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On the auxetic behaviour of metamaterials with re-entrant cell structures

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Abstract

In the present paper a two dimensional plate, made by structural straight elements and showing an auxetic behaviour, is considered. Theoretical, experimental and numerical analyses for the characterisation of its mechanical properties are presented and compared. The main geometrical parameters of an elementary cell affecting the deformation behaviour are highlighted, with emphasis to the negative values of the Poisson's ratio.

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Keywords: Metamaterials; Auxetic; Smart structures; Re-entrant cell structures.

1. Introduction

Auxetic materials are characterized by a counterintuitive mechanical behavior: when uniaxially stretched they tend to transversally expand as well as when compressed they contract transversally. This deformation behavior leads to an overall negative Poisson's ratio. The unique behavior has been investigated since the pioneering work of Lakes in 1987 [1] on some foam structures (the term 'auxetic' was coined by Evans in 1991 [2]). Auxetic materials have attracted the research interest of several authors in the literature, who studied them from the viewpoint of both material development [3-6] and mechanical modeling [7]. In addition, research interests have been driven by the application of these materials in relation to some of their own mechanical properties, including: indentation resistance, high energy absorption, fracture resistance, shear resistance, vibration absorption. Auxetic behavior is achieved by manufacturing the material with an inherent microstructure, which is itself responsible for such a

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behavior. These engineered materials (so-called metamaterials) present different features. A class of auxetic metamaterials (with high porosity) are characterized by a lattice microstructure, where negative Poisson's ratio is achieved by a lattice geometry with either reentrant corners [7] or chiral structure [8]. On the other hand, auxetic behavior can be also obtained by introducing in the hosting bulk material periodic arrays of holes with a low porosity level [9].

In the present paper, the geometrically nonlinear deformation behavior up to failure of a high porosity metamaterial is experimentally and theoretically investigated. In particular, a lattice microstructure with reentrant corners, fabricated using 3D printing with polylactic acid, is explored. A through measurement of the nonlinear strain field is performed using some digital image techniques. The main parameters affecting the deformation behaviour are highlighted.

2. An auxetic material with re-entrant lattice structure

In the present study a lattice-like 2D auxetic sheet is considered; its unit cell is characterised by a re-entrant double arrow shape as shown in Fig. 1.

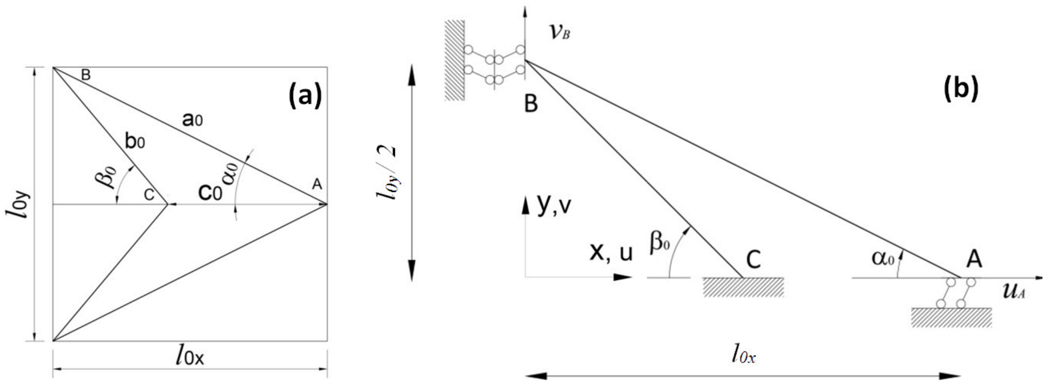

Fig. 1. Geometrical characteristics of the elementary auxetic cell (a). Structural scheme of the cell (b)

The geometrical properties of the elementary cell are defined by three parameters, e.g. the reference length a_0 and the two angles α_0, β_0. On the basis of these three geometrical parameters, all the relevant dimensions of the cell can be deduced:

$$l_{0x} = a_0 cos\alpha_0,\ l_{0y} = 2a_0 sen\alpha_0,\ b_0 = a_0 \frac{sen\alpha_0}{sen\beta_0},\ c_0 = a_0 cos\alpha_0 \left(1 - \frac{tan\alpha_0}{tan\beta_0}\right) \quad (1)$$

The particular case of an elementary square cell corresponds to:

$$l_{0x} = l_{0y} = 1 \rightarrow\ a_0 cos\alpha_0 = 2a_0 sen\alpha_0 \rightarrow tan\alpha_0 = 0.5$$

Due to the discrete nature of the lattice structure, the Poisson's ratio of the elementary cell is defined with respect to a gauge length related to the reference lengths of the cell itself. Therefore, the Poisson's ratio is related to the ratio between the vertical displacements at point B (v_B) and the horizontal one at point A (u_A), namely:

$$\nu = -\frac{\varepsilon_y}{\varepsilon_x},\ with \quad \varepsilon_x = \frac{u_A}{l_x}\ ,\ \varepsilon_y = \frac{v_B}{l_y} \quad (2)$$

where l_x, l_y are the reference lengths of the cell, i.e. $l_x = c_0$ in x-direction, while $l_y = l_{0y}/2$ in y-direction. The deformability of the elementary cell can be determined by introducing some hypotheses that can then be relaxed to get a better accuracy of the model.

In a first attempt, by assuming that the elements of the cell behave as rigid trusses, its motion can be studied through a trivial kinematic analysis under small displacements condition. Accordingly, the Poisson's ratio for an elementary cell is defined as:

$$\nu = -\frac{1}{tan\alpha_0 \, tan\beta_0} \tag{3}$$

that is, it depends only on the geometrical properties of the cell summarised by the two angles α_0, β_0. The variation of the Poisson's ratio as a function of the cell geometry is illustrated in Fig. 2 (note that the relative size of the elememtary cell, defined as the ratio between a reference length and the width of the lattice elements, e.g. a_0/s where s = width of the lattice elements, does not influence these kinematic analysis results). It can be observed that, irrespectively of the angle α_0, the Poisson's ratio is negative for $\beta_0 < 90°$ and tends to unlimited values when $\beta_0 \rightarrow 0°$. It is worth noticing that Eq. (3) can be applied to a general current configuration of the elementary cell under motion. In such cases, the angles α_0, β_0 have to be regarded as the current inclination angles of the cell elements and the Poisson's ratio as the tangent value related to the tangent displacement vector of the current configuration (note that in this case the reference lengths appearing in Eq. (2) are those related to the current configuration).

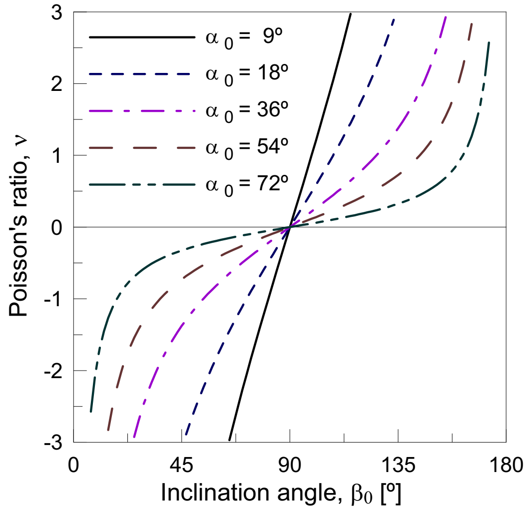

Fig. 2. Poisson's ratio vs the inclination angle β_0 for various values of the angle α_0 (for a square cell $\alpha_0 \approx 27°$)

An improvement of the above kinematic approach can be introduced by adopting for the elementary cell a statically indeterminate structural scheme, which accounts for its deformation. Due to the periodic symmetry conditions arising when several cells are assembled together, some rotation constraints can be imposed. The resulting mechanical system can be modelled as a statically undetermined framed structure under a prescribed horizontal displacement at one of its nodes. By writing the stiffness matrix according to the classical Euler-Bernoulli beam formulation, the vector of the unknowns of interest can be explicitly written as:

$$\begin{Bmatrix} u_B \\ v_B \\ Q_{B\phi} \end{Bmatrix} = \begin{bmatrix} \frac{EA}{a_0}d^2 + \frac{12EI}{a_0^3}c^2 + \frac{EA}{b_0}f^2 + \frac{12EI}{b_0^3}e^2 & \frac{EA}{a_0}cd - \frac{12EI}{a_0^3}cd + \frac{EA}{b_0}ef - \frac{12EI}{b_0^3}ef & 0 \\ \frac{EA}{a_0}cd - \frac{12EI}{a_0^3}cd + \frac{EA}{b_0}ef - \frac{12EI}{b_0^3}ef & \frac{EA}{a_0}c^2 + \frac{12EI}{a_0^3}d^2 + \frac{EA}{b_0}e^2 + \frac{12EI}{b_0^3}f^2 & 0 \\ \frac{6EI}{a_0^2}c + \frac{6EI}{b_0^2}e & -\frac{6EI}{a_0^2}d - \frac{6EI}{b_0^2}f & 1 \end{bmatrix}^{-1} \cdot \begin{Bmatrix} \frac{EA}{b_0}f^2 + \frac{12EI}{b_0^3}e^2 \\ \frac{EA}{b_0}ef - \frac{12EI}{b_0^3}ef \\ -\frac{6EI}{b_0^2}e \end{Bmatrix} \cdot u_A \tag{4}$$

where $c = \sin\alpha_0$, $d = \cos\alpha_0$, $e = \sin\beta_0$, $f = \cos\beta_0$, $A = s$ is the area of the beam cross-section, $I = s^3/12$ is the moment of inertial of the beam cross-section (a unit thickness is considered), E is the Young modulus of the material, $Q_{B\phi}$ is the reaction moment at node B. The ratio v_B / u_A corresponds to the Poisson's ratio of the elementary cell.

By considering a square cell from now onwards ($\tan\alpha_0 = 0.5$), the Poisson's ratio depends only on the angle β_0 and on a relative stiffness parameter defined as $\kappa = K_a / K_b$ (where $K_a = EA/a_0$ and $K_b = EI/a_0^3$), i.e. on the ratio between the axial and the bending stiffness of the beam elements composing the lattice cell. After some simple but tedious algeabric manipulations, the following expression for the Poisson's ratio of the cell can be obtained:

$$\nu = -\frac{2c_0}{l}\frac{v_B}{u_A} = -(2-\cot\beta_0)\frac{f_I(\beta_0) + \kappa \cdot f_{AI}(\beta_0) + \kappa^2 \cdot f_A(\beta_0)}{g_I(\beta_0) + \kappa \cdot g_{AI}(\beta_0) + \kappa^2 \cdot g_A(\beta_0)} \tag{5}$$

where $f_I(\beta_0), f_{AI}(\beta_0), f_A(\beta_0), g_I(\beta_0), g_{AI}(\beta_0), g_A(\beta_0)$ are known functions of the inclination angle β_0.

In the case of beam elements constituting the cell with an axial stiffness much greater than the bending one, $\kappa \to \infty$, Eq. (5) reduces to the expression considered in Eq. (3). The above expression (5) can be rewritten by using a relative density parameter ρ_r, defined as the ratio between the bulk density of the material and the theoretical density of the solid phase. Such a parameter can be calculated (with a first order approximation, i.e. by neglecting the overlapped area at the lattice nodes) as the ratio between the area occupied by the material and the whole area of the auxetic cell and it is the complent to unity of the porosity φ, namely

$$\rho_r = \frac{2(a_0 + b_0)}{c_0}\frac{s}{l} = 1 - \varphi \tag{6}$$

The parameter κ can be expressed as a function of ρ_r, that is

$$\kappa = \frac{T(\beta_0)}{\rho_r^2} \tag{7}$$

where $T(\beta_0)$ is the function of the inclination angle β_0. Finally, by means of Eq. (5) the Poisson's ratio of the square cell can be expressed through the relative density:

$$\nu = -\left(2 - cot\beta_0\right) \cdot \frac{f_I\left(\beta_0\right)\cdot\rho_r^4 + f_{AI}\left(\beta_0\right)\cdot T\left(\beta_0\right)\cdot\rho_r^2 + f_A\left(\beta_0\right)\cdot T\left(\beta_0\right)^2}{g_I\left(\beta_0\right)\cdot\rho_r^4 + g_{AI}\left(\beta_0\right)\cdot T\left(\beta_0\right)\cdot\rho_r^2 + g_A\left(\beta_0\right)\cdot T\left(\beta_0\right)^2} \tag{8}$$

Now, by removing the small displacement hypothesis while assuming the cell elements as rigid trusses (kinematic analysis), the cell size and the arrangement of its elements in the current configuration must be considered. The current geometrical features of the cell can be expressed as a function of the lengths a_0, b_0 of the rigid trusses and of the length c which is linearly dependent on the (longitudinal) normal strain along the x-axis ε_x :

$$c = c_0\left(1+\varepsilon_x\right), \quad \alpha = \arccos\left(\frac{a_0^{\,2} + c^2 - b_0^{\,2}}{2a_0 c}\right), \;\; \beta = \arccos\left(-\frac{b_0^{\,2} + c^2 - a_0^{\,2}}{2b_0 c}\right) \tag{9}$$

In the case of large deformation kinematic analysis, the definition of the Poisson's ratio is twofolds, depending on the strain measurements. A nominal (secant) Poisson's ratio can be defined by considering Eq. (2), where the displacement vector joins the reference configuration with the current one, and the lengths $l_x = c_0$ and $l_y = l_{0y}/2$ are related to the reference configuration. Accordingly exploiting Eq. (9), we have:

$$\nu = -\frac{\varepsilon_y}{\varepsilon_x} = \frac{sen\beta - sen\beta_0}{sen\beta_0}\,\frac{c_0}{c - c_0} \tag{10}$$

On the other hand, if one considers in Eq. (2) the tangent displacement vector and the lengths l_x and l_y related to the reference configuration, the incremental strains $\delta\varepsilon_x$ and $\delta\varepsilon_y$ can in turn be calculated and a tangent Poisson's ratio can be defined (again exploting Eq. (9)):

$$\nu_{tg} = -\frac{\delta\varepsilon_y}{\delta\varepsilon_x} = -\frac{c}{a_0\, sen\alpha}\,\frac{1}{tan\beta - tan\alpha} \tag{11}$$

It is worth noticing that Eq. (11) represents the first derivative of the function $\varepsilon_y = \varepsilon_y\left(\varepsilon_x\right)$. If the tangent Poisson's ratio is calculated for $\varepsilon_x \to 0$ (i.e. for the intial reference configuration) Eq. (11) coincides with Eq. (3).

According to the above expression in Fig. 3 the tangent and nominal values of the Poisson's ratio against the longitudinal deformation in x direction are plotted for the case of a square cell with $\beta_0 = 50°$.

Finally, by removing all the above simplificative hypotheses, the elementary cell is studied as a framed structure (see Fig. 1a) under a geometrically non linear large displacements analysis considering the actual axial and bending stiffness of the cell elements. Due to the complexity of the above defined problem, a Finite Element (FE) solution can be conveniently adopted.

3. Experimental tests

In order to verify the auxetic property of a metamaterial constituted by the assembly of repeated elementary arrow-shaped cells as described above, some experimental tests have been performed on 2D plates. A simple mechanical testing machine is used to study the experimental behaviour of the auxetic sheet under imposed displacements.

The auxetic specimens have been obtained by assembling two different elementary square cells (Fig. 4) that are characterised by a reference length of their edges equal to 20 (sample A) and 15 mm (sample B). The two samples A and B are related to a relative density ρ_r (see Eq. (6)) equal to 0.15 and 0.19, respectively. A 3D printer, based on the so-called FFF technology, is used to produce the specimens made of polylactic acid polymer (PLA). The main features of the printing process are: printing temperature of 196°C, printing speed equal to 35mm/s, thickness of the single printed layer thickness equal to 0.2 mm, nozzle diameter of 0.4 mm.

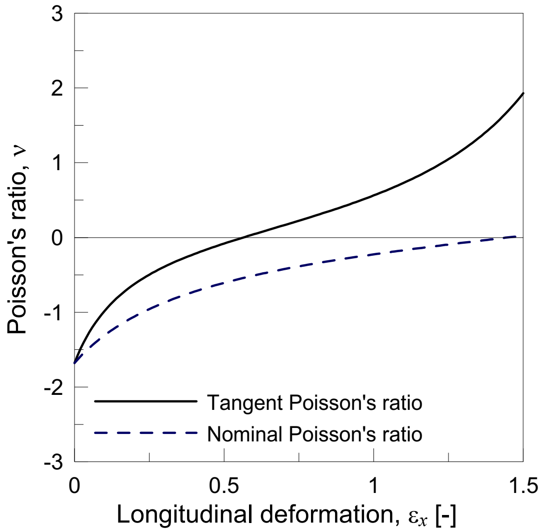

Fig. 3. Tangent and nominal values of the Poisson's ratio vs the longitudinal deformation for the case of a square cell with $\beta_0 = 50°$.

All the nodes located along the boundary on the right hand side of the plate are subjected to a monotonically increasing longitudinal (along the x-direction) displacement measured through a micrometer with a precision 10-5 m. A full map of deformed shape of the plate is evaluated through a photographic technique by quantifying the aobsolute displacements of the nodes of the lattice structure through high resolution pictures taken at given intervals of the applied deformation.

The deformation maps in the specimens at different deformation steps are obtained on the basis of the nodal displacements measured through photographic technique. Such displacements are interpolated through classical bilinear shape functions, referred to a background rectangular cell framework, in order to get the strain field in a continuous equivalent plate. The nodal displacements are evaluated with respect to their initial undeformed position, while an interior region of the plate is adopted for the numerical evaluation of the strains.

The map of the local values of the Poisson's ratio obtained by using Eq. (3) is illustrated at different deformation levels. In Fig. 5 the contour map of the Poisson's ratio for the sample A is displayed for an increasing value of the applied longitudinal strain. It can be noted as the Poisson's coefficient tends to attain a nearly uniform distribution inside the plate with a value that is equal to about -0.9.

4. Discussion

In order to assess for the suitability of FE models for the auxetic plates under study, some geometrically nonlinear numerical analyses are carried out (Fig. 6). In order to take into account for the finite size of the nodes of the analysed specimens, a proper rigid link option is introduced in the FE models (Fig. 6b).

As illustrated in Fig. 7, for both the two samples the average Poisson's ratio obtained from numerical analyses is lower (about 10 to 40%) than the corresponding experimental one, unless the effect of the finite node size is properly accounted for. This correction enables to estimate the Poisson's ratio with a good accuracy compared with the experimentally determined values.

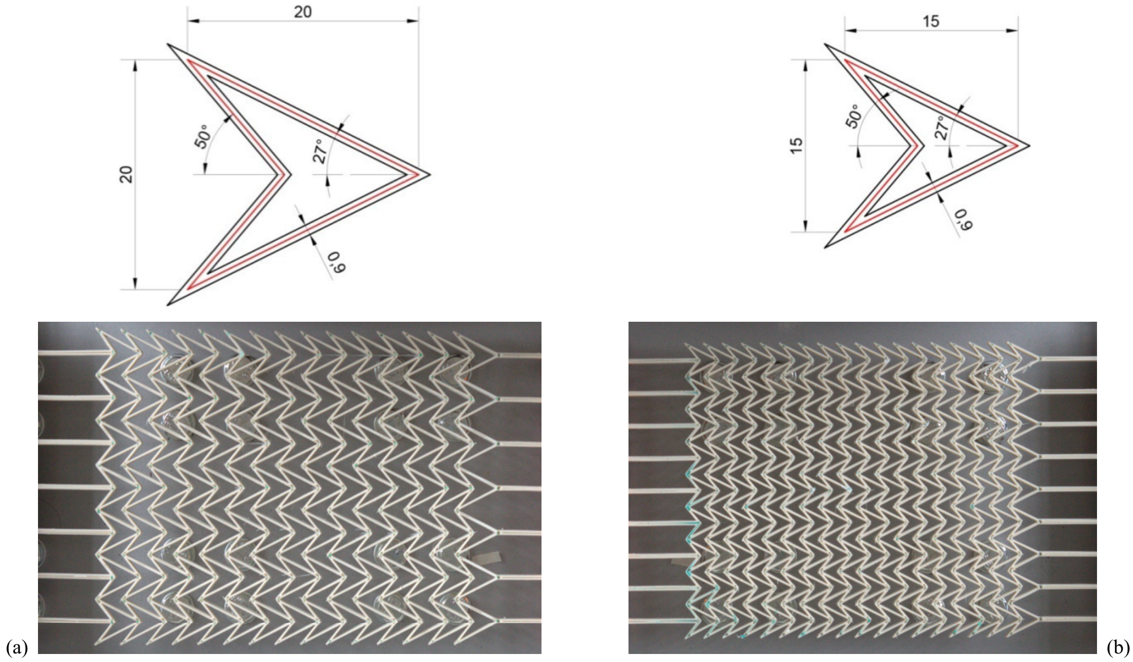

Fig. 4. Specimens of auxetic plate with elementary square cells (skecthes of the elementary cell and pictures of the assembled plate with details of the gripping system): case of cell length equal to 20 mm, sample A (a), and 15 mm, sample B (b). All dimensions are given in mm.

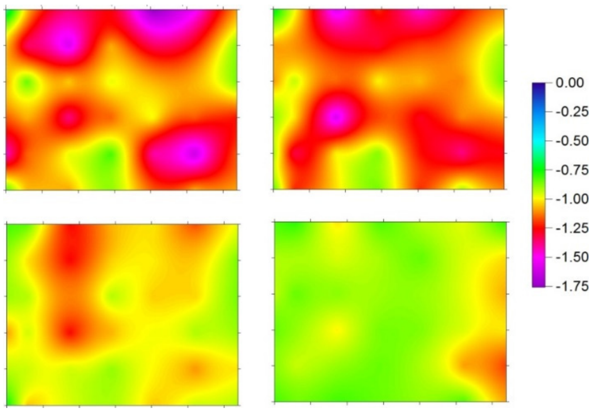

Fig. 5. Contour maps of the Poisson's ratio for four different levels of the applied longitudinal strain $\varepsilon_x = 2.8\%(a), 5.6\%(b), 8.5\%(c), 14\%(d)$ in the sample A.

5. Conclusions

So-called metamaterials, i.e. engineered materials having particular and unusual properties obtained thanks to their structure rather than composition, have gained a wide popularity in the last decades. In this context a class of materials known as auxetic are of particular interest due to their negative values of the Poisson's ratio. This enables to produce structural components with, in comparison to traditional couterparts, superior properties (high toughness, resilience, shear resistance, indentation resistance, improved fracture toughness and particular vibration absorption and acoustic properties), which can be exploted in a wide range of applications (biomedical, aerospace, automotive, etc.). In the present paper a two dimensional auxetic elementary cell, used to obatain 2D sheets by its repetitive assembly, is studied theoretically, experimentally and numerically. The deformability behaviour of such auxetic plates is investigated and the different approaches to its characterization are critically compared and discussed.

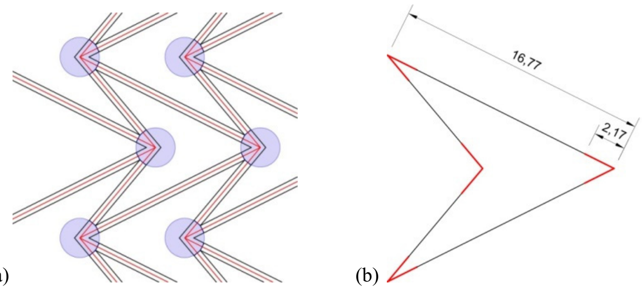

Fig. 6. FE beam model of the of the auxetic cells by considering rigid node features (a); detail of the rigid parts of the unit cell (b) (dimensions in mm).

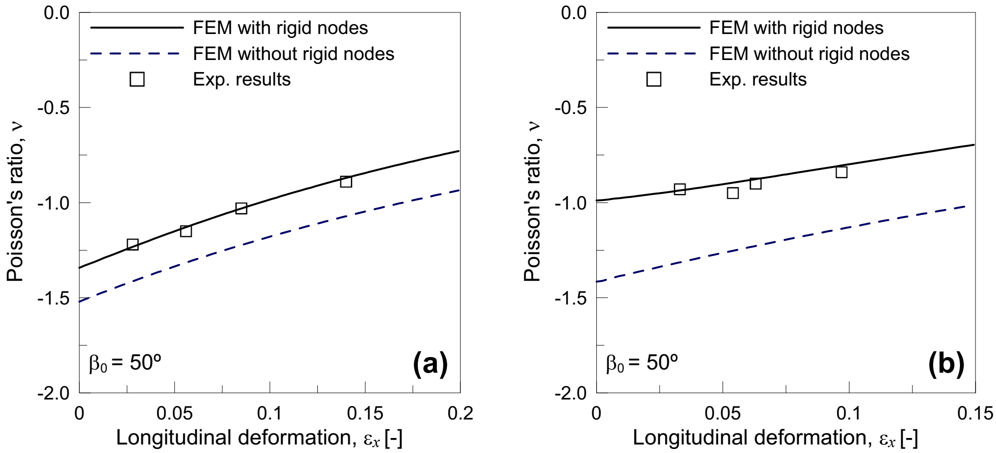

Fig. 7. Comparison of the experimental and FE Poisson's ratio for the sample A (a) and B (b) vs the applied longitudinal deformation with and without considering rigid node features in the FE analyses.

References

[1] R.S. Lakes, Foam structures with a negative Poisson's ratio, Science. 235 (1987) 1038.
[2] K.E. Evans, Auxetic polymers: a new range of materials, doi:10.1016/0160-9327(91)90123-S, 1991.
[3] K.E. Evans, K.L. Alderson, Auxetic materials: the positive side of being negative, Engng. Sci. Edu. J. 9 (2000) 148–154.
[4] E.A. Friis, R.S. Lakes, J.B. Park, Negative Poisson's ratio polymeric and metallic materials, J. Mater. Sci. 23 (1988) 4406-4414.
[5] G.N. Greaves, A.L. Greer, R.S. Lakes, T. Rouxel, Poisson's ratio and modern materials. Nature Materials. 10 (2011) 823–837.
[6] J.N. Grima, A. Alderson, K.E. Evans, An alternative explanation for the negative Poisson's ratios in auxetic foams, J Phys. Soc. of Japan. 74 (2005) 1341-1342.
[7] J.B. Choi, R.S. Lakes, Fracture toughness of re-entrant foam materials with a negative Poisson's ratio: experiment and analysis, Int. J. Fracture. 80 (1996) 73–83.
[8] D. Prall, R.S. Lakes, Properties of a Chiral honeycomb with Poisson's ratio of -1, Int. J. of Mech. Sci. 39 (1996) 305-314.
[9] M. Taylor, L. Francesconi, M. Gerendas, A. Shanian, C. Carson, K. Bertoldi, Low Porosity Metallic Periodic Structures with Negative Poisson's Ratio, Adv. Mater. (2013) DOI: 10.1002/adma.201304464.