Accurate stability prediction of one-bit higher-order delta-sigma (Δ-Σ) modulators for multiple-sinusoidal inputs

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Abstract-**The present approaches on predicting stability of Delta-Sigma (Δ-Σ) modulators are mostly confined to DC inputs. This poses limitations as practical applications of Δ-Σ modulators involve a wide range of signals other than DC such as multiple sinusoidal inputs for speech modeling. In this paper, a quasi-linear model for Δ-Σ modulators with nonlinear feedback control analysis is presented that accurately predicts stability of single-loop 1-bit higher-order Δ-Σ modulators for multiple sinusoids. Theoretical values are shown to match closely with simulation results. The results of this paper would enable optimization of the design of higher-order single-loop Δ-Σ modulators with increased dynamic ranges for various applications that require multiple-sinusoidal inputs or any general input composed of a finite number of sinusoidal components.**

***Index Terms*—delta-sigma, stability analysis, non-linear systems, multiple inputs**

I. Introduction

The stable input amplitude limits for Delta-Sigma (Δ-Σ) modulators are complicated to predict due to the non-linearity of the quantizer. The stable input amplitude limit decreases as the order of the Δ-Σ modulator increases. One technique is to model the quantizer as a threshold function in the state equations, which gets complicated for higher-order Δ-Σ modulators and is limited to 1^{st}- and 2^{nd}- order Δ-Σ modulators [1]. Another approach to simplify the analysis has been to assume a DC input to the Δ-Σ modulator [2]-[9]. The linearised modeling approach in [2] did not previously provide useful stability predictions until a new interpretation of the instability mechanism for Δ-Σ modulators based on the Noise Amplification Factor was given in [10]. However, this is also restricted to DC inputs. The approach of using separate signal and quantization noise gains in [2]; and of the noise amplification factor in [10] has been combined in [11] in which stability has been predicted for a single-sinusoidal input. In [12], the analysis is extended for predicting stability for dual-sinusoidal inputs. An in depth analysis of the approach in [11], [12] with detailed simulation results are given in [13]. As the approach in [11]-[13] is applicable to only low-pass Δ-Σ modulators, in [14], the analysis and results are given for predicting stability of band-pass Δ-Σ modulators. Although the analysis in [11]-[14] is similar, it is based on separate gains for each of the signal inputs and the quantization noise which are derived from the concept of the modified non-linearity. These result in different quantizer gains for the specific type of the input signal such as DC, single-sinusoid or dual-sinusoid. The analysis is therefore signal-specific and the approach gets complicated further as the number of inputs increases. As real applications often require multiple-sinusoidal inputs such as speech signals, it is not possible to continue with the approach given in [11]-[14] for quantifying stability. In this paper, a quasi-linear model of the Δ-Σ modulator that deploys nonlinear feedback control analysis with a single quantizer gain for multiple-sinusoidal inputs and quantization noise, is shown to accurately predict the stability of 1-bit higher-order Δ-Σ modulators. Multi-bit quantizers offer less quantization noise and more stability compared to their 1-bit counterparts. However, their complicated implementation as well as the feedback errors of the multi-bit digital-to-analog converter (DAC) in the feedback makes 1-bit quantizers more attractive. Numerous 3^{rd}-, 4^{th}- and 5^{th}-order 1-bit Δ-Σ modulator have been implemented for wireless, sensors and audio applications [15]-[23]. One-bit higher-order Δ-Σ modulators are also preferred for microphone applications in which 4^{th}-order Δ-Σ modulators have been implemented [24], [25].

The analysis has been done e by deploying the non-linear quantizer gain that simultaneously captures the statistics of the composite signal, consisting of multiple-sinusoids and quantization noise at the quantizer input. The simplified novel approach is based on the assumption that the input to the quantizer has a Gaussian probability density function (PDF). A comparative analysis is undertaken of the multiple- and the single-sinusoidal input to quantify the increase in dynamic ranges. The findings of this indicate far more accurate results by employing a single quantizer gain rather than using separate gains as in [11]-[14].

The novel theoretical analysis presented in this paper is well supported by detailed simulation results. Results from this novel model would enable optimization of the design of higher-order Δ-Σ modulators with increased dynamic ranges for applications that require multiple-sinusoidal inputs or any general input composed of a finite set of sinusoidal components. Section II describes the stability mechanism in terms of the quasi-linear model and the Noise Amplification Factor. Section III elaborates on the concept of the ratio of the signal variance to the quantization noise variance at the quantizer input followed by variation of the quantization noise in the Δ-Σ modulator. Subsequently a comparative analysis for the multiple- and the single-sinusoidal inputs is undertaken, which is followed by an insight into the analysis for a multi-bit 3-level quantizer. Simulations are given in Section IV, followed by conclusions in Section V.

II. QUASI-LINEAR Δ-Σ MODULATOR AND NOISE AMPLIFICATION FACTOR

A quasi-linear model of a Δ-Σ modulator is shown in Fig.1, where $G(z)$ is the input transfer function, $H(z)$ the feedback filter transfer function and the quantizer is replaced by a gain factor K followed by an additive white quantization noise source $q(n)$:

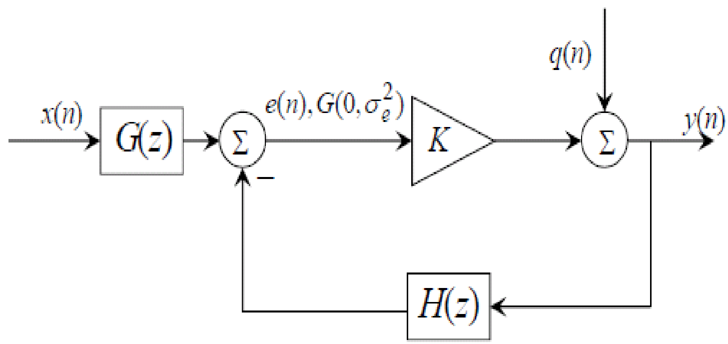

Fig. 1. Quasi-linear Δ-Σ modulator.

Assuming $q(n)$ to be white with zero mean and variance σ_q^2, and the Noise Transfer Function (*NTF*) between $q(n)$ and $y(n)$ to be known, the noise variance V_0 at the output of the Δ-Σ modulator is given by [10]:

$$V_o = \sigma_q^2 \int_0^1 \left| NTF\left(e^{j\pi f}\right) \right|^2 df = \sigma_q^2 A(K) \quad (1)$$

where, $A(K)$ is the Noise Amplification Factor. Using Parseval's theorem, $A(K)$ can also be found in the time-domain as [10]:

$$A(K) = \sum_{n=0}^{\infty} \left| ntf(n) \right|^2 \triangleq \| ntf \|_2^2 \quad (2)$$

where $ntf(n)$ is the impulse response corresponding to $NTF(z)$ and $\|ntf\|_2^2$ is the squared second-norm of $ntf(n)$. The variation of $A(K)$ with K can be plotted as $A(K)$ curves from (2) and is used to explain the stability of Δ-Σ modulators. A typical curve for a 4^{th}-order Δ-Σ modulator is shown in Fig. 2, where A_{min} is the global minimum value of the curve which is 2.19.

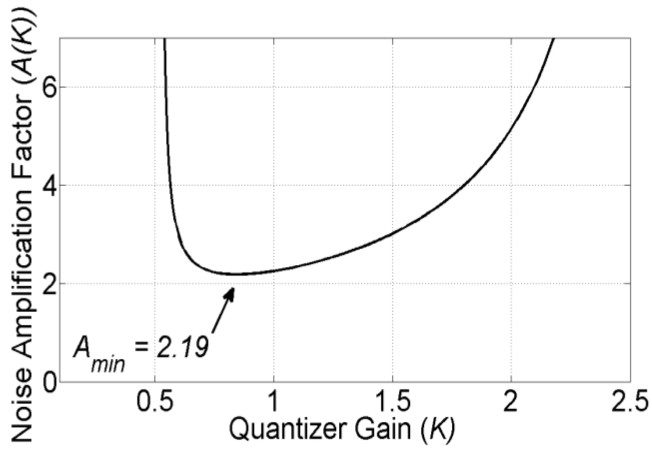

Fig. 2. Noise Amplification Factor variation with quantizer gain.

The Δ-Σ modulator is considered to be stable in the positive slope section of the $A(K)$ curve. If K increases slightly in this section, $A(K)$ increases which results in more circulating noise which in turn tends to decrease K; the system is therefore in equilibrium. As the input amplitude increases $A(K)$ continues to decrease thereby reaching a point when $A(K)$ reaches A_{min}, where the Δ-Σ modulator commences to operate in the negative slope region in which there is no equilibrium and hence becomes unstable. For stable operation of the Δ-Σ modulator therefore, $A(K) > A_{min}$.[10]. If one can quantify the variation of $A(K)$ with the input signal amplitude a for which $A(K) > A_{min}$, then the maximum stable input amplitude limits can be established. Quantifying $A(K)$ is done in two steps, first by estimating the ratio of signal variance to quantization noise variance at the input to the quantizer gain K and subsequently estimating the variation of quantization noise variance σ_q^2 with the input signal amplitude in the Δ-Σ modulator loop.

III. Signal and Noise Variance at Quantizer Input and Quantization Noise

A. The ratio of the signal variance to quantization noise variance at the quantizer input.

Consider an input signal, consisting of five incommensurate sinusoids with the variances σ_i^2, $i=1,2,\ldots5,$ which finds practical applications as in the case of synthetic speech consisting of five sinusoids. There is no upper limit on the maximum number of sinusoids as the input signal variance can be incremented accordingly. The input signal variance for the five sinusoids is given by:

$$\sigma_x^2 = \sum_{i=1}^{5} \sigma_i^2 \tag{3}$$

If $\sigma_{ex}{}^2$ and $\sigma_{eq}{}^2$ are variances of the signal and quantization noise at the input to the quantizer, as the signal and quantization noise are uncorrelated the combined variance at the quantizer input is given by:

$$\sigma_e^2 = \sigma_{ex}^2 + \sigma_{eq}^2 \tag{4}$$

The quantizer gain K of a Δ-Σ modulator for a Gaussian input with variance $\sigma_e{}^2$ for a single-bit output $\pm\Delta$ is given by [26]:

$$K = \frac{\Delta}{\sigma_e}\sqrt{\frac{2}{\pi}} \tag{5}$$

Defining ρ^2 as the ratio of the signal variance to the quantization noise variance at the quantizer input yields the equation below:

$$\rho^2 = \frac{\sigma_{ex}^2}{\sigma_{eq}^2} \tag{6}$$

The quantizer gain K can be found from (4), (5) and (6) as given by:

$$K = \frac{\Delta}{\sigma_{eq}} \frac{1}{\sqrt{1+\rho^2}} \sqrt{\frac{2}{\pi}} \tag{7}$$

The Signal Transfer Function (*STF*) of the Δ-Σ modulator should ideally be ≈ 1, therefore:

$$K^2 \sigma_{ex}^2 = \sigma_x^2 \tag{8}$$

From (3) the variance of the signal consisting of five sinusoids each with equal amplitudes a_i is given by:

$$\sigma_x^2 = \frac{1}{2}\sum_{i=1}^{5} a_i^2 = \frac{5}{2} a_i^2 \tag{9}$$

As Δ can be assumed to be equal to 1 for a single-bit quantizer, one gets the following relation from (7), (8) and (9):

$$\frac{2}{\pi}\left[\frac{\sigma_{ex}^2}{\sigma_{eq}^2}\right] \frac{1}{[1+\rho^2]} = \sigma_x^2 = \frac{5}{2} a_i^2 \tag{10}$$

$$\Rightarrow \frac{2}{\pi}\rho^2 \frac{1}{[1+\rho^2]} = \frac{5}{2} a_i^2 \tag{11}$$

By solving (11), we get ρ which is plotted in Fig. 3. The variation of ρ is plotted as a function of amplitude a such that $a = \sum_{i=1}^{5} a_i$. This has been done to facilitate the comparative analysis with that of a single-sinusoidal input with amplitude a. It is observed that ρ increases linearly with a.

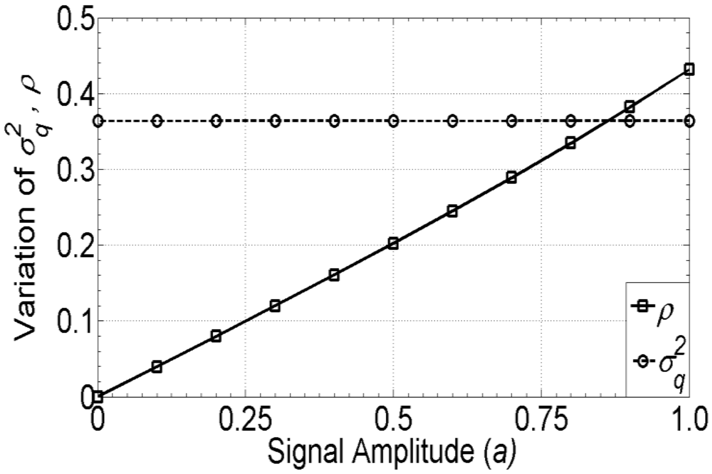

Fig. 3. Variation of σ_q^2 and ρ.

B. Quantization noise variance.

If $E(.)$ is the expectation operator, the power at the output of the Δ-Σ modulator is given by:

$$E[y^2(n)] = \sigma_q^2 + K^2\sigma_{ex}^2 + K^2\sigma_{eq}^2 = \Delta^2 \quad (12)$$

The quantization noise variance σ_q^2 can be found from (7), (8) and (12):

$$\sigma_q^2 = \Delta^2\left[1 - \left(\frac{2}{\pi}\right)\frac{\rho^2}{[1+\rho^2]} - \left(\frac{2}{\pi}\right)\frac{1}{[1+\rho^2]}\right] \quad (13)$$

Assuming Δ as ±1 for the single-bit quantizer, the quantization noise variance σ_q^2 is obtained from (13) and is plotted as in Fig. 3. The quantization noise variance remains constant as the signal amplitude increases.

C. Noise Amplification Factor.

The noise variance at the output of a Δ-Σ modulator is:

$$V_o = \sigma_q^2 + K^2\sigma_{eq}^2 \quad (14)$$

From (7) and (14) we get:

$$V_o = \sigma_q^2 + \frac{2}{\pi}\frac{1}{1+\rho^2} \quad (15)$$

The variation of $A(K)$ with a can be found by (1) and (15), which is given as:

$$A(K)=\frac{V_o}{\sigma_q^2}=\frac{\sigma_q^2+\frac{2}{\pi}\left[\frac{1}{1+\rho^2}\right]}{\sigma_q^2} \tag{16}$$

Using (16), $A(K)$ is plotted in Fig. 4. It decreases as the sum of sinusoids a increases reaching A_{min} at which point the Δ-Σ modulator becomes unstable. From (1) and (16), it is seen that as the Δ-Σ modulator order increases, so does V_o thereby increasing $A(K)$ in Fig. 2. Thus, the Δ-Σ modulator gets unstable at lower amplitudes for a higher *NTF* order. On the other hand, as the number of quantizer bits increases, the quantization noise σ_q^2 decreases, thereby lowering $A(K)$ and A_{min} and increasing the input stable amplitude limit for the same *NTF* order.

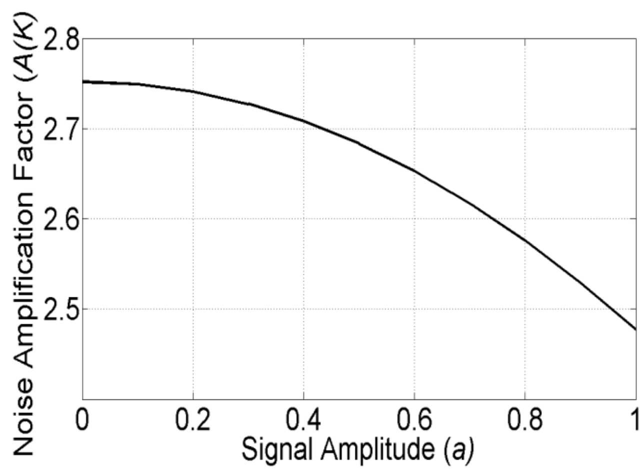

Fig. 4. Variation of $A(K)$ with signal input amplitude for multiple-sinusoids

D. Comparison with a single-sinusoidal input.

For a single-sinusoidal input with amplitude a the signal variance γ^2 is given by:

$$\gamma^2=\frac{1}{2}a^2 \tag{17}$$

From (11) one can get an equivalent relation:

$$\left(\frac{2}{\pi}\right)\frac{\rho^2}{[1+\rho^2]}=\frac{1}{2}a^2 \tag{18}$$

From (18) and (13), the plots of ρ and σ_q^2 with varying a, can be found for the single-sinusoidal input and are shown in Fig. 5. It is seen that σ_q^2 for the single-sinusoidal input is also constant and equal to the multiple-sinusoidal case as in Fig. 3. However, the ρ values obtained are higher for the same a.

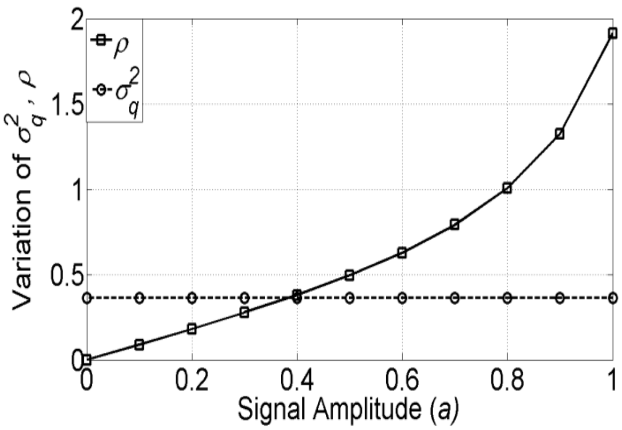

Fig. 5. Variation of σ_q^2 and ρ for single-sinusoidal input.

The ρ values obtained for the single-sinusoid input are higher than those for the multiple-sinusoidal inputs. This is because the signal variance at the input to the quantizer is higher for the single-sinusoidal with an amplitude a than for a multiple-sinusoid of equal amplitudes a_i such that $a = \sum_{i=1}^{5} a_i$ .This is illustrated by the variation of σ_q^2 and γ^2 with a_i for i=5 in Fig.6.

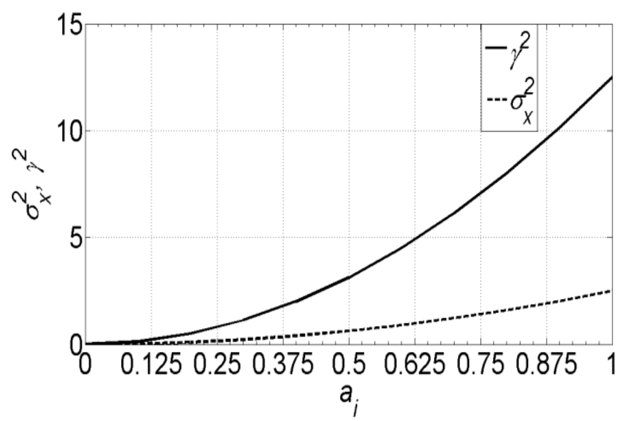

Fig. 6. Variation of σ_x^2 and γ^2 with a_i.

Assuming the stable input amplitude limit for the single-sinusoidal input for a 3rd-order Δ-Σ modulator to be typically 0.6, it would be incorrect to conclude that the stable limit for an input with five sinusoids would be of equal amplitudes of 0.6/5 = 0.12. As the signal variance at the input to the quantizer would be much lower for five sinusoids with equal amplitudes of 0.12 than for a single-sinusoidal input with an amplitude 0.6, the 3rd-order Δ-Σ modulator is likely to be stable for a > 0.6 for the multiple-sinusoidal input. This is observed from the $A(K)$ values for the single-sinusoidal input shown in Fig. 7, which are obtained from (16). The $A(K)$ values for the single-sinusoidal input are lower than the multiple-sinusoidal $A(K)$ values obtained in Fig.4. This results in the Δ-Σ modulator reaching the unstable limits at lower values of a for the single-sinusoidal input.

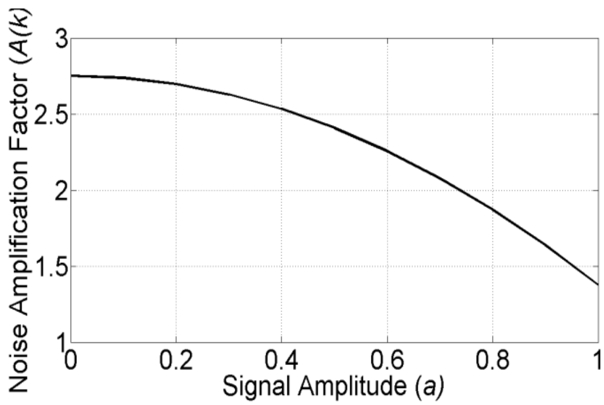

Fig. 7. Variation of $A(K)$ with signal amplitude for single-sinusoidal.

E. Multi-bit quantizer-present approach limitation.

This section concerns the limitations that are associated with the current approach for the multi-bit quantizer case. The concept of modified non-linearity can be used to find the mid-rise and mid-thread multi-bit quantizer gains from first principles and are given in [26]. Consider a simple case of a 3-level mid-thread multi-bit quantizer with the parameter δ , the point at which the quantizer step rises as shown in Figure 8.

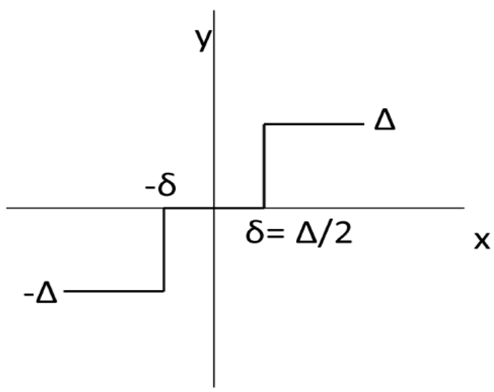

Fig. 8. 3-level multi-bit quantizer.

The multi-bit quantizer gain K_m for a mid-thread quantizer is given by [26]:

$$K_m = 2\frac{\Delta}{\sigma_e} PF\left(\frac{\delta}{\sigma_e}\right) \tag{19}$$

where,

$$PF(x) \underline{\underline{\Delta}} \frac{1}{\sqrt{2\pi}} e^{\left(-\frac{x^2}{2}\right)} \tag{20}$$

Assuming a uniform quantizer we have:

$$\delta = \frac{\Delta}{2} \tag{21}$$

Using equations (19), (20) and (21) the multi-bit quantizer gain is given by:

$$K_m = \frac{\Delta}{\sigma_e}\sqrt{\frac{2}{\pi}}\psi \tag{22}$$

where

$$\psi = e^{\left(-\frac{\Delta^2}{8\sigma_e^2}\right)} \tag{23}$$

Comparing (22) and (5), we observe that the multi-bit quantizer has an additional exponential term as a function of the input variance σ_e^2 as compared to the single-bit quantizer. It is therefore very mathematically challenging to split the variance of the signal and quantization noise at the input to the quantizer as done in the single-bit case using (6) and continue with similar analysis as before. One possible approach may be to model the exponential function as another function that may permit separation of the variances which would enable one to proceed with the analysis as has been done for the 1-bit quantizer using (7)-(16). Using (23), the variation in ψ for the 3-level uniform quantizer shown in Fig. 8 with $\Delta = \pm 1$ for a 5th-order Δ-Σ modulator is shown in Fig. 9. The variation does offer some insight into the stable nature of the 3-level quantizer. As ψ reduces the 3-level quantizer gain value K_m by almost a factor of 1/10 for a = 0.1 as compared to K , this along with the fact that the quantization noise variance is less for the 3-level quantizer as compared to the 1-bit quantizer, the variance at the quantizer input σ_e^2 is lower for the 3-level quantizer as compared to the 1-bit quantizer for the same input single amplitude a. For a 5th-order Δ-Σ modulator, this is shown in Figure 10 wherein the 3-level quantizer gets unstable at a = 0.75 where σ_e^2 = 0.34. The 1-bit quantizer has a comparable σ_e^2 = 0.41 for a much lower value of a = 0.24, at which point the Δ-Σ modulator gets unstable. Further work, however, would be required for a comprehensive explanation for the stability analysis of multi-bit quantizers based on the concept given in sections III A-D.

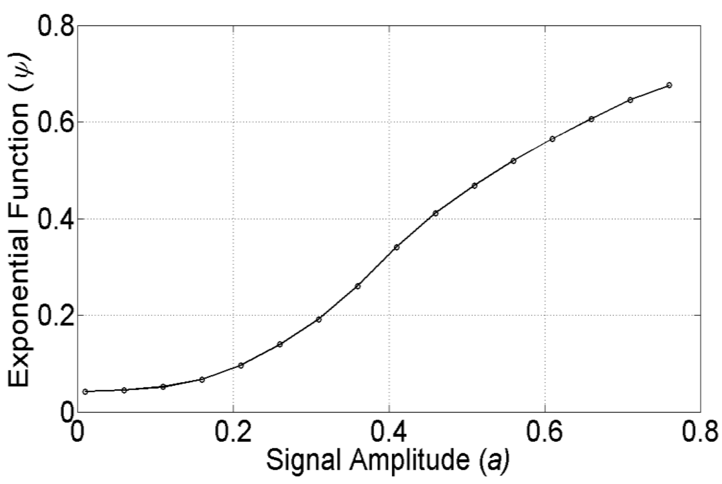

Fig. 9. Variation of exponential function with quantizer input variance.

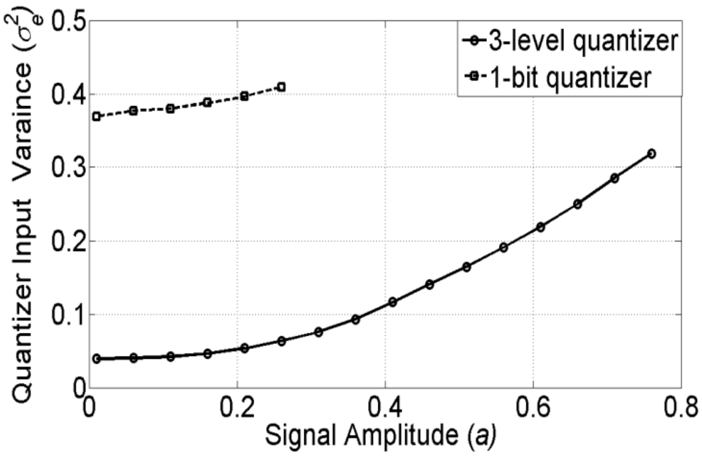

Fig. 10. Variation of quantizer input variance with signal amplitude.

IV. Simulation Results

A. Simulation Results.

Simulations were undertaken for 3rd-, 4th- and 5th-order single-loop single-bit Δ-Σ modulators. The corresponding $A(K)$ curves for these Δ-Σ modulators are shown in Fig 11.. The A_{min} values for the curves are 2.11, 2.41 and 2.67 respectively.

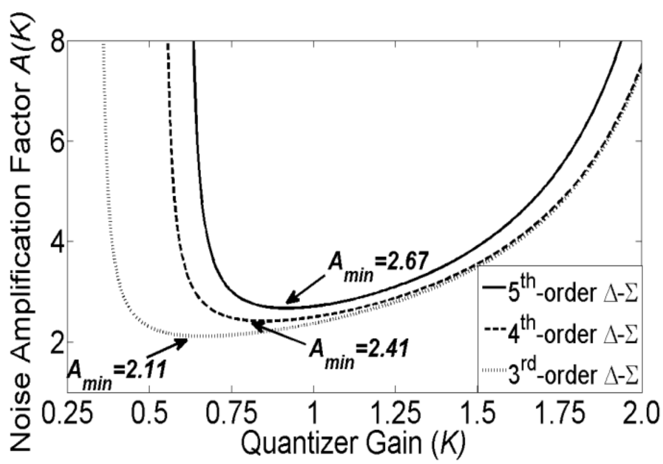

Fig.11. Variation of Noise Amplification Factor with quantizer gain.

The Δ-Σ modulators were implemented by deploying a cascade-of-accumulators feedback-form (CAFB) topology as shown in Fig. 12. for the 4th-order Δ-Σ modulator case.

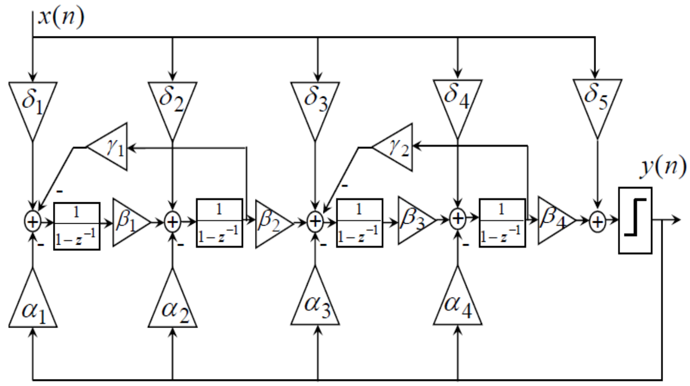

Fig.12. Fourth-order Δ-Σ modulator in CAFB topology.

The coefficient values for the three Δ-Σ modulators are shown in Table I.

TABLE I

COEFFICIENTS FOR Δ-Σ MODULATORS

Δ Σ	i	1	2	3	4	5	6
5th-order	δ_i	0.0028	0.0334	0.1852	0.5904	1.1120	1.0000
	α_i	0.0028	0.0334	0.1852	0.5904	1.1120	-
	β_i	1.0000	1.0000	1.0000	1.0000	1.0000	-
	γ_i	0.0007	0.002	-	-	-	-
4th-order	δ_i	0.0157	0.1359	0.5140	0.3609	1.0000	-
	α_i	0.0157	0.1359	0.5140	0.3609	-	-
	β_i	1.0000	1.0000	1.0000	1.0000	-	-
	γ_i	0.003	0.0018	-	-	-	-
3rd-order	δ_i	0.0751	0.0421	0.9811	1.0000	-	-
	α_i	0.0751	0.0421	0.9811	-	-	-
	β_i	1.0000	1.0000	1.0000	-	-	-
	γ_i	0.0014	-	-	-	-	-

The input signal consists of five incommensurate sinusoid amplitudes, which are increased in steps of 0.0003 that increases the overall amplitude a. The sinusoids are selected at random frequencies of 1 kHz, 3.25 kHz, 5.5 kHz, 7.5 kHz and 8 kHz. For comparison with the single-sinusoidal input, the frequency chosen is 8 kHz to ensure the same Over-Sampling Ratio (OSR). The Δ-Σ modulator clock frequency is 500 kHz, which results in an OSR of 32 for 2097152 output samples. The Signal-to-Noise Ratio (SNR) is obtained by plotting the FFT Power Spectral Density (PSD) using a Hanning window. The variation in the SNR with a for the 3rd-order Δ-Σ modulator is shown in Fig. 13.

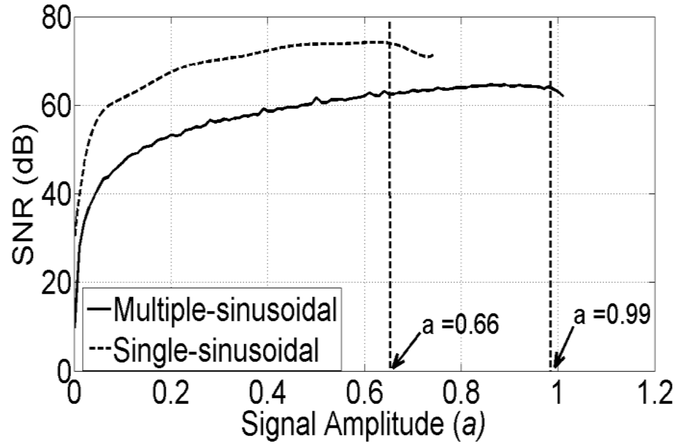

Fig. 13. 3rd-order variation of SNR with input signal amplitude.

The SNR increases linearly with an increase in *a*. The SNR for the single-sinusoidal input remains higher than the multiple-sinusoidal input due to the greater signal variance. As the variance of the signal at the quantizer input is also higher, the Δ-Σ modulator becomes unstable at the lower value of *a*= 0.66 *than* it does for the multiple-sinusoidal input for *a* = 0.99. At *a* = 0.66, it starts to fall showing the onset of instability for the single-sinusoidal input. The numerical stable value of *a* as predicted from Fig. 7 is 0.68, when A (K) = 2.11 = A_{min}. The predicted stable value of 0.68 is very close to 0.66 that is obtained via simulations. The SNR for the multiple-sinusoidal input falls at *a* = 0.99 indicating the onset of instability. The numerical stable value of *a* as predicted from Fig. 4 is 1.55 when A (K) = 2.11. The variation in the SNR with *a* for the 4th-order Δ-Σ modulator is shown in Fig. 14.

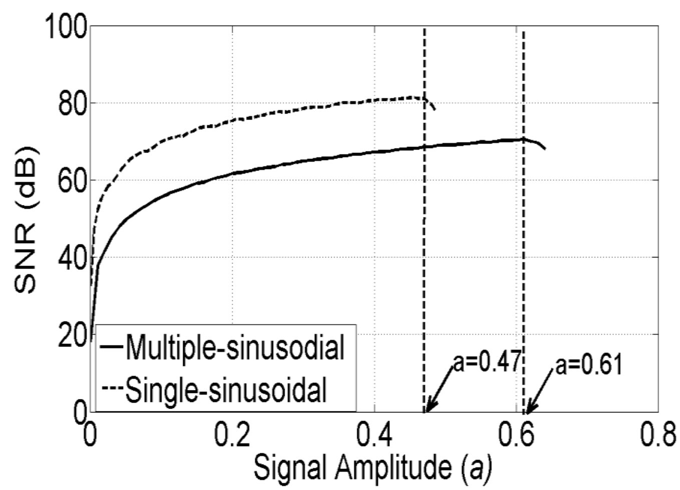

Fig. 14. 4th-order variation of SNR with input signal amplitude.

The 4th-order Δ-Σ modulator is stable up to a = 0.47 for the single-sinusoidal input. The numerical stable value of a as predicted from Fig. 7 is 0.49, when A (K) = 2.41 = A_{min}. For the multiple-sinusoidal case, the SNR starts to fall at a = 0.61, at which point the Δ-Σ modulator becomes unstable. The numerical stable value of a as predicted from Fig. 4 is 1.3, when A (K) = 2.41 = A_{min}. The variation in the SNR with a for the 5th-order Δ-Σ modulator is shown in Fig. 14.

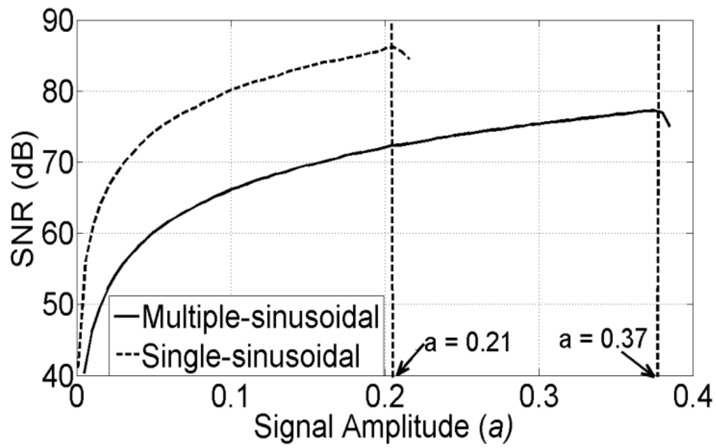

Fig. 15. 5th-order variation of SNR with input signal amplitude.

At a = 0.21, it starts to fall showing the onset of instability for the single-sinusoidal input. The numerical stable value of a as predicted from Fig. 7 is 0.23, when A (K) = 2.67 = A_{min}. For the multiple-sinusoidal input, the stable amplitude limit is 0.37. The numerical stable value of a from Fig. 4 is 0.38, for A_{min} = 2.67. The gain in the SNR is 5 dB as a increases from the stable input amplitude limit of 0.21 for the single-sinusoidal input to the 0.37 input amplitude limit for the multiple-sinusoidal input.

B. Accuracy of Results.

The numerically predicted stable amplitude limits along with the ones obtained via simulations are shown in Table II.

TABLE II

SIMULATION VALUES

Single-sinusoidal	Order	Stable Limit		Error in dB
		Simulated	Numerically Predicted	
	III	0.66	0.68	0.71
	IV	0.47	0.49	0.96
	V	0.21	0.23	0.76
Multiple-sinusoidal	III	0.99	1.55	2.00
	IV	0.61	1.30	4.00
	V	0.37	0.38	0.04

It is observed that the predicted values from the *A* (*K*) curves are more accurate for the single-sinusoidal input as compared to the multiple-sinusoidal input for the 3rd- and 4th-order cases. In addition the numerically predicted values for the single-sinusoidal input are more accurate with this simpler approach with a single quantizer gain in comparison to employing separate signal and noise gains as in [11]-[14]. Accurate results can be predicted for the 5th-order for both the single- and multiple-sinusoidal inputs. This can be attributed to the validity of the assumption that the input at the quantizer input has a Gaussian PDF. This assumption must hold for the quantizer gain given by (5) on which the analysis is based. The input to the quantizer, which is a combination of the quantization noise with a Uniform PDF and the signal PDF, tends to be less Gaussian for the multiple-sinusoidal input as compared to the single-sinusoidal input that tends to be more Gaussian, hence leading to more accurate results in the latter case. The is established by undertaking the Kolmogorov-Smirnov (KS) Gaussian test, according to which the test static (k_s) is given by [27]:

$$k_s = \max(|F(x) - G(x)|) \tag{16}$$

where $F(x)$ is the cumulative density function (CDF) of the quantizer input and $G(x)$ is the standard Gaussian CDF used for this test, the lower the value of k_s the closer is the quantizer input to having a Gaussian PDF. The variation of k_s values for the 3rd-, 4th- and 5th-order Δ-Σ modulator quantizer inputs with the signal amplitude *a* are shown in Fig.16, 17 and 18 respectively. For the 3rd- and 4th- order Δ-Σ modulators, it is observed that the single-sinusoidal input has lower k_s values as compared to the multiple-sinusoidal input indicating a more Gaussian PDF. For the 5th-order Δ-Σ modulator in Fig 18, the k_s values are almost the same and lower than those for the 3rd- and 4th- order Δ-Σ modulators.

This validates the assumption that the input to the quantizer, when it undergoes a triple- or higher-order integration tends to make the input a Gaussian distribution. More accurate results, therefore, are predicted in Table II for the 5th-order Δ-Σ modulator for the multiple-sinusoid input than for the 3rd- and 4th-order Δ-Σ modulator.

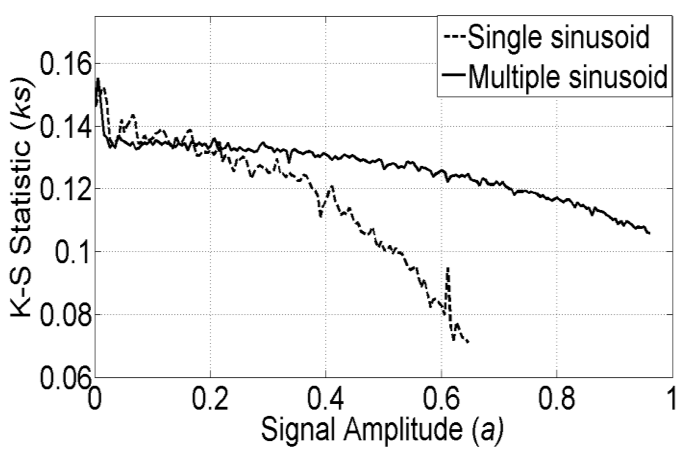

Fig. 16. 3rd-order variation of k_s with input signal amplitude.

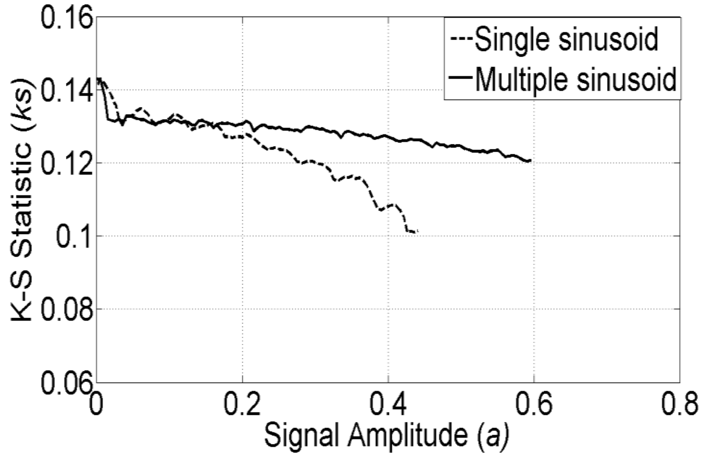

Fig. 17. 4th-order variation of k_s with input signal amplitude.

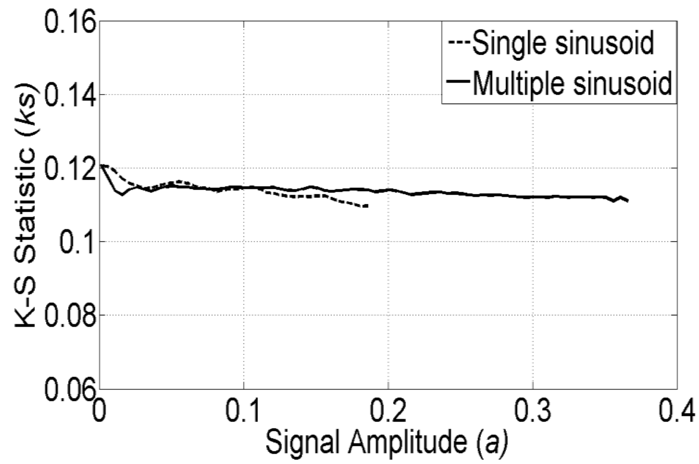

Fig. 18. 5th-order variation of k_s with input signal amplitude.

For any *NTF*, one can plot the $A(K)$ curve and accurately predict stability of the Δ-Σ modulator from Fig. 4 and Fig. 7 circumventing the need for elaborate and time consuming detailed simulations.

V. Conclusions

The stable input amplitude limits have been predicted for higher-order Δ-Σ modulators for a composite signal consisting of five sinusoidal inputs by employing a novel simplified approach of a single quantizer gain for the signal and quantization noise. The theoretical values are shown to be in very close agreement with the simulation results. The analysis can be extended to any number of sinusoids greater than five, for any *NTF*, be it low-pass or band-pass or any other. The Δ-Σ modulator relationship between stability, increase in the *NTF* order and the number of quantizer bits has been mathematically explained and novel results are reported. The novel results would enable optimizing the design of higher-order Δ-Σ modulators for various applications that require multiple-sinusoidal inputs such as speech processing and moreover for any general inputs that can be modeled as the Fourier series decomposition of individual sinusoids. The results from this work will also make it possible to design and commission Δ-Σ modulators for the applications mentioned without the need for exhaustive time consuming stability analysis and simulations. The analysis, although becomes more complicated can be extended to multi-level quantizers, the results of which would be reported in a future publication.

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