APPLICATION OF MULTIPLE CRITERIA GOAL PROGRAMMING FOR SOLVING SOME MANAGEMENT AND PRODUCTION PROBLEMS


PRIMENA VIŠEKRITERIJUMSKOG CILJNOG PROGRAMIRANJA ZA REŠAVANJE NEKIH PROBLEMA MENADŽMENTA I PROIZVODNJE

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SUMMARY

One method for solving multiple criteria fractional goal programing problem is proposed in this paper. In this method are introduced deviational variables that express increases or decreases of objective functions related to their given goals. The fractional objective functions are transformed to unfractional ones that contains deviational varijables. An auxiliary objective function, which contains deviational variables only, is formulated and the problem is solved as a nonlinear or linear progam. One example related to the optimal program of production in some production plant is presented in the paper.

Key words: *Multiple criteria programming, fractional programming, goal programming.*

REZIME

U ovom radu se predlaže jedan postupak za rešavanje problema mutikriterijumskog razlomačkog ciljnog programiranja. Uvode se devijacione promenljive koje izražavaju povećanje ili smanjenje funcija kriterijuma u odnosu na definisane ciljeve koji se žele postići. Funkcije cilja transformišu se u obične funkcije ograničenja koje sadrže devijacione promenljive. Formuliše se pomoćna funkcija kriterijuma koja sadrži samo devijacione promenljive i problem rešava kao zadatak nelinearnog ili linearnog programiranja. U radu je urađen jedan primer za određivanje optimalnog programa proizvodnje.

Ključne reči: *Višekriterijumsko programiranje, razlomačko programiranje, ciljno programiranje.*

1. INTRODUCTION

The idea of Goal programming (GP) apeared firstly in the work of Charnes, Cooper and Ferguson (1955) and gained popularity during the 1960s and 70s years of the last century as a method for modelling of single and multiple criteria problems. In GP are estabilished goals or thresholds that could be achieved to solve the given problem. Fractional programming with a linear and fractional objective function and linear constraints was solved by Isabell and Marlow (1956), Charnes and Cooper (1962), Martos (1964), Wagner and Yuan (1968), Tantawy (2007), Caballero and Hernandez (2010) and others. Some developments in generalized fractional programming are described by Barros at al. (1996) and Shaible and Shi (2000). Many authors have proposed different methods for solving programs with nonlinear fractional objective functions. Some of these methods are explained more detailly in the books and works written by Martos (1975), Polunin (1975), Schaible (1976), Bazaraa and Shetty (1979), Krčevinac *at al*. (1983), Zlobec and Petrić (1989), Winston (1994), Prascević (2009) and others.

As Steuer (1984, p. 282) emphasises, goal programming is distinguished from lineaar programming by:

1. The conceptualization of objectives as goals.
2. The assignment of *priorities* and/or *weights* to the achievement of the goals.
3. The presence of deviational variables d_i^+ and d_i^- to measure *overachievement* and *underachievement* from target (or threshold) levels $z_{0,i}$.
4. The minimization of weighted sum of deviational variables to find that best satisfy the goals.

Here are given several important applications of the fractional and goal fractional programming in the econ-

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omy and management as well as in other fields. These applications in which objective are described by the fractional function are:

1. Productivity *P,* which represents quantity of production *Q* devided by some factor of production *F* (costs, working power, etc).
2. Profitability of production *PF*, which represents realized profit *Pr* divided by capital *K* invested in the production.
3. Specific risk of investment *SR*, which represents expected risk of investment *R* divided by kapital *K* invested in the production.
4. Kvotient between output *O* from production and input into production *I*.

In this paper are considered multiple criteria problems with fractional objective functions, that apears in many different poblems of practice as efficiency, profitability, productivity, specific risk etc. In the previous author's paper (Praščevic, 2010) was considered and proposed method with deviational variables to solve the GP problem with one objective function. For all these objective functions z_i are prescribed or previously determined goals $z_{0,i}$ (i=1,2,…,k) which should be *overachieved* for *maximization* or *underachieved* for *minimization* of the objective function z_i.

2. MULTIPLE CRITERIA FRACTIONAL GOAL PROGRAMMING PROBLEM

A multiple criteria fractional programing problem is generally formulated to find values of criteria functions

$$\text{Max (min) } z_i = \frac{P_i(\mathbf{x})}{Q_i(\mathbf{x})} (\leq, \geq)\, z_{0,i}, \quad Q_i(x) > 0, \quad i =, 2, ...k, \tag{1}$$

subject to constraints

$$g_i(\mathbf{x}) \leq 0,\ \mathbf{x} \geq \mathbf{0},\ i = 1, 2, ...\ m; \quad \mathbf{x} = [x_1, x_2, ..., x_n]^T \tag{2}$$

where $z_{0,i}$ are targets (thresholds) or goal levels of achievement.

As Steuer (1986) points out, usually a solution that satisfies all goals is not possible, so shold be faund the compromise solution that satisfies these goals "as closelly as possibile". If some of criteria (1) has the sign "≤", than that criterion should be minimized, and if has the sign "≥" that criterion shoud be maximized.

To solve this problem in the literature exist several models: *Archimedian*, *Preemptive* and *Lexicographic*, which are described by Steuer (1985), Krčevinac at al. (Winston ,1985) and other authors. Here is used so called Archimedian model which is based on introduction of deviations from the prescribed goals values.

In this paper is proposed method to solve te problem by multipling criteria (1) by the functions $Q_i(\mathbf{x})$, which gives

$$P_i(\mathbf{x}) - z_{0,i} Q_i(\mathbf{x}) (\leq, \geq) 0. \quad i=1,2,...,k. \tag{3}$$

Conditions (3) in the further consideration are regarded as constraints. Introducing positive $d_i^+ \geq 0$ and negative $d_i^- \geq 0$ deviations from the goals, conditions (3) may be written as equations:

– for minimization of the criterion z_i with sign "≤",

$$P_i(\mathbf{x}) - z_{0,i} Q_i(\mathbf{x}) + d_i^+ - d_i^- = 0, \tag{4}$$

– for maximization of the criterion z_i with sign "≥".

$$P_i(\mathbf{x}) - z_{0,i} Q_i(\mathbf{x}) - d_i^+ + d_i^- = 0, \tag{5}$$

Deviational variables d_i^+ and d_i^- are shown in Fig. 1 as slack variables. The variable d_i^+ affects on an increase of objective function z_i to overachieve goal $z_{0,i}$, while d_i^- affects its decrease to underachieve goal $z_{0,i}$ when the objective function is maximised, and vice versa when tis function is minimised. Because of that, d_i^+ should have maximal value and d_i^- minimal value. From these resons, an auxiliary objective functin may be formulated in the form

$$\max y = \sum_{i=1}^{k} (d_i^+ - d_i^-), \tag{6}$$

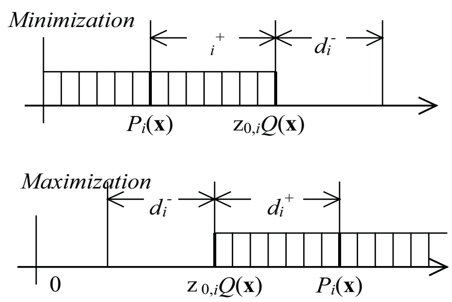

Figure 1. Deviational variables

In the practical applications of multicriteria programming usually considered criteria have different importance for decision makers. Because of that, the weight coefficients w_i are introduced for all prescribed criteria z_i, that form the vector of weights $\mathbf{w} = [w_1, w_2, ..., w_k]$, for which is valid $w_1 + w_2 + ... + w_k = 1$

Taking this into account, the objective function (6) becomes

$$\max y = \sum_{i=1}^{k} w_i (d_i^+ - d_i^-). \tag{7}$$

The linear auxiliary objective function (7) and constraints (2), (4) and (5) constitute a nonlinear program. Solving this program, the values of variables x_j (j=1,2,...,n) and deviational variables d_i^+ and d_i^- are obtained, and, after that, values of objective functions z_i (i=1,2,...,k) are calculated. If $d_i^+ = 0$ and $d_i^- > 0$, then the goal $z_{0,i}$ is not achievable.

3. MULTIPLE CRITERIA FRACTIONAL LINEAR GOAL PROGRAMMING PROBLEM

If functions $P_i(\mathbf{x})$ and $Q_i(\mathbf{x})$ (i = 1,2,...,k) are linear ones

$$P_i(\mathbf{x}) = \sum_{j=1}^{n} p_{ij} x_j + \mathsf{a}_i; \quad Q_i(\mathbf{x}) = \sum_{j=1}^{n} q_{ij} x_j + \mathsf{b}_i;$$

$$i = 1,2,...,k; \tag{8}$$

and constraints (2) are a system of linear inequalities

$$\sum_{j=1}^{n} a_{ij} x_j \le b_j; \; x_j \ge 0; \;\; i = 1,2,...,k; j = 1,2,...n; \tag{9}$$

then the multiple criteria fractional goal programming problem becomes the multiple criteria fractional linear goal programming (FLGP) problem. In this case equations (4) and (5) are

$$\sum_{j=1}^{n} (p_{ij} - z_{0,i} q_{ij}) x_j + \mathsf{a}_i - z_{0,i} \mathsf{b}_i + d_i^+ - d_i^- = 0$$

$$\sum_{j=1}^{n} (p_{ij} - z_{0,i} q_{ij}) x_j + \mathsf{a}_i - z_{0,i} \mathsf{b}_i - d_i^+ + d_i^- = 0$$

or written in the shorter form when the fununction of criterion z_i is minimized

$$\sum_{j=1}^{n} r_{ij} x_j + d_j^+ - d_j^- = c_i; \;\; \mathrm{i} = 1,2,...,\mathrm{k}; \tag{10}$$

and when fununction of criterion z_i is maximized

$$\sum_{j=1}^{n} r_{ij} x_j - d_j^+ + d_j^- = c_i; \;\; \mathrm{i} = 1,2,...,\mathrm{k}; \tag{11}$$

where

$$r_{ij} = p_{ij} - z_{0,i} q_{ij}; \quad c_i = -\mathsf{a}_i + z_{0,i} \mathsf{b}_i;$$

$$i = 1,2,...,k; j = 1,2,...,n. \tag{12}$$

The auxiliary objective function remains in the form (7).

The problem of solution of multiple critriteria fractional linear goal programming is transformed into the problem of linear programming with objective function (7) and constraints (10) or (11). Solving this linear program by the Simplex method real variables x_i (i = 1,2,...,n), deviational variables d_i^+ i d_i^- (i =1,2,...,k) and values of criteria functions z_i (i =1,2,...,k) are obtained.

In this case, fractional objective functions (1) represents *hyperbolic function*, so Martos in his works this type of matematical programming calls *hyperbolic programming* (Martos,1964, 1975). When are given two variables x_1 and x_2, then objective function z_i is a *helicoidal surface*, as it shown in Fig. 2.

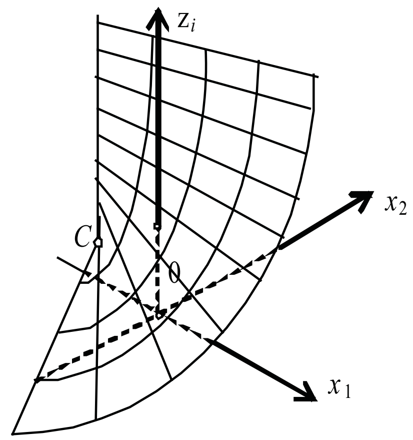

Fig 2. Helicoidal surface

According to proposed procedure a computer program is written in MATLAB programming language.

4. EXAMPLE – OPTIMAL PROGRAM OF PRODUCTION

In one production plant are produced two types of products PR_1 and PR_2 using machines M_1 and M_2. On te next table are given daily working time, times in working hours of processing of these products on the machines, planed profit, invested capital and risk.

Table 1. Input data

	PR1	PR2	Work. Time
M1 (h/prod)	0.16	0.16	8.00 h
M2 (h/prod)	0.10	0.20	8.00 h
Profit	40	60	-
Inv. Capital (€/prod)	100	120	-
Risk (%/prod)	8	12	-

During eight hours of production is planned profit 100 € with additional capital of 110 € independently of the realized production. Minimal summary deily production is 20 produkts PR_1 or PR_2. The problem is to formulate corresponding mathematical model and find optimal daily plan of production, i.e. number of products PR_1 and PR_2, to gain maximal profitability PF with goal $z_{0,1}$ = 0.4 and minimal specific risk SR with goal $z_{0,2}$ = 0.10.

Unknown numbers of products PR_1 and PR_2 are denoted as $x_1 \ge 0$ and $x_2 \ge 0$.

Maximal profitability PF, as quotient between the profit Pr and invested capital K, is

$$\max PF = \max z_1 = \frac{40x_1 + 60x_2 + 100}{100x_1 + 120x_2 + 110} \ge 0.40. \tag{a}$$

Minimal specific risk of investment, as quotient between the whole risk R and invested capital K is

$$\min SR = \min z_2 = \frac{0.08 \times 100x_1 + 0.10 \times 120x_2 + 0.5 \times (0.08 + 0.12) \times 110}{100x_1 + 120x_2 + 110} \le 0.10$$

or

$$\min SR = \min z_2 = \frac{8x_1 + 12x_2 + 11}{100x_1 + 120x_2 + 110} \le 0.10. \tag{b}$$

The constraints, that are related to technological conditions of production, are according to data given in Table 1

$$0.16x_1 + 0.16x_2 \le 8.00,$$
$$0.10_1x_1 + 0.20x_2 \le 8.00, \qquad (c)$$
$$x_1 + x_2 \ge 20.$$

According to proposed procedure, the auxiliary objective function (8) is for the same importance of both criteria $w_1 = 0.50$, $w_2 = 0.50$

$$\max y = 0.5d_1^+ - 0.5d_1^- + 0.5d_2^+ - 0.5d_2^-. \qquad (d)$$

Coefficients r_{ij} and c_i in constraints (10) and (11) according to expressions (12) are

$r_{11} = 40 – 0.40×100 = 0$,
$r_{12} = 60 – 0.40×120 = 12$,

$r_{21} = 8 – 0.10×100 = -2$,
$r_{22} = 12 – 0.10×120 = 0$.

$c_1 = -100 + 0.40×110 = – 56$,
$c_2 = 11– 0.10×110 = 0$.

Constraints (10) and (11) with these values of coefficients r_{ij} are:

$$12x_2 - d_1^+ + d_1^- = -56; \quad -2x_1 + d_2^+ - d_2^- = 0. \qquad (e)$$

After solving the auxiliary linear program with the objective function (*d*) and constraints (*c*) and (*e*), next values of variables and objective functions are obtained:

$x_1 = 0$ products PR_1, $x_2 = 40$ products PR_2,

$$d_1^+ = 536, \; d_1^- = 0, \; d_2^+ = 0, \; d_2^- = 0.$$

$P_1(\mathbf{x}) = 2\,500$, $Q_1(\mathbf{x}) = 4\,910$,
$z_1(\mathbf{x}) = 2\,500/4\,910 = 0.5092 > z_{0,1} = 0.400$;

$P_2(\mathbf{x}) = 491$, $Q_1(\mathbf{x}) = 4\,910$,
$z_2(\mathbf{x}) = 491/4\,910 = 0.1000 = z_{0,2} = 0.100$.

3. CONCLUSION

Proposed method, based on deviational functions, transforms multiple criteria fractional goal programming problem to one criteria nonlinear programing problem with real and deviational variables and the linear objective function which contains deviational variables only. In the case, when criteria function are linear ones, the multiple criteria fractional goal program is transformed to the linear programming problem, which is easy solvable by the Simplex method. According to the proposed procedure, the authors have wriiten out corresponding computer program in MATLAB programming language.

4. APPENDIX - COMPUTER PROGRAM "FRACT_GOAL_MCLP.M"

```
%==================================================================
% Program Fract_Goal_MCLP.m solves Multicriteria Fractional Goal Program    with linear constraints.
% max(min)[sum(P(i,j)*x(j))+alfa(i)]/[sum(Q(i,j)*x(j))+beta(i)](<=,>)b(i)
%  i =1,2,..., ncrit,
% with constraints
%   sum A(i,j)*x(j)(<=,>)b(i),  i=1,2,..nunk.
% Program written by N. Prascevic
clear all
%------------------------------------------------------------------
% INPUT DATA
%------------------------------------------------------------------
fid=fopen('FR_MCLP_REZULTS.txt'.'w')
%------------------------------------------------------------------
% Example 1 „OPTIMAL PLAN OF PRODUCTION“
%------------------------------------------------------------------
ncrit=2  %<---- number of criteria
nunk=2   %<---- number of unknowns
ncon=3   %<---- number of constraints
% Matrix P
  P=[40 60; 8 12]
% Matrix Q
  Q=[100 120; 100 120]
% Vector alfa
  alfa=[100 11]
% Vector beta
  beta=[110 110]
% Threshold vector z0
   z0=[0.4 0.1]
% Vector of criteria type: s(i)= 1 for maximization of criteria i
%                        s(i)=-1 for minimization of criteria i
  s=[1 -1]
% Weights of criteria
```

```
  w=[0.5 0.5]
% Matrix of constraints A
  A=[.16  .16; .10  .20; -1 -1]
% Vector of constraints
  b=[8 8 -20]'
%-----------------------------------------------------------------%  End  of  input
data
%-----------------------------------------------------------------%   Determination
of auxiliary objective function y(j)
%-----------------------------------------------------------------
for j=1:nunk
    y(j)=0;
end
for jp=1:ncrit
        y(nunk+2*jp)=-w(jp);
        y(nunk+2*jp-1)=w(jp);
end
%-----------------------------------------------------------------
% Calculation of matrices Pn, D and matrix An
%-----------------------------------------------------------------
for i=1:ncrit
    for j=1:nunk
        Pn(i,j)=P(i,j)-z0(i)*Q(i,j);
    end
end
for i=1:ncrit
    for j=1:2*ncrit
      D(i,j)=0;
    end
end
for i=1:ncrit
    if s(i)<0
        D(i,2*i)=-1;
        D(i,2*i-1)=1;
    else
        D(i,2*i)=1;
        D(i,2*i-1)=-1;
    end
end
lb=[zeros(nunk+2*ncrit,1)];
An=[Pn D;-Pn -D;A zeros(ncon,2*ncrit)];
%-----------------------------------------------------------------
% formulation of vector bn
%-----------------------------------------------------------------
for i=1:ncrit
    bp(i)=z0(i)*beta(i)-alfa(i);
end
%bn=[bp';-bp;b]
bn=[bp';-bp';b]
%-----------------------------------------------------------------
% Solving auxiliary linear programming program and determination of unknowns
%-----------------------------------------------------------------
%[xn,f]=linprog(-y,An,bn,[],[],lb)
for i=1:nunk
    x(i)=xn(i);
end
for i=1:ncrit
    dplus(i)=xn(nunk+2*i-1);
    dminus(i)=xn(nunk+2*i);
end
for i=1:ncrit
    z1(i)=x*P(i,:)'+alfa(i);
    z2(i)=x*Q(i,:)'+beta(i);
    z(i)=z1(i)/z2(i);
end
%=================================================================
% Printing of results
%=================================================================
fprintf(fid,'\n\n  OPTIMAL PLAN OF PRODUCTION\n')
fprintf(fid,'  ==================================================\n')
fprintf(fid,'\n  Inut data\n')
```

```
    fprintf(fid,'  ---------\n')
    fprintf(fid,'\n  Number of criteria = %2.0f\n',ncrit)
    fprintf(fid,'\n  Number of constraints = %2.0f\n',ncon)
    fprintf(fid,'\n  Number of unknowns = %2.0f\n',nunk)
    fprintf(fid,'\n  Matrix of coefficijents P(i,j)\n\n')
    for i=1:ncrit
        fprintf(fid,'   %6.2f',P(i,:))
        fprintf(fid,'\n')
    end
    fprintf(fid,'\n  Matrix of coefficients Q(i,j)\n\n')
    for i=1:ncrit
        fprintf(fid,'   %6.2f',Q(i,:))
        fprintf(fid,'\n')
    end
    fprintf(fid,'\n  Values alfa(i) and beta(i)\n\n')
    for i=1:ncrit
        fprintf(fid,'    alfa(%2.0f) = %6.2f      beta(%2.0f) = %6.2f\n',i,alfa(i),i,beta
(i))
    end
    fprintf(1,'\n   Type of criteria functions\n')
    for i=1:ncrit
        if s(i)==1
            fprintf(fid,'\n    Criterion (%2.0f) is maximized\n',i)
        else
            fprintf(fid,'\n    Criterion (%2.0f) is minimized\n',i)
        end
    end
    fprintf(fid,'\n  Coefficients of weghts for criteria\n\n')
    for i=1:ncrit
        fprintf(fid,'    w(%2.0f) = %7.3f\n',i,w(i))
    end
    fprintf(1,'\n\n  Conditions of constraints Ax <= b\n')
    fprintf(fid,'\n  Matrix of constraints A(i,j) and vector b(i)\n\n')
    for i=1:ncon
        fprintf(fid,'   %7.2f  %7.2f',A(i,:),b(i))
        fprintf(fid,'\n')
    end
    fprintf(fid,'\n\n  Output data\n')
    fprintf(fid,'  ------------\n')
    fprintf(fid,'\n  Values of variables x(j)\n')
    for j=1:nunk
        fprintf(1,'\n    x(%2.0f) = %8.3f\n',j,x(j))
    end
    fprintf(1,'\n  Deviational variables\n')
        for icr=1:ncrit
            fprintf(fid,'\n    Criterion %2.0f\n',icr)
            fprintf(fid,'\n     dplus(%2.0f) = %8.3f    dminus(%2.0f) = %8.3f\n',icr,dplu
s(icr),icr,dminus(icr))
        end
        for icr=1:ncrit
            fprintf(fid,'\n    Criterion (%2.0f)\n',icr)
    fprintf(fid,'\n    functions: z1(%2.0f) = %10.3f   z2(%2.0f) = %10.3f\n',icr,z1(icr),
icr,z2(icr))
            if s(icr)==1
                        fprintf(fid,'\n      Criteria  function   Max  z(%2.0f)  =  %10.5f\
n',icr,z(icr))
                fprintf(fid,'\n    Threshold value Min z0(%2.0f) = %10.4f\n',icr,z0(icr))
            else
    fprintf(fid,'\n    Criteria function  Min z(%2.0f)=      %10.5f\n',icr,z(icr))
    fprintf(fid,'\n    Threshold value Max z0(%2.0f) =  %10.4f\n',icr,z0(icr))
            end
       end
    %-------------------------------------------------------------------
    % Achievement of criteria thresholds
    %-------------------------------------------------------------------
    tr=0;
    for i=1:ncrit
        if dminus(i)>0
    fprintf(fid,'\n\n    Thresholds value %8.3f for the criterion %2.0f is not achievable\
n',z0(i),i)
          tr=tr+1;
```

```
    end
end
if tr==0;
fprintf(fid,'\n\n    All threshold values for the criteria are achievable\n')
end
%---------------------------------------------------------------------
% End of the program
%---------------------------------------------------------------------
```

NUMERICAL DATA

```
=====================================================================

  Inut data
  ---------

  Number of criteria =  2
  Number of constraints =  3
  Number of unknowns =  2

  Matrix of coefficijents P(i,j)

    40.00    60.00
     8.00    12.00

  Matrix of coefficients Q(i,j)

   100.00   120.00
   100.00   120.00

  Values alfa(i) and beta(i)

    alfa( 1) = 100.00      beta( 1) = 110.00
    alfa( 2) =  11.00      beta( 2) = 110.00

  Criterion (1) is maximized
  Criterion (2) is minimized

  Coefficients of weghts for criteria
    w( 1) =   0.500,  w( 2) =   0.500

  Matrix of constraints A(i,j) and vector b(i)

      0.16     0.16      8.00
      0.10     0.20      8.00
     -1.00    -1.00    -20.00


  Output data
  ------------

  Values of variables x(j)

  x(1)= 0.00  x(2)= 40.00

    Criterion  1
     dplus( 1) =  536.000    dminus(1) =    0.000

    Criterion  2
     dplus(2) =    0.000    dminus(2) =    0.000

    Criterion (1)
    functions: z1( 1) =   2500.000   z2( 1) =   4910.000
    Criteria function  Max z( 1) =    0.50916
    Threshold value Min z0( 1) =     0.4000

    Criterion (2)
    functions: z1(2) =    491.000   z2(2) =   4910.000
    Criteria function  Min z( 2) =    0.10000
    Threshold value Max z0( 2) =     0.1000
```

All treshold values for the criteria are achievable

Using this procedure and corresponding computer program many problems of construction management and production as optimal efficiency, profitability, productivity, specific risk, etc can be solved.

REFERENCES

[1] Barros, A. I., Frenk, J. B. G., Schaible, S. and Zhang, S. "A new algorithm for generalized fractional programs", *Mathematical Programming* Vol. 72, **2**, 1996, pp. 147 – 175.

[2] Bazaraa, M. S., Shetty, C. M., "*Nonlinear Programming, Theory and Algorithms*", John Wiley, New York, 1979.

[3] Caballero, R., Hernandez, M.,"Resolution of the Linear Fractional Goal Programming Problem", *Revista Electronica de Comunications y Trobayas de ASEPUMA*, Vol. 11, 2010, pp. 27 – 40.

[4] Charnes, A., Cooper, W. W., and Ferguson, R. O., "Optimal Estimation of Executive Compensation by Linear Programming" , *Management Science*, Vol. 1, No 2, 1955, pp. 138 – 151.

[5] Charnes, A. and Cooper, W. W., "Programming with linear fractional functions", *Naval Research Logistics Quarterly*, Vol. 9, 1962, pp. 181– 186.

[6] Isabell, J. R. and Marlow, W. H., "Attrition games". *Naval Research Logistics Quarterly*, Vol. 3, 1956, pp. 1– 9.

[7] Krčevinac, S., Čupić, M., Petrić, J., Nikolić, I., *Algoritmi i programi iz operacionih istraživanja*, Naučna knjiga, Beograd, 1983.

[8] Martos, B., „Hyperbolic programming", *Naval Research Logistics Quarterly*, Vol. 9, 1964, pp. 181–186.

[9] Martos, B.,"*Nonlinear Programming*", Theory and Methods", North-Holland, Amsterdam, 1975.

[10] Schaible, S., „Fractional Programming I, Duality", *Management Sciences*, Vol. 22, 1976, pp. 858 – 867

[11] Schaible, S. and Shi, J., "Recent Developments in Fractional Programming: Single Ratio and Max-min Case", *Report Japan Society for the Promotion of Sciences,* 2000, pp 1-11.

[12] Steuer, R. E. *Multiple Criteria Optimization*, John Wiley&Sons, New York., 1984.

[13] Winston, W. L., *Operations Research – Applications and Algorithms*, Third edition, Duxbury Press, Belmont, Cal., 1994.

[14] Praščević, N., "Jedan metod za rešavanje problema razlomljenog programiranja sa primenama u građevinskom menadžmentu", *Treći Int. Naučno-stručni skup Građevinarstvo nauka i praksa*, Zabljak, 2010, str.1793– 1807.

[15] Полунин, И. Ф., "*Курс математического програмирования*", Вышэйшая школа, Минск, 1975.

[16] Tantawy, S. F.,"A New Method for Solving Linear Fractional Programming Problems", Australian Journal of Basic and Applied Sciences, Vol. 1, No 2,2007, pp. 105-108.

[17] Praščević, Ž., Praščević, N., „Operaciona istraživanja u gradjevinarstvu", Čugura print, 2009.

[18] Zlobec, S., Petrić, J.,"Nelinearno programiranje", Naučna knjiga, Beograd, 1989.