PRIMENA MODIFIKOVANOG RASPLINUTOG TOPSIS METODA ZA VIŠEKRITERIJUMSKE ODLUKE U GRAĐEVINARSTVU

APPLICATION OF MODIFIED FUZZY TOPSIS METHOD FOR MULTICRITERIA DECISIONS IN CIVIL ENGINEERING

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1 UVOD

TOPSIS metod (*Technique for Order Preference by Similarity to Ideal Solution* – Tehnika za redosled prioriteta prema sličnosti sa idealnim rešenjem) za rešavanje višekriterijumskih problema (MCDMP) sa više alternativa predložili su i razvili Hwang i Yoon [7] 1981. godine.

Metod je baziran na činjenici da izabrana ili najbolja alternativa treba da ima najkraće rastojanje od pozitivnog idealnog rešenja (PIS) i najduže rastojanje od negativnog idealnog rešenja (NIS). Pozitivno idealno rešenje maksimizuje kriterijume koji se odnose na korisnosti, a minimizuje kriterijume koji se odnose na troškove ili gubitke. Negativno idealno rešenje minimizuje kriterijume koji se odnose na korisnosti, a maksimizuje kriterijume koji se odnose na troškove. Izabrana alternativa ima maksimalnu sličnost (bliskost) sa PIS i minimalnu sličnost (bliskost) sa NIS.

Chen i Hwang [3] su ovaj metod sa fiksnim (nerasplinutim) podacima transformisali u metod s rasplinutim podacima. U poslednjih više od trideset godina, mnogi autori učestvovali su u razvoju ovog metoda i predložili brojne modifikacije. Metod se često uspešno koristio kao pomoć donosiocima odluka za rešavanje mnogih praktičnih problema u različitim oblastima primene. Opricović [12] je predložio i razvio metod nazvan VIKOR, za višekriterijumsku optimizaciju složenih sistema. Ovaj metod koristi se za rangiranje i izbor alternativa u slučaju postojanja konfliktnih kriterijuma. On

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1 INTRODUCTION

TOPSIS method (*Technique for Order Preference by Similarity to Ideal Solution*) for solving multiple criteria decision problem (MCDMP) with several alternatives was proposed and developed by Hwang and Yoon [7] 1981.

The method is based on the fact that the chosen or most appropriate alternative should have the shortest distance from positive ideal solution (PIS) and the longest distance from negative ideal (anti ideal) solution (NIS). Positive ideal solution maximizes the criteria that are related to the benefits and minimizes the criteria that are related to the costs or losses. The negative ideal solution minimizes the criteria that are related to the benefits and maximizes the criteria that are related to the costs and losses. The chosen alternative has the maximum similarity (closeness) with PIS and minimum similarity (closeness) with NIS.

Chen and Hwang [3] have transformed this method with the crisp (nonfuzzy) data to the method with the fuzzy data. In more than last thirty years a lot of authors participated in development of this method and proposed numerous modifications. The method was applied successfully in the practice as a help to decision makers for solving many problems in different fields of application. Opricović [12] has proposed and developed a method, named VIKOR, or multiple criteria optimization of complex systems. This method focuses on ranking and selecting alternatives in the presence of conflicting

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je uveo indeks višekriterijumskog rangiranja, koji se određuje na osnovu blisjosti idealnom rešenju.

Opricović i Tzeng [13] upoređivali su osnovne karakteristike VIKOR i TOPSIS metoda u svim koracima rešavanja problema: proceduralna baza, normalizacija, agregacija i konačno rešenje. Kasnije je Opricović proširio VIKOR metod za rešavanje rasplinutih višekriterijumskih problema s konfliktnim i nekonfliktnim kriterijumima i razvio metod VIKOR-F [14]. Metod VIKOR više puta je korišćen za višekriterijumsko rangiranje alternativa prilikom rešavanja mnogih problema u građevinarstvu, hidrotehnici i saobraćaju, kao i u drugim inženjerskim oblastima. Pored ove dve metode, u literaturi postoji još metoda za višekriterijumsko donošenje odluka (AHP, PROMETHEE, ELECTRE i dr.).

Wang i Elhang [17] predložili su fuzzy TOPSIS metod, zasnovan na alfa presecima rasplinutih skupova za rešavanje problema upravljanja rizikom kod mostova. Za svaku alternativu i izabrani alfa nivo, definisali su nelinearni program (NLP) s gornjom i donjom vrednošću relativne bliskosti od NIS kao funkcijama cilja i u već definisanim gornjim i donjim vrednostima kao ograničenjima. Na taj način, relativne bliskosti posmatrane su kao rasplinuti brojevi, a kasnije, posle defazifikacije, rangirane su alternative. U stranoj I domaćoj literaturi postoje brojni primeri primene TOPSIS metode s fiksnim i rasplinutim brojevima u svim područjima građevinarstva i realizacije investicionih projekata za rangiranje alternativa ili subjekata imajući u vidu propisane kriterijume. Kraći prikaz jednog broja tih radova dali su autori u njihovom ranijem radu [16].

Procena rizika objekata (mostova, zgrada i ostalih objekata) najčešće se koristi za određivanje optimalnog plana ili rangiranja održavanja objekata u odnosu na rizik. Ovaj problem proučavali su mnogi autori, a u literaturi postoje različiti metodi procene rizika. Na primer, Adey, Hajdin i Brühwiler [1] predstavili su pristup određivanju optimalnog plana održavanja mostova, baziranog na rizicima koje je izazvalo više hazarda. Wang i Ehlang [18] predložili su pristup grupnom donošenju odluka za procenu rizika, koristeći rasplinuti TOPSIS metod.

Mnogi radovi u kojima je reč o oceni stanja, održavanju i sanaciji građevinskih objekata i naselja publikovani su u zbornicima radova s međunarodnih konferencija, čiji je editor Folić [9], [10].

U ovom radu razmatra se problem višekriterijumskog rangiranja objekata za rekonstrukciju na osnovu definisanih kriterijuma, korišćenjem modifikovane rasplinute TOPSIS procedure koju su predložili autori, a koja je detaljno prikazana u radu [16]. U ovom metodu, svi ulazni podaci predstavljeni su kao trougaoni rasplinuti brojevi. Za ove brojeve i njihove proizvode određene su generalisane očekivane vrednosti, varijanse, standardne devijacije i koeficijenti varijacija. Ove vrednosti, dalje, korišćene su u matematičkim formulama za određivanje relativnih rastojanja svake alternative do PIS i NIS za njihovo rangiranje. Predložena procedura je opštija od procedure u kojoj se koriste fiksni brojevi i donosiocu odluka pruža realnije podatke za donošenje najprihvatljivije odluke.

criteria. He introduced the multiple criteria ranking index based on the particular measure of closeness to the ideal solution.

Opricović and Tzeng [13] compared main features of VIKOR and TOPSIS metods in all steps of problem solution: procedural basis, normalization, aggregation and final solution. Opricović later extended VIKOR method for solving fuzzy multiple criteria problems with conflicting and non conflicting criteria and developed VIKOR-F [14] . VIKOR method has been used many times for multiple criteria ranking of alternatives for solving many problems in civil, hydro technical and transportation engineering and other branches of practice as well. Besides these two methods, there are more methods for multiple criteria decision making (AHP, PROMETHEE, ELECTRE, etc.) in the literature considering this field of research.

Wang and Elhang [17] proposed fuzzy TOPSIS method based on alfa level sets with application to the bridge risk management. For every alternative and chosen alfa level, they formulated nonlinear programs (NLP) with lower and upper value of relative closeness to NIS as the objective functions and with prescribed lower and upper values as the constrains. In such a way these relative closeness are considered as fuzzy numbers, and then after defuzzification, the alternatives are ranked according to these closeness. In the foreign and domestic literature there is large number of examples of application of the TOPSIS method in all area of civil engineering and construction project realization for ranking alternatives or subjects related to the prescribed criteria. Short review of these works is presented in the author's work [16].

The risk assessment of an object (bridge, building, etc) is usually performed to determine the optimal scheme or rank order of the object maintenance. This problem has been investigated by numerous authors and there are different methods for the risk assessment. For instance, Adey, Hajdin and Brühwiler [1] presented risk-based approach to the determination of optimal interventions for bridges affected by multiple hazards. Wang and Ehlang [18] proposed a fuzzy group decision making approach for the risk assessment using fuzzy TOPSIS method.

Numerous papers related to the assessment, maintenance and rebuilding of structures and settlements are given in the proceedings of international conferences, edited by Folić [9],[10].

This paper deals with a problem of multiple criteria ranking of objects for reconstruction against prescribed criteria using modified fuzzy TOPSIS procedure proposed by authors in the paper [16]. In this method all input data are presented as probabilistic triangular fuzzy numbers. Generalized expected values, variances, standard deviations and coefficients of variations are found for these fuzzy numbers and their products. These values are, further, used in mathematical formulas to determine relative closeness of every alternative to the PIS and NIS for their ranking. This proposed procedure is more general than the procedure based on crisp data and gives to the decision maker more realistic data to make the most acceptable decision.

2 DEFINICIJA PROBLEMA

U ovom radu razmatra se neka firma ili institucija (vlasnik) koja je odgovorna za održavanje *n* objekata (zgrada, mostova i drugih) koji su označeni sa $A_1, A_2, \ldots, A_m$. Da bi se smanjile posledice rizika koje utiču na sigurnost, funkcionalnost, održivost, raspoloživost, uticaje okoline i druge bitne faktore, neophodno je uložiti određenu količinu novca za održavanje tih objekata. Raspoloživa količina novca obično nije dovoljna za održavanje svih objekata, pa je zbog toga neophodno objekte rangirati prema nivou rizika, te novac uložiti u objekte shodno listi rangiranja. Pomenuti faktori, koji se zovu *kriterijumi*, označeni su sa $C_1, C_2, \ldots, C_n$, dok objekti predstavljaju *alternative* višekriterijumskog odlučivanja (MCDM). Svaku alternativu A_i u odnosu na kriterijum C_i numerički su ocenili eksperti vrednošću f_{ij} $(i = 1,2,...,m;\ j = 1,2,...,n)$. Ove vrednosti jesu elementi *matrice odlučivanja*, koja je označena sa $\mathbf{F} = [f_{ij}]_{m\times n}$.

Skup kriterijuma Ω sadrži dva disjunktna skupa Ω_b i Ω_c, tj.

2 DEFINITION OF THE PROBLEM

A firm or institution (owner) which is responsible for the maintenance of *n* objects (buildings, bridges or other objects) A_1, A_2, A_m is considered in this paper. To reduce consequences of a risk that influence safety, functionality, sustainability, availability, environmental and other important factors, a corresponding amount of money should be invested in the maintenance of these objects. The available amount of money usually is insufficient for all objects or projects, so that they should be ranked according to the risk rating, and the money should be invested in the objects according to this rank list. The mentioned factors are named as *criteria* denoted by $C_1, C_2, \ldots, C_n$, while the objects represent *alternatives* for multi-criteria decision making (MCDM). Each alternative A_i is numerically evaluated by experts with respect to the criterion C_j by values f_{ij} $(i = 1,2,...,m;\ j = 1,2,...,n)$. These values are elements of the decision matrix denoted by $\mathbf{F} = [f_{ij}]_{m\times n}$

The set of criteria Ω contains two disjunctive subsets Ω_b and Ω_c, i.e

$$\Omega = (C_1, C_2, ..., C_n) = (\Omega_b \cup \Omega_c),\ (\Omega_c \cap \Omega_b) = \emptyset. \tag{1}$$

Podskup Ω_b predstavlja *dobiti* ili *kriterijume s povoljnim efektima* koje treba maksimizovati, dok podskup kriterijuma Ω_c predstavlja *troškove* ili *kriterijume* s nepovoljnim efektima koje treba minimizovati.

Svaki kriterijum C_j eksperti ocenjuju *relativnom težinom* ili *faktorom značajnosti* w_j $(j = 1,2,\ldots,n)$. Ove vrednosti formiraju *vektor težina* $\mathbf{w} = [w_j]_{1\times n}$. Cilj rešavanja problema jeste da se odredi najprihvatljivija ili najbolja alternativa – A_c koja zadovoljava sve kriterijume i koja je najbliža *pozitivnom idelanom rešenju*, a najudaljenija od *negativnog idealnog rešenja*, kao i da se alternative rangiraju prema navedenom pravilu.

Idealno pozitivno rešenje F^* sadrži vrednosti f_{ij} koje predstavljaju maksimume kriterijuma dobiti i minimume kriterijuma troškova to jest

The subset of criteria Ω_b represents *benefits* or *criteria with favourable effects* that should be maximised, while subset of criteria Ω_c represents *costs* or *criteria with unfavourable effects* that should be minimized in the procedure.

Every criterion C_j is assessed by experts with relative *weight values* or *factors of importance* w_j $(j = 1,2,\ldots,n)$. These values form the *vector of weights* $\mathbf{w} = [w_j]_{1\times n}$. The goal of the problem solution is to find the most preferable or the best (compromise) alternative A_c that satisfies all criteria together and which is closest to the *positive ideal solution* and farthest to the *negative ideal solution*, and rank alternatives according to this rule as well.

The positive ideal solution F^* contains the values f_{ij} that are maximal for the benefit criteria and minimal for the cost criteria, i.e.

$$F^* = \{f_1^*, .. f_i^* ... f_n^*\} = \{(\max_j f_{ij},\ i \in \Omega_b), (\min_j f_{ij}, i \in \Omega_c)\}. \tag{2}$$

Idealno negativno rešenje F^- sadrži vrednosti f_{ij} koje odgovaraju minimumima kriterijuma dobiti i maksimumima kriterijuma troškova to jest

The ideal negative solution F^- contains values f_{ij} that are minimal for the benefit criteria and maximal for the cost criteria, i.e.

$$F^- = \{f_1^-, .. f_i^- ... f_n^-\} = \{(\min_j f_{ij},\ i \in \Omega_b), (\max_j f_{ij}, i \in \Omega_c)\}. \tag{3}$$

3 TOPSIS PROCEDURA SA FIKSNIM BROJEVIMA

Ako su elementi matrice odlučivanja f_{ij} $(i = 1,2,...,m;\ j = 1,2,...,n)$ i koeficijenti značajnosti ili težine kriterijuma w_j $(j = 1,2,\ldots,n)$, fiksni ili nerasplinuti brojevi, onda se primenjuje TOPSIS procedura s fiksnim brojevima, koja se izvršava u sledećim koracima.

1. *Normalizacija*. Pošto kriterijumi mogu imati različita značenja i različitu prirodu, elementi matrice odlučivanja izražavaju se različitim dimenzionalnim merama i skalama

3 TOPSIS PROCEDURE WITH CRISP NUMBERS

If elements of the decision matrix f_{ij} $(i = 1,2,...,m;\ j = 1,2,...,n)$ and coefficients of importance or weights of criteria w_j $(j = 1,2,\ldots,n)$, are crisp or non fuzzy numbers, then the TOPSIS procedure with crisp numbers is applied, which performs in the next steps.

1. *Normalization*. Since the criteria may have different meanings and nature, then elements of the decision matrix are expressed by different dimensional

vrednosti. Stoga, treba izvršiti normalizaciju elemenata matrice odlučivanja **F**. U literaturi postoji više predloga za normalizaciju, a ovde će biti prikazana dva koja se najčešće primenjuju.

Prema prvom postupku, za svaki kriterijum C_j ($j = 1,2,\ldots,m$) odredi se maksimalna vrednost

measures and scales of values. Because of that, the normalization of elements of the decision matrix **F** should be performed to obtain dimensionless values. There are several proposals for this normalization in literature and here will be presented two of them that are most frequently applied.

According to the first proposal for every criterion C_j ($j = 1,2,\ldots,m$) maximal value is determined

$$f_j^* = \max_i f_{ij} \quad (i = 1,2,...,m; j = 1,2,...,n) \tag{4}$$

i s tom vrednošću podele sve vrednosti f_{ij} u koloni C_j matrice **F**. Na taj način, dobijaju se normalizovane i bezdimenzionalne vrednosti a_{ij} koje sačinjavaju normalizovanu matricu odlučivanja $\mathbf{A} = [a_{ij}]$

and with this value all values f_{ij} in the column C_j of the matrix **F** are divided. Thus, normalized and non dimensional values a_{ij} that compose normalized decision matrix $\mathbf{A} = [a_{ij}]$ are obtained

$$a_{ij} = f_{ij} / f_j^*, \quad (i = 1,2,...,m; j = 1,2,...,n). \tag{5}$$

Drugi postupak naziva se *vektorski postupak* i u njemu se za svaki kriterijum nađu dužine odgovarajućih vektora po formuli

The second proposal is named a vector procedure in which lengths of corresponding vectors are determined for every criterion

$$f_j^* = \sqrt{f_{1j}^2 + f_{2j}^2 + ... + f_{mj}^2}, \quad (j = 1,2,...,n). \tag{6}$$

Kao i u prethodnom slučaju, sa ovim vrednostima dele se elementi matrice odlučivanja **F** i tako dobijaju elementi normalizovane matrice **A**.

2. *Određivanje težinske matrice* **C.** Svaki element normalizovane matrice **A** množi se sa odgovarajućim težinskim koeficijentom ili koeficijentom značajnosti kriterijuma w_j i tako se dobiju elementi c_{ij} težinske matrice $\mathbf{C} = [c_{ij}]$

As in the previous case, elements of the decision matrix **F** are divided by these values and obtained elements a_{ij} of the normalized decision matrix **A**.

2. *Determination of the weighted matrix* **C.** Every element of the normalized matrix **A** is multiplied by the corresponding *weighted coefficient* or *coefficient of significance* w_j to obtain elements c_{ij} of the weighted matrix $\mathbf{C} = [c_{ij}]$

$$c_{ij} = a_{ij} w_j, \quad (i = 1,2,...,m : j = 1,2,...,n). \tag{7}$$

3. *Određivanje pozitivnog idealnog rešenja (PIS) i negativnog idealnog rešenja (NIS)*

Za svaku alternativu A_i određuju se komponente c_i^* pozitivnog idealnog rešenja i c_j^- negativnog idealnog rešenja prema sledećim formulama

3. *Determination of the positive ideal solution (PIS) and negative ideal solution (NIS)*

For every alternative A_i are determined components c_i^* of positive ideal solution and components c_j^- of negative ideal solution according to the next formulas

$$c_i^* = \max_j c_{ij}: C_j \in \Omega_b \text{ or } c_j^- = \min_j c_{ij} : C_j \in \Omega_c; \; i = 1,2,...,m; \; j = 1,2,...,n. \tag{8}$$

$$c_i^* = \min_j c_{ij}: C_j \in \Omega_b \text{ or } c_j^- = \max_j c_{ij} : C_j \in \Omega_c; \; i = 1,2,...,m; \; j = 1,2,...,n. \tag{9}$$

4. *Određivanje udaljenosti i relativnih bliskostii alternativa A_i pozitivnom idealnom (PIS) i negativnom idealnom rešenua (NIS)*

Za svaku alternativu A_i određuje se distanca od pozitivnog idealnog rešenja D_i^* i negativnog idealnog rešenja D_i^{*-} prema sledećim formulama

4. *Determination of distances and relative closeness of the alternatives A_i to positive ideal solution (PIS) and negative ideal solution. (NIS)*

For every alternative A_i the distances D_i^* from the positive ideal solution and D_i^{*-} from the negative ideal solution are determined by the following formulas

$$D_i^* = \left[\sum_{j=1}^{m} (c_{ij} - c_i^*)^2 \right]^{1/2}; \quad D_i^- = \left[\sum_{j=1}^{m} (c_{ij} - c_i^-)^2 \right]^{1/2}; \; i = 1,2,...,m; j = 1,2,...,n; \tag{10}$$

i relativne bliskosti RC_i^* pozitivnom idealnom rešenju i RD_i^- i negativnom idealnom rešenju

and relative closeness RC_i^* to positive ideal solution and RC_i and to negative ideal solution

$$RC_i^* = D_i^* /(D_i^* + D_i), \quad RC_i = D_i /(D_i^* + D_i); \; i = 1,2,...,m; \; j = 1,2,...,n. \tag{11}$$

$$RC_i^* + RC_i = 1. \tag{12}$$

Ove relativne bliskosti nazivaju se još i *koeficijenti bliskosti* alterantive A_i pozitivnom idealnom rešenju i negativnom idealnom rešenju.

Alternative se rangiraju prema ovim koeficijentima. Alternative s manjom relativnom bliskosšću RC_i^* pozitivnom idealnom rešenju, i većim relativnom bliskosšću RC_i^- negativnom idealnom rešenju, bolje su rangirane. Najbolje rangirana alternativa jeste ona koja ima najmanji koeficijent bliskosti idealnom pozitivnom rešenju RC_i^*.

The relative closeness are named *coefficients of closeness* of the alternative A_i to the positive ideal solution and negative ideal solution respectively. The alternatives are ranked according to these coefficients. Alternatives with the smaller relative closeness RC_i^* to the positive ideal solution and greater relative closeness RC_i^- to the negative ideal solution are better ranked. The best ranked alternative has the smallest coefficient RC_i^*.

3.1 Primer

Radi lakšeg razumevanja ove procedure, razmotriće se jedan jednostavan primer. Neka postoje tri alternative A_1, A_2 i A_3 i dva kriterijuma C_1 i C_2, i neka se kriterijum C_1 odnosi na trošak, a kriterijum C_2 na korisnost (dobit), tako da su skupovi kriterijuma

3.1 Example

For the easiest understanding of this procedure, one simple example will be considered. Let exists three alternatives A_1, A_2, A_3 and two criteria C_1, C_2, and let the criterion C_1 is related to the cost, and criterion C_2 to the benefit (profit), so that the sets of criteria are

$$\Omega_b = C_2 , \; \Omega_c = C_1, (\Omega_c \cap \Omega_b) = \varnothing.$$

Neka su procenjene vrednosti matrice odlučivanja

Let assess values of the decision matrix

$$\begin{matrix} & C_1 & C_2 & \\ \mathbf{F} = & \begin{bmatrix} 1.00 & 2.00 \\ 3.00 & 1.00 \\ 4.00 & 3.00 \end{bmatrix} & & \begin{matrix} A_1 \\ A_2 \\ A_3 \end{matrix} \end{matrix}.$$

Kriterijum C_1 se minimizuje, a kriterijum C_2 se maksimizuje, tako da su prema (3) elementi pozitivnog idealnog rešenja (PIS)

The criterion C_1 is minimised, while criterion C_2 is maximised, so the elements of positive ideal solution (PIS), according to (3), are

$$f_1^{\#} = \min(1.00,\ 3.00,\ 4.00) = 1.00, \quad f_2^{\#} = \max(2.00, 1.00,\ 3.00) = 3.00,$$

i negativnog idealnog rešenja (NIS)

and negative ideal solution (NIS)

$$f_1^{-} = \max(1.00,\ 3.00,\ 4.00) = 4.00, \quad f_2^{-} = \min(2.00, 1.00,\ 3.00) = 1.00.$$

Ova rešenja predstavljaju dve zamišljene idealne alternative, PIS - $A^* = [1.00 \;\; 3.00]$ i NIS - $A^- = [4.00 \;\; 1.00]$.

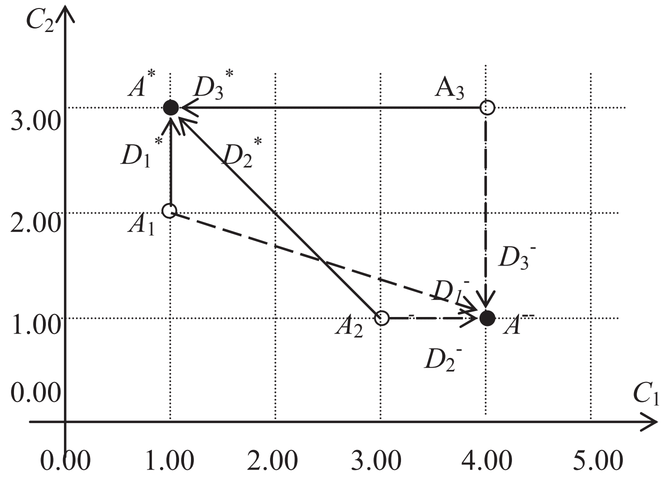
U koordinatnom sistemu kriterijuma C_1 i C_2, na slici 1, sve ove alternative prikazane su kao tačke.

Pod uslovom da su koeficijenti težina (značajnosti) oba kriterijuma isti i da su vrednosti elementata matrice $\mathbf{F}$ za oba kriterijuma izraženi u istim jedinicama mere nije neophodno vršiti normalizaciju ovih vrednosti. Određuju se udaljenosti od tačaka A_1, A_2 i A_3, koje predstavljaju alternative od tačaka A^* i A^-, koje predstavljaju pozitivno (PIS) i negativno (NIS) idealno rešenje respektivno se određuju u sledećem koraku..

These solutions represent imaginary ideal alternatives, PIS - $A^* = [1.00 \;\; 3.00]$ and NIS $A^- = [4.00 \;\; 1.00]$.

In the coordinate system of criteria C_1 and C_2, shown in Fig. 1, all the alternatives are presented as points.

Provided that the coefficients of weights (coefficients of importance) for the both of criteria are the same and that the values of elements of the matrix $\mathbf{F}$ for both of criteria are expressed in the same units of measure, it is unnecessary to perform normalisation of these values. Distances of the points A_1, A_2 and A_3, that represent alternatives, from the points A^* i A^-, that represent positive ideal solution (PIS) and negative ideal solution (NIS) respectively are determined in the next step.



Slika 1. Grafički prikaz alternativa i kriterijuma
Figure 1. Graphical presentation of the alternatives and criteria

Udaljenosti D_i^* (*i* = 1,2,3) alternativa od pozitivnog idealnog rešenja (PIS) jesu:

The distances D_i^* (*i* = 1,2,3) of the alternatives from the positive ideal solution (PIS) are according to (10):

$$D_1^* = [(1.00-1.00)^2 + (2.00-3.00)^2]^{1/2} = 1.00\,,$$
$$D_2^* = [(3.00-1.00)^2 + (1.00-3.00)^2]^{1/2} = 2.82\,,$$
$$D_3^* = [(4.00-1.00)^2 + (3.00-3.00)^2]^{1/2} = 3.00\,.$$

Udaljenosti d_i^{*-} (*i* = 1,2,3) alternativa od negativnog idealnog rešenja (NIS) jesu:

The distances D_i^- (*i* = 1,2,3) of the alternatives from the negative ideal solution (NIS) are according to (10):

$$D_1^- = [(1.00-400)^2 + (2.00-1.00)^2]^{1/2} = 3.16\,,$$
$$D_2^- = [(3.00-4.00)^2 + (1.00-1.00)^2]^{1/2} = 1.00\,,$$
$$D_3^- = [(4.00-4.00)^2 + (3.00-1.00)^2]^{1/2} = 2.00\,.$$

Na kraju, sračunavaju se relativne bliskosti alternativa od PIS RC_i^* (*i* = 1,2,3) i relativne udaljenosti alternativa od NIS RC_i^- (*i* = 1,2,3):

At the end, are calculated *relative closeness* to the alternatives RC_i^* (*i*=1,2,3) from PIS and RC_i^- (*i*=1,2,3) from NIS, according to (11), :

$$RC_1^* = 1.00/(1.00+3.16) = 0.24, \quad RC_1 = 3.16/(1.00+3.16) = 0.76,$$
$$RC_2^* = 2.82/(2.82+1.00) = 0.74, \quad RC_2 = 1.00/(2.82+1.00) = 0.26,$$
$$RC_3^* = 3.00/(3.00+2.00) = 0.60, \quad RD_3 = 2.00/(3.00+2.00) = 0.40.$$

Na osnovu ovih rezultata, može se zaključiti da alternativa A_1 ima najmanju udaljenost $D_1^* = 1.00$ i najmanju relativnu bliskost PIS pozitivnom idealnom rešenju $RC_1^* = 0.24$ koje je predstavljeno tačkom A^*. Ova alternativa ima najveću udaljenost $D_1^- = 3.16$ i najmanju relativnu bliskost NIS $RC_1^- = 0.76$ od NIS, koje je predstavljeno tačkom A^- na slici 1. Prema tome, alternativa A_1 je najbliža ili „najsličnija" pozitivnom idealnom rešenju A^*, pa je zbog toga – najprihvatljivija. Ako se izvrši rangiranje prema relativnoj udaljenosti od pozitivnog idealnog rešenja, onda je redosled alternativa A_1, A_3, A_2.

From these results may be concluded that alternative A_1 has the smallest distance $D_1^* = 1.00$ and smallest relative closeness from PIS $RC_1^* = 0.24$, which is represented by the point A^*. This alternative has the largest PIS distance $RC_1^* = 3.16$ and the largest relative closeness to NIS $RC_1^- = 0.76$, that is represented by the point A^- in Fig. 1. Therefore, the alternative A_1 is the nearest or "most similar" to the positive ideal solution A^*, and because of that it is most acceptable. If alternatives are ranked according to the relative distance of alternatives from the positive ideal solution, then order of alternatives is A_1, A_3, A_2.

4 POJAM RASPLINUTOG SKUPA I RASPLINUTOG BROJA

U mnogim realnim situacijama, elementi f_{ij} matrice odlučivanja $\mathbf{F}$ i elementi w_j vektora težina $\mathbf{w}$ ne mogu se tačno izmeriti ili proceniti i prikazati fiksnim realnim brojevima, nego se izražavaju približnim vrednostima. Neki od tih elemenata prikazuju se lingvističkim vrednostima, kao što su „dobar", „loš", „visok", „nizak" i slično. Zbog toga, za ulazne podatke treba koristiti rasplinute brojeve, kao posebnu klasu rasplinutih skupova, pa se tako problem transformiše u problem rasplinutog višekriterijumskog odlučivanja (FMCDMP).

Pojam i definiciju rasplinutog skupa uveo je Lotfi Zadeh 1965. godine, u svom čuvenom radu „Rasplinuti skupovi" [19]. On je postavio osnove teorije rasplinutih skupova, i rasplinute logike, teorije mogućnosti, teorije rasplinutih sistema i upravljanja ovim sistemima. Ove teorije su posle toga imale veoma intenzivan razvoj i našle široku primenu u razmatranju i rešavanju brojnih problema teorije i prakse u različitim disciplinama. U stranoj i domaćojoj literaturi, postoji veoma mnogo radova, knjiga i publikacija koje se odnose na rasplinute skupove i njihove primene. Ovde se daju kratke definicije pojma rasplinutog skupa i rasplintog broja i neke aritmetčke operacije s tim brojevima.

4 NOTION OF FUZZY SET AND FUZZY NUMBER

In many real situations elements f_{ij} of the decision matrix $\mathbf{F}$ and elements w_j of the vector of weights $\mathbf{w}$ cannot be measured or assessed precisely and expressed by the crisp numbers, since they are expressed by approximate values. Some of these elements sometimes may be quantified by linguistic values "good", "bad", "high", "low" and in some other similar way. For these reasons, the fuzzy numbers for input data should be used, and the problem transformed to the fuzzy multiple criteria decision making problem (FMCDMP).

The notion and definition of the fuzzy set has introduced Lotfi Zadeh in his famous paper "Fuzzy sets" [19]. He founded theory of the fuzzy sets and fuzzy logics, theory of possibility, theory of fuzzy systems and control of these systems. These theories have had very intensive development and found wide application in consideration and solution of numerous problems of the theory and practice in different disciplines. In the foreign and domestic literature there are numerous papers, books and other publications that are related on the fuzzy sets and their applications. Here are given some short mathematical definitions of the fuzzy set and fuzzy number and some arithmetic operations with these numbers.

4.1 Definicija rasplinutog skupa i rasplinutog broja

Neka $X=\{x\}$ označava neku kolekciju objekata ili tačaka prikazanih sa x, onda fuzzy skup $\widetilde{A}$ u X jeste skup uređenih parova x i $\mu_{\widetilde{A}}(x)$

4.1 Definition of the fuzzy set and the fuzzy number

Let $X=\{x\}$ represents some collection of objects or points denoted by x, then a fuzzy set $\widetilde{A}$ in X is the set ordered pairs x i $\mu_{\widetilde{A}}(x)$

$$\widetilde{A} = \{x, \mu_{\widetilde{A}}(x) \,|\, x \in X\}, \tag{13}$$

gde se $\mu_{\widetilde{A}}(x)$ naziva *funkcija pripadnosti* ili *stepen pripadnosti* objekta x skupu $\widetilde{A}$. (Zadeh,[19] , Zimmermann, [21]).

Skup X Zadeh naziva *univerzalni skup*, čije elemente x funkcija pripadnosti $\mu_{\widetilde{A}}(x)$ preslikava u elemente podskupa realnih brojeva [0, 1], što se simbolički piše

Where $\mu_{\widetilde{A}}(x)$ is named a *membership function* or *grade of membership* of the object x to the set $\widetilde{A}$. (Zadeh, [19], Zimmermann, [21]).

Zadeh has called the set X *universe of discourse*, whose elements x membership function $\mu_{\widetilde{A}}(x)$ copies into elements of a subset of the real numbers [0, 1], which is written symbolically

$$\mu_{\widetilde{A}}(x) : X \rightarrow [0,1].$$

Mogu se navesti mnogi primeri fuzzy skupova. Na primer „skup mladih ljudi" jeste fuzzy skup, čija funkcija ili stepen pripadnosti $\mu_{\widetilde{A}}(x)$ zavisi od starosti svakog člana toga skupa. Isto tako, „skup odličnih studenata" jeste fuzzy skup, jer funkcija ili stepen pripadnosti svakog studenta ovom skupu zavisi od ostvarene prosečne ocene i nekih drugih bitnih pokazatelja uspeha.

Ako je $\widetilde{A}$ fuzzy podskup skupa X, onda je njegov *α-nivo* ili *α-presek* nerasplinuti skup A_α koji sadrži sve elemente čija je funkcija pripadnosti veća ili jednaka broju α, tj.

Many examples of the fuzzy sets could be cited. For example, a "set of young people" is the fuzzy set, whose membership function or grade of membership $\mu_{\widetilde{A}}(x)$ depends on the age of every member of that set. In the same way, a set of excellent students is the fuzzy set, since its membership function of every student depends on an achieved average grade and other basic indicators of his success.

If $\widetilde{A}$ is a fuzzy subset of the fuzzy set X, then its *α-level* or *α-cut* is a crisp (non fuzzy) set A_α that contains all elements with the membership function which is more or equal to the number α, i.e.

$$A_\alpha = \{x \in X \,|\, \mu_{\widetilde{A}}(x) \geq \alpha,\ 0 < \alpha \leq 1\}. \tag{14}$$

Rasplinuti skup je *konveksan* ako su mu svi α - preseci A_α konveksni skupovi.

Ako univerzalni skup predstavlja skup realnih brojeva R, čiji su elementi *x* predstavljeni brojnom pravom, onda se rasplinuti skup $\widetilde{A} \in R$ naziva *rasplinuti* (*fuzzy*) *broj*, s funkcijom pripadnosti $\mu_{\widetilde{A}}(x): R \to [0,1]$, ako ispunjava sledeće uslove (Dubois and Prade, [5]):

- $\widetilde{A}$ je normalan broj, što znači da postoji broj $x_0 \in R$ za koji je $\mu_{\widetilde{A}}(x_0) = 1$;
- $\widetilde{A}$ je rasplinuto konveksan broj, tj. $\mu_{\widetilde{A}}(\lambda x + (1-\lambda)y) \geq \min\{\mu_{\widetilde{A}}(x), \mu_{\widetilde{A}}(y)\}$, za sve $x, y \in R,\ \lambda \in [0,1]$;
- $\widetilde{A}$ je odozgo semikontinualan broj (tj. $\mu_{\widetilde{A}}^{-1}([\alpha,1])$ zatvoren je za sve $\alpha \in [0,1]$),
- Osnova rasplinutog broja $\widetilde{A}$ $Supp(\widetilde{A})$) zatvoren je skup za koji je $\mu_{\widetilde{A}}(x) > 0,\ \ x \in R.$
- A_α – je α presek rasplinutog (fuzzy) broja je nerasplinuti broj
- $A_\alpha = \{x \in R \mid \mu_{\widetilde{A}}(x) \geq \alpha,\ 0 < \alpha \leq 1\}$, koji predstavlja zatvoreni *interval poverenja*

The fuzzy set is *convex* one if all its α - cuts A_α are convex sets.

If universe of discourse is the set of real numbers R, whose elements *x* are represented by the numeric straight line, then the fuzzy set $\widetilde{A} \in R$ is named a *fuzzy number*, with the membership function $\mu_{\widetilde{A}}(x): R \to [0,1]$, if satisfies the following conditions (Dubois and Prade, [5]):

- $\widetilde{A}$ is a normal number, which means that exists a number $x_0 \in R$ for which is $\mu_{\widetilde{A}}(x_0) = 1$;
- $\widetilde{A}$ is fuzzy convex number, i.e. $\mu_{\widetilde{A}}(\lambda x + (1-\lambda)y) \geq \min\{\mu_{\widetilde{A}}(x), \mu_{\widetilde{A}}(y)\}$, for all $x, y \in R,\ \lambda \in [0,1]$;
- $\widetilde{A}$ is upper semi continuous (i.e. $\mu_{\widetilde{A}}^{-1}([\alpha,1])$ is closed for all $\alpha \in [0,1]$),
- The support of $\widetilde{A}$ ($Supp(\widetilde{A})$) is bounded for which is $\mu_{\widetilde{A}}(x) > 0,\ \ x \in R.$
- A_α is α cut of the fuzzy number $\widetilde{A}$, that is a crisp number
- $A_\alpha = \{x \in R \mid \mu_{\widetilde{A}}(x) \geq \alpha,\ 0 < \alpha \leq 1\}$, which represents a closed *interval of confidence*

$$A_\alpha = [A_l(\alpha), A_u(\alpha)],\ \ 0 < \alpha \leq 1, \tag{15}$$

$A_l(\alpha)$ i $A_u(\alpha)$ jesu donja i gornja granica ovog intervala. Par funkcija $A_l(\alpha)$ i $A_u(\alpha)$ predstavlja parametarsku prezentaciju rasplinutog broja $\widetilde{A}$.

$A_l(\alpha)$ i $A_u(\alpha)$ are lower and upper limits of this interval. The pair of functions of this interval $A_l(\alpha)$ and $A_u(\alpha)$ expresses a parametric presentation of the fuzzy number $\widetilde{A}$.

$$A_l(\alpha) = \inf\{x \in R : \mu_{\widetilde{A}}(x) \geq \alpha\},\ \ A_u(\alpha) = \sup\{x \in R : \mu_{\widetilde{A}}(x) \geq \alpha\}. \tag{16}$$

Zavisno od funkcije pripadnosti, postoji više tipova rasplinutih brojeva: trougaoni, trapezoidni, broj oblika *S* i π i drugi. U teoriji i praksi, najčešće se zbog linearnosti funkcija pripadnosti koriste trougaoni i trapezoidni rasplinuti brojevi. U ovom radu se koristi trougaoni rasplinuti broj, prikazan na slici 1. Ovaj broj se obično prikazuje s tri karakteristične vrednosti: donjom a_l, modalnom a_m (za koju je $\mu(a_m)=1$) i gornjom a_u , tj.

There are several types of fuzzy numbers depending of membership function: triangular, trapezoidal, S and π shape and others. In the theory and practical applications triangular and trapezoidal fuzzy numbers are used most frequently. Triangular fuzzy number, shown in Fig. 1, is used in this paper. This number is usually represented by three characteristic values: lower a_l, modal a_m (for which is $\mu(a_m)=1$) and upper a_u, i. e.

$$\widetilde{A} = (a_l, a_m, a_u)\ . \tag{17}$$

Funkcije pripadnosti ovog broja jesu:

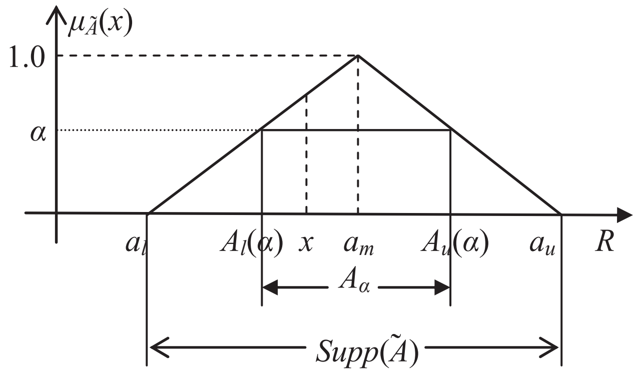
The membership functions of this number are:

$$\begin{aligned} &\mu_{\widetilde{A}}(x) = 0 \text{ za } x \leq a_l, \\ &\mu_{\widetilde{A}}(x) = (x - a_l)/(a_m - a_l) \text{ za } a_l \leq x \leq a_m, \\ &\mu_{\widetilde{A}}(x) = (a_u - x)/(a_u - a_m) \text{ za } a_m \leq x \leq a_u, \\ &\mu_{\widetilde{A}}(x) = 0 \text{ za } x \geq a_u. \end{aligned} \tag{18}$$

Za izabrani presek α, parametarska prezentacija rasplinutog broja $\widetilde{A}$ jeste

For the chosen α – cut , parametric presentation of the fuzzy number $\widetilde{A}$ is

$$A_l(\alpha) = a_l + (a_m - a_l)\alpha,\ A_u(\alpha) = a_u - (a_m - a_l)\alpha,\ 0 < \alpha \le 1. \tag{19}$$



Slika 2. Trougaoni rasplinuti broj
Figure 2. Triangular fuzzy number

4.2 Aritmetičke operacije s rasplinutim brojevima

Neka su data dva rasplinuta broja $\widetilde{A}$ i $\widetilde{B}$ pisana u parametarskoj formi

4.2 Arithmetic operations with fuzzy numbers

If are given two fuzzy numbers $\widetilde{A}$ and $\widetilde{B}$, written in the parametric form

$$A_\alpha = [A_l(\alpha), A_u(\alpha)] \text{ i } B_\alpha = [B_l(\alpha), B_u(\alpha)], \tag{20}$$

onda, primenjujući aritmetičke operacije s tim brojevima, dobija se rasplinuti broj $\widetilde{C}$ pisan u parametarskoj formi $C_\alpha = [C_l(\alpha), C_u(\alpha)]$

Aritmetičke operacije za $C_l(\alpha)$ i $C_u(\alpha)$, kada su $A_l(\alpha) > 0,\ B_l(\alpha) > 0$, jesu:

- za sabiranje:

then, a fuzzy number $\widetilde{C}$ written in the parametric form $C_\alpha = [C_l(\alpha), C_u(\alpha)]$ is obtained by applying arithmetic operations with these numbers.

Arithmetic operations for $C_l(\alpha)$ and $C_u(\alpha)$, when, $A_l(\alpha) > 0,\ B_l(\alpha) > 0$ are:

- for addition:

$$C_l(\alpha) = A_l(\alpha) + B_l(\alpha),\quad C_u(\alpha) = A_u(\alpha) + B_u(\alpha); \tag{21}$$

- za oduzimanje:
- for subtraction:

$$C_l(\alpha) = A_l(\alpha) - B_u(\alpha),\quad C_u(\alpha) = A_u(\alpha) - B_l(\alpha), \tag{22}$$

- za množenje:
- for multiplication:

$$C_l(\alpha) = A_l(\alpha)B_l(\alpha),\quad C_u(\alpha) = A_u(\alpha)B_u(\alpha); \tag{23}$$

- za deljenje:
- for division:

$$C_l(\alpha) = A_l(\alpha)/B_u(\alpha),\quad C_u(\alpha) = A_u(\alpha)/B_l(\alpha),\quad A_l(\alpha) > 0,\ A_u(\alpha) > 0. \tag{24}$$

4.3 Generalisana očekivana vrednost, varijansa i standardna devijacija slučajnog rasplinutog broja

Ovde predloženi metod za rangiranje alternative zasnovan je na generalisanoj očekivanoj (srednjoj) vrednosti, varijansi i standardnoj devijaciji slučajnog rasplinutog broja, koji predstavlja neki rasplinuti događaj, kako je to definisao Zadeh [20].

4.3 Generalized expected value, variance and standard deviation of a random fuzzy number

The proposed method for ranking alternatives is based on the generalized expected (mean) value, variance and standard deviation of a random fuzzy number that represents some probabilistic fuzzy event, as it is defined by Zadeh [20].

Neka je $P(\widetilde{A}) \geq 0$ mera verovatnoće na skupu realnih brojeva R, gde je $\widetilde{A}$ slučajni rasplinuti događaj predstavljen slučajnim rasplinutim brojem u skupu realnih brojeva R ($\widetilde{A} \in R$); onda je, prema Zadehu [20], verovatnoća slučajnog rasplinutog događaja (rasplinutog broja)

Let $P(\widetilde{A}) \geq 0$ be a probability measure over the measurable space of real numbers R, where $\widetilde{A}$ is random fuzzy event represented by the fuzzy number in R ($\widetilde{A} \in R$), then according to Zadeh [20], the probability of this random fuzzy event (fuzzy number) is

$$P(A) = \int_R \mu_{\widetilde{A}(x)} dP = E(\mu_{\widetilde{A}}(x)), \tag{25}$$

gde E označava operator očekivane vrednosti funkcije pripadnosti.

where E denotes an operator of expected value of the membership function.

Očekivana vrednost $E(\widetilde{A})$ rasplinutog broja $\widetilde{A}$ u odnosu na meru verovatnoće P jeste

The expected value (mean) $E(\widetilde{A})$ of the fuzzy number $\widetilde{A}$, related to the probability measure P, is

$$E(\widetilde{A}) = x_e = \frac{1}{P(\widetilde{A})} \int_R x \mu_{\widetilde{A}}(x) \frac{dP}{dx} dx. \tag{26}$$

Varijansa $V(\widetilde{A})$ slučajnog rasplinutog broja $\widetilde{A}$ u odnosu na meru verovatnoće P jeste

Variance $V(\widetilde{A})$ for the random fuzzy number $\widetilde{A}$, related to the probability measure P, is

$$V(\widetilde{A}) = \frac{1}{P(\widetilde{A})} \int_R (x - x_e)^2 \mu_{\widetilde{A}}(x) dP = \frac{1}{P(\widetilde{A})} \int_R x^2 \mu_A(x) \frac{dP}{dx} dx - x_e^2. \tag{27}$$

$\widetilde{A}$ je,dakle, rasplinuti broj koji ima funkciju pripadnosti $\mu_{\widetilde{A}}(x)$, funkciju raspodele verovatnoće $P(\widetilde{A})$ i funkciju gustine raspodele verovatnoće $g(x)$. Karakteristične statističke vrednosti $E(\widetilde{A})$ i $V(\widetilde{A})$, koje se sračunavaju pomoću ovih formula nazivaju se *generalisana očekivana (srednja) vrednost* i *generalisana varijansa slučajnog broja* $\widetilde{A}$.

$\widetilde{A}$ is, therefore, the random fuzzy number which has membership function $\mu_{\widetilde{A}}(x)$, probability distribution function $P(\widetilde{A})$ and probability density function $g(x)$. Characteristic statistical values $E(\widetilde{A})$ and $V(\widetilde{A})$ calculated from these formulas are called *generalized expected (mean) value* and *generalized variance* of the random fuzzy number $\widetilde{A}$.

Ako je funkcija raspodele verovatnoća uniformna za fuzzy broj $\widetilde{A} = (a_l, a_m, a_u)$

If the probability distribution function is uniform for the fuzzy number $\widetilde{A} = (a_l, a_m, a_u)$

$$P(\widetilde{A}) = 0 \text{ za } x < a_l,\ P(\widetilde{A}) = (x - a_l)/(a_u - a_l) \text{ za } a_l \leq x \leq a_u,\ P(\widetilde{A}) = 0 \text{ za } x > a_u, \tag{28}$$

onda je

then

$$g(x) = \frac{dP}{dx} = \frac{1}{a_u - a_l}.$$

Kada se ovaj izraz uvrsti u izraze (26) i (27), dobija se za uniformnu raspodelu verovatnoća:

- generalisana očekivana vrednost

Introducing this expression in the expressions (26) and (27), one obtains for the uniform probability distribution:

- generalized expected (mean) value

$$x_e^U(\widetilde{A}) = \frac{\int_R x \mu_{\widetilde{A}}(x) dx}{\int_R \mu_{\widetilde{A}}(x) dx} \tag{29}$$

– generalisana varijansa

– generalized variance

$$`V_e^U(\widetilde{A}) = \frac{\int_R x^2 \mu_{\widetilde{A}}(x)dx}{\int_R \mu_{\widetilde{A}}(x)dx} - (x_e^U)^2 . \tag{30}$$

Za trouglasti rasplinuti broj $\widetilde{A} = (a_l, a_m, a_u)$ ove vrednosti su

For the triangular fuzzy number $\widetilde{A} = (a_l, a_m, a_u)$ these values are

$$x_e^U(\widetilde{A}) = (a_l + a_m + a_u)/3, \tag{31}$$

$$V_e^U(\widetilde{A}) = (a_l^2 + a_m^2 + a_u^2 - a_l a_m - a_l a_u - a_m a_u)/18. \tag{32}$$

Ako je raspodela verovatnoće $P(x)$ trougaona, tako da je proporcionalna funkciji pripadnosti $\mu_{\widetilde{A}}(x)$

If the probability distributions function $P(x)$ is triangular, so that it is proportional to the membership function $\mu_{\widetilde{A}}(x)$

$$P(x) = k\mu_{\widetilde{A}}(x), \tag{33}$$

gde je *k* factor proporcionalnosti, onda su:

– generalisana očekivana vrednost rasplinutog broja

where *k* is factor of proportionality then is:

– generalized expected value

$$x_e^T(\widetilde{A}) = \frac{\int_R x(\mu_{\widetilde{A}}(x))^2 dx}{\int_R (\mu_{\widetilde{A}}(x))^2 dx}, \tag{34}$$

– generalisana varijansa

– generalized variance

$$V_e^T(\widetilde{A}) = \frac{\int_R x^2(\mu_{\widetilde{A}}(x))^2 dx}{\int_R (\mu_{\widetilde{A}}(x))^2 dx} - (x_e^T(\widetilde{A}))^2 . \tag{35}$$

Za trouglasti rasplinuti broj $\widetilde{A} = (a_l, a_m, a_u)$ ove vrednosti su

For the triangular fuzzy number $\widetilde{A} = (a_l, a_m, a_u)$ these values are

$$x_e^T(\widetilde{A}) = (a_l + 2a_m + a_u)/4, \tag{36}$$

$$V_e^T(\widetilde{A}) = (3a_l^2 + 4a_m^2 + 3a_u^2 - 4a_l a_m - 2a_l a_u - 4a_m a_u)/80. \tag{37}$$

5 MODIFIKOVANA RASPLINUTA (FUZZY) TOPSIS PROCEDURA

Elementi rasplinute matrice odlučivanja $\widetilde{\mathbf{F}}$ su trougaoni rasplinuti brojevi $\widetilde{f}_{ij} = (f_{ij,}^{(l)}, f_{ij}^{(m)}, f_{ij.}^{(u)})$, tako da se ova matrica može prikazati pomoću tri matrice s fiksnim (nerasplinutim) elementima $\widetilde{\mathbf{F}} = (\mathbf{F}_l, \mathbf{F}_m, \mathbf{F}_u)$. Rasplinuta TOPSIS procedura izvršava se u nekoliko koraka koji su objašnjeni u ovom radu, uz predloženu modifikaciju. Ovi koraci su: normalizacija, računanje generalisane očekivane vrednosti i standardne devijacije, rangiranje alternativa i izbor najbolje alternative.

MODIFIED FUZZY TOPSIS PROCEDURE

Elements of the fuzzy decision matrix $\widetilde{\mathbf{F}}$ are triangular fuzzy numbers $\widetilde{f}_{ij} = (f_{ij,}^{(l)}, f_{ij}^{(m)}, f_{ij.}^{(u)})$, so that this matrix can be expressed by three crisp matrices $\widetilde{\mathbf{F}} = (\mathbf{F}_l, \mathbf{F}_m, \mathbf{F}_u)$. Fuzzy TOPSIS procedure performs in several steps that will be explained in this paper with some proposed modification. These steps are: normalization, calculation of generalized expected values and standard deviations, ranking alternatives and choice of the best alternative.

5.1 Normalizacija

Normalizacija se i ovde vrši iz istih razloga iz kojih se to čini i u TOPSIS metodi s fiksnim (nerasplinutim) brojevima – da bi se dobile bezdimenzionalne vrednosti u matrici odlučivanja $\widetilde{\mathbf{F}}$. Međutim, zbog rasplinutosti njezinih elemenata, u literaturi postoji nekoliko predloga za normalizaciju (Wang i Elhang, [17]). Ovde će se koristiti metod koji su predložili Ertugrud i Karakasagly [6]. Normalizovane vrednosti elementa $\widetilde{f}_{ij}$ rasplinute (fuzzy) matrice odlučivanja $\widetilde{\mathbf{F}}$ označeni su sa $\widetilde{a}_{ij}$, i one sačinjavaju normalizovanu rasplinutu matricu odlučivanja $\widetilde{\mathbf{A}}$ i sračunavaju se po sledećoj formuli

5.1 Normalization

Normalization is performed here due to the same reasons as in TOPSIS method with the crisp numbers to obtain dimensionless values in the decision matrix $\widetilde{\mathbf{F}}$. However, due to fuzziness of its elements, there are several proposals for the normalization (Wang and Ehlang, [17]). Here, a method which is proposed by Ertugrud and Karakasagly [6] is used. Normalized values of elements $\widetilde{f}_{ij}$ are denoted as $\widetilde{a}_{ij}$, and they constitute the normalized fuzzy matrix $\widetilde{\mathbf{A}}$ and they are calculated by the following formula

$$\widetilde{a}_{ij} = (f_{ij}^{(l)} / f_i^{*(u)},\, f_{ij}^{m} / f_i^{*(u)},\, f_{ij}^{u} / f_i^{*(u)})\ ;\ i = 1,2,...,m;\, j = 1,2,...n; \tag{38}$$

gde za svaki kriterijum C_i

where for every criterion C_i

$$f_i^{*(u)} = \max_j f_{ij}^{(u)},\ i = 1,2,...,m. \tag{39}$$

5.2 Određivanje karakterističnih vrednosti elemenata težinski normalizovane matrice odlučivanja $\widetilde{\mathbf{C}}$

Elementi $\widetilde{c}_{ij}$ težinske normalizovane matrice odlučivanja $\widetilde{\mathbf{C}}$ računaju se kao proizvodi dva rasplinuta trouglasta broja $\widetilde{a}_{ij}$ i težine w_j koja u većini slučajeva predstavlja *koeficijent značajnosti* kriterijuma C_j

5.2 Determination of characteristic values of the weighted normalized decision matrix $\widetilde{\mathbf{C}}$

Elements $\widetilde{c}_{ij}$ of the weighted decision matrix $\widetilde{\mathbf{C}}$ are calculated as a product of two fuzzy numbers $\widetilde{a}_{ij}$ and the weight $\widetilde{w}_j$, which in many cases represents *coefficient of significance* of the alternative C_j

$$\widetilde{c}_{ij} = \widetilde{a}_{ij} v_i,\ i = 1,2,...,m;\ j = 1.2,...,n; \tag{40}$$

Rasplinuti brojevi $\widetilde{a}_{ij}$ i $\widetilde{w}_j$ se mogu prikazati prema (17)

Fuzzy numbers $\widetilde{a}_{ij}$ i $\widetilde{w}_j$ may be shown according to (17)

$$\widetilde{a}_{ij} = (a_{ij}^{(l)}, a_{ij}^{(m)}, a_{ij}^{(u)}), \quad \widetilde{w}_i = (w_i^{(l)}, w_i^{(m)}, w_i^{(u)}), \quad i=1,2,..,m;\ j=1,2,...n.$$

Lako se može zaključiti, iz pravila o množenju rasplinutih brojeva (23), da proizvod dva trougaona rasplinuta broja nije trougaoni rasplinuti broj, tako da elementi $\widetilde{c}_{ij}$ matrice $\widetilde{\mathbf{C}}$ nisu trouglasti rasplinuti brojevi. Međutim, mnogi autori pretpostavljaju, radi uprošćenja procedure, da ovi elementi jesu trouglasti brojevi i sračunavaju elemente težinske fuzzy matrice $\widetilde{\mathbf{C}}$ prema sledećoj formuli

It is easy to conclude from the rule of multiplication of fuzzy numbers (23), that product of two fuzzy triangular numbers is not a triangular fuzzy number, hence elements $\widetilde{c}_{ij}$ of the fuzzy matrix $\widetilde{\mathbf{C}}$ are not triangular fuzzy numbers. However many authors suppose, due to simplicity of the procedure, that these numbers are triangular ones and calculate elements of this matrix by the simplified formula

$$\widetilde{c}_{ij} = (\widetilde{a}_{ij}^{(l)} \widetilde{w}_i^{(l)},\ \widetilde{a}_{ij}^{(m)} \widetilde{w}_i^{(m)},\ \widetilde{a}_{ij}^{(u)} \widetilde{w}_i^{(u)}),\ i = 1,2,...,m;\ \ j = 1,2,...,n. \tag{41}$$

U svom prethodnom radu [16], autori su izveli obrasce za tačno određivanje ovih rasplinutih brojeva koji nisu trougaoni, i predložili proceduru s generalisanim očekivanim vrednostima e_{ij} i varijansama v_{ij} proizvoda rasplinutih brojeva $\widetilde{c}_{ij} = \widetilde{a}_{ij} \widetilde{w}_j$

In the earlier paper written by these authors [16], a procedure with the generalized expected values e_{ij} and variances v_{ij} of the fuzzy numbers products $\widetilde{c}_{ij} = \widetilde{a}_{ij} \widetilde{w}_j$ has been proposed.

$$e_{ij} = x_e(\widetilde{c}_{ij}), \quad v_{ij} = V(\widetilde{c}_{ij}); \quad i = 1,2,...,m; \; j = 1,2,...,n. \tag{42}$$

U toj proceduri, brojevi $\widetilde{c}_{ij}$ tretiraju se kao slučajni fuzzy događaji koji s jedne strane imaju odgovarajuću raspodelu verovatnoće događanja, a s druge funkciju pripadnosti rasplinutom (fuzzy) skupu $\widetilde{A}$.

Ove vrednosti su elementi matrica $\mathbf{E}$ i $\mathbf{V}$ respektivno, a računaju se prema formulama izvedenim u pomenutom radu [16], zavisno od izabrane raspodele verovatnoća fuzzy događaja, koje mogu biti uniformne ili trougaone (proporcionalne).

In this procedure, the fuzzy numbers $\widetilde{c}_{ij}$ are assumed as fuzzy events that have a corresponding probability distribution as well as membership function to the fuzzy set $\widetilde{A}$.

These values are elements of matrices $\mathbf{E}$ and $\mathbf{V}$ respectively and they are calculated by the formulas that are given in the mentioned paper [16], depending on the chosen probability distribution of fuzzy events, which may be uniform or triangular (proportional) one.

$$e_{ij} = x_e(\widetilde{c}_{ij}) = \frac{M_1(\widetilde{c}_{ij})}{F(\widetilde{c}_{ij})}, \quad v_{ij}(\widetilde{c}_{ij}) = \frac{M_2(\widetilde{c}_{ij})}{F(\widetilde{c}_{ij})} - (x_e(\widetilde{c}_{ij}))^2, \; \sigma_{ij} = (v_{ij}(\widetilde{c}_{ij}))^{1/2}. \tag{43}$$

Za uniformnu raspodelu slučajne rasplinute promenljive:

For the uniform distribution of a random fuzzy variable are:

$$F(\widetilde{c}_{ij}) = \frac{\overline{B}_l - \overline{B}_u}{2} + \frac{2(\overline{C}_l - \overline{C}_u)}{3}, \tag{44}$$

$$M_1(\widetilde{c}_{ij}) = \frac{\overline{A}_l\overline{B}_l - \overline{A}_u\overline{B}_u}{2} + \frac{\overline{B}_l^2 - \overline{B}_u^2}{3} + \frac{3(\overline{B}_l\overline{C}_l - \overline{B}_u\overline{C}_u)}{4} + \frac{2(\overline{A}_l\overline{C}_l - \overline{A}_u\overline{C}_u)}{3} + \frac{2(\overline{C}_l^2 - \overline{C}_u^2)}{5}, \tag{45}$$

$$\begin{aligned} M_2(\widetilde{c}_{ij}) = {} & \frac{\overline{A}_l^2\overline{B}_l - \overline{A}_u^2\overline{B}_u}{2} + \frac{\overline{B}_l^3 - \overline{A}_u^3}{4} + \frac{5(\overline{B}_l C_l^2 - \overline{B}_u\overline{C}_u^2)}{6} + \frac{2(\overline{A}_l\overline{B}_l^2 - \overline{A}_u\overline{B}_u^2)}{3} + \\ & + \frac{3(\overline{A}_l\overline{B}_l\overline{C}_l - \overline{A}_u\overline{B}_u\overline{C}_u)}{2} + \frac{4(\overline{B}_l^2\overline{C}_l - \overline{B}_u^2\overline{C}_u)}{5} + \frac{2(\overline{A}_l^2\overline{C}_l - \overline{A}_u^2\overline{C}_u)}{3} + \\ & + \frac{2(\overline{C}_l^3 - \overline{C}_u^3)}{7} + \frac{4(\overline{A}_l\overline{C}_l^2 - \overline{A}_u\overline{C}_u^2)}{5}. \end{aligned} \tag{46}$$

Za trougaonu (proporcionalnu) raspodelu slučajne rasplinute promenljive su:

For the triangular (proportional) distribution of a random fuzzy variable are:

$$F(\widetilde{c}_{ij}) = \frac{\overline{B}_l - \overline{B}_u}{3} + \frac{2(\overline{C}_l - \overline{C}_u)}{2}, \tag{47}$$

$$M_1(\widetilde{c}_{ij}) = \frac{\overline{A}_l\overline{B}_l - \overline{A}_u\overline{B}_u}{3} + \frac{\overline{B}_l^2 - \overline{B}_u^2}{4} + \frac{3(\overline{B}_l\overline{C}_l - \overline{B}_u\overline{C}_u)}{5} + \frac{\overline{A}_l\overline{C}_l - \overline{A}_u\overline{C}_u}{2} + \frac{\overline{C}_l^2 - \overline{C}_u^2}{3}, \tag{48}$$

$$\begin{aligned} M_2(\widetilde{c}_{ij}) = {} & \frac{\overline{A}_l^2\overline{B}_l - \overline{A}_u^2\overline{B}_u}{3} + \frac{\overline{B}_l^3 - \overline{A}_u^3}{5} + \frac{5(\overline{B}_l C_l^2 - \overline{B}_u\overline{C}_u^2)}{7} + \frac{\overline{A}_l\overline{B}_l^2 - \overline{A}_u\overline{B}_u^2}{2} + \\ & + \frac{6(\overline{A}_l\overline{B}_l\overline{C}_l - \overline{A}_u\overline{B}_u\overline{C}_u)}{5} + \frac{2(\overline{B}_l^2\overline{C}_l - \overline{B}_u^2\overline{C}_u)}{3} + \frac{\overline{A}_l^2\overline{C}_l - \overline{A}_u^2\overline{C}_u}{2} + \\ & + \frac{\overline{C}_l^3 - \overline{C}_u^3}{4} + \frac{2(\overline{A}_l\overline{C}_l^2 - \overline{A}_u\overline{C}_u^2)}{3}. \end{aligned} \tag{49}$$

U ovim izrazima su:

In these expressions are:

$$\overline{A}_l = a_{ij}^{(l)} w_i^{(l)}, \;\; \overline{B}_l = (a_{ij}^{(m)} - a_{ij}^{(l)}) w_i^{(l)} + (w_i^{(m)} - w_i^{(l)}) a_{ij}^{(l)}, \;\; \overline{C}_l = (a_{ij}^{(m)} - a_{ij}^{(l)})(w_i^{(m)} - w_{ij}^{(l)}), \tag{50}$$

$$\overline{A}_u = a_{ij}^{(u)} w_i^{(u)}, \;\; \overline{B}_u = (a_{ij}^{(m)} - a_{ij}^{(u)}) w_i^{(u)} + (w_i^{(m)} - w_i^{(u)}) a_{ij}^{(u)}, \;\; \overline{C}_u = (a_{ij}^{(m)} - a_{ij}^{(u)})(w_i^{(m)} - w_{ij}^{u)}), \tag{51}$$

$$i = 1,2,...,m; \; j = 1,2,...,n.$$

5.3 Sračunavanje očekivanog idealno pozitivnog i idealno negativnog rešenja

Po kolonama matrice očekivanih vrednosti $\mathbf{E}$, za svaki kriterijum C_j pronalazi se očekivano pozitivno idealno rešenje e_j^* i negativno idealno rešenje e_j^- po sledećim formulama

5.3 Calculation of the expected ideal positive and ideal negative solutions

For every criterion C_j are found the best expected positive ideal solution e_j^* and the worst negative ideal solution e_j^- in the columns of the matrix of expected values $\mathbf{E}$ by the following formulae

$$e_i^* = \{{}^{\max}_{i} e_{ij} : j \in \Omega_b \text{ or } {}^{\min}_{i} e_{ij} : j \in \Omega_c\}, \quad (52)$$

$$e_i^- = \{{}^{\min}_{i} e_{ij} : j \in \Omega_b \text{ or } {}^{\max}_{i} e_{ij} : j \in \Omega_c\}. \quad (53)$$

Ove vrednosti su elementi vektora očekivanog idealno pozitivnog (*EPIS*) A_e^* i očekivanog idealno negativnog rešenja (*ENIS*) A_e^-

These values are elements of vectors of the *expected positive ideal solution* (*EPIS*) A_e^* and *expected negative ideal solution* (*ENIS*) A_e^- solution

$$A^* = [e_1^*, e_2^*, ..., e_n^*], \quad A^- = [e_1^-, e_2^-, ..., e_n^-] \quad (54)$$

Varijanse koje odgovaraju očekivanim vrednostima označene su sa v_i^* i v_i^- i one čine vektore

Variances that correspond to these expected values are denoted as v_i^* and v_i^- and they constitute vectors

$$V^* = [v_1^*, v_2^*, ..., v_n^*], \quad V^- = [v_1^-, v_2^-, ..., v_n^-]. \quad (55)$$

5.4 Određivanje očekivanog Euklidoveog rastojanja i njegove varijanse od *EPIS* i *ENIS*

Očekivana Euklidova rastojanja ED_i^* i ED_i^- za svaku alternativu A_i od očekivanog pozitivnog rešenja *EPIS* A_e^* i očekivanog negativnog idealnog rešenja *ENIS* A_e^- određuju se prema sledećim formulama

5.4 Calculation of the expected Euclidean distances and its variance from *EPIS* and *ENIS*.

The expected Euclidean distances ED_i^* i ED_i^- for every alternative A_i from the expected positive ideal solution *EPIS* A_e^* and from the expected negative ideal solution *ENIS* A_e^- are calculated by formulas

$$ED_i^* = [\sum_{j=1}^{n} (e_{ij} - e_j^*)^2]^{1/2}, \quad i = 1,2,...,m; \quad (56)$$

$$ED_i^- = [\sum_{j=1}^{n} (e_{ij} - e_j^-)^2]^{1/2}, \quad i = 1,2,...,m. \quad (57)$$

Varijanse V_j^* rastojanja alternative A_j od očekivanog pozitivnog idealnog rešenja *EPIS* A_e^* i varijanse V_j^- od očekivanog negativnog idealnog rešenja *ENIS* A_e^-, sračunavaju se prema sledećim formulama, uzimajući u obzir pravilo o sabiranju i oduzimanju varijansi međusobno nezavisnih slučajnih vrednosti

Variance V_j^* of the distance of alternative A_j from the expected positive ideal solution *EPIS* A_e^* and variance V_j^- from the expected negative ideal solution *ENIS* A_e^-, are calculated by the following formulas, taking into account rule for summation and subtraction of variances for the mutually independent random variables

$$V_i^* = \sum_{j=1}^{n} (v_{ij} + v_j^*), \quad V_i^- = \sum_{j=1}^{n} (v_{ij} + v_j^-), \quad i = 1,2,...,m. \quad (58)$$

Odgovarajuće standardne devijacije σ_i^* udaljenosti svake alternative A_i od pozitivnog idealnog rešenja A^* i standardne devijacije σ_i^- svake alternative A_i od negativnog idealnog rešenja A^- jesu

Corresponding standard deviation σ_i^* of the distance of each alternative A_i from the expected ideal positive solution A^* and standard deviation σ_i^- of each alternative A_i from the expected negative ideal solution A_e^- are

$$\sigma_i^* = [V_i^*]^{1/2}, \quad \sigma_i^- = [V_i^-]^{1/2}; \quad i = 1,2,...,m. \tag{59}$$

Dobijena očekivana rastojanja svake alternative A_i od pozitivnog idealnog i negativnog idealnog rešenja, kasnije se koriste za formulisanje pravila za rangiranje alternativa i za izbor najbolje alternative. Očekivana rastojanja od očekivanog pozitivnog idealnog i očekivanog negativnog idealnog rešenja predstavljena su kao rasplinuti brojevi ili kao slučajni (probabilistički) rasplinuti događaji koje opisuju sračunate vrednosti.

These characteristic values of expected distance of each alternative A_i from the expected positive ideal solution A_e^* and the expected negative ideal solution A_e^- are further used to formulate rules for ranking alternatives and for choice of the best alternative. The expected distances from these solutions are assumed as the random fuzzy numbers or probabilistic fuzzy events described by these values.

5.5 Očekivana relativna blizskost i relativna standardna devijacija do *EPIS* and *ENIS* i rangiranje alternativa

Slično kao u TOPSIS metodi s fiksnim podacima, očekivana relativna bliskost svake alternative A_i do očekivanog pozitivno idealnog rešenja ERC_i^* i očekivanog negativnog idealnog rešenja ERC_i su bitni indikatori za rangiranje alternativa. Ove vrednosti računaju se prema sledećim formulama

5.5 Expected relative closeness and relative standard deviation to *EPIS* and *ENIS* and ranking alternatives

Like in the TOPSIS method with crisp data, expected relative closeness ERC_i^* of each alternative A_i to the expected positive ideal solution and expected negative ideal solution ERC_i are important indicators for ranking alternatives. These values are calculated by the following formulae

$$ERC_i^* = ED_i^* / (ED_i^* + ED_i^-), \quad i = 1,2,...,m; \tag{60}$$

$$ERC_i^- = ED_i^- / (ED_i^* + ED_i^-), \quad i = 1,2,...,m. \tag{61}$$

Alternativa s manjom vrednošću ERC_i^* i većom vrednošću ERC_i^- bolje je rangirana.

Za rangiranje rasplinutih brojeva, Lee i Li [11] upotrebili su generalisanu srednju vrednost i standardnu devijaciju, koje su zasnovane na merama verovatnoće rasplinutih događaja. Cheng [4] je poboljšao ovaj metod koristeći *koeficijent varijacije* CV, kao relativnu meru varijanse koja povezuje, kako je to poznato iz Statistike, standardnu devijaciju i srednju vrednost. Prema ovom postupku, sračunavaju se koeficijenti varijacije CV_i^* i CV_i^- za distancu alternative A_i (i = 1,2,…,m) od očekivanog pozitivnog idealnog rešenja i očekivanog negativnog idealnog rešenja respektivno

Alternative with smaller ERC_i^* and bigger ERC_i^- are better ranked.

For ranking fuzzy numbers Lee and Li [11] used the generalized mean and standard deviation based on the probability measure of fuzzy events. Cheng [4] improved this method using *coefficient of variation* CV, as a relative measure of the variance that relates, as it is known from Statistics, the standard deviation and the mean value. According to this method coefficients of variation CV_i^* and CV_i^- for the distance of the alternative A_i (i = 1,2,…,m) are calculated from the positive expected ideal solution A_e^* and expected negative ideal solution A_e^-, respectively

$$CV_i^* = \sigma_i^* / ED_i^*, \quad CV_i^- = \sigma_i^- / ED_i^-, \quad i = 1,2,...,m. \tag{62}$$

Alternativa koja ima veću CV_i^* vrednost, a manju CV_i^- ima bolju poziciju na rang-listi. Rangiranje alternativa na ovaj način je jednostavno, ali nekad ima određene nedostatke. Moguć je slučaj poređenja alternativa A_i i A_k koje imaju očekivana rastojanja od

Alternative with bigger CV_i^* and smaller CV_i^- has the better rank on the rank list. Ranking alternatives in this way are simple, but sometimes has some disadvantages. It is possible when comparing two alternatives A_i and A_k which have expected distances from positive ideal solutions $ED_i^* > ED_k^*$ and pozitivnog idealnog rešenja $ED_i^* > ED_k^*$ i $CV_i^* < CV_k^*$. Prema ovom pravilu rangiranja, alternativa A_k je bolje rangirana od alternative A_i. slučaju, alternativu A_k treba bolje rangirati od alternative Ovaj zaključak neće biti prihvaćen od strane donosioca odluka ako je razlika između CV_i^* i CV_k^* mala. U tom A_i, naročito kada alternativa A_k ima manju očekivanu relativnu blizinu od alternative A_i, tj. $ERC_k^* < ERC_i^*$. Rangiranje prema očekivanoj relativnoj blizskosti ima prednost u odnosu na druga pravila rangiranja. Međutim, u praksi treba koristi sva pravila, a potom analizirati dobijene rezultate i donosiocu odluke predložiti alternativu koja maksimalno zadovoljava ova pravila.

$CV_i^* < CV_k^*$. According to this ranking rule, alternative A_k is better ranked then alternative A_i. This conclusion will not be accepted by the decision maker if differences between CV_i^* and CV_k^* are small. In such a case alternative A_k will be ranked better then alternative A_i, especially when alternative A_k has smaller expected relative closeness then alternative A_i, i.e. $ERC_k^* < ERC_i^*$. Ranking according to the expected relative closeness have advantage over other rules. However, in practice all the rules should be applied, then, the obtained results analyzed and the alternative which best satisfies these rules should be proposed to the decision maker.

6 RASPODELA RASPOLOŽIVE KOLIČINE NOVCA ZA ODRŽAVANJE OBJEKATA

Raspoloživa količina novca Q, opredeljena za održavanje objekata, može se raspodeliti na osnovu dobijene rang-liste, prema sledećim formulama

– za rang-listu prema ERC_i^*

6 DISTRIBUTION OF AVAILABLE AMMOUNT OF MONEY FOR OBJECTS MAINTENANCE

An available amount of money Q, which is assigned for the maintenance of considered objects, should be delivered according to the obtained rank list by the following formulae

– for the rank list according to ERC_i^*,

$$Q_{ci} = (KIC)_i Q, \quad i = 1,2,\ldots m; \tag{63}$$

– za rang listu prema CV_i^*

– for the rank list according to CV_i^*,

$$Q_{vi} = (KIV)_i Q, \quad i = 1,2,\ldots m; \tag{64}$$

gde su $(KIC)_i$ i $(KIV)_i$ koeficijenti raspodele količine novca Q, koji se sračunavaju prema sledećim formulama

where $(KIC)_i$ and $(KIV)_i$ coefficients of distribution of the amount of money Q that are calculated according to the following formulas

$$(KIC)_i = \frac{ERC_i^*}{\sum_{i=1}^{m} ERC_i^*}, \qquad (KIV)_i = \frac{CV_i^*}{\sum_{i=1}^{m} CV_i^*}, \quad i = 1,2,\ldots,m. \tag{65}$$

Prema izloženoj proceduri, autori su napisali odgovarajući računarski program FUZZY_TOPSIS korišćenjem MATLAB programskog sistema.

According to this procedure, the authors have written a corresponding computer program FUZZY_TOPSIS in MATLAB programming system.

7 PRIMER

Ovaj primer, koji je u vezi s procenom rizika mostova, preuzet je iz rada Wang i Ehlang [17],[18], u kome je problem rešen na sasvim drugačiji način.

Prema Britanskoj agenciji za auto-puteve [2], rizik mosta definiše se kao bilo koji događaj ili hazard koji može onemogućiti postizanje poslovnih ciljeva ili ostvarivanja očekivanja zainteresovanih strana (vlasnika, deoničara, korisnika i dr.) i definiše se kao proizvod verovatnoće i posledice ostvarenog događaja.

U primeru je analizirano pet mostovskih konstrukcija $BS_1, BS_2, \ldots, BS_5$ koje su predstavljene kao alternative $A_1, A_2, \ldots, A_5$. Sve posledice i verovatnoće rizičnih događaja procenjene su na osnovu evidencija i procena tri inženjera eksperta, imajući u vidu četiri kriterijuma: *sigurnost* (C_1), *funkcionalnost* (C_2), *održivost* (C_3) i *okruženje* (C_4). Eksperti su takođe procenili i koeficijenate značaja alternativa. Ove vrednosti izražene su kao lingvističke i numeričke vrednosti, koje su

7 EXAMPLE

This example, which is related to the bridge risk assessment, is taken from papers written by Wang and Ehlang [17],[18] where this problem is solved in quite different way.

According to British Highway Agency [2] bridge risk is defined as any event or hazard that could hinder the achievement of business goals or the delivery of stakeholder expectations and it is defined as product of the likelihood (probability) and consequence of the occurred event.

In this example five bridge structures BS_1, BS_2,..., BS_5 are considered which represent alternatives A_1, A_2,…, A_5. All consequences and probabilities of the risk events are assessed on the base of evidence and engineering judgment by three experts against four criteria: *safety* (C_1), *functionality* (C_2), *sustainability* (C_3) and *environment* (C_4). The significance coefficients of alternatives are also assessed by experts. These values konačno transformisane u trougaone fuzzy brojeve. Dobijene vrednosti su elementi fuzzy matrice odlučivanja $\widetilde{\mathbf{F}} = (\mathbf{F}_l, \mathbf{F}_m, \mathbf{F}_u)$ i predstavljaju nivo rizika konstrukcije mosta BS_i u odnosu na kriterijum C_j (i=1,2,..,5; j=1,2,..,4). Zadatak je odrediti optimalnu shemu (redosled rangiranja) i koeficijente raspodele količine novčanih sredstava Q za održavanje mostova.

are assessed as linguistic and numeric variables that are finally transformed into triangular fuzzy numbers. These values are elements of the fuzzy decision matrix $\widetilde{\mathbf{F}} = (\mathbf{F}_l, \mathbf{F}_m, \mathbf{F}_u)$ and denotes levels of risk of bridge structure BS_i against criterion C_j (i=1,2,..,5; j=1,2,..,4). The task is to determine optimal scheme (rank order) and coefficients of distribution of available amount of money Q for the bridge maintenance.

$$\mathbf{F}_l = \begin{bmatrix} 73 & 38 & 62 & 15 \\ 62 & 62 & 38 & 22 \\ 27 & 73 & 10 & 15 \\ 0 & 62 & 62 & 27 \\ 0 & 0 & 62 & 73 \end{bmatrix}, \quad \mathbf{F}_m = \begin{bmatrix} 85 & 73 & 85 & 50 \\ 85 & 85 & 73 & 50 \\ 62 & 85 & 38 & 50 \\ 0 & 85 & 85 & 62 \\ 0 & 0 & 85 & 85 \end{bmatrix}, \quad \mathbf{F}_u = \begin{bmatrix} 100 & 95 & 100 & 85 \\ 100 & 100 & 95 & 78 \\ 90 & 100 & 73 & 85 \\ 5 & 100 & 100 & 90 \\ 5 & 10 & 100 & 100 \end{bmatrix}.$$

$$\mathbf{w}_l = [0.77 \;\; 0.50 \;\; 0.30 \;\; 0.13], \quad \mathbf{w}_m = [0.93 \;\; 0.70 \;\; 0.50 \;\; 0.30], \quad \mathbf{w}_u = [1.00 \;\; 0.87 \;\; 0.70 \;\; 0.50].$$

Pošto se rangira prema najvećem riziku, podskupovi Ω_b i Ω_c jesu

Since the rank order is calculated according to high level of risk, the subsets Ω_b and Ω_c are

$$\Omega_b = (C_1, C_2, C_3, C_4), \quad \Omega_c = \varnothing.$$

Korišćenjem računarskog programa FUZZY TOPSIS, koji su razvili autori ovog rada, dobijeni su odgovarajući rezultati, sumirani u sledećoj tabeli.

The corresponding results summarized in the following table are obtained by using computer program FUZZY TOPSIS developed by the authors of this paper.

Tabela 1. Sumarni rezutati
Table 1. Summary results

Rang alternative Rank of alternative	Očekivana udaljenost. alter. ED_i^* Expected distance of altern. ED_i^*		Očekivana relat. blisk. altern. ERC_i^* Expect. relat closen. of altern. ERC_i^*		$(KIC)_i$ %	Koef. varijanse alternative CV_i^* Coeffic. of var. of alternat. CV_i^*		$(KIV)_i$ %
1	$A_2=BS_2$	0.1203	$A_2=BS_2$	0.1142	28.7	$A_2=BS_2$	0.9089	28.6
2	$A_1=BS_1$	0.1402	$A_1=BS_1$	0.1322	28.1	$A_1=BS_1$	0.8455	26.5
3	$A_3=BS_3$	0.3141	$A_3=BS_3$	0.2846	23.1	$A_3=BS_3$	0.7510	23.5
4	$A_4=BS_4$	0.7684	$A_4=BS_4$	0.5651	14.1	$A_4=BS_4$	0.4795	15.0
5	$A_5=BS_5$	0.9584	$A_5=BS_5$	0.8129	6.0	$A_5=BS_5$	0.2028	6.4

Na osnovu dobijenih rezultata, datih u ovoj tabeli, može se zaključiti:

- Konstrukcija mosta BS_2 (alternativa A_2) ima najmanju očekivanu udaljenost od pozitivnog idealnog rešenja, tj. rešenja s najvećim nivoom rizika;
- Konstrukcija mosta BS_1 (alternativa A_1) ima sve karakteristične vrednosti koje su vrlo bliske vrednostima konstrukcije BS_2, pa tako ove dve konstrukcije imaju praktično isti nivo rizika i zahtevaju iste količine novca za održavanje;
- Konstrukcije mostova BS_4 i BS_5 imaju manje karakteristične vrednosti i manji nivo rizika, pa samim tim zahtevaju manju količinu novca za održavanje od konstrukcija BS_1 i BS_2;
- Redosledi na osnovu očekivane relativne bliskosti ERC_i^* i generalizovanog koeficijenta varijacije CV_i^* u ovom slučaju su isti;
- Koeficijenti raspodele investicija $(KIC)_i$ i $(KIV)_i$ u ovom slučaju su vrlo bliske vrednosti za sve konstrukcije mostova.
-

According to the obtained results, given in this table, the following may be concluded:

- Bridge structure BS_2 (alternative A_2) has the smallest value of the expected distance from ideal positive solution, i.e. solution with the highest values of degree of risk;
- Bridge structure BS_1 (alternative A_1) has all characteristic values that are very close to BS_2, so that these two structures have practically the same degree of risk and require the same amount of money for the maintenance;
- Bridge structures BS_4 and BS_5 have smaller characteristic values and smaller level of risk, so that they require smaller amount of money for the maintenance in comparison with structures BS_1 and BS_2;
- Rank list made by the expected relative closeness ERC_i^* and generalized coefficient of variation CV_i^* in this case are the same;
- Coefficients of investment distribution $(KIC)_i$ and $(KIV)_i$ are very close for all bridge structures in this case.

8 ZAKLJUČAK

Rasplinuti TOPSIS metod omogućava kompletnije, fleksibilnije i realnije modeliranje višekriterijumskog odlučivanja od nerasplinutog TOPSIS metoda s fiksnim vrednostima. U rasplinutom TOPSIS metodu moguće je uvesti neprecizne ulazne podatke za matricu odlučivanja i težine kriterijuma. Metod predložen u ovom radu zasnovan je na generalisanoj očekivanoj vrednosti i varijansi proizvoda elemenata matrice odlučivanja i težina kriterijuma. Za ove proizvode izvedene su odgovarajuće formule za njihovo tačno sračunavanje. Stoga, predloženi metod pruža donosiocu odluka tačnije i relevantnije rezultate nego klasični TOPSIS, što je važno za donošenje korisnih odluka. Ovaj metod, uz korišćenje pomenutog računarski programa, je upotrebljen za rangiranje alternativa, odnosno varijanti trase za buduću železničku prugu Pljevlja – Bijelo Polje – granica sa Kosovom i Metohijom, kao I za još neke investicione projekte. Metod može biti korisno upotrebljen za rangiranje alternativa i optimalnu raspodelu investicionih sredstava na projekte, optimalnu procenu rizika različitih tipova objekata, optimalan izbor objekata za rekonstrukciju, izbor najpovoljnijeg izvođača radova na tenderskim procedurama i u mnogim drugim slučajevima višekriterijumskog odlučivanja u građevinarstvu.

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8 CONCLUSION

Fuzzy TOPSIS method enables more complete, flexible and realistic modelling of the multiple criteria decision making problems then crisp TOPSIS method with crisp values. In the Fuzzy TOPSIS it is possible to introduce imprecise input data for the decision matrix and weights of criteria. The method proposed in this paper is based on the generalised expected values and variances of products of the decision matrix elements and weights of criteria. Thus, corresponding formulas and their exact calculation are derived for these products. Therefore, proposed method provides more accurate and relevant results for the decision maker in comparison with classic TOPSIS, which is important for useful decision making. This method with corresponding computer program is used for ranking traces of the future railway Pljevlja – Bijelo Polje – Border with Kosovo and Metohija, as for some ather investment projects. The method may be used successfully for ranking of alternatives and optimal distribution of investments on the projects, optimal risk assessment of different types of objects, optimal choice of objects for reconstruction, choice of the most acceptable contractor in tender procedures and in many other cases of multicriteria decision making in the Civil Engineering.

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REZIME

PRIMENA MODIFIKOVANOG RASPLINUTOG TOPSIS METODA ZA VIŠEKRITERIJUMSKE ODLUKE U GRAĐEVINARSTVU

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Nataša PRAŠČEVIĆ

U ovom radu predlaže se i primenjuje jedan modifikovani rasplinuti TOPSIS metod za višekriterijumsko rangiranje objekata za rekonstrukciju i održavanje. Na početku se daje kratak osvrt na nastanak i razvoj ovog metoda i opisuje se TOPSIS procedura s fiksnim (nerasplinutim) ulaznim podacima koji sačinjavaju matricu odlučivanja i težinske koeficijente kriterijuma. Ova procedura se ilustruje jednim jednostavnim brojčanim primerom. Objašnjava se neophodnost prikazivanja ovih parametara - kao trougaonih rasplinutih brojeva - zbog nemogućnosti njihovog preciznog određivanja ili procenjivanja u praksi. U radu se daju tačni izrazi, koje su autori ranije izveli, za određivanje proizvoda elemenata matrice odlučivanja i težinskih koeficijenata kao trougaonih rasplinutih brojeva. Ovi parametri za svaku alternativu (objekat) tretiraju se kao slučajne rasplinute veličine, za koje se određuju tačne generalisane očekivane vrednosti, varijanse i standardne devijacije. Iz normalizovane matrice očekivanih vrednosti određuju se očekivana idealna pozitivna i očekivana idalna negativna rešenja. Za svaku alternativu određuju se generalisane očekivane distance i relativne bliskosti ovim rešenjima, kao i odgovarajuće varijanse i koeficijenti varijacije. Alternative se rangiraju prema ovim vrednostima. U radu se predlažu izrazi za sračunavanje koeficijenta raspodele investicionih sredstava na (alternative) objekte. Na kraju, dat je jedan primer rangiranja mostovskih konstrukcija u odnosu na rizik i formulisani su odgovarajući zaključci.

Ključne reči: Rasplinuti (fuzzy) TOPSIS, rasplinuti broj, održavanje objekata, raspodela investicionih sredstava,upravljanje rizikom.

SUMMARY

APPLICATION OF MODIFIED FUZZY TOPSIS METHOD FOR MULTICRITERIA DECISIONS IN CIVIL ENGINEERING

Živojin PRASCEVIC
Nataša PRASCEVIC

In this paper is presented and applied one fuzzy TOPSIS method for the multicriteria rancing of objects for reconstruction and maintenance. At the beginning is given short review on the genesis and development of this method and described a TOPSIS procedure with crisp input data that constitute a decision matrix and weights of criteria. This procedure is illustrated by one simple numerical example. The necessity of presentation of these parameters as triangular fuzzy numbers due to impossibility of their precise determination or assessment in the practice. The exact expressions for the determination of these products of the decision matrix and weights coefficients as triangular fuzzy numbers, that authors of this paper are derived earlier, are given in the paper. For every alternative (the object) these parameters are assumed as random fuzzy numbers for which are determined generalised expected values, variances and standard deviations. From the normalised matrix of the expected values are determined expected ideal positive and ideal negative values. For every alternative are determened generalized expected distances and relative closenesses to the ideal positive and ideal negative solution. The ranking of alternatives is performed according to these values. Matematical expressions for coefficients of investments distribution on the alternatives (objects) are proposed in the work. One example of ranking of the bridge structures according to the risk is given at the end of the work and formulated corresponding conclusions.

Key words: Fuzzy TOPSIS, fuzzy number, maintenance of objects, distribution of investments, risk management.