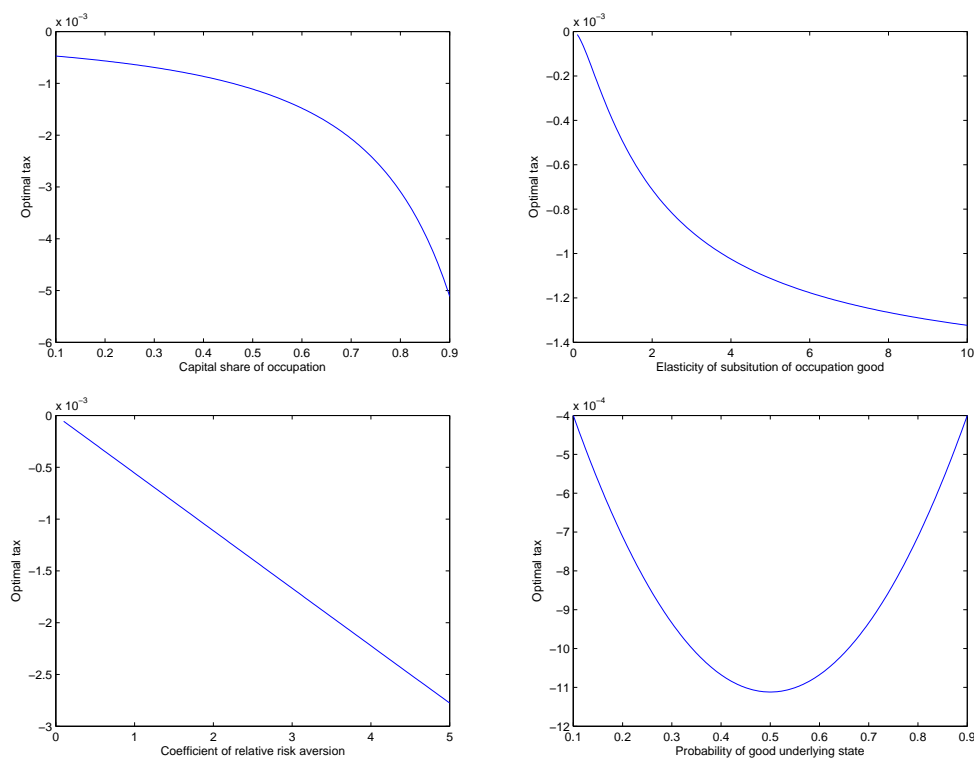


Figure 1.2: Optimal Tax Rate for Various Parameter Values

Remark

In the one-period model, there are many tax schemes that implement the unconstrained planner's allocation. Labor is supplied inelastically after the occupation choice. The relative attractiveness of occupations i and j is affected by the factor $\left(\frac{1-\tau_i}{1-\tau_j}\right)^{1-\rho}$. By changing the tax rates and keeping this ratio constant, the government can in principle collect positive tax revenues and still implement the optimal allocation. Of course, in a model where taxation would reduce labor supply this effect would disappear. A more formal treatment of this argument is given in the Appendix.

1.3.8 Planner's Problem: Constrained Efficiency

The unconstrained planner's problem implicitly assumes that the planner is able to transfer income from the lucky occupations to the unlucky ones. This does not seem particularly realistic. An alternative way to approach the planner's problem is to solve for a *constrained efficient* allocation. That is, an allocation where the planner is not able to re-distribute income after the occupation-specific shocks. This type of problem has been studied in a neoclassical growth model by Davila et al. (2012). The concept of constrained efficiency itself was introduced by Diamond (1967). A general framework is developed in Greenwald and Stiglitz (1986).

The first welfare theorem states that when markets are complete, the competitive equilibrium is Pareto-optimal. This happens because ‘pecuniary externalities’, i.e. externalities transferred through general equilibrium wages, exactly offset each other with complete markets. However, since markets are not complete in this paper, there is no reason to believe that the pecuniary externalities would offset each other.

I will now solve for the constrained efficient allocation. That is, I assume that the planner is not able to overcome the incomplete markets problem and chooses the allocation that is efficient in the presence of the incomplete markets problem. After the idiosyncratic productivities are drawn, it is not possible to change the planner’s cutoffs in a way which would make everyone better off. Therefore, it turns out that the constrained efficient allocation is equivalent to an allocation which maximizes utilitarian welfare across all individuals¹ in the economy. Alternatively, one can consider this to be the *ex ante* utility of an individual before any productivity realizations. Since the Fréchet distribution has the property that the distribution of the maximum is also Fréchet, we get the following proposition:

Proposition 1.3.1. *The utilitarian welfare can be written as*

$$\left(\int_0^1 |v_i|^{\frac{\psi}{1-\rho}} di \right)^{\frac{1-\rho}{\psi}} \Gamma \left(1 + \frac{\rho-1}{\psi} \right)$$

Proof. See the Appendix. □

¹That is, the sum of individual utilities weighted by the density of individuals.

The constrained efficient allocation is the solution to the following problem:

$$\begin{aligned}
& \max_{h_i} \left(\int_0^1 |v_i|^{\frac{\psi}{1-\rho}} di \right)^{\frac{1-\rho}{\psi}} \\
& \text{s.t.} \\
& w_i = c_1 \gamma_i^{\frac{\sigma}{\kappa}} L_i^{-\frac{1}{\kappa}} Y^{\frac{1}{\kappa}} \\
& L_i = h_i^{\psi-1} \left(\int_0^1 h_j^{\psi} dj \right)^{\frac{1-\psi}{\psi}} \Gamma \left(1 - \frac{1}{\psi} \right) \\
& Y = c_2 \left(\int_0^1 \gamma_i^{\frac{\sigma}{\kappa}} L_i^{\frac{\kappa-1}{\kappa}} di \right)^{\frac{\kappa}{\kappa-1}}
\end{aligned} \tag{1.13}$$

Therefore, the constrained planner maximizes utilitarian welfare (dropping the constant $\Gamma \left(1 + \frac{\rho-1}{\psi} \right)$) subject to wages being determined competitively, labor units being determined by the planner's cutoffs and total production being consistent with the labor allocation. When the cutoffs are adjusted, wages change which impacts pecuniary externalities. The functional form of the objective function hints that the constrained planner would like to keep the variance of the $|v_i|$'s small.

As shown in the Appendix, the constrained efficient allocation can be written as:

$$L_{i,ce} = \frac{\left(\frac{E(\gamma_i^{\frac{(1-\rho)\sigma}{\kappa}})^{\frac{\psi}{1-\rho}}}{E(\gamma_i^{\frac{\sigma}{\kappa}})} \right)^{\frac{\kappa}{\kappa+\psi-1}}}{\left(\int_0^1 \left(\frac{E(\gamma_j^{\frac{(1-\rho)\sigma}{\kappa}})^{\frac{\psi}{1-\rho}}}{E(\gamma_j^{\frac{\sigma}{\kappa}})} \right)^{\frac{\kappa}{\kappa+\psi-1} \frac{\psi}{\psi-1}} dj \right)^{\frac{\psi-1}{\psi}}} \Gamma \left(1 - \frac{1}{\psi} \right) \tag{1.14}$$

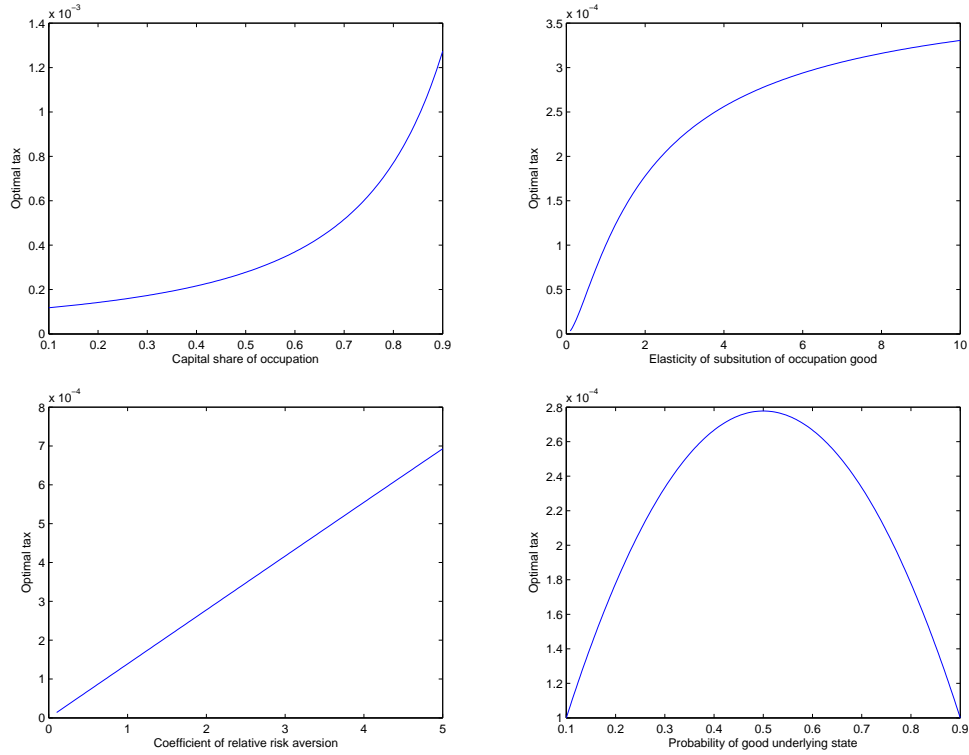


Figure 1.3: Optimal Taxes for the Constrained Planner.

Taxes that Implement the Constrained Efficient Allocation

A tax that implements the constrained efficient allocation is obtained by comparing the competitive equilibrium allocation and the constrained efficient allocation:

$$\tau_{i,ce} = 1 - \left(\frac{E(\gamma_i^{(1-\rho)\frac{\sigma}{\kappa}})^{\frac{1}{1-\rho}}}{E(\gamma_i^{\frac{\sigma}{\kappa}})} \right)^{\frac{1}{\psi-1}} \quad (1.15)$$

These taxes are shown graphically in Figure 1.3.

The optimal tax of the constrained planner has the opposite pattern compared to the unconstrained planner. While the unconstrained planner wants to subsidize

more risky occupations., the constrained planner instead wants to tax those occupations. The constrained planner understands that when less people go to the risky occupations, the general equilibrium wage of those occupations will go up. Since taxes are proportional and the general equilibrium wage schedule is convex, after-tax wages will eventually increase when less people go to the risky occupations. This increase in wage will compensate the individuals for the additional risk which brings the $|v_i|$'s closer to each other across i .

Note that the planner is not able to eliminate wage inequality *within* occupations, because people are paid according to their idiosyncratic productivity. However, the constrained planner can minimize wage inequality *across* occupations. In such an allocation, the changes in pecuniary externalities help especially individuals who get such draws that they have to go the less preferred risky occupations. This result can be contrasted to Davila, Hong, Krusell and Ríos-Rull (2012) who show that in a neoclassical growth model with incomplete markets the ‘consumption-poor’ are especially important in the constrained planning problem.

1.3.9 An Example

Let us use a simple example to analyze the output losses caused by the incomplete markets problem. Assume that there are two types of occupations: ‘risky’ and ‘safe’ occupations. The table below describes risky and safe occupations.

Occupation type	Fraction of all occupations	Productivity realizations
Safe	ω	$\gamma_i = 1$ for sure.
Risky	$1 - \omega$	$\gamma_i = \gamma_g$ with probability π . $\gamma_i = \gamma_b$ with probability $1 - \pi$.

Table 1.1: A Simple Example

Therefore the safe occupations have a predictable productivity² whereas the risky occupations have two possible productivity realizations such that $\gamma_g > \gamma_b$.

The output of the economy is obtained by rewriting equation (1.5):

$$Y = c_2 \left(\omega L_{i, safe}^{\frac{\kappa-1}{\kappa}} + (1 - \omega) E(\gamma_i^{\frac{\sigma}{\kappa}}) L_{i, risky}^{\frac{\kappa-1}{\kappa}} \right)^{\frac{\kappa}{\kappa-1}}$$

Here $L_{i, safe}$ and $L_{i, risky}$ denote the labor units going to each of the safe and risky occupations. Because all risky occupations are identical, the labor units are the same for all those occupations. The same is true for the safe occupations. Substituting the competitive equilibrium labor choices to this expression gives

²Because the coefficients in the final good production sum up to one, the law of large numbers in principle implies that $\omega + (1 - \omega)(\pi\gamma_g + (1 - \pi)\gamma_b) = 1$. However, I relax that assumption in this section in order to be able to better understand the comparative statics.

$$Y_{eq} = \frac{c_2 \left(\omega + (1 - \omega) E(\gamma_i^{\frac{\sigma}{\kappa}}) E(\gamma_i^{(1-\rho)\frac{\sigma}{\kappa}})^{\frac{1}{1-\rho}} \frac{(\psi-1)(\kappa-1)}{\kappa+\psi-1} \right)^{\frac{\kappa}{\kappa-1}}}{\left(\omega + (1 - \omega) E(\gamma_i^{(1-\rho)\frac{\sigma}{\kappa}})^{\frac{1}{1-\rho}} \frac{\kappa\psi}{\kappa+\psi-1} \right)^{\frac{\psi-1}{\psi}}} \Gamma \left(1 - \frac{1}{\psi} \right)$$

Similarly, substituting the unconstrained planner's output maximizing allocation to the production function gives

$$Y_{om} = \frac{c_2 \left(\omega + (1 - \omega) E(\gamma_i^{\frac{\sigma}{\kappa}})^{1 + \frac{(\psi-1)(\kappa-1)}{\kappa+\psi-1}} \right)^{\frac{\kappa}{\kappa-1}}}{\left(\omega + (1 - \omega) E(\gamma_i^{\frac{\sigma}{\kappa}})^{\frac{\kappa\psi}{\kappa+\psi-1}} \right)^{\frac{\psi-1}{\psi}}} \Gamma \left(1 - \frac{1}{\psi} \right)$$

Finally, the constrained efficient output level is

$$Y_{ce} = \frac{c_2 \left(\omega + (1 - \omega) E(\gamma_i^{\frac{\sigma}{\kappa}}) \left(\frac{E(\gamma_i^{(1-\rho)\frac{\sigma}{\kappa}})^{\frac{\psi}{1-\rho}}}{E(\gamma_i^{\frac{\sigma}{\kappa}})} \right)^{\frac{\kappa-1}{\kappa+\psi-1}} \right)^{\frac{\kappa}{\kappa-1}}}{\left(\omega + (1 - \omega) \left(\frac{E(\gamma_i^{(1-\rho)\frac{\sigma}{\kappa}})^{\frac{\psi}{1-\rho}}}{E(\gamma_i^{\frac{\sigma}{\kappa}})} \right)^{\frac{\kappa}{\kappa+\psi-1}} \frac{\psi}{\psi-1} \right)^{\frac{\psi-1}{\psi}}} \Gamma \left(1 - \frac{1}{\psi} \right)$$

The output levels depend on parameter values. Figures 1.4 and 1.5 show output for various levels of π and ω . The assumption here is that $\rho = 2$, $\psi = 10$, $\theta = 0.8$, $\sigma = 50$, $\gamma_g = 1.1$ and $\gamma_b = 0.9$.

Figures 1.4 and 1.5 are sensitive to parameter values and have been drawn using somewhat 'extreme' parameter values.

Figure 1.4 top-left panel shows that when $\pi = 0$ or $\pi = 1$, the unconstrained planner's output is the same as in competitive equilibrium. This of course happens because the riskiness of all occupations disappears and the incomplete markets problem disappears. The difference between the unconstrained planner's allocation is

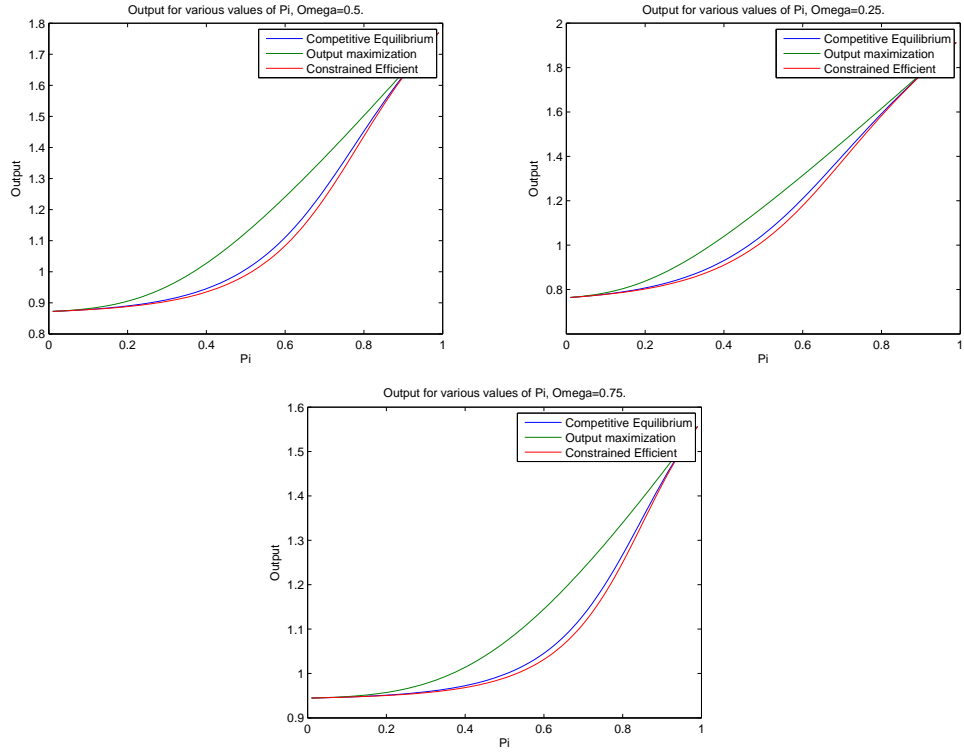


Figure 1.4: Outputs for various values of π when $\omega = 0.5$ (top-left), $\omega = 0.25$ (top-right) and $\omega = 0.75$ (bottom).

maximized around $\pi = 0.5$ since this is where the wage risk is highest for individuals.

Figure 1.4 also shows output in the constrained efficient allocation. The constrained efficient allocation yields a lower output than the competitive equilibrium.

These observations highlight the main feature of the occupational choice in this paper: labor is not perfectly substitutable across occupations and putting a lot of labor in certain occupation decreases total efficiency units in the economy. As

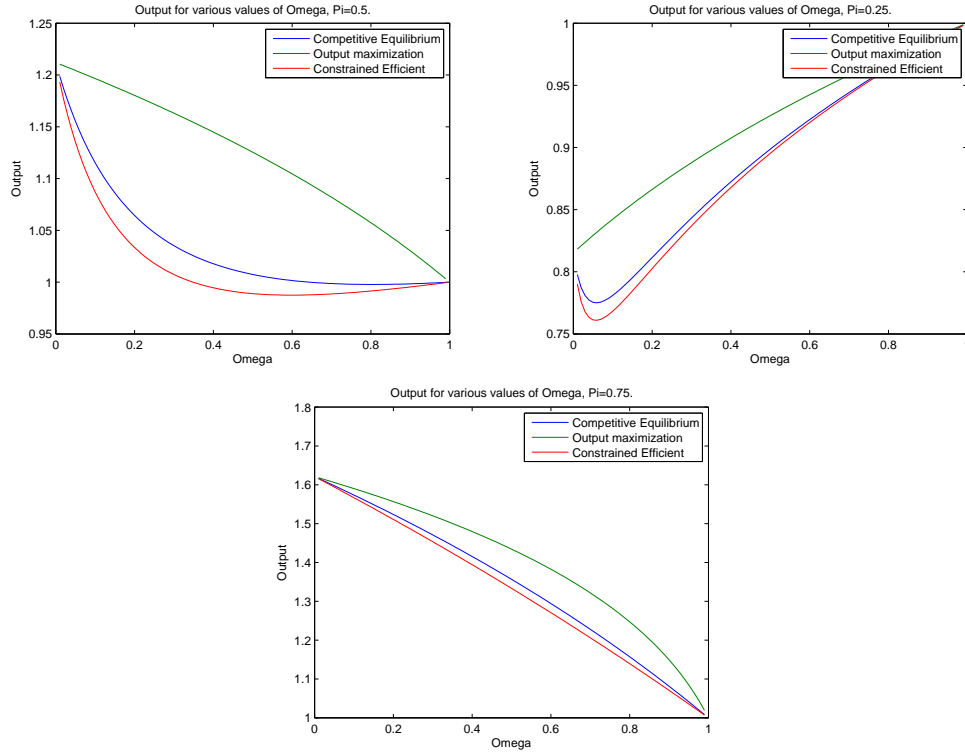


Figure 1.5: Outputs for various values of ω when $\pi = 0.5$ (top-left), $\pi = 0.25$ (top-right) and $\pi = 0.75$ (bottom).

different individuals are good in different occupations, it is beneficial to always put some amount of people in each occupation.

Varying the proportion of risky occupations does not change the pattern of the graphs in Figure 1.4.

Figure 1.5 top-left panel shows that when $\pi = 0.5$ (risky occupations are risky but have the same mean as the safe occupations), the competitive equilibrium out-

put is at a very low level when the proportion of safe occupations is relatively low ($\omega \approx 0.3$). An interesting feature of the model is that when all occupations are risky, the competitive equilibrium output also maximizes output. Since the incomplete markets problem distorts the occupation choice equally much for all occupations, the workers make the correct occupation choices. This happens because workers choose their occupation in relation to all other occupation, and in this case the distortion across all occupations cancel out.

Figure 1.5 also shows output when the risky occupations have a lower mean than the safe occupation (top-right) and the risky occupations have a higher mean than the safe occupation (bottom). Interestingly, the top-right panel shows that the equilibrium output may decrease when the fraction of more productive safe occupations is increased in the economy.

The utilitarian welfares are shown in Figures 1.6-1.7. The constrained efficient gives the highest level of utilitarian welfare by definition. The equilibrium gives the second highest level of utilitarian welfare.

1.4 Dynamic Model

Next a dynamic version of the model is developed. To be able to take the model to the data, the Fréchet distribution is assumed to take a more general form:

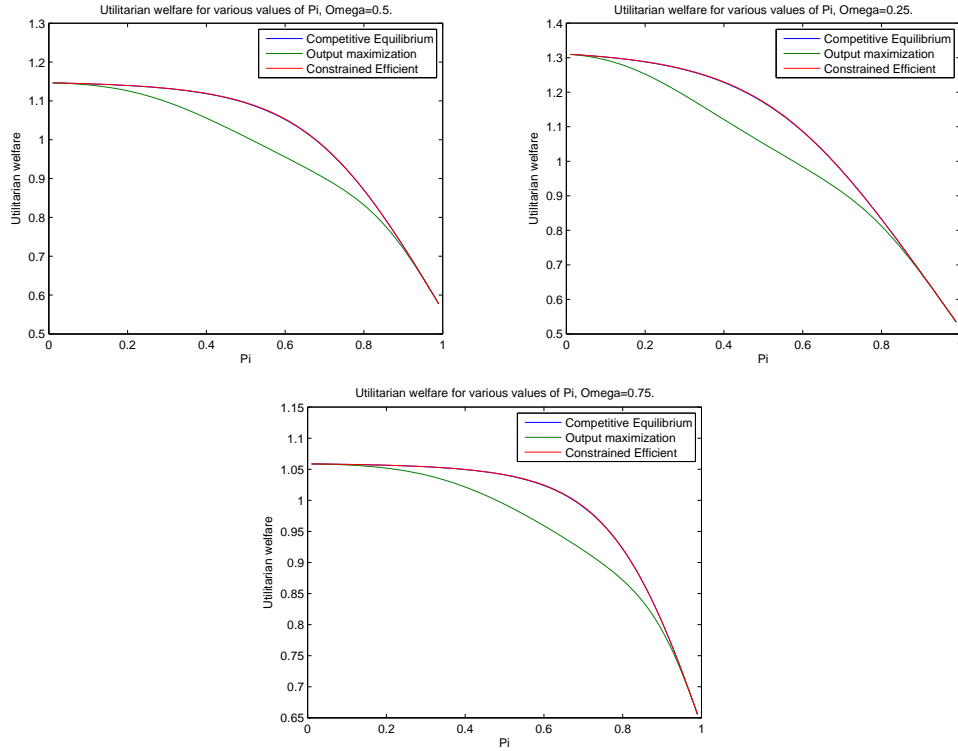


Figure 1.6: Utilitarian welfare for various values of π when $\omega = 0.5$ (top-left), $\omega = 0.25$ (top-right) and $\omega = 0.75$ (bottom).

$$F(z) = \exp(-T_i z^{-\psi})$$

Here $\psi > 1$ is still constant across occupations but T_i is occupation-specific.

As shown in the one-period model, the means and the variances of occupation-specific wages are important determinants of potential output losses caused by the incomplete markets problem. To build a tractable dynamic model where the means and variances change stochastically, I use a learning process where the state of

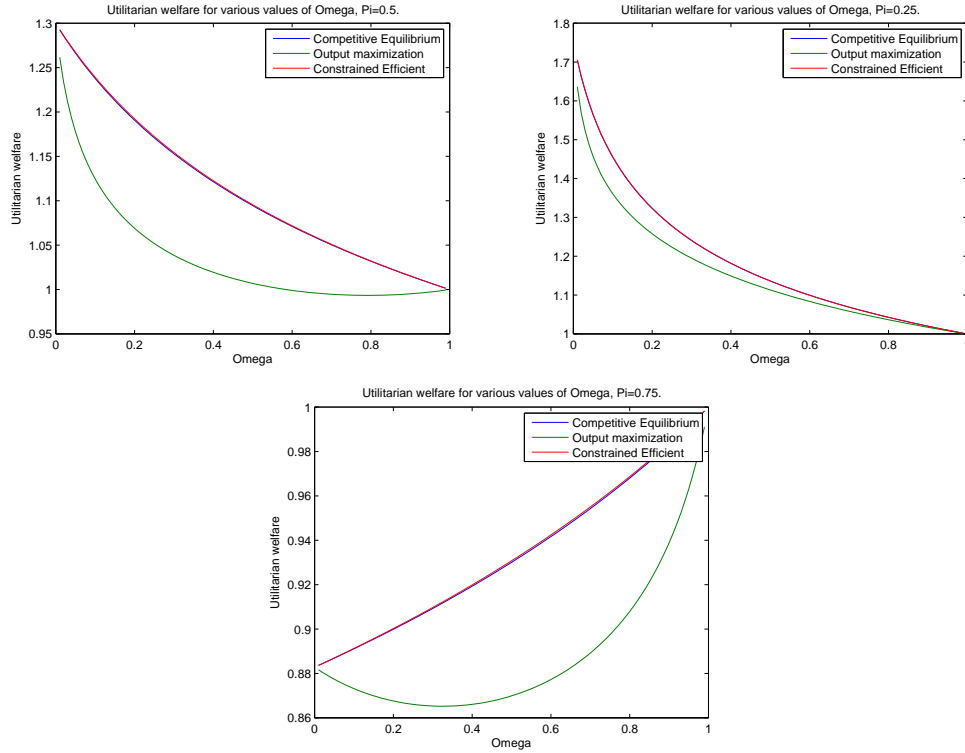


Figure 1.7: Utilitarian welfare for various values of ω when $\pi = 0.5$ (top-left), $\pi = 0.25$ (top-right) and $\pi = 0.75$ (bottom).

each occupation changes stochastically. Such a learning process helps to match the model to the petroleum, chemical and mechanical engineering data which I use in the calibration.

1.4.1 Value Functions

If a person provides one efficiency unit of labor to sector i , his value function is:

$$v_i(\phi_i, L_i, \Omega) = \sum_{s \in \{good, bad\}} \pi_s (u(w_{i,s}) + (1 - \delta)v_i(\phi'_i, L'_i, \Omega')) \quad (1.16)$$

where π_s refers to the probability of realization s occurring, ϕ is the probability that the sector is in the good underlying state (see below), L_i is the total number of efficiency units of labor in that occupation and Ω is a variable that summarizes the rest of the economy. ϕ_i is updated according to the Bayes rule, as explained below. The efficiency units in each occupation are the sum of the labor force that remains from previous periods and the new entrants:

$$L'_i = (1 - \delta)L_i + L_{i,new} \tag{1.17}$$

Due to the general equilibrium wages and prices, the occupation choices and productivity realizations for other occupations affect the wages in occupation i . The more productive other occupations are, the higher the wage will be in occupation i . These pecuniary externalities enter through state variable Ω . I assume that Ω is constant across years which corresponds to the steady state distribution across occupations.

Finally, the continuation value is discounted by the probability of death δ such that the discount factor is $\beta = 1 - \delta$.

1.4.2 Learning about Shock Persistence

The learning process is reminiscent of the process in Veldkamp (2005) and Ordoñez (2013). In each period, productivity in sector i can take two possible values $\gamma_{i,g}$

and $\gamma_{i,b}$ such that $\gamma_{i,g} > \gamma_{i,b}$. These values correspond to positive and negative productivity shocks, respectively. New entrants to the labor force can observe the full history of productivity realizations and use these when they make their occupation choice.

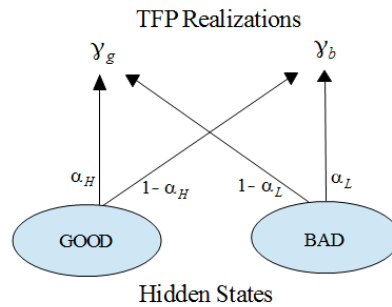


Figure 1.8: Hidden States

Each occupation can be in one of two hidden states. The workers do not observe the hidden state but can use the history of productivity realizations to learn about the hidden state. Figure 1.8 shows the relationship between the hidden states and productivity realizations.

Let ϕ_i be the probability that sector i is in good hidden state. In good hidden state, productivity will be $\gamma_{i,g}$ with probability α_H . In bad hidden state, productivity will be $\gamma_{i,g}$ with probability α_L where $\alpha_L < \alpha_H$. The *ex ante* probability of a good productivity realization is $\phi_i \alpha_H + (1 - \phi_i) \alpha_L$.

At the end of the period, the *ex post* probability of being in good hidden state is computed as follows:

$$\begin{aligned}\hat{\phi}'_i &= \frac{\phi_i \alpha_H}{\phi_i \alpha_H + (1 - \phi_i) \alpha_L} \text{ if realized productivity is high} \\ \hat{\phi}'_i &= \frac{\phi_i (1 - \alpha_H)}{\phi_i (1 - \alpha_H) + (1 - \phi_i) (1 - \alpha_L)} \text{ otherwise.}\end{aligned}\tag{1.18}$$

The hidden state of each occupation follows a two-state Markov process with persistence parameter $1 - \hat{\lambda}$. This means that the hidden state changes over time with a positive probability. To account for the possible change of state, the next period's prior is

$$\phi'_i = (1 - \hat{\lambda}) \hat{\phi}'_i + \hat{\lambda} (1 - \hat{\phi}')\tag{1.19}$$

An important special case of this setup is the case where learning is perfect so that $\alpha_H = 1$ and $\alpha_L = 0$. In that case, the model corresponds to a business cycle model where the state changes with probability $\hat{\lambda}$. However, these parameter values imply that occupation choices change instantly after a state change. The petroleum engineering data instead shows a slow increase in enrollment. Therefore slow learning in the model is more consistent with the data.

1.4.3 Definition of Equilibrium

For $i \in [0, 1]$, a recursive equilibrium consists of value functions $v_i : [0, 1] \times \mathbb{R}_+ \times \mathbb{R}_{++} \rightarrow \mathbb{R}$, policy functions of new entrants $L_{i,new} : [0, 1] \times \mathbb{R}_+ \times \mathbb{R}_{++} \rightarrow \mathbb{R}_+$,

pricing functions $p_i, w_i : [0, 1] \times \{\mathbb{R}_{++}\}_{i \in [0,1]} \rightarrow \mathbb{R}_{++}$ and an aggregate law of motion $H : \{[0, 1] \times \mathbb{R}_+\}_{i \in [0,1]} \rightarrow \{[0, 1] \times \mathbb{R}_+\}_{i \in [0,1]}$ such that

- Each occupation good producer chooses K_i and L_i to maximize profits.
- Given expected wages and realized ξ_{ij} , worker j optimally chooses discipline in the beginning of his life.
- Given expected wage paths and prices, each worker j chooses consumptions $c_{ij} = \xi_{ij}w_i$.
- Goods market clears: $\int_j \int_0^1 c_{ij} di dF(j) = Y$.
- Market for labor clears, $l_i = L_i$.
- Given r , capital is chosen optimally for each i .
- Beliefs about the probability of high-productivity states evolve according to the Bayes' rule and the Markov transition matrix.

1.4.4 Worker's problem

There is no saving technology in the economy. The only nontrivial problem that each worker faces is his occupation choice before entering the labor market. To recap, the worker draws ξ_j for all j and goes to sector i if $\xi_i |v_i|^{\frac{1}{1-\rho}} \geq \xi_j |v_j|^{\frac{1}{1-\rho}} \quad \forall j \neq i$. As before, the total efficiency units of entrants to occupation i is given by

$$L_{i,new} = \frac{T_i |v_i|^{\frac{\psi-1}{1-\rho}}}{\left(\int_0^1 T_j |v_j|^{\frac{\psi}{1-\rho}} dj\right)^{\frac{\psi-1}{\psi}}} \Gamma\left(1 - \frac{1}{\psi}\right) \quad (1.20)$$

where v_i is the value function of a worker going to occupation i with one efficiency unit of labor, as specified in equation (1.16). The law of motion for the labor in occupation i is given by (1.17).

1.4.5 Numerical Solution of the Model

This section describes the solution method for the dynamic version of the model. The employed numerical method is value function iteration. I assume that there is no aggregate uncertainty in the economy which makes the model reminiscent of the Aiyagari (1994) model. That is, each occupation faces idiosyncratic risk in terms of shocks it faces and also people's response to these shocks. However, on aggregate the measure of firms in each state remains constant over time.

1.4.6 The Value Function Iteration Procedure

The exact method of value function iteration requires a bit of explanation. Theoretically the optimal policy solves

$$L_{i,new} = T_i |v_i|^{\frac{\psi-1}{1-\rho}} \left(\int_0^1 T_j |v_j|^{\frac{\psi}{1-\rho}} dj\right)^{\frac{1-\psi}{\psi}} \Gamma\left(1 - \frac{1}{\psi}\right)$$

In the numerical estimation of the model I guess a value for Ω such that

$$L_{i,new} = T_i |v_i|^{\frac{\psi-1}{1-\rho}} \Omega$$

where

$$\Omega = \left(\int_0^1 T_j |v_j|^{\frac{\psi}{1-\rho}} dj \right)^{\frac{1-\psi}{\psi}} \Gamma \left(1 - \frac{1}{\psi} \right) \quad (1.21)$$

One iteration using this procedure is sufficient to solve for the *relative numbers* of entrants to various disciplines. However, it does not give the correct *levels* of entrants which will be important since we want to contrast the results to the planner's problem. The correct levels are obtained by iteration.

Finding a fixed point To find the fixed point, I use the following algorithm:

1. Guess a value for Ω . Denote the guess by Ω_g .
2. Using value function iteration, solve for the optimal policies.
3. Construct a Markovian matrix where each ϕ - L_i combination is listed on both the vertical and horizontal axis. The evolution of ϕ 's is obtained from the Bayesian learning process and the evolution of L_i 's is obtained from the policy function.
4. Find the stationary distribution of the Markovian matrix.
5. Using the stationary distributions for all occupations, construct the realized Ω using (1.21). denote this by Ω_r .

6. If $|\Omega_g - \Omega_r| < 10^{-8}$, stop. Otherwise adjust the guess and go back to step 1.

Remark 1. When computing the fixed point for competitive equilibrium, it is possible to replace Step 6. by adjusting the measure of firms such that the economy is in the fixed point after the first iteration. The required measure of firms can be computed from equation (1.21). This adjustment is without loss of generality and significantly reduces the time needed for computation.

Remark 2. The algorithm requires that the measures of occupations in (1.21) are specified. In matching the model to the data, I let each of the three occupations (petroleum, chemical and mechanical engineering) be of equal importance. This is roughly consistent with the GDP shares of the sectors where these occupations primarily work. In other words, I assume that there are actually three continua of occupations and weight these continua equally.

Remark 3. The model contains pecuniary externalities that come through Y . Fortunately, these externalities cancel out in the occupational choice problem, as all occupations are subject to the same pecuniary externalities. However, the pecuniary externalities will affect the aggregate productivity in the economy. As before, total production is computed using equation (1.5).

1.4.7 Solving for the Social Planner's Problem in the Dynamic Version of the Model

It is necessary to solve the unconstrained planner's problem to see how far off the optimum the economy is. The social planner's problem is greatly simplified by noting that the outcome is equivalent to the competitive equilibrium when $\rho = 0$. I provide a formal proof of this result in the Appendix.

The social planner's allocation differs from the competitive equilibrium. A partial equilibrium example of the difference between the competitive equilibrium and the planner's allocation is depicted in Figure 1.9.

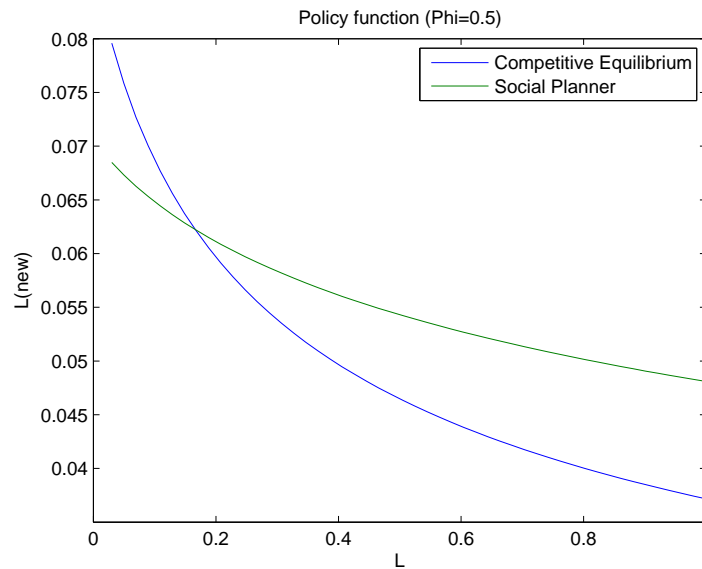


Figure 1.9: Policy Function

In stationary equilibrium the policy functions look different from Figure 1.9 depending on the riskiness of the occupation and also the distribution parameter T_i . A general result of my calibration is that uncertainty tends to be high when wages are low and vice versa. Hence, people will strongly ‘over-react’ to changes in perceived states of each occupation. In other words, the observed sharp changes in enrollment to the engineering disciplines cannot be explained by wages only.

Figure 1.10 shows a simulated boom-bust cycle where an occupation sequentially experiences 10 positive productivity shocks followed by a bust. Figure 1.10 is drawn based on the assumption that $\alpha_H = 1$, which implies that one negative productivity shock is enough to infer that the occupation is in a bad hidden state. Therefore the occupation choices change drastically after the negative shock.

During a boom-bust cycle, the aggregate labor units in equilibrium become smaller than in the unconstrained planner’s problem. Figure 1.11 illustrates why this is the case. For this discussion I assume only two occupations but the principle itself is without loss of generality.

In Figure 1.11, productivity realizations for occupation 1 and 2 are displayed on the horizontal and vertical axis, respectively. The comparative advantage cut-offs are lines going through the origin. The cutoff has to be a straight line, reflecting

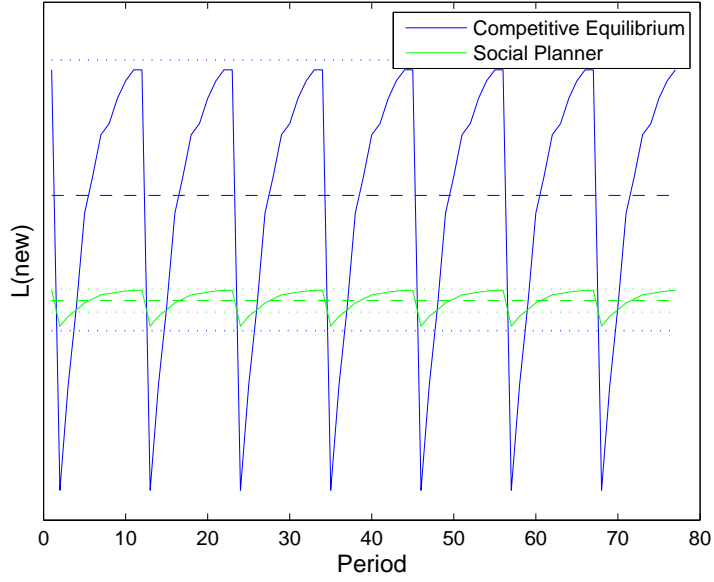


Figure 1.10: Boom-Bust Cycle: Chemical engineering ($\gamma_i \in \{1.0875, 0.9125\}$). Occupation hit with sequences of 10 positive shocks followed by one negative shock. Mean and std shown by horizontal lines.

the fact that individuals are sorted by their comparative advantage. All individuals below the cutoff line go to occupation 1 and the rest go to occupation 2. The unconstrained planner's cut-off line has a slope of $(\frac{h_1}{h_2})$. The slope of the equilibrium cut-off line is $(\frac{v_1}{v_2})^{\frac{1}{1-\rho}}$.

Consider a scenario in which occupation 1 is risky and occupation 2 is safe at $t = 1$. Assume also that the opposite is true at $t = 2$. At $t = 1$, all individuals whose productivity realization lies below the green solid line go to occupation 1 and others go to occupation 2. At $t = 2$, all individuals whose productivity realization

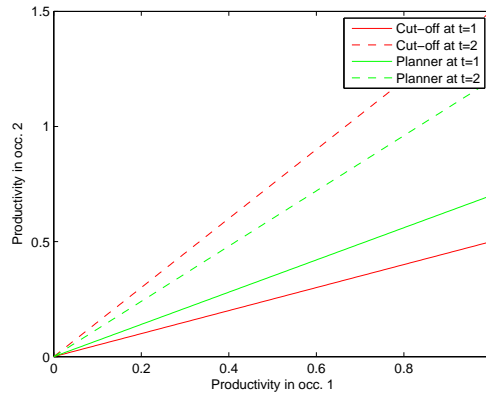


Figure 1.11: The Comparative Advantage Cut-Off Lines

is below the dashed green line go to occupation 1 and others go to occupation 2. However, the occupation choice of individuals whose productivity realization lies between the two green lines will then depend on the period in which they are born. This creates a possibility to increase total labor efficiency units by changing the cutoffs across periods.

The red lines in Figure 1.10 are the unconstrained planner's cut-off lines. The area between the two lines is smaller than in equilibrium. At $t = 1$ the planner allocates all the same individuals to occupation 1 and some additional ones as well. At $t = 2$, more individuals are allocated to occupation 2. This allocation rule implies that total efficiency units of labor will be higher in the unconstrained planner's allocation compared to the equilibrium allocation. If the future would be certain, the comparative advantage line would be constant for all periods and the slope of the line would be determined by the marginal productivity of each occupation. This

would yield the maximal amount of efficiency labor units to the economy. When the comparative advantage line fluctuates over time, the total amount of efficiency units of labor decreases.

1.5 Empirics

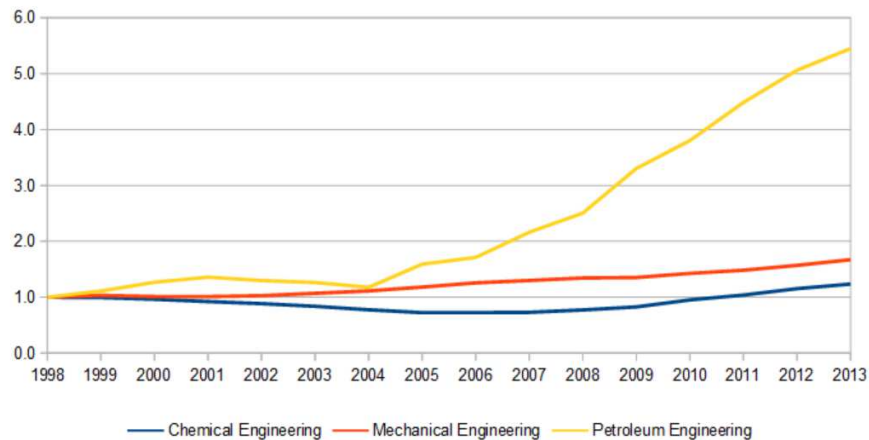
1.5.1 Empirical Motivation

Empirical motivation for this paper comes from the data on the evolution of the number of engineers by field of specialization. It seems that similar arguments could be made for many other academic disciplines if data were available. Using data on engineering degrees awarded by discipline, I document drastic changes in academic specialization over time. In addition to data availability, the decision to focus on engineering education has two major advantages. First, engineers are often considered to be important for technological progress and growth and are therefore relevant for the topics studied in this paper. Second, the fields of specialization in engineering are well-defined and they provide students with highly specialized skills. This makes the assumption of occupation-specific human capital plausible.

The data on engineering education reveals astonishingly large changes over time. Figure 1.12 plots the degrees awarded in petroleum, chemical and mechanical en-

gineering in 1998-2013. The primary interest is in the evolution of petroleum engineering enrollment. Since chemical and mechanical engineering are closely related disciplines, they are used for the quantitative section. As the sizes of various disciplines vary, the time series are normalized so that they all start from one. The total number of engineering degrees awarded is on the rise and grew by 38% over this time period. At the same time, degrees awarded in petroleum engineering grew by a factor of five. These changes may be due to additional demand for these types of skills caused by the discovery of new oil mining techniques. On the other hand, the number of degrees awarded in chemical and mechanical engineering has remained relatively constant.

Figure 1.12: Degrees Awarded in Selected Engineering Disciplines for 1998-2013 (Normalized)



The background of the petroleum engineering profession is as follows. This oc-

cupation has been around in the U.S. for more than a century. However, after the oil crises of the 1970's university enrollment in petroleum engineering plummeted and many universities ended up closing down the program. Since the late 1990's the U.S. has experienced a huge boom in the Gas and Oil Extraction industry which grew by +242% in 1997-2012, making it the fastest growing sector in the U.S. Meanwhile, the low supply of petroleum engineers has been a major problem for many oil firms looking to expand their operations. As a response, oil firms have hired workers from closely related engineering fields (mechanical and chemical engineering) and retrained them to gas and oil extraction. However, this retraining has led to a loss of production. As discussed in the Appendix, it is possible to run Mincer regressions on the engineering wage data to estimate the effect the petroleum engineering major has on wages in the petroleum engineering occupation. In particular, if we focus only on engineers working in the petroleum engineering occupation, the petroleum engineering majors received on average 33% higher wages, controlling for all other factors.

1.5.2 Calibration Strategy

I will proceed to calibrate the model using the engineering data. The structural parameters of the model are $\{T_i\}_{i=1}^3$, α_H , α_L , λ , ρ and ψ . It is also necessary to choose the initial values $\phi_{i,0}$ and $L_{i,0}$.

The data set consists of observations of wages for each discipline in each period and the engineering degrees awarded by discipline. I will make the following assumptions regarding the data:

1. Since the productivity realization is unobserved and binary, I choose the productivity shocks to maximize the model fit.
2. Since the data only includes observations on the degrees awarded, I assume that the major choice was made four years prior to the awarding of the degree. This assumption is consistent with the fact that most engineering undergraduate students in the U.S. graduate in four years.
3. Nominal wages of each discipline are converted to real values using the Consumer Price Index.

I take the following parameter values from existing literature:

For the speed of learning I use $\alpha_H = 0.8$ and $\alpha_L = 0.24$. These parameters determine the speed of learning in the model. They will also pin down the relationship between wages and uncertainty in each occupation. In the Appendix I show that these values of α_H and α_L are able to replicate the wage data of the three occupations. A model without occupational uncertainty is not able to replicate the increase in enrollment to petroleum engineering, as petroleum engineering wage data does not show sufficiently large increase over time. Therefore, part of

Parameter	Value	Justification
ρ	2	Typical value from the literature.
λ	0.1	Corresponds to hidden state changing once in 10 years on average.
ψ	5	Close to the baseline value of Hsieh et al. (2013)
σ	5	No good micro-data estimates available.
δ	0.025	Corresponds to a average career length of 40 years.

Table 1.2: Parameter Values for Calibration

the increase in enrollment is explained by a decrease in occupational uncertainty.

The remaining parameters are estimated as follows:

$$\min_{\{T_i\}_{i=1}^3} \sum_{i,t} (Pr_{i,t} - \hat{P}r_{i,t})^2 \quad (1.22)$$

where $Pr_{i,t}$ is the observed fraction of new students who choose to study discipline i in period t and $\hat{P}r_{i,t}$ is the model prediction of fraction of new students choosing to study discipline i in period t . In words, the model parameters are estimated to minimize the sum of squared residuals between the model predictions and data.

The value functions are solved by a value function iteration. Since the productivity realizations are binary in the model, I choose productivity realizations for

each i and t to maximize the model fit. Therefore, part of the unobserved deviations from the trend are explained by the changes in the beliefs of the workers. To re-iterate the estimation procedure in detail, I use the following steps:

1. Make a guess for the vector of structural parameters and define the initial values $\phi_{i,0}$ and $L_{i,0}$.
2. Solve the value functions for these parameter values using the Aiyagari methodology.
3. Starting from $t = 1$, use the model prediction of the number of entrants and choose the productivity realization that maximizes model fit for that period.
4. Solve for $\phi_{i,t+1}$ and $L_{i,t+1}$
5. Recursively do this for all $t > 1$.
6. Construct the sum of squared errors in equation (1.22).
7. Iterate by choosing new vector of structural parameters, until it is no longer possible to decrease the sum of squared residuals.

The estimated parameters values are $T_1 = 0.0116$, $T_2 = 1.3563$ and $T_3 = 4.7565$. The model fit is quite good, as expected due to the low number of data points compared to the estimated parameters. The fit of the model is visually good, as can be confirmed by looking at the model predictions and data presented in Figures 1.13-1.15.

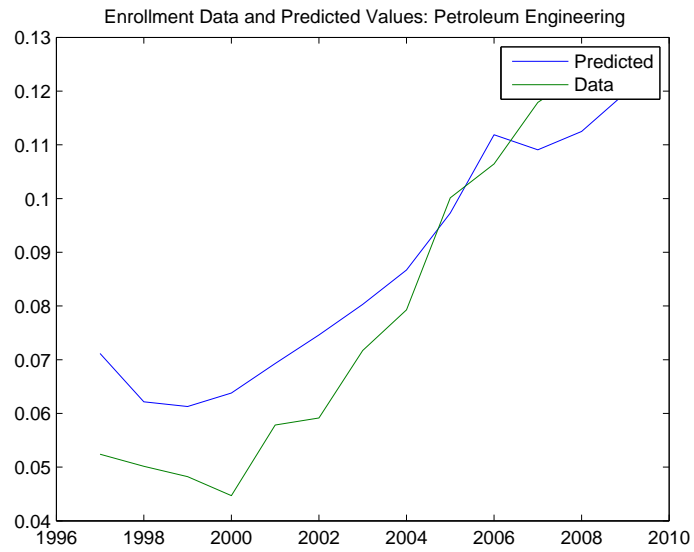


Figure 1.13: Fraction of New Petroleum Engineering Students

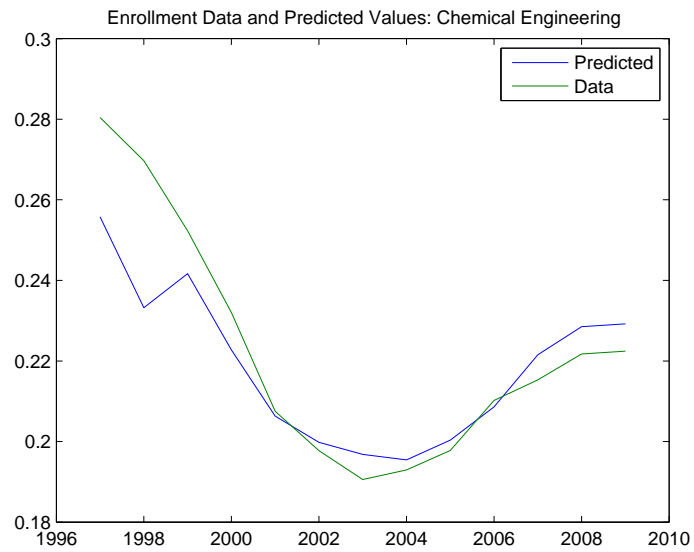


Figure 1.14: Fraction of New Chemical Engineering Students

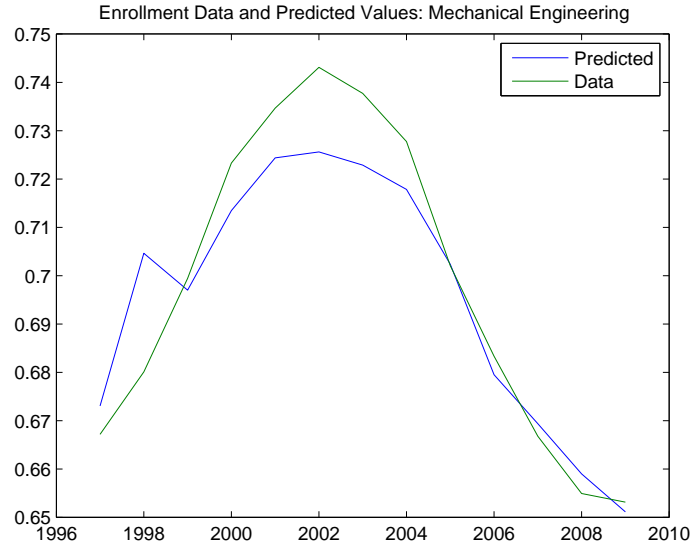


Figure 1.15: Fraction of New Mechanical Engineering Students

Overall, the fit seems very good for chemical and mechanical engineering students. The fit is less good for petroleum engineering students, although in general the drastic increase in the popularity of petroleum engineering is captured by the model.

The response of occupational choices to changes in hidden states elasticity depends heavily on ψ , a parameter that is not often estimated in the literature. In my estimation I used $\psi = 5$ which is roughly similar to the estimate of Hsieh, Klenow et al (2013). These authors try a wider range for ψ in their robustness checks. My model, on the other hand, is not able to capture the increase in enrollment in petroleum engineering if ψ is very different from 5. In general, if ψ takes a lower value, the responses of enrollment in a given discipline will be too modest and for a

larger parameter value they will be too prominent. The Appendix derives an expression for the elasticity of occupational choices with respect to changes in the value functions. The conclusion is that ψ is indeed a crucial variable in this sense.

I use TFP realizations that are roughly consistent with the historical standard deviation of the sectors where each occupation primarily works. These sectors are determined using the the Bureau of Labor Statistics Industry-Occupation matrices. I quantify the deviation of each sector's GDP around the average growth rate of the economy and pick numbers that are roughly able to generate this deviation. As a starting point I use the documented TFP realizations from the GGDC 10-Sector Database for manufacturing industry. Since the manufacturing industry and the machinery industry (mechanical engineering) behave very similarly in the historical data, I use that TFP number for mechanical engineering. The TFP realizations for chemical engineering and petroleum engineering are then determined so that the proportional standard deviations are consistent with the historical sector GDP data. The numerical values are in Table 1.3.

The fact that the TFP values differ across occupations creates exogenous differences between each occupation. In addition, the estimated values for the T_i 's play a role in the quantitative results.

Occupation	γ_g	γ_b
Petroleum	1.14	0.86
Chemical	1.0875	0.9125
Mechanical	1.07	0.93

Table 1.3: TFP Values for Calibration

1.5.3 Effect on the Aggregate Economy

An important caveat of the quantitative section is the fact that this paper studies only a subset of all occupations. While there certainly are many occupations that require specific university training and credentials, there are many occupations that do not require such training. If occupational switches are easy, the incomplete markets problem becomes less severe.

How could one estimate the impact of a subset of occupations on the aggregate economy? A very parsimonious way is to assume a Constant Elasticity of Substitution production function. Let Y denote the production of a typical occupation that requires specialized university training and let Z be the production of a typical occupation that does not require such training. Let μ be the proportion of occupations that require university training. The production in the aggregate economy can then be written as $(\mu Y^{\frac{\nu-1}{\nu}} + Z^{\frac{\nu-1}{\nu}})^{\frac{\nu}{\nu-1}}$ where ν is the elasticity of substitution. The impact of a change in Y on the aggregate production could then be estimated

using empirical estimates of Z and μ . However, the estimate will depend crucially on the elasticity of substitution ν . This approach will not be pursued further in this paper. The implication is that all estimates should be interpreted to apply only to a subset of all occupations.

1.6 Quantitative Results

Since there are no reliable micro-data estimates for the elasticity of substitution between the goods of these occupations, I experiment with alternative values of elasticity of substitution σ and the capital share of labor θ . Note that there is no reason to believe that θ would be close to the typical estimate 0.3. For one, the model does not attempt to estimate the capital share of the aggregate economy. Secondly, the occupations studied in this paper are occupations where a lot of expensive equipment is needed.

The following discussion refers to the table above. Model 1 corresponds to a calibration where $\sigma = 5$ and $\theta = 0.3$. The output loss is then estimated to be quite modest (0.1%). Model 2 corresponds to a calibration where $\sigma = 30$ and $\theta = 0.7$. These parameter values imply that the wage risk is much higher in the model. Correspondingly, the output loss is estimated to be 3.7% of the GDP.

The purpose of Model 3 is to see how our different productivity distributions

Parameter	Model 1	Model 2	Model 3	Model 4
ρ	-	-	-	-
λ	-	-	-	-
ψ	-	-	-	-
σ	5	30	30	30
θ	0.3	0.7	0.7	0.7
α_H	-	-	-	-
α_L	-	-	-	-
δ	-	-	-	-
T_i	estimated	estimated	$T_i = 0.32 \forall i$	$T_i = 0.32 \forall i$
γ_i values	from data	from data	from data	the same $\forall i$
Output loss	.1%	3.7%	6.8%	1.2%

Table 1.4: Quantitative Results

affect the results. Since the model is calibrated so that T_i is the lowest for petroleum engineering and highest for mechanical engineering, the implication is that labor in petroleum engineering seems to be less productivity on average. Since petroleum engineering also is the most risky occupation, the output maximizing planner has to take people away from the mechanical engineering and put these individuals to petroleum engineering. Since this implies that people transfer to less productive tasks, the labor force decreases and hence the estimate for model 2 is only 3.7%. Without this reduction in labor force, Model 3 shows that the output loss would have been 6.8% of the GDP. The conclusion here is that the productivity distribution matters for the estimates.

Model 4 aims to assess the output loss caused by the decrease in labor units which results from the fluctuation of the comparative advantage line (see Section 1.4.7). That model sets all T_i and also all TFP realizations equal. As a consequence, all occupations are equally risky and labor is equally productive in all occupations. The remaining inefficiency should hence only be due to the ‘over-reaction’ in occupational choices over time. The estimate is that people’s over-reaction causes a 1.2% GDP loss.

Figures 1.16-1.18 show the policy functions in the competitive equilibrium and the social planner’s problem.

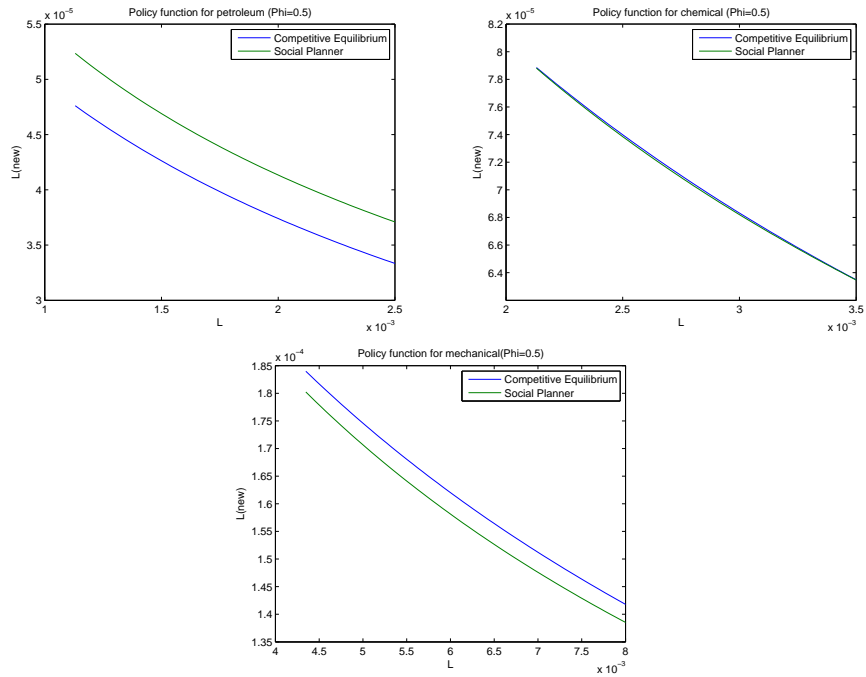


Figure 1.16: Policy Functions for $\phi = 0.5$

Total efficiency units of labor by occupation are displayed for different models in the following tables. These tables show that there is indeed a change in labor units, as discussed above.

The steady state distributions corresponding to Model 1 are shown in Figures 1.19-1.21.

1.7 Policies

Thus far it has been argued that the competitive equilibrium outcome differs from the planner's solution because of the risk aversion and the resulting incomplete mar-

Efficiency Units of Labor: Model 1		
Occupation	Equilibrium	Planner
Petroleum	0.17076	0.17556
Chemical	0.27794	0.27777
Mechanical	0.61478	0.61028
Total	1.0635	1.0636

Table 1.5: Efficiency Units of Labor: Model 1

Efficiency Units of Labor: Model 2		
Occupation	Equilibrium	Planner
Petroleum	9.5558	15.182
Chemical	16.82	15.624
Mechanical	32.559	27.083
Total	58.934	57.889

Table 1.6: Efficiency Units of Labor: Model 2

Efficiency Units of Labor: Model 3		
Occupation	Equilibrium	Planner
Petroleum	12.898	20.81
Chemical	16.82	14.72
Mechanical	17.286	13.796
Total	47.004	49.326

Table 1.7: Efficiency Units of Labor: Model 3

Efficiency Units of Labor: Model 4		
Occupation	Equilibrium	Planner
Petroleum	16.82	17.413
Chemical	16.82	17.413
Mechanical	16.82	17.413
Total	50.46	52.24

Table 1.8: Efficiency Units of Labor: Model 4

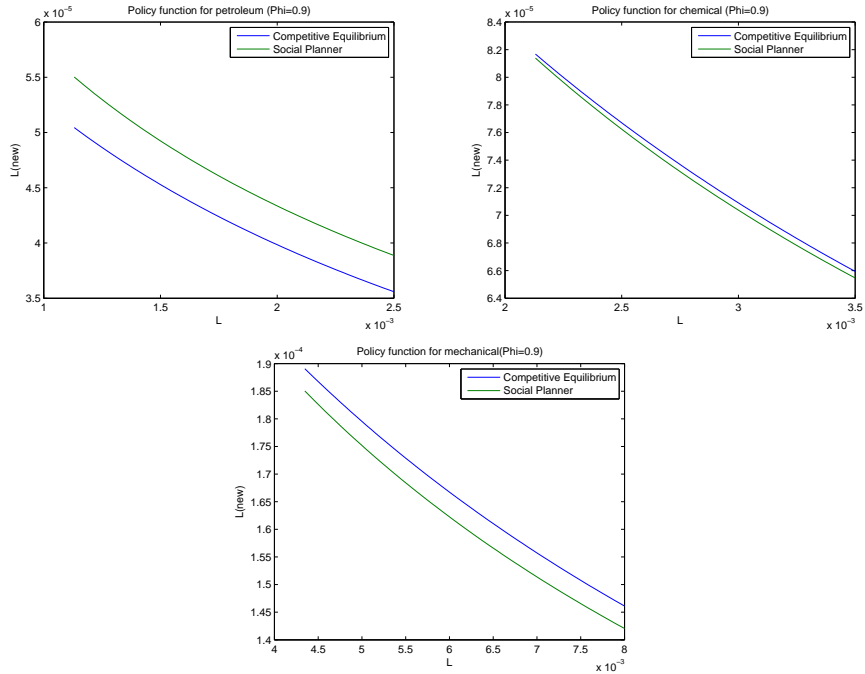


Figure 1.17: Policy Functions for $\phi = 0.9$

kets problem. The question is therefore: what policies could make the competitive equilibrium allocation identical to the unconstrained planner's allocation? In this section I discuss two possible policies: (i) education quotas (ii) insurance contracts.

1.7.1 Education Quotas

Based on the model setup, the planner's allocation is easy to obtain by simply restricting the total number of individuals who can choose a particular major in a given year. As long as education quotas are such that they preserve the sorting based on comparative advantage, the planner's allocation is obtained. A possible way to achieve this would be to first determine the number of slots in each discipline

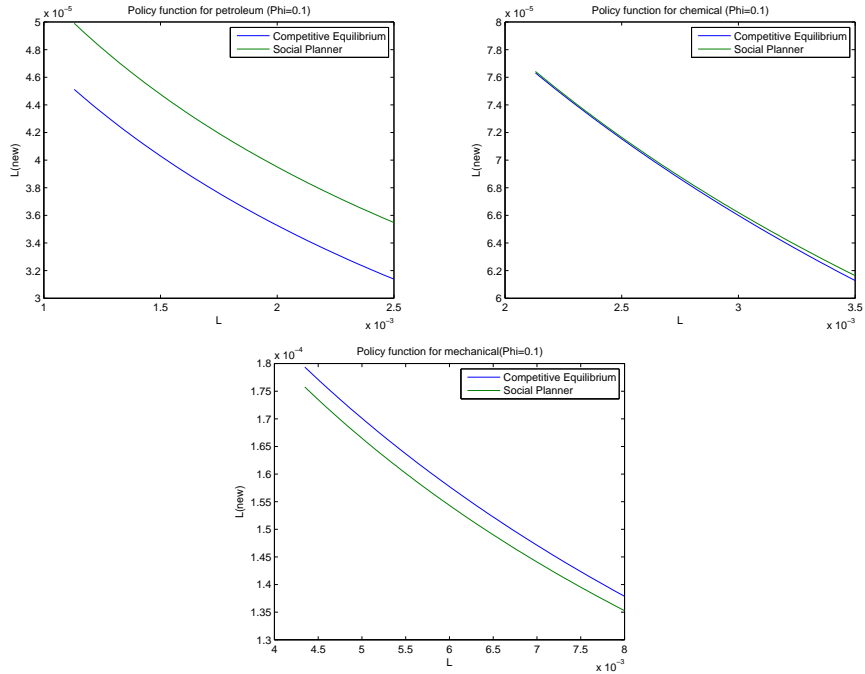


Figure 1.18: Policy Functions for $\phi = 0.1$

for a given year and then test the individuals' skills and talent in that particular discipline. The obvious caveat to this procedure and also the model presented in this paper is the assumption that people actually know their comparative advantages and that the comparative advantages do not change during the studies. However, the utilitarian welfare will be very low if the insurance against adverse occupation-specific shocks is not available.

In a broader picture, the paper raises questions about the organization of education systems. For example, without presenting any evidence, it seems intuitively clear that the United States has a fairly decentralized education system where the

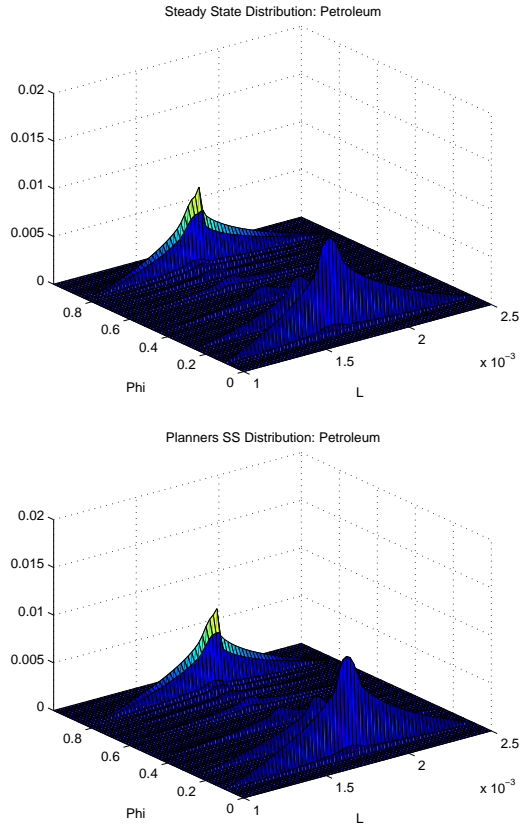


Figure 1.19: Stationary Distributions of Petroleum Engineers (Top: competitive equilibrium Bottom: planner)

proportion of private universities is higher than in most other countries and the education system is likely to respond to changes in demand for each major. That is, if a lot of people want to enroll in a particular discipline, some universities in the country are likely to increase slots in that discipline. This is contrasted to other countries where the university system is more centralized and a government agency creates employment projections and the slots in each major are based on these projections.

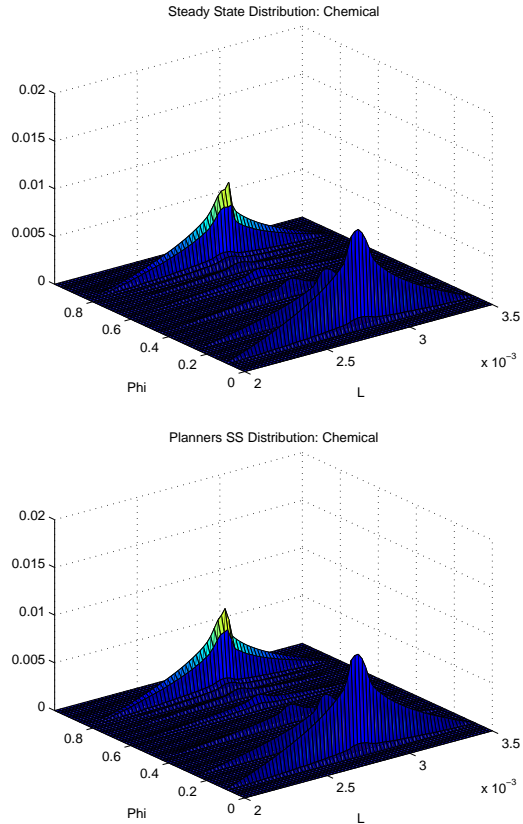


Figure 1.20: Stationary Distributions of Chemical Engineers (Top: competitive equilibrium Bottom: planner)

1.7.2 Insurance Contracts

Since the output loss in these occupations is fairly large, it is surprising that financial markets do not provide insurance against adverse occupation-specific shocks. In theory it could be possible to start an insurance company that collects money from individuals who are about to start their career and then distribute payments to those whose occupation ended up experiencing an adverse shock. The problems with this type of arrangement are quite obvious, though. Typically people face

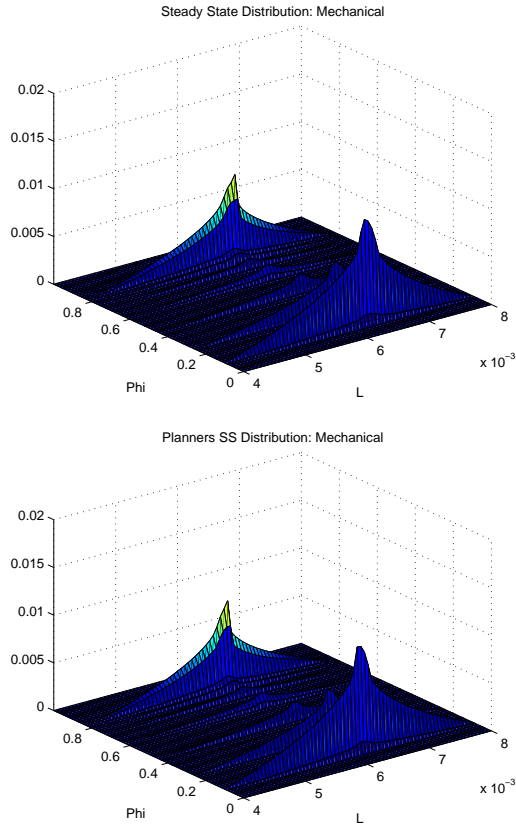


Figure 1.21: Stationary Distributions of Mechanical Engineers (Top: competitive equilibrium Bottom: planner)

severe credit constraints when they are starting their career and they might not have enough funds to invest in this type of insurance. On the other hand, if the insurance fund was financed *ex post* by the payments of those individuals whose occupation experienced a positive shock, there would be a serious commitment problem. Therefore, it seems that the financial markets will have a hard time providing this type of insurance.

1.8 Conclusions

This paper develops a model in which individuals have heterogeneous productivities in various occupations. The occupational choices are distorted because the model embeds a market failure since individuals are not able to purchase insurance against adverse occupation-specific shocks. I provide explicit form solutions to the equilibrium allocation and two planning problems: (i) the unconstrained planning problem (ii) the ‘constrained efficient’ planning problem. The paper makes two contributions. First, the theoretical model is used to study how the competitive equilibrium allocation could be improved in a world where there is no insurance against adverse occupation-specific shocks. The main result is that the direction of the market failure is the opposite from what one might expect: the constrained planner should put *less* people in the risky occupations. Second, a dynamic version of the model is used to quantify how much more output could be produced if the insurance was available. I use data on petroleum, chemical and mechanical engineers for this purpose. The potential increase in output is estimated to be about 3.7% for these occupations.

The analysis in this paper is subject to several limitations. For one, the model assumes that people choose their occupations once and for all. This is obviously unrealistic as people often switch occupations. Nevertheless, Kambourov and Manovskii (2008,2009) show that occupational switching is quite low in occupations that re-

quire a lot of university level education. Moreover, the theoretical results apply also if occupation switches are possible, as long as there are sufficiently high costs to occupation switches. The model also makes very particular functional form assumptions. First, the Fréchet distribution is used extensively due to its special properties. Even though this is a very specific distribution, it seems that the main theoretical arguments should hold for small perturbations around this distribution. A second and perhaps more severe criticism is the assumption that workers are not able to save in my model. Including saving would render it impossible to get closed form solutions but the consequences of adding saving are easily predictable. If agents were able to save without credit constraints, the inefficiencies would be mitigated, as agents would simply borrow money after an adverse occupation-specific shock. However, part of the inefficiency would still remain. On the other hand, if credit constraints are such that workers are able to save but not borrow, the outcome would be somewhat different. Under such circumstances, new entrants would care a lot about their first paycheck. Therefore we would expect that even more workers would enter the occupations that are experiencing a boom. This would amplify the ‘over-reaction’ in booms and busts.

The model lends itself to several interesting extensions. For one, the stationarity of the model helps with computations but is somewhat unrealistic. In reality new occupations are born and old occupations die over time. The model could be mod-

ified to account for this by choosing a Bayesian process in which new occupations are more uncertain than the established ones but as workers learn about the new occupations over time, the uncertainty disappears. This modification of the model seems appropriate for modeling many interesting occupations in the economy.

1.9 Appendix

1.9.1 Details of Final Good and Occupation Good Producer's Problem

The derivation of (1.2) goes as follows. The FOC of the final good producer's problem is:

$$\frac{\sigma}{\sigma - 1} \left(\int_0^1 \gamma_j Y_j^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}-1} \gamma_i \frac{\sigma - 1}{\sigma} Y_i^{\frac{1}{\sigma}} = p_i$$

This is the same as equation (1.2).

To solve the occupation good producer's problem, note that the first-order conditions of the cost-minimizing problem lead to

$$\frac{w_i}{r} = \frac{1 - \theta}{\theta} \frac{K_i}{L_i}$$

The zero-profit condition requires that $p_i Y_i - r K_i - w_i L_i = 0$. Using the inverse demand (1.2) and $Y_i = K_i^\theta L_i^{1-\theta}$, we can eliminate p_i , Y_i and K_i from the zero-profit condition. Solving for w_i gives equation (1.4).

To derive the production function in (1.5), write the occupation good production function as

$$Y_i = \left(\frac{w_i}{r} \frac{\theta}{1-\theta} \right)^\theta L_i$$

Plugging in the expression for w_i (equation (1.4)), we get

$$Y_i = \left(c_1 \frac{1}{r} \frac{\theta}{1-\theta} \right)^\theta Y^{\frac{\theta}{\kappa}} L_i^{1-\frac{\theta}{\kappa}} \gamma_i^{\frac{\sigma\theta}{\kappa}}$$

Using this to eliminate Y_i in the final good production function and solving for Y gives equation (1.5).

1.9.2 Derivation of $L_{i,new}$

First compute the efficiency units $L_{i,new}$ of those who go to discipline i :

$$\begin{aligned} L_{i,new} &= \int_0^\infty \xi_i \prod_{j \neq i} Pr \left(\xi_j < \left(\frac{|v_i|}{|v_j|} \right)^{\frac{1}{1-\rho}} \xi_i \right) dF_i(\xi_i) \\ &= \int_0^\infty \xi_i \prod_{j \neq i} \exp \left(-T_j \left(\frac{|v_i|}{|v_j|} \right)^{\frac{-\psi}{1-\rho}} \xi_i^{-\psi} \right) \exp(-T_i \xi_i^{-\psi}) T_i \psi \xi_i^{-\psi-1} d\xi_i \\ &= \int_0^\infty \xi_i \exp \left(- \left(\int_0^1 T_j \left(\frac{|v_i|}{|v_j|} \right)^{\frac{-\psi}{1-\rho}} dj \right) \xi_i^{-\psi} \right) T_i \psi \xi_i^{-\psi-1} d\xi_i \\ &= \int_0^\infty \xi_i \exp(-\tilde{T} \xi_i^{-\psi}) T_i \psi \xi_i^{-\psi-1} d\xi_i \end{aligned}$$

where $\tilde{T} = \left(\int_0^1 T_j \left(\frac{|v_i|}{|v_j|} \right)^{\frac{-\psi}{1-\rho}} dj \right)$. Using a change of variable $x = \tilde{T} \xi_i^{-\psi}$ such that $dx = -\psi \tilde{T} \xi_i^{-\psi-1} d\xi_i$, the previous expression becomes

$$\begin{aligned}
L_{i,new} &= \int_0^\infty \left(\frac{x}{\tilde{T}}\right)^{\frac{-1}{\psi}} \exp(-x) \frac{T_i}{\tilde{T}} dx \\
&= \tilde{T}^{\frac{1}{\psi}-1} T_i \Gamma\left(1 - \frac{1}{\psi}\right) \\
&= \frac{T_i}{\left(\int_0^1 T_j \left(\frac{|v_j|}{|v_i|}\right)^{\frac{\psi}{1-\rho}} dj\right)^{\frac{\psi-1}{\psi}}} \Gamma\left(1 - \frac{1}{\psi}\right) \\
&= T_i \left(\int_0^1 T_j \left(\frac{|v_j|}{|v_i|}\right)^{\frac{\psi}{1-\rho}} dj\right)^{\frac{\psi-1}{\psi}} \Gamma\left(1 - \frac{1}{\psi}\right)
\end{aligned}$$

Here the second equation follows from the definition of the gamma function:

$$\Gamma(t) = \int_0^\infty x^{t-1} e^{-x} dx.$$

1.9.3 Derivations for the One-Period Model

Competitive Equilibrium I solve the competitive equilibrium allocation with an arbitrary tax rate to avoid duplication. The ‘real’ competitive equilibrium is obtained by setting $\tau_i = 0$ for all i . The after-tax income is

$$(1 - \tau_j)w_j = (1 - \tau_j) \left(c\gamma_j L_j^{-\frac{1}{\sigma}} Y^{\frac{1}{\sigma}}\right)^{\frac{\sigma}{\kappa}}$$

Plugging this into the expected utility $|v_i|$ allows us to write

$$\left(\frac{|v_j|}{|v_i|}\right)^{\frac{\psi}{1-\rho}} = \left(\frac{L_j}{L_i}\right)^{\frac{-\psi}{\kappa}} \frac{(1 - \tau_j)^\psi}{(1 - \tau_i)^\psi} \left(\frac{E(\gamma_j^{(1-\rho)\frac{\sigma}{\kappa}})}{E(\gamma_i^{(1-\rho)\frac{\sigma}{\kappa}})}\right)^{\frac{\psi}{1-\rho}}$$

Write the law of motion for labor as

$$L_i = \left(\int_0^1 \left(\frac{|v_j|}{|v_i|}\right)^{\frac{\psi}{1-\rho}} dj\right)^{\frac{1-\psi}{\psi}} \Gamma\left(1 - \frac{1}{\psi}\right)$$

Substitute in $\left(\frac{|v_j|}{|v_i|}\right)^{\frac{\psi}{1-\rho}}$ and re-arrange:

$$L_i \frac{(L_i)^{\frac{\psi-1}{\kappa}}}{(1-\tau_i)^{\psi-1} E\left(\gamma_i^{\frac{(1-\rho)\sigma}{\kappa}}\right)^{\frac{\psi-1}{1-\rho}}} = \text{constant across } i$$

Set the left-hand side equal for occupations i and j and re-arrange:

$$\frac{L_j}{L_i} = \left(\frac{(1-\tau_j)^{\psi-1} E\left(\gamma_j^{\frac{(1-\rho)\sigma}{\kappa}}\right)^{\frac{\psi-1}{1-\rho}}}{(1-\tau_i)^{\psi-1} E\left(\gamma_i^{\frac{(1-\rho)\sigma}{\kappa}}\right)^{\frac{\psi-1}{1-\rho}}} \right)^{\frac{\kappa}{\kappa+\psi-1}}$$

and

$$\left(\frac{|v_j|}{|v_i|}\right)^{\frac{\psi}{1-\rho}} = \left(\frac{1-\tau_j}{1-\tau_i}\right)^{\frac{\psi\kappa}{\kappa+\psi-1}} \left(\frac{E\left(\gamma_j^{\frac{(1-\rho)\sigma}{\kappa}}\right)}{E\left(\gamma_i^{\frac{(1-\rho)\sigma}{\kappa}}\right)}\right)^{\frac{1}{1-\rho} \frac{\psi\kappa}{\kappa+\psi-1}}$$

Substitute this in the law of motion for labor:

$$L_i = (1-\tau_i)^{\frac{(\psi-1)\kappa}{\kappa+\psi-1}} E\left(\gamma_i^{\frac{(1-\rho)\sigma}{\kappa}}\right)^{\frac{1}{1-\rho} \frac{(\psi-1)\kappa}{\kappa+\psi-1}} \times \left(\int_0^1 (1-\tau_j)^{\frac{\psi\kappa}{\kappa+\psi-1}} E\left(\gamma_j^{\frac{(1-\rho)\sigma}{\kappa}}\right)^{\frac{1}{1-\rho} \frac{\psi\kappa}{\kappa+\psi-1}} dj \right)^{\frac{1-\psi}{\psi}} \Gamma\left(1 - \frac{1}{\psi}\right)$$

Planner's Solution The FOC with respect to K_i is

$$Y_i^{\frac{1}{\sigma}} \gamma_i \theta \frac{Y_i^{\frac{\sigma-1}{\sigma}}}{K_i} = r$$

The second FOC leads to

$$\frac{h_i}{E\left(\frac{dY}{dY_i} \frac{dY_i}{dL'_i}\right)} = E\left(\int_0^1 \frac{dY}{dY_j} \frac{dY_j}{dL'_j} T_j h_j^{\psi-1} \left(\int_0^1 T_k h_k^{\psi} dk\right)^{-1} dj\right)$$

Since the RHS is constant, we get

$$\frac{h_j}{h_i} = \frac{E\left(\frac{dY}{dY_j} \frac{dY_j}{dL'_j}\right)}{E\left(\frac{dY}{dY_i} \frac{dY_i}{dL'_i}\right)}$$

Substitute optimal capital to the production function and write $\kappa = (1 - \theta)\sigma + \theta$:

$$Y_i = \left(\left(Y^{\frac{1}{\sigma}} \gamma_i \theta r^{-1} \right)^\theta L_i^{1-\theta} \right)^{\frac{\sigma}{\kappa}}$$

Then we can solve for

$$\begin{aligned} \frac{dY}{dY_i} \frac{dY_i}{dL_i} &= Y^{\frac{1}{\sigma}} \gamma_i (1 - \theta) \frac{Y_i^{\frac{\sigma-1}{\sigma}}}{L_i} \\ &= (1 - \theta) \left(\frac{\theta}{r} \right)^{\frac{\theta(\sigma-1)}{\kappa}} Y^{\frac{1}{\kappa}} \gamma_i^{\frac{\sigma}{\kappa}} L_i^{\frac{-1}{\kappa}} \end{aligned}$$

Using this the optimality condition becomes

$$\left(\frac{h_j}{h_i} \right)^\psi = \left(\frac{L_j}{L_i} \right)^{-\frac{\psi}{\kappa}} \left(\frac{E(\gamma_j^{\frac{\sigma}{\kappa}})}{E(\gamma_i^{\frac{\sigma}{\kappa}})} \right)^\psi$$

Write the law of motion for labor as

$$L_{i,new} = \left(\int_0^1 \left(\frac{h_j}{h_i} \right)^\psi dj \right)^{\frac{1-\psi}{\psi}} \Gamma \left(1 - \frac{1}{\psi} \right)$$

Substitute in $\frac{h_j}{h_i}$ and re-arrange:

$$L_i \frac{L_i^{\frac{\psi-1}{\kappa}}}{E(\gamma_i^{\frac{\sigma}{\kappa}})^{\psi-1}} = \text{constant across } i$$

Set the left-hand side equal for occupations i and j :

$$\frac{L_j}{L_i} = \left(\frac{E(\gamma_j^{\frac{\sigma}{\kappa}})^{\psi-1}}{E(\gamma_i^{\frac{\sigma}{\kappa}})^{\psi-1}} \right)^{\frac{1}{1+\frac{\psi-1}{\kappa}}}$$

Therefore

$$\left(\frac{h_j}{h_i}\right)^\psi = \left(\frac{E(\gamma_j^{\frac{\sigma}{\kappa}})}{E(\gamma_i^{\frac{\sigma}{\kappa}})}\right)^{\frac{\kappa\psi}{\kappa+\psi-1}}$$

and

$$L_i = E(\gamma_i^{\frac{\sigma}{\kappa}})^{\frac{\kappa(\psi-1)}{\kappa+\psi-1}} \left(\int_0^1 E(\gamma_j^{\frac{\sigma}{\kappa}})^{\frac{\kappa\psi}{\kappa+\psi-1}} dj\right)^{\frac{1-\psi}{\psi}} \Gamma\left(1 - \frac{1}{\psi}\right)$$

1.9.4 Utilitarian Welfare

Proposition (1.1) argues that the utilitarian welfare can be written as $\left(\int_0^1 |v_i|^{\frac{\psi}{1-\rho}}\right)^{\frac{1-\rho}{\psi}} \Gamma\left(1 + \frac{\rho-1}{\psi}\right)$.

To prove this result, I formally assume that the unit interval is divided to n occupations. After the derivation, I let $n \rightarrow \infty$ to generalize the proof for the case of a continuum.

The probability that *ex ante* utility from any occupation is less than z is

$$\begin{aligned} Pr(\xi_i^{1-\rho} v_i \leq z, \forall i) &= Pr\left(\xi_i \leq \left(\frac{|z|}{|v_i|}\right)^{\frac{1}{1-\rho}}, \forall i\right) \\ &= \prod_i \exp\left(-T_i \left(\frac{|z|}{|v_i|}\right)^{\frac{-\psi}{1-\rho}}\right) \\ &= \exp\left(\tilde{T}|z|^{\frac{-\psi}{1-\rho}}\right) \end{aligned}$$

where $\tilde{T} = \int_0^1 T_i |v_i|^{\frac{\psi}{1-\rho}} di$. Since v_i can be either positive or negative depending on ρ , I use absolute values to cover both cases. I use notation (-) as a negative

sign which applies only if $\rho > 1$. Taking the derivative with respect to z gives the density of the utility from the most preferred occupation:

$$f(z) = \exp\left(-\tilde{T}|z|^{\frac{-\psi}{1-\rho}}\right) \tilde{T} \frac{\psi}{|1-\rho|} |z|^{\frac{-\psi}{1-\rho}-1}$$

The *ex ante* utility of a worker is then the expected value of z :

$$\begin{aligned} Eu &= \int z \exp\left(-\tilde{T}|z|^{\frac{-\psi}{1-\rho}}\right) \tilde{T} \frac{\psi}{|1-\rho|} |z|^{\frac{-\psi}{1-\rho}-1} dz \\ &= (-) \int \exp\left(-\tilde{T}|z|^{\frac{-\psi}{1-\rho}}\right) \tilde{T} \frac{\psi}{|1-\rho|} |z|^{\frac{-\psi}{1-\rho}} dz \end{aligned}$$

Using a change of variables $x = \tilde{T}|z|^{\frac{-\psi}{1-\rho}}$, we get $dx = \tilde{T} \frac{\psi}{1-\rho} |z|^{\frac{-\psi}{1-\rho}-1} dz$. The expected utility becomes

$$\begin{aligned} Eu &= (-) \int \exp(-x) |z| dx \\ &= (-) \int \exp(-x) \left(\frac{x}{\tilde{T}}\right)^{\frac{1-\rho}{-\psi}} dx \\ &= (-) \tilde{T}^{\frac{1-\rho}{\psi}} \Gamma\left(1 + \frac{\rho-1}{\psi}\right) \\ &= (-) \left(\int_0^1 T_i |v_i|^{\frac{\psi}{1-\rho}} di\right)^{\frac{1-\rho}{\psi}} \Gamma\left(1 + \frac{\rho-1}{\psi}\right) \end{aligned}$$

where the definition of the gamma function was used after the second equality.

This proves Proposition (1.1).

1.9.5 Constrained Efficient Allocation

The constrained efficient allocation is derived as follows. Substituting the wage constraint into the absolute value of expected utility gives

$$|v_i| = \frac{1}{|1-\rho|} c_1^{1-\rho} \left(\frac{Y}{L_i} \right)^{\frac{1-\rho}{\kappa}} E(\gamma_i^{(1-\rho)\frac{\sigma}{\kappa}})$$

Using the total production constraint we get

$$\frac{Y}{L_i} = c_2 \left(\int_0^1 \gamma_j^{\frac{\sigma}{\kappa}} \left(\frac{L_j}{L_i} \right)^{\frac{\kappa-1}{\kappa}} dj \right)^{\frac{\kappa}{\kappa-1}}$$

Substituting the previous two expressions into the constrained planner's objective function and dropping constants gives an alternative form of the problem:

$$\max_{h_i} \int_0^1 \left(\int_0^1 \gamma_j^{\frac{\sigma}{\kappa}} \left(\frac{L_j}{L_i} \right)^{\frac{\kappa-1}{\kappa}} dj \right)^{\frac{\psi}{\kappa-1}} E(\gamma_i^{(1-\rho)\frac{\sigma}{\kappa}})^{\frac{\psi}{1-\rho}} di$$

s.t.

$$L_i = h_i^{\psi-1} \left(\int_0^1 h_j^{\psi} dj \right)^{\frac{1-\psi}{\psi}} \Gamma \left(1 - \frac{1}{\psi} \right)$$

The remaining constraint implies that $\frac{L_j}{L_i} = \frac{h_j^{\psi-1}}{h_i^{\psi-1}}$. Substituting this into the objective function and moving all terms with j in them outside the integral gives the unconstrained problem:

$$\max_{h_i} \left(\int_0^1 \gamma_j^{\frac{\sigma}{\kappa}} h_j^{(\psi-1)\frac{\kappa-1}{\kappa}} dj \right)^{\frac{\psi}{\kappa-1}} \left(\int_0^1 h_i^{\frac{-(\psi-1)\psi}{\kappa}} E(\gamma_i^{(1-\rho)\frac{\sigma}{\kappa}})^{\frac{\psi}{1-\rho}} di \right)$$

The first-order condition of this problem characterizes the optimal choice of h_i .

The first-order condition can be simplified to

$$\gamma_i^{\frac{\sigma}{\kappa}} h_i^{(\psi-1)\frac{\kappa-1}{\kappa}-1} \left(\int_0^1 h_j^{\frac{-(\psi-1)\psi}{\kappa}} E(\gamma_j^{(1-\rho)\frac{\sigma}{\kappa}})^{\frac{\psi}{1-\rho}} dj \right) = \left(\int_0^1 \gamma_j^{\frac{\sigma}{\kappa}} h_j^{(\psi-1)\frac{\kappa-1}{\kappa}} dj \right) h_i^{\frac{-(\psi-1)\psi}{\kappa}-1} E(\gamma_i^{(1-\rho)\frac{\sigma}{\kappa}})^{\frac{\psi}{1-\rho}}$$

Collecting all terms with i on them on the left-hand side, we conclude that

$$\frac{E(\gamma_i^{\frac{\sigma}{\kappa}})h_i^{(\psi-1)\frac{\kappa+\psi-1}{\kappa}}}{E(\gamma_i^{(1-\rho)\frac{\sigma}{\kappa}})^{\frac{\psi}{1-\rho}}} = \text{constant across } i.$$

Setting the left-hand side equal for two arbitrary occupations i and j , and solving for $\frac{h_j}{h_i}$ gives

$$\frac{h_j}{h_i} = \left(\frac{E(\gamma_j^{(1-\rho)\frac{\sigma}{\kappa}})^{\frac{\psi}{1-\rho}} E(\gamma_i^{\frac{\sigma}{\kappa}})}{E(\gamma_i^{(1-\rho)\frac{\sigma}{\kappa}})^{\frac{\psi}{1-\rho}} E(\gamma_j^{\frac{\sigma}{\kappa}})} \right)^{\frac{\kappa}{(\psi-1)(\kappa+\psi-1)}}$$

Substituting this expression to the labor constraint $L_i = \left(\int_0^1 \left(\frac{h_j}{h_i} \right)^\psi dj \right)^{\frac{1-\psi}{\psi}} \Gamma \left(1 - \frac{1}{\psi} \right)$ and simplifying gives the constrained efficient allocation in (1.14).

1.9.6 Utilitarian Welfare in One-Period Model

By substituting in the constraints we get

$$\begin{aligned} |v_i| &= \frac{1}{|1-\rho|} E(w_i^{1-\rho}) \\ &= \frac{1}{|1-\rho|} c_1^{1-\rho} E(\gamma_i^{(1-\rho)\frac{\sigma}{\kappa}}) \left(\frac{Y}{L_i} \right)^{\frac{1-\rho}{\kappa}} \\ &\propto E(\gamma_i^{(1-\rho)\frac{\sigma}{\kappa}}) \left(\int_0^1 \gamma_j^{\frac{\sigma}{\kappa}} \left(\frac{L_j}{L_i} \right)^{\frac{\kappa-1}{\kappa}} dj \right)^{\frac{1-\rho}{\kappa-1}} \end{aligned}$$

Letting \tilde{L}_i denote the numerator of the expression for the labor force (denominator is just a constant and cancels out), we can conclude that

$$|v_i|^{\frac{\psi}{1-\rho}} \propto E(\gamma_i^{(1-\rho)\frac{\sigma}{\kappa}})^{\frac{\psi}{1-\rho}} \tilde{L}_i^{-\frac{\psi}{\kappa}} \left(\int_0^1 \gamma_j^{\frac{\sigma}{\kappa}} \tilde{L}_j^{\frac{\kappa-1}{\kappa}} dj \right)^{\frac{\psi}{\kappa-1}}$$

The utilitarian welfare is then proportional to

$$\left(\int_0^1 |v_i|^{\frac{\psi}{1-\rho}} di \right)^{\frac{1-\rho}{\psi}} \propto \left(\int_0^1 E(\gamma_i^{(1-\rho)\frac{\sigma}{\kappa}})^{\frac{\psi}{1-\rho}} \tilde{L}_i^{-\frac{\psi}{\kappa}} di \right)^{\frac{1-\rho}{\psi}} \left(\int_0^1 \gamma_j^{\frac{\sigma}{\kappa}} \tilde{L}_j^{\frac{\kappa-1}{\kappa}} dj \right)^{\frac{1-\rho}{\kappa-1}}$$

This expression is used to draw the graphs for the one-period model.

1.9.7 Model without Idiosyncratic Productivity Differences

Assume that people are homogeneous with respect to productivity in each occupation. This means that they are able to switch occupations with one-to-one rate of transformation. This section will discuss the relevant problem for competitive equilibrium, unconstrained planner and constrained planner.

Let there be only safe and risky occupations, as in the example discussed in text. Fraction ω of the occupations is safe. The unconstrained planner's problem is:

$$\begin{aligned}
& \max_{L_s, L_r} \omega L_s v_s + (1 - \omega) L_r v_r \\
& \text{s.t.} \\
& v_s = \frac{1}{1 - \rho} \left(c_1 L_s^{-\frac{1}{\kappa}} Y^{\frac{1}{\kappa}} \right)^{1 - \rho} \\
& v_r = \frac{1}{1 - \rho} \left(c_1 L_r^{-\frac{1}{\kappa}} Y^{\frac{1}{\kappa}} \right)^{1 - \rho} E(\gamma_r^{(1 - \rho) \frac{\sigma}{\kappa}}) \\
& Y = c_2 \left(\omega L_s^{\frac{\kappa - 1}{\kappa}} + (1 - \omega) E(\gamma_r^{\frac{\sigma}{\kappa}}) L_r^{\frac{\kappa - 1}{\kappa}} \right)^{\frac{\kappa}{\kappa - 1}} \\
& \omega L_s + (1 - \omega) L_r = 1
\end{aligned} \tag{1.23}$$

Plugging the constraints in the objective function gives:

$$\begin{aligned}
& \min_{L_s, L_r} \left(\omega L_s^{-\frac{1 - \rho}{\kappa} + 1} + (1 - \omega) L_r^{-\frac{1 - \rho}{\kappa} + 1} E(\gamma_r^{(1 - \rho) \frac{\sigma}{\kappa}}) \right) \left(\omega L_s^{\frac{\kappa - 1}{\kappa}} + (1 - \omega) E(\gamma_r^{\frac{\sigma}{\kappa}}) L_r^{\frac{\kappa - 1}{\kappa}} \right)^{\frac{1 - \rho}{\kappa - 1}} \\
& \text{s.t.} \\
& \omega L_s + (1 - \omega) L_r = 1
\end{aligned}$$

The solution to this problem is the constrained efficient labor allocation. The competitive equilibrium is obtained by setting $v_s = v_r$ in (1.23). Doing so leads to a pair of equations:

$$L_s^{-\frac{1 - \rho}{\kappa}} = L_r^{-\frac{1 - \rho}{\kappa}} E(\gamma_r^{(1 - \rho) \frac{\sigma}{\kappa}})$$

$$\omega L_s + (1 - \omega) L_r = 1$$

The solution to this pair of equation is the competitive equilibrium allocation.

Finally, the constrained planner simply wants to maximize output with respect to the labor constraint:

$$\max_{L_s, L_r} \left(\omega L_s^{\frac{\kappa-1}{\kappa}} + (1-\omega) E(\gamma_r^{\frac{\sigma}{\kappa}}) L_r^{\frac{\kappa-1}{\kappa}} \right)^{\frac{\kappa}{\kappa-1}}$$

s.t.

$$\omega L_s + (1-\omega) L_r = 1$$

The solution to this problem gives the allocation of the unconstrained planner.

1.9.8 Budget Balance

So far I have talked about necessary subsidies which may give the impression that the government will run a deficit to implement the planner's allocation. This is not true. In the one-period version of the model, it is possible to reach the Pareto optimum with a balanced budget. This happens because all workers are only interested in relative wages and it is possible to decrease the after-tax income proportionally and still obtain the same allocation, as long as workers keep supplying their labor inelastically. To show this analytically, assume that the subsidies given by (1.12) are such that the government runs a deficit $G < 0$. The government's constraint is

$$\int_0^1 \tau_i w_i L_i' di = G \tag{1.24}$$

Consider replacing tax rates τ_i by $\tau_i + \Delta\tau_i$ and keeping the ratio of taxes constant across all occupations such that

$$\frac{1 - \tau_i - \Delta\tau_i}{1 - \tau_j - \Delta\tau_j} = \frac{1 - \tau_i}{1 - \tau_j} \quad (1.25)$$

for all i and j . It is straightforward to see that the worker's occupation choice with the new taxes will remain the same. Equation (1.25) is simplified to get

$$\Delta\tau_i = \Delta\tau_j \frac{1 - \tau_i}{1 - \tau_j} \quad (1.26)$$

We can use the new tax rates into the government's budget to get a new deficit G' :

$$\int_0^1 (\tau_i + \Delta\tau_i) w_i L'_i di = G' \quad (1.27)$$

Holding j constant and using (1.24), (1.26) and (1.27)

$$G + \frac{\Delta\tau_j}{1 - \tau_j} \int_0^1 (1 - \tau_i) w_i L'_i di = G' \quad (1.28)$$

Finally, to obtain a balanced budget where $G' = 0$, the government can choose $\Delta\tau_j$ such that

$$\Delta\tau_j = \frac{-(1 - \tau_j)G}{\int_0^1 (1 - \tau_i) w_i L'_i di} \quad (1.29)$$

Hence we have shown that the government can always achieve the efficient outcome with a balanced budget in one-period version of the model. In this setup some occupations are subsidized and some are taxed. However, the same principle does not apply in the dynamic version of the model.

1.9.9 Proof that Competitive Equilibrium Coincides with Planner's Problem when $\rho = 0$.

For any set of estimated parameters, the planner's problem can be solved by setting $\rho = 0$ and solving for the competitive equilibrium. This section provides a formal proof of this useful result.

The unconstrained planner's sequential problem can be solved by taking the first-order conditions. The planner's cut-offs h_i satisfy

$$\frac{h_i}{\frac{d\Omega_t}{dL'_{i,t}}} = \int_0^1 \frac{d\Omega_t}{dL'_{j,t}} T_j h_j^{\psi-1} \left(\int_0^1 T_k h_k^\psi dk \right)^{-1} dj \quad (1.30)$$

where I have used the short-hand notation

$$\frac{d\Omega_t}{dL'_{j,t}} = E \left(\sum_{t=1}^{\infty} \beta^{t-1} \frac{dY_t}{dY_{j,t}} \frac{dY_{j,t}}{dL'_{j,t}} (1-\delta)^{t-1} \right) \quad (1.31)$$

and

$$\frac{dY_t}{dY_{j,t}} \frac{dY_{j,t}}{dL'_{j,t}} = \gamma_{j,t}^{\frac{\sigma}{\kappa}} \left(\frac{\theta}{r} \right)^{\theta \frac{\sigma-1}{\kappa}} Y_t^{\frac{1}{\kappa}} L_{j,t}^{\frac{-1}{\kappa}} (1-\theta) \quad (1.32)$$

The expression in (1.30) is the same as

$$h_i \left(\int_0^1 T_k h_k^\psi dk \right) = \frac{d\Omega_t}{dL'_{i,t}} \int_0^1 \frac{d\Omega_t}{dL'_{j,t}} T_j h_j^{\psi-1} dj \quad (1.33)$$

From this expression it is straightforward to see that the social planner's problem is solved by

$$h_i = \frac{d\Omega_t}{dL'_{i,t}} \quad (1.34)$$

Furthermore, choice rule of the planner is specified so that multiplying h_i for all i by any positive constant does not change the allocation. Therefore the choice rule can be normalized by dividing by an appropriate constant. As a consequence,

$$h_i \propto E \left(\sum_{t=1}^{\infty} \beta^{t-1} (1 - \delta)^{t-1} \gamma_{j,t}^{\frac{\sigma}{\kappa}} L_{j,t}^{\frac{-1}{\kappa}} \right) \quad (1.35)$$

where \propto means 'is proportional to'. We proceed by showing that in the competitive equilibrium with $\rho = 0$, the cut-off rule in (1.6) is similar to (1.35). Let us start with wages. Since constant parameters do not affect occupation choice, the wage in (1.4) is conveniently written as

$$w_j \propto \gamma_j^{\frac{\sigma}{\kappa}} L_j^{\frac{-1}{\kappa}} \quad (1.36)$$

Since $\rho = 0$, periodic utility is linear in wage. The lifetime expected utility is then

$$v_i \propto E \left(\sum_{t=1}^{\infty} \beta^{t-1} (1 - \delta)^{t-1} \gamma_{j,t}^{\frac{\sigma}{\kappa}} L_{j,t}^{\frac{-1}{\kappa}} \right) \quad (1.37)$$

This is the same as (1.35) which completes the proof.

1.9.10 Response of Occupation Choice to Changes in Value Functions

It is worthwhile to study the model predictions a bit further theoretically. In particular, the elasticity of enrollment in a given discipline with respect to changes in value function seems like a key variable.

The percentage of individuals going to sector i in the model is

$$\text{Pr}_i = \frac{T_i |v_i|^{\frac{\psi-1}{1-\rho}}}{\sum_k T_k |v_k|^{\frac{\psi-1}{1-\rho}}}$$

It is then possible to show that the elasticity of occupation choice (with respect to a change in $|v_i|$) is:

$$\frac{\frac{\partial \text{Pr}_i}{\text{Pr}_i}}{\frac{\partial |v_i|}{|v_i|}} = \frac{\psi - 1}{1 - \rho} (1 - \text{Pr}_i)$$

The model prediction is that the elasticity is larger for smaller sectors (small Pr_i). This seems consistent with the observation that in the engineering data the largest change in degrees awarded happens for petroleum engineering which is a relatively small discipline.

1.9.11 Petroleum Engineering Wage Data

This Appendix provides additional proof that college major matters in the job market. PUMS (Public Use Microdata Sample) is a survey of the Census and contains detailed data of workers in the United States. For four years now, the data has included both the individual's college major and current occupation. This data can be used to support the claim that major choices matter. If we look at individuals who work in the petroleum engineering occupation, only 24% of those individuals actually majored in petroleum engineering. The rest are engineers who studied other disciplines, the most prominent majors being chemical engineering (14.4%) and mechanical engineering (18.8%). This is consistent with the discussion in the text.

To analyze this further, I first estimate a Mincer equation on all engineering workers regardless of discipline or occupation. Since all engineers have a bachelor's degree, the Mincer equation measures education by including dummies for Master's level, MBA level (professional degree) and PhD level work. Additionally, I include a variables that measures work experience and also the square of this variable. The results are summarized in Figure 1.22.

I save the residuals from this regression. The residual for those individuals who have a degree in petroleum engineering is .56 on average whereas the average residual

Figure 1.22: Results from a Mincer Equation

```

. reg logwage masters profdegree phd exper exper2

```

Source	SS	df	MS			
Model	4270.38048	5	854.076097	Number of obs =	120078	
Residual	25046.4487120072		.208595249	F(5,120072) =	4094.42	
Total	29316.8292120077		.244150247	Prob > F =	0.0000	
				R-squared =	0.1457	
				Adj R-squared =	0.1456	
				Root MSE =	.45672	

logwage	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
masters	.18338	.0028849	63.56	0.000	-.1777255	.1890344
profdegree	-.3434027	.0077208	44.48	0.000	-.3282701	-.3585352
phd	.2695653	.0059603	45.23	0.000	-.2578832	.2812475
exper	.0434429	.0004989	87.08	0.000	-.0424651	.0444207
exper2	-.0007396	.0000115	-64.30	0.000	-.0007621	-.000717
_cons	-2.936618	.0048892	-600.64	0.000	-2.9462	-2.927035

for non-petroleum engineers is .27. The difference between these observations is statistically significant. Since wages is measured in logs, the difference in means corresponds to $\exp(.56 - .27) - 1 \approx 33\%$ higher wages for the individuals with petroleum engineering degrees as compared to the other engineers in the petroleum engineering occupation. These wage differences can be interpreted as economic loss from wrong major choices.

Chapter 2

Career Concerns in Teams and the Possibility of Multiple Equilibria

Abstract

The objective of this paper is to study career concerns in teams and the possibility of multiple equilibria. I use an information structure where only the joint output of the team, rather than signals for each team member, is observed by the principal. As opposed to the previous literature on the topic, I show the existence of multiple equilibria if either (i) the labor market exhibits increasing returns to perceived talent (ii) there is complementarity in hidden effort. In one equilibrium, both workers exert little effort and have bad career prospects. In the other equilibrium, both workers exert high effort and have good career prospects. I show that linear wage contracts will eliminate the bad equilibrium in case (i) but not in case (ii).

2.1 Introduction

A lot of production happens in teams and it is crucially important to ensure that all team members have proper incentives to exert effort. This paper studies the optimal provision of incentives in a model of career concerns in teams. Many papers have addressed this issue (see for example Jeon (1996), Auriol et al. (2002), Meagher and Prasad (2016) and Ortega (2003)). However, the literature has not addressed the possibility of multiple equilibria. In many professions, careers may lead to very high positions and in such positions even small differences in perceived talent may lead to considerable differences in firm performance and wages (see for example the literature on executive pay, such as Gabaix and Landier (2008) and related literature). I show that if wages exhibit increasing returns to perceived talent of the worker, the career concerns model in teams may lead to multiple equilibria.

The literature on career concerns has been popular for quite some time now. The seminal paper by Holmström (1999) outlines a model where worker's output is a function of her innate talent, the effort level and a random term. The innate talent is initially unknown to everyone but as the worker works, both the worker and principal learn about this innate talent. By exerting an appropriate amount of effort the worker can positively influence the principal's view of her talent. Doing so leads to a higher future wage if wages are determined competitively. As a result, the worker has incentives to exert effort even if there are no explicit performance-based

wage contracts.

An important extension of Holmström's work is to study career concerns in teams, as most production happens in teams. This extension has been pursued by Jeon (1996), Auriol, Friebe and Pechlivanos (2002), Meagher and Prasad (2016) and Ortega (2003). While each of these papers studies an important aspect of team production, they do not address the plausible existence of multiple equilibria. The main contribution of this paper is to show the possibility of multiple equilibria in the context of career concerns in teams. Such multiplicity arises with very plausible assumptions. In particular, if the job market exhibits increasing returns to perceived talent, there may be multiple equilibria. Alternatively, if effort levels enter production in a complementary manner, multiple equilibria may arise. Under these circumstances there will be two equilibria: (i) a bad equilibrium where both workers exert little effort and have poor career prospects (ii) a good equilibrium where workers exert high effort and have good career prospects.

Following Jeon (1996), I use a model specification where only team output is observed. Many papers in the literature assume that the principal observes a separate signal for each team member. While such specification is useful for answering certain questions, in general the information structure seems a bit too generous. In reality there are many situations where it is not possible to observe separate signals

for each worker. For example, reading a co-authored academic paper does not give any hints about the contribution of each author. Similarly, reading financial results of a cost center of a firm does not give any hints about the individual contribution of each team member. While additional information may be obtained by contacting relevant individuals, doing so is likely to be costly and individuals may not have correct incentives to report truthfully. Therefore, I use a model where only the total output of the team, rather than a separate signal for each team member, is observed by the principal. The principal and workers apply Bayesian updating using their observation on team output. Even though information is limited, it is still sufficient for learning something about each individual.

Since multiple equilibria are a possible outcome of my model, how could the bad equilibrium be eliminated? In general, there are two ways to provide incentives for workers (i) explicit incentives in which wages are based on realized production (ii) implicit incentives in which wages are flat but workers find it optimal to exert effort since this has a positive impact on future wages. If using only implicit incentives lead to multiple equilibria, explicit incentives in the form of linear wage contracts may eliminate the bad equilibrium. I study this by solving the model with linear wage contracts. The result suggests that linear wages are useful if the market exhibits increasing returns to perceived talent but irrelevant if effort levels enter production in a complementary manner.

The baseline learning process of the model operates in the following way. Each worker's innate talent is drawn from a normal distribution with a known mean and variance. However, the realized talent is unobserved. When a team works together, the output of the team is observed. Team output is a sum of individuals' talents, a function of exerted efforts and error terms. Since each error term is drawn from a normal distribution, the distribution of the sum of these error terms is also normal. After each observation, Bayesian updating is applied to form the best guess of each worker's talent and next period wages are determined. This helps to solve for the optimal effort levels exerted by each individual.

As in the existing literature, the incentives to exert effort depend on the extent of uncertainty about a worker's talent. Additionally, due to the used information structure, also uncertainty about the other team members' talent affects these incentives. If output turns out to be unexpectedly low, the individuals with most uncertain talent will be blamed. Since everybody knows this, the individuals with most uncertain talent exert excessive effort and the individuals with more certain talent shirk responsibility. As a result, the effort is sub-optimally allocated across team members. However, using performance-based wage contracts may eliminate part of the inefficiency.

This paper focuses on analyzing the normal distribution for a couple of reasons. For one, normal distribution is viewed as the most important probability distribution and it is plausible that talent is normally distributed among population. Additionally, two-dimensional worker-specific priors give the problem a very interesting economic interpretation. In any firm there is a wide range of workers from different backgrounds and it seems reasonable to assign priors based on demographic averages. For example, a Harvard graduate may be expected to be more talented than a graduate from an average university. The model of this paper enables one to accommodate this feature by assigning a higher prior mean for the sub-population of Harvard graduates. Of course this does not guarantee that workers from that sub-population are always more talented; the Bayesian approach leaves room for under-performers regardless of the initial preconception. Prior variance has a useful interpretation as well. It can be set equal to the variance of abilities of that particular demographic group. Alternatively, one can model senior workers as having a smaller prior variance than junior workers. This way, normal distribution allows a rich prior structure.

The chapter is organized as follows. Section 2.2 gives a brief overview of related literature while Section 2.3 describes the model. Section 2.4 characterizes the equilibrium and Section 2.5 demonstrates the possibility of multiple equilibria. Section 2.6 adds linear wage contracts to the model. Section 2.7 studies an extension where

the principal does not know which equilibrium was played in the first period and uses the best guess based on the realized output. Section 2.9 concludes and the proofs are in the Appendix.

2.2 Related Literature

Holmström (1999) outlined the original career concerns model for a single agent framework. He shows that even without performance-based wage contracts workers will have an incentive to exert effort if this affects the principal's view of their talent and leads to higher future wages. Holmström's model confirms the conjecture of Fama (1980). Jeon (1996) extends the analysis of Holmström (1999) to the framework of team production and uses a similar information structure as the present paper. He shows that the incentives to exert effort are affected not only by uncertainty about the worker himself but also by uncertainty about other team members. He also shows that it is beneficial to use Positive Assortative Matching (PAM) with respect to prior variances when matching workers into teams. However, Jeon does not consider the possibility of multiple equilibria.

Auriol, Friebel and Pechlivanos (2002) also extend the Holmström model to cover production in teams. Their assumption is that the principal observes a signal for each team member separately and workers can help their colleagues. The authors show that when workers anticipate that their wage contracts will be rene-

gotiated, their willingness to support each other decreases. As a result, workers have incentives to sabotage their peers and the optimal contract has to reduce incentives to sabotage. Meagher and Prasad (2016) consider career concerns in teams when there is both individual talent and team talent. As a result of including team talent, workers have less of an incentive to free-ride. Ortega (2003) describes a related model where workers have varying amounts of power in the organization. The unequal distribution of power make more powerful workers more prominent in the organization and the firm owner will learn more about their talent. This affects incentives to exert effort.

For a single-agent career concerns model, Gibbons and Murphy (1992) provide a model which aims to characterize the optimal linear wage contract in a career concern model. As career concerns mitigate the moral hazard problem, the wage contract has to provide incentives only for the situations where career concerns are not sufficient to overcome the moral hazard. In fact, one of the main insights of Gibbons and Murphy (1992) is that performance-pay is not very important in the beginning of a worker's career, since career concerns encourage effort at that point. On the contrary, performance-pay is more important at the later periods of the career, since career concerns are less prominent at that stage. Dewatripont et al. (1999) use various information structures to study how these affect incentives in a model of career concerns.

Since only joint production of the team is observed in this paper, there are similarities to the literature on moral hazard in teams. It has been shown that in such a setting workers will have an incentive to shirk even when the production is determinate, as the principal has no way of knowing which worker is shirking. In an early paper, Alchian and Demsetz (1972) discuss the problems caused by the fact that typically individual contributions to team production are not observed. Also Holmström (1982) studies moral hazard in teams. In that paper, only joint production rather than individual contributions is observed, and this is the driving factor behind moral hazard. For learning in teams, Meyer (1994) contains a model where junior and senior employees are being matched optimally to maximize learning. However, that paper assumes linear production and the statistical setup is very different from the present paper.

The model of this paper suggests that matching of workers to a team is affected considerably by the extent of uncertainty about each worker's talent. In fact, a finding of Jeon (1996) is that with fairly weak assumptions, PAM with respect to workers' prior variances is optimal. The literature on assortative matching was started by Becker (1973) and more recent work with search frictions is surveyed in Burdett and Coles (1999). Shimer and Smith (2000) and Atakan (2006) study matching with different types of search frictions, and Legros and Newman (2007)

study matching with search and nontransferable utility (see also Legros and Newman (2002)).

In terms of information structure and learning, another related paper is Anderson and Smith (2010). These authors study dynamic matching with evolving reputations (types). They show existence results and characterize general properties of their model in general form. Later on they specialize to the case where there are only two possible types for each agent and only total production instead of individual contribution to production is observed. Using this particular distribution, Anderson and Smith show that PAM may not be an equilibrium outcome because of informational concerns. In particular, in their model it may be beneficial to pair agents with unknown types and agents with known types, since that allows one to get a cleaner signal which facilitates learning. Anderson (2011) builds on Anderson and Smith (2010) and proves additional general properties of dynamic matching with evolving human capital using contraction mapping techniques. Eeckhout and Weng (2010) also study matching with learning. As opposed to Anderson and Smith, Eeckhout and Weng conclude that PAM is an equilibrium outcome of their model of matching with learning. The difference is that there is no uncertainty regarding firm types in their model (Eeckhout and Weng study matches between workers and firms). Other papers on matching with uncertainty are Hoppe et al. (2009) who allow agents to send costly signals before matching, and Lee and Schwarz (2012)

who present an interview model. Evolving productivity characteristics are crucial also in the model of Postel-Vinay and Robin (2002). Moreover, Nagypál (2005) makes the case that worker's reputation evolves over time and this crucially affects incentives to search on the job. An early paper on learning about match-specific productivity is Jovanovic (1979). Moscarini (2005) studies a general equilibrium search model where match-specific productivity is learned over time as in Jovanovic (1979).

2.3 Model

This section outlines the model which is based on the career concerns models of Holmström (1999) and Jeon (1996). There are two periods and two workers in the team. Workers have the option to change employer after the first period, as specified below. This effectively creates an outside option for workers at $t = 2$. Apart from this outside option, the firm has all the bargaining power.

Each worker i has talent η_i which is unknown to everyone. There are prior beliefs about η_i . One can imagine the initial prior beliefs to be formed based on the worker's demographic group, previous work experience, etc. The prior of worker i in the beginning of period t is distributed as Normal with mean μ_{it} and variance σ_{it} . The joint prior beliefs regarding the talent of the two workers is summarized

by a vector of prior means μ_t and a covariance matrix Σ_t .

Learning about the η_i 's happens through the public history of outputs using a Bayesian updating procedure.

Team production at t is given by

$$y_t = \eta_{1t} + \eta_{2t} + \left(a_{1t}^\theta + a_{2t}^\theta\right)^{\frac{1}{\theta}} + \epsilon_{1t} + \epsilon_{2t}$$

where a_{it} is worker i 's unobservable labor input, $\theta \leq 1$ and ϵ_{it} is a stochastic noise term. The random component of production is modeled as normally distributed error terms $\epsilon_{it} \sim N(0, \sigma^2)$ for all i, t . The expression for production embeds the idea that efforts may be complementary. In particular, $\theta = 1$ represents the standard linear production function used in the literature. However, if $\theta < 0$, efforts are complementary. The only observable variable is y_t .

To build the standard trade-off between providing insurance and incentives, I assume that workers are risk-averse. Preferences of worker i are given by

$$U_i = -\exp\left(-r \sum_{t=1}^2 (w_{it} - g(a_{it}))\right)$$

Here w_{it} is the wage of worker i at time t and g represents the cost of exerting effort. This function satisfies $g'(\cdot) > 0$, $g''(\cdot) > 0$ and $g(0) = 0$. For most of the

paper I assume that $g(a) = \frac{1}{2}a^2$.

The utility function is widely used in the literature and has the benefit of being tractable. However, the downside is that the workers do not value consumption-smoothing in this model.

Workers are allowed to frictionlessly change workers after $t = 1$. I assume that the market values talent according to function h . That is, if worker i chooses to change employer after $t = 1$, the other firms on the market are going to offer him wage $w_{i2} = h(\mu_{i2})$.

The firm owner has all the bargaining power apart from the outside options that are 0 in period 1 and $h(\mu_{i2})$ in period 2.

2.4 Characterization of Equilibrium

2.4.1 Bayesian Learning

The effort levels of individual team members are unobserved. Nevertheless, the history of team production contains information that can be used for learning. The expected posterior means are given by

$$E \begin{pmatrix} \mu_{12} \\ \mu_{22} \end{pmatrix} = \begin{pmatrix} \mu_{11} \\ \mu_{21} \end{pmatrix} + \frac{(a_{1t}^\theta + a_{2t}^\theta)^{\frac{1}{\theta}}}{\sigma_{11}^2 + \sigma_{12}^2 + 2\sigma^2} \begin{pmatrix} \sigma_{11}^2 \\ \sigma_{21}^2 \end{pmatrix} \quad (2.1)$$

2.4.2 Optimal Behavior

Apart from the Bayesian learning process, the solution method is straightforward since backward induction can be used to solve for optimal behavior.

Let w_{i1} and $w_{i2}(y_1)$ be the wage in periods 1 and 2, respectively. Similarly, let a_{i1} and $a_{i2}(y_1)$ be worker i 's effort levels in those periods.

Period 2 Since the firm owner has all the bargaining power, he will pay as little as possible. However, incentive compatibility and individual rationality constraints have to be satisfied. Hence the second period wage conditional on observed first period production y_1 solves

$$\begin{aligned} \min w_{i2} \\ \text{s.t.} \\ \text{CE}_i = w_{i2} - g(a_{i2}) - \frac{r}{2} \text{Var}(w_{i2}) \geq h(\mu_{i2}) \\ a_{i2} \in \text{argmax}_{a_i} \text{CE}_i \end{aligned} \quad (2.2)$$

Let \hat{a}_{it} be the conjectured labor supply in period t . At $t = 2$, the workers no longer care about the principal's view of their talent. As a result, there are no incentives to exert effort and $a_{i2} = \hat{a}_{i2} = 0$ for all i . As a result, $w_{i2} = h(\mu_{i2})$,

Period 1 The firm owner's problem is

$$\begin{aligned}
& \min w_{i1} \\
& \text{s.t.} \\
& w_{i2} = h(\mu_{i2}) \\
& a_{i2} = 0 \\
& \text{CE}_i = \sum_{t=1}^2 (Ew_{it} - g(a_{it})) - \frac{r}{2} \text{Var}(w_{i1} + w_{i2}) \geq 0 \\
& a_{i1} \in \text{argmax}_{a_{i1}} \text{CE}_i
\end{aligned} \tag{2.3}$$

Since the first period wage is predetermined, it is not necessary to solve this optimization problem. The incentive compatibility constraint reduces to

$$\max_{a_{i1}} w_{i1} - g(a_{i1}) + \beta E(w_i(\mu_{i2}(y_1)))$$

where w_{i1} is predetermined.

The critical term here is $E(w_i(\mu_{i2}(y_1)))$ which is the expected wage at $t = 2$ conditional on the team output at $t = 1$. By exerting effort at $t = 1$, worker i may positively affect his second period expected wage. However, in equilibrium he will not be able to fool the market. The principal can predict the actions of agents and applies Bayesian updating using $y_1 - (\hat{a}_{11}^\theta + \hat{a}_{21}^\theta)^{\frac{1}{\theta}} = \sum_i (\eta_i + \epsilon_{i1})$ where \hat{a}_{it} refers to the conjectured effort level. Since $\sum_i \epsilon_{i,t} \sim N(0, 2\sigma^2)$, we can obtain an expression for $E(\eta_i|y_1) = \mu_{i2}$.

The first-order condition for the optimal effort at $t = 1$ is given by

$$\beta E (h'(\eta_{i2}(y_1))) \frac{a_{i1}^{\theta-1}}{(a_{11}^\theta + a_{21}^\theta)^{\frac{\theta-1}{\theta}} \hat{\sigma}_1^2} \sigma_{i1}^2 = g'(a_{i1}) \quad (2.4)$$

where I have used the short-hand notation $\hat{\sigma}_1^2 = \sigma_{11}^2 + \sigma_{21}^2 + 2\sigma^2$.

2.4.3 Relationship to Jeon (1996)

This first-order condition generalizes the result of Jeon (1996). The resulting asymmetrical equilibrium shows an important source of inefficiency. This inefficiency is worthwhile to reiterate with a proposition:

Proposition 2.4.1. *Assume that*

- *Production is linear in efforts ($\theta = 1$).*
- *There is more uncertainty about worker 1's talent ($\sigma_{11}^2 > \sigma_{21}^2$).*

Under these assumptions,

- *Worker 1 exerts more effort than worker 2.*
- *If there are several workers with differing variances that have to be paired in teams, the surplus $a_{11} + a_{12} - g(a_{11}) - g(a_{12})$ is maximized by PAM with respect to prior variances.*

The proofs of the proposition are in Jeon (1996).

2.4.4 Efficiency

The efficient level of effort is characterized by $g'(a_{i,1}) = 2^{\frac{1-\theta}{\theta}}$ for all i . In equilibrium the first period efforts are distorted in two ways. First, the total level of effort is below optimum for both workers even though career concerns provides some incentives to exert effort. Second, the worker whose talent is known more precisely exerts less effort than the other worker and the effort levels are thus misallocated across workers.

Both workers exert zero effort in the second period effort since career concerns are no longer relevant. If there were more periods, the incentives provided by career concerns would decrease over time, as the workers get closer to the last period. Implicit incentives do not exist in the last period but the inefficiency can still be partially corrected using linear wage contracts along the lines of Gibbons and Murphy (1992). This extension is pursued shortly.

2.5 Multiple Equilibria

The main purpose of this paper is to demonstrate the possibility of multiple equilibria in the framework of career concerns in teams. Multiple equilibria may happen in two different ways: (i) the labor market exhibits increasing returns to perceived talent (ii) there are complementarities in hidden effort. In the next two subsections I will analyze these in detail.

2.5.1 Increasing Returns to Perceived Talent

There are many professions where careers of some people tend to lead to very high positions. Even small differences in talent level of individuals in such high positions may lead to huge differences in the result of the firm (see literature on CEO pay, such as Gabaix and Landier (2008)). The wage compensation for these high positions are typically very large. Therefore, the outside option for talented individuals is likely to exhibit strongly increasing returns to talent. I will next analyze the presence of multiple equilibria in such a case.

I will demonstrate the existence using an example. Even though the following example is very particular, the principle itself is more general. Let $h(\mu) = \exp(\mu)$ so that the market value of talent is strictly convex. Let $g(a) = \frac{1}{2}a^2 + za$ for some $z > 0$. The second term in this expression guarantees that the solutions to the first-order conditions exist and are interior. Let $\theta = 1$ so that efforts enter production in a linear manner. This assumption implies that there are no complementarities coming through the production process itself. Furthermore, I assume that $\hat{a}_{i1} = 0$ for simplicity. As will become clear, this does not affect the multiplicity of equilibria and hence does not distort the qualitative properties of the model. The first-order condition characterizing optimal behavior is given by

$$\beta \exp \left(\left(\mu_{i1} + \frac{\sigma_{i1}^2}{\hat{\sigma}_1^2} (a_{11} + a_{21}) \right) + \frac{\tilde{\sigma}_{i2}^2}{2} \right) \frac{\sigma_{i1}^2}{\hat{\sigma}_1^2} = a_{i1} + z$$

Here $\hat{\sigma}_1^2 = \frac{\sigma_{i1}^2(\sigma_{j1}^2+2\sigma^2)}{\sigma_{11}^2+\sigma_{21}^2+2\sigma^2} + 2\sigma^2$ is the posterior variance of worker i 's talent.

Assume that $\mu_{i1} = \mu_{j1} = 0$. If $\sigma_{i1}^2 = \sigma_{j1}^2$, the resulting equilibria will be symmetric. The best-response functions are displayed in Figure 2.1.

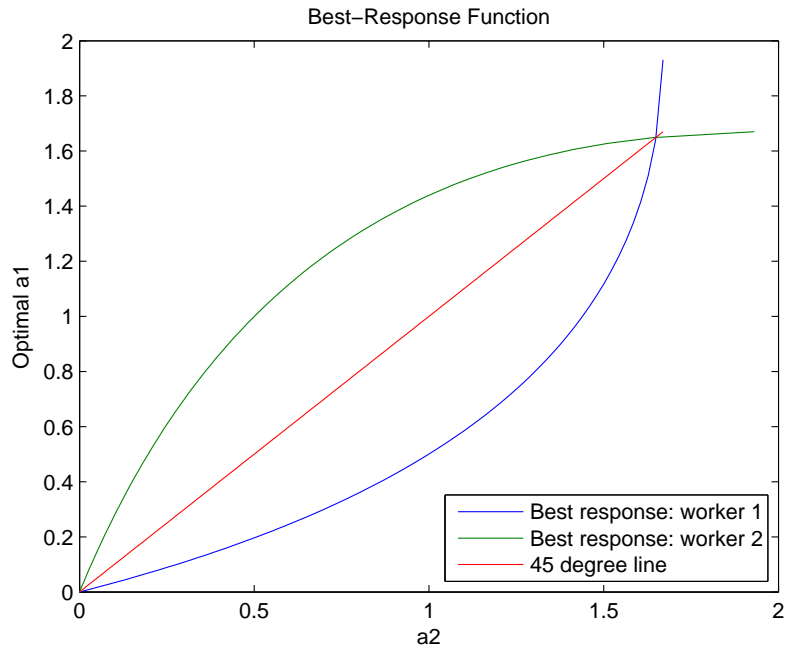


Figure 2.1: Best-response function intersects the 45 degree line twice. Therefore there are two symmetric equilibria in this special case.

On the other hand, if $\sigma_{11} < \sigma_{21}$, there will be one symmetric equilibrium and one asymmetric. The best-response functions for this case are in Figure 2.2. The asymmetric equilibrium is such that the worker whose talent is more uncertain

exerts more effort.

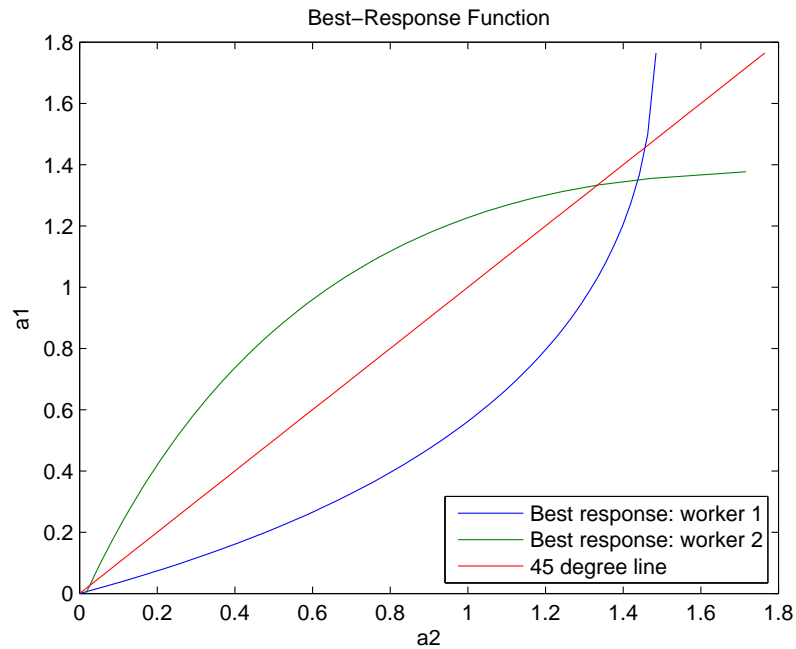


Figure 2.2: There are two equilibria: one symmetric and one asymmetric.

2.5.2 Complementaries in Hidden Effort

The model has a CES (Constant Elasticity of Substitution) functional form for the hidden efforts. If team effort is more than a sum of individual efforts, one can justify assuming $\theta < 1$. When $\theta < 0$, we get two pure strategy equilibria. In the first equilibrium, both workers exert low effort (zero to be exact). As a result, they will incur very little dis-utility from production but their career do not have good prospects. In the second equilibrium, both workers exert high effort.

To illustrate this type of multiplicity, assume that the market values talent according to a linear function: $h(\mu) = \mu$. Let $\theta < 0$. Let $g(a) = \frac{1}{2}a^2$. The best-response functions are displayed in Figure 2.3.

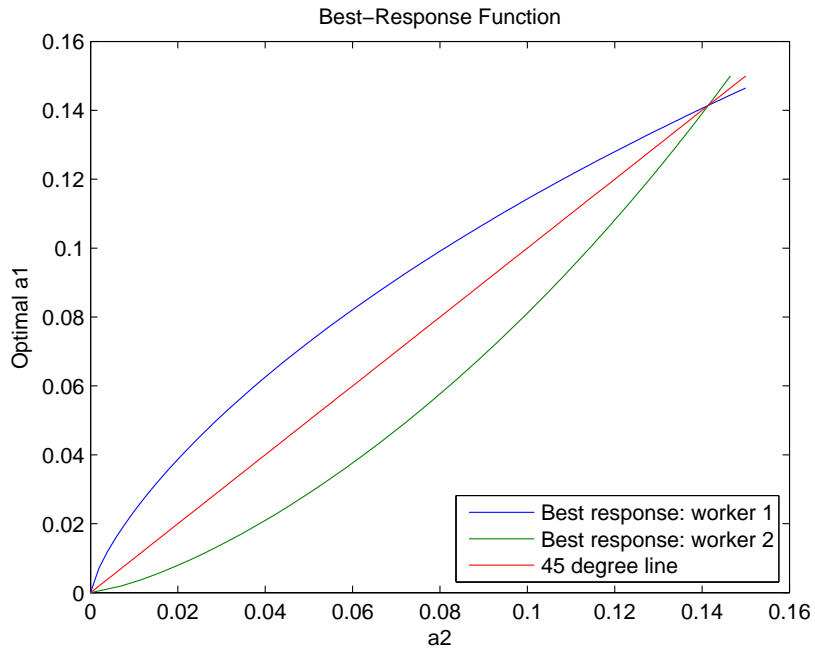


Figure 2.3: Best-response function intersects the 45 degree line twice. Therefore there are two symmetric equilibria in this special case.

The best-response functions for the case where $\sigma_{11} < \sigma_{21}$ are in Figure 2.4. The worker whose talent is more uncertain again exerts more effort in the asymmetric equilibrium.

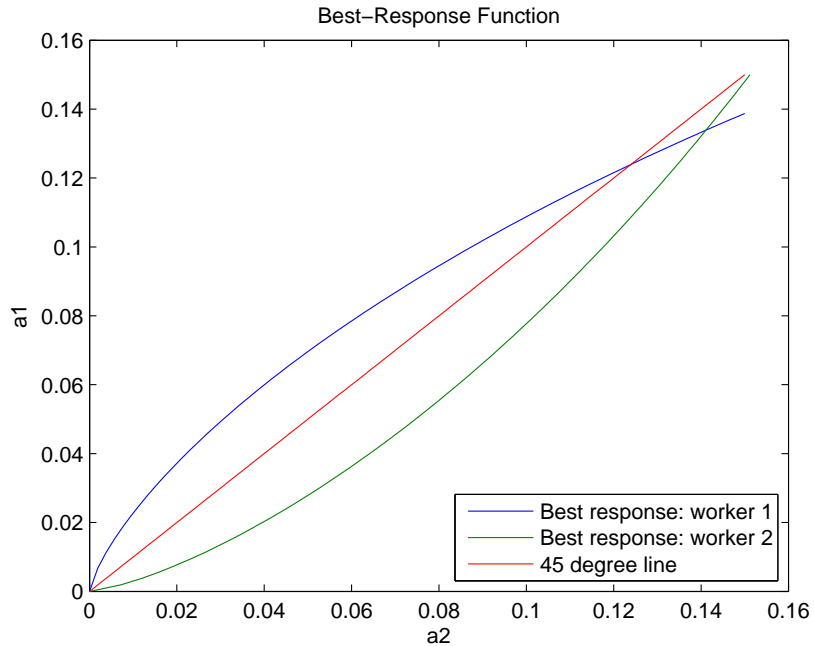


Figure 2.4: There are two equilibria: one symmetric and one asymmetric.

2.6 Explicit Incentives: Solution for Multiple Equilibria?

Thus far I have shown that multiple equilibria may arise in this model due to two mechanisms: (i) the labor market exhibits increasing returns to perceived talent (ii) there is complementarity in hidden effort. Under these cases, there are two equilibria: one where both workers exert little effort and one where both of them exert more effort. The firm would of course prefer the equilibrium with more effort.

One may wonder whether explicit incentives in the form of linear wage contracts would make the problem of multiple equilibria disappear. This section analyzes this

question.

Let us try contracts of the form $w_{it} = \beta_{it} + \alpha_{it}y_t$. In words, the firm will pay workers a fixed payment of β_{it} and a bonus payment $\alpha_{it}y_t$ where the latter depends on the realized team output. The firm owner is then choosing parameters β_{it} and α_{it} to maximize its profits subject to the incentive compatibility (IC) and individual rationality (IR) constraint for each worker. As before, the problem can be analyzed by backward induction starting from period 2.

Second Period The firm owner's second period maximization problem is

$$\begin{aligned} \max_{\alpha_{i2}, \beta_{i2}} & \left(1 - \sum_{i=1}^2 \alpha_{it}\right) E y_t - \sum_{i=1}^2 \beta_{it} \\ \text{s.t.} & \\ & \alpha_{i2} E(y_2|y_1) + \beta_{i2} - g(a_{i2}) - \frac{r}{2} \text{Var}(w_{i2}|y_1) \geq h(\mu_{i2}) \quad (2.5) \\ & a_{i2} \in \text{argmax}_{a_i} \alpha_{i2} E(y_2|y_1) + \beta_{i2} - g(a_{i2}) - \frac{r}{2} \text{Var}(w_{i2}|y_1) \\ & \text{Var}(w_{i2}|y_1) = \alpha_{i2}^2 \text{Var}(y_2|y_1) \end{aligned}$$

The incentive compatibility constraint is solved by taking the first-order condition:

$$\alpha_{i2} \frac{a_{i2}^{\theta-1}}{\left(a_{12}^{\theta} + a_{22}^{\theta}\right)^{\frac{\theta-1}{\theta}}} - g'(a_{i2}) = 0$$

Here I assume that the solution is interior. The individual rationality constraint

is satisfied with equality and can be solved for β_{i2} . Substituting this constraint into the objective function leads to an alternative expression of the maximization problem :

$$\begin{aligned} \max_{\alpha_{i2}} E(y_2|y_1) - \sum_{i=1}^2 g(a_{i2}) - \frac{r}{2} \sum_{i=1}^2 \alpha_{i2}^2 \text{Var}(y_2|y_1) \\ \text{s.t.} \\ \alpha_{i2} = \frac{a_{i2}^{1-\theta} g'(a_{i2})}{(a_{12}^\theta + a_{22}^\theta)^{\frac{1-\theta}{\theta}}} \end{aligned} \quad (2.6)$$

If the efforts are chosen symmetrically (see the Appendix for details), the optimal incentive component of wage is characterized by

$$\alpha_{i2} = \frac{1}{1 + r \text{Var}(y_2|y_1) 2^{2\frac{\theta-1}{\theta}}} \quad (2.7)$$

Equation (2.7) generalizes the incentive contracts found in the literature. If $\theta = 1$, the term $2^{2\frac{\theta-1}{\theta}}$ disappears from the denominator and we get the same expression as in Auriol, Friebel and Pechlivanos (2002). A lower value of θ leads to a lower α_{i2} . Hence the incentive component of wages becomes lower when production becomes more complementary.

The firm owner chooses the fixed wage component β_{i2} to satisfy the individual rationality condition with equality:

$$\beta_{i2} = h(\mu_{i2}) + g(a_{i2}) + \frac{r}{2} \text{Var}(w_{i2}|y_1) - \alpha_{i2} E(y_2|y_1) \quad (2.8)$$

In period 1, worker i realizes that his effort will affect both period 1 wage and also β_{i2} through the first term $h(\mu_{i2})$.

First Period Differentiating (2.8) gives an expression for implicit incentives:

$$\frac{d\beta_{i2}}{da_{i1}} = (Eh'(\mu_{i2}) - \alpha_{i2}) \frac{a_{i1}^{\theta-1}}{(a_{11}^{\theta} + a_{21}^{\theta})^{\frac{\theta-1}{\theta}}} \frac{\sigma_{i1}^2}{\hat{\sigma}_1^2}$$

In words, increasing first period effort increases the career concerns term $Eh'(\mu_{i2})$. However, this effect is partially offset by the α_{i2} term since the worker dislikes the incentive component of the second period pay.

The IC constraint for period 1 gives the following first-order condition:

$$\left(\alpha_{i1} + (Eh'(\mu_{i2}) - \alpha_{i2}) \frac{\sigma_{i1}^2}{\hat{\sigma}_1^2} \right) \frac{a_{i1}^{\theta-1}}{(a_{11}^{\theta} + a_{21}^{\theta})^{\frac{\theta-1}{\theta}}} - g'(a_{i1}) = 0 \quad (2.9)$$

The worker benefits from exerting effort directly in the form of first period bonus payment (first term) and indirectly in terms of career concerns (second term). These equal the marginal dis-utility of exerting effort. Next I will discuss the impact of linear wage contracts in the two cases discussed in the previous section.

Discussion: Increasing Return to Perceived Talent If production is linear ($\theta = 1$), equation (2.9) reads as

$$\alpha_{i1} + (Eh'(\mu_{i2}) - \alpha_{i2}) \frac{\sigma_{i1}^2}{\hat{\sigma}_1^2} - g'(a_{i1}) = 0$$

Even if the other worker exerts zero effort, worker 1 still has incentives to set $a_{11} > 0$ because it increases first period wage through incentive coefficient α_{i1} . This

eliminates the second equilibrium.

Discussion: Complementary Production From worker 1's perspective, if $a_{21} = 0$ and $\theta < 0$, the first term in (2.9) will always be zero regardless of a_{11} . Therefore, if one of the workers exerts zero effort the other worker finds it optimal to exert zero effort as well despite the first period bonus. There will be two equilibria even with linear wage contracts.

2.7 Extension

So far I have implicitly assumed that the principal (firm owner) conjectures *ex post* that the high effort equilibrium was played in the case of multiple equilibria. The principal used this information to adjust future wages by the conjectured effort levels. It may however not always be possible for the principal to know which equilibrium was played. An alternative to that assumption is discussed in this section.

Let us consider what happens in the model if the principal does not know which equilibrium was played. Since realized output is observable, the principal can form a best guess of the played equilibrium as follows. Let $f(y_1|a^L)$ be the density of y_1 conditional on both workers exerting an effort level that corresponds to the low effort equilibrium. Similarly, let $f(y_1|a^H)$ be the density of y_1 corresponding to the high effort equilibrium. After the realization of y_1 , the principal believes that the

low effort equilibrium was played with probability

$$\Pr(a^L|y_1) = \frac{f(y_1|a^L)}{f(y_1|a^L) + f(y_1|a^H)}$$

The principal will adjust¹ the estimate of each worker's talent by subtracting the conjectured effort levels $\Pr(a^L|y_1)(\hat{a}_{11}^L + \hat{a}_{21}^L) + (1 - \Pr(a^L|y_1))(\hat{a}_{11}^H + \hat{a}_{21}^H)$ where the superscripts L and H denote the low effort and high effort equilibrium, respectively. Given this specification, the expected talent of worker i in period two is given by:

$$E\mu_{i2} = \int_{-\infty}^{\infty} \mu_{i2}(y_1, a^L) dF(y_1|a^L) - \frac{\sigma_{i1}^2}{\hat{\sigma}_1^2} \left(\Pr(a^L|y_1)(\hat{a}_{11}^L + \hat{a}_{21}^L) + (1 - \Pr(a^L|y_1))(\hat{a}_{11}^H + \hat{a}_{21}^H) \right) \quad (2.10)$$

where

$$\mu_{i2}(y_1, a^L) = \frac{1}{\hat{\sigma}_1^2} \left(\sigma_{i1}^2 y_1 + (\sigma_{j1}^2 + 2\sigma^2)\mu_{i1} - \sigma_{i1}^2 \mu_{j1} \right)$$

The workers know the principal's inference problem and choose their effort optimally. The key term is the derivative of $E\mu_{i2}$ with respect to a_{i1} . To compute this derivative, I use a change of variables $\epsilon_1 = y_1 - \sum_{i=1,2}(\mu_{i1} + a_{i1}^L)$ in the second and third term of (2.10). As a result, we get

¹I assume in this section that $\theta = 1$. Remember also that $\hat{\sigma}_1^2 = \sigma_{11}^2 + \sigma_{12}^2 + 2\sigma^2$.

$$\begin{aligned}
\frac{dE\mu_{i2}}{da_{i1}} &= \frac{\sigma_{i1}^2}{\hat{\sigma}_1^2} + \frac{\sigma_{i1}^2}{\hat{\sigma}_1^2} \int_{-\infty}^{\infty} \frac{d\Pr(a^L|y_1)}{da_{i1}} \sum_{i=1,2} (\hat{a}_{i1}^H - \hat{a}_{i1}^L) dF(\epsilon_1) \\
&= \frac{\sigma_{i1}^2}{\hat{\sigma}_1^2} - \frac{\sigma_{i1}^2}{\hat{\sigma}_1^2} \Phi \sum_{i=1,2} (a_{i1}^H - a_{i1}^L)
\end{aligned} \tag{2.11}$$

where $\Phi = \int_{-\infty}^{\infty} \Pr(a^L|y_1)(1 - \Pr(a^L|y_1)) \frac{1}{\hat{\sigma}_1^2} \sum_{i=1,2} (\hat{a}_{i1}^H - \hat{a}_{i1}^L) dF(\epsilon_1)$. Repeating similar computations for the workers who intend to play the high effort equilibrium, we get an identical expression. The first-order conditions that characterize the optimal behavior are thus given by:

$$\beta E \left(h'(\mu_{i2}(y_1|a^L)) \right) \frac{\sigma_{i1}^2}{\hat{\sigma}_1^2} \left(1 - \Phi \sum_{i=1,2} (a_{i1}^H - a_{i1}^L) \right) = g'(a_{i1}^L) \tag{2.12}$$

$$\beta E \left(h'(\mu_{i2}(y_1|a^H)) \right) \frac{\sigma_{i1}^2}{\hat{\sigma}_1^2} \left(1 - \Phi \sum_{i=1,2} (a_{i1}^H - a_{i1}^L) \right) = g'(a_{i1}^H) \tag{2.13}$$

Since future wages are based on imperfect information, the effort levels in both the low-effort and high-effort equilibrium enter both of these first-order conditions. Therefore it is necessary to solve these two equations jointly.

The first-order conditions are similar to the ones showed earlier except for the term $-\Phi \sum_{i=1,2} (a_{i1}^H - a_{i1}^L)$ which reduces career concerns. The economic interpretation is straightforward. Because the workers can only influence the principal's view of them with regard to the equilibrium they play, there is less effort. In particular, since $E \left(h'(\mu_{i2}(y_1|a^H)) \right) > E \left(h'(\mu_{i2}(y_1|a^L)) \right)$, the workers who intend to play

the high-effort equilibrium reduce their effort by more than the low-effort equilibrium types compared to the baseline model. However, multiple equilibria are still possible even in this case. For fixed effort levels corresponding to equilibrium (2.4), we can always increase the convexity of h to obtain two equilibria.

2.8 Signaling Effort to the Firm Owner

The model assumes that team members can observe each other's effort level but these efforts are hidden from the firm owner. What happens if the workers are able to signal their effort levels to the firm owner and also their future employers for a fixed cost?

To answer this question, it is necessary to specify how this additional signaling process works. If the firm owner is able to perfectly observe the effort levels, it becomes possible to specify contracts that pay a positive wage only for a given effort level. If the effort differs from this specified level, the wage is zero. Such an information structure would force the workers to exert a surplus maximizing effort and most of the content of the career concerns model would become irrelevant. Therefore, that extension is not pursued further.

A better alternative is to assume that by paying a cost of c the workers are able to signal their own contribution to the production process. That is, the firm owner

will observe² $y_{it} = a_{it} + \epsilon_{it}$ for worker i . As a result, the learning process is similar to the standard single-agent career concern model. As in Holmström (1999), the optimal first period effort is characterized by the first-order condition

$$\beta E(h'(\eta_{i2}(y_{i1}))) \frac{\sigma_{i1}^2}{\sigma_{i1}^2 + \sigma^2} = g'(a_{i1})$$

This pins down the optimal effort level a_{i1}^* . The main difference to the baseline model is that the other worker's effort level no longer affects the market's view of a worker's talent. The expected utility of worker i is given by

$$CE_i^{single-agent} = \sum_{t=1}^2 (Ew_{it} - g(a_{it}^*)) - \frac{r}{2} \text{Var}(w_{i1} + w_{i2})$$

It is then in the best interest of worker i to pay the fixed cost c if:

$$-c + CE_i^{single-agent} \geq CE_i \tag{2.14}$$

In words, the expected utility with the finer information structure minus the cost of information is weakly higher than the expected utility with the coarser information structure.

Assume that worker $j \neq i$ is choosing the low effort strategy. Since this reduces the right-hand side of equation (2.14), sending the signal to the firm owner becomes more attractive. Since the bad equilibrium is caused by both workers exerting less effort than conjectured by the firm owner, there is no similar force in the case when

²In this section I assume that $\theta = 1$ so that efforts enter production in a linear manner.

each worker's contribution is observed separately. One can also easily notice that for low enough c the bad equilibrium disappears. As one of the workers chooses to send the signal to the firm owner, it becomes a best-response for the other worker to exert a higher effort level. Of course, the single-agent framework leads to a better outcome in the case of a very low c , since free-riding becomes less prominent.

2.9 Conclusions

This paper developed a model of career concerns in teams to demonstrate the possibility of multiple equilibria. The principal was only able to observe total output of the team rather than a separate signal of each worker's contribution. It was argued that such information structure is more plausible in many real-life settings. As in Jeon (1996), the information structure has the implication that the prior variance of the partner directly affects incentives to exert effort.

I showed that under plausible assumptions there will be two pure strategy equilibria: one where both workers exert little effort and have poor career prospects, and one where both workers exert high effort and have good career prospects. As higher effort is preferable from everyone's point of view, it is important to consider how we could coordinate on the good equilibrium. For this purpose, I introduced linear wage contracts into the model to see if the bad equilibrium can be eliminated using explicit incentives provided by bonus payments.

More specifically, I showed that the possibility of multiple equilibria arises if either (i) the labor market exhibits increasing returns to perceived talent or (ii) effort levels enter the production function in a complementary manner. The main result was that bonus payments eliminate the bad equilibrium in case (i) but not in case (ii). The explanation is intuitive: if case (ii) prevails and the other worker does not exert effort in the first period, the first period bonus payments can not be affected by the worker. On the other hand, in case (i) the first period bonus payments are sufficient to make the bad equilibrium an unattractive option.

The speed of learning about a worker's talent depends on his initial prior variance. For those workers whose talent is most uncertain, the learning will be fastest. This encourages those workers to exert a high effort and therefore there is less need for explicit performance-based wage contracts. If team members have different prior variances, effort levels will be sub-optimally allocated across workers. In the efficient allocation all workers exert the same level of effort.

The fact that there is only one observation but several random variables in the underlying learning process implies that the learning process is unidentified. That is, the principal learns about the talent of the team rather than the talent of team members separately. Asymptotically, if the team works together forever, the princi-

pal will only learn the sum of team members' talents rather than individual talents separately. This specification has the economic implication that the principal wants to keep the team members working together, as a reduced variance increases surplus in a setup where there is a trade-off between providing incentives and insurance.

The baseline model of this paper implicitly assumed that the principal knows *ex post* which equilibrium was played and adjusts wages to eliminate the effect of conjectured effort levels. Thus in equilibrium the workers are not able to fool the market and talent levels are inferred correctly given the information structure. However, as an extension I also considered a possibility that the principal does not know which equilibrium was played and has to form a best guess about the equilibrium using the observed realized production. I demonstrated that in such a setup the workers have lower career concerns and the effort levels in the high-effort equilibrium are reduced more than the effort levels in the low-effort equilibrium. However, the possibility of multiple equilibria still prevails under this alternative model specification.

2.10 Appendix

Derivation of Posterior

Since f is a one-to-one mapping, the principal can infer the value of $\sum_i \eta_{it}$ based on the observed production. Conditional on η , $y_t - (\hat{a}_{1t}^\theta + \hat{a}_{2t}^\theta)^{\frac{1}{\theta}}$ is distributed as

$$y_t | \eta \sim N(\mu_{1t} + \mu_{2t}, 2\sigma^2)$$

The posterior parameters satisfy

$$f(\mu_2 | y_1) \propto f(y_1 | \mu_1) f(\mu_1) \quad (2.15)$$

where μ_t is a 2×1 vector and y_1 is a scalar.

Let vector μ_t and matrix Σ_t be the prior mean and variance, respectively. Let e be a 1×2 vector of ones. Since $f(\eta | y_1) \propto f(y_1 | \eta) f(\eta)$, the exponential term on the right-hand side of (2.14) equals

$$(y_1 - e\eta)' \frac{1}{2\sigma^2} (y_1 - e\eta) + (\eta - \mu_1)' \Sigma_1^{-1} (\eta - \mu_1)$$

Multiply out these terms and re-arrange:

$$\begin{aligned} & (y_1 - e\eta)' \frac{1}{2\sigma^2} (y_1 - e\eta) + (\eta - \mu_1)' \Sigma_1^{-1} (\eta - \mu_1) \\ &= \frac{1}{2\sigma^2} y_1^2 + \mu_1' \Sigma_1^{-1} \mu_1 + \eta' \left(-\frac{1}{2\sigma^2} e' y_1 - \Sigma_1^{-1} \mu_1 \right) \\ &+ \left(-y_1 \frac{1}{2\sigma^2} e - \mu_1' \Sigma_1^{-1} \right) \eta + \eta' \left(\frac{1}{2\sigma^2} e' e + \Sigma_1^{-1} \right) \eta \end{aligned}$$

On the other hand, the exponential term in the posterior is:

$$\eta' \Sigma_2^{-1} \eta - \eta' \Sigma_2^{-1} \mu_2 - \mu_2' \Sigma_2^{-1} \eta + \mu_2' \Sigma_2^{-1} \mu_2$$

By matching coefficients in the previous two expressions, we note immediately that

$$\Sigma_2^{-1} = \frac{1}{2\sigma^2} e' e + \Sigma_1^{-1}$$

Furthermore,

$$-\Sigma_2^{-1} \mu_2 = -\frac{1}{2\sigma^2} e' y_1 - \Sigma_1^{-1} \mu_1$$

This implies that

$$\mu_2 = \Sigma_2 \left(\frac{1}{2\sigma^2} e' y_1 + \Sigma_1^{-1} \mu_1 \right)$$

These are the posterior parameters as a function of the prior and y_1 .

Let $\rho_1 = 0$ (the priors are independent in the beginning). Write out the posterior variance using the matrix inversion rule for 2×2 matrices:

$$\Sigma_2 = \left(\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \frac{1}{2\sigma^2} + \frac{1}{\det(\Sigma_1)} \begin{pmatrix} \sigma_{21}^2 & 0 \\ 0 & \sigma_{11}^2 \end{pmatrix} \right)^{-1}$$

The determinant of the term in parentheses is:

$$\det(\Sigma_2^{-1}) = \left(\frac{1}{2\sigma^2} + \frac{\sigma_{21}^2}{\det(\Sigma_1)} \right) \left(\frac{1}{2\sigma^2} + \frac{\sigma_{11}^2}{\det(\Sigma_1)} \right) - \left(\frac{1}{2\sigma^2} \right)^2$$

Since $\det(\Sigma_1) = \sigma_{11}^2 \sigma_{21}^2$, this can be written as:

$$\det(\Sigma_2^{-1}) = \frac{1}{\sigma_{11}^2 \sigma_{21}^2} \left(\frac{\sigma_{11}^2 + \sigma_{21}^2}{2\sigma^2} + 1 \right)$$

Therefore:

$$\Sigma_2 = \frac{1}{\det(\Sigma_2^{-1})} \left(\begin{pmatrix} \frac{1}{2\sigma^2} + \frac{\sigma_{11}^2}{\det(\Sigma_1)} & -\frac{1}{2\sigma^2} \\ -\frac{1}{2\sigma^2} & \frac{1}{2\sigma^2} + \frac{\sigma_{21}^2}{\det(\Sigma_1)} \end{pmatrix} \right)$$

Now we can use the previous results to write

$$\begin{aligned} E\mu_2 &= E \begin{pmatrix} \mu_{12} \\ \mu_{22} \end{pmatrix} = \Sigma_2 \left(e' \frac{Ey_1}{2\sigma^2} + \Sigma_1^{-1} \mu_1 \right) \\ &= \Sigma_2 \left(\frac{1}{2\sigma^2} \begin{pmatrix} y_1 \\ y_1 \end{pmatrix} + \begin{pmatrix} \frac{\mu_{11}}{\sigma_{11}^2} \\ \frac{\mu_{21}}{\sigma_{21}^2} \end{pmatrix} \right) \\ &= \frac{\sigma_{11}^2 \sigma_{21}^2}{\frac{\sigma_{11}^2 + \sigma_{21}^2}{2\sigma^2} + 1} \left(\frac{1}{2\sigma^2} \begin{pmatrix} \frac{\mu_{11}}{\sigma_{11}^2} - \frac{\mu_{21}}{\sigma_{21}^2} \\ -\frac{\mu_{11}}{\sigma_{11}^2} + \frac{\mu_{21}}{\sigma_{21}^2} \end{pmatrix} + \frac{1}{2\sigma^2} \begin{pmatrix} \frac{Ey_1}{\sigma_{21}^2} \\ \frac{Ey_1}{\sigma_{11}^2} \end{pmatrix} + \begin{pmatrix} \frac{\mu_{11}}{\sigma_{11}^2 \sigma_{21}^2} \\ \frac{\mu_{21}}{\sigma_{11}^2 \sigma_{21}^2} \end{pmatrix} \right) \\ &= \frac{1}{\sigma_{11}^2 + \sigma_{21}^2 + 2\sigma^2} \left(\begin{pmatrix} \sigma_{11}^2 \\ \sigma_{21}^2 \end{pmatrix} Ey_1 + \begin{pmatrix} (\sigma_{21}^2 + 2\sigma^2)\mu_{11} - \sigma_{11}^2\mu_{12} \\ (\sigma_{11}^2 + 2\sigma^2)\mu_{21} - \sigma_{21}^2\mu_{11} \end{pmatrix} \right) \\ &= \begin{pmatrix} \mu_{11} \\ \mu_{21} \end{pmatrix} + \frac{(a_{11}^\theta + a_{21}^\theta)^{\frac{1}{\theta}}}{\sigma_{11}^2 + \sigma_{21}^2 + 2\sigma^2} \begin{pmatrix} \sigma_{11}^2 \\ \sigma_{21}^2 \end{pmatrix} \end{aligned}$$

By writing out the posterior variance we get

$$\Sigma_2 = \frac{1}{\sigma_{11}^2 + \sigma_{21}^2 + 2\sigma^2} \begin{pmatrix} \sigma_{11}^2(\sigma_{21}^2 + 2\sigma^2) & -\sigma_{11}^2\sigma_{21}^2 \\ -\sigma_{11}^2\sigma_{21}^2 & \sigma_{21}^2(\sigma_{11}^2 + 2\sigma^2) \end{pmatrix}$$

2.10.1 Distribution of y_t

Since $\eta \sim N(\mu_t, \Sigma_t)$, we can write

$$\eta = \mu_t + v_t$$

where $v_t \sim N(0, \Sigma_t)$.

Furthermore, since

$$y_t = e(\eta + a_t^* + \epsilon)$$

we get

$$y_t = e(\mu_t + v_t + a_t^* + \epsilon)$$

where $\epsilon \sim N(0, \sigma^2 I)$. Therefore

$$y_t \sim N(e(\mu_t + a_t^*), e\Sigma_t e' + 2\sigma^2)$$

2.10.2 Linear Wages at $t = 2$

The firm owner's problem (2.6) is

$$\begin{aligned} \max_{\alpha_{i2}} E(y_2|y_1) - \sum_{i=1}^2 g(a_{i2}) - \frac{r}{2} \sum_{i=1}^2 \alpha_{i2}^2 \text{Var}(y_2|y_1) \\ \text{s.t.} \end{aligned} \tag{2.16}$$

$$\alpha_{i2} = \frac{a_{i2}^{1-\theta} g'(a_{i2})}{(a_{12}^\theta + a_{22}^\theta)^{\frac{1-\theta}{\theta}}}$$

This problem can be solved by differentiating logarithmic versions of the two constraints with respect to a_{i2} to solve for $\frac{\alpha_{i2}}{a_{i2}}$ and $\frac{\alpha_{j2}}{a_{i2}}$ and substituting these into the derivative of the objective function. Doing so gives

$$\begin{aligned} (a_{12}^\theta + a_{22}^\theta)^{\frac{1-\theta}{\theta}} a_{i2}^{\theta-1} - g'(a_{i2}) \\ - r \text{Var}(y_2|y_1) \left(\alpha_{i2}^2 \left((1-\theta)a_{i2}^{-1} + \frac{g''(a_{i2})}{g'(a_{i2})} \right) - (1-\theta)(a_{12}^\theta + a_{22}^\theta)^{-1} (\alpha_{i2}^2 a_{i2}^{\theta-1} + \alpha_{j2}^2 a_{j2}^{\theta-1}) \right) = 0 \end{aligned} \tag{2.17}$$

Notice that both workers enter the objective function symmetrically. In addition, each term is concave. Therefore, the solution has to satisfy $a_{i2} = a_{j2}$. Substituting this into the previous equation gives

$$2^{\frac{1-\theta}{\theta}} - g'(a_{i2}) - r \text{Var}(y_2|y_1) \alpha_{i2}^2 \frac{g''(a_{i2})}{g'(a_{i2})} = 0$$

Solving this for $g'(a_{i2})$ and noting that the constraint satisfies $\alpha_{i2} = \frac{g'(a_{i2})}{2^{\frac{1-\theta}{\theta}}}$ gives the solution:

$$\alpha_{i2} = \frac{1}{1 + r \text{Var}(y_2|y_1) 2^{2\frac{\theta-1}{\theta}}}$$

If $\theta = 1$, this term is reduced to the expression $\alpha_{i2} = \frac{1}{1+r\text{Var}(y_2|y_1)}$ which is familiar from the literature. The lower θ , the smaller will α_{i2} be. At the extreme, when $\theta \rightarrow -\infty$, the limiting value is $\alpha_{i2} = \frac{1}{1+4r\text{Var}(y_2|y_1)}$. Therefore, as complementarity increases, it becomes less necessary to pay bonus payments.

Bibliography

- Aiyagari, S. Rao.** (1994). ‘Uninsured idiosyncratic risk and aggregate saving’, *Quarterly Journal of Economics* pp. 659–684.
- Alchian, Armen and Demsetz, Harold.** (1972). ‘Production, information costs, and economic organization’, *American Economic Review* 62(5), 777–795.
- Anderson, Axel.** (2011). ‘A dynamic generalization of Becker’s assortative matching result’, *Georgetown University Working Paper* .
- Anderson, Axel and Smith, Lones.** (2010). ‘Dynamic matching and evolving reputations’, *Review of Economic Studies* 77, 3–29.
- Atakan, Alp E.** (2006). ‘Assortative matching with explicit search costs’, *Econometrica* 74(3), 667–680.
- Atkeson, Andrew and Kehoe, Patrick J.** (2007). ‘Modeling the transition to a new economy: lessons from two technological revolutions’, *American Economic Review* 97(1), 64–88.

- Atkeson, Andrew and Phelan, Christopher.** (1994). ‘Reconsidering the costs of business cycles with incomplete markets’, *NBER Macroeconomics Annual 1994* 9, 187–218.
- Auriol, Emmanuelle, Friebel, Guido and Pechlivanos, Lambros.** (2002). ‘Career concerns in teams’, *Journal of Labor Economics* 20(2), 289–307.
- Becker, Gary.** (1973). ‘A theory of marriage: Part I’, *Journal of Political Economy* 81, 813–846.
- Borjas, George.** (1987). ‘Self-selection and the earnings of immigrants’, *American Economic Review* pp. 531–553.
- Burdett, Kenneth and Coles, Melvyn.** (1999). ‘Long-term partnership formation: Marriage and employment’, *The Economic Journal* 109, F307–F334.
- Chari, Varadarajan V. and Hopenhayn, Hugo.** (1991). ‘Vintage human capital, growth, and the diffusion of new technology’, *Journal of Political Economy* pp. 1142–1165.
- Cole, Harold L. and Obstfeld, Maurice.** (1991). ‘Commodity trade and international risk sharing: How much do financial markets matter?’, *Journal of Monetary Economics* 28(1), 3–24.
- Comin, Diego and Hobijn, Bart.** (2004). ‘Cross-country technology adoption: making the theories face the facts’, *Journal of Monetary Economics* 51(1), 39–83.

- Davila, Julio, Hong, Jay, Krusell, Per and Ríos-Rull, José-Víctor.** (2012). ‘Constrained efficiency in the neoclassical growth model with uninsurable idiosyncratic shocks’, *Econometrica* 80(6), 2431–2467.
- Dewatripont, Mathias, Jewitt, Ian and Tirole, Jean.** (1999). ‘The economics of career concerns, part I: Comparing information structures’, *Review of Economic Studies* 66(1), 183–198.
- Diamond, Peter A.** (1967). ‘The role of a stock market in a general equilibrium model with technological uncertainty’, *American Economic Review* pp. 759–776.
- Eaton, Jonathan and Kortum, Samuel.** (2002). ‘Technology, geography, and trade’, *Econometrica* pp. 1741–1779.
- Eeckhout, Jan and Weng, Xi.** (2010). ‘Assortative learning’, *University of Pennsylvania Working Paper* .
- Fama, Eugene F.** (1980). ‘Agency problems and the theory of the firm’, *Journal of Political Economy* pp. 288–307.
- Fieler, Ana Cecilia.** (2011). ‘Nonhomotheticity and bilateral trade: Evidence and a quantitative explanation’, *Econometrica* 79(4), 1069–1101.
- Fuller, Winship C., Manski, Charles F. and Wise, David A.** (1982). ‘New evidence on the economic determinants of postsecondary schooling choices’, *Journal of Human Resources* pp. 477–498.

- Gabaix, Xavier and Landier, Augustin.** (2008). ‘Why has CEO pay increased so much?’, *Quarterly Journal of Economics* 123(1).
- Gibbons, Robert and Murphy, Kevin J.** (1992). ‘Optimal incentive contracts in the presence of career concerns: theory and evidence’, *Journal of Political Economy* 100(3), 468–505.
- Gould, Eric D.** (2002). ‘Rising wage inequality, comparative advantage, and the growing importance of general skills in the United States’, *Journal of Labor Economics* 20(1), 105–147.
- Greenwald, Bruce C. and Stiglitz, Joseph E.** (1986). ‘Externalities in economies with imperfect information and incomplete markets’, *Quarterly Journal of Economics* pp. 229–264.
- Heckman, James J. and Honore, Bo E.** (1990). ‘The empirical content of the Roy model’, *Econometrica* pp. 1121–1149.
- Heckman, James J., Lochner, Lance and Taber, Christopher.** (1998). ‘Explaining rising wage inequality: Explorations with a dynamic general equilibrium model of labor earnings with heterogeneous agents’, *Review of Economic Dynamics* 1(1), 1–58.
- Heckman, James J. and Scheinkman, Jose.** (1987). ‘The importance of bundling in a Gorman-Lancaster model of earnings’, *Review of Economic Studies* 54(2), 243–255.

- Heckman, James J. and Sedlacek, Guilherme.** (1985). ‘Heterogeneity, aggregation, and market wage functions: an empirical model of self-selection in the labor market’, *Journal of Political Economy* 93(6), 1077–1125.
- Holmström, Bengt.** (1982). ‘Moral hazard in teams’, *The Bell Journal of Economics* pp. 324–340.
- Holmström, Bengt.** (1999). ‘Managerial incentive problems: A dynamic perspective’, *Review of Economic Studies* 66(1), 169–182.
- Hoppe, Heidrun, Moldovanu, Benny and Sela, Aner.** (2009). ‘The theory of assortative matching based on costly signals’, *Review of Economic Studies* 76(1), 253–281.
- Hsieh, Chang-Tai, Hurst, Erik, Jones, Charles I. and Klenow, Peter J.** (2013). ‘The allocation of talent and us economic growth’, (No. w18693).
- Imrohoroglu, Ayse.** (1989). ‘Cost of business cycles with indivisibilities and liquidity constraints’, *Journal of Political Economy* pp. 1364–1383.
- Jeon, Seonghoon.** (1996). ‘Moral hazard and reputational concerns in teams: Implications for organizational choice’, *International Journal of Industrial Organization* 14(3), 297–315.
- Jovanovic, Boyan.** (1979). ‘Job matching and the theory of turnover’, *Journal of Political Economy* 87(5), 972–990.

- Kambourov, Gueorgui and Manovskii, Iourii.** (2008). ‘Rising occupational and industry mobility in the United States: 1968-97’, *International Economic Review* 49(1), 41–79.
- Kambourov, Gueorgui and Manovskii, Iourii.** (2009). ‘Occupational specificity of human capital’, *International Economic Review* 50(1), 63–115.
- Krueger, Dirk and Kumar, Krishna B.** (2004). ‘Skill-specific rather than general education: A reason for US-Europe growth differences?’, *Journal of Economic Growth* 9(2), 167–207.
- Krusell, Per, Mukoyama, Toshihiko, Şahin, Ayşeçül and Smith, Anthony A.** (2009). ‘Revisiting the welfare effects of eliminating business cycles’, *Review of Economic Dynamics* 12(3), 393–404.
- Krusell, Per and Smith, Anthony A.** (1999). ‘On the welfare effects of eliminating business cycles’, *Review of Economic Dynamics* 2(1), 245–272.
- Lee, Donghoon and Wolpin, Kenneth I.** (2006). ‘Intersectoral labor mobility and the growth of the service sector’, *Econometrica* 74(1), 1–46.
- Lee, Robin and Schwarz, Michael.** (2012). ‘Interviewing in two-sided matching markets’, *NYU Working Paper 22* .
- Legros, Patrick and Newman, Andrew.** (2002). ‘Monotone matching in perfect and imperfect worlds’, *Review of Economic Studies* 69(4), 925–942.

- Legros, Patrick and Newman, Andrew.** (2007). ‘Beauty is a beast, frog is a prince: Assortative matching with nontransferabilities’, *Econometrica* 75(4), 1073–1102.
- Lucas, Robert. E.** (1987), *Models of Business Cycles*, Basil Blackwell Ltd., Oxford.
- Matsuyama, Kiminori.** (1992). ‘A simple model of sectoral adjustment’, *Review of Economic Studies* 59(2), 375–387.
- McFadden, Daniel.** (1974). ‘The measurement of urban travel demand’, *Journal of Public Economics* 3(4), 303–328.
- Meagher, Kieron and Prasad, Suraj.** (2016). ‘Career concerns and team talent’, *Journal of Economic Behavior & Organization* .
- Meyer, Margaret A.** (1994). ‘The dynamics of learning with team production: Implications for task assignment’, *Quarterly Journal of Economics* 109(4), 1157–1184.
- Moscarini, Giuseppe.** (2005). ‘Job matching and the wage distribution’, *Econometrica* 73(2), 481–516.
- Nagypál, Éva.** (2005). ‘On the extent of job-to-job transitions’, *Northwestern University Working Paper* .

- Ordoñez, Guillermo.** (2013). ‘The asymmetric effects of financial frictions’, *Journal of Political Economy* 121(5), 844–895.
- Ortega, Jaime.** (2003). ‘Power in the firm and managerial career concerns’, *Journal of Economics & Management Strategy* 12(1), 1–29.
- Parente, Stephen L. and Prescott, Edward C.** (1994). ‘Barriers to technology adoption and development’, *Journal of Political Economy* pp. 298–321.
- Postel-Vinay, Fabien and Robin, Jean-Marc.** (2002). ‘Equilibrium wage dispersion with worker and employer heterogeneity’, *Econometrica* 70(6), 2295–2350.
- Rothschild, Casey and Scheuer, Florian.** (2013). ‘Redistributive taxation in the Roy model’, *Quarterly Journal of Economics* 128(2), 623–668.
- Roy, Andrew Donald.** (1951). ‘Some thoughts on the distribution of earnings’, *Oxford Economic Papers* 3(2), 135–146.
- Shimer, Robert and Smith, Lones.** (2000). ‘Assortative matching and search’, *Econometrica* 68(2), 343–369.
- Storesletten, Kjetil, Telmer, Chris I. and Yaron, Amir.** (2001). ‘The welfare cost of business cycles revisited: Finite lives and cyclical variation in idiosyncratic risk’, *European Economic Review* 45(7), 1311–1339.
- Veldkamp, Laura L.** (2005). ‘Slow boom, sudden crash’, *Journal of Economic Theory* 124(2), 230–257.