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ON THE ROLE OF CIRCULAR SECTIONS OF QUADRIC SURFACES

The elaboration of the topic by two creative geometric student's tasks

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ABSTRACT: As a part of new advanced course curriculum of descriptive geometry, in the Faculty of Civil engineering, creative 3D modeling of specific structures is included. These structures, inspired by contemporary architecture achievements, relay on the basic theory of quadric surfaces (also elaborated in the course - Mathematics 2). Throughout two given 3D modeling tasks, students elaborate two separate subtopics: discretization of the surface elliptic hyperboloid of one sheet (ELHY) and spiral surfaces, both connected by key elements – circular sections of quadrics. The subtopic's methodology designed for student's tasks is guided by 3D structures modeling process through applications (the first topic-Auto Lisp app./ the second topic-Visual Basic app.) implemented in engineering software *Auto CAD* and final advanced 3D modeling operations.

Keywords: students geometry task, circular sections of quadric surfaces, polygonal mesh structure, spiral surfaces.

1. INTRODUCTION

The educational necessity of any technical i.e. engineering faculty nowadays concerns 3D modeling of various structures, depending on the field of interest. Various investigations on this subject [2, 9, 10] were carried out. The 3D modeling has an important prior – designing role before final prototyping [2]. The 3D modeling is significantly present when considered a variety of available parametric modelers: Rinoceros 3D, 3Dmax, Solid Works, Catia and many others. The inconvenience of such software solutions, when issue of Descriptive Geometry is of the matter [10], is that real understanding, i.e. recognizing of the geometric procedures is reduced, almost neglected, in order to obtain the final "product".

The 2nd year curriculum of the basic studies on the Civil Engineering Faculty enables the knowledge earning and linking, concerning several areas: descriptive geometry, algebraic geometry, computer drawings (graphics) and computer programming. Regarding several reasons, especially to get closer the theory and engineering practice [8], the advanced descriptive geometry course, in the Faculty of Civil Engineering in Belgrade, implemented the topics which elaborate challenging 3D modeling tasks. This initiative should be encouraging for the students in their individual modeling design of further scholar projects (buildings, bridges, or other civil engineering structures). Some extraordinary examples from engineering practice make this topic worth of attention (Fig.1).

Fig.1 Spiral bridge, Singapore [12]

In the course, the modeler is Auto CAD software, commonly used in engineering practice, in the role of teaching instrument, as well as a very precise and efficient tool. Since this is the first students' experience with modeling, significant technical support is provided by two applications written in Auto-Lisp and Visual Basic programming languages.

In order to be actual in design, the chosen topics concern the quadric surfaces (simultaneously elaborated in students' course Mathematics 2) and as an extension – the spiral surfaces. The prior descriptive geometry approach emphasizes the idea of better spatial understanding of the quadric surfaces' characteristics along with spiral structures, commonly present in the form of algebraic (the quadrics) and parametric (the spiral surfaces) equations. Both are supported with computer applications aimed for modeling process where the final results are creative 3D structures

2. **TWO GEOMETRIC TOPICS**

The first creative theme is *discretization of ELHY surface*, i.e. generation of polygonal mesh structure, offering three solutions: from simplest – triangular, quadrilateral, up to the concave – hexagonal "ornaments" [9]. This topic has a rather simple explanation and graphical presentation (wire-frame and surface models) of geometrical procedures, i.e. constructions, in opposite to some complex analytical geometry solutions aimed for the parametric modeling [5, 6].

The second topic concerns various solutions of *spiral surfaces* deriving from geometric origins - quadric surfaces: cone, sphere, ellipsoid, elliptic hyperboloid and paraboloid. The regular polygons, inscribed in circular sections of the quadrics in spiral "motion" obtain the spiral surface/structure model. Two types of structures are offered, depending on the line of centers of inscribed polygons.

Both topics are elaborated through student's tasks, where the final result is 3D Auto CAD model (prototype) of the structure with specified (practical) aim.

2.1 Task 1 - ELHY surface discretization

The method of generating the ELHY's wire-frame model relies on settings of two arbitrary circular sections (center points - C and C_1) in two parallel horizontal planes and one generatrix *AB* (Fig.2). Detail elaboration of the procedure is given in previous author's research, with accompanying basic CAD application [3, 4]. In accordance to intention of enabling efficient tool for discretization of ELHY shaped structure, the extensions of basic application were created for the purposes of the student's practicing. The basic application generates ELHY's net (two series) of generatrices and stores entity handles in the external file, that is used by the application extension. The extension offer three different discretization solutions-three polygonal meshes: triangular, quadrilateral and hexagonal, with planar elements.

The *first option*, the simplest one, concerns triangulated ELHY's mesh obtained by circular sections and the net of generatrices (Fig. 3) The ELHY surface is divided on horizontal "rings" by circular sections. The intersection points of generatrices – nodes are connected into planar triangles, varying in dimensions. All the triangle's edges contained in separate "ring" are equal. Their dimensions depend on the number of generatrices and the radius of the circular section as well.

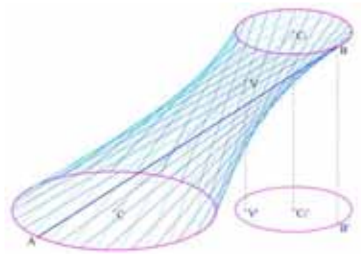

Figure 2: Wire-frame model of ELHY

Figure 3: Triangular ELHY's mesh

The modeling task for mesh prototype is supported by basic Auto Lisp application - wireframe model of generatrices and circular sections accompanied with surface model.

The *second option* relies on the wireframe model also. In this case circular sections are obtained by the coplanar midpoints of neighbor nodes on each generatrix. The ELHY surface is divided on horizontal strips by circular sections. In each stripe trapezoids and parallelograms alternate each other. In fact, a tangential plane of each node is bounded by trapezoid; between trapezoids a sequence of variable parallelograms lays. (Figs. 4, 4a)

The application for generation of quadrilateral ELHY's mesh uses nodes from triangular mesh to calculate circular sections which contain sides of trapezoids and parallelograms. When all nodes for mesh generation are calculated, application rearranges them in specific order to provide automation of generation process. Application offers the possibility to choose different layer (color) for parallelograms, so they can be easily discerned in the mesh.

The *third option* offers hexagonal (concave) solution of ELHY's mesh. The specific disposition of nodes is set, obtaining one structural cell, which is multiplied. One central node is surrounded by 6 neighbor nodes (convex hexagon disposition) incorporated in the predefined basic ELHY's net of generatrices. Here, 7 tangential planes are bounded by 7 concave hexagons. (Figs. 5, 5a)

Each hexagon stretches through two horizontal ELHY's "rings". Intersection lines – the edges of hexagon are imagery diagonals stretching through three "rings" while connecting basic nodes. Besides, the geometric regularity of alternation of tangent line and secant, passing through the coplanar nodes (contained in separate circular sections) is utilized for practical Auto Lisp application.

The application for generation of hexagonal ELHY's mesh "walks" through each circular section and, using the above mentioned principles, calculates nodes for the mesh generation.

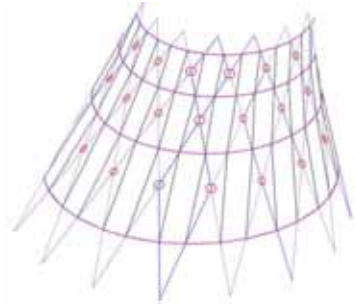

Fig. 4 Detail of quadrilateral ELHY's mesh

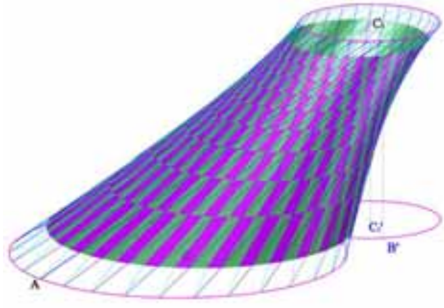

Figure 4a: Quadrilateral ELHY's mesh

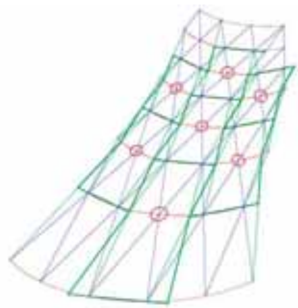

Figure 5: Detail of hexagonal ELHY's mesh

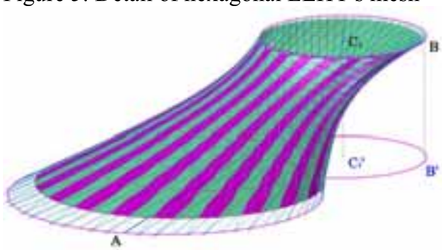

Figure 5a: Hexagonal ELHY's mesh

This application also rearranges nodes in specific order to provide automation of mesh generation process.

2.2 Task 2 - Spiral structure

Spiral surfaces of the specific type, with geometric origins: cone, sphere, ellipsoid, elliptic hyperboloid and paraboloid are chosen "set" for the modeling process in Auto CAD software. Each surface has two series of circular sections, in this case used as a pattern for inscribed regular polygons – parallel sections of the final surface's model.

The *first solution* for the spiral surface wireframe model considered the option that the line of centers for rotated polygons coincides with the line of centers of the origin surface's circular sections. The designer's choice of parameters is only relevant for the resulting spiral surface model. (Fig. 6)

The *second solution* varied the line of centers for rotated polygons. Geometric relation of the first and the last position of the basic polygon, i.e. geometric concept, is predefined in application by adding a new center of rotation in the polygon's plane. Hence, depending on geometric concept, line of centers may be the "other" straight line (origin surface - cone), or a curve (origin surface-sphere (Fig. 7), ellipsoid, ELHY, etc.).

Technical supportive "tool" for geometric construction of the wireframe surface model is created for each type of the origin surface, i.e. each surface has its own VBA subroutine, which has to be uploaded as a separate application and run from the command line. The "user" – a student, after the task introduction and theoretical explanations, is guided through the settings of the initial parameters of the primary surface (e.g. values for three axes of the ellipsoid) with circular sections, and afterwards, the spiral surface (the polygon type and fill rotation angle, i.e. rotation "step" angle). The application routine is presented in Fig. 8.

Precondition for the application execution is VBA module installation. It has to be adequate for Auto CAD software's version, as well as for operative system's architecture.

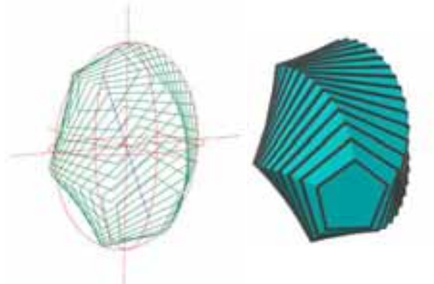

Fig.6 Wireframe and solid model of spiral structure E5 (ellipsoid/inscribed pentagons)

Fig. 7 Wireframe and solid model of spiral structure S4a (sphere/inscribed squares)

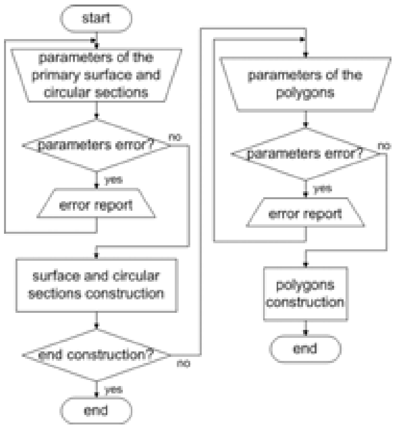

Figure 8: Application algorithm

No additional settings are needed for running the application inside drawing document. All constructive elements are set in separate layers. That makes easier further modeling procedure.

The procedure contains dialog boxes with textual descriptions of entry parameters and their ranges. If entered parameters out of the defined range, the user gets error response being returned to the entry (Fig. 9).

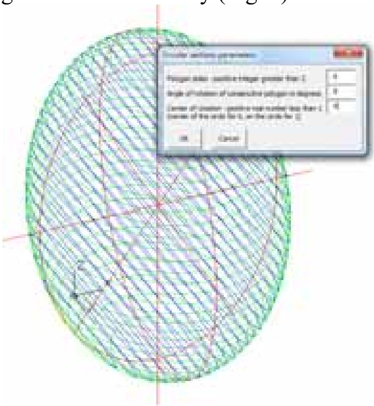

Figure 9: Dialog box for entering the spiral surface's parameters

Further modeling process includes utilization of advanced Auto CAD's modeling tools (e.g. "loft" command for twisted solid structure, or "sweep" command for the frames extrusion, Boolean operations, etc.)

3. DIDACTICAL PRINCIPLES

Geometrical principles presented in the first student task elaborate the interesting case of the ruled surface containing two series of straight lines – generatrices (rulings), where the solutions of discretization can be carried out by simple intersections of tangent planes, defined by nods in various dispositions. Thus, geometric solution is "visible", i.e. obvious, and can be manually drawn directly in 3D computer surroundings, on the basis of the wire-frame surface (ELHY's) model. This is an important issue when the students' spatial abilities are of the matter. The final "product" in this task should be a complex structure obtained by combinations of shapes (identical or various, multiplied, cut, etc.), where spatial imagination and creation have an important role. Thereby, a student should present his modeling skills, by proper utilization of Auto CAD's modeling tools.

Manual geometric construction of mesh structure unfortunately requires a lot of patience and time, and hence, for this purposes it is not convenient and practical. However, the possibility of making the "paper" surface model exists, regarding developable "rings", or "strips" on ELHY surface. Students' efforts in direction of exploring the possibilities in exercising with real (exact) models, recommended by P. H. Meier [11], should be stimulated and additionally estimated. In this case, the advanced task relies on the possibility of creating a new discretization pattern, i.e. engagement of new geometrical ornaments (regarding disposition of nods) which is offered optionally for excellent students.

The didactical reasons, regarding understanding of descriptive geometric principles in solving of the second presented task, imply explanation of complete constructive and modeling procedure including the detail explanation of geometric procedure for obtaining the circular cross sections of 2^{nd} order surfaces. Regarding practical reasons, such as minimization of manual drawing of circular sections, inscribed polygons, with numerous repetitions of necessary operations (rotation of the separate objects, UCS positioning, etc.) in Auto CAD's environment procedures, the design of appropriate VBA application is created.

The intention of such approach was to provide the continuity between spatial understanding of geometric procedures, analytical expressions of the geometric problem, computer programming, and finally, model designing and visualization. This concept affected the programming of the applications. The possibility to "stop" the "algorithm" is enabled, during separate parts explanations of the creative procedure. After each part of the construction the dialog box contains necessary details, i.e. coordinates and supportive tutorials.

The main reasons for the VBA programming language (Fig. 10) choice, in order to implement the application, are its possibility of running on various software versions and user (student) acquaintance with object-oriented programming, which enable better understanding and modification of the application.

Figure 10: The part of application for construction and calculation of parameters for the circular sections

4. CONCLUSIONS

Contemporary possibilities concerning 3D modeling of geometric structures are the matter of various investigations, varying from elementary school level, to complex engineering practice purposes. Here presented research offers specific descriptive geometry approach to the specific modeling topics, technically supported with computer applications in engineering software Auto CAD, aimed for the technically orientated students. It is included in the curriculum of advanced course on descriptive geometry for second year students in the Faculty of Civil Engineering in Belgrade. Two attractive topics both connected with key element – circular sections of the quadric surfaces: discretization of the ruled surface –ELHY and spiral surface modeling are elaborated regarding descriptive geometry principles and implemented in the whole of studies' curriculum. The contribution concerning automation of the geometric procedures for both topics is independent with respect to existing mathematically based software packages. Although the programming procedure requires analytical expressions, they strictly relay on the constructive geometric concept.

The expectations, within results of implementation of such challenging tasks in education process, are concerning of obtaining the continuity of various knowledge acquisitions towards practical applications in the future engineering practice.

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