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IMPROVED COGGING CALCULATION METHODS FOR SURFACE MOUNTED PERMANENT MAGNET SYNCHRONOUS MOTORS

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Abstract: As part of the design process of surface mounted permanent magnet synchronous motors (PMSM), a combination of analytical calculation and finite element analysis (FEA) is proposed for the cogging torque calculation. The analytical methods are recommended for the initial design iterations in view of their high computational speed. In general, however, finite element analysis is more accurate and is therefore recommended for the final design iterations. In order to obtain continuity when switching from analytical calculations to FEA, two modifications are made to the equations upon which the analytical methods are based in order to improve accuracy. This is demonstrated by comparing the results from the unmodified and modified analytical method with those using the finite element method through their application using the nominal parameters of a Control Techniques Dynamics CTD 142UMC300 motor. Air-gap flux density calculations are compared as well as cogging torque calculations.

1 Introduction

Cogging torque is an unwanted but unavoidable characteristic in permanent magnet synchronous motors. Consequently, it is usual to design a PMSM with a cogging torque less than 1.5% of the stall torque. To carry out the necessary calculations, analytical and finite element methods are available. In general, the finite element methods are more precise than the analytical methods but are more time consuming. In order for the design engineer to minimise the computational time without compromising accuracy, it is proposed to use an analytical method to arrive at a rough design through some iterations using design parameters such as the permanent magnet dimensions, followed by fine tuning with finite element analysis.

The well known formula of Zhu and Howe (Zhu, et. Al. [2] for the cogging calculation is arguably the most accurate available to date but it is restricted to the design of PMSM with parallel magnetised arc magnets. Such magnets limit the extent to which the cogging torque can be minimised, further improvements being attainable by employing instead surface parallel, surface radial, tapered or bread-loaf magnets. Up to now, similar formulae have not been developed for motors with such magnets and modifications to the standard formulae of Zhu and Howe to solve this problem in the cases of tapered and bread-loaf magnets constitute the main contribution of this paper. With the

knowledge that a finite element package is capable of more accurate results, the analytical calculation is verified by means of finite element analysis.
The intention is not to improve the analytical method to the extent of replacing the finite element method but to improve it to a level where it is able to help the motor designer achieve a good 'first cut' design prior to use of the finite element method for refinement. It should be noted, however, that computational errors become more significant (as a percentage of the peak cogging torque) as the design is improved to reduce the peak cogging torque. Although the calculation can be improved by better geometry and mesh creation, this will not eliminate the problem entirely. There are two main methods upon which finite element programmes are based. The first is the Maxwell stress method and the second is the energy method. The Maxwell stress method is the most common and its accuracy depends on the contour location as well as the judiciously chosen mesh sizing. The energy method does not have contour dependent accuracy but suffers from the drawback of numerical differentiation and the associated errors. In this paper, the Maxwell stress method is employed.

2. Standard Formulae for Analytical Methods

The well known cogging torque formulae of Zhu and Howe is in common use [1] [2] [3] and is used in this study. It is formulated in three stages. First, the air gap flux density distribution formula for the ideal slot-less machine is taken. Then it is modified to take account of the air gap permeance. Finally the cogging torque is calculated from modified air gap flux density. Thus, the equation for the flux density at the surface of the stator bore of an ideal slot-less machine is as follows:

$$B_{rl}(\theta) = \sum_{n=1,3,5,\ldots}^{\infty} 2\frac{\mu_0 M_n}{\mu_r}\frac{np}{(np)^2-1}\left(\frac{R_m}{R_s}\right)^{np+1} \cdot \left\{\frac{(np-1)+2\left(\frac{R_r}{R_m}\right)^{np+1}-(np+1)\left(\frac{R_r}{R_m}\right)^{2np}}{\frac{\mu_r+1}{\mu_r}\left[1-\left(\frac{R_r}{R_s}\right)^{2np}\right]-\frac{\mu_r-1}{\mu_r}\left[\left(\frac{R_m}{R_s}\right)^{2np}-\left(\frac{R_r}{R_m}\right)^{2np}\right]}\right\} cos(np\theta) \quad (1)$$

where $M_n = 2\left(B_r/\mu_0\right)\alpha_p \frac{sin\left(\frac{n\pi\alpha_p}{2}\right)}{\frac{n\pi\alpha_p}{2}}$, B_r is the remanence flux density of the magnet, μ_0 is the permeability of free space, R_m is the magnet outer radius of curvature, R_s is the stator bore radius, R_r is the rotor radius, p is the number of pole pairs and μ_r is the relative permeability of the magnetic stator and rotor material.

Then the slotting effect is taken into account by means of the permeance equation

$$\lambda_{rel} = \frac{g+\frac{l_m}{\mu_r}}{g+\frac{\pi}{2}w_1+\frac{l_m}{\mu_r}} \quad (2)$$

Finally, the cogging torque is given by

$$T_{cog}(\theta_1) = \frac{L}{2\mu_0}\sum_{m=1}^{N_s}\left[\int_0^{\frac{ws}{2}} B_{PM}^2(w_1).(R_s + w_1)dw_1 - \int_{\frac{ws}{2}}^{ws} B_{PM}^2(w_2).(R_s + w_2)dw_2\right] \quad (3)$$

and computed by means of the equivalent discrete algorithm obtained by numerical approximations to the integrals:

$$T_{cog}(\theta_1) = \frac{\pi L R_s}{2\mu_0 N}\sum_{m=1}^{N}\left[B_{PM}^2\left(\frac{2\pi}{N}m + \theta_1\right).\left(R_M + g_\alpha\right).ssg\right] \quad (4)$$

where

$$\begin{cases} g_\alpha = 0 \text{ and } ssg = 0 & \text{outside the slot opening} \\ g_\alpha = w_1 + g \text{ and } ssg = 1 & \begin{cases}\text{on the left side} \\ \text{of the slot opening}\end{cases} \\ g_\alpha = w_2 + g \text{ and } ssg = -1 & \begin{cases}\text{on the right side} \\ \text{of the slot opening}\end{cases} \end{cases} \quad (5)$$

3. Modified Formulae

3.1 Tapered Magnet Profile

A common measure taken to reduce cogging torque is decreasing the radius of curvature of the outer surface of the permanent magnet to obtain an increase of the air gap towards the edges of the magnet so as to achieve a closer approximation to the ideal sinusoidal flux distribution around the air gap [3] [4] [5]. This yields a tapered magnet profile as illustrated in Figure 1. Here, the centre of curvature of the magnet outer surface is offset by M_{off} from the rotor centre and increase of this parameter yields increase of the tapering.

The standard cogging formulae presented in section 2 becomes inaccurate once magnet tapering is introduced. To overcome this problem, the following formula for R_m has been implemented, based on the geometry of Figure 1.

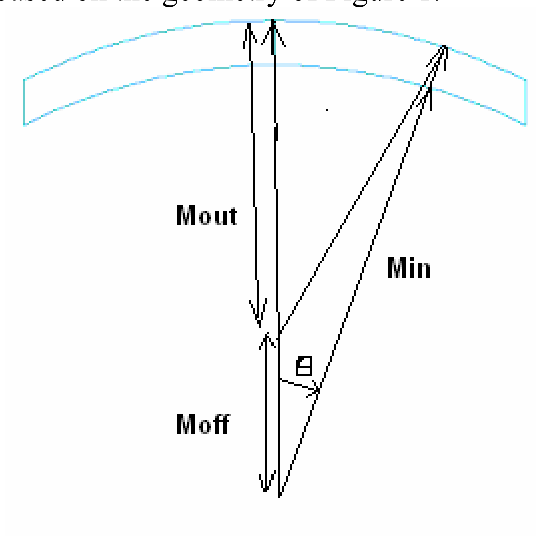

Figure 1 Geometry of tapered magnet profile

$$R_m = \frac{M_{out}\sin\left\{\theta + \sin^{-1}\left(M_{off}\sin(\theta)\big/M_{out}\right)\right\}}{\sin(\theta)} \quad (6)$$

3.2 Bread-loaf Magnet Profile

An alternative method to the magnet tapering for cogging torque reduction is to maintain coincidence of the centre of curvature of the outer magnet surface and the rotor centre to keep a constant air gap but introduce a flat inner magnet surface, as illustrated in Figure 2.

This also yields a closer approximation to the ideal sinusoidal flux distribution around the air gap than attainable with parallel magnetised arc magnets. The modification of the standard formulae to cater for this case is the following equation for R_r according to the geometry of Figure 2:

$$R_r = \frac{\sqrt{M_{in}^2 - \frac{M_{wid}^2}{4}}}{cos(\theta)} \qquad (7)$$

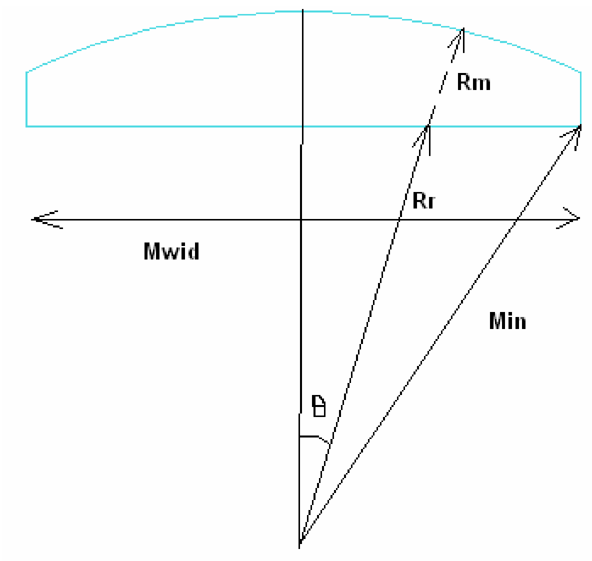

Figure 2 Geometry of bread-loaf magnet profile

4. Computational Results

A C++ program was developed for the analytical method based on equations (1), (2) and (4) for the non tapered magnet design and additionally equation (6) for the magnet tapering by variation of the outer magnet radius of curvature and equation (7) for the bread-loaf magnet.

For the finite element calculation, Opera 2d was employed. The Maxwell integration is performed along a circular contour going through the air gap. The integration is undertaken near to the rotor and near to the stator and then the average is taken.

In all cases, the air gap flux density and the corresponding cogging torque is plotted against rotor angle in degrees, two plots being displayed, one for the analytical method and the other for the finite element method.

4.1 Standard Formulae

Figure 3 shows the results obtained with the standard formulae assuming magnetised arc magnets.

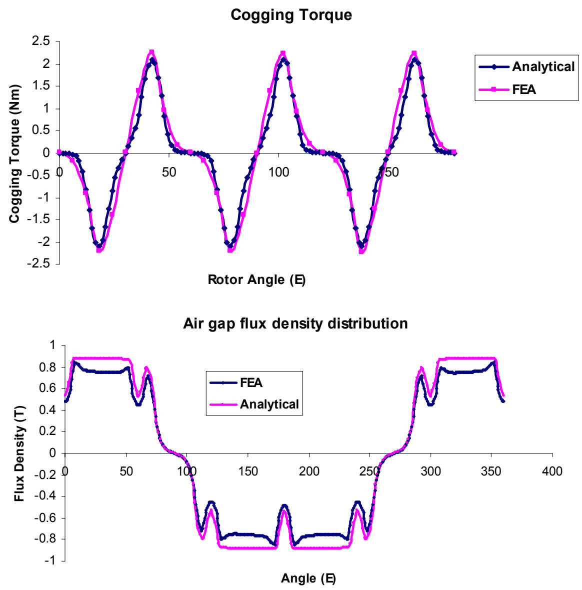

Figure 3, Comparison of Finite Element and analytical results for non-tapered magnet

Although the errors between the analytical and FEA plots are significant, they are not so large that they would entail an excessive number of design iterations with the FEA after the initial iterations using the analytical method in order to arrive at an optimal design. Hence, if similar levels of errors between the analytical and FEA plots are obtained for the modified formulae in the following two sections, then the results will be considered satisfactory.

4.2 Modified Formulae for Tapered Magnet Profile

Figures 4 and 5 show plots corresponding to Figure 3 for the analytical formulae modified to cater for the tapered magnet profile. As expected, the peak cogging torque is reduced compared to non tapered case of Figure 3. Comparing Figures 4 and 5 shows that increasing the offset, M_{off} significantly reduces the peak cogging torque.

In Figures 4 and 5, the percentage errors between the analytical and FEA plots are of a similar order to those obtained in Figure 3. The modified formula for the tapered magnet profile therefore produces satisfactory results.

It must be noted, however, that a severe taper reduces the magnet thickness so much at its ends that it will adversely affect the demagnetisation characteristics of the motor.

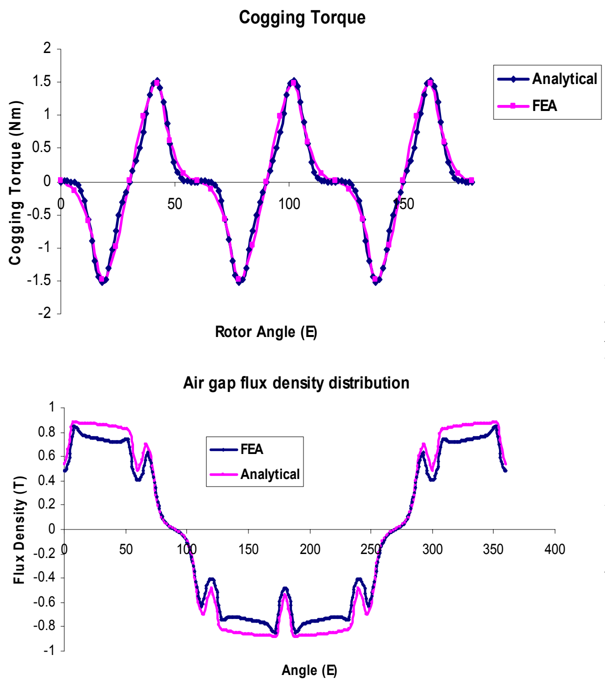

Figure 4. Comparison of Finite Element and analytical results for taper with Moff = 3.35mm

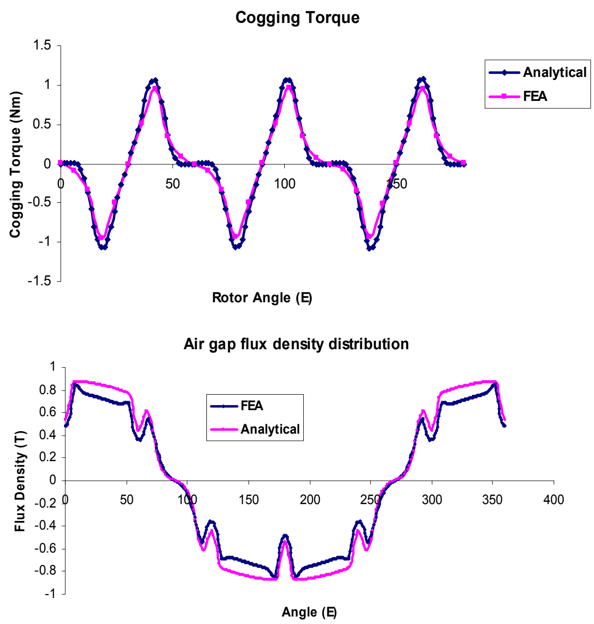

Figure 4 Comparison of Finite Element and analytical results for taper with Moff = 6.35mm

4.3 Modified Formulae for Bread-loaf Magnet Cogging Torque Profile

Figure 6 shows plots corresponding to Figure 3 for the analytical formulae modified to cater for the bread-loaf magnet profile. Comparison with Figure 3 reveals that in this case no reduction of the peak cogging torque has resulted but the main objective here is to compare the analytical and FAE results for the same case. From this point of view, the results are again satisfactory. Although the percentage error between the analytical and FAE plots has increased, it would still be sufficiently small to enable the FAE method to take over from the analytical method in a design scenario without an excessive number of iterations.

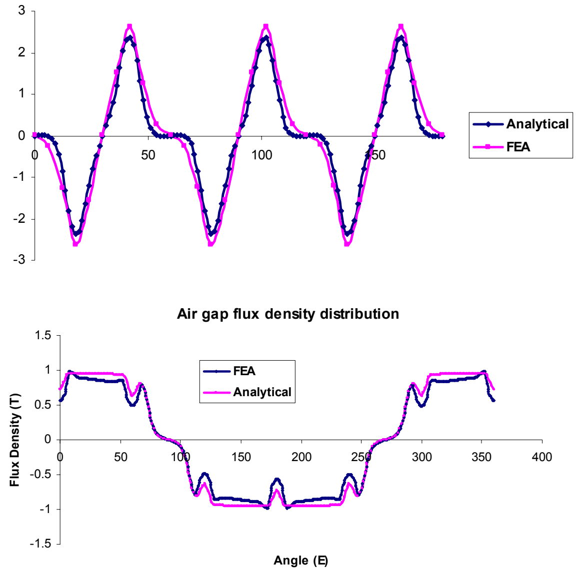

Figure 6 Comparison of Finite Element and analytical results for bread-loaf magnet

5. Conclusions

The standard formulae of Zhu and Howe for calculating the cogging torque of a PMSM with magnetised arc magnets by the analytical method has been modified to enable the method to be applied in the design of motors with outer surface tapered magnets and bread-loaf magnets. In all cases, sufficiently small errors between the analytical and FEA plots of air gap flux density and cogging torque are obtained for the analytical method to produce an adequate ‘first cut’ motor design to initiate the final design stage using FEA without an excessively large number of iterations. It is suggested that similar modifications are produced to enable the analytical method to be applied to PMSM with other magnet geometries.

6. Acknowledgements

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7. References

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