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Essays In The Economics Of Human Capital

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Abstract
This dissertation deals with two different aspects of human capital accumulation: early childhood development and tertiary education. Specifically, it analyzes the role that public policies or changes in regulations affect incentives of agents in a way that ends up affecting the aggregate endowment of human capital in an economy.

The first chapter is related to early childhood development. Recent literature has shown that skills shaped during childhood have long lasting consequences later in life. This fact has promoted a large number of programs aimed at stimulating the skill formation process for children in disadvantage. However, little is known about how cost-effective are these policies. In this chapter I evaluate the cost-effectiveness of three alternative policies aimed at improving the living standards of families in disadvantage: cash transfers, childcare subsidies, and subsidies for child investments. I find that subsidies promoting child investments are much more productive than the other two alternatives.

In the second chapter, co-authored with David Zarruk, we analyze the consequences that subsidized loans for higher education have on the quality of education offered by colleges in the context of a developing country. We find that subsidized student loan policies lead to a widening gap in the quality of services provided by higher education institutions. This happens because the demand for elite institutions unambiguously increases when individuals can borrow. This does not happen in non-elite institutions, since relaxing borrowing

constraints makes some individuals move from non-elite to elite institutions. The higher increase in demand for elite institutions allows them to increase prices and investment per student. If investment and average student ability are complementary inputs in the quality production function, elite universities also increase their acceptance cut-offs. In this new equilibrium, the differentiation of the product offered by colleges increases, where elite universities provide higher quality education to high-ability students and non-elite universities offer lower quality to less-able students. We calibrate the model to Colombia, which implemented massive student loan policies during the last decade and experienced an increase in the gap of quality of education provided by elite and non-elite universities. We show that the increase in the quality gap can be a by-product of the subsidized loan policies. Such results show that, when analyzed in a general equilibrium setting, subsidized loan policies can have negative effects in equilibrium.

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Rodrigo Azuero Melo

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ESSAYS IN THE ECONOMICS OF HUMAN CAPITAL

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ABSTRACT

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This dissertation deals with two different aspects of human capital accumulation: early childhood development and tertiary education. Specifically, it analyzes the role that public policies or changes in regulations affect incentives of agents in a way that ends up affecting the aggregate endowment of human capital in an economy.

The first chapter is related to early childhood development. Recent literature has shown that skills shaped during childhood have long lasting consequences later in life. This fact has promoted a large number of programs aimed at stimulating the skill formation process for children in disadvantage. However, little is known about how cost-effective are these policies. In this chapter I evaluate the cost-effectiveness of three alternative policies aimed at improving the living standards of families in disadvantage: cash transfers, childcare subsidies, and subsidies for child investments. I find that subsidies promoting child investments are much more productive than the other two alternatives.

In the second chapter, co-authored with David Zarruk, we analyze the consequences that subsidized loans for higher education have on the quality of education offered by colleges in the context of a developing country. We find that subsidized

student loan policies lead to a widening gap in the quality of services provided by higher education institutions. This happens because the demand for elite institutions unambiguously increases when individuals can borrow. This does not happen in non-elite institutions, since relaxing borrowing constraints makes some individuals move from non-elite to elite institutions. The higher increase in demand for elite institutions allows them to increase prices and investment per student. If investment and average student ability are complementary inputs in the quality production function, elite universities also increase their acceptance cut-offs. In this new equilibrium, the differentiation of the product offered by colleges increases, where elite universities provide higher quality education to high-ability students and non-elite universities offer lower quality to less-able students. We calibrate the model to Colombia, which implemented massive student loan policies during the last decade and experienced an increase in the gap of quality of education provided by elite and non-elite universities. We show that the increase in the quality gap can be a by-product of the subsidized loan policies. Such results show that, when analyzed in a general equilibrium setting, subsidized loan policies can have negative effects in equilibrium.

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Chapter 1

Evaluating Early Childhood Policies: An Estimable Model of Family Child Investments

Abstract

There is extensive evidence showing that skills developed early in life have important consequences for adult life outcomes. Such findings have motivated a large literature analyzing the production of skills in young children. However, little is known about how families make decisions about investments in their children. In this paper, I estimate a production function of skills in young children, nested within a collective model of household behavior, using data from Chile. The estimated model is used to simulate the effects of various policies aimed at increasing skills of children in disadvantaged households that are popular in developing countries. The data reveals substantial disparities in the skills of poor and rich children when they are five years old. I find that to close this gap in skills, it is more effective to design policies that subsidize the acquisition of skill-enhancing goods for children than policies providing unconditional cash transfers or childcare subsidies.

1.1 Introduction

Research in medicine, psychology and economics shows that skills shaped during the first years of life have significant consequences for adult life outcomes.[1] This fact has motivated a large amount of research in economics aimed at understanding the skill formation process. The results of these studies allow a better understanding of the key inputs that promote skills in young children.[2] For instance, they showed that parenting and general family environment are among the most relevant inputs in the production of skills (Heckman and Mosso, 2014; Schoellman, 2014).

Gaps in skills between rich and poor children emerge very early in life, even before they start their formal education. Duncan and Magnuson (2013) find that differences in reading and math test scores between children in the top and bottom quartile of the income distribution are about one standard deviation when they start kindergarten in the US. Schady et al. (2015) report similar quantitative results for five Latin American countries, using a vocabulary test for children younger than five. Research in neuroscience shows that malleability of skills decreases with age (Nelson and Sheridan, 2011). To close gaps in skills between the rich and poor population, we need to develop policies addressing this issue during early childhood.

The goal of this paper is to evaluate the effects that early childhood policy interventions have on the skill gaps between rich and poor children. Knowledge of

[1] For a review, see Conti and Heckman (2012)

[2] See, for example, Cunha, Heckman and Schennach (2010)

the skill production function is not enough to assess the effectiveness of policies aimed at improving children's skills. Families administer resources and make the relevant decisions that determine the allocation of inputs for young children. Family investments in children might react to policy interventions. To analyze how early childhood policies affect resources allocated to children and skill formation, I develop and estimate a skill production function nested within a collective model of household behavior using data from Chile. I evaluate the effects of cash transfers, childcare subsidies and in-kind transfers, which are transfers of goods that can be used in the skill formation process in children (for example, books, toys, puzzles, and guides about early childhood development). I find that in-kind transfers provide the most cost-efficient way to reduce the gaps in skills between rich and poor children.

This paper makes several contributions to the literature on family investments and child outcomes. First, there are not many papers estimating a model of household behavior where parents allocate time and money to their children to enhance their skills (Bernal, 2008; Del Boca, Flinn and Wiswall, 2014, 2016; Gayle, Golan and Soytas, 2015). This is the first paper that empirically evaluates and compares the effects that cash transfers, childcare subsidies and in-kind transfers have on the gaps in skills between rich and poor children.

Cash transfer programs have been widely implemented in developing countries. In Latin America, they constitute the largest social assistance programs, covering

millions of households in countries such as Brazil, Mexico, Nicaragua and Colombia (Fiszbein, Schady and Ferreira, 2009). Additionally, governments in both developing and developed countries have invested a large amount of resources in the provision of preschool services. In 2011, the United States federal government spent US$ 8.1 billion on Head Start, the largest childcare program. In Chile, firms employing more than twenty people are required to provide childcare services to their female employees. During the last ten years, Chile has experienced a massive expansion in the number of childcare providers. Between 2006 and 2010, the network of childcare providers increased its capacity, measured as the maximum number of children for whom the system could provide coverage, by approximately 500% (Chile, 2010).

A limitation of cash transfers is that it is not possible not possible to guarantee that a given amount of money will be spent on goods that can actually translate into better child outcomes. However, when the transfer is done in-kind via puzzles, toys, guides about child development, or specific types of food that can improve children's nutritional status, governments can enrich the environment and thus promote skills for children. These transfers are usually implemented by governments through their early childhood development programs. Currently, the program "Chile Grows with You"[3], which is the main early childhood program in Chile, delivers a basket of goods to families for such purposes. Given the large amount of resources that governments spend on enriching childhood environments, and given the fact that events during childhood heavily influence adult outcomes, it is important to un-

[3]In Spanish, "Chile Crece Contigo".

derstand the most cost-effective way of allocating these resources, whether through cash transfers or in-kind.

This paper also contributes to the literature of household decisions and child outcomes by allowing individual family members to have different preferences. First, modeling household behavior through the collective approach has proven to result in better empirical predictions than the unitary framework. Second, from a policy perspective, it is common to see interventions targeting individual household members. For instance, most cash transfer programs in developing countries state as an explicit condition that, in households with children, mothers should be the sole recipients of such subsidies (Fiszbein, Schady and Ferreira, 2009). It is often argued that mothers have stronger preferences for meeting the needs of children and therefore cash in the hands of mothers translates into better child outcomes (Blundell, Chiappori and Meghir, 2005). By estimating a technology of skill formation within a collective setting, I am able to assess the extent to which targeting individual household members translates into different child outcomes.

The dataset used in this article allows me to overcome some empirical limitations that the literature has previously faced. For instance, studies have shown that parental skills largely determine children's skills (Heckman and Mosso, 2014). By having information on parental IQ tests and personality assessments, I am able to incorporate parental skills into my estimation strategy. Additionally, we know that there is a multiplicity of skills that are relevant to determining adult life outcomes

(Cunha, Heckman and Schennach, 2010). I incorporate multiple measures of skills across various dimensions, such as motor, communication, cognitive and behavioral abilities in children.

The productivity of time investments in children depends on the interactions between parents and children. Fiorini and Keane (2014) find that, when evaluating information about the time parents spend with children, it is important to differentiate among activities such as watching TV, educational activities with parents, and educational activities with other adults, as each of these translates differently into skill formation. By using data on the frequency with which parents perform fourteen different types of activities with their children, I am able to incorporate not only the time component but also the quality of interactions between parents and children. Additionally, I use geocoded datasets matching all the nationally registered childcare providers with the households in the survey to obtain information about the cost of investing in children. Households that have a relatively large supply of childcare services within their neighborhood might, in principle, find it easier to invest in their children through preschool services. Additionally, households living in neighborhoods with a large number of preschool providers might live in a children friendly environment, where the availability of goods to increase skills in children is relatively high.

The survey used for this study is the Early Childhood Longitudinal Survey from Chile (ECLS). This survey was developed with the goal of precisely characterizing

the skill formation process in children. Therefore, I am able to provide a unique empirical description of parental investments in children. I observe the weekly frequency of consumption of different types of food for children, as well as availability of toys, books, and puzzles, as well as a precise characterization of which specific skills such elements might promote.

This paper also makes a methodological contribution to the estimation of dynamic microeconomic models with unobserved and continuous state variables. By implementing an efficient simulation-based estimator using particle filtering techniques from the machine learning and financial econometrics literature (Murphy, 2012; Creal, 2012), I propose a feasible computational approach for dealing with the high dimensionality integration problem that arises in such models. Moreover, this is the first paper in the literature of household choices and child development that estimates a technology of skill formation through a dynamic latent-factor approach a-là Cunha, Heckman and Schennach (2010). This allows me to obtain non-parametric identification of the skill production technology by using a large number of skill measures. Most of the prior research analyzing the child skill formation process uses data from the United States. By analyzing this process in the context of Chile, I bring new insights regarding the skill formation process and the effect that policies and programs have on the skills of children in a situation where poor children face significant disadvantages.

There has been extensive study of the theoretical properties of the collective

model of household behavior related to goods that are "public" within the context of the household (Blundell, Chiappori and Meghir, 2005; Chiappori and Donni, 2009; Browning, Chiappori and Weiss, 2014). However, there are still very few empirical studies (Cherchye, De Rock and Vermeulen, 2012). The main challenge of estimating collective models of household behavior is that of identifying the bargaining power, or Pareto weight, of each household member. The common approach to deal with this issue is to observe the consumption of private goods within the household, such as gender-specific clothing, together with distribution factors. Distribution factors are variables that affect the final outcomes of households, exclusively by modifying the bargaining power of each member. Examples of distribution factors commonly used in the literature include local sex ratios, the proportion of non-labor income in the household that is in the hands of women, and the differences in ages between husband and wife. This approach assumes that the good observed is purely private (i.e., a husband does not care about his spouse's clothing) and that all the bargaining power is explained by the consumption of a single good.

In this paper, I propose a new framework for estimating collective models of household behavior. I use information from questionnaires related to female empowerment and gender roles to assess the bargaining power of each household member. The use of answers to such questions, combined with exogenous variation in the distribution factors, allows me to identify the Pareto weight of each member in the household. Following such an identification strategy, I am also able to allow for

unobserved heterogeneity.

The data from test scores show significant large gaps in skills between rich and poor children at age 5. The skill gap between children in the lowest quintile of the income distribution and children in the highest quintile, are in between 0.3 and 0.7 standard deviations in tests measuring cognitive abilities, socio-emotional development, and vocabulary skills, among others. These inequalities are mostly explained by differences in parental skills and monetary investments. Additionally, , the model parameter estimates show that fathers' time spent with children is 50% as productive as mothers' time and that mothers have stronger preferences for children. However, the higher productivity and the stronger preferences for children do not by themselves explain the observed disparities in time investments between mothers and fathers. Given that women have lower bargaining power, they contribute more to the provision of public goods within the household. This particular mechanism explains 15% of the differences in time investments between mothers and fathers.

I use the estimated behavioral model to simulate the effects that cash transfer programs, free childcare subsidies, and in-kind transfers have on the skill gap between rich and poor children. Although less prevalent than the other two programs, in-kind transfers are currently being implemented in Chile through the "Chile Grows with You" program. I find that in-kind transfers are much more effective than the other alternatives when it comes to closing the gaps in skills between rich and poor

children.

The remainder of this article is structured as follows: In Section 1.2, I briefly review the literature in order to identify the main contributions of this article. I describe the data in Section 1.3. In Section 1.4, I present some preliminary evidence motivating the economic model, which will be described in Section 2.3. The estimation procedure, together with the relevant identification arguments, are introduced in Section 1.6. The main results of the paper are in Section 2.6. I summarize the main points of this paper in Section 1.8.

1.2 Review of the literature

This article is related to four areas of the literature in economics. First of all, this paper is related to the literature analyzing how household behavior affects the production of skills in children. One of the most important decisions families make relevant to the production of child skills, is that of labor supply. As household members increase their participation in the labor market, this will bring more monetary resources to the household but will reduce the amount of time parents interact with their children. For this reason, the impact of labor force participation on the skills of children is not obvious at first glance.

The question of how labor supply decisions affect the production of skills in young children has been explored in the literature. Bernal (2008) estimates a structural model of female labor force participation, taking into account that skills are

affected by family income and also by the total amount of time that mothers interact with their children. Due to data limitations, she does not incorporate paternal time as a potential input in the skills of children. Taking into account the overall effect of an increase in income but a decrease in the amount of time that mothers interact with their children,Bernal (2008) finds that one year of full employment decreases the skills in children by approximately 0.13 of a standard deviation.

Del Boca, Flinn and Wiswall (2014) extend the results of Bernal (2008) and take into account that both parents participate in the production of skills. The authors estimate a unitary dynamic model of household behavior where each parent allocates time to labor market, leisure, or interaction with their children. Additionally, they incorporate decisions about how much money to allocate to monetary investments in children versus consumption. Results show that, when mothers increase time spent in the labor force, the potential negative effect is not only alleviated by the increase in the amount of resources due to wages but also by the fact that the father starts to spend more time with the children at home. One of the main conclusions is that time of both fathers and mothers are relatively more important than monetary investments in the production of child skills. Gayle, Golan and Soytas (2015) extends the modeling framework of Del Boca, Flinn and Wiswall (2014) to incorporate endogenous fertility. However, they do not observe test scores in their data or monetary investments by parents and ignore the role of preschool education.

The article that is most related to this paper is Del Boca, Flinn and Wiswall

(2014). This paper extends Del Boca, Flinn and Wiswall (2014) in several ways. First, I incorporate the decision to enroll a child in preschool services. This is important given that subsidizing preschool is one of the most important policies to improve the conditions in which children develop, and to increase female labor force participation.

Additionally, a major point in which this article departs from the analysis of Del Boca, Flinn and Wiswall (2014) is that I estimate a collective model of household behavior, allowing parents to have different preferences, as opposed to using the unitary approach. There are two reasons why this is important. First of all, in most developing countries, cash transfers to families with children are given to their mothers, motivated by the findings that cash in the hands of women seems to translate into better child outcomes than cash in the hands of men (Duflo, 2000; Attanasio and Lechene, 2014; Thomas, 1994). To assess the effect that targeting individual household members has on child outcomes and to identify the extent to which additional resources should be spent on targeting, I estimate a collective model of household behavior, where parents have different preferences. Additionally, the empirical regularity that there is a positive correlation between women's empowerment and child development (Haddad et al., 1997) cannot be explained by considering the household as a single entity with one utility function. This has motivated a large literature analyzing the relationship between female empowerment and child outcomes (Doepke and Tertilt, 2014). By modeling household behavior

using a collective approach, I am able to assess the extent to which empowering women translate into better child outcomes.

Third, this is the first paper that estimates a model of parental investments and child outcomes using observations not only on time investments but also on in-kind investments. The data I use includes a detailed description of the environment in which children grow. Enumerators who visited the households were trained to provide a precise characterization of the child's environment. For instance, not only I do observe the availability of toys, but also whether the toys are ideal for the promotion of specific skills, such as motor skills or behavioral skills, or toys that help develop free expression in children. I observe the availability puzzles, costumes, and children's books and music. Additionally, I have detailed information about the frequency with which children consume different types of food, such as fruits, vegetables, and fish, among others. This information is used to assess the effect of in-kind investments.

The dataset I use allows me to incorporate several facts about the skill formation process in children that were not incorporated in Del Boca, Flinn and Wiswall (2014). First of all, there is a consensus in the literature that skills are multiple (emotional, physical, cognitive). In this paper, rather than using one cognitive test score as a measure of skills, I use various indicators of motor development, cognitive achievement and emotional attainment in young children as broad measures of skills. Additionally, an important element in the skill formation process is their

dependence on parental skills (Francesconi and Heckman, 2016). Ignoring parental skills when estimating a production function might bias the effect of other inputs, such as time or in-kind investments. I overcome this limitation by using various assessments of cognitive achievement and personality traits of parents.

By implementing a dynamic latent factor structure in the estimation of the skill production function for children, I am able to obtain non-parametric identification of the skill production function in children. This is accomplished by using identification results from the literature of skill formation (Cunha, Heckman and Schennach, 2010). Because of that, the results of the estimation are less sensitive to the specific parametric form assumed for the skill formation technology, and the bias arising from measurement error is reduced, making the results more robust. This, along with the fact that a latent factor structure can be interpreted as unobserved heterogeneity (Carneiro, Hansen and Heckman, 2003) and potentially improves the accuracy of the estimates, has made factor analysis a popular tool to get accurate estimates of the skill production function (Cunha, Heckman and Schennach, 2010; Cunha and Heckman, 2008; Heckman, Stixrud and Urzua, 2006). This paper is the first to estimate the production function of skills via a latent-factor approach, nested within a collective model of household behavior.

The second area of related literature the empirical implementation of collective models of household behavior. The income pooling assumption establishes that, in a household composed of various members, it does not make a difference if transfers

are given to one member or the other. Ultimately, what matters is the overall resources of the household. This assumption has been rejected in contexts as diverse as Sweden (Cesarini et al. (2013)), South Africa (Duflo, 2000)), Mexico (Attanasio and Lechene, 2014), Brazil, the US and Ghana (Thomas, 1994). The rejection of this assumption has motivated a significant amount of research aimed at exploring alternatives. The collective model of household behavior assumes that each parent has his/her own preferences and that the decision reached in the household is Pareto efficient (Chiappori and Donni, 2009). The collective approach has resulted in better empirical predictions than the unitary framework.

Although there is an extensive literature exploring the properties of the collective model of household behavior, there are still very few empirical implementations of the model, one exception being Cherchye, De Rock and Vermeulen (2012). In their model, the authors assume that each parent has his or her own preferences and each parent derives utility from spending time with their children. They do not model how the time parents spend on their children impacts child skills. In this paper, I asume parents spend time with their children in part to augment their skill set.

Additionally, this paper provides a new framework for identifying collective models of household behavior. The usual identification strategy of such models relies on observing the consumption of a given number of private goods, clothing being the most popular choice. Once the decisions about consumption of such private goods

are observed, there is a one-to-one mapping from these decisions into the Pareto weight given to each agent. However, such arguments ignore the fact that every good consumed within the household has a public component. For example, it is also possible that couples care about each other's clothing. In this paper, rather than using private goods, I use answers provided from questionnaires about female empowerment and gender roles as noisy measures of the bargaining power within the household.

This article also contributes to the literature on optimal design of policies for disadvantaged households in developing countries. Currently, Conditional Cash Transfers (CCT) are one of the most important policies to alleviate poverty and reduce inequality in most developing countries. Every country in Latin America has a CCT program. In some cases, such as in Brazil and Mexico, this program accounts for the largest social assistance program executed by the central government (Fiszbein, Schady and Ferreira, 2009). In most countries, the design of such programs establishes that, in households with children, the mother of the child receives the monetary transfers. This is supported by findings such as those in Bobonis (2009) and Duflo (2000), where the authors explore whether or not the gender of the recipient of a monetary transfer matters in terms of child development. In both cases, it is found that transfers to women translate into better child outcomes than those made to men. The common interpretation of this fact is that women's preferences are more aligned with child outcomes and, therefore, making transfers

to them is more efficient. However, to establish the mechanism that is generating such an outcome, it is necessary to estimate an economic model able to identify all possible channels.

The finding that transfers made to women result in better child outcomes deserves additional analysis. One interpretation is that women spend their own income on public goods, as explained by Bobonis (2009), or that they have stronger preferences for child outcomes than men. However, there are multiple possible explanations. Blundell, Chiappori and Meghir (2005) show that, as long as the marginal willingness to pay for child outcomes is higher for women than for men, we will have such a result. Having women with stronger preferences for child outcomes is not a necessary condition. Basu (2006) provides an example where, even in the case in which women care more for their children, there might be an inverted-U relationship between the bargaining power of the women and the welfare of children, because, as women become relatively more powerful, they can devote resources derived from child labor into their own private consumption. It is important for the design of policies to identify and explain the mechanism generating the positive relationship between women's empowerment and child outcomes. In this paper, I allow parents to have different preferences for children. By estimating the structural parameters of the model, I can analyze which mechanisms generate such a relationship.

Finally, this paper is related to the literature exploring the production of skills in children. Todd and Wolpin (2007) present alternative ways of estimating the pro-

duction function depending on the type of data available to the researcher. Cunha, Heckman and Schennach (2010) estimate a production function of skills in children taking into account the joint condition of multiple skills and that the productivity of inputs might vary with age. As both inputs and outputs are observed with error, the authors estimate the production function via a dynamic latent factor structure. In this article, I use the estimation methods presented in Todd and Wolpin (2007), taking into account that the availability of data allows me to use a value-added specification. For the econometric implementation, I use a latent factor structure as in Cunha, Heckman and Schennach (2010). However, to account for the endogeneity of inputs, I use an economic model of household behavior. Although Cunha, Heckman and Schennach (2010) is considered a seminal contribution to the skill production function literature, there is little scope for counterfactual analysis because the inputs are hard to interpret. The measures of investments do not map to any possible effort levels or monetary investment in the family. In this paper, by embedding the skill production function within model of household behaviors, counterfactual analysis can be performed with easy interpretation of findings.

This is one of the few articles that have attempted to estimate a production function of skills in a developing country. Much attention has been focused on the United States and Europe due to the availability of data. I use a unique dataset from Chile. A final contribution of this paper relies on the estimation strategy. Estimating dynamic models with continuous state variables is a huge challenge in mi-

croeconomics. Different solutions such as discretization (Keane, Todd and Wolpin, 2011) have been proposed. I bring to the table a new alternative commonly used in macroeconomics and macro econometrics: particle filtering techniques.

1.3 Data

I use a rich longitudinal dataset from Chile. Chile is the country of Latin America with the highest GDP per capita -$US 20,000 PPP- and is often considered a case of economic success in the region due to good economic performance during the last twenty years.[4] Two of the most distinctive facts about the Chilean economy are its high level of inequality and the low levels of female labor force participation. Women's participation in the labor market has been historically low, not only when the comparison is made with countries that are similar in terms of income and geographic location.

The dataset used for this project comes from the Early Childhood Longitudinal Survey from Chile (ECLS). The first wave of this survey was collected in 2010 and includes a nationally representative sample of all households in Chile with a child under 5 years of age, which accounts for 15,175 households. The second wave was implemented in 2012 and included 85% of the households in the original sample and a new sample of 3,135 new households with children younger than 2 years of age. In each wave, information about labor force participation for every member older than

[4]Since 2012, Chile has been considered as a developed country for the World Bank. However, most of the literature treats it as a developing country, especially when dealing with data pre-2012. The International Monetary Fund does not include Chile in the list of advanced economies.

15 was collected, together with income, educational background, knowledge about the process of early childhood development and productive routines performed with the child, such as reading books, teaching letters and taking children to the park.

The dataset includes multiple test scores for children and questionnaires answered by the primary caregiver of the child in order to assess the skills level of children, for different domains such as socio-emotional development, behavioral problems and development of vocabulary. Not every test was answered by all the children, as all of them include different age specifications.[5] The description of the tests included in the sample is included in Tables 1.1 and 1.2. I use these test scores as noisy information about children's skills

Given that I want to identify how families make decisions about investments in young children, I restrict the sample to children living with both biological parents. I do this because the main goal of the article is to identify how parents reach such decisions in a context where there are multiple members with plausibly different preferences.

In the economic model, I consider the case of families with only one child under the age of five. For that reason, I take into account families with only one child or with multiple ones so long as the child being analyzed has no siblings within a

[5]For instance, the Batelle Index of Development, a questionnaire included in the 2010 survey to be answered by the primary caregiver of the child, is designed for children between 6 and 24 months of age. Given that most children are older than 24 months in the 2010 survey, I do not include this test when performing the analysis of skills in young children.

five-year age range.[6] The reason for doing this is that allowing for multiple children in the economic model would imply solving additional questions that are not the main goal of this paper. For instance, I would need to identify or take a stance on whether parents have the same preferences for boys and girls, or whether they have preferences for equality of skills among children, as opposed to devoting more resources to the most promising child. Moreover, we also would need to understand to what extent there is a quality-quantity tradeoff in fertility decisions: do parents prefer to have more children and devote fewer resources to each of them or to terminate their childbearing early and devote most resources to a limited number of children.

In Table 1.3, I report the summary statistics of families in the survey. We see that fathers, whose average age is 35, are on average four years older than mothers, whose average age is 31. There is not much difference in terms of schooling, as both fathers and mothers attain on average 11 grades of education. We do observe significant differences between fathers and mothers in labor market variables. Fathers participate in the labor force on average 44 hours a week, which is more than twice the average of mothers, at 18 hours. As will be discussed in the preliminary evidence section, unemployment rates do not explain a great deal of the low level of hours that mothers participate in the labor market. This is due to women being actively out of the labor force, not looking for a job but rather reporting that they

[6] A similar data restriction is implemented in Bernal (2008) and in the main analysis of Del Boca, Flinn and Wiswall (2014).

don't work because they have to take care of their children.

There are differences in the wages of men and women on a weekly basis. The weekly wage of a woman is $83,890 Chilean Pesos (CLP) whereas men make $104,220 CLP.[7] In terms of ages of children, we see that they are on average 50 months old.

The survey also reports the frequency with which parents perform different types of activities with their children. The description of each of these activities is presented in Tables 1.4 and 1.5. In Figure 1.3 I present the average frequency for each activity that parents report performing with the child for the activities reported in 2012. As can be seen, in every activity, fathers report a lower frequency than mothers. The most common activities that parents perform with their children are sharing a meal, talking to them and teaching them the numbers or letters. The least common activities are taking the children to cultural activities or parks or reading to them.

In Tables 1.6 and 1.7, I report all the subdomains of the test scores and parental assessments used for the skills of children in the two waves of the survey. As can be seen, in both waves I use information about test scores related to vocabulary tests and cognitive abilities, and also parental assessments related to overall child development, together with behavioral and emotional skills.

The dataset also contains information about other important inputs into the production of skills in children. For instance, there is significant information about issues for the child during pregnancy and the health conditions at birth. This infor-

[7]The exchange rate for 2012 corresponds to 1 Chilean peso for 0.002 USD

mation will be used in order to assess the skills of children at birth. The indicators of health at birth and conditions during pregnancy are reported in Table 1.8.

To incorporate the fact that parental skills affect skills of children, I use scores of different tests performed to mothers of the children selected. In Table 1.9, I report all the test scores used, which include two widely-used test scores assessing general cognition (Wais Test Scores), together with the Big Five personality traits scores (BFI).

A relevant input into the production of skills is the amount of monetary investments that parents make in their children. This type of investment can include any type of materials that can improve the living conditions of children or that can stimulate their learning experiences, such as toys, food investments, physical space exclusively used by the child, and so on. Previous studies such as Del Boca, Flinn and Wiswall (2014) and Bernal (2008) take into account such factors in the production of skills in children but do not observe such measures of investments. The identification of how monetary investments affect the production of skills in children in their studies relies, then, on functional forms assumptions. Going beyond previous studies, I use some indicators of parental investments in children that will give some idea of how parents invest in their children. Some of these measures are exactly the same as those used in Cunha, Heckman and Schennach (2010), which come from the HOME inventory test score. The details of the measures used to asses the level of monetary investment in the children can be found in Tables 1.10

and 1.11.

A novel feature of this dataset is the inclusion of questions regarding female empowerment and gender roles within the household. For instance, there is information on whether it is the mother or the father who manages the income and whether the mother considers that it is better to have a bad marriage than to remain single. These variables allow us to identify the extent to which the woman has a say in the household and whether she has any power when making decisions of economic relevance. The variables used to assess the degree of a woman's empowerment in the household are presented in Table 1.12. Tables 1.13 and 1.14 include summary statistics of the answers provided on the empowerment questionnaires. It is interesting to see, for instance, that 64% of men think that women should devote all their time to taking care of children and should work only in the case of extra time. However, as noted in Table 1.14, women also consider that they should be more in charge of children involved in the workforce. For instance the question related to "A woman in charge of chores should not work" receives an average score of 2.62 out of 4. These facts show that female empowerment should be an important concern for policymakers in this subpopulation.

In addition to the ECLS, I use information about the location of every preschool provider in Chile and I compute the distance from each center to each household. I use the relative availability of preschool providers near each household as a shifter in the cost of childcare and monetary investments in children. In Figure 1.1, I report

summary statistics about the availability of childcare providers for households. We see that, on average, the nearest preschool provider is 0.61 kilometers away from the household. Additionally, in Figure 1.2 I report an example of how the information about availability of preschool allows me to geographically locate each center.

Finally, I use information from the household survey (CASEN) in 2011, together with the CENSUS dataset in order to obtain some of the distribution factors. I use as distribution factors the share of non-labor income in the hands of men, the difference in ages between fathers and mothers, and the sex ratio in the city of residence, as well as the gender wage gap and the gender unemployment ratio in each region. The descriptive statistics of the distribution factors can be found in Table 1.15.

1.4 Preliminary Evidence

In this section, I present four facts found in the dataset that motivate the economic model developed in the next section.

1.4.1 Gaps in skills emerge early in life

When analyzing height at birth, weight at birth and the incidence of pre-term births[8], for different income groups, we do not observe huge differences between poor and rich children, as can be seen in Figure 1.5. However, we do observe differences in various dimensions of development, such as vocabulary, communication skills,

[8]These are variables that have often been used as a measure of health at birth Sørensen et al. (1999).

motor skills and cognitive achievement, when children are five years old. This can be seen in Figure 1.6. The figure reports the scores in different tests and parental assessments. All of them are standardized to be mean zero and variance one. We see, for instance, that children in the lowest income quintile score 0.1 of a standard deviation below the mean on the Battelle test score for Motor Skills, whereas children in the richest quintile score 0.15 of a standard deviation above the mean. The most dramatic case is vocabulary, where children in the lowest income quintile score 50% of a standard deviation below children located in the richest income quintile. This early emergence of gaps in the development of children is consistent with the literature Schady et al. (2015); Cunha, Heckman and Schennach (2010).

1.4.2 Mothers spend more time with children than do fathers

As shown previously in Figure 1.3, mothers spend more time with their children, in every activity, than fathers do. One possible explanation is the difference in labor supply. Fathers specialize in remunerated activities in the labor market, whereas mothers specialize in taking care of children. In Tables 1.16 and 1.17, I analyze the relationship between labor supply of both spouses and time spent with the child. In order to simplify the analysis, I construct a measure of time investment via principal component analysis and I regress the predicted factor with other covariates of the family. We observe that there is a negative correlation between time spent

with the child and labor supply decisions for both fathers and mothers, in the two waves of the dataset being used, as can be seen in Tables 1.16 and 1.17.

Additionally, we observe a positive correlation between each parent's own effort and the labor supply of his/her spouse. This might be evidence of compensating behavior by parents. For example, when one parent increases his/her labor supply, that parent decreases the amount of time spent with the child and thus the other parent might react by increasing the amount of time spent interacting with the child. This compensating behavior might diminish the plausible negative impact on child development of an increase in female labor force participation.

The evidence from these regressions is complemented with the estimates of regressions in differences reported in Table 1.18. The results again seem to suggest that, as members participate more in the labor market, they decrease the amount of time spent with their child, but this is compensated by an increase in the spouse's time with their child.

Although labor market behavior might explain part of the differences in the time investments between mothers and fathers, there are other stories consistent with such a result. The differences might be due to preferences, as mothers find it less costly to invest time in their children, or due to differences in productivity, as the amount of time that mothers spend with their children might be more efficient in enhancing children's skills than that of fathers. Moreover, there is a possible explanation related to the fact that the utility derived from children's skills is a public

good but the time investments are privately exerted. As women are relatively less empowered than men, the cost of effort exerted by women is less than the cost of effort exerted by men. This implies that, even with the same preferences and resources, women would spend more time taking care of children. In the economic model, I allow all these aforementioned factors to be a possible explanation of the differences in time investment between fathers and mothers.

1.4.3 Female empowerment and child outcomes

The last point to be mentioned in the preliminary evidence section is the correlation between female empowerment and child outcomes. There is evidence in the literature pointing to the fact that women's empowerment is associated with better child outcomes in various contexts Attanasio and Lechene (2014); Thomas, Contreras and Frankenberg (2002).

We do observe evidence of a positive relationship between female empowerment and child outcomes. Table 1.19 presents the results of various regressions showing positive correlations between child outcomes and the share of income earned by women. Even after controlling for variables such as the IQ level of the primary caregiver, total household income, grades of schooling of both parents and their ages, we observe a positive relationship between the share of the total household income earned by mothers and children's outcomes.

When analyzing the responses to the female empowerment questionnaires, we

also observe a positive relationship between female empowerment and investments in children. In Table 1.20, some regressions of child investments and female empowerment are presented. I show again that, even after controlling for the same variables as mentioned before, those households where women are relatively less empowered make fewer investments in their children. Those households where the woman manages the income are more likely to have toys for the development of children, and the frequency of consumption of fruits and vegetables is higher whereas that of bread is smaller. Similarly, households that are more accepting of the opinion that women should not work and should exclusively take care of their children are more likely to have the children sharing their bed with someone else, which might be an indicator of lower investments in children.

The results of these regressions cannot be interpreted as incorruptible evidence of a causal relationship between female empowerment and child outcomes. Nonetheless, they suggest that there are either some unobservables that are not captured in the regressions, which are also correlated with female empowerment, and which positively affect child outcomes, or that it is indeed female empowerment that improves the conditions of children in the households. In order to incorporate such findings in the economic model, I allow parents to have different preferences regarding leisure, consumption, and skills of children, among other preferences, so that we can understand whether the relationship between female empowerment and child outcomes arises from such patterns or either due to unobserved heterogeneity.

1.4.4 Female Labor Force Participation

As mentioned before, mothers participate in the labor market 19 hours a week on average, whereas fathers do so 44 hours a week. One plausible explanation can be due to involuntary unemployment: it is harder for women to find a job offering a wage higher than their reservation wage, and because of that they do not actively participate in the labor market. However, it turns out to be the case that female unemployment in the population analyzed is low, below 5%. The main reason for observing these low levels of female participation in the labor market is due to voluntary unemployment: women with young children decide not to participate in the labor market. As can be seen in Figure 1.4, this is characteristic of women across all age groups. Most of them are not working or looking for a job and 83% of them state that the main reason is that they do not do it is because they are taking care of children.

The fact that unemployment plays a small role in explaining the low levels of female activity in the labor market should guide the economic model as to how to approach the problem of deciding whether or not to work. Including frictions in the model, as is usually done in the literature in order to explain unemployment and variation in earnings for observationally equivalent agents, would complicate the model and the gains from doing so might not be significant. Because of this, I will simplify the usual decision about labor force participation, as is usually done in the neoclassical model of household behavior, where people decide whether or not

to work at a given wage determined by the market.

1.5 Economic Model

In this section I, describe the economic model used to rationalize investments in children together with household behavior. Each household (h) is composed of two agents (j), namely the father (f) and the mother (m). In each household, there is also a child with a level of skills denoted by (s), who is not a decision maker.[9] In each period t, parents make decisions of time investments in their children (e_t^j) and monetary investments for the child (I_t), private consumption (c_t^j) and labor market (h_t^j) decisions. I assume that the decision of labor market participation is made only at the extensive margin, that is, members decide whether or not to participate in the labor market: $h_t^j \in \{0, 1\}$. Additionally, during the first period, parents need to decide whether or not the child attends preschool (a_t) and then a_t can take the value of zero or one depending on whether the child goes to preschool.

There is a preference shock $\boldsymbol{\epsilon_t}$ associated with each decision about labor supply and preschool. Because there are two decisions about labor supply and two possible decisions about preschool, this shock is four-dimensional. In particular, the choice set for labor supply and childcare decisions is given by $D_t = \{(h_t, a_t) : h_t \in \{0, 1\}, a_t \in \{0, 1\}\}$. $q_t^{j,d}$ is an indicator function for individual j in period t taking the value of 1 if decision $d \in D_t$ is taken and 0 otherwise. I assume the preference

[9] This is a common assumption in the literature Del Boca, Flinn and Wiswall (2014); Bernal (2008) that seems reasonable given the little influence that children under six years of age can have on the resource allocation of the household.

shock follows a multivariate normal distribution with mean zero and variance Ω. The flow utility derived for each parent j in time t is given by the following utility function:

$$u_t^j(c_t^j, h_t^j, e_t^j, d_t^j, s_t) = \alpha_{1,t}^j \ln(c_t^j) + \alpha_{2,t}^j \ln(s_t) - \alpha_{3,t}^j(h_t^j) - (1 + h_t^j)\alpha_{4,t}^j e_t^j - \alpha_{5,t}^j h_t^j(1 - a_t) + \epsilon_{d,t}^j q_t^{j,d} \tag{1.5.1}$$

where $\epsilon_{d,t}^j$ is the d-th element of the vector $\boldsymbol{\epsilon_t}$. Additionally, I allow the cost of time investments in children $\alpha_{4,t}^j$ to change if there is an additional person helping with household chores such as cleaning the house, cooking or taking care of the child. Specifically, I set $\alpha_{4,t}^j = \alpha_{4,0,t}^j + \alpha_{4,1,t}^j HM_t$, where HM_t takes the value of one if there is a person helping with the household chores, and zero otherwise.

At period t, the skills of the child depend on monetary investments (I_t), time investments from both parents (e_t^j), preschool attendance (a_t), the skills of the mother (PG), which are constant over time[10], the previous level of skills (s_{t-1}) and the age of the child in months (τ_t). I allow for unobserved heterogeneity in the production of skills denoted by ($\eta_{s,t}$). The distribution of the unobserved heterogeneity term $f_{\eta_{s,t}}$ is gender-specific. The variable Members_t denotes the number of household members present in period t in the household. This captures the idea that, by

[10]There is evidence pointing to the fact that cognitive skills remain stable at around age 8 and non-cognitive skills are stable during adult life Borghans et al. (2008); Roberts et al. (2007). For this reason, assuming that skills of the mother are stable is not unreasonable.

having additional household members, not only might the production of skills be affected but also the productivity of each input. The production of skills is specified in the following equation:

$$s_t = r_t s_{t-1}^{\theta_0} \tilde{I}_t^{\theta_1} e_t^{\theta_2} \tag{1.5.2}$$

where r_t denotes the total factor productivity, specified as:

$$r_t = \underbrace{\exp\left(\delta_0 + \delta_1 \tau_t + \delta_2 a_t + \delta_{3,t} PG + \delta_4 \text{Members}_t + \eta_{s_t}\right)}_{\text{Total Factor Productivity}} \tag{1.5.3}$$

e_t is the total time effort invested in the child, given by the production function:

$$e_t = \underbrace{\left[\gamma_0 \left(\tilde{e}_t^f\right)^{\phi} + \gamma_1 \left(\tilde{e}_t^m\right)^{\phi}\right]^{1/\phi}}_{\text{Total time investment}} \tag{1.5.4}$$

where

$$\tilde{e}_t^j = e_t^j \exp\left(\eta_{e_t^j}\right) \tag{1.5.5}$$

and

$$\tilde{I}_t = I_t \exp\left(\eta_{I_t}\right) \tag{1.5.6}$$

The terms $\eta_{e_t^j}$ and η_{I_t} are unobserved heterogeneity. This term captures the fact that parents can differ in unobserved ways in how productive they are in terms of the time and monetary investments in their children. That is, even with the same amount of effort and monetary investment, the productivity of these inputs might be different across households. The terms $\eta_{I_t}, \eta_{e_t^j}$ and $\eta_{s_t^j}$ reflect complete information in the sense that parents make decisions knowing the productivity of their own inputs at every point in time.

1.5.1 Dynamic problem

I assume that parents need to make investment decisions for two periods. Each period lasts for two years and the first period starts when children are on average three years old. After the two periods, children enter a different stage in which parents and children face a different set of incentives in the process of skills production. Parents face a different set of incentives given that children start the formal schooling years and start behaving more as agents making their own decisions, which might have consequences for their own skills. For this reason, I only model childhood lasting for two periods: birth to age 3 and age 3 to age 5. This assumption is commonly made in the literature. Bernal (2008) assumes that early childhood relevant decisions are made until age 5. Del Boca, Flinn and Wiswall (2014) model household behavior until children are 16 years old but only use information on two periods to estimate their model, that is, when children are on average four and nine

years old.

$$V_2(\Psi_2) = \max_{\{I_2, \{c_2^j, e_2^j, c_2^j, h_2^j\}_{j=m,f}\}} \mu_2 u_2^f(c_2^f, h_2^f, e_2^f, d_2^f, s_2) + (1-\mu_2) u_2^m(c_2^m, h_2^m, e_2^m, d_2^m, s_2) \tag{1.5.7}$$

Ψ_2, which will be defined below, includes the state variables relevant to the decisions made in the second period. $\mu \in [\underline{\mu}, \bar{\mu}] \subseteq [0,1]$ represents the Pareto weight or bargaining power of the father. The solution for the problem of the household should satisfy the technological constraint given in 1.5.2, which is the time constraint for each agent:

$$h_2^j \in \{0,1\}, \text{ for } j = m, f \tag{1.5.8}$$

the non-negativity constraint:

$$c_2^f, c_2^m, I_2, e_2^f, e_2^m \geq 0 \tag{1.5.9}$$

and the budget constraint

$$c_2^f + c_2^m + P_{I,2} I_2 = Y_2^f + Y_2^m + w_2^m h_2^f + w_2^f h_2^f + \Xi_2 \tag{1.5.10}$$

where w_2^j represents the wage offer for individual j, Y^j is the corresponding non-labor income, and Ξ_2 is the total non-labor income that cannot be attributed to

any specific household member.[11] $P_{I,2}$ is the price of monetary investments in children for the second period. Note that in the second period parents don't make decisions regarding childcare attendance as virtually every child in the sample goes to preschool during the second period. sample goes to preschool during the second period.

The problem of the household during the first period is given by:

$$V_1(\Psi_1) = \max_{\{I_1,\{c_1^j,e_1^j,c_1^j,h_1^j\}_{j=m,f}\}} \mu_1 u_1^f(c_1^f, h_1^f, e_1^f, d_1^f, s_1) + (1-\mu_1)u_1^m(c_1^m, h_1^m, e_1^m, d_1^m, s_1) +$$

$$\beta V_2(\Psi_2) \tag{1.5.11}$$

subject to the skill production technology given in 1.5.2, the budget constraint:

$$c_1^f + c_1^m + P_{I,1}I_1 + P_a a = Y_1^f + Y_1^m + w_1^m h_1^f + w_1^f h_1^f + \Xi_1 \tag{1.5.12}$$

where P_a is the price of taking the child to preschool and a can take the value of zero or one depending on whether or not the child goes to preschool.

I assume that wages follow a Mincer equation:

$$\ln(w_t^j) = \beta_0^j + \beta_1^j yrschool_t^j + \beta_2^j age_t^j + \beta_3^j (age_t^j)^2 + \varepsilon_{t,w^j} \tag{1.5.13}$$

[11] Examples of elements included in the Ξ_2 term are subsidies for water consumption for the household.

where $\varepsilon_{t,w^j} \sim N(0, \varepsilon_{w_j})$ is measurement error.[12] Additionally, the relative importance of each household member will depend on characteristics of the household. In particular, I assume the following parametrization of μ_t:

$$\mu_t(E_t) = \frac{\exp(\Lambda' E_t + \eta_{\mu_t})}{1 + \exp(\Lambda' E_t + \eta_{\mu_t})} \tag{1.5.14}$$

where $\Lambda \in \mathbb{R}^L$ is a vector of coefficients; X are variables affecting the the relative bargaining power of each member in the household; and $\eta_{\mu,t}$ is unobserved heterogeneity. $\underline{\mu}$ and $\bar{\mu}$ are the lower and upper bounds for the Pareto weight.[13] In the E_t variables, I include the ratio of offered wages, the difference in ages between spouses, the difference in grades of schooling and the father's share in non-labor income. Additionally, I include conditions of the local labor market, which include the relationship between male and female unemployment, the sex ratio and the wage ratio in the region of residence of the household. Similar specifications to this

[12]Note that I am imposing a separate distribution for men and women. We could assume that all the correlation is yet given by assortative mating and is no necessary to assume a bivariate distribution in their wages. The only difference will be to estimate an additional parameter which will be the correlation between wage offers.

[13]The assumption that μ is bounded, given by $\mu \in \left[\underline{\mu}, \bar{\mu}\right] \subseteq [0, 1]$ is made without loss of generality.

one have been used previously in the literature.[14]

$$E_t = \Bigg[\frac{w_t^f}{w_t^m}, \frac{Y_t^f}{Y_t^f + Y_t^m}, age_t^f - age_t^m, yrschool_t^f - yrschool_t^m, \\ \frac{\bar{Female}_t}{\bar{Male}_t}, \frac{U^{Male_t}}{U^{Female_t}}, \frac{w^{Male_t}}{w^{Female_t}} \Bigg] \tag{1.5.15}$$

where $\bar{U}$ denotes the unemployment rate for each gender, $\frac{\bar{Female}_t}{\bar{Male}_t}$ is the sex ratio in the region of residence of the household, and $\frac{w^{Male_t}}{w^{Female_t}}$ is the wage ratio between women and men in the region of residence. These variables are what the literature refers to as distribution factors, variables that affect the behavior of the household only through its impact on the bargaining power. Descriptive statistics of these variables can be found in Table 1.15. The price of investments and the price of childcare depend on the availability of preschool services in the neighborhood through the following specification:

$$P_a = \text{Pchildcare}_{a,0} + \text{Pchildcare}_{a,1} DChildcare \tag{1.5.16}$$

$$P_I = \text{Price}_{I,0} - \text{Price}_{I,1} Dens \tag{1.5.17}$$

where $DChildcare$ is the distance to the nearest preschool provider and $Dens$ is the number of preschool providers within 5km of the household.

[14] Again, this determinant of bargaining power has been previously used in the literature Cherchye, De Rock and Vermeulen (2012), Bruins (2015) and Browning, Chiappori and Lewbel (2013).

The state variables are given by:

$$\Psi_t = \{r_t, s_{t-1}, \boldsymbol{\eta}, \boldsymbol{\epsilon_t}, \Xi_t, E_t, \{\epsilon^j_{d,t}, Y^j_t, w^j_t\}_{j=m,f}, P_a, P_I\} \tag{1.5.18}$$

where the vector $\boldsymbol{\eta}_t$ collects the unobserved heterogeneity:

$$\boldsymbol{\eta}_t = \{\eta_{I_t}, \eta_{e^f_t}, \eta_{e^m_t}, \eta_{\mu_t}, \eta_{s_t}\} \tag{1.5.19}$$

I assume that household members have perfect information regarding the terms related to unobserved variables at all moments. That is, in the first period they know the levels of their preference shocks and unobserved heterogeneity in the second period.

1.5.2 Model solution

Note that the model involves a set of discrete choices -childcare and labor supply- together with continuous decisions such as investment, effort and consumption. I solve this by first finding the optimal decisions about investment, consumption and effort, for each labor supply-childcare decision, and then choosing the discrete alternatives that derives the highest utility. Given the dynamic nature of the problem, I first solve for the second-period problem. The solution is given by:

$$e_2^{m,*} = \frac{\kappa_2^2(\mu_2)\theta_2\gamma_1}{(1-\mu)\alpha_{4,2}^m(1+h_2^m)}\xi_2(m)\exp\left(-\eta_{e_2^m}\right) \tag{1.5.20}$$

$$e_2^{f,*} = \frac{\kappa_2^2(\mu_2)\theta_2\gamma_0}{\mu\alpha_{4,2}^f(1+h_2^f)}\xi_2(f)\exp\left(-\eta_{e_2^f}\right) \tag{1.5.21}$$

$$I_2^* = \frac{\kappa_2^2(\mu_2)\theta_1\left(h_2^f w_2^f + h_2^m w_2^m + Y_2^f + Y_2^m + \Xi\right)}{\kappa_2^1(\mu_2) + \kappa_2^2(\mu_2)\theta_1 P_I}\exp\left(-\eta_{I_2}\right) \tag{1.5.22}$$

$$c_2^{f,*} = \max\{\frac{\alpha_{1,2}^f\mu_2 I_2}{\theta_1\kappa_2^2(\mu)}, \zeta\} \tag{1.5.23}$$

$$c_2^{m,*} = \max\{\frac{\alpha_{1,2}^f\mu_2 I_2}{\theta_1\kappa_2^2(\mu)}, \zeta\} \tag{1.5.24}$$

$$e_1^{m,*} = \frac{\left[\kappa_2^2(\mu_2)\theta_2 + \beta\kappa_2^2(\mu_2)\theta_2\theta_0\right]\gamma_1}{(1-\mu)\alpha_{4,2}^m(1+h_2^m)}\xi_1(m)\exp\left(-\eta_{e_1^m}\right) \tag{1.5.25}$$

$$e_1^{f,*} = \frac{\left[\kappa_1^2(\mu_1)\theta_2 + \beta\kappa_2^2(\mu_2)\theta_2\theta_0\right]\gamma_0}{\mu\alpha_{4,2}^f(1+h_2^f)}\xi_1(f)\exp\left(-\eta_{e_1^f}\right) \tag{1.5.26}$$

$$I_1^* = \frac{\left[\kappa_1^2(\mu_1)\theta_1 + \kappa_2^2(\mu_2)\theta_0\theta_1\beta\right]\left(h_2^f w_2^f + h_2^m w_2^m + Y_2^f + Y_2^m + \Xi - P_a a\right)}{\kappa_1^1(\mu_1) + \kappa_1^2(\mu_1)\theta_1 + \beta\theta_0\theta_1\kappa_2^1(\mu_2)}\exp\left(-\eta_{I_1}\right) \tag{1.5.27}$$

$$c_1^{f,*} = \max\{\frac{\alpha_{1,2}^f\mu_2 I_2}{\theta_1\kappa_1^2(\mu_1) + \beta\theta_0\theta_1\kappa_2^2(\mu_2)}, \zeta\} \tag{1.5.28}$$

$$c_1^{m,*} = \max\{\frac{\alpha_{1,2}^f\mu_2 I_2}{\theta_1\kappa_1^2(\mu_1) + \beta\theta_0\theta_1\kappa_2^2(\mu_2)}, \zeta\} \tag{1.5.29}$$

where

$$\xi_t(j) = \frac{\left(\gamma_j \mu \alpha_{4,t}^f (1 + h_t^f)\right)^{\frac{\phi}{1-\phi}}}{\gamma_0 \left[\gamma_0 (1-\mu)\alpha_{4,t}^m (1 + h_t^m)\right]^{\frac{\phi}{1-\phi}} + \gamma_1 \left[\gamma_1 \mu \alpha_{4,t}^f (1 + h_t^f)\right]^{\frac{\phi}{1-\phi}}} \tag{1.5.30}$$

$$\kappa_t^i(\mu) = \mu \alpha_{i,t}^f + (1-\mu)\alpha_{i,t}^m \tag{1.5.31}$$

$$\zeta = 1.0e-5 \tag{1.5.32}$$

and

$$\gamma_j = \begin{cases} \gamma_0 \text{ if } j = f \\ \\ \gamma_1 \text{ if } j = m \end{cases} \tag{1.5.33}$$

The optimal decisions of labor supply and childcare are given by:

$$(h_2^{f,*}, h_2^{m,*}) = \max_{\{h_2^f, h_2^m\}} \mu_2 u_2^f(c_2^{f,*}(h_2^f, h_2^m), h_2^f, e_2^{f,*}(h_2^f, h_2^m),$$

$$d_2^f(h_2^f, h_2^m), s_2(h_2^f, h_2^m)) +$$

$$(1-\mu_2) u_2^m(c_2^m(h_2^f, h_2^m), h_2^m(h_2^f, h_2^m), e_2^{m,*}(h_2^f, h_2^m), d_2^m(h_2^f, h_2^m),$$

$$s_2(h_2^f, h_2^m)) \tag{1.5.34}$$

$$(h_1^{f,*}, h_1^{m,*}, a) = \max_{\{h_1^f, h_1^m, a\}} \mu_1 u_1^f(c_1^{f,*}(h_1^f, h_1^m, a), h_1^f, e_1^{f,*}(h_1^f, h_1^m, a)),$$

$$d_1^f(h_1^f, h_1^m, a), s_1(h_1^f, h_1^m, a)) +$$

$$(1-\mu_1) u_1^m c_1^m(h_1^f, h_1^m, a), h_1^m(h_1^f, h_1^m, a), e_1^{m,*}(h_1^f, h_1^m, a),$$

$$d_1^m(h_1^f, h_1^m, a), s_1(h_1^f, h_1^m, a))$$

$$+ \beta \left[V_2(\Psi_2(h_1^f, h_1^m, a) \right] \tag{1.5.35}$$

1.6 Estimation

The main challenge in the estimation of this model is that we do not observe the main features of the model in the dataset. Rather, we observe measures about the relevant factors of the model that are contaminated by measurement error.

Specifically, I define the set K to include the latent variables in the model:

$$K = \{\{\ln(s_t), \ln(e_t^{f,*}), \ln(e_t^{m,*}), \ln(I_t^*), \mu\}_{t=1,2}, \ln(PG), \ln(s_0)\} \tag{1.6.1}$$

Rather than observing them directly, we have a set of measures that give some information about the true latent level of each variable. Such relationships between the measures and the latent factors can be described in the following system:

$$Z_m^k = \iota_{m,0}^k + \iota_{m,1}^k k + \varepsilon_m^k \text{ for } m = 1...N_k \tag{1.6.2}$$

where Z_m^k denotes the measure m for the latent variable k and N_k denotes the number of measures available for the latent factor k. The variables used as measurements for each factor are described in Tables 1.4 - 1.11. I assume the ε_m^k are uncorrelated across observations and follow a distribution $\mathcal{N}(0, \sigma_{km})$. However, as will be shown later, this assumption is not necessary for identification.

Given the structure of the model, there is a well-defined likelihood function denoted by:

$$P(O|X;\Theta) = \mathcal{L}(\Theta|O;X) \tag{1.6.3}$$

where (O) denotes the observed outcomes in the three periods: $O = \{O_0, O_1, O_2\}$ and X is the set of exogenous characteristics in the model. The set of outcomes for

the period 0 are composed exclusively of the measures of the primary caregiver's skills and birth outcomes. The set of observed outcomes for the first and second period are the measures corresponding to the specified factors in addition to the labor supply decision and the observed wages. Formally:

$$O_0 = \{\{z_m^{PG}\}_{m=1}^{N_{PG}}, \{z_m^{S_0}\}_{m=1}^{N_{S_0}}\}$$

for t=1,2:

$$O_t = \{h_t^f, h_t^m, a_t, \mathcal{Z}_t\} \cup \underbrace{\{w_t^f\}}_{\text{if } h_t^f>0} \cup \underbrace{\{w_t^m\}}_{\text{if } h_t^m>0}$$

$$\mathcal{Z}_1 = \{\ln(s_1), \ln(\hat{e}_1^f), \ln(\hat{e}_1^m), \ln(\hat{I}_1)\}$$

$$\mathcal{Z}_2 = \{\ln(s_2), \ln(\hat{e}_2^f), \ln(\hat{e}_2^m), \ln(\hat{I}_2), \mu_2\} \tag{1.6.4}$$

Note that I have measures of μ_2 available only for the second period. The exogenous characteristics are given by the age, grades of schooling, age of parents and distribution factors in E_t.

Given that we need to integrate over the the distribution of the unobserved factors (because they are not observed), the expression of the likelihood function becomes a high-dimensional integral with no closed form solution. The natural approach to estimate such likelihood is to approximate the integral via Monte-Carlo

methods - that is, drawing shocks from the distribution of the unobserved factors, estimating the likelihood and averaging over these draws. However, note that the time-dependency arising in the production of skills generates an additional difficulty for this approach, because, for each draw in period 0, we would have to generate multiple draws in the first period and for each draw in the first period we would have to draw multiple draws in the second period. The curse of dimensionality makes it infeasible to estimate this likelihood with the usual simulation techniques.

A pure simulation strategy to estimate the model would be computationally infeasible. We use particle filtering techniques in order to be able to estimate the model via simulated methods. The full description of the estimation technique and the derivation of the likelihood function are described in Appendix A.0.2.

For purposes of estimation, I assume that the preference shocks $\boldsymbol{\epsilon_t}$ are distributed according to a normal distribution with no correlation between choices. The unobserved heterogeneity terms, $\eta_{e_t^j}$ also follow a normal distribution. Although I do not allow for correlation between these shocks, I do allow for correlation between the underlying factors in the model (e.g., Pareto weight and skills of mother). The assumption about normality in these terms is not an identifying assumption; as I describe in the next section that I can obtain non-parametric identification of such distribution under some independence conditions. The same applies to the error terms in the measurement system of Equation 1.6.2. I assume they are distributed according to a normal distribution and that they are independent of each other but

this is not an identifying assumption.

The sample used for the estimation of the model includes only families with children, in which both parents live together and where the child has no siblings within a five-year age range. Moreover, given that I use test scores and measures of health at birth in order to estimate the production of skills, I drop from the sample families that did not complete such questionnaires. The description of how the sample is selected is in Table 1.21. The sample considered for the analysis consists of 950 families. Some descriptive statistics of the sample used, for the 2012 wave, are included in Table 1.22 and some details about the age distribution of the children included, for the 2012 wave, are included in Table 1.23. The preliminary evidence section uses all the information available in the survey. However, the results from the preliminary section also hold when using the sample used for the model. These results are available in the online appendix.

1.6.1 Identification

The identification argument is divided into three parts. First, I show how the parameters of the measurement system described in 1.6.2 are identified. Secondly, I show what variation in the data allows us to recover the distribution of the latent factors. Finally, I show how the parameters of the economic model are recovered.

Measurement System

The general measurement system in a factor model can be written as:

$$Z = \iota_0 + \iota_1 K + \boldsymbol{\varepsilon} \tag{1.6.5}$$

where $Z \in \mathbb{R}^M$ contains all the measures available, M is the total number of measurements for all the factors, $K \in \mathbb{R}^{11}$ is the vector of 11 factors and $\boldsymbol{\varepsilon} \in \mathbb{R}^M$ is measurement error. $\iota_1 \in \mathbb{R}^{M \times 11}$ is the matrix of factor loadings. As is common in factor analysis, a location and scale normalizations are necessary to ensure identification of the system. The first step is to normalize the first element of ι_1 for each measure to one, which corresponds to setting $\iota_{1,1}^k = 1$ for every factor $k \in K$ in Equation 1.6.2. The location normalization corresponds to setting the mean of each factor to a specified level. The arbitrary scale is set to be:

$$\begin{aligned} \mathbb{E}[\ln(s_t)] =& \mathbb{E}[\ln(PG)] = 0 \text{ for } t = 0, 1, 2 \\ \mathbb{E}[\mu] =& 0.5 \end{aligned} \tag{1.6.6}$$

I also set normalizations for effort levels and investments, which I will explain in full detail in Section 1.6.1. This normalization is irrelevant given that we can re-define new measures $Z - \iota_0$ and the analysis will remain unchanged. From the observed

measures Z, I can obtain the covariances by noting that:

$$\Sigma_Z = \iota_1 \Sigma_K \iota_1' + \Sigma_\varepsilon \tag{1.6.7}$$

where Σ_x is the variance covariance-matrix of x. Note that we have $M \times (M+1)/2$ moments in order to identify $M \times 11$ factor loadings, $11 \times (11+1)/2$ elements in Σ_k and $M \times (M+1)/2$ elements in Σ_ε. As is often the case in factor analysis, it is necessary to make further assumptions in order to identify the relevant parameters of the model. The normalization $\iota^k_{1,1} = 1$ implies that the number of factor loadings to estimate becomes $M - 11$.

I still need to make further assumptions to recover all the relevant parameters. By making the assumption that the measurement error of the skills at birth is independent of the measurement error of the measures corresponding to the remaining factors, I have enough moments to identify all the parameters. Formally, the assumption is given by $\varepsilon^{\ln(s_0)}_m \perp\!\!\!\perp \varepsilon^{k'}_{m_1}$ for $m = 1...N_{ln(s_0)}$, $k \neq \ln(s_0)$, $m' = 1...N_k$. The details of why this is enough to identify the parameters in the measurement system are described in Appendix A.0.1.

I can recover ι^k_m for $k \neq \ln(s_0)$ by noting that:

$$\frac{Cov(Z^k_m, Z^{\ln(s_0)}_1)}{Cov(Z^k_1, Z^{\ln(s_0)}_1)} = \iota^k_{m,1} \tag{1.6.8}$$

and the factor loadings of $\ln(s_0)$ are obtained simply by changing the roles of k by $\ln(s_0)$:

$$\frac{Cov(Z_m^{\ln(s_0)}, Z_1^k)}{Cov(Z_1^{\ln(s_0)}, Z_1^k)} = \iota_{m,1}^{\ln(s_0)} \tag{1.6.9}$$

Distribution of latent factors

Once the identification of the factor loadings is ensured, we can non-parametrically estimate the distribution of the latent factors using a version of the Kotlarsky Theorem. Define:

$$ME_j = \{\frac{Z_j^k}{\iota_{j,1}^k}\}_{k \in K} \tag{1.6.10}$$

$$me_i = \{\frac{\varepsilon_j^k}{\iota_{j,1}^k}\}_{k \in K} \tag{1.6.11}$$

as long as, for at least two measures $j = 1, 2$, the following holds:

$$\mathbb{E}\left[me_1 | K, me_2\right] = 0 \tag{1.6.12}$$

$$me_2 \perp\!\!\!\perp \theta \tag{1.6.13}$$

Theorem 1 in Schennach (2004) provides a non-parametric estimator for the joint density of the latent factors. The theorem notes that the distribution of factors can be expressed as a function of the Fourier transformation of the distribution of

measures under the aforementioned assumptions:

$$p(K) = \frac{\int_{-\infty}^{\infty} e^{-i\chi K} e^{\left(\int_0^{\chi} \frac{E\left[iME_1 e^{i\psi ME_2}\right]}{\left[e^{i\psi ME_2}\right]} d\psi\right)} d\chi}{2\pi} \tag{1.6.14}$$

Once the distribution $p(K)$ has been identified, we can recover the second-order moments $Cov(k, k')$ for any $k, k' \in K$. Once we recover the second-order moments, we can identify the remaining elements of Σ_ε from the system of equations:

$$Cov(Z_m^l, Z_{m'}^{k'}) = \iota_{m,1}^k \iota_{m',1}^{k'} Cov(k, k') + Cov(\varepsilon_m^k, \varepsilon_{m'}^{k'}) \tag{1.6.15}$$

Technology of Skill Formation

Because we have ensured identification of $p(K)$, we can recover the conditional distribution:

$$p\left(\ln(s_{t+1})|\ln(s_t), \ln(\tilde{e}_{t+1}^f), \ln(\tilde{e}_{t+1}^m), \ln(\tilde{I}_{t+1}), \mu, \ln(PG)\right) \tag{1.6.16}$$

from $p(K)$ for $t = 0, 1$. We can define the following function:

$$s_{t+1} = f_s\left(s_t, \tilde{e}_t^f, \tilde{e}_t^m, \tilde{I}_t^m\right) =$$

$$\mathbb{E}\left[\exp\left(\ln(s_{t+1})|\ln(s_t), \ln(\tilde{e}_{t+1}^f), \ln(\tilde{e}_{t+1}^m), \ln(\tilde{I}_{t+1}), \mu, \ln(PG)\right)\right] \tag{1.6.17}$$

where the expectation is taken with respect to the distribution in 1.6.16. However, note that we are interested in a function s_{t+1} that has as an additional argument the term η_{s_t} corresponding to heterogeneity. Matzkin (2007) has negative identification results in this case and shows that, in order to be able to non-parametrically identify the function in which we are interested, we need to impose some restrictions. In particular, if we assume that the term η_{s_t} enters additively in 1.6.17, I can trivially identify the production of skills. Additionally, the distribution of η_s is identified as:

$$
\begin{aligned}
&F_{\left(s_{t+1}|\ln(s_t),\ln(\tilde{e}_t^f),\ln(\tilde{e}_t^m),\ln(\tilde{I}_t^m)\right)}\left(\tilde{s}_{t+1}|\ln(s_t),\ln(\tilde{e}_t^f),\ln(\tilde{e}_t^m),\ln(\tilde{I}_t^m)\right) = \\
&P\left(s_{t+1}\leq\tilde{s}_{t+1}|\ln(s_t),\ln(\tilde{e}_t^f),\ln(\tilde{e}_t^m),\ln(\tilde{I}_t^m)\right) = \\
&P\left(f_s\left(s_t,\tilde{e}_t^f,\tilde{e}_t^m,\tilde{I}_t^m\right)+\eta_{s,t}\leq\tilde{s}_{t+1}|\ln(s_t),\ln(\tilde{e}_t^f),\ln(\tilde{e}_t^m),\ln(\tilde{I}_t^m)\right) = \\
&P\left(\eta_{s,t}\leq\tilde{s}_{t+1}-f_s\left(s_t,\tilde{e}_t^f,\tilde{e}_t^m,\tilde{I}_t^m\right)|\ln(s_t),\ln(\tilde{e}_t^f),\ln(\tilde{e}_t^m),\ln(\tilde{I}_t^m)\right) \qquad (1.6.18)
\end{aligned}
$$

and thus we can identify the cdf of $\eta_{s,t}$ conditional on factors other than s_{t+1}. With similar arguments we can identify the distribution of the remaining factors.

Preferences

The parameters of the economic model are identified by a combination of exclusion restrictions, exogenous sources of variations and functional form specifications. The main argument used to identify preferences of fathers and mothers follows standard

procedures from the literature on collective models of household behavior Chiappori and Donni (2009). The use of distribution factors -variables that affect the behavior of the household but do not modify household behavior in any other way- allows us identify preferences of mothers and fathers. The main idea is that variation in such instruments will cause a movement along the Pareto frontier that is exclusively generated by the change in bargaining power. The distribution factors used in this article have been previously used in the literature Cherchye, De Rock and Vermeulen (2012); Attanasio and Lechene (2014); Blundell, Chiappori and Meghir (2005).

First, I describe identification of the Pareto weight function specified in Equation 1.5.14 because, through this function, we can separately identify preferences of fathers and mothers. To identify parameters in Λ, I use exogenous variation in the gender wage gap, the unemployment gender gap and the sex ratio. The key assumption is that we have enough variation in the data for these factors, and variation is given in a way that is exogenous to the household. In Table 1.15 I report the descriptive statistics of the distribution factors, where we see that there is some variability that is used to secure the identification of the model. Additionally, I impose the exclusion restriction that differences in ages and schooling do not affect the behavior of the household other than in the Pareto weight. Finally, we need to have exogenous variation in the share of non-labor income earned by the man to secure identification of all the parameters in Equation 1.5.14. I describe how I get

such variation in the following paragraph.

The way in which the Chilean social security system schedules monetary transfers to households generates variation in the proportion of income earned by men in the household. The "Social Protection Card"[15] assigns a score to each household corresponding to its socioeconomic status. This score is used as the main targeting device through which monetary transfers are assigned to households, and all subsidies are given to mothers of children whenever there is a child in the household. The amount of the subsidy depends on an additional set of characteristics of the households, such as the number of children under 18 living in the household. There are seven different programs giving monetary transfers to families in Chile, but the basic ones correspond to the "Unique Family Subsidies" and "Family Assignments". Under these programs, a mother who earns less than $187,515 CLP and has a score under 11.734 on the Social Protection Card, is eligible to receive a transfer of $7,179 CLP per month, for each child under 18 and for herself. Additionally, families with a lower score on the Social Protection Card are eligible for subsidies, all received by the mother, depending on their score, the months they have currently been beneficiaries of the programs and the demographic composition of the household.

The discontinuities in the monetary transfer programs, as well as the variation in elements such as the number of members in the household, gives me variation in the proportion of non-labor income in the hands of women. Using variation in responses to the female empowerment and gender roles questionnaires, we can identify

[15] "Ficha de Proteccion social" in Spanish

the extent to which non-labor income affects the process of decision-making within the household. The structure of the basic monetary transfers in Chile is reported in Figure 1.7. A description of how the monetary subsidies scheduling system has evolved over time is available in the Appendix in Section A.0.6.

At this point, it is important to normalize the remaining factors that were not normalized in Section 1.6.1. Effort and investment units do not have natural units. I impose the following normalizations:

$$\mathbb{E}\left[e_t^{f,*} \mid \mu = 0.5, h^f = 1\right] = 1 \tag{1.6.19}$$

$$\mathbb{E}\left[I_t^* \mid \mu = 0.5, d = 10\right] = 1 \tag{1.6.20}$$

The average effort of fathers in families with a Pareto weight of 0.5 and who participate in the labor market is normalized to one. Similarly, the average investments for families who have a Pareto weight of 0.5 and who have 10 childcare providers within 5 kilometers is normalized to one. Once this normalization is done, we can identify sources of variation in the data that allow me to identify the key parameters.

Because I see variation in effort levels in both, fathers and mothers, due to changes in distribution factors, this allow me to identify preferences for children of both parents. For instance, variation in distribution factors might increase the bargaining power of the mother. If we see that effort levels increase as a consequence of

the variation in the distribution factors, this gives us information about the relative preferences for children between fathers and mothers. Similarly, changes in investments due to changes in distribution factors allow me to identify the preferences for consumption of mothers and fathers.

Identification of the remaining parameters follows standard arguments in the literature. For wages, as long as we have enough variation in education and age, we can identify the β coefficients. Similarly, the price elasticity of investments, with respect to the availability of preschool providers $P_{I,1}$, is identified as long as we have variation in the number of preschool providers within five kilometers of households. In Figure 1.1, I show that there is significant variation in the data regarding this variable. The fact that Chile saw a massive expansion in the number of providers between 2006-2010 gives us significant variation in the data, as the system increased its capacity, measured in the number of children that the system can provide services for, by 450%. Following the normalization in Equation 1.6.20, and with the corresponding variation in childcare providers, we can identify the parameters $P_{I,1}, P_{I,0}$. Similar arguments are used to identify price of childcare.

1.7 Results

The results of the parameters estimated, together with the corresponding standard errors, are presented in Tables 1.24 - 1.30. As we see, childcare services tend to liberate more time resources for mothers than for fathers. In the same regard, having

one additional member in the household decreases the cost of time investments more for mothers than for fathers. We observe that mothers have stronger preferences for children and that fathers find it more costly to spend time with their child than mothers do. Having an additional person in the household helping with childcare or with household chores decreases the utility penalty of investing time in children, more for mothers than for fathers.

Regarding the estimates of the production of skills, we see some evidence of differences in the productivity of time investments of mothers and fathers. It is not possible to make comparisons between the productivities of different inputs because they are measured in different units (except father's and mother's effort). Nonetheless, we see that monetary investments, childcare attendance, skills of primary caretaker and having adequate birth conditions all seem to have positive effects on the skills of a child. We also observe that availability of childcare services decreases both the price of childcare and the price of monetary investments in children. This coefficients are estimated with high precision.

Looking at the estimates of the determinants of the Pareto weight, we see there is an effect of the wage ratio on the Pareto weight. This is important because the relationship holds even when we control for differences in education, age and in non-labor income. We observe that, as the age gap between the man and woman decreases, the bargaining power of the man decreases as well. Interestingly, we find a negative relationship between gender ratio, unemployment ratio and wage ratio

at the province level and the man's bargaining power.

Regarding the measurement system, we can compute the extent to which each measure contributes to the signal extraction problem. Every measure is contaminated by measurement error. With the estimation results I am able to extract the proportion of the variance due to true signal and the proportion due to noise.

$$\text{Signal-noise ratio}_{m,k} = \frac{\iota_{m,1}^2 Var(k)}{\iota_{m,1}^2 Var(k) + Var(\varepsilon_m^k)} \tag{1.7.1}$$

In Figures 1.14 - 1.15, I present the signal to noise ratio of the measurement system of the model for measures of effort and investments.[16] We find that cultural activities are the most informative about time investment in children, while sharing a meal or performing household chores are within the group of less informative activities. We should be careful with the interpretation of these results: it does not mean that cultural activities are the most productive ones but rather they are the most informative ones. It can certainly be the case that there is an underlying activity that is not reported in the dataset that is performed more often by those parents who perform cultural activities and that such an activity is the one that is really productive, rather than performance of cultural activities itself. Making inferences about which activities are more productive requires more analysis.

[16]The signal to noise ratio of all the remaining measures for all the latent factors is available in the online appendix. The estimates of the factor loadings are also available in the online appendix.

1.7.1 Model fit

The model does a good job when predicting labor force participation and childcare decisions of the household. In Tables 1.33 and 1.34, I report the means of labor force participation for both mothers and fathers in 2010 and 2012. The model does a good job in predicting the average levels of participation. Moreover, in Figure 1.8, I compare the predicted and observed levels of female employment by grade of schooling attained. I predict the labor force participation when the terms corresponding to unobserved heterogeneity are located at their mean. The model is able to replicate the gradient in female labor force participation related to education. More educated women participate more in the labor market both in the data and in the simulated results of the model. No significant gradient between education and male labor force is observed in either the model or the data.

I report the predicted levels of childcare demand and how they compare with what is observed in the data in Table 1.35. The model does a good job at predicting the demand for childcare services according to female labor force participation. 67.7% of children living in families where the mother works attend preschool services, whereas the corresponding number for children living in families where the mother does not participate in the labor market is 42.9%. The corresponding proportions predicted in the model are 68.4% and 41.6%.

The simulated patterns from the model are generated assuming unobserved heterogeneity variables are at their means. An alternative way of reporting the model

fit is to generate draws from their distributions and report the corresponding distribution of model fit. I report the results of such model fit alternative using 200 draws in Figures 1.10 - 1.11. As we can see, in both cases the model fits the data well. Finally, the model does a good job at predicting the wages for men and women. In Figure 1.12, I report the estimated distribution of wages for women and men, both those predicted and those observed in the data. I report only the estimated wages for agents who participate in the labor market. The model does a good job of predicting not only the average wage but also the distribution.

With the information about measures and the information about the production of skills, we can get a more precise estimate of the distribution of skills for each individual. The estimated smoothing distribution of skills, which uses all information available in order to make inference about the skills of each individual in the sample, is estimated and the results are reported in Figure 1.13. The details for the construction of the smoothing distribution are presented in Appendix A.0.5. The results confirm huge disparities in skills between rich and poor kids.

1.7.2 Evaluating the Effects of Government Programs on the Skills of Young Children

In this section, I describe the effects that different policy programs would have on the skills of young children. Additionally, I consider the effects of such policies on female labor force participation and preschool attendance. The policies considered

are: 1. increasing the amount of monetary transfers that poor households receive from the central government in the form of subsidies; 2. same as 1. but having the father, rather than the mother, as the recipient of such transfers; 3. setting up a system of free childcare services for children older than three and; 4. using the resources of the first policy counterfactual in order to perform in-kind transfers where poor families receive goods that can be used to enhance the skills of young children, such as books, toys and puzzles.

Cash transfers are a widely-used program in developing countries. Every country in Latin America has a form of cash transfer that varies by the amount given to the households and the type of conditions that families need to fulfill in order to be beneficiaries (Fiszbein, Schady and Ferreira, 2009). Policymakers often invoke the effect of such programs on the promotion of skills of young children as one of the many benefits of these policies. Moreover, the vast majority of these programs establish that, for families with children, the mother should always be the beneficiary. The main argument for this is that cash in the hands of women is associated with better child outcomes than cash in the hands of men (Doepke and Tertilt, 2014).

Given the high use of cash transfers as a policy tool in developing countries, and given the explicit condition that transfers go to mothers rather than fathers or other adult members, the first counterfactual policy that I consider is to increase the amount of cash transfers given to mothers of young children. Since 2010, the value of transfers that poor families with children receive has increased significantly.

Between 2012 and 2016, families in the lowest quintile of the income distribution have seen an increase of 72.8%, in real terms, in the cash transfers that they receive from the central government. The details of these programs and how such increase was distributed among various policies are described in Appendix A.0.6. Given that governments seem to increasingly spend more resources in these type of policies, the first counterfactual simulated in this paper consists on doubling the amount of monetary transfers that families located in the lowest quintile of the income distribution receive. Such a policy would imply a transfer equivalent to 18% of the average income for families in the the lowest quintile, which corresponds to $23,056 CLP a month.

The Chilean government states explicitly that mothers should be the recipients of such transfers. In order to identify the extent to which this condition is justified, and to get an idea of whether it makes sense to spend additional resources in targeting an individual household member as the recipient of such transfers, in the second counterfactual I simulate what would happen if we set the father, rather than the mother, as the recipient of the transfers.

Free childcare and preschool policies have also been very popular not only as a way to promote skills in young children but also as a tool to promote female employment. In 2013, the government of Chile established free and mandatory preschool services for children older than five years of age. Partly due to this policy, Chile is now the country with the highest expenditure on preschool education as

a share of total government expenditure, among countries in the OECD. [17] Due to the increasing importance of such public policies, in the third counterfactual I simulate the effects of setting up free childcare services for families located in the lowest quintile of the income distribution.

Finally, in the fourth counterfactual I simulate the effects of a system of in-kind transfers where the families receive goods that can potentially increase skills in young children. Although probably less prevalent than childcare subsidies or cash transfers, in-kind transfer programs are starting to become more popular in developing countries. In Chile, for example, such transfers are being done through the "Chile Crece Contigo"[18] (ChCC) program, established in 2009. ChCC is composed of a set of services for poor families with children younger than five years of age. The goal of the program is to guarantee that every child has the necessary resources so that they can achieve their full developmental potential during childhood. The program offers resources to parents such as a 24-hour phone line for inquiries about child development, and the distribution of books, toys, songs and story books for children, as well as providing learning materials to parents in order to increase their knowledge about child development. ChCC is the most important child development public program currently operating in Chile. Due to its growing importance, I simulate the effect of extending one of ChCC's benefits: that of transfers of goods to improve children's skills, such as toys appropriate for cognitive stimulation, as

[17] Out of the total government expenditures, 2.3% go to the preschool system compared to the average of other OECD countries, which is 1.1% (Chile, 2013).

[18] Chile Grows with You, in Spanish

well as musical material to increase their vocabulary. In the fourth counterfactual, I analyze the effects of spending the same amount of resources as in counterfactual 1 -i.e. $$23,056 CLP a month per family- for families in the lowest income distribution, but doing so as in-kind transfer.

The effects of such policies on the gaps in skills between children in the highest quintile of the income distribution and children in the lowest quintile can be found in Figure 1.16. Initially the gaps in skills between rich and poor children are estimated at approximately of 60% of a standard deviation. We see that in-kind transfer is the most effective policy, decreasing the gap by 8%. Cash transfers and childcare subsidies decrease this gap in approximately 2%. There are no differences between cash in the hands of women and cash in the hands of men, as these two policies have virtually the same effect.

Cash in the hands of women, however, increases their bagaining power so that women have a stronger say in the household. This can be seen as part of the estimation results of the Pareto weight function reported in Table 1.32. Additionally, women have stronger preferences for children. However, the two effects combined -the increase in their bargaining power and having stronger preferences for children- are not strong enough to justify that it actually makes a difference to target specific members in the household as the sole recipients of monetary transfers from the central government.

The effects of the policies being implemented are decomposed in Tables 1.36-

1.39. Both cash transfers and childcare subsidies have an effect on employment levels. Cash transfer decreases both female and male labor force participation by less than one percentage point. Childcare subsidies have an effect only in female employment, which is due to the fact that preschool services decreases the penalty of participating in the labor market more for mothers than for fathers. Regarding monetary investments in children, cash transfers and childcare subsidies do not significantly affect this variable. Childcare subsidies increase it for two reasons. First parents spend less resources on preschool fees. Additionally mothers participate slightly more in the labor market, increasing the amount of resources available for child investments. However, in-kind transfers have by far the largest effect on monetary investments. This particular mechanism explains most of the reason why in-kind transfer are most effective when it comes to decreasing gaps in skills between rich and poor children.

The fact that cash transfers are not very effective at closing the gap in skills between rich and poor children is consistent with the results from the literature. As pointed out by Heckman and Mosso (2014), evidence seems to suggest very limited effect of cash transfers on skills of disadvantaged children. Paxson and Schady (2010) evaluate a cash transfer program in Ecuador using a strategy of random assignment to the treatment. They find that such transfers had no effect on cognitive development for children, except for the poorest, where a modest effect is found. However, the authors suggest that the mechanism driving this effect might

be through improvement in nutrition and health outcomes. Such a mechanism is unlikely to operate in Chile, where the incidence of stunting and wasting in children is below 1%, whereas in the sample used by Paxson and Schady (2010), the corresponding proportions are 10% for stunting and 23% for wasting. Macours, Schady and Vakis (2012) find a positive effect of a cash transfer program in Nicaragua. However, the mechanisms suggested by the authors include improvement in nutritional status, which might not necessarily operate in Chile for the aforementioned reasons, in addition to the fact that the cash transfer program included educational activities for parents that might modify their behavior. In summary, cash transfers by themselves seem to have modest effects on children's skills.

With regard to implementation of the policies, I find that childcare subsidies are cheaper than the other policies. Providing free childcare service to families is cheaper than implementing the increase in monetary or in-kind transfers to families. The information about the cost of each policy is explicitly described in Table 1.40. Although it is cheaper to provide childcare subsidies, at the same time, I assume that there are no general equilibrium effects as a result of the increase in demand for child care, which might generate an increase in price. Moreover, I am implicitly assuming that the available infrastructure is enough to absorb the increment in the demand. However, when we set the amount devoted to each program to be the same as the cost of providing free childcare services, the ranking in the performance of each policy is preserved. The effects of performing the same counterfactuals with

the same amount of expenditure for each policy are reported in Figure 1.17.

1.7.3 Child Investments and Bargaining Power

As shown in the preliminary evidence, women spend more time with their children even when controlling for labor supply. This, together with the evidence that cash in the hands of women translates into better child outcomes than cash in the hands of men, is often used as evidence that women have stronger preferences for children and thus monetary transfers should be given to women if the objective is to invest more in children. Nonetheless, this evidence is explained by several other factors.

First of all, mothers' time seems to be more productive than fathers' time, as shown by the estimation results of the model. Additionally, mothers have stronger preferences for children and the utility penalty of time investments is lower for mothers than for fathers. However, in addition to these facts, the relative empowerment of each member distorts time decisions, which explains part of the differences in time investments. Given that both parents are making investments in a public good (skills of their child) and that effort is costly and privately exerted, the fact that women spend more time with children is also a consequence of their relative disempowerment in the household rather than simply a result of different preferences.[19]

[19]Doepke and Tertilt (2014) develop a non-cooperative model of household behavior to answer the question of how female empowerment might promote economic development. The authors argue that the reason to develop a non-cooperative model of household behavior lies in the fact that the only mechanism capable of generating differences in investments in children in a collective approach would be that of preferences. However, in this paper I present a collective model of household behavior where differences in investment can arise for a variety of reasons other than

The allocation of time investments is a result of maximizing the household's welfare, which includes the skills of children, taking into account the utility penalty of time investments. The time cost of each member is not equally weighted, it depends on the relative empowerment of each household member. If the mother is relatively less empowered, the cost of her time is lower than that of the father. This difference in empowerment levels distorts the cost of providing effort and implies inefficiencies in the allocation of resources for children. Put it differently, with the same amount of total effort being provided, we can find an alternative allocation of time investments that would make the child better off.

Consider the centralized problem of choosing the effort levels for the second period in order to maximize the skills of children -taking all other inputs as fixed- subject to the fact that the total amount of effort exerted should not exceed the total amount of effort found in the problem of the household described in 1.5.20-1.5.21. We are basically asking whether or not it is possible to find an alternative allocation of time that would make children better off, whitout modifying the total amount of effort exerted by both parents. The problem is formally defined as:

$$\max_{e^f, e^m} s_2(e^f, e^m, .) \text{ subject to } e^f + e^m = e^{f,*} + e^{m,*} \tag{1.7.2}$$

where $e^{j,*}$ is the optimal solution to the maximization of the household welfare problem described in Equation 1.5.7. Define the solution to the problem in 1.7.2 as

preferences.

$\left(e^{f,c_1}, e^{m,c_1}\right)$.

Similarly, we can define an alternate centralized problem where we maximize skills subject to the fact that the total time-cost exerted in the production of skills should not exceed that found in the household's problem defined in 1.5.1-1.5.19. Formally:

$$\max_{e^f, e^m} s_2(e^f, e^m, .) \text{ subject to } c\left(e^f\right) + c\left(e^m\right) = c\left(e^{f,*}\right) + c\left(e^{m,*}\right) \tag{1.7.3}$$

where the cost of effort is given by $c^j(e^j) = \alpha^j_{4,2} e^j(1 + h^j)$. I call the solution to 1.7.3 $\left(e^{f,c_2}, e^{m,c_2}\right)$. In both cases, for $l = 1, 2$, we do find that:

$$\frac{\left(\frac{e^{f,c_l}}{e^{m,c_l}}\right)}{\left(\frac{e^{f,*}}{e^{m,*}}\right)} \propto \left[\frac{(1-\mu)}{\mu}\right]^{\phi/(1-\phi)} \tag{1.7.4}$$

The difference of ratios of effort in the centralized solutions and in the household problem originally defined in 1.5.1-1.5.19 depends on the Pareto weight and the degree of substitutability between parental efforts. If the Pareto weight heavily favors one member, and if there is some degree of substitutability between parental effort, there would be an inefficient allocation in time investments given that we can find an alternative allocation with the same amount of cost, or the same amount of total effort, that will yield better child outcomes. I find that this mechanism explains 15% of the differences in time investments between mothers and fathers.

It is often argued in the literature that, in a collective model of household be-

havior, observing different child outcomes when there is a shift in the bargaining power can only be explained by differences in preferences or productivities between parents (Doepke and Tertilt, 2014). Nonetheless, if we take into account that child skills are a public good produced with effort, the cost of which is privately exerted, shifts into bargaining power can translate in changes in child skills even when parents are identical in terms of preferences and productivities.

This result can be interpreted as an additional argument for female empowerment within households, not invoking an argument of equality but one of efficiency: disparities in bargaining power lead to inefficient allocations within the household. Taking this into account, and with the estimates of the economic model, we can quantify the extent to which the differences observed in time spent with children are due to productivity, preferences or empowerment differences.

1.8 Conclusions

The way in which skills are shaped during the first years of life has significant consequences for adult life outcomes. This fact has motivated a large amount of research aimed at understanding the skill formation process in children. Some of the key facts that we have learned from the literature about the skill formation process in children are:

1. **Malleability decreases with age**. As children age, it is harder to modify, or improve, children's cognitive and non-cognitive skills.

2. **Inequalities in skills emerge very early in life**. In developing and developed countries, disparities in the process of skill formation are evident as early as when children are three years old. Children who come from poor families score systematically worse than their richer counterparts in tests and parental assessments measuring cognitive and non-cognitive skills.

3. **Parenting and general family environment largely determine the skill formation process in children**. Early stimulation in children, coming directly from parents, has been shown to be one of the most relevant inputs in the skill formation process. Such stimulation can come in the form of time investments in children or by improving the environment of the children by improving their housing situation, the quality of the food being provided and the availability of items that promote skills, such as toys, puzzles, books and music.

4. **Parental investments in children can be complements or substitutes of public policy programs**. Given that inequalities in skills emerge very early in life, there have been multiple attempts from central governments to improve the conditions in which children live. Programs such as cash transfers, preschool subsidies and in-kind transfers have been developed with the goal of improving the quality of the environment in which children develop. However, such programs might have consequences for the way parents invest in their children, in ways that can be beneficial or detrimental for them. As

an example, cash transfers might discourage female labor force participation without further increasing the amount of investments made in children, as there is no guarantee that such money will be used to improve skills in children.

Developing programs to improve the skill formation process for children in disadvantaged households should be a priority for central governments in developing and developed economies. This is one of the most efficient ways to reduce crime, improve educational outcomes and increase productivity (Cunha, Heckman and Schennach, 2010). However, when developing such programs, we need to understand what is the most efficient way to do so and how such programs affect parental investments in their children. This is the first article in the literature that empirically evaluates the effect of cash transfers, childcare subsidies and in-kind transfers on the acquisition of skills for children from disadvantaged backgrounds.

To have an accurate assessment of how such policies affect skills in children, I develop a model of household behavior and child outcomes. One of the main features of cash transfers is that they are targeted to women exclusively, with the argument that cash in the hands of women translate into better child outcomes than cash in the hands of men. To incorporate this feature, I allow parents to have different preferences about skills for children. I also allow cash transfers to have an effect on the bargaining process in the household. Additionally, I take into account that parents can invest in their children either by improving the quality of their environment via monetary investments, or by spending time with them performing

activities such as reading, counting, going to cultural activities, among others.

The dataset used for this article was collected exclusively with the goal of getting a better understanding about the skill formation process in children. Thanks to that, I am able to incorporate several features of the skill formation process in children that have been ignored previously in the literature, such as parental skills, information about multiple dimensions of skills in children, quality and quantity of time and monetary investments by parents in their children, and preschool attendance, among others. This is the first paper in the literature that estimates a skill production technology via a dynamic-latent-factor structure a-là Cunha, Heckman and Schennach (2010), nested within a model of household behavior. This allows me to obtain non-parametric identification of the skill production function. By endogenizing the investment decisions of parents, I am able to perform counterfactual policy analysis, taking into account that parents' investments in their children are distorted by government interventions.

This article proposes a new framework to estimate models of household behavior with unobserved and continuous state variables. By implementing particle filtering techniques from Machine Learning and Financial Econometrics, I demonstrate an efficient algorithm to circumvent the high-dimensionality problem. Additionally, I introduce a new estimation strategy for collective models of household behavior. Rather than using the consumption of semi-private goods within the household, I use questionnaires about female empowerment and gender roles as noisy measures

of bargaining power for adult members in the household.

The results of this paper show that cash transfers have a very limited effect on reducing the gaps in skills between rich and poor children. Moreover, giving the transfers to fathers or mothers does not seem to make a significant difference. Consistent with most of the literature, I find that cash transfers have a very limited effect on female labor force participation. Childcare services have a positive but modest effect on skill promotion in children, as well as on female labor force participation. The main result suggests that the most effective way to close the gaps in skills between rich and poor children is by giving in-kind transfers. These are transfers that are given to households through a basket of goods that can be used to increase skills in their children, such as books, toys, puzzles and music. Whereas doubling the amount of monetary transfers to poor households reduces the gap in skills between children from the lowest quintile of the income distribution and their richer counterparts by about 2%, spending the same amount of resources on in-kind transfers decreases the gap by about 8%. These results are important for the design of policies to promote skills in children. Cash transfers and childcare subsidies have received significant attention in both the literature and the design of government policies. Programs that directly affect the physical environment in which children live have been less studied but seem to be more promising when it comes to increasing skills in young children.

1.9 Figures and Tables

Table 1.1: 2010 Tests-Measures of child skills

Test	Scoring Interpretation	Ages (in months)	Abbreviation
TEPSI	Higher score indicates a higher level of psychomotor development.	24-60	$MS_{1,10}$-$MS_{3,10}$
CBCL	A higher score indicates more persistence of behavioral problems.	18-60	$MS_{5,10}$-$MS_{11,10}$

Table 1.2: 2012 Tests-Measures of child skills

Test	Scoring Interpretation	Ages (in months)	Abbreviation
TADI	Higher scores indicate higher levels of childhood development	6-84	$MS_{1,12}$-$MS_{4,12}$
BT	Higher score indicates a higher level of child development-Batelle test score .	6-84	$MS_{5,12}$-$MS_{10,12}$
TVIP	Higher scores indicate higher levels of verbal intelligence for children	30-84	$MS_{13,12}$

Table 1.3: Summary statistics

Variable	**Mean**	**(Std. Dev.)**	**Min.**	**Max.**	**N**
Mother's age	30.87	(7.16)	14	56	15754
Father's age	35.11	(7.76)	17	84	10564
Mother's years of schooling	11.49	(2.89)	0	21	15699
Father's years of schooling	11.41	(3.06)	0	21	10418
Mother's hours of work (week)	18.53	(21.45)	0	112	15743
Father's hours of work (week)	43.93	(15.59)	0	120	10530
Mother's weekly wage (1,000 CLP)	83.89	(92.17)	1.16	1744.19	7382
Mother's weekly wage (USD)	167.78	(184.33)	2.33	3488.37	7382
Father's weekly wage (1,000 CLP)	104.22	(144.38)	2.91	5755.81	9813
Father's weekly wage (USD)	208.44	(288.76)	5.81	11511.63	9813
Household's total Income (Weekly-CLP)	102.86	(121.75)	0	1867.44	15754
Household's total Income (Weekly (USD))	205.71	(243.49)	0	3734.88	15754
Age of child (months)	49.94	(18.04)	7	83	14183

All summary statistics are reported for the survey used in 2012.

Abbreviation	Activity
$MS1_{EF,12}$	Reads Children's storybooks or drawing books
$MS2_{EF,12}$	Tells her stories
$MS3_{EF,12}$	Sings to child
$MS4_{EF,12}$	Takes her to parks
$MS5_{EF,12}$	Takes her to museums, zoos, libraries or other cultural activities
$MS6_{EF,12}$	Spends time with her chatting or drawing
$MS7_{EF,12}$	Invites her to participate in household chores
$MS8_{EF,12}$	Takes her to the supermarket
$MS9_{EF,12}$	Shares a meal with her
$MS10_{EF,12}$	Teaches the animals and their sounds
$MS11_{EF,12}$	Teaches her the colors
$MS12_{EF,12}$	Goes with her to visit friends or family members
$MS13_{EF,12}$	Teaches her the numbers and how to count
$MS14_{EF,12}$	Teaches her words

For each question parents reply how often, during the last seven days, they perform each activity. The possible answers are: Never, 1-3 times, 4-6 times.

Table 1.4: Measures used for parental effort in 2012

Abbreviation	Activity
$MS1_{EF,10}$	Reads Childre's storybooks or drawing books
$MS2_{EF,10}$	Tells her stories
$MS3_{EF,10}$	Sings to her
$MS4_{EF,10}$	Takes her to parks
$MS5_{EF,10}$	Takes her to museums, zoos, libraries or other cultural activities
$MS6_{EF,10}$	Plays with her
$MS7_{EF,10}$	Spends time with her talking or drawing

Table 1.5: Measures used for parental effort in 2010

Abbreviation	Outcome
$MS1_{12}$	TADI-Cognitive subdomain
$MS2_{12}$	TADI-Motor skills subdomain
$MS3_{12}$	TEPSI-Motor skills subdomain
$MS4_{12}$	TADI-Language subdomain
$MS5_{12}$	Battelle-I
$MS6_{12}$	Battelle-II
$MS7_{12}$	Battelle-III
$MS8_{12}$	Battelle-IV
$MS9_{12}$	Battelle-V
$MS10_{12}$	Battelle-T
$MS11_{12}$	PPVT-Vocabulary Test

All test scores are standardized to be mean zero and variance one.

Table 1.6: Measures used for Skills in 2012

Abbreviation	Outcome
$MS_{1_{10}}$	TEPSI-Coordination subdomain
$MS_{2_{10}}$	TEPSI-Language subdomain
$MS_{3_{10}}$	TEPSI-Motor skills subdomain
$MS_{4_{10}}$	CBCL-Emotional intelligence
$MS_{5_{10}}$	CBCL-anxiety -depression
$MS_{6_{10}}$	CBCL-somatic complaints
$MS_{7_{10}}$	CBCL-Isolation
$MS_{8_{10}}$	CBCL-Sleeping disorder
$MS_{9_{10}}$	CBCL-Attention deficit
$MS_{10_{10}}$	CBCL-Aggressive behavior

All test scores are standardized to be mean zero and variance one.

Table 1.7: Measures used for Skills in 2010

Abbreviation	Outcome
$MS_{1_{BIRTH}}$	Mother diagnosed with Preeclampsia during pregnancy
$MS_{2_{BIRTH}}$	Mother diagnosed with Cholestasis during pregnancy
$MS_{3_{BIRTH}}$	Mother diagnosed with Urinary infections during pregnancy
$MS_{4_{BIRTH}}$	Mother diagnosed with Hemorrages during pregnancy
$MS_{5_{BIRTH}}$	Mother diagnosed with Hipertension during pregnancy
$MS_{6_{BIRTH}}$	Mother diagnosed with Placenta Previa during pregnancy
$MS_{7_{BIRTH}}$	Mother diagnosed with Diabetes G during pregnancy
$MS_{8_{BIRTH}}$	Mother diagnosed with Anemia during pregnancy
$MS_{9_{BIRTH}}$	Mother diagnosed with Toxoplasmosis during pregnancy
$MS_{10_{BIRTH}}$	Mother diagnosed with Depression during pregnancy
$MS_{11_{BIRTH}}$	Mother diagnosed with Bipolar D. during pregnancy
$MS_{12_{BIRTH}}$	Mother diagnosed with Anxiety D. during pregnancy
$MS_{13_{BIRTH}}$	Mother diagnosed with Obsesive compulsive D. during pregnancy
$MS_{14_{BIRTH}}$	Mother diagnosed with Fobia during pregnancy
$MS_{15_{BIRTH}}$	Mother diagnosed with Panic D. during pregnancy
$MS_{16_{BIRTH}}$	Mother diagnosed with PTSD during pregnancy
$MS_{17_{BIRTH}}$	Cigarrettes consumed during pregnancy
$MS_{18_{BIRTH}}$	Cigarrettes consumed during the first six months of life of child
$MS_{19_{BIRTH}}$	Alcohol consumption during pregnancy*
$MS_{20_{BIRTH}}$	Substance abuse during pregnancy*
$MS_{21_{BIRTH}}$	Child was born pre-term
$MS_{22_{BIRTH}}$	Weight at birth (grams)
$MS_{23_{BIRTH}}$	Height at birth (cm)

*Possible answers are never (0), rarely (1) and often (2).

Table 1.8: Measures used for Skills at birth

Abbreviation	Outcome
$MS1_{PG}$	WAIS-Numerical test
$MS2_{PG}$	WAIS-Vocabulary test
$MS3_{PG}$	BFI-Agreeableness
$MS4_{PG}$	BFI-Openness
$MS5_{PG}$	BFI-Extroversion
$MS6_{PG}$	BFI-Neuroticism
$MS7_{PG}$	BFI-Conscientiousness

All test scores are standardized to be mean zero and variance one.

Table 1.9: Measures used for Skills of primary caregiver

Abbreviation	Question
$MS1_{INV,10}$	Child has a special place where to store toys and belongings
$MS2_{INV,10}$	Child has at least one toy that involves muscular activity
$MS3_{INV,10}$	Child has toys to pull and push
$MS4_{INV,10}$	Child has at least one toy with wheels
$MS5_{INV,10}$	Availability of plush toys-stuffed animals
$MS6_{INV,10}$	Availability of mobiles for child
$MS7_{INV,10}$	Availability of musical or literary toys
$MS8_{INV,10}$	Child has three or more books of his own

Table 1.10: Measures used for Investments in 2010

Abbreviation	Outcome
$MS1_{INV,12}$	Consumption of hamburger-pizza-fries*
$MS2_{INV,12}$	Consumption of Fish-Beef-Chicken*
$MS3_{INV,12}$	Consumption of bread-rice-pasta
$MS4_{INV,12}$	Consumption of legumes*
$MS5_{INV,12}$	Consumption of Chocolate-Candy*
$MS6_{INV,12}$	Consumption of juice*
$MS7_{INV,12}$	Consumption of snacks in bags*
$MS8_{INV,12}$	Consumption of milk*
$MS9_{INV,12}$	Consumption of water*
$MS10_{INV,12}$	Consumption of cookies*
$MS11_{INV,12}$	Consumption of fruits and vegetables*
$MS12_{INV,12}$	There are two or more toys in the household where child can learn colors, sizes and shapes
$MS13_{INV,12}$	Child has three or more puzzles
$MS14_{INV,12}$	There is a music device where child can listen children's music
$MS15_{INV,12}$	There are two or more toys for free expression or impersonations such as tools and customs
$MS16_{INV,12}$	There are two or more toys in the household that can help with learning numbers
$MS17_{INV,12}$	There are at least ten children's books available in the house
$MS18_{INV,12}$	There are at least ten books for adults
$MS19_{INV,12}$	At first sight, there is very little evidence that there is a child living in the household
$MS20_{INV,12}$	Number of people with whom child shares bed
$MS21_{INV,12}$	Number of people with whom child shares room

*: The possible answers are 1: never, 2: one to two times a month; 3: one to three times a week; 4: four to six times a week; 5: once a day; 6: two or more times a day.

Table 1.11: Measures used for Investment in 2012

Abbreviation	Question
$MS1_{BARG}$	A woman who is in charge of most part of tasks of the household has no time to work*
$MS2_{BARG}$	Both spouses should contribute to household income*
$MS3_{BARG}$	It is better for everyone if the man goes to work and the woman takes care of the household and the family*
$MS4_{BARG}$	Men should assume a more active role in the household chores and childcare than what they actually do*
$MS5_{BARG}$	If my spouse earned enough there is no reason for me to work*
$MS6_{BARG}$	After having children, the best for a woman is to develop her carreer*
$MS7_{BARG}$	Taking into account the pros and cons, it is very important for me to have a paying job*
$MS8_{BARG}$	Having a payed job is the best way for a woman to become independent*
$MS9_{BARG}$	Father's and mother's time is equally important for the children*
$MS10_{BARG}$	It is better to have a bad marriage than to remain single*
$MS11_{BARG}$	Woman participates in the process of administering income (yes-no)
$MS12_{BARG}$	Man participates in the process of administering income (yes-no)
$MS13_{BARG}$	Both, father and mother participate in the process of administering income (yes-no)
$MS14_{BARG}$	(Mother) Who should take care of children (Father-Mother-Both-Other)
$MS15_{BARG}$	(Man) Women should only be in charge of taking care of children (yes-no)
$MS16_{BARG}$	(Man) Women should take care of children and work part time (yes-no)
$MS17_{BARG}$	(Man) Women should work full-time and delegate childcare to someone else (yes-no)
$MS18_{BARG}$	(Man) Men are better at childcare than women (yes-no)

*: For each question the woman provides an answer between 1 to 5 with the following scale:
Disagrees very much; disagrees; doesn't know; agrees; agrees very much.

Table 1.12: Measures used for Pareto weight

Table 1.13: Father's opinion on gender roles

Item	Number	Per cent
Women should only spend time taking care of chidlren	3,925	34.34
Women should take care of children and work if there is remaining time	6,930	60.62
Women should work full time	525	4.59
Men take care better of children than women	51	0.45
Total	11,431	100.00

Table 1.14: Summary statistics-Measures of bargaining power

Variable	**Mean**	**(Std. Dev.)**
A woman in charge of chores should not work	2.59	(1.03)
Both parents should contribute equally to household income	1.85	(0.95)
It is better if the man goes to work and the woman stays at home	2.57	(1.06)
Men should be more involved in household chores	1.84	(0.99)
If husband earned enough there is no reason for woman to work	2.55	(1.6)
It is better if woman has children after having a succesful carreer	2.45	(1.25)
It is very important for a woman to have a job	1.94	(1.06)
Having a job is the best way for a woman to achieve independence	1.86	(0.98)
Father's time is as important as mother's time for children	1.56	(0.87)
It is better to have a bad marriage than being single	3.57	(1.4)
N	15754	

All questions are answered by the mother of the child. The possible answers are 1: strongly agrees; 2: agrees; 3: disagrees; 4: strongly disagrees.

Table 1.15: Summary statistics-Variables determining Pareto weight

Variable	**Mean**	**(Std. Dev.)**	**N**
Father's non-labor income share	0.27	(0.34)	4470
Age difference (Father-Mother)	3.01	(5.18)	4470
Difference in grades attained (Father-Mother)	-0.21	(2.82)	4470
Sex ratio in region (Women/Men)	1.01	(0.06)	4470
Unemployment ratio in region (Men/Women)	0.67	(0.11)	4470
Wage ratio in region (Men/Women)	1.41	(0.07)	4470
Distance to women protection center (km)	19.15	(31.79)	4465

The ratio of wages offered is not reported in these table as is the results of the parameters estimated in the model. The share of father's non-labor income, as well as the age difference and the differences in grades attained are all obtained from the ECLS dataset. The sex ratio in the city is computed using information from the CENSUS dataset. The last CENSUS available for Chile is from 2002. I use information about female-male ratio based on the population projections from the National Institute of Statistics fro Chile. The unemployment and wage information is obtained from the CASEN household survey of 2011.

Table 1.16: Time investments and labor supply (2010)

VARIABLES	(1) Mother's effort (2010)	(2) Father's effort (2010)
Mother: hours worked weekly	-0.00***	0.00***
	(0.00)	(0.00)
Father: hours worked weekly	0.00***	-0.00***
	(0.00)	(0.00)
Total household income	0.00	0.00***
	(0.00)	(0.00)
Age of child (months)	0.01***	0.00*
	(0.00)	(0.00)
BFI-Extraversion	0.05***	0.07***
	(0.02)	(0.02)
BFI-Kindness	0.05**	0.04*
	(0.02)	(0.02)
BFI-Responsibility	0.06***	0.05**
	(0.02)	(0.02)
BFI-Neuroticism	-0.05***	-0.02
	(0.01)	(0.02)
BFI-Openness	0.15***	0.02
	(0.02)	(0.02)
Wais-digits	0.01	0.01*
	(0.01)	(0.01)
Wais-Vocabulary	-0.00	-0.00
	(0.00)	(0.00)
Number of siblings	-0.07***	-0.06***
	(0.01)	(0.01)
Observations	7,058	7,058
Adjusted R-squared	0.07	0.04

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1
Additional controls include age of child, race, age of both parents and test scores of primary caregiver.

The measure of effort is constructed via Principal component analysis, extracting one factor for the variables used as measures of time investments by parents. The measures of parental effort, together with the big five personality test scores and the Wais cognitive assessments are all standardized to have mean zero and one standard deviation. In the regression the measure of effort is in hundreds.

Table 1.17: Time investments and labor supply (2012)

VARIABLES	(1) Mother's effort (2012)	(2) Father's effort (2012)
Mother: hours worked weekly	-0.01***	0.00***
	(0.00)	(0.00)
Father: hours worked weekly	0.00	-0.01***
	(0.00)	(0.00)
Total household income	0.00	0.00
	(0.00)	(0.00)
Age of child (months)	0.01***	0.00***
	(0.00)	(0.00)
BFI-Extraversion	0.01	0.05*
	(0.03)	(0.03)
BFI-Kindness	0.06	-0.00
	(0.04)	(0.03)
BFI-Responsibility	0.11**	0.11***
	(0.04)	(0.03)
BFI-Neuroticism	-0.05	-0.04
	(0.03)	(0.03)
BFI-Openness	0.19***	0.05*
	(0.04)	(0.03)
Wais-digits	-0.02	-0.00
	(0.01)	(0.01)
Wais-Vocabulary	0.01***	0.01***
	(0.00)	(0.00)
Number of siblings	-0.09***	-0.06***
	(0.02)	(0.02)
Observations	8,020	7,956
Adjusted R-squared	0.04	0.03

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1
Additional controls include age of child, race, age of both parents and test scores of primary caregiver.

The measure of effort is constructed via Principal component analysis, extracting one factor for the variables used as measures of time investments by parents. The measures of parental effort, together with the big five personality test scores and the Wais cognitive assessments are all standardized to have mean zero and one standard deviation. In the regression the measure of effort is in hundreds.

Table 1.18: Regressions of effort in differences

VARIABLES	(1) ΔEffort father	(2) ΔEffort mother
ΔHours worked mother	0.03***	-0.02***
	(0.01)	(0.01)
ΔHours worked father	-0.03***	0.01**
	(0.01)	(0.01)
ΔEffort mother	0.37***	
	(0.01)	
ΔEffort father		0.36***
		(0.01)
Observations	4,531	4,531
R-squared	0.14	0.15

*** p<0.01, ** p<0.05, * p<0.1.
Standard error in parentheses.

$\Delta X = X_{2012} - X_{2010}$. The measure of effort is the same used as in Table 1.17 but in differences. The same controls as in Table 1.17 are used.

Table 1.19: Child outcomes in 2012 and share of income earned by women

VARIABLES	(1) Motor skills 2 (B3)	(2) Cognitive test (B5)	(3) Batelle Total
Mother's income share	0.09*	0.09*	0.10**
	(0.05)	(0.05)	(0.05)
Total household income	0.00	0.00**	0.00**
	(0.00)	(0.00)	(0.00)
Mother's years of schooling	0.01**	0.02***	0.03***
	(0.01)	(0.01)	(0.01)
Father's years of schooling	0.02***	0.01**	0.02***
	(0.01)	(0.01)	(0.00)
Number of siblings	0.02	-0.00	-0.03*
	(0.01)	(0.01)	(0.01)
Age of child (months)	0.00***	0.00***	0.01***
	(0.00)	(0.00)	(0.00)
BFI-Extraversion	0.06***	0.04**	0.04***
	(0.02)	(0.02)	(0.02)
BFI-Kindness	-0.00	0.09***	0.02
	(0.02)	(0.02)	(0.02)
BFI-Responsibility	0.10***	0.08***	0.07***
	(0.02)	(0.02)	(0.02)
BFI-Neuroticism	-0.02	-0.03*	-0.01
	(0.02)	(0.02)	(0.02)
BFI-Openness	0.07***	0.03	0.03
	(0.02)	(0.02)	(0.02)
Wais-digits	0.01	0.01	0.02***
	(0.01)	(0.01)	(0.01)
Wais-Vocabulary	0.00	0.00***	0.00***
	(0.00)	(0.00)	(0.00)
Observations	6,823	6,823	6,822
Adjusted R-squared	0.03	0.05	0.08

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Additional controlsi include age of child, race, age of both parents, test scores of primary caregiver and number of siblings. +: lower scores indicate lower incidence of behavioral problems.

Table 1.20: Female empowerment and Child outcomes

VARIABLES	(1) Toys for development	(2) Fruits and vegetables	(3) People sharing bedroom with child
Total household income	0.00***	0.00	-0.00***
	(0.00)	(0.00)	(0.00)
Mother's years of schooling	0.01***	0.01**	-0.03***
	(0.00)	(0.01)	(0.00)
Father's years of schooling	0.01***	0.01**	-0.02***
	(0.00)	(0.00)	(0.00)
Number of siblings	0.00	0.04**	0.08***
	(0.01)	(0.01)	(0.01)
People in household	-0.01**	0.01	0.13***
	(0.01)	(0.01)	(0.01)
Woman administers+	0.03**	0.09***	-0.00
	(0.01)	(0.02)	(0.02)
Gender roles -Woman++	-0.01	-0.03**	0.02*
	(0.01)	(0.01)	(0.01)
Gender roles - Man++	-0.01	-0.05*	0.06**
	(0.01)	(0.03)	(0.02)
Observations	6,344	8,245	8,246
Adjusted R-squared	0.04	0.03	0.19

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Consumption of bread, fruits and vegetables and cookies and candies is related to the frequency of consumption of this food on a weekly basis. More details can be found in Table 1.11. + dummy variable indicating whether the mother is the person in charge of administering the resources of the household (1) or no (0). ++ opinion of gender roles according to the man and the woman. A value of one indicates that the person agrees with the sentence "Women should not work and should only take care of children".

Filter	Number of households
Initial sample	18,310
Household not surveyed in 2012	16,033
Household not surveyed in 2010	12,898
Parent not living in household	7,855
Siblings within five years of age in the household	4,125
Children with incomplete skills questionnaires	2,247
Households with incomplete questionnaires	950

Table 1.21: Description of sample used in the analysis

Variable	Mean	25%	75%	Sd
Mother's age	34.52	29.00	39.00	6.94
Father's age	37.41	32.00	43.00	7.96
Mother's years of schooling	11.27	10.00	12.00	2.97
Father's years of schooling	10.72	8.00	12.00	3.13
Mother's hours of work (week)	24.22	0.00	45.00	21.34
Father's hours of work (week)	43.20	45.00	48.00	16.03
Mother's weekly wage (1,000 CLP)	82.73	41.86	95.24	92.78
Mother's weekly wage (USD)	165.46	83.72	190.49	185.55
Father's weekly wage (1,000 CLP)	85.48	42.62	93.02	88.19
Father's weekly wage (USD)	170.95	85.23	186.05	176.39
Household's total Income (Weekly-CLP)	124.55	59.88	151.16	108.83
Household's total Income (Weekly (USD))	249.10	119.76	302.33	217.66
Age of child (months)	64.60	58.00	72.00	8.40

Table 1.22: Descriptive statistics - Families in 2012

All summary statistics are reported for the survey used in 2012.

Table 1.23: Age distribution (2012)

Item	Number	Per cent
4	310	32.63
5	397	41.79
6	243	25.58
Total	950	100.00

Parameter	Estimate	Standard Error
$\alpha^m_{1,12}$	0.6312	0.0028
$\alpha^m_{2,12}$	0.0517	0.0001
$\alpha^m_{3,12}$	0.3035	0.2208
$\alpha^m_{4,0,12}$	0.0136	0.0001
$\alpha^m_{4,1,12}$	0.0012	0.0001
$\alpha^m_{1,10}$	0.0554	0.0003
$\alpha^m_{2,10}$	0.0038	0.0001
$\alpha^m_{3,10}$	0.1026	0.2437
$\alpha^m_{4,0,10}$	0.0001	0.0001
$\alpha^m_{4,1,10}$	0.0001	0.0001
$\alpha^m_{5,10}$	0.8381	0.3831

Table 1.24: Estimates: Utility function. Mother's preferences

Parameter	Estimate	Standard Error
$\alpha^f_{1,12}$	0.1587	0.0026
$\alpha^f_{2,12}$	0.0339	0.0001
$\alpha^f_{3,12}$	0.8042	0.3610
$\alpha^f_{4,0,12}$	0.0032	0.0001
$\alpha^f_{4,1,12}$	0.0016	0.0001
$\alpha^f_{1,10}$	0.6157	0.0026
$\alpha^f_{2,10}$	0.1407	0.0005
$\alpha^f_{3,10}$	0.8042	0.4496
$\alpha^f_{4,0,10}$	0.0114	0.0001
$\alpha^f_{4,1,10}$	0.0001	0.0001
$\alpha^f_{5,10}$	0.0057	1.0415

Table 1.25: Estimates: Utility function. Father's preferences

Parameter	Estimate	Standard Error
$\sigma^m_{W,A}$	3.6627	0.8352
$\sigma^m_{NW,A}$	0.9095	0.1140
$\sigma^m_{NW,NA}$	0.0794	0.2469
$\sigma^f_{W,A}$	0.5020	0.4519
$\sigma^f_{NW,A}$	0.0851	0.4550
$\sigma^f_{NW,NA}$	0.0020	0.0777
Preference shocks for work-no childcare are standardized to zero		

Table 1.26: Estimates: Preference shock

Parameter	Estimate	Standard Error
β_0^m	5.7874	0.4394
β_1^m	0.2757	0.0251
β_2^m	0.0732	0.0379
β_3^m	-0.0006	0.0006
σ_{w_m}	0.8280	0.0606

Table 1.27: Estimates: Mothers wages

Parameter	Estimate	Standard Error
β_0^f	5.8103	0.2997
β_1^f	0.1260	0.0055
β_2^f	0.1875	0.0156
β_3^f	-0.0022	0.0002
σ_{w_f}	0.6894	0.0130

Table 1.28: Estimates: Fathers wages

Parameter	Estimate	Standard Error
θ_0	0.2128	0.0011
θ_1	0.2673	0.0017
θ_2	0.5199	0.0032
ϕ	0.4688	0.0007
γ_f	0.3647	0.0006
γ_m	0.6353	0.0016
δ_0	-0.8000	0.0051
δ_1	-0.0000	0.0001
δ_2	0.0010	0.0004
$\delta_{3,10}$	3.5038	0.0172
$\delta_{3,12}$	5.3000	0.0408
δ_4	0.0130	0.0001
σ_s	1.5754	0.0065

Table 1.29: Estimates: Production of Skills

Parameter	Estimate	Standard Error
σ_{ef}^m	2.5133	0.0039
σ_{ef}^f	3.3754	0.0025
σ_{inv}	2.1896	0.0144

Table 1.30: Estimates: Distribution of latent factors

Parameter	Estimate	Standard Error
Price_{I_0}	966.2378	1.8225
Price_{I_1}	1.0537	0.0019
Pchildcare_0	2440.6020	1.1684
Pchildcare_1	622.6098	1.2417

Table 1.31: Estimates: Prices

Parameter	Estimate	Standard Error	Description
λ_0	-2.7321	0.0136	Intercept
λ_1	0.0023	0.0143	Wage ratio
λ_2	0.0527	0.0006	Non-labor income ratio
λ_3	-0.1194	0.0001	Age difference
λ_4	0.0036	0.0026	Educational difference
λ_5	-2.5325	0.0039	Gender ratio
λ_6	-0.0069	0.0328	Unemployment ratio
λ_7	-0.7722	0.0006	Wage ratio (region)
σ_μ	0.5179	0.0074	Standard deviation

Table 1.32: Estimates: Pareto weight

Table 1.33: Model Fit - I

Female Labor Force Participation	Predicted	Data
2010	57.2%	60.28%
2012	62.6%	61.47%

Table 1.34: Model Fit - II

Male Labor Force Participation	Predicted	Data
2010	91.8%	94.6%
2012	96.1%	93.2%

Table 1.35: Model Fit - III

Childcare Attendance	Predicted	Data
Working Mothers	68.4%	67.7%
Not-working Mothers	41.6%	42.9%

Counterfactual	Effect on Female employment
1	-0.63
2	-0.63
3	0.63
4	0.00

Table 1.36: Effects of Policy counterfactuals. Change in Female employment (percentage points)

Counterfactual	Effect on Male employment
1	-0.21
2	-0.21
3	0.00
4	0.00

Table 1.37: Effects of Policy counterfactuals. Change in Male employment (percentage points)

Counterfactual	Change in Money Invested
1	11.36
2	11.36
3	34.08
4	333.59

Table 1.38: Effects of Policy counterfactuals. Change in Money invested

Counterfactual	Change in Money Invested
1	11.36
2	11.36
3	34.08
4	333.59

Table 1.39: Effects of Policy counterfactuals. Change in Money invested

Counterfactual	Expenditure per capita (USD)
Transfers to Mother	449.59
Transfers to Father	449.59
Childcare Subsidy*	221.64
In-kind transfers	449.59

Table 1.40: Cost of policy interventions

Figure 1.1: Information on Preschool Providers

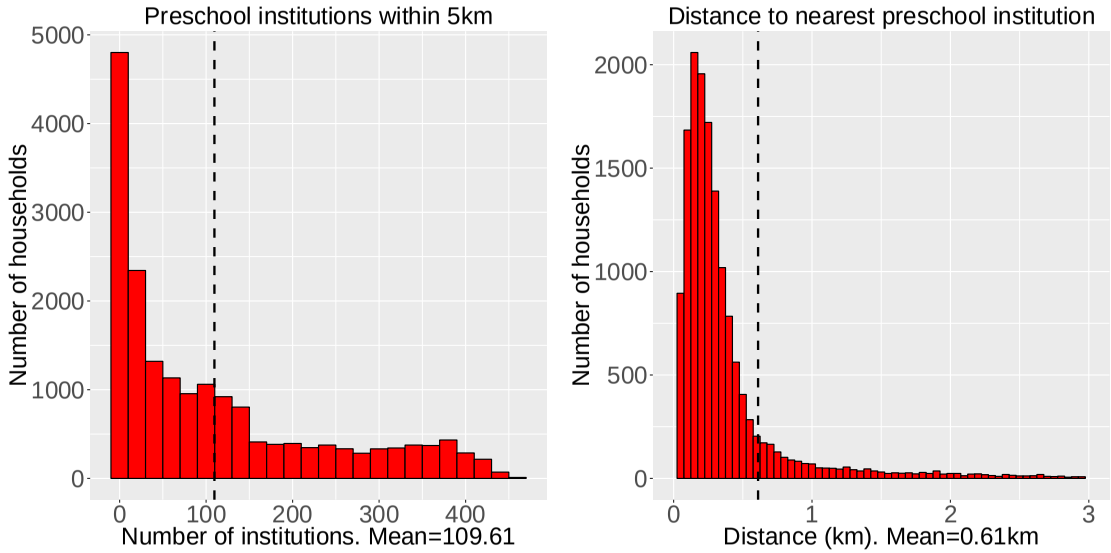
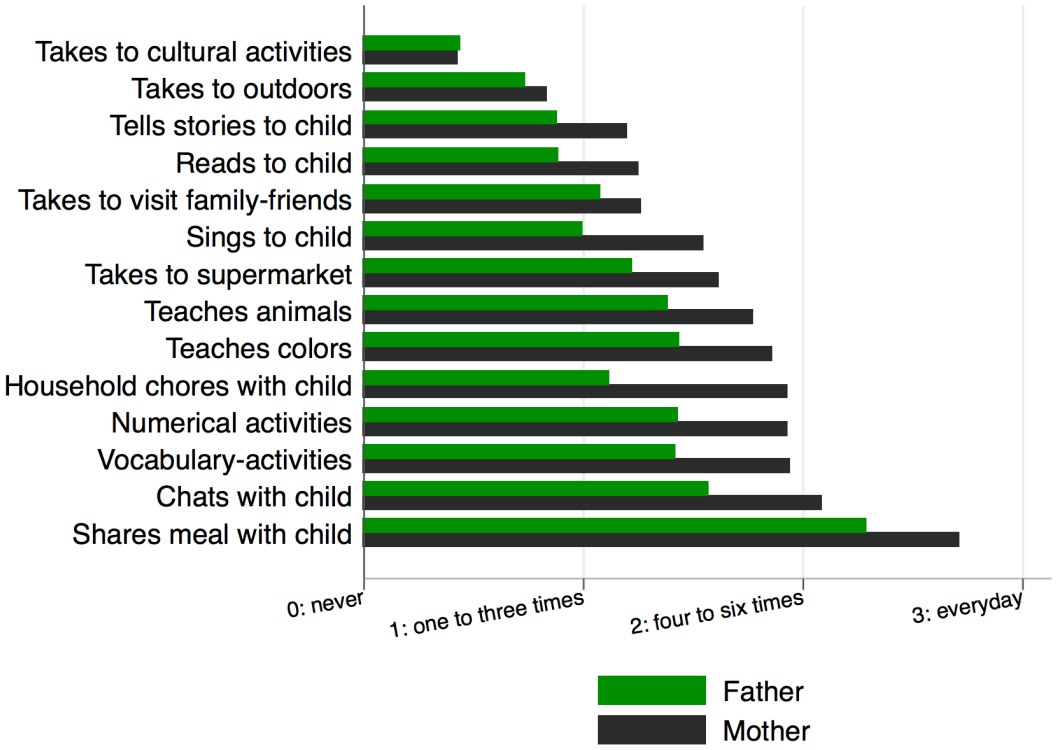


Figure 1.2: Example of distribution of childcare providers. City of "La Serena", Chile

Figure 1.3: Weekly frequency of activities between parents and children



For each activity there are possible answers: 0: never, 1: one to three times a week; 2: four to six times a week; 3: everyday.

Figure 1.4: Female Labor Force Participation (%)

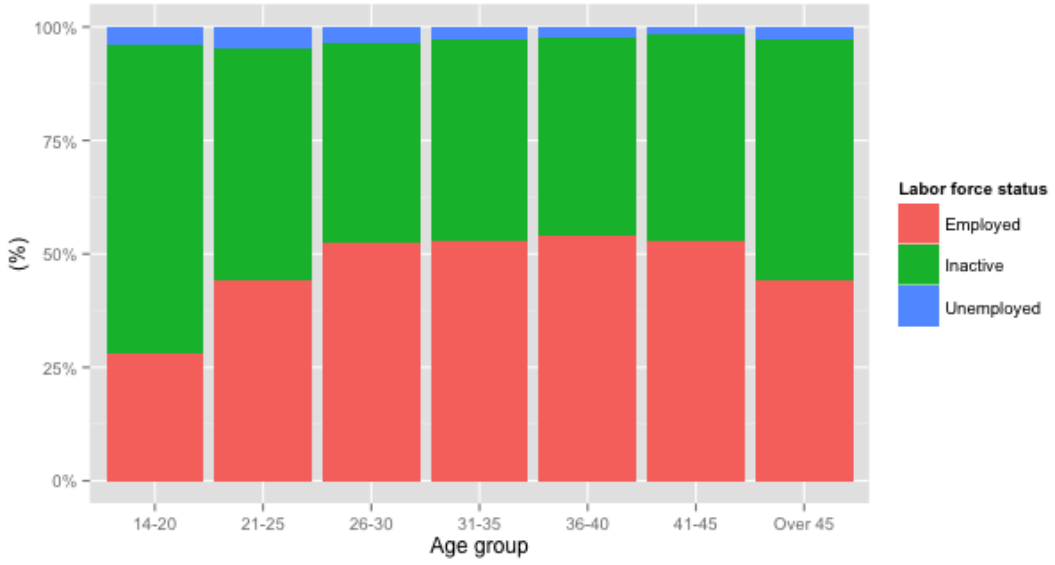


Figure 1.5: Gaps in health at birth (%)

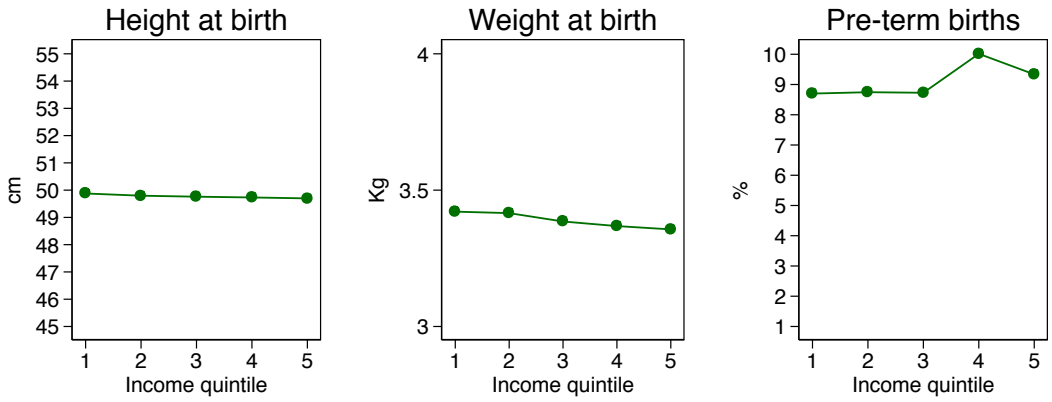
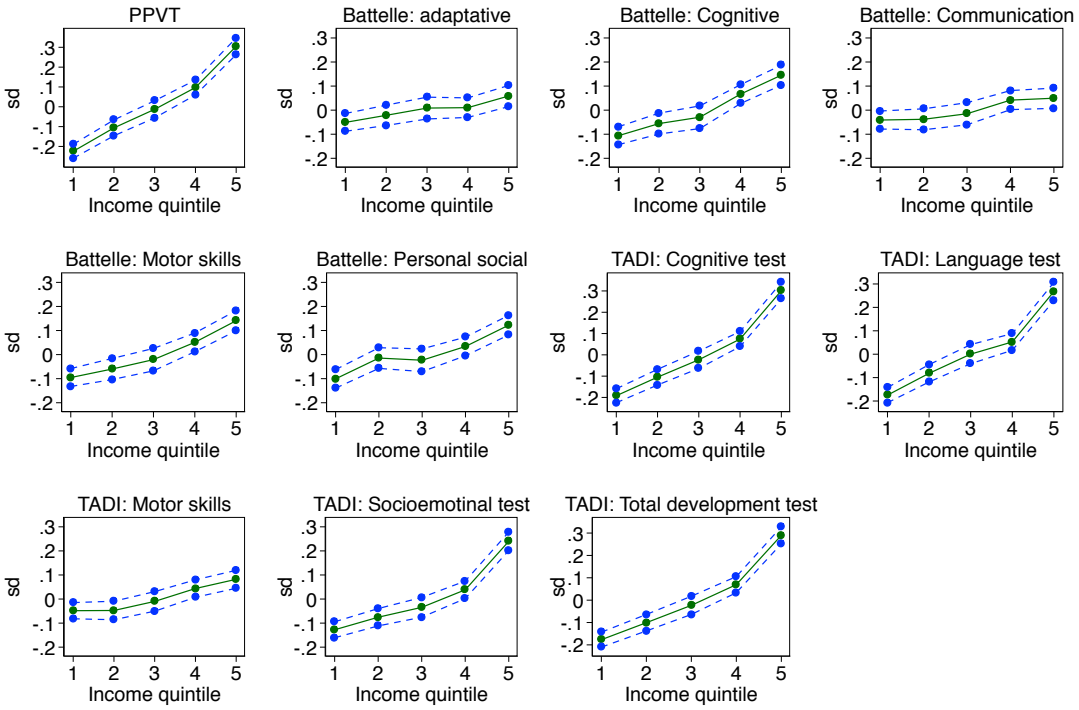
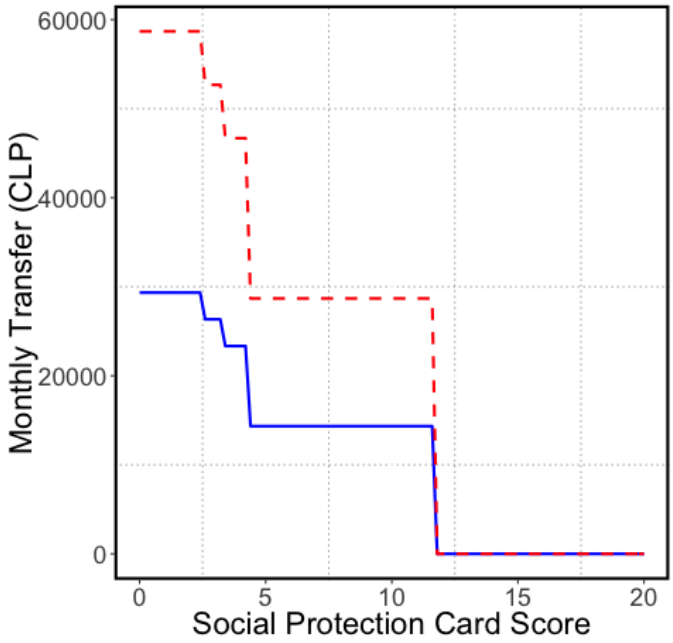


Figure 1.6: Gaps in skills at age 5



The green (solid) line is the mean score, the blue (dashed) line is the 95% confidence interval. All test scores and parental assessments are normalized to have mean zero and variance one. PPVT stands for Peabody Picture Vocabulary Tests. Battelle is an instrument containing different scales to measures development of children. TADI is a test of learning and child development[20]. In all tests, differences between the scores of children in the lowest quintile of the income distribution is statistically different to those children who are in the highest quintile of the income distribution.

Figure 1.7: Monetary Transfers to Families in Chile



The graph reports the relationship between monetary transfers that families receive from the central government, on a monthly basis, from three main programs: "Unique Family Subsidies"[21], "Family Assignments"[22] and "Social Protection Transfer"[23] and their score in the Social Protection Card[24]. The conditions to be eligible for these programs are to have a score in the Social Protection Card below 11.734 and for those who work, having a monthly income of less than $187,515 CLP. The final amount being transferred to the household also depends on the size of the household and the time they have been beneficiaries of such programs. The solid line represents the schedule for a family composed of two adults and one child who has been in the program for 50 months. The dashed line corresponds to a bi-parental household with three children under 18 who have been in the program for less than six months.

Figure 1.8: Model fit: Female labor force participation according to education

Figure 1.9: Model fit: Male labor force participation according to education

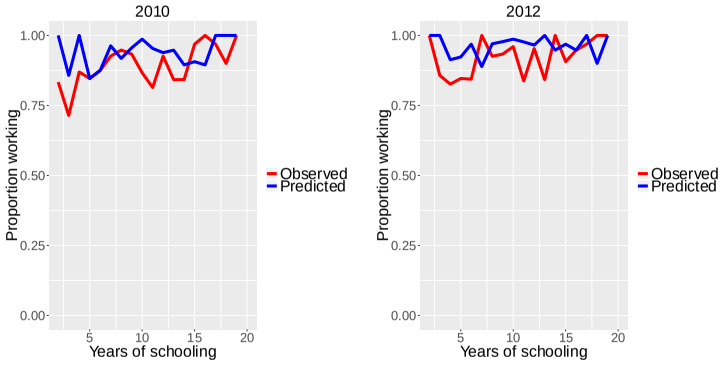
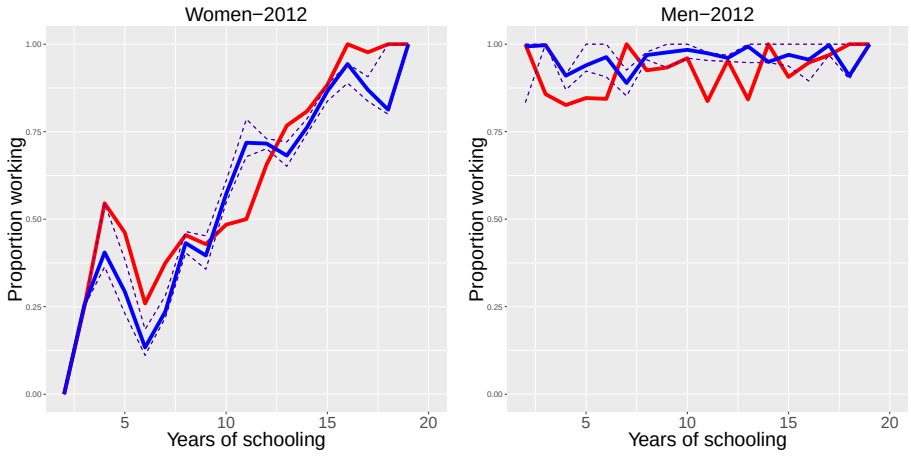
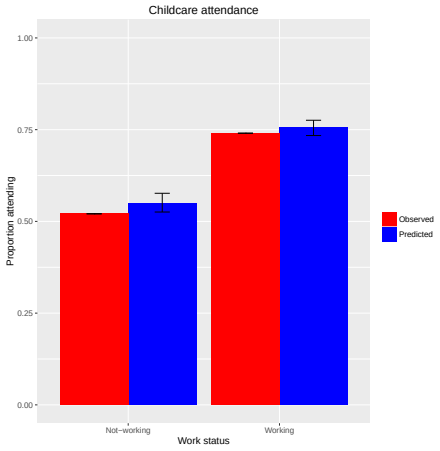


Figure 1.10: Bootstrap fit: Parents' Labor Force Participation in 2012



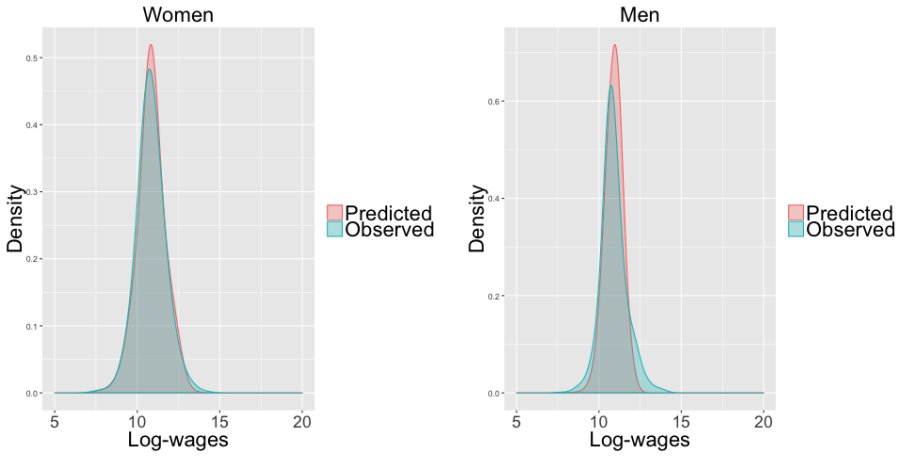
Dashed lines represent the 95% confidence interval

Figure 1.11: Bootstrap fit: Childcare decisions (%)



Brackets include the 95% confidence interval

Figure 1.12: Model fit: distribution of wages



Kernel density estimates of predicted and observed wages. Bandwidth chosen is 3.

Figure 1.13: Distribution of skills. Smoothing distribution

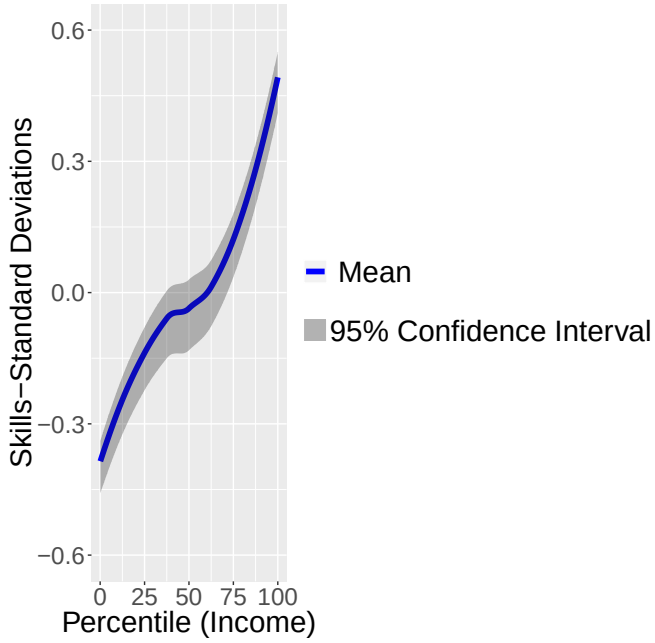


Figure 1.14: Signal to noise ratio. Mother's effort (2012)

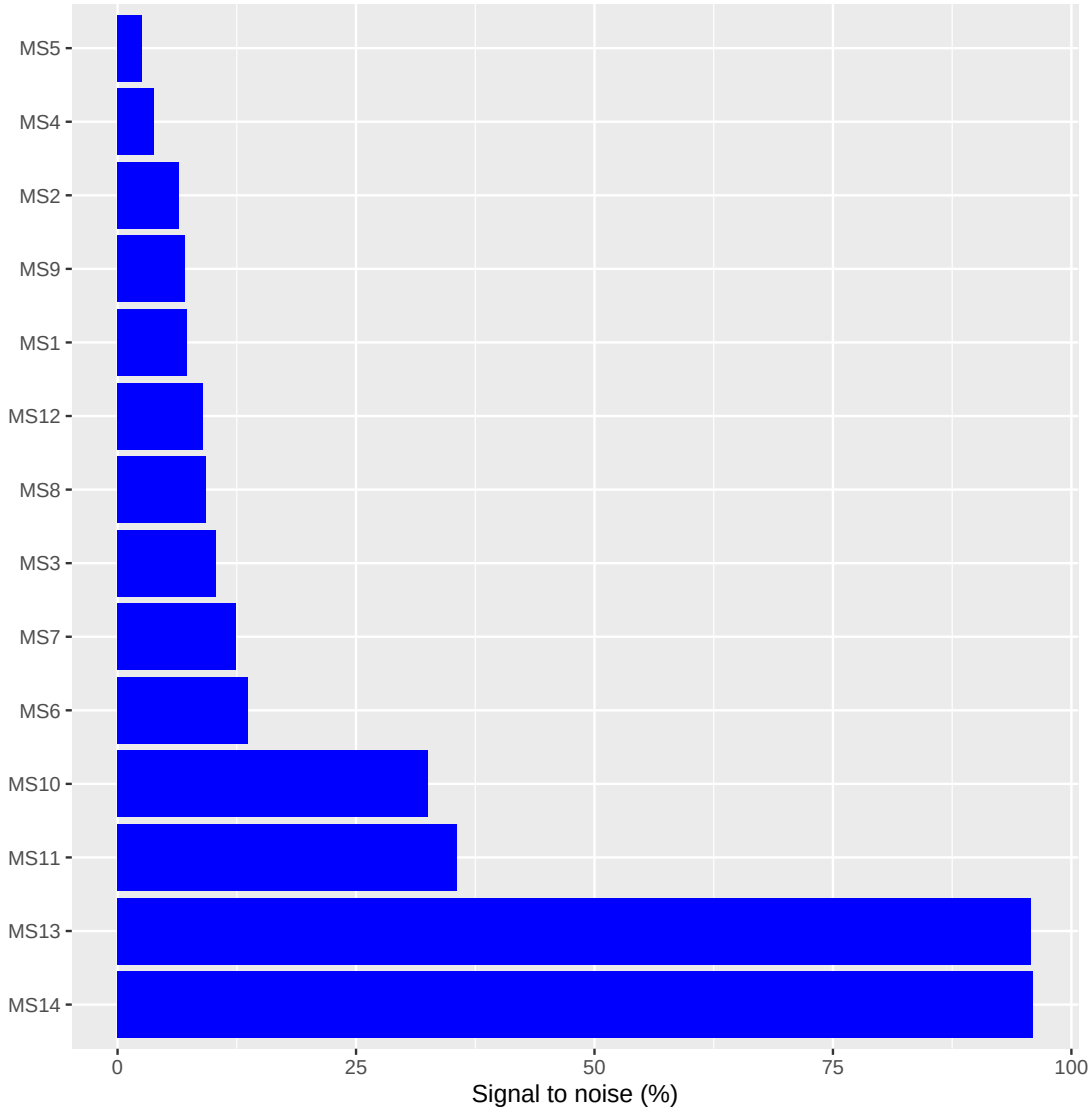


Figure 1.15: Signal to noise ratio. Monetary Investment (2012)

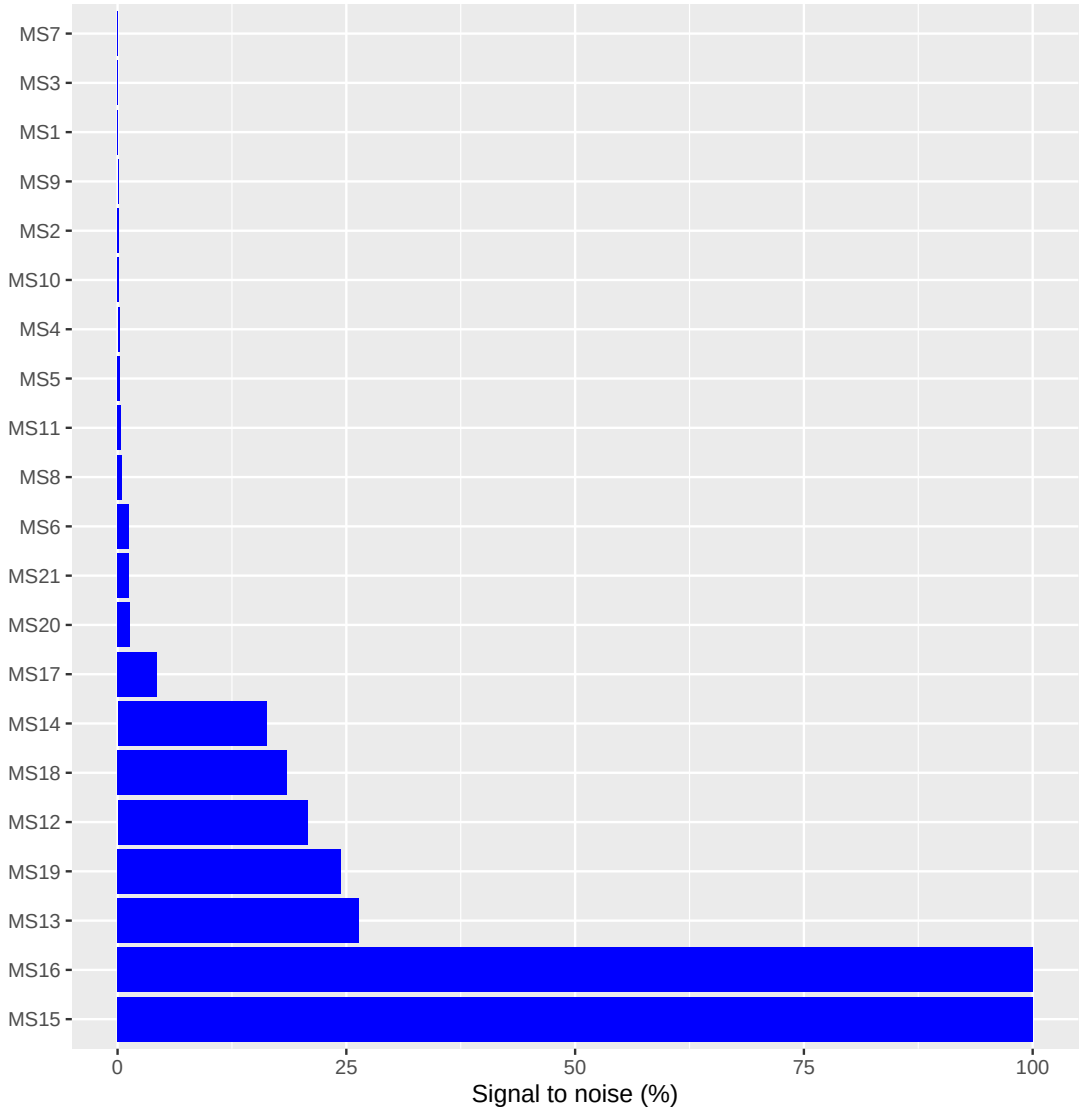


Figure 1.16: Effects of Policy Interventions

Effects on the gap in skills between the top 20% richest households and the poorest 20% of the households

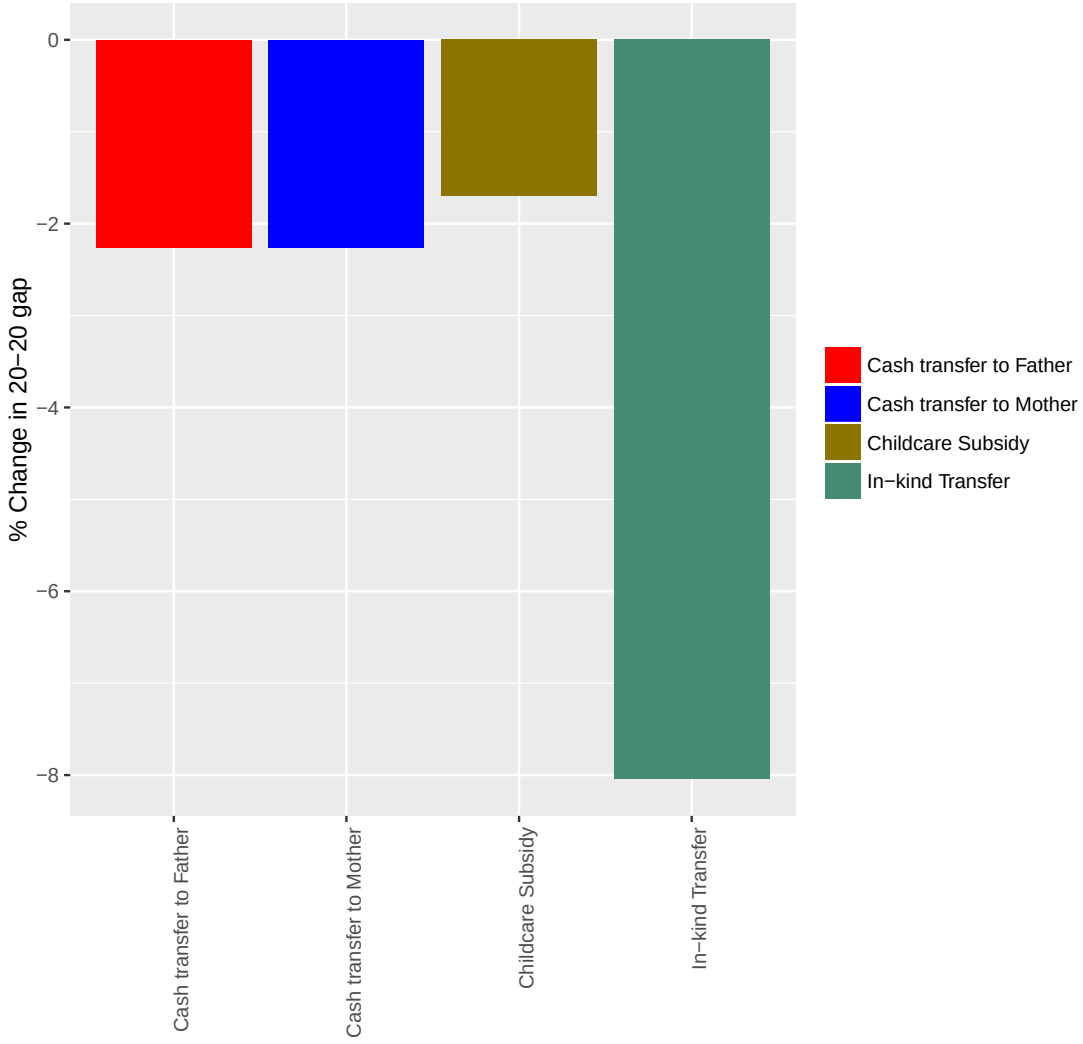
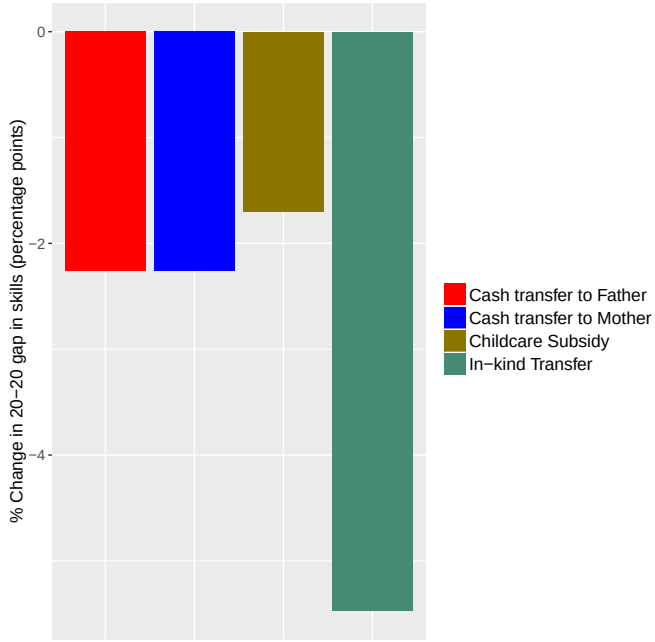


Figure 1.17: Effects of Policy Interventions

Effects on the gap in skills between the top 20% richest households and the poorest 20% of the households when money spent in each policy is the same.



Chapter 2

The Effects of Student Loans on the Provision and Demand for Higher Education

This chapter is co-authored with David Zarruk.

Abstract
We characterize the outcomes of the tertiary education market in a context where borrowing constraints bind, there is a two-tier college system operating under monopolistic competition in which colleges differ by the quality offered and returns to education depend on the quality of the school attended. College quality, tuition prices, acceptance cut-offs and education demand are all determined in a general equilibrium model and depend on the borrowing constraints faced by households. Our main finding shows that subsidized student loan policies can lead to a widening gap in the quality of services provided by higher education institutions. This happens because the demand for elite institutions unambiguously increases when individuals can borrow. This does not happen in non-elite institutions, since relaxing borrowing constraints makes some individuals move from non-elite to elite institutions. The higher increase in demand for elite institutions allows them to increase prices and investment per student. As investment and average student ability are complementary inputs in the quality production function, elite universities also increase their acceptance cut-offs. In this new equilibrium, the differentiation of the product offered by colleges increases, where elite universities provide higher quality to high-ability students and non-elite universities offer lower quality to less-able students. We illustrate the main results through a numerical exercise applied to Colombia. We show that the observed increase in the quality gap can be a by-product of the subsidized loan policies. Such results show that, when analyzed in a general equilibrium setting, subsidized loan policies can have regressive effects on the income distribution.

2.1 Introduction

The market for higher education has received significant attention in the economics literature. In particular, the effects that subsidized loan policies have on the demand side of the market have been widely studied, given the dramatic increase in student debt during the last two decades in the U.S. Overall, there seems to be a consensus in the literature on the fact that credit constraints explain only a small fraction of enrollment decisions in higher education in the U.S. However, this is not necessarily the case in developing countries, where student financial aid systems are weak and evidence suggests that college enrollment is highly determined by family wealth (World Bank, 2003, 2012). In this context, the implementation of subsidized student loan policies can potentially affect the demand for education, generating equilibrium effects such as increases in tuition prices and changes in the quality of services offered by colleges.

Understanding the effects of these policies is of central importance given the massive investments that have been made in student credit programs during the last two decades in the developing world, in Latin America and some African countries[25]. The demand side effects of these policies in a context where borrowing constraints determine enrollment decisions have been studied by the literature and the conclusions are certainly appealing: an expansion in student loans will lead

[25]See Salmi (1999) and World Bank (2009) for a review on student loan programs in Latin American and African countries.

to an increase in the demand for higher education among the most able students (Canton and Blom, 2004). This will reduce the inefficiency that exists when very high-ability individuals with low initial wealth cannot access tertiary education. A partial equilibrium perspective would unambiguously show that such policies have welfare improving effects on it's beneficiaries. As a consequence, these programs have often received the support of multilateral institutions, such as the World Bank and the Inter-American Development Bank[26].

However, the equilibrium effects of such credit programs have not been widely studied. In particular, not much has been said about the effect of subsidized student loans on tuition prices and on the quality of services provided by educational institutions. Such policies can have negative effects on a subset of the agents in the economy that might offset the overall benefits from introducing such policies[27].

This paper contributes to the literature studying the consequences of subsidized loan policies, by analyzing the general equilibrium effects that such programs have on the quality of education provided by different tiers of colleges, which we will denote henceforth as low and high-quality, or elite and non-elite colleges. We develop a

[26]These institutions have contributed to different student loan projects in the developing world. For example, the World Bank has been financing the Colombian ACCES program since 2002 and committed in 2014 to lend $200 million during the period 2014-2019. Recently, the IDB provided a $10 million dollar loan to the Higher Education Finance Fund in 2012, to finance student loan programs in 4 Latin American countries.

[27]Obiols-Homs (2011) argues that in an incomplete markets setting, although increasing borrowing limits increases the welfare of borrowing constrained individuals, in equilibrium this also leads to an increase in the interest rate paid by the borrowers. The two effects oppose each other, so the effect of loosening borrowing limits on welfare is ambiguous and follows a U-shape. Although we do not take into account the effect of borrowing constraints on the interest rate and assume government student loans are subject to an exogenous interest rate, his findings strengthen our theory that student loan policies might have negative effects on welfare, in equilibrium.

general equilibrium model where families decide whether or not to go to college, and if they go they need to decide which college they will attend. The market of higher education operates under monopolistic competition, as is common in the literature (Epple, Romano and Sieg, 2006), in a two-tier system where students who graduate from elite colleges have higher rewards in the labor market than students who go to the non-elite system. Colleges maximize the quality of the services offered where such quality depends on the skills of their student body and on the investment per student made by the university. Their objective is to chose skills thresholds for admission, tuition rates, and investment per students, while maintaining a balanced budget in order to maximize the quality of the services offered. We characterize the demand for higher education, the incentives to invest and admit students by each tier-college, as well as the consequences of loan policies in the equilibria arising from such setting.

We find a set of equilibria such that once the subsidized loan policy is implemented, colleges change the quality of services provided in a way such that the gap in quality provided by elite and non-elite institutions becomes wider. Elite universities will face a higher demand for services as a consequence of the subsidized loan policy. They will be able to increase the minimum level of skills required for acceptance while maintaining budget balancedness. Increasing the quality of their student body will make elite-universities invest more in their students so long as these two inputs are complements in the provision of quality provided by col-

leges.. This will lead to a higher segmentation of the market for higher education as high-ability students will attend elite colleges and low-ability students will be concentrated in the non-elite system. In such case, least able students will not be exposed to high-ability students, limiting the extent to which they benefit from their peer effects. Additionally, the marginal productivity of investments per students will decrease in non-elite colleges as the quality of their student body deteriorates, decreasing the incentives by non-elite colleges to invest in their students. This ends up decreasing the gains from higher education for students attending non-elite universities, making subsidized-loan policies a regressive policy in the overall economy.

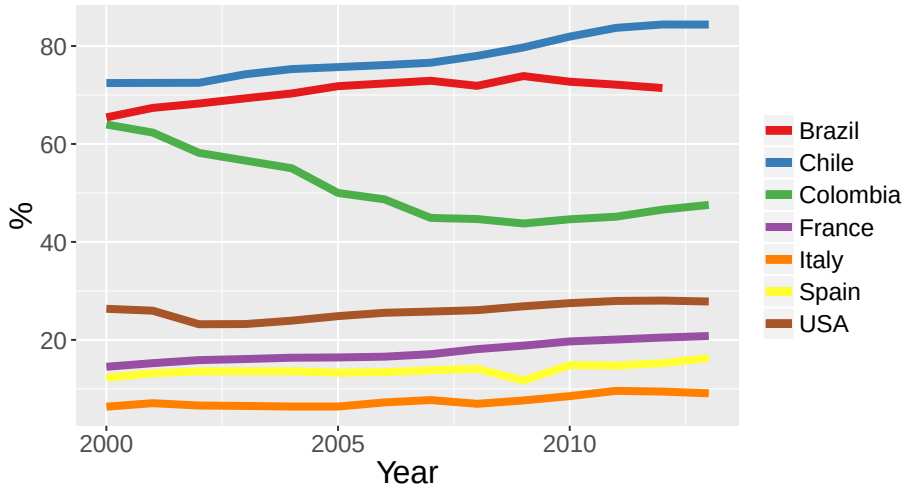
The rationale behind the widening gap in quality comes from the fact that the market for higher education operates under monopolistic competition and each tier will serve a different segment of the market once the subsidized loan policy has been implemented. Under such circumstances, universities have incentives to differentiate their product -quality offered-, given the differential effect that such policy has on the segments of the market that each university serves. On one hand, the size of the segment that attends the low-quality college is ambiguous after the policy is implemented. It can expand, as people that could not study before can now afford to attend the low-quality college, but can also contract, given that some individuals that prefer the high-quality college can now afford to pay for better education. On the other hand, the demand for the elite college unambiguously expands, since now some individuals can afford its services and no individuals will stop attending it

because of the policy. The larger increase in the demand for the elite college, as opposed to the non-elite college, allows the former to increase prices per student more than the latter. With more resources, the high-quality college will find it optimal to increase investment per student. Moreover, if the peer effects and the investment per student are complements in the quality production function, the elite institution will increase the cut-offs, so that the average student ability is higher in equilibrium. In this way, the quality gap supplied in equilibrium widens.

Our analysis is also novel, given our focus on developing countries and the fact that the structural literature has only explored the context of the U.S., as far as our knowledge goes. The educational sector in developing economies is particularly different from that of developed economies, for three main reasons. First, in developing countries there is evidence that credit constraints play a role in determining college enrollment decisions among households (Melguizo, Sanchez and Velasco, 2015), as opposed to the case of developed countries. Second, in many developing countries the private institutions own a larger share of the market for higher education, as compared to European countries or even the U.S. (see Figure 2.1). This is important because public institutions may not be as responsive to market incentives, but rather follow the social planner's objectives. In contrast, private institutions are potentially more responsive to market signals, so any change in demand will generate stronger equilibrium effects in developing economies. Third, enrollment rates in developing countries are very low, when compared to enrollment in developed coun-

tries. As documented by Mestieri (2016), there is an existing positive correlation between enrollment rates and income per capita at a cross-country level.

Figure 2.1: % of enrollment in private institutions by country.



We use the results of the equilibria to rationalize what we observe happened in Colombia after the introduction of a massive subsidized loan policy. Colombia is a developing country that undertook massive expansions of publicly supplied student loan availability during the last decade. A contribution of this paper is the construction of a novel dataset that allows us to analyze the evolution of various measures of quality of education before and after the policy was implemented. After the introduction of such policy, we see that the number of students increased. However, with the dataset constructed we also find evidence consistent with a widening gap in the quality of elite and non-elite universities. The data reveals that the gap between elite and non-elite colleges increased in measures such as average test

scores in entry and exit examination tests, the number of professors per students and various measures of academic production such as articles published per faculty. The main purpose of this paper is to rationalize how this expansion in the gap can arise as a consequence of the subsidized loan policy previously implemented. This would imply that student loan programs also have downsides when studied in general equilibrium, and all of this should be taken into account for future policy design.

Finally, our analysis will give us tools to discuss the design of the optimal student loan policy in a context where the government has outside funds that has to allocate within the existing population. From a partial equilibrium perspective, we find that the student loan policy that maximizes the average utility of society (and enrollment) is one that gives priority to the lowest-ability individuals that are borrowing constrained and would like to study. The reasoning behind this is the following: high-ability individuals will receive higher incomes over their lifetime, regardless of their education level. Since marginal utility is decreasing, the benefits of studying to have additional income are relatively small. In addition, those that are borrowing constrained and study will not be able to smooth consumption. In this sense, they face a higher opportunity cost of education. Therefore, relaxing the constraint for high ability individuals will change the study decision of fewer individuals, than if the constraint were relaxed among lower ability individuals.

In a general equilibrium setting, there is an opposing force in action. Individuals

choose whether and where to study according to the quality offered by each college, since their future earnings will depend on it. The quality offered by colleges, in turn, is a composite of the average ability of the student body (to resemble peer effects) and investments per student made by the college. As assumed by the literature, these two inputs are complements in the quality production function. In this regard, the best quality-enhancing student loan policy would maximize the average student ability, which would increase returns to investments per student, and would lead to higher quality offered. That is, loosening credit constraints to the most able students would lead to higher education quality.

In this regard, there are two opposing forces shaping the optimal student loan policy. Relaxing the borrowing constraints of lower ability households will have the highest impact on school enrollment, but will reduce the returns to investments per student done by schools and the quality of education. In contrast, relaxing the borrowing limits of high ability households maximizes the education quality offered, but enrollment is not as large. The issue becomes even more complex once we incorporate a two-tier education system, in which colleges might respond differently in their pricing, admissions and investment policies when faced to a demand shock of this nature.

The rest of the paper is organized as follows. Section 2.2 describes the literature relevant to our paper. Section 2.3 describes an model of the market for higher education, characterizes the demand for a two-tiered education system and explains

the mechanism through which borrowing constraints affect equilibrium quality supplied. In this section we illustrate the main theoretical results of the paper. That is, a policy leading to subsidized loans can increase the gap of quality of education. Section 2.4 describes the case of Colombia and illustrates how this case is consistent with what we predict in the theoretical model. Section 2.5 estimates the parameters of the quality function, using data on average test scores, professors per student and average wages of students from each university in Colombia. Section 2.5.2 describes the computation of the model and the parameters used for calibration. Section 2.6 shows the main results of the estimations and some counterfactual experiments. Section 2.7 concludes.

2.2 Literature Review

Different strands of the literature are related to the present research. First, our paper is related to the literature that studies the importance of borrowing constraints in the schooling decisions. Given that we are studying the welfare effects of government loan policies in developing countries, knowing whether borrowing constraints matter is of central importance. Although there is evidence suggesting that borrowing constraints do not determine school attendance of students in developed countries, the opposite is the case for countries in the developing world. Second, our paper is related to the literature that studies general equilibria in the market for education. This literature has mostly studied what is known as the

"Bennett Hypothesis"[28]. Our paper adds to this literature because we study equilibrium welfare effects that go beyond prices in the context of binding borrowing constraints, whereas this literature has been concerned only with studying equilibria in the United States, where the inefficiency caused by the existence of borrowing constraints in education is, at best, small. Finally, our paper models the educational sector following the treatment that the industrial organization literature has given to universities, as monopolistically competitive firms that maximize the quality provided, subject to a budget balance constraint.

2.2.1 The Role of Borrowing Constraints

Using data from the United States, there is a wide literature that argues that the effects of borrowing constraints on the post-secondary decisions of youngsters are negligible. Using the 1979 of the National Longitudinal Survey of Youth (NLSY79), Carneiro and Heckman (2002) find evidence that borrowing constraints and family income account for a very small fraction of post-secondary school attendance decisions, while early childhood differences are determinant. Once the estimations control for family background and ability, current family wealth becomes unimportant in the schooling decision. According to their estimates, only between 0% and 8% of high school graduates are actually borrowing constrained. Similarly, Keane and Wolpin (2001) find that, although borrowing constraints are tight in the U.S.

[28]The *"Bennett Hypothesis"* states that an expansion in the number of grants provided to students are almost totally appropriated by colleges through increases in tuition prices.

and individuals cannot even borrow the amount to cover one year of schooling, their existence does not determine the decision to study. In counterfactual experiments, when the authors remove the borrowing constraints, the educational attainment does not change significantly. Borrowing constraints only affect labor supply and savings decisions of students.

Dinarsky (2003) measures the impact of the exogenous elimination of the Social Security Student Benefit Program in 1982 on school attendance in the U.S. This program provided students that came from families with deceased, disabled, or retired Social Security beneficiaries with monthly payments while enrolled in college. The paper finds that the exogenous reduction in aid led to a decrease in the probability of being enrolled by students at the margin. However, the author argues that this cannot be interpreted as existence of borrowing constraints, since grants do not only relax the borrowing constraints of households, but change also the relative price of education.

More recent studies argue that, although credit constraints did not seem to affect the schooling decision some decades ago, during the last two decades they might be playing an important role in post-secondary schooling in the United States. Using data from the 1979 and 1997 National Longitudinal Survey of Youth (NLSY79 and NLSY97), Belley and Lochner (2007) find a dramatic increase in the importance of family income on school attainment, after controlling for family background and ability as in the previous studies. Similarly, Lochner and Monge-Naranjo (2011)

estimate a structural model that suggests that, although American households were not borrowing constrained during the 1980s, during the last decade family income has been determinant in schooling decisions. They argue that in the last two decades there have been rising costs and returns to education, while government student loan programs have not grown at the same pace, so people have become borrowing constrained.

Although there is not much research that studies the role of credit constraints in educational choices in developing countries, the existing evidence seems to unambiguously point towards the importance of borrowing constraints in the educational decisions. As Attanasio and Kaufmann (2009) state, *"one important difference between Mexico and the U.S., for instance, might be the wider availability of scholarships and student loans in the U.S., cannot be found in Mexico for higher education."* Attanasio and Kaufmann (2009) and Kaufmann (2014) provide evidence suggesting that liquidity constraints do determine the post-secondary schooling decision in Mexico. They use data that characterizes the expected returns of education for every household in their sample. If credit constraints were not binding, there should exist a positive gradient between subjective expected returns from schooling and school attendance. Their results show that this gradient breaks for the lowest income households in their sample. Under their interpretation, this is evidence of existing borrowing constraints. Solis (2013) studies the existence of borrowing constraints in Chile. Using administrative data on the entire sample of individuals

that participate in the college admissions' process, he uses a regression discontinuity approach to study the impact of providing educational loans. After controlling for socio-economic covariates, individuals right above the eligibility threshold for receiving educational loans have a significantly higher probability of enrolling in college than those right below the threshold. The author finds evidence suggesting a positive gradient between income and enrolment among those households that have no access to the government loans. This gradient disappears for individuals that access the program. Also using a regression discontinuity approach, Gurgand, Lorencau and Melonio (2011) find evidence that the enrolment to college of households without access to student loans is 20 percentage points lower in South Africa. Regarding the Colombian case, Melguizo, Sanchez and Velasco (2015) find evidence that the implementation of a massive government loan program in the past decade, which is the topic of this paper, did increase student enrolment.

2.2.2 General Equilibrium Effects and the Bennett Hypothesis

During the last decade, the literature that has tried to explain what has become to be known as the *Bennett Hypothesis*: expansions of government-supplied student aid for education have been almost almost entirely appropriated by colleges through an increase in tuition prices. As the former U.S. Secretary of Education stated in 1987, *"If anything, increases in financial aid in recent years have enabled colleges and universities blithely to raise their tuitions, confident that Federal loan subsidies*

would help cushion the increase."[29] Singell and Stone (2007) study the effect that Pell Grants have had on tuition prices of public and private schools. They study the Pell Program, which has been the biggest post-secondary educational loan program in the United States. In 1999, the Pell Grants were awarded to 3 million students across more than 6000 colleges, out of a total of 9 million students. The authors estimate the impact of Pell Grants per student on tuition charged by universities, using a panel of 1554 colleges from 1989 to 1996. They find that the increase in Pell Grants caused an almost one-to-one increase in the price of tuition charged by private and public out-of-state colleges. However, they find no such a causality on the in-state tuition charged by public schools. In contrast, Rizzo and Ehrenberg (2002) find evidence that private and public out-of-state tuition prices were not affected by government loans, while in-state tuition by public colleges were. Finally, Lucca, Nadauld and Shen (2016) use exogenous variation in the legislation that rules Pell Grants, to study the relationship between student aid and tuition.

Gordon and Hedlund (2015) study the increase in tuition prices by estimating a structural model in which universities provide human capital and households decide their investments in education. They study the rise in college tuition over the last decades, as a reaction to cuts in state appropriations, an increase in the costs of skilled labor in other industries, and an increase in government supplied loans. The authors find out that the increase in government loans explains 102% of the tuition increase, as opposed to only 16% of the other two hypotheses. This result

[29] William Bennett to the New York Times, 1987.

provides evidence in favor of the Bennett Hypothesis. Our paper differs from theirs in the sense that we want to study the equilibrium effects on quality provided and welfare effects of relaxing borrowing constraints in a context in which they matter. The authors study increases in the borrowing limits in the context of the U.S. As has been already argued, there is evidence that these constraints are of secondary importance on the decision to attend school. Therefore, relaxing these limits does not improve efficiency. In contrast, in countries in which the borrowing constraints are binding, relaxing them does generate efficiency improvements.

2.2.3 The Education firms

Our paper makes part of the literature that models universities as firms in the educational sector. Universities produce human capital and use households both as inputs and costumers. This approach has been used to study different questions regarding post-secondary education. For instance, Chade, Lewis and Smith (2014) model the universities as an oligopoly with a fixed number of universities (firms), in which the goods produced by universities (education) are ranked exogenously by all households in the same way. Universities only choose admission standards, so as to fill a fixed capacity of students and maximize the ability of the student body. The paper studies the role of frictions in the application process on the student sorting between universities. Namely, the model has information frictions and fixed costs of application. The authors, as Caucutt (2001), treat the utility that households

receive of attending each of the universities as exogenous and independent from the product offered by each university. We endogenize the valuation of households as a function of the equilibrium quality offered. The authors do not include tuition prices as a policy of universities, assume an exogenous valuation for the universities and take the size of universities as fixed. We depart from all of these assumptions, but assume there are no frictions in the application process. The reason is that our purpose is not to study the outcome of the application process but, instead, model the strategic interactions in the post-secondary education sector between universities and households.

The educational sector in our model closely mimics Epple, Romano and Sieg (2006). In their paper, the authors model the supply side of the educational sector as an oligopoly sector in which a fixed amount of colleges interact to attract students and maximize the quality of the education they offer, subject to a balanced budget constraint. Quality by universities is a composite of average student ability, to resemble peer effects in schooling, and the average investments per student. This treatment of quality has been standard in the literature that models schools (Caucutt, 2001). Households value quality as an input on their utility function. In their model, households play a passive role in their model, since their purpose is not to estimate equilibrium interactions between households and firms. Rather, they concentrate in studying thoroughly the supply side. Furthermore, they estimate their model by using a "club goods" approach, instead of explicitly solving

the Nash equilibrium of the monopolistically competitive market. We depart from this approach, since we consider that the strategic interactions between colleges are of first order importance to explain the different reaction of elite and non-elite institutions to subsidized student loan policies.

Finally, we treat wages of college graduates as a function of the quality supplied by the school attended. To the best of our knowledge, this approach has not been used in structural estimations in the past, but there is empirical evidence that relates future wages to the quality of the education. Black and Smith (2006) estimate a latent model in which quality is a latent variable, and there are "signals" of quality. They find out that SAT scores, faculty-student ratio, rejection rate, freshmen retention rate, and faculty salaries are significant signals of quality. Furthermore, the latent variable of quality significantly affects post-college wages of individuals. Similarly, Black, Smith and Daniel (2005) find evidence that quality increases post college earnings, driven by higher wages. Leaving quality aside, there is extensive evidence that estimates positive returns to college attendance in terms of higher future wages (Zimmerman, 2014; Patrinos, Ridao-Cano and Sakellariou, 2006). OECD and World Bank (2012) estimates that average starting earnings for individuals with a bachelor's degree were 4 times higher than those of individuals with high-school degree. Although these estimates do not control for unobservable household characteristics, other estimates find that people with post-secondary degrees earn significantly higher wages in Latin America (Gasparini et al., 2011).

2.3 A Model of the Market for Higher Education with Credit Constraints

There are two types of agents in the economy: households and universities. There is a government that offers educational credits to high-ability individuals that decide to attend college, at an exogenous interest rate $R \geq r$, where r is the risk free interest rate. In addtition, the government subsidizes the interest paid by the poorest households that access the credit, at a subsidy rate s. In order to finance these subsidies, the government levies a marginal tax, τ, to every household in the economy. The government policies are exogenous, fixed before the economy starts and satisfy budget balance. Given these policies, the market of higher education operates under monopolistic competition. Universities supply human capital in the market for education, by choosing a tuition price, a minimum ability level for admission and a level of investment per student. Given government and university policies, the households decide if they want to study in any university at the prevailing market prices.

2.3.1 Households

Households are born with innate ability and wealth (θ, b), according to a bivariate distribution $F(\theta, b)$ over the space $[0, 1] \times [\underline{b}, \bar{b}]$. Individuals live for two periods, after which they die with probability equal to one. In period 1, individuals choose either to study at the university or work in the non-skilled labor market at a wage

w per efficiency unit of labor. Individuals that do not study receive a wage θw, do not have access to credit markets and can save at the risk-free rate r. There are two universities in the economy denoted by h and l. Each university sets a threshold $\underline{\theta}^j$ for $j = h, l$ such that only students that have ability $\theta \geq \underline{\theta}^j$ are admitted to university j, and we assume that this information is public[30]. Therefore, individuals with $\theta < \min\{\underline{\theta}^h, \underline{\theta}^l\}$ cannot study and have to work. Individuals who decide to study at university j cannot work, and have to pay a tuition, P^j, set by the university.

In order to finance education, the government offers student loans of up to the price of the tuition, P^j, at the interest rate R to people that decide to study and have an ability level $\theta \geq \theta_{min}$. In addition, students with low wealth, $b \leq b_{max}$, that decide to study and have access to the loan will receive a subsidy on the interest rate, s. Loans are given conditional on studying, and individuals that study and are eligible for the loan choose whether to borrow from the government or not. In order to finance these subsidies, the government levies a proportional tax, τ, to every individual in the economy. Individuals for which $\theta < \theta_{min}$ are borrowing constrained and can only finance education with their initial wealth. Therefore, in the first period the household decides its level of consumption, c, whether to study or not in any university, h, l, and the level of savings, a, which can be potentially negative for households that study and satisfy the government conditions for the

[30]We assume that $\underline{\theta}^j, j = h, l$ is a public threshold, since our purpose is not to study the frictions in the college application process, as opposed to some papers in the literature that model explicitly these information frictions (Chade, Lewis and Smith, 2014; Fu, 2014).

educational loans.

In the second period, the households are either non-, low- or high-skilled, depending on whether they decided to study in the first period and which college they attended. Those who decided to study in period 1, will enter the j-skilled labor market in period 2, and receive a wage equal to $w\theta(1+z^j)$, where z^j is a skill premium that is university specific. This quality is an equilibrium object that depends on the quality of the student body and investments per student, and is fully characterized in the next section. We assume that individuals have perfect foresight of the value of z^j for $j=h,l$ when they optimize. Individuals who do not study will become part of the non-skilled labor force at a wage $w\theta$. We exclude the possibility of default in the model by assuming that repayment is fully enforced, so in the second period individuals that have government debt will repay their student loan. Given prices R, r, w, government policies τ, s, university policies $\{\underline{\theta^j}, P^j\}_{j=h,l}$, and perfect foresight about education quality $\{z^l, z^h\}$, a household that is eligible for studying at the university j, $\theta \geq \underline{\theta}^j$, and decides to study gets a utility equal to:

$$V^j(\theta, b) = \max_{c,a} \quad u(c) + \beta u(c'), \quad \text{s.t.} \tag{2.3.1}$$

$$c + a + P^j = b \cdot (1-\tau) \tag{2.3.2}$$

$$c' = a(1+r) \cdot \mathbb{1}_{\{a \geq 0\}} + a(1+\tilde{R}) \cdot \mathbb{1}_{\{a<0\}} + w\theta(1+z^j) \tag{2.3.3}$$

$$\tilde{R} = \begin{cases} R(1-s) & \text{if } b \leq b_{max} \\ R & \text{if } b > b_{max} \end{cases} \tag{2.3.4}$$

$$a \geq -\mathbb{1}_{\{\theta \geq \theta_{min}\}} \cdot P^j, \quad c \geq 0, \quad c' \geq 0 \tag{2.3.5}$$

Individuals that decide not to study, get the following utility:

$$V^N(\theta, b) = \max_{c,a} \quad u(c) + \beta u(c'), \quad \text{s.t.} \tag{2.3.6}$$

$$c + a = b \cdot (1 - \tau) + w\theta \tag{2.3.7}$$

$$c' = a(1 + r) + w\theta \tag{2.3.8}$$

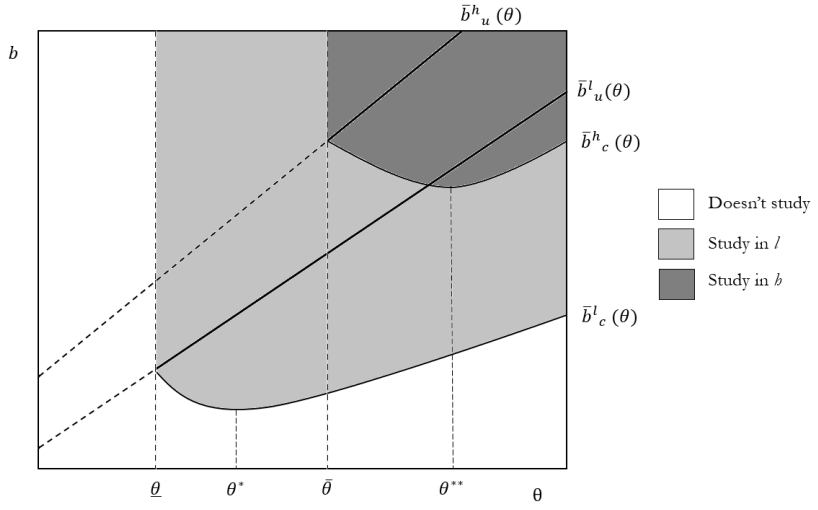
$$a \geq 0, \quad c \geq 0, \quad c' \geq 0 \tag{2.3.9}$$

The individual with ability and wealth (θ, b) decides to study at university j whenever $\theta \geq \underline{\theta}^j$ and $V^j(\theta, b) \geq V^N(\theta, b)$, and $V^j(\theta, b) \geq V^{-j}(\theta, b)$ if they can attend to the other university $-j$, i.e. $\theta \geq \underline{\theta}^{-j}$. Otherwise, the individual decides not to study. Therefore, the household's value function is given by:

$$V(\theta, b) = \begin{cases} \max\{V^h(\theta, b), V^l(\theta, b), V^N(\theta, b)\} \text{ if } \theta \geq \max\{\underline{\theta}^h, \underline{\theta}^l\} \\ \max\{V^j(\theta, b), V^N(\theta, b)\} \text{ if } \underline{\theta}^{-j} > \theta \geq \underline{\theta}^j \\ V^N(\theta, b) \text{ if } \theta < \min\{\underline{\theta}^h, \underline{\theta}^l\} \end{cases}$$

The following section gives a detailed characterization of the demand for both tiers of schools in the state space. This characterization will allow us to give insights on the optimal student loan policy on a monopolistically competitive market.

Figure 2.2: Representation of the education decisions on the state space.



Characterization of the Demand

For a given set of initial parameters, the shaded region in Figure 2.2 illustrates the individuals that choose to study in the state space when both universities set their acceptance threshold to 0 and there are no government-supplied student loans. The following sequence of theorems characterize the demand for college education on the state space, and its close relationship with borrowing constraints. This will let us derive some results about the socially optimal student loan policy. First, we describe the college decision for households that are unconstrained.

Theorem 1. *Among the unconstrained households, the decision of whether and where to study is independent of initial wealth, b, and follows a cut-off rule on θ. That is, there exist $\underline{\theta}$ and $\bar{\theta}$ such that:*

- *If $\theta \leq \underline{\theta}$, the individual will not study.*

- *If $\underline{\theta} \leq \theta \leq \bar{\theta}$, the individual will attend the low-quality college.*

- *If $\bar{\theta} \leq \theta$, the individual will attend the high-quality college.*

where:

$$\bar{\theta_l} = \frac{1+r}{w}\left(\frac{P_l}{z_l - (1+r)}\right), \quad \bar{\theta_h} = \frac{1+r}{w}\left(\frac{P_h - P_l}{z_h - z_l}\right)$$

Proof. See Proof B.0.1 □

Theorem 1 is a result of the fact that ability θ, unskilled labor w and quality of the school attended z_j are complements. In particular, this complementarity implies: *a*) among the unconstrained individuals, those with higher ability face higher marginal returns of education, so will choose, *ceteris paribus*, a higher quality school for a given wealth, *b*) as the wages of unskilled labor w increase, the marginal returns to education rise for every θ, so marginal individuals will shift to higher levels of education, *c*) if college j, for $j \in \{l, h\}$, increases its price P_j or reduces its quality z_j, marginal individuals will change their schooling decision in the expected direction. That is, if P_j increases or z_j decreases, marginal individuals will change their decision of attending school j. Finally, *d*) if the interest rate r increases, present consumption becomes more valuable than future consumption, so marginal individuals will reduce their present expenditures in education. Theorem 2 characterizes the individuals that, given their decision to attend college j, are borrowing constrained.

Theorem 2. *Given an ability θ, there exist cut-offs, $\bar{b}_u^j(\theta)$, $j \in \{N, l, h\}$, on the initial wealth, such that individuals with $b \geq \bar{b}_u^j(\theta)$ that attend college j will not be borrowing constrained. Individuals that attend college j and have $b < \bar{b}^j(\theta)$ will be borrowing constrained and will not be able to smooth consumption over time. The cut-offs are linear, increasing in θ and take the form:*

$$\bar{b_u^N}(\theta) = -\bar{A}(1 + (\beta(1+r))^{-1/\sigma}(1+r)) - w\theta(1 - (\beta(1+r))^{-1/\sigma})$$

$$\bar{b_u^l}(\theta) = P_l + (\beta(1+r))^{-1/\sigma} w\theta(1+z_l) - \bar{A}(1 + (\beta(1+r))^{-1/\sigma}(1+r))$$

$$\bar{b_u^h}(\theta) = P_h + (\beta(1+r))^{-1/\sigma} w\theta(1+z_h) - \bar{A}(1 + (\beta(1+r))^{-1/\sigma}(1+r))$$

Proof. See Proof B.0.2 □

Given a level of education and initial wealth, individuals with a higher θ have higher lifetime income and in an unconstrained world would consume more in every period of their lives. Given the existence of a borrowing limit $\bar{A}$, for a sufficiently high θ individuals will be borrowing constrained. As a consequence, the initial wealth that individuals must have not to be borrowing constrained is increasing in ability. Figure 2.2 illustrates the cut-off functions $\bar{b}_u^j(\theta)$ on the state space. As illustrated, individuals above the $\bar{b}_u^j(\theta)$ function, will decide to study in college j whenever her θ falls inside the corresponding interval in the cut-offs defined in Theorem 1. Note also that individuals that are borrowing constrained when studying at

college l will also be borrowing constrained when studying in h, assuming a higher price of education in the high-quality college (which, of course, is an equilibrium object). Moreover, the functions $\bar{b}_u^j(\theta)$ are steeper when the quality z_j increases, since quality of schooling and ability are complements. Finally, we do not consider the case in which individuals are borrowing constrained when they do not study. Since in our context, individuals that do not study earn the same wage in every period, they will only be borrowing constrained when the interest rate $\beta(1+r) << 1$. However, for a reasonable calibration, individuals will be able to smooth consumption. The following two theorems illustrate the study decision of individuals that are borrowing constrained.

Theorem 3. *Given ability θ, the decision to study in the low-quality college, l, or not study at all, follows a cut-off strategy on b, such that individuals with $b \geq \bar{b}_c^l(\theta)$ will attend college l, and those with $b < \bar{b}_c^l(\theta)$ will not study. The cut-off is characterized implicitly by equation (B.0.1) in the proof. Moreover, if the intertemporal elasiticity of substitution is lower than 1 the cutoff is U-shaped and there exists a θ^* such that $\bar{b}_c^l(\theta)$ is:*

- *decreasing in θ for $\theta \leq \theta^*$*
- *increasing in θ for $\theta \geq \theta^*$*

where θ^* solves:

$$\left(\frac{1}{1-\sigma}\right)(b(\theta^*) - P_l + \bar{A})^{1-\sigma} + \left(\frac{\beta}{1-\sigma}\right)(w\theta^*(1+z_l) - (1+r)\bar{A})^{1-\sigma} -$$

$$\Phi(w\theta^*(2+r)+b(\theta^*)(1+r))^{1-\sigma}=0$$

$$b(\theta)=\theta\left[\frac{wX(1+z_l)-w(2+r)}{1+r}\right]-X\bar{A}$$

$$X=\left[\frac{\Phi(1-\sigma)(2+r)}{\beta(1+z_l)}\right]^{1/\sigma}$$

Proof. See Proof B.0.3. □

The cut-off $\bar{b}_c^l(\theta)$ is illustrated in Figure 2.2, where we assume that the utility function is CRRA with $\sigma=2$, as is common in the literature, so the intertemporal elasticity is lower than 1. The individuals who are constrained (below $\bar{b}_u^l(\theta)$) will choose either to study at l or not, if their initial wealth exceeds $\bar{b}_c^l(\theta)$. The cut-off is U-shaped because two effects are in action. First, the "complementarity" effect means that, given a b, individuals with higher θ will have higher marginal returns from studying, so are willing to study even though they will not be able to smooth consumption. Therefore, the cut-off is initially decreasing. However, the "constrainedness" effect dominates after some point: given an initial wealth b, individuals with higher θ will face a larger wedge in their Euler equation, meaning that they will be able to smooth consumption to a lower extent. When the wedge is large enough, individuals will prefer not to study and smooth consumption by deciding not to study. Of course, this results strongly depends on the value of σ chosen, and continues to hold for any $\sigma>1$. For the sake of exposition, in Appendix B.0.7 we characterize the demand for education with a linear utility function (that is, when $\sigma=0$ and there is an infinite elasticity of substitution). Figure B.1 illustrates

the decision of individuals in the state space. As can be expected, in the linear case individuals derive no utility from consumption smoothing, so there does not exist such a "constrainedness" effect. In this case, the threshold is never increasing. The next theorem characterizes the cut-off for individuals that are constrained when studying at h. The results are parallel to Theorem 3.

Theorem 4. *Given ability θ, the decision to study in h or l, follows a cut-off strategy on b, such that individuals with $b \geq \bar{b}_c^h(\theta)$ will attend college h, and those with $b < \bar{b}_c^h(\theta)$ will attend l. The cut-off is characterized implicitly by equation (B.0.2) in the proof. Moreover, if the intertemporal elasiticity of substitution is lower than 1 the cutoff is U-shaped and there exists a θ^{**} such that $\bar{b}_c^h(\theta)$ is:*

- *decreasing in θ for $\theta \leq \theta^{**}$*
- *increasing in θ for $\theta \geq \theta^{**}$*

where θ^{**} solves:

$$\left(\frac{1}{1-\sigma}\right)\left(b^*(\theta^{**}) - P_h + \bar{A}\right)^{(1-\sigma)} + \left(\frac{\beta}{1-\sigma}\right)\left(w\theta^{**}(1+z^h) - (1+r)\bar{A}\right) -$$

$$\Phi \times (w\theta^{**}(1+z_l) + b(1+r) - P_l(1+r)) = 0$$

$$b^*(\theta) = \theta w\left(X^*(1+z^h) - (1+z_l)\right) - X^* + P_l$$

$$X^* = \left(\frac{\Phi \times (1-\sigma)(1+z_l)}{\beta(1+z_h)}\right)^{1/\sigma}$$

Proof. See Proof B.0.4. □

Having characterized the demand for education in the state space, we can say a couple of things about the relationship between borrowing constraints and the demand. The following result describes the differential effect of relaxing the borrowing limits to households, $\bar{A}$.

Theorem 5 (Borrowing constraints)**.** *If the intertemporal elasticity of substitution is lower than 1, for any given θ the cut-offs $\bar{b}_c^l(\theta)$ and $\bar{b}_c^h(\theta)$ are decreasing on $\bar{A}$. Moreover, the elasticities of $\bar{b}_c^l(\theta)$ and $\bar{b}_c^h(\theta)$ with respect to the borrowing limit $\bar{A}$ are decreasing on θ, meaning that a relaxation of the borrowing constraint has a higher impact on enrollment among the marginal individuals that have lower θ.*

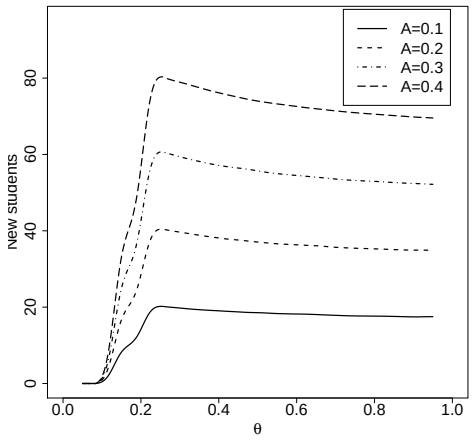
Proof. See Proof B.0.5. □

Theorem 5 states that among the constrained individuals, those with lower θ are more sensitive to relaxing the borrowing constraints. That is, if the borrowing constraints were relaxed by the same amount to all the individuals, more low-θ individuals would change their study decision. This result is a consequence of the decreasing marginal utility. Individuals with high θ and sufficiently low initial wealth have a trade-off between earning relatively high wages in every period and smoothing consumption it they do not study, or studying to earn large wages in the second period at the expense of a very low consumption in the first period. However, because of decreasing marginal utility, the utility of a very large wage in the second period is not as large as for lower θ individuals, so individuals will optimally decide to study only when there is a large increase in the borrowing limits

of the first period.

This result has very important implications on the design of an optimal student loan policy in a partial equilibrium setting. If the objective of the government is to maximize enrollment, the policy should target the lower ability individuals. As a matter of illustration of Theorem 5, Figure **??** illustrates the number of individuals of ability θ that change their study decision as the borrowing constraint is relaxed from $\bar{A} = 0$. As stated in Theorem 5, the individuals in the state space with low ability that would study in the unconstrained world (those with $\theta \in [\bar{\theta}_l, \bar{\theta}_h]$) are more sensitive to relaxing borrowing constraints. Therefore, increasing the borrowing capacity increases enrollment more among the low ability individuals.

Figure 2.3: Number of students that change their study decision when borrowing constraints change from $\bar{A} = 0$ to $\bar{A}$, by ability θ.



2.3.2 Universities

Universities act as firms that maximize an objective function. Given that university systems in most countries are non-profit firms, we follow the literature on education and industrial organization and assume that universities maximize a composite of the quality they offer to students, denoted by z, and the economic diversity of their student body, subject to a budget constraint. Quality offered by universities is an abstract concept. The literature has argued that the quality offered by a school is determined both by the quality of the student body and the investments per student done by the school. Epple, Romano and Sieg (2006), for instance, model the objective function of the university as a composite of the average ability of the student body, the investment per student and the inverse of the mean income. They argue that there is empirical and anecdotal evidence that shows that colleges engage in policies to attract low income students. Universities take as given the values of τ, s, R, w and the distribution F. Additionally, we assume that universities set their policies simultaneously and so, the pricing and admission policies set in equilibrium should satisfy the no profitable one shot deviation principle.

University j takes as given $(\tau, r, s, R, w, P^{-j}, \underline{\theta^{-j}})$ and will set the pricing and admission policies $(P^j, \underline{\theta^j})$ in order to solve the following problem:

$$\max_{\{P^j, \underline{\theta^j}, I^j\}} \left(z^j\right)^{\alpha} \left(\sigma_b^j\right)^{1-\alpha} \tag{2.3.10}$$

subject to:

$$z^j = \tilde{\theta^j}^{\alpha_1}(I^j)^{\alpha_2} \tag{2.3.11}$$

$$\tilde{\theta^j} = \int_{\Theta \times B} \theta \cdot e^j(\theta, b) dF(\theta, b) \tag{2.3.12}$$

$$I^j \cdot N^j + V^j(N^j) + C^j = P^j \cdot N^j + E^j \tag{2.3.13}$$

$$N^j = \int_{\Theta \times B} e^j(\theta, b) dF(\theta, b) \tag{2.3.14}$$

where $\tilde{\theta^j}$ is the average ability of the individuals that attend school j. σ_b^j is the inverse of the average income of the student body and reflects the fact that universities care about the diversity in their student body. $e^j(b, \theta)$ indicates with values zero or one if a student with ability θ and wealth b decides to study or not. I^j is the monetary amount that the university invests per student, V^j is a convex cost function, N^j is the size of the student body, C^j is a fixed cost and E^j the university's endowment. Note that the policy P^j does not depend on student's characteristics such as wealth or skills. This is not only a simplifying assumption but also follows closely the case of Colombia where private universities do not price-discriminate students based on ability or wealth. As will be discussed in the relevant section, the extent of financial aid provided by such institutions is very limited in the period of analysis.

2.3.3 Discussion

Although in principle the solution to the problem of the university might seem simple given that there are only two variables of choice, there are several elements of the model that increase the complexity of such decision. First of all, both policies are interdependent. When a university changes one decision variable -either the price or the admission threshold- this will distort the incentives faced when setting the other policy. For instance, a change in tuition price will not only change the revenue of the university but will change the demand in a way that we expect to see a change in the average ability of the student body. Such a change in the average ability of the student body will affect the marginal productivity of investments made by the university, which in turn will affect its pricing decisions.

Moreover, we need to deal with the fact that in equilibrium no university should have incentives to deviate. Given that both universities make the decision simultaneously and that there are no elements of incomplete information in the model, the relevant equilibrium concept is Nash Equilibrium: no university will have incentives to deviate given the decisions made by the other university. Note that given the nature of the problem we cannot be sure of the existence of such equilibrium -university payoffs are not continuous- and moreover, uniqueness cannot be guaranteed.

The aforementioned elements make it clear why analyzing the consequences that subsidized loan policies will have in the market of higher education is a complex

problem. Let's suppose that the government imposes such policy by subsidizing the interest rate of student loans. The first effect such policy will have is an increase in the number of students going to college. Note, however, that it is also not unreasonable to assume that the quality of the student body will change. This is because people who changed their decision to go to college are either those who were credit constrained or those having low ability levels that now decide to go to college given the decrease in the opportunity cost.

We can expect that after imposing such a policy, households will react by changing their decision of studying and universities should expect a change not only in the size of their student body but also in their quality. Given such changes, universities might want to change the prices charged to their students. This is due to the fact that as the quality of the student body changes, the productivity of investment will also be affected. Additionally, the willingness to pay for educational services is affected by such policy and universities will react to that. Moreover, universities might want to change the admission threshold either to improve the quality of their student body or to attract less able students that are willing to pay more for education. The overall effect depends on how sensitive is the demand for education with respect to the quality of services being provided.

Finally, note that -as said previously- the decisions of universities need to be analyzed in equilibrium. When deciding what is optimal, each college needs to take into account what their competitor is doing in the market and there should be no

room for profitable deviations. After imposing a subsidized loan policy we might end up in an equilibrium where one college serves a specific part of the population. For instance one college serves a large demand for students with relatively low levels of ability whereas the other one specializes in providing high quality education for a reduced number of high ability students. Additionally, we can have a symmetric equilibrium where both firms are indistinguishable from one another or one in which only one firm operates in the market.

2.3.4 Government

We do not model the government as a welfare maximizing agent in the economy. We abstract from this fact and simply analyse the impact of the change in the government policies on the higher education market. However, we do interpret the student loan policy implementation as a way of the government to reduce the existent inefficiency in the educational market.

In a social planner's solution, the efficient outcome would be one in which the high ability individuals decide to study, independent of their wealth. Thus, the role of the student loan policy can be interpreted as a way to reduce the existing inefficiency in the educational sector, although we do not model it as an optimal decision. We assume that the government has a borrowing constraint in the international borrowing markets, so is only able to finance the education of some fraction of the individuals in the economy. For now, we assume that the government finances

individuals that have $\theta \geq \theta_{min}$, and of those that can access the loans, subsidizes the interest rate on the loan for those individuals that have $b \leq b_{max}$. The government sets thresholds $\bar{b}$ and θ_0, such that

$$s \cdot (R - 1 - r) \cdot \int_{\Theta_2 \times B_2} \left(e^l(b,\theta) + e^h(b,\theta)\right) \times dF(\theta, b) = \tau \int_{\Theta \times B} b dF(\theta, b) \quad (2.3.15)$$

where $\Theta_2 \times B_2 = (\Theta_1 \times B_1) \cap ([\theta_0, 1] \times [0, \bar{b}])$ is the set of households who study and decide to take the subsidy.

Definition 6 (Competitive Equilibrium)**.** Given a set of government policies, $\tau, s, b_{max}, \theta_{min}$, and prices R, r, w, a competitive equilibrium is a set of university policies $(P^j, \underline{\theta^j}, I^j)_{j=h,l}$ and household's value function $V(\theta, b)$ and policy functions $c(\theta, b), a(\theta, b), e^h(\theta, b), e^l(\theta, b)$, such that:

1. Given $\tau, s, b_{max}, \theta_{min}$, R, r, w, and university policies $\{P^j, \underline{\theta^j}, I^j\}_{j=h,l}$, the value function $V(\theta, b)$ solves the household's problem, with $c(\theta, b), a(\theta, b)$, $e^h(\theta, b)$ and $e^l(\theta, b)$ being the corresponding policy functions.

2. For each university $j = h, l$, it should hold that given $\tau, s, b_{max}, \theta_{min}$; prices, R, r, w; policy functions $c(\theta, b), a(\theta, b), e^h(\theta, b), e^l(\theta, b)$; and policies from university $-j$, $(P^{-j}, \theta^{-j}, I^{-j})$, university j chooses policies $(P^j, \underline{\theta^j}, I^j)$ that solve the university's problem described in 2.3.10-2.3.14.

3. The government's budget is balanced (equation 2.3.15 holds).

The nature of the problem makes it hard not only to compute the competitive equilibrium but also to show its existence. Note that, by only analyzing the supply side of the market, we cannot be sure that such an equilibrium will exist in this economy. In order to compute the Nash equilibrium of the supply side of the market, we need to find pricing and admission policies that are profit-maximizing given what the policies of the other university.

The computation of such equilibrium is more involved when we note that there is an additional fixed-point problem in the computation of the equilibrium. Universities offer their students a given level of quality that needs to be self-fulfilled: the quality offered by universities will attract certain students to the market but the quality of students going to universities determines the quality offered by universities. It is not possible to use any fixed-point theorem to show existence of a fixed point in this quality self-fulfilling problem given that the necessary assumptions are not satisfied. In particular, note that the fixed-point quality problem is not continuous as whenever the low-quality university offers the same quality as the high one, all students who are beyond the ability threshold will go to the cheapest one, generating a massive exit from one university to the other one, generating a discontinuous jump in the quality being offered.

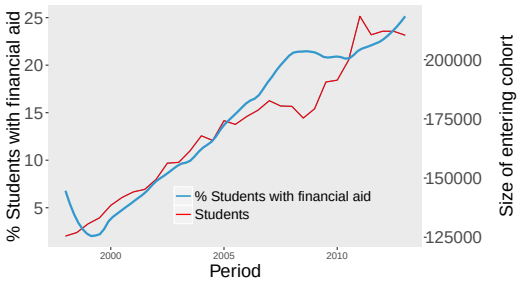
In order to illustrate this point extensively, we show in appendix B.0.7 the failure to prove existence of the equilibrium in the case of a linear utility function.

2.4 The "Revolución Educativa" of Colombia

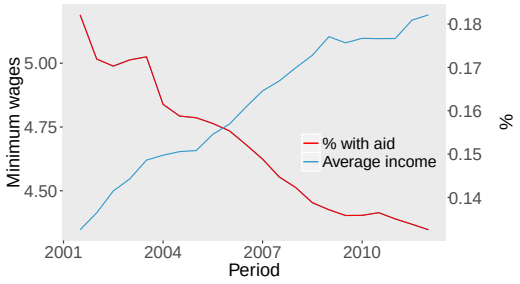
In the present research, we will use Colombia as a natural experiment of a country that implemented a rapid credit expansion program to alleviate credit constraints. Colombia is a developing country which by the beginning of last decade had low enrollment rates in post-secondary education, and significant differences in enrollment by quintiles of income. As will be argued, the majority of students came from high-income families, and the existence of financial constraints kept high-ability individuals from the lowest quintiles out of the education market. During the last decade, the government engaged into the strategy *Revolucion Educativa*, aimed at increasing the education coverage at all levels. During the decade, there were substantial increases in enrollment and educational credit access (see Figure 2.4).

Figure 2.4: Enrollment, income and financial aid.

(a) Enrollment and % of students with financial aid.



(b) Average income and % of students with financial aid.



2.4.1 Enrollment and inequality

At the beginning of last decade, college enrollment in Colombia was among the lowest in Latin America and a student financial aid system was almost non-existent. In 2000, 23.2% of the people between 18 and 23 years old enrolled in tertiary education, below the enrollment rates of Bolivia, Peru, Brazil, Chile and Venezuela, and very close to the enrollment rates of Mexico. Because of a lack of a well-functioning financial aid system, less than 5%[31] of the entering cohorts had any

[31]Extracted from the dataset of indicators for tertiary education, SPADIES, from the Ministry of Education.

kind of public or private financial support (World Bank, 2003, 2012). By the end of the decade, the enrollment rates grew to 37%, and reached 50% in 2015. The fraction of students with some type of credit increased to almost 25% of the entering cohorts (see Figure 2.4(a)).

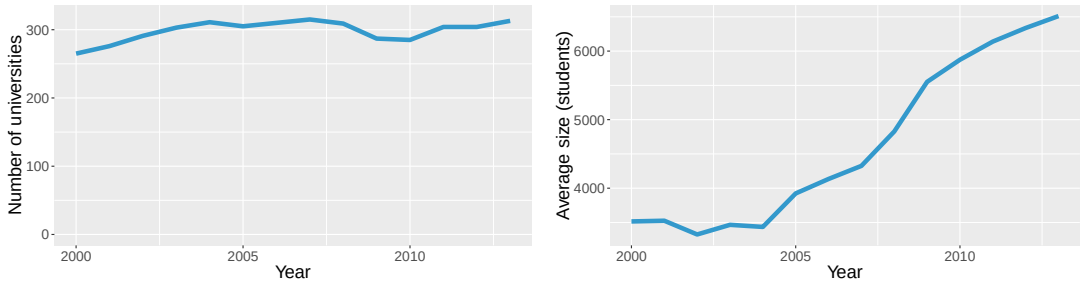
Access to education has always been unequal and, despite the fast growth of enrollment, many disparities persist. In 2013, only 45% of the low-income students graduated from high school, and only 25% of them enrolled in tertiary education. Of the high-income households, 60% graduated from high school and 54% of them enrolled in a post-secondary institution (Melguizo, Sanchez and Velasco, 2015). According to World Bank (2003, 2012), the enrollment gap between the lowest and the highest quintiles of wealth widened throughout the decade: in 2001, the enrollement rates were of 8% in the lowest quintile and 41% in the highest, while in 2010 these numbers grew to 10% and 52%, respectively. If quality is taken into account, disparities are even larger as a larger proportion of the low-income students attend non-professional institutions, which have less resources and offer lower expected income in the future. Many theories have been used to explain the low enrollment of low-income students, such as disparities in the quality of public and private high school education, the high costs of tertiary education and the lack of a well-functioning financial aid system (Melguizo, Sanchez and Velasco, 2015).

2.4.2 Higher Education institutions

The university system in Colombia functions as a monopolistically competitive market in which there are significant institutional barriers to entry, and universities do not have fixed "production capacities", as assumed by Chade, Lewis and Smith (2014) (Figure 2.5). There are approximately 300 tertiary education institutions, of which around 190 are universities, and the rest offer non-professional degrees (mainly technical and technological). Despite the growing size of the entering cohorts throughout the decade, the number of institutions remained almost constant, while the average size of each institution doubled, on average. It is important to note that around $45 - 50\%$ of the total student body is enrolled in private tertiary education institutions (OECD and World Bank, 2012). Private institutions do not have any regulations regarding the price or investment per student they offer, although they have to satisfy a minimum quality requirement in terms of the programs and degrees offered. Therefore, the education market in Colombia can be studied as a monopolistically competitive market with barriers to entry and not subject to much government regulation.

Figure 2.5: Evolution of Higher Education in Colombia

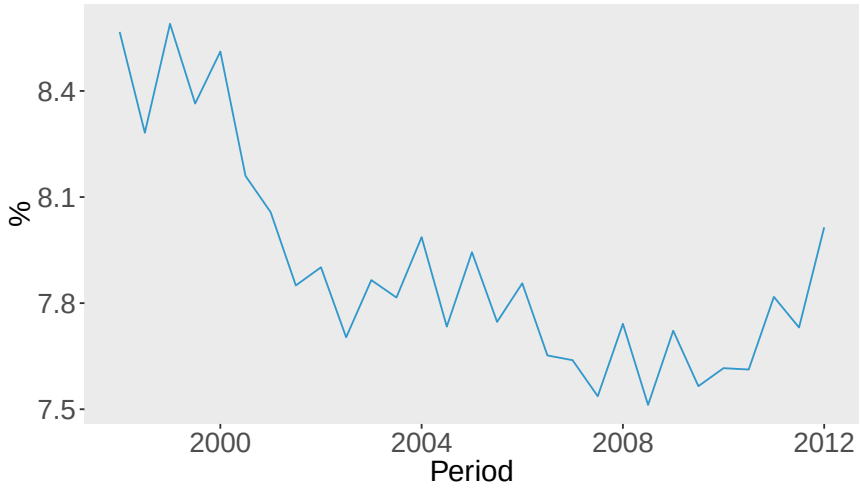
(a) Left: number of universities. (b) Right: average size of universities



In Colombia, every student that wants to graduate from high school has to present an exam called SABER11 set by the Colombian Institution for Education Evaluation (ICFES), similar to the SAT test in the U.S. Although not every tertiary education institution takes into account the results of the SABER11 in their admission decision, 78% use it as a criterion for admission (OECD and World Bank, 2012). As SABER11 has no pass-mark, each institution sets its own minimum threshold for admission. In contrast to what happens in Chile and some European countries, in Colombia there is not any institution that clears the market for admissions, so individuals apply to as many institutions as they like and universities choose their admission standards independently (Melguizo, Sanchez and Velasco, 2015). Although not perfect, the results in the SABER11 exam can be used as a proxy for the quality of the student body at universities. Figure 2.6 illustrates the average decile of the SABER11 scores of the entering cohorts to tertiary education institutions. Throughout the decade, universities seem to have adjusted their ad-

missions standards in such a way that led to a reduction in the ability of the student body, as measured by relative position in the test scores.

Figure 2.6: Average decile of ability of entering student body, measured by test scores



2.4.3 The ACCES Program

To alleviate the low access, in 2002 the government implemented the credit program *Access with Quality to Higher Education*, ACCES, with the support of the World Bank, that massively increased the available credit to students. The credit is awarded to students that have test scores above a threshold set by the government, and covers up to 75% of the tuition for the lowest income students, and up to 50% for the rest. The credit has a subsidized zero-real interest rate for the poorest households, and a real interest rate of 8% for the high-income students. Students that graduate from their programs have twice the time of their study period to repay the loan. The ACCES program has full coverage, in the sense that any student that has

test scores on the highest deciles of their region can access this credit line. The test score cut-offs vary by region, to account for disparities in the quality of secondary education across regions with different infrastructure and economic development. Given that the credit is awarded according to regional cutoffs, the disparities in the ability of people accessing the credit are large. The best students from the poorest regions might not have high ability and preparation when compared to the best students of the principal cities, so the credit is not awarded to the highest ability individuals in absolute terms.

Using a regression discontinuity approach, Melguizo, Sanchez and Velasco (2015) find evidence that the ACCES program had a positive impact on the enrollment rates, especially for individuals that come from poor households. Although the growth in the number of students enrolled in college may have been a consequence of other factors, such as better economic activity, the massive increase in financing seems to be a driving factor of such a trend.

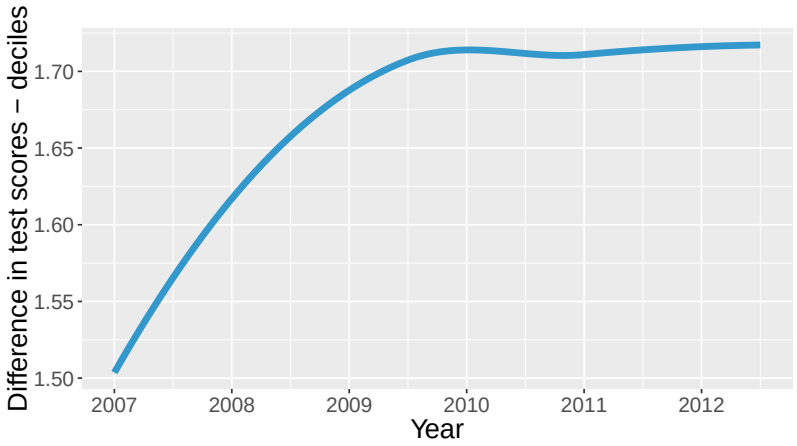
2.4.4 Product Differentiation in the Market for Higher Education

In this subsection we introduce the dataset constructed to analyze the behavior of colleges before and after the introduction of the subsidized loan program. We use administrative data from the Ministry of Education including the SABER-PRO examination scores of each college. These are major-specific examinations that are mandatory in Colombia in order to receive the equivalent of a Bachelor's degree.

Additionally, we use publicly available information scrapped from the internet regarding the academic production of professors, as well as the academic credentials of the professorial body for each university. Moreover, we build information regarding the major-specific tuition charged by each college in order to track its behavior during the last ten years.

The analysis suggests that, after the introduction of the subsidized loan policy, elite institutions engaged in significant efforts to improve the quality of services provided. All the evidence suggest that once the subsidized loan policy was introduced, the gap in quality between elite and non-elite institutions increased. Figure 2.9 shows how the composition of the entering student body in elite institutions changed during the period of reference when compared to non-elite institutions. When ranked according to the decile in the distribution they are located by the SABER-11 examination score, we find that in 2007, students entering to elite institutions where, on average, located 1.5 deciles above the average student entering to non-elite universities. After five years we see that such gap increased to 1.7 and has remained constant until the last period of data available.

Figure 2.7: Differences in quality of student body



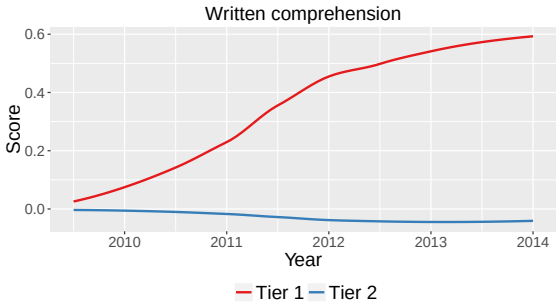
Note: Differences in the average decile of entering cohort in SABER-11 examination scores. This dataset is constructed using publicly available information provided by the Colombian Ministry of Education on its official website: http://www.mineducacion.gov.co/1759/w3-channel.html

Differences in the quality of student body are also observed when analyzing exit-level examination scores. Figure 2.8 shows the evolution of average test scores in written comprehension and reading comprehension, for students attending elite and non-elite colleges. The test scores are standardized to be mean zero and standard deviation one for every year in the dataset. Although in 2009 there were negligible differences between test scores of elite and non-elite colleges, we observe that in 2014 the average student graduating from an elite institution would score 60% of a standard deviation above the mean whereas students in non-elite institutions would score slightly below the mean. Taking into account that the average length of a bachelor's degree program lasts 4.5 years, the score for 2014 corresponds to students who were entering in 2008, approximately. This fact is consistent with the

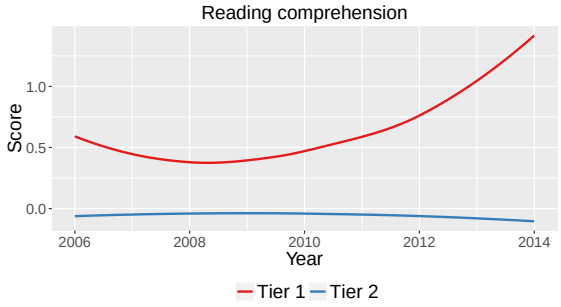
scores for reading comprehension presented in panel B of the corresponding figure. Moreover, reading comprehension exams were being done since 2006 and thus we have a longer panel allowing us to infer that no significant changes were observed until cohorts graduating after 2010.

Figure 2.8: Quality supplied by colleges.

(a) Writing score.



(b) Reading comprehension score.

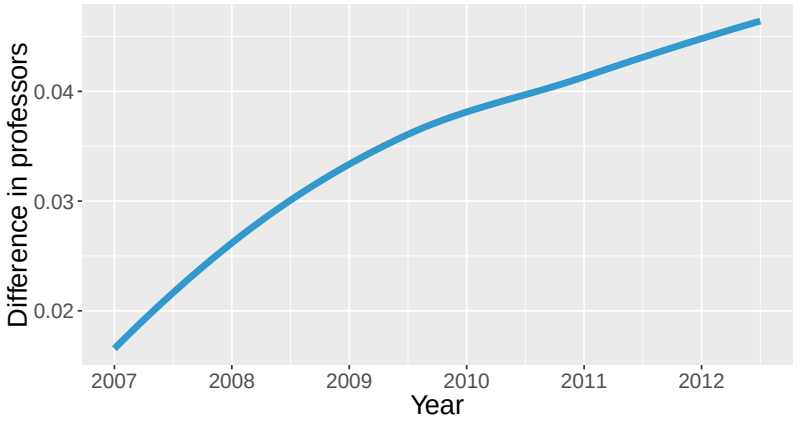


Note: Test scores are standardized to be mean zero and standard deviation 1 in every year. This information comes from the official statistics provided by the Colombian Ministry of Education.

So far we have provided evidence suggesting that the gap between elite and non-elite institutions, when it comes to the the quality of entering and exiting cohorts, increased after the introduction of the subsidized loan policy. However, the evidence suggest that the behavior of universities changed beyond the quality of their student

body. Figure 2.9 shows that during the same period, elite universities engaged in significant efforts to increase the ratio of professors per students when compared to non-elite institutions. In 2007, the difference in the ratio of professors per student between elite and non-elite institutions, was under 0.02. However, for 2013 the difference more than doubled beyond 0.05.

Figure 2.9: Difference in professors per student

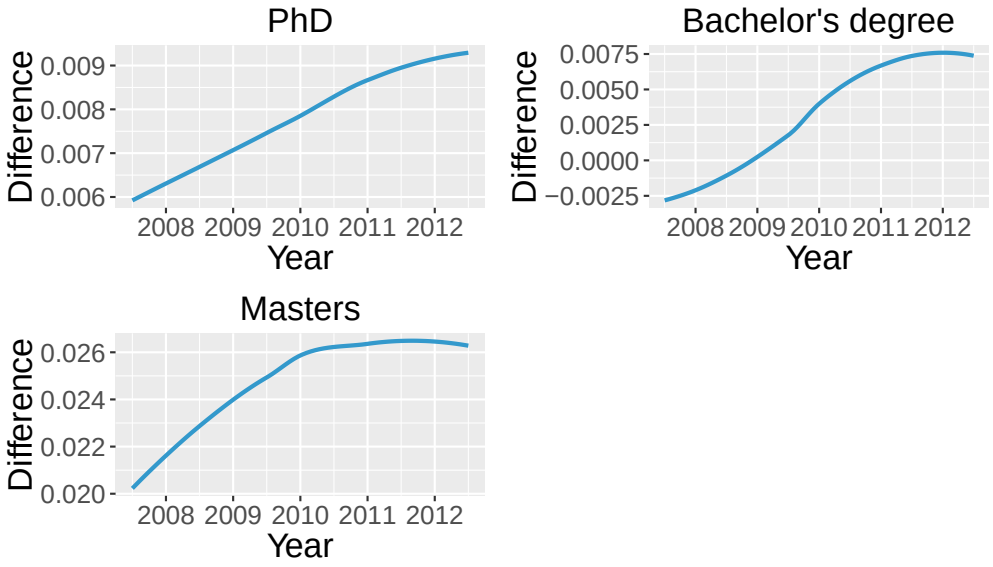


Note: This data is publicly available at the National System for Information on Higher Education website: http://www.mineducacion.gov.co/sistemasdeinformacion/1735/w3-propertyname-2672.html

We can go beyond the gross statistics of professors per student and analyze the academic credentials of the faculty composition of elite and non-elite colleges. In Colombia, it is not uncommon to see new hired faculty whose highest academic credential corresponds to a Bachelor's or a Master's degree. Taking into account this fact, the trend observed in Figure 2.9 would not imply by itself that elite institutions are making significant efforts to improve the quality of their faculty body. They might be substituting PhD professors by faculty whose highest academic credential

is a Bachelor's degree. However, In Figure 2.10 we find that the professors-student ratio of elite institutions increased when compared to non-elite institutions for every category of professors: those with a PhD, with a Master's degree, and with a Bachelor's degree.

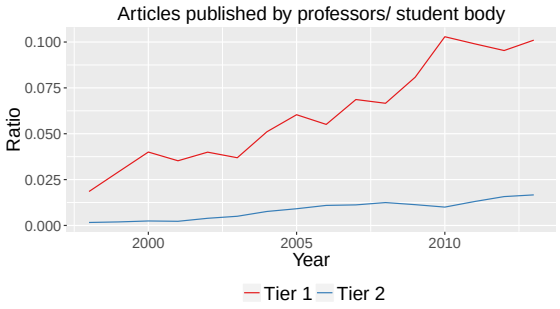
Figure 2.10: Evolution of Faculty composition. Professors per Student



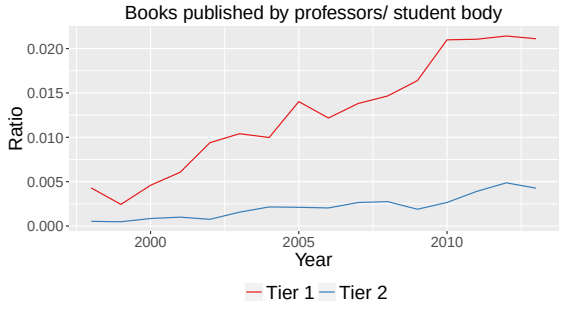
Finally, the dataset also allows us to analyze the academic production of faculty from every college in Colombia. We construct a dataset of articles published in refereed journals by authors' affiliation as well as total number of books by faculty. The results are presented in Figure 2.11. When we analyze the academic production per students, as measured by articles and books published, we also find evidence suggesting that the gap in academic production between elite and non-elite universities increased dramatically after the introduction of the subsidized loan policy.

Figure 2.11: Gap in academic production.

(a) Writing score.



(b) Reading comprehension score.

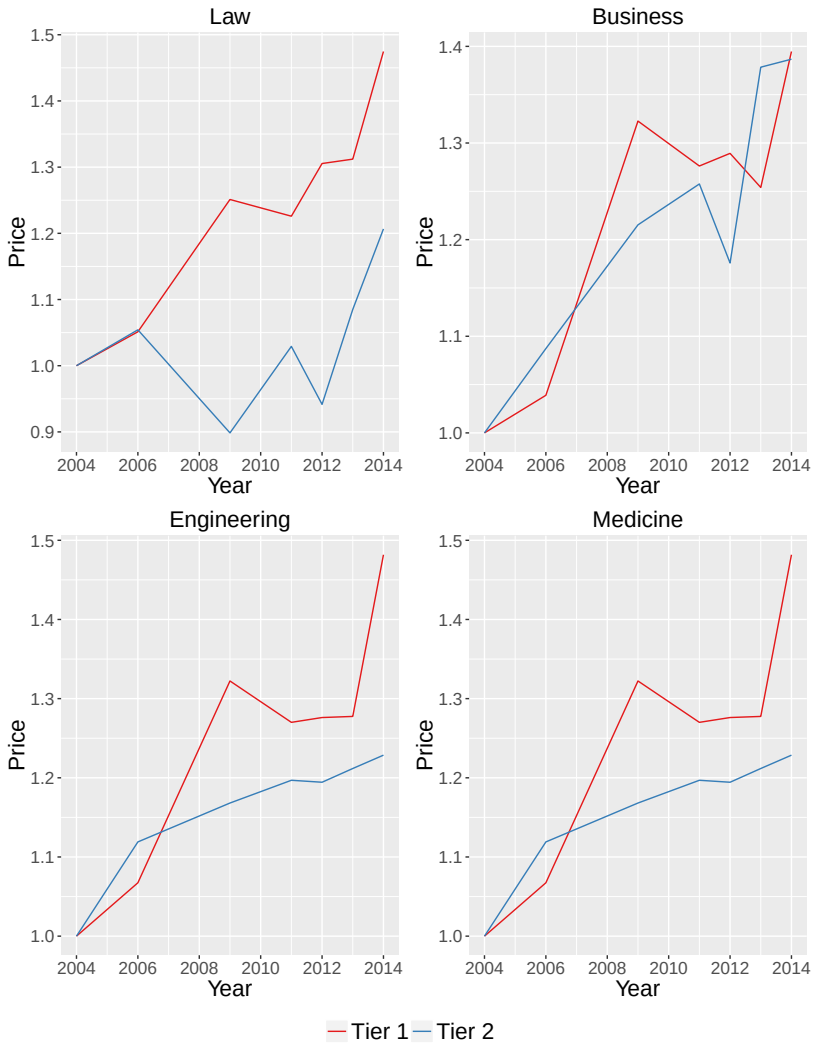


Note: We constructed this dataset by scrapping information available online at the Administrative Department of Science, Technology and Innovation (Colciencias) http://www.colciencias.gov.co/. A more detailed description of how this dataset was constructed is available in Spanish at http://laramaciudadana.com/universidades.html

Finally, we analyze the evolution of tuition being charged by higher education institutions. Figure 2.12 illustrates the behavior of the average real price of tuition during the decade, in terms of 2004 pesos. As can be observed, there has been a steady increase in the real price of education throughout the decade for all universities in Colombia. Additionally, the price of the high-quality colleges seems to have peaked at a higher pace for Law, Engineering and Medicine schools. This increasing trend suggests that the Bennett Hypothesis might also be taking place

in the Colombian context, given the fast increase in the government provided loans to education.

Figure 2.12: Evolution of average tuition prices over the decade



Finally, we conclude this subsection by summarizing the main findings we observe from the data. We find that after the subsidized loan policy program was implemented in Colombia, the degree of product differentiation between elite and

non-elite universities in Colombia increased. We conclude this after analyzing the trend of four key characteristics of universities in Colombia. First, the gap in the quality of student body increased dramatically when analyzing it via entering (SABER-11) or exiting (SABER-PRO) test scores. Second, we observe a gap in the professors per student ratio for every possible category of professorship (PhD, Master's and Bachelor's degree). Third, the gap in academic production, measured as number of peer-reviewed articles and books published, per student, increased during the same period. Finally, we observe that in both, elite and non-elite institutions, there was a significant increase in the tuition being charged for some of the most popular degrees of study. All this evidence is consistent with the fact that after the introduction of subsidized loan policies, the gap in quality between elite and non-elite institutions increased significantly.

2.5 Numerical Analysis

In this section we perform a numerical analysis of the economic presented in Section 2.3 illustrating that the increased gap in quality between elite and non-elite institutions observed in the data, can be rationalized as a consequence of the introduction of subsidized loans for higher education.

2.5.1 Evolution of Quality

According to the specifications assumed in the model, we are able to identify the parameters of the wage equation. For this, we will use data on the average wages of graduates from each university in Colombia through 2007-2012, and the minimum wage, as a measure of w, to estimate the parameters of the quality production function of universities. Per-efficiency unit wages are given by:

$$w_h = w \cdot (1 + z_h), \quad w_l = w \cdot (1 + z_l)$$

Where w_h and w_l are the wages of high- and low-quality college graduates, given equilibrium qualities of education z_h and z_l, respectively, and w is the wage of non-skilled labor per-efficiency unit. The quality of education, z, is given by equation (2.3.11) in the universities' problem. We have a panel of data for 50 universities in Colombia from 2007 to 2012. We have the average ability of students in the entering cohorts, number of professors per student and average wages during the first year after graduation. For every university i in our sample, the following equation holds:

$$w_i = w \cdot (1 + \kappa \bar{\theta_i}^{\alpha_1} I_i^{\alpha_2})$$

Rearranging and taking logarithms:

$$\log\left(\frac{w_i}{w} - 1\right) = \log \kappa + \alpha_1 \bar{\theta_i} + \alpha_2 I_i$$

Assuming that there is measurement error in the wages of each of the universities, and assuming an exclusion restriction that the measurement error is uncorrelated with the explanatory variables, we can estimate the following equation:

$$\log\left(\frac{w_{i,t}}{w_t} - 1\right) = \log\kappa + \alpha_1\bar{\theta_{i,t}} + \alpha_2 I_{i,t} + \eta T_{i,t} + \phi_t + \psi_i + \epsilon_{i,t}$$

where T_i is an indicator function that takes the value of one when the university i is a low-quality institution, and zero otherwise. Under this specification, we can estimate possible differences in the technology parameter, κ, between top and second tier schools. In order to isolate possible omitted variable bias, we estimate the above model under three different specifications, with and without time and geographic fixed effects, ϕ_t and ψ_i, respectively.

For the estimation, we constructed a panel of the top 50 universities in Colombia, according to a quality ranking published by the Ministry of Education in 2014[32]. This panel includes data on average wages during the first year after graduation for graduates of every school, as a measure of $w_{i,t}$, the average test scores for the entering cohorts, as a measure of $\theta_{i,t}$, and the number of professors per student, as a measure for $I_{i,t}$. We also have data on total operational expenditures by each school for 2014. However, with only one year we are not able to construct the evolution of quality of universities over time. Since the number of professors per student are a good indicator of the total expenditures per student, we will use that

[32]The ranking is published in the website of the Ministry of Education, and can be found in the following link: http://www.mineducacion.gov.co/cvn/1665/w3-article-351855.html

Parameter	OLS	OLS	OLS
$\hat{\alpha_1}$	0.211	0.228	0.168
	(0.026)	(0.026)	(0.026)
$\hat{\alpha_2}$	.358	0.478	0.414
	(0.361)	(0.357)	(0.403)
$\hat{\eta}$	-0.029	0.008	-0.046
	(0.047)	(0.043)	(0.046)
$\hat{\log(\kappa)}$	-0.84	-0.957	-0.163
	(0.232)	(0.228)	(0.198)
Time fixed effects		Yes	Yes
City fixed effects			Yes
N	382	382	382
R-squared	0.353	0.444	0.567
Prob > F	0.000	0.000	0.000

Robust standard errors in parenthesis

$***p < 0.01, **p < 0.05, *p < 0.1$

Table 2.1: Estimates for the quality production function.

variable, instead. For the non-skilled labor wages, w_t, we will use the values of the real minimum wage (in 2007 pesos). The average wages of college graduates are strictly above the minimum wage during the period, so the dependent variable is well defined for every college in every period. In addition, we have information about the municipality of the school, to control for regional differences. The results of the estimation are displayed in Table 2.1.

The estimates show that the elasticities α_1 and α_2 are fairly robust to different specifications and do not change dramatically when including control variables. Moreover, the parameter η is negative in two of the specifications, although non statistically significative. This means that, on average, tier 2 universities have a lower technology parameter, κ, on their quality production function. This will be

one of the main differences between tier 1 and tier 2 universities in our calibration of the model.

2.5.2 Calibration of the Model

In order to draw conclusions about the relevance of our model, we calibrate the parameters to values that are relevant to the Colombian context. To achieve this, we will map a life-cycle model to a two period model, so the conclusions of Section 2.3 hold. We follow an approach similar to the one used by Lochner and Monge-Naranjo (2011), but in a discrete time economy. The environment is as follows.

Individuals live for T periods, after which they die with certainty. Individuals start their adult life at $t = 0$, when they must choose whether to attend the low- or high-quality school, or not study at all. Studying lasts for S periods, so those individuals that attend college will not receive any income during $t \in \{0, \ldots, S-1\}$ and have to pay a per-period price of P_j for attending school j. Moreover, during the first S periods individuals are borrowing constrained. Those that decide not to study cannot borrow at all. Those that decide to study, can borrow up to the exogenous limit set by the government student loan policy, $\bar{A}$. After period $S-1$, the individual enters the labor market and earns a per-period wage $\theta w(1+z_j)$, that depends on the quality of the school attended. During periods $S, \ldots, T$ individuals only consume and save. We assume that from period S onwards, individuals enter into perfect financial markets where debt repayments are fully enforced. In this

context, individuals can borrow any amount they want.

Clearly, individuals that are not borrowing constrained during their study period will perfectly smooth consumption along the life-cycle. However, those individuals that are constrained during the first S years of life will exhibit a jump in their consumption once they graduate from college. This setting can be easily embedded into the two-period model described in last section, by setting the discount factors and budget constraints appropriately. Namely, the problem for the household becomes:

$$\max_{c,c'} \frac{c^{1-\sigma}}{1-\sigma} + \tilde{\beta}\frac{(c')^{1-\sigma}}{1-\sigma}, \quad s.t.$$

$$c + c'\left(\frac{\Phi_S}{\Phi_0(1+r)^S}\right) + (P_H h + P_l l)\left(\frac{\Phi_r^y}{\Phi_0}\right) =$$

$$w\theta(1-h)(1-l)\left(\frac{\Phi_r^y}{\Phi_0}\right) + w\theta(1+z_j)\left(\frac{\Phi_r^0}{\Phi_0(1+r)^S}\right) j + \frac{b}{\Phi_0}$$

$$a \geq \bar{A}$$

The derivation of the parameters $\tilde{\beta}, \Phi_0, \Phi_S, \Phi_r^o, \Phi_r^y$ is explained in detail in the Appendix B.0.8. In this environment, all the results from Section 2.3 hold.

2.5.3 Parameterization

In our calibration, we set one period to be exactly one year. We will set some parameter values to match the Colombian educational market. All parameter values are reported in Table 2.2.

We set $S = 5$, so that the individuals that choose to attend a college study during 5 periods, since most professional degrees in Colombia take exactly 5 years. In Colombia, life expectancy at birth is 73.95 years of life[33]. Although the National Statistics Department of Colombia (DANE) does not publish the life expectancy by age, we estimate the life expectancy at 18 years to be 55 more years of life[34]. That is, we set $T = 55$ to match the life expectancy in Colombia for high-school graduates.

We set $\sigma = 2$, which a standard parameter in the literature (Fernandez-Villaverde and Krueger, 2011; Lochner and Monge-Naranjo, 2011). For the real interest rate, we choose $r = 8.9\%$, which is the value for Colombia in 2014 published by the World Bank[35]. We do not claim that this value is representative of developing countries, since the real interest rate for most Latin American countries has a huge variation, ranging from negative values in Argentina (-4.1%) and Venezuela (-14.5% in 2013), to very high values like Brazil (23.5%). We choose $\beta = 0.92$ such that $\beta = 1/(1+r)$. With these parameter values, the discount factor in our two-period model becomes $\tilde{\beta} = 1.89$. This reflects the fact that the post-college period is much longer than the study period, even though individuals discount time at a high rate.

As for the university parameters, we use the estimations of Section 2.5. In particular, we choose $\alpha_1 = 0.211$, $\alpha_2 = 0.358$, $\kappa_l = 0.8$ and $\kappa_h = 0.85$, obtained

[33]See life expectancy tables here.

[34]For instance, in the U.S. life expectancy at 18 is only 0.79 more years than life expectancy at birth. Therefore, we will set life expectancy at 18 in Colombia to be 1.05 years above life expectancy at birth, as a conservative estimate.

[35]See the real interest rates for all the countries in this link.

from the wage regressions displayed in Table 2.1.

Parameter	Value	Source
Utility and discount		
β	0.97	Literature
σ	2	Literature
r	2%	Colombia
w	2	Normalization
Time parameters		
T	78	Colombia
S	5	Colombia
University parameters		
α_1	0.211	Estimation
α_2	0.358	Estimation
κ_l	1.4	Estimation
κ_h	1.2	Estimation
$E^h - C^h$	-12	Estimation
$E^l - C^l$	-7	Estimation

Table 2.2: Parameter values

2.6 Results

In this section we show the results of the numerical computation of the equilibria without the subsidized-loan policy being implemented and once it was implemented. In order to mimic as closely as possible the post-reform equilibrium, we set up a tax rate of 10% used to fund a subsidized loan policy offering credits for higher education for people whose income is below the median income in the economy. The policy implemented in Colombia is designed as a subsidy to the interest rate paid by students. In the model we set up the subsidy in such a way that students that have access to it only have to pay 50% of the interests accumulated in students debts.

In addition to having an income below the median, a student who wants to qualify for the policy must have an ability level in the top 30%[36]. Table 2.3 illustrates the results before and after the implementation of the student loan policies. As can be observed, after the reform there is a widening gap in the quality offered by each university. Elite universities offer a higher quality, while non-elite universities reduce it. There is also a market segmentation, where better students attend the elite institution, and the ability of the students attending the low-quality institutions falls.

Table 2.3: Equilibria computations

		Pre-reform	Post-reform
Elite institutions	Students attending	5,863	9,431
	Average ability of student body	0.48	0.64
	Quality offered	1.01	1.19
Non-elite institutions	Students attending	6,971	6,753
	Average ability of student body	0.41	0.38
	Quality offered	0.53	0.42

[TO DO: WELFARE ANALYSIS]

2.7 Conclusion

Subsidized loan policies have been used widely in both developing and developed economies, as a policy tool to increase college attendance. Such policies are particularly relevant in a context where credit constraints explain a significant amount in the college non-attendance rate. However, when implementing such policies, it is

[36]The institutional details of the policy implemented in Colombia are fully described in Melguizo, Sanchez and Velasco (2015)

important to observe not only the distortion it imposes in the demand side of the market but also the way it affects the incentives of the providers of higher education.

We show that subsidized loan policies can distort the incentives of colleges providing services of higher education in a way that can be harmful for a group of households in an economy. Taking into account that the market for higher education operates under a monopolistic competition setting, granting subsidized loans does not translate into an expansion of the providers of higher education services but rather on the same colleges facing a new set of incentives. As elite institutions unambiguously observe an increase in their demand, they can use their pricing and admission policies to be more selective in their admission process and to spend more per student, which translates into providing better services for their student body. On the contrary, the universities in the low-quality tier will observe a migration to the high-quality group when such policies are implemented. The result is a new equilibrium in the market for higher education where the quality gap between elite and non-elite institutions is widened as a result of the implementation of subsidized loan policies.

Our model is consistent with what we observe in the market for higher education in Colombia: an expansion of the gap in the quality offered by different institutions. In such scenario, subsidized policy loans can make some households worse off as, although the attendance to higher education institutions becomes easier, the gains from attending low-quality universities is not offset by the amount households have

to pay in taxes in order to pay for the policy implemented.

Appendices

Appendix Chapter 1

A.0.1 Identification of Measurement System

The measurement system is described by:

$$Z = \iota_0 + \iota_1 K + \boldsymbol{\varepsilon} \tag{A.0.1}$$

We normalize $E[k] = 0$ for every factor. The variance-covariance matrix of the measurement system is given by:

$$\Sigma_Z = \iota_1 \Sigma_K \iota_1' + \Sigma_\varepsilon \tag{A.0.2}$$

The matrix of moments Σ_Z contains $M(M+1)/2$ moments in order to identify the necessary parameters of the models. M is the total number of measures available and is equal to the sum of measures for each factor:

$$M = \sum_{k \in K} N_k = 151 \tag{A.0.3}$$

as we have

$$N_{ln(S_0)} = 23$$

$$N_{ln(S_1)} = 11$$

$$N_{ln(S_2)} = 13$$

$$N_{ln(PG)} = 8$$

$$N_{\mu} = 19$$

$$N_{ln(I_1)} = 8$$

$$N_{ln(I_2)} = 21$$

$$N_{ln(\hat{e}_1^f)} = 10$$

$$N_{ln(\hat{e}_2^f)} = 14$$

$$N_{ln(\hat{e}_1^m)} = 10$$

$$N_{ln(\hat{e}_2^m)} = 14$$

The dedicated factor structure assumed imposes that each measure loads exclusively to one factor. This implies that rather than $11 \times M$ factor loadings to obtain we only have to estimate M elements in ι_1 to be estimated. Given that the scale of the factor is irrelevant for the analysis, we can normalize one factor loading for each factor to be 1. In total, we have $M - 11 = 140$ factor loadings to be estimated.

The matrix Σ_K contains $(11 \times (11 + 1)/2)$ covariances to be estimated and Σ_ε has $M \times (M + 1)/2$. We see that it is necessary to make some assumptions about

the correlation structure of the factors or of the measurement error system in order to be able to identify the system. If we assume that the measurement error in the system for skills at birth is independent of measurement error in the remaining systems $\varepsilon_m^{\ln(s_0)} \perp\!\!\!\perp \varepsilon_{m'}^k$ for $m = 1...N_{ln(s_0)}$, $k \in K, k \neq \ln(s_0)$, $m' = 1...N_k$ we have enough moments to identify the system. By doing this assumption, we are assuming that the elements in Σ_ε that correspond corresponding to $ln(s_0)$ and other factors are zero. With this, we have enough moments to identify the system.

A.0.2 Estimation

In this section I will derive the full likelihood function of the model as well as the filtering procedure to estimate it.

Likelihood function

The likelihood of the model is:

$$\mathcal{L}(\Theta|O;X) = P(O|X;\Theta) = P(O_1, O_2, O_3|X;\Theta)$$

$$p_0(O_0|\Theta, X)p_1(O_1|O_0, \Theta, X)p_2(O_2|O_1, \Theta, X) \tag{A.0.4}$$

Now, inspecting every element. The first term is composed by the observed outcomes in period zero. Given that the only one observed in this case is the first

period of skills, this is composed then by that.

$$p_0(O_0|\Theta, X) = \int p_0(O_0, K_0|\Theta, X)dK_0 =$$

$$\int p_0(O_0|K_0, \Theta, X)p(K_0|\Theta, X)dK_0 =$$

$$E_{p(K_0|\Theta,X)}\left[P_0(O_0|K_0, \Theta, X)\right] \approx$$

$$\sum_{rr=1}^{RR} P_0(O_0|K_0^{\{rr\}}, \Theta, X) \tag{A.0.5}$$

for RR large, and for the $\{K_0^{\{rr\}}\}_{rr=1}^{RR}$ being drawn from the distribution $p(K_0|\Theta, X)$. K_0 is the set of unobserved factors relevant for period zero given by

$$K_0 = \{\ln(s_0), \ln(PG)\} \tag{A.0.6}$$

Note that in the model the distribution $p(K_0|\Theta, X)$ is not specified. I will assume that both factors are independent and each follow a normal distribution with mean zero and variance $\sigma_{s_0}^2$ and σ_{PG}^2 respectively. This way, evaluating the likelihood for period 0 ends up being a process of drawing shocks from the distribution $p(K_0|\Theta, X)$, computing the likelihood of each shock given by the measurement system of the unobserved latent factors and averaging such likelihoods over the RR shocks.

For the first period the set of relevant factors is given by:

$$K_1 = \{\ln(s_1), \ln(e_1^{f,*}), \ln(\hat{e}_1^{m,*}), \ln(I_1^*), \mu_1\} \tag{A.0.7}$$

and the likelihood can be expressed as:

$$p_1(O_1|O_0, \Theta, X) = \int p_1(O_1, K_1|O_0, \Theta, X)dK_1 =$$

$$\int\int p_1(O_1, K_1, K_0|O_0, \Theta, X)dK_1dK_0 =$$

$$\int\int p_1(O_1|K_1, K_0, O_0, \Theta, X)p(K_1|O_0, K_0, \Theta, X)p(K_0|O_0, \Theta, X)dK_1dK_0 \tag{A.0.8}$$

Note that

$$p(K_1|O_0, K_0, \Theta, X) = p(K_1|K_0, \theta, X) \tag{A.0.9}$$

as O_0 would not carry more information beyond that in K_0 that is relevant for K_1. Also, note that

$$p_1(O_1|K_1, K_0, O_0, \Theta, X) = p_1(O_1|K_1, \Theta, X) \tag{A.0.10}$$

Taking into account the facts presented in Equations A.0.9 and A.0.10 we can express A.0.8 as:

$$\int\int p_1(O_1|K_1,\Theta,X)p(K_1|K_0,\Theta,X)p(K_0|O_0,\Theta,X)dK_1dK_0 =$$

$$\int p(K_0|O_0,\Theta,X)\left[\int p(O_1|K_1,\Theta,X)p(K_1|K_0,\Theta,X)dK_1\right]dK_0 =$$

$$E_{p(K_0|O_0,\Theta,X)}\left[\int p(O_1|K_1,\Theta,X)p(K_1|K_0,\Theta,X)dK_1\right] = \tag{A.0.11}$$

in Equation A.0.11 $p(O_1|K_1,\Theta,X)$ is given by the measurement system of factors, the likelihood of wages (for those that are observed) and the preference shocks cdf. We can re-write such expression as:

$$p(O_1|K_1,\Theta,X) =$$

$$p(\mathcal{Z}_1|K_1,\Theta,X)\times p(w^f|K_1,\Theta,X)^{(1-h^{f,*})}\times p(w^m|K_1,\Theta,X)^{1-h^{m,*}}$$

$$\times\, p(h^{f,*},h^{m,*},a|w^f,w^m,K_1,\Theta,X) \tag{A.0.12}$$

As specified previously, $p(\mathcal{Z}_1|K_1,\Theta,X)$ is given by the measurement system.

$p(w^f|K_1, \Theta, X)$ is given by the measurement error associated to the observed wages:

$$\ln\left(w^j\right) = \beta_0^j + \beta_1^j yrschool^j + \beta_2^j Age^j + \beta_3^j (Age^j)^2 + \varepsilon_{w^j} \tag{A.0.13}$$

where ε_{w^j} is measurement error following a distribution $\varepsilon_{w^j} \sim N(0, \sigma_{\varepsilon^j})$.

Finally, $p(h_f^*, h_m^*, a^*|w^f, w^m, K_1, \Theta, X)$ is given by the probability of having the observed decisions as the optimal ones:

$$\begin{aligned} & p(h_f^*, h_m^*, a^*|w^f, w^m, K_1, \Theta, X) = \\ & p_{(.)}\Big(W(u^f(h^{f*}, h^{m,*}, a^*), u^m(h^{f,*}, h^{m,*}, a^*)) \\ & \in \arg\max_{\{h^f, h^m, a\}} W(u^f(h^f, h^m, a), u^m(h^f, h^m, a))|K_1, \Theta, X\Big) \end{aligned} \tag{A.0.14}$$

where $p_{()}$ is the distribution of the preference shocks.

$p(K_1|K_0, \Theta, X)$ is given by the transition equation. Note, however, that the dynamics of the system are only given through the skills of the child, the remaining factors do not have any dynamics carried from the previous period. This implies that such expression will be given by the skills production function and the distribution

of heterogeneity in the remaining factors. Being explicit:

$$p(K_1|K_0,\Theta,X) = p(\ln(s_1),\ln(e_1^{f,*}),\ln(\hat{e}_1^{m,*}),\ln(\hat{I}_1^*),\mu_1|\ln(PG),\ln(s_0),\Theta,X)$$

$$= p(\ln(s_1),\ln(e_1^{f,*}),\ln(e_1^{m,*}),\ln(I_1^*),\mu_1|\ln(PG),\ln(s_0),\Theta,X) =$$

$$p(\ln(s_1)|\ln(e_1^{f,*}),\ln(e_1^{m,*}),\ln(I_1^*),\mu_1,\ln(PG),\ln(s_0),\Theta,X) \tag{A.0.15}$$

$$\times\, p(\ln(e_1^{f,*})|\mu_1\Theta,X) \tag{A.0.16}$$

$$\times\, p(\ln(e_1^{m,*})|\mu_1,\Theta,X) \tag{A.0.17}$$

$$\times\, p(\ln(I_1^*)|\mu_1,\Theta,X) \tag{A.0.18}$$

$$\times\, p(\mu_1|\Theta,X) \tag{A.0.19}$$

The term A.0.15 is given by the production of skills and the remaining A.0.16-A.0.18 are given by the distribution of heterogeneity in each factor: η_{e^f}, η_{e^m} and η_I. The term A.0.19 is given by the distribution of heterogeneity in 1.5.14. Note that we can also use Monte-Carlo techniques to approximate the expression in A.0.11 by:

$$\sum_{rr=1}^{RR} \hat{w}_0^{\{rr\}} \left[\int p(O_1|K_1,\Theta,X) p(K_1|K_0^{\{rr\}},\Theta,X) dK_1 \right] \tag{A.0.20}$$

where $\{K_0^{\{rr\}}\}_{rr=1}^{RR}$ are drawn from an importance distribution $g_0(K_0|\mathcal{Z}_0, \Theta, X)$ and the weights are given by:

$$\hat{w}_0^{rr} = \frac{w_0^{rr}}{\sum_{rr=1}^{RR} w_0^{rr}} \tag{A.0.21}$$

and the individual weights are defined:

$$w_0^{rr} \propto \frac{p(K_0|O_0, \Theta, X)}{g_0(K_0|\mathcal{Z}_0, \theta_0, \Theta, X)} \tag{A.0.22}$$

Note that after some algebra, we can define:

$$\tilde{w}_1 = \frac{p(O_1|K_1, \Theta, X)p(K_1|K_0, \Theta, X)}{g_t(K_0|, O_0, O_1, \Theta, X)} \tag{A.0.23}$$

where $g_t(K_0|, O_0, O_1, \Theta, X)$ is the proposal -importance- distribution from which the particles are going to be drawn. We will explain below what this distribution is. Note that replacing A.0.23 into A.0.11 we obtain:

$$\sum_{rr=1}^{RR} \hat{w}_0^{\{rr\}} \left[\int p(O_1|K_1, \Theta, X)p(K_1|K_0^{\{rr\}}, \Theta, X)dK_1 \right] =$$

$$\sum_{rr=1}^{RR} \hat{w}_0^{\{rr\}} \left[\sum_{rr'=1}^{RR} \tilde{w}_1^{rr'}(rr) \right] \tag{A.0.24}$$

And finally note that the dependence given between rr and rr' generates a *dirac*

measure in dependence (all that follow from rr different in the dependence path go to zero in rr'). Then, we can write the expression of the likelihood in the first period as:

$$p_1(O_1|K_1, K_0, O_0, \Theta, X) = \sum_{rr=1}^{RR} \hat{w}_0^{rr} \tilde{w}_1^{rr} \tag{A.0.25}$$

The computation of the likelihood for the second period is identical to that of the first period with the exception that we need to change the measurement system for the corresponding measures available in the second period and the childcare decision is not available in the behavioral model.

In this case we will use as importance distribution the same transition equation. The literature refers to this type of filtering as the bootstrap filter Creal (2012).

A.0.3 Estimates of Measurement System

Parameter	Estimate	Standard Error
$MS_{1,10}$	0.1679	0.0002
$SDS_{1,10}$	2.5059	0.0040
$MS_{2,10}$	0.1305	0.0003
$SDS_{2,10}$	2.4928	0.0027
$MS_{3,10}$	0.1117	0.0002
$SDS_{3,10}$	2.4283	0.0019
$MS_{4,10}$	-0.6097	0.0008
$SDS_{4,10}$	2.0847	0.0022
$MS_{5,10}$	-0.5080	0.0003
$SDS_{5,10}$	2.2865	0.0014
$MS_{6,10}$	-0.3238	0.0002
$SDS_{6,10}$	2.6349	0.0019
$MS_{7,10}$	-0.4028	0.0003
$SDS_{7,10}$	2.4463	0.0011
$MS_{8,10}$	-0.3325	0.0002
$SDS_{8,10}$	2.2173	0.0020
MS_{9_10}	-0.5363	0.0003
SDS_{9_10}	2.2473	0.0012
MS_{10_10}	-1.0000	0.0000
SDS_{10_10}	0.0010	0.0000

Table A.1: Estimates: Measurement system -Skills in 2010

Parameter	Estimate	Standard Error
MS_{1_12}	1.000	0.000
SDS_{1_12}	2.754	0.010
MS_{2_12}	0.951	0.048
SDS_{2_12}	3.102	0.083
MS_{3_12}	1.097	0.012
SDS_{3_12}	2.943	0.032
MS_{4_12}	1.059	0.033
SDS_{4_12}	3.192	0.106
MS_{5_12}	0.990	0.012
SDS_{5_12}	3.533	0.134
MS_{6_12}	1.086	0.019
SDS_{6_12}	2.292	0.039
MS_{7_12}	1.102	0.012
SDS_{7_12}	2.794	0.041
MS_{8_12}	1.131	0.036
SDS_{8_12}	2.595	0.112
MS_{9_12}	0.977	0.100
SDS_{9_12}	3.055	0.005
MS_{10_12}	1.244	0.036
SDS_{10_12}	0.003	0.016
MS_{11_12}	1.116	0.002
SDS_{11_12}	4.810	0.084

Table A.2: Estimates: Measurement system -Skills in 2012

Parameter	Estimate	Standard Error
$MS_{1,10}$	0.1679	0.0002
$SDS_{1,10}$	2.5059	0.0040
$MS_{2,10}$	0.1305	0.0003
$SDS_{2,10}$	2.4928	0.0027
$MS_{3,10}$	0.1117	0.0002
$SDS_{3,10}$	2.4283	0.0019
$MS_{4,10}$	-0.6097	0.0008
$SDS_{4,10}$	2.0847	0.0022
$MS_{5,10}$	-0.5080	0.0003
$SDS_{5,10}$	2.2865	0.0014
$MS_{6,10}$	-0.3238	0.0002
$SDS_{6,10}$	2.6349	0.0019
$MS_{7,10}$	-0.4028	0.0003
$SDS_{7,10}$	2.4463	0.0011
$MS_{8,10}$	-0.3325	0.0002
$SDS_{8,10}$	2.2173	0.0020
MS_{9_10}	-0.5363	0.0003
SDS_{9_10}	2.2473	0.0012
MS_{10_10}	-1.0000	0.0000
SDS_{10_10}	0.0010	0.0000

Table A.3: Estimates: Measurement system -Skills in 2010

Parameter	Estimate	Standard Error
$MS_{1_{BARG}}$	-0.2155	0.0667
$SDS_{1_{BARG}}$	0.6144	0.0141
$MS_{2_{BARG}}$	0.0875	0.0684
$SDS_{2_{BARG}}$	0.6301	0.0145
$MS_{3_{BARG}}$	-0.0703	0.0694
$SDS_{3_{BARG}}$	0.6395	0.0151
$MS_{4_{BARG}}$	0.0155	0.0668
$SDS_{4_{BARG}}$	0.6151	0.0141
$MS_{5_{BARG}}$	0.1219	0.0694
$SDS_{5_{BARG}}$	0.6395	0.0147
$MS_{6_{BARG}}$	-0.1325	0.0675
$SDS_{6_{BARG}}$	0.6223	0.0142
$MS_{7_{BARG}}$	0.1015	0.0657
$SDS_{7_{BARG}}$	0.6056	0.0139
$MS_{8_{BARG}}$	-0.0151	0.0672
$SDS_{8_{BARG}}$	0.6191	0.0142
$MS_{9_{BARG}}$	0.0941	0.0685
$SDS_{9_{BARG}}$	0.6307	0.0145
$MS_{10_{BARG}}$	-0.0283	0.0671
$SDS_{10_{BARG}}$	0.6180	0.0144
$MS_{11_{BARG}}$	0.0166	0.0040
$SDS_{11_{BARG}}$	0.0292	0.0059
$MS_{12_{BARG}}$	0.0445	0.0020
$SDS_{12_{BARG}}$	0.0117	0.0005
$MS_{13_{BARG}}$	0.6507	0.0889
$SDS_{13_{BARG}}$	0.6429	0.0879
$MS_{14_{BARG}}$	-0.5486	0.0546
$SDS_{14_{BARG}}$	0.5027	0.0116
$MS_{15_{BARG}}$	0.2877	0.0230
$SDS_{15_{BARG}}$	0.1605	0.0128
$MS_{16_{BARG}}$	-0.7954	0.0907
$SDS_{16_{BARG}}$	0.6504	0.0744
$MS_{17_{BARG}}$	1.0000	0.0416
$SDS_{17_{BARG}}$	0.1868	0.0078
$MS_{18_{BARG}}$	0.6205	0.0376
$SDS_{18_{BARG}}$	0.0725	0.0044

Table A.4: Estimates: Measurement system -Pareto weight

Parameter	Estimate	Standard Error
$MS_{1_{INV,10}}$	0.123	0.020
$SDS_{1_{INV,10}}$	0.176	0.181
$MS_{2_{INV,10}}$	1.000	0.000
$SDS_{2_{INV,10}}$	2.241	1.349
$MS_{3_{INV,10}}$	0.383	0.211
$SDS_{3_{INV,10}}$	0.617	0.373
$MS_{4_{INV,10}}$	0.334	0.247
$SDS_{4_{INV,10}}$	0.404	0.316
$MS_{5_{INV,10}}$	0.047	0.043
$SDS_{5_{INV,10}}$	0.052	0.089
$MS_{6_{INV,10}}$	0.041	0.002
$SDS_{6_{INV,10}}$	0.171	0.162
$MS_{7_{INV,10}}$	0.115	0.096
$SDS_{7_{INV,10}}$	0.242	0.181
$MS_{8_{INV,10}}$	0.074	0.062
$SDS_{8_{INV,10}}$	0.259	0.175

Table A.5: Estimates: Measurement system -Investments 2010

Parameter	Estimate	Standard Error
$MS_{1_{EF,10}}$	0.368	0.084
$SDS_{1_{EF,10}}$	1.101	0.081
$MS_{2_{EF,10}}$	0.280	0.008
$SDS_{2_{EF,10}}$	0.887	0.058
$MS_{3_{EF,10}}$	0.212	0.049
$SDS_{3_{EF,10}}$	0.325	0.181
$MS_{4_{EF,10}}$	0.280	0.091
$SDS_{4_{EF,10}}$	0.917	0.003
$MS_{5_{EF,10}}$	0.299	0.045
$SDS_{5_{EF,10}}$	0.391	0.026
$MS_{6_{EF,10}}$	1.000	0.000
$SDS_{6_{EF,10}}$	0.962	0.109

Table A.6: Estimates: Measurement system -Parental effort 2010

Parameter	Estimate	Standard Error
$MS_{1_{BARG}}$	-0.2155	0.0667
$SDS_{1_{BARG}}$	0.6144	0.0141
$MS_{2_{BARG}}$	0.0875	0.0684
$SDS_{2_{BARG}}$	0.6301	0.0145
$MS_{3_{BARG}}$	-0.0703	0.0694
$SDS_{3_{BARG}}$	0.6395	0.0151
$MS_{4_{BARG}}$	0.0155	0.0668
$SDS_{4_{BARG}}$	0.6151	0.0141
$MS_{5_{BARG}}$	0.1219	0.0694
$SDS_{5_{BARG}}$	0.6395	0.0147
$MS_{6_{BARG}}$	-0.1325	0.0675
$SDS_{6_{BARG}}$	0.6223	0.0142
$MS_{7_{BARG}}$	0.1015	0.0657
$SDS_{7_{BARG}}$	0.6056	0.0139
$MS_{8_{BARG}}$	-0.0151	0.0672
$SDS_{8_{BARG}}$	0.6191	0.0142
$MS_{9_{BARG}}$	0.0941	0.0685
$SDS_{9_{BARG}}$	0.6307	0.0145
$MS_{10_{BARG}}$	-0.0283	0.0671
$SDS_{10_{BARG}}$	0.6180	0.0144
$MS_{11_{BARG}}$	0.0166	0.0040
$SDS_{11_{BARG}}$	0.0292	0.0059
$MS_{12_{BARG}}$	0.0445	0.0020
$SDS_{12_{BARG}}$	0.0117	0.0005
$MS_{13_{BARG}}$	0.6507	0.0889
$SDS_{13_{BARG}}$	0.6429	0.0879
$MS_{14_{BARG}}$	-0.5486	0.0546
$SDS_{14_{BARG}}$	0.5027	0.0116
$MS_{15_{BARG}}$	0.2877	0.0230
$SDS_{15_{BARG}}$	0.1605	0.0128
$MS_{16_{BARG}}$	-0.7954	0.0907
$SDS_{16_{BARG}}$	0.6504	0.0744
$MS_{17_{BARG}}$	1.0000	0.0416
$SDS_{17_{BARG}}$	0.1868	0.0078
$MS_{18_{BARG}}$	0.6205	0.0376
$SDS_{18_{BARG}}$	0.0725	0.0044

Table A.7: Estimates: Measurement system -Pareto weight

A.0.4 Filtering

Now that we have an expression for the likelihood function in a way that can be computed via simulation, I will present the algorithm used to evaluate the likelihood function at a given point:

Filtering Algorithm

1. Set t=0.

   (a) For rr=1....RR:

      i. draw $K_0^{\{rr\}}$ from proposal distribution $g(K_0|\Theta, X)$

      ii. Compute the weights $\hat{w}_0^{\{rr\}} = \frac{1}{RR}$

   (b) Compute likelihood for measurement system

   in $t = 0$: $\frac{1}{RR} \sum_{rr=1}^{RR} P_0(O_0|K_0^{\{rr\}}, \Theta, X)$

2. Set t=t+1

   (a) For rr=1....RR:

      i. Draw θ_t from proposal distribution (transition equation):

      $p(K_t^{\{rr\}}|K_{t-1}^{\{rr\}}, \Theta, X)$

      ii. Compute the weights $\tilde{w}_t^{\{rr\}} = p(O_t|K_t^{\{rr\}}, \Theta, X)$

      iii. Define $w_t^{\{rr\}} = \hat{w}_{t-1}^{\{rr\}} \tilde{w}_t^{\{rr\}}$

   (b) For rr=1...RR

i. Define $\hat{w}_t^{\{rr\}} = \frac{w_t^{\{rr\}}}{\sum_{rr=1}^{RR} w_t^{rr}}$

(c) Compute the likelihood for period t: $\sum_{rr=1}^{RR} \tilde{w}_t^{rr} \hat{w}_{t-1}^{rr}$

(d) For rr=1....RR

i. Re-sample RR particles $\theta_t^{\{rr\}}$ from step $(2.i)$ with probabilities $\hat{w}_t^{\{rr\}}$

ii. Set $w_t^{rr} = \frac{1}{RR}$

It is usually assumed that it is costly to sample from the original distribution $p(K_t|K_{t-1}, \psi, X)$. Such is not the case of this article and then as importance distribution we will use the transition system as the importance distribution. When such distribution is used, the algorithm implemented receives the name of the bootstrap filter.

Figure A.1: Particle Filtering Algorithm

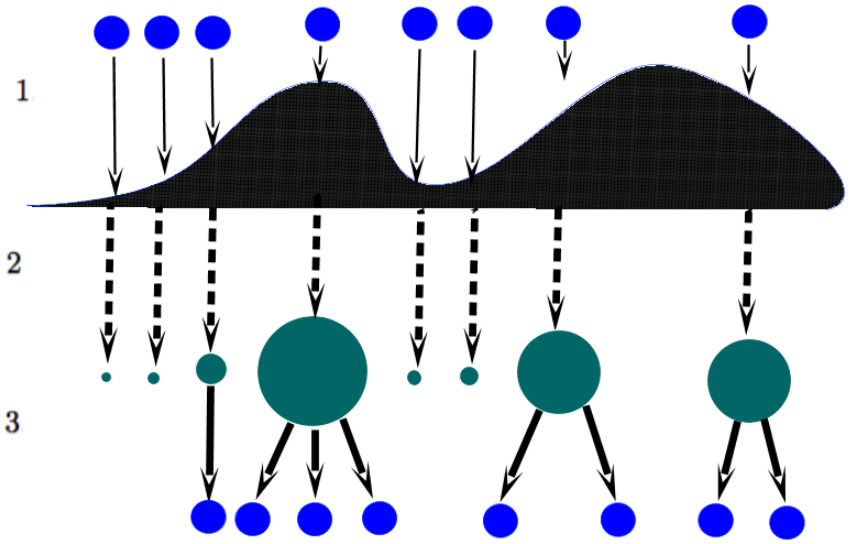


Figure A.1 illustrates the particle filtering algorithm with eight particles. In

the first step, particles are drawn from the proposal distribution $g(K_0|\Theta, X)$. In the second step, the likelihood of each particle is evaluated through the likelihood system $P_0\left(O_0|K_0, \Theta, X\right)$. In the third step, a new set of particles are drawn with the corresponding weight given by the measurement system. Some particles might die and some others are drawn multiple times.

A.0.5 Smoothing distribution

The smoothing distribution is useful if we are interested in making inference about the state of the unobserved factors. In this case, it is particularly interesting to make inference about the skills of children. The following procedure describes how to use the information provided in the model and in the data in order to derive the smoothing distribution of the unobserved latent factors. This procedure is adapted from Klaas et al. (2006):

I use as main input for this file the article "Fast Particle Smoothing: If I had a Million Particles". I translate the notation in the one used in the paper. Define $O_{0:t} = \{O_0, O_1..., O_t\}$. The smoothed density is:

$$p(K_t|O_{0:2}) \tag{A.0.26}$$

where we basically condition on all the measures we have. Note that we can write Equation A.0.26 as:

$$p(K_t|O_{0:2}) = p(K_t|O_{0:t}) \int \left(\frac{p(K_{t+1}|O_{0:2})p(K_{t+1}|\theta_t)}{\int p(K_{t+1}|\theta_t)p(K_1|O_{0:t})dK_t} \right) dK_{t+1} \tag{A.0.27}$$

And then we can approximate this distribution by $\hat{p}(\theta_t|O_{0:2})$ with:

$$\hat{p}(K_t|O_{0:2}) = \sum_{rr=1}^{RR} w_{t|T}^{(rr)} \delta_{K_t^{(rr)}}(K_t) \tag{A.0.28}$$

where $\delta_{K_t^{(rr)}}(K_t)$ is the Dirac distribution and

$$w_{t|T}^{(mm)} = w_t^{(mm)} \left[\sum_{rr=1}^{RR} w_{t+1|T}^{(rr)} \left(\frac{p\left(K_{t+1}^{(rr)} | K_t^{(mm)}\right)}{\sum_{kk=1}^{KK} w_t^{(kk)} p\left(K_{t+1}^{(rr)} | K_t^{(kk)}\right)} \right) \right] \tag{A.0.29}$$

where $w_{T|T} = w_T$

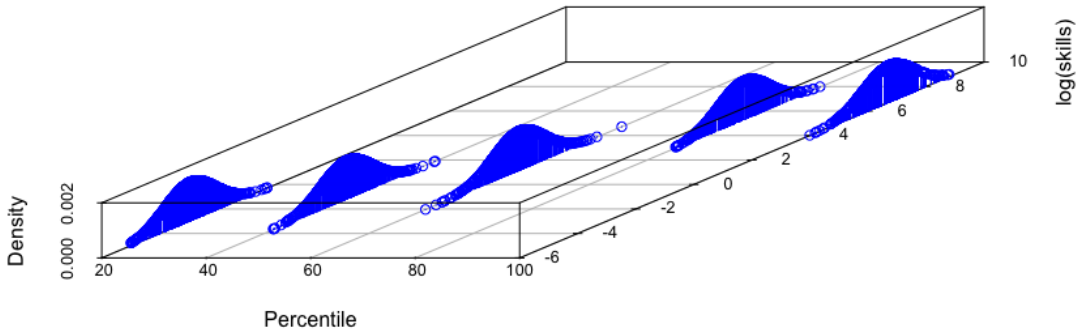
Smoothing algorithm

1. For t=0,1,2 perform the particle filtering to obtain $\{K_t^{rr}, w_t^{rr}\}_{rr=1}^{RR}$

2. Set $w_{2|2}^{rr} = w_2^{rr}$ for $rr = 1...RR$

3. For t=1,0 define $w_{t|2}^{(mm)} = w_t^{(mm)} \left[\sum_{rr=1}^{RR} w_{t+1|2}^{(rr)} \left(\frac{p\left(x_{t+1}^{(rr)} | x_t^{(mm)}\right)}{\sum_{kk=1}^{KK} w_t^{(kk)} p\left(x_{t+1}^{(rr)} | x_t^{(kk)}\right)} \right) \right]$

Figure A.2: Smoothing Distribution of Skills According to Household's Income Percentile



The distribution of $log(skills)$ is plotted for representative households. Households located in the 20th, 40th, 60th, 80th and 100th percentile of total household income. The smoothed distribution of all the households is presented in Figure 1.13

A.0.6 Cash Transfer Programs in Chile

The basic program through which poor families receive cash transfers from the central government is the "Unique Family Subsidy".[37] Such program established a monthly transfer of $14,340 CLP in 2012, for a family in conditions of vulnerability[38] with one child.[39] The recipient of the transfer is always set to be the mother of the children who generate the transfer. In addition to be within the 40% most vulnerable, in order for the mother should be economically inactive in order to receive the transfer. However, the alternate program "Family Assignment" cash transfers of the same value for those mothers who were working, with a fadeout scheme.[40]

In 2016, the basic amount of a transfer in the programs "Unique Family subsidy" and "Family Assignments" corresponded to $10,577. When compared to the $7,170 CLP of 2012, this represents an increase of 29% in real terms. Additionally, in 2014 the government of Michelle Bachelet implemented the implemented the "Permanent Family Contribution Program". In 2016, those families who were eligible to either "Unique Family Subsidy" or "Family Assignments" were automatically eligible to be part of the "Permanent Family Contribution Program". which consisted in a

[37] Subsidio Unico familiar in Spanish.

[38] The condition of vulnerability corresponds to a score below 11.734 in the "Ficha de Proteccin Social". Approximately 40% of Chilean families lie below this threshold

[39] The $14,340 CLP were generated by the mother and the child, each generating a transfer of $7,170 CLP.

[40] The transfer scheme consisted of $7,179CLP for women with monthly wages below $187,515 CLP; $5,054 CLP for women whose wages was in between $187,515 CLP and $307,863 CLP; and $1,600 CLP for women whose wages was between $307,863 CLP and $480,163.

transfer of $43,042 anually for each children and one for the family as a whole. Thus, a family one child would be eligible to receive $86,084 CLP.

Overall, a family of one child that was receiving transfers from the "Unique Family Subsidy" program in 2012, would see an increase in the monetary transfers from the central government equivalent to 72.8% in real terms.

Appendix Chapter 2

The problem of the households is:

$$\max_{c,l,h,a} \frac{c^{1-\sigma}}{1-\sigma} + \beta \frac{c'^{1-\sigma}}{1-\sigma}, \quad s.t.$$

$$a + c + hP_h + lP_l = w\theta(1-h)(1-l) + b$$

$$c' = w\theta + w\theta z_h h + w\theta z_l l + (1+r)a$$

B.0.1 Solution of the unconstrained households:

Proof of Theorem 1. The unconstrained consumptions are:

$$c^N = \frac{(\beta(1+r))^{-1/\sigma}\,(w\theta(2+r) + (1+r)b)}{1 + (\beta(1+r))^{-1/\sigma}\,(1+r)},$$

$$c'^N = \frac{(w\theta(2+r) + (1+r)b)}{1 + (\beta(1+r))^{-1/\sigma}\,(1+r)}$$

$$c^l = \frac{(\beta(1+r))^{-1/\sigma}\,(w\theta(1+z_l) + (1+r)b - P_l(1+r))}{1 + (\beta(1+r))^{-1/\sigma}\,(1+r)},$$

$$c'^l = \frac{(w\theta(1+z_l) + (1+r)b - P_l(1+r))}{1 + (\beta(1+r))^{-1/\sigma}\,(1+r)}$$

$$c^h = \frac{(\beta(1+r))^{-1/\sigma}\left(w\theta(1+z_h)+(1+r)b-P_h(1+r)\right)}{1+(\beta(1+r))^{-1/\sigma}(1+r)},$$

$$c'^h = \frac{\left(w\theta(1+z_h)+(1+r)b-P_h(1+r)\right)}{1+(\beta(1+r))^{-1/\sigma}(1+r)}$$

The utilities of each of the options are:

$$u^N = \Phi \times \left(w\theta(2+r)+b(1+r)\right)^{1-\sigma}$$

$$u^l = \Phi \times \left(w\theta(1+z_l)+b(1+r)-P_l(1+r)\right)^{1-\sigma}$$

$$u^h = \Phi \times \left(w\theta(1+z_h)+b(1+r)-P_h(1+r)\right)^{1-\sigma}$$

where

$$\Phi = \left(\frac{1}{1-\sigma}\right)\left(\frac{1}{1+(\beta(1+r))^{-1/\sigma}(1+r)}\right)^{1-\sigma}\left((\beta(1+r))^{(\sigma-1)/\sigma}+\beta\right)$$

The household's decision of whether and where to study follows a *cut-off* rule on θ, and the decision is independent of initial wealth, b. The cut-offs are:

$$\bar{\theta_l} = \frac{1+r}{w}\left(\frac{P_l}{z_l-(1+r)}\right), \quad \bar{\theta_h} = \frac{1+r}{w}\left(\frac{P_h-P_l}{z_h-z_l}\right)$$

□

B.0.2 Wealth cutoff rules for households:

Proof of Theorem 2. The debt levels of the unconstrained households are:

$$a^N = \frac{w\theta(1-(\beta(1+r))^{-1/\sigma})+b}{1+(\beta(1+r))^{-1/\sigma}(1+r)}$$

$$a^l = \frac{b-P_l-(\beta(1+r))^{-1/\sigma}w\theta(1+z_l)}{1+(\beta(1+r))^{-1/\sigma}(1+r)}$$

$$a^h = \frac{b-P_h-(\beta(1+r))^{-1/\sigma}w\theta(1+z_h)}{1+(\beta(1+r))^{-1/\sigma}(1+r)}$$

Given the exogenous borrowing constraint $\bar{A}$, for a given θ we can construct a cut-off $\bar{b}(\theta)$ on the initial wealth such that individuals with $b < \bar{b}(\theta)$ are constrained and $b \geq \bar{b}(\theta)$ are unconstrained. These are given by:

$$a^N \geq \bar{A} \quad \Leftrightarrow \quad b \geq -\bar{A}(1+(\beta(1+r))^{-1/\sigma}(1+r)) - w\theta(1-(\beta(1+r))^{-1/\sigma})$$

$$a^l \geq \bar{A} \quad \Leftrightarrow \quad b \geq P_l + (\beta(1+r))^{-1/\sigma}w\theta(1+z_l) - \bar{A}(1+(\beta(1+r))^{-1/\sigma}(1+r))$$

$$a^h \geq \bar{A} \quad \Leftrightarrow \quad b \geq P_h + (\beta(1+r))^{-1/\sigma}w\theta(1+z_h) - \bar{A}(1+(\beta(1+r))^{-1/\sigma}(1+r))$$

That is, the cut-offs are:

$$\bar{b}_u^N(\theta) = -\bar{A}(1+(\beta(1+r))^{-1/\sigma}(1+r)) - w\theta(1-(\beta(1+r))^{-1/\sigma})$$

$$\bar{b_u^l}(\theta) = P_l + (\beta(1+r))^{-1/\sigma} w\theta(1+z_l) - \bar{A}(1 + (\beta(1+r))^{-1/\sigma}(1+r))$$

$$\bar{b_u^h}(\theta) = P_h + (\beta(1+r))^{-1/\sigma} w\theta(1+z_h) - \bar{A}(1 + (\beta(1+r))^{-1/\sigma}(1+r))$$

□

This subdivides the state space in three subregions, as shown in the following Figure 2.2.

B.0.3 Solution of the constrained households:

Next, we have to consider the decision of studying of those households that are constrained. Note that, although if an individual is borrowing constrained when he decides to study, he might prefer to study and not smooth consumption, than not studying and being able to smooth consumption. Therefore, we must compare the utility of studying while being constrained, with the utility of not studying and being unconstrained. The constrained consumptions are given by:

$$c_c^N = w\theta + b + \bar{A}, \quad c_c^{'N} = w\theta - (1+r)\bar{A}$$

$$c_c^l = b - P_l + \bar{A}, \quad c_c^{'l} = w\theta(1+z_l) - (1+r)\bar{A}$$

$$c_c^h = b - P_h + \bar{A}, \quad c_c^{'h} = w\theta(1+z_h) - (1+r)\bar{A}$$

There are three decisions to characterize:

1. Whether to study in l or not study, for individuals that are constrained when

studying in l. These individuals will study in l whenever:

$$\left(\frac{1}{1-\sigma}\right)(b - P_l + \bar{A})^{1-\sigma} + \left(\frac{\beta}{1-\sigma}\right)(w\theta(1+z_l) - (1+r)\bar{A})^{1-\sigma} -$$

$$\Phi \times (w\theta(2+r) + b(1+r))^{1-\sigma} \geq 0$$

2. Whether to study in l or in h, for individuals that are constrained when studying in h but not constrained when studying in l. These individuals will study in h whenever:

$$\left(\frac{1}{1-\sigma}\right)\left(b - P_h + \bar{A}\right)^{1-\sigma} + \left(\frac{\beta}{1-\sigma}\right)\left(w\theta(1+z_h) - (1+r)\bar{A}\right)^{1-\sigma} -$$

$$\Phi \times (w\theta(1+z_l) + b(1+r) - P_l(1+r))^{1-\sigma} \geq 0$$

3. Whether to study in l or in h, for individuals that are constrained when they decide to study in h or l. These individuals will study in h whenever:

$$\left(\frac{1}{1-\sigma}\right)\left(b - P_h + \bar{A}\right)^{1-\sigma} + \left(\frac{\beta}{1-\sigma}\right)\left(w\theta(1+z_h) - (1+r)\bar{A}\right)^{1-\sigma}$$

$$\left(\frac{1}{1-\sigma}\right)\left(b - P_l + \bar{A}\right)^{1-\sigma} - \left(\frac{\beta}{1-\sigma}\right)\left(w\theta(1+z_l) - (1+r)\bar{A}\right)^{1-\sigma} \geq 0$$

The cut-offs that define the college decision for constrained individuals are defined in the following theorem proofs:

Proof of Theorem 3. Define the following function:

$$G(\theta, b) = \left(\frac{1}{1-\sigma}\right)(b - P_l + \bar{A})^{1-\sigma} + \left(\frac{\beta}{1-\sigma}\right)(w\theta(1+z_l) - (1+r)\bar{A})^{1-\sigma} -$$

$$\Phi \times (w\theta^*(2+r) + b(1+r))^{1-\sigma} \tag{B.0.1}$$

Let the function $\bar{b}_c^l(\theta)$ be implictly defined by the equality $G(\theta, \bar{b}_c^l(\theta)) = 0$. By the implicit function theorem,

$$\frac{\partial \bar{b}_c^l(\theta)}{\partial \theta} = -\frac{\partial G/\partial \theta}{\partial G/\partial b}$$

Setting $\partial G/\partial \theta = 0$ gives the result in Theorem 3. □

B.0.4 Proof of Theorem 4

Proof of Theorem 4. The proof is similar to Proof B.0.3. Define:

$$G^*(\theta, b) = \left(\frac{1}{1-\sigma}\right)(b - P_h + \bar{A})^{1-\sigma} + \left(\frac{\beta}{1-\sigma}\right)(w\theta(1+z_h) - (1+r)\bar{A})^{1-\sigma} -$$

$$\Phi(w\theta^*(1+z_l) + b(1+r) - P_l(1+r))^{1-\sigma} \tag{B.0.2}$$

and setting $\partial G/\partial \theta = 0$ gives the result in Theorem 4.

□

B.0.5 Proof of Theorem 5

Proof of Theorem 5. By implicit function theorem, $\partial b/\partial \bar{A} = -\frac{\partial G/\partial \bar{A}}{\partial G/\partial b}$.

$$\frac{\partial G}{\partial \bar{A}} = (b - P_l + \bar{A})^{-\sigma} + \beta(1+r)(w\theta(1+z_l) - (1+r)\bar{A})^{-\sigma} \geq 0$$

Since $\partial G/\partial b > 0$, the first result follows.

For the second result, note that:

$$\begin{aligned}
\frac{\partial b}{\partial \bar{A}\partial\theta} = & \frac{1}{(\cdot)^2}\left[\left(\sigma\beta(1+r)w(1+z_l)(w\theta(1+z_l) - (1+r)\bar{A})^{-(1+\sigma)}\right)\right. \\
& \left.\cdot\left((1+r)((b - P_l + \bar{A})^{-\sigma} - \Phi(1-\sigma)(1+r)(w\theta(2+r) + b(1+r))^{-\sigma})\right)\right] \\
& + \frac{1}{(\cdot)^2}\left[\left((b - P_l + \bar{A})^{-\sigma} + \beta(1+r)(w\theta(1+z_l) - \bar{A}(1+r))^{-\sigma}\right)\right. \\
& \left.\cdot\left(\sigma\Phi(1-\sigma)w(1+r)(2+r)(w\theta(2+r) + b(1+r))^{-\sigma}\right)\right] \\
\geq & 0
\end{aligned} \tag{B.0.3}$$

This proves Theorem 5.

□

B.0.6 Computation of Nash Equillibrium

In this section we will describe the algorithm used to compute the Nash Equilibrium between elite and non-elite universities. The Nash Equilibrium is composed by a

tuple $(P_h^*, \underline{\theta_h}^*, P_l^*, \underline{\theta_l}^*)$ such that:

$$(P_i^*, \underline{\theta_i}^*) \in \arg \max_{(P_i, \underline{\theta_i}) \in \mathcal{R}^+ \times [0,1]} \left(z_i(P_i, \underline{\theta_i}, P_{-i}^*, \underline{\theta_{-i}}^*) \right)^{\alpha} \left(\sigma_{b,i}(P_i, \underline{\theta_i}, P_{-i}^*, \underline{\theta_{-i}}^*) \right)^{1-\alpha} \tag{B.0.4}$$

Note that the problem defined in B.0.4 involves solving for a fixed point nested within another fixed point problem. In particular, the universities will offer a given level of z_l, z_h to the households and, conditional on such offer households will demand education services that need to fulfill the promised levels of z_l, z_h. This implies that when solving for the optimal of the universities we need to take into account that the offered level of productivities need to be satisfied by the demand of educational services. The full procedure to find the Equilibrium is described below:

Computation of the Nash Equilibrium

1. Start algorithm with some initial guess $\langle P_h^g, \underline{\theta_h^g}, P_l^g, \underline{\theta_l^g} \rangle$. Set $E = 10$.
2. Find $\langle P_h^T, \underline{\theta_h^T} \rangle \in \arg\max_{(P_h, \underline{\theta_h}) \in \mathcal{R}^+ \times [0,1]} \left(z_h(P_h, \underline{\theta_h}, P_l^g, \underline{\theta_l}^g) \right)^{\alpha} \left(\sigma_{b,h}(P_h, \underline{\theta_h}, P_l^g, \underline{\theta_l}^g) \right)^{1-\alpha}$
   (a) Set $\langle P_h^r, \underline{\theta_h^r} \rangle = \langle P_h^g, \underline{\theta_h^g} \rangle$
   (b) Given $\langle P_h^r, \underline{\theta_h^r}, P_l^g, \underline{\theta_l^g} \rangle$, go to 5. to compute $\langle z_h, z_l \rangle$
   (c) Given $S1 = \langle P_h^r, \underline{\theta_h^r}, P_l^g, \underline{\theta_l^g}, z_h, z_l \rangle$ compute the objective function of the university $H(S1)$.
   (d) Update for a new guess of the optimal $\langle P_h^r, \underline{\theta_h^r} \rangle = \langle P_h^{new}, \underline{\theta_h^{new}} \rangle$ according to some rule.
   (e) Repeat $(b) - (d)$ until optimal $\langle P_h^T, \underline{\theta_h^T} \rangle$ is found
3. Find $\langle P_l^T, \underline{\theta_l^T} \rangle \in \arg\max_{(P_l, \underline{\theta_l}) \in \mathcal{R}^+ \times [0,1]} \left(z_l(P_h^g, \underline{\theta_h^g}, P_l, \underline{\theta_l}) \right)^{\alpha} \left(\sigma_{b,l}(P_h^g, \underline{\theta_h^g}, P_l, \underline{\theta_l}) \right)^{1-\alpha}$

(a) Set $\langle P_l^r, \underline{\theta_h^l} \rangle = \langle P_l^g, \underline{\theta_l^g} \rangle$

(b) Given $\langle P_h^g, \underline{\theta_h^g}, P_l^r, \underline{\theta_l^r} \rangle$, go to 5. to compute $\langle z_h, z_l \rangle$

(c) Given $S1 = \langle P_h^g, \underline{\theta_h^g}, P_l^r, \underline{\theta_l^r}, z_h, z_l \rangle$ compute the objective function of the university $L(S1)$.

(d) Update for a new guess of the optimal $\langle P_l^r, \underline{\theta_l^r} \rangle = \langle P_l^{new}, \underline{\theta_l^{new}} \rangle$

(e) Repeat $(b) - (d)$ until optimal $\langle P_l^T, \underline{\theta_l^T} \rangle$ is found

4. Set $E = ||\langle P_h^g, \underline{\theta_h^g}, P_l^g, \underline{\theta_l^g} \rangle - \langle P_h^T, \underline{\theta_h^T}, P_l^T, \underline{\theta_l^T} \rangle||$. If E is smaller than a tolerance level, stop the algorithm, the NE is given by the tuple $\langle P_h^T, \underline{\theta_h^T}, P_l^T, \underline{\theta_l^T} \rangle$. Otherwise, set $\langle P_h^g, \underline{\theta_h^g}, P_l^g, \underline{\theta_l^g} \rangle = \langle P_h^T, \underline{\theta_h^T}, P_l^T, \underline{\theta_l^T} \rangle$ and go to 2.

5. Computation of $\langle z_h, z_l \rangle$ given $\langle P_h, \underline{\theta_h}, P_l, \underline{\theta_l} \rangle$

(a) Start algorithm with some initial guess $\langle z_h^g, z_l^g \rangle$ and set $\varepsilon = 10$

(b) Given $\langle P_h, \underline{\theta_h}, P_l, \underline{\theta_l} \rangle$, the guess $\langle z_h^g, z_l^g \rangle$ and the policy functions of the households, compute the realized values of $\langle z_h^r, z_l^r \rangle$

(c) set $\varepsilon = (z_h^r - z_h^g)^2 + (z_l^r - z_l^g)^2$.

(d) If ε is smaller to a tolerance level, the algorithm is complete. Otherwise, set $\langle z_h^g, z_l^g \rangle = \langle z_h^r, z_l^r \rangle$ and go to (b).

B.0.7 Analysis in the linear case

In order to get a clear idea of how credit constraints affect the market for higher education, we illustrate the linear case where $\sigma = 1$. Furthermore, we need to distinguish scenarios where households would like to substitute future for current consumption and the other way around. This is given by the inequality $\beta(1 + r) <$

1. Whenever this inequality is satisfied, households would prefer to get as much debt during the first period. The opposite case, when $\beta(1+r) \geq 1$ will motivate households to save as much as possible given that the returns to savings, in terms of utility, are more than one to one.

Case 1.$\beta(1+r) \geq 1$

In this case, households will prefer to save as much as they want and then the value functions for each case (not study, study in low quality university or study in high quality university) are given by:

$$V^N(b,\theta) = \beta\left[b(1-\tau)(1+r) + w\theta(2+r)\right] \tag{B.0.5}$$

(B.0.6)

The value function for households going to the low quality university is only defined whenever they can afford it. That is, whenever $P_l - b(1-\tau) \leq \min\{\bar{A}, \frac{w\theta(1+z^l)}{1+r}\}$. In particular, consider the case where $P_l - b(1-\tau) \leq 0$. If this holds, then households are able to afford the price of education with their income after taxes and thus we have no concerns about they not getting enough debt to fund their education.

However, when students should get positive debt in order to attend the low

quality university, the amount of debt should satisfy two constraints:

$$P_l - b(1-\tau) \leq \bar{A} \tag{B.0.7}$$

$$P_l - b(1-\tau) \leq \frac{w\theta(1+z^l)}{1+r} \tag{B.0.8}$$

The constraint given in B.0.7 states that the amount of debt students get should not exceed the upper limit given exogenously in the economy. The inequality given in B.0.8 guarantees that students have enough funds to get the necessary debt to attend college. The two aforementioned inequalities give bounds in b and θ for students to being able to pay the tuition in the low quality college:

$$b \geq b_{p_l} = \frac{\bar{A} - P_l}{1-\tau} \tag{B.0.9}$$

$$b \geq L(\theta) = \frac{P_l}{1-\tau} - \frac{w\theta(1+z^l)}{(1-\tau)(1+r)} \tag{B.0.10}$$

Now, for households with state variables (b, θ) such that low quality education is affordable, we can define the value of going to the low university as:

$$V^L(b,\theta) = \beta\left[(b(1-\tau) - Pl)(1+r) + w\theta(1+z^l)\right] \tag{B.0.11}$$

Similarly, in order to be able to go to the high quality institutions, it should be the

case that:

$$b \geq b_{p_h} = \frac{\bar{A} - P_h}{1 - \tau} \tag{B.0.12}$$

$$b \geq H(\theta) = \frac{P_h}{1 - \tau} - \frac{w\theta(1 + z^h)}{(1 - \tau)(1 + r)} \tag{B.0.13}$$

For those households, we can define the value of going to the high quality college as:

$$V^H(b, \theta) = \beta \left[(b(1 - \tau) - P_h)(1 + r) + w\theta(1 + z^h) \right] \tag{B.0.14}$$

Consider the case of a person who is deciding whether to go to the low quality college or not study. In such case, granted that he could afford to pay tuition, he will decide to attend whenever $V^L(b, \theta) \geq V^N(b, \theta)$. This implies that the decision will be to go to the low quality college whenever:

$$\theta_l \geq \theta_L = \frac{P_l(1 + r)}{w[z^l - r - 1]} \tag{B.0.15}$$

Similarly, when a person is deciding whether to go to the high quality college or to the low quality one, and granted he could afford both, the relevant decision rule will be to go to the high quality college whenever $V^H(b, \theta) \geq V^L(b, \theta)$. This

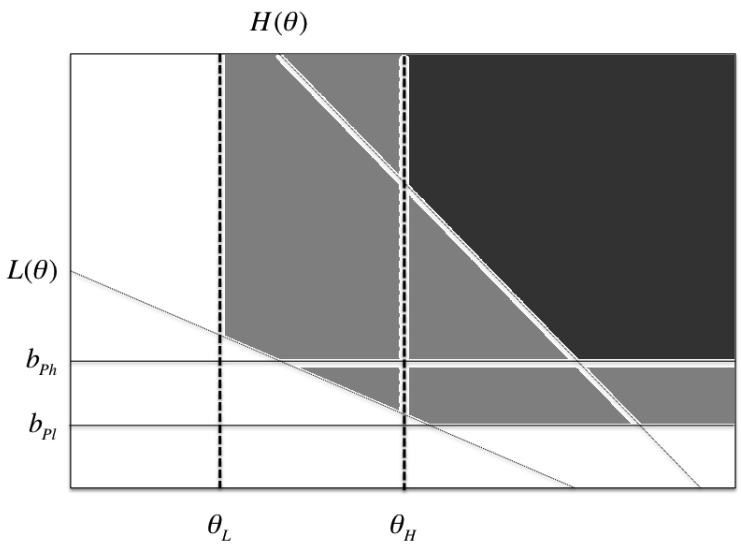


Figure B.1: Representation of the education decisions on the state space.

inequality generates the decision rule of going to college whenever:

$$\theta \geq \theta_H = \frac{(P_h - P_l)(1+r)}{w(z^h - z^l)} \tag{B.0.16}$$

The decision rules can be represented in the state space according to the following graph:

Note that we can express N^H in terms of elements that we have found previously:

$$N^H = \int_{\theta^H}^{\theta^{Ih}} \int_{H(\theta)}^{\bar{b}} dF(b,\theta) + \int_{\theta^{Ih}}^{1} \int_{b_{Ph}}^{\bar{b}} dF(b,\theta) \tag{B.0.17}$$

where $\bar{b}$ is the maximum level of bequests in the state space and

$$\theta^{Ih} = \frac{(1+r)\bar{A}}{(1+z^h)w} \tag{B.0.18}$$

For the sake of simplicity, we will assume a uniform distribution for (b, θ). As long as $P^h > P^l$ and $z^h > z^l$ we can express the measure of people going to the high quality university as:

$$N^H = \frac{1}{\bar{b}} \left[\left(b - \frac{P_h}{1-\tau} \right) \left(\frac{(1+r)\bar{A}}{(1+z^h)w} - \frac{(P_h - P_l)(1+r)}{w(z^h - z^l)} \right) \right. \tag{B.0.19}$$

$$\left. + \frac{w(1+z^h)}{(1-\tau)(1+r)} \left[\left(\frac{(1+r)\bar{A}}{w(1+z^h)} \right)^2 - \left(\frac{(P_h - P_l)(1+r)}{w(z^h - z^l)} \right)^2 \right] \right]$$

Similarly, the average level of skills of people attending such college is given by:

$$\begin{aligned} \tilde{\theta}^H = & \frac{1}{\bar{b}} \left[\left(\left(\frac{(1+r)\bar{A}}{(1+z^h)w} \right)^2 - \left(\frac{(P_h - P_l)(1+r)}{w(z^h - z^l)} \right)^2 \right) \left(\frac{\bar{b}}{2} - \frac{P_h}{2(1-\tau)} \right) + \right. \\ & \frac{w(1+z^h)}{3(1-\tau)(1+r)} \left[\left(\frac{(1+r)\bar{A}}{w(1+z^h)} \right)^3 - \left(\frac{(P_h - P_l)(1+r)}{w(z^h - z^l)} \right)^3 \right] + \\ & \left. \frac{1}{2} \left[\bar{b} - \frac{\bar{A}}{1-\tau} + \frac{P_h}{1-\tau} \left(1 - \left(\frac{(1+r)\bar{A}}{1(1+z^h)} \right)^2 \right) \right] \right] \end{aligned} \tag{B.0.20}$$

We can express the relevant variables for low quality college, granted $P_h > P_l$ and $z_h > z_l$, as:

$$\begin{aligned} N^L = & \int_{\theta_L}^{\theta_H} \int_{L(\theta)}^{1} dF(b, \theta) + \int_{\theta_H}^{\theta^{Il}} \int_{L(\theta)}^{H(\theta)} dF(b, \theta) + \\ & \int_{\theta^{Il}}^{\theta^{Ih}} \int_{b_{Pl}}^{H(\theta)} dF(b, \theta) + \int_{\theta^{Ih}}^{1} \int_{b_{Pl}}^{b_{Ph}} dF(b, \theta) \end{aligned} \tag{B.0.21}$$

$$\tilde{\theta}^L = \int_{\theta_L}^{\theta_H} \int_{L(\theta)}^{1} \theta dF(b,\theta) + \int_{\theta_H}^{\theta^{Il}} \int_{L(\theta)}^{H(\theta)} \theta dF(b,\theta) +$$

$$\int_{\theta^{Il}}^{\theta^{Ih}} \int_{b_{Pl}}^{H(\theta)} \theta dF(b,\theta) + \int_{\theta^{Ih}}^{1} \int_{b_{Pl}}^{b_{Ph}} \theta dF(b,\theta) \tag{B.0.22}$$

$$\mu_{b^L} = \int_{\theta_L}^{\theta_H} \int_{L(\theta)}^{1} b dF(b,\theta) + \int_{\theta_H}^{\theta^{Il}} \int_{L(\theta)}^{H(\theta)} b dF(b,\theta) +$$

$$\int_{\theta^{Il}}^{\theta^{Ih}} \int_{b_{Pl}}^{H(\theta)} b dF(b,\theta) + \int_{\theta^{Ih}}^{1} \int_{b_{Pl}}^{b_{Ph}} b dF(b,\theta) \tag{B.0.23}$$

It is important to note that throughout this analysis we have not implemented the fact that both colleges are able to set a threshold rule such that people with a level of skills below such threshold will not be admitted. In such a case, we will simply modify the regions of integration to consider that only people with ability beyond the threshold will be able to attend.

Existence of equilibrium

The expressions found in B.0.19, B.0.20, B.0.21 and B.0.22 can be used to express the necessary conditions that the offered qualities need to satisfy in equilibrium. In

particular, we need to find z^h, z^l such that:

$$\begin{bmatrix} z^h \\ z^l \end{bmatrix} = \begin{bmatrix} \kappa^h \left(\tilde{\theta}^h(\underline{\theta^h}, \underline{\theta^l}, P_h, P_l, z^h, z^l)\right)^{\alpha_1} \left(I(\underline{\theta^h}, \underline{\theta^l}, P_h, P_l, z^h, z^l)\right)^{\alpha_2} \\ \kappa^l \left(\tilde{\theta}^h(\underline{\theta^l}, \underline{\theta^l}, P_h, P_l, z^h, z^l)\right)^{\alpha_1} \left(I(\underline{\theta^h}, \underline{\theta^l}, P_h, P_l, z^h, z^l)\right)^{\alpha_2} \end{bmatrix} \tag{B.0.24}$$

We need to prove existence of a fixed point in the qualities offered by universities before proving the existence of the Nash Equilibrium. Note, however, that difficulty arises in this point given the fact that there is no natural way to bound the set of qualities offered by the universities. Additionally, note that equations B.0.19, B.0.20, B.0.21 are not continuous in $z^h = z^l$. The inability of proving the existence of a fixed point in the qualities offered by universities shows that it is not possible to prove existence of the Nash Equilibrium. We rely purely on the computational analysis to find a Nash Equilibrium in this case that might not be unique.

Case 2. $\beta(1+r) < 1$

This case is more involved as households value more current consumption than future and will try to get as much debt as possible. The difficulty arises as even when students can afford to pay college, they might be constrained given that they want to substitute future by current consumption. Additionally, we need to establish which is the relevant constraint that households face when getting the desired level of debt, either the exogenously given level of credit constraint or they reach a point where they can't fund the debt with their resources.

We start analyzing the case of a person who is not going to university. In this case, the person will get as much debt as possible and he will be constrained whenever $\frac{w\theta}{1+r} > \bar{A}$. If this is the case, the person will get the maximum level of debt $\bar{A}$. Taking into account this case when computing the value of not going to college, we see that:

$$V^N(b,\theta) = \begin{cases} b(1-\tau) + w\theta\frac{2+r}{1+r} \text{ if } \theta \leq \frac{\bar{A}(1+r)}{w} \\ \\ b(1-\tau) + w(\theta)(1+\beta) + \bar{A}[1-\beta(1+r)] \text{ if } \theta > \bar{A}\frac{1+r}{w} \end{cases} \tag{B.0.25}$$

Now, let's consider a household that goes to the low-quality university. Evidently, the value function will only be defined for the case when it is possible to pay tuition price via endowment or debt. For people whose income is below the tuition price ($b(1-\tau) < P_l$) and who are constrained either by the exogenous level $\bar{A}$ or by their earning capacity $\frac{w\theta(1+z^l)}{1+r}$, the value of going to the low quality college will not be defined.

An individual who is not constrained and takes as much debt as he can, will derive utility given by $b(1-\tau) - P_l + \frac{w\theta(1+z^l)}{1+r}$. The first term, $b(1-\tau) - P_l$ corresponds to net income after tuition and the remaining part $\frac{w\theta(1+z^l)}{1+r}$ is simply the amount they will make in the second period taken to the present value of the first period.

If the net income after tuition is negative, an individual will not be credit con-

strained so long as:

$$P_l - b(1-\tau) \leq \min\{\bar{A}, \frac{w\theta(1+z^l)}{1+r}\} \tag{B.0.26}$$

However, it is possible to have individuals who are borrowing constrained even if the net income after tuition is positive. These individuals are those who would like to borrow against their future income, given that current consumption is more valuable than future consumption, but they are not able to borrow as much as they want given the exogenous limit $\bar{A}$. Those are individuals such that:

$$\frac{w\theta(1+z^l)}{(1+r)} < \bar{A} \tag{B.0.27}$$

and they are forced to borrow no more than $\bar{A}$. This implies that we can define the value of going to low-quality college as:

$$V^L(b,\theta) = \begin{cases} b(1-\tau) - P_l + \frac{w\theta(1+z^l)}{1+r} \text{ if } \begin{cases} b(1-\tau) - P_l \geq 0 \quad \theta \leq \frac{\bar{A}(1+r)}{w(1+z^l)} \\ \text{or} \\ b(1-\tau) - P_l < 0 \quad P_l - b(1-\tau) \leq \min\{\bar{A}, \frac{w(\theta)(1+z^l)}{1+r}\} \end{cases} \\ \\ b(1-\tau) - P_l + \bar{A}[1-\beta(1+r)] + w\beta(1+z^l) \text{ if } b(1-\tau) - P_l > 0 \text{ and } \theta > \frac{\bar{A}(1+r)}{w(1+z^l)} \end{cases} \tag{B.0.28}$$

Finally, doing the same analysis but with P_h and z^h we can find the value of

going to the high quality college:

$$V^H(b,\theta) = \begin{cases} b(1-\tau) - P_h + \frac{w\theta(1+z^h)}{1+r} \text{ if } \begin{cases} b(1-\tau) - P_h \geq 0 \quad \theta \leq \frac{\bar{A}(1+r)}{w(1+z^h)} \\ \text{ or} \\ b(1-\tau) - P_h < 0 \quad P_h - b(1-\tau) \leq \min\{\bar{A}, \frac{w(\theta)(1+z^h)}{1+r}\} \end{cases} \\ \\ b(1-\tau) - P_h + \bar{A}[1-\beta(1+r)] + w\beta(1+z^h) \text{ if } b(1-\tau) - P_h > 0 \text{ and } \theta > \frac{\bar{A}(1+r)}{w(1+z^h)} \end{cases} \tag{B.0.29}$$

B.0.8 Life-cycle Model

In this section we embed a life-cycle model into a two-period model, so our calibration of Section 2.5.2 is realistic. We solve the household's problem in two parts: 1) during the study periods, $t = 0, \ldots, S-1$, and 2) after college age, $S, \ldots, T$, and leave the problem expressed as a two-period maximization problem in which households decide how much to save for post-college periods. First, we start by solving the post-college optimization problem. We assume that after college graduation, individuals enter perfect financial markets, so there is perfect consumption smoothing. The problem of the households is:

$$\max_{c_t} \sum_{t=S}^{T} \beta^{t-S} \frac{c_t^{1-\sigma}}{1-\sigma}, \quad s.t.$$

$$c_S = b + a_{S+1} + w(1+z_j)\theta$$

$$c_t + a_t(1+r) = a_{t+1} + w(1+z_j)\theta, \quad t \in \{S, \ldots, T\}$$

where a_{t+1} is the debt at period t to be repaid next period, and b are the savings that the individual carries from the college years. In here, we assume that there are no borrowing constraints, since households enter perfect financial markets. Solving this problem, yields the present value budget constraint in period S:

$$\sum_{t=S}^{T} \frac{c_t}{(1+r)^{t-S}} = b + \sum_{t=S}^{T} \frac{w\theta(1+z_j)}{(1+r)^{t-S}}$$

Combining this with the Euler equation, the optimal consumption path is given by:

$$c_S = \frac{1}{\Phi_S}\left(b + w(1+z_j)\theta\Phi_r^o\right)$$

$$c_t = \left((1+r)\beta\right)^{\frac{t-S}{\sigma}} c_S, \quad t \in \{S, \ldots, T\}$$

where Φ_S and Φ_r^o are given by the following expressions:

$$\boxed{\Phi_S = \frac{1 - \left(\frac{\beta}{(1+r)^{\sigma-1}}\right)^{\frac{T-S+1}{\sigma}}}{1 - \left(\frac{\beta}{(1+r)^{\sigma-1}}\right)^{\frac{1}{\sigma}}}} \qquad \boxed{\Phi_r^o = \frac{1 - \left(\frac{1}{1+r}\right)^{T-S+1}}{1 - \left(\frac{1}{1+r}\right)}}$$

The present value utility at time S of this consumption path is given by:

$$\sum_{t=S}^{T} \beta^{t-S} u(c_t) = \Phi_S u(c_S)$$

Note that c_S is determined for every given savings b carried from the college

period, so without solving the problem for periods $\{0, \ldots, S-1\}$, it will not be completely pinned down. Now, we solve for the households' problem during periods $0, \ldots, S-1$. Given that during college, there exist exogenous borrowing constraints given by $\bar{A}$, there are two cases: a) individuals are unconstrained, and b) individuals are constrained. The unconstrained solution of the problem in periods $\{0, \ldots, S-1\}$ yields:

$$c_0\Phi_0 + (P_h h + P_l l)\Phi_r^y + \frac{a}{(1+r)^S} = w\theta\Phi_r^y(1-l)(1-h) + b$$

$$c_t = ((1+r)\beta)^{\frac{t}{\sigma}} c_0, \quad t \in \{1, \ldots, S-1\}$$

where b is the initial wealth of individuals, and Φ_0, Φ_r^y are given by:

$$\boxed{\Phi_0 = \frac{1 - \left(\frac{\beta}{(1+r)^{\sigma-1}}\right)^{\frac{S}{\sigma}}}{1 - \left(\frac{\beta}{(1+r)^{\sigma-1}}\right)^{\frac{1}{\sigma}}}} \qquad \boxed{\Phi_r^y = \frac{1 - \left(\frac{1}{1+r}\right)^S}{1 - \left(\frac{1}{1+r}\right)}}$$

Utility in period 0 is given by

$$\sum_{t=0}^{S} \beta^t u(c_t) = \Phi_0 u(c_0)$$

Note that now, the problem can be perfectly embedded in the two-period model described in Section 2.3. Households solve the following two-period problem:

$$\max_{c_0, c_S} \quad u(c_0) + \tilde{\beta} u(c_S), \quad s.t.$$

$$c_s\Phi_S = a + w\theta(1+z_j)\Phi_r^o$$

$$c_0\Phi_0 + (P_h h + P_l l)\Phi_r^y + \frac{a}{(1+r)^S} = w\theta\Phi_r^y(1-l)(1-h) + b$$

$$a \geq -\bar{A}$$

where:

$$\boxed{\tilde{\beta} = \frac{\beta^S\Phi_S}{\Phi_0}}$$

These two budget constraints can be rewritten as a single lifetime budget constraint:

$$c_0\Phi_0 + (P_h h + P_l l)\Phi_r^y + \frac{c_s\Phi_s}{(1+r)^S} = w\theta\Phi_r^y(1-l)(1-h) + \frac{w\theta(1+z_j)\Phi_r^o}{(1+r)^S} + b$$

The unconstrained consumptions are given by:

$$c_n = \frac{(\beta(1+r))^{(-S/\sigma)}\left[w\theta\left(\frac{\Phi_r^o+(1+r)^S\Phi_r^y}{\Phi_S}\right) + \frac{b(1+r)^S}{\Phi_S}\right]}{1 + \frac{(\beta(1+r))^{(-S/\sigma)}\Phi_0(1+r)^S}{\Phi_S}}$$

$$c_h = \frac{(\beta(1+r))^{(-S/\sigma)}\left[w\theta(1+z_h)\frac{\Phi_r^o}{\Phi_S} + \frac{b(1+r)^S}{\Phi_S} - \frac{P_h\Phi_r^y(1+r)^S}{\Phi_S}\right]}{1 + \frac{(\beta(1+r))^{(-S/\sigma)}\Phi_0(1+r)^S}{\Phi_S}}$$

$$c_l = \frac{(\beta(1+r))^{(-S/\sigma)}\left[w\theta(1+z_l)\frac{\Phi_r^o}{\Phi_S} + \frac{b(1+r)^S}{\Phi_S} - \frac{P_l\Phi_r^y(1+r)^S}{\Phi_S}\right]}{1 + \frac{(\beta(1+r))^{(-S/\sigma)}\Phi_0(1+r)^S}{\Phi_S}}$$

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