Synergistic and Interdisciplinary Approaches for the Conservation of Monumental Heritage: Cupola of Santa Maria del Fiore in Florence, Italy

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Abstract: This paper presents the results of an interdisciplinary study carried out on Brunelleschi's Cupola of Santa Maria del Fiore in Florence Italy, one of the most emblematic masonry domes in the world. The Cupola has been affected since the beginning by a widespread cracking phenomenon, and several studies were done over the centuries to clarify its safety conditions. To have a direct and indirect record of the cracks opening or closing, a complex monitoring system was installed on the monument during the last century. An accurate analysis of crack widths and global displacements, performed considering both historical and recent monitoring data, has allowed for the identification of the movements developed in the monument over centuries and evaluating their relation with environmental and seismic events. In line with the interdisciplinary approach strongly recommended in the field of assessment and conservation of monumental heritage, this paper reconsiders some issues concerning the causes of the actual damage to the Cupola. In particular, in light of the obtained results, the famous seventeenth century Viviani's conclusions about the Cupola's damage (horizontal thrusts worsened by seismic response), confirmed by Chiarugi in the 1980s, are compared with other hypotheses proposed by other scholars over the centuries, such as the differential settlement of pillars (Cecchini in 1698 and Ximenes in 1757) and the influence of temperature variations (Nervi in 1934). The large amount of measured data and the results of the last numerical models of the Cupola, combined with recent dynamic measurements, allowed the updating of some previous conclusions on damage causes and trends. Starting from these conclusions, a more reliable forecasting model of the monument can be set up that could be useful in identifying effective conservation strategy for this outstanding monument. **DOI:** 10.1061/(ASCE)CF.1943-5509.0000831.

Author keywords: Structural models; Geotechnical and geophysical investigations; Structural monitoring; Conservation; Historic and instrumental monitoring.

Introduction

Brunelleschi's Cupola of the Cathedral of Santa Maria del Fiore in Florence, Italy, is one of the most deeply investigated architectural masterpieces in the world. It represents an excellent example in the field of structural conservation, both for the importance of the monument itself and for the number and complexity of the different studies carried out on it over time (e.g., historical studies, material analyses, geotechnical investigations, dynamic tests, structural analyses, and the installation of monitoring systems). Indeed, its damage (i.e., the system of cracks on and between the webs) has brought into question the stability of the dome over the centuries. Its severe crack pattern—large cracks (*screpoli*) primarily concentrated on the Cupola—started at the end of its construction, after the earthquake of the year 1453, before the erection of the Lantern, and has increased in size over the centuries. Cracks have been the object of different observations and studies, starting from the precise survey performed by Gherardo Silvani in 1693 up to the works of the last Scientific Committees (R. Sabatini and P.L. Nervi in 1934; G. De Angelis D'Ossat and C. Cestelli Guidi in 1985) (Di Pasquale 1977; Gurrieri 1994; Chiarugi et al. 1998; Fanelli and Fanelli 2004). Each of these investigations on the Cupola provided many enhancements in the knowledge of its mechanical behavior.

New improvements about the structural behavior of the Cupola require an interdisciplinary approach in which the approximations of the numerical models, belonging to the modern assumptions of the building's structural mechanics, must be compared and cross-correlated with historic and conservation sciences. In fact, difficulties and carefulness in numerical modelling of ancient masonry structures are well known by literature, especially analyzing the seismic response (Roca 2004; Del Coz Díaz et al. 2007, 2013; Bartoli and Betti 2013). In keeping with this, an interdisciplinary approach could represent a critical instrument for a more reliable assessment of the actual structural behavior of a monumental building. An integrated study was undertaken that reconsiders the different damage hypotheses proposed over the centuries, at the light of

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Note. This manuscript was submitted on January 21, 2015; approved on July 27, 2015No Epub Date. Discussion period open until 0, 0; separate discussions must be submitted for individual papers. This paper is part of the ***Journal of Performance of Constructed Facilities***, © ASCE, ISSN 0887-3828.

an accurate analysis of the crack width and of the global displacements (horizontal and vertical) of the Cupola—performed considering both historic and recent monitoring data—in relation to environmental phenomena and seismic events. The integrated analysis of monitoring data, geotechnical investigations, and numerical analyses allows for a better understanding of the real mechanical behavior of the Cupola, and the definition of a more precise intervention for the conservation of this monument.

Brunelleschi's Cupola

Geometry and Structure

The Cupola, with its 116 m height, stands approximately 80 m above the Florence skyline (still stopped at approximately 30 m of height by a Municipality Rule of the year 1250). Brunelleschi conceived and vaulted the Cupola from 1417 to 1436; its construction lasted 20 years (growing approximately 2.5 m a year) with an average staff of 50 workers (*maestri muratori*, freemasons). The complex geometry of Brunelleschi's Cupola has been well known since the 1980s (Di Pasquale 1977; Fanelli and Fanelli 2004), and the technical tricks (the *corda blanda* and the *spinapesce apparatus*) employed by Brunelleschi to achieve the best structural behavior (Chiarugi et al. 1983), which has assured the Cupola's resistance for over six centuries. Some new acquisitions (Giorgi and Matracchi 2008)—which testifies to some irregularities in Brunelleschi's perfection (e.g., discontinuities and variable thickness of the mortar joints, variability in bricks inclination)—do not change these features.

Brunelleschi's Cupola consists of two domes: An inner thick masonry dome with an even thickness of approximately 2.2 m that spans the diameter of the octahedral ring beam (the tambour), and a thinner outer dome that becomes gradually thinner from the base, in which the thickness is approximately 80 cm, to the oculus with a thickness of approximately 40 cm. These two domes have cylindrical surfaces with elliptic sections, and are structurally linked by 24 huge and articulated ribs—as was used in the Roman Pantheon—all focused to the center of the Cupola. The domes rise up from the tambour, the octagonal structure which gets height to the Cupola, and links the four huge pillars, which confine the aisles with the transect and the apse.

Brunelleschi used two particular techniques while building the Cupola (Di Pasquale 1977; Gurrieri 1994; Fanelli and Fanelli 2004; Ottoni 2012). He adopted the *corda blanda* (slack cable) layout for bricks alignments: The bricks are lying along surfaces of conical sections, avoiding any discontinuities in the corners of the octagonal sections, with the main aim of circumventing the creation of weak areas in the angle spurs (Fig. 1). The second technique was the *spinapesce* (herringbone) apparatus, which consists of courses of bricks placed flat but interrupted at regular intervals (approximately 1.20 m) by sets of vertical bricks that are radially oriented. These have the function of not only containing the courses of bricks and allowing for the build up the dome without *centinas*, but also for creating a system of radial helixes in the thickness of the walls. In addition he inserted in the dome a triple encircling system constituted by:

1. Three girding belts of *macigno*, a strong sandstone, inserted at the level of the first, second and third galleries, cramped by iron brackets;
2. An encircling belt of wood (whose actual stiffness and efficiency is difficult to determine) inserted above the first gallery level; and

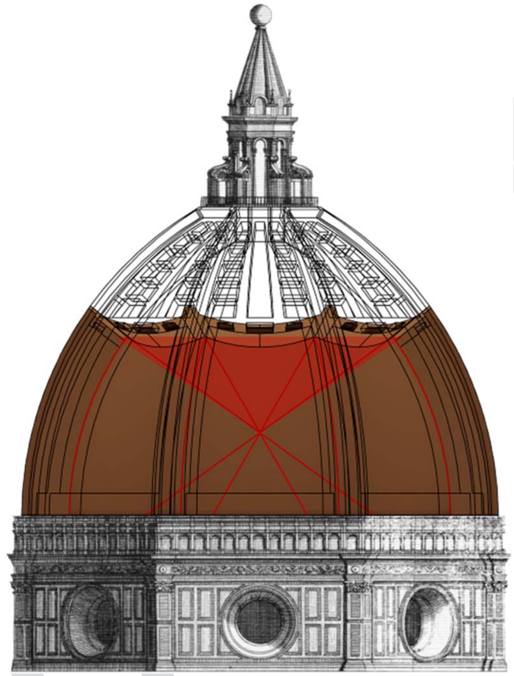


Fig. 1. (ikonternal) structure of the Cupola of Santa Maria del Fiore cathedral

3. A hidden system of encircling constituted by the structural behaviour known as *piattabanda* (flat arch) of each side of the octagonal ring.

These technical solutions adopted by Brunelleschi are well known today and, despite the current cracking pattern, have assured the resistance and safety of the Cupola for centuries, allowing for the achievement of good structural behavior under dead loads.

Materials and Foundations

The tambour and the pillars of the Cupola are made by stone masonry (mortar and *pietraforte* stone blocks). Approximately 8 million bricks of specific sizes and quality (called *quadroni*, which respectively measure $17 \times 34 \times 5$ cm and $22 \times 44 \times 5$ cm for the *corda blanda* apparatus, and $22 \times 22 \times 5$ cm for the *spinapesce*), and a high quality mortar were used in the construction of the Cupola. The ribs were riveted by Carrara Marble blocks to increase their static load carrying capacity.

The foundations of the pillars were made by huge and massive concrete; their thickness is approximately 6 m and they are settled in the main body of the Arno river gravels. The bedrock is approximately 18 m below the ground level, and is composed of shale of the *Sillano Fm* (Ghinelli and Vannucchi 1991; Coli et al. 2008). The bedrock is demonstrated in Fig. 2, in which the black line depicts the extension of the foundations, and the black block represent the remains of the ancient cathedral of S. Reparata. Geotechnical parameters of the Arno gavels are as follow: Unified soil classification system (USCS) = poorly graded gravel (*GP*); c'(cohesion) ≈ 10 kPa; φ'(angle of internal friction) $\approx 40°$; Vs (shear wave velocity) ≈ 420 m/s. Assuming seismic input from the bedrock with $a_g = 0.15$ g (as stated by the Italian recommendation for the Florence site given a return period of approximately 500 years) the calculated seismic amplification factor at the surface is between 1.4 and 1.5 ($a_g = 0.22$ g).

The water table is approximately 7 m below the ground level and has a seasonal excursion of up to ± 1.5 m, with a general direction

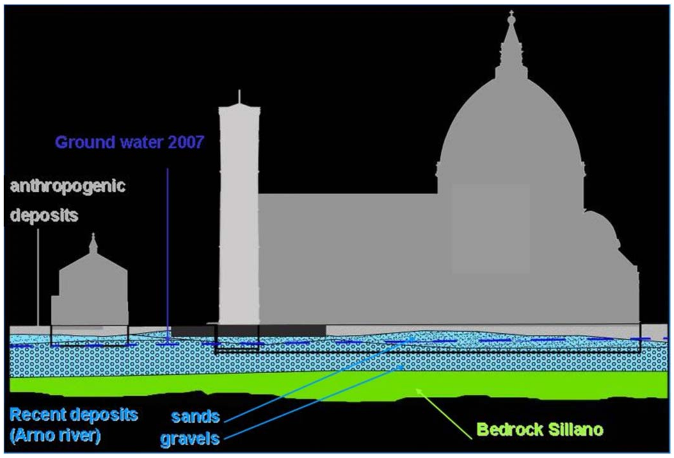


Fig. 2. Geological setting of the Baptistery, Giotto's bell-tower, and Santa Maria del Fiore cathedral complex

Table 1. Physical and Mechanical Data (γ = Weight per Unit Volume; UCS = Unconfined Compressive Strength; f_t = Tensile Strength; E = Young's Modulus; ν = Poisson's Ratio; p = Porosity; pe = Permeability)

Material	γ (kN/m^3)	UCS (MPa)	f_t (MPa)	E (GPa)	ν	p (%)	pe
Bricks	15.5	27.6	2.66	11	0.18	35	—
Mortar	17.5	19.6	2.05	7.85	0.27	28.2	—
Pietraforte	27	140	—	—	—	4–6	—
Carrara marble	27	130[a]—96[b]	—	49.5	0.274	—	0.2
Macigno	26.7	90	—	—	—	4÷6	—

[a]Cubic samples.
[b]ISRM standard.

of the water flow from east to west, parallel to both the Arno river and the nave of the cathedral. The available physical-mechanical data of the main materials employed to build the Cupola are reported in Table 1.

Cracks and Damage

Despite the constructive solution utilized by Brunelleschi, the Cupola is affected by a complex and widespread system of cracks, whose main peculiarity is summarized together with the main hypothesis developed over the centuries concerning its origin.

Cracking Pattern

The cracking pattern, as visible today, is composed of almost symmetrical cracks (Blasi and Ceccotti 1984) which, according to the last commonly recognized classification (Petrini 1984; Bartoli et al. 1996), is illustrated in Fig. 3 and can be classified as follows:

1. Major passing cracks (on both the domes)—approximately 5–6 cm wide—with vertical direction, laying at the centre of webs, on even sides (webs n. 2, 4, 6 and 8 in Fig. 3)(A).
2. Other major cracks on the tambour, in odd webs (1, 3, 5, and 7 in Fig. 3) —1–2 cm wide—starting from the central oculi, proceeding at 60° to the horizontal, to the intrados of the arch below (B).
3. Vertical non-passing cracks in the intrados of the internal dome, laying on the eight edges, starting from the base and reaching an intermediate level between the second and the third gallery (C).

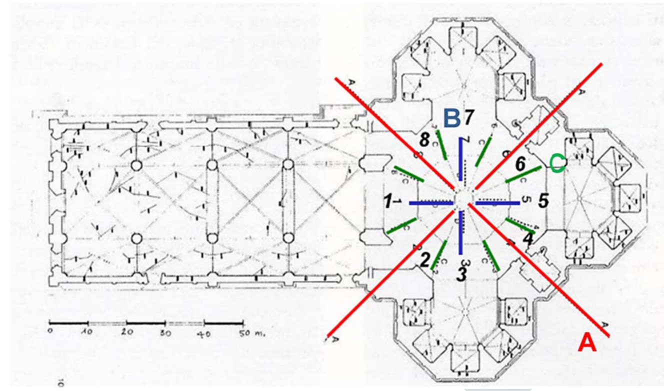


Fig. 3. Severe crack

Some minor cracks that are 1–2 mm wide, lay at the center of the odd webs at the intrados of the inner dome, and between the second and the third gallery level (Fig. 3, D); other minor fissures are visible on the semi-domes (E), on the tribune (F), and on the central nave (G), representing a secondary crack system (Petrini 1984).

This complex cracking pattern has been evolving during the centuries and has demonstrated a substantial symmetry, which finds notable variations with a concentration on the even webs. This phenomenon has been explained by considering the different levels of stiffness of the underlying bearing structures (the huge pillars instead of the wide arches). However, the global crack pattern that is now visible is consistent with the well-known damage mechanisms typical of domed structures: A vertical deflection of the top of the structure under its own weight with significant horizontal thrusts on the bearing elements (Chiarugi et al. 1993).

Main Hypotheses Spanning the Centuries on the Origin of the Damage

Previous studies (Ottoni et al. 2010) have extensively reported on the debates that have taken place over the centuries on the issue of the stability of Brunelleschi's Cupola, and three hypotheses can be considered as the most significant among those proposed overtime. The main conclusions reached by the scholars that have been charged with solving the issue of the Cupola's stability are briefly reported in the following text. Thanks to the combined analysis of monitoring data, numerical model results, and geotechnical investigations, some final conclusions can be advanced on the basis of their reliability.

The first hypothesis dates back to the late seventeenth century and was proposed by the Scientific Committee charged by the Grand Duke of Tuscany Cosimo III (1642–1723) to solve the question of the Cupola's stability. Giovan Battista Nelli and Vincenzo Viviani (the latter being a disciple of Galileo Galilei), after having observed the results of the primeval monitoring system which they had installed on the Cupola—a stone spy positioned on a major crack that had broken after the earthquake of September 22, 1695—hypothesized that the main cause of Brunelleschi's monument damage could be the weight of the dome itself, producing horizontal thrusts on the pillars; this is illustrated in Fig. 4. According to this hypothesis, they proposed the installation of four order of encircling tie rods, which was the traditional and commonly applied solution for dome masonry strengthening at the time. This intervention, rejected by public opinion, was never realized (Chiarugi 1996) and Brunelleschi's Cupola remains as the only great coeval masonry dome without a steel chain system. The same hypothesis was proposed two centuries later by the latest

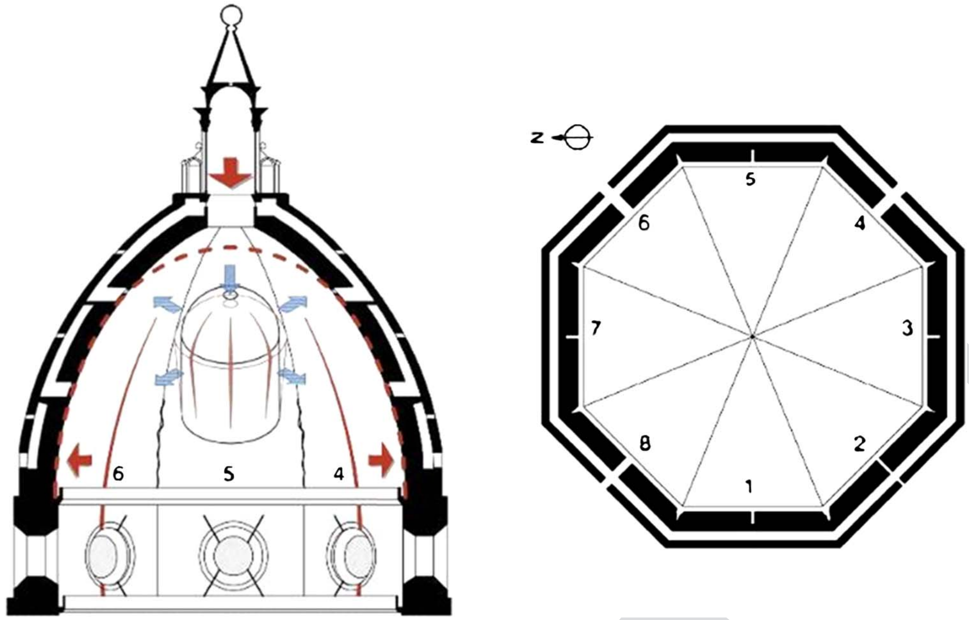


Fig. 4. Scheme of the hypothesis on the main cause of dome damage advanced by Viviani in 1695 and Chiarugi in 1988

Ministerial Committee (lead by G. De Angelis D'Ossat and C. Cestelli Guidi, in 1985) (Ministero 1985) which set up the first numerical model of the whole dome (Fanelli and Fanelli 2004). The Commission finally reached the conclusion that the main cause of the cracking pattern was the self-weight of the dome combined with the lack of tensile resistance of masonry; this conclusion confirmed the hypothesis of Vincenzo Viviani (Chiarugi et al. 1995).

The second main hypothesis was proposed one year after Viviani's conclusion, by the "obscure architect from Prato" (Chiarugi 1996) Alessandro Cecchini. He attributed the cause of the cracks to a differential settlement of the foundations. One century later the famous astronomer Leonardo Ximenes, after his precise primeval "topographic survey" of the dome, seemed to confirm this hypothesis (Ximenes 1757). Ximenes (1757) found a lowering at the base of the pillar under the web n. 4 which was consistent with a general movement of the whole structure south, as demonstrated in Fig. 5. Moreover, this hypothesis was supported by the strong asymmetry of the crack pattern at that time: According to the survey of Ximenes (1757), only two main cracks (on web n. 4 and 6) were present at that time on the dome.

In 1934 the first modern Ministerial Committee (chaired by R. Sabatini and P.L. Nervi) was charged of verifying and evaluating

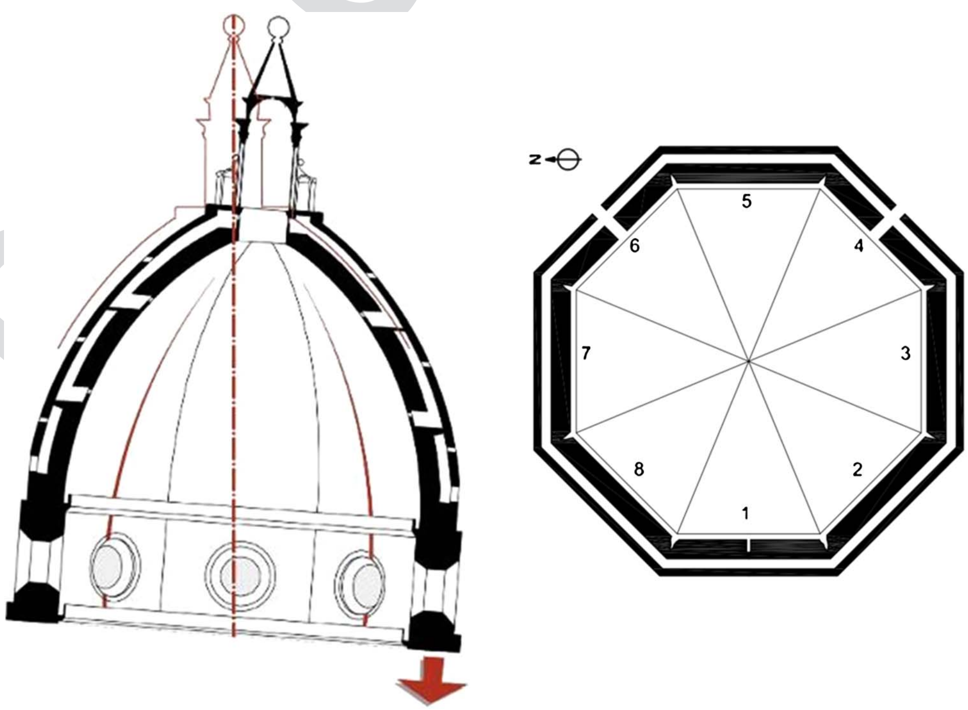


Fig. 5. Scheme of the hypothesis on the main cause of dome damage proposed by Cecchini in 1696 and Ximenes in 1757

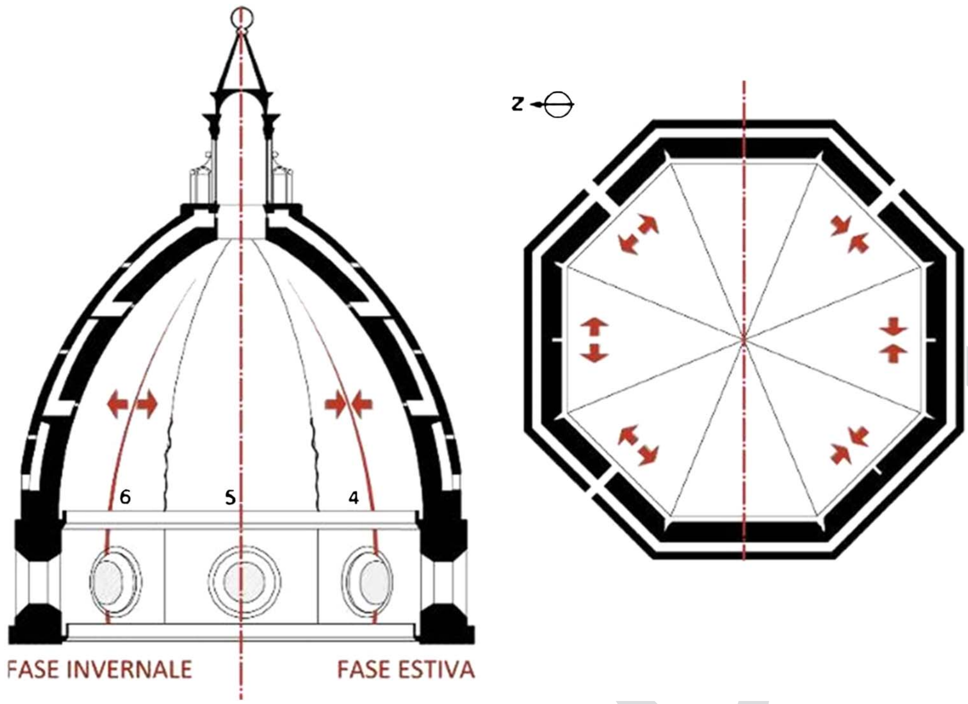


Fig. 6. Scheme of the hypothesis on the main cause of dome damage proposed by P.L. Nervi in 1934 and 1951

Cupolas' damages and cracks causes, and the relation between crack width and temperature variations (both seasonal and daily) was investigated (Nobili et al. 1934). After three years of studies, precise surveys, and monitoring operations lead by Padre Alfani (seismologist and Director of the *Osservatorio Ximeniano* in Florence), the Committee's conclusions on the causes of damage were unequivocal. In the structural report Pier Luigi Nervi finally refutes the hypothesis of initial breaking of the dome because of its dead weight, and identified that the issue lies in thermal variations as the main cause of crack evolution (Nobili et al. 1934; Di Pasquale 1977). The temperature variations (both seasonal and daily) were identified as the principal and unavoidable cause of Cupola damage (the so-called "Cupola's breath") is demonstrated in Fig. 6. The question of the Cupola was not yet settled, and in 1939 the Sabatini-Nervi Committee strongly suggested monitoring the evolution of cracks to clarify their effective behavior. Two different monitoring systems were installed on the Cupola to record its own real response on its stability question, and these monitoring have been integrated and enlarged over the years.

Damage Monitoring

As discussed in the previous section, Brunelleschi's Cupola damage began at the end of its construction and has evolved during the centuries. Over time, primeval monitoring apparatuses and more recent modern monitoring systems were installed on the Cupola to investigate the behavior of the cracks in relation to the environmental and mechanical events occurring to the structure. When concerning historic buildings, it is important to collect, by indirect observation, the information about the response of the structure with respect to past strong events (Ottoni 2012) to correlate these historical data with the trends obtained by the modern monitoring system. In this way, the whole life of a monument can be reconstructed, giving fundamental indication for its potential future behavior, possible conservation strategies, and for proper structural analyses.

The different data recorded by these monitoring systems are discussed and analyzed next.

Different Monitoring Systems

Over the centuries, numerous and different devices (e.g., gauges made by marbles, stones, alloys, iron wedges) were installed on the Cupola to control the evolution of its damage. Monitoring systems measure crack width variations, which are normally strictly connected to seasonal and daily cycles.

There are two main monitoring systems currently working on the Cupola: (1) a mechanical one installed by the *Opera del Duomo* (O.D.)(the legal entity founded by the Florentine Republic in 1296 to supervise the construction of the cathedral) in 1955; and (2) a digital one commissioned by the *Soprintendenza* (the local authority in charge of the conservation of the monument) and installed by the Experimental Institute for Models and Structures (ISMES) in 1987.

Despite the exceptional precision of these two systems, the quantity and quality of recorded data are concerning (Bartoli et al. 1996; Fanelli and Fanelli 2004); they have recorded only the last part of the entire life of the monument: The last 60 years out of the previous 6 centuries, or only the last 10%. It is clear that the correlation of these data with the often indirect evidences of previous damage evolutions provide crucial information in reconstructing the global behavior of the monument. In this respect, the identification and the dating of any repair or strengthening intervention made on the Cupola over the centuries, like the knowledge of the traumatic events suffered during centuries (e.g., earthquakes, fires) is extremely important. A number of indirect signs are able to tell the whole story of the building, giving important information on the evolution of its damage. This process, which is known as historical monitoring, constitutes a third monitoring system "installed" over centuries on the dome that allows researchers to trace the global graph of the building damage.

Mechanical Monitoring System

The first monitoring system installed by O.D. in 1955 was aimed at recording the opening and closing of the major cracks of the inner dome. It was installed following the directions of the coeval Ministerial Committee (R. Sabatini and P.L. Nervi) and is still working. This monitoring system is composed of 22 mechanical deformometers, recording crack-width variations four times a year. The collected data (approximately 270 for each instrument since the installation) allows researchers to reconstruct the last 60 years of crack-width evolution (together with dome internal and external temperatures). During this long period—the longest lasting period to the best of the authors' knowledge—different events have occurred, such as earthquakes, groundwater level variations, windstorms, and the 1966 flood of Florence.

Digital Monitoring System

The second and more articulated system, the one installed in 1987 by ISMES (Castoldi et al. 1989), is composed of 166 instruments registering crack width variations and the most significant structural parameters (temperature, vertical displacements, inclination, and underground water levels) to achieve an entire description of the dome's conditions. 72 inductive-displacement transducers (named as DFi-jj, where i is the web on which the instrument is placed and jj is the number identifying its position on the web) were placed on the main cracks of the inner and outer domes at five different levels, measuring the displacement of the cracks edges with a precision of ± 0.02 mm. Eight plumb-lines were placed at all the internal edges among the webs (intercepted by telecoordinometers at three different levels: Lower tambour, first galley level and ground level) for the purpose of measuring global displacements of both the pillars and the tambour. The evolution of the pillars over time that caused a vertical subsidence phenomenon was measured by a hydraulic levelling system, which was placed corresponding to the center of the lower part of the oculi (at approximately the second gallery level). Two piezometers, located near web n. 4 and below the nave, were placed to register the variation of the underground water level. As previously discussed, former studies assumed temperature to be the main cause of the crack-width evolution. Because of this, both air and masonry temperatures in the two domes have been monitored during the last 20 years as the monitoring system includes as many as 60 thermometers, measuring masonry and air temperature; instruments are indicated by TMi-jj and TAi-jj, for masonry and air temperatures, respectively, and they record temperatures on each web, primarily at the second corridor level (with a precision of ± 0.05°C). The acquisition system logs data every six hours during every day, starting at 6:00 a.m.

20 years of data, recorded from January 8, 1987, to July 31, 2007 (approximately 31,373 measurements for each instrument, more than five million measurements of data), have been analyzed.

"Historical Monitoring"

Despite their exceptionality, the two systems previously discussed have only registered the last 60 years of the last six centuries; the correlation of these data with the often indirect evidence of damage evolution over centuries can provide crucial information in reconstructing the current response of the monument. At the end of the twentieth century, Roberto Di Stefano said "The strengthening intervention on historical buildings has not to be a closed and myopic 'technical' vision: A complete study of structural behavior of a monument … must always start from historical analysis" (Di Stefano 1990). To reconstruct the historical evolution of the damage of the Cupola of the cathedral of Santa Maria del Fiore, a careful analysis of the historical archives has been carried out. The first information about cracks in Brunelleschi's Cupola can be related to the first great earthquake [estimated between 6 and 7 on the Mercalli-Cancani-Sieberg (MCS) scale] which had struck the Cupola at the end of its construction: It hit the monument on September 28th, 1453, a few years before the completion of the Lantern which, in the concept of Brunelleschi, had to play a significant role in topping and closing the Cupola. According the historical chronicles, the first cracks in the Cupola (from which some stones had fallen) can be dated back to this event. It is impossible to be sure that these cracks correspond to the current ones, but it is reasonable to assume that after considering the report by G. B. Nelli (Nelli 1753) after a much smaller seismic event (two centuries later) that the opening of the main cracks (on web n. 4 and 6) was actually triggered by this seismic event.

The successive information about cracks evolution is obtainable by the comparison, made by Nelli in his report, between the measurements of the crack at the South-East of the tambour, at the cornice level, and in the loggia of Baccio D'Agnolo. Observing the marbles of the so-called *gabbia de' grilli* (the marble construction externally adorning one of the eight sides of the Dome at its springing), Nelli deduced that in 1515, just 50 years after the completion of the Lantern, the crack in web n. 4 had already reached a width of approximately 1.7 cm. It is then possible to determine the width of the same crack in 1579, the year of completion of the Vasari's frescoes, in the inner dome of the Cupola by recording the difference—approximately 3.3 cm—between the crack in the plaster and in the underlying masonry. Such historical information can be employed to build a kind of primeval "monitoring system" in Fig. 7, in which (1) the linear regression stresses a crack width increasing of approximately 3 mm per century in the whole period and (2) it passes from 6 mm/century to 2 mm/century in the split analysis.

At the beginning of the seventeenth century, Gherardo Silvani refers in his report (dated September 18th, 1639) about hairs (*peli*) "through which air and wind can penetrate inside the dome" (Nelli and Sgrilli 1733); this is the first direct information on the presence of cracks on the Cupola. After the survey carried out in 1694 by the Grand Duke Commission—the one composed by G.B. Nelli and V. Viviani—two major cracks in webs n. 4 and 6, reaching a 2–9 cm maximum width (*un soldo di braccio*) were referred to. The first monitoring system was installed on the Cupola on that occasion; during the survey G. B. Nelli put stone spies on the main cracks, and after the seismic event of September 22nd, 1695, this primeval "deformometers" testified, with their breaking, the crack width evolution, which was measured for the first time.

The historical investigation suggests a hypothesis about the evolution of the damage, which is fundamental for the interpretation of the modern monitoring data. The broken stone detected by Nelli (1733) not only tells on the evolution of the cracks; it offers guidance for the understanding of the Cupola static behavior and, primarily, of its response to seismic events. The rapid widening of the fracture, which recorded an enlargement of 12 times in a month and a half, is the forerunner recording of what is now called *creep*. The information concerning the past behavior of the Cupola offers a crucial element for the analysis of the modern monitoring data: The effects of earthquakes on masonry structures have to be evaluated by also considering their time delay.

In Nelli's report (as in Silvani's one), major cracks are referred to, which implies that some minor cracks should have been present on the Cupola at those times. In his accurate measurements in 1757, Ximenes (1757) detected 13 cracks, but only two of them were referred to as major. Ximenes (1757) attributed these cracks to the differential settlements of the pillars (in particular pillar n. 4, of

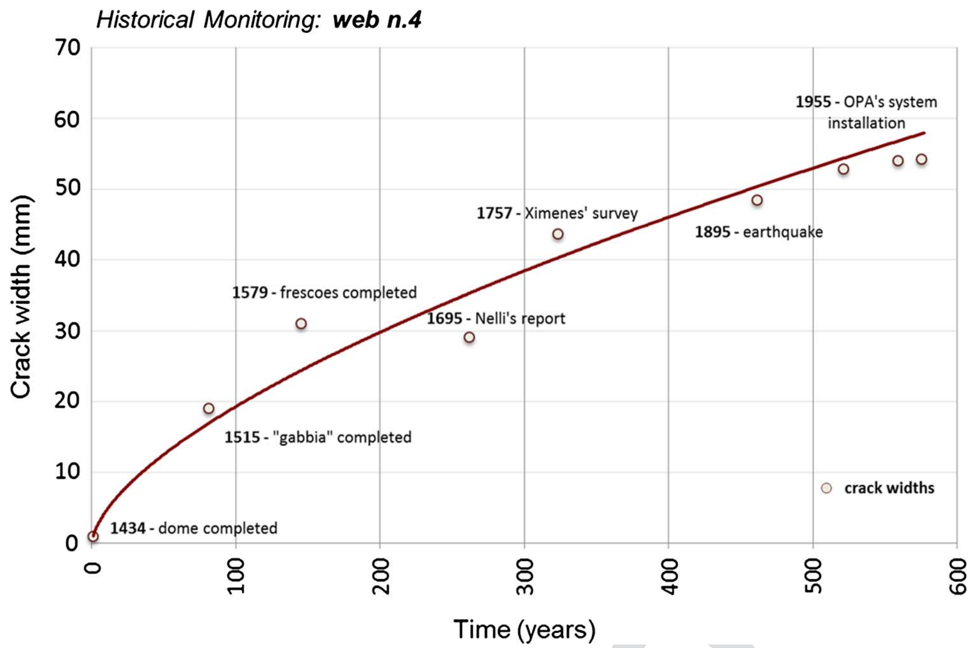


Fig. 7. "Historical monitoring": graph of crack evolution in web n. 4, as derived by indirect observation during centuries

which he had registered a certain inclination). From the examined reports, the appearance of cracks in the webs n. 2 and 8 emerges as clearly successive in nature, and the exact position and amplitude are reported in 1934 by the first modern Ministerial Committee (Nobili et al. 1934). It is impossible to establish when these two further cracks (web n. 2 and 8) were formed, but literature finds probable cause in a seismic event (Blasi and Ceccotti 1984). After the devastating earthquake on May 18, 1895, Luigi del Moro, architect in charge of the study of the Cupola at that time, registered a strong evolution of cracks. He wrote that "... the major cracks on the web n. 2 and 8, which also existed before, [were] made visible inside, after the falling of the grout." By reassembling the documents, it is possible to identify seismic events as the primary trigger and cause for the acceleration of the Cupola's damage. Assuming that the cracks in webs n. 2 and 8 have been formed because of the earthquake of 1895, if they had followed the same trend as those of webs n. 4 and 6 (after the 1453 earthquake) they would now measure approximately 2.5 cm. This amplitude is the real measure of these two cracks, and is recorded by the modern instruments in use.

Some Results of Static Monitoring Data Analysis

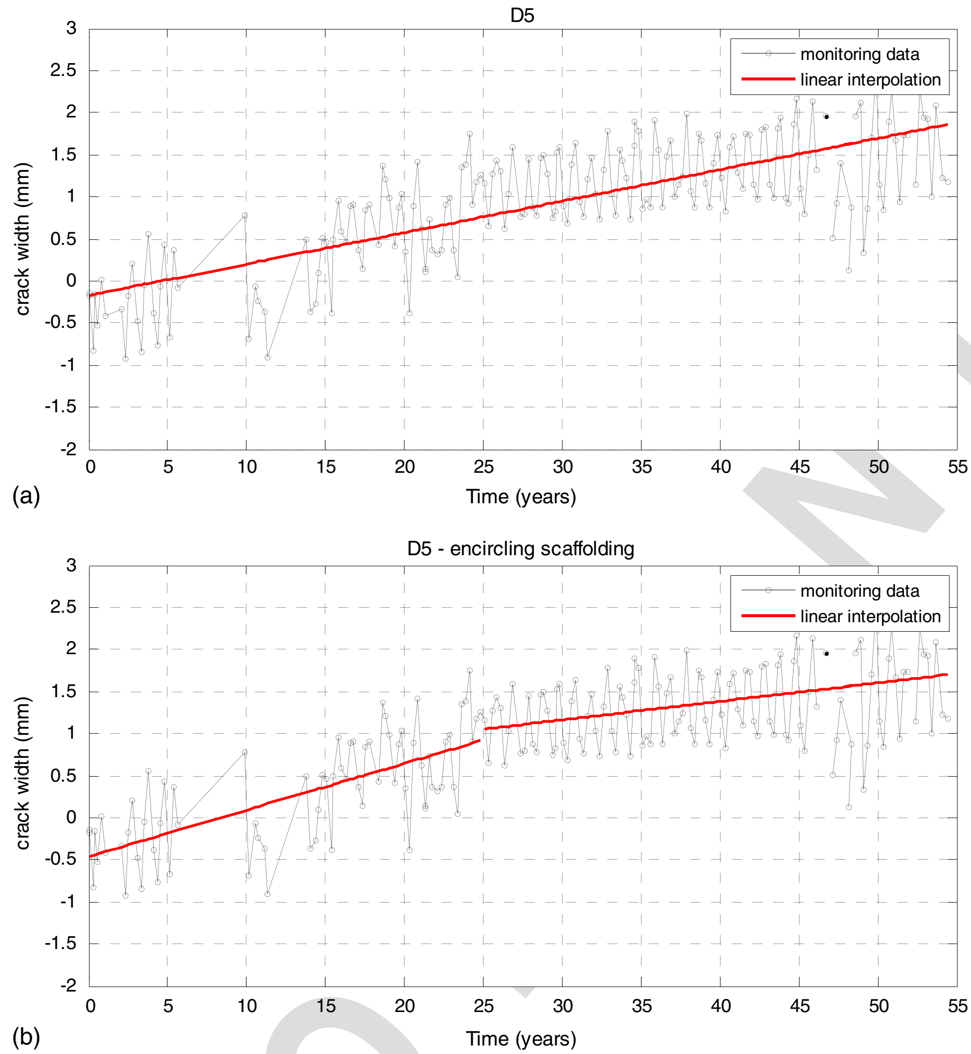
The three systems previously discussed constitute one of the most significant and complex monitoring systems now present on a historical monument, not only for the huge number of installed instruments but also for the exceptional duration of measurement. Previous studies have already examined the data recorded by the instrumental monitoring systems until 1996 (Chiarugi and Foraboschi 1995; Bartoli et al. 1996; Chiarugi et al. 1998; Gabbanini et al. 2004), analyzing the trends of the cracks and demonstrating a strict relation with temperature variation. In this paper, the data analysis has been extended to 2007, and the elaboration made so far has shown some interesting evidence on the reliability of the previously summarized hypotheses.

Horizontal Thrust

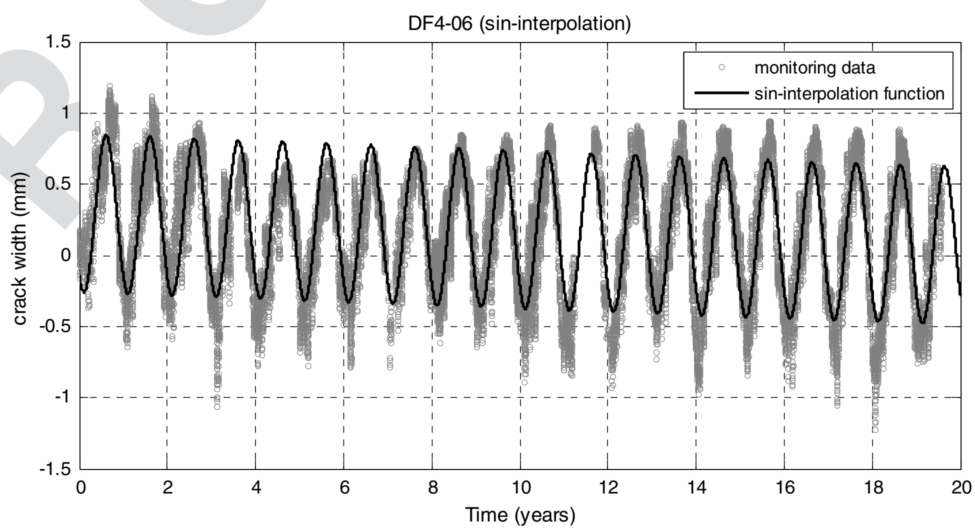
The linear regression applied to the 22 instruments of the mechanical monitoring system has highlighted a global increasing trend of major crack (deformometer D5 in web n. 4) width of approximately 3 mm per century [Fig. 8(a)]. This result partially confirms the conclusions of previous studies, which have underlined a steady pejorative increasing in crack size estimable at approximately 5 mm for century. However, it is clear by the figure that the agreement between the plotted regression and the registered data are quite poor. The presence on the Cupola in the years from 1980 to 1996 of (contested) encircling scaffoldings—put in for work for the restoration of the frescos (Dalla Negra 1995)—suggested a different interpretation of data. By splitting the records into two different periods—before (from 1955 to 1980) and during the presence of the scaffoldings (1980–1996)—a significant variation in the width trend can be registered. The trend passes from 6 mm per century (before the installation of the scaffoldings) to almost 2 mm per century during the "encircling" of the Cupola with the scaffoldings [Fig. 8(b)]. Here, only the analysis of the most significant deformometer (D5 on web n. 4) is plotted, considering its easier and direct comparison with the digital system, but similar results have been obtained also for the other instruments.

"Cupola Breath"

The strict relation between crack width and temperature variation clearly results by the analysis of the data recorded by the digital monitoring system. Despite the evidence observed periodically, and because of the thermal variations that are demonstrated in Fig. 9, it is possible, after this analysis, to finally understand the context of the conclusion of 1934's committee. Indeed, whether the temperature certainly contributes and significantly influences the crack-width variations, a residual trend is shown by experimental data analysis after the "purging operation" (Ottoni et al. 2012) (Fig. 10). In short, the data show an opening trend totally independent of temperature variation which conversely—unless a seasonal and daily periodicity—has remained constant over the years of observation; this fact confirms that other phenomena (such as dead weight combined to seismic events) have to be retraced as the main causes of dome damage.



F8:1 **Fig. 8.** Two different regressions of the same deformometer (D5 in web n. 4) in the last 55 years, from 1955 to 2009



F9:1 **Fig. 9.** Experimental data of DF4-06

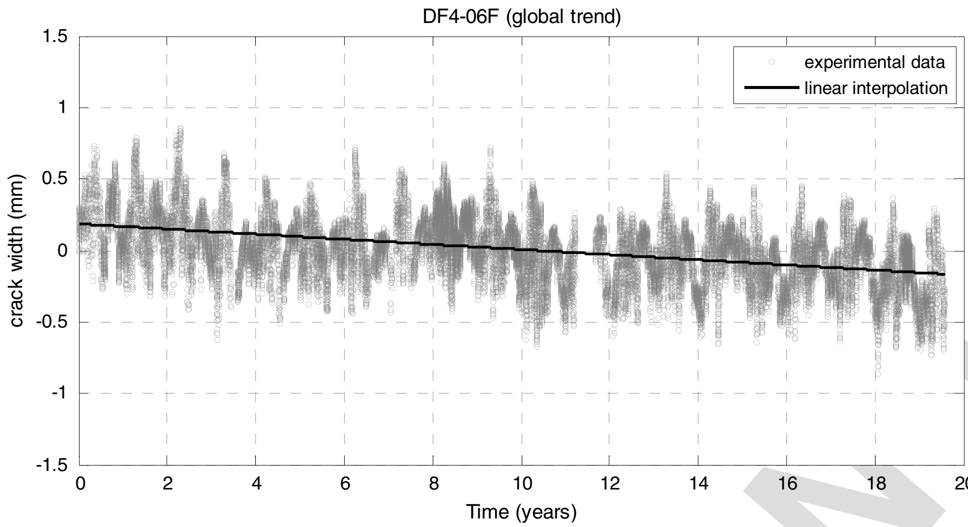


Fig. 10. Linear regression of the experimental "purged" data recorded by DF4-06 in the whole period (showing a residual linear trend)

Differential Settlements

The data recorded by the level meters and by the telecoordinometers were analyzed concerning the differential settlements of the pillars. The horizontality of the plane of impost of the Cupola was recorded through a precision topographic levelling carried out by the Military Geographic Institute (IGM) of Florence. The analysis of the results led to considering the differences in the vertical measurements of the Cupola as absolutely irrelevant on the overall behavior of the monument, given their extremely low values.

Previous studies (Chiarugi et al. 1996) have stressed that the data acquired by the level meters during the first years were actually unusable because of the presence of air bubbles in the hydraulic circuit, which had often provided unreliable results. Indeed, from 1988 to 1992, the system was consequently reduced from 8 to 6 level meters vessels to isolate the part affected by anomalies; the system can now be trusted to provide accurate data. In the graph reported in Fig. 11, the data recorded by the eight level meters installed on the eight webs are plotted. It is possible to observe that, from a structural point of view, there are no significant variations in level among the different points. Substantially the whole instrument exhibits the same trend, which is incompatible with the differential settlement of the pillar n. 4 as hypothesized by Ximenes in 1757. The same result can be observed analyzing the data recorded by telecoordinometers. The installation of the plumb lines in the corners of the octagon, at the lower part of the Cupola, has allowed the possible variations in the verticality of its bearing structures (the tambour) to be kept under constant control, and to evaluate the displacements of the underlying pillars. Despite some significant operational difficulties which have produced unreliable data during the first period of measurement and were highlighted in a previous work (Chiarugi et al. 1996), the signals recorded in the last years have shown some negligible irregularities. Fig. 12 shows the graph of the displacements of the top of the pillars between the web n. 4 and the two adjacent ones (n. 3 - on the south side, and n. 5 - on the east side; Fig. 13 shows the y-axis corresponds to geographical north. A shift of the top of the pillar under the web n.4 towards

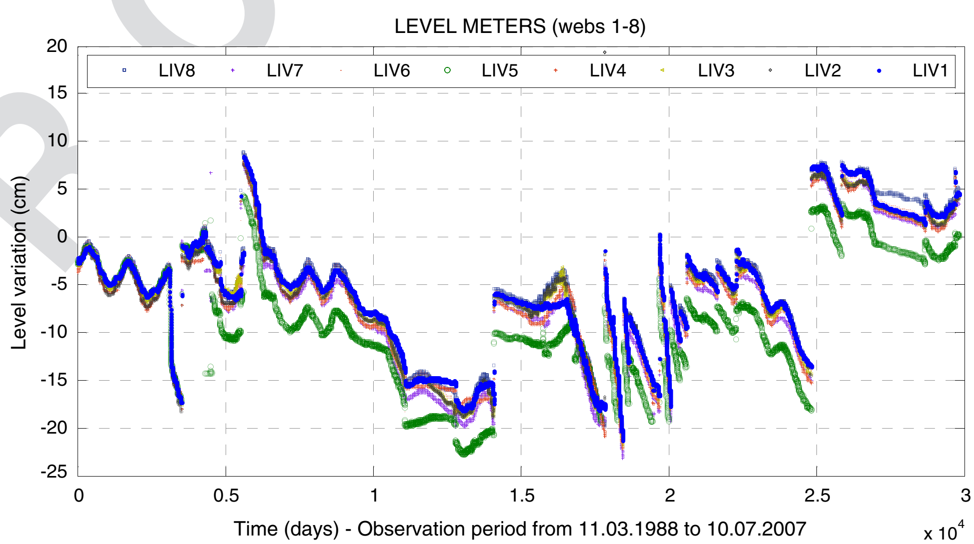


Fig. 11. Plot of the data recorded by the 8 level meters installed on the Cupola webs. The measures do not evidence significant variations among the four pillars

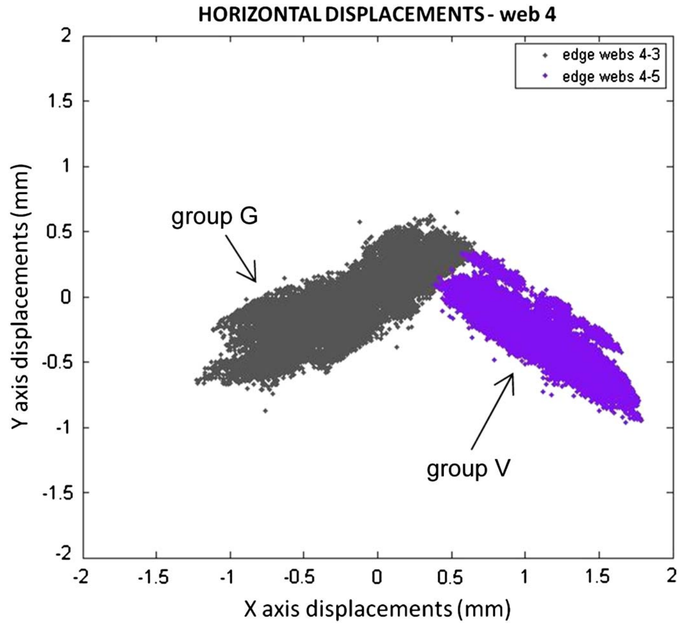


Fig. 12. Horizontal displacements of the top of the pillars between the web n. 4 and the two adjacent

the south-east direction should be observed in the graph, which is fully compatible with the crack detected and with the structural configuration of the monument. However, the overall entity of this displacement is not significant from a structural point of view.

As a conclusive remark, it can be confirmed that the displacement components of the top of the same pillar show a clear periodicity, further confirming the strong dependence of monument behavior with temperature variation. Almost the same behavior has been registered for the other pillars. Overall, the very small recorded displacements and their negligible trends (Fig. 13), joined to the lack of substantial differences between the eight webs, which have all shown a coherent behavior, lead to confirm after centuries the exclusion of the ancient hypothesis of differential settlement as the origin of the Cupola progressive damage.

Measured Seismic Response

The dynamic response function of the Cupola was analyzed by using four 3-component seismic stations installed from the ground up to the top (Fig. 14). The Lennartz seismometers (Le 3D/5 s) were sampled at 100 Hz by a Guralp CMG-DM24 24 bit digitizer. The used sensors have an eigenperiod of 5 seconds and a sensitivity of 400 V/m/s. All measurement stations were synchronized by using a GPS time code. With the aim of providing uniform results, all four stations were located along the web n. 7 towards north, facing the street *via dei Servi*, following the vertical profile of the structure as shown in Fig. 14. The first station (blue box) was at the ground level within the *Sacrestia Vecchia*, the second (red box) was outside on the terrace, the third (green box) was placed inside in the gallery at the base of the web, and the fourth (black box) was at the summit of the Cupola within the *Serraglio*. The four stations are identified by different colors, which are used to compare the spectral response of the structure reported in Fig. 15. The peak at 0.1–0.5 Hz is generated by the ocean microseism and is well recorded by the stations with the same amplitude, which indicate no amplification at such frequency range. At frequencies higher than 0.5 Hz it is evident the amplification effect related to the structure with a clear peak at 1.7 Hz.

The frequency response of the Cupola was evaluated by using both the ambient seismic noise and a local tectonic earthquake (the ML 4.2 local earthquake, Mugello, March 1, 2008, Figs. 16 and 17). From the ratio of the spectral components, it is showed that the Cupola is characterized by a main frequency at approximately 1.7 Hz (Fig. 18). This frequency, highlighted by the analysis of all of the horizontal components, confirmed the results obtained in a previous in-situ experimental campaign performed in the 1980s (ISMES 1987) when the following main frequencies were obtained: f_1 (north–south) = 1.7 Hz and f_2 (east–west) = 1.8 Hz (Table 2). The recording of the ML 4.2 earthquake reveals that the Cupola itself seems to slightly reduce the seismic amplitude with respect to the rest of the Santa Maria del Fiore cathedral (Fig. 17).

Numerical Model

Thanks to the attention driven by the last Ministerial Committee lead by G. De Angelis D'Ossat and C. Cestelli Guidi in 1985

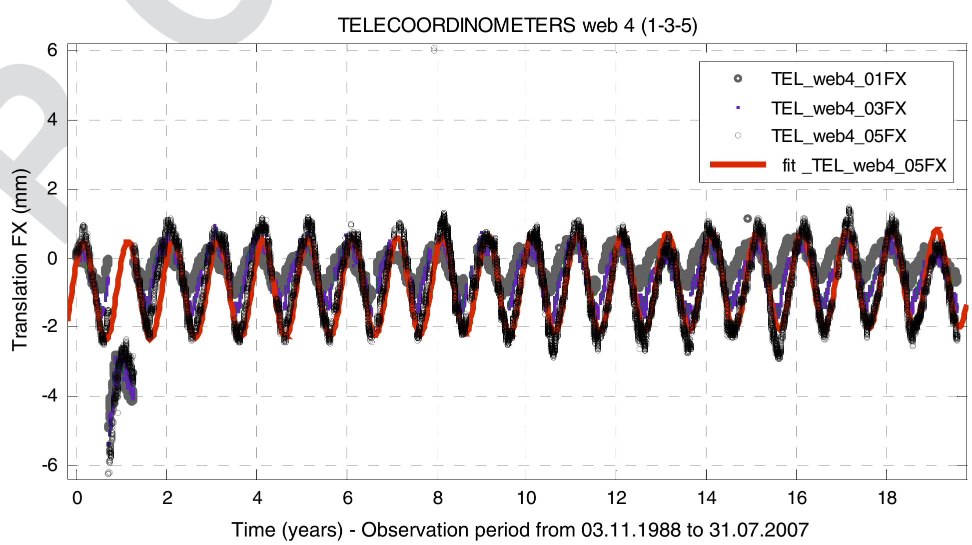


Fig. 13. Horizontal displacements (*x*-axis direction)

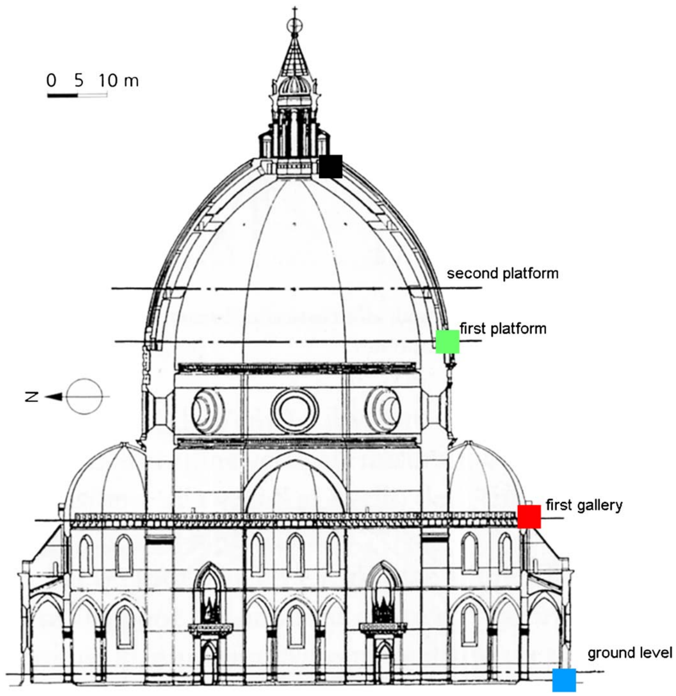


Fig. 14. Location of the four seismic stations along the vertical profile

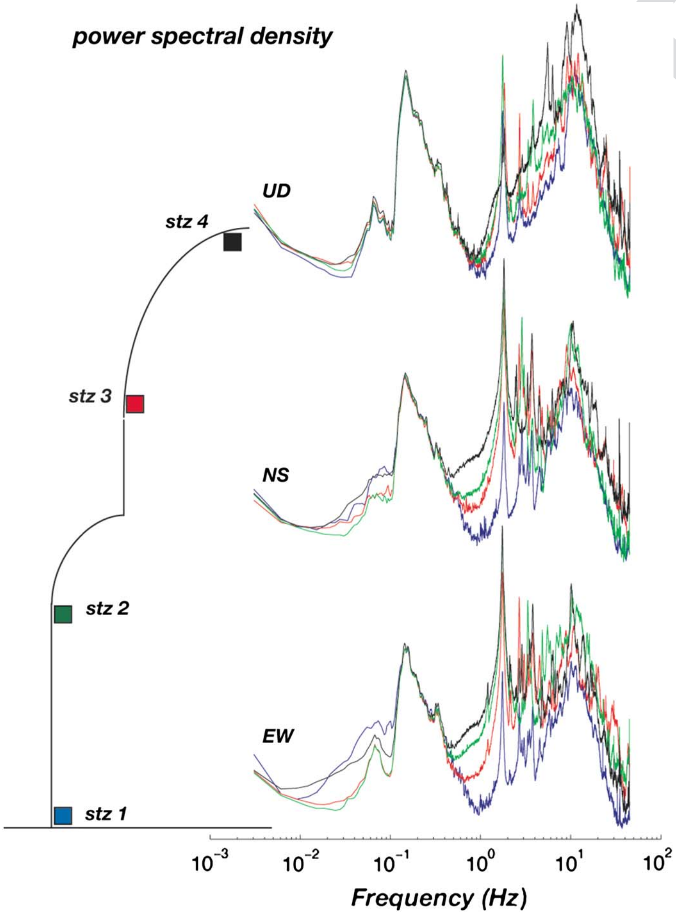


Fig. 15. Power spectral density of the seismic displacement in the North-South (NS), East-West (EW) and up-down (UD) directions measured at the four stations

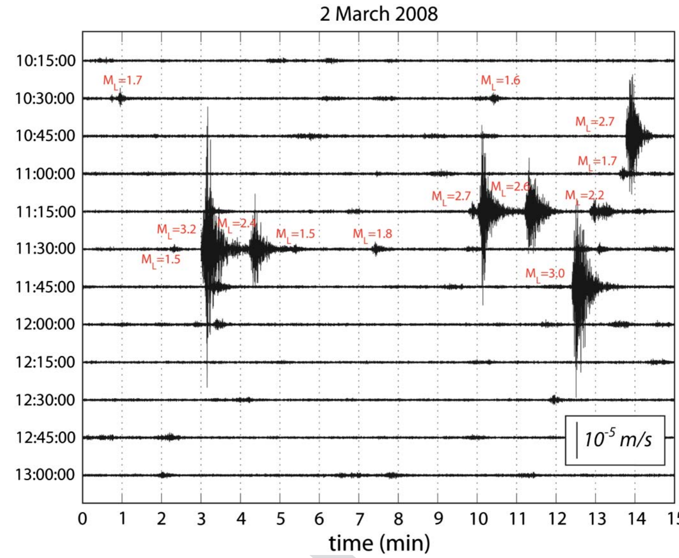


Fig. 16. Seismic sequence of March 1st, 2008 occurred in Mugello (at a distance of ∼35 km)

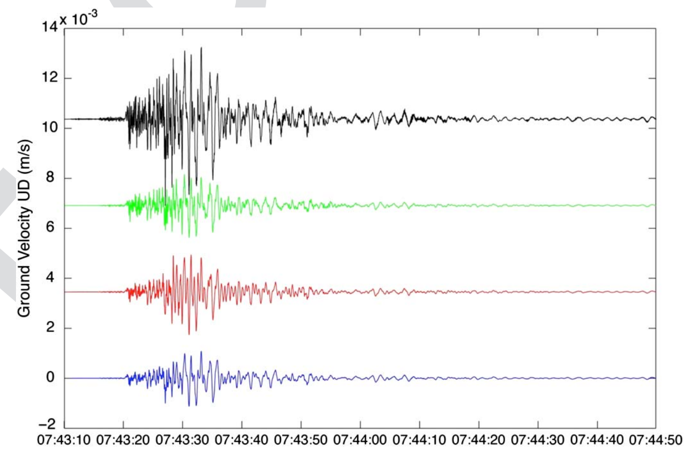


Fig. 17. Vertical component of the seismic signals recorded into the cathedral during the ML 4.2 earthquake on 1st March 2008

(Ministero 1985), the 1990s saw the development of a series of studies and numerical models of increasing complexity of the Cupola that followed both the evolution of numerical techniques and the growing knowledge on the structure (e.g., monitoring, in-situ experiments, knowledge of structural details.). The first models (Chiarugi et al. 1983, 1993, 1995; Fanelli and Fanelli 2004) were built assuming a simplified geometry of the structure, taking advantage of the radial symmetry, allowing for an insight into the comprehension of the static and thermal behavior of the Cupola and to examine the origin of the actual cracking pattern in depth.

On the basis of the results of a 3D topographic survey of the whole Cupola that was used to build a new detailed numerical model, recent results concerning the static identification are reported. The new numerical model was built with the commercial code ANSYS using solid hexahedral iso-parametric finite elements to discern all of the main geometrical components. The aim of this new modelling was to identify a numerical model to assess possible development of the cracks over the centuries (through staged construction analysis) and to move the first steps towards the study of seismic vulnerability of the Cupola. The damage and the cracking pattern were first analyzed and, after calibrating the numerical

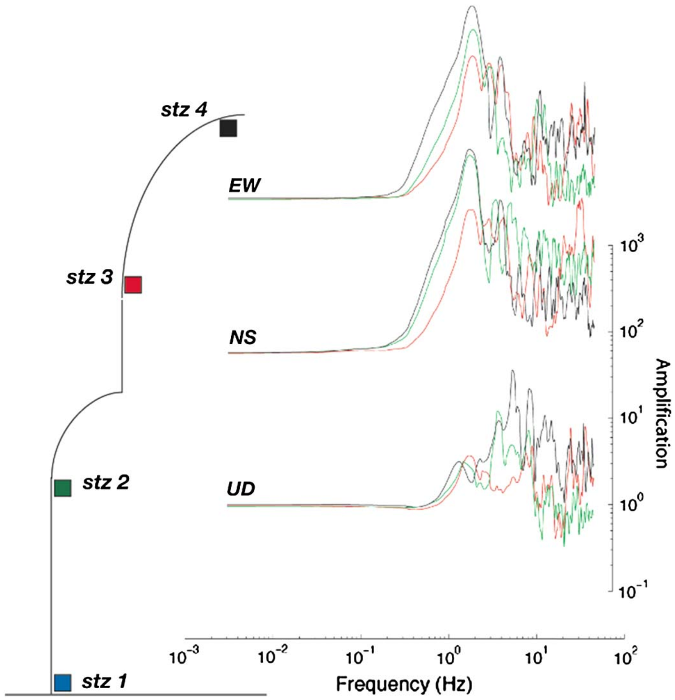


Fig. 18. Spectral response of the Cupola calculated for the components

Table 2. Experimental and Numerical Frequencies

Frequencies (Hz)	Experimental results	Undamaged model	Model D with cracks
f_1 (North-South)	1.700	1.290	1.700
f_2 (East-West)	1.800	1.403	1.805

model to fit the actual damage, nonlinear analyses were next performed to assess the potential seismic vulnerability of the structure (through a simplified pushover approach).

Static Analysis

To account for the current crack state, and to identify the numerical model, an iterative staged construction procedure was adopted aimed at assessing the likely cracks time evolution. The analysis of the Cupola under its own weight was performed through a step-by-step application of the self-weight to reproduce the effective stages of construction of the Cupola. To this aim, the BIRTH & DEATH feature of the finite element (FE) code ANSYS was used. The sequence of own weight application, i.e., the element BIRTH, is reported in Fig. 19 (e.g., first the effects of the main pillars have been considered, next the archesin). A final step, not reported in Fig. 19, is STEP 7 that corresponds to the construction of the Lantern, whose weight is approximately 800t. The Lantern was not included in the model, and the loads that it transfers to the Cupola were included in the FE model as distributed loads applied to the finite elements that model the oculus. The structure changes at each step (according to the sequence reported in Fig. 19), and at the end of each load step through the BIRTH option, the new loads are applied during the deformed geometry so the nonlinear geometric effects are activated between each load step.

To analyze the static behavior of the Cupola, two numerical models differing for the employed cracks modelling technique were developed. The first model (Model D) was built adopting a discrete crack modelling strategy (Betti et al. 2010; Bartoli and Betti 2013). Starting from the results of a preparatory model (undamaged model) a first refinement was made by introducing nonlinear contact elements along the area in which a non-admissible tensile stress state arose; the solid elements were disconnected and along the discontinuity the corresponding nodes were doubled and connected with the following typology of elements: (1) element *contact52* (compression only elements); and (2) element *link10* (tension only element, with a tensile cut-off of approximately 0.2 N/mm^2). This procedure was repeated iteratively to look for a possible time evolution of the cracks along the webs, primarily focusing on cracks A and C (Fig. 3). The final 3D identified model consisted of 82,492 nodes, 60,572 3D *solid45* elements, 1,503 1D *contact52* elements, and 1,503 1D *link10* elements corresponding to 241,635 DOFs. A second model (Model S) was built adopting a smeared-crack modelling approach. Model S allows for an effective and immediate representation of the cracks that develops in the Cupola between each load step. When analyzing the results in terms of cracks, it is possible to compare the cracking pattern at the end of STEP 6 (Fig. 20) with the cracking pattern at the end of STEP 7 (Fig. 21). The results clearly showed that the cracks developed in the Cupola are a consequence of the increase of the weight attributable to the construction of the Lantern. Before the erection of the Lantern, Fig. 20 shows the presence of small cracks near the central oculi of the tambour. After the erection of the Lantern, Fig. 21 shows a widespread cracking pattern develops at the center of each even web that cross the whole section of the Cupola (crack A). Vertical cracks in the intrados of the internal dome that start from the tambour and reach an intermediate level between the second and the third gallery are laying on the eight edges (crack C).

The results obtained with Model D in particular show that because of the self-weight and geometry of the Cupola, the first cracks likely to appear are the A cracks they develop starting from the oculi at the tambour level (on the even webs), under and beneath the oculi themselves. The appearance of these cracks modifies the structural behavior of the Cupola and facilitates the development of the C cracks. At the end of the iterative procedure, the numerical model matches the different width of the A cracks in webs n. 2, 8, 4, and 6. The maximum opening recorded in webs n. 4 and 6 is approximately 5.5 cm against the 5.4 cm numerically estimated; the maximum opening recorded in webs n. 2 and 8 is approximately 2.5 cm against the 2.3 cm which is the result obtained by the numerical model. Despite the coincidence of these values, the difference between the opening on the two groups of webs is originated by the presence close to webs n. 2 and 8, of the main nave of the cathedral, which in turn originates an effective constraint against horizontal displacement as already observed through the analysis of the monitoring system data.

The FE models D and S provided complementary information about the static behavior of the Cupola; taking into account the limited information about the material properties, the discrete-crack modelling (Model D) was preferred over the smeared-crack modelling (Model S) as the former is less demanding from a computational point of view. Subsequent analyses were performed employing Model D.

Modal Analysis

The FE model was also used to evaluate the main frequencies and modal shapes of the Cupola, taking into account the experimental results obtained during the in-situ survey. The first two modal shapes found with the identified model are reported in Fig. 22. The first modal shape is transversal (north-south direction, orthogonal to the main nave direction) whereas the second one is longitudinal (east-west direction, parallel to the main nave direction). The ratio

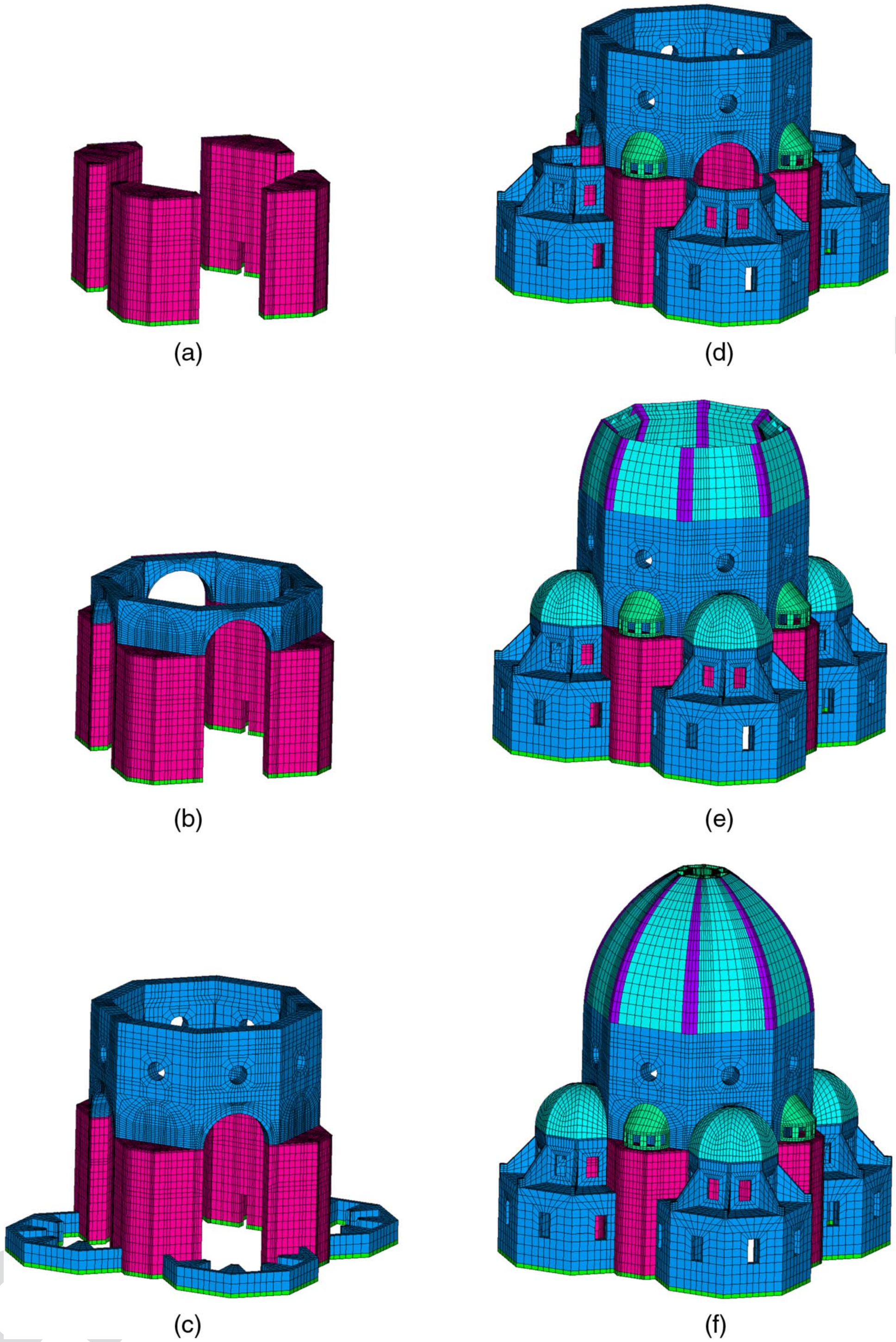


Fig. 19. Elements self-weight application: (a)-(f) BIRTH sequence

between the first two modal shapes is very close to the experimental results (0.942 against the experimental value of 0.944, Table 2). By comparing the results of the undamaged model with those of the damaged one, it is possible to observe, as expected, that cracks play a fundamental role from a dynamical point of view.

Pushover Analysis

The identified numerical model (Model D) was used to move toward the estimation of the Cupola's seismic behavior by a pushover approach. Accordingly, the effects of the seismic loads were evaluated through the application of two systems of orthogonal forces lying in the horizontal plane. These forces, which are assumed to not be acting simultaneously, were determined as follows: (1) a first load distribution was assumed directly proportional to the masses (uniform); and (2) a second one was assumed to be proportional to the product of the masses times the displacements of the corresponding Cupola modal shape (modal). These load configurations could be considered as two limit states for the Cupola's capacity.

The procedure starts from the identified static cracking configuration (in which the cracks are attributable to the self-weight only)

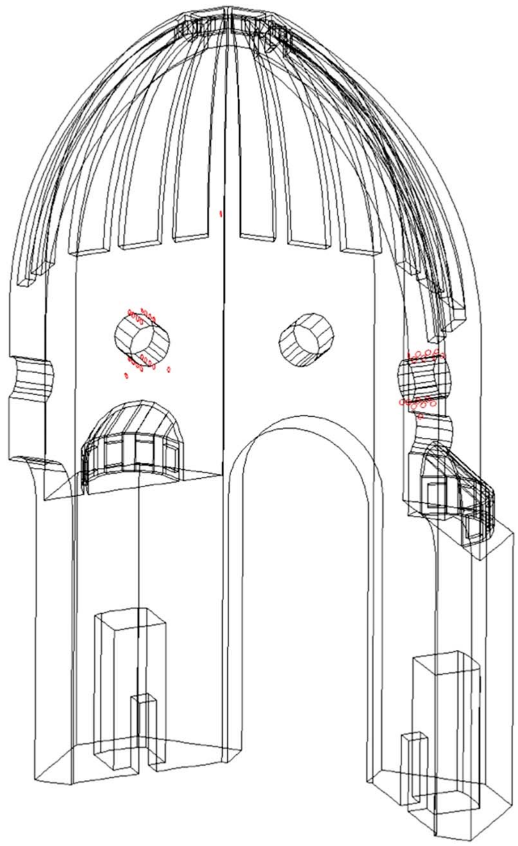


Fig. 20. Model D: cracking pattern STEP 6

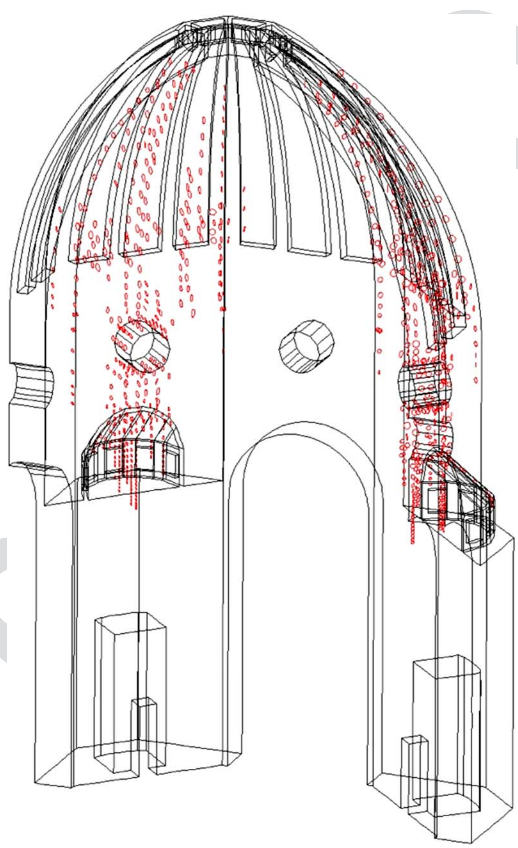


Fig. 21. Model D: cracking pattern STEP 7

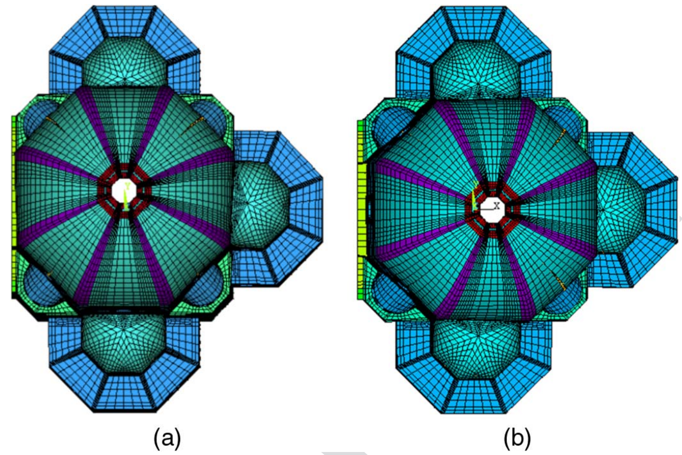


Fig. 22. Modal shapes: (a) first mode $f_1 = 1.70$ Hz; (b) second mode $f_2 = 1.80$ Hz

and subsequently, as the external horizontal pushover load increases, new unilateral contact elements were inserted (updating the FE model correspondingly, as for the static staged construction analysis). At each load step, the circumferential stresses were checked and connections between adjacent nodes were released in those areas in which the tensile stresses were not admissible for masonry (that is, tensile stresses arise more than 0.2 N/mm^2) by inserting unilateral contact interfaces. One must observe that, by means of this iterative procedure, the development of the cracked area follows the load application, i.e. the area in which the cracking appears is not imposed (*a priori*).

Focusing the attention to the north-south direction (y-direction) in case of uniform loading (attributable to the radial symmetry of the Cupola the behavior in east-west direction is quite similar, but is not identical because of the presence of the main nave) a general sketch of the seismic behavior is shown in Fig. 23. As the load is acting in the $+y$-direction, it is possible to observe that a crack opening arises in webs n. 6 and 8, with a corresponding crack closure in webs n. 2 and 4. The cracks in the webs in the direction

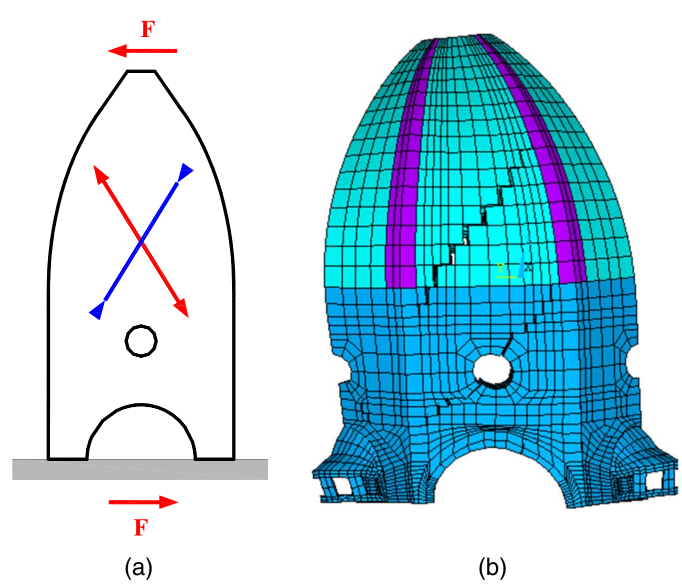


Fig. 23. (a) Principal tensile and compressive stresses directions; (b) model with cracks updated

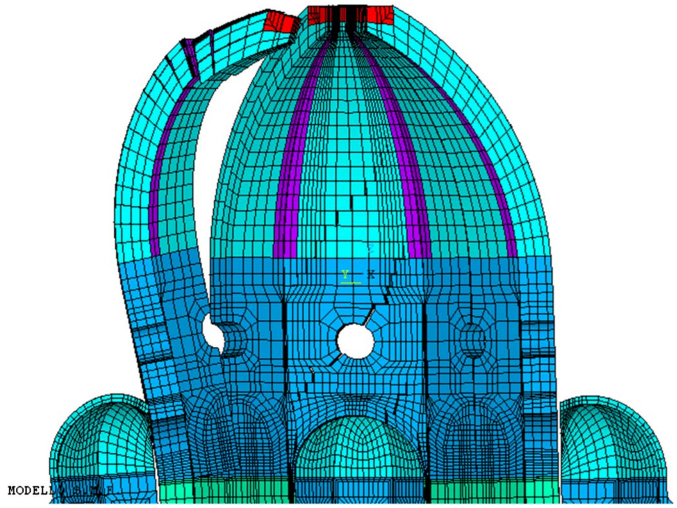


Fig. 24. Collapse configuration: global view

of seismic forces show a trend to close, whereas the cracks in the opposite webs tend to widen themselves.

In the event of a seismic load, the crack opening on the webs originates a crack A propagation over the top level of the Cupola (the oculus). As a reference, the load corresponding to the PGA in Florence with a return period of approximately 500 years has been assumed ($a_g = 0.22$ g). When the load reaches approximately 5% of the maximum seismic load, the first observed phenomenon is represented by a concentration of tensile stresses at the top level of the cracks (both A and C). At approximately 15% of the seismic load, tensile stresses arise on the major arches beneath the tambour, and it is possible to recognize the typical mechanism affecting the triumphal arches, with tensile stresses in the intrados. This stress state justifies the increase in new sub-vertical fissures. To proceed with the adaptive upgrading of the internal static conditions, new unilateral contact elements were added in this area. Next, load steps (approximately 20% of the maximum seismic load) show the development of a shear-type behavior on the webs parallel to the load direction. From a qualitative point of view, it is possible to observe that both the principal tensile stresses and the principal compressive stresses are positioned at 45° with respect to the horizontal directions [Fig. 23(a)]. The model was then updated accordingly through the insertion of new unilateral contact elements [Fig. 23(b)]. The last step of the iterative procedure concerns the webs positioned in the direction orthogonal to the seismic loads. The effect of the horizontal load is to enhance the flexural behavior that the arches already show under dead load (attributable to the widening of cracks A). To take it into account and introduce an internal stress state which is statically admissible for masonry, new contact elements were introduced in the corresponding area. The final load step is shown in Fig. 24, which depicts the numerical collapse configuration of the Cupola. It also suggests that the corresponding collapse mechanism can be investigated by a kinematic approach.

The analyses discussed in this paper represent a first step in the evaluation of the monument behavior under seismic-like forces, assessing possible ultimate collapse mechanisms. A more exhaustive interpretation of the overall structural response under seismic loads will require an interaction between several modelling strategies (e.g., including limit analysis) and, even if still a long way off, a time-history analysis with seismic ground-motion inputs reflecting the seismogenic characteristics of the Florence area through a realistic rheological model of the dynamic properties of the Cupola masonry.

Conclusion

Monitoring systems, combined with historical studies, material analysis, dynamic investigations, and numerical models allow modern researchers to reliably forecast the normal behavior of the Cupola, evidencing cracks-width variations which are strictly connected to seasonal and daily cycles. Despite the fact that temperature certainly contributes and significantly influences crack width variations, residual trend is shown by the analysis of monitoring data, which is fully consistent with the results of the numerical model. The conclusion on the structural behavior of this great monument—reached by Chiarugi in the 1980 s following the first Viviani's hypothesis—that the crack pattern is simply caused by the presence of the horizontal thrusting forces exerted by the Cupola because of its self-weight, seems to find further confirmation by the monitoring data analysis combined with the results of numerical models. The study presented in this paper, gathering all the information needed for a correct analysis of the damage evolution, can be used to further calibrate, and validate, possible strengthening and reinforcement interventions. In addition, it represents a general methodology, particularly recommended for ancient monument retrofitting strategy, which finds it primary guarantee of success in adopting a multidisciplinary approach.

Acknowledgments

Heartfelt thanks are expressed to the Opera del Duomo of Florence, which, together with the Soprintendenza (SBAA, Soprintendenza ai Beni Ambientali e Architettonici di Firenze, Prato e Pistoia), kindly allowed us to use the data of the mechanical monitoring system (1955–2007) and those of the digital one (1987–2007) for the present study.

References

Bartoli, G., and Betti, M. (2013). "Cappella dei principi in Firenze, Italy: Experimental analyses and numerical modeling for the investigation of a local failure." *J. Perform. Constr. Facil.*, 10.1061/(ASCE)CF.1943-5509.0000315, 4–26.

Bartoli, G., Chiarugi, A., and Gusella, V. (1996). "Monitoring systems on historic buildings: The Brunelleschi dome." *J. Struct. Eng.*, 10.1061/(ASCE)0733-9445(1996)122:6(663), 663–673.

Betti, M., Bartoli, G., and Orlando,, M. (2010). "Evaluation study on structural fault of a Renaissance Italian palace." *Eng. Struct.*, 32(7), 1801–1813.

Blasi, C., and Ceccotti, A. (1984). "La Cupola del Brunelleschi: Indagine sullo stato di fessurazione." *Proc., INARCOS IngegneriArchitetti-Costruttori*, Bologna, Italy, 228–236 (in Italian).

Castoldi, A., Anesa, F., Imperato, F., and Gamba, F. (1989). "Cattedrale di S. Maria del Fiore, Firenze: Sistema di monitoraggio strutturale della Cupola e del suo basamento." I Quaderni dell'ISMES, 262, Bergamo, Italy (in Italian).

Chiarugi, A. (1996). "Orazione in onore di Vincenzo Viviani." *Bollettino degli Ingegneri*, 9, 8–10 (in Italian).

Chiarugi, A., Bartoli, G., and Bavetta, F. (1995). "La meccanicadella Cupola." *La Cupola di Santa Maria del Fiore, il cantiere di restauro 1980-1995*, Istituto Poligrafico e Zecca dello Stato, Roma, Italy, 47–62, (in Italian).

Chiarugi, A., Bartoli, G., and Bavetta, F. (1996). "Elaborazione ed interpretazione dei dati provenienti dal sistema di monitoraggio installato sulla cupola di S. Maria del Fiore a Firenze." *Proc., Evoluzionenella-Sperimentazione per le Costruzioni*, Malta.

Chiarugi, A., Bartoli, G., and Morano, S. G. (1998). "The surveillance of Brunelleschi dome in Florence." *Proc., 2nd Int. Conf. on Structural Analysis of Historical Constructions*, Barcelona, Spain, 337–354.

Chiarugi, A., Fanelli, M., and Giuseppetti, G. (1983). “Analysis of a Brunelleschi-type dome including thermal loads.” *Proc., IABSE Symp.*, Venice, Italy, 169–178.

Chiarugi, A., Fanelli, M., and Giuseppetti, G. (1993). “Diagnosis and strengthening of the Brunelleschi Dome.” *Proc., IABSE Symp.*, Rome, Italy, 441–448.

Chiarugi, A., and Foraboschi, P. (1995). “Monitoraggio ed identificazione strutturale.” *Il monitoraggiodellecostruzionicivili*, CISM, Collana di Ingegneria Strutturale, Udine, Italy, 1-47, (in Italian).

Coli, M., Tanini, C., Haines, M., Pandeli, E., Pini, G., and Bencini, F. (2008). “The “Pietra Serena” stones of Brunelleschi’s Cupola.” *J. Cult. Heritage*, 9(2), 214–221.

Dalla Negra, R. (1995). “La cupola del Brunelleschi: Il cantiere, le indagini, i rilievi.” *La Cupola di Santa Maria del Fiore, il cantiere di restauro 1980-1995*, Istituto Poligrafico e Zecca dello Stato, Roma, Italy (in Italian).

Del Coz Díaz, J. J., Adam, J., Martínez-Luengas, A., and Alvarez Rabanal, F. (2013). “Collapse of a masonry wall in an industrial building: Diagnosis by numerical modeling.” *J. Perform. Constr. Facil.*, 10.1061/(ASCE)CF.1943-5509.0000310, 65–76.

Del Coz Díaz, J. J., García Nieto, P. J., Lozano Martínez-Luengas, A., and Alvarez Rabanala, F. P. (2007). “Evaluation of the damage in the vault and portico of the pre-Romanesque chapel of San Salvador de Valdediós using frictional contacts and the finite-element method.” *Int. J. Comput. Math.*, 84(3), 377–393.

Di Pasquale, S. (1977). *Primo rapporto sulla Cupola di Santa Maria del Fiore*, CLUSF, Firenze, Italy (in Italian).

Di Stefano, R. (1990). *Il consolidamento strutturale nel restauro architettonico*, Edizioni Scientifiche Italiane, Napoli, Italy (in Italian).

Fanelli, G., and Fanelli, M. (2004). “Brunelleschi’s Cupola.” *Past and present of an architectural masterpiece*, Mandragora, Firenze, Italy (in Italian).

Gabbanini, F., Vannucci, M., Bartoli, G., and Moro, A. (2004). “Wavelet packet methods for the analysis of variance of time series with application to crack widths on the Brunelleschi Dome.” *J. Comput. Graph. Stat.*, 13(3), 639–658.

Ghinelli, A., and Vannucchi, G. (1991). “Condizioni stratigrafiche e fondazioni della zona absidale di Santa Maria del Fiore in Firenze.” *Bollettino degli Ingegneri*, 1–2, 28-29 (in Italian).

Giorgi, L., and Matracchi, P. (2008). “New studies on Brunelleschi’s Dome in Florence.” *Proc., 6th Int. Conf. on Structural Analysis of Historical Constructions*, Bath, U.K., 191–198.

Gurrieri, F. (1994). *La Cattedrale di Santa Maria del Fiore a Firenze*, Cassa di Risparmio di Firenze, Firenze, Italy (in Italian).

ISMES (Istituto Sperimentale Modelli e Strutture). (1987). *Rilievo di vibrazioni sulla cupola del Brunelleschi in S. Maria del Fiore (Firenze)*, Milano, Italy (in Italian).

Ministero per i Beni Culturali e Ambientali. (1985). “Rapporto sulla situazione del complesso strutturale cupola-basamento della cattedrale di Santa Maria del Fiore in Firenze.” *Commissione di studio. Soprintendenza per i Beni Ambientali e Architettonici per le provincie di Firenze e Pistoia*, Firenze, Italy (in Italian).

Nelli, G. B. (1753). “Discorsi di architettura del senatore Giovan Batista Nelli con la vita del medesimo e due Ragionamenti sopra le Cupole di Alessandro Cecchini Architetto, Per gli Eredi Paperini, Firenze, Italy.” ⟨http://www.archive.org/details/discorsidiarchit00nell⟩ (Sept. 2014).

Nelli, G. B., and Sgrilli, B. S. (1733). *Descrizione e studi dell’insigne fabbrica di S. Maria del Fiore*, Metropolitana Fiorentina, Firenze, Italy (in Italian).

Nobili, M., Sabatini, R., Alfani, G., Nervi, P. L., and Padelli, G. (1934). *Rilievi e studi sulla Cupola del Brunelleschi, eseguiti dalla Commissione nominata il 12 gennaio 1934*, Tipografia Ettore Rinaldi, Firenze, Italy (in Italian).

Ottoni, F. (2012). *Dalle cupole e del loro tranello. La lunga vicenda delle fabbriche cupolate tra dibattito e sperimentazione*, Aracne Editrice, Roma, Italy (in Italian).

Ottoni, F., Coisson, E., and Blasi, C. (2010). “The crack pattern in Brunelleschi’s dome in Florence. Damage evolution from historical to modern monitoring system analysis.” *Adv. Mater. Res.*, 133–134, 53–64.

Petrini, G. (1984). *Catalogo dei plessi fessurativi della Cupola di Santa Maria del Fiore in Firenze*, Italian Ministry of Cultural Heritage, Florence, Italy, 31–37 (in Italian).

Roca, P. (2004). “Considerations on the significance of history for the structural analysis of ancient constructions.” *Proc., 4th Int. Conf. on Structural Analysis of Historical Constructions*, Padua, Italy, 63–73.

Ximenes, L. (1757). *Del vecchio e nuovo Gnomone fiorentino*, Stamperia Imperiale, Firenze, Italy (in Italian).

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