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A comparison of Value at Risk methods in portfolios with linear and non-linear financial instruments

A dissertation submitted in partial fulfilment of the requirements of the School of Business and Law, University of East London for the degree of Finance and Risk MSc

September 2016

Word count: 13,114

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A comparison of Value at Risk methods in portfolios with linear and non-linear financial instruments

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September 2016

Abstract

This paper intends to critically evaluate and compare the most used Value at Risk (VaR) methods, whilst also presenting the strengths and weaknesses of each model. The analysis is based on a stock (linear) portfolio and an option (non-linear) portfolio. The methodologies applied are the delta-normal, delta gamma, historical simulation and Monte Carlo simulation, computed to one and five days time horizon with 95% and 99% of confidence level.

The results demonstrate that Monte Carlo simulation provided the most accurate risk measure and consistent results for both portfolios which reinforce the flexibility of the model to estimate VaR. Although the Delta Gamma also showed an accurate VaR for the option portfolio, it is complex and demands a high level of calculation which can become complicated and costly. The historical simulation for both portfolios were overestimated because of the fact that the historical simulation is strongly based on historical data. Additionally, the delta normal was shown to be a weak model as it does not properly present accuracy even for the linear portfolio. This is because this model is heavily based on normal distributions, and in practice fat tails are more frequent than predicted by the model.

The benefit of portfolio diversification in the VaR measure was also proved in this paper. It presented a substantial improvement in the VaR measure when considering in the calculations the correlation between instruments. Once more, the Monte Carlo simulation presented a higher efficiency in VaR measures with the diversified portfolio.

Finally, to deal with the problem of extreme values in the sample, this work suggests an approach to improve the VaR measures, the Extreme Value Theory (EVT).

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1. Introduction

The last global economic disasters led to the bankruptcies of large and robust financial institutions that in many cases were considered systemically key for the industry. The interconnection between these financial institutions allowed an increase in the size of the disasters in the market. Most crises were caused as a result of a failure of risk management systems. The 2007-2008 crisis highlighted serious deficiencies in risk management and demonstrated the importance of an efficient risk management process (Jorion, 2009). Thus, the weaknesses of the world regulatory system were exposed and criticised by the financial market which raised the necessity of a more rigorous international regulation to control and monitor the financial market. As a consequence, the financial regulation was reformed and new measurements and controls were established, for instance, the Dodd-Frank Act and the Systemically Important Financial Institution – SIFI regulation.

In order to adhere to international agreements defined by the regulators, financial institutions had to develop tools and systems to measure and control their risk exposure. Using statistical and mathematical approaches to measure risk, JP Morgan developed the RiskMetrics which was published in 1994 through a technical document available for all market participants. It increased the popularity of the Value at Risk – VaR methodology that became a reference in estimating market risk. As a result, other financial institutions started developing improvements and variants of the VaR.

VaR reflects the maximum expected loss an institution can obtain given a time horizon and confidence level. It is widely used by banks and financial regulators as a standard measure to monitor and compare risks in different sectors. The VaR popularity is a result of a combination of factors. Firstly, the improvement of the banking regulation on risk management such as the Basel Accord. Secondly, the globalisation of the financial market that increases volatility and exposure to several risks. Lastly, the technology which enhances the risk control (Jorion, 2007). With regards to managing and supervising risk exposure, banks and regulators are able to assess the likely loss of a portfolio through the VaR.

The importance of the VaR is reinforced by the enormous volatility of the financial market through interest rates, exchange rates, stock prices and the widespread derivatives market over the last decades. On top of that, the sharp growth of over the counter (OTC) derivatives market allows an increase in security trades, funding management, and foreign transactions by enhancing interconnections among companies worldwide. As a consequence, it raises the number of instruments in the companies’ portfolios and their complexity. Because most portfolios are frequently traded, their risk positions change over time. So, it is essential for a risk model to provide managers with a proper level of controls, monitoring and escalation of their risk exposures. Although VaR methodology can provide it, its application is not effortless.

On the last BCBS (Basel Committee on Banking Supervision) standard publication about capital requirement for market risk, it is recommended the VaR application for measuring market and default risks (BCBS, 2016). There are also other BCBS's publications suggesting VAR methodology to compute different types of risk exposures. Additionally, BCBS constantly issues guidelines about capital requirement which might be followed by central banks and financial institutions as best practices. For this reason, the decision of what model should be used for risk measurement becomes crucial for all banks and financial institutions. This is because, besides managing risks prudently, banks need to be in compliance with regulation in terms of holding capital according to their risk measured. Cuoco and Liu (2005) suggest VaR as an efficient methodology to estimate risk for capital allocation, they also find that it can contribute to reveal this risks. All financial institutions aim to be efficient in managing risk. In order to achieve it, they invest effort in choosing an adequate VaR methodology among the numerous approaches available. Moreover, they can create their own proprietary model that often better fits their portfolios, risk exposures and capital requirements. These internal models usually are based on VaR methodology.

Due to the importance of the VaR as a risk measure recommended by regulation and its widespread reach all over the international market, an important question arises: which VaR method is able to deliver the most accurate risk exposure estimation? The theory says that there is no single winner and that it depends on the portfolio composition and strategy. Lambadrais *et al* (2000) present a comparison between Monte Carlo and historical simulations for linear and non-linear portfolios, and conclude that the Monte Carlo simulation shows more accurate performances in the linear portfolio; however, for the non-linear portfolio any model performs well. Chapter 2, the literature review, is illustrated with more papers that compare different VaR methodologies.

The aim of this paper is to critically evaluate and compare different VaR approaches for linear and non-linear portfolios and presents the strengths and weaknesses of each model. Figure 1 shows an overview of the traditional methodologies used to calculate the VaR divided by parametric and non-parametric models. This paper calculates and compares the four methods: delta normal, delta gamma, historical and Monte Carlo simulation~~s~~. These methods are computed for linear (stock) and non-linear (option) portfolios, as the VaR methods analysed have different applications with regards to the linearity of the instruments. The first portfolio is composed by 20 stocks and the second by two stock options and one index option. The approaches are calculated for one and five days of time horizon with 95% and 99% of confidence level that is the standard of the financial market. Additionally, this paper suggests a model called Extreme Value Theory to improve the VaR methodology when there are extreme values among the observations.

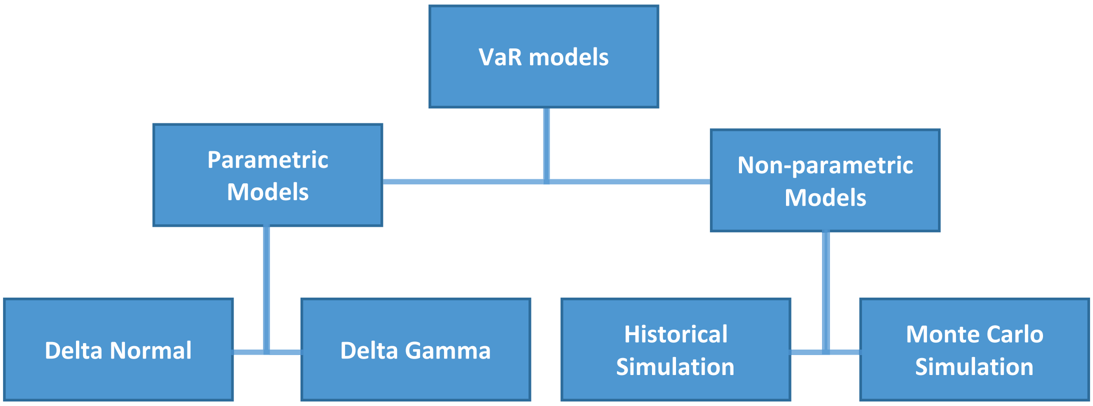

Figure 1 – Different VaR methodologies

This work is organised as follows. The next section, chapter 2, presents a review of the traditional and more recent literature related to VaR. Chapter 3 describes the main VaR methodologies which are used later to evaluate the portfolio's market risk. In chapter 4, a comparison of four VaR methods is made by applying them to a linear stock portfolio and a non-linear option portfolio, also the advantages and weaknesses of each model are presented. Chapter 5 suggests an alternative VaR approach based on extreme values. Chapter 6 presents the general conclusion. Finally, chapter 7 suggests the recommendations.

2. Critical Literature Review

One of the most robust pieces of research about the VaR methods emanates from Duffie and Pan (1997). They compare the delta gamma method with the Monte Carlo simulation for an option portfolio randomly selected. It is composed of 10,996 options with 418 different underlying ones exposed by different risk factors: commodities, equity, fixed income and foreign exchange. This study uses some identical parameters applied in the RiskMetrics (1996), such as standard deviation. This research finds that the Monte Carlo simulation, with a stochastic process and a lognormal distribution of the returns, presents higher VaR than the delta gamma model. This is based on a 99% confidence level, and also considering long and short positions. This difference between the approaches is 0.1% of the portfolio value for 1 day VaR and 1.8% for 10 days VaR. This is due to the non-normality (fat tails) and non-linearity of the options.

A highly significant contribution to spreading the VaR methodology is the RiskMetrics (1996) by JP Morgan and Reuters. This technical document provides enhancement in the parameters estimation to calculate the VaR analytically for a portfolio. The assumptions used makes the VaR application simpler and available for any financial institution. Regarding the distributions, this study assumes that the standardised returns have a conditional normal distribution although they might not be normal because, in practice, fat tails are common. The standardised returns are estimated as the return divided by the standard deviation. The research's attention to this standardised returns allows seeing the importance of the proportion of the return associated with standard deviation. So, a high profit or loss due to a great volatility might lead to a low standardised return while a great return in a low volatile scenario, might result in a high standardised return. The assumption of the standardised returns allows increase the possibility of outliers in the distribution increasing more than what could exist in a normal distribution. In this way the RiskMetrics can be applied to normal mixture distributions that enables the inclusion of larger probability for outliers. However, in this approach what is needed is to estimate the probability of the jumps and their standard deviations, which is neither simple nor effortless as they are rare.

RiskMetrics (1996) also explains some fundamental procedures in the VaR system. Firstly, it presents three necessary parameters to estimate the VaR: the time horizon, the confidence level and the currency used to measure risk. Then, it shows the importance of identifying the cash flow and Marked to Market (MtM) of the portfolio positions, and applying the mapping process where the positions are aggregated in risk factors. Therefore, this paper demonstrates that a crucial step is to choose the method to estimate the VaR. If it is expected that the portfolio will have an approximated normal condition, the RiskMetrics approach (delta normal) should be applied. On the other hand, if the portfolio is exposed to non-linearity and the assumption of normality is not expected, two methodologies should be chosen: the delta gamma and Monte Carlo.

Additionally, Hull and White (1998) recommend alternatives to measuring VaR when the observations are not normally distributed. It enables the calculation of any probability distribution for these observations, but the distribution still has to be a multivariate normal distribution. There are also other studies that create relevant variations of the distribution, however they might lead with some issues. Assessing inputs for non-normal methods is likely to be hard, mainly when using historical data. Also, it is gradually harder to measure losses with asymmetric and fat tailed distribution than with normal distributions.

Furthermore, plenty of research has developed improvements in techniques to estimate variance and covariance, which is an important parameter in the VaR calculations. Some papers argue about the relevance of sampling data and creating different ways to do it. Also, there are paper that develop new statistical models to create more precise estimates from the data in order to give the VaR a more realistic number. The most traditional models believe in the hypothesis that the standard deviation of the returns does not change during the given time. This is the homoscedasticity process. On the other hand, according to Engle (2001), more accurate VaR calculation can be realised when considering possible changes in the standard deviation over time. Using this approach, he argues for the autoregressive conditional heteroscedasticity (ARCH) and the generalised autoregressive conditional heteroscedasticity (GARCH) to deal with standard deviation and variance more precisely. However, the disadvantage of applying these techniques to calculate the variance and covariance of the VaR is that it is only used in portfolios with linear market risk exposure not including options.

Regarding to accuracy and computational time, Pritsker (1997) analyses and compares six VaR models for non-linear instruments and 500 random portfolios. The instruments are call and put options classified in long and short positions, in/out the money and different maturities. The portfolios comprehend underlying options. The results show that 25% of the 500 portfolios under/overestimate the VaR by 10%. When the underlying options of the portfolios are deep out the money or with a short time to expiration the estimations are weaker. For the call options, alternatives models of the delta normal and delta gamma are also applied and the results are highly overestimated. For the put options, the VaR models are underestimated. In terms of the trade-off between accuracy and time consumption, the delta gamma Monte Carlo presents the best performance. Moreover, Krause and Paolella (2014) present a VaR approach based on GARCH for return distributions that contain leptokurtosis, asymmetry and conditional heteroscedasticity. This paper illustrates that this approach works properly and deliver higher results than traditional methods. This is achievable due to the use of quicker and simpler ways to get the risk measure. Also, this work calculates the VaR using samples of 250 observations or less.

Moreover, Castellacci and Siclari (2003) measure VaR methods using 5 approaches for non-linear instruments (option strategies) modelling the return distribution as multi normal random observations. The models applied are: full Monte Carlo, delta gamma Monte Carlo, delta normal, delta gamma normal and Cornish Fisher. Although the theory suggests the delta gamma approach for option portfolios, this paper presents a better performance in the delta normal VaR measure. However, it shows relevant improvements in the delta gamma approach instead of Monte Carlo as it considers the gammas. This is a result of the non-diagonal function in the gamma matrix, also the effect of correlation between the instrument of the portfolios. The delta gamma Monte Carlo approach shows a good performance with moderate computational time. Broadly speaking, this paper demonstrates that the parametric models overestimate the VaR while delta gamma Monte Carlo slightly underestimates the VaR.

There are many literatures which compare and contrast the VaR methodologies, also they provide different views, strengths and weaknesses of the VaR. For instance, Hendricks (1996) critically evaluates 12 methods to calculate the VaR. These methods are alternatives based on the variance-covariance model and the historical simulation. He uses 1000 portfolios randomly selected which are exposed to foreign exchange. The application of 9 methods to measure the VaR is done by comparing the real loss with its estimate. Finally, he does not recommend any method. He argues that extreme values are more frequent than assumed by the normal distribution and the volatility is not constant over time.

Additionally, Lambadrais *et al* (2000) compare VaR methods applying Monte Carlo and historical simulations in the Greek bond and stock markets. For the two methods, portfolios with linear and non-linear instruments are applied. The findings are different for linear and non-linear portfolios. For the linear portfolio, the Monte Carlo simulation performs well but in the historical simulation the VaR is overestimated. For the non-linear portfolio, both methods do not demonstrate clear results as their numbers differ in the measurement test. The paper also shows that for linear portfolios, the accuracy of the models depends on the confidence level. Emmer *et al* (2015) compare VaR among other approaches to estimate market risk. This analysis finds positive and negative aspects of VaR. The negative point is that VaR does not account for tail risks beyond the VaR which can be worse when leading with risks that have different tails. In terms of robustness, they argue that VaR has superior results.

Kuester et al. (2006) also compare different VaR models applying more than 30 years of daily price for a stock index portfolio. This paper's results show that despite most models being accepted by the regulator, they are not satisfactory. The most accurate results are from a GARCH hybrid model with Extreme Value Theory, a variant of a filtered historical simulation and a heteroskedastic mixture distribution. The model shows that for the parametric approaches a significant result in violation frequency is likely to be seen when considering scale dynamics, and also, when considering heteroskedastic, it results in unclustered VaR violations. This paper demonstrates that normality is reached with innovation distribution that includes skewness and fat tails.

Another relevant paper from Giot and Laurent (2003) measures different VaR approaches and compares their outcomes. This study focuses on commodity market through portfolios with long and short positions using data series of 5 years. It analyses the RiskMetrics, skewed student APARC (ARMA model with GARCH) and skewed student ARCH methods. The results show more precise VaR using the APARCH approach even though the ARCH approach achieves important results without being a necessary non-linear optimization step. According to Christoffersen *et al.* (2001) a good way to validate a VaR model is to use a robust measure. His suggestion is that the measure should follow some steps: to attend the efficient VaR premises and to compare two less accurate VaR methods. Then, this research develops a new approach to measure VaR models and applies it for traditional VaR measures and VaR based on implied volatilities, and it shows a significant result.

Reuse (2010) compares the traditional delta normal VaR method with more modern VaR method such as the historical simulation with copula functions, during a financial crisis scenario using 10 years data. Surprisingly, his results show that the crisis has no effect in the portfolio optimization but the VaR methods present considerable different estimates even more when increasing time horizon. The ideas he presents behind the results is that while in the historical simulations risk is captured by the difference between expected loss and historical data; in the delta normal, risk is captured by the expected loss and the current value, also, in the delta normal, there is a linear approximation which not necessarily illustrate the real data. Regarding the portfolio selection, he argues that the historical simulation gives a better combination of assets, and the delta normal method increases risk exposure. Consequently, the diversification factor is more efficient in the historical simulation. Additionally, the results do not change when raising confidence level. However, this paper shows that the weaknesses in both models is being dependent on historical data, and that is difficult to define the period accurately to have a precise VaR estimation. He also argument that VaR models should be less complicated to understand and apply by the risk managers; it sometimes is more important than accuracy.

According to Trenca *et al.* (2011), the portfolio characteristics are very important factor to consider when selecting which VaR model to use. This work captures parametric (delta normal) and non-parametric models (historical simulation, filtered historical simulation - FHS and Monte Carlo), also it deals with volatility and correlation calculations (GARCH model). They argue that even though the delta normal is the simplest to apply, it usually underestimates the VaR and capital allocation as it does not take account the fat tails that, in practice, is quite common. They reinforce that VaR is one of the most popular approach to control, measure and that it can prevent market risk as well.

Bozkaya (2013) includes high frequency intraday data in the volatility to calculate the VaR more precisely. Also, he computes the historical simulation and the parametric VaR with realised volatility forecasting power of the EWMA (exponential weighted moving average), MA (moving average) and GARCH, the popular approaches to estimate volatility. These models give precise estimations as they consider the possibility of changing volatility over time. This paper results show that the methodologies applied to calculate volatility in general work well even though they present a slightly small difference between the benchmark and realized volatility, mainly caused by the time horizon; also, the EWMA model performs better than the GARCH model. After back testing, the MA demonstrates to be the most accurate model for 5 minutes and 10 minutes returns in the intraday VaR. According to the paper, this is because the data is highly volatile with extreme values and the MA VaR deals well with this assumption. In general, this study says that the moving average approach to calculate the volatility is a better model to compute the market risk through VaR. However, when calculating this model for another portfolio or instrument, it can provide a different outcome.

Regarding the historical simulation approaches, Cabedo and Moya (2003) develop a VaR method called historical simulation with ARMA (autoregressive moving average model) forecast - HSAF. This model does not use a simple time series of historical prices but rather the distribution of forecasting errors. In order to measure VaR, this model captures the autocorrelation of the historical prices and, estimates the historical return in absolute values, applying the autocorrelation functions to forecast precisely the future returns. Using oil prices, the researchers compare their own model with the other two VaR methodologies: the standard historical simulation and the delta normal method. The results show a better performance in the HSAF model rather than in the two traditional models. A positive aspect of the HSAF is that it does not need statistical assumption in the distribution of the historical prices as in the ARCH and GARCH approaches. However, if the distribution of returns is non-stationary or non-statistically significant, the VaR calculation of HSAF is the same as the standard historical simulation.

Additionally, Boudoukh *et al.* (1998) investigate the limitations of the variants in historical simulations. They develop a hybrid approach combining the RiskMetricks and the historical simulation to improve the VaR for fat tailed distributions. This approach consists of calculating the percentile of the return distribution by declining weights in the historical data through a decay factor. As the data is more recent, it has more weight in the analysis. This model is applied to a stock portfolio with historical data of 250 days including equally the period before and after the stock market crash of 19th October 1987. The results demonstrate that the VaR using the hybrid approach for this portfolio is not over/underestimated even on the day after the crash because the days are equally weighted according to the model. As the recent past has more weight in the data, is does not cause an outlier in the return distribution.

In a different way but with the same aim of updating historical data, Hull and White (1998) propose improvements in historical VaR by adjusting the data to include changes in volatility over the time. For instance, they argue that when using series with different volatilities during the time, if the series are updated before calculating the VaR, it can improve the risk measure. Using GARCH and exponentially weighted moving average models, the simulation is applied to 9 years of historical data, 12 different exchange rates and 5 stock indices and shows relevant results in estimating the VaR.

On the other hand, Pritsker (2006) evaluates and criticizes the previous papers concerning adjusting the historical data to calculate the VaR. He evaluates the standard historical simulation, the historical simulation suggested by Boudoukh *et al.* (1998) - BRW and the FHS of Barone-Adesi *et al.* (1999). For the standard historical simulations and the BRW methodologies, the results show that there is an improvement in the VaR measure when considering a high loss in the portfolio return but not when there is a large profit. For the FHS the results show a necessity of adjustment in the model to consider time-varying correlations and to select an adequate length of the past sample. Also, previous tests illustrate that applying the FHS for 2 years of daily returns a 99% confidence level and 10 days of time horizon, might not have sufficient outliers to estimate the VaR precisely.

Regarding to the Monte Carlo approach, the greatest weakness of the Monte Carlo simulation is computational time as the model requires plenty of random scenarios to compute the VaR. Jamshidian and Zhu (1997) suggest a scenario simulation approach for the Monte Carlo VaR for large portfolios with exposure to multi market risks. This method applies a limited and manageable quantity of scenarios instead of multivariate distribution of returns. This method also can be used in credit risks. The findings show a large computational efficiency to estimate VaR precisely for portfolios highly exposed to risk factors. Moreover, Frye (1997) presents a method to reduce the quantity of random scenarios by pre-selecting and computing shocks in the data analysis.

Furthermore, Glasserman *et al.* (2000) propose a model to simplify the sampling of scenarios in Monte Carlo simulation by using the delta-gamma approach. As deltas and gammas are used in valuations and other analyses, they are often accessible without further effort. The results show that this model decreases the quantity of simulations to achieve accuracy, and that consequently it is more efficient in terms of computational time and cost to generate the Monte Carlo VaR. Additionally, Botev *et al.* (2010) suggest a new Monte Carlo model through generalised splitting algorithm which they say provides superior results regards to accuracy and time consumption.

Despite being the most widespread methodology to measure risk, there is relevant research that does not recommend VaR. Beder (1995) argues that VaR is highly seductive but dangerous as a method to estimate risk. After applying 8 methods for 3 portfolios, this work finds significant inaccuracies in the VaR estimations. Also, it points out that VaR is heavily dependent on parameters, data and assumptions and concludes that VaR is not a sufficient tool to control risk. Other papers show that it is totally possible to find different outputs from the same portfolio and underlying asset considering the same approach. Berkowitz, O'Brien (2002), Ju and Pearson (1998) argue that the VaR estimation for some banks are highly imprecise. Artzner et al (1997) also criticise VaR. They mention examples where a portfolio is divided into 2 sub-portfolios and the calculation of their VaR demonstrates that the sum of the VaR of the 2 sub-portfolios is lower that the VaR of the portfolio before being divided. It should account for the correlation of the instruments that compose the portfolio. This would be an issue for VaR limits in individual books. Also Artzner et al (1997) say that VaR fails in neglecting the size of the loss that exceeds it.

According to Yamai and Yoshiba (2005), a substantial problem of the VaR methodology is that it does not deal with the loss size that exceed the VaR, they called it as tail risk. They analyse a credit portfolio and foreign exchange rates in a stressed scenario. They also discuss the issues can be caused by this risk and make a comparison between VaR and an alternative risk measure, known as Expected Shortfall. Their findings show that this alternative in many cases can properly replace VaR; however, it needs more sample size to result a precise measure.

Similar to any other mathematical or statistical method, VaR can fail if its assumptions or parameters are not precise. Alexander and Sarabia (2012) present a model to deal with the problem of imprecise VaR due to the inadequate choice of the VaR approach or inaccurate VaR parameter calculations. They say this model provides a key to avoid model risk and to attend the capital regulation that requires institutions to control it. This is based on a comparison between a benchmark VaR and the daily VaR computed by the institution. They argue that the benchmark VaR should consider the total information, beliefs and the maximum possible distribution. It should also be determined by the local regulator and applied for institutions. The difference in the quantile between the VaR and its benchmark would determine the model risk and it should be adjusted in the capital requirement. They also illustrate their model by applying an example. However, according to Barrieu and Scandolo (2015) VaR has higher level of model risk than expected shortfall, the other methodology used to measure market risk. It considers 99% and 95% of confidence level. This finding is contrary to the theory which says that expected shortfall is more impacted by model risk than VaR. This test was performed using.

3. Research Methodology

3.1 Definitions

VaR is a market risk statistical measure for investments and/or portfolios. VaR represents the maximum possible loss a company/investor can have due to the asset price movements considering two important parameters: the time horizon and the confidence level. Following the VaR's concept, losses higher than the VaR has a low chance of occurring so, VaR is a standard measure of loss in a normal market condition. In other words, VaR illustrates the quantile of the projected distribution of the profit or loss over the time horizon (Jorion, 2007).

An important concept is that the VaR considers a variety of different assets and risks. The VaR allows the calculation of the risks related to equities, fixed income and/or derivatives in a consistent approach making simpler the risk control. Additionally, one more consideration in the VaR methodology is the correlation between the assets in a portfolio. When these assets have opposite risk factors, symmetric negative correlation for example, the overall portfolio risk is lower as the risk is being offset. On the other hand, if the correlation between the assets is positive and symmetric the portfolio VaR tends to be greater.

The functions of the VaR is the aggregation at the largest level, which implicates a plenty of positions. However, an important process into the VaR system, called mapping, provides a shortcut. VaR mapping is the procedure of replacing the values of the portfolio position with the market risk exposure. This process helps the VaR calculation because it aggregates the individual positions of the portfolio by market risk exposure otherwise modelling the positions one by one would be extremely difficult. Additionally, most instruments often share the same market risk exposure. So, mapping the risk factors is a simpler way to perform the VaR. Another advantage is that once mapped, it can be used to perform different VaR methods and to make the gain/loss distributions.

VaR has its limitations and weaknesses. VaR methodology uses the historical data to predict the future, according to the concept that the past tendency will happen again in the future. This is called backward-looking assumption. However, there is always the possibility of an unpredictable event to occur in the future. For this reason, it is important to reinforce that VaR should have several hypothetical scenarios analyses, such as a stress and back tests. Another issue is that the VaR assumptions may not be applied to any moment or environment, so VaR outcomes can be impacted. This can be prevented by comparing the results with others models. Another limitation of the VaR is that its application should be undertaken by people who have the experience and knowledge to apply it and are always updated about new methodologies.

3.1.1 The time horizon

The purpose of the VaR application defines an adequate choice of the time horizon. Trading desks compute their gain and loss every day in order to control their liquidity position. In this situation, the most suitable VaR approach is one day time horizon. It helps to quickly adjust the portfolio if needed. As traders tend to change their portfolio positions constantly, VaR with a long time period might not be effective in this situation. However, in portfolios with less liquidity and lower volume of trading one month VaR might be more common, a pension fund for example. Regardless of the purpose, one day of time horizon is the most common in the VaR calculation because the following equation is often applied (Hull, 2012).

$$\mathrm{T\ day\ VaR} = 1\ \mathrm{day\ VaR} \,.\, \sqrt{\mathrm{T}} \tag{1}$$

Where **T** is the time horizon. This equation is more accurate when the portfolio returns, over successive days, are normally distributed with mean zero; otherwise this formula is an estimation.

3.2 VaR and capital requirement

Financial regulators around the world have recommended VaR as an efficient measure for risk management and capital requirement. Starting in 1999 with Basel II, the Basel Committee on Banking Supervision (BCBS) has issued guidelines where VaR is recommended as the market risk measure to keep the capital required by regulators. So, most financial institutions have applied VaR to estimate the amount of capital they have to hold. Regulators have estimated the compulsory capital for market risk applying a multiple of the VaR computed for 10 days and considering 99% of the confidence level. For this estimation, the instruments have to be risk-weighted accounting to keep more capital for riskier instruments. According to BCBS publications, when financial institutions are impacted by an unexpected event which is not captured in the 10 days VaR, 99% of confidence level, they must ensure that the effect of this events is covered by the internal capital assessment. This can be done by applying a stress testing scenario (BSCS, 2005).

Although VaR is recommended by BCBS, Artzner (1999) says that VAR should be rejected as a risk measure to capital requirement. He examines the properties of VaR for capital requirements in insurance companies. According to his results, VaR does not react satisfactorily when increasing risks but it starts aggregation issues. Also, VaR does not encourage diversification as it does not consider economic consequences of events or how VaR would react to it. He argues that this issue acts in contrast to the regulator policies and worries. Additionally, he recommends a more coherent measure called tail conditional expectation, which is more conservative than VaR and the cheapest to apply among others measures. This model is the expected size of a loss that exceeds VaR.

On the other hand, Cuoco and Liu (2005) recommend VaR to estimate the market risk for capital allocation. This paper analyses the behaviour of financial institutions that use VaR as internal risk measurement, and finds that VaR can be an effective market risk tool not only to cover portfolio risk but also to contribute to rev ling this risk. Also, the paper demonstrates that mandatory capital requirements make financial institutions review their portfolios to be in compliance with regulatory risk weights in regards to high systematic risks.

There are several different methods of applying the VaR. In the next section, four models are briefly summarized. The delta-normal method, the historical simulation, the Monte Carlo simulation and the delta-gamma method.

3.3 Delta-normal method

The delta-normal method uses linear or delta exposures. Thus, changes in the asset values are linear with the risk factors. It has a normal probability distribution and requires variance and covariance calculations, for this reason it is also called variance-covariance VaR or parametric VaR. The following formula shows the simplicity of this model (Hull, 2012):

$$VaR = \sigma \, \mathrm{N}^{-1} \, (X) \tag{2}$$

The confidence level is represented by **X**, the portfolio standard deviation is **σ** and N^{-1} is the inverse cumulative normal distribution (the NORMSINV formula in Excel). It is clear in this equation that the VaR is equivalent to the portfolio's standard deviation.

The daily variance is defined by the square root of the daily standard deviation, usually historical data. The volatility of a multiple asset portfolio is calculated by using a variance-covariance matrix. The diagonal variables of the matrix are variances as the covariance between an asset and itself is its variance, symmetric and perfectly correlated. The variance-covariance matrix can be written as follows.

$$\begin{bmatrix} var_1 & cov_{1,2} & cov_{1,3} & \cdots & cov_{1,n} \\ cov_{2,1} & var_2 & cov_{2,3} & \dots & cov_{2,n} \\ cov_{3,1} & cov_{3,2} & var_3 & \dots & cov_{3,n} \\ \dots & \dots & \dots & \dots & \dots \\ cov_{n,1} & cov_{n,2} & cov_{n,3} & \cdots & var_n \end{bmatrix} \tag{3}$$

According to the variance-covariance matrix, the equation of the portfolio's standard deviation is

$$\sigma_p^2 = w^T \, Cw \tag{4}$$

where ***w*** is the column vector of asset weight (amount), **C** is the variance-covariance matrix, and w^T is the transposed asset weight (Hull, 2012).

To summarize the delta-normal method by steps:

- Identify the risk factors that will be adequate to evaluate the portfolio.
- Define the sensitivity of the assets in the portfolio against their risk factors.
- Collect historical data of each risk factor to estimate the volatility of the returns (standard deviations) and the covariances between each asset.
- Calculate the standard deviation of the portfolio considering the sensitivity and the covariances. If the portfolio has multiple assets, the variance-covariance matrix application is essential.
- Under the normal distribution assumption, calculate the VaR according to the confidence level that is usually 95% or 99%.

3.4 Historical simulation

The historical simulation implicates getting the daily historical data to estimate the probability distribution of the portfolio change in the future through the movement of the market price. The steps to calculate the historical simulation method is as follows:

- Collect the historic return over some observation period by risk factor (exchange rate, interest rate, stock price etc.) and calculate the percent change day-by-day. In practice, most banks consider a period of 250 to 750 days to balance the precision of the calculation with non-stationarity.
- Create hypothetical scenarios based on the past by multiplying the current weight/amount by asset with its respective daily percent change. The sum of all asset scenarios results in the total portfolio position. The alternative scenarios represent hypothetical returns that a bank would have if it had held the same position over the observation period.
- According to the confidence level, the relevant percentile (PERCENTILE function in Excel) from the distribution of the hypothetical portfolio returns lead to the expected VaR. For instance, considering 99% of confidence level, one day of time horizon and a sample of 250 days/observations, the VaR is the third worst daily scenario.

3.5 Monte Carlo simulation

The Monte Carlo approach is used by financial institutions to value complex and exotic derivatives and to manage risk. Using computer simulations, the Monte Carlo approach generates repeatedly random prices for financial instruments that lead to diverse portfolio values. This method is the most widespread and successful among the VaR approaches, as it is very flexible. The Monte Carlo assumes that there is a known probability distribution for the risk factors. The most common application assumes a stable, joint-normal distribution. This method can be applied for different financial instruments and types of risks such as market, credit, operational and so on. It is also applied to non-linear risk exposures, non-normal distribution and complex pricing models like prices with more than one stochastic variable. Additionally, simulations can be created with long time horizon. However, this method requires a large financial investment in high performance computing and intellectual employment.

The Monte Carlo VaR requires some particular steps:

- Select the pricing model then calculate its parameters like volatility and correlation for each instrument that composes the portfolio according to the market data accessible.

- Using random number generator (the RAND() function in Excel), build multiple hypothetical scenarios of the market returns. Each random number creates a fictitious price for each instrument by multiplying the current price and its random number. The sum of all instruments results in the different portfolio values.
- Repeating this simulation for the portfolio many times is possible, to create a more realistic distribution of the portfolio value.
- Finally, the percentile function can be applied to find the VaR, considering the confidence level.

3.6 Delta-gamma method

When changes in the portfolio value are not linearly dependent on the asset prices that compose the portfolio, the distributions generally tend to be non-normal. Several examples of portfolios that contain options demonstrate that the distributions of changes in the portfolio value have skewness and kurtosis. The reason for that is the option delta which tend to change dynamically. This situation makes the application of delta-normal VaR unreliable. Figure 2 shows the delta gamma approximation of an European long call option. It is clear that the delta gamma estimation is closer to the actual price so, is more accurate than the delta estimation. The graph clearly demonstrates that the delta normal approach just works for small movements therefore, for great movements the delta gamma method is a more precise model.

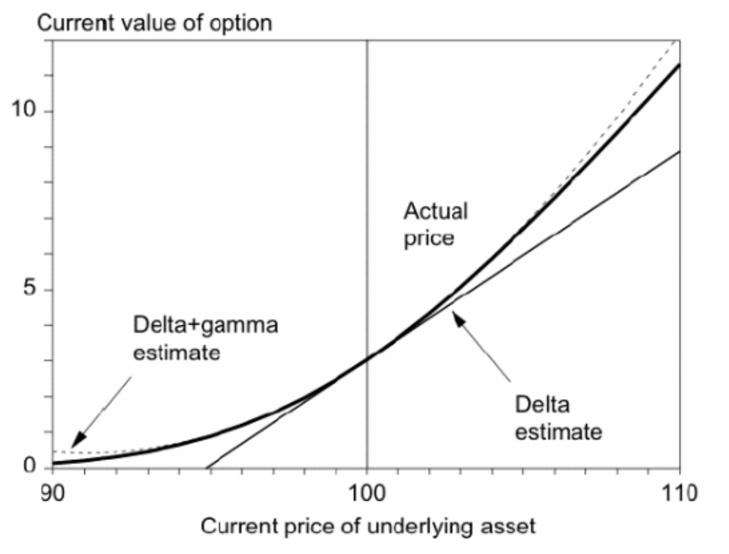

Figure 2 – Delta gamma approximation for a long call option (Jorion, 2007)

The gamma measures the level of change in delta when there is a movement in the price of the underlying instrument. Strategies involving selling positions have negative gammas while long positions have positive gammas. A positive gamma means that prices of underlying instruments change at the same direction as delta, otherwise with negative gamma price movements impact the delta in an opposite direction. The following formula is used to calculate the gamma:

$$\Gamma = \frac{N'(d1)}{S\sigma\sqrt{t}} \tag{5}$$

where

$$N'(d1) = \frac{1}{\sqrt{2\pi}} e^{-d1^2/2} \tag{6}$$

also

$$d_1 = \frac{\ln\left(\frac{s}{k}\right) + t\left(r - q + \frac{1}{2}\sigma^2\right)}{\sigma\sqrt{t}} \tag{7}$$

where $\boldsymbol{\Gamma}$ represents the gamma, **S** is the current price of the underlying risk factor, **t** is the yearly time to expiration, **K** is the strike price and **q** is the yearly dividends.

Hence, for a more precise estimate of VaR, using the second-order Taylor series approximation, the following formula can be applied to calculate the VaR of portfolios with calls and puts with long or short positions (Jorion, 2007).

$$VaR = |\Delta|\,(\alpha\sigma S) - \frac{1}{2}\Gamma(\alpha\sigma S)^2 \tag{8}$$

Where delta (Δ) is the asset sensitivity to changes in prices and α is the standard normal deviate based on the confidence level.

The gamma defines the curvature of the relationship among the portfolio value and the market price of the underlying risk factor. A positive gamma leads to a positive skewed probability distribution also it reduces the VaR. However, a negative gamma leads to a negative skewed probability distribution which raises the VaR. An example of a positive gamma is an option with a net long option and a negative gamma is an option with net short position. So, the portfolio VaR is completely dependent on the left tail of the probability distribution.

3.7 Comparison of different VaR methods

The choice of which method is better depends on different aspects like the portfolio composition, the risk factors, the cost of implementing the model and the flexibility. The delta normal model seems to be the simplest among the VaR methodologies. Despite having limitations such as the fact that it is just applied to normal distributions, the delta normal model is quicker, easier and cheaper to implement as it does not require intellectual and computational investment like the other VaR models. For small companies that do not have complex instruments in their portfolio, the delta-normal method seems to be the most suitable choice.

On the other hand, large financial institutions generally apply the Monte Carlo simulation because it is a more flexible model and it is applied to portfolios with different types of risk factors such as options and complex derivatives. Historical simulation would be applied for portfolios whereby the risk factors are not highly volatile and have consistent historical data. Additionally, for portfolios with instruments that are not linear or with non-normal distribution, the delta-gamma method can also be a good option.

Table 1 summarizes the main difference between the four VaR methods. According to Linsmeier and Pearson (2000), the VaR methodologies diverge in their flexibility of considering options in the portfolio, the level of difficulty regarding implementation, the facility of explaining the method to the senior managers, the efficiency of considering changes in the assumptions and the assurance of the results.

Assumptions	Delta normal	Historical simulation	Monte Carlo	Delta gamma
Can it capture non-normality?	No	Yes	Yes	Yes
Can it capture non-linearity?	No	Yes	Yes	Yes
Can it be applied to options?	No	Yes	Yes	Yes
Is it easy to compute and implement?	Yes	Yes	No	No
Can it be implemented without historical data?	Yes	No	Yes	Yes
Is it generally a flexible method?	Yes	No	Yes	Yes

Table 1. Comparison of different VaR methods

3.7.1 Options and the non-linearity

The VaR methods distinguish themselves in terms of linear and non-linear exposure to market risks. The expressive development of the derivative markets brings hedging and speculation strategies and consequently, non-linear exposures to the portfolios. As discussed in the delta-gamma method section, it considers the curvature of the non-linear risk exposure according to the second- order of the Taylor series. So, with regards to non-linear risk exposure, the delta-gamma results in a more accurate risk measure than the simple delta-normal method.

Additionally, in portfolios with options, the Monte Carlo and historical simulation also perform better than the delta-normal method. The delta-normal model deal with non-linearity of options as a linear approximation, which is not adequate to catch up changes in option price due to underlying price movements. However, when the time horizon is one day, great movements in prices are less probable and the linear approach might perform. But for long time horizon, changes in prices are more probable to happen and the delta-normal VaR can result in an inaccurate performance. Monte Carlo and historical simulation capture the non-linearity of options because they consider the value of each risk factor in each scenario and consequently the right portfolio value.

Another concern about the accuracy of VaR methods is the possibility of including the fact that option volatility is constant and changes frequently. Although it is not often used in practice, if the option volatility is included in the VaR calculation as a risk factor, it would be captured by the simulation models and also the delta-normal and delta-gamma methods.

3.7.2 Models' implementation

There is plenty of software available to calculate the delta-normal, delta-gamma, historical and Monte Carlo simulations which might make the VaR implementation easy. However, the software will not necessarily apply to all types of instruments and risk factors and the implementation in portfolios that have risk exposure not included in the system can be hard. As mentioned before, there is a significant quantity of over-the-counter derivatives like exotic options and their parameters are not standardised.

Regarding portfolios with instruments or risk factors not included in the available software, the Monte Carlo simulation can be simpler or more complicated to perform. The positive characteristic is that in calculating the portfolio VaR, the mapping process, the procedure of mapping positions on the selected market risk factors, is not mandatory in the Monte Carlo method. On the other hand, the method needs the selection of the distribution of the random vectors and the choice of the parameter for this distribution might require intellectual knowledge. Another negative aspect is that the Monte Carlo model also requires a considerable amount of time to compute portfolios with diverse instruments and market factors.

Historical simulation seems to be simple to implement if the historical data of all risk factors that compose the portfolio is accessible. It can be simple because it is possible to perform easily in Excel. On the other hand, if the past data is not available over all the time horizon analysed it is not likely to apply the historical simulation. This is a relevant weakness of this model, being heavily dependent on historical data. Moreover, even when the past data is available, this model can be complicated for large portfolios with exposure to great risk factors, such as different exchange and interest rates, as the data cannot be easily accessible on-line.

Finally, the complexity of implementing the four VaR methods analysed depends on the kind of product and risk exposure which composes the portfolio. The fact that these VaR methods need the pricing models to evaluate the products included in their portfolios, makes the VaR performances more complicated to portfolios with non-standard products such as complex derivatives.

3.7.3 Flexibility of the VaR methods

As the financial market changes over time, the possibility of adapting the parameters of the VaR models into the calculations is a concern among financial institutions. A specific event can cause a relevant impact on historical data and distort the VaR calculation. If this event that happened in the past is not expected, it can cause a significant distortion in the VaR numbers.

To this possibility to change parameters, the VaR methodologies respond differently. For instance, historical simulation is totally dependent on past data and consequently past events. So, it cannot be applied to historical data with outliers that are not forecast for the future. However, the delta-normal, delta-gamma and Monte Carlo methods are more flexible in this question as they are able to replace the parameters of the past data by a more suitable parameter in practice.

4 Data Analysis

4.1 The data and procedures

The aim was to evaluate the VaR methods described in section 3. This application took into account the delta normal, delta gamma, historical and Monte Carlo simulations. The parameters used were 99% and 95% of confidence level as they are the most applied by the financial institutions. Also, the VaR methods were estimated for 1 and 5 days. In order to apply the methodologies, a stock portfolio and an option portfolio were created. Thus, it allowed testing different VaR methodologies applied to instruments with distinct concepts and characteristics such as the linearity of the market risk factors.

The premise in selecting the instruments was that they should have liquidity (daily traded). This was essential to the application of the market risk measures otherwise; the prices could not change on a daily basis. So, all stocks selected are from the Financial Times Stock Exchange 100 Index – FTSE 100. This index is composed of 100 companies listed on the London Stock Exchange with the largest market capitalization.

All the data such as historical share prices, strike prices and risk free rate were imported from the Bloomberg terminal. The selected window for the analysis was from the 8th August 2013 to the 27th July

2016 which supposes a sample of 750 days. In practice, most financial institutions use windows from 250 to 750 days to lead with the non-stationary issue. The risk-free rate used is the 10 years UK treasury bond (GILT), considered one of the safest investments. The tests were performed in Excel software. The Monte Carlo simulation was based on 10,000 random scenarios.

4.1.1 The stock portfolio

The first portfolio was composed of 20 stocks from the FTSE 100 index. Table 2 lists the assets chosen for this portfolio and the amount invested. The total value of the portfolio was £100,000. It consisted of £5,000 invested in each company in an equal proportion.

Code	Company	Amount
AV/ LN Equity	Aviva PLC	5,000.00
BARC LN Equity	Barclays PLC	5,000.00
BDEV LN Equity	Barratt Developments PLC	5,000.00
BLT LN Equity	BHP Billiton PLC	5,000.00
BP/ LN Equity	BP PLC	5,000.00
BT/A LN Equity	BT Group PLC	5,000.00
GSK LN Equity	GlaxoSmithKline PLC	5,000.00
HSBA LN Equity	HSBC Holdings PLC	5,000.00
IAG LN Equity	International Consolidated Airlines Group SA	5,000.00
ITV LN Equity	ITV PLC	5,000.00
LGEN LN Equity	Legal & General Group PLC	5,000.00
LLOY LN Equity	Lloyds Banking Group PLC	5,000.00
PRU LN Equity	Prudential PLC	5,000.00
RBS LN Equity	Royal Bank of Scotland Group PLC	5,000.00
RDSB LN Equity	Royal Dutch Shell PLC	5,000.00
SAB LN Equity	SABMiller PLC	5,000.00
TSCO LN Equity	Tesco PLC	5,000.00
TW/ LN Equity	Taylor Wimpey PLC	5,000.00
ULVR LN Equity	Unilever PLC	5,000.00
VOD LN Equity	Vodafone Group PLC	5,000.00
Total		100,000.00

Table 2. Composition of the stock portfolio

In order to get the portfolio return, the individual returns were calculated applying the following formula to the closing prices

$$R_t = (P_t - P_{t-1}) \,/\, P_{t-1} \tag{9}$$

where **R** is the daily return and **P** is the price of each share.

Then, the mean, standard deviation and variance for each stock were calculated. The variance-covariance matrix (formula 3) was applied to estimate the delta normal VaR according to the formula (2). Firstly, the delta normal method was calculated for 1 day then, it was computed for 5 days using the formula 1 described earlier. Additionally, the historical and Monte Carlo VaR were estimated. The delta gamma method was not applied for this stock portfolio as it is more suitable for derivatives or non-linear instruments. After estimating the 3 VaR methods for this linear portfolio, a comparison of the results was made. This is described and detailed in the chapter 4.2.1.

4.1.2 The option portfolio

Using the same criteria, Table 3 shows that the second portfolio was composed of 3 options. One put and one call with underlying stocks from FTSE 100 and one more call with the FTSE 100 index itself as underlying. These 3 options have long positions that are the right to buy or sell the underlying assets. The issuers of the underlying stocks were from different economic sectors which allowed an analysis of the effect of the correlation. The same expiration date of the options was another parameter established for a nil impact on the VaR numbers. Thus, the expiration date was the 20th January 2017.

Option	Underlying	Company	Amount
Put	BP/ LN Equity	BP PLC	20,000.00
Call	RBS LN Equity	Royal Bank of Scotland Group PLC	20,000.00
Call	FTSE100	UK Index	20,000.00
Total			60,000.00

Table 3. Composition of the option portfolio

Before estimating the VaR methods, the options needed to be priced. So, the Black-Scholes model was calculated for the call and put options that composed the portfolio. The following formula (Hull, 2012) was applied.

$$c = SN(d_1) - Ke^{-rt}N(d_2) \tag{10}$$

And

$$p = Ke^{-rt}\,N(-d_2) - SN(-d_1) \tag{11}$$

where ***c*** is the call premium, ***p*** is the put premium, d_1 is shown in the formula (7) and d_1 is demonstrated below.

$$d_2 = d_1 - \sigma\sqrt{t} \tag{12}$$

After calculating the call and put prices, the delta normal (formula 2), delta gamma (formula 8), historical and Monte Carlo simulations were estimated and compared. The results are shown in chapter 4.2.2.

4.2 Empirical results

4.2.1 The stock portfolio

Figure 3 shows the comparison among the delta-normal method, historical and Monte Carlo simulations for the stock portfolio composed of shares of 20 companies. This chart presents the portfolio VaR amounts for 1 and 5 days of time horizon also considering 99% and 95% of confidence level.

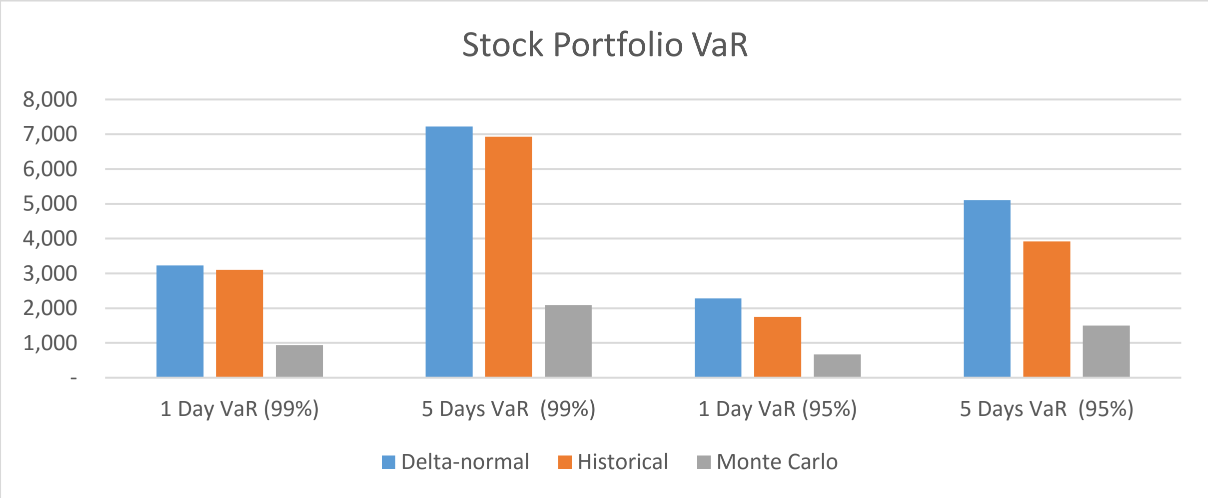

Figure 3 – The VaR comparison of the stock portfolio

As expected in the three models, when increasing the time horizon, the three models illustrate a greater VaR amount which is explained by more market risk exposure in 5 days than in a single day. Moreover, when assuming 99% of confidence level the portfolio VaR is higher than when assuming 95%. This is because a higher confidence level means a more precise measure. However, the larger VaR requires more capital allocation to cover potential losses as mentioned in the section 3.3.

It is possible to realize in Figure 3 that there is less discrepancy between the 3 VaR methods when considering 95% of confidence level. According to Hendricks (1996), a lower confidence level can calibrate the VaR measure. In his research, the risk measure is more accurate for the 95th percentile than 99th. He says that when there is a fat tail in the distribution, it can be adjusted by decreasing the confidence level parameter which is in line with the analysed data of this stock portfolio. Lambadrais *et al* (2000) also demonstrate that the accuracy of the methods depends on the confidence level used.

It is notable that the historical simulation and the delta-normal VaR are the highest between the three models. Firstly, in historical simulations the historical data is the most important assumption and the volatility is totally based on the past performance of the stocks. So, when stocks had a high volatility in the past, the historical simulation VaR tends to be high. The stocks selected for the analysis are liquid and highly volatile as it composes the FTSE100 index. Figure 4 demonstrates the monthly return of each share that composes the portfolio between 2013 and 2016. Secondly, the delta normal approach is completely based on the standard deviation of the returns which explains the portfolio VaR being considerably higher than the Monte Carlo simulation. As figure 4 can illustrate, unexpected outliers in the stock returns distribution makes the assumption of normality in the delta normal VaR unreliable.

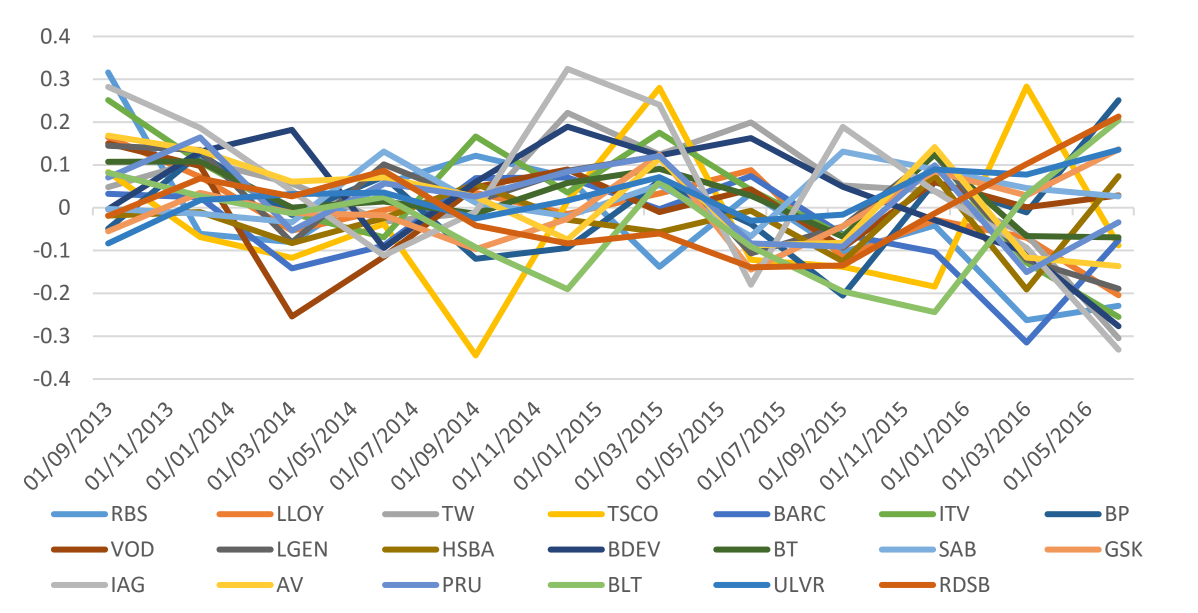

Figure 4 – The share volatilities of the portfolio over the time

Following the Monte Carlo approach mentioned in the section 3.6, it seems to be the most adequate and suitable model for this stock portfolio as it is not based on historical simulation. The Monte Carlo model assumes that the returns are lognormally distributed, which is different from delta normal that assumes a normal distribution. As this portfolio is composed only by shares and non-linearity is not assumed, the VaR methodologies should get the same or similar numbers, but this is not the case. This result can be a consequence of a lognormal distribution rather than a normal distribution for the stock returns that compose this portfolio.

4.2.2 The option portfolio

Figure 5 demonstrates the comparison among the delta-normal, delta-gamma, historical and Monte Carlo simulations for an option portfolio composed by 2 long calls and 1 long put. This chart presents the portfolio VaR amounts for 1 and 5 days regarding to 99% and 95% of confidence level. As mentioned before and based on the theory, an increase in the portfolio VaR amount is expected as the time horizon and the confidence level increase. A greater confidence level and time horizon imply a larger possibility of changes in the underlying asset. Moreover, the theory predicts that a higher confidence level increases the underestimation/overestimation of the VaR (Jorion, 2007).

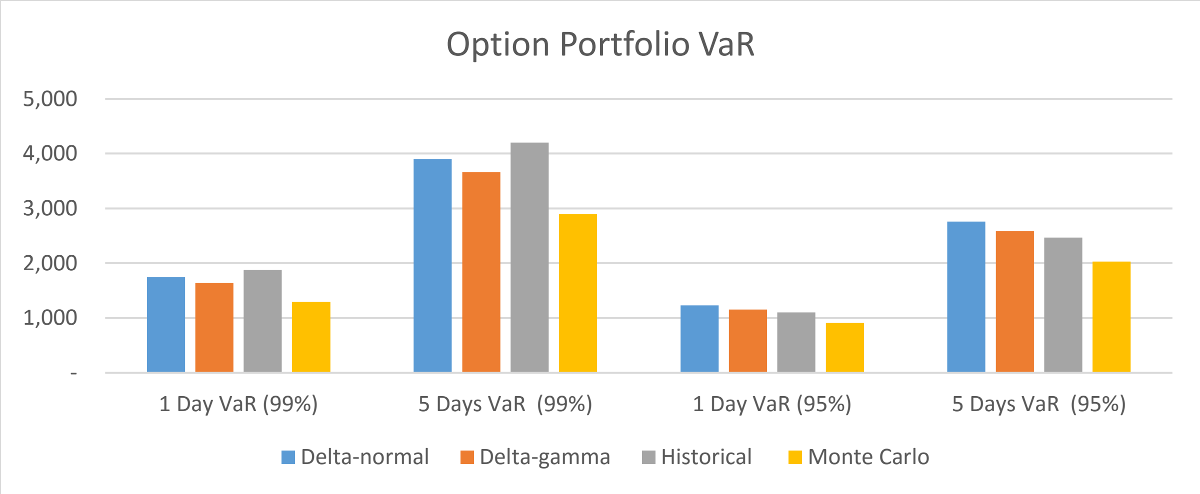

Figure 5 – The VaR comparison of the option portfolio

Again, the historical simulations and the delta normal method have the most representative VaR numbers among the four methodologies for 99% of the confidence level. As shown in Figure 6, the historical return of the underlying instruments between 2013 and 2016 is highly volatile which impacts the forecasted historical VaR. Additionally, the delta normal method for the option portfolio is expected to have a large discrepancy. This result is caused by the linear approximation of this model. As the option price has a non-linear function of the stock price (non-linear payoffs), the delta normal VaR for this option portfolio is not an accurate measure. This finding reflects the weakness of the delta normal approach that it is designed only for instruments or portfolios with linear relationship between the risk factors and the instrument prices. On the other hand, the delta gamma and Monte Carlo simulation represents more accurately the option premium behaviour as a function of the underlying stock price. Regarding to the fact that the delta gamma and Monte Carlo approaches do not require only linear approximation, the expected maximum loss is more precise in practice.

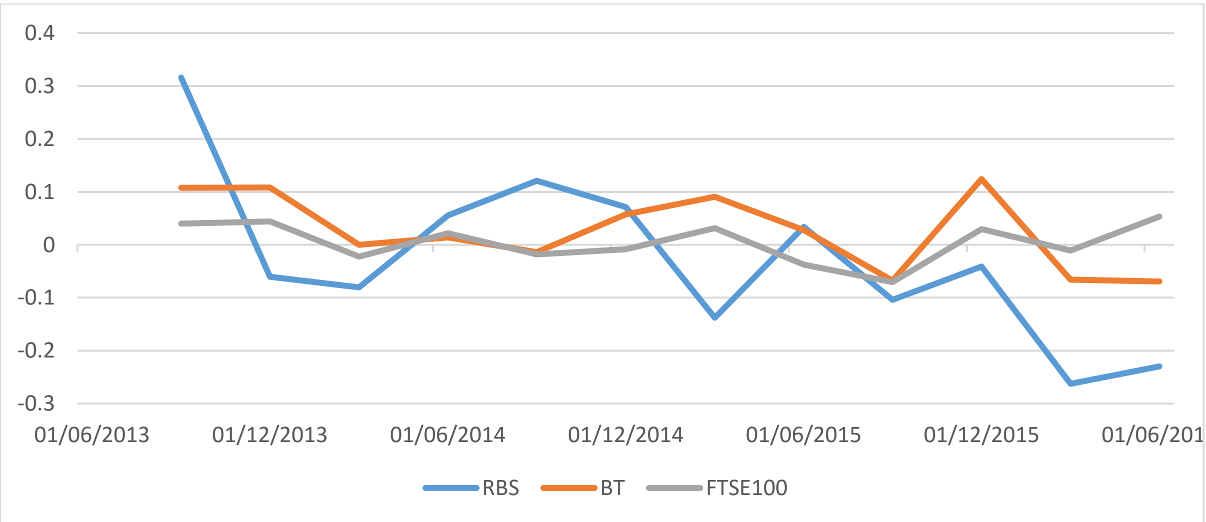

Figure 6 – The volatilities of the underlying (option portfolio)

The gamma

An important variable to be analysed is the gamma, the rate of change in delta when the price of the underlying asset changes. Figure 7 illustrates the gamma behaviour of the call option with the underlying RBS and strike price £160 versus the hypothetical stock price as time to maturity decreases. The long call position is in the money (ITM), that is when the price of the underlying stock of a long call is higher than the strike price. Options that are in the money and close to the expiration have very high gammas. So, the option price is highly sensitive to changes in price of the underlying asset. On the other hand, when options are out the money (OTM) or have a further expiration date, they have lower gammas. In the option portfolio analysed here, there are two ITM options (underlying BT and RBS stocks) and one OTM option (underlying FTSE 100 index). This comparison is demonstrated in Table 4. Following the theory, the higher gammas in the portfolio are from the ITM options. As the weight invested in each option is equal, the impact of the gamma in the portfolio tends to increase the delta gamma VaR. Clearly, the higher gammas are from underlying assets with higher standard deviation according to the equation (5).

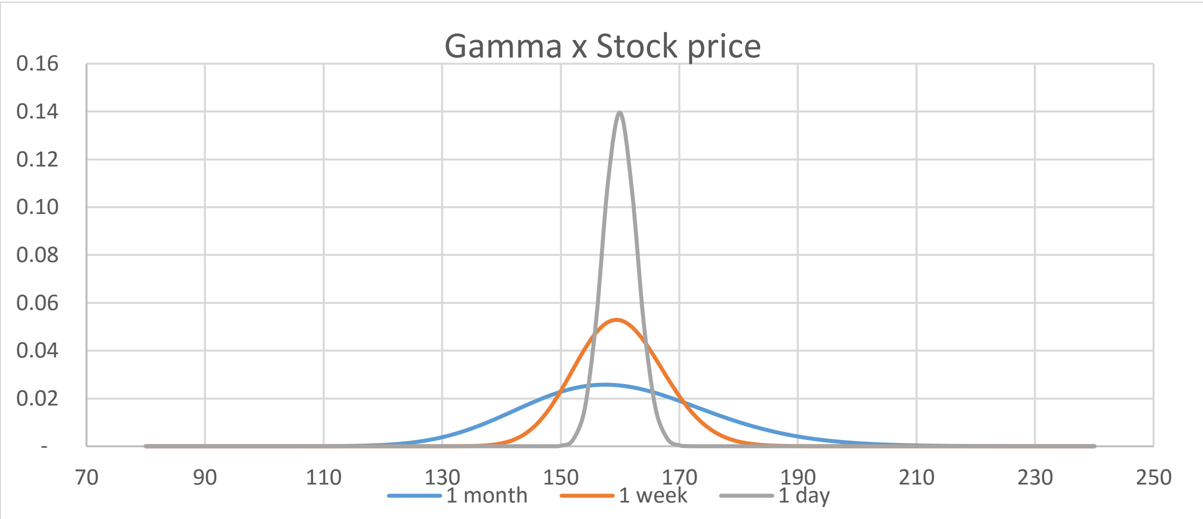

Figure 7 – Call option gamma x underlying stock price

Position	Underlying	Gamma	Standard Deviation
ITM Long call	RBS stock	0.86%	33.90%
ITM Long put	BT stock	0.51%	22.62%
OTM Long call	FTSE 100 index	0.06%	15.30%

Table 4 – The gamma of the underlying (option portfolio)

The correlation

According to the theory, there is a gain in the risk measure when considering a portfolio compound by correlated instruments. The correlation between financial instruments represents how they react between themselves to the market price movements, positively or negatively and the magnitude of this movement. Meucci (2010) demonstrates the benefits and performance of diversification in market risk measures.

Figure 8 shows the comparison between the diversified and non-diversified option portfolio VaR for the delta normal and gamma, historical and Monte Carlo models. The blue bars represent the VaR of a non-diversified portfolio and the orange bars the diversified portfolios. The Monte Carlo simulation presents higher efficiency with diversification as it has larger difference between undiversified and diversified VaR portfolios. On the other hand, the delta gamma is less efficient in considering correlation in the risk measure.

It is clear the benefits of the diversification due to the interaction of the underlying stock prices with different risk factors. In the undiversified portfolio the options are totally correlated then, the portfolio VaR is simply the sum of the individual option VaRs. The higher the correlation, the lower the difference between the undiversified and diversified portfolios. The correlation between the options of this portfolio is an advantage as it can limit or stop the loss of money with unexpected change in prices.

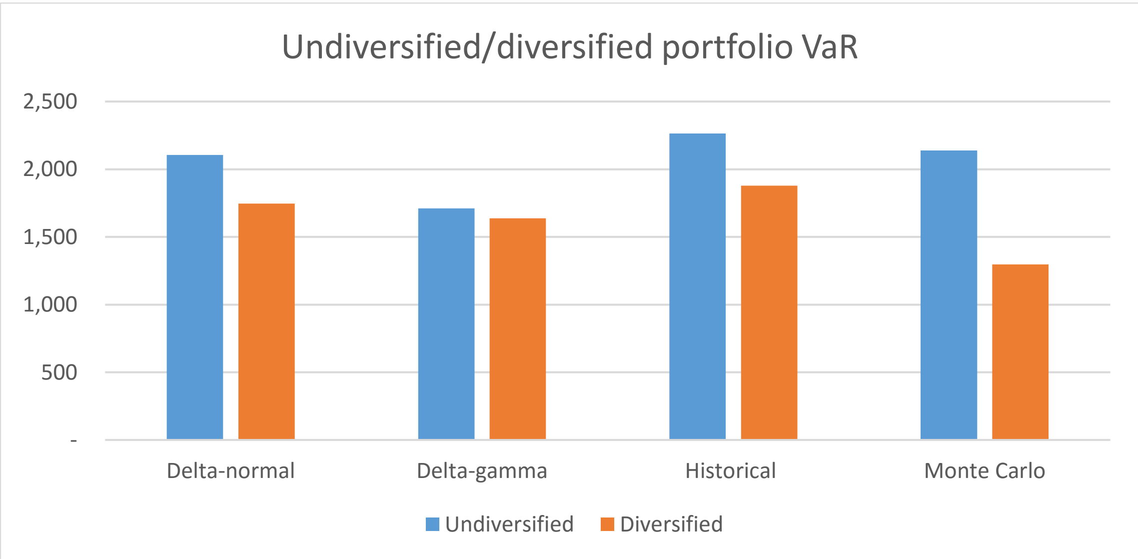

Figure 8 - The 1 day (99%) VaR for the undiversified and diversified option portfolio.

4.3 Results comparison

In the historical simulation, the VaR is totally based upon time series of past returns. Consequently, it can underestimate the VaR whether the historical data is stable or overestimate the VaR when the historical data is not stable. The results showed that both portfolios analysed in this study are highly liquid and volatile. For these 2 portfolios, the historical simulation is not a good indicator of forecasting losses. Boudoukh *et al.* (1998), Hull and White (1998) propose alternative models based on historical simulation that leads with the problem of outliers in the return distribution by weighting the data.

Regarding the delta normal VaR, this is heavily based on the standard deviation of the returns and consequently, it has a normal distribution. However, catastrophic events are quite likely to happen according to the financial historical facts. Engle (2001) suggests a more accurate VaR calculation when considering possible changes in the standard deviation over time. Also, volatile instruments can have outliers in their return distribution that might make the normal distribution inapplicable. It is clear that the delta normal VaR cannot be considered the best outfit model for the stock portfolio. For the option portfolio analysed, the higher delta normal VaR is more conservative in terms of estimating the expected loss because it does not consider the non-linear premise of options. So, this is not applicable to the option portfolio.

For both portfolios, the Monte Carlo simulations show to be a more flexible and fair method to estimate the VaR. The consistent results for portfolios with different peculiarities illustrate the flexibility and preference of this model among financial institutions. On the other hand, Jamshidian and Zhu (1997), Frye (1997), Glasserman *et al.* (2000) and Botev *et al.* (2010) suggest improvements in Monte Carlo to deal with the computational time required to simulate plenty of random scenarios to compute the VaR.

The delta-gamma model also seems to be an accurate measure for the option portfolio allowing to estimate the VaR for the non-linear instruments. However, this method requires mathematics so intensively that can become extremely complicated and costly to some institutions. Despite most literature recommending the delta-gamma approach for leading with non-linearity, Castellacci and Siclari (2003) find a more precise measure in the delta normal than in the delta gamma method for non-linear option portfolios.

4.4 Advantages and weaknesses of the VaR methods

The delta normal approach is computationally easy and fast to apply even for a portfolio with a considerable quantity of instruments. It just requires a simple variance-covariance matrix. On the other hand, the delta normal VaR has weaknesses that can make the model unreliable. Most part of the financial instruments have outliers in their historical data that can cause fat tails in the return distributions. As the delta normal approach is based on normal distribution, it can estimate the VaR inaccurately. This fact was stated in the stock portfolio application (chapter 4.2.1). This error increases with greater confidence levels. Another important factor is that the model is no applied to non-linear instruments.

Historical simulation can be the simplest model to estimate losses if the historical time-series for each risk factor are available, the marking to market share price for example. A strength of this method is that it considers the fat tails of the past data. Additionally, as the full valuation can be used to apply this method, it can deal with the gamma and vega risks (the delta and volatility effects on option prices). However, the historical simulation has weaknesses. The fact that the model is heavily dependent on the past, means that it can consider or omit events that could or not happen in the immediate future. Another weakness is the sampling variation. It is difficult to find a great trade-off between non-stationarity and accuracy when defining the data sample to apply the historical simulation. Additionally, this model assumes a stationary distribution during the chosen period but it can change during that time. Hull and White (1998) presents more accurate measure in historical simulation by adjusting the data, before calculating the VaR, to include changes in volatility over the time.

The Monte Carlo model is the most efficient measure to estimate market risk. It is flexible when leading with different assumptions such as non-linearity, non-normality, time variation in volatility or expected return, extreme values and fat tails. Also, the model allows the inclusion of specific characteristics of instruments for instance, time decay of options, daily settlement or contractual cash flows. Nevertheless, like other methodologies Monte Carlo has its limitations. The model considers particular stochastic procedures that could be unreliable. The model requires a great deal of computational time to create various random scenarios which makes itself the most expensive model to apply due to the investment in infrastructure and intellectual. Pritsker (1997) analyses the trade-off between speed and precision in an option portfolio. As expected, the delta normal model shows the highest absolute error in predicting VaR with 5,4% but the model is the fastest to compute, 0.08 seconds. However, the Monte Carlo simulation demonstrates the most precise number with 0% of VaR error but it spends much more time, 66 seconds.

An important advantage of the delta gamma method is that it is applied to non-normal and non-linear portfolios because the second order approximation takes account of the gamma risk. However, the reliance on the second order approximation cannot be sufficiently precise to predict losses. Additionally, when applying the delta gamma method, the normality is rejected even if the portfolio return distribution is normal. The effect of the non-linearity in the portfolio can result in a chi-squared distribution rather than a normal distribution. This loss of normality is unavoidable as the model use a second order approximation. Jamshidian and Zhu (1997) and Fallon (1996) apply the delta gamma model and find a more precise risk measure than in the delta normal VaR application. On the other hand, Dowd (1998) says that the delta gamma model is complicated to implement and not necessarily more precise than the delta normal. He makes a comparison of the methods after estimating both models for an option portfolio.

5 Extreme Value Theory

5.1 The theory and model

An important concern among risk managers is the possibility of an extreme event occuring and if the risk model implemented allows them to deal with rare but catastrophic events and their outcomes. The extreme value theory - EVT allows an estimate of market risk considering a possibility of an extreme event happening. It uses a different approach to calculate the VaR and consequently, EVT can improve the VaR models. EVT focuses on estimating the shape of the tail of the probability distribution. So, it is possible to estimate the losses with very low probabilities. Despite the fact that EVT comes from hydrology industry, it has been applied constantly in financing specially in VaR methodologies.

There are 2 types of approaches for EVT: the block maxima and the peaks-over-threshold. The block maxima approach deals with the highest observations taken from samples of equally distributed observations. The peaks-over-threshold is a more modern and useful approach which applies for all observation that exceed a certain high threshold. This approach is more applied in practice as it is more efficient in terms of using the data on extreme values. Regarding the peaks-over-threshold approach there are 2 models to be considered, the semi-parametric and the fully parametric models. The theory posits that the 2 models are efficient if well applied.

As the aim of the EVT is to deal with sample extrema, the highest or lowest so, the Generalised Extreme Value distribution – GEV application is essential. According to Fisher Tipplet theory, it can be observed that for a large class of distribution the normalised sample maxima tends to converge to the GEV distribution with larger sample. The GEV refers to 3 extreme value distributions, the Frechet distribution ($\xi > 0$), the Weibull distribution ($\xi < 0$) and the Gumbel distribution ($\xi = 0$). According to the ξ parameter, the distribution can be categorized as fat tail ($\xi > 0$), thin tail ($\xi = 0$) and short tail ($\xi < 0$).

The generalised Pareto distribution – GPD is a two-parameter distribution with distribution function (Embrechts, 2000).

$$G_{\xi,\beta}(x) = \begin{cases} 1 - \left(1 + \frac{\xi x}{\beta}\right)^{-\frac{1}{\xi}} & \xi \neq 0 \\ 1 - \exp\left(-\frac{x}{\beta}\right) & \xi = 0 \end{cases}$$

Where $\beta > 0$ and $x \geq 0$ when $\xi \geq 0$ and $0 \leq x \leq -\beta/\xi$ when $\xi < 0$. ξ is the shape of the distribution and β is a scaling parameter. In the context of this work, the first equation is the most important for estimating losses since there is a heavy-tailed GPD, shape higher than zero. This distribution is generalised as it allocates other distributions into a general parametric equation.

Estimation of the tail distributions

There are plenty of approaches to calculate the estimate the tail of the distribution. Embrechts (2000) proposes the equation. It is an aggregation between the historical simulation and EVT and this approach is better fitted when data is independent or low dependent.

$$F\,(x) = 1 - \frac{N_u}{n}\left(1 + \xi\,\frac{x-u}{\beta}\right)^{\frac{-1}{\xi}}$$

Where ***u*** is the threshold, ***n*** is the sample size N_u is the number of exceedances. This equation is only applied when x > u.

A relevant step in determining the parameters of the tail estimation is to set the value of the threshold that heavily impact on the results. This is because there is a trade-off between bias and variance. As more data is used (lower threshold), the number of observations rises which leads to more accurate estimation. However, it makes the estimation biased with larger number of observations from the centre of the distribution. Danielsson et al (2001), present a bootstrap model to lead with the sampling estimation. The model allows an improvement in the sample fraction determination by reducing the asymptotic mean squared deviation.

5.2 VaR estimation

Financial institutions have grown their market risk exposure with sophisticated transactions which have led to an increase in the awareness of very high losses. In this context the EVT plays an important function in the VaR methodologies. This section discusses how to include EVT in the market risk control. The method of a fully aggregated position is introduced by Longin (2000). It consists in some important procedures to compute the VaR with the EVT. Firstly, having been chosen as the instruments of the portfolio, is essential to observe their liquidity to determine the frequency of the data return. The frequency needs to be very large to capture extreme prices. After defining the sample size and time series, the data should be divided into sub-periods with equal number of observations. Another important decision to make is about the selection of the parameters of the asymptotic distribution of the returns. As mentioned before, these parameters are used in the EVT calculations. In order to certify that the parameters are good estimators, a test of asymptotic distribution can be applied. This will show if the parameters explain well the statistical behaviour of the returns.

According to (Embrechts, 2000), after estimating of the shape and scale parameters, the EVT can be used to calculate the VaR. This equation represents a quantile (***q***) estimation, where the quantile is an unknown parameter of an unknown distribution.

$$VAR_q = u + \frac{\beta}{\xi}\left[\left(\frac{n}{N_u}(1-q)\right)^{-\xi} - 1\right]$$

In order to perform a full valuation to calculate the VaR, it is necessary to build the historical returns of the entire position. If the portfolio consists of complex instruments or time-changing premises, it can be time consuming to obtain the historical return and re-calculate the asymptotic distribution every time the VaR needs to be estimated. Therefore, it might to be simpler if the positions are decomposed so the VaR can be estimated individually with a risk aggregation formula. This is a weakness of this theory, it can be complicated to implement if more than a single risk factor is included. Additionally, this model can be extremely difficult to parametrize the assumptions as there is no much observations of extreme events.

6 Conclusion

Over the last decades, financial disasters and crises have caused large losses and bankruptcies in the financial industry. These events highlighted the importance of managing risk adequately and exposed deficiencies in the regulation. As a result, more robust and international regulations were established. At the same time, the marked growth of the OTC derivatives market contributed to an increase in security trades internationally raising the exposure to exotic and complex instruments and different risk factors (foreign exchange, interest rates, stock prices and so forth). Thus, banks and financial institutions have had more risks to control and manage.

In this context, VaR is currently one of the most used methodologies to access market risk beside the point of being recommended by the central bank regulations. Over time, many variants of the VaR methodology have been developed as alternatives to deal with complex instruments and OTC derivatives. However, there is a question in the market about which VaR approach can estimate a more precise risk measure. So, the aim of this paper is to critically evaluate and compare the most popular VaR methods and present their advantages and disadvantages. The VaR methods applied are: delta-normal, delta gamma, historical simulation and Monte Carlo simulation. This analysis is based on two portfolios: stock (linear) portfolio and option (non-linear) portfolio. The first portfolio is composed of 20 stocks and the second by two stock options and one index option. The methods are computed for one and five days time horizon with 95% and 99% of confidence level.

The findings show that most results are consistent in the stock and option portfolios. Firstly, when increasing the time horizon, a higher VaR amount is demonstrated for all cases which is explained by the growth of market risk exposure. Secondly, when assuming 99% of confidence level, the VaR is higher than when assuming 95%. As the theory says, a higher confidence level represents a more accurate measure. In both portfolios, there are more discrepancies between the four VaR methods compared when considering 95% of confidence level. This output reflects the literature which says that a higher confidence level raises the VaR misestimating and/or a lower confidence level can calibrate the VaR measure (Hendricks, 1996; Lambadrais *et al,* 2000 and Jorion, 2007).

The historical VaR simulation for both portfolios is overestimated. This is a consequence of the most relevant weaknesses in this model which is being strongly based on historical data. Because the portfolios are composed of FTSE 100 stocks and the own index, they are highly liquid and volatile. For the stock and option portfolios, the historical simulation is not a good indicator for forecasting losses. Boudoukh *et al.* (1998), Hull and White (1998) find the same issue and propose alternative models based on historical simulation that leads with the problem of outliers in the return distribution by weighting the data.

Additionally, the delta normal VaR in both portfolios do not demonstrate itself to be a good estimation. The large VaR amounts are influenced by the high standard deviations of the returns as the model is totally based in this assumption. This is in line with Hendricks's (1996) and Engle's (2001) suggestion about including changes in the standard deviation (conditional volatility) over time. Also, unexpected outliers in the stock returns distribution contribute to an inaccurate VaR estimation because this model works only for normal distributions and linear instruments, so in the option portfolio a large misestimating was expected as the result is a linear approximation. Hendricks (1996) says that extreme outcomes are more common and frequent than predicted by the delta normal model (fat tails).

The delta gamma method was calculated only for the option portfolio because the model is indicated for non-linear instruments and includes the variable gamma. The result of this model presents a good estimation as it is not dependent on linear approximation and the expected maximum loss is more precise in practise. However, it is a complex model and demands high level of calculation which can become complicated and costly. Although most literature recommend the delta-gamma approach for leading with non-linearity, Castellacci and Siclari (2003) find a more accurate measure in the delta normal than in the delta gamma method for non-linear portfolios.

Finally, the Monte Carlo simulations were demonstrated to be a more accurate, flexible and fair method to estimate the VaR in the stock and option portifolios. The consistent results for instruments with different peculiarities prove the flexibility of this model and why Monte Carlo simulation is the most used among financial institutions. Moreover, Jamshidian and Zhu (1997), Frye (1997), Glasserman *et al.* (2000) and Botev *et al.* (2010) suggest improvements in Monte Carlo method to deal with the computational time required to simulate a higher number of random scenarios to calculate the VaR.

The impact of portfolio diversification in the VaR measure was also analysed in the option portfolio and it showed substantial improvement in the risk measure when considering the correlation between instruments in the calculations. Once more, the Monte Carlo simulation presented a higher efficiency in VaR measures with the diversification factor. This is in line with the theory that says there is a gain in the risk measure when considering a portfolio compound by correlated instruments. Meucci (2010) also demonstrates the benefits and performance of diversification in market risk measures.

Furthermore, this paper suggests a model to improve the VaR measures, the Extreme Value Theory (EVT). It applies to different VaR methods including a possibility of an extreme event to happen. EVT focuses on estimating the shape of the tail in the probability distribution. So, it allows the estimation of losses with very low probabilities. The literature shows that EVT improves the risk measures considerably.

7 Recommendations

An extension of this work could be applied in future research where VaR methodologies would be compared in a more realistic scenario through computing them for current portfolios of a group of banks and/or companies from different sectors. This comparison could allow the inclusion of different strategies and risk factors in a practical situation improving the accuracy of the numbers. Also, it would better at capturing the challenges faced by risk managers.

Additionally, as there are plenty of variants and improvements of the traditional VaR models, the number of VaR approaches analysed and compared could be expanded in the future. It might make the estimations more precise, for instance, the application of EVT method in VAR measure or models that deal with changing in volatility such as ARCH and GARCH.

Furthermore, other models to measure market risk could be contrasted with the VaR methodology in future research, for example, the Expected Shortfall. Yamai and Yoshiba (2005) endorse the utilization of Expected Shortfall arguing that it can bring better results as it leads with the VaR weakness of tail risk. However, a larger sample is needed in the expected shortfall model to achieve the same accuracy provided by VaR.

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