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New Model for the Effective Permeability of Ferrite Microstrip

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Abstract: Ferrite microstrip has found many uses in phased array radar systems. Since the discovery of high temperature superconductor (HTS) [1] in 1986, HTS microstrip ferrite phase shifters [2-6] have been an attractive option for phased array radar systems because of their low loss. This work is to develop a new model for the effective permeability of ferrite microstrip which is based on Wheeler's [7] microstrip impedance model. The newly developed model for the effective permeability will have some improvements over the well-known model developed by Pucel [8, 9], about three and a half decades ago. In this model, the transition error (or the discontinuity) from narrow to wide ferrite microstrip has been removed. Unlike Pucel's model, where one equation is derived for the narrow ferrite microstrip and another for the wide ferrite microstrip, this model will only use a single equation to predict the effective permeability of the ferrite microstrip for the entire range of width.

1. INTRODUCTION

Ferrite theories have been established well over half a century because of the importance to phased array radar systems and communication systems. Since the introduction of the planar microstrip structures about four decades ago, the ferrite microstrip has also found use in planar microwave devices, e.g. planar ferrite phase shifters, circulars and isolators.

Pucel [8, 9] had developed a set of equations for the effective permeability for ferrite microstrip about three and a half decades ago. His equations have been used as the main reference for most ferrite microstrip devices. These equations are based on two sets of narrow and wide microstrip equations developed by Wheeler [10, 11]. Therefore, there is a discontinuity at the transition, from narrow ferrite microstrip to wide ferrite microstrip, in Pucel's model.

This work is to introduce a new set of equations for effective permeability of ferrite microstrip by using Wheeler's [7] microstrip equation which merges the narrow and the wide microstrip equations into one. This new equation will remove the discontinuity in the transition range from narrow to wide microstrip. Therefore, this will remove the ambiguity of the decision point, e.g. the critical width separating the narrow and the wide regimes used in Pucel's calculations

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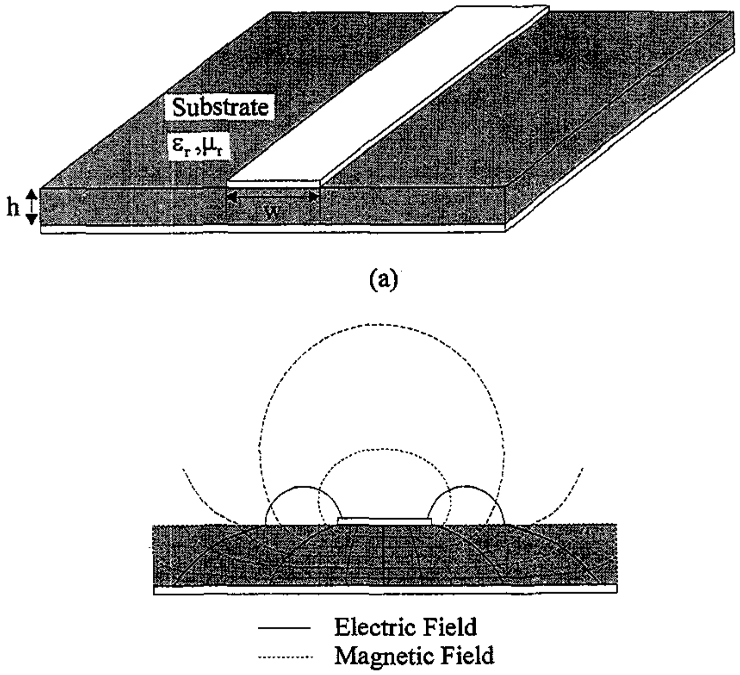
2. FORMULAS DERIVATION

Microstrip is a transmission line which operaties in quasi-TEM mode. Fig. 1 shows a circuit layout of a microstrip on a ferrite substrate and its fields distribution. It is classified as quasi-TEM because the propagation fields are not perfectly confined in the substrate as shown in Fig. 1(b).



(a)

(b)

Figure 1: Microstrip (a) Layout (b) Fields Distribution

Characteristic impedance, Z_o, of a dielectric microstrip can be determined by using Wheeler's [7] analytical expression for Z_o for the entire range of widths. For a ferrite substrate microstrip, an additional parameter which is known as effective permeability has to be determined in order to determine the characteristic impedance. The characteristic impedance of ferrite substrate microstrip can be written as [12]:

$$Z_o(\varepsilon_r, \mu_r) = Z_o(1)\sqrt{\frac{\mu_{eff}(\mu_r)}{\varepsilon_{eff}(\varepsilon_r)}} = Z_o(\varepsilon_r)\sqrt{\mu_{eff}(\mu_r)} \qquad (1)$$

where $Z_o(1)$ is the characteristic impedance of microstrip with air as the substrate, $Z_o(\varepsilon_r)$ is Wheeler's analytical expression for characteristic impedance of dielectric substrate microstrip, with relative permittivity of ε_r, and $\mu_{eff}(\mu_r)$ and $\varepsilon_{eff}(\varepsilon_r)$ are effective permeability and permittivity, respectively.

In order to derive an expression for the effective permeability, we much first derive an expression for the effective permittivity. From (1) and Wheeler's equation [7], the effective permittivity can be written as [12]:

$$\varepsilon_{eff}(\varepsilon_r)=\left[\frac{Z_o(1)}{Z_o(\varepsilon_r)}\right]^2$$

$$=\frac{\varepsilon_r+1}{2}\left[\frac{\ln\left(1+2a^2+a\sqrt{4a^2+\pi^2}\right)}{\ln\left(1+a^2b+a\sqrt{a^2b^2+c\pi^2}\right)}\right]^2 \tag{2}$$

where

$$a=\frac{4h}{w};\; b=\frac{14+\frac{8}{\varepsilon_r}}{11};\; c=\frac{1+\frac{1}{\varepsilon_r}}{2} \tag{3}$$

For a TEM approximation model, which is assumed in Wheeler's microstrip analysis, the permeability and permittivity form a duality relationship as in Maxwell's equations, e.g. [8]:

$$\mu\rightarrow\frac{1}{\varepsilon} \tag{4}$$

Therefore, the effective permeability can be derived from the effective permittivity based on this duality relationship. The relationship between the effective permeability and the effective permittivity can be shown as:

$$\mu_{eff}(\mu_r)\rightarrow\frac{1}{\varepsilon_{eff}\left(\frac{1}{\varepsilon_r}\right)} \tag{5}$$

Thus, the new model of effective permeability can be shown to give:

$$\mu_{eff}(\mu_r)=\frac{2\mu_r}{\mu_r+1}\left[\frac{\ln\left(1+a^2b'+a\sqrt{a^2b'^2+c'\pi^2}\right)}{\ln\left(1+2a^2+a\sqrt{4a^2+\pi^2}\right)}\right]^2 \tag{6}$$

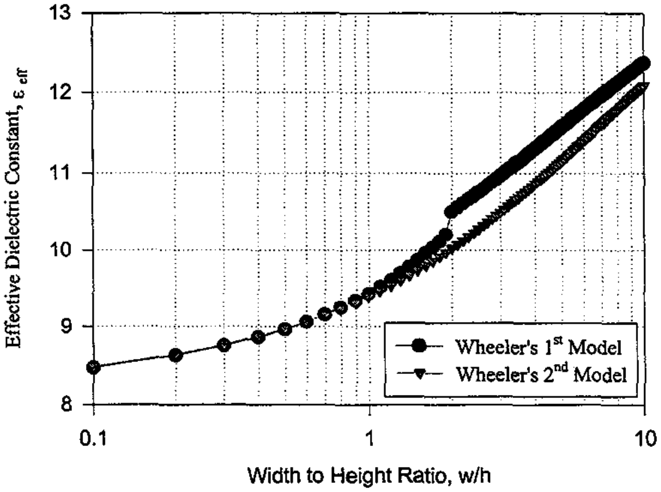
where

$$b'=\frac{14+8\mu_r}{11};\; c'=\frac{1+\mu_r}{2} \tag{7}$$

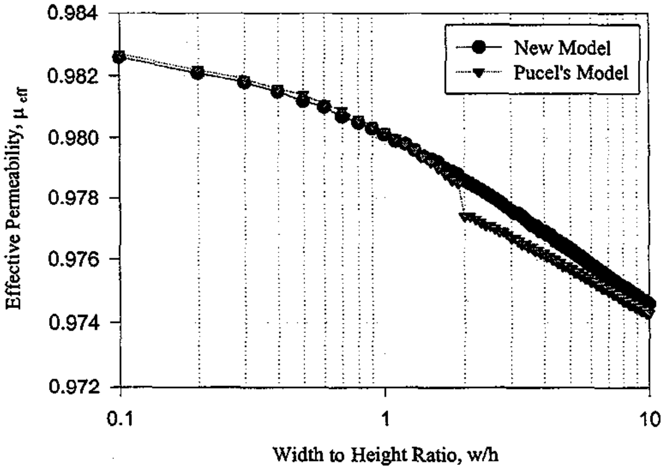
and a is given in (3). Both (2) and (6) are valid for the entire range of microstrip widths.

3. GRAPHICAL RESULTS

To show the improvement of this model compared to Pucel's model, two plots are given in Fig. 2. Fig. 2(a) shows the plot of effective permittivity for the two models derived by Wheeler. It is shown that the second model, (2), has no discontinuity whereas in the original model there is a transition error from narrow to wide microstrip.



(a)



(b)

Figure 2: Plot of (a) Effective Permittivity (b) Effective Permeability of a Typical Ferrite Microstrip

Fig. 2(b) shows a comparison between the new model discussed in this paper and the one derived by Pucel. Again, it is shown that the transition error has been removed. The effective permittivity and the effective permeability will converge to the relative permittivity and the relative permeability, respectively, when the width of the microstrip becomes very large. These have shown to be true in Fig. 2.

4. CONCLUSION

A new model for the effective permeability for ferrite microstrip has been derived and presented. The improvement of this model over the existing model is also discussed. Calculation has been made simple because there is only one equation describing the effective permeability in this model.

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