PARAMETRIC STABILITY ANALYSIS OF STEEL FRAME STRUCTURES

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UDK: 624.046.3.072.2
DOI:10.14415/konferencijaGFS 2016.032

Summary: *In this paper it is presented parametric stability analysis of frame structures with accuracy assessment of solutions given in codes for design of steel structures. Method for calculating the critical load and the effective buckling length of the frame structures that is based on the global stability analysis is formulated. According to results of analysis of the whole structure, the critical load and effective buckling length for each column member can be obtained. The numerical analysis in this paper is based on the calculation of the critical load in elaso-plastic domain.*

Keywords: *energy efficiency of buildings, renovation, mesurements*

1. INTRODUCTION

Research of the stability of linear structures, starting from the first Euler's investigations until recently, was mainly based on solving the differential equation of buckling according to the second order theory. In order to find simple solutions for the more complex structures, the researchers performed some approximations in their calculation. It means that isolated members with different boundary conditions were analyzed.
Numerical expressions and graphical representation of the obtained results for the critical can be found, for example in [1], [2]. Such approximations were used to formulate procedures for calculations of multi-story frames. The most used methods for this type of calculation are Slope deflection method and Stiffness distribution method, and they can be found in the literature, for example [1], [3]. The above procedures for calculation of one-bay multi-storey frames can be applied for more complex multi-bay frames. Namely, in this case, the calculation of multi-bay framework can be reduced to calculation of equivalent one-bay frame, as it is shown in [4], [5]. Sometimes, the framework because of its irregularity can not be reduced to an equivalent single-bay frame in order to calculate their critical load. In that case, some other procedure, such as energy method [6], are suggested.
Theoretical approach based on the calculation of isolated member was applied in the national and European regulation for the stability of frame structures [7] - [9]. However, the application of these codes shows that such calculation, in some cases, may lead to the substantial errors, because obtained results are approximate [10]. Therefore, in recent years, considerable effort has been made in order to improve these approximate

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calculation methods. So, for example, the objective of the analysis [11], [12] is to propose improved input parameters for the determination of the effective buckling length coefficient of columns in multi-story frames.
In this analysis, the finite element method, as the most effective method for numerical analysis of stability of frame structures is applied. Application of finite element method in the stability analysis of the linear frames was investigated by many authors, for example [13], [14].
The aim of this paper is to formulate the exact matrix stability analysis. So, the interpolation functions should be derived from the solution of the differential equation of bending according to the second order theory. Also, when the buckling of structure occurs in plastic domain, constant modulus of elasticity E should be replaced with the tangent modulus Et. More details about this approach can be found in [15].

2. DETERMINATION OF THE EFFECTIVE BUCKLING LENGTH OF THE COMPRESSED MEMBERS

Calculation based on the theory of elastic stability is widely applied in engineering practice. It is because it can be assumed that building structures generally have elastic behavior when they are subjected to standard exploitation load, even in the case when the critical load is reached. Namely, in order to design structures with high slenderness, designers often make tall structures with low stiffness, so buckling occurs in the elastic domain. Therefore it is reasonable that this type of elastic calculation is the basis of the regulations for the stability analysis of frame structures. Such calculation procedure is defined through the determination of the effective buckling length of frame columns.
From the physical point of view, “effective buckling length” is a length of the equivalent member with constant cross-section that is pined at the both ends and is subjected to the compressive axial force. Critical force is defined by:

$$P_{cr} = \frac{\pi^2 EI}{(\beta \cdot l)^2} \qquad (1)$$

and it is equal to the critical force of the analyzed member with arbitrary characteristics.
From the mathematical (geometrical) point of view, “effective buckling length” is a distance between inflection points of the bended member.
The effective buckling length is given as the product of the column's effective length factor „β“ and the geometric length of the column „*l* “:

$$l_i = \beta \cdot l \qquad (2)$$

Procedure for calculation of the effective buckling length for the members in steel frame structures is given in national JUS standards [9] and European regulations EC3 [7], [8]. On the basis of such obtained buckling lengths, the calculation based on the buckling curves is applied for the axially compressed members.

Effective buckling lengths for the members with different boundary conditions are shown in Figure 1. According to obtained buckling lengths, critical load is derived from the equation (1).

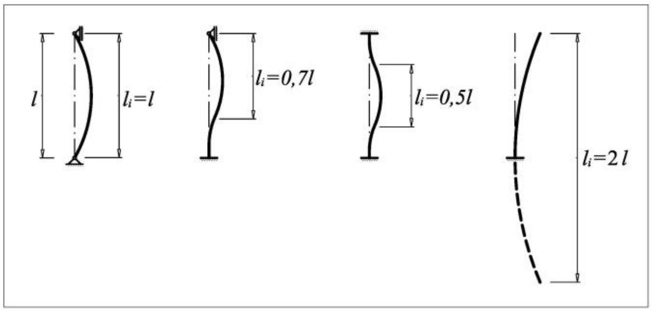
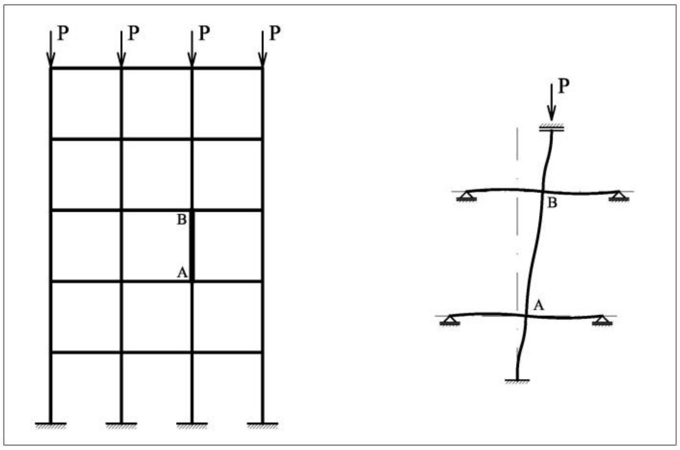


Figure 1. Effective buckling lengths in function of the boundary conditions

This approach to calculate the critical load, when the effective buckling length is obtained, is applied also to the columns of the frame structures. For example, analysis of the whole frame structure (Figure 2a) is performed using the equation (1), and critical load and effective buckling lengths should be defined for each column individually. The reason for this is the fact that until recently it was considered that global stability analysis of frame structures is too complicated for engineering practice. That is the reason why the codes for the stability analysis of plane frame columns [7] - [9], use simplified static scheme, as presented in Figure 2b. Practically, this means that codes consider only columns which are isolated from the frame structure. These isolated columns are supported only by the adjacent columns and beams. Basically, presence of the other structural elements connected to the considered one is introduced by the corresponding boundary conditions.



a) b)

Figure 2. Simplified static scheme for the stability analysis according to the codes

This approximate, simplified calculation has its advantages because results can be obtained very easily and they are shown by adequate diagrams and approximate formulas. However, very important question is how these approximate solutions are correct and how they can be used for the various examples in engineering practice. Also, there is important question should such solutions still be used, due to the fact that fast development of computing possibilities is in the progress and the programs for stability analysis of frame structures become generally available.

The methodology of calculation that is used in this analysis is related to the global stability analysis of frame structures. This means that critical load for the whole structure ($P_{cr,gl}$,) should be calculated first. When this critical load is calculated, the critical load for each column (P_{cr}) can be obtained. Based on these results, effective length factor of individual columns is given by:

$$\beta = \sqrt{\frac{\pi^2 EI}{P_{cr} \cdot l^2}} \tag{3}$$

In the case of buckling in the plastic domain, the expression for the coefficient β has the same form as (3), but critical load (P_{cr}) is related to the critical force that is computed in the inelastic analysis of the frame ($P_{cr,inel}$). Also, modulus of elasticity E is no longer constant, but it is stress dependent and it is replaced by the tangent modulus E_t:

$$\beta = \sqrt{\frac{\pi^2 E_t I}{P_{cr,inel} \cdot l^2}} \tag{4}$$

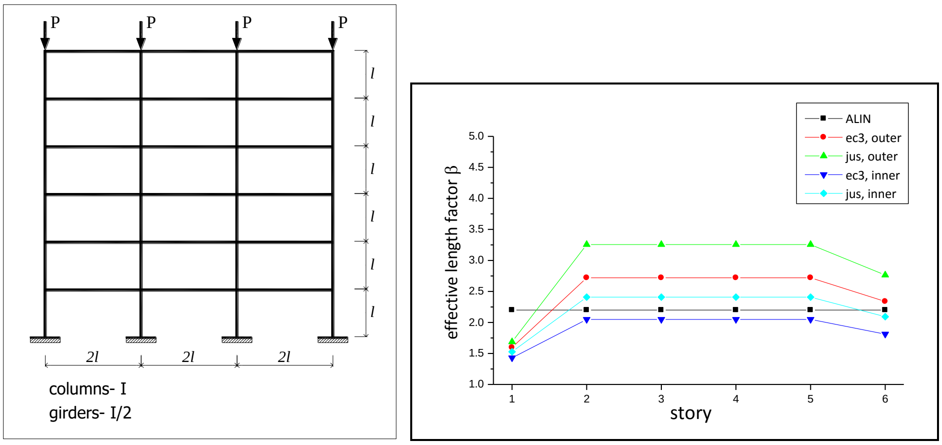
In this analysis it is used an empirical relationship between the two moduli [15] in a form:

$$E_t = 4E \cdot \left[\frac{\sigma}{\sigma_y} \left(1 - \frac{\sigma}{\sigma_y} \right) \right] \tag{5}$$

3. NUMERICAL EXAMPLES

The determination of the buckling length of frame structures is presented in this section. The calculation is performed using the corresponding computer program ALIN developed in the C++ programming language [16], [17]. Obtained results are compared with the results from the European EC3 and national JUS standards [7] - [9].

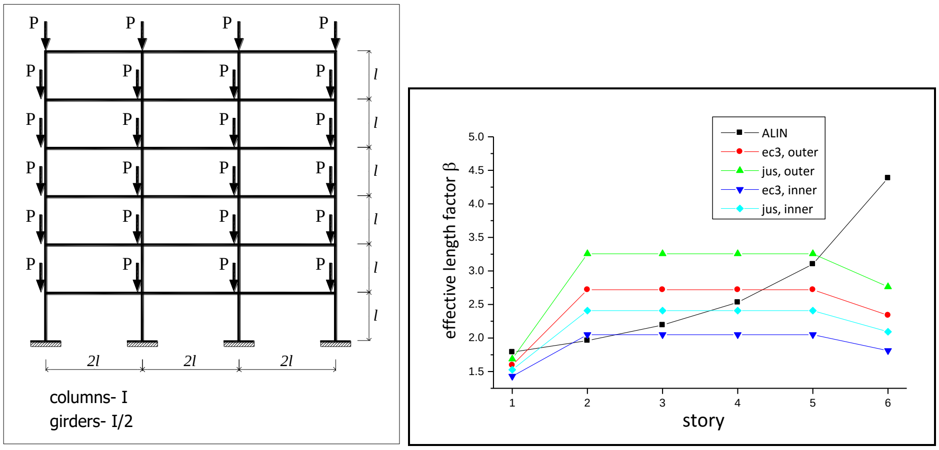
A six-story three-bay plane frame, clamped at the base, is first considered in this analysis (Figure 3a). Graphical presentation of the results for the coefficient β is shown in Figure 3b. These results are obtained using the code ALIN, as well as European and national codes for steel structures. It can be seen that in this case there is a quite well coinciding of the results.



a) b)

Figure 3 a) Six-story sway frame with constant axial force in the columns
b) The values of the coefficient β for the analyzed frame

A analysed six-story three-bay frame with the load that is exerted on each column at each story in the frame is shown in Figure 4a). Results for the effective buckling length coefficient are given in Figure 4b.

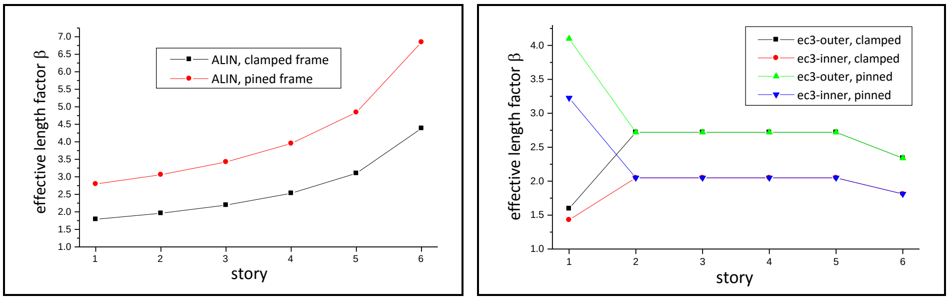


a) b)

Figure 4 a) Six-story sway frame with load at each story
b) The values of the coefficient β for the analyzed frame

This graph shows that errors in the application of regulations increase when compared to the previous example. One of the main reasons for this is that the calculations that are based on the isolated treatment of considered compressed elements (as in the regulations) do not take into consideration the axial force value in the element. They only consider stiffness of the other structural elements connected to the considered one. Thus, in analyzed numerical example obtained errors are larger than in the previous case. It is clear that that these differences in the determination of the buckling length affect the calculation of the buckling resistance of axially compressed members.

Although the clamped columns are more present in the engineering practice, this paper also considers the frames that are pinned at the base. The characteristics and the geometry of the frames are assumed the same as in the previous numerical examples, except for the boundary conditions at the bottom. The external loads are exerted on each column at each story in the frame. The modulus of elasticity for the buckling analysis in the elastic range is adopted to be constant (E). In the inelastic range, tangent modulus aproach is applied. Comparative review of the results for the coefficient β for the clamped and hinged frame is given herein. Results obtained using the code ALIN and Eurocode 3 are presented in Figure 5.



a) b)

Figure 5 Coefficient β for the columns of the frames which are clamped and pinned at the base a) obtained using the program ALIN; b) using the EC3 standard

Regarding the fact that hinged frames have lower stiffness when compared to clamped frames, it is clear that smaller critical load is obtained for them. Thus their effective buckling length coefficient has a higher value for all floors. Also, it is clear that, by applying the standards, the results for analyzed frames are different only at the first floor. Neglection of the stability treatment of the structure as a whole, is just one of the main reasons for the errors that are obtained using the standards.

The six-story three-bay plane frame, clamped at the base, with the load on each column at each story in the frame is also analyzed. In order to make comparison of the results simpler, all data is given in the parametric form. Considered frame has the same characteristics as the frame given in Figure 4a. In this paper are presented the comparative results for the effective buckling length coefficient for the considered sway and non-sway frames (Figure 6).
Since the non-sway frame (considered in this numerical example) has six times higher critical force with respect to the same sway frame (given in Figure 4) these results for the coefficient β are expected.

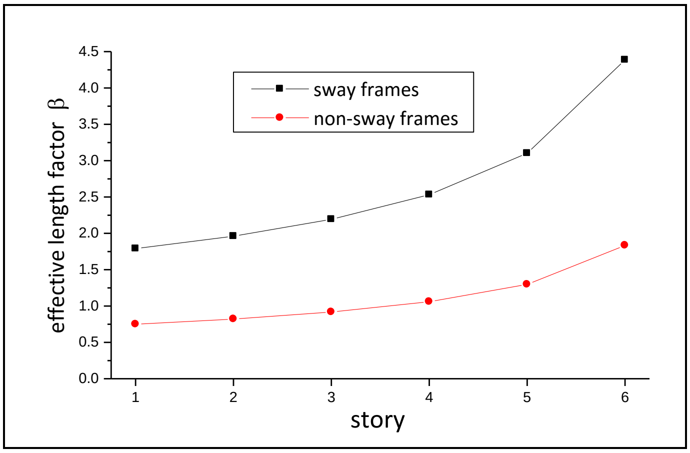


Figure 6 Comparison of the values β for the six-story sway and non-sway frames

4. CONCLUSIONS

In this paper it was investigated the accuracy of the solutions given in JUS and EC3 standards in details, related to the effective buckling length determination. Multi floor sway and non-sway frames were analyzed. The effect of the various parameters to the critical load value, such as different supports, loads, etc. was investigated. Applying the proposed method it was shown that substantial errors can be made when the approximate solutions from the codes are used. That means that there is a need for the innovation of these standards in the part where the effective length of frame columns is considered. It can be done, for example, in the way as it is proposed herein. It should be emphasized that some steps have already been taken in EC3 standards, in the part related to the complex deformable structures which require the calculation according to second order theory, however, no more details are given in EC3.

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ПАРАМЕТАРСКА АНАЛИЗА СТАБИЛНОСТИ ЧЕЛИЧНИХ ОКВИРНИХ НОСАЧА

Резиме: *У овом раду је приказана параметарска анализа стабилности оквирних носача са оценом тачности решења датих у стандардима за прорачун челичних оквирних конструкција. Формулисана је метода за прорачун критичне силе, односно дужине извијања оквирних конструкција, а која се заснива на анализи глобалне стабилности конструкције. Тако срачуната сила омогућује да се даље срачунају и критичне силе, односно дужине извијања појединих притиснутих штапова у конструкцији. Нумеричка анализа у овом раду се заснива на прорачуну критичног оптерећења у еласто-пластичној области.*