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Joint DOA, Range, and Polarization Estimation for Rectilinear Sources with a COLD Array

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***Abstract*—In this paper, a novel localization method for near-field (NF) rectilinear or strictly noncircular sources with a symmetric uniform linear array of cocentered orthogonal loop and dipole (COLD) antennas is proposed. Based on the rank reduction (RARE) principle, the multiple parameters including direction of arrival (DOA), range and polarization parameters are separated. Furthermore, a closed-form solution for polarization parameters and noncircular phases is also provided. The deterministic Cramer-Rao bound (CRB) of the estimation problem under consideration is also derived as a benchmark. Numerical simulations are provided to demonstrate the effectiveness of the proposed method.**

***Index Terms*—DOA estimation, near-field, noncircular signals, rank reduction, COLD array.**

I. Introduction

As an important topic in the area of array signal processing and wireless communications, direction of arrival (DOA) estimation with diversely polarized antenna arrays has attracted much attention [1–3]. Compared with a scalar antenna array, a diversely polarized array can extract both the spatial-time and the polarization information of an incident electromagnetic wave. Therefore, algorithms based on such a vector antenna array usually outperform their conventional scalar antenna array based counterparts. Most existing methods assume that all incident signals are far-field (FF) plane waves, and their locations can be characterized by DOA only. However, in practice, near-field (NF) signals can be present around the array with spherical wavefronts. In this case, both the DOA and range parameters need to be characterized, and some methods have been developed specifically for NF source localization [4, 5].

For NF source localization based on vector antenna arrays, an ESPRIT-like algorithm was proposed in [6] based on polarimetric fourth-order cumulant (FOC) for closed-form DOA and range estimation. Using FOC matrices, localization of multiple NF sources with a linear tripole array was studied in [7] with an additional advantage of extended aperture. To avoid the high computational complexity of constructing FOC matrices, two second-order statistics (SOS) based methods were presented in [8, 9], where a sparse linear array was employed in [8] and an array of cross-dipoles studied in [9].

All the above mentioned methods assumed that the incoming signals are circular. However, strictly noncircular signals [10–12], such as amplitude modulation (AM) and binary phase shift keying (BPSK) signals, are widely used in modern radio communications, for which the DOA and range estimation performance can be improved significantly by exploiting both covariance matrix and conjugate covariance matrix of noncircular signals. Therefore, in this paper, based on a uniform linear array of cocentered orthogonal loop and dipole (COLD) antennas, a novel noncircularity based localization algorithm for NF polarized sources is proposed. Based on the principle of rank reduction (RARE) [4, 5], the DOA and range parameters of NF noncircular sources can be successively estimated through two one-dimensional (1-D) spectral searches. Meanwhile, estimating the polarization parameters and noncircular phase are also achieved with a closed-form expression.

Notation: $[\cdot]^*$, $[\cdot]^T$,$[\cdot]^H$,$[\cdot]^{-1}$ represent operations of conjugation, transpose, conjugate transpose, and inverse, respectively; $E[\cdot]$ is the expectation operation; $diag\{\cdot\}$ stands for the diagonalization operation; $\mathbf{I}_p$ denotes the p-dimensional identity matrix; $Re\{\cdot\}$ denotes the real part, while $tr\{\cdot\}$ and $det\{\cdot\}$ denote the trace and determinant of a matrix, respectively. $\otimes$ and $\odot$ are the Kronecker product and Hadamard product operations, respectively. $\mathbf{1}_p$ denotes an all-one $p\times 1$ row vector.

II. Signal and Array Models

Consider a symmetric uniform linear array (ULA) of $N = 2M+1$ COLD antennas, as shown in [Fig.1, 13]. There are K narrowband completely polarized signals impinging on the array from $(\theta_k, r_k), k = 1,\cdots,K$, where $\theta_k \in (-\pi/2, \pi/2)$ represents the angle between the kth source and the z-axis and r_k denotes the range from the source to the origin of coordinates. The completely polarized signals can be decomposed into a polarization electric component and a polarization magnetic component. The COLD composite antenna measures each polarization component separately, with the loop measuring the magnetic component and the dipole measuring the electric component. Referring to [2], the polarization-space steering vector of the kth source can be expressed as

$$\boldsymbol{\xi}_k = \begin{bmatrix} -\cos\gamma_k \\ -\sin\gamma_k e^{i\eta_k} \end{bmatrix} \quad (1)$$

where γ_k and η_k represent the polarization angle and polarization phase difference respectively.

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Use the array center indexed by 0 as the phase reference point, and then received signal $\mathbf{x}_m(t)$ at the mth antenna at time t can be expressed as

$$\mathbf{x}_m(t) = \sum_{k=1}^{K} a_{m,k}(\theta_k, r_k)\boldsymbol{\xi}_k s_k(t) + \mathbf{n}_m(t) \tag{2}$$

$m = -M, \dots, -1, 0, 1, \dots, M$, where $a_{m,k}(\theta_k, r_k) = e^{j\frac{2\pi r_k}{\lambda}(\sqrt{1+(\frac{md}{r_k})^2-\frac{2md\sin\theta_k}{r_k}}-1)}$ is the spatial phase factor for the kth source to the mth antenna. $s_k(t)$ is the complex envelope of the kth signal, $\mathbf{n}_m(t)$ is white Gaussian noise which is independent and identically distributed (I.I.D) with zero mean and uncorrelated with the sources. The range of the NF signals is within the Fresnel region [4–10], which means $r_k \in (0.62\sqrt{D^3/\lambda}, 2D^2/\lambda)$ with D being the array aperture and λ being the wavelength.

The observed data vector $\mathbf{x}(t) = [\mathbf{x}_{-M}(t), \cdots, \mathbf{x}_0(t), \cdots, \mathbf{x}_M(t)]^T$ at time t can be expressed as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{3}$$

where $\mathbf{A} = [\mathbf{a}_1 \otimes \boldsymbol{\xi}_1, \mathbf{a}_2 \otimes \boldsymbol{\xi}_2, \cdots, \mathbf{a}_K \otimes \boldsymbol{\xi}_K] \in \mathbb{C}^{2N\times K}$ is the array manifold matrix, $\mathbf{a}_k = [a_{-M,k}, \cdots, a_{0,k}, \cdots, a_{M,k}]^T$ is its k-th column vector, $\mathbf{s}(t) = [s_1(t), s_2(t), \cdots, s_K(t)]^T$ is the signal vector and $\mathbf{n}(t) = [\mathbf{n}_{-M}(t), \cdots, \mathbf{n}_0(t), \cdots, \mathbf{n}_M(t)]^T$ is the noise vector with $E\{\mathbf{n}(t)\mathbf{n}^H(t)\} = \sigma_n^2\mathbf{I}_{2N}$.

III. THE PROPOSED METHOD

In this section, a novel algorithm to estimate the DOAs, ranges, polarization parameters and noncircular phases of the NF strictly noncircular sources is proposed.

For the kth rectilinear or strictly noncircular signal, it holds that [12]

$$E[s_k(t)s_k(t)] = \alpha_k e^{j\beta_k} E[s_k(t)s_k^*(t)] \tag{4}$$

where β_k is the noncircularity phase and $\alpha_k = 1$ is the noncircularity rate of the kth strictly noncircular signal. For signal vector $\mathbf{s}(t)$ consisting of K uncorrelated signals, its unconjugated covariance matrix is given by

$$E[\mathbf{s}(t)\mathbf{s}^T(t)] = \mathbf{P}\mathbf{B}E[\mathbf{s}(t)\mathbf{s}^H(t)] = \mathbf{P}\mathbf{B}\mathbf{R}_s \tag{5}$$

where $\mathbf{B} = diag\{e^{j\beta_1}, e^{j\beta_2}, \cdots, e^{j\beta_K}\}$ and $\mathbf{P} = \mathbf{I}_K$.

In order to exploit the noncircular information, we construct a new vector $\mathbf{z}(t)$ as follows

$$\mathbf{z}(t) = \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{x}^*(t) \end{bmatrix} \tag{6}$$

Then, the covariance matrix of $\mathbf{z}(t)$ is given by

$$\mathbf{R}_z = \begin{bmatrix} \mathbf{A} \\ \mathbf{A}^*\mathbf{B}^* \end{bmatrix} \mathbf{R}_s \begin{bmatrix} \mathbf{A} \\ \mathbf{A}^*\mathbf{B}^* \end{bmatrix}^H + \sigma_n^2\mathbf{I}_{4N} \tag{7}$$

The eigenvalue decomposition of $\mathbf{R}_z$ leads to

$$\mathbf{R}_z = \mathbf{U}\boldsymbol{\Lambda}\mathbf{U} = \mathbf{U}_S\boldsymbol{\Lambda}_S\mathbf{U}_S + \mathbf{U}_N\boldsymbol{\Lambda}_N\mathbf{U}_N \tag{8}$$

where the $4N \times K$ martix $\mathbf{U}_S$ and the $4N \times (4N-K)$ martix $\mathbf{U}_N$ are the signal subspace and noise subspace, respectively. The $K \times K$ martix $\boldsymbol{\Lambda}_S$ and the $(4N-K) \times (4N-K)$ martix $\boldsymbol{\Lambda}_N$ are diagonal matrices consisting of the eigenvalues of $\mathbf{R}_z$.

A. Solution Based on the Accurate Signal Model

Base on the principle of the MUSIC algorithm, we have

$$\begin{bmatrix} \mathbf{A} \\ \mathbf{A}^*\mathbf{B}^* \end{bmatrix}^H \mathbf{U}_N = 0 \tag{9}$$

Then, a cost function $f(\theta_k, r_k, \gamma_k, \eta_k, \beta_k)$ can be constructed as follows

$$f(\theta_k, r_k, \gamma_k, \eta_k, \beta_k) = \| \begin{bmatrix} \mathbf{a}_k \otimes \boldsymbol{\xi}_k \\ \mathbf{a}_k^* \otimes \boldsymbol{\xi}_k^* e^{-j\beta_k} \end{bmatrix}^H \mathbf{U}_N \|_F^2 \tag{10}$$

According to the property of Kronecker product, we have

$$\mathbf{a}_k \otimes \boldsymbol{\xi}_k = (\mathbf{a}_k \otimes \mathbf{I}_2)\boldsymbol{\xi}_k \tag{11}$$

Substituting (11) into (10), we have

$$\begin{bmatrix} \mathbf{a}_k \otimes \boldsymbol{\xi}_k \\ \mathbf{a}_k^* \otimes \boldsymbol{\xi}_k^* e^{-j\beta_k} \end{bmatrix} = \begin{bmatrix} \mathbf{a}_k \otimes \mathbf{I}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{a}_k^* \otimes \mathbf{I}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{\xi}_k \\ \boldsymbol{\xi}_k^* e^{-j\beta_k} \end{bmatrix} \tag{12}$$

The cost function can be simplified as

$$f(\theta_k, r_k, \gamma_k, \eta_k, \beta_k) = \mathbf{p}^H(\gamma_k, \eta_k, \beta_k)\mathbf{M}(\theta_k, r_k)\mathbf{p}(\gamma_k, \eta_k, \beta_k) \tag{13}$$

where the 4×4 matrix $\mathbf{M}(\theta_k, r_k)$ has a form of

$$\mathbf{M}(\theta_k, r_k) = \begin{bmatrix} \mathbf{a}_k \otimes \mathbf{I}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{a}_k^* \otimes \mathbf{I}_2 \end{bmatrix}^H \mathbf{U}_N\mathbf{U}_N^H \begin{bmatrix} \mathbf{a}_k \otimes \mathbf{I}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{a}_k^* \otimes \mathbf{I}_2 \end{bmatrix} \tag{14}$$

$$\mathbf{p}(\gamma_k, \eta_k, \beta_k) = \begin{bmatrix} \boldsymbol{\xi}_k \\ \boldsymbol{\xi}_k^* e^{-j\beta_k} \end{bmatrix} = \begin{bmatrix} -\cos\gamma_k \\ -\sin\gamma_k e^{j\eta_k} \\ -\cos\gamma_k e^{-j\beta_k} \\ -\sin\gamma_k e^{-j\eta_k} e^{-j\beta_k} \end{bmatrix} \tag{15}$$

In general, $\mathbf{p}(\gamma_k, \eta_k, \beta_k) \neq 0$, and based on the RARE principle, we know that $\mathbf{M}(\theta_k, r_k) = 0$ means $f(\theta_k, r_k, \gamma_k, \eta_k, \beta_k) = 0$. Then, the cost function can be changed to

$$f(\theta_k, r_k) = det[\mathbf{M}(\theta_k, r_k)] \tag{16}$$

It is clear that the $rank\{\mathbf{M}(\theta_k, r_k)\} < 4$ if and only if $(\theta, r) = (\theta_k, r_k)$ are the true DOA and range. The K DOA and range estimates $(\hat{\theta}_k, \hat{r}_k), k = 1, \cdots, K$ can be acquired by searching for the K minima of $f(\theta_k, r_k)$.

Substituting the estimated DOAs and ranges into (13), we have $\mathbf{M}(\hat{\theta}_k, \hat{r}_k)\mathbf{p} = 0$, and then the estimation of polarization parameters (γ_k, η_k) and the noncircularity phase $\beta_k, k = 1, \cdots, K$ is transformed to a least square problem.

Defining two new matrices

$$\mathbf{Q}_1 = \begin{bmatrix} -M_{11} \\ -M_{21} \\ -M_{31} \\ -M_{41} \end{bmatrix}, \mathbf{Q}_2 = \begin{bmatrix} M_{12} & M_{13} & M_{14} \\ M_{22} & M_{23} & M_{24} \\ M_{32} & M_{33} & M_{34} \\ M_{42} & M_{43} & M_{44} \end{bmatrix} \tag{17}$$

where $M_{ij}, i, j = 1, 2, 3, 4$ is the element of $\mathbf{M}(\theta_k, r_k)$, we have

$$(\mathbf{Q}_2^H\mathbf{Q}_2)^{-1}\mathbf{Q}_2^H\mathbf{Q}_1 = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} \tag{18}$$

where

$$m_1 = \tan\gamma_k e^{j\eta_k}, m_2 = e^{-j\beta_k}, m_3 = \tan\gamma_k e^{-j\eta_k} e^{-j\beta_k} \tag{19}$$

Thus, the polarization parameters and the noncircular phase can be expressed as

$$\gamma_k = \arctan(|m_1|), \eta_k = \arg(m_1), \beta_k = -\arg(m_2) \quad (20)$$

B. Solution Based an Approximate Signal Model

However, the estimation of DOAs and ranges based on the above algorithm still needs 2-D peak searching. In order to reduce the computational effort, we can replace the spatial phase factor defined in (2) with an approximate second-order expansion which is known as Fresnel approximation in forms of $a_{l,k}(\theta_k, r_k) \approx e^{j(m\mu_k + m^2\varphi_k)}$ where $\mu_k = -2\pi d \sin\theta_k/\lambda$ and $\varphi_k = \pi d^2 \cos^2\theta_k/\lambda r_k$.

Then the steering vector $\mathbf{a}_k$ can be re-expressed as [5, 8]

$$\mathbf{a}_k = \varsigma(\mu_k)\boldsymbol{\upsilon}(\varphi_k) = \begin{bmatrix} e^{j(-M)\mu_k} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \\ \vdots & \iddots & \vdots \\ e^{jM\mu_k} & \cdots & 0 \end{bmatrix} \begin{bmatrix} e^{jM^2\varphi_k} \\ \vdots \\ 1 \end{bmatrix} \quad (21)$$

Substituting (21) into (14), $\mathbf{M}(\theta_k, r_k)$ can be written as

$$\mathbf{M}(\theta_k, r_k) = \begin{bmatrix} \boldsymbol{\upsilon}_k \otimes \mathbf{I}_2 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\upsilon}_k^* \otimes \mathbf{I}_2 \end{bmatrix}^H \mathbf{C}(\theta_k) \begin{bmatrix} \boldsymbol{\upsilon}_k \otimes \mathbf{I}_2 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\upsilon}_k^* \otimes \mathbf{I}_2 \end{bmatrix} \quad (22)$$

where

$$\mathbf{C}(\theta_k) = \begin{bmatrix} \varsigma_k \otimes \mathbf{I}_2 & \mathbf{0} \\ \mathbf{0} & \varsigma_k^* \otimes \mathbf{I}_2 \end{bmatrix}^H \mathbf{U}_N \mathbf{U}_N^H \begin{bmatrix} \varsigma_k \otimes \mathbf{I}_2 & \mathbf{0} \\ \mathbf{0} & \varsigma_k^* \otimes \mathbf{I}_2 \end{bmatrix} \quad (23)$$

Based on the RARE principle, $\mathbf{C}(\theta_k)$ and $\mathbf{M}(\theta_k, r_k)$ are of full column rank unless $\theta = \theta_k$, and then the cost function defined in (16) can be changed to

$$f(\theta_k) = det[\mathbf{C}(\theta_k)] \quad (24)$$

The K DOA estimates $\hat{\theta}_k$ can be obtained through 1-D angle search. Substituting the K DOAs $\hat{\theta}_k$ into (22) and searching for the peaks of $\mathbf{M}(\theta_k, r_k)$ about range r, we finally have the ranges of the K sources and no parameter pairing is needed.

Remark 1: The computational complexity of the proposed method is analyzed in terms of the number of complex-valued multiplications, mainly including spectral searching, construction of the extended covariance matrix ($(4N)^2 T$ flops, with T being the number of snapshots) and performing EVD ($(4N)^3$ flops). For the accurate signal model, the proposed method has to conduct direct two-dimensional (2-D) spatial spectrum searching which needs $\frac{\pi}{\triangle\theta}\frac{2D^2/\lambda - 0.62(D^3/\lambda)^{1/2}}{\triangle r}(4N)^2$ flops by defining the scanning interval for DOA $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, and range $r \in \left[0.62(D^3/\lambda)^{1/2}, 2D^2/\lambda\right]$ parameters as $\triangle\theta$, and $\triangle r$ respectively, while for the approximate signal model, it involves $\frac{\pi}{\triangle\theta}(4N)^2 + K\frac{2D^2/\lambda - 0.62(D^3/\lambda)^{1/2}}{\triangle r}(4N)^2$ flops since it only needs two 1-D searches. The proposed method has higher complexity than methods in [8, 10] by jointly exploiting the polar and noncircular properties.

Remark 2: A closed-form solution for source localization is achievable using a COLD antenna array [13] through parallel factor (PARAFAC) analysis. However, the noncircular information is not included in PARAFAC, and how to incorporate the noncircular information into PARAFAC will be a topic of research in our further work to avoid the spectrum searching process.

IV. DETERMINISTIC CRAMER-RAO BOUND

In this section, the deterministic CRB for the estimates of DOA, range, polarization and noncircular phase parameters is derived for NF strictly noncircular signals.

First, define a real-valued vector of the unknown parameters as $\boldsymbol{\xi} = \begin{bmatrix} \boldsymbol{\theta}^T & \mathbf{r}^T & \boldsymbol{\gamma}^T & \boldsymbol{\eta}^T & \boldsymbol{\beta}^T \end{bmatrix}^T$ with $\boldsymbol{\theta} = [\theta_1, \theta_2, ..., \theta_K]^T$, $\mathbf{r} = [r_1, r_2, ..., r_K]^T$, $\boldsymbol{\gamma} = [\gamma_1, ..., \gamma_K]^T$, $\boldsymbol{\eta} = [\eta_1, ..., \eta_K]^T$, and $\boldsymbol{\beta} = [\beta_1, ..., \beta_K]^T$. Under the deterministic assumption, $\mathbf{z}(t)$ are circularly Gaussian distributed with mean $\mathbf{A}_e\mathbf{s}(t)$ and covariance $\sigma_n^2\mathbf{I}_{4N}$, where $\mathbf{A}_e = [\mathbf{A}^T, \mathbf{A}^H\mathbf{B}^H]^T$. Then, the (p,q)th entry of the $5K \times 5K$ CRB matrix for the parameters in $\boldsymbol{\xi}$ is given by [11, 12]

$$[\text{CRB}^{-1}(\boldsymbol{\xi})]_{p,q} = \frac{L}{\sigma^2}\text{Re}\left\{\frac{\partial\mathbf{A}_e^{\text{H}}}{\partial\boldsymbol{\xi}_{\text{p}}}\mathbf{P}_{\mathbf{A}_e}^{\perp}\frac{\partial\mathbf{A}_e}{\partial\boldsymbol{\xi}_{\text{q}}}\mathbf{R}_{\text{s}}\right\} \quad (25)$$

where $\mathbf{P}_{\mathbf{A}_e}^{\perp} = \mathbf{I}_{4N} - \mathbf{A}_e(\mathbf{A}_e^H\mathbf{A}_e)^{-1}\mathbf{A}_e^H$.

Define

$$\mathbf{D} = [\mathbf{D}_\theta, \mathbf{D}_r, \mathbf{D}_\gamma, \mathbf{D}_\eta, \mathbf{D}_\beta] \quad (26)$$

with $\mathbf{D}_\theta = \left[\frac{\partial\mathbf{A}_e}{\partial\theta_1}, ..., \frac{\partial\mathbf{A}_e}{\partial\theta_K}\right]$, $\mathbf{D}_r = \left[\frac{\partial\mathbf{A}_e}{\partial r_1}, ..., \frac{\partial\mathbf{A}_e}{\partial r_K}\right]$, $\mathbf{D}_\gamma = \left[\frac{\partial\mathbf{A}_e}{\partial\gamma_1}, ..., \frac{\partial\mathbf{A}_e}{\partial\gamma_K}\right]$, $\mathbf{D}_\eta = \left[\frac{\partial\mathbf{A}_e}{\partial\eta_1}, ..., \frac{\partial\mathbf{A}_e}{\partial\eta_K}\right]$, $\mathbf{D}_\beta = \left[\frac{\partial\mathbf{A}_e}{\partial\beta_1}, ..., \frac{\partial\mathbf{A}_e}{\partial\beta_K}\right]$, and after some simplification, the closed-form expression for the CRB is given by

$$\text{CRB}(\boldsymbol{\xi}) = \frac{\sigma^2}{L}\left\{ \text{Re}[(\mathbf{D}^H\mathbf{P}_{\mathbf{A}_e}^{\perp}\mathbf{D}) \odot (\mathbf{1}_5 \otimes \mathbf{1}_5^T \otimes \mathbf{R}_s^T)] \right\}^{-1} \quad (27)$$

V. SIMULATION RESULTS

In this section, the proposed method is compared with some existing methods including Tao's method [8], Xie's method [10] and the deterministic CRB for the scenario of NF strictly noncircular sources. For the simulations, a ULA consists of nine antennas ($M = 4$) with a quarter-wavelength inter-antenna spacing ($d = \lambda/4$) is employed. The impinging sources are equal-power, uncorrelated BPSK signals and all the sources are located in the Fresnel region $(1.75\lambda < r < 8\lambda)$ of the array. The root mean square error (RMSE) $RMSE(\vartheta_k) = \sqrt{\frac{1}{500K}\sum_{k=1}^{K}\sum_{q=1}^{500}(\hat{\vartheta}_{qk} - \vartheta_k)^2}$, where $\hat{\vartheta}_{qk}$ is the estimate of the parameters $\hat{\theta}_k, \hat{r}_k, \hat{\gamma}_k, \hat{\eta}_k$ at the qth Monte Carlo simulation, ϑ_k is the true value.

In the first set of simulations, we examine the performance of the proposed method using both the exact signal model and the approximate signal model versus the SNR. Four NF signals are located at $(-30°, 2.1\lambda)$, $(-5°, 2.3\lambda)$, $(20°, 2.5\lambda)$ and $(45°, 2.7\lambda)$, respectively. The polarization angles and polarization phase differences of the sources are $(20°, 70°)$, $(30°, 85°)$, $(55°, 100°)$ and $(75°, 115°)$. The RMSEs of DOA, range, polarization angle, and polarization phase difference estimation are shown in Fig. 1(a)-(d), where the SNR varies from -6 dB to 20 dB, with the number of snapshots fixed

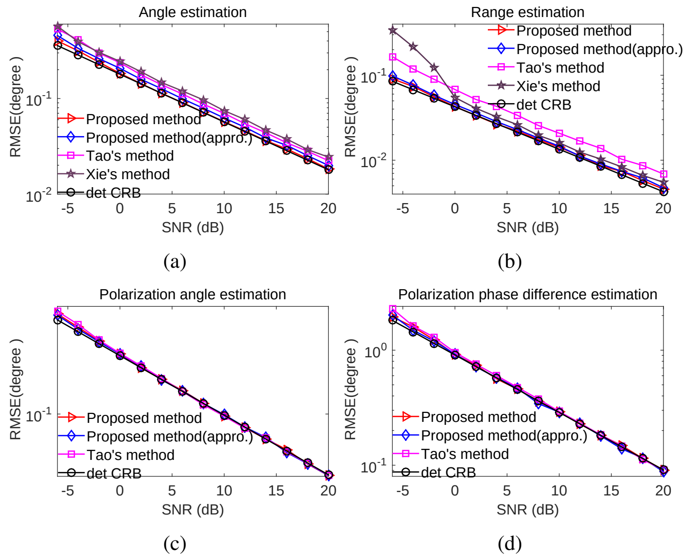

Fig. 1. RMSEs of four near-field sources versus SNR.

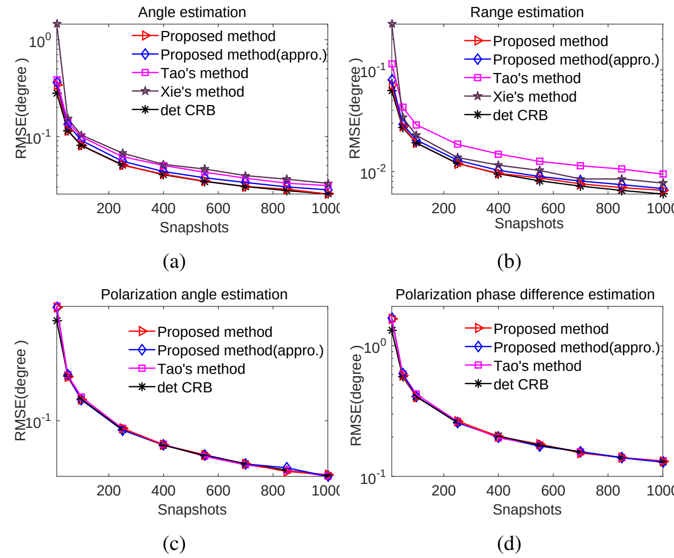

Fig. 2. RMSEs of four near-field sources versus snapshots.

at 2000. In Fig. 1, it is clear that the proposed algorithm outperforms the existing methods for both DOA and range estimation of NF sources by simultaneously utilizing the non-circular and polarization information at low SNRs. However, because of the Fresnel approximation error, the performance of the proposed algorithm using the exact signal model is better than the counterpart using the approximate signal model. In addition, the RMSEs of the proposed method are very close to the CRBs.

In the second simulation, we examine the performance of the proposed method versus the number of snapshots. The simulation conditions are similar to those in the first example, except that the SNR is set at 20 dB, and the number of snapshots varies from 10 to 1000. The results are shown in Fig. 2(a)-(d). As expected, the proposed method is always superior to the existing methods, especially for DOA and range estimation, and the RMSEs of the proposed method decrease monotonically and are very close to the CRBs.

VI. Conclusion

An effective localization method for NF rectilinear sources with a COLD array has been proposed. Based on the RARE principle, the separation of DOA, range and polarization parameters can be realized in the estimation process. By exploiting the approximate signal model, we can obtain the estimation of DOA and range through two 1-D searches, and a closed-form solution for polarization parameters and noncircular phases is provided. As demonstrated by computer simulations, the proposed method has outperformed existing methods and led to a performance very close to the derived deterministic CRB.

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