GRAĐEVINSKI MATERIJALI U SAVREMENOM GRADITELJSTVU

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MATEMATIČKA INTERPRETACIJA NELINEARNE VEZE NAPONA I DILATACIJA KOD NERĐAJUĆIH ČELIKA

Rezime: *Osnovne specifičnosti nerđajućih čelika ogledaju se u nelinearnoj vezi napona i dilatacija, izraženoj duktilnosti, efektima ojačanja usled hladne deformacije, asimetriji i anizotropiji materijala. Ove osobine određuju drugačije ponašanje konstruktivnih elemenata od ovog materijala u odnosu na ekvivalentne elemente od ugljeničnog čelika. Primena proračunskog koncepta koji se zasniva na idelanom elasto-plastičnom modelu materijala kakav je ugljenični čelik, u slučaju nerđajućeg čelika često daje konzervativne rezultate, što dodatno otežava njegovu nekonkurentnu poziciju u građevinarstvu. Pravilno definisanje preporuka za proračun i njihova implementacija u tehničke propise zahteva preciznu i tačnu matematičku interpretaciju nelinearnosti nerđajućeg čelika. Ovaj rad prikazuje najznačajnije analitičke modele za opisivanje veze napona i dilatacija kod različitih legura nerđajućeg čelika koji su razvijeni poslednjih godina u svetu. Većina ovih modela zasnovana je na izvornom Ramberg-Osgood-ovom analitičkom izrazu.*
Ključne reči: *Nerđajući čelik, materijalna nelinearnost, napon, dilatacija, analitički model materijala.*

MATHEMATICAL INTERPRETATION OF NONLINEAR RELATIONSHIP OF STAINLESS STEEL STRESS AND STRAIN

Abstract: *Basic distinctive characteristics of stainless steels are reflected in the nonlinear relationship of stress and strain, prominent ductility, strain hardening due to cold forming, asymmetry and anisotropy of material. These properties lead to a different behavior of structural elements of this material than the equivalent elements made of carbon steel. Implementation of a design concept based on an ideal elasto-plastic model of material such as carbon steel, often produces conservative results in case of stainless steel, which is already at a disadvantage due to its cost in construction engineering. Proper defining of recommendations for design and their implementation in technical codes requires a precise and accurate mathematical interpretation of nonlinearity of stainless steel. This paper presents the most important analytical models for description of the relationship of stress and strain of various alloys of stainless steel which have lately been developed worldwide. Most of these models are based on the original Ramberg-Osgood analytical expression.*
Key words: *Stainless steel, material nonlinearity, stress, strain, analytical material model.*

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1. INTRODUCTION

Stainless steel is a contemporary civil engineering material whose distinctive features are superior appearance, high resistance to corrosion, nonlinearity and prominent ductility, strain hardening due to cold formation, sustainability of mechanical properties at high temperatures, environmental friendliness and potential of recycling.

During tensile tests, stainless steel exhibits a prominently nonlinear relationship between stress and strain. This property is illustrated by the comparative presentation of σ-ε curves of austenitic stainless steel 1.4301 and carbon steel S275, in Figure 1 [1]. The stainless steel curve indicates gradual yield of material, it is rounded, without clearly prominent yield point and with a small value of stress at the limit of proportionality. The degree of roundness of the curve depends on the type and percentage of alloys present in the stainless steel, heat treatment of material and level of cold working of the finished products. With the increase of strain, the stress increase is considerable. The occurrence of material strain hardening is a consequence of the structural changes of metal during plastic deformation. Due to the abrupt contraction of the specimen section, the further increase of strain is followed by a "short" decrease of stress, up to the onset of failure. The austenitic stainless steels have the most prominent nonlinearity and yield capacity in the family of stainless steels.

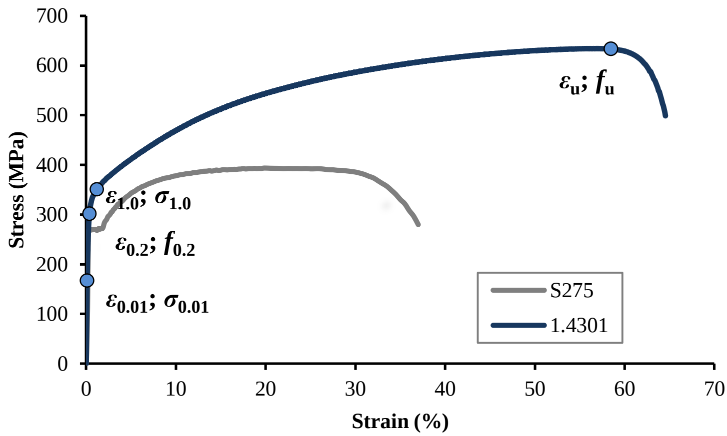

Figure 1. Tensile stress-strain curves for stainless steel grade 1.4301 and carbon steel grade S275

Basic stress-strain parameters describing the material response, presented in Figure 1 [1] and Figure 2 [2] are:

$f_{0.2}$ is the 0.2% proof stress (conventional yield point), value of the stress corresponding to the permanent strain of 0.2%;

$\sigma_{0.01}$ is the value of the stress corresponding of the permanent strain of 0.01%, is marked in literature as proportionality limit σ_p. While in case of carbon and low-alloyed steels, the proportionality limit is no less than 70% of the yield point value, in case of stainless steels, this range is from 36% to 60%. The low value of proportionality limit has negative consequences for the local and global stability of the structural element;

$\sigma_{1.0}$ is the 1.0% proof stress, value of the stress corresponding of the permanent strain of 1.0%;

f_u is the ultimate tensile strength. Load carrying capacity of axially loaded members and members loaded by bending most often depends on the value of yield point or stress at which buckling occurs, and this value is usually lower than the tensile strength. The exceptions are the members and joints of members under tension where local concentration of stress may occur, so the load carrying capacity does not only depend on the yield point but on the tensile strength as well;

E is the modulus of elasticity which represents the gradient of the initial, elastic part of the curve σ-ε in respect to the abscissa;

E_t is the tangent modulus which defines the gradient of the stress-strain curve, that is, the strain of the tangent on the curve for the certain value of stress in the non-elastic region in respect to the abscissa. In the stress region higher than the value of proportionality limit $\sigma_{0.01}$, tangent modulus E_t becomes progressively lower than the modulus of elasticity E. This characteristic reduces the resistance of axially loaded stainless steel members in the domain of medium slenderness.

$E_{0.2}$ is the tangent modulus corresponding to the 0.2% proof stress $f_{0.2}$;

E_s is the secant modulus representing relationship of values of stress and total strain on the curve σ-ε;

n, $n_{0.2,1.0}$, $n_{0.2,u}$ are the strain hardening exponents which define the degree of curve roundness in the corresponding regions of stress.

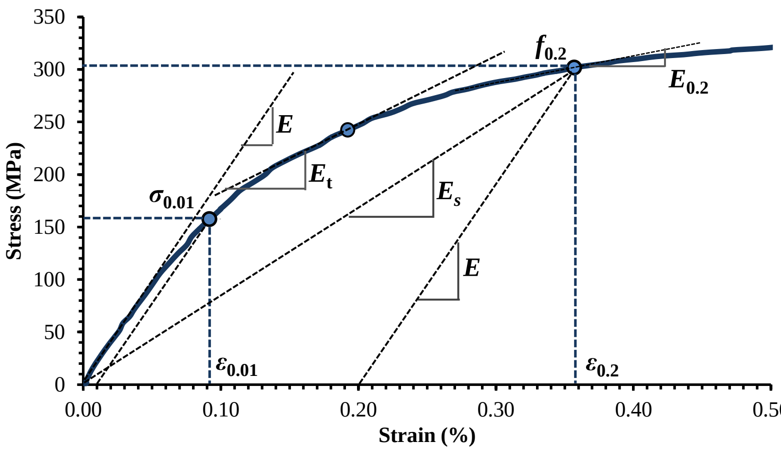

Figure 2. Stress and strain parameters in the initial part of the stress-strain curve

Precise mathematical formulation of nonlinear stress-strain relationship is a basis for analytical description of overall behavior of differently loaded structural elements. The need for the analytical model is particularly prominent in numerical structural analyses, when there are no experimental data about the mechanical properties of material in the entire stress range. In this paper are presented and analysed contemporary material models which with a high accuracy evaluate and describe the real stress-strain curve of various stainless steel alloys.

2. THE PREDICTIVE MATERIAL MODELS

2.1. Ramberg-Osgood model

The first analytical form of the relationship between stress and strain of nonlinear models were defined by Holmquist and Nadai [3] in 1939. especially for the elastic and plastic areas, using a polynomial expression to describe the material behaviour beyond the proportional limit:

$$\varepsilon = \frac{\sigma}{E} \quad \text{for } \sigma \leq \sigma_p \tag{1}$$

$$\varepsilon = \frac{\sigma}{E} + \varepsilon_y \left(\frac{\sigma - \sigma_p}{f_y - \sigma_p} \right)^n \quad \text{for } \sigma > \sigma_p \tag{2}$$

Four years later, while examining mechanical properties of aluminum alloys under compression and tension, Ramberg and Osgood [4] have defined the equation with three parameters in plastic (nonelastic) area, which in a similar way as the previous model, represents the total strain in the element as the sum of elastic and plastic strain:

$$\varepsilon = \frac{\sigma}{E} + K \left(\frac{\sigma}{E} \right)^n \tag{3}$$

where *K* and *n* are the coefficients which are presented by the authors in the function of the secant modulus E_s. By analysing the Ramberg-Osgood analytical model [4], Hill [5] concluded that the values of the coefficients *K* and *n* can be determined in a simpler way in the function of the conventionally determined values of stress obtained at the intersection of the curve with the straight lines parallel to the initial elastic part of the curve. In this way the equation (3) obtains the following form:

$$\varepsilon = \frac{\sigma}{E} + 0.002 \left(\frac{\sigma}{f_{0.2}} \right)^n \tag{4}$$

where *n* is the coefficient of nonlinearity defined as a relationship of conventionally defined stress values corresponding to the permanent plastic strains of 0.2% and 0.1%, respectively.

2.2. Mirambell-Real model

By researching mechanical properties of stainless steels, Mirambell and Real [6] concluded that implementation of Ramberg-Osgood equation (4) produces satisfactory results in the strain area below the 0.2% proof stress $f_{0.2}$, but also that there are significant deviations from the experimental results in the case of high stress values. As a part of own research of deformability of beams loaded to bending, the authors defined a new analytical model for describing the relationship between the stress and strain, Figure 3.

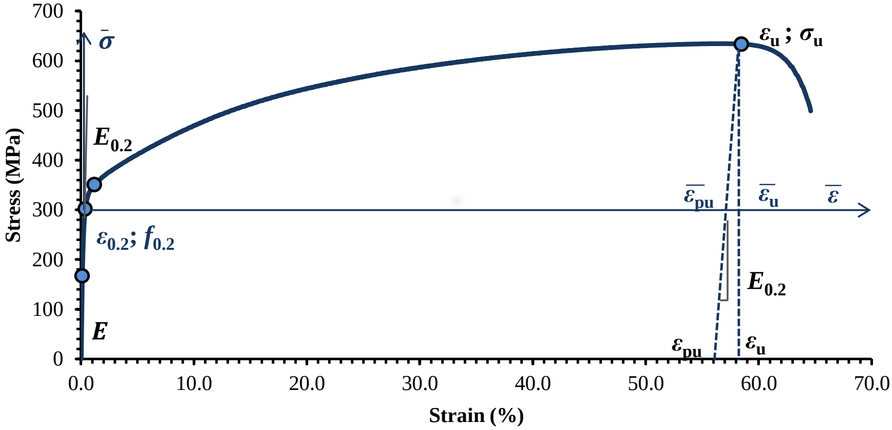

Figure 3. Parameters in Mirambell-Real model [6]

For the stress values lower than the conventional yield point, the authors proposed implementation of Ramberg-Osgood equation (4), where the coefficient of nonlinearity n is determined in the function of the stress values corresponding to the permanent plastic strain in the range between 0.05% and 0.2%. In the stress area above the conventional yield point $f_{0.2}$, the authors modified Ramberg-Osgood equation, analysing σ-ε curve in a new referential coordinate system with the origin in the point ($\varepsilon_{0.2}$, $f_{0.2}$), where $\varepsilon_{0.2}$ is the total strain corresponding to strain $f_{0.2}$:

$$\bar{\varepsilon} = \frac{\bar{\sigma}}{E_{0.2}} + \bar{\varepsilon}_{pu}\left(\frac{\bar{\sigma}}{\bar{f}_u}\right)^{n_{0.2,u}} \qquad \sigma > f_{0.2} \tag{5}$$

and where $\bar{\varepsilon}_{pu}$ is the permanent plastic strain corresponding to stress $\bar{f}_u$ in the new origin:

$$\bar{\varepsilon}_{pu} = \varepsilon_u - \varepsilon_{0.2} - \frac{f_u - f_{0.2}}{E} \tag{6}$$

Stress-strain values in the new origin can be defined in a following way:

$$\bar{\varepsilon} = \varepsilon - \varepsilon_{0.2};\ \bar{\sigma} = \sigma - f_{0.2} \tag{7}$$

Using the mathematical transformations, equation (6) obtains the definite form:

$$\varepsilon = \frac{\sigma - f_{0.2}}{E_{0.2}} + \left(\varepsilon_u - \varepsilon_{0.2} - \frac{f_u - f_{0.2}}{E_{0.2}}\right)\left(\frac{\sigma - f_{0.2}}{f_u - f_{0.2}}\right)^{n_{0.2,u}} + \varepsilon_{0.2} \qquad \sigma > f_{0.2} \tag{8}$$

In the equation (8) the modulus of elasticity of the second range has to be equal to the tangent modulus $E_{0.2}$ that corresponds to the stress $f_{0.2}$, which is determined as the first derivative of the stress function by strain:

$$E_{0.2} = \frac{E}{1 + 0{,}002n\frac{E}{f_{0.2}}} \tag{9}$$

The strain hardening exponent n is valid for the stress range lower than the 0.2% proof stress and the exponent $n_{0.2,u}$ is valid in stress range beyond the 0.2% proof stress.

2.3. Rasmussen model

On the basis of further, extensive research of austenitic, ferrous and duplex stainless steels, Rasmussen [7] proposed a calculation model based on the Mirambell-Real [6] model, which reduced a number of function parameters. For the stress values lower than the conventional yield point, the Ramberg-Osgood equation was valid (4), in which the strain hardening exponent n is determined in the function of the $\sigma_{0.01}$ and $f_{0.2}$ stress value:

$$n = \frac{\ln(20)}{\ln(f_{0.2} / \sigma_{0.01})} \tag{10}$$

Considering the prominent ductility of stainless steel, the author ignores the error in the assumption that the (transformed) permanent plastic strain $\bar{\varepsilon}_{pu}$ which corresponds to the (transformed) ultimate stress $\bar{f}_u$ is equal to the total strain ε_u, and it introduces a simplified expression for this parameter:

$$\bar{\varepsilon}_{pu} = \varepsilon_u \tag{11}$$

With this assumption, equation (9) obtains the following form:

$$\varepsilon = \frac{\sigma - f_{0.2}}{E_{0.2}} + \varepsilon_u \left(\frac{\sigma - f_{0.2}}{f_u - f_{0.2}} \right)^{n_{0.2,u}} + \varepsilon_{0.2} \quad \sigma > f_{0.2} \tag{12}$$

The strain hardening exponent $n_{0.2,u}$ for the non-elastic part of the curve, above the stress value $f_{0.2}$, is determined implementing the following expression:

$$n_{0.2,u} = 1 + 3{,}5\frac{\sigma_{0.2}}{\sigma_u} \tag{13}$$

By using various statistical methods in result analysis, the author proposed implementation of the following equations in determining the relationship of conventional yield point and tensile strength:

$$\frac{f_{0.2}}{f_u} = 0.2 + 185 f_{0.2} / E \quad \text{for austenite and duplex steels} \tag{14}$$

$$\frac{f_{0.2}}{f_u}=\frac{0.2+185f_{0.2}/E}{1-0{,}0375(n-5)} \quad \text{for other alloys} \tag{15}$$

$$\varepsilon_u=1-\frac{f_{0.2}}{f_u} \tag{16}$$

The Rasmussen model [7] describes the relationship between stress and strain with only three parameters: E, $f_{0.2}$ and n, and demonstrates an extremely high level of agreement with the results of experimental curves obtained during tensile tests. This analytical model is included in the Annex C of theexisting Eurocode EN 1993-1-4.

2.4. Gardner-Nethercot model

Through analysis of the Mirambell-Real model [6], Gardner, Nethercot and Ashraf [8], [9] made a conclusion that its use is limited only to the cases of tension. During compressive testes, the parameters f_u and ε_u, are missing, regarding the absence of contraction and section failure. In that sense, the authors modified the equation (8), introducing, instead of the tensile stress f_u and corresponding total strain ε_u, the parameters corresponding to the permanent plastic strain in the amount of 1%:

$$\varepsilon=\frac{\sigma-\sigma_{0.2}}{E_{0.2}}+\left(\varepsilon_{1.0}-\varepsilon_{0.2}-\frac{\sigma_{1.0}-\sigma_{0.2}}{E_{0.2}}\right)\left(\frac{\sigma-\sigma_{0.2}}{\sigma_{1.0}-\sigma_{0.2}}\right)^{n0.2,1.0}+\varepsilon_{0.2} \quad \sigma>\sigma_{0.2} \tag{17}$$

where $n_{0.2,1.0}$ is the coefficient of nonlinearity on the part of the curve ($\varepsilon_{0.2}$, $\sigma_{0.2}$)-($\varepsilon_{1.0}$, $\sigma_{1.0}$).

This approach is supposed to be more convenient because it can also represent the compressive behaviour with a relatively good agreement up to 10% strain. In the all analytical models, the strain hardening exponent n, is determined in the function of the value of stress and corresponding strain in two selected points on the stress-strain curve. This method produces good agreement of experimental and analytical curves in the proximity of the selected points, but deviations occur outside this range, which necessitates an expansion of the measuring points range and an application of regression analysis on as many available test results as possible.

2.5. Abdella explicit stress equation

As opposed to the majority of analytical formulations where nonlinear behavior of material is expressed in the function of the stress, Abdella [10] defined an inverse form of the equation which presentes the stress value in the function of the strain. The formulation is applied to both tension and compression and it is an approximation to the closed form inversion of an existing two-stage stress-strain relation which is based on a modified Ramberg-Osgood equation. In the initial part of the stress below 0.2% proof stress, the author adopted, as an initial parameter, the step function which representes deviation of stress-strain curve from the linearly elastic behavior. By implementing the differentiation method to the function defined in this way, an explicit form of the stress equation in the strain range $\varepsilon \leq \varepsilon_{0.2}$ was developed:

$$\sigma_n = \frac{r\varepsilon_n}{1+(r-1)\varepsilon_n{}^p} \qquad \varepsilon \le \varepsilon_{0.2} \tag{18}$$

In the strain area higher than 0.2%, the author relies on the Rasmussen and Gardner-Nethercot analytical model, respectively. In the new, reference system whose origin is in the point ($\varepsilon_{0.2}$, $f_{0.2}$), the stress-strain curve is also approximated using the step function. By applying the mathematical analogy with the procedures in the initial area of the strain onto the Gardner-Nethercot analytical model, the author defined the following expression which was valid in the strain range $\varepsilon > \varepsilon_{0.2}$:

$$\sigma_n = 1 + \frac{r_2(\varepsilon_n - 1)}{1+(s-1)\left(\dfrac{\varepsilon_n - 1}{\varepsilon_{1n} - 1}\right)^{p_1}} \qquad \varepsilon > \varepsilon_{0.2} \tag{19}$$

In case of the Rasmussen model, the author provided an equation which approximated the nonelastic part of the stress-strain curve in the function of total ultimate strain ε_u:

$$\sigma_n = 1 + \frac{r_2(\varepsilon_n - 1)}{1+(r^*-1)\left(\dfrac{\varepsilon_n - 1}{\varepsilon_{1n} - 1}\right)^{p^*}} \qquad \varepsilon > \varepsilon_{0.2} \tag{20}$$

where σ_n and ε_n are normalized stress-strain values defined by the expressions:

$$(\varepsilon_n, \sigma_n) = (\varepsilon / \varepsilon_{0.2}, \sigma / f_{0.2}) \tag{21}$$

The coefficients r, r_2, r^*, p, p_1, p^* and s are determined implementing the following equations:

$$r = E_0 \varepsilon_{0.2} / \sigma_{0.2}; \; r_2 = \frac{E_{0.2}\varepsilon_{0.2}}{\sigma_{0.2}}; \; r^* = E_{0.2}\frac{\varepsilon_u - \varepsilon_{0.2}}{\sigma_u - \sigma_{0.2}}; \; r_u = E_u \frac{\varepsilon_u - \varepsilon_{0.2}}{\sigma_u - \sigma_{0.2}} \tag{22}$$

$$s = E_{0.2}\frac{\varepsilon_{1.0} - \varepsilon_{0.2}}{\sigma_{1.0} - \sigma_{0.2}}; \; s_1 = E_{1.0}\frac{\varepsilon_{1.0} - \varepsilon_{0.2}}{\sigma_{1.0} - \sigma_{0.2}} \tag{23}$$

$$p = r\frac{1-r_2}{r-1}; \; p_1 = s\frac{1-s_1}{s-1}; \; p^* = r^*\frac{1-r_u}{r^*-1} \tag{24}$$

3. CONCLUSIONS

The use of metallic materials such as stainless steels in modern civil engineering structures brought up the difficulty of their mechanical behavior implementation in design codes. Indeed, the nonlinear stress-strain relationship of such material requires a proper analytical material model to be available. This paper presented the analytical material models which facilitate description of stainless steel behavior under tension and compression in two characteristic stress phases which are determined by the conventional yield point. In case of the widely accepted Rassmusen [7] or Gardner-Nethercot model [9], the strain values are determined in the function of 0.01% proof stress $\sigma_{0.01}$, 0.2% proof stress $f_{0.2}$, 1.0% proof stress $\sigma_{1.0}$, the ultimate tensile stress f_u and nonlinearity coefficient n. These values are obtained as a result of the standard material tensile test and in a general case they are provided in the corresponding standards or in the mill certificates. Some of these models have been implemented in the standards for design of stainless steel structures, which provide more precise and accurate analysis of distinctive features of unequally loaded elements and connections behavior, without a need for preliminary experimental material behavior tests.

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