# Green and Fast DSL via Joint Processing of Multiple Lines and Time-Frequency Packed Modulation 

Stefano Buzzi and Chiara Risi<br>University of Cassino and Lazio Meridionale / CNIT<br>I-03043 Cassino, Italy<br>Email: \{buzzi,chiara.risi\}@unicas.it

Giulio Colavolpe<br>University of Parma / CNIT<br>I-43100, Parma, Italy<br>Email: giulio@unipr.it


#### Abstract

In this paper strategies to enhance the performance, in terms of available data-rate per user, energy efficiency, and spectral efficiency, of current DSL lines are proposed. In particular, a system wherein a group of copper wires is jointly processed at both ends of the communication link is considered. For such a scenario, (a) the statistical multiplexing gain for the generic end user is analyzed; (b) a resource allocation scheme aimed at energy efficiency maximization is proposed; and, finally, (c) time-frequency packed modulation schemes are investigated for increased spectral efficiency. Results show that a joint processing of even a limited number of wires at both ends of the communication links brings remarkable performance improvements with respect to the case of individual point-topoint DSL connections; moreover, the considered solution does represent a viable means to increase, in the short term, the data-rate of the wired access network, without an intensive (and expensive) deployment of optical links.


## I. Introduction

Residential broadband internet access is nowadays mainly based on Digital Subscriber Lines (DSL) technology [1]. Indeed, although there is a wide agreement that the ultimate technology to increase the data rates of the access network is represented by optical networks, bringing a fiber in every house and/or in its close proximity - an approach called Fiber-to-theCurb (FTTC) - is a long and extremely expensive process. This is the main reason why optical fibers are nowadays widely used in core networks, and for backhauling in wireless networks, but their use for the access network is proceeding at a limited pace (of course with few notable exceptions). There is thus common consensus that DSL connections will be the main and dominant access technology still for some decades. Accordingly, in recent years, both academia and industry have been active in finding methods to boost the performance of DSL connections. Initially, asymmetric DSL (ADSL) connections had a data rate of few Mbit/s, but over the years they have improved and nowadays they are able to offer data rates that in some cases may be around 20Mbit/s (ADSL2+ standard), or larger [2]. While most research on DSL connections has focused on the case in which the lines departing from the central office (CO) are terminated at the customer premises, and they can be thus jointly managed only at one side of the communication links, some papers in recent years have also considered the situation in which a bunch of lines departing from the CO and arriving in the same location (i.e., the basement of a building), can be

[^0]jointly managed at both ends of the communication links [3][5], showing the advantages of such approach. In particular, the paper [3] designs joint transmit-receive linear processing schemes to minimize the transmit power subject to a quality-of-service constraint. The study [4], using methodologies from circuit theory, shows that for a copper DSL binder of 200 line connections the ultimate available shared bandwidth is on the order of $100 \mathrm{Gbit} / \mathrm{s}$, while the paper [5] shows that data-rates up to 1Gbit/s can be achieved through proper joint processing of four twisted pairs (category 3) over short distances (up to 300m).

This paper is focused on a scenario similar to that studied in the papers [3]-[5]: we consider the joint processing of a bunch of lines departing from the CO and arriving in the same physical location. We believe that the potential of this architecture has not yet been fully understood by telecom operators. Indeed, considering that it will take several years (and a huge amount of money) to extensively deploy fibers in close proximity of the end users, the proposal to jointly manage bunchs of copper wires arriving in the same location can guarantee to end users data-rates well larger than those currently available. In this paper, thus, the following contributions are provided: (a) the statistical multiplexing gain deriving from the joint management of the lines will be analyzed; (b) a resource allocation algorithm will be proposed in order to increase the system energy efficiency; and (c) motivated by the fact that the DSL access multiplexer (DSLAM) installed at the CO is no longer connected to the remote end users DSL modems, compliance to the standard DSL modulation formats is no longer required, and the use of alternative modulation schemes, based on time-frequency packing, will be investigated, with the aim of achieving larger values of spectral efficiency.

## II. System Model

Consider a group of $N$ lines departing from the DSLAM and arriving in the same physical location. According to the DSL standard, frequency-division duplexing (FDD) is used to separate uplink and downlink transmission, so that the transmissions in the two opposite directions do not interfere each other. As a consequence, near-end crosstalk (NEXT) disappears and far-end crosstalk (FEXT) is the only source of disturbance. Discrete Multi-Tone (DMT) modulation permits separating the channel into orthogonal carriers, so that no intercarrier interference is experienced. The discrete-time baseband equivalent of the downstream signal received in a symbol interval on the $k$-th carrier, can be written as the following
$N$-dimensional vector ${ }^{1}$

$$
\begin{equation*}
\mathbf{y}_{k}=\mathbf{H}_{k} \mathbf{P}_{k}^{1 / 2} \mathbf{x}_{k}+\mathbf{n}_{k} \tag{1}
\end{equation*}
$$

In (1), the vector $\mathrm{x}_{k}$ contains the data symbols; its $j$-th entry, say $x_{k}(j)$, is the information symbol transmitted on carrier $k$ and line $j$, with $E\left[\left|x_{k}(j)\right|^{2}\right]=1(E[\cdot]$ denotes statistical expectation); note that, due to FEXT, the signal observed on a given line contains contributions from the symbols transmitted on all the lines. The diagonal matrix $\mathbf{P}_{k}$ is $(N \times N)$-dimensional and its $(j, j)$-th entry contains the transmitted power $p_{k}(j)$ on the $k$-th carrier of the $j$-th line. The matrix $\mathbf{H}_{k} \in \mathbb{C}^{N \times N}$ contains the channel gains on tone $k$. Its $(i, j)$-th entry, say $h_{k}(i, j)$, is the complex gain of the channel from transmitter $j$ to receiver $i$; note that the diagonal elements of $\mathbf{H}_{k}$ contain the direct channels whilst the offdiagonal elements contain the crosstalk channels. This matrix is usually column-wise diagonally dominant. Finally, $\mathbf{n}_{k}$ is the vector containing the thermal noise and is a white zeromean complex Gaussian random vector with covariance matrix $\sigma^{2} \mathbf{I}_{N}$, with $\mathbf{I}_{N}$ being the identity matrix of order $N$.

## III. The statistical multiplexing gain

In this section we briefly dwell on the statistical multiplexing gain that might be achieved by sharing the $N$ ADSL connections among the $N$ subscribers served by these lines. To fix the ideas, we focus on the downstream link; the following assumptions are made:
(1) Each ADSL line can support a data-rate of $R \mathrm{bit} / \mathrm{s}$, so that the whole offered data-rate on the group of $N$ lines is $R N$; we are thus neglecting the fact that, should all the $N$ lines be simultaneously active, the increase of the crosstalk level would lead to a reduction of the effective rate.
(2) Only $P<N$ lines are switched on; this may be due to the fact that not all the $N$ users have their connections simultaneously switched on (business users are active during working hours of working days, while home users are active in the late afternoon and evening of weekdays and during weekends).
(3) Switched-on connections may be either idle or active. We denote by $\alpha$ the probability that a switched-on connection is active.

Based on the above assumptions, the number of lines $Q$ that are active at a given instant is a discrete random variable taking values in the set $\{0,1,2, \ldots, P\}$ and with the following probability distribution:

$$
\begin{equation*}
\operatorname{Prob}(Q=q)=\binom{P}{q} \alpha^{q}(1-\alpha)^{P-q}, \quad q=0,1, \ldots, P \tag{2}
\end{equation*}
$$

Assuming that the whole offered data-rate $R N$ is equally divided among the active lines, the available rate per user is expresses as $R N / Q$. In Fig. 1 it is reported the probability distribution of the said available rate per user for the following choice of the parameters: (a) the number of lines jointly processed is $N=30$; (b) the base data-rate of each ADSL connection is $R=7 \mathrm{Mbit} / \mathrm{s}$; (c) the probability that a switchedon connection is active is $\alpha=0.3$; and (d) the fraction of switched-on connections (i.e., the load $P / N$ ) is set to 0.1 , 0.5 and 0.9 . The legend of the figure also reports the average

[^1]

Fig. 1. Probability distribution of the available rate per user for $N=30, R=$ $7 \mathrm{Mbit} / \mathrm{s}$ and $\alpha=0.3$. Three different fractions of switched-on connections $(P / N)$ are plotted: $0.1,0.5$ and 0.9 .
available rate per user for the three considered loads. Results clearly show that sharing the ADSL connections may lead to data-rates that are well superior to the $7 \mathrm{Mbit} / \mathrm{s}$ limit available in the benchmark scenario of individual use of the lines. Even in the heavily loaded scenario $(P / N=0.9)$, the average available data rate is $28.8 \mathrm{Mbit} / \mathrm{s}$, with a four-fold increase with respect to the benchmark.

As a final remark, note also that in situations of small load and with low traffic, the possibility of a joint processing and management of the $N$ lines at both ends of the link would permit turning off some unused lines, so as to save energy. When traffic increases, these lines may then be turned on again.

## IV. EnERgy-EFFICIENT RESOURCE ALLOCATION

In this section we propose a resource allocation algorithm aimed at minimizing the transmit power subject to a QoS constraint. Since we are jointly considering $N$ copper wires, and assuming that on each wire there are $C$ available carriers, there are a total of $N C$ physical channels, wherein by physical channel we mean the pair (line, carrier). For each assigned data-rata that is to be transmitted through the $N$ jointly processed lines, and for a target BER, we have thus to choose the physical channels to be used, and, also, the transmit power and the modulation cardinality on each chosen channel, so that we are able to support the required data-rate $\widetilde{R}$ with the required BER $\widetilde{\beta}$.

Before giving a formal definition of the described optimization problem, we need to add further details on the modulation scheme and reception algorithm. We assume that on each subcarrier a QAM modulation scheme can be employed whose cardinality belongs to the set $\mathcal{M}=$ $\{4,16,64,256,1024,4096,16384\}$ (note that these numbers are compliant with the ADSL standard, wherein each carrier can be loaded with up to 15 bits in each signaling interval.). At the receiver side a soft estimate of the transmitted data symbols is obtained through a linear minimum mean square error (LMMSE) receiver.

Let now $p_{\text {max }}$ be the maximum allowed transmit power on each physical channel, and define the system sum-power
as the overall transmitted power, i.e. the sum of the powers transmitted on all the active physical channels. We aim at minimizing the system sum-power, subject to the following constraints:

$$
\left\{\begin{array}{l}
\text { supported rate }=\widetilde{R}  \tag{3}\\
\mathrm{BER} \leq \widetilde{\beta}
\end{array}\right.
$$

We assume here that the target data-rate $\widetilde{R}$ is an integer multiple of $2 / T$, with $T$ the inverse of the bandwidth of each physical channel ${ }^{2}$. The minimization of the sum-power is made with respect to the choice of the physical channels, and of the transmit power (to be taken not larger than $p_{\max }$ ), and cardinality of the modulation (to be taken in the set $\mathcal{M}$ ) on each chosen active channel. Following [6], the following bound can be used for the BER of an $M$-QAM modulation system:

$$
\begin{equation*}
B E R \leq 2 \exp \left(\frac{-1.5 \gamma}{M-1}\right), \quad M \geq 4 \tag{4}
\end{equation*}
$$

with $\gamma$ the Signal-to-Noise Ratio. Although expression (4) holds for the AWGN channel, it is a reasonable choice also for the case in which there is co-channel interference and an LMMSE receiver is adopted. We will thus use Eq. (4) with $\gamma$ replaced by the output SINR of the LMMSE receiver. Accordingly, the BER constraint in (3) can be replaced by the following constraint on the received SINR:

$$
\begin{equation*}
\mathrm{SINR} \geq \frac{M-1}{1.5} \ln \left(\frac{1}{\widetilde{\beta}}\right) \tag{5}
\end{equation*}
$$

It can be easily shown that solution of the considered problem requires a prohibitive computational complexity, given the fact that an exhaustive search over all possible allocations of the $\widetilde{q}$ bits among the available physical channels, along with the choice of the relative modulation cardinality is a combinatorial problem. In what follows we thus propose a simple suboptimal algorithm, which, despite its simplicity, will be shown to achieve good performance results. The algorithm is sequential, in the sense that data bits are sequentially allocated to the physical channels, in groups of two bits each. Given $\widetilde{q}=2$, we start by choosing the best physical channel, i.e. we choose the pair (line, carrier) with the largest channel gain, and allocate two bits on this channel; otherwise stated, we start by choosing line $i$ and carrier $k$ iff:

$$
\begin{equation*}
\left\|\mathbf{H}_{k}(:, i)\right\| \geq \max _{i^{\prime} \neq i, k^{\prime} \neq k}\left\|\mathbf{H}_{k^{\prime}}\left(:, i^{\prime}\right)\right\| \tag{6}
\end{equation*}
$$

Then, we have to allocate the next group of two bits. We have the following choices:
a) We can use the already active $(i, k)$-th physical channel by switching from a QPSK to a 16-QAM modulation;
b) We can use one of the remaining $N C-1$ physical channels by using a QPSK modulation;

We evaluate the sum-power needed to reach the target SINR for configuration a) and for the $N C-1$ configurations of choice b), and choose the configuration with the smallest sum-power.

[^2]Note that at this step we have thus a complexity linear in $N C$, the number of available physical channels. The computational complexity can be made even smaller if, in performing this step, we neglect interference, i.e. we choose the most convenient configuration nulling the crosstalk contribution in the received $\operatorname{SINR}^{3}$. Now that we have allocated the first 4 bits we can proceed in a similar way to allocate the next group of 2 bits. In general, at the generic step of the algorithm we will have a certain number, say $\Gamma$, of active physical channels (and each active channel will be using a modulation with a certain cardinality), and $N C-\Gamma$ empty physical channels. To proceed, we have thus to allocate additional two bits and, again, this can be done either by multiplying by 4 the cardinality of the modulation on one of the channels already in use, or turning on a new physical channel with a QPSK modulation. Of course, the solution corresponding to the minimum sumpower will be taken. In the simplified form of the algorithm, we have just to compare the best (i.e. with the largest channel coefficient) unused channel with the best active channels for each modulation cardinality in the set $\mathcal{M}$. The following remarks can be now done. First of all, since at each step we have to choose among $N C$ different configurations, and since the number of steps is $\widetilde{q} / 2$, we have that the overall complexity of the proposed algorithm scales linearly with $N$, $C$ and $\widetilde{q}$. The computational complexity savings are based on the sequential nature of the algorithm: bits are allocated in groups of two, and at each allocation the channels already in use cannot be dismissed, they can only be upgraded to a modulation with larger cardinality. Finally, it is worth noting that, of course, it might also happen that, for too large target data-rates, the system is not able to meet the required target SINR. This occurrence is detected by the fact that, at a given step, in any of the possible allocations of the two additional bits, the target SINR cannot be reached at least for one active physical channel ${ }^{4}$.

## V. Time-Frequency Packing for Improved Spectral Efficiency

In this section, inspired by [8], we investigate on the use of time-frequency packed modulations aiming at systems with increased spectral efficiency. While [8] considered a singleuser transmission link (i.e. a system impaired by the thermal noise only), in the following we extend the concepts developed in [8] to the considered multiuser scenario, wherein FEXT disturbance exists, including the adoption of an extended window LMMSE receiver.

To begin with, assume a linear modulation with base pulse $p(t)$ of duration $T_{p}$ which is shifted of multiples of $T$ in the time domain and of multiple of $F$ in the frequency domain ${ }^{5}$; the baseband equivalent of the transmitted signal on the $N$ wires can be written as the following $(N \times 1)$ vector-valued function:

$$
\mathbf{x}(t)=\sqrt{E_{s} T F} \sum_{m} \sum_{l} \mathbf{x}_{l}(m) p(t-m T) e^{j 2 \pi l F t}
$$

[^3]where $x_{l}^{i}(m)$ is the symbol transmitted on the $i$-th wire in the $m$-th symbol interval and on the $l$-th subcarrier. For the sake of simplicity, in the following we will restrict our attention to the case in which QPSK modulation is used on all the carriers; of course extension to the case of adaptive cardinality of the modulation can be done with ordinary efforts. The received signal can be modeled now as the following vector-valued signal:
$\mathbf{y}(t)=\sqrt{E_{s} T F} \sum_{m} \sum_{l} \mathbf{H}_{l}(m) \mathbf{x}_{l}(m) p(t-m T) e^{j 2 \pi l F t}+\mathbf{n}(t)$,
where $\mathbf{H}_{l}(m)$ is the DSL channel matrix in the $m$-th time interval and on subcarrier $l$. Due to the stationary nature of the copper wires, the channel is assumed to be constant in time. In the frequency domain, instead, in keeping with assumptions of the previous sections, it is considered flat over each subcarrier of bandwidth $F$, while it changes from subcarrier to subcarrier. Finally, $\mathbf{n}(t)$ represents the thermal noise; its $i$-th entry is the additive noise received on the $i$-th wire, and is modeled as a circularly symmetric zero-mean white Gaussian noise with PSD (power spectral density) $\sigma^{2}$.

In order to convert the received signals to discrete-time, filters matched to the time-frequency shifted replicas of the base pulse are employed at the receiver side. We thus obtain, for the case of filters matched to the $(n, k)$-th time-frequency pair, the following test statistics:

$$
\begin{aligned}
& \mathbf{y}_{k}(n)=\int \mathbf{y}(t) p^{*}(t-n T) e^{-j 2 \pi k F t} d t=\sqrt{E_{s} T F}\left[\mathbf{H}_{k}(n) \mathbf{x}_{k}(n)\right. \\
& \left.+\sum_{m \neq n} \sum_{l \neq k} A_{m, l}(n T, k F) \mathbf{H}_{l}(m) \mathbf{x}_{l}(m)\right]+\mathbf{z}_{k}(n)
\end{aligned}
$$

where $A_{m, l}(n, k)=\int p(t-m T) p^{*}(t-n T) e^{-j 2 \pi(k-l) F t} d t$ is called ambiguity function and $\mathbf{z}_{n, k}=\int p^{*}(t-$ $n T) e^{-j 2 \pi k F t} \mathbf{n}(t) d t$.

In order to detect the data vector $\mathbf{x}_{k}(n)$, a square processing window of length $(2 P+1)$ and $(2 L+1)$ in the time and frequency domain, respectively, is considered.

It can be shown (details are omitted due to lack of space) that the LMMSE estimate, say $\widehat{x}_{k}^{i}(n)$, of the data symbol $x_{k}^{i}(n)$ is written as

$$
\begin{equation*}
\widehat{x}_{k}^{i}(n)=\mathbf{v}_{k}^{i}(n)^{H} \widetilde{\mathbf{h}}_{k}^{j}(n) x_{k}^{j}(n)+\mathbf{v}_{k}^{i}(n)^{H} \widetilde{\mathbf{i}}_{k}^{j}(n)+\mathbf{v}_{k}^{i}(n)^{H} \widetilde{\mathbf{z}}_{k}^{j}(n) . \tag{7}
\end{equation*}
$$

wherein $\mathbf{v}_{k}^{i}(n)$ is the LMMSE detector, $\widetilde{\mathbf{h}}_{k}^{j}(n)$ is the "signature" of the desired data symbol, $\widetilde{\mathbf{i}}_{k}^{j}(n)$ is the crosstalk term and $\widetilde{\mathbf{z}}_{k}^{j}(n)$ is the additive noise contribution.

Now, we are willing to have a reliable approximation of the spectral efficiency of a communication system whose output is given by Eq. (7). To this end, in keeping with [8] and, in turn, [9], instead of simply neglecting the interference terms in eq. (7) due to adjacent terms in time and frequency, we model such interference as an additional zero-mean Gaussian disturbance. Letting $N_{I}$ denote the variance of the overall disturbance in Eq. (7), i.e. crosstalk plus thermal noise, we introduce the following auxiliary channel model

$$
\begin{equation*}
\widehat{x}_{k}^{i}(n)=\sqrt{E_{s} T F} \mathbf{v}_{k}^{i}(n)^{H} \widetilde{\mathbf{h}}_{k}^{i}(n) x_{k}^{i}(n)+w_{k}^{i}(n) \tag{8}
\end{equation*}
$$

wherein $w_{k}^{i}(n)$ is assumed to be a zero-mean Gaussian random variate with variance $N_{I}$. We are now interested in evaluating the ultimate performance limits (in terms of spectral efficiency) when using a symbol-by-symbol receiver designed for the auxiliary channel model (8) when the actual channel model is the one in (7). This is an instance of mismatched decoding and the achievable information rate in bit per channel use can be shown to be expressed as [9]:

$$
\begin{align*}
& I\left(x_{k}^{i}(n) ; \widehat{x}_{k}^{i}(n)\right)= \\
& \mathrm{E}_{x_{k}^{i}(n) ; \widehat{x}_{k}^{i}(n)}\left[\log _{2}\left(\frac{M p_{\widehat{X}_{k}^{i}(n) \mid X_{k}^{i}(n)}\left(\widehat{x}_{k}^{i}(n) \mid x_{k}^{i}(n)\right)}{\sum_{x} p_{\widehat{X}_{k}^{i}(n) \mid X_{k}^{i}(n)}\left(\widehat{x}_{k}^{i}(n) \mid a\right)}\right)\right] \tag{9}
\end{align*}
$$

where $p_{\widehat{X}_{k}^{i}(n) \mid X_{k}^{i}(n)}\left(\widehat{x}_{k}^{i}(n) \mid x_{k}^{i}(n)\right)$ is a Gaussian probability density function (pdf) with mean $\mathbf{v}_{k}^{i}(n)^{H} \widetilde{\mathbf{h}}_{k}^{i}(n) x_{k}^{i}(n)$, and variance $N_{I}$, while the outer statistical average, with respect to $x_{k}^{i}(n)$ and $\widehat{x}_{k}^{i}(n)$, is carried out according to the real channel model of Eq. (7), [8], [9]. Note that Eq. (9), which can be computed via Montecarlo simulations, represents the information rate on the $k$-th subcarrier of the $i$-th line in the $n$-th signaling interval; due to the frequency selectivity of the channel, the average information rate of the $i$-th line is obtained as

$$
\begin{equation*}
\mathrm{AIR}=\frac{1}{C} \sum_{k} I\left(x_{k}^{i}(n) ; \widehat{x}_{k}^{i}(n)\right) \tag{10}
\end{equation*}
$$

where, we recall, $C$ is the number of subcarriers. Given Eq. (10), the spectral efficiency, measured in bit/s/Hz, is finally expressed as

$$
\begin{equation*}
\eta=\frac{\mathrm{AIR}}{F T} \tag{11}
\end{equation*}
$$

which is to be now maximized with respect to $T$ and $F$.
Similar reasoning can be used to evaluate the spectral efficiency in the case in which full-duplex transmission over the entire line bandwidth is adopted. Mathematical details are however omitted for the sake of brevity.

## VI. Simulation results

In the following, we will refer to papers [10], [11], and [12] to model the channel matrices $\mathbf{H}_{k}$ and $\mathbf{H}_{k}^{\text {NEXT }}{ }^{6}$ Since Fig. 1 has already been commented in Section III, we begin with the simplified resource allocation algorithm proposed in Section IV. In Fig. 2 we report the rate versus sum-power for the proposed solution, and for comparison purposes, we also show a curve corresponding to the case in which ADSL lines are treated separately, and the rate is randomly split among them. There are $N=20$ lines jointly processed, the number of subcarriers per line is $C=480$ (this number is compliant with the ADSL2 standard) and $\widetilde{\beta}=10^{-6}$. The simulated loop length is 200 m . While in an underloaded scenario the two solutions exhibit the same performance, as the requested datarate increases, our solution is capable of delivering, for a fixed amount of transmit power, larger data-rates, or, equivalently, for a given delivered data-rate, our solution requires smaller values of transmit power.

[^4]

Fig. 2. Total power consumed by the system versus the rate. The total number of possible active user is 20 .

Consider now the use of time-frequency packed modulations. Figure 3 depicts the average spectral efficiency (ASE) versus the time spacing $T$ and the frequency spacing $F$ with a Signal-to-Noise Ratio equal to 70 dB and $N=25$ lines jointly processed. There are several pairs $(T, F)$ allowing to obtain an ASE larger than that achieved by the orthogonal signaling ( $F=4000$ and $T=T p$ ), and gains about $20 \%$ may be achieved. The figure was obtained for a DSL link of 800 m length, and with a unit-length time-frequency processing window (i.e., $L=P=0$ ). The results so far shown have not considered the use of full-duplex. Let us assume now to cancel the NEXT contribution. In Table I, we report, for some loops lengths, the optimal values of the time-frequency spacings, the ASE when FDD and orthogonal signaling is used (ASE1), the ASE when FDD and time-frequency packing is used (ASE2), and, finally, the ASE when full-duplex transmission, NEXT cancellation and time-frequency packing is used (ASE3). With NEXT cancellation, the overall ASE is practically doubled with respect to the FDD configuration.

## VII. Conclusions

This paper has shown that a joint processing of copper wires at both ends of the link permits achieving a considerable statistical multiplexing gain, enables the use of resource allocation procedures taking advantage from the structure of the crosstalk, and, also, permits using of modulation schemes better than OFDM in terms of spectral efficiency (which is more than doubled in the full-duplex case with echo cancellation). Although copper lines cannot compete with fiber-based optical channels, the solutions proposed in this paper may be useful in the short-to-medium term, while we wait for a thorough deployment of the fiber, as well as in developing countries wherein optical fibers are not yet at the horizon.

## REFERENCES

[1] C. Summers, ADSL Standards, Implementation, and Architecture. CRC, 1999, vol. 4.
[2] "Asymmetric digital subscriber line transceivers 2 (ADSL2) extended bandwidth (ADSL2plus)," ITU-T Reccommendation G.992.5, 2005.

| Loop length $[\mathrm{m}]$ | 800 | 1600 | 2400 | 3200 |
| :---: | :---: | :---: | :---: | :---: |
| optimal T $[\mathrm{s}]$ | $2.500 \cdot 10^{-4}$ | $2.375 \cdot 10^{-4}$ | $2.375 \cdot 10^{-4}$ | $2.375 \cdot 10^{-4}$ |
| optimal F $[\mathrm{Hz}]$ | 3050 | 3200 | 3200 | 3200 |
| ASE 1 $[\mathrm{bit} / \mathrm{s} / \mathrm{Hz}]$ | 2.0000 | 1.9990 | 1.1676 | 0.6049 |
| ASE $[\mathrm{bit} / \mathrm{s} / \mathrm{Hz}]$ | 2.3832 | 2.3176 | 1.2888 | 0.6616 |
| ASE 3 $[\mathrm{bit} / \mathrm{s} / \mathrm{Hz}]$ | 4.7664 | 4.6352 | 2.5681 | 1.3231 |

TABLE I. ASE 1: ASE when FDD and OFDM are used. ASE 2: ASE When FDD and the optimal $T$ and $F$ values are considerd. ASE 3: ASE WHEN THERE IS NO FDD, THE OPTIMAL $T$ and $F$ values are used and the NEXT cancellation is performed.
[3] D. Palomar, M. Lagunas, and J. Cioffi, "Optimum joint transmit-receive linear processing for vectored dsl transmission with QoS requirements," in Signals, Systems and Computers, 2002. Conference Record of the Thirty-Sixth Asilomar Conference on, vol. 1. IEEE, 2002, pp. 388392.
[4] J. Cioffi, S. Jagannathan, M. Mohseni, and G. Ginis, "CuPON: the copper alternative to PON $100 \mathrm{~Gb} / \mathrm{s}$ DSL networks," Communications Magazine, IEEE, vol. 45, no. 6, pp. 132 -139, june 2007.
[5] B. Lee, J. Cioffi, S. Jagannathan, and M. Mohseni, "Gigabit DSL," Communications, IEEE Transactions on, vol. 55, no. 9, pp. 1689-1692, 2007.
[6] A. Goldsmith and S. Chua, "Variable-rate variable-power MQAM for fading channels," Communications, IEEE Transactions on, vol. 45, no. 10, pp. 1218-1230, 1997.
[7] S. Ulukus and R. Yates, "Stochastic power control for cellular radio systems," Communications, IEEE Transactions on, vol. 46, no. 6, pp. 784-798, 1998.
[8] A. Barbieri, D. Fertonani, and G. Colavolpe, "Time-frequency packing for linear modulations: spectral efficiency and practical detection schemes," Communications, IEEE Transactions on, vol. 57, no. 10, pp. 2951-2959, 2009.
[9] N. Merhav, G. Kaplan, A. Lapidoth, and S. Shamai Shitz, "On information rates for mismatched decoders," Information Theory, IEEE Transactions on, vol. 40, no. 6, pp. 1953-1967, 1994.
[10] A. N. S. F. Telecommunications, "Spectrum management for loop transmission systems," Tech. Rep., 2000.
[11] J. Maes, M. Guenach, and M. Peeters, "Statistical MIMO channel model for gain quantification of DSL crosstalk mitigation techniques," in Communications, 2009. ICC '09. IEEE International Conference on, june 2009, pp. 1-5.
[12] K. Kerpez and S. Galli, "Methods of summing crosstalk from mixed sources. II. performance results," Communications, IEEE Transactions on, vol. 50, no. 4, pp. 600-607, 2002.


Fig. 3. ASE versus $T$ and $F .10 \log _{10}\left(E_{s} / \sigma^{2}\right)=70 \mathrm{~dB}$ Loop lenghts: 800 m . $\mathrm{N}=25$.


[^0]:    The work of Chiara Risi and Stefano Buzzi has received funding from the European Union Seventh Framework Programme (FP7/2007-2013) under grant agreement n. 257740 (Network of Excellence "TREND").

[^1]:    ${ }^{1}$ A similar model holds for the upstream signal too.

[^2]:    ${ }^{2}$ Such a bandwidth equals the carrier spacing and in the ADSL standard is in turn equal to 4.3125 khz , so we are assuming that $\widetilde{R}$ can be increased in steps of about $8 \mathrm{kbit} / \mathrm{s}$.

[^3]:    ${ }^{3}$ Indeed in this case we have to compare configuration a) with only one configuration b), i.e. the one with the largest channel coefficient.
    ${ }^{4}$ In this case we are dealing with an unfeasible power control problem; details on this can be found in the papers [7].
    ${ }^{5}$ While in conventional OFDM we have $T=T_{p}$ and $F=1 / T$, here the parameters $T$ and $F$ are to be properly optimized in order to increase the system spectral efficiency.

[^4]:    ${ }^{6}$ Note that the matrix $\mathbf{H}_{k}^{\text {NEXT }}$ appears only when considering the full duplex case; we will show in the following some results for this scenario as well, although the mathematical details have been omitted due to lack of space.

