Applying causality and bicausality to multi-port elements in Bond Graphs

Eric Bideaux, Diana Martin de Argenta, Wilfrid Marquis-Favre, Serge Scavarda

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Laboratoire d'Automatique Industrielle
E. Bideaux, D. Martin de Argenta, W. Marquis-Favre and S. Scavarda
INSA de Lyon
20, Avenue A. Einstein
F-69621 Villeurbanne Cedex
Tel : (33) 0-4.72.43.89.78 / Fax : (33) 0-4.72.43.85.35
Email : bideaux@lai.insa-lyon.fr / favre@lai.insa-lyon.fr / scavarda@lai.insa-lyon.fr

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Abstract :
The introduction of the bicausality concept in the bond graph language has allowed new analytical methodologies of a system, for instance in the context of model inversion, mechatronic system sizing and control. The causality assignment generally imposes the way these constitutive relations have to be used. In the case of linear multi-port elements, derivative causality or of bicausality is not necessarily possible. The conditions for the existence of a causal configuration are related to the form of the constitutive relation of the multi-port element. In this paper, we propose to inspect this condition and then to focus on the use of the causality applied to the linear multi-port elements. We show that the constitutive relations of any linear multi-port element may be used to determine quickly what kind of causality assignment does exist and what could be determined using different schemes of calculus. It clearly appears that this approach may be applied in other contexts and may have interesting applications on system sizing, identification and control.

1 - INTRODUCTION

The bicausality concept introduced implicitly by Cornet and Lorenz [Cornet and Lorenz 88] and more formally by Gawthrop [Gawthrop 95, Gawthrop 97], has initiated a new philosophy in regards to a bond graph model. It allows to force the value of the power by imposing effort and flow on a bond or the value of one power variable on a junction (the other being zero). This approach has to be related to a mathematical research in an oriented graph. In "standard" causality, the imposed variable on an element is either the effort, or the flow variable. The meaning of this statement is that we assume that we know or impose the behaviours of the flow and the effort at a specific place in the system, and this approach allows the study of some kind of inverse problems. This principle has been successfully applied in design or sizing problems [Fotsu Ngwompo et al. 96, Fotsu Ngwompo et al. 97] and in control synthesis [Gawthrop et al. 99].

The causal stroke of "bicausal" bond is seen as half strokes; each of these half stroke is associated to an effort and a flow variable that can be placed independently at each end of the bond and impose the corresponding variable (using the convention of the normal causality). Causal half strokes indicate the fixed or known variables of the bond and therefore determine the way to solve the modeled system. This approach is also consistent for normal causality where it is still possible to split the causal stroke in one effort and one flow half causal stroke.

In this paper, this approach is used to solve the problem of the existence of a causal and bicausal configuration in the case of multi-port elements. In a first part, the problem is limited to the existence of a causal configuration in the case of 2-port elements, this condition is then extended to n-port elements. In the following part, the use of the bicausality is developed and the use of the obtained bicausal configuration are discussed in the case of 2-port elements. These results are still usable in the case of n-port elements. The study of a physical application will illustrate the consequences of this work in solving the reversibility of systems with multi-port elements.

2 – EXISTENCE OF PARTIAL DERIVATIVE FORM OF 2-PORT ELEMENTS

Any linear multi-port element is characterized by its constitutive relations, which exist, and are known in a preferred causality assignment [Karnopp et al.90, Breedveld 84]. As the condition of existence of a causal configuration may be generalized to any kind of multi-port element, the constitutive relation will firstly be represented by an input-output relation. For an R-element depending on the preferred causal configuration the input variable may be the flow (respectively the effort) and the output the effort (respectively the flow). In the case of a C-element (respectively I-element), the input variable to the constitutive relation is the generalized displacement (respectively the generalized momentum) and the output the effort (respectively the flow). The proposed approach is also usable for any other multi-port element, although in each case the results may be adapted and/ or restricted. The problem is then identical and we will consider the following formulation:

$$\mathbf{\Psi} = A\mathbf{\Xi} \text{ i.e. } \begin{bmatrix} \psi_1 \\ \vdots \\ \vdots \\ \psi_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & \cdots & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \vdots \\ \vdots \\ \xi_n \end{bmatrix} \tag{1}$$

Let us firstly limit this approach to 2-port elements, that is to say to consider a 2x2 system of linear equations :

$$\mathbf{\Psi} = A\mathbf{\Xi} \text{ with } A = \begin{bmatrix} \mathrm{a}_{11} & \mathrm{a}_{12} \\ \mathrm{a}_{21} & \mathrm{a}_{22} \end{bmatrix} \tag{2}$$

Non-preferable causal form

The existence of the non-preferable causal form simply corresponds to the possibility to express the inputs as a function of the outputs, that is to inverse the system:
Let us note B the matrix obtained in non-preferred causality. B exists if and only if $Det(A) \neq 0$.

$$\mathbf{\Xi} = B\mathbf{\Psi} = A^{-1}\mathbf{\Psi} \text{ with } B = \frac{1}{Det(A)} \begin{bmatrix} \mathrm{a}_{22} & -\mathrm{a}_{21} \\ -\mathrm{a}_{12} & \mathrm{a}_{11} \end{bmatrix} \tag{3}$$

Hybrid causal form

In this case, one of the input became an output and its conjugate power variable an input. Let us assume the existence of this form. The new formulation is for example:

$$\begin{bmatrix} \xi_1 \\ \psi_2 \end{bmatrix} = \mathrm{B} \begin{bmatrix} \psi_1 \\ \xi_2 \end{bmatrix} \text{ with } B = \frac{1}{a_{11}} \begin{bmatrix} 1 & a_{12} \\ a_{21} & a_{11}a_{22} - a_{12}a_{21} \end{bmatrix} \tag{4}$$

The condition of existence is then that $a_{11} \neq 0$. For the second hybrid form, the formulation would have been $a_{22} \neq 0$.

Conclusion

If only one equation has to be reversed, it is not necessary that the system is reversible. In the case of a 2-port element, the condition of the existence of hybrid causality is the coefficient linking the output to the input in the equation which has to be reversed, being non-zero. This is still correct in the case of an n-port element if only one bond is not in preferred causality. If the i^{th} output (respectively the i^{th} input) became an input (respectively an output), the resolution consists in changing the form of the i^{th} equation (5) using ξ_i as a new output and ψ_i as a new input. The new relation is then given by (6) and has to be introduced in the other equations as shown in (7).

$$\psi_i = a_{i1}\xi_1 + \cdots + a_{ii}\xi_i + \cdots + a_{in}\xi_n \tag{5}$$

$$\xi_i = -\frac{1}{a_{ii}}\left(a_{i1}\xi_1 + \cdots - \psi_i + \cdots + a_{in}\xi_n\right) \tag{6}$$

$$\begin{bmatrix} \psi_1 \\ \vdots \\ \xi_i \\ \vdots \\ \psi_n \end{bmatrix} = \begin{bmatrix} a'_{11} & \cdots & \frac{a_{1i}}{a_{ii}} & \cdots & a'_{1n} \\ \vdots & \ddots & & & \vdots \\ -\frac{a_{i1}}{a_{ii}} & \cdots & \frac{1}{a_{ii}} & \cdots & -\frac{a_{n1}}{a_{ii}} \\ \vdots & & & \ddots & \vdots \\ a'_{n1} & \cdots & \frac{a_{ni}}{a_{ii}} & \cdots & a'_{nn} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \vdots \\ \psi_i \\ \vdots \\ \xi_n \end{bmatrix} \tag{7}$$

with $a'_{kl} = \left(a_{kl} - a_{ki}\frac{a_{il}}{a_{ii}}\right) \quad k \neq i,\ l \neq i$

3 – EXTENSION TO N-PORT ELEMENTS

In the case of an n-port element, it may be requested that a subpart of the system is forced in non-preferred causality. Let us note $r = \{r_1, \cdots, r_m\} \subset \{1, \cdots, n\}$ the indices of these equations. The causal form exists if and only if it is possible to express the new input set as a function of the new output set. This is equivalent to express in each equation in the subset r, the output as a function of the new input set, that is to inverse this subpart of the system. As the variables associated to the equations which remain in preferred causality, do not influence the reversibility of the equations in the subset r, the condition of existence is given by the non-nullity of the determinant of the matrix R (8). The square matrix R is constituted of the elements of the initial matrix A after the cancellation of the column and the row corresponding to the complementary subset of r.

Theorem 1

Let A be the matrix corresponding to the constitutive relations in the preferred causality.

Let us note $r = \{r_1, \cdots, r_m\} \subset \{1, \cdots, n\}$ the subset of the indices of the outputs that became known variables, i.e. the new inputs (that is also the subset of the indices of the inputs that became unknown variables, i.e. the new outputs).

Let us note $r' = \bar{r}$ the complementary set.

The causal form corresponding to put the bond of the subset *r* in non preferred causality exists if and only if the determinant of the square matrix R constituted by the elements of A after the cancellation of the columns and the rows corresponding to the subset *r'* is not zero.

$$\begin{bmatrix} \psi_1 \\ \vdots \\ \psi_{r_1} \\ \vdots \\ \psi_i \\ \vdots \\ \psi_{r_m} \\ \vdots \\ \psi_n \end{bmatrix} = \begin{bmatrix} a_{11} & \cdots & a_{1r_1} & \cdots & a_{1i} & \cdots & a_{1r_m} & \cdots & a_{1n} \\ \vdots & \ddots & & & \vdots & & & & \vdots \\ a_{r_1 1} & \cdots & a_{r_1 r_1} & \cdots & a_{r_1 i} & \cdots & a_{r_1 r_m} & \cdots & a_{r_1 n} \\ \vdots & & & \ddots & \vdots & & \vdots & & \vdots \\ a_{i1} & \cdots & a_{ir_1} & \cdots & a_{ii} & \cdots & a_{ir_m} & \cdots & a_{in} \\ \vdots & & & & \vdots & \ddots & \vdots & & \vdots \\ a_{r_m 1} & \cdots & a_{r_m r_1} & \cdots & a_{r_m i} & \cdots & a_{r_m r_m} & \cdots & a_{r_m n} \\ & & & & \vdots & & & \ddots & \vdots \\ a_{n1} & \cdots & a_{nr_1} & \cdots & a_{ni} & \cdots & a_{nr_m} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \vdots \\ \xi_{r_1} \\ \vdots \\ \xi_i \\ \vdots \\ \xi_{r_m} \\ \vdots \\ \xi_n \end{bmatrix}$$

avec $i \notin r$

that leads to the reduced matrix $R = \lfloor a_{ij} \rfloor \quad \{i, j\} \in r \times r$ (8)

4 – BICAUSAL ASSIGNMENT FOR MULTI-PORT ELEMENTS

If bicausal bonds are used in the case of multi-port elements, the input and the output are known in the corresponding relation of the constitutive relation of the element. This knowledge of the power in one bond, may be used in several ways :

- Firstly to propagate this knowledge to another bond, i.e. to impose the power (flow and effort variable). The bicausality is then propagated to the rest of the system by applying the algorithm described in Gawthrop [Gawthrop 95] or Fotsu-Ngwompo [Fotsu Ngwompo et al. 96].
- Secondly to determine some part of the constitutive relation of the element in order to size the system applying the method introduced by Fotsu-Ngwompo.

If n is the dimension of the system, the situation of the problem is the following. There are n equations and $n \times n$ parameters in the matrix, n inputs and n outputs (that is $n^2 + 2n$ potential parameters). That is why $n^2 + n$ potential parameters have to be known parameters, and only n may be unknown parameters. Each bond imposing effort and flow (bicausal bond) fixes two variables as known variables.

Let us note p the number of imposed variables chosen either in the input variable set or in the output variable set. There are three different cases considering any combination of the imposed variable set.

- Case 1 : If $p < n$, then there are less imposed variables than inputs to the constitutive relation. This means that the number of known variables is smaller than the dimension of the system. The system is then under constrained and it is not possible to solve it as the number of known parameters is $n^2 + p < n^2 + n$.
- Case 2: If $p = n$, then there are as many bicausal bonds that impose the power to the element as bicausal bonds, which force the power to their connected element. It is also the case when there is no bicausal bond connected to the multi-port element. The system has a chance to be solved. But it is not possible to set a coefficient of the constitutive matrix as an unknown parameter because the number of required known parameters is $n^2 + n - p = n^2$, and is exactly the amount of coefficients.

- Case 3 : If $p > n$, then the number of imposed variables is greater than the number of constitutive relations and the number of the required known variables is now $n^2 + n - p < n^2$. The system is over constrained and 3 new cases may be considered:
  - $k > p - n$ coefficients of the constitutive relation are set as unknown parameters. This means that the number of known parameters is $n^2 - k + p < n^2 + n$. The system is then under constrained and it is not possible to solve it.
  - $k = p - n$ coefficients of the constitutive relation are set as unknown parameters in the subset of equations where all the input and output variables are known. The system has a chance to be solved as the number of known parameters is $n^2 - k + p = n^2 + n$.
  - All the coefficients of the constitutive relation are known or $k < p - n$ coefficients are set as unknown parameters. If, after reducing the system, the dependent equations are verified, the system has a solution. If not, there is no solution. Anyway this case seems an ill-defined problem.

This discussion shows that only case 2 and 3 are to be considered. The case 2 corresponds to the problem of the propagation of bicausality through a multi-port element. In order to determine one or more coefficients of the constitutive relation of a multi-port element, the case 3 has to be taken into consideration. The number of coefficients, which may be determined using this method, is therefore equal to the difference between the number of variables imposed by causal or bicausal bonds and the dimension of the multi-port element. The both interesting cases are developed in the next section.

5 - APPLICATION TO A 2-PORT ELEMENT AND GENERALIZATION

The two last cases are now studied for a 2 port element. The results are then extended to any multi-port element.

Case 2 : $p = n$

Here the number of bicausal bonds, which impose effort and flow to the multi-port element, is the same as the number of bicausal bonds, which impose effort and flow to their connected element. The theorem 1 (section 3) is therefore a particular case of the following development as the number of bicausal bond is zero.

In the case of a 2-port element, only one bicausal bond imposes the power to the 2-port element. This assumption means that one input variable and its conjugate output variable are known, and that the other input variable and its conjugate output variable have to be determined. The bicausality is propagated through the element. If, for example, the output and its conjugate input are known in the first equation of the constitutive relation, the resolution consists to express the second input as a function of the first input and output:

$$\xi_2 = \frac{1}{a_{12}}\left(\psi_1 - a_{11}\xi_1\right) \tag{8}$$

$$\begin{bmatrix} \xi_2 \\ \psi_2 \end{bmatrix} = \mathrm{B}\begin{bmatrix} \xi_1 \\ \psi_1 \end{bmatrix} \text{ with } B = \frac{1}{a_{12}}\begin{bmatrix} -a_{11} & 1 \\ a_{21}a_{12} - a_{11}a_{22} & a_{22} \end{bmatrix} \tag{9}$$

The condition is then that $a_{12} \neq 0$ is required for the existence of this bicausal configuration.

The situation is then very similar to the one described for the hybrid causal form, although it is now necessary to compute the determinant of the reduced matrix after canceling the rows corresponding to the outputs which remain unknowns, and the columns corresponding to the inputs which remain imposed variables. The following theorem is a generalization of this result.

Theorem 2

Let A be the matrix corresponding to the constitutive relation in the preferred causality.

Let us note $r = \{r_1, \cdots, r_m\} \subset \{1, \cdots, n\}$ the subset of the indices of the outputs that remain unknown variables (i.e. outputs in the preferred causality) and $t = \{t_1, \cdots, t_m\} \subset \{1, \cdots, n\}$ the subset of the indices of the inputs that remain known variables (i.e. inputs in the preferred causality).

The new causal form corresponding to this new set of known and unknown variables exists if and only if the determinant of the square matrix R constituted of the elements of A after the cancellation of the columns corresponding to the subset t and the row corresponding to the subset r is not zero.

The removed columns and rows are indicated on equation (10) by the vertical ($t_k \in t = \{t_1, \cdots, t_m\}$) and horizontal lines ($r_l \in r = \{r_1, \cdots, r_m\}$).

$$\begin{bmatrix} \psi_1 \\ \vdots \\ \psi_{r_1} \\ \vdots \\ \psi_i \\ \vdots \\ \psi_{r_m} \\ \vdots \\ \psi_n \end{bmatrix} = \begin{bmatrix} a_{11} & \cdots & a_{1j} & \cdots & a_{1t_1} & \cdots & a_{1t_m} & \cdots & a_{1n} \\ \vdots & \ddots & & & \vdots & & & & \vdots \\ a_{r_1 1} & \cdots & a_{r_1 j} & \cdots & a_{r_1 t_1} & \cdots & a_{r_1 t_m} & \cdots & a_{r_1 n} \\ \vdots & & & \ddots & \vdots & & \vdots & & \vdots \\ a_{i1} & \cdots & a_{ij} & \cdots & a_{it_1} & \cdots & a_{it_m} & \cdots & a_{in} \\ \vdots & & & & \vdots & \ddots & \vdots & & \vdots \\ a_{r_m 1} & \cdots & a_{r_m j} & \cdots & a_{r_m t_1} & \cdots & a_{r_m t_m} & \cdots & a_{r_m n} \\ \vdots & & & & \vdots & & & \ddots & \vdots \\ a_{nn} & \cdots & a_{nj} & \cdots & a_{nt_1} & \cdots & a_{nt_m} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \vdots \\ \xi_j \\ \vdots \\ \xi_{t_1} \\ \vdots \\ \xi_{t_m} \\ \vdots \\ \xi_n \end{bmatrix}$$

with $i \notin r$ and $j \notin t$

that leads to the reduced matrix $R = \lfloor a_{ij} \rfloor \quad \{i, j\} \in r \times t$ (10)

Remark : It appears that theorem 2 is a generalization for theorem 1. If theorem 2 is applied with identical subsets r and t, this case corresponds to put a subset of bonds in non-preferred causality without bicausal bonds.

Case3 : $p > n$ and $k = p - n$ coefficients of the constitutive relation are unknowns.

In case of a 2 port element with the first bond being a bicausal bond and the second bond in non reversed causality, the system to be solved is the following one :

$$\begin{cases} a_{12}\xi_2 = \psi_1 - a_{11}\xi_1 \\ a_{22}\xi_2 = \psi_2 - a_{21}\xi_1 \end{cases} \tag{11}$$

The left-hand side represents the unknown terms and the right hand side the known ones. It is then necessary to choose one coefficient as unknown in order to relax the constraint on the system. If a_{11} is chosen as unknown, we obtain :

$$\begin{cases} a_{11}\xi_1 + a_{12}\xi_2 = \psi_1 \\ a_{22}\xi_2 = \psi_2 - a_{21}\xi_1 \end{cases} \tag{12}$$

This system has a solution if :

$$\det\left(\begin{bmatrix} \xi_1 & a_{12} \\ 0 & a_{22} \end{bmatrix}\right) \neq 0 \tag{13}$$

That is :

$$\det\left(\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\right) = \det([a_{22}]) \neq 0$$

and $\xi_1 \neq 0$ (14)

Theorem 3:

For an n-port element, if $p > n$ variables in the input or output set are defined as known variables, there are $k = p - n$ over constraint equations. If $k = p - n$ coefficients of the constitutive relations are set as unknown parameters, the condition for solving the problem is that:

i. the k coefficients of the constitutive relations chosen as unknowns are related to the subset of known variables in the input set (else the system becomes non linear),
ii. only one coefficient per line (or relation) is set as unknown (else some relations remain over constraints),
iii. the known variables associated to the unknown coefficients are different from zero,
iv. the matrix of the constitutive relations in preferred causality, where the relations in which coefficients of the matrix are unknown have been removed, verifies theorem 2.

Remark 1 : Some care have to be taken if the matrix is symmetric or antisymmetric (I, C), it is then possible to set 2 coefficients per line (or relation) as unknowns.

Remark 2 : Theorem 3 shows that there are only n coefficients in the constitutive relations, which may be determined using this method.

5 – APPLICATION OF THE BICAUSALITY TO MULTI-PORT ELEMENTS

In order to illustrate the obtained results, the variable capacity (figure 1) is an interesting application. We note :

- x the distance in between the plates and v the velocity.
- F the force applied to one plate, the other one being fixed.
- U the voltage between the plates.
- i the current in the capacity and q the charge.
- S the section of the plates.
- ε the dielectric constant of the medium.

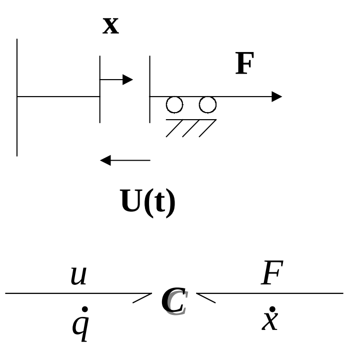


Figure 1. Variable Capacity

The variable capacity is a non-linear system that may be represented in bond graph by a multi-port C element. One bond is in the electrical field, the other one in the mechanical field. It is defined by the non-linear constitutive relations (15) and the linearization around the equilibrium point (x_e, q_e, u_e, F_e) gives the form (16).

$$\begin{cases} u(q,x) = \dfrac{qx}{\varepsilon S} \\ F(q,x) = \dfrac{q^2}{2\varepsilon S} \end{cases} \tag{15}$$

$$\begin{bmatrix} u^* \\ F^* \end{bmatrix} = \begin{bmatrix} \dfrac{x_e}{\varepsilon S} & \dfrac{q_e}{\varepsilon S} \\ \dfrac{q_e}{\varepsilon S} & 0 \end{bmatrix} \begin{bmatrix} q^* \\ x^* \end{bmatrix} \quad \Leftrightarrow \quad \mathbf{\Psi} = A\mathbf{\Xi} \tag{16}$$

where x^* notes the variation of x around the equilibrium point x_e : $x^* = (x - x_e)$

In this case, the preferred causality, that is the integral causality, is obtained when the flows are imposed to the element. Therefore the form (16) is in preferred causality, the inputs are the generalized displacements (q^*, x^*) and the outputs the efforts (u^*, F^*).

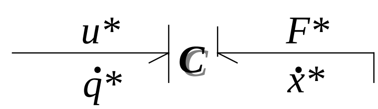
Causality assignment

It clearly appears that the conditions of existence of the different causality are related to the values of q_e and x_e (to stay in the physical reality, we assume that ε and S are not zero). The table 1 summarizes these results. Applying theorem 2 according to the considered causality assignment, the removed columns and rows are indicated in table 1 by the vertical and horizontal lines in order to obtain the condition of existence of the causal form.

Sizing problem

Although the problem of the design is not very useful in this case, it seems interesting to look at its application. If, for example, the design problem is to size the coefficient $\dfrac{x_e}{\varepsilon S}$, the crucial question is what are the required trajectory (or measurements) ? The result straightforwardly comes by applying the proposed method (Table 2). To determine this coefficient, the knowledge of at least 3 on the 4 variables (inputs and outputs) is necessary and one of these variables has to be q^*. One bicausal bond has to impose the flow and the effort, the other may have any causality, either effort or flow, but q^* must anyway be imposed. There are then 3 cases, which are shown on Table 2.

Remark : In this example, q^* is only related to F^*, it is possible to avoid the non linearity and to use the following causality to solve the problem:



$$\text{that gives} \begin{cases} q^* = \dfrac{\varepsilon S}{q_e} F^* \\ \dfrac{x_e}{\varepsilon S} = \dfrac{q_e}{\varepsilon S}\left(u^* - \dfrac{q_e}{\varepsilon S} x^*\right)\dfrac{1}{F^*} \end{cases} \quad \text{if } F^* \neq 0$$

Table 1. Condition of existence for the different causalities

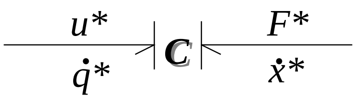
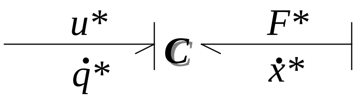
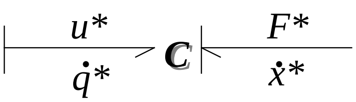
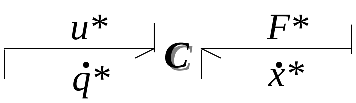
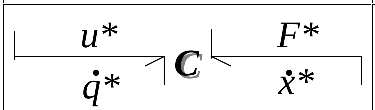
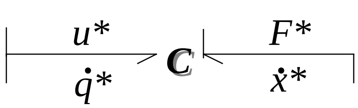
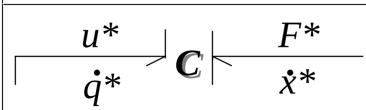
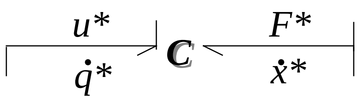
Causality	Condition of existence	Existence	Calculus scheme
	$\det\left(\begin{bmatrix} \frac{x_e}{\varepsilon S} & \frac{q_e}{\varepsilon S} \\ \frac{q_e}{\varepsilon S} & 0 \end{bmatrix}\right) \neq 0$	If $q_e \neq 0$	$\begin{bmatrix} q^* \\ x^* \end{bmatrix} = \left(\frac{\varepsilon S}{q_e}\right)^2 \begin{bmatrix} 0 & \frac{q_e}{\varepsilon S} \\ \frac{q_e}{\varepsilon S} & -\frac{x_e}{\varepsilon S} \end{bmatrix} \begin{bmatrix} u^* \\ F^* \end{bmatrix}$
	$\det\left(\begin{bmatrix} \frac{x_e}{\varepsilon S} & \frac{q_e}{\varepsilon S} \\ \frac{q_e}{\varepsilon S} & 0 \end{bmatrix}\right) \neq 0$	If $x_e \neq 0$	$\begin{bmatrix} q^* \\ F^* \end{bmatrix} = \frac{\varepsilon S}{x_e} \begin{bmatrix} 1 & -\frac{q_e}{\varepsilon S} \\ \frac{q_e}{\varepsilon S} & -\left(\frac{q_e}{\varepsilon S}\right)^2 \end{bmatrix} \begin{bmatrix} u^* \\ x^* \end{bmatrix}$
	$\det\left(\begin{bmatrix} \frac{x_e}{\varepsilon S} & \frac{q_e}{\varepsilon S} \\ \frac{q_e}{\varepsilon S} & 0 \end{bmatrix}\right) \neq 0$	no	
	$\det\left(\begin{bmatrix} \frac{x_e}{\varepsilon S} & \frac{q_e}{\varepsilon S} \\ \frac{q_e}{\varepsilon S} & 0 \end{bmatrix}\right) \neq 0$	If $q_e \neq 0$	$\begin{bmatrix} x^* \\ F^* \end{bmatrix} = \frac{\varepsilon S}{q_e} \begin{bmatrix} -\frac{x_e}{\varepsilon S} & 1 \\ \left(\frac{q_e}{\varepsilon S}\right)^2 & 0 \end{bmatrix} \begin{bmatrix} q^* \\ u^* \end{bmatrix}$
	$\det\left(\begin{bmatrix} \frac{x_e}{\varepsilon S} & \frac{q_e}{\varepsilon S} \\ \frac{q_e}{\varepsilon S} & 0 \end{bmatrix}\right) \neq 0$	If $q_e \neq 0$	$\begin{bmatrix} u^* \\ q^* \end{bmatrix} = \frac{\varepsilon S}{q_e} \begin{bmatrix} \frac{x_e}{\varepsilon S} & \frac{q_e}{\varepsilon S} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} F^* \\ x^* \end{bmatrix}$

Table 2. Condition of existence for the different causalities

Causality	Condition of existence	Existence	Calculus scheme
	$F^* = \frac{q_e}{\varepsilon S} q^*$ is over constraint	no	
	$F^* = \frac{q_e}{\varepsilon S} q^*$ is over constraint	no	

	$\det\left(\begin{bmatrix} \frac{x_e}{\varepsilon S} & \frac{q_e}{\varepsilon S} \\ \frac{q_e}{\varepsilon S} & 0 \end{bmatrix}\right) \neq 0$ and $q^* \neq 0$	If $q^* \neq 0$	$\begin{cases} F^* = \frac{q_e}{\varepsilon S} q^* \\ \frac{x_e}{\varepsilon S} = \frac{1}{q^*}\left(u^* - \frac{q_e}{\varepsilon S} x^*\right) \end{cases}$

6 – CONCLUSION

In the proposed approach, the causal stroke or half-stroke clearly appears as a symbolism for the orientation of the calculus model. We show that the constitutive relations of any linear multi-port element may be used to determine quickly what kind of causality assignment does exist and what could be determined using different schemes of calculus, that is causality. Our original goal was to establish the subsets of state variables and inputs that enable to define the equilibrium set for a multi-port storage element. The solution to this problem is now straightforward by applying the proposed approach. The inspection of the condition of existence for a causality assignment in linear multi-port shows that the results are potentially relevant in different contexts and have interesting applications on problems such as system design, identification and control.

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