On Integer Additive Set-Sequential Graphs

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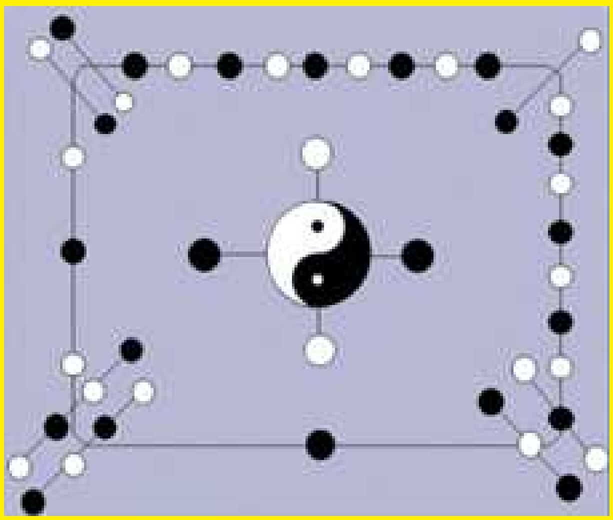

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It is at our mother's knee that we acquire our noblest and truest and highest ideals, but there is seldom any money in them.

By Mark Twain, an American writer.

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On Integer Additive Set-Sequential Graphs

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Abstract: A set-labeling of a graph G is an injective function $f: V(G) \to \mathcal{P}(X)$, where X is a finite set of non-negative integers and a set-indexer of G is a set-labeling such that the induced function $f^{\oplus}: E(G) \to \mathcal{P}(X) - \{\emptyset\}$ defined by $f^{\oplus}(uv) = f(u) \oplus f(v)$ for every $uv \in E(G)$ is also injective. A set-indexer $f: V(G) \to \mathcal{P}(X)$ is called a set-sequential labeling of G if $f^{\oplus}(V(G) \cup E(G)) = \mathcal{P}(X) - \{\emptyset\}$. A graph G which admits a set-sequential labeling is called a set-sequential graph. An integer additive set-labeling is an injective function $f: V(G) \to \mathcal{P}(\mathbb{N}_0)$, $\mathbb{N}_0$ is the set of all non-negative integers and an integer additive set-indexer is an integer additive set-labeling such that the induced function $f^+: E(G) \to \mathcal{P}(\mathbb{N}_0)$ defined by $f^+(uv) = f(u) + f(v)$ is also injective. In this paper, we extend the concepts of set-sequential labeling to integer additive set-labelings of graphs and provide some results on them.

Key Words: Integer additive set-indexers, set-sequential graphs, integer additive set-labeling, integer additive set-sequential labeling, integer additive set-sequential graphs.

AMS(2010): 05C78.

§1. Introduction

For all terms and definitions, not defined specifically in this paper, we refer to [4], [5] and [9] and for more about graph labeling, we refer to [6]. Unless mentioned otherwise, all graphs considered here are simple, finite and have no isolated vertices.

All sets mentioned in this paper are finite sets of non-negative integers. We denote the cardinality of a set A by $|A|$. We denote, by X, the finite ground set of non-negative integers that is used for set-labeling the elements of G and cardinality of X by n.

The research in graph labeling commenced with the introduction of β-valuations of graphs in [10]. Analogous to the number valuations of graphs, the concepts of set-labelings and set-indexers of graphs are introduced in [1] as follows.

Let G be a (p, q)-graph. Let X, Y and Z be non-empty sets and $\mathcal{P}(X)$, $\mathcal{P}(Y)$ and $\mathcal{P}(Z)$ be their power sets. Then, the functions $f: V(G) \to \mathcal{P}(X)$, $f: E(G) \to \mathcal{P}(Y)$ and $f: V(G) \cup E(G) \to \mathcal{P}(Z)$ are called the *set-assignments* of vertices, edges and elements of G respectively. By a set-assignment

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of a graph, we mean any one of them. A set-assignment is called a *set-labeling* or a *set-valuation* if it is injective.

A graph with a set-labeling f is denoted by (G, f) and is referred to as a *set-labeled graph* or a *set-valued graph*. For a (p, q)- graph $G = (V, E)$ and a non-empty set X of cardinality n, a *set-indexer* of G is defined as an injective set-valued function $f : V(G) \to \mathcal{P}(X)$ such that the function $f^{\oplus} : E(G) \to \mathcal{P}(X) - \{\emptyset\}$ defined by $f^{\oplus}(uv) = f(u) \oplus f(v)$ for every $uv \in E(G)$ is also injective, where $\mathcal{P}(X)$ is the set of all subsets of X and $\oplus$ is the symmetric difference of sets.

Theorem 1.1([1]) *Every graph has a set-indexer.*

Analogous to graceful labeling of graphs, the concept of set-graceful labeling and set-sequential labeling of a graph are defined in [1] as follows.

Let G be a graph and let X be a non-empty set. A set-indexer $f : V(G) \to \mathcal{P}(X)$ is called a *set-graceful labeling* of G if $f^{\oplus}(E(G)) = \mathcal{P}(X) - \{\emptyset\}$. A graph G which admits a set-graceful labeling is called a *set-graceful graph*.

Let G be a graph and let X be a non-empty set. A set-indexer $f : V(G) \to \mathcal{P}(X)$ is called a *set-sequential labeling* of G if $f^{\oplus}(V(G) \cup E(G)) = \mathcal{P}(X) - \{\emptyset\}$. A graph G which admits a set-sequential labeling is called a *set-sequential graph*.

Let A and B be two non-empty sets. Then, their *sum set*, denoted by $A + B$, is defined to be the set $A + B = \{a + b : a \in A, b \in B\}$. If $C = A + B$, then A and B are said to be the *summands* of C. Using the concepts of sum sets of sets of non-negative integers, the notion of integer additive set-labeling of a given graph G is introduced as follows.

Let $\mathbb{N}_0$ be the set of all non-negative integers. An *integer additive set-labeling* (IASL, in short) of graph G is an injective function $f : V(G) \to \mathcal{P}(\mathbb{N}_0)$ such that the induced function $f^+ : E(G) \to \mathcal{P}(\mathbb{N}_0)$ is defined by $f^+(uv) = f(u) + f(v)$ for $\forall uv \in E(G)$. A graph G which admits an IASL is called an IASL graph.

An *integer additive set-labeling* f is an integer additive set-indexer (IASI, in short) if the induced function $f^+ : E(G) \to \mathcal{P}(\mathbb{N}_0)$ defined by $f^+(uv) = f(u) + f(v)$ is injective(see [7]). A graph G which admits an IASI is called an IASI graph.

The following notions are introduced in [11] and [8]. The cardinality of the set-label of an element (vertex or edge) of a graph G is called the *set-indexing number* of that element. An IASL (or an IASI) is said to be a k-uniform IASL (or k-uniform IASI) if $|f^+(e)| = k \ \forall \ e \in E(G)$. The vertex set $V(G)$ is called *l-uniformly set-indexed*, if all the vertices of G have the set-indexing number l.

Definition 1.2([13]) *Let G be a graph and let X be a non-empty set. An integer additive set-indexer $f : V(G) \to \mathcal{P}(X) - \{\emptyset\}$ is called a* integer additive set-graceful labeling *(IASGL, in short) of G if $f^+(E(G)) = \mathcal{P}(X) - \{\emptyset, \{0\}\}$. A graph G which admits an integer additive set-graceful labeling is called an* integer additive set-graceful graph *(in short, IASG-graph).*

Motivated from the studies made in [2] and [3], in this paper, we extend the concepts of set-sequential labelings of graphs to integer additive set-sequential labelings and establish some results on them.

§2. IASSL of Graphs

First, note that under an integer additive set-labeling, no element of a given graph can have $\emptyset$ as its set-labeling. Hence, we need to consider only non-empty subsets of X for set-labeling the elements of G.

Let f be an integer additive set-indexer of a given graph G. Define a function $f^* : V(G) \cup E(G) \to \mathcal{P}(X) - \{\emptyset\}$ as follows.

$$f^*(x) = \begin{cases} f(x) & \text{if } x \in V(G) \\ f^+(x) & \text{if } x \in E(G) \end{cases} \tag{2.1}$$

Clearly, $f^*[V(G) \cup E(G)] = f(V(G)) \cup f^+(E(G))$. By the notation, $f^*(G)$, we mean $f^*[V(G) \cup E(G)]$. Then, f^* is an extension of both f and f^+ of G. Throughout our discussions in this paper, the function f^* is as per the definition in Equation 2.1.

Using the definition of new induced function f^* of f, we introduce the following notion as a sum set analogue of set-sequential graphs.

Definition 2.1 *An IASI f of G is said to be an* integer additive set-sequential labeling *(IASSL) if the induced function $f^*(G) = f(V(G)) \cup f^+E(G)) = \mathcal{P}(X) - \{\emptyset\}$. A graph G which admits an IASSL may be called an* integer additive set-sequential graph *(IASS-graph).*

Hence, an integer additive set-sequential indexer can be defined as follows.

Definition 2.2 *An integer additive set-sequential labeling f of a given graph G is said to be an integer additive set-sequential indexer (IASSI) if the induced function f^* is also injective. A graph G which admits an IASSI may be called an* integer additive set-sequential indexed graph *(IASSI-graph).*

A question that arouses much in this context is about the comparison between an IASGL and an IASSL of a given graph if they exist. The following theorem explains the relation between an IASGL and an IASSL of a given graph G.

Theorem 2.3 *Every integer additive set-graceful labeling of a graph G is also an integer additive set-sequential labeling of G.*

Proof Let f be an IASGL defined on a given graph G. Then, $\{0\} \in f(V(G))$ (see [13]) and $|f^+(E(G))| = \mathcal{P}(X) - \{\emptyset, \{0\}\}$. Then, $f^*(G)$ contains all non-empty subsets of X. Therefore, f is an IASSL of G. □

Let us now verify the injectivity of the function f^* in the following proposition.

Proposition 2.4 *Let G be a graph without isolated vertices. If the function f^* is an injective, then no vertex of G can have a set-label $\{0\}$.*

Proof If possible let a vertex, say v, has the set-label $\{0\}$. Since G is connected, v is adjacent to at least one vertex in G. Let u be an adjacent vertex of v in G and u has a set-label $A \subset X$. Then, $f^*(u) = f(u) = A$ and $f^*(uv) = f^+(uv) = A$, which is a contradiction to the hypothesis that f^* is injective. □

In view of Observation 2.4, we notice the following points.

Remark 2.5 Suppose that the function f^* defined in (2.1) is injective. Then, if one vertex v of G has the set label $\{0\}$, then v is an isolated vertex of G.

Remark 2.6 If the function f^* defined in (2.1) is injective, then no edge of G can also have the set label $\{0\}$.

The following result is an immediate consequence of the addition theorem on sets in set theory and provides a relation connecting the size and order of a given IASS-graph G and the cardinality of its ground set X.

Proposition 2.7 *Let G be a graph on n vertices and m edges. If f is an IASSL of a graph G with respect to a ground set X, then $m+n = 2^{|X|} - (1+\kappa)$, where κ is the number of subsets of X which is the set-label of both a vertex and an edge.*

Proof Let f be an IASSL defined on a given graph G. Then, $|f^*(G)| = |f(V(G)) \cup f^+(E(G))| = |\mathcal{P}(X) - \{\emptyset\}| = 2^{|X|} - 1$. But by addition theorem on sets, we have

$$\begin{aligned} |f^*(G)| &= |f(V(G)) \cup f^+(E(G))| \\ \text{That is,}\quad 2^{|X|} - 1 &= |f(V(G))| + |f^+(E(G))| - |f(V(G)) \cap f^+(E(G))| \\ &= |V| + |E| - \kappa \\ \Longrightarrow &= m + n - \kappa \\ \text{Whence}\quad m+n &= 2^{|X|} - 1 - \kappa. \end{aligned}$$

This completes the proof. □

We say that two sets A and B are of *same parity* if their cardinalities are simultaneously odd or simultaneously even. Then, the following theorem is on the parity of the vertex set and edge set of G.

Proposition 2.8 *Let f be an IASSL of a given graph G, with respect to a ground set X. Then, if $V(G)$ and $E(G)$ are of same parity, then κ is an odd integer and if $V(G)$ and $E(G)$ are of different parity, then κ is an even integer, where κ is the number of subsets of X which are the set-labels of both vertices and edges.*

Proof Let f be a integer additive set-sequential labeling of a given graph G. Then, $f^*(G) = \mathcal{P}(X) - \{\emptyset\}$. Therefore, $|f^*(G)| = 2^{|X|} - 1$, which is an odd integer.

Case 1. Let $V(G)$ and $E(G)$ are of same parity. Then, $|V| + |E|$ is an even integer. Then, by Proposition 2.7, $2^{|X|} - 1 - \kappa$ is an even integer, which is possible only when κ is an odd integer.

Case 2. Let $V(G)$ and $E(G)$ are of different parity. Then, $|V| + |E|$ is an odd integer. Then, by Proposition 2.7, $2^{|X|} - 1 - \kappa$ is an odd integer, which is possible only when κ is an even integer. □

A relation between integer additive set-graceful labeling and an integer additive set-sequential labeling of a graph is established in the following result.

The following result determines the minimum number of vertices in a graph that admits an IASSL with respect to a finite non-empty set X.

Theorem 2.9 *Let X be a non-empty finite set of non-negative integers. Then, a graph G that admits an IASSL with respect to X have at least ρ vertices, where ρ is the number of elements in $\mathcal{P}(X)$ which are not the sum sets of any two elements of $\mathcal{P}(X)$.*

Proof Let f be an IASSL of a given graph G, with respect to a given ground set X. Let $\mathcal{A}$ be the collection of subsets of X such that no element in $\mathcal{A}$ is the sum sets any two subsets of X. Since f an IASL of G, all edge of G must have the set-labels which are the sum sets of the set-labels of their end vertices. Hence, no element in $\mathcal{A}$ can be the set-label of any edge of G. But, since f is an IASSL of G, $\mathcal{A} \subset f^*(G) = f(V(G)) \cup f^+(E(G))$. Therefore, the minimum number of vertices of G is equal to the number of elements in the set $\mathcal{A}$. □

The structural properties of graphs which admit IASSLs arouse much interests. In the example of IASS-graphs, given in Figure 1, the graph G has some pendant vertices. Hence, there arises following questions in this context. Do an IASS-graph necessarily have pendant vertices? If so, what is the number of pendant vertices required for a graph G to admit an IASSL? Let us now proceed to find the solutions to these problems.

The minimum number of pendant vertices required in a given IASS-graph is explained in the following Theorem.

Theorem 2.10 *Let G admits an IASSL with respect to a ground set X and let $\mathcal{B}$ be the collection of subsets of X which are neither the sum sets of any two subsets of X nor their sum sets are subsets of X. If $\mathcal{B}$ is non-empty, then*

(1) $\{0\}$ *is the set-label of a vertex in G;*

(2) *the minimum number pendant vertices in G is cardinality of $\mathcal{B}$.*

Remark 2.11 Since the ground set X of an IASS-graph must contain the element 0, every subset A_i of X sum set of $\{0\}$ and A_i itself. In this sense, each subset A_i may be considered as a *trivial sum set* of two subsets of X.

In the following discussions, by a sum set of subsets of X, we mean the non-trivial sum sets of subsets of X.

Proof Let f be an IASSL of G with respect to a ground set X. Also, let $\mathcal{B}$ be the collection of subsets of X which are neither the sum sets of any two subsets of X nor their sum sets are subsets of X. Let $A \subset X$ be an element of $\mathcal{B}$. then A must be the set-label of a vertex of G. Since $A \in \mathcal{B}$, the only set that can be adjacent to A is $\{0\}$. Therefore, since G is a connected graph, $\{0\}$ must be the set-label of a vertex of G. More over, since A is an arbitrary vertex in $\mathcal{B}$, the minimum number of pendant vertices in G is $|\mathcal{B}|$. □

The following result thus establishes the existence of pendant vertices in an IASS-graph.

Theorem 2.12 *Every graph that admits an IASSL, with respect to a non-empty finite ground set X, have at least one pendant vertex.*

Proof Let the graph G admits an IASSL f with respect to a ground set X. Let $\mathcal{B}$ be the collection of subsets of X which are neither the sum sets of any two subsets of X nor their sum sets are subsets of X.

We claim that $\mathcal{B}$ is non-empty, which can be proved as follows. Since X is a finite set of non-negative integers, X has a smallest element, say x_1, and a greatest element x_l. Then, the subset $\{x_1, x_l\}$ belongs to $f^*(G)$. Since it is not the sum set any sets and is not a summand of any set in $\mathcal{P}(X)$, $\{x_1, x_l\} \in \mathcal{B}$. Therefore, $\mathcal{B}$ is non-empty.

Since $\mathcal{B}$ is non-empty, by Theorem 2.10, G has some pendant vertices. □

Remark 2.13 In view of the above results, we can make the following observations.

(1) No cycle C_n can have an IASSL;

(2) For $n \geqslant 2$, no complete graph K_n admits an IASSL.

(3) No complete bipartite graph $K_{m,n}$ admits an IASL.

The following result establish the existence of a graph that admits an IASSL with respect to a given ground set X.

Theorem 2.14 *For any non-empty finite set X of non-negative integers containing* 0*, there exists a graph G which admits an IASSL with respect to X.*

Proof Let X be a given non-empty finite set containing the element 0 and let $\mathcal{A} = \{A_i\}$, be the collection of subsets of X which are not the sum sets of any two subsets of X. Then, the set $\mathcal{A}' = \mathcal{P}(X) - \mathcal{A} \cup \{\emptyset\}$ is the set of all subsets of X which are the sum sets of any two subsets of X and hence the sum sets of two elements in $\mathcal{A}$.

What we need here is to construct a graph which admits an IASSL with respect to X. For this, begin with a vertex v_1. Label the vertex v_1 by the set $A_1 = \{0\}$. For $1 \leqslant i \leqslant |\mathcal{A}|$, create a new vertex v_i corresponding to each element in $\mathcal{A}$ and label v_i by the set $A_i \in \mathcal{A}$. Then, connect each of these vertices to V_1 as these vertices v_i can be adjacent only to the vertex v_1. Now that all elements in $\mathcal{A}$ are the set-labels of vertices of G, it remains the elements of $\mathcal{A}'$ for labeling the elements of G. For any $A'_r \in \mathcal{A}'$, we have $A'_r = A_i + A_j$, where $A_i, A_j \in \mathcal{A}$. Then, draw an edge e_r between v_i and v_j so that e_r has the set-label A'_r. This process can be repeated until all the elements in $\mathcal{A}'$ are also used for labeling the elements of G. Then, the resultant graph is an IASS-graph with respect to the ground set X. □

Figure 1 illustrates the existence of an IASSL for a given graph G.

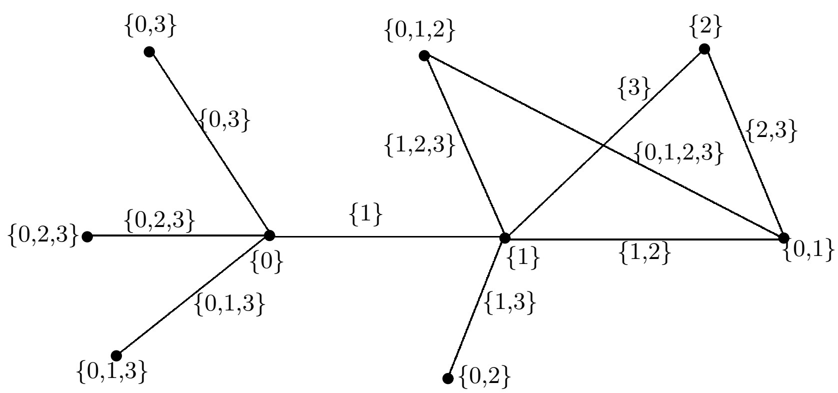

Figure 1

On the other hand, for a given graph G, the choice of a ground set X is also very important to have an integer additive set-sequential labeling. There are certain other restrictions in assigning set-labels to the elements of G. We explore the properties of a graph G that admits an IASSL with respect to a given ground set X. As a result, we have the following observations.

Proposition 2.15 *Let G be a connected integer additive set-sequential graph with respect to a ground set X. Let x_1 and x_2 be the two minimal non-zero elements of X. Then, no edges of G can have the set-labels $\{x_1\}$ and $\{x_2\}$.*

Proof In any IASL-graph G, the set-label of an edge is the sum set of the set-labels of its end vertices. Therefore, a subset A of the ground set X, that is not a sum set of any two subsets of X, can not be the set-label of any edge of G. Since x_1 and x_2 are the minimal non-zero elements of X, $\{x_1\}$ and $\{x_2\}$ can not be the set-labels of any edge of G. □

Proposition 2.16 *Let G be a connected integer additive set-sequential graph with respect to a ground set X. Then, any subset A of X that contains the maximal element of X can be the set-label of a vertex v of G if and only if v is a pendant vertex that is adjacent to the vertex u having the set-label $\{0\}$.*

Proof Let x_n be the maximal element in X and let A be a subset of X that contains the element x_n. If possible, let A be the set-label of a vertex , say v, in G. Since G is a connected graph, there exists at least one vertex in G that is adjacent to v. Let u be an adjacent vertex of v in G and let B be its set-label. Then, the edge uv has the set-label $A+B$. If $B \neq \{0\}$, then there exists at least one element $x_i \neq 0$ in B and hence $x_i + x_n \notin X$ and hence not in $A+B$, which is a contradiction to the fact that G is an IASS-graph. □

Let us now discuss whether trees admit integer additive set-sequential labeling, with respect to a given ground set X.

Theorem 2.17 *A tree G admits an IASSL f with respect to a finite ground set X, then G has $2^{|X|-1}$ vertices.*

Proof Let G be a tree on n vertices. If possible, let G admits an IASSI. Then, $|E(G)| = n-1$. Therefore, $|V(G)|+|E(G)| = n+n-1 = 2n-1$. But, by Theorem 2.9, $2^{|X|}-1 = 2n-1 \implies n = 2^{|X|-1}$. □

Invoking the above results, we arrive at the following conclusion.

Theorem 2.18 *No connected graph G admits an integer additive set-sequential indexer.*

Proof Let G be a connected graph which admits an IASI f. By Proposition 2.4, if the induced function f^* is injective, then $\{0\}$ can not be the set-label of any element of G. But, by Propositions 2.15 and 2.16, every connected IASS-graph has a vertex with the set-label $\{0\}$. Hence, a connected graph G can not have an IASSI. □

The problem of characterizing (disconnected) graphs that admit IASSIs is relevant and interesting in this situation. Hence, we have

Theorem 2.19 *A graph G admits an integer additive set-sequential indexer f with respect to a ground set X if and only if G has ρ' isolated vertices, where ρ' is the number of subsets of X which are neither the sum sets of any two subsets of X nor the summands of any subsets of X.*

Proof Let f be an IASI defined on G, with respect to a ground set X. Let $\mathcal{B}$ be the collection of subsets of X which are neither the sum sets of any two subsets of X nor the summands of any subsets of X.

Assume that f is an IASSI of G. Then, the induced function f^* is an injective function. We have already showed that $\mathcal{B}$ is a non-empty set. By Theorem 2.10, $\{0\}$ must be the set-label of one vertex v in G and the vertices of G with set-labels from $\mathcal{B}$ can be adjacent only to the vertex v. By Remark 2.5, v must be an isolated vertex in G. Also note that $\{0\}$ is lso an element in $\mathcal{B}$. Therefore, all the vertices which have set-labels from $\mathcal{B}$ must also be isolated vertices of G. Hence G has $\rho' = |\mathcal{B}|$ isolated vertices.

Conversely, assume that G has $\rho' = |\mathcal{B}|$ isolated vertices. Then, label the isolated vertices of G by the sets in $\mathcal{B}$ in an injective manner. Now, label the other vertices of G in an injective manner by other non-empty subsets of X which are not the sum sets of subsets of X in such a way that the subsets of X which are the sum sets of subsets of X are the set-labels of the edges of G. Clearly, this labeling is an IASSI of G. □

Analogous to Theorem 2.14, we can also establish the existence of an IASSI-graph with respect to a given non-empty ground set X.

Theorem 2.20 *For any non-empty finite set X of non-negative integers, there exists a graph G which admits an IASSI with respect to X.*

Figure 2 illustrates the existence of an IASSL for a given graph with isolated vertices.

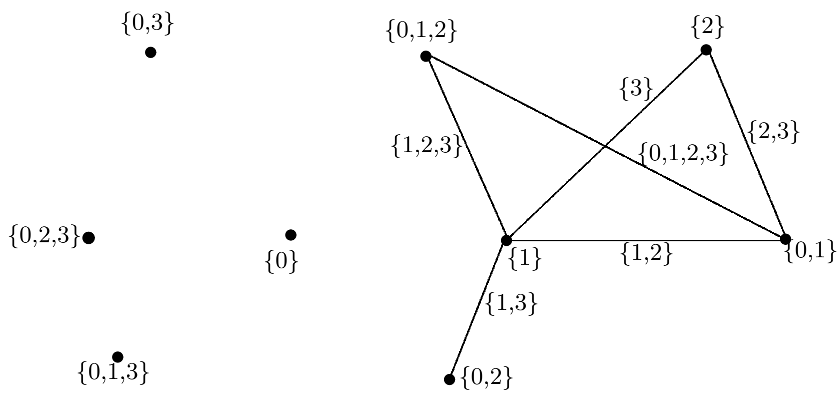

Figure 2

§3. Conclusion

In this paper, we have discussed an extension of set-sequential labeling of graphs to sum-set labelings and have studied the properties of certain graphs that admit IASSLs. Certain problems regarding the complete characterization of IASSI-graphs are still open.

We note that the admissibility of integer additive set-indexers by the graphs depends upon the nature of elements in X. A graph may admit an IASSL for some ground sets and may not admit an IASSL for some other ground sets. Hence, choosing a ground set is very important to discuss about IASSI-graphs.

There are several problems in this area which are promising for further studies. Characterization of different graph classes which admit integer additive set-sequential labelings and verification of the existence of integer additive set-sequential labelings for different graph operations, graph products and graph products are some of them. The integer additive set-indexers under which the vertices of a given graph are labeled by different standard sequences of non-negative integers, are also worth studying.

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In silence, in steadiness, in severe abstraction, let him hold by himself, add observation to observation, patient of neglect, patient of reproach , and bide his own time , happy enough if he can satisfy himself alone that the day he has seen something truly.

By Ralph Waldo Emerson, an American thinker.

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[4]Linfan Mao, *Combinatorial Geometry with Applications to Field Theory*, InfoQuest Press, 2009.
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Research papers

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[9]Kavita Srivastava, On singular H-closed extensions, *Proc. Amer. Math. Soc.* (to appear).

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