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Prestressed Saddle Shaped Tensile Structures, An Analysis

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PRESTRESSED
SADDLE SHAPED
TENSILE
STRUCTURES

AGUILAR
MELANCON

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ALBUQUERQUE, NEW MEXICO 87106

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MASTER OF ARCHITECTURE

PRESTRESSED SADDLE SHAPED TENSILE STRUCTURES,
AN ANALYSIS

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Department

Committee

PRESTRESSED SADDLE SHAPED TENSILE STRUCTURES, AN ANALYSIS

By
LORENZO H. AGUILAR MELANÇON
B.F.A., University of New Mexico, 1971

THESIS

Submitted in Partial Fulfillment of the
Requirements for the Degree of
Master of Architecture
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Problem

In today's field of architecture, a great deal of emphasis is placed upon the use of new forms and new materials by a great number of architects. Yet the vast majority of these persons are not equipped to carry out this task. It is not that they do not possess the technical knowledge, it is that they do not have the right approach in tackling the problem.

This is often the case when dealing with prestressed tensile structures; more specifically, saddle shaped surfaces.

Scope

This project represents an investigation of the structural principles involved in the design of prestressed saddle shaped tensile structures, the stripping down of these principles to their barest and simplest expressions, and an application of these same principles to model analysis involving the basic prestressed saddle surface structural prototype.

Procedure

This analysis is undertaken with the help of a machine which collects several types of information from the models. The machine, described in pages 64 and 65 of this work, was essential in the overall understanding of the basic structural conditions present in each particular model.

A process of observation and conclusion was carried out through each individual model test and each particular model..

Observations were made on each test's graphic data and were

due to the author's relating of these same observations to basic tensile principles. This process was carried out and was refined to the point where varying situations of structural complexity were easily related to these same principles. Conclusions were made from each individual test observation, were compared to those of other tests, and were synthesized into an overall conclusion or conclusions.

Results

The results arrived at by following this process of investigation enables the reader to make value judgements as to the condition of stress distribution on a particular model, and also gives the reader a way in which to cope with this situation. An indirect result arrived at in the process of investigation was that the reader acquires a common sense approach,not only to prestressed saddle shaped surfaces, but to the field of tensile structures as a whole.

Acknowledgements

To my wife Margarita, my children, my parents, brothers; and to my professors, William Gafford, Michel Pillet and Edie Cherry. I owe all of my thanks to these people whose cooperation, understanding, and in some cases sacrifice brought about this work.

Lorenzo H. Aguilar M.

Lorenzo H. Aguilar Melancon, Dec 1974 .

Problem

In today's field of architecture, a great deal of emphasis is placed upon the use of new forms and new materials by a great number of architects. Yet the vast majority of these persons are not equipped to carry out this task. They seem to be convinced that a midnight "brainstorm" and a pen is enough to create a fantastic and novel structure. This attitude on the part of the architects is utterly irresponsible. It is not that they do not possess the technical knowledge, it is that they do not have the right approach in tackling the problem.

Inspiration does play a big part, but with a very minimal role in the overall process of design and analysis. Any kind of innovation, in any field, is the product of long and painstaking work, and more often than not, very slow progress.

This is often the case when dealing with prestressed tensile structures; more specifically, saddle-shaped surfaces. This project is not an attempt to give the architect an "easy way" out of the process, nor can the architect build a tent structure after he reads this work; instead, it gives the architect a tool by which he can enable himself to better understand prestressed saddle-shaped tensile structures and their potential as a structural system.

SCOPE

This project represents an investigation of the structural principles involved in the design of prestressed saddle shaped tensile structures and an application of these same principles to model analysis involving various tensile surface prototypes. These principles are explained simply and concisely so as to permit the reader to have a firm common sense approach to tensile structure analysis and to apply this knowledge more readily in the analysis of the saddle surface models.

Actual "pen in hand" structural analysis of the structural prototypes will be held to a minimum; instead, the gross amount of analysis will be focused on extensive dabbling into the basic structural conditions derived from the testing of the different structural prototypes. This analysis will be furnished in order to help the reader gain a basic understanding of the structural "goings on" that take place in the different families of prestressed saddle surfaces.

PREFACE

This project [illegible] the [illegible] of the [illegible] types [illegible] in the [illegible]. [illegible] of [illegible] in [illegible] [illegible]. Three prototypes are [illegible] singly and [illegible] in [illegible]. The reader is [illegible] a [illegible] in terms of [illegible] and to apply this [illegible] in the analysis of the saddle surface [illegible].

Actual, "one to one" structural analysis of the structural system types will be left to a engineer. Instead, the gross aspect of analysis will be focused on "stress" or "detailing" into the basic structural conditions derived from the design of the different structural prototypes. This analysis will be developed in order to help the reader gain a basic understanding of the structural "goings on" that take place in the different families of prototypes of saddle surfaces.

Objectives

The main objectives of this work are to give an account of what has already been done in the order of prestressed saddle surfaces, to analyze and to divide this field of structures into different families and subfamilies that follow the same structural principles; to derive conclusions from the analysis of the different family model prototypes; and to provide a way in which to utilize and implement this information for an actual project.

These objectives will be carried out through the use of written material, drawings, photographs and graphs.

The analysis carried out by the use of a "testing apparatus", later explained in the text, will provide the reader with a basic knowledge of the workings of the structure, which will in turn be applied to the understanding of an actual structure.

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Introduction

Primitive requirements to satisfy the basic needs for shelter were by no means limited to humans. The animal world gives us vast numbers of examples in dwellings that, apart from providing the basic needs for shelter, vary from the rough branches and leaves that the gorilla uses for shelter to the admirable building skill which the sparrow uses in the building of her incredibly complex nest. Not only does this show us that there are highly developed ranges of skills in this domain, but that there is also that feeling of place, of belonging, that comes through; a kind of rude sense of aesthetics.

These abilities that animals exhibit in their shelters would seem to justify that these traits were also abilities present in primitive man. Slowly, man discovered the possibilities inherent in his brain. This development affected his dealings with his family, his skills and abilities, his tools, his dealings with the supernatural, his living habits, and finally, the development of his shelter.[1]

The ability of man to adapt his shelter seems to be one of primitive man's most singular accomplishments, and also the primitive beginning of what we now call architecture.

There is evidence that early man first sought shelter and protection in caves; as game became scarce he was forced to be nomadic; and there had to be an adaptation of his shelter to his way of life. Forced by his circumstances, nomadic man's first form of shelter was a tentlike structure consisting of animal

skins or reeds as a cover and a number of poles for support.

As the discovery of agriculture gave man an opportunity to settle and live off the land, it gave him an incentive to react and perfect his shelter using the materials that were available to him. The form of these shelters varied depending on the materials available in the region.[2]

It is from these beginnings that we trace the origin of architecture's two basic types of construction; the first, a type of light tent construction; and the second, a very solid type of architecture.

Up until about the beginnings of the Twentieth Century almost all of man's achievements in architecture had been in the field of solid construction. The aesthetic and architectural elements that were perfected and used in the house of man, and the eventual applications of these same principles in reverse, gave rise to movements and countermovements in the field of architecture all through the ages. Even today, the Twentieth Century, there are people who consider that Greek elements, applied to the entrance of their house, to the house of their god, or to the house of their money, is a sign of good taste, afluence,....... a kind of ancient and mystical security symbol.

The end of this architectural era was mainly brought about by great technological and sociological developments started during the middle of the last century by the mass production of concrete, steel, and other materials; and by the demand of society for a

new and more efficient architecture.

The second type of architecture, light tent architecture, did not evolve in the same manner as did solid construction; the invention of knotting and weaving assured some degree of progress. From its beginnings, efficiency, lightness and protability proved to be tension construction's most unique and important qualities; and it is these qualities that, after being adapted as a shelter, and refined to the point of perfection, stopped progressing as shelters. The basic type of tent, the central mast surrounded by a conical membrane, scarcely changed over the centuries; indeed, progress in the field took other routes and implementations.

Progress in the field of light materials for various functions was mainly restricted to the development of cheap and useful functional elements such as sails, canvas and ropes.

The commercial and economic factor, and the stringent requirements involved in shipping, gave the field of stronger and lighter materials for sails a big push, and led to a greater perfection in design.[3]

Time passed and so did large scale sailing and navigation,and yet the advances that had take place in technology and in materials during this span of time, and the new considerations of economics, efficiency, and simplicity in construction,, has resulted in a resurgence of interest, unprecedented in scale, and a serious consideration for tensile structure, specifically prestressed tension structures.

FOOTNOTES

[1] Talbot Hamlin, *Architecture through the ages* (New York,1953) p.3.

[2] *Ibid*., pp.19-23.

[3] Frei Otto, *Tensile Structures Vol. II* (Boston, Mass.,1969) pp. 16-17.

History

Information dealing with the history of tent construction is sparse and inaccurate, due to the fact that tent construction did not use an individual shape, material, or method of erection.

Some outstanding possibilities in shapes and functions achieved by man in the way of tents are better explained through the following examples:

In the Nineteenth Century, the tepee of the Sioux Indians served as a fantastic model for an early form of tent construction, and showed the versatility that can be achieved in the employment of such a structure. It consisted of a one piece covering of animal hides held together on one side with toggles, placed on top of a supporting structure of poles. This structure was purposedly designed to be as flexible in its use as possible. The toggle closure leaves a vent open at the top and an entrance at the front. The air vent has two flaps which could be moved with poles and adjusted to suit the wind, or could be closed completely.[1]

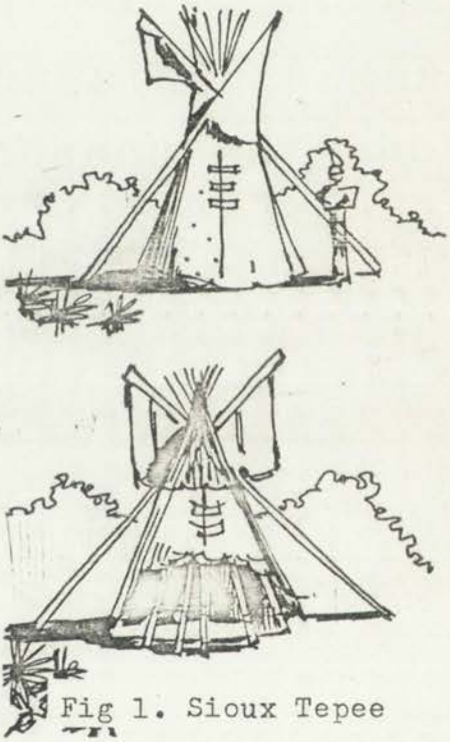


Fig 1. Sioux Tepee

The tents of the great North Arabian Nomads have a great advantage, in that these people, after having a great number of problems due to the eternal presence of the desert wind, provided their dwellings with the unique characteristic of being able to withstand

great magnitudes of wind loads. In a storm, the side facing the wind can be given a favorable aerodynamic form in which the poles are laid down and the supporting strips of the fabric lie at a steep angle.[2]

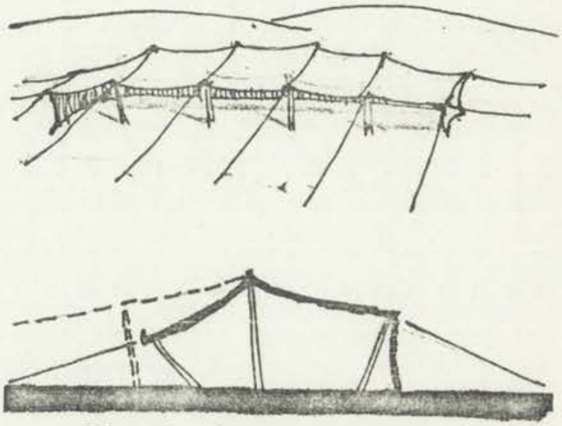


Fig 2. Arabian Tent

Roofs over streets using awnings have long been popular in the Latin countries of the Mediterranean and have taken other forms and functions in countries in America. For centuries this form was widely used in the covering of social gathering places such as market places, outdoor theaters, etc.

These so-called "Toldos", "Velas", or awnings which in the case of Seville, still in use over street, have definetly existed since the Sixteenth Century, although their use is so ancient, they definetly spring from an early origin.

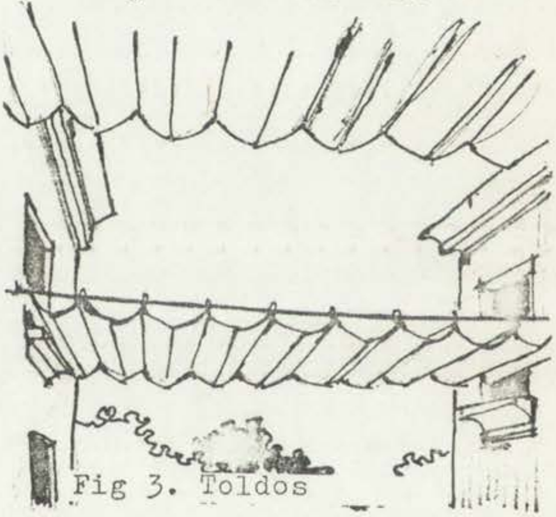


Fig 3. Toldos

"Toldos" consist of cloth strips hung from rings on cables parallely bunched, which are then strung accross the street.[3]

Roman Theater velums with very large spans were also in existence; the Romans used these roofs to cover their amphitheaters and

theaters against the sun; therefore these tension roofs required dimensions not attainable again until tents of modern times.

Our knowledge of these roofs stems from pictorial representations of the velums, the writings of Roman authors and traces found in the ruins of Roman theaters and amphiteaters. However none of these sources provides a detailed description of these roofs, for in all cases this information has been recorded in reference to some other subject.

Velum covers were originally used in 65 B.C., when Caesar was sponsoring gladiatorial battles; sun vellums were placed all along the Via Sacra from his house to the Capitol. After this time, the installation of different and novel types of vellums became fashionable for different varieties of public gatherings.[4]

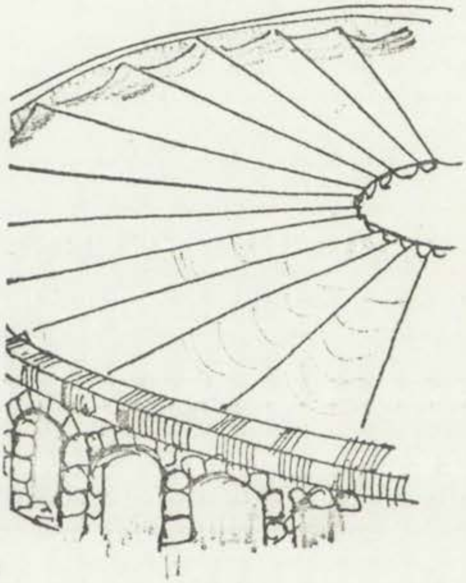


Fig 4. Vellum

Nero alledgedly had installed in the amphitheater a deep blue vellum decorated with stars, while in the Pompeii amphitheater in Rome, he installed a purple vellum which showed his image as a godlike charioteer surrounded by stars.[5]

The only known pictorial representation of a vellum roof is in a fresco found in the ruined city of Pompeii; even so in this fresco the velum roof is only roughly represented. The expert's

general analysis of this view is that only part of the roof was shown so as to show a view of the action taking place inside the arena.

This little information in any case gives us the general knowledge that the structures consisted of cables and massive beams and some manner of tow devisc because:

> "the erection required.... excellent design and mechanical engineering ability." 6 "

Another piece of information is that sailors were responsible for the extension and retraction of the vellum.

From research in these examples it can be assumed that for thousands of years, fabrics refined in design and made of varying types of strong fibers were used and implemented, but not along the lines of their use in prestressed shelter structures.

Up until the Nineteenth century no outstanding work involving the use of tension membranes appeared.

Large circus tents did not appear until the last century, when the use of a tent suspended at the center from four large masts became a type of trademark of the circus world for the business of entertainment.

The workings of this tent consisted usually of four central masts supporting a large canvas surface. The

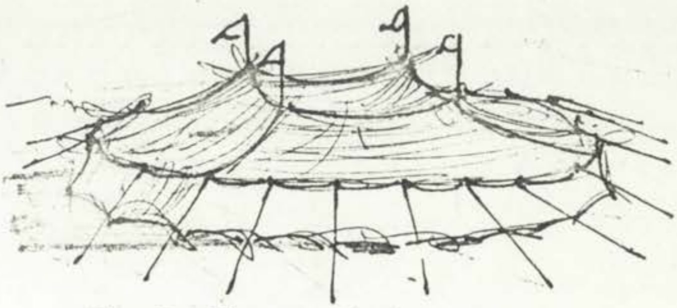


Fig 5. Circus Tent

purpose of this covering and the structural requirements rendered the surface to being subjected to very large tension stresses due to loading and relatively large spanning. Therefore the success of the system depends directly on the additional reinforcement present along the principal points of stress, which in actuallity consist of sewn-on ropes.[7]

FOOTNOTES

[1] Blumel P., Cracke R., Hennicke J., Kumel F., Pankoke U., Otto F., Schock H. J., Wagner J., <u>Convertible Roofs. (IL5)</u> (Berlin, 1972) p. 13.

[2] *Ibid*., pp. 14-15.

[3] *Ibid*., pp. 19-26.

[4] *Ibid*., pp. 29.

[5] *Ibid*., pp. 29.

[6] *Ibid*., pp. 29.

[7] Frei Otto, *Tensile Structures Vol II* (Boston, Mass., 1969) p. 16.

General Trends

Prestressed membranes and tension structures could have been implemented in ancient times and arrived at the same shapes, stresses, structural conditions and accuracy, using the materials that were available at the time.

Before the production of steel cables, tension bridges were used to span relatively large distances; their materials consisted of very strong fibres having a relatively short life. They therefore were in need of constant maintenance and replacement. The structure of these bridges consisted of a pathway along a catenary suspended from two points, with additional cables for handrails; highly efficient examples which we know of existed in the Andes region of Peru and were constantly utilized by the Inca civilization.[1]

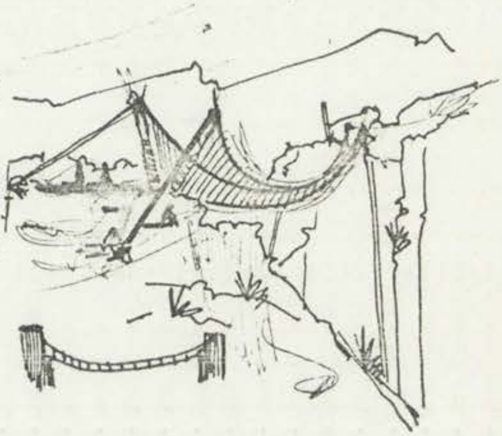


Fig 6. Suspended Bridge

Not until the Nineteenth century and the implementation of steel cables did level paths along with the use of suspension bridges come into existence. The structure of these bridges consisted of a level roadway suspended from the catenary by means of cable members subjected to tension, creating a separation between the roadway and the main structure, the catenary.

The design of these bridges created such innovations in bridge design and construction, that the early works executed around 1816 served as a base for later large spanning bridge design.

The perfection of the design of these types of bridges was accomplished by John Roebling, and consisted of cable members in tension suspended from both the compression members and the catenary. These cables formed a cable mesh from which the roadway was held rigid in place by these two mesh walls, as seen in the elevation in figure ; we can see the application of this principle in Roebling's two major works: the Brooklyin Bridge in New York and the Ohio Bridge in Cincinnati.[2]

— Fig 7. Roebling's Suspension Bridge.

Until recently, the basic structural priciples involved in the design of tension bridges had suffered no drastic changes; the idea of three dimensional prestressing in bridges is an idea but recently developed. Although three dimensional suspension bridges have been employed in the past (Elyzabeth Mock, Wich Bridge, England, built around 1741), true three dimensional prestressing design in bridges is a product of recent times. (Fig.8)

The structural principle of three dimensional prestressing in bridges consists of a roadway connected by tension members to three or more catenaries, regular and inverted, forming a three dimensional system, in which the roadway is secured against

The suspension bridge [illegible] [illegible] [illegible] [illegible] structures [illegible] bridges
perfected by John Roebling, [illegible] [illegible] [illegible] [illegible] [illegible] in tension [illegible] from [illegible] the [illegible] [illegible] and the catenary. These [illegible] forces [illegible] [illegible] [illegible] which the cable [illegible] [illegible] [illegible] [illegible] to [illegible] the [illegible] as much as the elevation in Figure 7 we can see the application of this principle in Roebling's two major works: the Brooklyn Bridge in New York and the Ohio Bridge in Cincinnati,7.

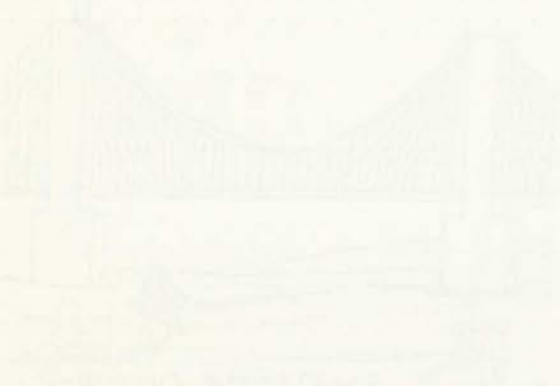


Fig. 7. Roebling's Cincin-nati Bridge.

Until recently, the main structural principle involved in the design of tension structures had suffered no drastic changes. The idea of three dimensional prestressing in bridges is [illegible] not readily developed. Although three dimensional suspension bridges have been designed in the past ([illegible] Work, High bridge, England, built around 17[illegible]), true three dimensional prestressing design in bridges is a product of recent times [17, 8].

The structural principle of three dimensional prestressing in bridges consists of a bridge supported by tension members so there is more [illegible], [illegible] and [illegible], forming a three dimensional system, as is the [illegible] in [illegible] [illegible] lateral or vertical displacements caused by either winds or heavy loading.

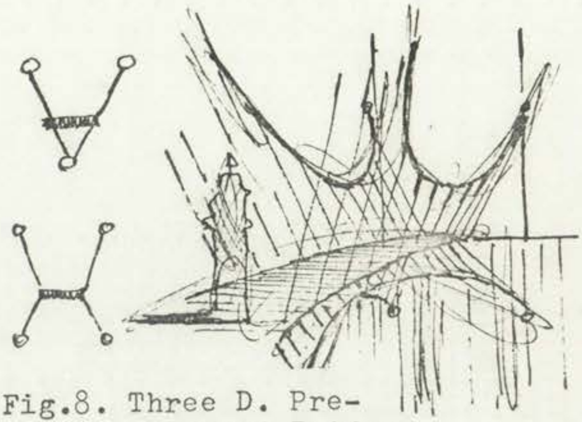


Fig.8. Three D. Prestressed Bridge.

Suspended roofs are also a product of recent times, and are a different design problem from that of suspension bridges. The difference resides in the fact that whereas bridges spanning large distances support heavy loads, roofs have smaller loads acting on them.

Tension-loaded surface structures became the object of acute interest and study around the middle of the 1930's when attempts to cover grain silos by the use of tension loaded steel sheathing of small curvature were made in the Eastern part of the United States.

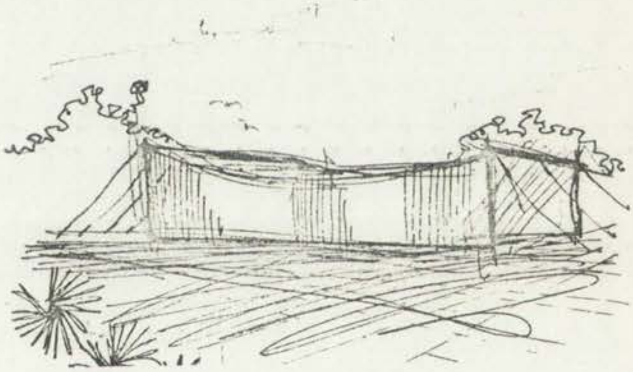


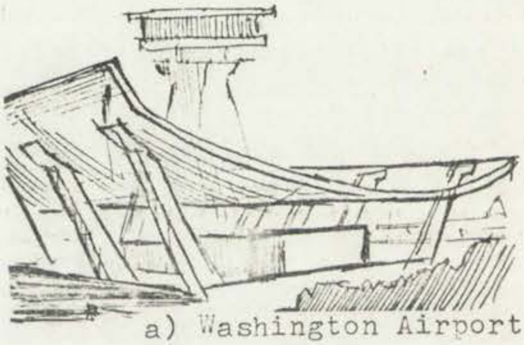
Fig 9. Suspended Roof.

By the beginning of the 1950's, a great deal had been accomplished in the way of tension-surface structures, demanding a study and survey of the field. This work was undertaken by the German architect Frei Otto in his dessertation <u>Das Hangende Dach</u> (Hanging Roofs). This work encompassed all works construc-

ted up to the 1950's and comprized an extensive description of non-prestressed tension loaded, centrally supported, undulating, prestressed cable nets and membranes; arches stabilized by cable nets, etc.

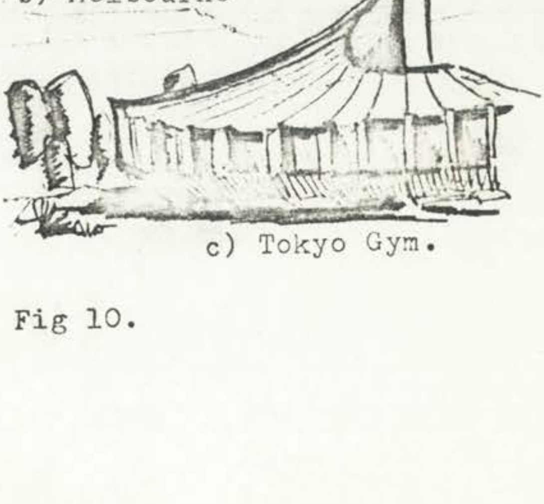
The information that the author condensed in his book influenced the field of architecture in such a way that tension structures began to be built all around the globe. Outstanding achievements in the field were done by such men as Borges, Rudolph, Freeman, Saarinen; and many others whose contributions to the field in the order of practical theoretical application to the construction of tension structures, have provided a way for more varied improvements in the field.

The more important structures in this field attributed to these people comprise works such as: the Dulles Airport in Washington, the Olympic Gymnasium in Tokyo, the Sydney Meyer Music Bowl in Melbourne, and have the same basic regard for the basic principles of two- and three- dimensional tension structures, but have applied these same principles in different and novel ways.



a) Washington Airport

b) Melbourne



c) Tokyo Gym.

Fig 10.

Development in the field of tensile structures has been very great in the last decade; most of this development has been due to the tremendous surge of interest that had occupied the minds of architects and engineers all through the preceeding decade, for new forms of expression through the utilization of different materials and different design concepts.

Most of the advancements accomplished during the past several years on structures which have forces from tension elements concentrated on a few members has been due to what can be called the career goal of the German architect Frei Otto.

Otto's work in tension structures has remained the reference point from which to begin to get acquainted with tensile structures; ever since Das Hangende Dach, every architect's introduction into the field of light- weight tension structures has been due, in some way or another, to the accomplishments he has acquired in this field. World wide prominence as an architect was achieved in 1972, when the Twentieth Olympic Games were held in the Olympic Stadium in Munich,a design by Frei Otto whose complexity at first sight is more baffling when compared to the simplicity of its structural princile.

His three books, Das Hangende Dach (Hanging Roofs), two volumes of Zugbeanspruchte Constructionen (Tension Structures), and the innumerable quantity of published articles gives us a detailed picture of a man whose determination , insight, and experience in dealing with the vast variety of problems and tasks engaging

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his attention has proved him to be the vanguard of the field, marking new trails through the virgin territory of tension structures.

It is an arduous task to ennumerate the tremendous contributions that Frei Otto has had on the field of architecture and advancements in construction technique; from his investigations in pneumatic structures, space frames with rigid joints in compression, structures loaded in tension and variations thereof; it can be said that his chief preoccupation, and the one in which he has had the most prominence , is the field of tension structures, particularly tents.

The basic work method and the one in which Frei Otto has acquired a great part of his knowledge of tensile structures, is a slow and tedious process of investigation. When embarking in an investigation of a particular structural design, the denoting of a specific structural type of family, which is the design base of the structural concept, is necessary for the denoting of possibilities and variations of such a structure. The incorporations or additions of different types of structural concepts to the structure also happens in the process of investigation; different structural cases are never considered in isolation,but in conjunction with different structural tension systems and the affinities of the particular structure to them.

This method gives the advantage of giving the designer a vast selection of design possibilities with the employment of combinathat structural principles can be recognised in the [illegible] [illegible] and can be [illegible] [illegible] the [illegible] structures of [illegible] [illegible].

[illegible] [illegible] [illegible] [illegible] the [illegible] [illegible] [illegible] that Frei Otto has set in the field of tensile [illegible] and the achievements in construction techniques. From his investigations of pneumatic structures, other fields have [illegible] benefited in various ways, especially [illegible] in tension and [illegible] structures. It can be said that his chief contribution, and the one for which he has had the most recognition, is in the field of tension structures, particularly [illegible].

The research method and the way in which Frei Otto has acquired a great part of his knowledge of tensile structures, is a slow and tedious process of investigation. This, according to an investigation of a particular structural design, the [illegible] of a specific structural type of design, which is the [illegible] [illegible] of the structural concept, is necessary for the [illegible] of possibilities and variations of such a structure. The investigation of solutions of different types of structural concepts to the structure also happens in the process of investigation; different structural types are never considered in isolation, but in combination with different structural [illegible] systems and the attributes of the particular materials to [illegible].

This will [illegible] the [illegible] of [illegible] for [illegible] a [illegible] selection of [illegible] [illegible] with the [illegible] of [illegible] tions and variations of different structural types. A great number of this architect's works have been a development of slow and methodical studies in design and structure by the utilization of scale models of the design in question. The usual sequence of investigations of models in the design stage consists of the following: a design model made of highly elastic fabric or material for checking scale, alterations in design, etc.; a measuring model of large scale, utilizing springs and chains for measuring forces acting upon the structure; and a structural testing model, for actual structural tests.

After following this regimented approach in dealing with the investigation of an actual structure, it is necessary to mention that the results achieved by the process are approximate; actual structural efficiency is only achieved after tremendous ammount of work in correlating the results from model measurements to those of the real project.

The reason for the utilization of models in the design process resides in the fact that when dealing with complex spatially curved shapes, it becomes increasingly hard to record or develop with any accuracy, the results by the utilization of a drafting board. The model alone gives the designer a clear conception of what is happenning. Space, form, a clear understanding of the forces acting on a tent, and a clarified path towards maximum utilization of time and materials in the actual construction, all can be readily modified and verified by the use of models in the design stage.[3]

It has been stated that the purpose of this work is to provide the reader with the tools essential to acquire a basic knowledge of the working of prestressed saddle shaped tension surfaces, and tension structures in general; also to provide the reader with this basis in order to use the knowledge acquired in the process and implement it on to an actual project by a method of analysis not so complex as that mentioned above.

To gain this objective it is necessary besides providing an historical outline and the trends the movement has acquired to date, to provide the reader with the basic structural principles involved in the analysis of one, two and three dimensional tension systems, and to relate this knowledge to the proposal at hand, prestressed saddle-shaped tensile structures and to provide methods of analysis of the structural conditions that exist in examples of basic structural conditions.

FOOTNOTES

[1] Victor W. Von Hagen, El imperio de los Incas, (Mexico, D.F., 1964), pp. 189-195.

[2] D. B. Steinman, Famous Bridges of the World (New York, 1953) p. 17.

[3] Conrad Roland, Frei Otto: Tension Structures (New York, 1970) pp. 2-3.

General Principles

In introducing the reader to the basic principles of tensile structures, it is necessary to start with a clarification of what is meant by the term structure. Structure as we interpret it means something already in existence or something already built, something that owes its existence to its own capacity of transmitting forces and moments within its own framework.

In animate and inanimate nature, all matter is in some way or another a structure. From the miniscule to the colosal, all matter is subjected to forces that act within the body and/or outside it; for example, a cell. A cell or element is composed of small particles which are subjected to forces of repulsion and attraction that function in such a way as to hold the electron particles in check around the neutron nucleus. It is these actions of these particles among themselves, that make the entity, the cell itself.[1]

The solar system acts in much a similar way; where the electrons are held in check by the forces exerted by the neutron nucleus, the planets circling aound the sun are held in these orbits by a combination of centripetal forces; keeping the celestial bodies in a checked orbit around the sun, much like a tension cable. The examples to which we have referred and the forces encountered acting in each, are not synonymous to other nature forms. Variation exists in the way these forces act, and in the ways each system is kept in balance.

Forces and moments are therefore an integral part of all nature.

When the capacity to control and transmit them is used by man, the result becomes a material expression of technicality and engineering.

Structures are therefore material and generally utilize matter that can transmit these forces; this principal applies to stones, earth, water, all the elements and their effects.

Frei Otto writes in an article entilted "Fundamental Concepts of Structures" :

> " Structures are a means of transmitting forces and moments." [2] .

Types of Structures

Material structures are best subdivided by categorically classifying them under the following criteria.

1-The types of structural members of which the structures are composed.

2-The nature of the loading of their structural members.

3-The directions in which the forces act: one, two or three dimensions.

4-The condition or characteristic of the material employed.

Types of Structural Supporting Members

Structural supporting members or systems are classified according to their dimensions , if a structural member has one predominant

dimension and two small ones, it is considered one-dimensional; if it has two large and one small, it is considered two-dimensional, and if all three dimensions predominate, it is considered three-dimensional.

One-Dimensional Supporting Systems

One -dimensional supporting systems are characterized by having one very large dimension and two small ones. The material of such a system is concentrated along a line, and it is along this line that all the stresses occurr.

The force exerted should be in direct correlation with the strength of the material utilized and the cross section of the structural member itself. As the strength of the material increases, the cross sectional dimension decreases.[3]

Cables in tension, girders subjected to bending and beams in compression, all are considered one-dimensional supporting systems in that they permit large forces to be exerted on them along a single axis.

Two-Dimensional Supporting Systems

The main characteristic of two-dimensional support systems or surface members is that they have two large dimensions and one small one. The surface of the member takes the shape of a plane has single or double curvature (sinclastic or anticlastic); depending on the nature of the material and on the loading; these types of members sometimes perform three dimensional actions and

have single or double curvature. The surface itself, depending on its condition, can be subjected to tension, as in the case of membranes, compression, as in the case of shells; or to bending, as in the case of folded structures composed of plates.

Two dimensional supporting systems have the ability, as in the case of one dimensional supporting systems, to withstand monoaxial stresses, and need not perform in one plane. Moreover these type of systems can perform in three dimensions as in the case of saddle or anticlastically curved surfaces.

Yet due to the tremendous spanning potential of these types of surfaces, it is a well known fact that two dimensional supporting members composed of crisscrossed linear members working in a three dimensional plane are more common than pure two dimensional support members. This can be seen by observing the workings of the German Pavillion at the 1967 Exposition in

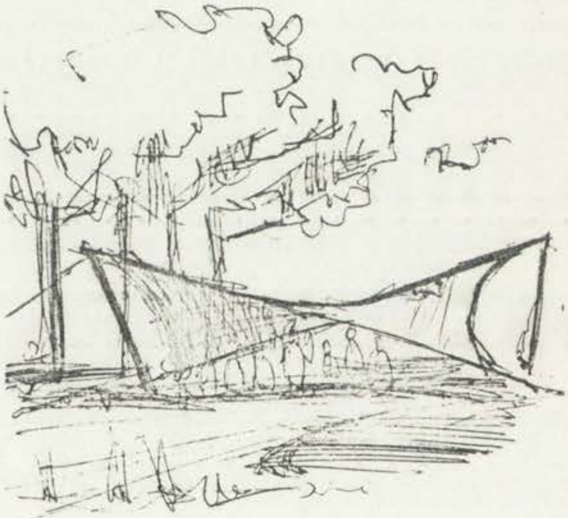

Fig 11. Bandstand at Kassel.

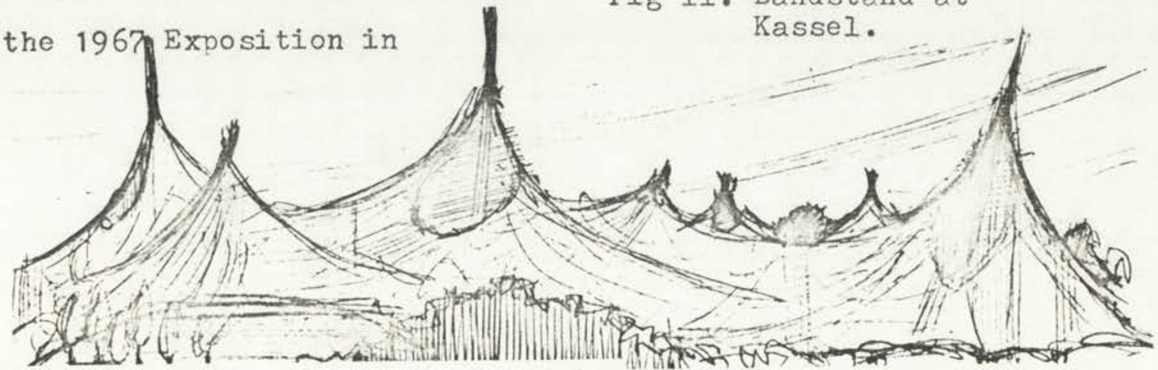

Fig 12. German Pavillion, Expo 67.

Montreal, Canada, as opposed to the Bandstand at the 1955 Bundesgartenschau (Federal Garden Exhibition) in Cassel, Germany. (Fig. 11 & 12)

Three-Dimensional Supporting Systems

Three-dimensional supporting systems have as characteristics, three predominating dimensions and are capable of taking up bending, tension and compression while being subjected to different kinds of stresses.

As in the case of two-dimensional supporting systems, true three-dimensional supports have a restricted use. With the exception of structural foam, which resists both bending stresses and compression , three-dimensional components' only ability in compression permits components solely of the type having a minimal volume, such as bricks, to be used; this observation is due to the fact that this type of component retains its strength and volume even after perforation, and permits the use of low-strength structural materials to be incorporated in the systems.[4]

Three-dimensional supporting systems are subdivided into geodesic, space and spatial cable net structures. Geodesic is a term which refers to systems whose rigid structural rods meet at nodes all through the system and are subjected to bending, as in the case of a three-dimensional roof frame.

"Space structures" is a term which refers to structures whose members meet at nodes and are subjected to both bending and compression as in the case of a skeleton structure of a skyscraper.

Spatial cable nets refers to systems whose members also meet at nodes, but are flexible, as in the case of a three-dimensional cable structure whose one dimensional elements meet at nodes but are subjected to tension.

Types of Stresses

The basic types of stresses that materials can be subjected to are compression, bending and tension stresses , or any combination of these.

The capacity of the material to withstand tension are called tension stresses.

The same analogy can be applied to compressive stresses.

Bending stresses are "moment stresses" and can be analyzed as a result of a combination of both compression and tension, although moment stresses act independently of both.

An analogy can be applied by identifying repulsion to compression; attractive forces acting outside the material to tension and rotation to torsion.

Stresses are subdivided into three categories; an identification between one, two and three dimensional stresses, and monoaxial, biaxial and triaxial stresses , is necessary for the identification of the different structural systems in which stresses act in one or several different directions.

A stress is one-dimensional or monoaxial if it acts in one direc-

tion or axis. A cable is a proper example of this condition as it is subjected solely to single directional stress and is one-dimensional.

A stress is biaxial or two-dimensional if the stresses act simultaneously in a plane. This creates a surface stress condition. An example of this is a drum, in which the skin is tensioned at several points along a plane.

Stresses in three dimensions or triaxial stresses are stresses which act in all directions.

Solid matter can be subjected to any type of stress in three dimensions and any combination of stresses is plausible.

It is by the insight acquired in the process of subdivision and classification of structures in general, that the reader will have a sound base from which to make value judgements as to the advantages of a particular structural system.

In today's field of architecture, the principal criteria for the design and construction of a certain project rests in achieving a correlation between design needs and structural accomodation, and the performance or efficiency of the system.

Efficiency in structures works in direct correlation with cost in terms of material consumption and time or erection. It is the contention of the author that although the use of tensile structures is not the appropriate solution for all types of projects, tensile structures' capacities by today's standards, in terms of

economy, make the field one of the most feasible.

Types of Tensile Systems

Tensile structures are divided into linear, surface and space structures, depending on the dimensions of the structural system.

Linear Tension Structures

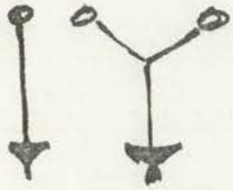

Fig 13. Vertical, Inclined.

1) Vertical or inclined suspension systems.
   Ex. Cranes, suspended buildings, directly suspended bridges.

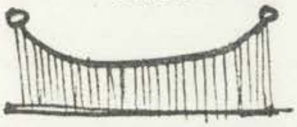

Fig 14. Catenary.

2) Catenary shaped suspension systems.
   Ex. Suspension bridges, suspended roofs, power and telephone cables, etc.

Surface Tension Structures

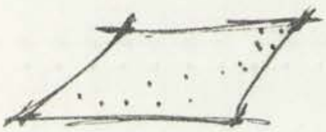

Fig 15. Plane

1) Plane prestressed surfaces. These structures consist of a very tensioned surface forming a plane; because of the stresses involved, these types of structures are not suitable for major projects.

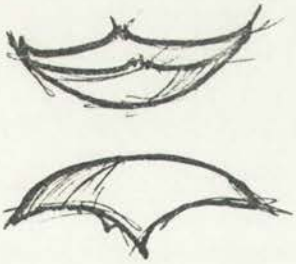

Fig 16. Sinclastic.

2) Surfaces curved in two directions, known as dome-shaped or sinclastic. These types of tensile structures can be either concave or convex, depending on the structure being pneumatic or suspended.

Fig 17. Curved Surfaces.

3) Surfaces curved in one direction only.
   Ex. Heavy suspended roofs consisting of cable networks.

Fig 18. Anticlastic

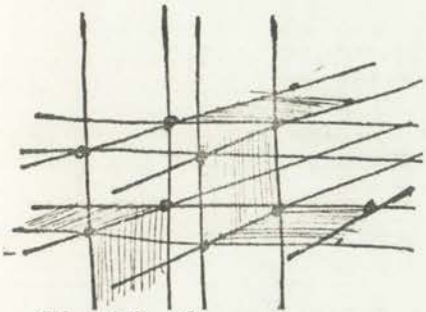

Fig 19. Space Structures

4) Surfaces curved in two opposite and perpendicular directions, known as saddle shaped or anticlastic surfaces.

   Ex. Prestressed membrane and cable network saddle surfaces.

Tensile Space Structures

1) Networks in three dimensions made up of linear tensile members.

   Ex. Suspended steel frame buildings.

Prestressed and Non-prestressed Tension Systems

Tensile structural systems are of two types; freely suspended systems and prestressed systems. Non-prestressed or freely suspended systems are systems which undergo stress only when loaded. No danger of buckling or bending, as in cases where compression or complicated states of stress are present, is associated with freely suspended structures. Such systems have as a distinct characteristic a direct relationship between shape and structure; flexible tension loaded components readily assume an equilibrium shape most suited for the transmission of forces acting on them.[5]

Prestressed tension systems have as characteristic the presence of stresses in the unloaded condition. This prestressing is due to forces which act in the system in the direction of the axis or at right angles to it, as in the case of prestressed structural concrete, where tension members or steel reinforcing cables are stressed in such a way as to place the concrete at the bottom

half of the member in compression.

Non-prestressed One-dimensional Systems

In order to understand the principles introduced in complex cases of one two and three dimensional tension systems, it is necessary first to have a thorough comprehension of the most basic of tension systems, one-dimensional cable members.

The principles involved in the analysis of any light-weight tension structure is in most cases linked directly to the structural principles present in any one-dimensional flexible tension member under a variety of loading conditions.

In relating the components of any tension system to one-dimensional tension systems (cables), the reader will arrive at a clearer comprehension of the workings of the structure in question. Cables are one-dimensional tension systems which have the capability of taking the most suitable shape in order to take up the forces acting on them, as shown in the following cases.

- Any cable member in tension takes the form of a straight line if no load acts upon it.
- It can resist any load acting perpendicularly or obliquely to its axis only if it undergoes some degree of deformation, depending on the material of the member; this is true for fabrics and cable nets.

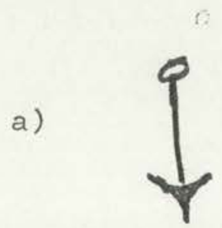

Fig 20. Cable Conditions in tension.

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- If no deformation is desired, the member will have to be tensioned a great deal; the magnitude of such a force is infinitely large.
- Cable loads can be classified by the actions performed by the linear member when dealing with different types of loading.
- If loading is applied along the axis of a prestressed vertical cable, a force load results, causing deformation at the upper portion of the cable. The magnitude of the deformation depends directly on the cable material.[6]
- If a load is applied normal to the axis of the linear member, a moment load results, causing deformation on both halves of the cable, and, depending on the prestressing, horizontal displacement where the load is applied.[7]
- A combination of force and moment load results on a cable if a load is applied in any direction other than the normal, as in the case of a

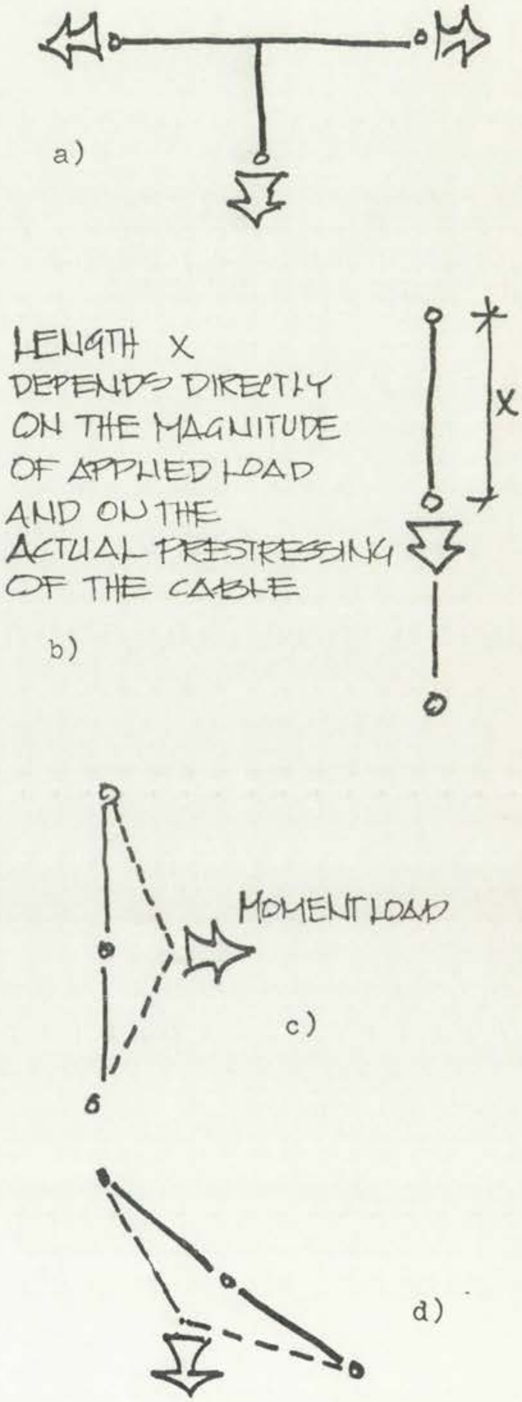


Fig 21. Cable Conditions in tension.

cable suspended between two points not in the same horizontal plane. If a load is applied at the mid-point of the span, the force on the cable is not distributed evenly; the greatest stress is concentrated on the top half of the member causing some deformation along its axis. Also in this case, horizontal and vertical displacements, where the load is concentrated, are apparent. This displacement varies depending on the angle at which the cable is suspended. By comparison, a cable suspended at a steep angle shows greater horizontal and less vertical displacements than a similar cable under the same load but at a lesser angle. This observation provides us with a non-arbitrary rule of thumb: when dealing with any material or member in tension, the greater the inclination with respect to the direction of the load, the greater its capabilities to withstand superimposed loading with a minimum of deflection.[8]

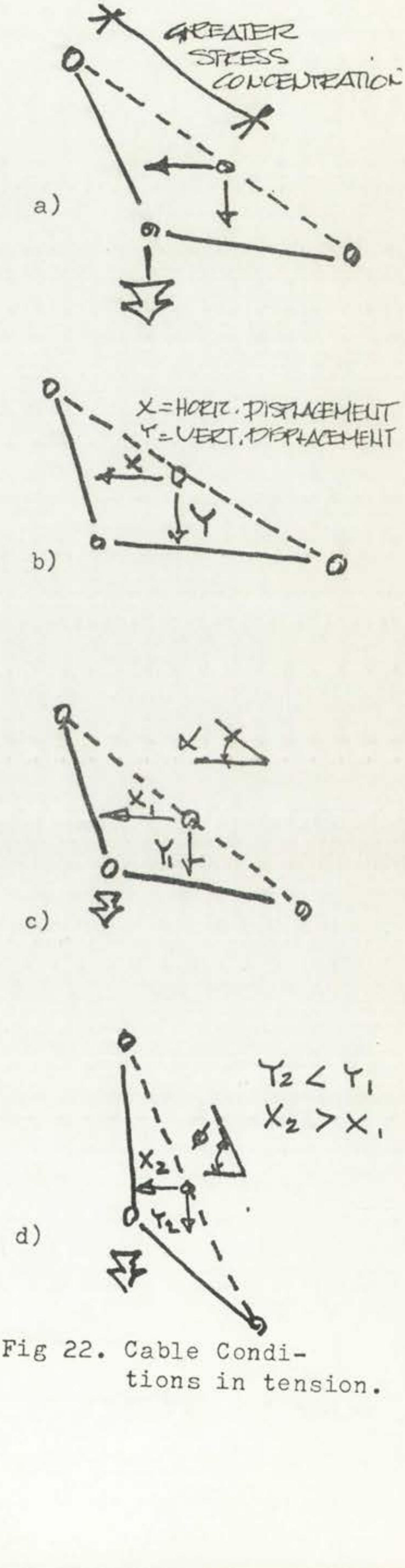


Fig 22. Cable Conditions in tension.

If a cable is uniformly loaded, it forms a catenary(Formula: $y= \frac{H}{W} \left[\cos H(\frac{W}{H} x)-1\right]$.

The shape of the catenary curve has no relationship to the magnitude of the loading.

By virtue of the laws governing equilibrium, the forces acting on a catenary are of two types, horizontal, in direct relationship to the span of the catenary, and vertical, directly related to the sag; both of these forces in conjuction result in a moment load. The horizontal tension component as well as the vertical component is greatest at the points of suspension, causing maximum tension stresses to occurr at the supports.

The resultant forces at the supports also have a direct relationship to the sag, and the slope at the supports, minimizes the horizontal component and therefore the resultant stresses at the supports. This

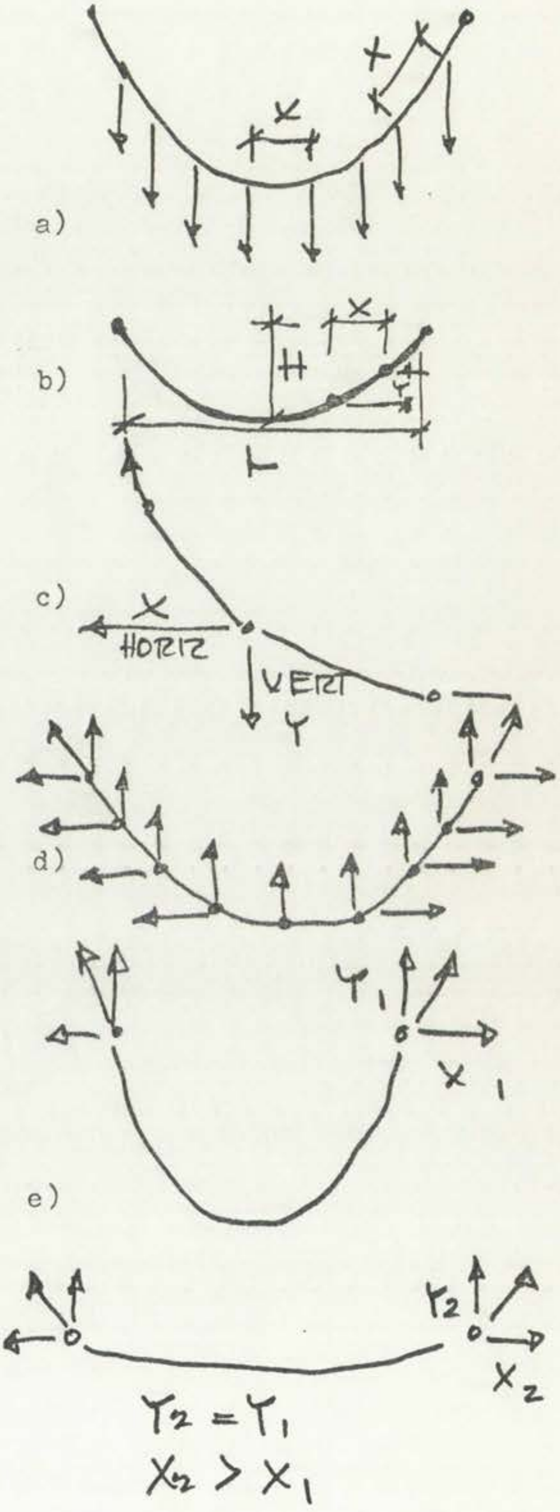


Fig 23. Catenary.

observation gives the reader another fact in which to base tensile analysis: In order to deal effectively with any concentration of forces, tension members should have a greater slope at the points of greater stress; the greater the slope the better the stress is being dealt with. As can be seen in soap bubble analysis, the film adopts a steeper angle where greater concentration of stresses are present.[9] On the other hand, if a cable is loaded uniformly at equal horizontal intervals, the cable forms a parabola, and follows the formula:
$y= wx^2 / 2h= 4hx^2 / l^2$.
The forces acting on a parabola follow the same principles as those of catenaries, also causing the greatest tension stresses to act at the points of suspension.[10]

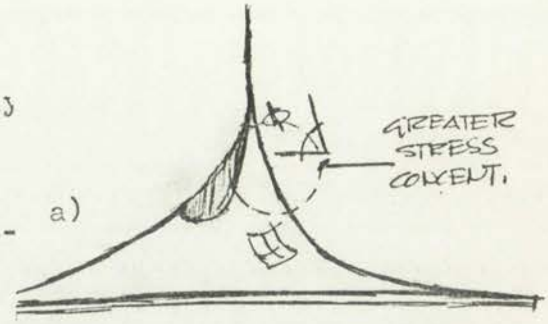
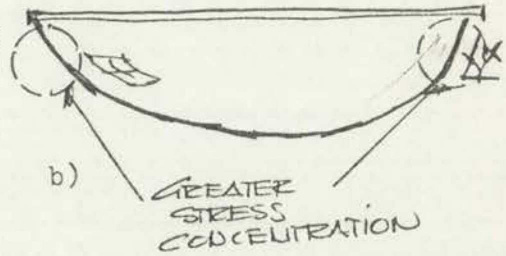


Fig 24. Stress Concentration.



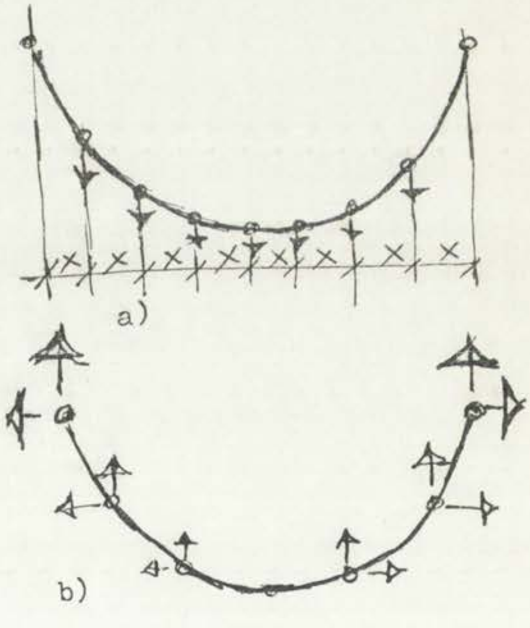


Fig 25. Parabola.

Surface Structures

The term surface structure in most cases relates to structures or structural members which have a working or functional volume

consisting of two large and one small dimension. In most cases, the tasks performed by these members have a three-dimensional range of function, as in cases where the curvature is anticlastic, sinclastic or have a catenary or a parabolic cross section.

Such structures can be formed of one continuous material such as membranes, be perforated and consist of networks made of cables, or have a mixture of cables and members rigid in bending.

Minimal Surfaces

In the study of tensile structures, a membrane that has the same ammount of surface tension or force per unit length at any point is called a minimal surface. Minimal surfaces are surfaces which span any closed linear configuration with a minimum amount of surface area. 11

Between any one-dimensional linear configuration on one plane, only plane surfaces of minimal area can be formed. In any other case, where one-dimensional elements lie in any three-dimensional configuration, only saddle shaped surfaces of minimal surface area can be formed.

Soap films provide the designer with a tool by which he can produce the surface most suited for spanning a linear configuration with a minimum of surface area. This is becouse the film in the soap solution always tends to adopt to the configuration most suited for an equal distribution of the fluid and therefore an equal distribution of the stresses. In this way peak stresses are

avoided by the shape or cinfiguration assumed by the film. See fig. 26.

The principal drawback to soap film model analysis is the fact that only models of very limited size can be employed; this is an outcome of the quick bursting of the soap film due to the excess of span or to prolonged exposure. This limits the size of such models to approximately four centimeters.

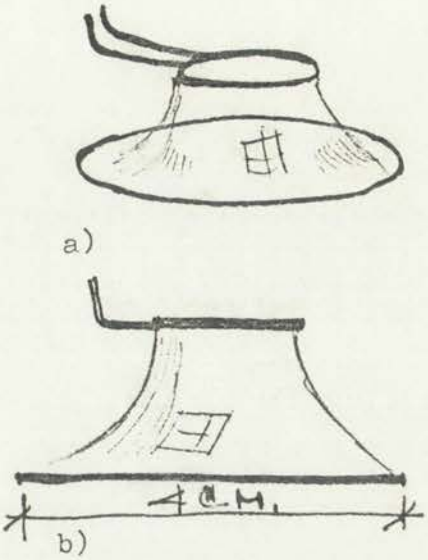


Fig 26. Film Models.

Because of this, no satisfactory analysis can be accomplished except with the use of very sophisticated photographic equipment. If such a model can be accomplished, it is regularly used to provide corrections in the design stage.

On the other hand, minimal surfaces can also be represented and analyzed using materials of high extensibility in all directions, i.e.,rubber membranes.

Prestressing on these types of models can be accomplished because of their high extensibility, and if the prestressing is done correctly by stretching the membrane with the same force in all directions the shape of the structure does not change noticeably.

Minimal surfaces in some cases are not suitable for projects which have a high concentration of stresses in certain areas;

in these cases, a greater curvature should be applied on the membrane in the areas where there is the greater stress concentration in order for the tent material to handle the load effectively and with less strain.[12]

Membranes and Cable Networks

The distinctive feature differentiating membranes and cable networks is the structural function they perform; where membranes have a limited efficient span of around fifty meters, cable latticeworks and networks have an efficient working span starting at about thirty meters, performing the same functions as a membrane if infilling is provided in between the cable mesh; membranes reinforced by steel, nylon or several layers of material and attaining efficient working spans of several hundred meters should be classified in between membranes and networks.

Membranes are structural members whose material of construction is tautly tensioned; they can be subdivided into anisotropic and isotropic membranes, depending on the stretching properties of the material. Rubber membranes and plastic sheets are considered isotropic becouse they stretch evenly in all directions when subjected to load.

Anisotropic membranes, as in the case of sheets or any other type of woven fabric, are membranes which exhibit greater stretching or deformation in one of two directions called the warp and the weft. Woven materials generally stretch less in the warp

direction than in the weft; this is due to the loose weaving of the weft threads in between the tautly stretched warp threads.

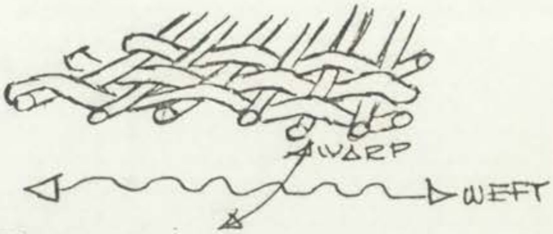


Fig 27. Warp and Weft.

Cable networks on the other hand can be analyzed as loosely woven fabrics made of strong linear members forming squares, triangles, hexagons,or other shapes depending on the way the mesh is designed.

Taking into account this information, the reader will find that there exist no analytical difference between anisotropic membranes and cable meshes; quantitative differentiation exists only with regard to mesh size and strength.

The span of any tensile structure is directly dependant on the strength of the material utilized i.e., the stronger the material employed, the greater the span. At this time the strongest material is high strength structural steel achieving a tensile strength of 260 Kg/mm^2 , and can be produced as wire.[13]

In the field of fabric, natural and synthetic materials of higher strength than structural steel can be imbeded into the membrane enabling the structure to achieve spans greater than ordinary tent fabric.

Isotropic Membranes

Isotropic membranes can be of several types: plastic sheets,

metal membranes, rubber membranes, resin-bonded fibres, wood membranes, and other forms of tent construction which have as a feature an equal deformation in all directions under load.

Due to their low tensile and breaking length and their extremely elastic actions under load, rubber membranes and plastic sheets made of polyester or polyethylene are suitable for small spans as in the cases of swimming pools, infilling for cable networks, etc.

Reinforcing these types of materials is possible by means of applying various sheets of the material, or by imbedding fabric networks consisting of materials of greater tensile strength and resistance to tearing in the plastic sheet.

Resin bonded fleeces i.e. glass and synthetic fibres glued together by polyvinyl chloride, are similar to paper in that the materials which they are composed of are of random length and strength; yet because of the fibres in their composition, they are generally of greater strength than plastic membranes,

Metal and wood membranes are the strongest materials that fall into the isotropic membrane category. They are generally not completely isotropic and have vary low extensibility under load, therefore components made of these materials have to be very accurately made in order to compensate for their low extensibility. These members are usually employed as infilling in cable lattice networks; because of their durability different types of layers of materials can be applied or glued on to them in order to provide

greater insulation, acoustics and fireproofing.

As was the case in ancient times, fabrics are generally the most used material in tent construction. Linen, silk, plastic, and other materials not developed until recently, comprize the different materials of which tents can be made.

Cotton fabrics have an approximate strength per unit length of around three thousand kilograms per meter, a measure achieved by most fabrics today; and exposed, have an approximate efficient life span of around three to five years.

Fabric generally stretches on wetting and shrinks on drying, and has a variable transparency depending on the thickness of the fabric. Cotton can vary in tone or color depending on the source of acquisition, and can also be reinforced at spots or all along the fabric by means of applying several layers of the fabric or providing reinforcing materials in the weave of the fabric.

Plastic fabrics have generally the same weight and strength factors as cotton fabrics, and are generally made of polyvinyl, polyacrilic or polyester. The only disadvantage in the use of plastic fabrics which links directly to their life span is the necessity of protection from the sun and in some cases from the elements.[14]

Methods of Assembly

In most cases where tightly woven materials are involved, strips of fabric ranging from three to five feet in width are woven. In most cases in order to create a three dimensional saddle-shaped

membrane, these strips are sewn or glued together forming wedge or fish belly shapes at the joints, making the fabric to double where these strips join.

By continually making these fish belly overlappings, the tent will be enabled to take a varying ammount of stress, generally greater, where these overlappings occur. This can also be turned to the advantage of the structure when they are applied at points of maximum stress, e.g., at points of support or at membrane peaks or humps .

In the cases of humped membranes, no special cutting is necessary to achieve a three dimensional effect; three dimensionality is acquired by the actual deformation of the tent under load.

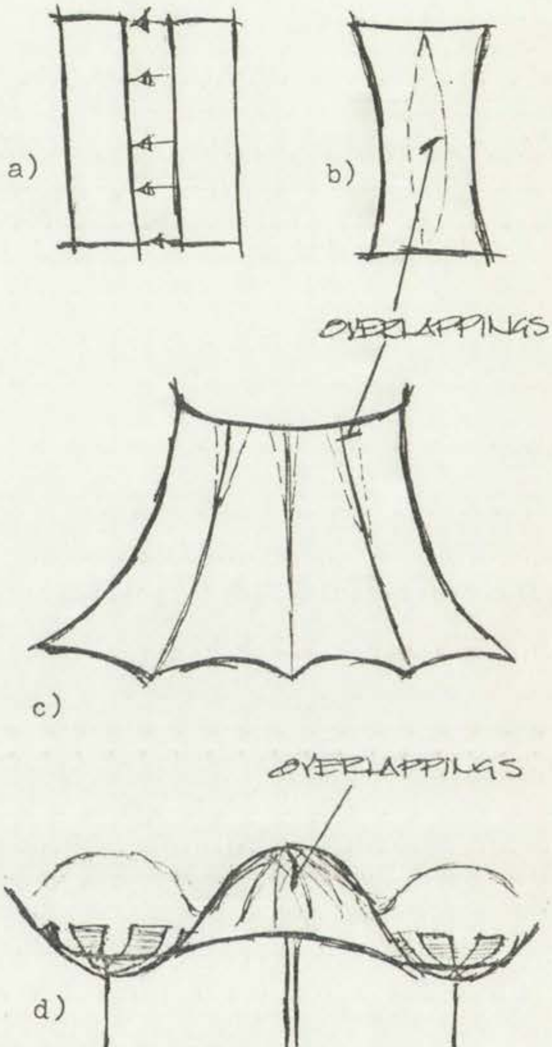


Fig 28. Fish Belly
Overlappings.

Cable Networks

Cable networks provide the field of tensile surface structures with a suitable structural system by which the project can span great distances with the insurance of a long life.

Whereas membrane surfaces require some form of maintenance, repair or replacement, in cable networks the cables themselves are made anticorrosive by means of galvanization.

The problem of making the infilling of the networks long lasting, which is not part of the structure itself, depends entirely on the material which is employed.

Cable networks are of a variety of types and vary also in methods of erection. Although spanning any three-dimensional edge configuration does not warrant the use of any set geometrical cable arrangement, for the sake of economy and simplicity in the structure, most surface structures have hexagonal, tetragonal, triangular, or octagonal meshes.

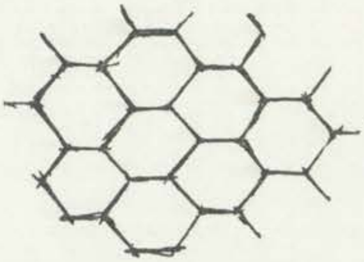


Fig 29. Hexagonal Cable Mesh.

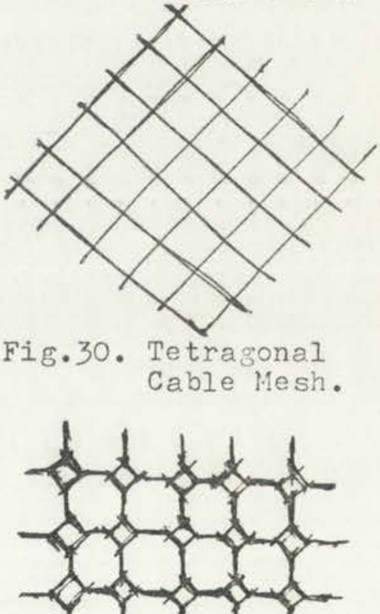


Fig.30. Tetragonal Cable Mesh.

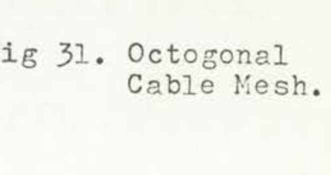


Fig 31. Octogonal Cable Mesh.

The most well known nets have tetragonal or quadrangular geometric configurations consisting of squares, rectangles, parallelograms, or rhomboids of regular or irregular shape. Hexagonal and tetragonal meshes can form any type of three-dimensional surface. Whereas

tetragonal surfaces stretch only diagonally, hexagonal meshes can easily stretch in any direction. Stiffening of quadrangular meshes can be accomplished by inseting additional cables in the configuration of the mesh.

Uniform triangular meshes are the strongest of any, but their ability to span with only three-dimensional planes of simple curvature makes them difficult to use. However, by subdividing the mesh into hexagons or squares, the triangular mesh can take the most varied of shapes and take up stresses in every direction.

Irregular networks whose members extend along the shortest possible path between the supports and the points of suspension approximate minimal surfaces. In spanning the surface with a minimum ammount of material, the project will lose economically in the outcome because of the complexity of erection.

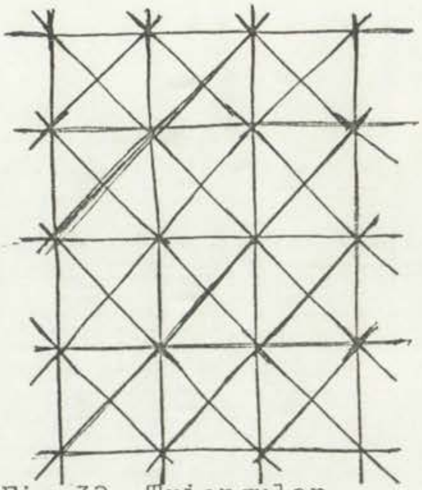


Fig 32. Triangular Mesh in Squares.

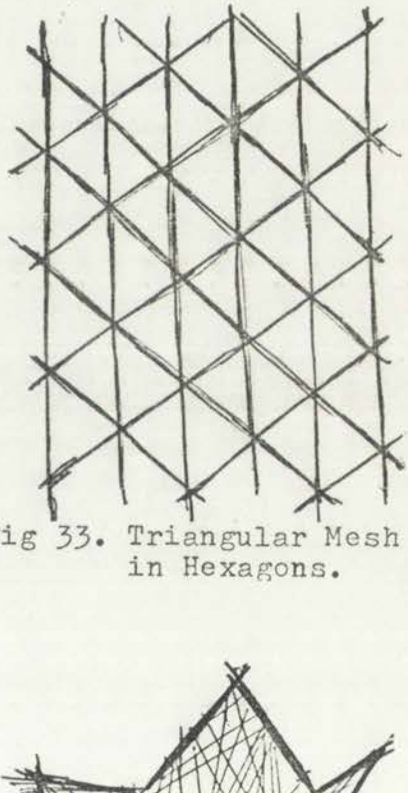


Fig 33. Triangular Mesh in Hexagons.

Fig 34. Irregular Cable Network.

The advantages provided in the way of reinforcement and stiffening by such methods of cable arrangement, make them suitable for areas where great stress concentrations occurr, e.g., supports, or points of suspension.

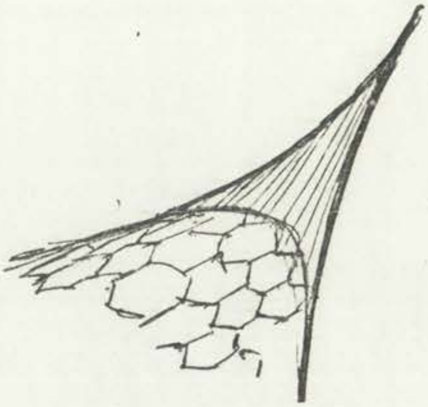


Fig 35. Irregular Reinforcing Cable Arrangement.

Methods of Assembly

For large spanning structures having heavy cable networks or networks of varying cable size, assembly can either be made on the site, hoisted in place and then tensioned; or assembled by hanging individual cables from the supports (arches, cables, frames, etc.), then interconnecting them by means of clamps or clips, and tensioning the network.

If a heavy network is assembled by either of these methods, the size of the mesh should have a consideration for further construction activity, such as walking on the cablemesh. A recommended size of mesh would be around two to three feet.

For short spanning structures, prefabricated netting of around twelve inches in size, and of different configurations, is available commercially and is suitable for medium span prestressed structures.

The method of erection is similar to that employed by fabric membranes, although strips of greater magnitude can be employed.

Advantages to this type of structural system lie in the fact that infilling is relatively easy to apply and can even be infilled by spraying light weight materials such as foam.

The disadvantages to this type of system lie in the assembly process itself; cables have to be spliced to be connected, thus ruining the continuous flow of geometric configuration of the mesh, and therefore its strength.

Edge Supports for Tension Surfaces

The shape of any three-dimensional surface in tension if particularly affected by the configuration of the edge supports.

The stresses that ocurr at the edges are directly dependant upon three things: 1) the spanning directions of the weave of the cable network or fabric; 2) the dead loads and live loads affecting the surface; 3) the prestressing achieved in the structure; Therefore the edges have acting on them, all of the stresses transmitted by the surface. Cables, straight or curved bars and arches are most suitable for edge supports.

Frames which have to support these loads and are rigid in bending have a high rate of material consumption precisely becouse of the great strain at the edges of the surface structure. Therefore from the purely economical point of view, the large forces exerted by the surface can be handled with the least amount of material consumption by giving a curved or arch shape to the edge configuration, i.e. between purely tension loaded flexible non-planar

lines that form frames.

In two dimensional surface structures which have uniform tension distribution all through the membrane, flexible edge supports form circles. By the same token, if the same two dimensional surface structure has a variation of tension in any direction, elyptical edge configurations result at the edge supports.

If a surface is stretched between the four vertexes of a square frame and if the radius of curvature of the edge cables is .7071 of the distance of the suspension points, the cables are tangential at the supports.

In three dimensional tension surfaces of uniform tension, the edge cables form lines of constant curvature and are not circles as in the case of two dimensional or plane surface structure.

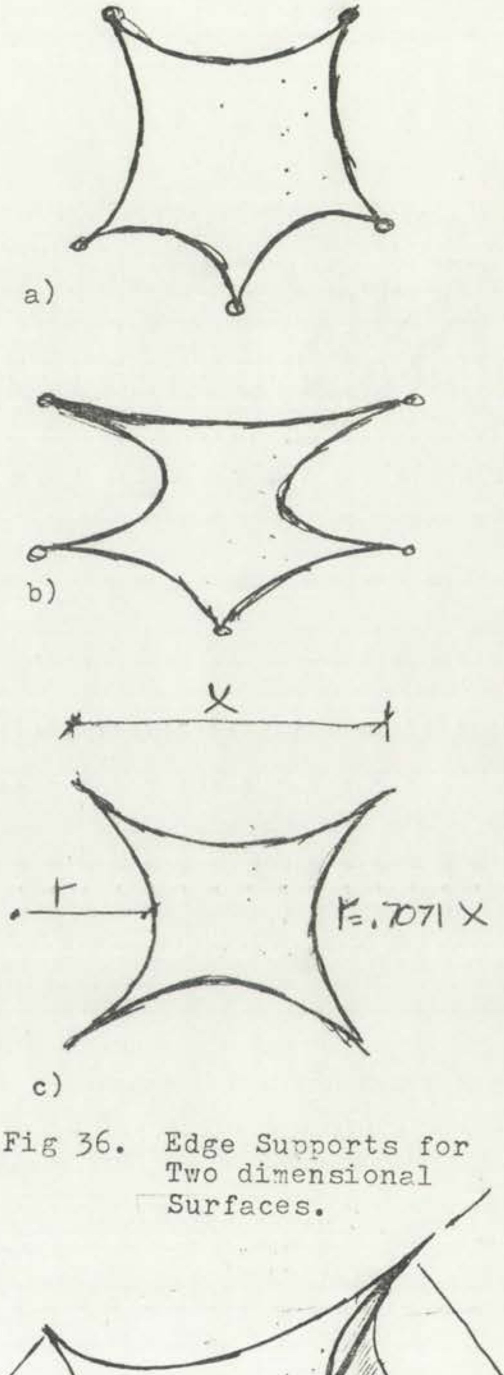


Fig 36. Edge Supports for Two dimensional Surfaces.

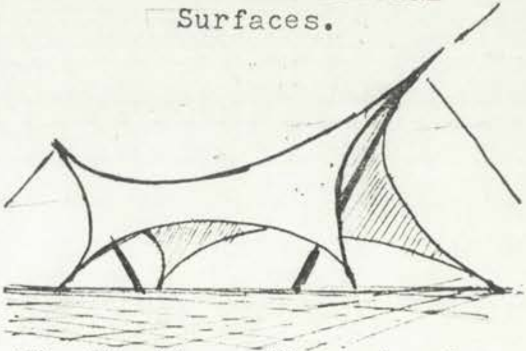


Fig 37. Three Dimensional Edge Supports.

Prestressed Saddle Surfaces

If two cables not in the same plane intersect at 90°, the intersection permits a fixing of a particular point in space. If two initially parallel cables intersect in the same manner with another pair of cables, a system of four nodes is fixed in space. If more cables are added in both directions, and if the stress distribution all along the tent is harmonic, a curved continuous cable net results, with the points of intersections being held rigid in place by the prestressing in all cable members.

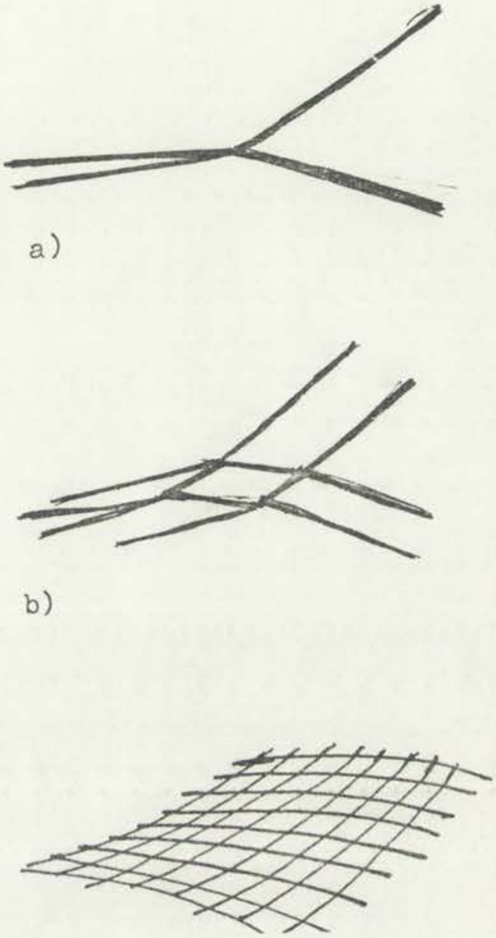


Fig 38. Cable Intersection Principle for Saddle Surfaces.

If the cables involved at the intersections perform an equal prestressing action and an equal but opposite curvature from each other, the surface formed by such actions is called an anticlastic or saddle surface.

The prestressing of cables involved in the intersections should be such that if a maximum concentrated load

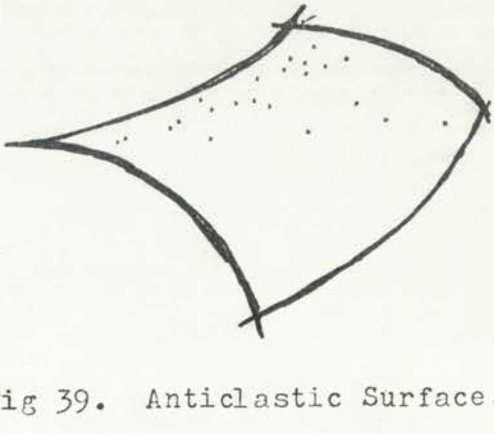


Fig 39. Anticlastic Surface.

is applied at any intersection, no cable taking part in the intersections will slacken and loose its rigidity.

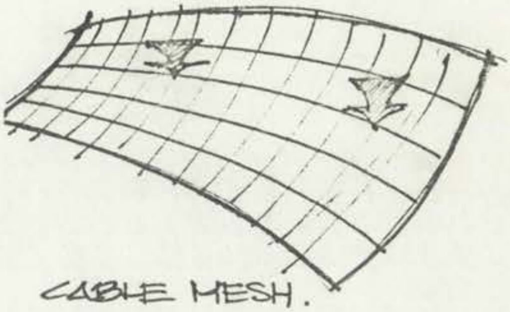


Fig 40. Cable Mesh

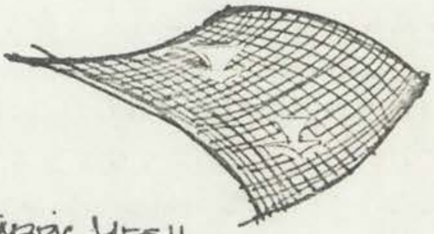


Fig 41. Fabric Mesh

By the same token, the principles involved in intersecting cables, can also be applied to intersecting threads as in the case of fabrics. Such actions as that of prestressing of the fabric threads, make saddle shaped surface structures particularly suitable when dealing with high wind loads and any actions derived from this load such as fluttering due to suction.

The common assumption that gravity, associated with suspension structures and surfaces also affects prestressed saddle surfaces is obliterated in this case. The actual prestressing of the material renders the effect of gravity and self weight of the structure to be a minor part of the surface stress. Thus we can assuredly say that prestressed saddle surfaces are tension structures in

the purest sense of the word.

At any point in a simple saddle surface, the principal directions of curvature are the directions of greatest curvature; if the saddle surface is a minimum surface, these directions are directly perpendicular to each other and are characterized as having the smallest radius of curvature.

Between the two principal directions of curvature and at half angles to them are found the minimal directions of curvature.

By means of individual cable analysis, it is the directions of principal curvature as opposed to the directions of minimal curvature that will be able to better deal with any load acting on the tent with a maximum rigidity and a minimum of deflection..

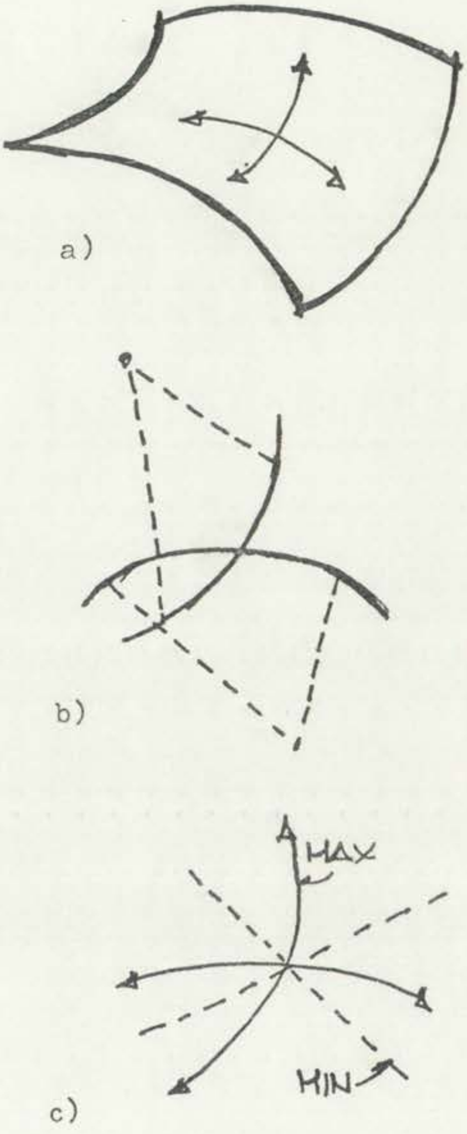


Fig 42. Maximum and Min. Curvature.

Also in order to regulate the prestress of the structure in accordance with the tensile structure's

expected structural performance, curvatures of small radii are advantageous.

In the case of any saddle surface, the positive loads arising from snow, dead weight and wind loads are dealt with by the downward or "carrying" direction; and the negative loads arising from wind suction are dealt with by the spanning direction.

Both directions working with each other in the sense that if more load is applied on one direction, the other experiences less loading.

The basic shape of a saddle surface is well known and is mostly because of its association with hyperbolic paraboloids of reinforced concrete shell construction. On the other hand, the structural actions performed by tension loaded surfaces is very different from the compression and tension loaded hyperbolic paraboloids; admittedly the shape of

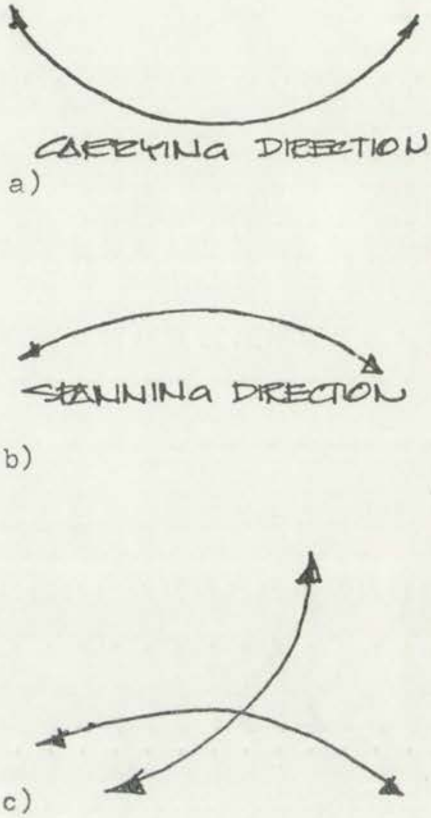


Fig 43. Carrying Direction and Spanning Direction of Anticlastic Surfaces.

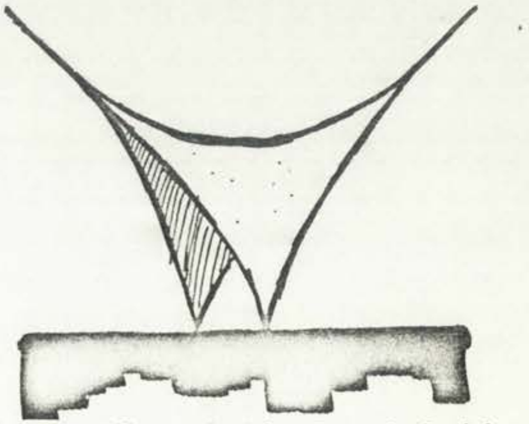


Fig 44. Hyperbolic Paraboloid shaped Saddle Surface.

tension loaded saddle surfaces are of a very striking similarity to these, but they are not identical.

The discrepancies in shapes is very noted; whereas the generating basis for the shapes of paraboloids are straight lines and could be generated from straight line supports, saddle surfaces show a slight warping of the fabric at the edge support; the condition then becomes acute and is most prominent at the vertices or the principal points of support.

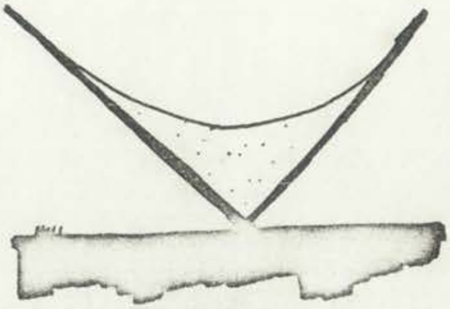


Fig 45. Hyperbolic Paraboloid.

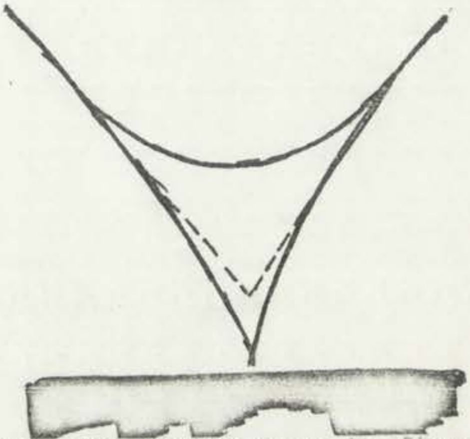


Fig 46. Difference in Shapes

State of Stress

If a circle is drawn on the unstretched skin of a drum, when the skin is stretched to cover the mouth of the drum, a greater circle will appear; if the skin is stretched evenly in all directions, the circle will not vary in shape but in diameter, forming a minimal surface in doing so.

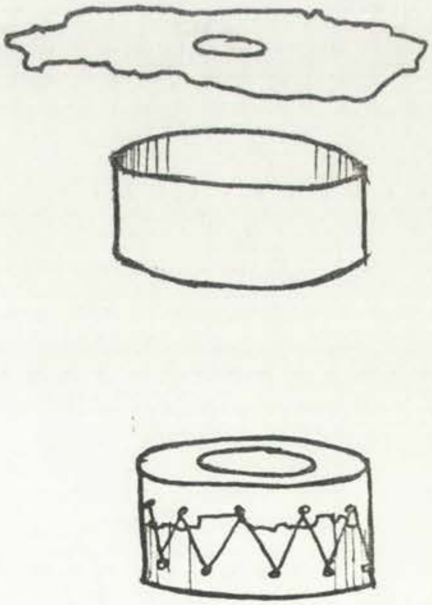


Fig 47. Tensioned Skin of Drum.

The difference of how the stress

affects the membrane of a saddle surface depends on the properties of the material the tent is made of. For example,a circle is drawn on a material in the unloaded state; if the material is loaded uniformly in all directions, different deformations will take place on the circle; depending on the material, the circle will form an elypse as in the case of anisotropic fabric membranes,or might form a circle of greater diameter as in the case of completely isotropic membranes.

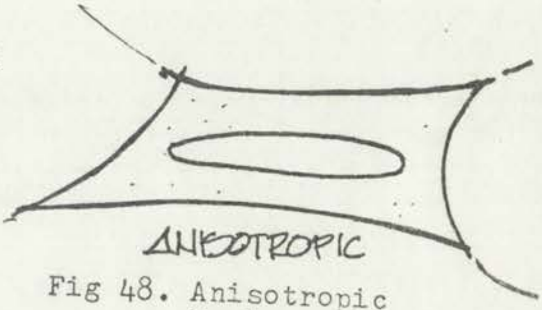


Fig 48. Anisotropic Membrane.

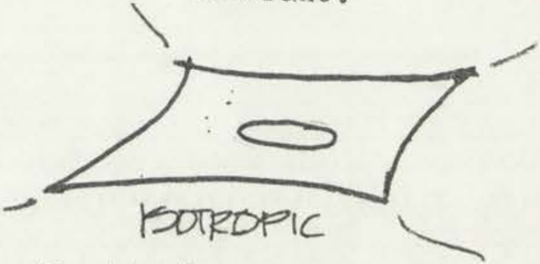


Fig 49. Isotropic Membrane.

In either case, testing of the material, and a measurement of the magnitude of deformations in all directions should take place before any degree of analysis as to the magnitude and directions of the principal stress components is undertaken.

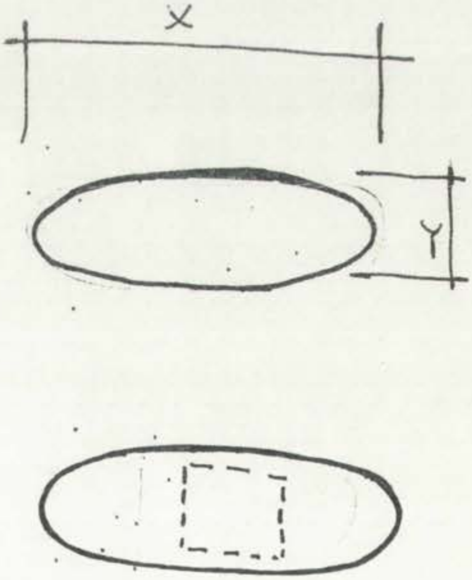


Fig 50. Material Deformation

Knowing some of the forces acting on the circle, we draw a square

inside the figure and cut an imaginary line at a certain angle.

As the sections of the square become separated, we hold these portions together by three clips. These clips assume a certain direction. This direction is the resultant tension vector, or the principal direction of stress: S.

When the angle of cutting is not in the same direction as S, the force that acts to diplace the parts from each other is O. O is the normal surface tension which acts at right angles with the angle of cutting.

The force per unit length of section is the shearing surface tension T.

T, S, and O are measured as force length lbs / inch, and when T and O are added vectorially, the resultant force is S.[15]

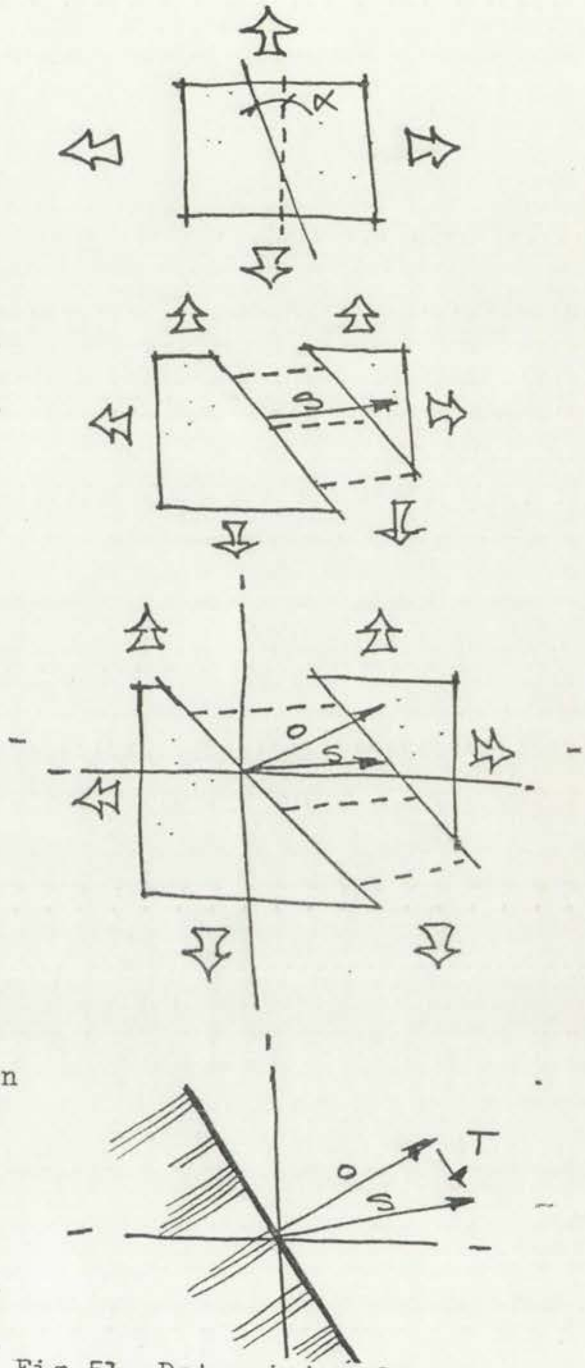


Fig 51. Determining Surface Tension (graphics)

A graphic method of analysis as to the magnitude and direction of the normal surface tension, the shearing surface tension and the surface tension resultant is presented in page 54; the pro-

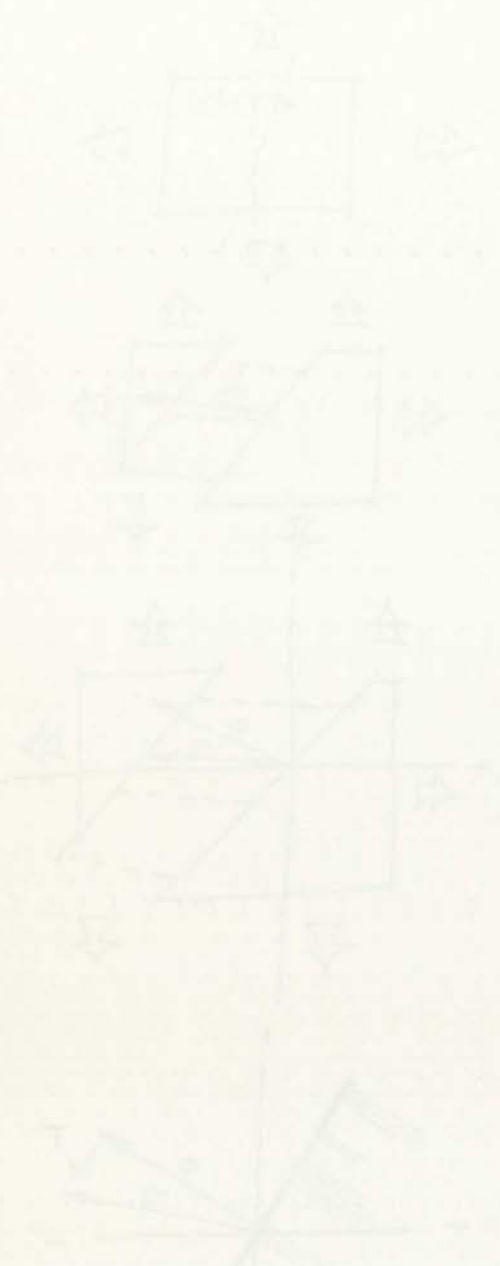


Fig. 31. Schematic [illegible] tension lines

[illegible]

[illegible] length.

in the directions of the arrows as

[illegible] respectively, as [illegible]

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These either causes a [illegible]

tion. This direction is the [illegible]

[illegible] vertical, or the prin-

ciple direction of stress S.

When the angle of cutting is at

in the same direction as S, the

force that acts to displace the

parts from each other is N. N is

the normal surface traction which

acts at right angles with that

angle of cutting.

The force per unit length of section

is the specific surface tension T.

T, S, and N are measured as force,

length. [illegible] and where T and

U are [illegible] respectively the result-

ant force is [illegible]

A graphic method of analysis, as to the magnitude and direction of the [illegible] surface [illegible], the [illegible] and the surface [illegible] is indicated in Fig. [illegible], the [illegible] cedure is as follows:

- Plot components of main force on the circle and reduce it to the unit of measurement. Plot on X and Y axis corresponding to the sides of the square. Call them forces A and B.
- Plot a line for O, not the magnitude.
- Draw one quarter circle through A until it touches the Y axis.
- Unite both intersecting points.
- Intersect circle with continuation of MG and plot the intersecting point with letter D.
- Call intersecting point along X axis E.
- Plot a line parallel to BA and pass it through point E.
- The intersection of the line that follows the Y axis and the above mentioned line call letter F.
- The conncection between F and M is S.
- Draw a line perpendicular to MD and pass it through the intersecting point F. Call out as G.

For graphic presentation see figure in the following page.

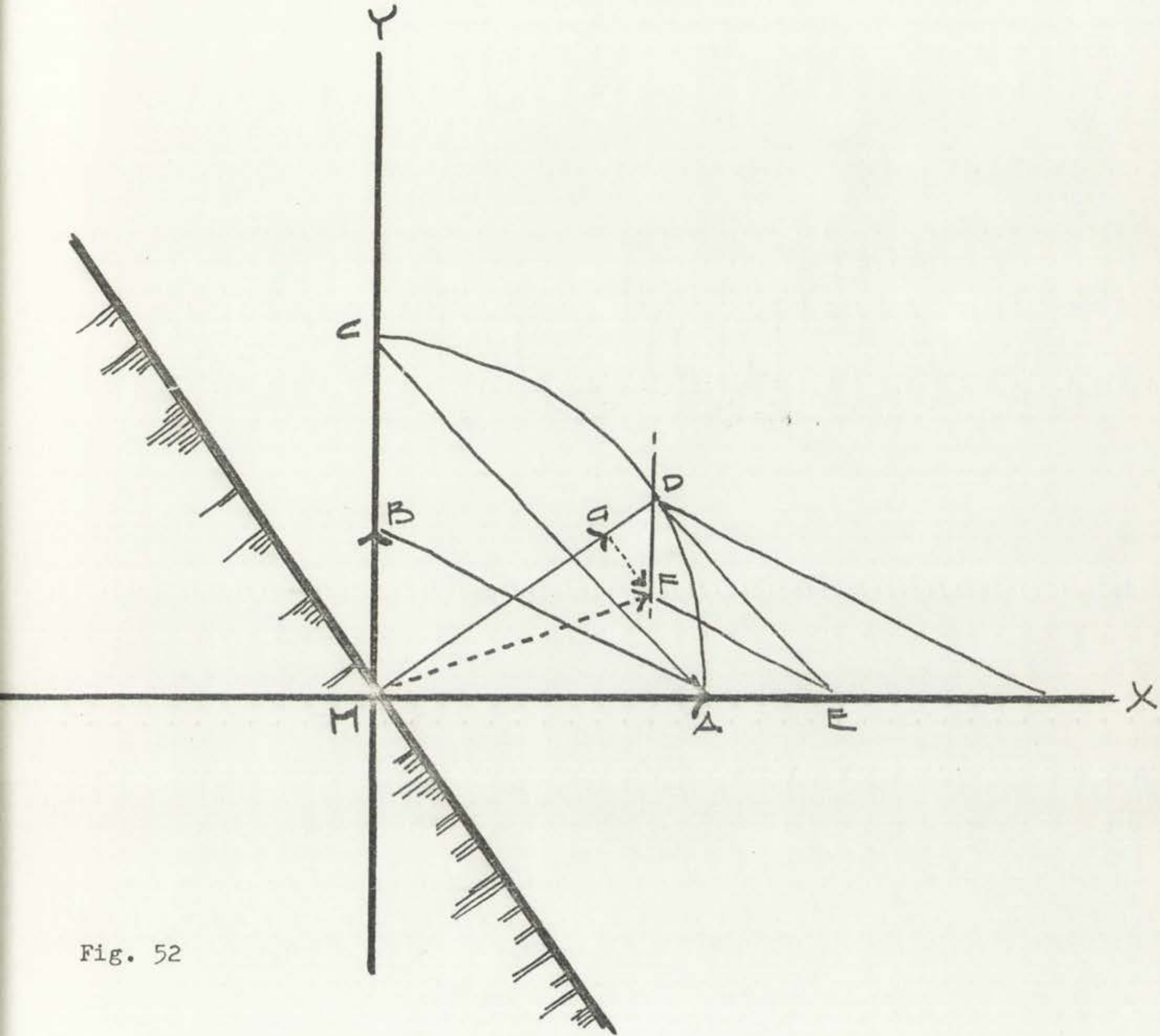


Fig. 52

MG..O..magnitude and direction of the normal surface tension.

GF..T..magnitude and direction of shearing surface tension.

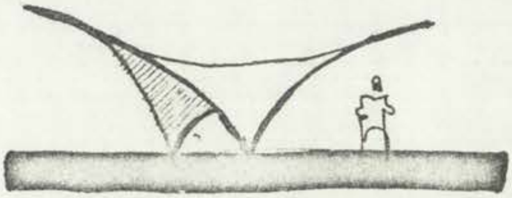
MF..S..magnitude and direction of surface tension resultant.

Types of Prestressed Saddle Surfaces

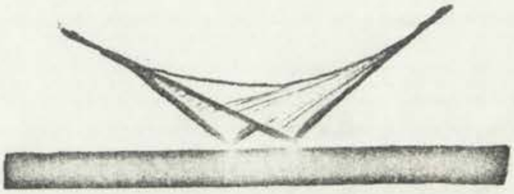
Saddle surfaces can be subdivided into different types of families. These families are characterized as being separate because of their affinity to different structural shapes, support elements and their lending themselves as particular design solutions.

1) Simple saddle surfaces

.Flexible linear supports at the edges of the surface.

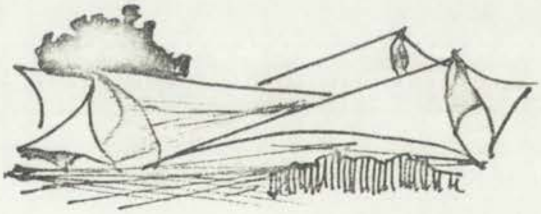


.Rigid linear supports at the edges.



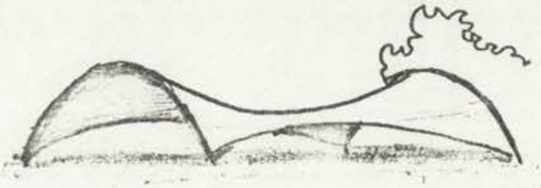
2) Undulating surfaces. These types of structures can also be regarded as addtions of simple saddle surfaces.

.Flexible linear supports at the principal points of suspension.



3) Saddle surfaces between arches.

.Linear supports at the edges.



.Linear support on the surface.

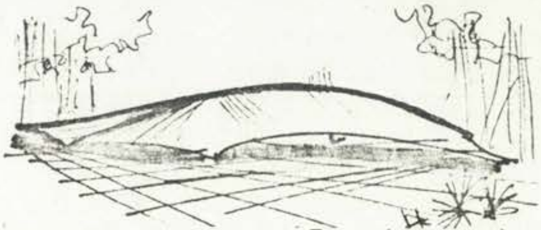


Fig 53. Types of Prestressed Saddle Surfaces.

4) Humped surfaces. These types of surfaces can either be point shaped or area supported as in the case of humps.

- Surfaces with high and low points.
- Surfaces with high points.
- Surfaces with low points.

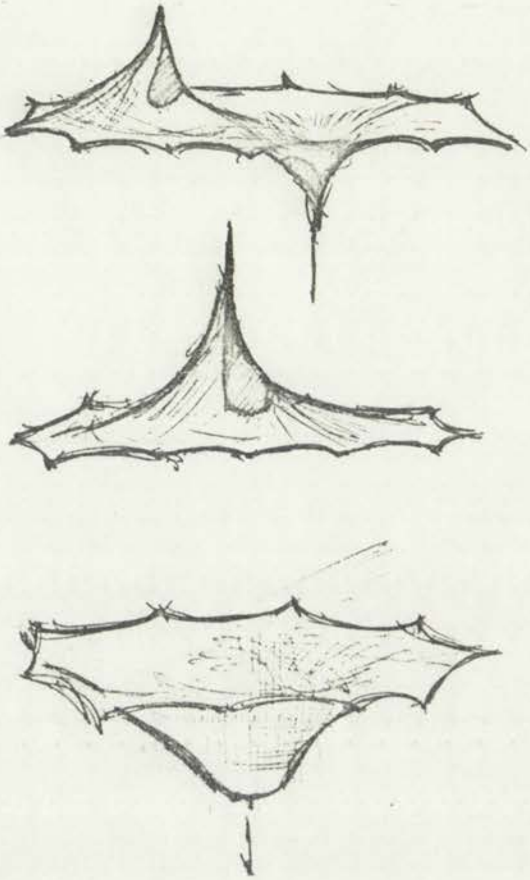


Fig 54. Types of Prestressed Saddle Surfaces.

FOOTNOTES

[1] Frei Otto, Tensile Structures Vol. II (Boston, Mass.,1969) p.11.

[2] Conrad Roland, Frei Otto: Tensile Structures (New York, 1970) p. 3.

[3] Frei Otto, Tensile Structures Vol. II (Boston, Mass.; 1969) p. 12.

[4] Ibid., p.13.

[5] Ibid., p. 34.

[6] Ibid., p. 36.

[7] Ibid., p. 35.

[8] Ibid., pp. 36-37.

[9] Ibid., pp. 77-78.

[10] Dulciani and Burman, Modern Algebra and Trigonometry (Boston, Mass., 1965) pp. 303-306.

[11] Frei Otto, Tensile Structures Vol. II (Boston. Mass., 1969) p.49.

[12] Ibid., p. 50.

[13] Conrad Roland, Frei Otto: Tensile Structures (New York, 1970) P. 5.

[14] Ibid., pp. 13-14.

[15] Frei Otto, Tensile Structures Vol.II (Boston, Mass., 1969) pp. 44-49.

Methods of Analysis

In the process of this work, the reader has so far acquired a basic knowledge of the principles involved in tensile structures and their roots; in order to test this knowledge, an analysis of specific structural types of saddle surfaces should be a part of the learning process. This analysis should take into account all that has been presented so far, and should also take into account projects already built that fall into the same category.

In order to permit the reader to acquaint himself with a precise method of analysis involving a particular family structural type, the author has undertaken the task of analyzing individual models of structural types or members of a particular family.

Method of Analysis

In order to proceed into an analysis of a tensile structure or family type which will provide positive results; one that will be complete, accurate and not very time consuming; the reader will have to take into consideration several requisites in the testing or analysis which will further the scope of the analysis and which will give understandable results.

First, a model of the structural type is necessary, a model which will approximate the shape of a minimal surface and at the same time provide some degree of rigidity when subjected to additional concentrated loading.

Second, a method by which the reader can test and record the shape and structural condition of a particular tension surface, one

which will keep a record of any change due to loading or additional stress in the surface of the model.

Third, a method by which the reader will find the directions of maximum and minimum curvature on the surface of the model, in order to provide a clear way in which to establish the two main directions of spanning a saddle surface with a tetragonal cable mesh. The tetragonal mesh that follows the two directions of minimum curvature, as would be the case in hyperbolic paraboloids, are relatively straight. This condition renders the surface flexible when subjected to any degree of superimposed loading. The other, following the two directions of maximum curvature, provide the structure with the advantage of being much more rigid when subjected to different types of stress as in the case of concentrated loading, wind loading, wind suction, etc.

To fulfill these requirements, the author constructed a machine on which several analysis functions could be carried out on a particular model. This machine consists of two surfaces, one on top of the other, at a distance of eighteen inches, with a movable attachment on the top surface, and fulfills the following functions. (Fig. 55).

The model is mounted on the bottom surface of the machine. The surface consists of a rigid plane having a grid formed by openings at every inch. The purpose of this plane is to facilitate loading on the particular tent model by attaching a series of weights on to the principal points of support by means of cable attachments of waxed string passing through openings in the gridded plan. (Fig. 55 & 56).

The second function that this machine performs centers on the top surface of the machine. This plane is located directly above the lower grid for the purpose of drawing the shape of the tent, and particular sections through the tent.

These particular funtions are accomplished by using an additional movable attachment on the top surface. This attachment consists of a plumb bob on the bottom of the surface which can be regulated to any height. (Fig. 56 a,b); an orifice is located on the top surface directly on top of the bob, through which the point of a pencil can mark the exact spot the bob is positioned and the height at which it is located. These actions permit the reader to produce a schematic record of the analysis. (Fig. 56 f)

Material

Several types of cloth were analyzed in the process of picking out one which would facilitate construction and measurement; one that could be molded and stretched evenly in any or all directions. The material finally chosen if of a particular fabric variety, consisting of elastic threads which when tested can be stretched greatly and evenly in all directions, and when unloaded assumes its original conditions. (Fig. 57)

Fig. 57 Sample Fabric of Models.

Procedure

To create a prototype of a tension surface whose basic design, or stress distribution could be manipulated in order to create a state of stress that would bear aproximation of minimal surface, circles of a diameter of one inch were drawn on the membrane when in the unloaded state. These circles were located in areas where the author considered stress to be an important consideration. When stressed, the magnitude of the loading at the principal points of support were manipulated in such a way as to approximate, by observations as to the shape of the circles, a minimal surface condition. True cases were recorded in this analysis where the circles, instead of increasing in diameter, become elypses and have greater forces acting at distinct axes; this deformation, depending on the shape of the model, permits the reader to get a visual representation as to the direction of the principal stress, and, by comparing the deformation of one individual circle to that of another in another area, a comparative magnitude of the forces acting on the tent (Fig. 55).

As was explained, this information is mainly acquired by having a visual contact with the model. In this analysis a representation of the magnitude and approximate direction of stress was made possible by representing the magnitude or size of circles by drawing sections through the tent where circles are present, and by representing a plan of the structure where the several points of the circumference of the circles were drawn. (Fig. 65 & 69). A graphic representation as to the relative stresses occurring

all through the tent were made possible by means of graphing the contour by following regular vertical intervals using the machine.

The author obtained a series of plans very similar to topographical layouts, and observed that by using the rule:

Inclination= Surface Tension Concentration (Referr pp. 32,33) major stresses occur when the contour lines are close together, i.e. when the horizontal displacement diminishes while following a regularly distanced vertical pattern. This condition is very marked in areas where there is supposed to be greater concentration of stresses. (Fig. 67).

The directions of maximum and minimum curvature at different point intersections were obtained by following a very simple procedure. Points at which these directions of curvature were obtained were not chosen at random, but were located at points where previously drawn circles were located, i.e. where directions and magnitudes of stresses were investigated by the use of circles. The points of intersection were located in places where the author considered them to be related to maximum stress directions.

By definition, directions of maximum curvature are directions at any point on the tent model which have the greatest amount of curvature and therefore the smaller radii. By this same analogy, the directions of minimum curvature are those which have lesser radii of curvature and also approximate straight lines. In the case of minimum surfaces, such directions indeed form straight lines and intersect the directions of maximum curvature at half angles. (Fig. 42).

In cases where models approximate minimum surface conditions, i.e. where stresses at all points are of the same magnitude; directions of maximum and minimum curvature can be easily found by following this procedure. Using a straight edge of minimum length and rotating it around a central axis until there is no difference between the curvature of the tent on the straight edge, the reader will find that two directions of minimum curvature will be encountered.

The directions of maximum curvature on the other hand can be found by intersecting the minimum curvature lines at half angles. The central axis or as in this case, the center of the circle, is the point of intersection of both the maximum and minimum curvature lines. (Fig 68, 69).

Additional testing of the model surface was accomplished by means of direct concentrated loading at different points on the membrane. These points were picked for the purpose of comparing the effect of loading in areas where greater slopes were apparent and also where the author considered the location of major stress concentrations.

This type of information is gained by analysis of the different sections under concentrated load. Again these points were not chosen at random but rather were situated at or near places where a greater of lesser stress was apparent due to the inclination of the material with respect to supports or to the displacement of the circles.

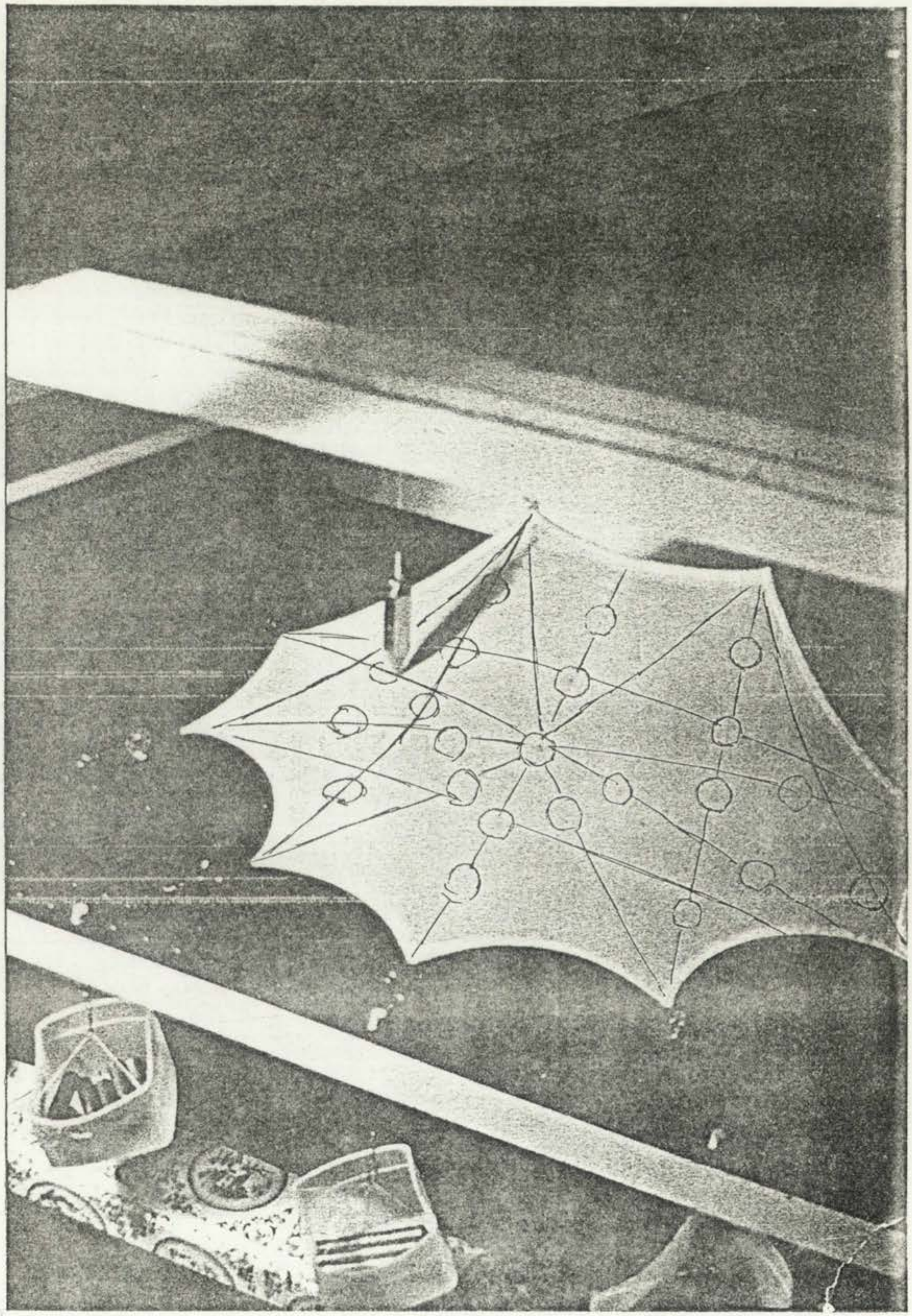


Fig. 55 | Model testing apparatus

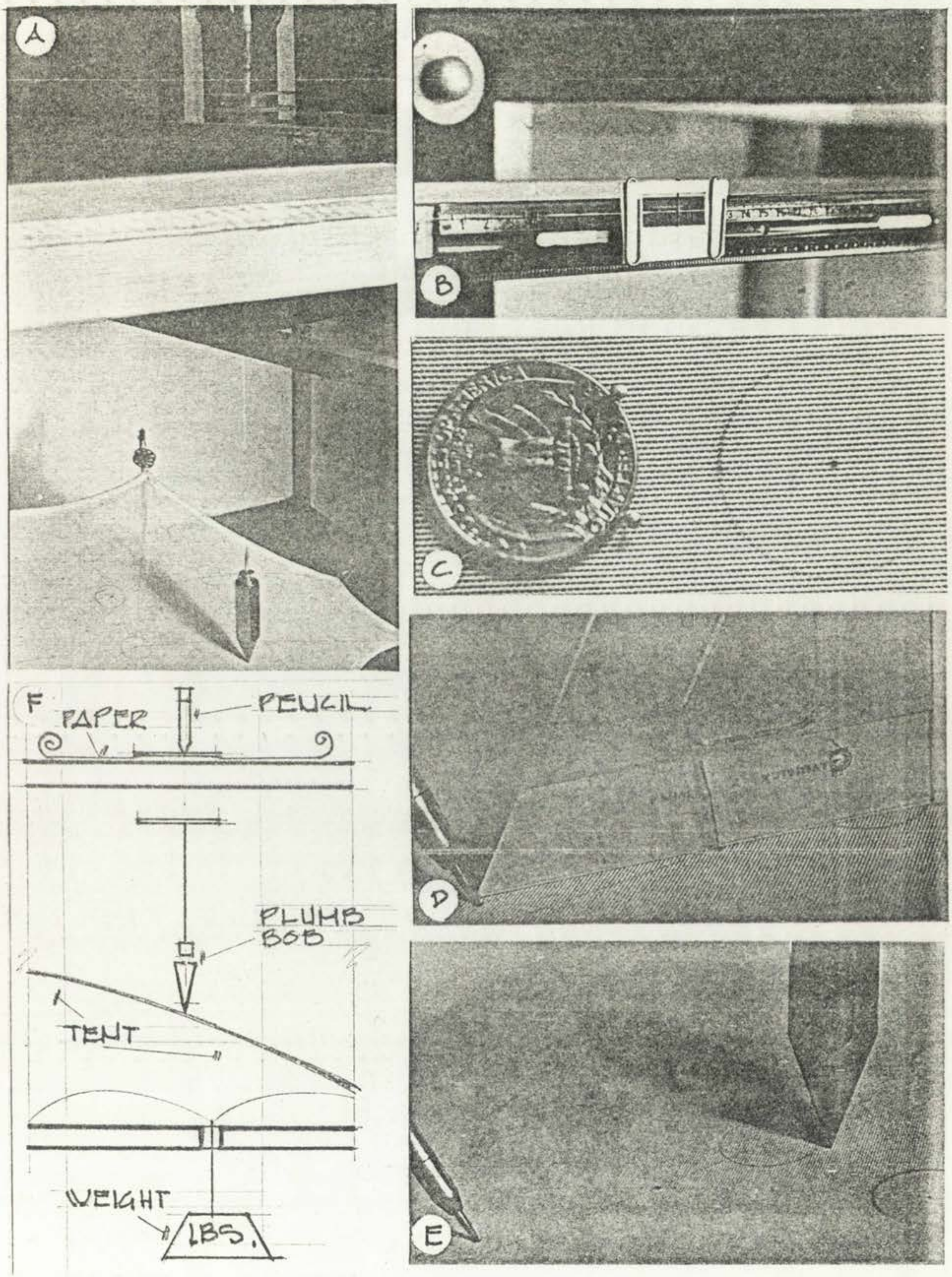


Fig. 56 Testing apparatus, details

Simple Saddle Surfaces

Simple Saddle Surfaces

The field of saddle surfaces is subdivided into very many types, the simplest of which consists of a membrane surface suspended between four interconnected flexible edge members whose points of suspension do not lie in the same plane. This condition gives the shape of the structure the appearance of a hyperbolic paraboloid and gives the surface two main directions of curvature between the four supporting edges.

As a design, this principal can be regarded as a point of reference from more complicated forms of tensile surfaces and their design. Its feasibility as a structural concept has one serious drawback. Due to the tremendous heights involved in the making of large spanning projects, simple saddle surfaces are not meant to be used in projects of large spans.

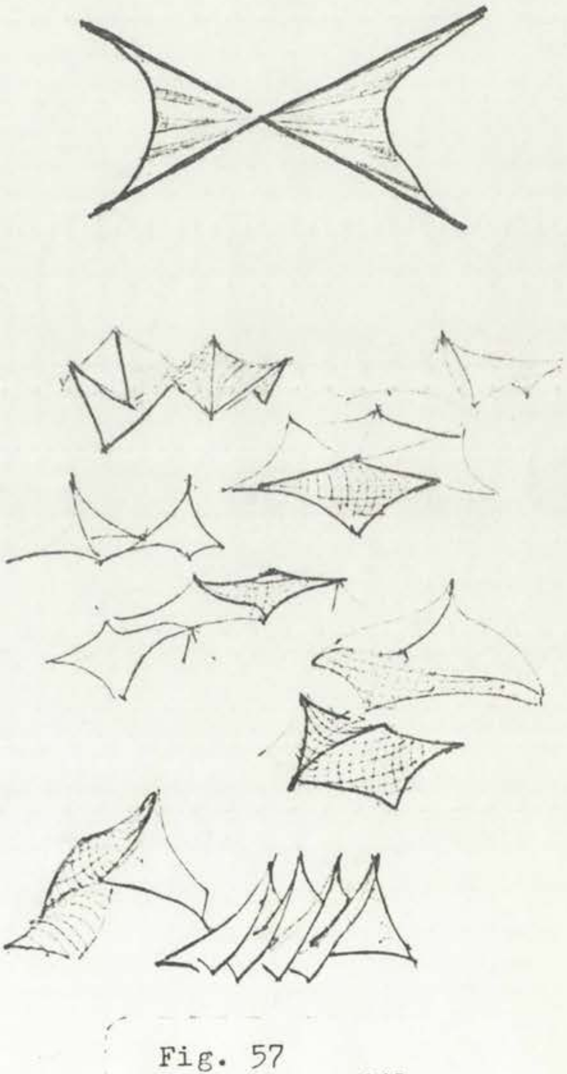


Fig. 57
Classical saddle surface and additions

The simple saddle surface's most noted feature is its adaptability for addition. Innumerable tensile structure forms stemming from simple saddle sur-

faces in their additive state come to mind as the subject presents itself. Indeed all saddle surfaces in some way or another can be analyzed for purposes of design interpretation as additions or variations of simple saddle surfaces.[1]

By this same token, variants of simple saddle surfaces do not have to have the regimented four supporting sides and the two main directions of curvature that follow generally the same directional pattern, these can be analyzed as a variations of the matrix form and therefore belong to the simple saddle surface family.

In context, simple saddle surfaces should be subdivided not according to shape, instead the subdivision should depend on the type of supports the structure utilyzes,the manner in which they are arranged and the materials utilized.

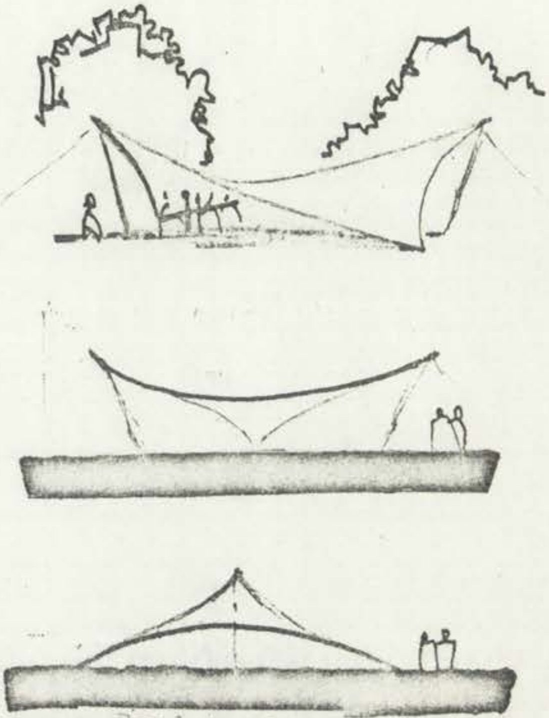
As was mentioned earlier in this work,the types of edge supports most utilized in any type of tensile surface consist of flexible members, members rigid in bending, and arches rigid in compression and any combination of the same.

In the process of investigation, two variations of simple saddle surfaces were analyzed; one having flexible edge supports and the other having rigid edge supports.

Both are classified as being simple saddle surfaces. The first, is a simple saddle surface with flexible edge supports. The second is a simple saddle surface with rigid edge supports.

Previously Constructed Simple Saddle Surfaces

Bandstand. Cassel, Germany. 1955.



Bandstand

Federal Garden Exhibition

Architect- Frei Otto

Description : Classical saddle shaped membrane with two high and two low points. It is a minimal surface having equal stresses and equal curvature in both principal directions of curvature.

Diagonal Length: 59 feet.

Lateral Length: 41 feet.

Height (maximum): 17 feet.

Material consisted of translucent cotton canvas strips of 0.04 inches of thickness and approximately 25 feet in length, bordered by a sewn in 16 mm. supporting cable. The structure is tensioned to support a suction wind load of approximately 16.32 lbs./ sq. ft.

It has outstanding acoustic properties proving that stretched membranes are excellent for their sound reflection. [2]

Shelter Pavillion. Cologne, Germany. 1957.

Federal Garden Exhibiton

Architect- Frei Otto

Description: This tent is a saddle like membrane with one support point and six anchor points.

Two intersecting edge cables stretch the tent taut and transversely to the directions of the seams and serve to secure the pole. The edge cables, membrane and pole merge almost tangentially at the support. Support at this point was made by means of overlappings of the material. These overlappings at the seams render the tensions all through the tent as being relatively constant. The tangentiality of the material to the support was achieved by making fish belly cuts or concave cuts in the strips. This spatially curved condition renders the structure to be most adaptable to wind pressure, wind suction, and any alteration of the same.

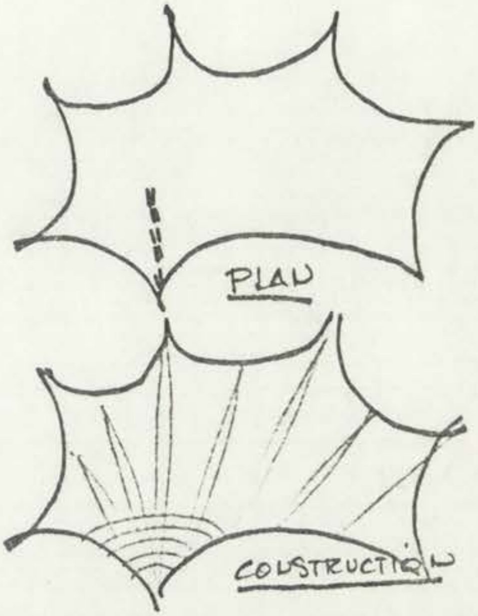


Fig. 59
Shelter Pavillion

Open Air Theater.

Wunsiedel, Germany. 1962-1970.

Architect- Frei Otto.

Description: Complex form of cable-bordered simple saddle surface; evolved to meet the requirements of the particular weather situation and of the acoustic requirements.

It is suspended from five suspension points, eight anchor points,

and six lateral stabilization points.

It is designed to use a translucent synthetic fabric, edge and suspension cables, and two lattice steel masts 69 feet in height. Also it is designed to cover an area of approximately 12000 square yards.[3]

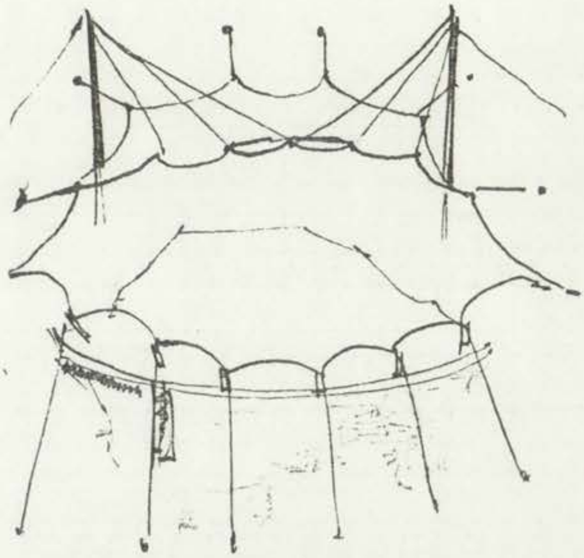


Fig. 60

Open Air Theater

Roof for Olympic Stadium.

Munich, Germany, 1974.

Architect- Frei Otto

Description: Suspension roof comprised of a series of nine variations of simple saddle surfaces; each saddle surface has two suspension points, two understayed support points, four anchor points and one continuous anchor edge for all units.

Maximum span, 213 feet.

Lenght of edge cable, 1,443 feet.

Maximum height, 190 feet.

Roof area, 366000 square feet.

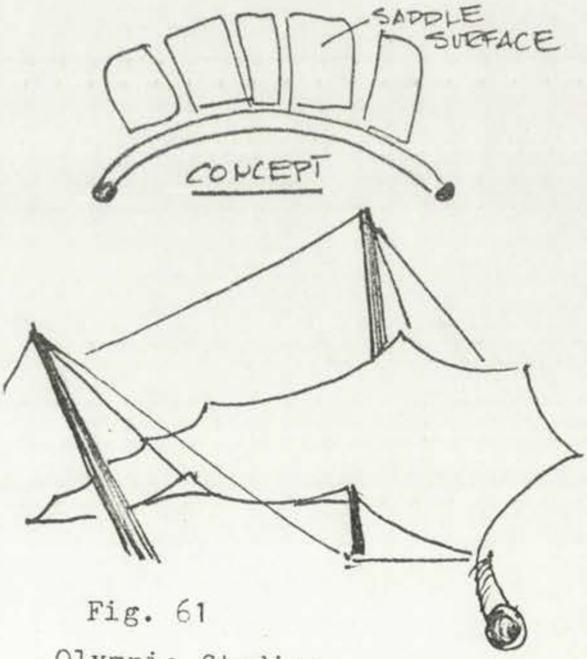


Fig. 61

Olympic Stadium

Materials: Made of net wire rope following the main directions of curvature, translucent acrylic sheeting, and tubular steel mats.

Model Analysis

Simple Saddle Surface with flexible linear supports at edges.

Description:

This model consisted of a saddle surface, having two high points seven anchor points.

Loading:

It was observed by the magnitude of the loading that the greater amount of load was concentrated at the points and next to the high points.

This analysis provides the reader with the observation that as the greater amount of the model's tensioning weight is concentrated at or next to the two principal supports, the cable interconnecting these four points provide some sort of lateral stabilization as well as provide the reader with the following deduction. The conclusion in this case is that the greatest concentration of stress is at the high points (due to the presence of principal supports) and at the anchors next to these supports.

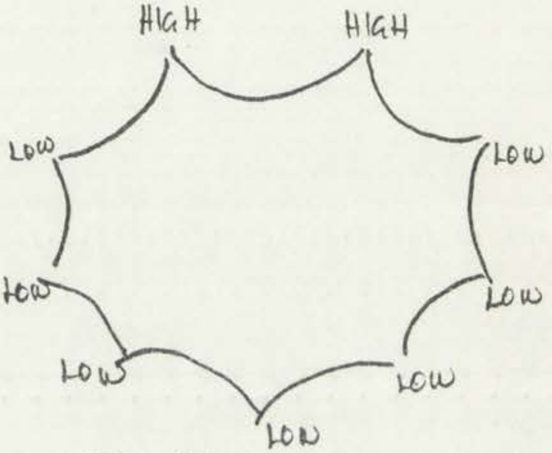


Fig. 62
High and low points

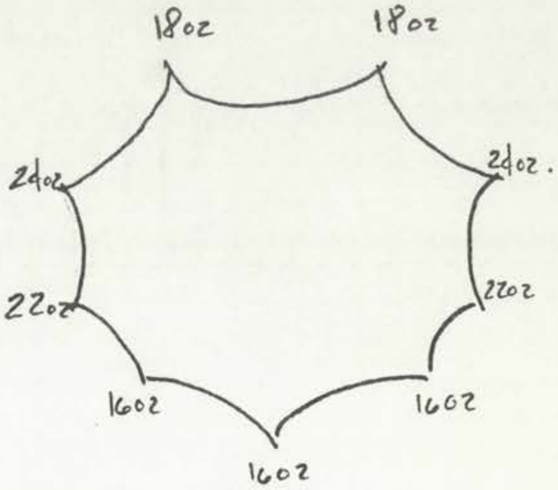


Fig. 63
Loading

Contour Diagram:

The contour diagram provides the reader with this observation. The contour lines become closer together toward the high points and become separated toward the bottom. This condition creates a higher slope as the lines become closer together. (Fig. 67)
From this observation and the conclusions arrived at in the loading stage, the reader will find that both conclusions in this case are the same: Greater stress occurs toward the high points.

Sections and Diagram:

The surface diagram as such gives the reader a basic knowledge as to the approximate directions of circular displacement or the directions of force, and, when correlated with the unloaded sections passing through the points in question, the magnitude of tension stress. In this case no significant difference as to size and deformation of circles was present in sections A-A & B-B. (Fig. 66 & 70)
This was due to the author's effort in making the size of the circles on these models uniform by manipulating the magnitude of the loading at the supports.
One observation in section E-E shows that as the section approaches the anchor support at the left of the section, there is a hump or a notable deformation with greater slope on the left hand side. This marked curvature on left hand side shows a discrepancy as to the disparity of the concentration of forces favouring the left hand side.

[illegible]

The [illegible] that the [illegible] the [illegible] (Fig. [illegible]).

[illegible] in the [illegible] stage, [illegible] the [illegible].

[illegible]

The [illegible] is the approximate [illegible] of the [illegible] directions of [illegible] with the [illegible] through the points in question. The [illegible] and [illegible] of [illegible] in sections [illegible] (Fig. 56 & 59)

This was due to the author's effort in making the size of the [illegible] of the loading at the supports.

[illegible] that [illegible] the [illegible] on the left hand side. This [illegible] hand side.

Maximum and Minimum Curvature.

The directions of maximum curvature give the reader these observations from which to further his analysis. It was observed that the spanning directions of maximum curvature approximate a radiation pattern about the high points of the structure, whereas the spanning directions of curvature and the lines of the contour diagram approximately follow the same path. (Fig. 67 & 68)
If the saddle surface was a perfect minimal surface, the carrying direction would in fact be the directions of principal or maximum stress, whereas the spanning directions would be the directions of minimum stress. In this saddle surface, there are several variations of stress magnitudes as well as curvature variations. Therefore it is an approximation of a minimal surface; the lines of maximum curvature are not representative of maximum or minimum stress and are approximations.

Additional Investigation.

Given the condition of equal stress and deformation on the simple saddle surface, additional testing was performed on this model in order to see what type of formula would approximate the lines formed by sections A-A, B-B, and C-C, under the presumption that these curves approximated either a parabolic or a catenary function. Therefore an effort was made to establish a formula that would follow the curve of a parabola or a catenary. After several vain attempts, it was made clear to the author that the stresses on the saddle surface do not function the same way as do the stress or loads present on a parabola or a catenary. A saddle surface

section has forces acting at different angles depending mostly on the shape of the structure and on the differing stresses acting in three dimensions throughout the section. This differs from the idea behind the catenary or parabola which in both cases have loads acting directly parallel to each other, all in a two dimensional plane of reference.

Conclusions.

It is the contention of the author, after having reviewed the information acquired during this analysis that the following suggestions in the construction design stage should be observed:

1) Due to the great stress conditions existing at the high points of the tent, additional layers of fabric should be placed on the membrane as the two principal points of support are approached.
2) The placing of fish belly cuts in the same direction as the carrying directions of maximum curvature at the left and right hand sides as one approaches the high points.
   This action would minimize the stress at these points and would also permit the equal distribution of stress throughout the surface.

Fig. 64

Stress concentrations

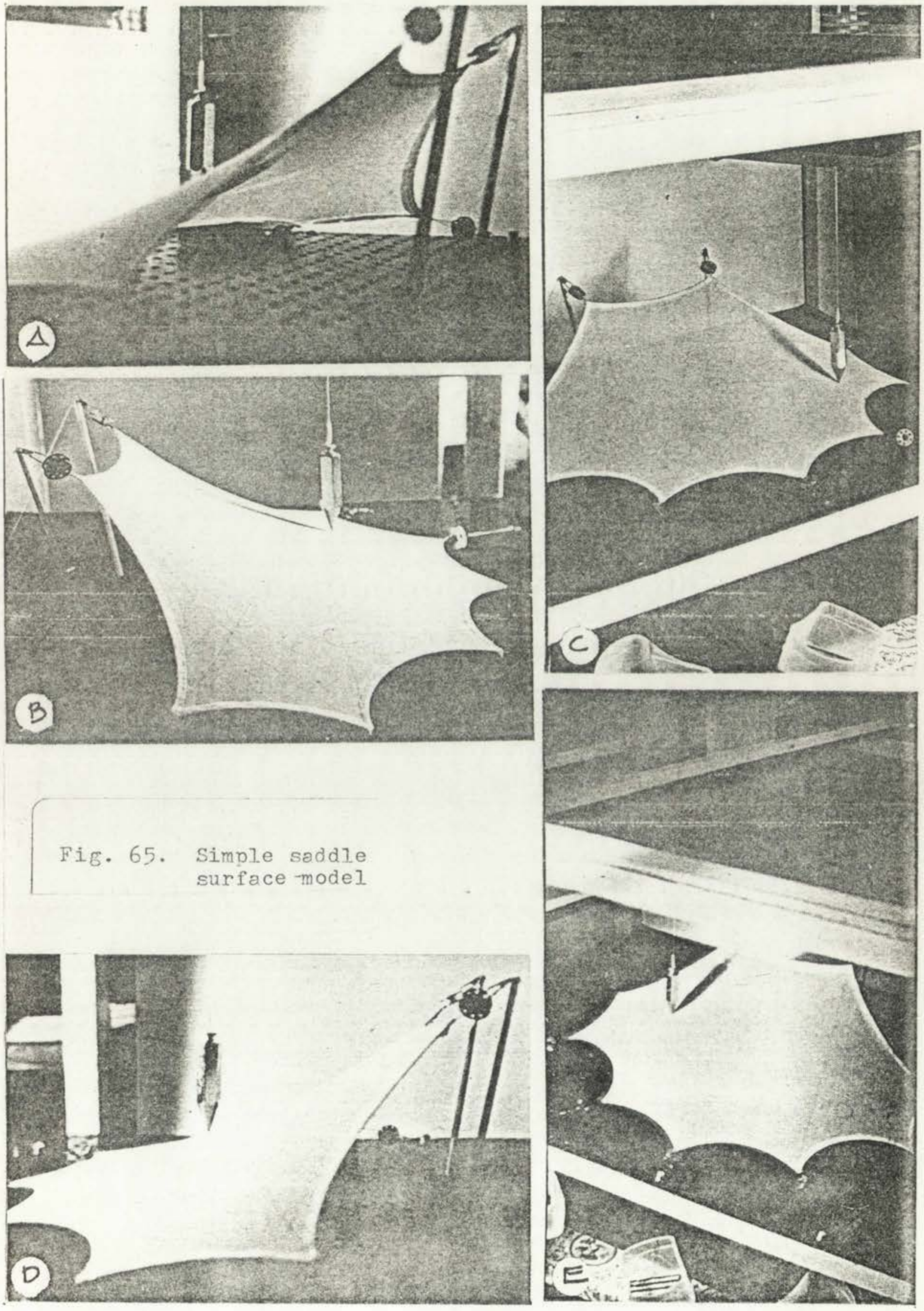


Fig. 65. Simple saddle surface model

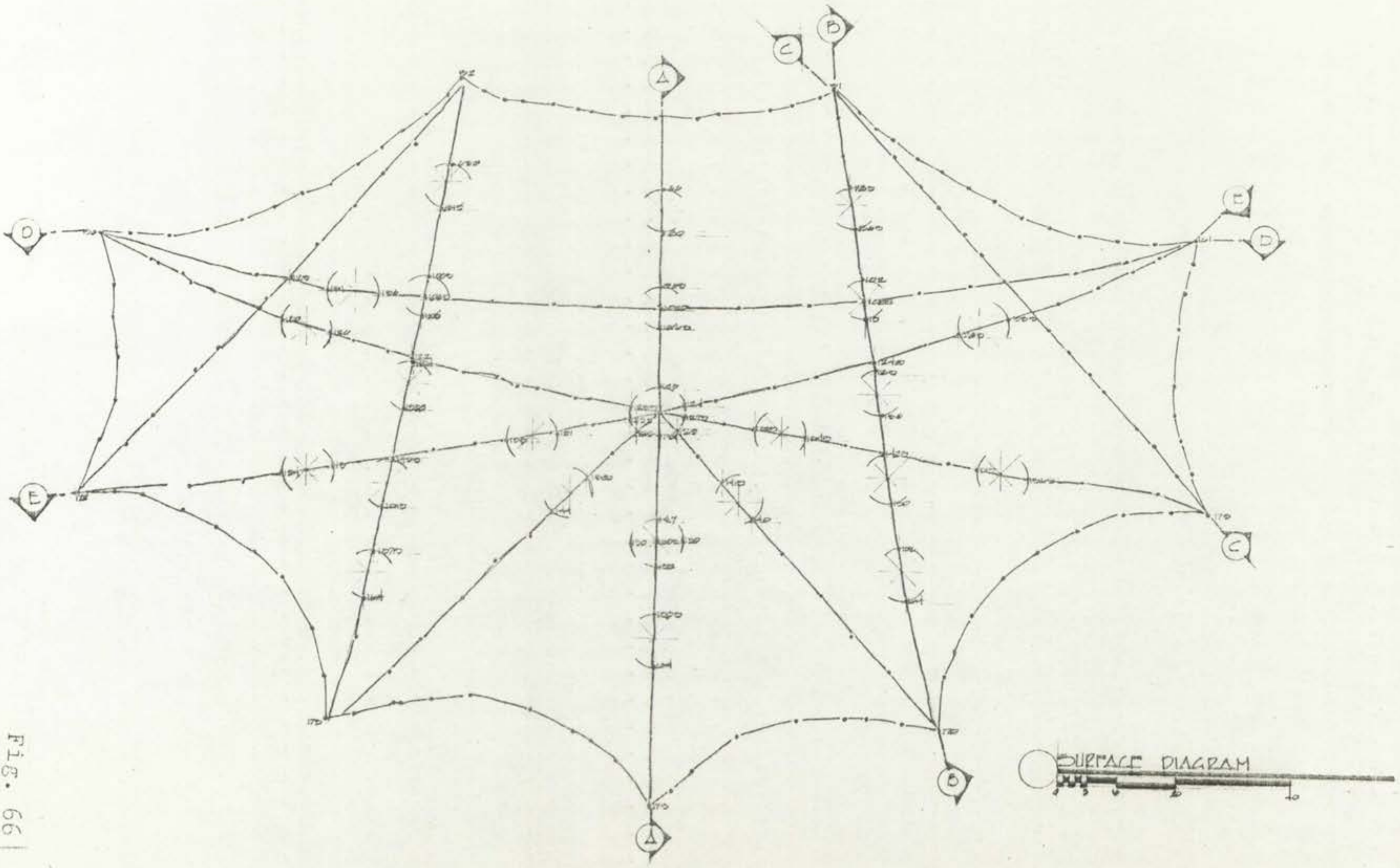


Fig. 66

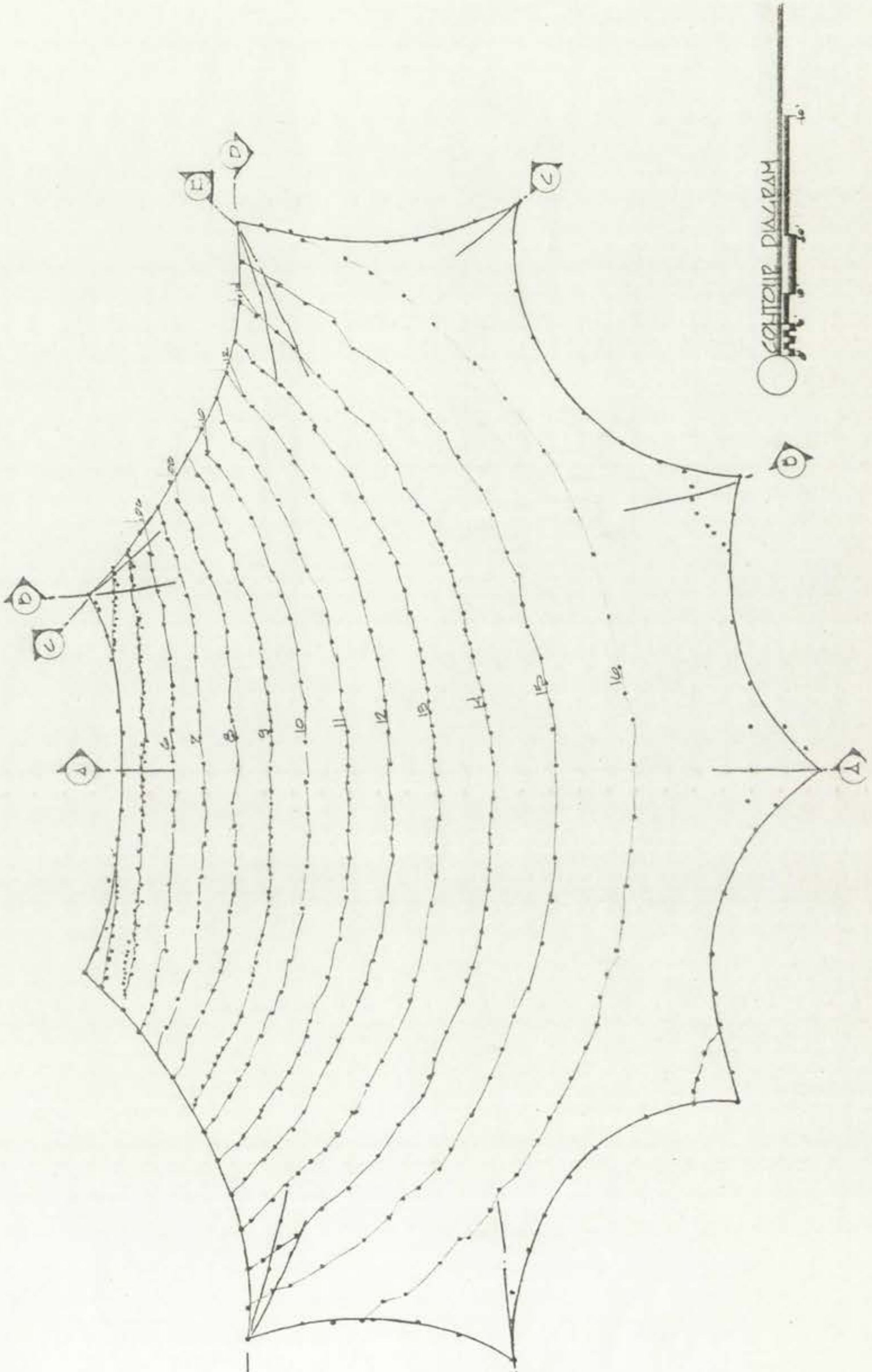


Fig. 67

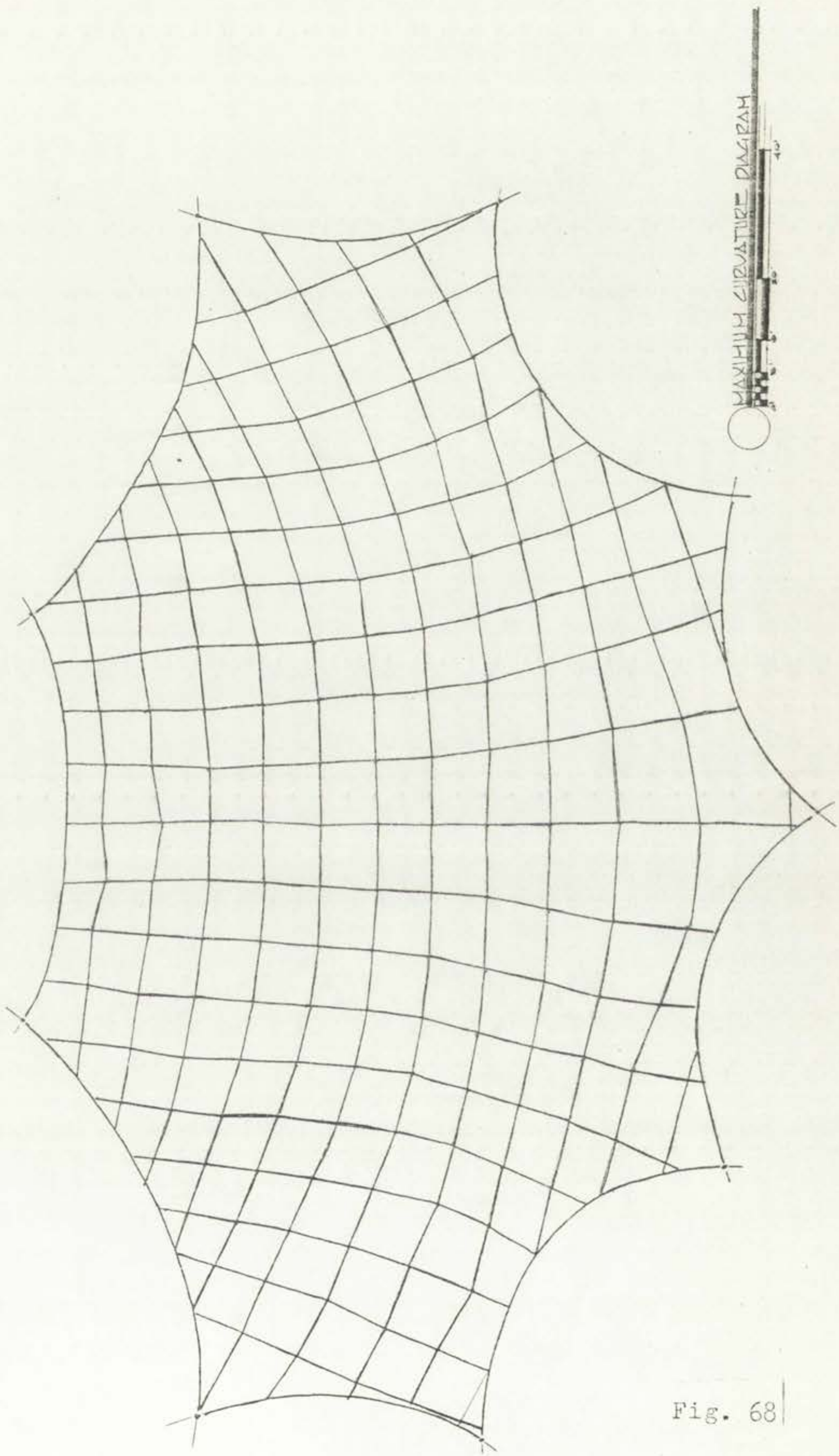


Fig. 68

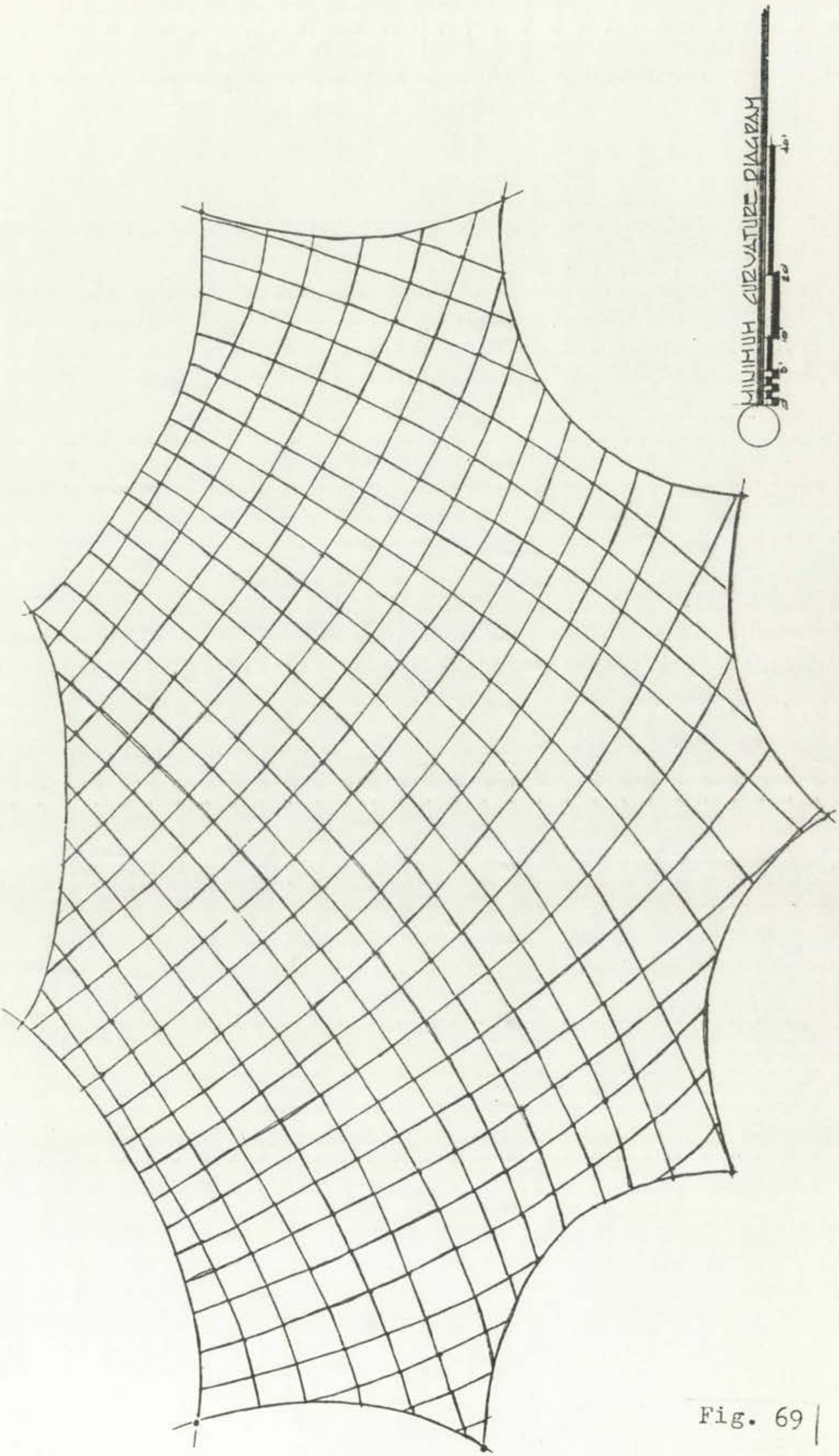


Fig. 69

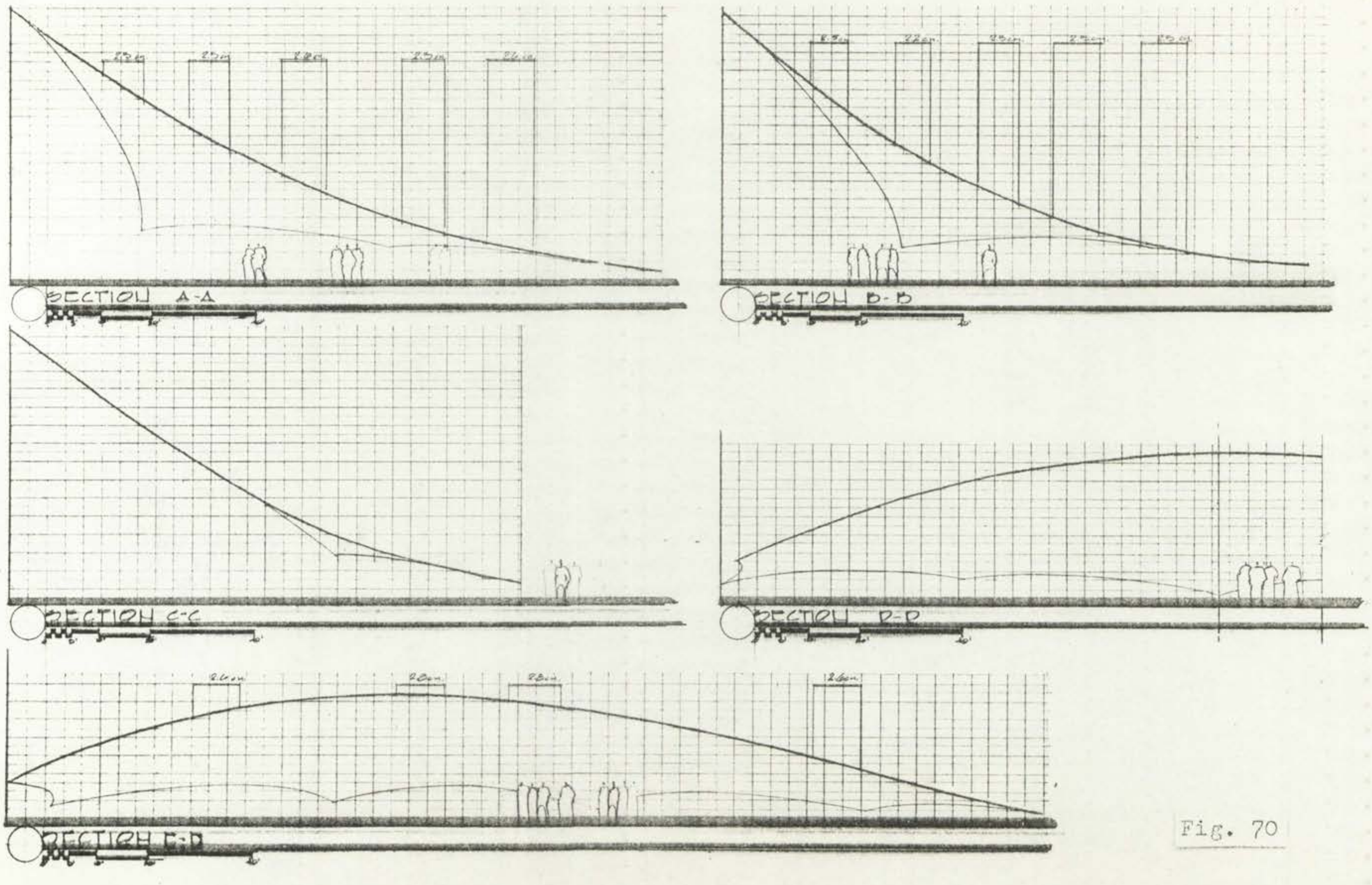

SECTION A-A
23m
25m
28m
25m
27m
SECTION B-B
23m
22m
25m
27m
25m
SECTION C-C
SECTION D-D
SECTION E-E
26m
28m
28m
26m

Fig. 70

Simple Saddle Surfaces with Rigid Supports at the Edges.

Description:

This saddle surface is a variation or a complex form of a simple saddle surface in that it is an addition of three simple saddle surfaces, three hyperbolic paraboloid shapes side by side revolving around a center.

The model in the unloaded state was a six sided figure having six alternating angles of 96° and 144°, the wider angle converging at a point to be supported by a compression member, the angle of 96° would be intersecting at a low point.

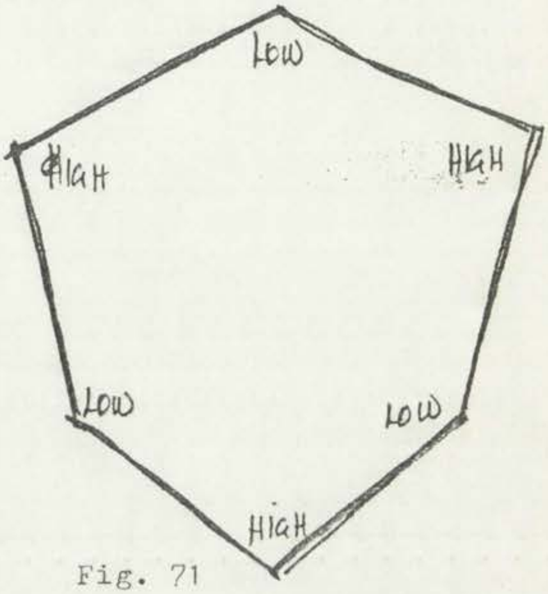


Fig. 71
High and low points

The sides of the tent are equal and are, in the analysis stage, semi-rigid. This semi-rigidness was achieved by overlapping additional cloth in thickness at the latteral supports. This condition permits the reader to have an idea of where deformation occurs at these supports. This action is due to the fact that when the actual structure having rigid nonflexible su-

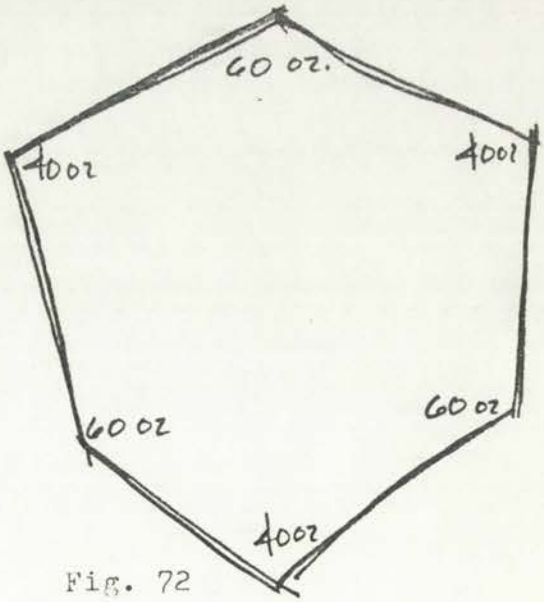


Fig. 72
Loading

pports is loaded , the amount of deflection occuring at the design stage should be considered for later use in the construction design stage. (Fig. 75 & 76).

Loading.

The loading of the structure shows the reader that the maximum concentration of loads occurs at the low point supports; 1/3 greater than the load at the high points, yet the angle of the compression member at the high points is of 64° from the horizontal. The information shows the reader that there is approximately an equal magnitude of forces at each of the supports of the tent. (Fig. 75e and 72)

Contour Diagram:

Observations: the contour diagram gives the reader another fact in which to base further analysis. These become closer as they get near the low point. This condition is not present toward the high points, showing a greater ratio of slope towards the points of anchorage. Although the conclusion that the author reached in the loading diagram was that of equal loading in high and low points, greater concentration of stresses on the membrane are present at the low points; it is important to underline the fact that the angle at the supports in the unloaded state were much smaller in the points of anchorage than in the high points. (Fig. 78.)

Conclusions: This shows the reader that there is some correlation between equal support load and greater linear stress concentration in places where there is less area to be considered; in this case greater stress concentration due to the different magnitudes of

the angles.

Also following the lines from the high points toward the low, one finds that there is an area of relatively no slope or a kind of prolonged point of inflection. This could be considered the area of the tension structure which has the least amount of surface stress due to its non curvature or inclination. (Fig. 76 d&e-Fig. 75 b).

Surface Diagram.

The surface in question was conceived as a saddle surface supported by rigid lateral supports. As was mentioned earlier, the model was constructed with semi rigid lateral supports. This consideration was essential in that the cutting of the tent was going to depend on the degree of deformation or stress concentration occurring at the points of support.

It was observed that overall, the lines drawn in the unloaded state suffered no lateral deformation; yet the edge of the structure suffered some displacement as the points of support were observed; there is an angularity that is accentuated as the edge approaches the points of support. (Fig. 76 b) .

Conclusion: This observation should be taken into account in relation to reinforcing due to stress concentration and the cutting of the fabric at the support points, when involved in the construction design stage of analysis.

Sections Not Under Load.

Observations: It was observed in Sections B-B, C-C, D-D, E-E and F-F, that a great deal of visual information as to the magni-

tude and directions of stress was present as the sections approach the high points. (Fig. 76 b- Fig.80).

In these sections the reader will observe an increased slope and curvature as the sections approach the points of support. This permits the reader to get an intuitive knowledge as to the degrees of tension by correlating the deformations to the spacing between sections.

Section A-A (Fig 80.) In this section the reader observes that as the section passes through the center of the tent or the area of no slope, the tension does not increase markedly as the section approaches the supports. This is due to the manipulation of the loads at the supports in order to achieve, as close as possible, an equal stress distribution all through the tent. This was done by directly observing the size and deformations of the circles.

Conclusions: Visual as well as quantitative data regarding the stresses present at different sections through the tent lead the author to form this preliminary conclusion: stresses increase markedly in the area where a line is drawn interconnecting the three high points. (Fig. 76 a).

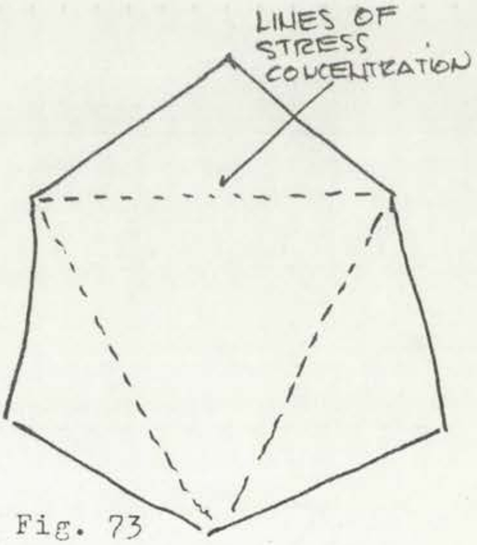


Fig. 73
Stress concentrations

Sections Under Load.

It was observed in the testing of section A-A under a variety of load locations, (Fig. 82), that: greater vertical deflections

occur as the load was applied at increasing distances from the supports. Horizontal deflection toward the supports occurred as the load became closer to the supports. There was a marked increase in curvature in some locations while in others the deformation was constant. (Fig. 74). This curvature was due to the presence of a concentrated transverse tension, which affects both the magnitude of vertical displacement and the displacement of the point of loading towards the supports.

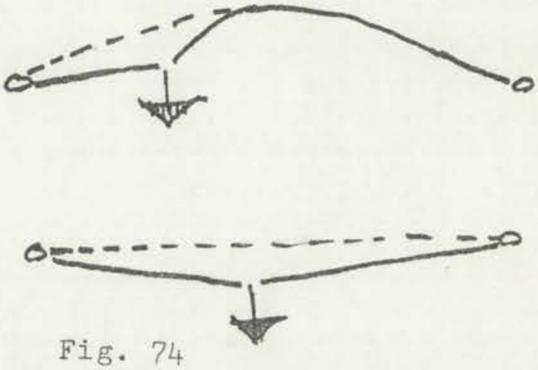


Fig. 74

Sections underload

Conclusions: By the accumulation of facts gathered in the analysis, the author has drawn the following conclusions: 1) Data should be accumulated as to the maximum deformation and the allowable prestressing the actual tent should be subjected to at all points in the tent, with particular attention to the areas at or near the points of support. 2) Consideration should be given to the reinforcement of the material at the principal points of support by overlapping, so as to convorm the stresses at the points to the stresses in the rest of the structure. 3) Additional consideration should be given to the minimizing of tension radiated by these high and low points of support. This can be accomplished by minimizing the angle of the compression or by minimizing the magnitude of the load. This action is due to the excesive concentration of forces along the three high points, the deformation of the lateral

supports at the high and low principal points of and at the endeavour to make this structure, as equal as possible to a minimal surface having equal magnitudes of stresses an all the tent.

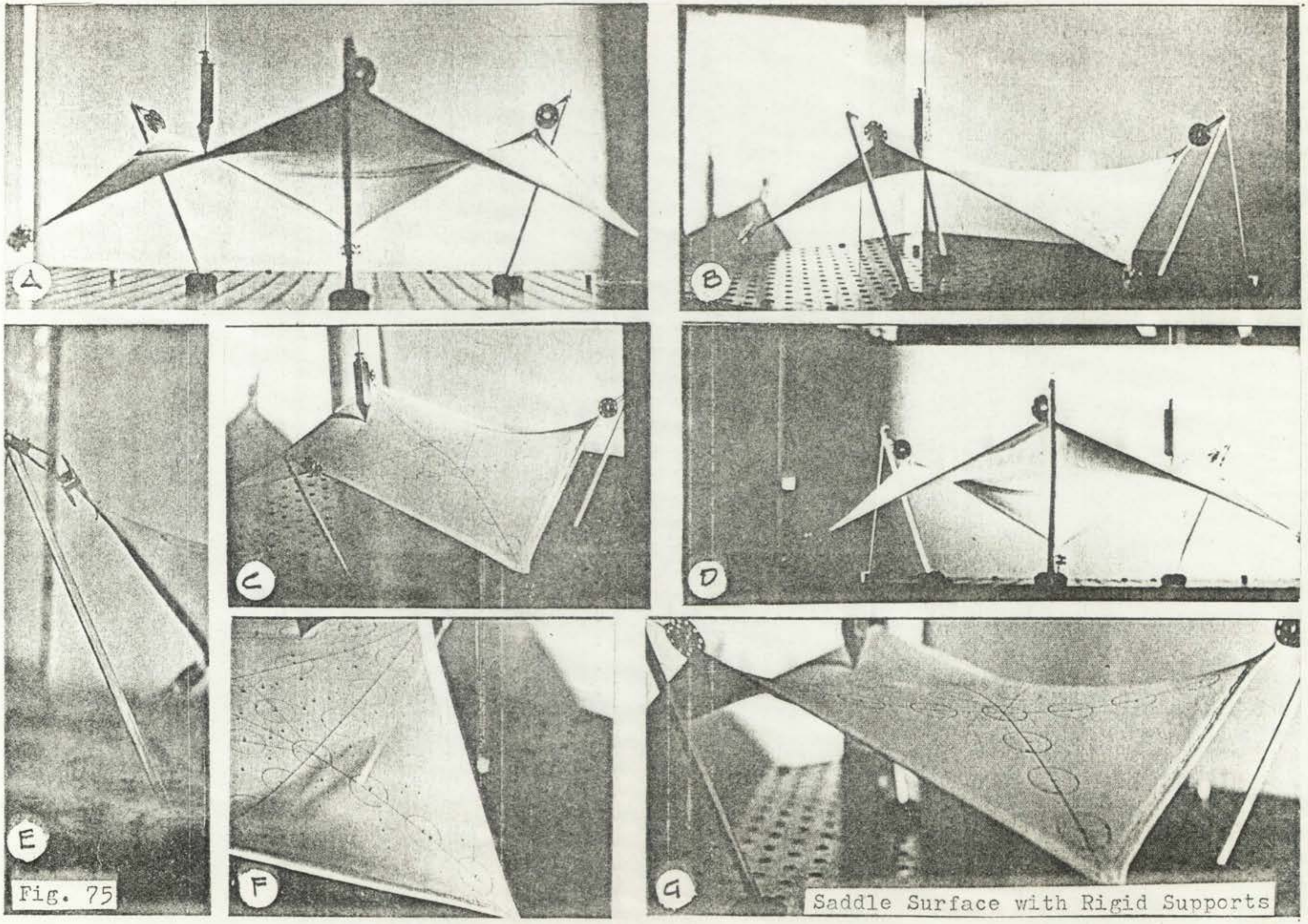


Fig. 75

Saddle Surface with Rigid Supports

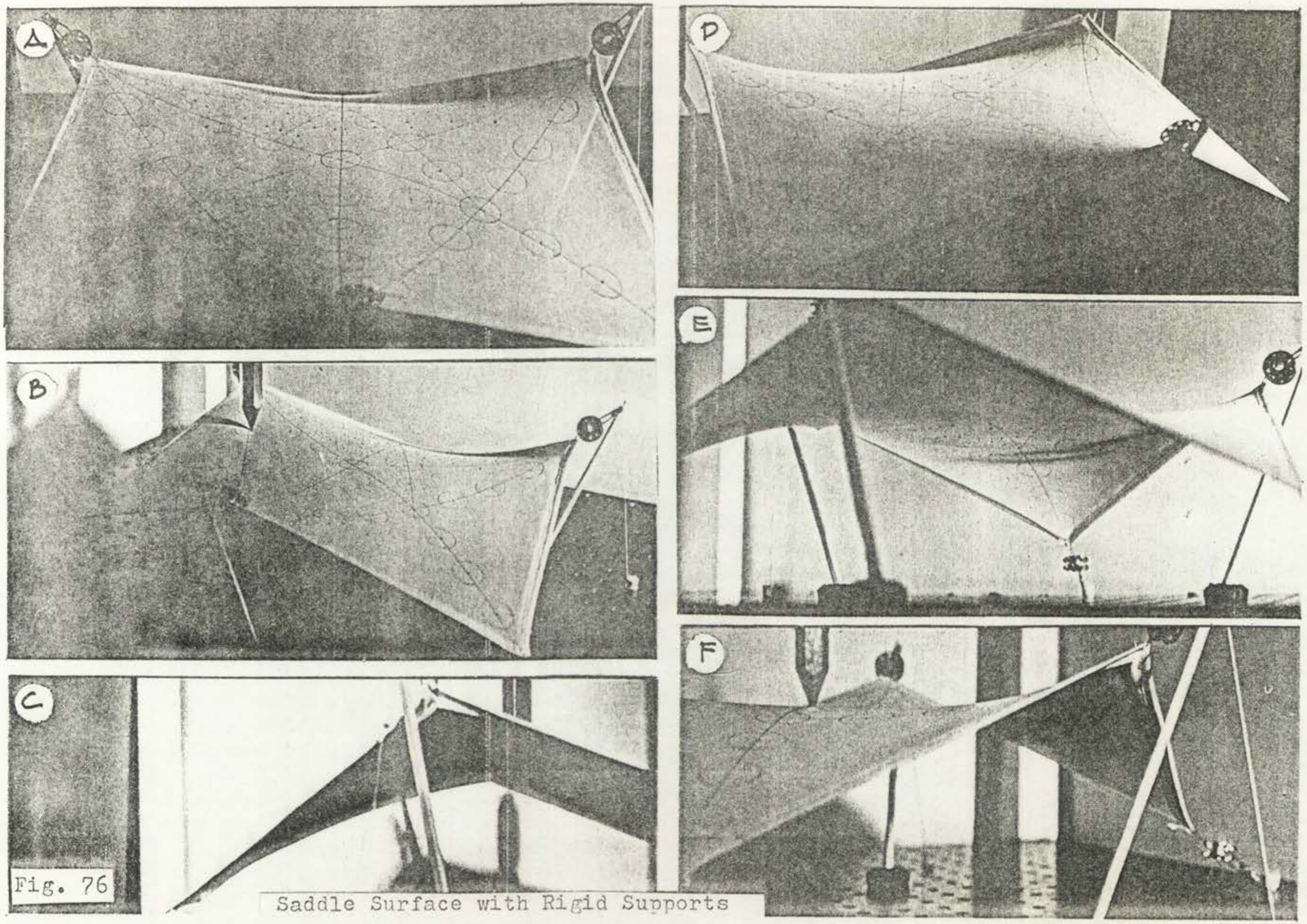


Fig. 76

Saddle Surface with Rigid Supports

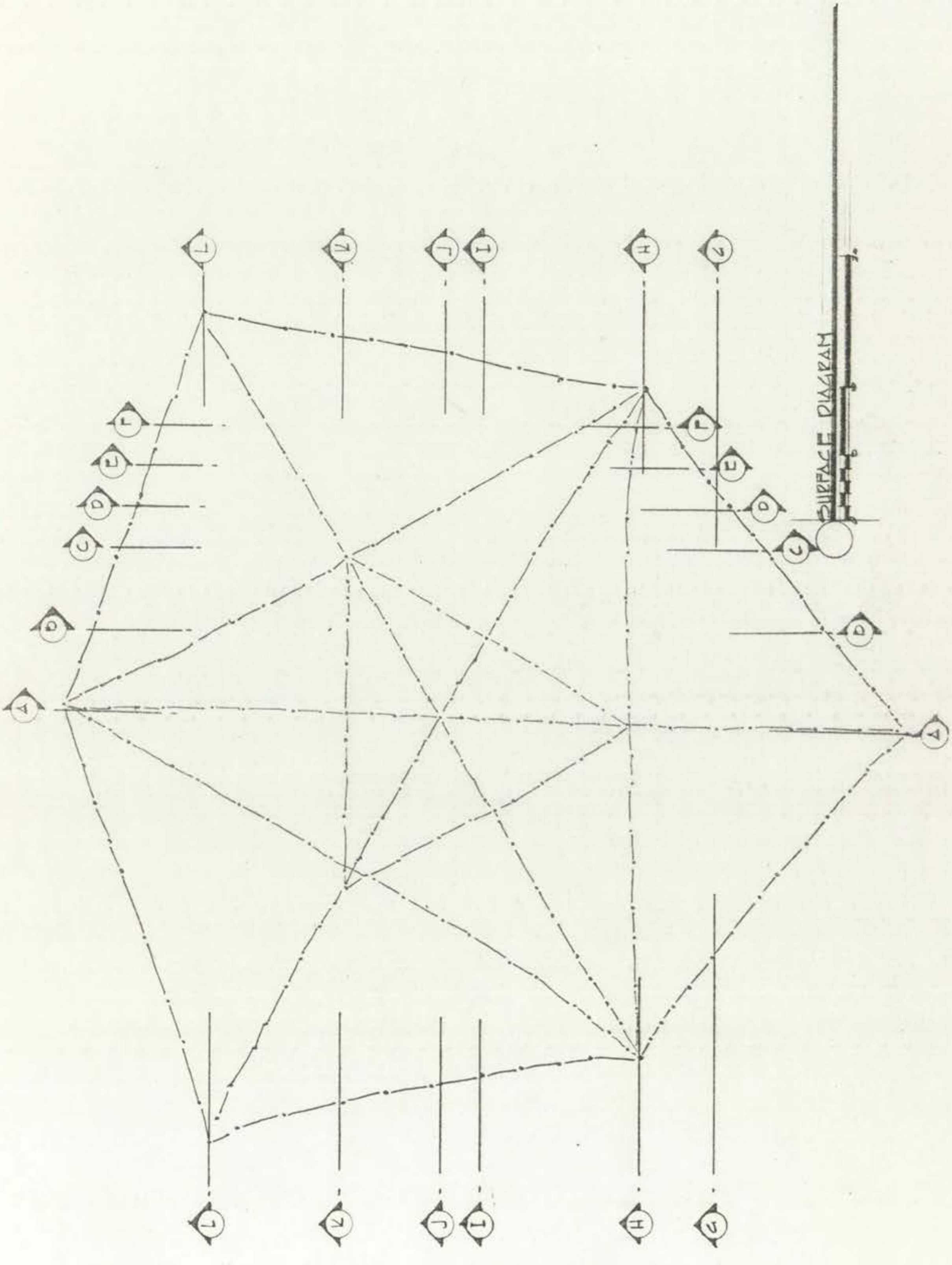


Fig. 77

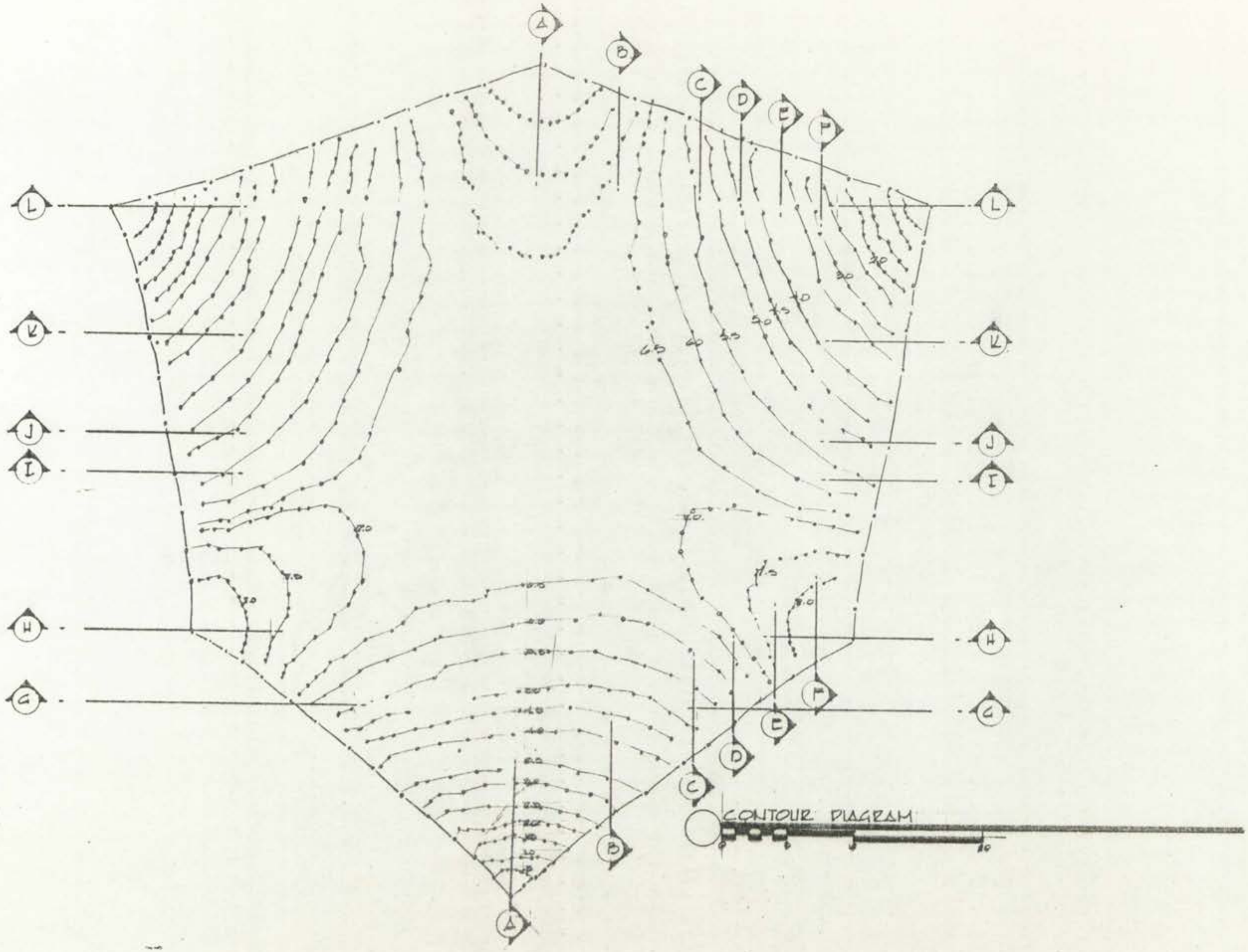


Fig. 78

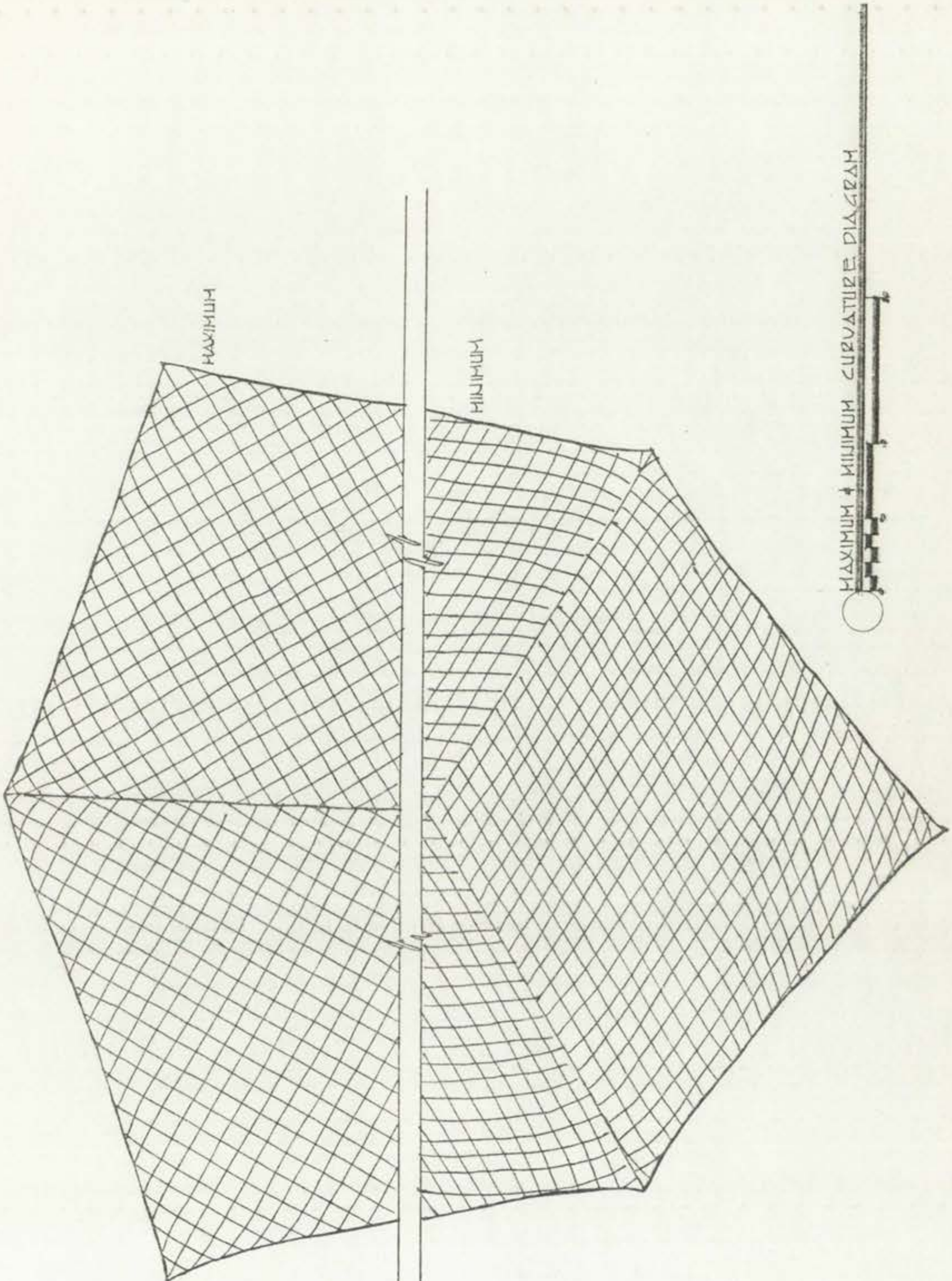
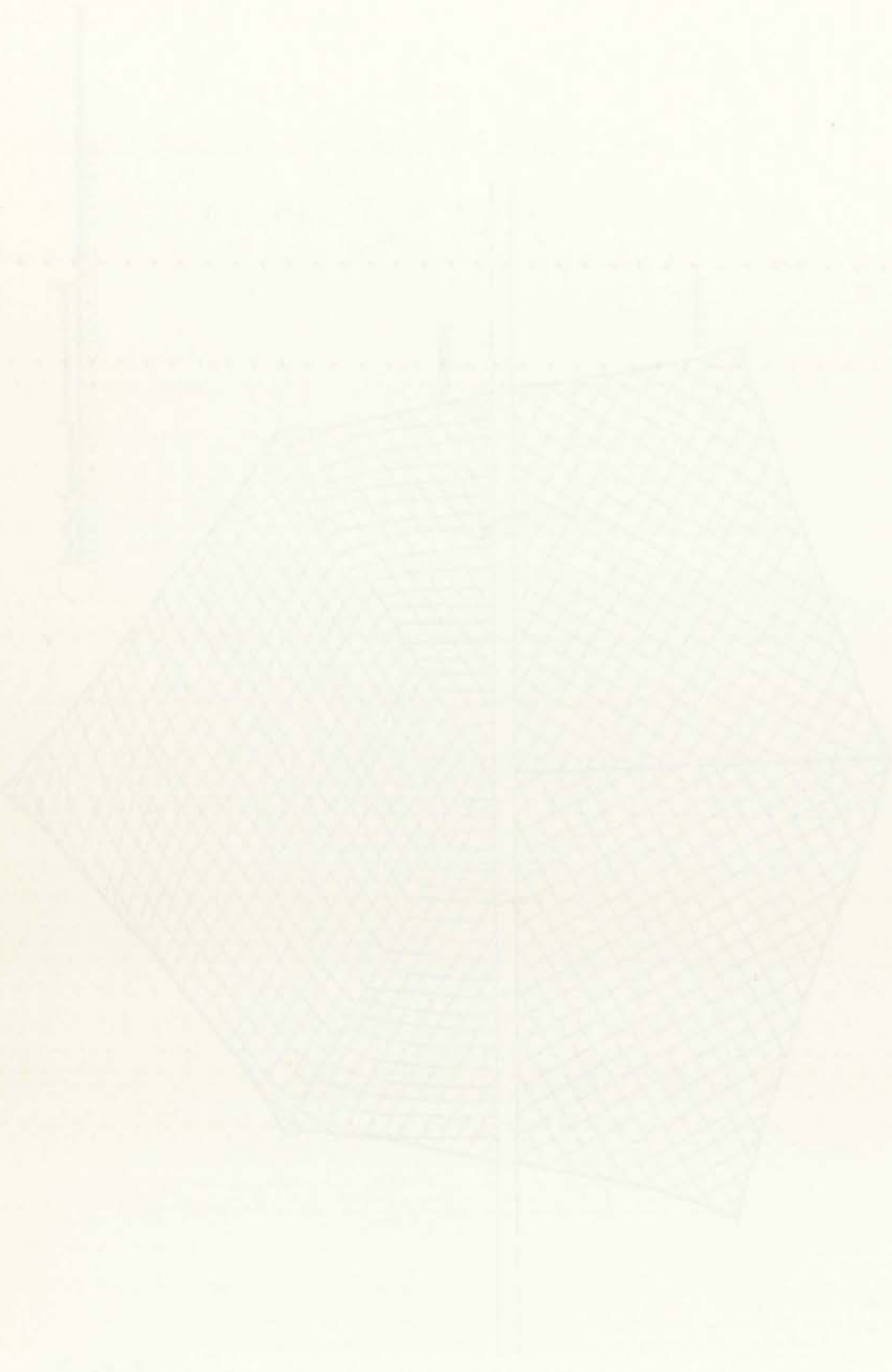


Fig. 79



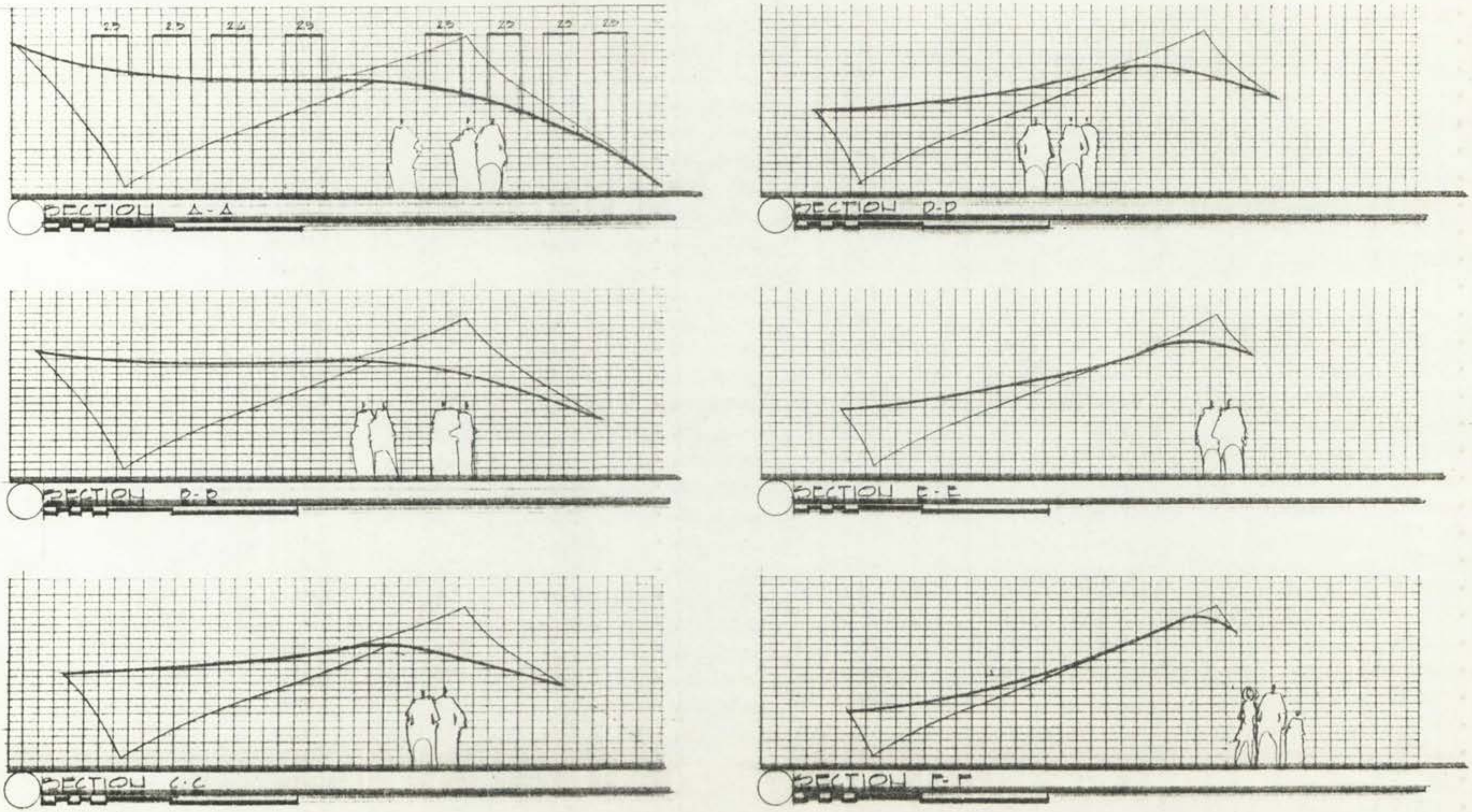
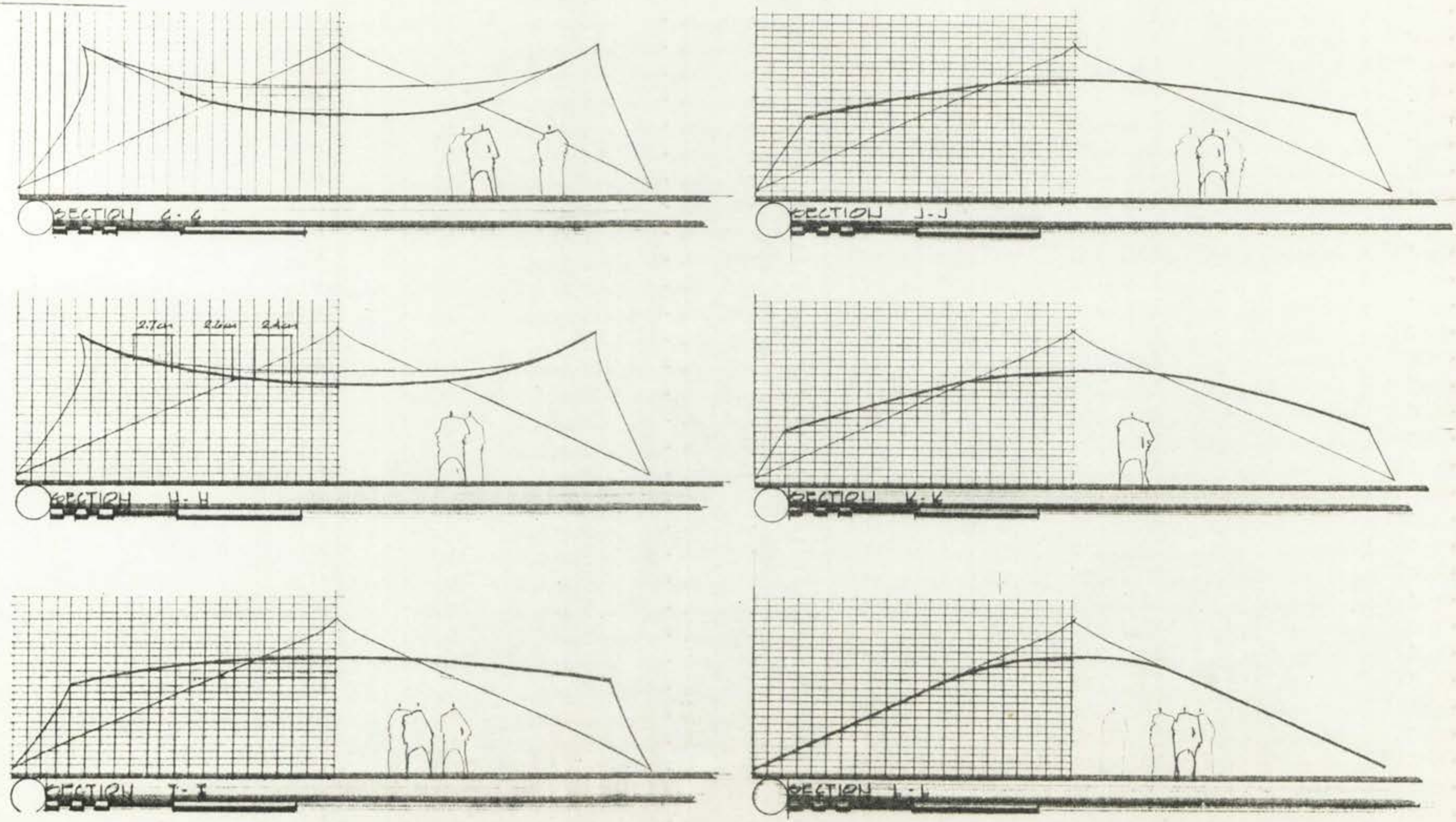

2.5
2.5
2.5
2.5
2.5
2.5
2.5
2.5
SECTION A-A
SECTION D-D
SECTION B-B
SECTION E-E
SECTION C-C
SECTION F-F

Fig. 80

Fig. 81


2.7cm
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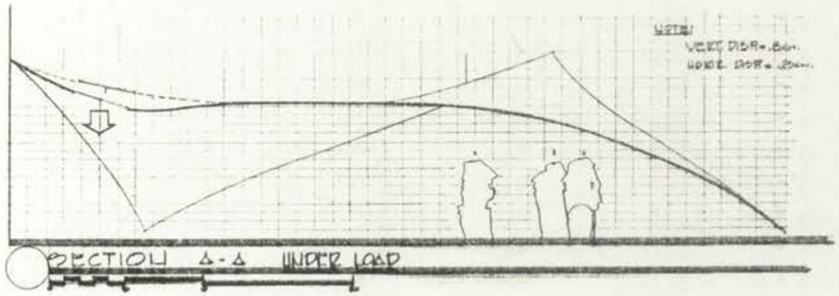
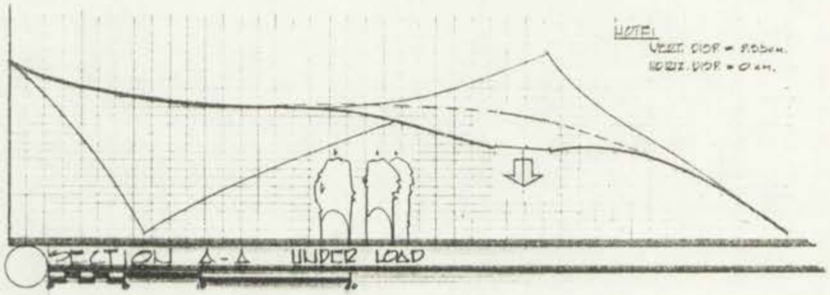
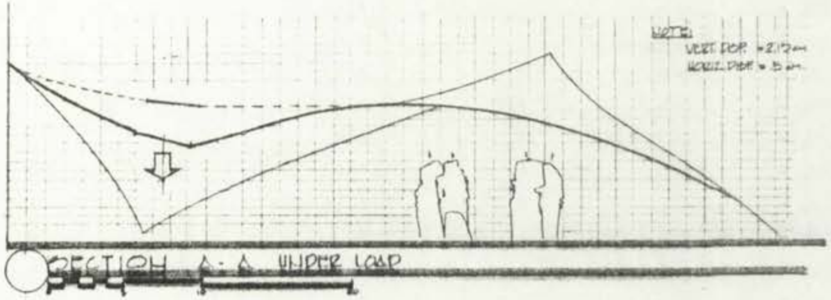
2.4cm
SECTION G-G
SECTION J-J
SECTION H-H
SECTION K-K
SECTION I-I
SECTION L-L


NOTE:
VERT. DISP. = .8 cm.
HORIZ. DISP. = .2 cm.
SECTION A-A UNDER LOAD


NOTE:
VERT. DISP. = 2.05 cm.
HORIZ. DISP. = 0 cm.
SECTION A-A UNDER LOAD


NOTE:
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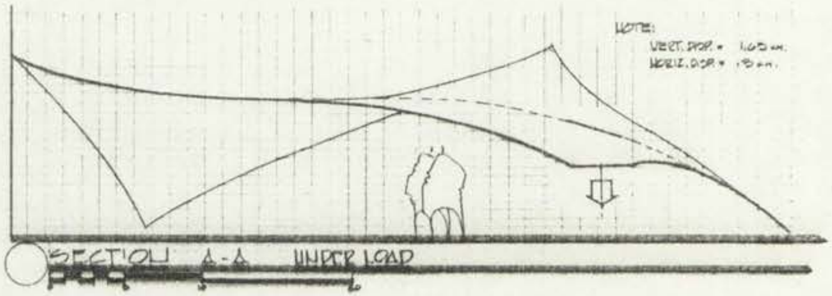
HORIZ. DISP. = .5 cm.
SECTION A-A UNDER LOAD


NOTE:
VERT. DISP. = 1.65 cm.
HORIZ. DISP. = .15 cm.
SECTION A-A UNDER LOAD

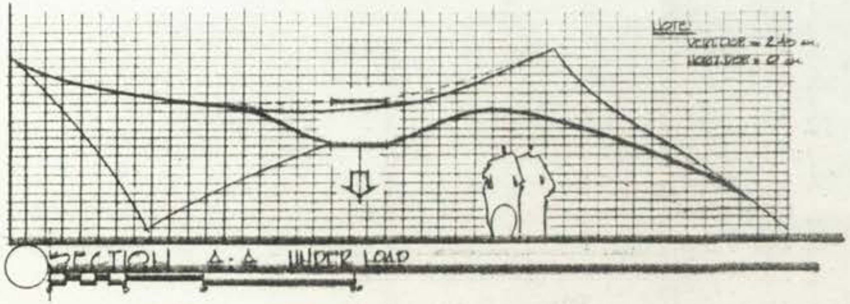

NOTE:
VERT. DISP. = 2.45 cm.
HORIZ. DISP. = 0 cm.
SECTION A-A UNDER LOAD

Fig. 82

FOOTNOTES

[1] Conrad Roland, Frei Otto: Tension Structures, (New York, 1970) pp. 16 - 17.

[2] Ludwig Glaesser, The Work of Frei Otto, (New York, 1972) pp. 12-13.

[3] Ibid. pp. 14 - 15.

Undulating Surfaces

Undulating Surfaces.

Undulating surfaces differ from simple saddle surfaces in that the prestress in the tent is not originated by transverse cables as in the case of hyperbolic paraboloid shapes, but by parallel or approximately parallel cables curved in opposite directions, and are characterized by having a repeating pattern of very strongly curved features.[1]

Undulating surfaces in relation to simple saddle surfaces are larger and have relatively smaller membrane stresses as well as cable support stresses, due to their strong curvature.
Membranes of cable meshes can be extended from the zig-zag spanning supports and can take a variety of shapes. Indeed, undulating surfaces can be added together parellel to one another and thus form roofs for buildings which have an elongated rectangular plan.

Previously Constructed Undulating Tents.

Wave Hall.

International Horticultural Exhibition.
Hamburg Germany, 1963
Architect- Frei Otto.

Description: It is a parallel undulating membrane of six bays having a total of six supports and seven restraining ridges.
It is not only asymetrical in section but comprizes two different elements. The hall itself employs saddle surfaces between parallel ridges that alternately support and restrain the membrane. The second element which is only connected for half its length to the Wave Hall is comprized of simple saddle surfaces.

Length: 269 feet

Width: 41 feet

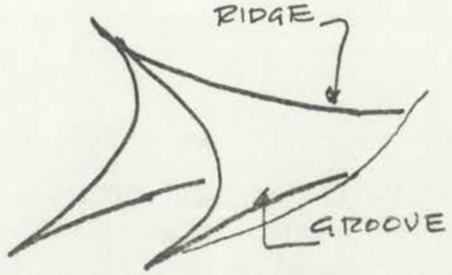
Span between masts: 66 feet

Height at center: 17 feet

Material: Coated cotton canvas strips, 3 feet in width, radiating about main compression supports increasing rigidity of surface and permitting some degree of reinforcing by overlapping of the material at or near main supports.[2]

Transverse and edge wire ropes.

Steel lattice masts and guyed poles.



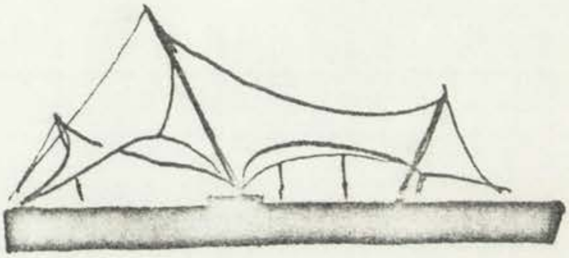
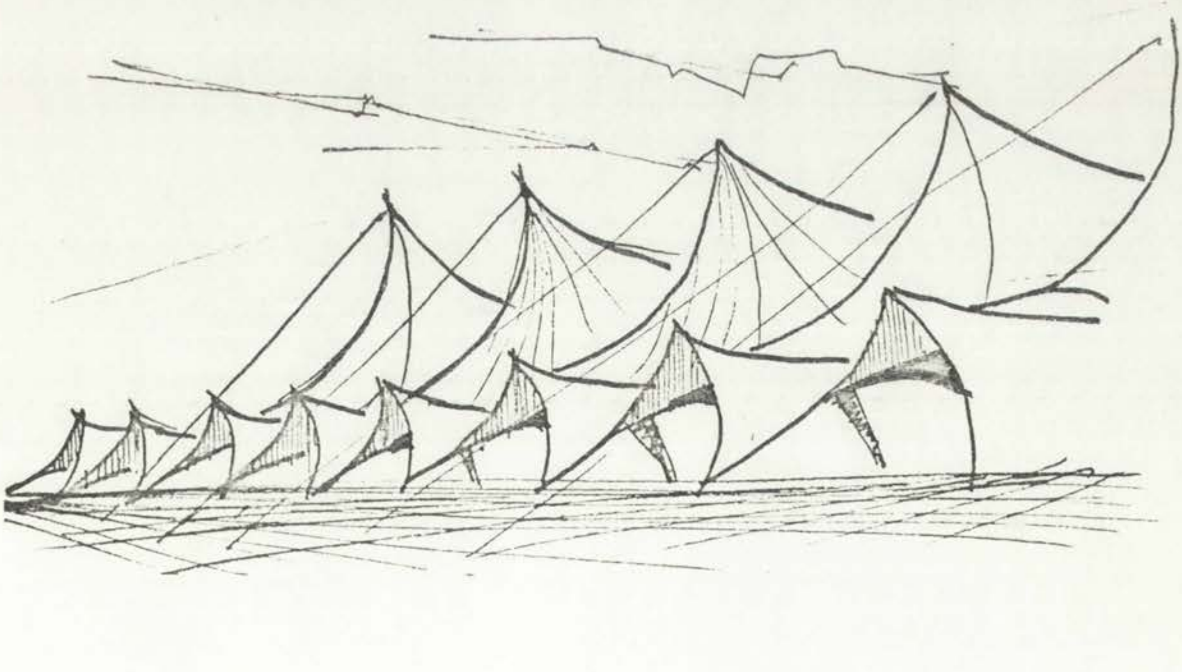


Fig. 83

Wave Hall



Federal Garden Exhibition. Dance Pavillion.

Cologne Germany, 1957

Architect- Frei Otto

Description: Radial undulating tent with each high and low edges support point directly connected to a central tension ring. It has six principal support points and six anchor points. Although the membrane is open at all points the membrane deflects outside sound and light into the central area and protects it from rain.

Maximum diameter 103 feet

Maximum height 33 feet

Materials: It is coated, translucent cotton canvas with radial and edge wire ropes, suspended from six lattice steel masts.[3]

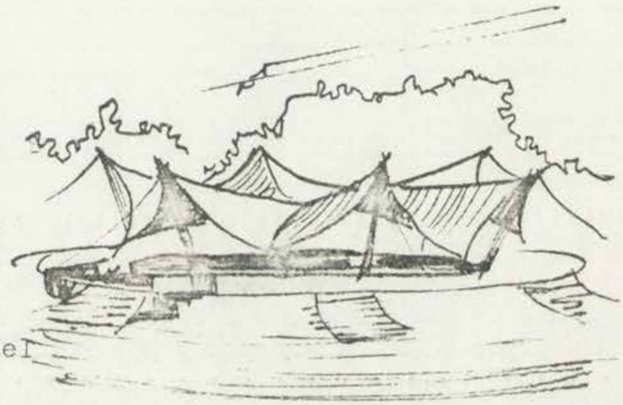


Fig. 84

Dance Pavillion

Model Analysis.

Undulating tent.

Description: This model is an undulating membrane of three bays and having four alternating high points as well as four alternating anchor points.

This condition creates a zig-zag pattern in that, in analysis, the structure consists of 3 four point surfaces added together to form an undulating assembly in such a manner that each high point is connected to the opposite low point by a carrying cable or ridge, and each low point is connected to the opposing two low points by a tensioning cable or groove.

A secondary structure consisting of a four point surface of non symetrical stress is attached to each alternating high point and anchor of the guy cable stabilizing the post of the high point.

Loading.

Due to the angle at which the masts are held, the magnitudes of load at the supports, and the effort of the author to stabilize the magnitude of deformation of the various circles, that the undulating surface was subjected to an equal magnitude of load at the supports.

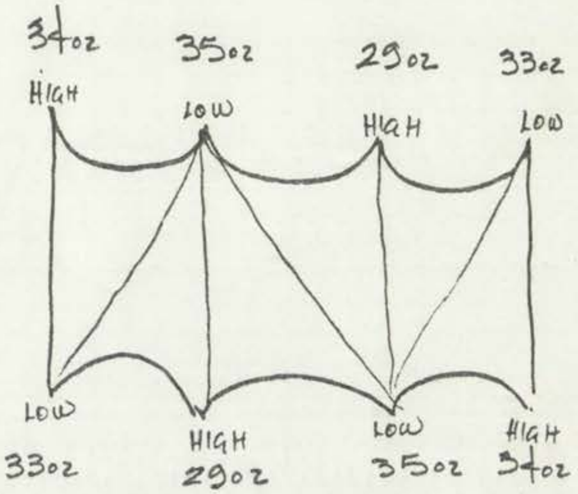


Fig. 85
Loading

Contours.

In this particular model the contour lines at or around the principal points of support do not give any great insight as to the magnitude of the forces confronted at the supports. There was observed that a slight increase in the slope towards the low points of the surface was present, classifying the low points towards the middle of the tent to be the points of greatest stress concentration. (Fig. 89).

Surface Diagram.

Information rendered to the reader from the surface diagram is mostly visual and can be found in the plan, and is as follows:

There is a certain circular deformation towards the center of the tent in the middle groove support (Section J-J) , which is accentuated as the line approaches the anchor point. (Fig. 86 h).

There is a certain increase in the spanning distance of the two middle ridges. The condition becomes more pronounced as the edges become almost tangential toward the low points, whereas no edge deformation is present at the high points.

There is also deformation present on the linear edge supports at the right and left hand sides. This deformation is very apparent since the line is straight in the unloaded state.

The concluding argument arrived at the loading stage of this analysis seems to be strengthened by the observation present in this stage.

Confronted by the facts previously exposed, the author arrived at the following conclusion : There is a presence of greater stress next to the minimum points. This is due to the fact that there are three major tension members anchored at these points: one supporting ridge and two tensioning grooves, whereas the high points are confronted only with one main tension member, the ridge support.

Sections not Under Load

Observations:

Sections C-C, D-D and E-E show marked points of inflection as the lines approach the center of the tent (Fig 91).

It was also observed that the deformations of the circles become accentuated as the sections draw closer to the high points, or principal points of support.

Sections B-B and F-F show a marked approximation to straight lines as the sections draw toward the center of the supports. (Fig. 91). Section E-E shows a marked increase in the magnitudes of the circles as they approach the high point and the area where the ridge and the groove meet, i.e. the area between the two principal members of support.

Sections A-A and C-C(Fig. 91) show a marked increase in the deformation of the circles as the section nears the supports.

Conclusions: Taking into account the observations previously noted, the author has arrived at the following conclusions: First, there is a marked increase in the membrane stresses, directional in nature, radiating from both the high and low points. Second, there is a higher membrane stress present in low points due to the accumulation of primary and secondary linear supports, i.e. the ridge and the groove.

Sections Under Load.

There is a marked reduction of vertical displacement present as the section approaches the two principal points of support (Fig. 93 & Fig. 87 f,g).

There is also a marked resumption of the original curvature as the load is applied near a secondary point of support.

Conclusion: Greater stress is present towards the main points of support (the high and low points). Greater stress present at or near the secondary linear members of support.

After receiving the observations and conclusions presented in the different stages of the model analysis, the author presents these

final conclusions to be kept in mind when approaching the construction design stage:

Great consideration given to the stresses present at both the high and low points of the membrane.

Consideration to be given to the fabric strips or cable arrangement; preferably in a radiating pattern stemming form the high and low points of the surface. See maximum and minimum curvature diagram. (Fig. 90). for patterns of arrangement.

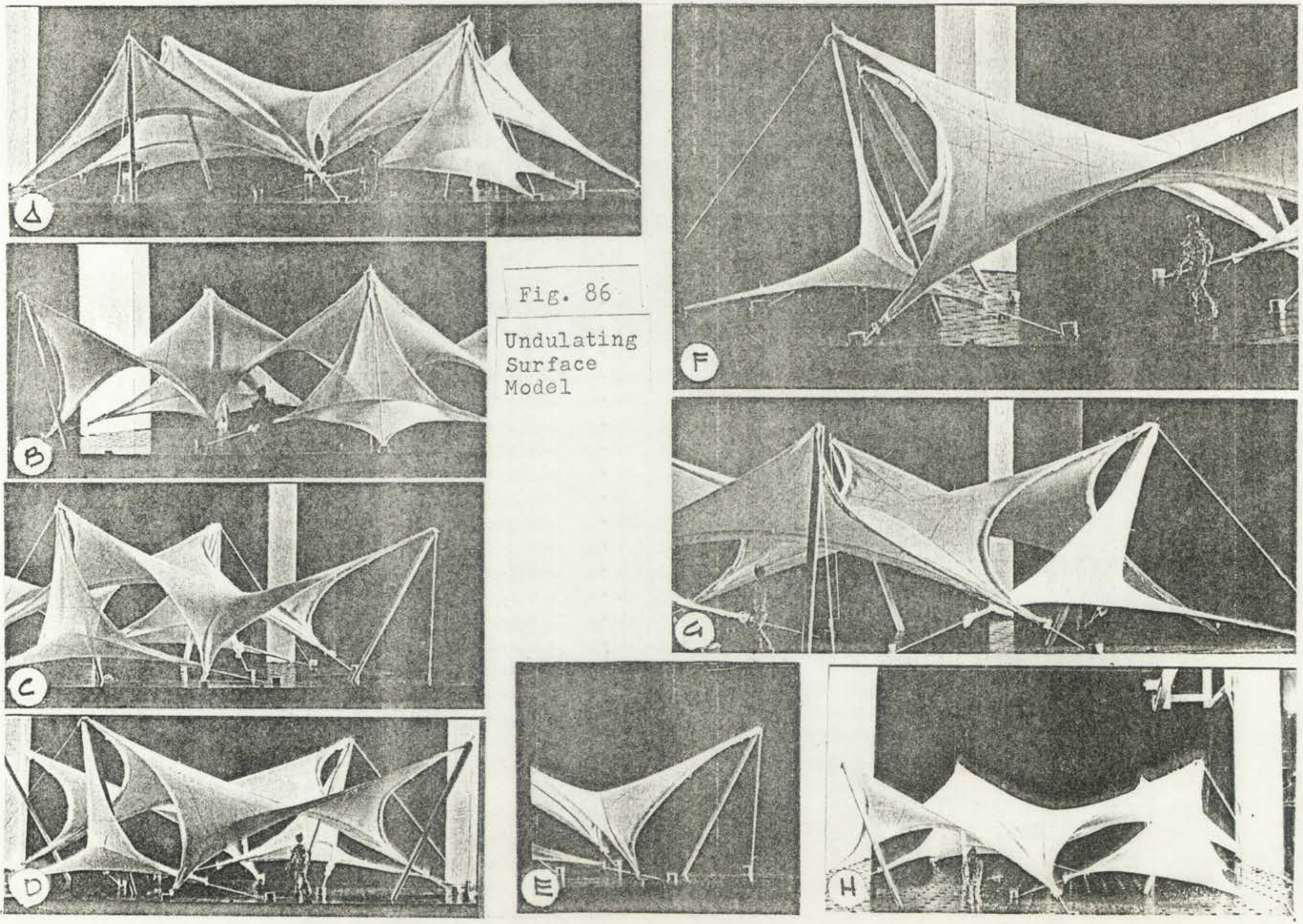


Fig. 86

Undulating
Surface
Model

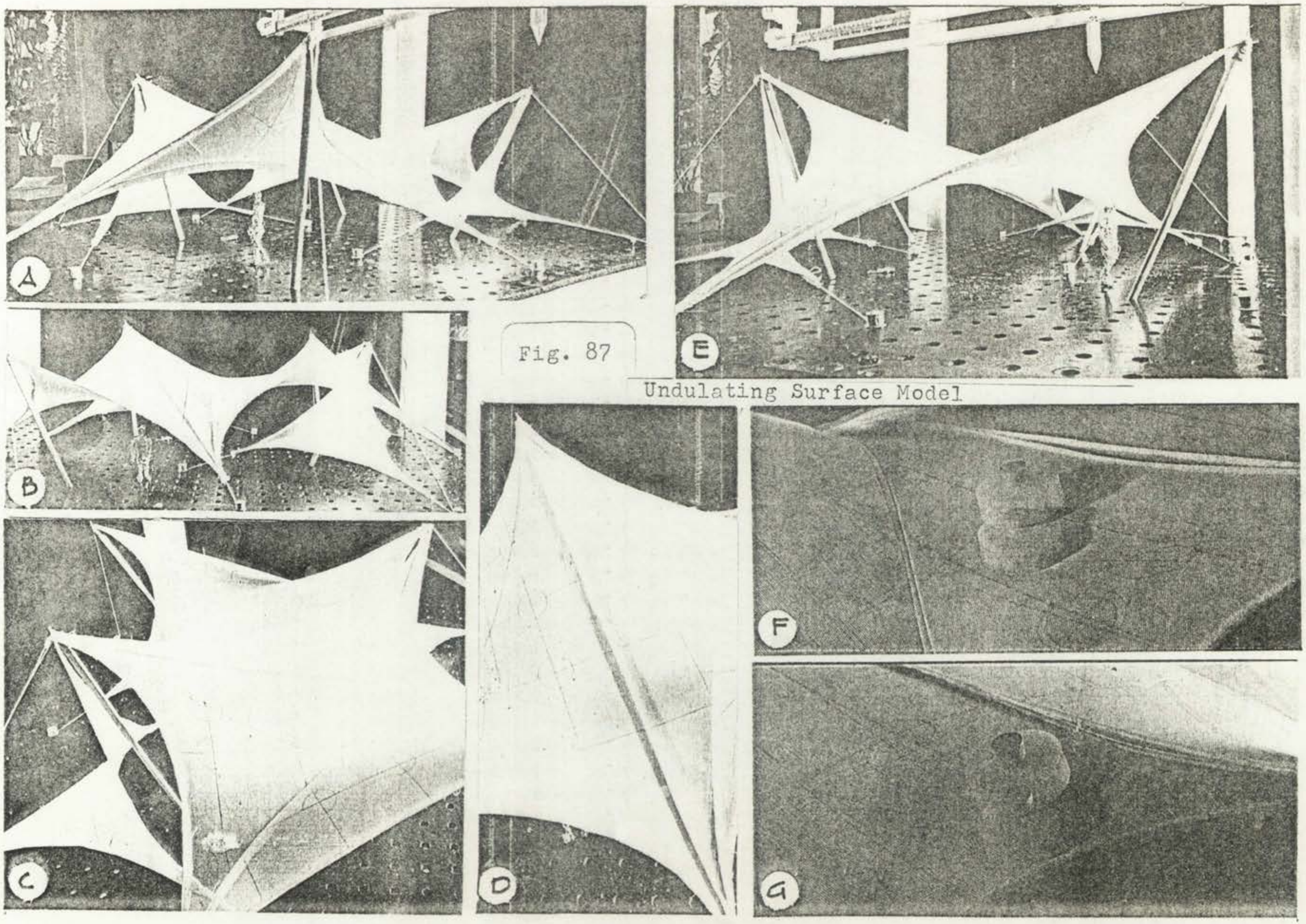


Fig. 87

Undulating Surface Model

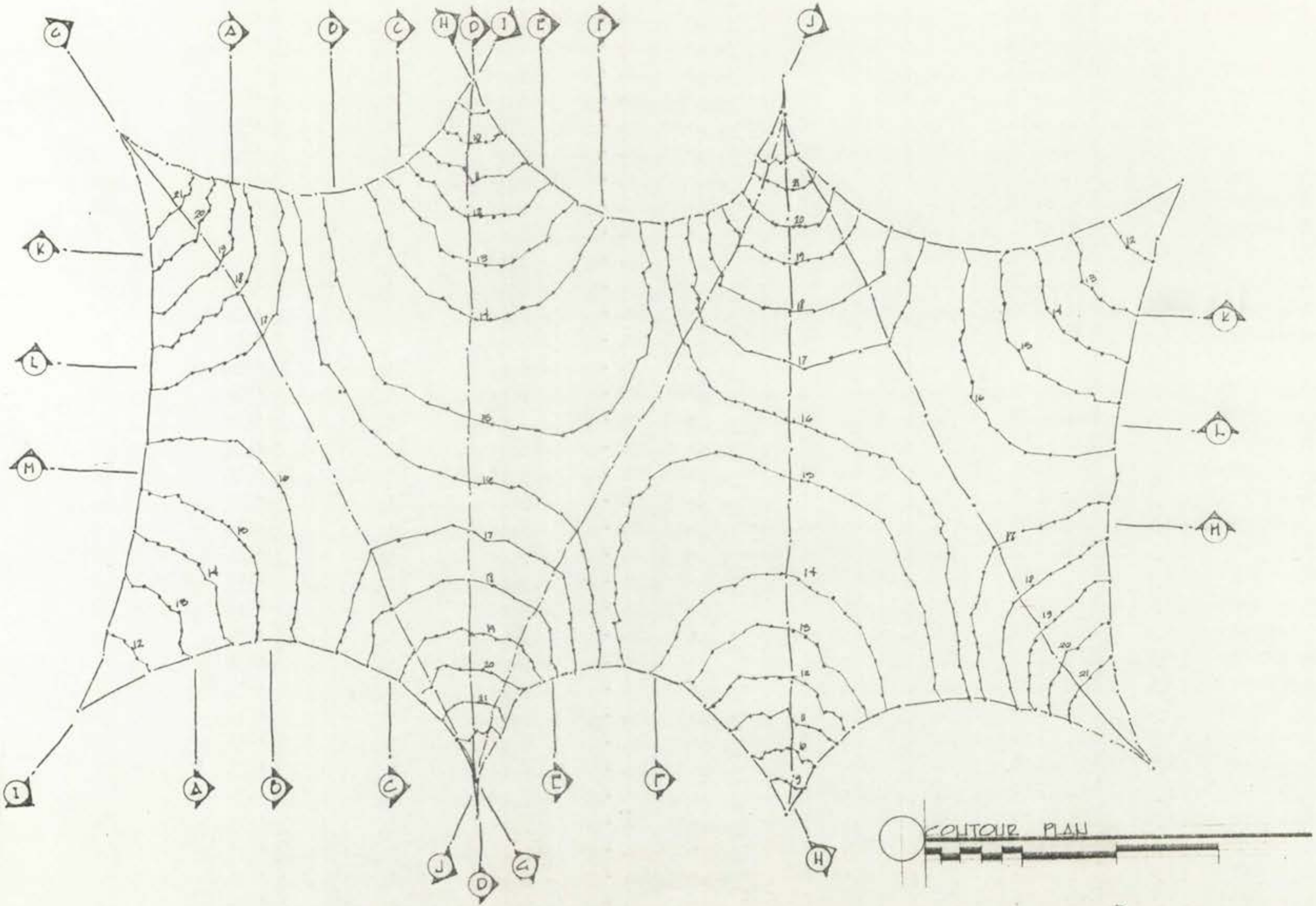


Fig. 88

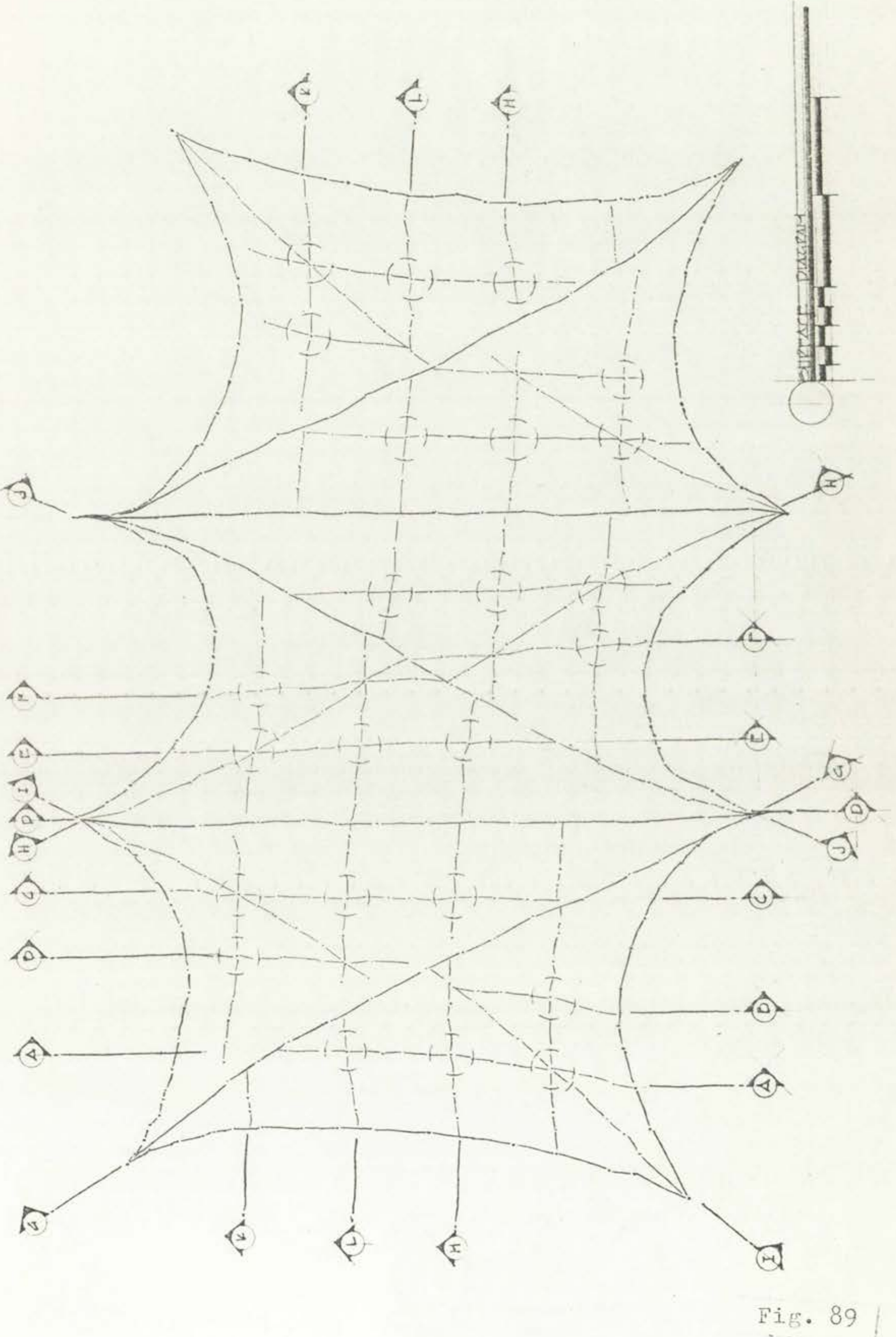


Fig. 89

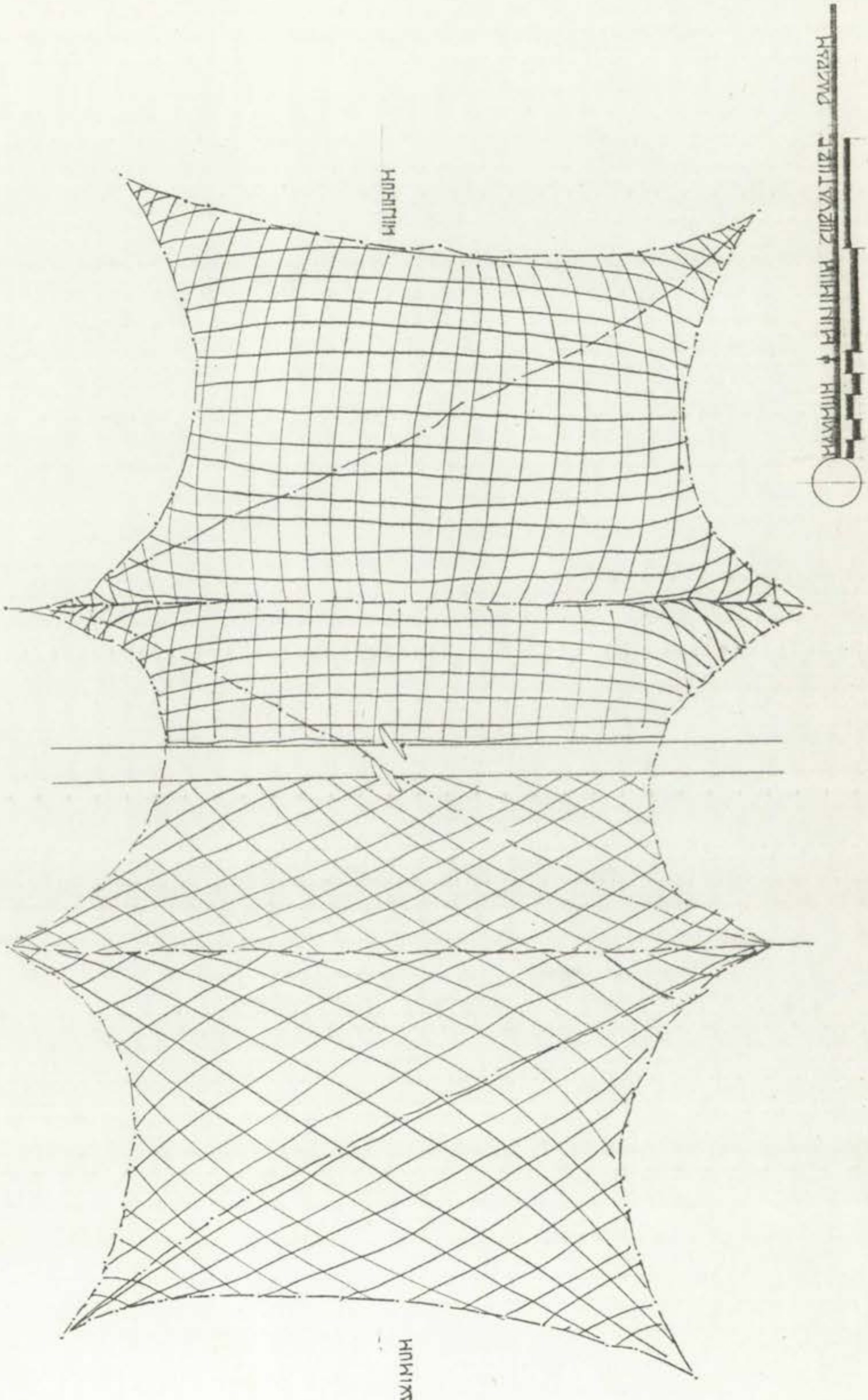
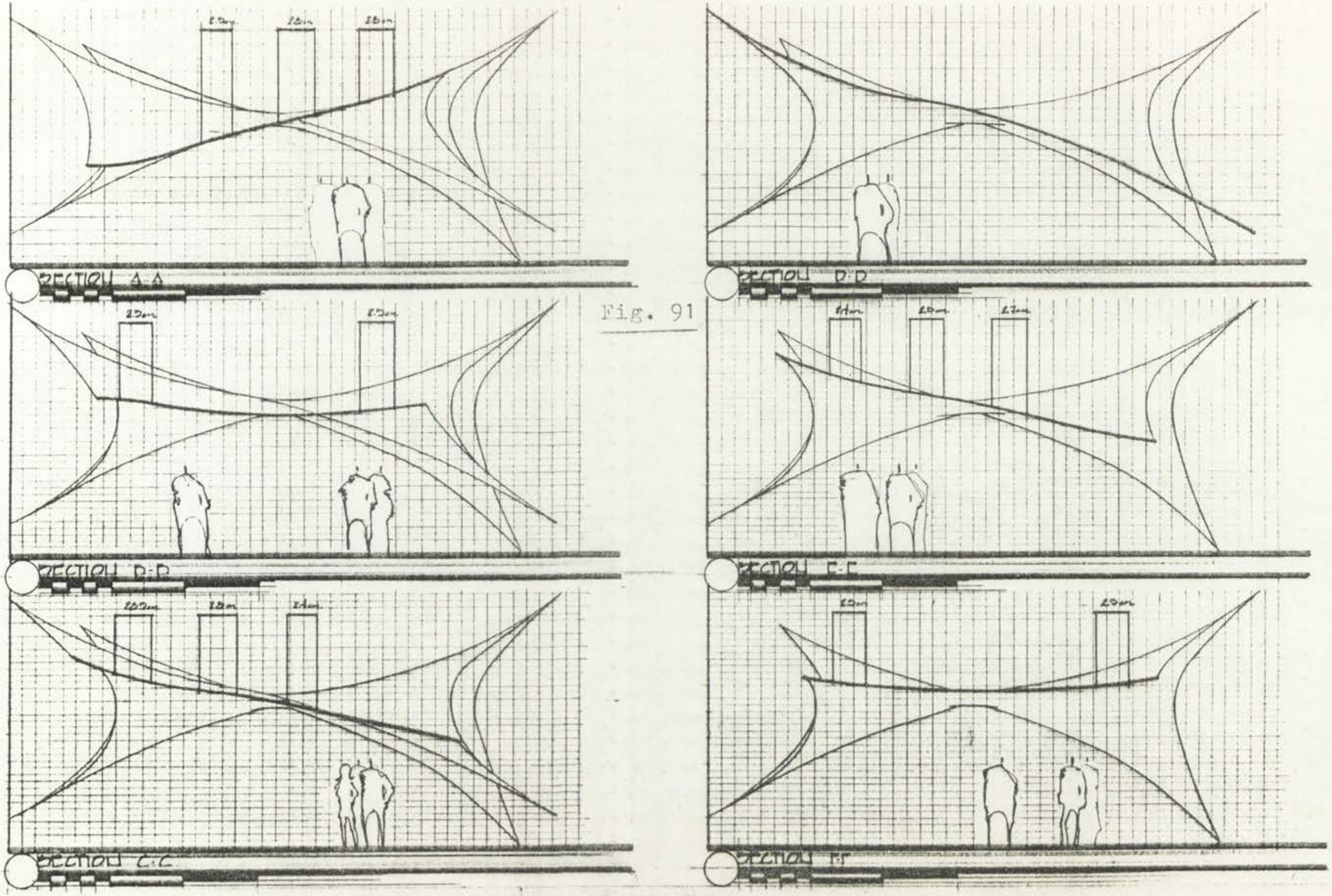


Fig. 90



SECTION A-A
SECTION D-D
SECTION D-D
SECTION E-E
SECTION C-C
SECTION F-F

Fig. 91

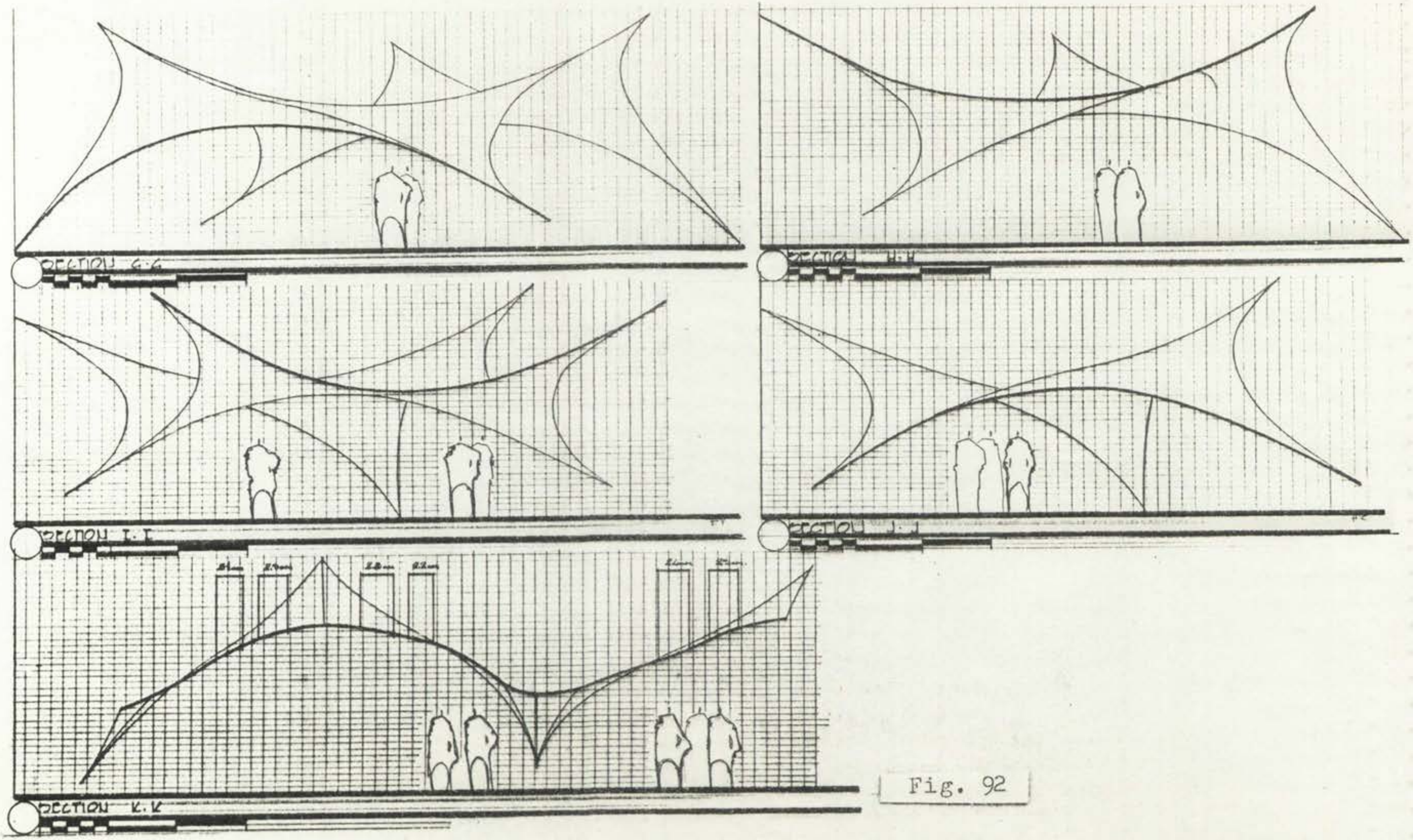
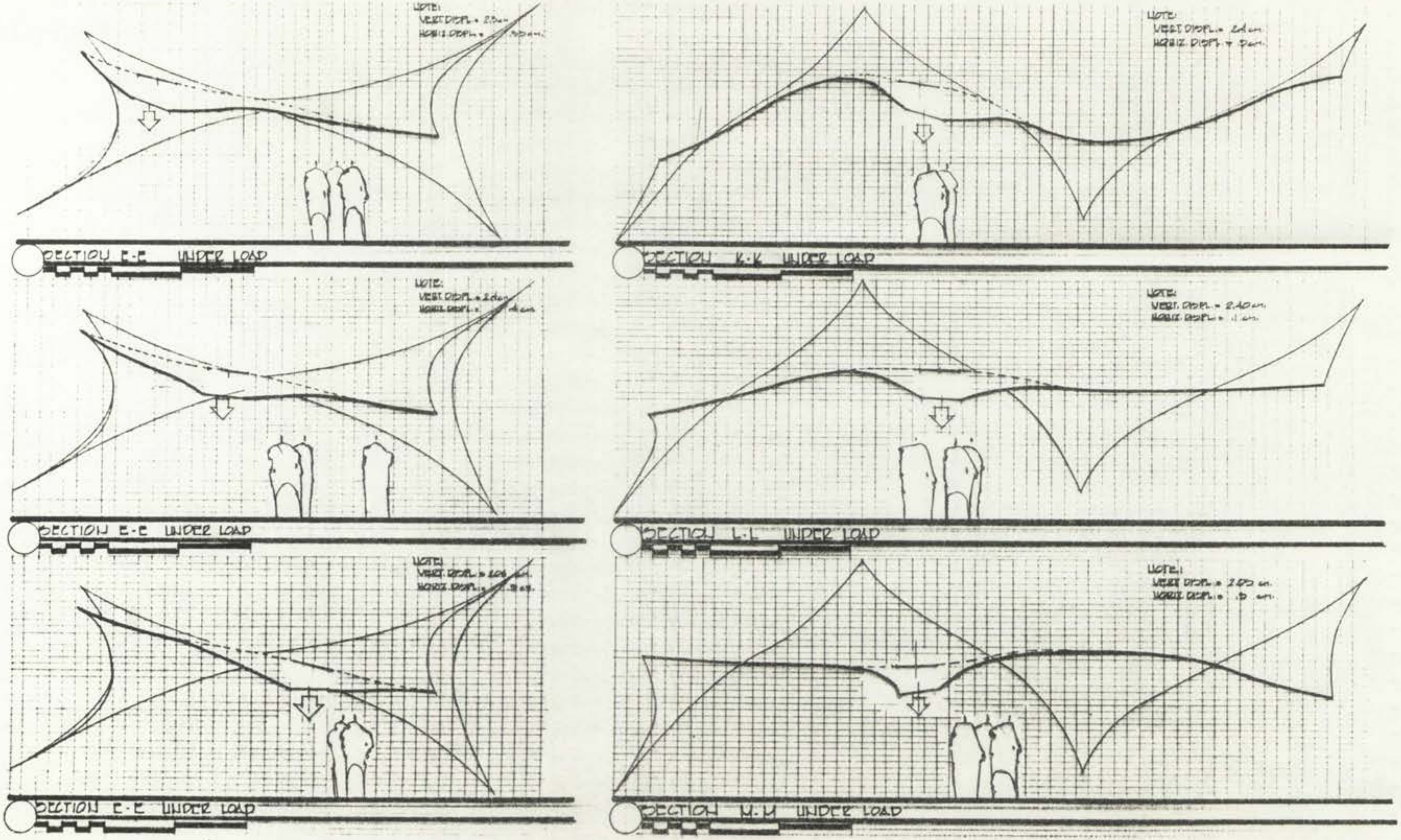


Fig. 92

Fig. 93


NOTE:
VERT. DISPL. = 2.5 cm
HORIZ. DISPL. = .50 cm
SECTION E-E UNDER LOAD
NOTE:
VERT. DISPL. = 2.4 cm
HORIZ. DISPL. = .5 cm
SECTION K-K UNDER LOAD
NOTE:
VERT. DISPL. = 2.4 cm
HORIZ. DISPL. = .4 cm
SECTION E-E UNDER LOAD
NOTE:
VERT. DISPL. = 2.40 cm
HORIZ. DISPL. = .1 cm
SECTION L-L UNDER LOAD
NOTE:
VERT. DISPL. = 2.06 cm
HORIZ. DISPL. = .3 cm
SECTION E-E UNDER LOAD
NOTE:
VERT. DISPL. = 2.05 cm
HORIZ. DISPL. = .5 cm
SECTION M-M UNDER LOAD

FOOTNOTES

[1] Conrad Roland, Frei Otto: Tension Structures, (New York, 1970) pp. 26-27.

[2] Ludwig Glaesser, The Work Of Frei Otto, (New York, 1972) pp. 23-25.

[3] Ibid. pp. 19-20.

Saddle Surface Supported by Arches

Saddle Surfaces Between Arches.

In tensile structure design, a great deal of emphasis is placed upon the feasibility, in economic terms, of the design. As experience as shown, the weight of the compression member or supporting structure and its engineering complexibility is almost as great as that of the tent superstructure itself. Therefore different means of supporting a tensile structure should be investigated. In terms of economics and simplicity of design, the most feasible compression member is the arch.

From experimentation with regard to the shape of a saddle surface structure in relation to the angle or position of the supporting arches, there has been no result where there has been any marked difference in the essential shape of the saddle surface.[1]

It is essential to point out that the shape or size of the arch does not have any set geometrical standards. Asymetrical arches are possible if the particular stress situation requires it.

The only requisite in arch support systems is the fact that arches have to be stabilized or tensioned on the side resisting the surface tension.

Previously Constructed Arch Supported Saddle Surfaces.

Entrance Arch.

Federal Garden Exhibition

Cologne, Germany, 1957

Architect- Frei Otto.

Description:

Arch supported saddle surface type membrane stabilized by four

anchor points.

Maximum span: 112 feet

Maximum height: 20 feet

Arch tube diameter: 7 1/2 inches

Materials:

coated glass fiber fabric, more durable but not as deformable as canvas. The fabric strips had to be cut with the greatest precision to conform with the established curvatures. Edge and suspension wire ropes. Tubular steel arch, and four guyed poles for the anchor points.[2] (Fig. 94).

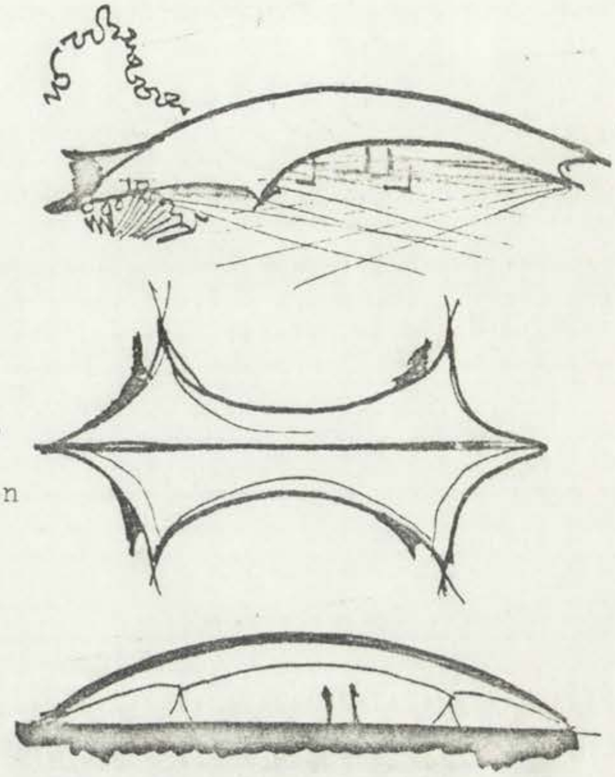


Fig. 94 Entrance Arch

Model Analysis

Simple Saddle Surface Bordered by Arches.

Description:

The model analyzed is a simple saddle surface which in the involved state had a length of 46.7 inches. It is supported at the edges by two arches having a radius of 14.9 cm. placed parallel to each other. The reason for the positioning of these arches in such a manner was in effect to visualize a continuation of these barrel like saddle surfaces, and what effect the stress would have in one individual case.

Loading:

The observation that major stresses placed lenghtwize on the tent

have an effect on the deformation of the circles in those directions. (Fig. 100)

Contour Diagram and Surface Diagram.

Observations: Major stress concentrations are at the arch supports. Contours curve in an elyptical manner around the high point of the arch creating a nipple effect, with major slope ratios occuring not at the high points but at the edges , and minor slopes at the midpoint of the span of both height and width.

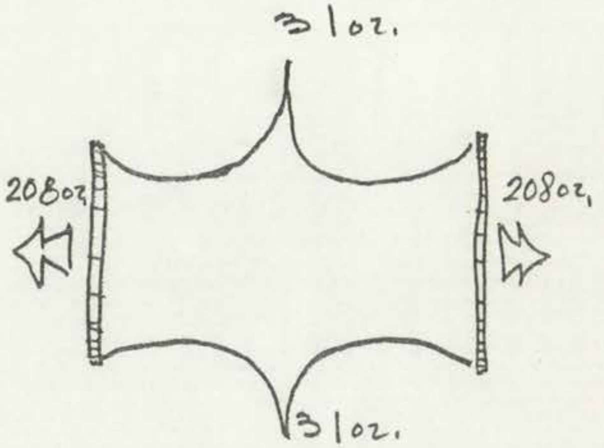


Fig. 95 Loading

Conclusions: This nipple effect seems to radiate with marked impact on the top of the flexible curved openings. (Fig.99 a and Fig. 98 f). This condition is very marked by the deformation of the line next to the top of the arch. (Fig. 99 a,b, and Fig. 98 f,g).

Sections.

Observation: There is a greater stress in the directions of rigid supports by observation of the deformation of the arches. (Fig. 103) The flexible curved openings have an effect on the lateral stress placed upon the two, stemming from the top of the openings. This can be seen in figures F-Fand G-G. (Fig.103).

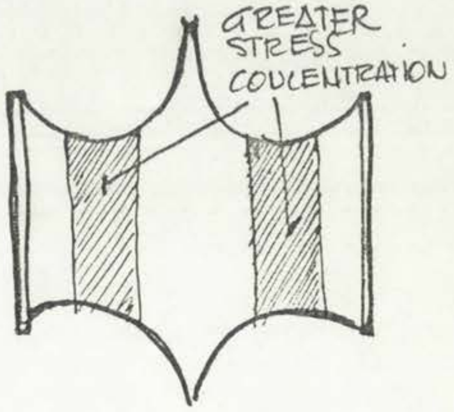


Fig. 96 Stress Concentration

Sections Under Load.

Observation: It was observed from the form section under load that there was a greater vertical displacement on the tent as the load was applied toward the middle of the tent. (Fig. 104).

It was also observed that greater rigidity in the tent was present at the lenght's quarter points on axis with the high points of the lateral openings.

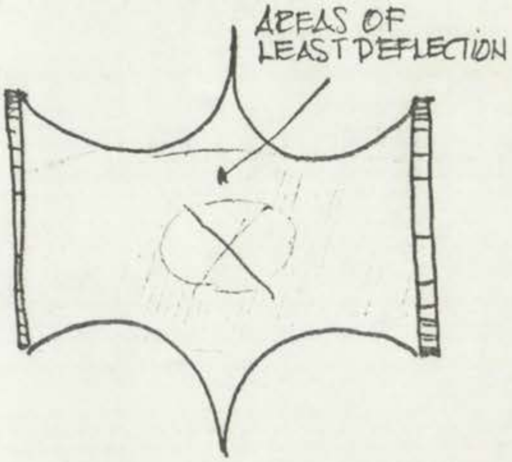


Fig. 97 Stress Concentrations

Conclusions: From direct observations of the saddle surface and conclusions gathered in the different analysis stages of the model, the author would like to mark two conclusions: First, least tension presented at the base of the arch as observed by the ruffles in photographs, (Fig. 98 a and d). Second, at the secondary points of load aplication. This was due to direct observation of ruffles present in photograph. (Fig. 98 g and c).

Maximum tension occurs in a radiating pattern and could be represemted by following the carrying directions of maximum curvature. (Fig. 102).

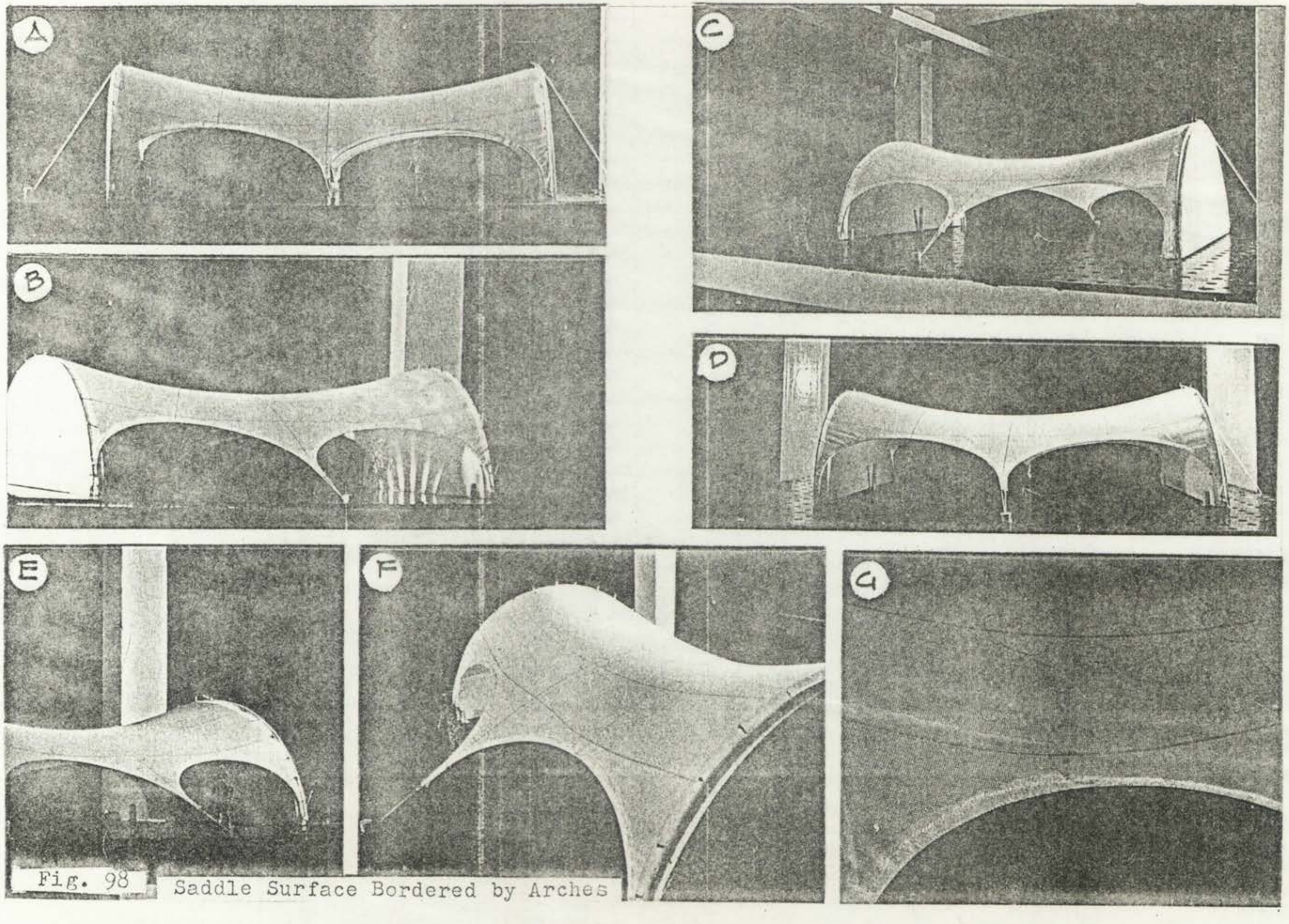


Fig. 98 Saddle Surface Bordered by Arches

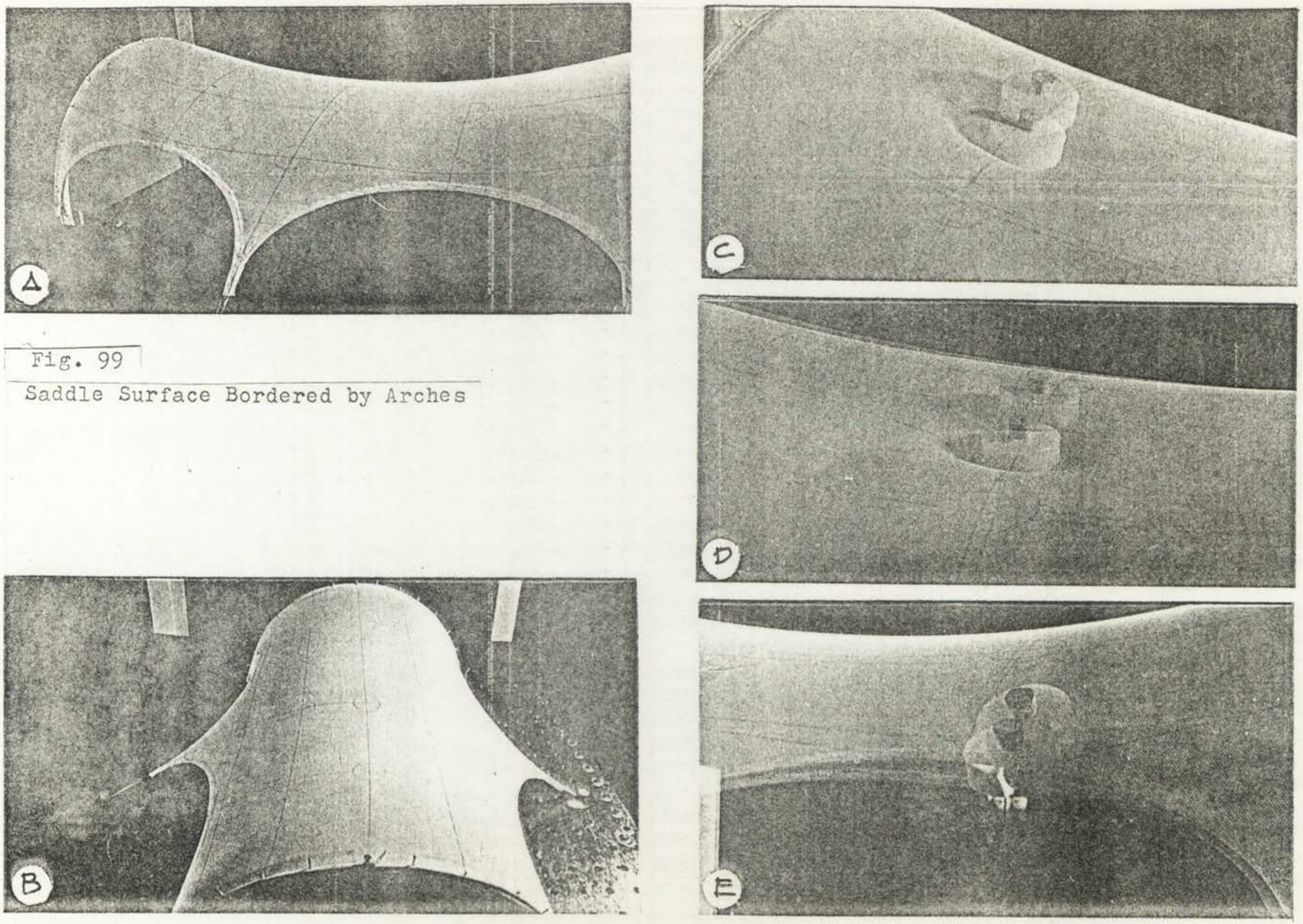


Fig. 99

Saddle Surface Bordered by Arches

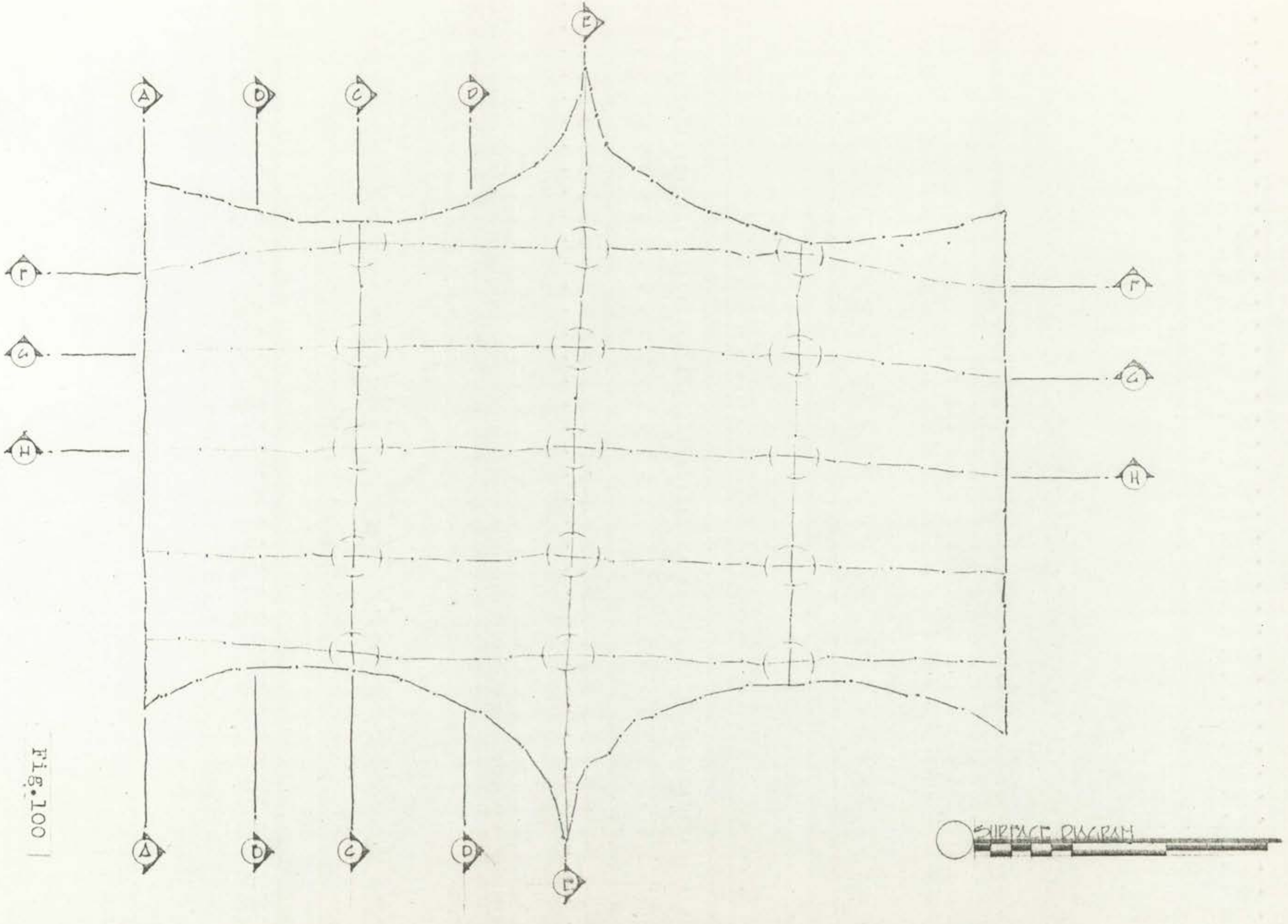


Fig.100

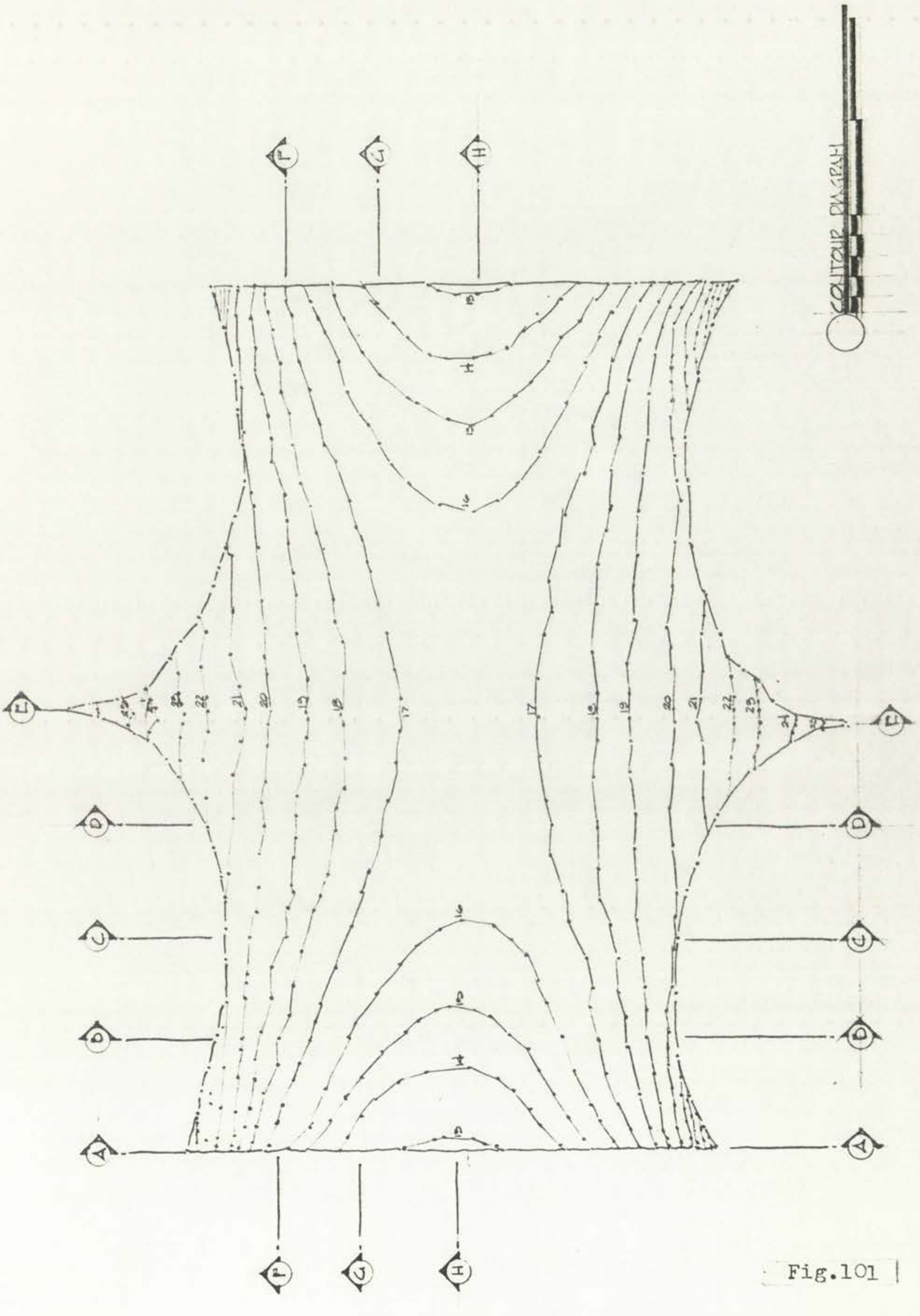

CONTOUR DIAGRAM

Fig.101

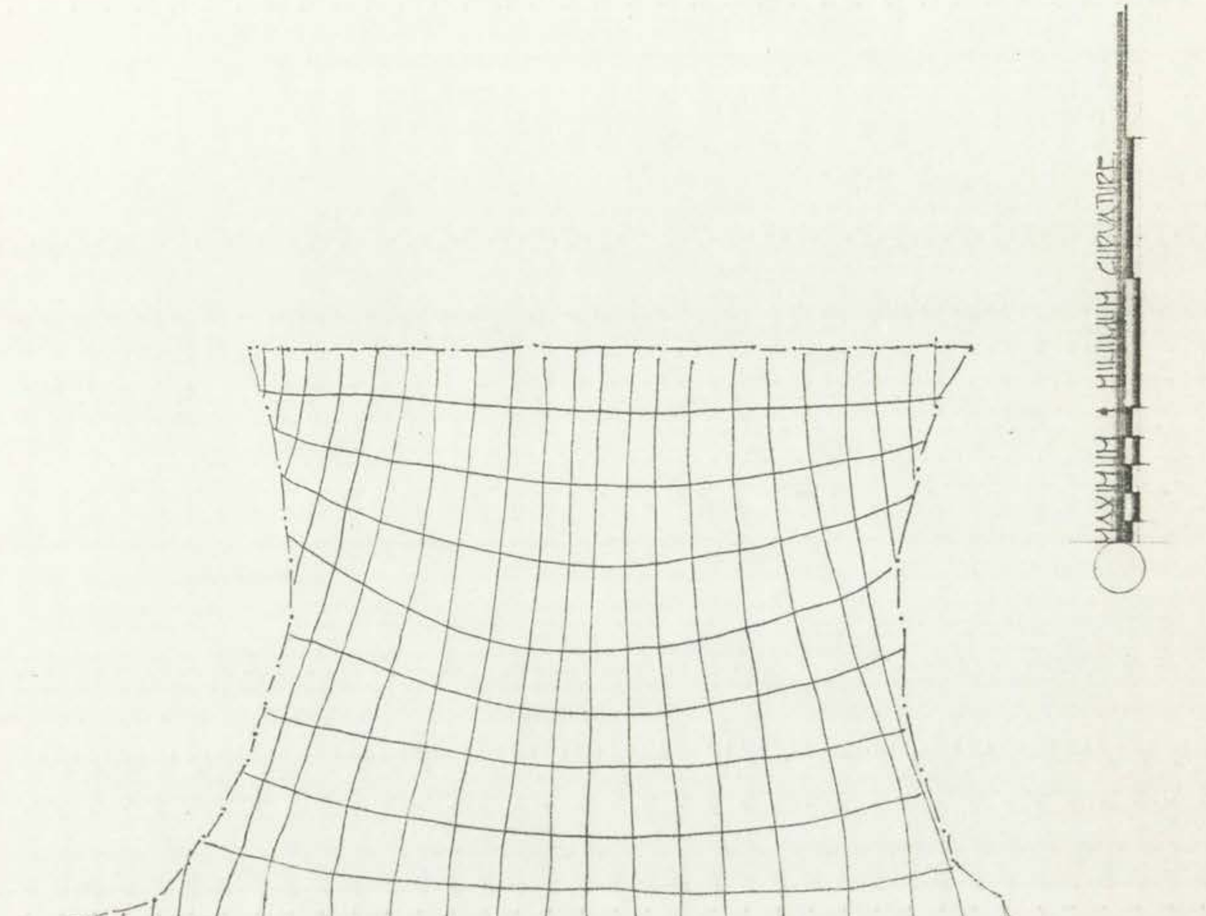


Fig. 102

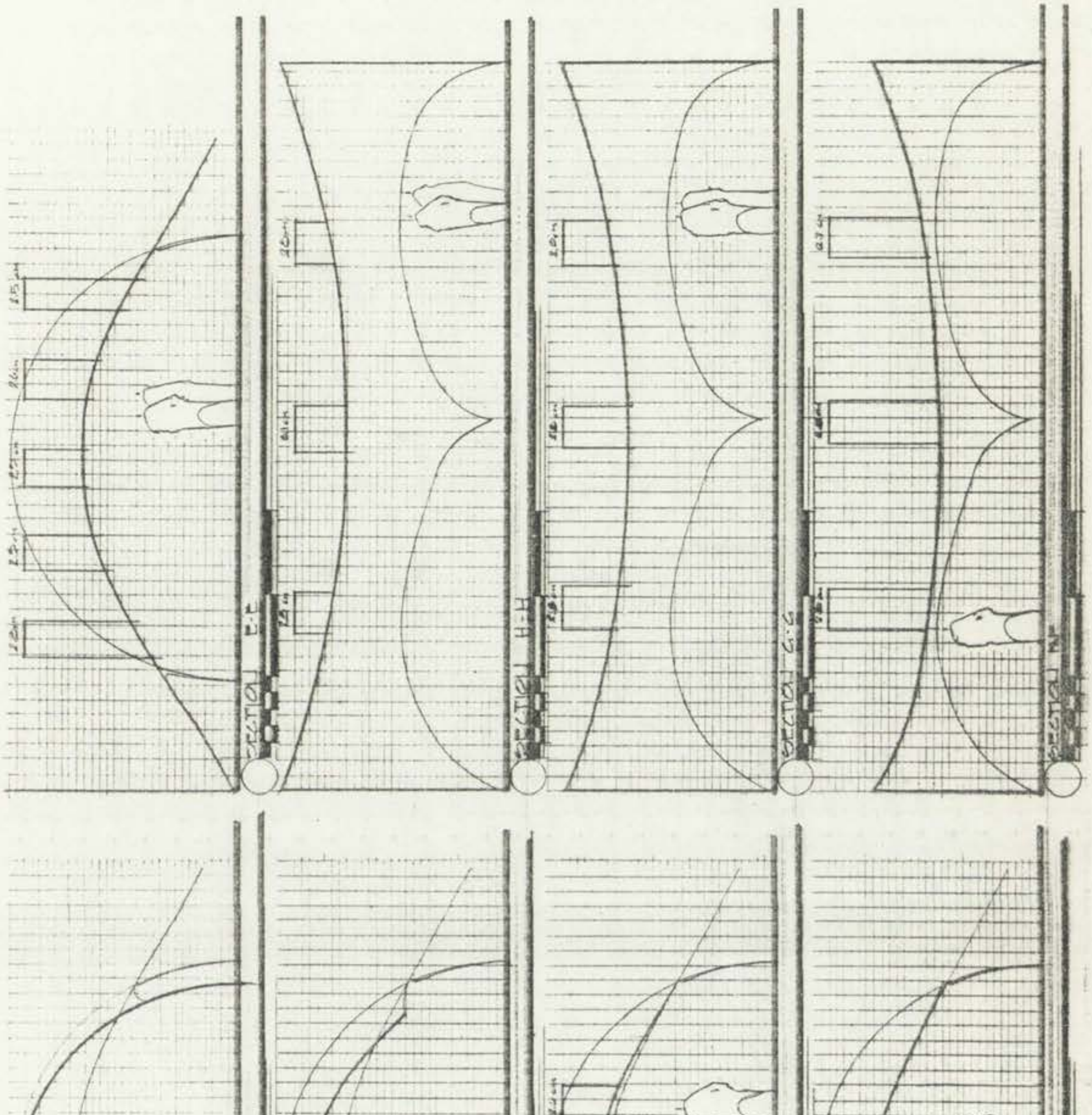

SECTION E-E
SECTION H-H
SECTION G-G
SECTION F-F
SECTION A-A
SECTION B-B
SECTION C-C
SECTION D-D

Fig.103

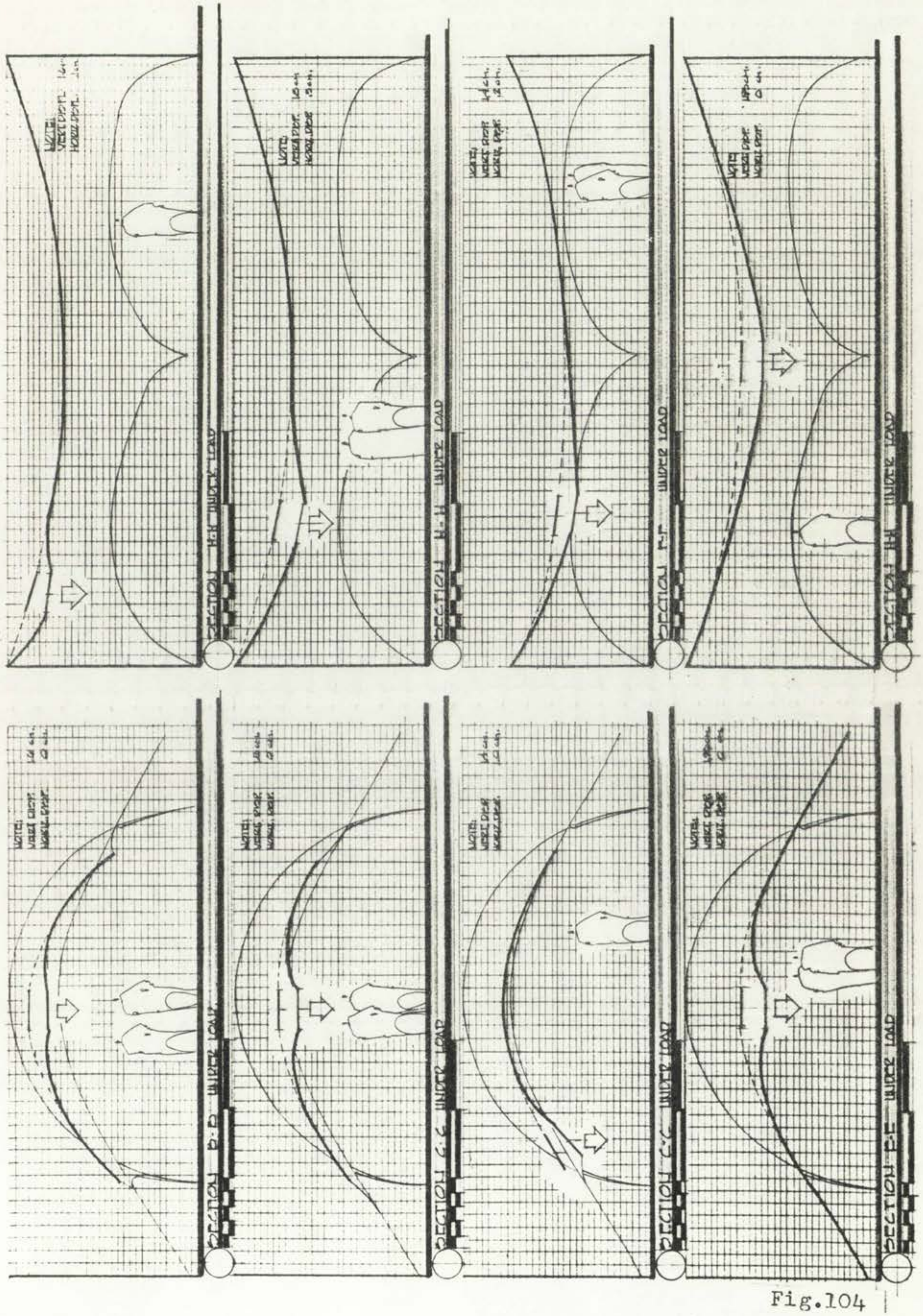

SECTION H-H UNDER LOAD
SECTION H-H UNDER LOAD
SECTION F-F UNDER LOAD
SECTION H-H UNDER LOAD
SECTION D-D UNDER LOAD
SECTION C-C UNDER LOAD
SECTION C-C UNDER LOAD
SECTION E-E UNDER LOAD

Fig.104

Saddle Surfaces Supported by Arches.

Description:

This particular model consisted of a surface comprised of four variations of saddle surfaces tensioned in between three circles radiating from one common compression point at 45° intervals, and has ten anchor points. The spans of the outside arches are of 15 inches whereas the middle is of 17 inches.

Loading:

Observations: Major concentration of load takes place at the point of arch convergence; therefore the greatest amount of stresses would be concentrated in that immediate area. (fig. 107 e,f, and c).

The secondary points of load concentration are located at the other end of the three radiating arches.

Conclusions: Information gives the reader two locations of stress concentration in the tent: at both ends of the supporting arches.

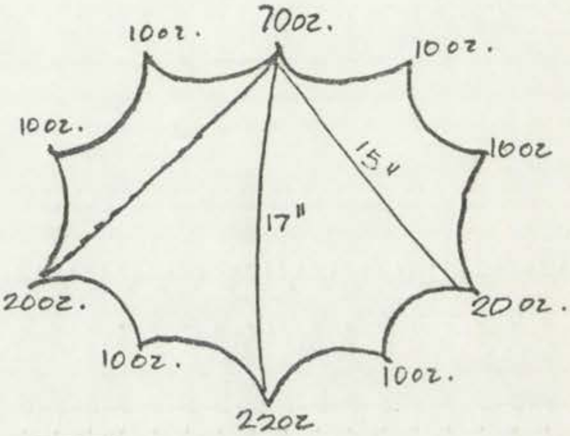


Fig. 105 loading

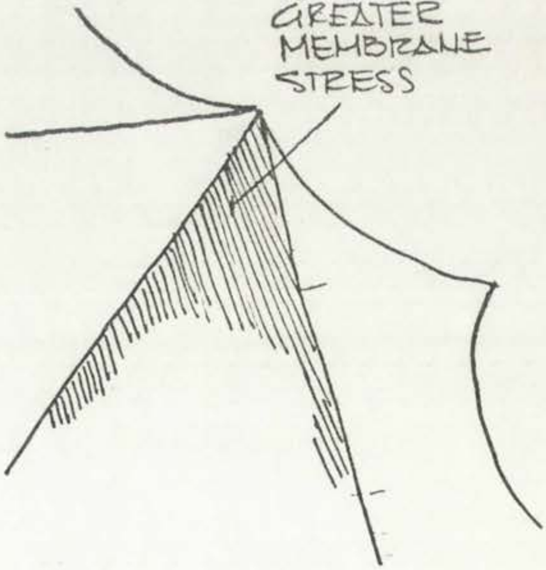


Fig. 106 Membrane Stress

Contour Diagram:

Observations: The greatest concentration of contour lines and therefore the greatest slope is concentrated at the conver-

gence point and at the high points of the arches; by further observation as to the location of contour lines, the reader observes that the least amount of tension is located at the opposite side of the model, in between the opposite compression points. (Fig. 106). This information reinforces the argument presented in the loading stage of analysis, which has the maximum areas of stress concentration located at the convergence points of the arch.

Surface Diagram.

Lateral deformation was encountered in the lines radiating from the convergence point, although some deformations on the lines drawn transversely to these show some degree of deformation; this condition is largely due to the three dimensionality of curvature of the saddle surface. (Fig. 108 e).

Sections Not Under Load.

Observation, Section E-E: The most marked fact gathered in the observation of this section is that the arch deforms and becomes asymmetrical as it nears the point of arch convergence; the curvature of the arch becomes more accentuated toward this support. (Fig. 108 b). Also, the displacement of the circles in the arch increase as the section reaches the point of highest curvature, i.e. where maximum stresses are present (Fig. 104 e).

This condition is also present, though not in such an exemplary form, in the various other sections also radiating from the conversion point.

Another type of deformation of the same magnitude is present in the

sections crossing transversely to the radiating lines.
This deformation consists of a section curvature increasing in deformation as the sections approach the arch's point of maximum curvature. This deformation is not only limited to curvature; deformation is also markedly expressed in the deformation or increased size of the circles.(Fig. 108 e).

Conclusion:

It is the contention of the author that maximum stress directions radiate from the convergence point of the arches; these stresses become accentuated as the lines approach the arches' greatest curvature points, then diminish as the curvature of the arch lessens.

Sections Under Load:

Observations: There is a marked difference as to the vertical displacement of the two sections subjected to concentrated loading. The magnitude of vertical displacement at the point of lower slope is almost twice the size as the concentrated load magnitude occurring in between the high point of the arches. (Fig. 114)

Maximum and Minimum Curvature:

Observation: The carrying directions of Maximum curvature follow an elyptical pattern about the point of convergence, wheras the spanning directions of maximum curvature radiate from the points of arch conversion.

Conclusion: After having analyzed the preceding facts gathered in the model analysis, it is the contention of the author that in this case, the lines of maximum and minimum curvature follow

with great approximation, the directions of maximum stress, and should be taken into account in later investigation regarding mesh construction, cable or fabric.

Overall Conclusions:

The construction design stage should consider the following conclusions gathered in this stage of analysis before proceding into an in depth structural investigation.

First. consideration should be given to the reinforcement of the tent by additional fabric overlappings or by a favorable cable arrangement for stress concentrations in places where there are the following stress concentrations:1) in the membrane area from which the arches radiate, in particular where the supporting arches have a higher degree of curvature.2) Transversely to this radiative pattern, in areas where the arches gain in height or where they gain in curvature.

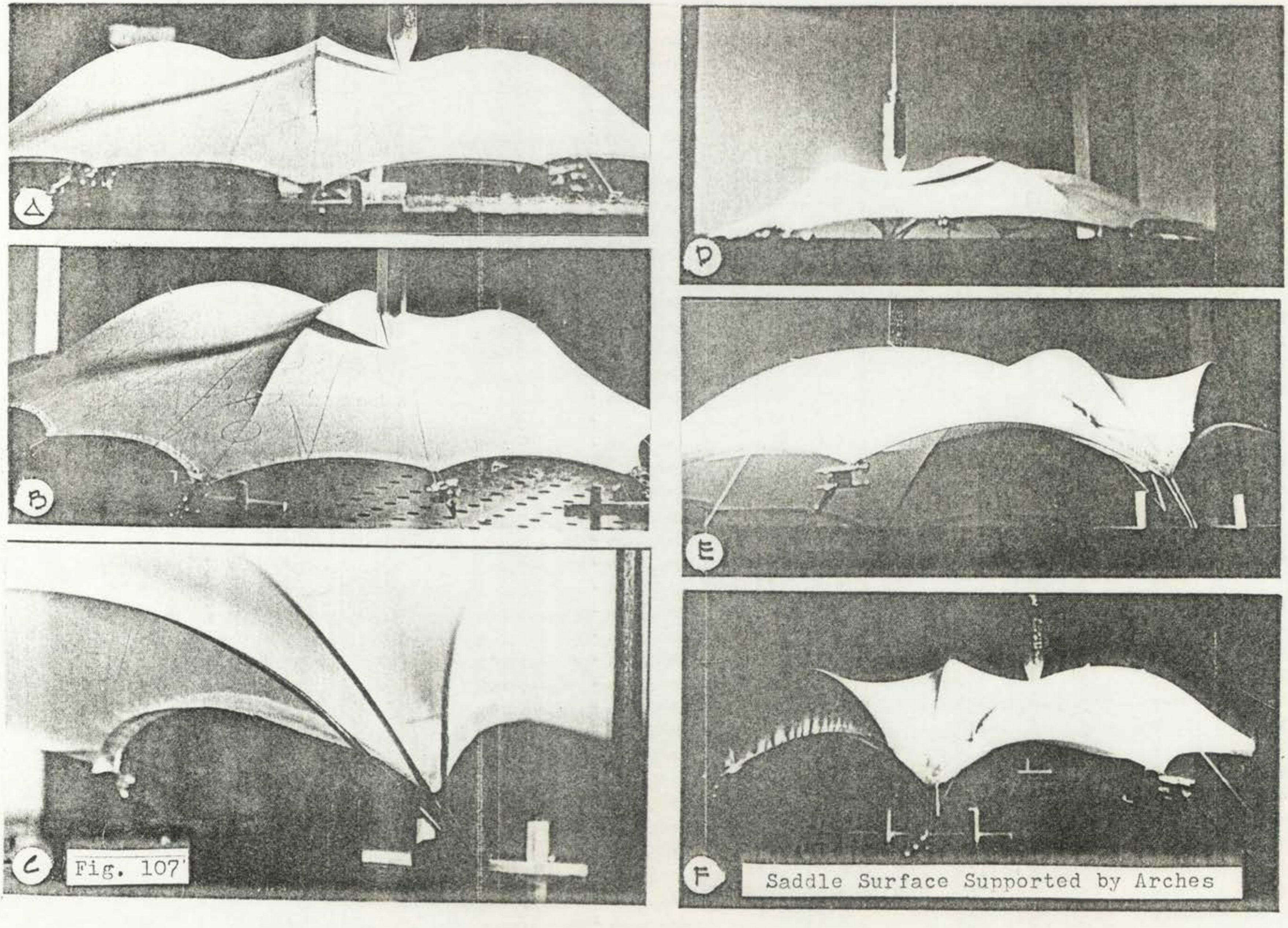


Fig. 107

Saddle Surface Supported by Arches

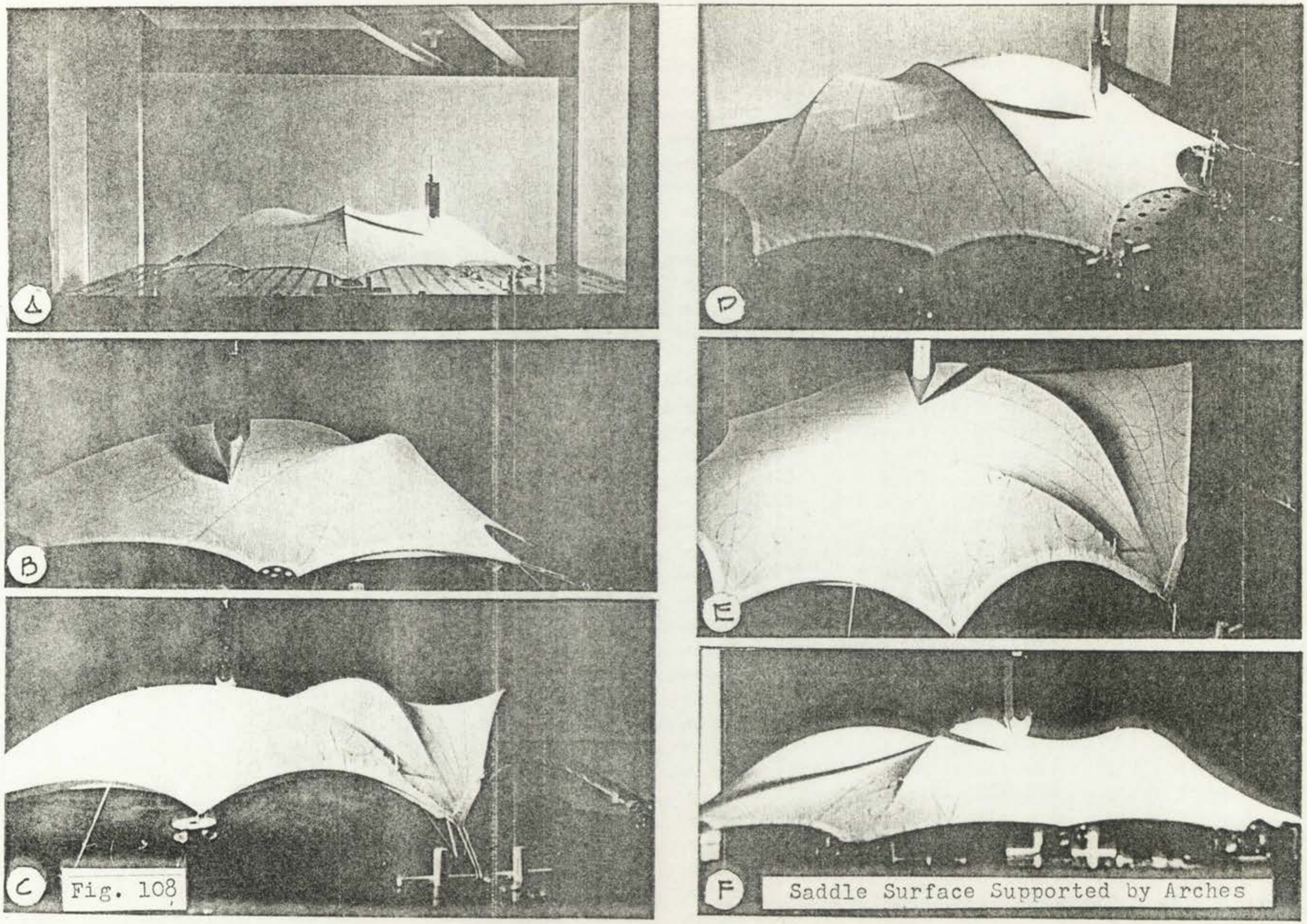


Fig. 108

Saddle Surface Supported by Arches

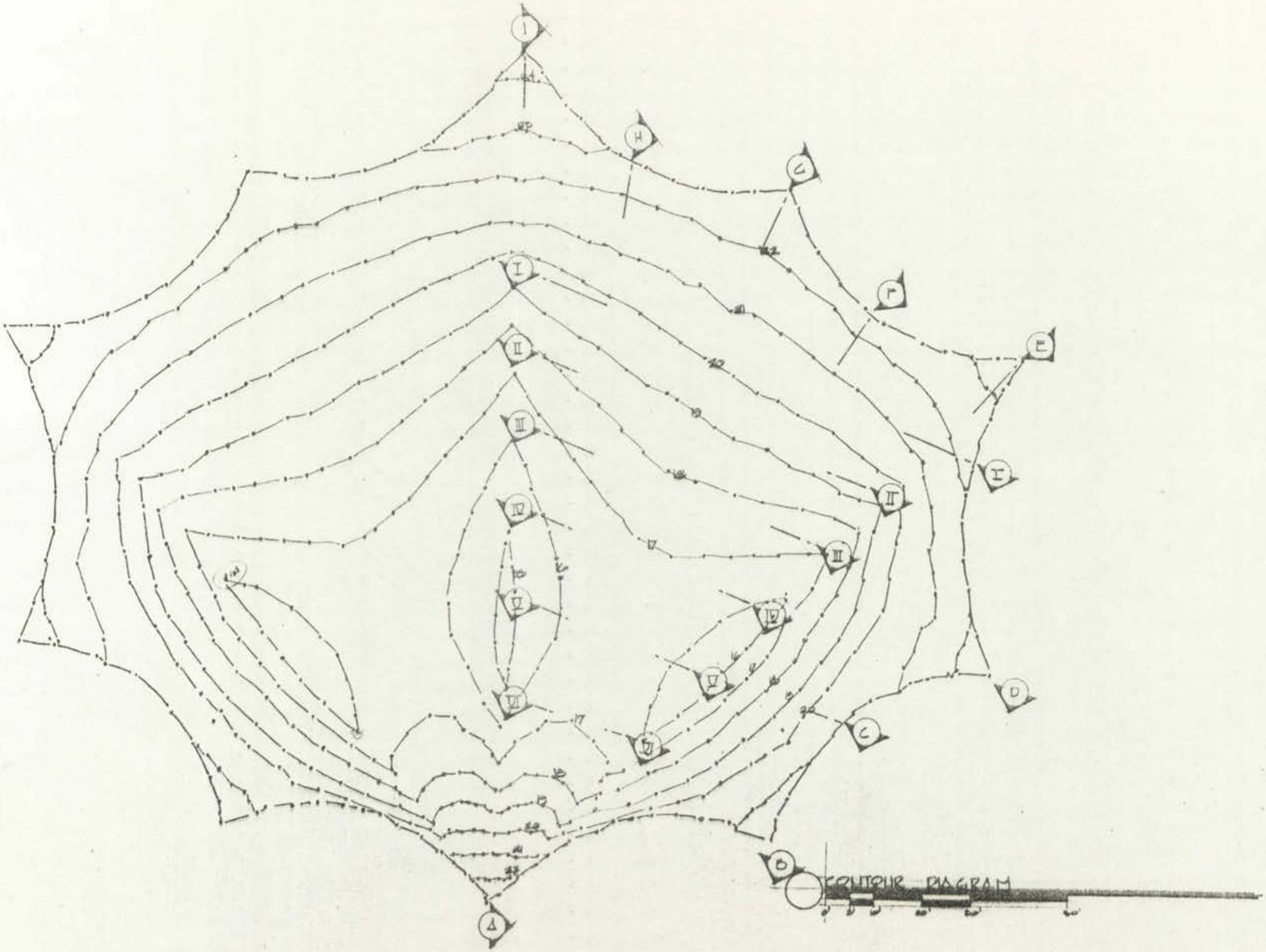


Fig. 109

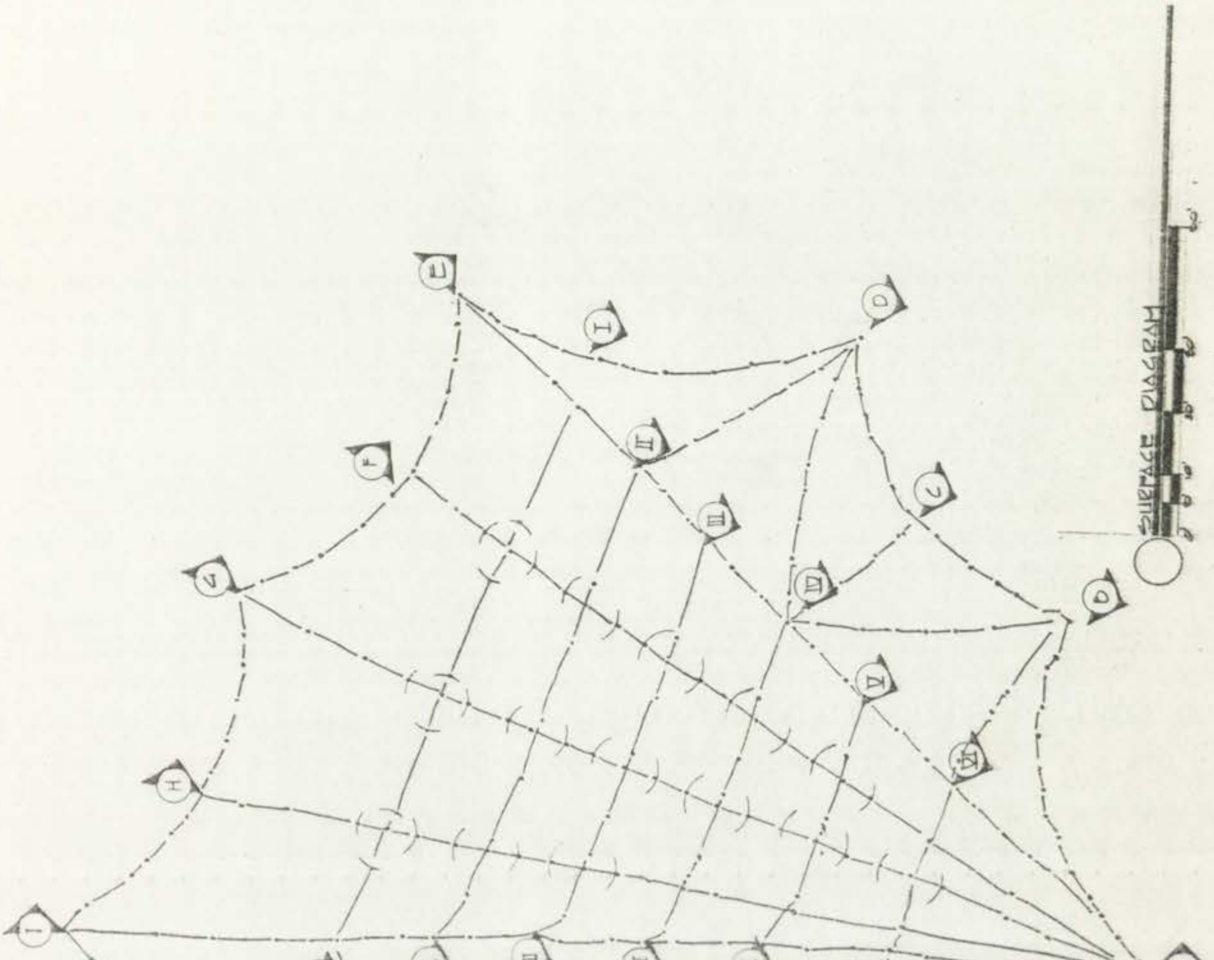


Fig. 110

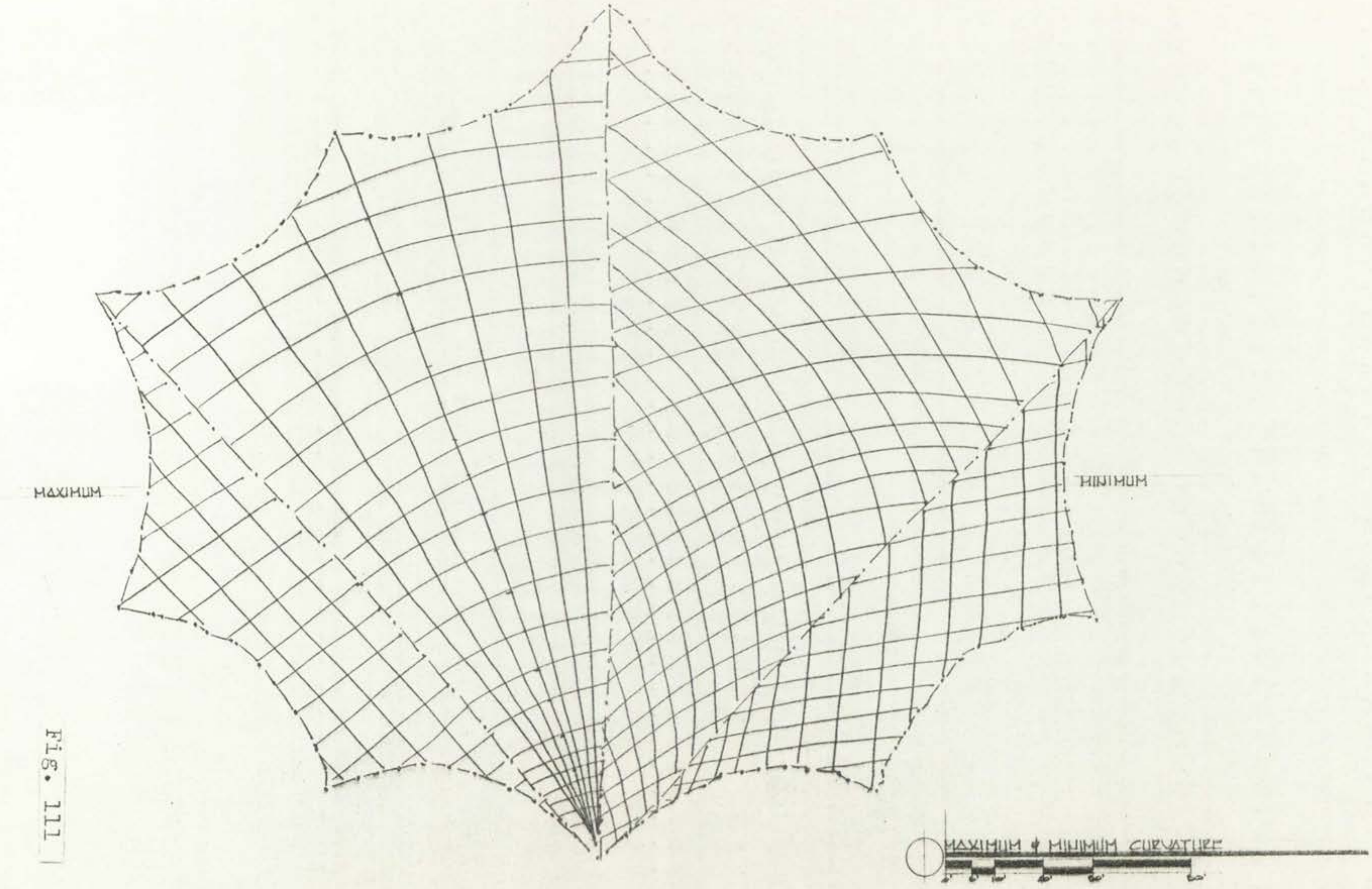


Fig. 111

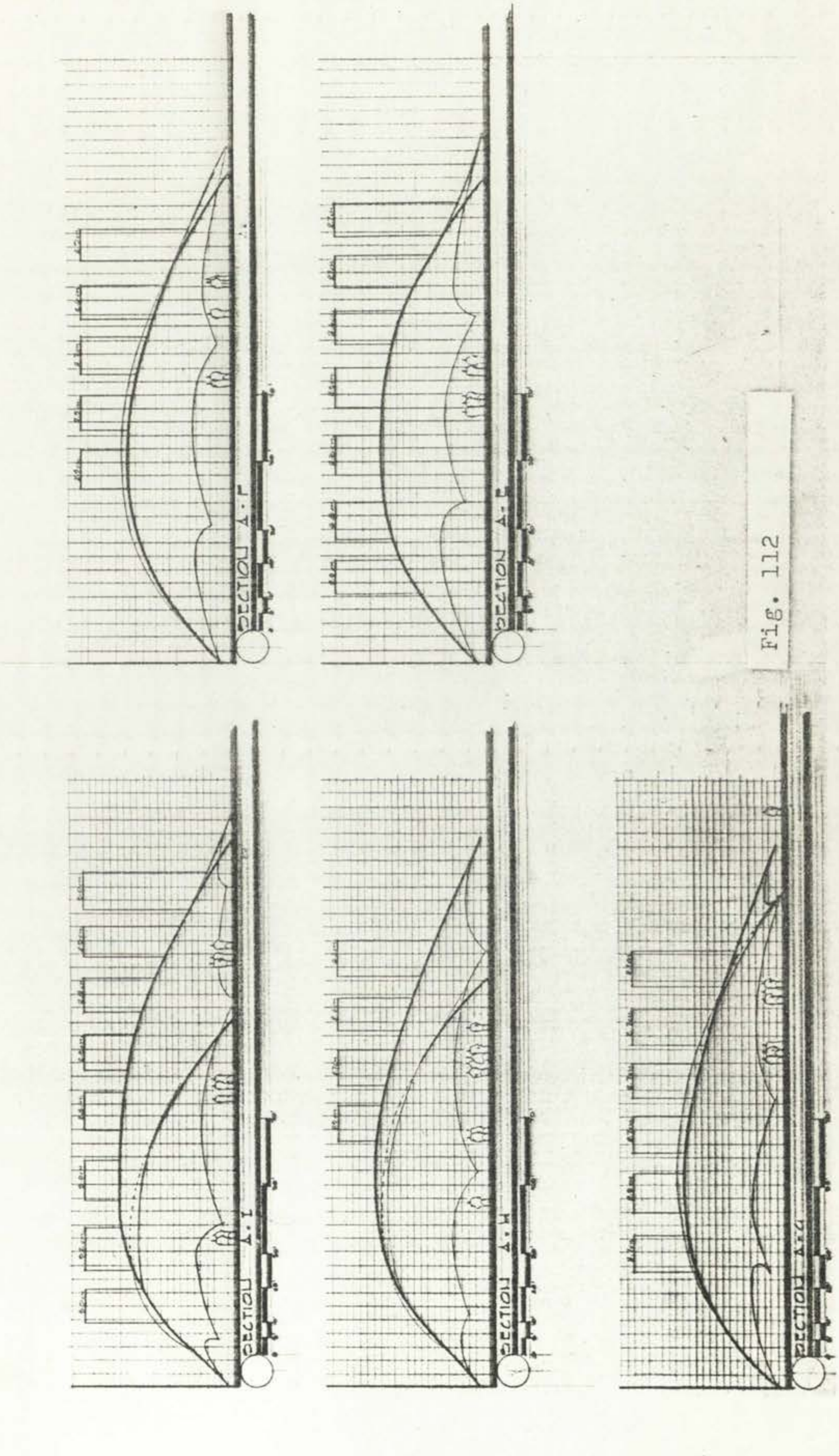
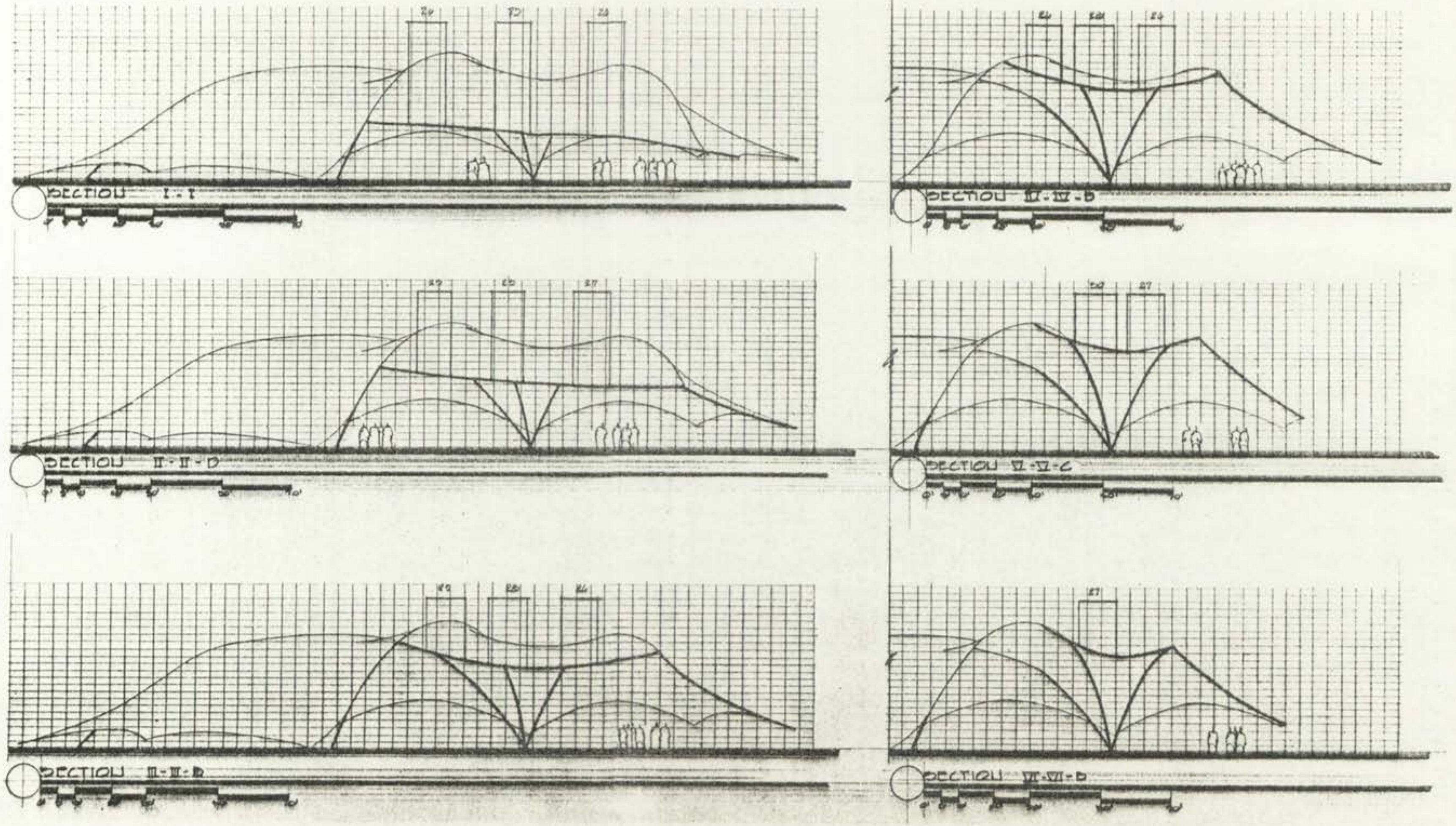


Fig. 112

Fig. 113


SECTION I-I
SECTION II-II-D
SECTION III-III-D
SECTION IV-IV-B
SECTION V-V-C
SECTION VI-VI-D

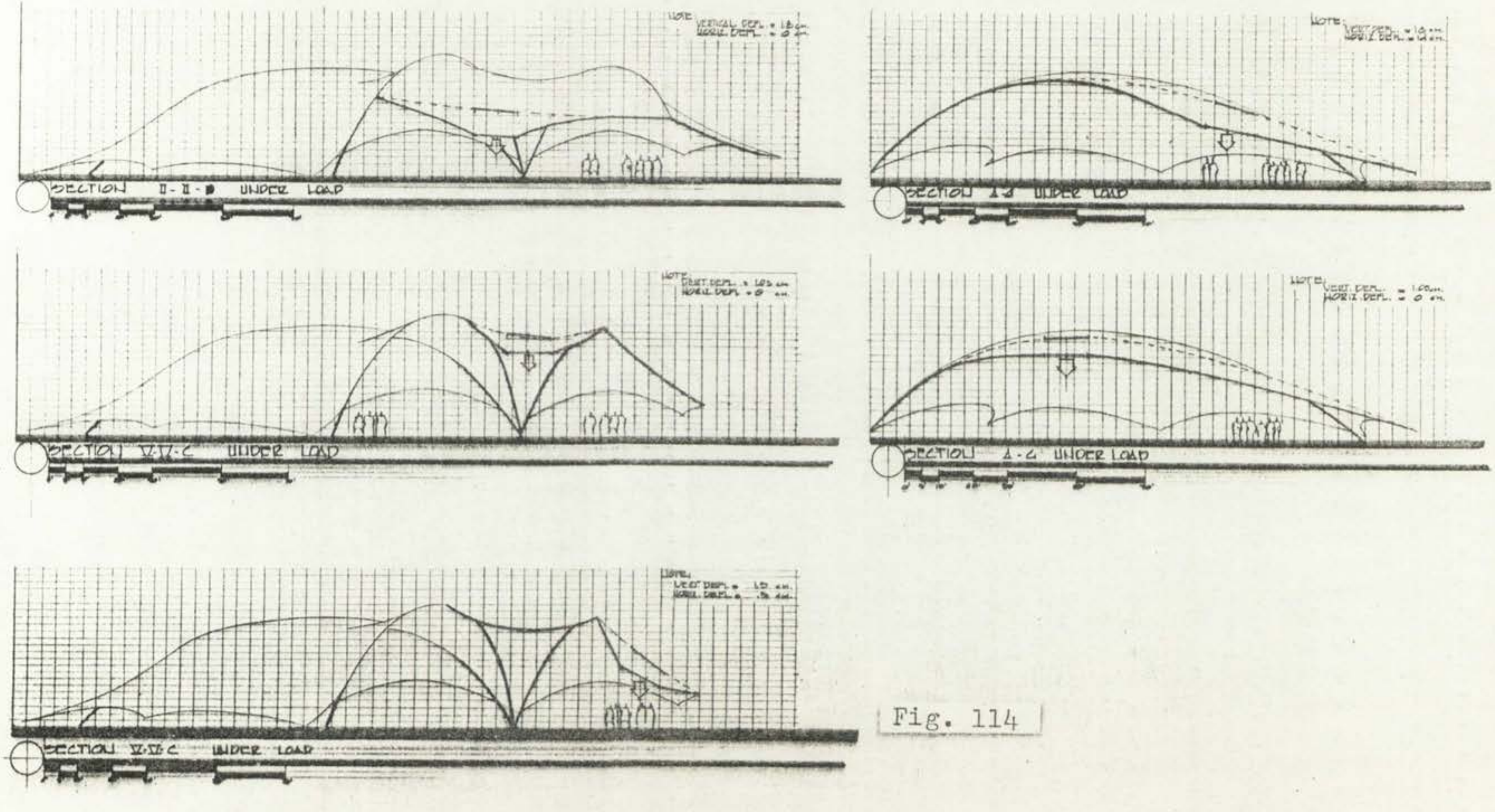

NOTE
VERTICAL DEFL. = 1.8 cm.
HORIZ. DEFL. = 0 cm.
SECTION II-II-B UNDER LOAD
NOTE:
VERT. DEFL. = 1.6 cm.
HORIZ. DEFL. = 1.4 cm.
SECTION I-I UNDER LOAD
NOTE:
VERT. DEFL. = 1.65 cm.
HORIZ. DEFL. = 0 cm.
SECTION V-V-C UNDER LOAD
NOTE:
VERT. DEFL. = 1.60 cm.
HORIZ. DEFL. = 0 cm.
SECTION A-A UNDER LOAD
NOTE:
VERT. DEFL. = 1.5 cm.
HORIZ. DEFL. = .5 cm.
SECTION VI-VI-C UNDER LOAD

Fig. 114

FOOTNOTES

[1] Conrad Roland, Frei Otto: Tension Structures, (New York, 1970) pp. 52-53.

[2] Ludwig Glaesser, The Work of Frei Otto, (New York, 1972) pp. 29.

Humped Surfaces

Humped Surfaces

Surfaces With High Points

Humped surfaces are divided into two families: the classical point shaped surface and the hump shaped surface. These types of structures are quite practical for structures of short span with internal supports. The interesting fact about these structures is that the point of suspension is minimal in comparizon to the size of the surface area and membrane thickness. The advantage of humped surfaces is that the stresses caused by this type of support can be resisted by having the largest possible bearing areas, curved to relatively rounded shapes, attached to the supports. If a large enough area is utilized, the membrane should be pushed up a little; this is due to the relationship : size or area of hump is directly proportional to the horizonal deflection occurring on the tent i.e. the bigger area of the support the smaller the deflection is going to be.[1]

The disadvantage to point shaped surfaces is that the surface tension at or near the supports is so great that the membrane would have to have a tremendous amount of reinforcing materials in those areas in order to make such a structure feasable. Therefore in order to avoid such extreme load concentrations at the support points, fabric membranes should be reinforced or strengthened at the vecinity of these high points by providing large, gently rounded supports in particular by spatially curved intersecting cables.

Surfaced with Low Points

This same principle applied in reverse produces a saddle shaped surface having low points as the area of greater stress concentration.

Surfaces with High and Low Points

By applying deformations in alternately opposite directions in relation to the horizontal; surfaces with high and low points are formed. The deformation of this membrane need not be perpendicular, indeed supports can be placed obliquely to the horizontal. (Fig 116 f).

Another feature of this type of saddle shaped surface is its ability for addition. Great spanning structures are possible with a minimum and stress concentration at any single point on the surface.

Previously Constructed Humped Surfaces

Humped Pavillion

Federal Garden Exhibition
Cologne Germany, 1957
Architect: Frei Otto.

Description: The structure consist of two intersecting humped surface with a saddle shaped configuration each having one high point. The structure as a whole has nine anchor points.

Maximum Length: 79 Feet.
Maximum Height: 14 feet.

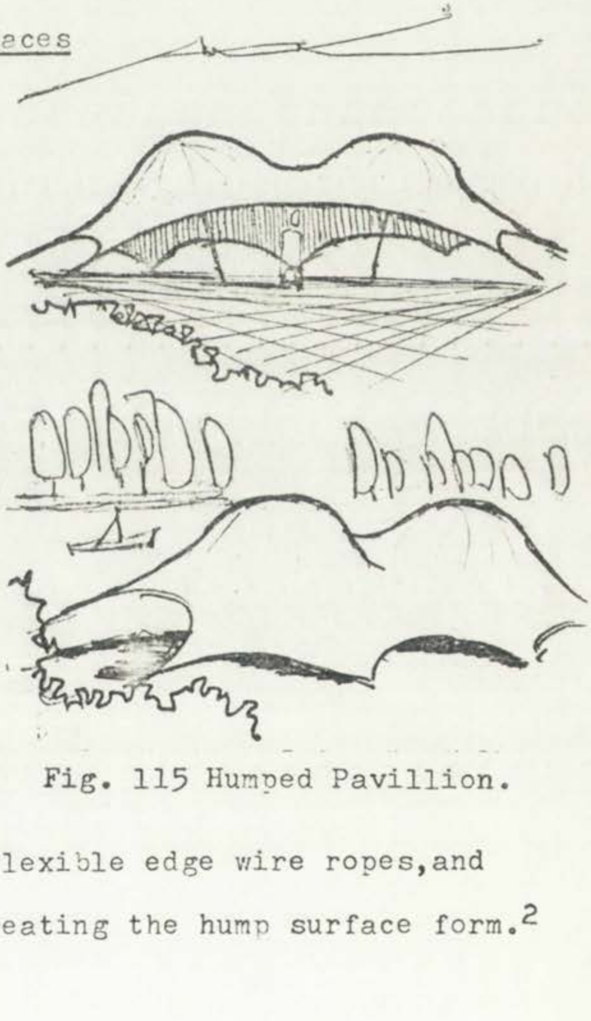


Fig. 115 Humped Pavillion.

Materials: Yellow cotton canvas, flexible edge wire ropes,and poles capped with spreader bars creating the hump surface form.[2]

Interbau Cafe

Interbau International Building Exhibition.

Berlin, Gemany 1957

Architect: Frei Otto

Description: The structure consists of eight offset high points formed by poles capped with star shaped heads made of plywood, which are flexible when in compression and form light hump supports. It also has three restraining grooves formed by transverse cables, and four anchor points.

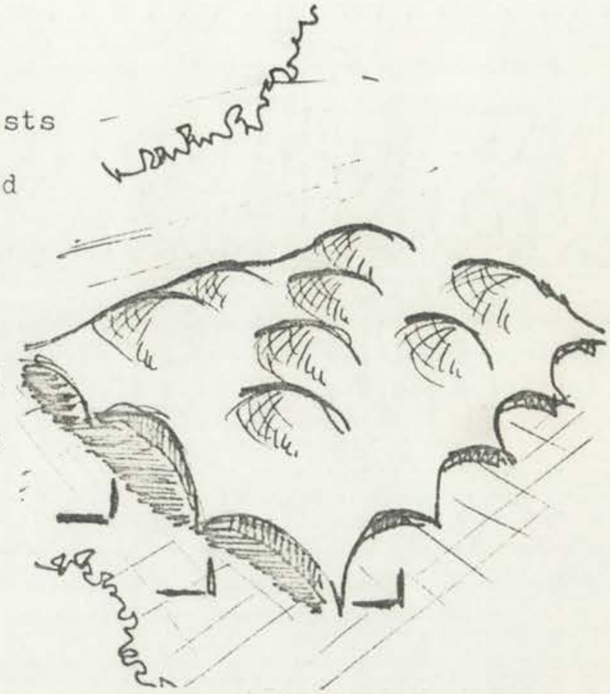


Fig. 116 Interbau Cafe

Length: 92 feet.

Width: 79 feet.

Compression at each support: 1,873 lbs.

Material: Synthetic fabric, edge cables, telescopic poles of varying heights.[3]

German Pavillion - Expo 67.

Montreal Canada, 1967.

Architect: Frei Otto, Rolf Gutbord

Description: Composite of high and low point surface with eight support points, three restraining points in combination with three continuous ridges, and 31 perimeter points.

Maximum Length: 427 feet.

Maximum Width: 346 feet.

Covered Area: 86,000 sq. ft.

Mast heights : 46 - 125 feet.

Materials: Net wire rope; edge, eye and ridge wire ropes; hung translucent sythetic fabric and tapered tubular steel masts.[4]

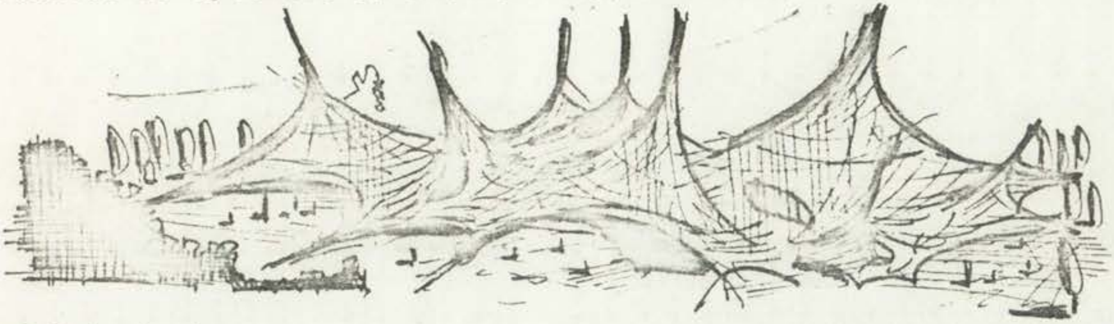


Fig. 117 German Pavillion

Model Analysis

Humped Surface with a High Point

Description: High point surface with twelve anchor points arranged in a circle. (Fig 120 c).

Loading

Observation: It was observed that the major concentration of forces radiated at 180 behind the high support.

Conclusion: A greater concentration of stress is present in an area 180° in back of the high point of support.

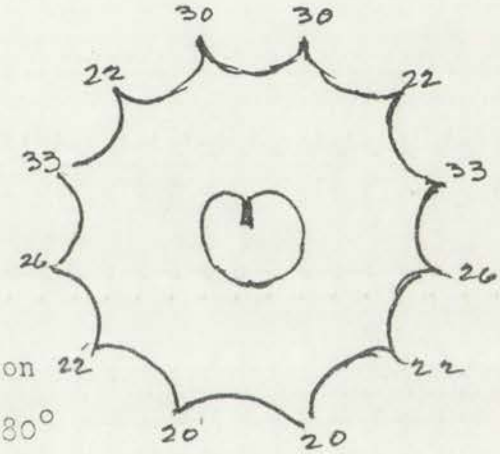


Fig. 118 Loading

Contour Diagram

Observation: There is a greater concentration of contour lines directly behind the high point of support becoming closer together as the lines approach the principal point of support. (Fig 121).

Sections Not Under Load.

Observation: this series of sections , A-A, B-B, C-C & D-D(Fig.124), provide the reader with a very concise knowledge as to the magnitudes of stress present at almost all points on the tent. This

is due to: first, the inclination of the sections, and second, to the displacement or deformations of the individual circles toward the center. These circles were placed at three individual distances from the center, each distance being equidistant from the next. The following figure explains in greater detail these conditions.

Lower Slope	Inner Circle Size	Middle Circle Size	Outer Circle Size
Section D-D	3.2 cm.	2.8 cm.	2.7 cm.
Section C-C	2.8 cm.	2.8 cm.	2.8 cm.
Section B-B	2.8 cm.	2.7 cm.	2.6 cm.
Section A-A	3.0 cm.	2.9 cm.	2.9 cm.
Section B-B	3.1 cm.	3.1 cm.	2.8 cm.
Section C-C	3.3 cm.	2.8 cm.	3.0 cm.
Section D-D	4.6 cm.	2.8 cm.	2.9 cm.
Higher Slope			

Fig. 119 Ratio Between Circle Magnitude and Slope

The observation that Fig. 119 provides us with, does not mean that the tensions of the membrane are symetrical; if one observes Sections G-G, H-H and I-I (Fig. 125), one sees the asymmetrical sections produced with major curvature concentrated toward the back of the principal point of support.

Conclusions: Taking into account the slope of all sections, the deformations of the circles taken at each section, one sees that as the sections approach the back of the point of support i.e. where the slope is greatest, surface tension increases

toward the point of support.

Sections Under Load

Conclusion : From the observation that greater vertical displacement occurrs in front of the principal point of support is apparent when the reader compares it to the displacement apparent in the back of the support point, the reader can safely assume that this condition is due to the concentration of greater stress at the back of the point of support.

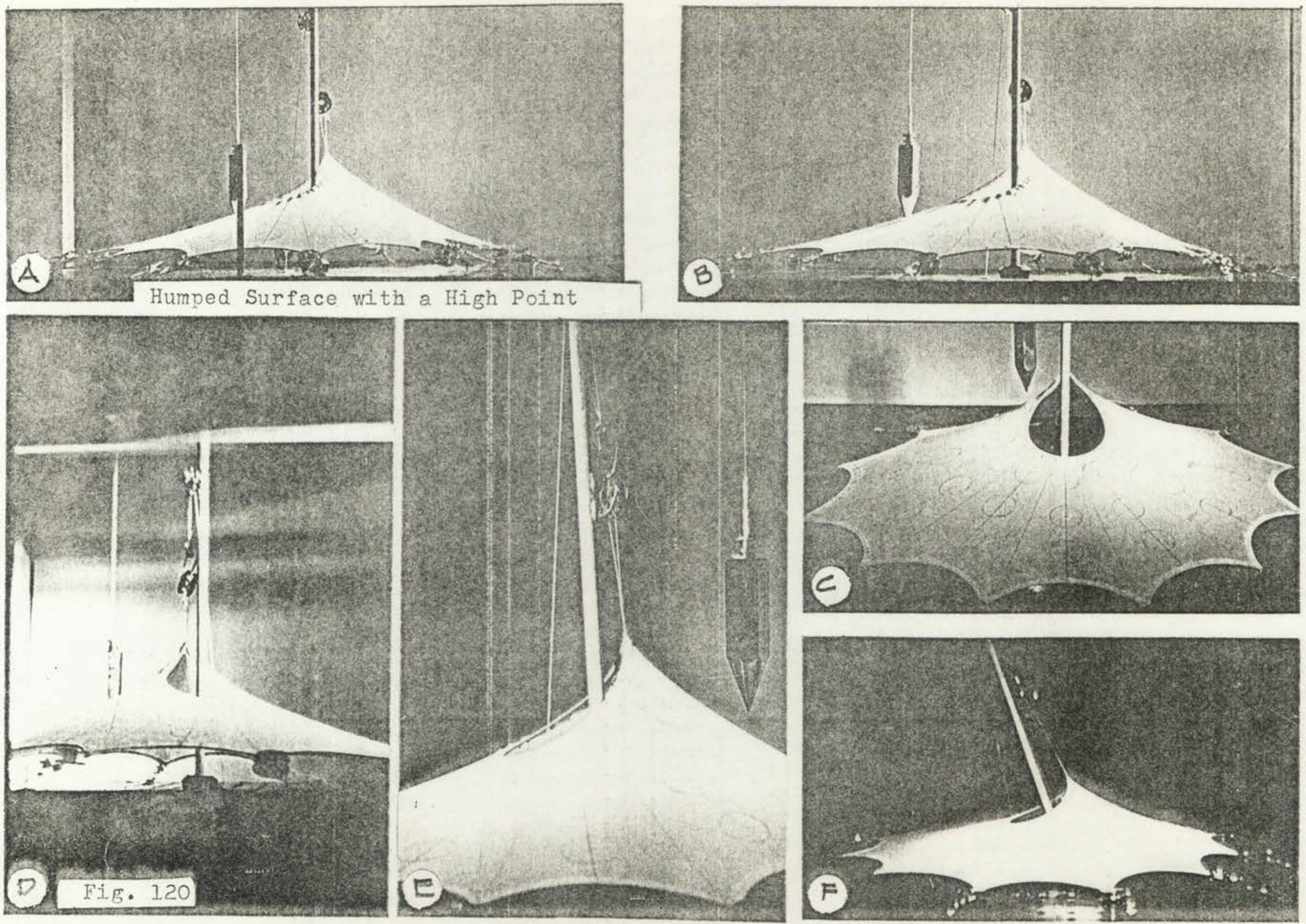
Maximum Curvature Diagram

Observation: It was observed that the carrying directions of curvature radiate from the principal points of support, wheras the spanning directions of curvature circle around it. (Fig. 123)

Conclusions: After studying the facts in the loading, contour and sections diagrams, it is the contention of the author that the maximum directions of curvature are the directions followed by maximum stresses.from the principal points of support, this condition becoming maximized in the same manner as the slope increases toward the back of the principal support.

Overall conclusions:

Considerations for stress design should center on the magnitude of stresses that follow the carrying direction of maximum curvature, with additional reinforcement placed at or near the principal point of support.



Humped Surface with a High Point

Fig. 120

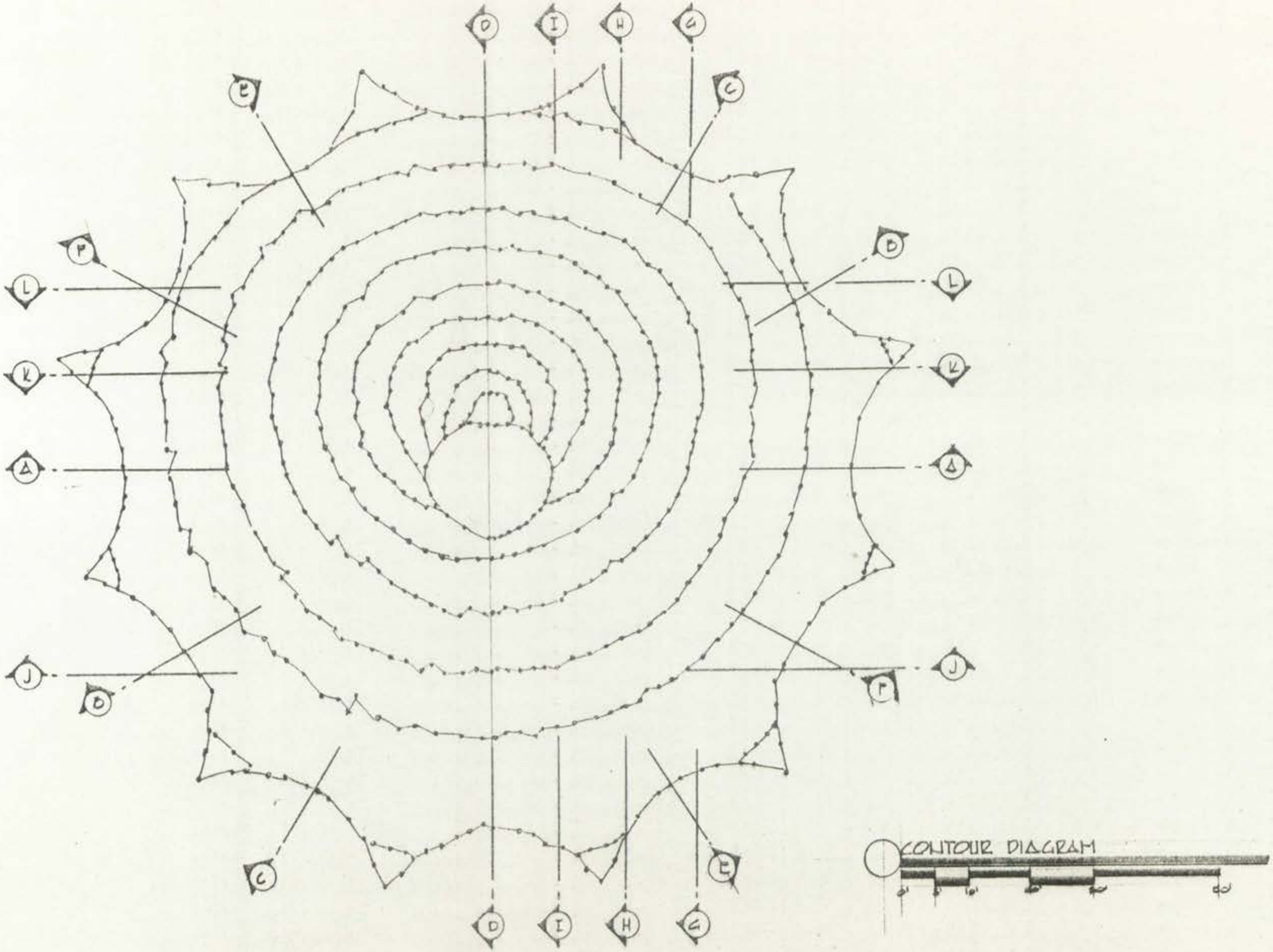


Fig. 121

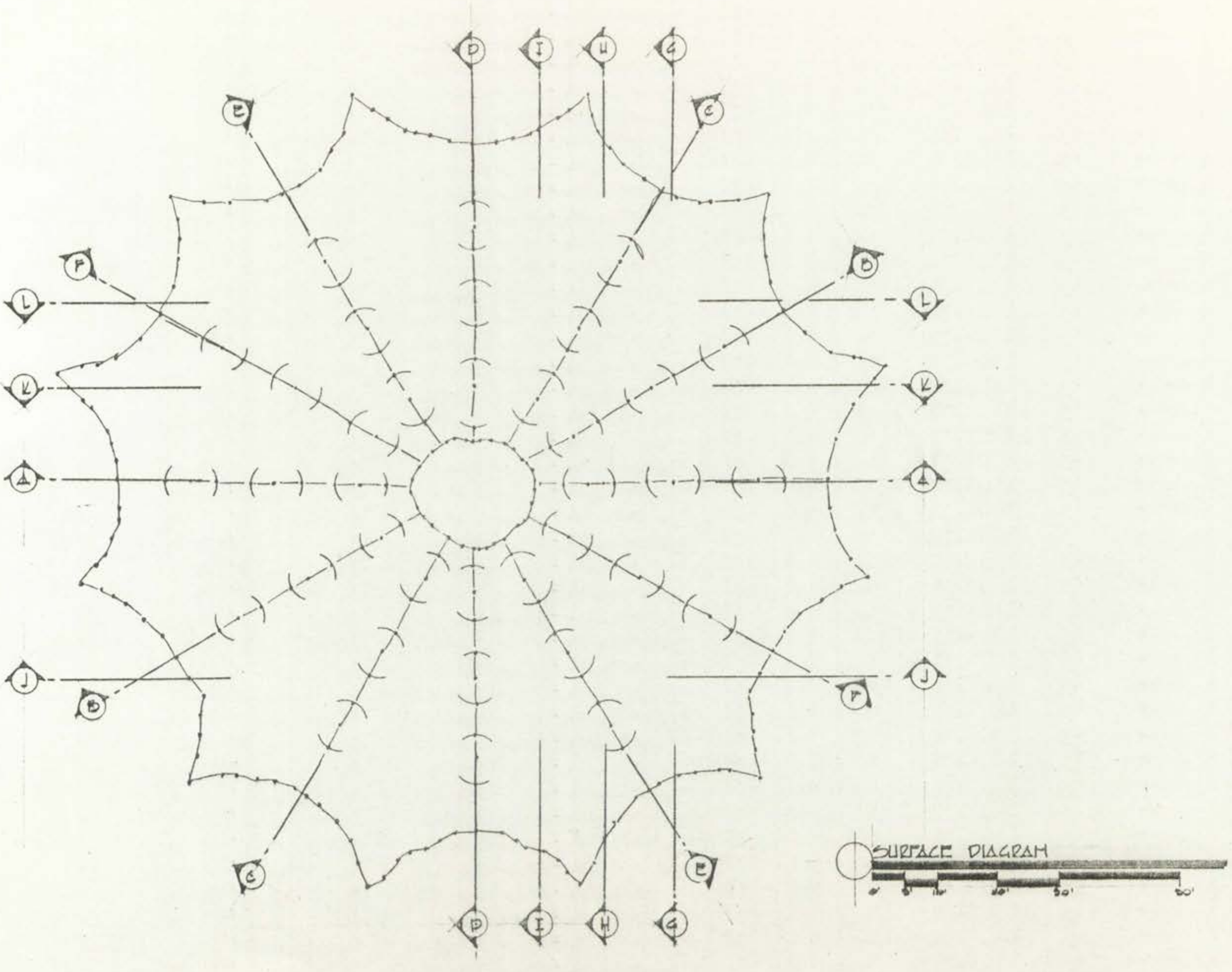


Fig. 122

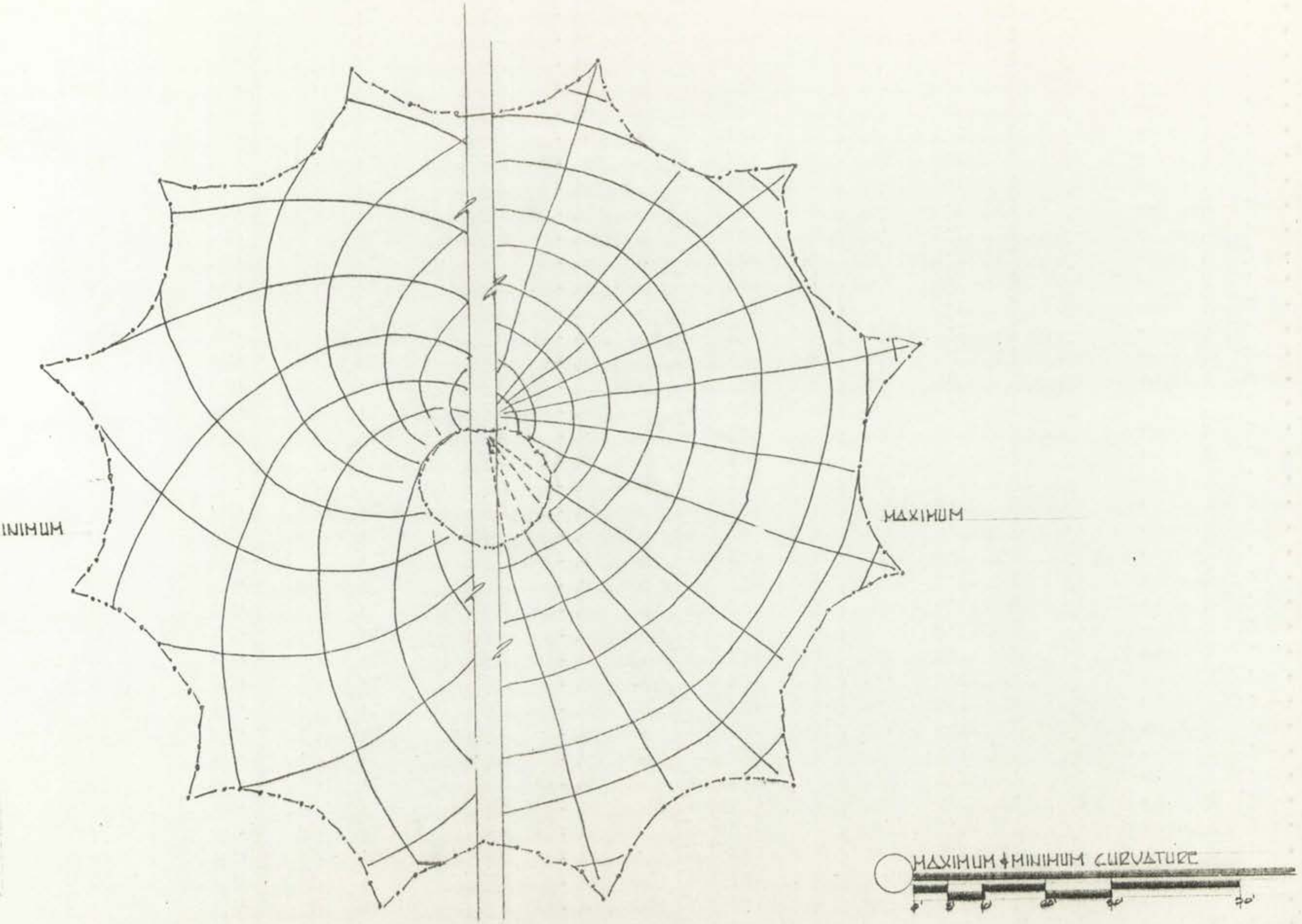


Fig. 123

Fig. 124

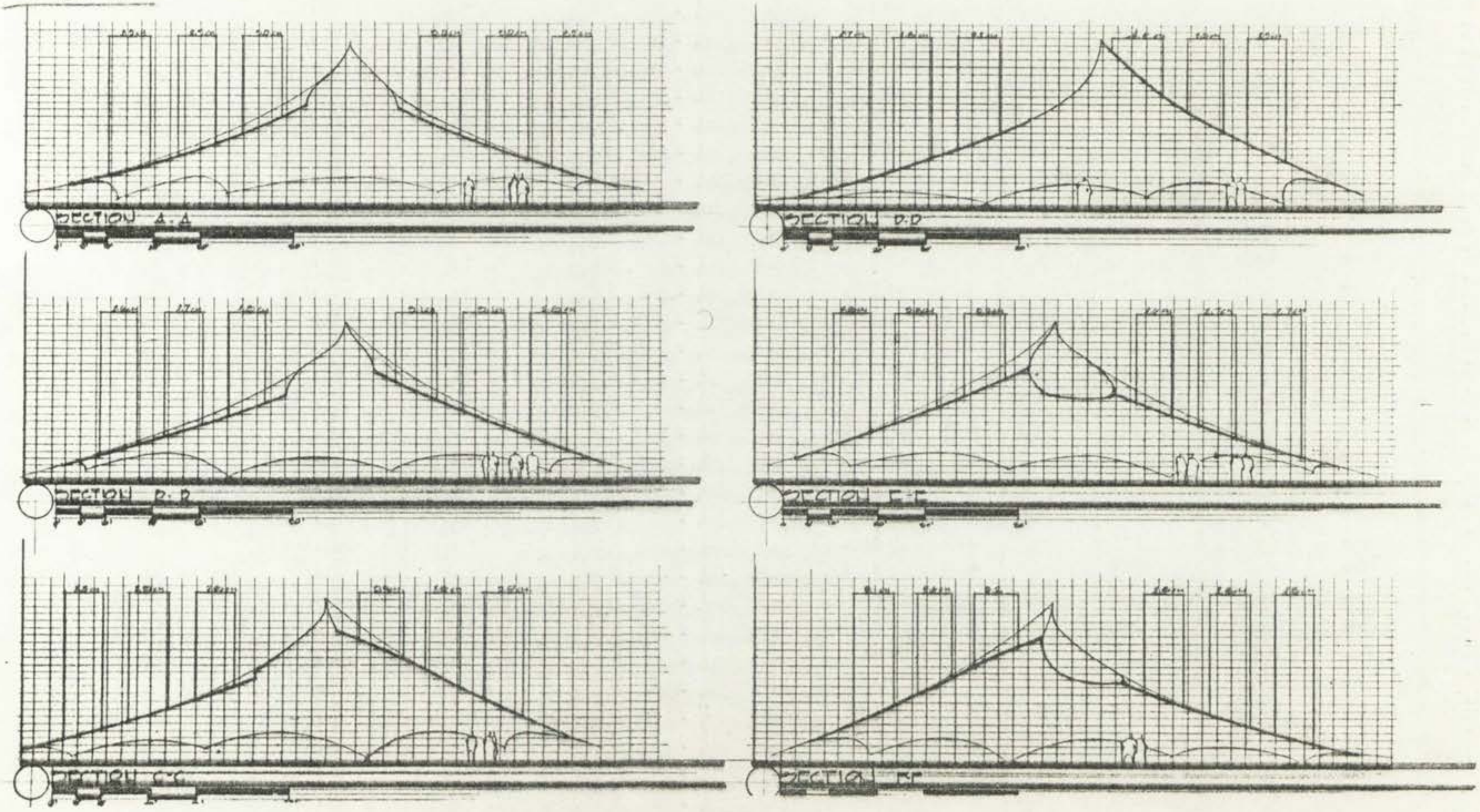

SECTION A-A
SECTION D-D
SECTION B-B
SECTION E-E
SECTION C-C
SECTION FF

Fig. 125

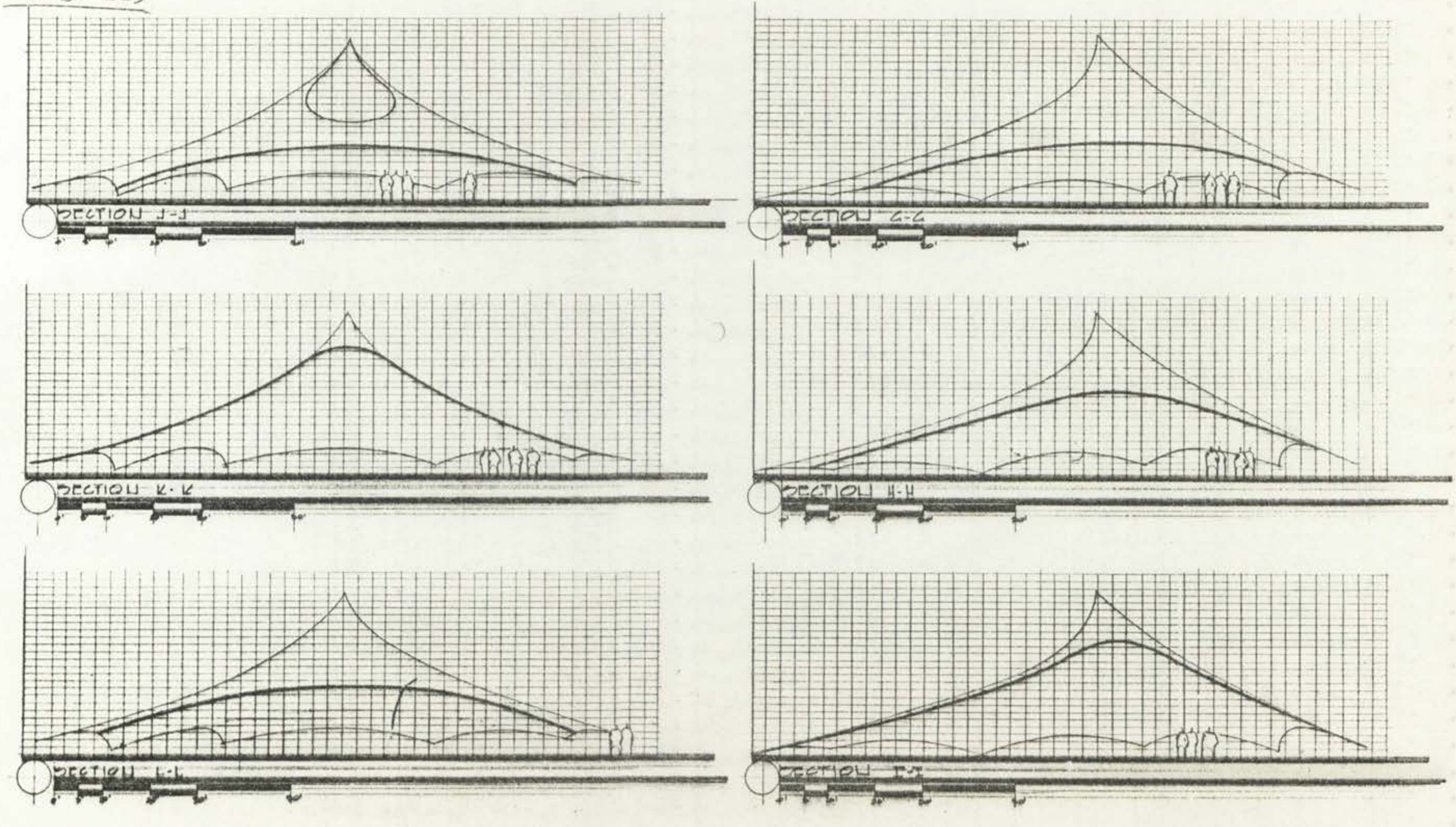
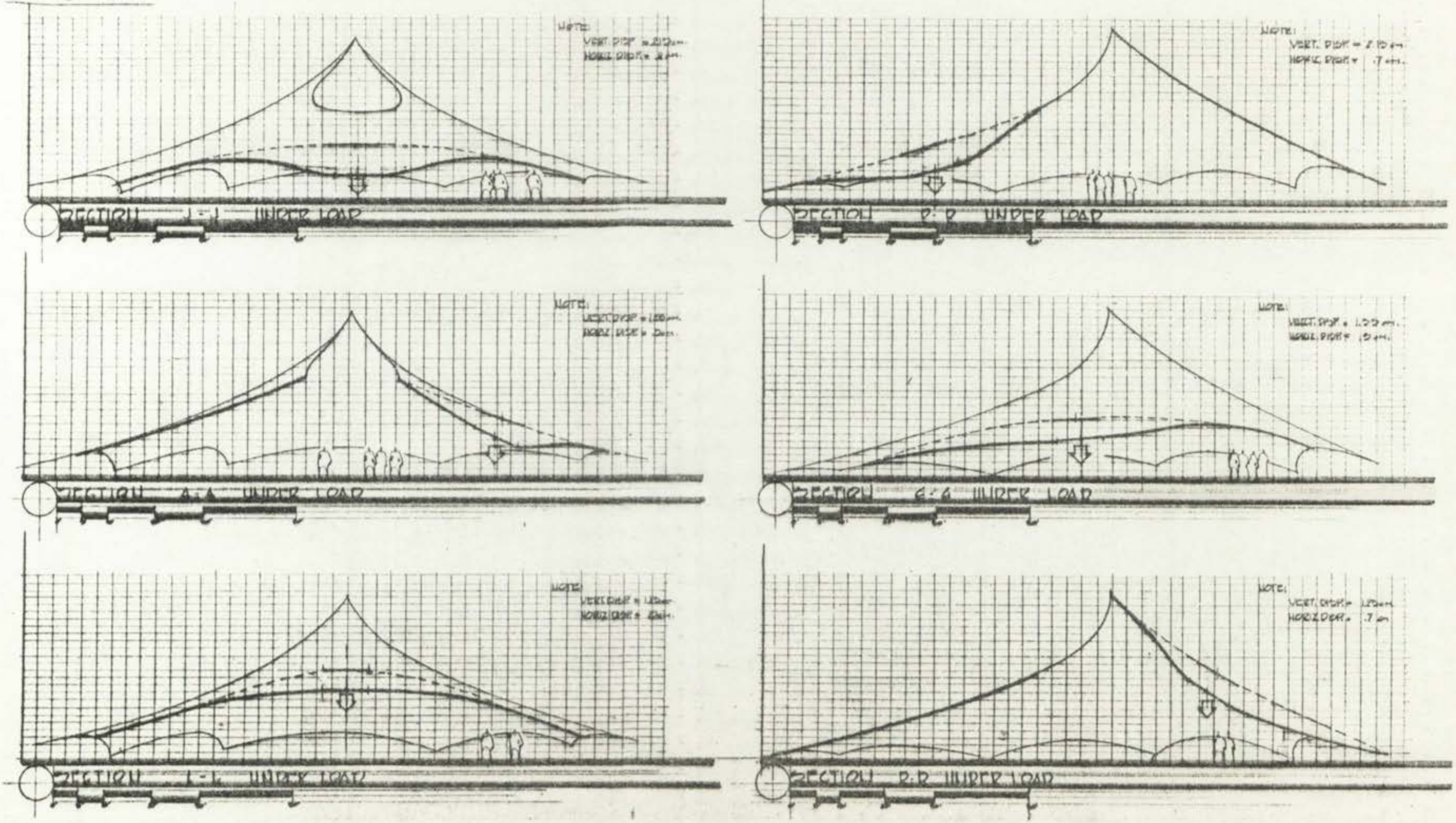

SECTION J-J
SECTION G-G
SECTION K-K
SECTION H-H
SECTION L-L
SECTION I-I

Fig. 126


NOTE:
VERT. DISP. = 2.12 m.
HORIZ. DISP. = .3 m.
SECTION 1-1 UNDER LOAD
NOTE:
VERT. DISP. = 2.15 m.
HORIZ. DISP. = .7 m.
SECTION P-P UNDER LOAD
NOTE:
VERT. DISP. = 1.20 m.
HORIZ. DISP. = .2 m.
SECTION A-A UNDER LOAD
NOTE:
VERT. DISP. = 1.22 m.
HORIZ. DISP. = .5 m.
SECTION G-G UNDER LOAD
NOTE:
VERT. DISP. = 1.20 m.
HORIZ. DISP. = .2 m.
SECTION L-L UNDER LOAD
NOTE:
VERT. DISP. = 1.25 m.
HORIZ. DEF. = .7 m
SECTION R-R UNDER LOAD

Humped Surface with High and Low Points.

Description:

The structure consists of an undulating surface having four humped compression points arranged symetrically, and four humped anchor points arranged alternately so as to create a continuous surface. It has eight guyed compression poles and eight anchor points at the edges.

Loading:

The loading on this surface was applied irregularly with greater magnitudes of load favouring the left of the model. This was done in order to investigate the changes in the deformation of the humps and the area in between the humps.

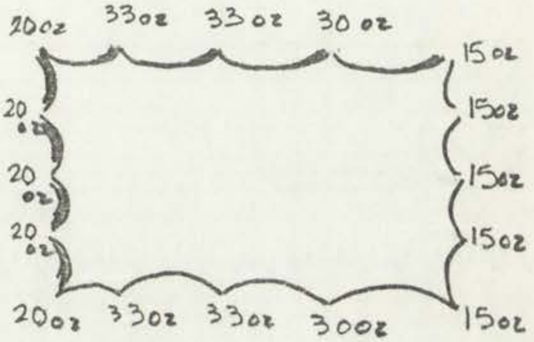


Fig. 127 Loading

Surface Diagram:

In this model, load was applied as was said, favouring the right of the model. This action did not have any noticeable effect on the overall shape of the model. Deformations do occurr enabling the reader to get an idea of the relationship between lateral tension and symmetry of plan. In this case deformations were only visible in plan,(Fig.130) but were visually unnoticeable. in the model.

Contour Diagram

This model, as was the case in the previous humped surface model,

shows a marked increase in the incrementation of contour lines as these approach the center of the positive or negative point of support. This condition is radiated in a succession of circular patterns indicating that as the contours become separated, a lesser slope is exhibited and therefore a lesser stress concentration in these areas. (Fig. 129)

There is also a marked increase in the concentration of contour lines or slope in the areas between the closer spaced supports, as opposed to the areas which have the supports further apart. This observation leads the reader to conclude that the ratio of tension from closer spaced to the separated is greater.

This action in areas of higher stress concentration is due to the fact that there is less area to which the same ammount of support stress is applied.(Fig.129)

Sections

In all of the sections encountered there is one detail that they all have in common: three points of inflection as the section cuts through the center of the supports (Detail Fig.132). This is due to the size of the humps at the supports; if bigger humps would have been used in the same points of support, less would have been the height of the supports and dimmer the representation of the three points of inflection.

These points of inflection represent the points on the surface of the tent were the following actions occurr: first, there is a change as to the degree of stress. The points of inflection next to the supports are the points at which the hump ceases

to touch the membrane and therefore starts to loose stress. This is observed by the changing of direction at this point, and the beginning of slope decreasment. Second, the point of minimum membrane stress; the point of inflection in the middle of both supports represents the point at which the membrane has minimum stress. Another observation in the sections through the tent is the ratio of slope from sections acting perpendicularl to each other. In cases where the supports were closer to each other, there is a marked increase in the rate of slope or inclination, whereas the cases where the supports were further apart, the angle of slope is less.

Conclusions.

After taking into account the information gathered from the surface diagram, the contour diagram and the sections, the author has arrived at the following conclusions regarding the stresses present in the model.

When in the design structural stage of a project of this same nature, the designer should place principal importance to reinforcement of the material in regions where humps are present. Also, consideration should be given to the arrangement of the positive and negative supports and the distance they should be placed from each other. This has a maximum bearing as to regulation of stresses on the membrane, not including the supports. Consideration should also be given to the size of the hump and the area to be supported by the hump as the size of the hump has a direct bearing on the shape of the tent and the concentration of stresses at these points.

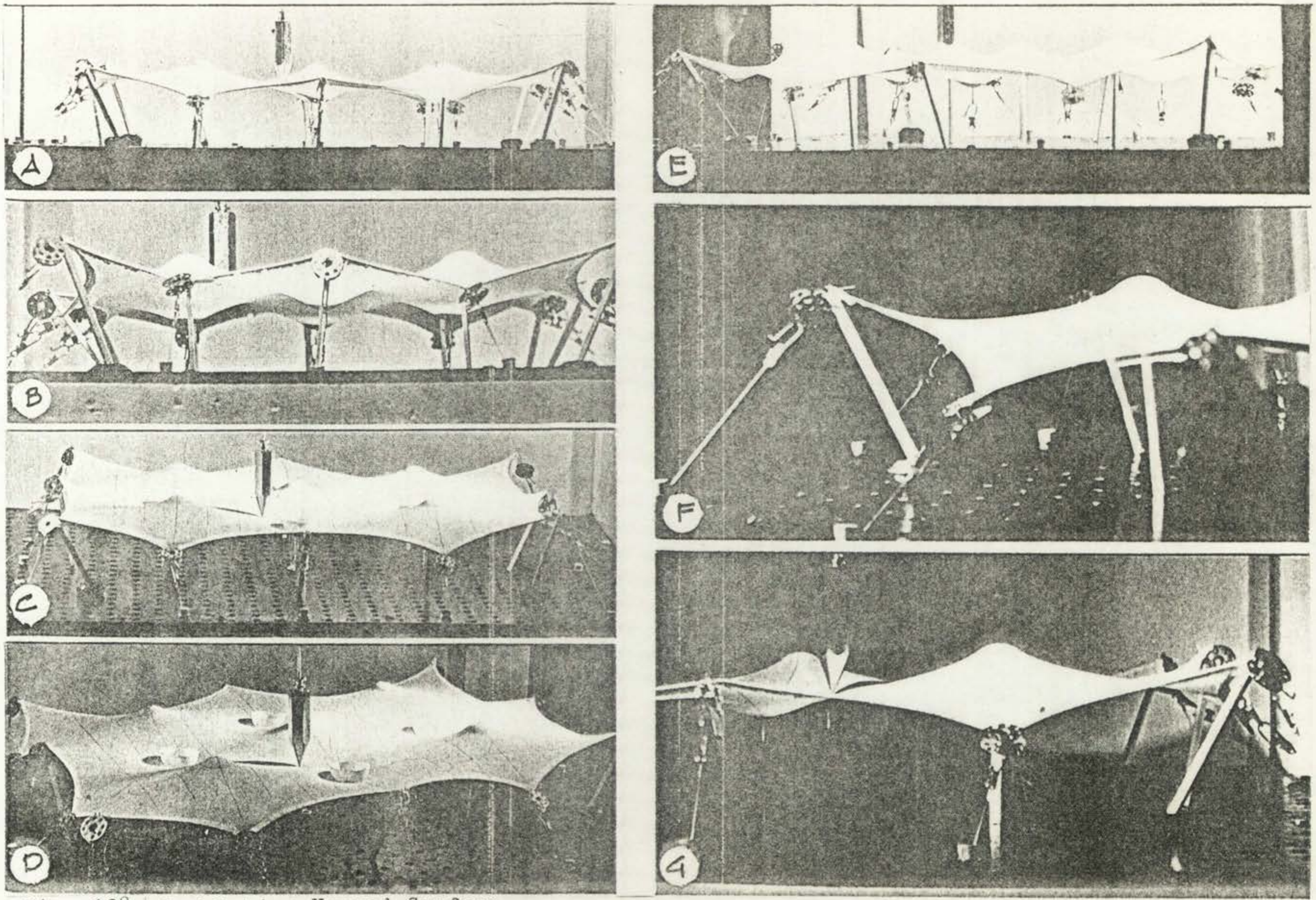


Fig. 128 | Undulating Humped Surface

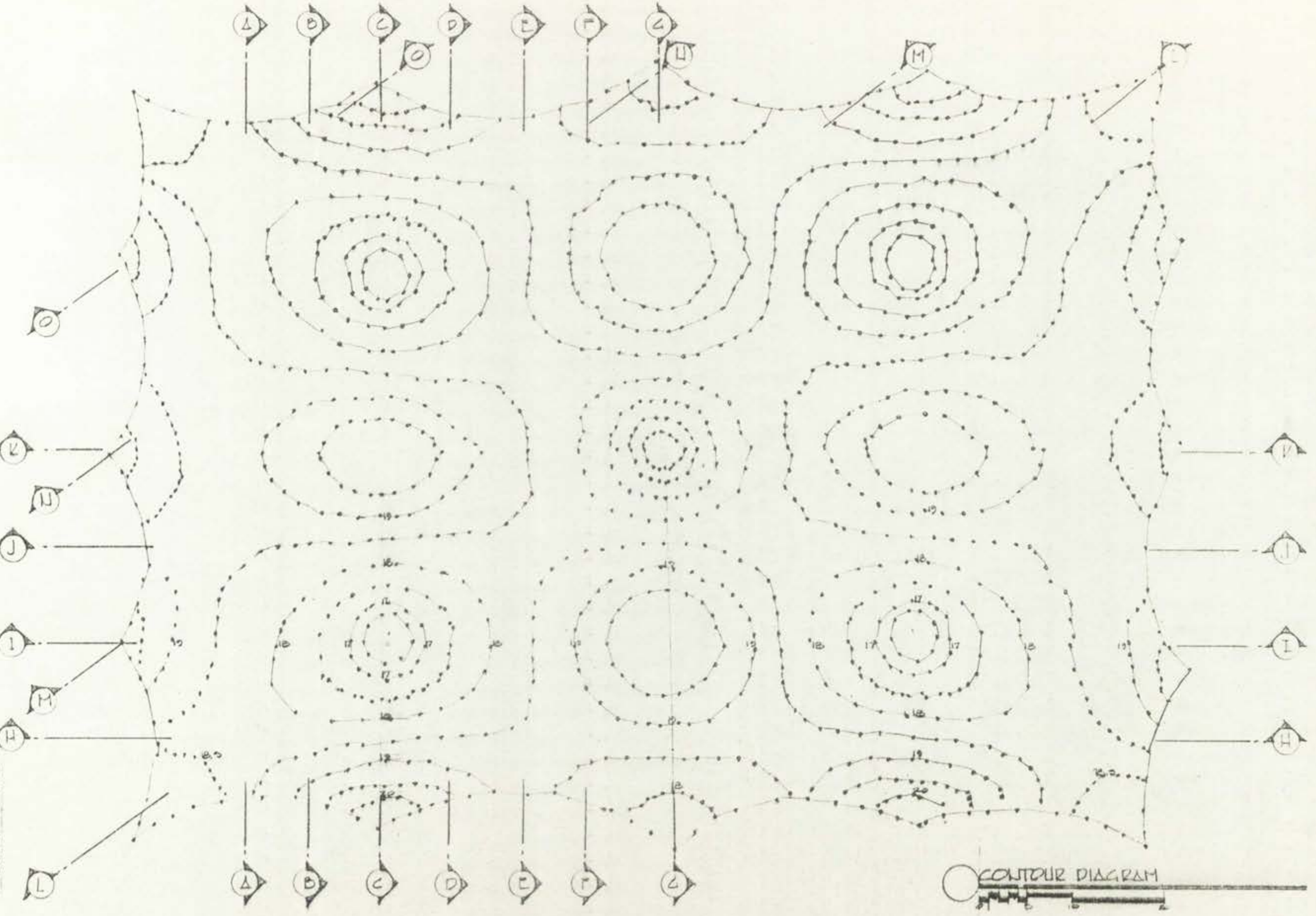


Fig. 129

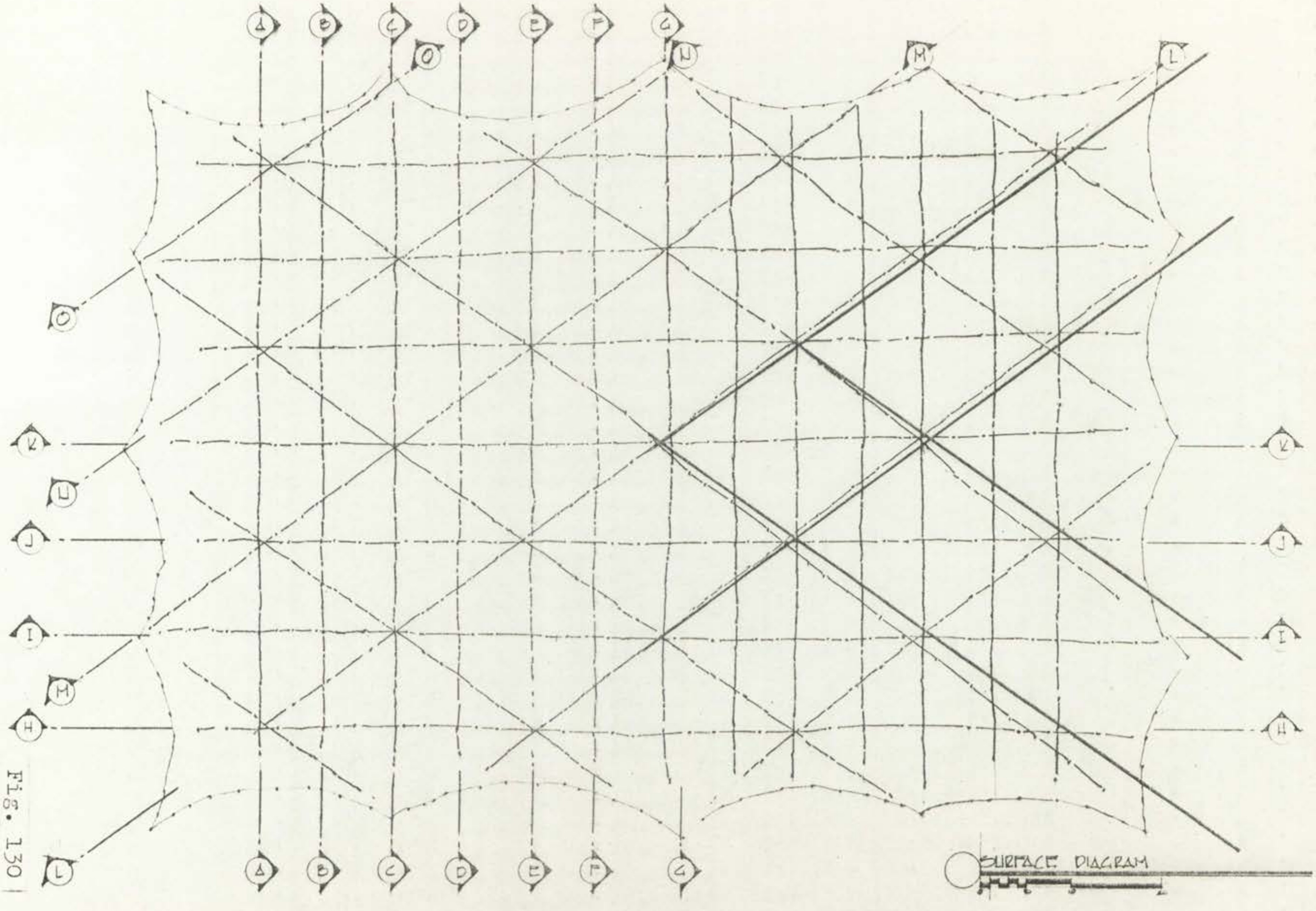


Fig. 130

130

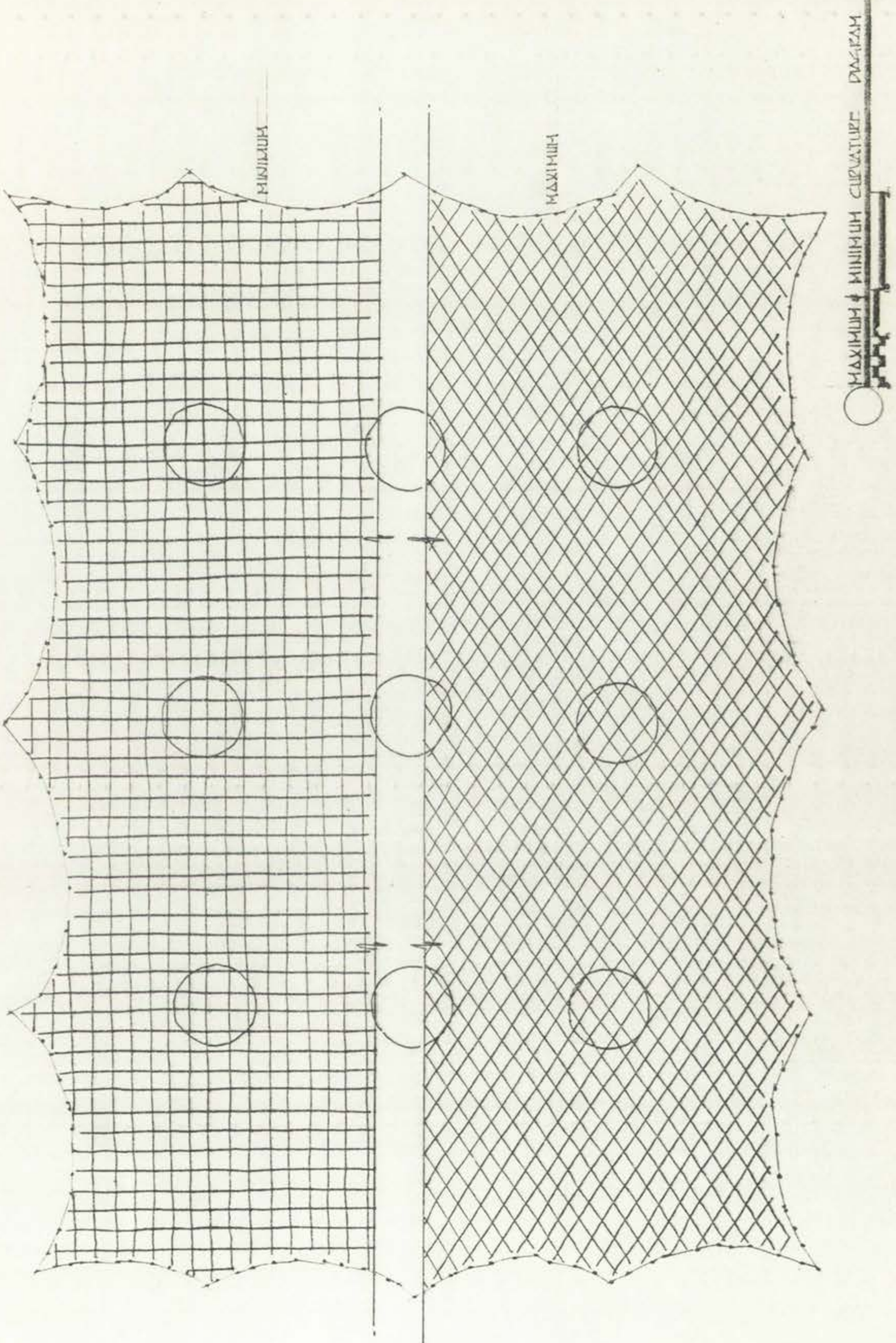


Fig. 131

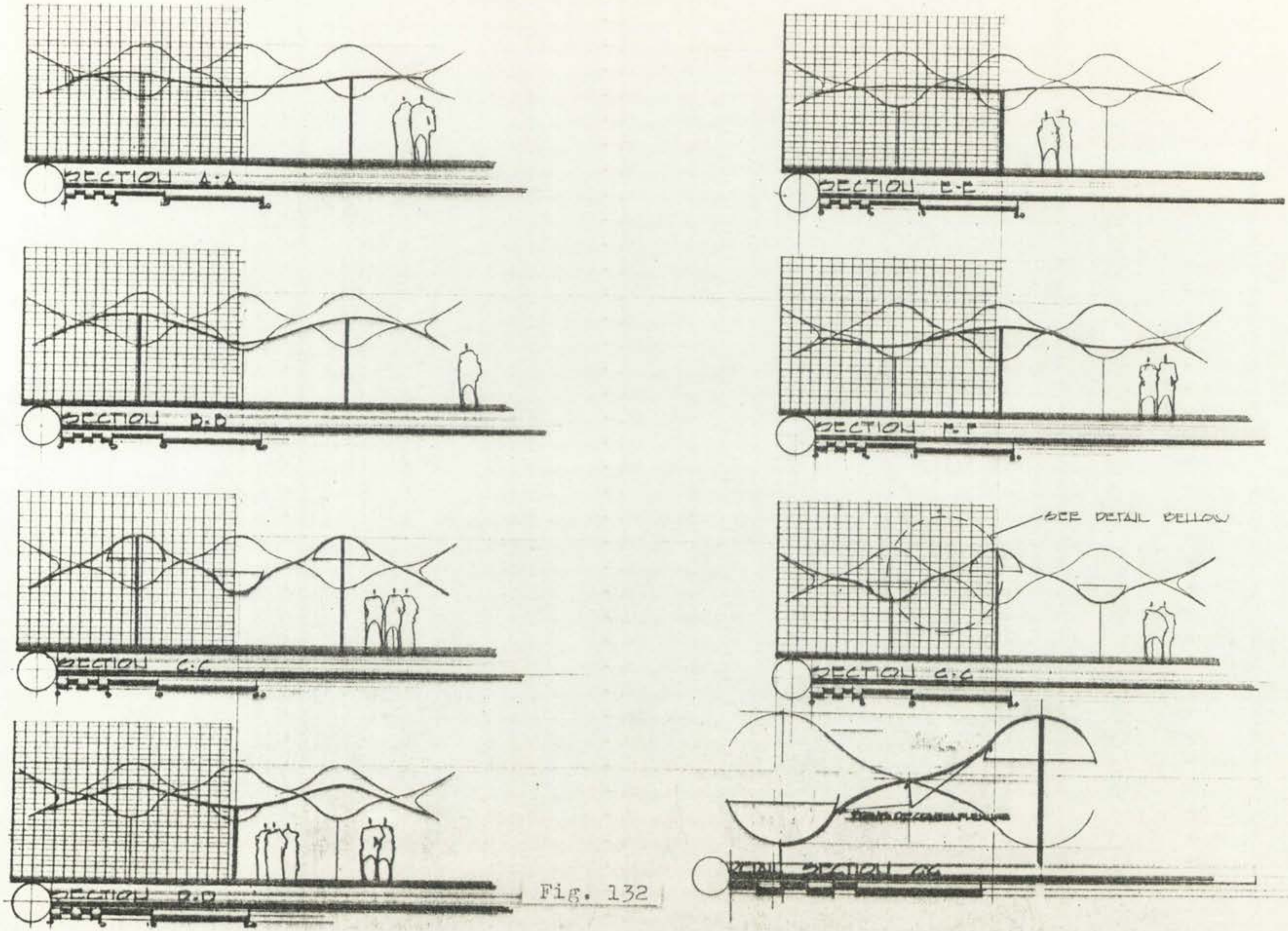
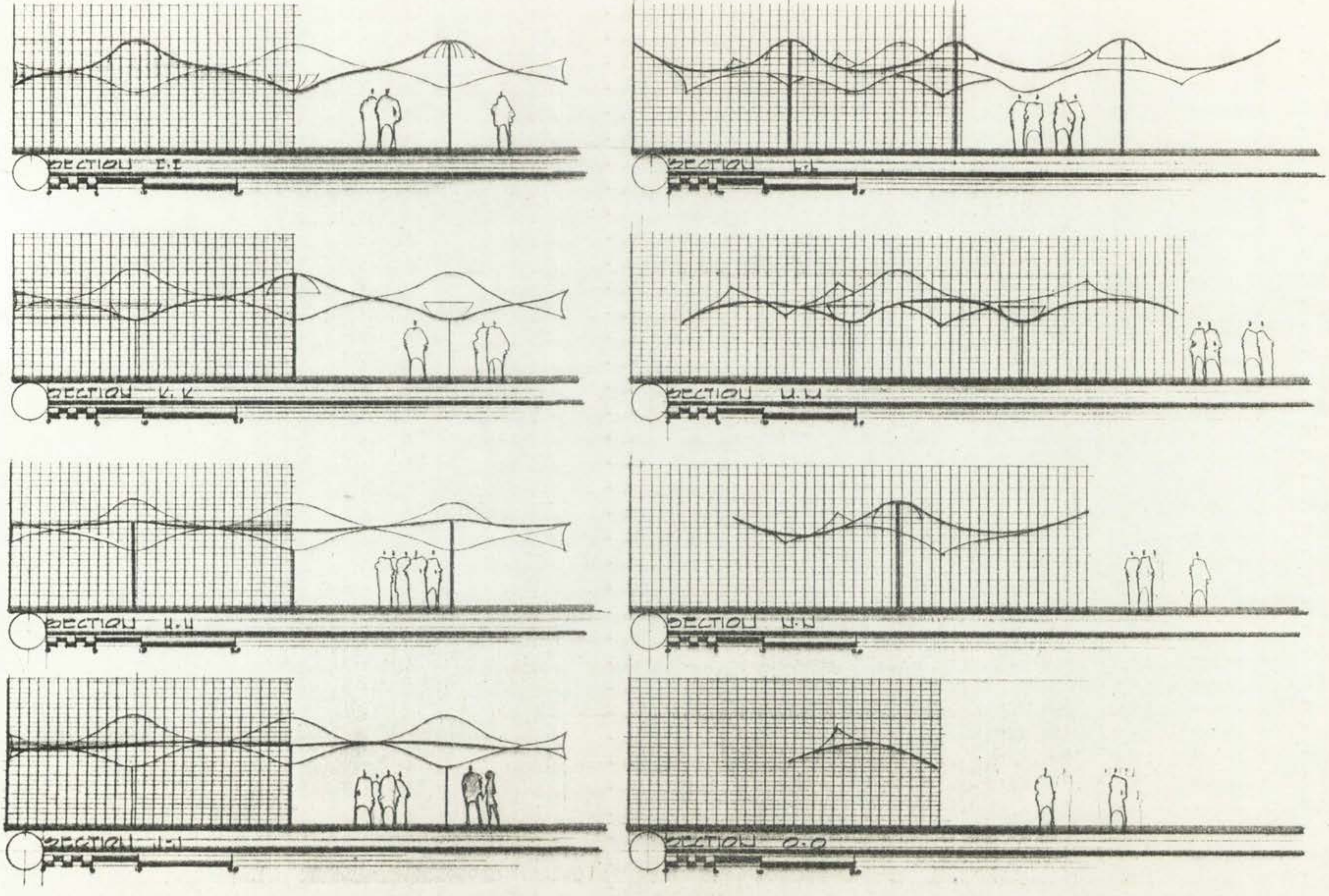

SECTION A-A
SECTION B-B
SECTION C-C
SECTION D-D
SECTION E-E
SECTION F-F
SEE DETAIL BELLOW
SECTION G-G
DETAIL SECTION G-G

Fig. 132

Fig. 133


SECTION I.I
SECTION L.L
SECTION K.K
SECTION M.M
SECTION H.H
SECTION N.N
SECTION J.J
SECTION O.O

FOOTNOTES

[1] Conrad Roland, Frei Otto: Tension Structures, (New York, 1970) pp. 62-63.

[2] Ludwig Glaesser, The Work Of Frei Otto, (New York, 1972) pp. 82-83.

[3] Ibid. pp 34.

[4] Ibid. pp 44-53.

Conclusion

Conclusion

In summary it is necessary to point out that this work was not meant to be a complete description of the field of saddle shaped tensile structures, nor does it even begin to probe into the intricacies of tensile structure analysis of these types of structures. The author has mainained all through the work that simplicity in analysis, the ability to relate complex cases of stress and curvature to simple tension principles is the basis for all forms of structural analysis.

The author's contention that a greater knowledge concerning tensile structures is only possible by having a firm basic knowledge of the original principles of the subject, and a personal involvement in the testing procedure of individual saddle surfaces, was reinforced by the author's ability to cope with situations of varying structural complexity, after starting with a basic understanding of the structural principles of tensile structures.

These same principles of procedural structural and design analysis; stemming from observations and conclusions of basic saddle types, are offered to the reader in the hope that they may serve to simplify the subject, and intice the reader into performing this procedure to clarify any structural conditions present in any prestressed saddle shaped tensile structure.

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