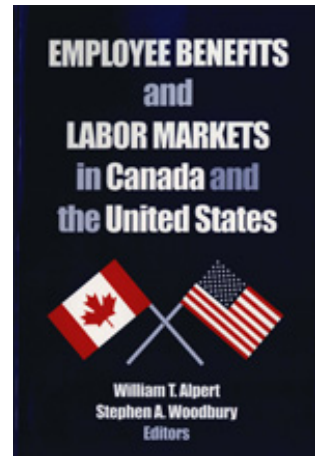




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Fringe Benefits and Employment

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5 Fringe Benefits and Employment

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The past few decades have witnessed dramatic growth in the fringe benefits component of total labor compensation in the United States and other countries in the Organization for Economic Cooperation Development (Hart et al. 1988). According to the conventional wisdom, an exogenous increase in nonwage labor costs adversely impacts employment. For instance, researchers have linked increases in quasi-fixed labor costs with reductions in employment.¹ Often overlooked, however, is the fact that voluntarily provided fringe benefits have grown by as much as legally required fringe benefits over the past 40 years, with the voluntary component in fact outpacing the legally required component after the early 1980s. Although legally required fringe benefits may be viewed as exogenous for the purposes of modeling, voluntarily provided fringe benefits must be treated as endogenous in any comprehensive analysis.² In this chapter, we undertake just such an analysis. In particular, we discuss a market level model in which wages and employment are permitted to respond to changes in the demand for and the cost of fringe benefits and in which increases in legally mandated fringe benefits are permitted to alter the market equilibrium.

Anticipating the results, the employment and wage effects of increases in nonwage payments vary with the source of the increase. We analyze the following three such sources: 1) an increase in worker demand for benefits, 2) a reduction in the cost of providing benefits, and 3) an increase in legally mandated benefits. Although these are obvious sources, the literature does not seem to have taken them into account. Our analysis is conducted primarily in a model in which no distinction is drawn between the number of workers and the hours of work or between straight-time and overtime hours. The final sections discuss how the results are altered by incorporating such distinctions and offer some concluding remarks.

BACKGROUND

Figure 1 shows that, in the United States, both legally required and voluntary fringe benefits grew significantly over the past 40 years, with the voluntary component outpacing the legally required component after the early 1980s. As Table 1 makes clear, growth in Social Security was the single most important factor behind the upsurge in legally required fringe benefits between 1951 and 1994. As for voluntary fringe benefits, growth in health and medical insurance far outpaced growth in any other component. Table 2 depicts industry differences in the changes in nonwage payments from 1966 to 1994. In general, the percentage of nonwage labor costs in total labor costs rose steadily during this period, with growth for all industries combined reaching almost 48 percent. This rate of increase varied among industries, ranging between 1.2 percent in finance to 67.9 percent in wood products. It was higher for manufacturing industries overall (over 50 percent) than for nonmanufacturing industries (39 percent).

The observed increase in the importance of nonwage labor costs most likely affected the relative attractiveness of various labor inputs, for example, part-time versus full-time workers or additional hiring of full-time workers versus additional hours worked by incumbent workers. In particular, the existing analyses predict that part-time workers should have become more attractive than full-time workers and that additional hours should have become more attractive than additional hiring.

It is unclear whether these predictions are supported by the data, at least on the aggregate correlation level. Figure 2 indicates that there is no discernible relationship across industries between changes in the ratio of full-time to total employment and changes in the importance of nonwage labor costs. Figure 3 shows that, if anything, hours worked per employee decreased in industries that experienced increases in nonwage labor costs, a finding that seems at odds with the theoretical prediction. Although a multivariate analysis using a more comprehensive data set is needed to test these predictions rigorously, the market-level theory discussed in this essay suggests that such ambiguous findings are to be expected.

**Figure 1 Fringe Benefits of Workers in Firms Surveyed by the Chamber of Commerce
(Indices of Benefits as a Percent of Total Compensation, 1951=100)**

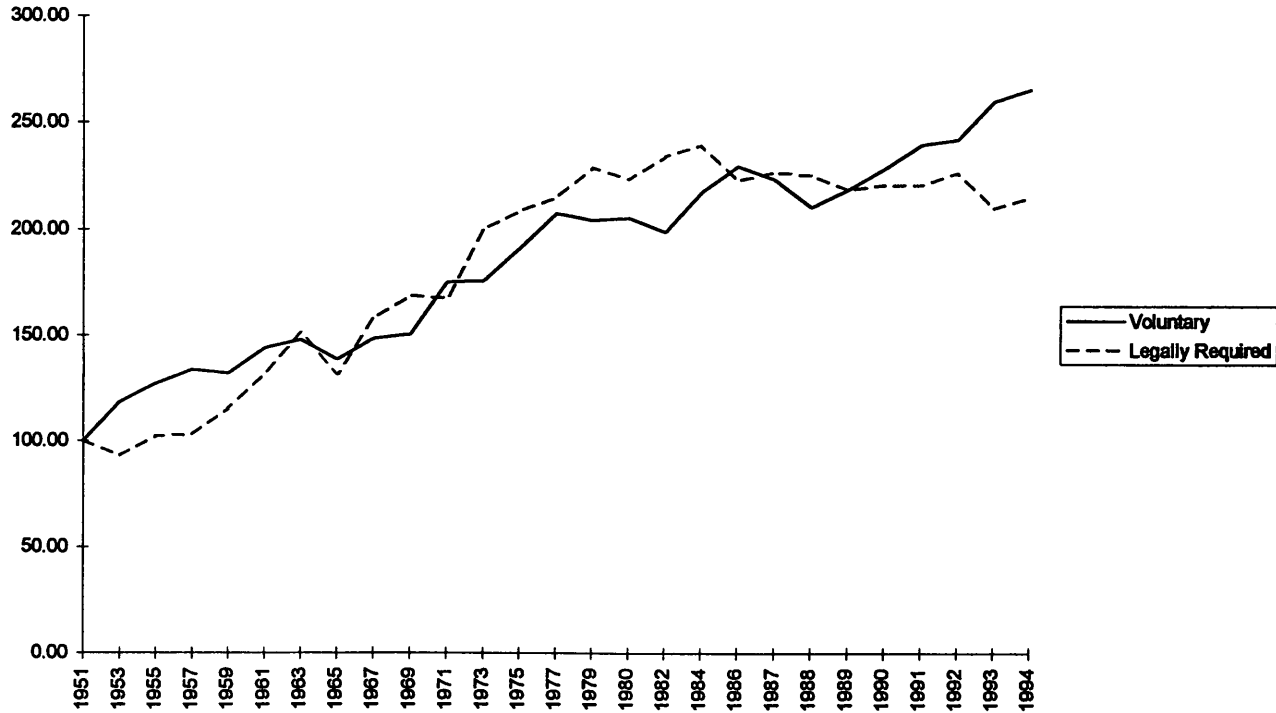


Table 1 Components of Fringe Benefits in Firms Surveyed by the Chamber of Commerce. Indices of Benefits as a Percent of Total Compensation (1951=100)

| | Legally required | | | Voluntary | | | Inside payroll | |
|------|------------------|----------------|--------|------------|-----------|---------|----------------|------------|
| | Social Security | Workers' comp. | Other | Retirement | Insurance | Misc. | Paid rest | Paid leave |
| 1951 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| 1955 | 174.81 | 87.41 | 58.27 | 102.04 | 174.81 | 145.741 | 16.54 | 104.93 |
| 1959 | 172.74 | 138.16 | 69.11 | 115.20 | 207.33 | 115.16 | 138.22 | 117.50 |
| 1963 | 240.98 | 120.49 | 100.44 | 110.50 | 240.98 | 160.65 | 140.60 | 126.55 |
| 1967 | 315.88 | 157.89 | 52.63 | 114.10 | 263.25 | 140.41 | 157.96 | 126.40 |
| 1971 | 360.71 | 160.34 | 40.10 | 126.95 | 360.71 | 146.93 | 173.68 | 144.31 |
| 1975 | 439.47 | 198.50 | 56.70 | 141.81 | 425.28 | 132.33 | 189.04 | 155.97 |
| 1979 | 435.71 | 249.06 | 82.83 | 138.36 | 477.16 | 152.13 | 179.82 | 143.18 |
| 1984 | 494.36 | 179.51 | 89.91 | 117.42 | 621.52 | 144.17 | 92.92 | 138.52 |
| 1988 | 522.56 | 166.73 | 43.80 | 126.29 | 711.84 | 46.37 | 117.11 | 160.07 |
| 1992 | 517.48 | 221.80 | 34.59 | 152.80 | 805.83 | 54.20 | 123.18 | 155.25 |
| 1993 | 495.39 | 172.18 | 30.33 | 157.98 | 839.94 | 76.50 | 110.03 | 149.35 |
| 1994 | 514.29 | 165.98 | 30.39 | 175.65 | 788.06 | 93.98 | 107.83 | 141.87 |

SOURCE: See Appendix A.

Table 2 Indices of Nonwage Labor Costs as a Proportion of Total Labor Compensation by Industry (1966=100)

| Industry | 1966 Values | 1966 | 1971 | 1976 | 1981 | 1985 | 1988 | 1992 | 1993 | 1994 |
|-----------------------|----------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| All | (0.198) | 100.0 | 118.7 | 135.4 | 137.4 | 138.4 | 136.4 | 144.8 | 147.6 | 146.1 |
| Manufacturing | (0.191) | 100.0 | 122.5 | 142.4 | 144.5 | 148.7 | 139.7 | 152.8 | 150.4 | 153.3 |
| Food, tobacco | (0.214) | 100.0 | 116.4 | 125.2 | 127.6 | 132.7 | 123.4 | 145.5 | 121.4 | 127.4 |
| Textiles, apparel | (0.159) | 100.0 | 120.1 | 141.5 | 149.1 | 153.5 | 157.1 | 146.6 | 158.9 | 143.6 |
| Wood products | (0.169) | 100.0 | 130.8 | 150.3 | 157.4 | 154.4 | 149.1 | 164.2 | 167.9 | 161.4 |
| Printing & publishing | (0.174) | 100.0 | 123.6 | 148.9 | 154.6 | 146.6 | 134.0 | 165.7 | 164.2 | 169.1 |
| Chemicals | (0.215) | 100.0 | 118.6 | 140.5 | 140.9 | 140.0 | 118.5 | 134.8 | 144.1 | 130.7 |
| Petroleum | (0.219) | 100.0 | 120.1 | 132.4 | 140.6 | 128.3 | 144.1 | 132.3 | 132.5 | 130.2 |
| Rubber and plastics | (0.201) | 100.0 | 115.9 | 136.8 | 134.8 | 142.3 | 144.9 | 166.7 | 148.6 | 145.9 |
| Stone, glass | (0.188) | 100.0 | 126.6 | 142.0 | 146.3 | 135.1 | 139.1 | 144.5 | 143.9 | 152.8 |
| Metals: | | | | | | | | | | |
| Primary | (0.200) | 100.0 | 129.0 | 151.0 | 151.0 | 166.0 | 149.9 | 149.6 | 144.6 | 127.4 |
| Fabricated | (0.186) | 100.0 | 119.9 | 147.3 | 147.8 | 159.1 | 152.2 | 169.1 | 158.5 | 149.2 |
| Machinery | | | | | | | | | | |
| Electrical | (0.192) | 100.0 | 118.8 | 139.6 | 142.2 | 146.4 | 135.3 | 150.7 | 148.0 | 163.4 |
| Other | (0.194) | 100.0 | 119.1 | 140.7 | 143.3 | 147.4 | 137.3 | 149.9 | 144.1 | 154.2 |

(continued)

Table 2 (continued)

| Industry | 1966 Values | 1966 | 1971 | 1976 | 1981 | 1985 | 1988 | 1992 | 1993 | 1994 |
|-----------------------|----------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Transportation equip. | (0.188) | 100.0 | 137.8 | 149.5 | 148.9 | 155.9 | 149.0 | 163.3 | 162.0 | 187.6 |
| Instruments, other | (0.192) | 100.0 | 123.4 | 132.3 | 145.8 | 152.6 | 134.7 | 127.2 | 139.8 | 144.2 |
| Nonmanufacturing | (0.212) | 100.0 | 111.8 | 124.5 | 125.5 | 125.5 | 128.4 | 134.3 | 138.8 | 136.0 |
| Utilities | (0.212) | 100.0 | 115.6 | 134.0 | 139.2 | 137.7 | 140.2 | 140.0 | 148.6 | 148.4 |
| Trade | | | | | | | | | | |
| Department stores | (0.188) | 100.0 | 103.7 | 128.7 | 124.5 | 125.0 | 136.4 | 128.9 | 123.4 | 152.2 |
| Other | (0.194) | 100.0 | 101.5 | 121.1 | 120.6 | 123.7 | 153.2 | 143.8 | 153.7 | 144.6 |
| Finance | (0.243) | 100.0 | 108.2 | 117.3 | 114.4 | 105.8 | 99.5 | 100.5 | 101.2 | 98.3 |
| Insurance | (0.213) | 100.0 | 113.1 | 99.1 | 130.5 | 128.2 | 120.2 | 132.5 | 136.8 | 134.6 |
| Hospitals | (0.204) | – | – | 100.0 | 116.2 | 125.5 | 121.6 | 123.8 | 134.7 | 131.9 |
| Other | (0.252) | – | – | 100.0 | 98.4 | 100.4 | 107.4 | 116.8 | 116.6 | 109.5 |

SOURCE: Calculated from Table 6 of *Employee Benefits* (Chamber of Commerce of the United States, various years) by taking $1/(1+(100/x))$, where x is employee benefits.

Figure 2 Manufacturing Industries (U.S. Chamber of Commerce Survey)

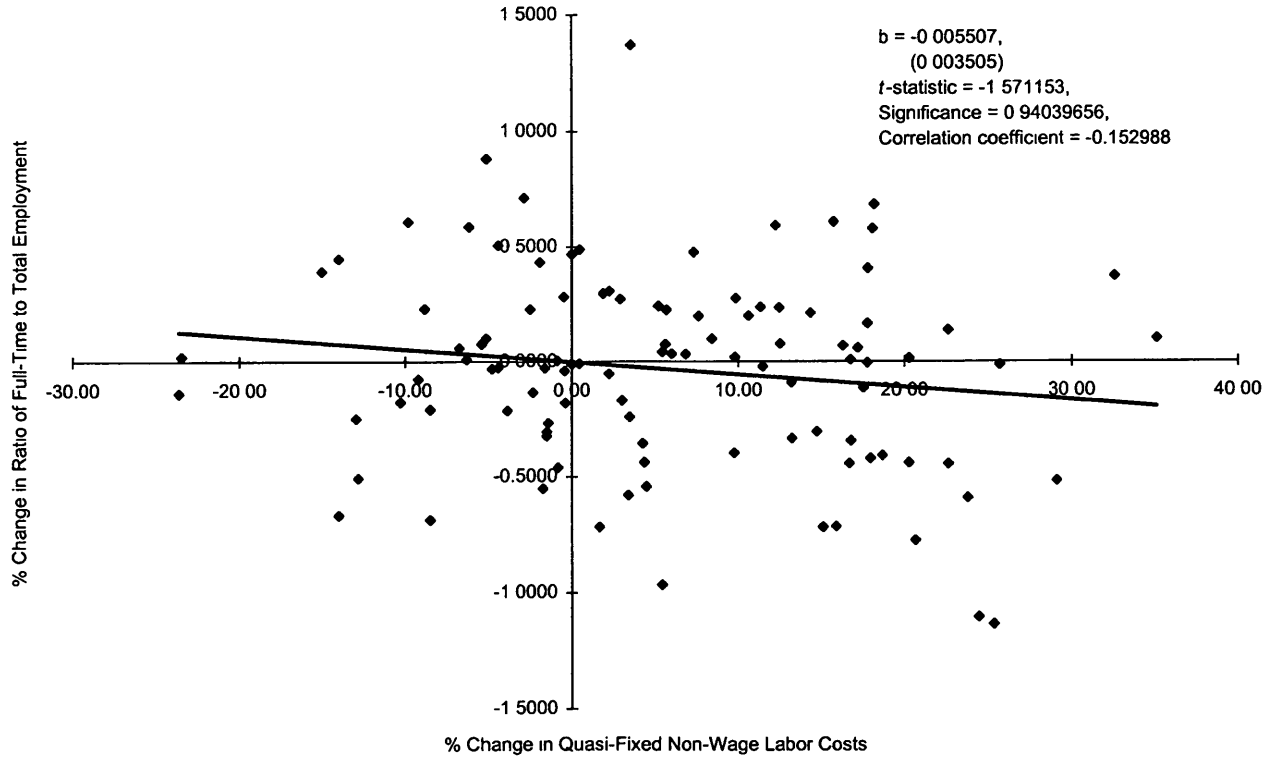
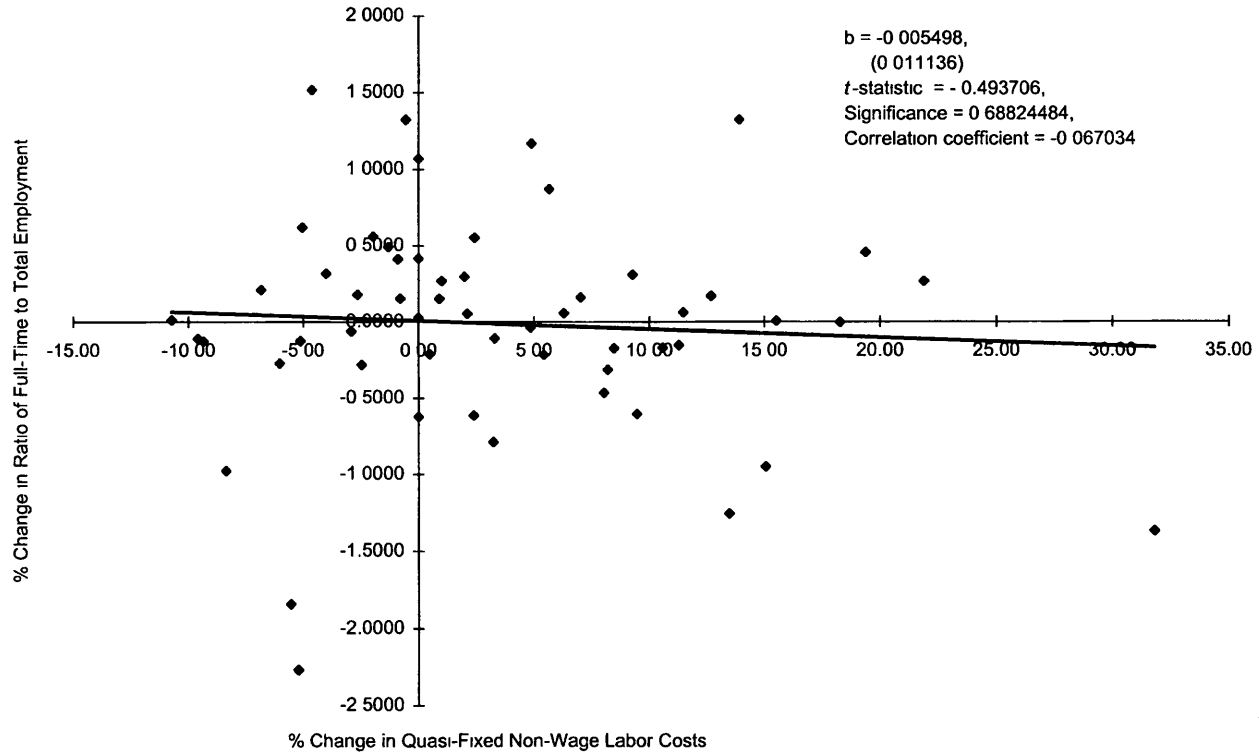


Figure 3 Nonmanufacturing Industries (U.S. Chamber of Commerce Survey)



THEORY AND PREDICTIONS

In contrast to the conventional approach of analyzing a single firm's decision, we consider a model that addresses the effects of increased fringe benefits on market wages and employment. To focus on the bare essentials of the analysis, we restrict ourselves to a simple model that abstracts from the distinction between the size of the workforce and the hours of work per worker. Following this simple model, we consider the implications from an extended model that includes such a distinction.

Beginning first with a firm level analysis and then extending to the market level analysis, suppose that an industry consists of identical firms whose production functions are given by

$$Q = F(n) \tag{1}$$

where Q is output and n is a firm's level of employment. The market level of employment is then given by $E = kn$, where k is the total number of firms. Each firm faces a labor expense function,

$$\theta = \theta(n, h, G) = whn + C(G, n)n, \tag{2}$$

where w is the wage and G is the quantity of fringe benefits per worker. Equation 2 is assumed to satisfy

$$C_1 > 0, C_{11} > 0, C_2 \geq 0, C_{12} = C_{21} \geq 0, C_{22} = 0,$$

so that the marginal cost of G is positive and rising ($C_1 > 0, C_{11} > 0$). Equation 2 allows for the existence of either internal diseconomies or economies in providing fringe benefits. Thus, as the firm expands its workforce, the cost per worker of providing fringe benefits might increase or decrease, depending upon whether scale diseconomies ($C_2 > 0$) or economies ($C_2 < 0$) exist. More importantly, the sign of $C_{12} = C_{21}$ is critical to the analysis. We interpret $C_{12} < 0$ as an indicator of cross-economies of scale and $C_{12} \geq 0$ as an indicator of cross-diseconomies of scale. Cross-economies of scale (diseconomies of scale) imply

that the marginal cost of G falls (rises) with the size of the workforce. For simplicity, we assume that $C_{22} = 0$.

The i th employee is assumed to view G as having a constant marginal value of λ_i dollars, making her indifferent between receiving λ_i dollars in w or in G . To simplify, we assume that all employees have the same marginal value, λ . As a result, $\partial w / \partial G = -\lambda$ from the employer's perspective; if G is increased by one unit, all employees are willing to work for λ dollars less in w .

An employer selects the optimum n and G by solving the following problem:

$$\text{Max}_{n,G} \pi(n, G) = pF(n) - [w + C(G, n)]n, \quad (3)$$

where p is the product price. The first order conditions are given by

$$\partial \pi / \partial n = pF'(n) - [w + C(G, n) + nC_2] = 0 \quad \text{and} \quad (4a)$$

$$\partial \pi / \partial G = (-\lambda + C_1)n = 0. \quad (4b)$$

A firm's labor demand is traced by Eq. 4a, rewritten as:

$$w = pF'(n) - [C(G, n) + nC_2]. \quad (4a^*)$$

The optimum quantity of G is given by Eq. 4b, implying that the marginal cost of G is equal to λ dollars at the optimum point.

Allowing for the product price changes that occur as all firms change outputs, the market demand curves for labor can be obtained by horizontally summing the firms' demand curves. To facilitate the analysis, we linearize the market demand curves as follows (see Appendix B for the details of this linearization).

$$w^d = \alpha - (\eta G + \rho G^2) + \beta(G)E, \quad (5)$$

where w^d is the employers' wage offer, w is employment, and α , η , ρ , and β are parameters. The expression represents the cost of providing G , and β depends on G if there are cross-scale effects in providing the benefits. Equations 2 and 4a* imply the following restrictions:

$$\alpha, \eta, \rho > 0, \beta < 0, d\beta / dG \leq 0, \text{ and } d^2\beta / dG^2 = 0, \tag{6}$$

where the condition $d\beta / dG \leq 0$ corresponds to $C_{12} \geq 0$ and the condition $d^2\beta / dG^2 = 0$ corresponds to $C_{12} = 0$ (see Appendix B). Figure 4 depicts the demand curves associated with three different quantities of G . If the marginal cost of G slopes upward, the demand curves diverge as G is increased, reflecting the rising cost of fringe benefits.

The supply side of the model is straightforward. As emphasized earlier, workers are assumed to be homogeneous while w and G are assumed to be perfectly substitutable at a rate of λ dollars per unit of G . As a result, the market supply of labor depends upon $w + \lambda G$. Because workers are assumed homogeneous, the supply curve depicted as a function of w is horizontal and shifts down as G is increased according to the following equation.

$$w^s = \gamma - \lambda G, \tag{7}$$

where w^s is the asking wage and γ (>0) and λ (>0) are parameters. Figure 5 depicts the supply curves.

The competitive market equilibrium is the solution that maximizes the sum total of the surpluses for both employers and employees. The process of reaching this equilibrium involves two steps. First, the market optimizes on the E associated with various quantities of G . This optimization generates a locus of the intersections of demand and supply curves corresponding to different values of G . Second, the market chooses the optimum quantity of G by equating the marginal cost of G with its marginal value. By doing so, the market in effect selects a point on the intersection locus that maximizes the sum of the surpluses. Figure 6 depicts the demand and supply curves together for three different levels of G as well as the corresponding intersection points on the intersection locus L .

The market equilibrium—the intersection of demand and supply that maximizes the joint surplus Z —is obtained by solving the following optimization problem:

$$\begin{aligned} \text{Max}_{E, G} Z &= \int_0^E [w^d(\epsilon) - w^s(\epsilon)] d\epsilon \\ &= [\alpha - (\eta G + \rho G^2) - \gamma + \lambda G]E + 0.5\beta E^2 + \text{Constant}. \end{aligned} \tag{8}$$

Figure 4 Demand Curves

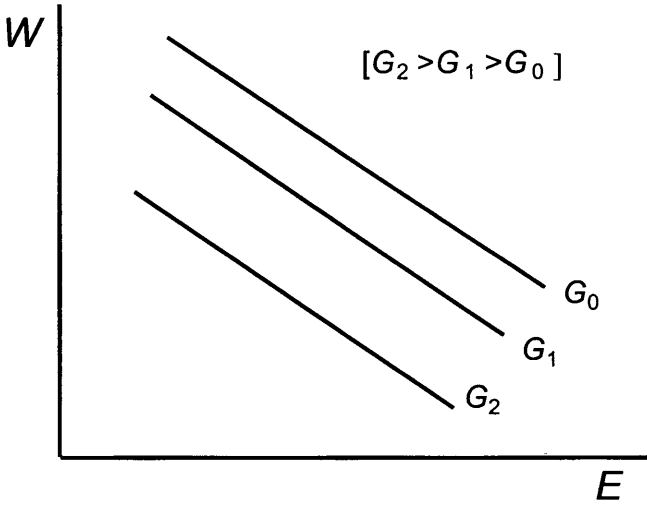
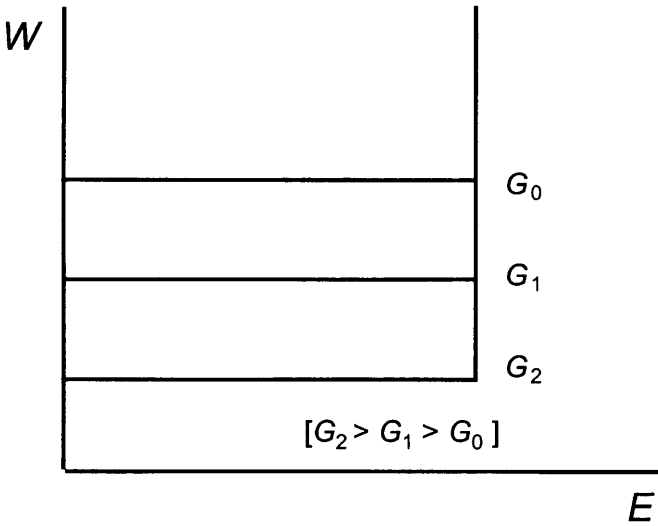


Figure 5 Supply Curves



The first order conditions are given by

$$\partial Z / \partial E = \alpha - (\eta G + \rho G^2) - \gamma + \lambda G + \beta E = 0 \quad \text{and} \quad (9a)$$

$$\partial Z / \partial G = -[\eta + 2\rho G - 0.5(d\beta/dG)E]E + \lambda E = 0 \quad (9b)$$

Equation 9a optimizes Z with respect to E , resulting in a locus of intersections between the demand and supply curves for different values of G . Equation 9b optimizes on G , equating the marginal cost (the first term) with the marginal value (the second term). Equations 9a and 9b together describe the point of market equilibrium.³

Figure 7 portrays the market equilibrium for three cases: no cross-scale effects ($d\beta/dG = 0$) cross-economies of scale ($d\beta/dG > 0$), and cross-diseconomies of scale ($d\beta/dG < 0$). The L curve is the locus of the intersections of the demand and supply curves for the different levels of G . There is a unique L curve associated with each of these three values of $d\beta/dG$; however, in Figure 7, only one L curve is depicted to conserve space. Moving downward along this locus, G is increased and w is decreased. We should point out that the market equilibrium level of employment is not necessarily at its maximum attainable level. In particular, Figure 7 shows that if there are cross-scale effects ($d\beta/dG \neq 0$), equilibrium employment is less than the maximum feasible level of employment on the relevant L curve.⁴ Only in the absence of cross-scale effects ($d\beta/dG = 0$) is equilibrium employment at its maximum feasible level (see Appendix C for a proof).

We are now in a position to evaluate the effects of nonwage payments on employment. Since G is endogenous in our model, changes in its magnitude must be traced to changes in worker demand for, and the cost of, G . These exogenous factors are represented by λ and η , respectively. In addition, we evaluate the effects of changes in legally required fringe benefits on employment and wages.

Changes in the Demand for G

A secular increase in nonwage payments can arise as a result of an increase in the demand for fringe benefits. Such an increase in demand occurs if, for example, a new law taxes nonwage benefits less heavily than wage earnings or if real income grows and fringe benefits are

Figure 6 Intersection Locus

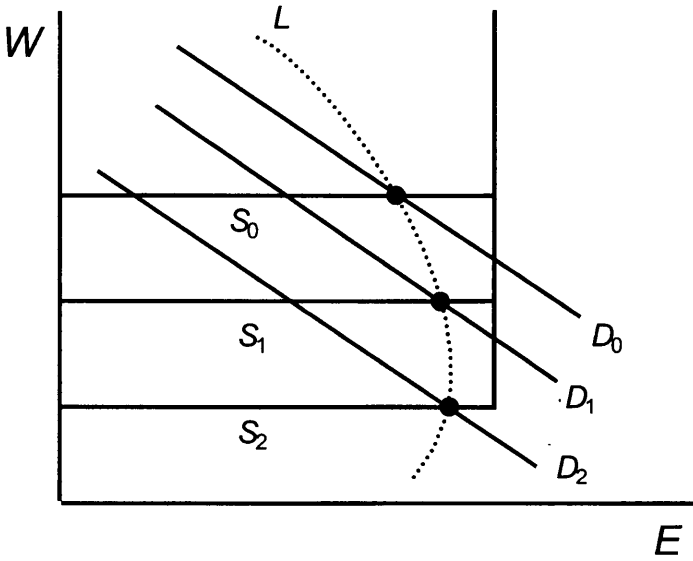
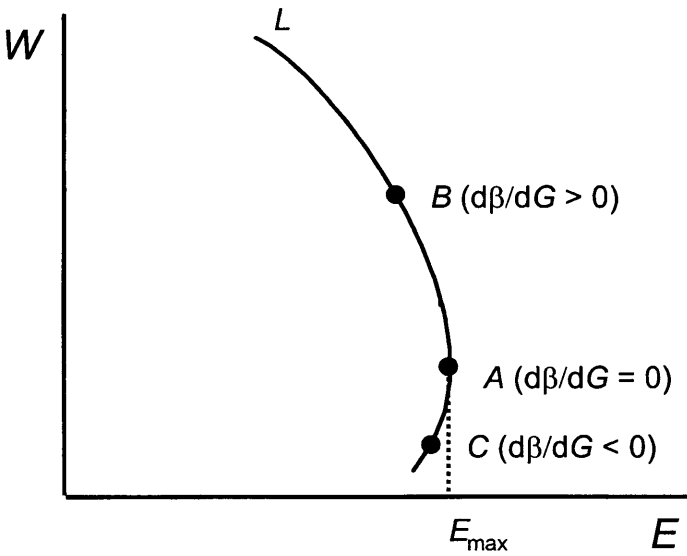


Figure 7 Market Equilibria



superior goods. An increased demand for G is represented by an increase in λ , yielding the following comparative statics results:

$$dG/d\lambda = (1/|H|)[- \beta + 0.5G(d\beta/dG)]E \quad (10a)$$

$$dE/d\lambda = (1/|H|)[2\rho G + 0.5E(d\beta/dG)]E, \quad (10b)$$

Clearly, if there are no cross-scale effects ($d\beta/dG = 0$) or if there are cross-economies of scale ($d\beta/dG > 0$), both equations are positive so that both G and E increase with λ . In other words, an increase in worker demand for fringe benefits increases both the amount of benefits provided and the level of employment. If a secular increase in non-wage payments is the result of an increased demand for these payments, such an increase in benefits should have the effect of stimulating employment.

If there are cross-diseconomies of scale ($d\beta/dG < 0$), conditions 10a and 10b seem to suggest that either G or E or both could decrease when λ increases. Such an outcome seems implausible, however, because it would imply that, as employers expand G in response to the increased demand for it, cross-diseconomies cause the cost of providing fringe benefits to rise, forcing employers to reduce the quantities of both G and E . For cross-diseconomies to remain operative, however, the aggregate amount of G must rise. On the basis of this argument, we conjecture that G , and possibly E , increase even in this case.

Because $w^d = w^s = w$ at the point of equilibrium, the effect on the wage is ascertained by evaluating the effect of a change in λ on w^s , or

$$dw/d\lambda = -\lambda(dG/d\lambda) - G \quad (10c)$$

Equation 10 implies that, if G and E increase in response to an increase in the demand for G , then $dw/d\lambda$ is negative and the wage falls. Employees, in effect, trade their wages for larger benefits.

Changes in the Cost of G

Nonwage payments may also increase as a result of a decrease in the cost of providing fringe benefits. A change in the cost of G is represented here by a change in η . Not surprisingly, the comparative statics analysis reveals that cost effects are mirror images of demand effects. In other words,

$$dG / d\eta = -(1/|H|)[- \beta + 0.5G(d\beta / dG)]E = -dG / d\lambda \quad \text{and} \quad (11a)$$

$$dE / d\eta = -(1/|H|)[2pG + 0.5G(d\beta / dG)]E = -dE / d\lambda. \quad (11b)$$

If $(d\beta/dG \geq 0)$ both G and E increase when η falls. Thus, if the observed increase in nonwage benefits is the result of a decrease in the cost of providing benefits, employment as well as benefits should rise. As in the case of increased demand for fringe benefits, even if there are cross-diseconomies $(d\beta/dG < 0)$, we conjecture that G , and possibly E , increase when costs fall.

The wage effect is evaluated from the following equation:

$$dw / d\eta = \lambda(dG / d\eta). \quad (11c)$$

The term $dw / d\eta$ is positive when $(d\beta/dG \geq 0)$. If a decrease in η is the cause of the observed increase in nonwage payments, then, as G and E increase in response, w should decrease.

Effects of Government Control of G

As Figure 1 demonstrates, legally required benefits have risen over time. If a government regulates the quantity of employer provided fringe benefits, then G in the previous analysis is replaced by the mandated quantity, \bar{G} . Given that $dw / d\bar{G} = -\lambda < 0$, it is clear that the wage will fall unambiguously when \bar{G} is increased. The effect on employment is not clear-cut, however.

To begin, E is now the only endogenous variable, making $\partial Z / \partial E = 0$ the only first-order condition. This first-order condition yields the following optimum level of employment:⁵

$$(d\beta / dG < 0),$$

$$E^* = \left[\alpha - (\eta \bar{G} + \rho \bar{G}^2) - \gamma + \lambda \bar{G} \right] / (-\beta). \quad (12)$$

The effect of an increased \bar{G} on employment is given by

$$\begin{aligned} dE^* / d\bar{G} &= (\eta + 2\rho\bar{G} - \lambda) / \beta \\ &+ (d\beta / dG) \times (\text{a positive term}). \end{aligned} \quad (13)$$

Suppose the market is initially at its competitive equilibrium so that $\bar{G} = G^*$, where G^* is the competitive equilibrium level of the benefits. In this case, the term $(\eta + 2\rho\bar{G} - \lambda) / \beta \gamma$ is zero because $(\eta + 2\rho\bar{G})$ and λ are, respectively, the marginal cost and marginal value of G . We already know that in the absence of cross-scale effects ($d\beta / dG = 0$), the competitive equilibrium corresponds to the maximum feasible level of employment.⁶ It is clear, therefore, that the introduction of legally required benefits lowers employment regardless of whether the mandated \bar{G} is larger or smaller than G^* . In other words, there is little that the government can do to increase employment by regulating G .

If there are cross-economies of scale ($d\beta / dG > 0$), employment increases because $dE^* / d\bar{G} = (d\beta / dG) \times (\text{a positive term}) > 0$. The government in effect forces the market to experience cross-economies of scale beyond what is efficient. If there are cross-diseconomies of scale ($d\beta / dG < 0$), employment decreases because $dE^* / d\bar{G} = (d\beta / dG) \times (\text{a positive term}) < 0$. In this case, the government in effect forces the market to experience cross-diseconomies beyond what is efficient.

Now assume that legally required fringe benefits already exist. What happens to employment if \bar{G} is increased? Consider the case of no cross-scale effects ($d\beta / dG = 0$). If \bar{G} is already set above the market equilibrium level, then the marginal cost is above the marginal revenue so that $(\eta + 2\rho\bar{G}) > \lambda$. As a result, $dE^* / d\bar{G} < 0$, implying that employment decreases when \bar{G} is increased. On the other hand, if \bar{G} is initially set below the market equilibrium level, then the marginal cost is lower than the marginal revenue so that $(\eta + 2\rho\bar{G}) < \lambda$. As a result, $dE^* / d\bar{G} > 0$, implying that employment increases when \bar{G} is increased. With respect to the latter of these policy moves, the government forces G to move closer to the market equilibrium level.

Allowing cross-scale effects to exist complicates the analysis. Suppose there are cross-economies of scale ($d\beta/dG > 0$). If \bar{G} is initially set above the competitive market level, then the effect of changes in employment of changes in \bar{G} is ambiguous given that $dE^*/d\bar{G}$ cannot be signed. In this case, there are opposing forces at work. On one hand, the government forces \bar{G} to increase beyond its already inefficiently high level, thereby adversely affecting employment. On the other hand, an increase in \bar{G} forces the market to enjoy cross-economies of scale, thereby positively affecting employment. The net outcome depends upon the relative strength of the opposing forces. If \bar{G} is initially below the competitive market level, then employment unambiguously increases because $dE^*/d\bar{G} < 0$. In this case, the government forces the market to move towards the competitive level of G , thereby reinforcing the stimulating effect on employment originating from cross-economies of scale.

Turning to the case of cross-diseconomies of scale ($d\beta/dG < 0$), if \bar{G} is initially above the competitive market level, then employment declines unambiguously when \bar{G} is increased because ($d\beta/dG < 0$). In this case, the government forces \bar{G} to move further away from the competitive equilibrium, thereby reinforcing the disemployment effect caused by cross-diseconomies of scale. If \bar{G} is initially below the competitive market level, then employment effects are ambiguous given that $dE^*/d\bar{G}$ cannot be signed. The government forces \bar{G} closer to the market equilibrium level, causing employment to expand, but cross-diseconomies of scale cause employment to decline. The net effect is uncertain.

To summarize, if an increase in nonwage payments is caused by an increase in the legally required benefits, wages fall unambiguously; however, employment effects are ambiguous. An important result is that, even in the case of an exogenous increase in legally required fringe benefits, employment can increase rather than decrease as conventionally thought. Whatever happens to employment, an increase in legally mandated fringe benefits tends to be inefficient. An exception is when the fringe benefits level is initially set below the competitive equilibrium level. In this case, it is obvious that an increase in fringe benefits increases efficiency so long as the increase does not overshoot the competitive equilibrium level.

AN EXTENSION: A MODEL WITH OVERTIME HOURS

Previous analyses assume that the relevant range of hours of work includes only the standard hours, omitting consideration of overtime hours and the potential ramifications of the overtime wage premium. Our model may be extended by assuming that the equilibrium number of hours of work incorporates overtime hours. Such an extension is important if exogenous changes in nonwage payments affect the marginal cost of increasing the labor input via increases in the hours of work beyond the standard hours. We are also interested in the effects of changes in the standard hours (or in the overtime wage premium) on employment and fringe benefits. For a fuller exposition on the technical aspects of such an extended model, the reader is referred to a companion paper (Hashimoto and Zhao 1996). Here, we simply outline some of the key predictions that emerge from this extended analysis.

The predictions discussed in the preceding section are generally unchanged in the extended model. We do, however, obtain additional predictions. First, suppose the government increases the standard hours of work. If there are no cross-scale effects, neither fringe benefits nor hours of work are affected by the changes in standard hours. Employment increases if the positive effect of the increased standard hours on labor demand dominates the negative effect on labor supply; it decreases otherwise. The effects on hours of work and fringe benefits depend on how the slope of the labor demand curve, β , changes. The straight-time wage rate rises as a result of an upward shift of the worker supply curve. The effects of an increase in the overtime wage rate are opposite of the effects of an increase in standard hours.

CONCLUSION

The importance of nonwage payments has risen noticeably in the United States over the past 40 years. Contributing to this trend are increases in both voluntarily provided and legally required fringe benefits. Furthermore, since the early 1980s, the growth of the voluntary

component has outpaced that of legally required component. These developments suggest the importance of evaluating how employment and wages are affected by the demand and supply forces that lead to increases in voluntarily provided fringe benefits. This chapter has addressed this issue. We find that predictions based upon the conventional firm-level analysis in which nonwage payments are assumed to be exogenous are misleading. In particular, contrary to the conventional wisdom, an increase in nonwage payments does not necessarily imply any adverse effects on employment and wages.

This outcome depends jointly on the source of the increase and the existence of cross-scale effects in the cost of providing fringe benefits. If the increase in nonwage payments is the result of either an increase in employee demand for fringe benefits or a decrease in the cost of providing benefits, employment may increase and the wage rate may decrease. More importantly, employment effects are ambiguous even when legally mandated fringe benefits are involved. To be sure, wages always fall when legally required fringe benefits are increased; however, employment may fall or rise depending on the initial condition and the existence and the nature of cross-scale effects.

In the special case in which there are no cross-diseconomies of scale, there is no presumption that an increase in nonwage benefits reduces employment so long as competitive market forces are responsible for such an increase. If new legally required fringe benefits are introduced into a labor market that is already at a competitive equilibrium, employment decreases regardless of whether the mandate is to increase or decrease such benefits. In this case, a government cannot increase employment by manipulating the levels of legally required fringe benefits.

Incorporating the distinction between standard hours of work and overtime hours of work does not change these results. Not surprisingly, we find that there is a symmetry of effects with respect to the standard hours and the overtime premium. In particular, the effects of increased standard hours of work on employment, wages, and fringe benefits are opposite of the effects of an increased overtime premium.

We end with a discussion of some of the restrictions and limitations imposed on the analysis of this paper. Relaxing these would undoubtedly make the model more complete. Given, however, that

our objective is to demonstrate that making nonwage payments endogenous changes some of the conventional results, we have chosen to use a simplified model here. In any event, four limitations warrant mention.

First, we abstract from the worker's choice of the number of hours to work. Incorporating such a decision, while making the model more complete, would greatly complicate our analysis. The same may be said with respect to the second limitation of our analysis—namely, our treatment of all nonwage labor as quasi-fixed benefits that are independent of the number of hours of work. Thus, we are talking about a quasi-fixed wage component that is approximately 20 percent of total labor compensation. Third, we abstract from higher order terms in the linearly specified demand and supply functions. More complicated specifications of the demand and supply curves may be desirable, although such extensions are likely to make the predictions ambiguous. Fourth, we assume that all employees are homogeneous with respect to the marginal value of fringe benefits. If they were made heterogeneous, the supply specification would need to incorporate distribution parameters determining the taste for fringe benefits. Relaxing these four restrictions is the subject of future research. In this chapter, however, our goal is simply to demonstrate that some of the conventional predictions are modified once analysis is conducted in a more general equilibrium framework.

Notes

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1. When models of employment-hours decisions are expanded to allow for changes in capital, many of the results concerning hours become ambiguous; however, the fixed-cost effect on employment remains intact (Hamermesh 1993; Hart 1984).
2. Almost all existing analyses focus on the behavior of firms for which it is reasonable to assume that fringe benefits are strictly exogenous. For example, see Rosen (1968), Ehrenberg (1971), Hamermesh (1993), and Hart (1984).

3. The second order conditions are given by $\partial^2 Z / \partial G^2 = -2\rho E < 0$ and $|H| = -2\beta\rho E - [0.5E(d\beta/dG)]^2 > 0$, and where $|H|$ is the determinant of the Hessian matrix. It is straightforward to show that if the demand curves are all parallel to one another, i.e., $d\beta/dG = 0$, the above conditions are implied by the assumptions made with regard to the demand and supply curves. If they are not parallel, these conditions must be imposed on the model.
4. Since the L curve is unique to the value of dB/dG , the maximum employment level is different for each case.
5. The equilibrium point corresponds to the intersection of the respective demand and supply curves. The second order condition is satisfied because $\partial^2 Z^* / \partial E^2 = \beta < 0$.
6. This can be seen from $d^2 E / (d\bar{G})^2 = 2\rho/\beta < 0$ when $\bar{G} = G^*$.

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Appendix A

Table A, an extension of Table 1 in Woodbury (1983), gives the data on which Figure 1 is based. This appendix describes the procedures used to compute the entries in Table A and in Table 1 of the text. To simplify our exposition, we begin with a discussion of the construction of Table A.

Woodbury used two sources in demonstrating the growth of employee benefits from 1965 to 1978: the U.S. Chamber of Commerce publication *Employee Benefits* (various years) and the Bureau of Labor Statistics (BLS) bulletin *Employee Compensation in the Private, Nonfarm Economy* (1974). It is helpful to discuss each of these sources individually.

The data available in *Employee Compensation in the Private, Nonfarm Economy* is the product of the Employer Expenditures for Employee Compensation survey (EEEC). This survey was discontinued in 1977; however, beginning in March of 1987, the BLS started publication of *Employment Cost Indexes and Levels* (ECI), which includes a measure of "Employer Costs for Employee Compensation."¹ The data provided under this heading appears to be comparable to that provided in the older publication, from which Table 1 in Woodbury (1983) is derived. The one significant difference stressed in Nathan (1987) concerns its means of measuring these costs. In particular, the EEEC focuses upon *past expenditures*—or, the actual money an employer spends on compensation during a specified time. The compensation levels given in the new BLS publication rely upon *current costs*—or, the annual costs based upon the current price of benefits under current plan provisions. Aside from this measurement difference, however, the ECI and EEEC appear quite similar, with both covering virtually the same benefits and, more importantly, reporting costs on the same per hour basis.² In addition, the ECI preserves the scope of the EEEC by reporting survey coverage of the private, nonfarm workforce.

Derivation of the entries given in the last three columns of Table A simply entails the application of the per hour costs reported in the ECI to the definitions utilized by Woodbury in his calculations. These per hour costs are subsequently expressed as a percent of total compensation per hour and indexed to equal 100 in 1966.

With respect to the Chamber of Commerce data, a comparison of this table with that of Woodbury shows that the pre-1983 entries have been recalculated. This is done for reasons of data availability. In particular, the Table 19 that Woodbury used to construct his numbers is no longer included in *Employee Benefits*. To construct similar numbers for this table, it is necessary to use other sources within the publication. Two of these are selected. The first is a table giving wage data by industry (Table 17 in 1967 and 1969, Table 18 through

Table A Trends in Wage and Nonwage Compensation, 1951–1995
(benefits expressed as indices of % total compensation)

| | Compensation (Chamber of Commerce, 1951=100) | | | Compensation (BLS, 1966=100) | | |
|-------------------|---|---------------------|------------------------|--|---------------------|------------------------|
| | Total comp. per hr. (\$) ^a | Benefits | | Total comp. per hr. (\$) ^a | Benefits | |
| | | Legally required | Voluntary ^b | | Legally required | Voluntary ^b |
| 1951 | 1.88 | 100.00 | 100.00 | | | |
| 1953 | 2.02 | 93.10 | 118.46 | | | |
| 1955 | 2.15 | 102.19 | 127.18 | | | |
| 1957 | 2.43 | 103.13 | 133.68 | | | |
| 1959 | 2.72 | 115.36 | 131.97 | | | |
| 1961 | 2.85 | 131.97 | 143.93 | | | |
| 1963 | 3.12 | 150.78 | 147.86 | | | |
| 1965 | 3.33 | 131.66 | 138.63 | | | |
| 1966 | | | | 3.43 | 100.00 | 100.00 |
| 1967 | 3.57 | 157.99 | 148.38 | | | |
| 1968 | | | | 3.90 | 92.76 | 115.45 |
| 1969 | 4.09 | 168.65 | 150.43 | | | |
| 1970 | | | | 4.54 | 96.57 | 127.68 |
| 1971 | 4.69 | 167.08 | 174.87 | | | |
| 1972 | | | | 5.23 | 105.52 | 139.48 |
| 1973 | 5.65 | 199.69 | 175.56 | | | |
| 1974 | | | | 6.32 | 120.57 | 156.22 |
| 1975 ^c | 6.63 | 208.15 | 190.77 | | | |
| 1977 | 7.60 | 214.42 | 207.01 | | | |
| 1979 | 9.06 | 228.53 | 203.76 | | | |
| 1980 | 9.85 | 222.88 | 204.79 | | | |
| 1982 | 12.08 | 233.54 | 198.12 | | | |
| 1984 | 13.00 | 238.87 | 216.92 | | | |
| 1986 | 15.89 | 222.38 | 229.06 | | | |

| | Compensation (Chamber of Commerce, 1951=100) | | | Compensation (BLS, 1966=100) | | |
|------|---|---------------------|------------------------|--|---------------------|------------------------|
| | Total comp. per hr, (\$) ^a | Benefits | | Total comp. per hr. (\$) ^a | Benefits | |
| | | Legally required | Voluntary ^b | | Legally required | Voluntary ^b |
| 1987 | 16.49 | 226.33 | 222.91 | 13.40 | 157.71 | 195.28 |
| 1988 | 17.44 | 224.76 | 209.74 | 13.77 | 165.90 | 194.85 |
| 1989 | 18.41 | 217.87 | 218.29 | 14.26 | 167.05 | 194.21 |
| 1990 | 19.64 | 220.38 | 228.03 | 14.93 | 168.38 | 197.00 |
| 1991 | 20.51 | 220.06 | 239.15 | 15.37 | 169.71 | 203.86 |
| 1992 | 20.81 | 226.02 | 241.54 | 16.11 | 171.43 | 213.09 |
| 1993 | 22.67 | 209.09 | 259.32 | 16.68 | 173.52 | 221.24 |
| 1994 | 22.66 | 214.42 | 264.79 | 17.09 | 177.14 | 225.97 |
| 1995 | | | | 17.07 | 174.10 | 212.45 |

^a Total Compensation includes legally required contributions to Social Security, federal and state unemployment insurance, and Workers' Compensation.

^b Benefits provided voluntarily by the employer include private insurance (life, health, and accidental), privately sponsored retirement and savings plans (pensions, savings and thrift plans), as well as other items (severance pay, supplemental unemployment benefits, and other miscellaneous benefits).

^c Comparable BLS benefits data are unavailable for the period extending from 1975-1986.

1984, Table 16 after 1988); the second is a chart detailing average annual employee benefits and earnings (Chart 2 in all publications). The first of these, Table 16, gives average gross payroll for all private industries included in the survey not only on an annual basis but also on an hourly basis. That gross payroll is expressed on a per hour basis is important because such a frequency makes it possible to construct entries that are compatible with those provided by Woodbury (1983). Chart 2 categorizes employee benefits and earnings in the following manner: 1) benefits are the sum of outside payroll and inside payroll. Inside payroll encompasses paid vacations and holidays, employee rest periods, and lunch breaks; outside payroll is made up of legally required payments, pensions, insurance, and other agreed upon items, and other benefits; and 2) earnings include total pay for all time worked; they comprise straight-time and premium-time pay, a shift differential, production bonuses, and other agreed upon items.

It should be noted that Woodbury could easily make comparisons among his entries for the various years because the Table 19 he used in their construction was a summary of employee benefits for only those companies submitting data over the entire interval of 1957–1977. The entries given in Table A are constructed for all companies reporting data in the various years listed. Because of changes in the number and composition of companies reporting benefits between 1951 and 1992 ($N = 736$ and $N = 1194$, respectively), these entries are not strictly comparable. They do, however, indicate the trend in benefits over time. In addition, Woodbury's calculations for supplements (% of total) in 1967 through 1977 are larger because the average benefits of the few companies included in the old Table 19 are somewhat higher than those for the full sample. The reason for this lies in the fact that those companies reporting over the entire period have larger, more established benefits programs than those companies included in the full sample but excluded from the Table 19 sample.

A two-step procedure is utilized to construct the entries shown in the first three columns of Table A. To begin, the information in Table 16 regarding average annual and average hourly gross payroll is used to determine the average number of hours for which an employee is paid. Given this information, Chart 2 is employed to determine the average benefits received per hour per employee.³ After calculating such benefits on per hour basis, these are applied to Woodbury's definitions of the three entries; the results reported in Table A are expressed as a percent of total compensation per hour and are indexed to equal 100 in 1951.

The entries given in Table 1 of the text are derived in a similar manner. In this case, however, we take from Table 7 of the *Employee Benefits* publications estimates of the average hourly employer contributions to the following components of legally required, voluntary, and inside payroll labor costs:⁴ 1) Social Security, workers' compensation, and other legally required benefits (unemployment insurance, state sickness benefits, etc.); 2) pension plan premiums and retirement savings plan contributions; contributions to employee life, death, and medical (and medically related) insurance, as well as miscellaneous voluntary benefits (supplemental unemployment insurance, employee discounts on company goods and services, employee meals, childcare, and other benefits payments); and 3) paid rest (coffee and meal breaks, setup and wash up time, travel time, etc.) and paid leave (paid vacations and holidays, sick leave, parental leave, etc.). These benefit costs per hour are in turn expressed as a percent of total compensation per hour using data from the first column of Table A; the results are subsequently indexed to equal 100 in 1951.

Notes

1. The ECI was only implemented in stages. Beginning in 1976, published statistics covered only quarterly changes in wages and salaries of private, nonfarm workers. In 1978, the BLS expanded the survey to include 13 additional statistical series (e.g., union/nonunion, manufacturing/nonmanufacturing); by 1980, it had incorporated into the survey the publication of quarterly changes in total employee compensation. What the *BLS Handbook of Methods* (Chapter 8, p. 56) terms the third stage in the development of the ECI involved the expansion of the survey to state and local (not federal) government employees. Finally, the most recent development in the ECI involves the inclusion of actual compensation costs on a per hour basis; the BLS has included these measures in the ECI since March of 1987.
2. These benefits include paid leave (vacations, holidays, sick leave, and other), supplemental pay (premium pay for overtime and for work on weekends and holidays, nonproduction bonuses), insurance benefits (life, health, sickness and accident insurance), retirement and savings benefits (pension and other retirement plans, savings and thrift plans), legally required benefits (Social Security, Workers' Compensation, Unemployment Insurance, and other), and other benefits (severance pay, supplemental unemployment plans, and employee merchandise discounts in department stores).
3. In other words, average benefits received per hour equal the ratio of average annual benefits to average hours for which the employee is paid per year.
4. All entries after 1979 are computed from Table 7 of the *Employee Benefits* publication (Chamber of Commerce of the United States, various years) for the corresponding year; all pre-1980 entries are computed from Table 7 of *Employee Benefits Historical Data, 1951-1979* (Chamber of Commerce of the United States 1981).

Appendix B

Derivation of Market Labor Demand Function

Assume that F and C are both quadratic as follows:

$$F(n) = 1/2 F_{11}n^2 + \theta_1n + \theta_0,$$

$$C(G, n) = 1/2 C_{11}G^2 + C_{12}Gn + \phi_1G + \phi_2n + \phi_0.$$

We first show that the individual labor demand functions have the following form:

$$n = \frac{1}{\Delta}(w - \Omega), \quad (\text{B1})$$

where

$$\Delta = pF_{11} - 2(C_{12}G + \phi_2),$$

$$\Omega = p(F_n - F_{11} \cdot n) - (1/2 C_{11}G^2 + \phi_1G + \phi_0).$$

To prove the above, note that we have the following expressions:

$$F_n = \frac{\partial F}{\partial n} = F_{11}n + \theta_1 > 0,$$

$$C_1 = \frac{\partial C}{\partial G} = C_{11}G + C_{12}n + \phi_1 > 0,$$

$$C_2 = \frac{\partial C}{\partial n} = C_{12}G + \phi_2 \begin{matrix} > \\ < \end{matrix} 0.$$

Now, the firm's labor demand function given by Eq. 4a* is expanded so that

$$\begin{aligned}
 w &= pF_n - (C + nC_2) \\
 &= \left\{ pF_n - \left[1/2 C_{11}G^2 + C_{12}Gn + \phi_1G + \phi_2n + \phi_0 + n(C_{12}G + \phi_2) \right] \right\} \\
 &= \left\{ pF_n - (1/2 C_{11}G^2 + \phi_1G + \phi_0) - 2(C_{12}G + \phi_2)n \right\} \\
 &= \left\{ p(F_n - F_{11}n) - (1/2 C_{11}G^2 + \phi_1G + \phi_0) + [p \cdot F_{11} - 2(C_{12}G + \phi_2)]n \right\} \\
 &= (\Omega + \Delta n).
 \end{aligned}$$

As a result,

$$\begin{aligned}
 n &= \frac{1}{\Delta} (w - \Omega) \tag{B2} \\
 &= \frac{w - p(F_n - F_{11} \cdot n) + (1/2 C_{11}G^2 + \phi_1G + \phi_0)}{pF_{11} - 2(C_{12}G + \phi_2)}
 \end{aligned}$$

is the firm's new labor demand function. Note that

$$F_n - F_{11}n = \theta_1,$$

so that the right hand side of Eq. B2 is a function of w , parameterized by G and p .

Next, we show that if all K firms are identical and if the price feedback is given by $p = p(E)$, $p' < 0$, where $E = Kn$, then the inverse market labor demand function has the form

$$w = \tau(G) + \beta(G)E + \varepsilon(G, E), \tag{B3}$$

where

$$\tau(G) = [const. - 1/2 C_{11}G^2 - \phi_1G], \tag{B4}$$

$$\beta(G) = [const. - 2C_{12}G + Kp'\theta_1], \tag{B5}$$

and $\varepsilon(G, E)$ is an error term.

To prove the above, note that since $p = p(E)$ is a function of E , the right-hand side of Eq. B2 contains the term E . This implies that the market labor demand function cannot be obtained by simply multiplying Eq. B2 by K . Instead, one must first gather all the E terms on the left-hand side as follows:

Multiplying Eq. B2 by K , we have

$$E = n \cdot K = \frac{K}{\Delta} (w - \Omega).$$

As such,

$$\begin{aligned} Kw &= \Delta E + K\Omega \\ &= E[pF_{11} - 2(C_{12}G + \phi_2)] + K[p(F_n - F_{11}n) - (1/2 C_{11}G^2 + \phi_1G + \phi_0)] \\ &= -(1/2 C_{11}G^2 + \phi_1G + \phi_0)K - 2(C_{12}G + \phi_2)E + KF_n \cdot p(E). \end{aligned}$$

Substituting $F_n = F_{11}n + \theta_1$, and $p(E) = p_0 + p'E + \dots$ into the above expression, we obtain

$$\begin{aligned} w &= \text{const.} - 1/2 C_{11}G^2 - \phi_1G \\ &\quad + \frac{(\text{const.} - 2C_{12}G + K \cdot p'\theta_1)E}{K} + \varepsilon(G, E) \\ &= \tau(G) + \beta(G)E + \varepsilon(G, E), \end{aligned}$$

where $\varepsilon(G, E)$ is an error term containing all higher order terms of E . Letting $\tau(G) = a_0 + a(G)$, we have $a(G) = (\text{const.} - 1/2 C_{11}G^2 - \phi_1G)$. Thus

$$\frac{\partial \tau}{\partial G} = -(C_{11}G + \phi_1), \quad \frac{\partial \tau^2}{\partial G^2} = -C_{11} < 0, \quad (\text{B6})$$

$$\frac{\partial \beta}{\partial G} = \frac{-2C_{12}}{K} \quad (\text{B7})$$

Note that Eq. B3 is approximated by Eq. 5 as $w^d = \alpha - (\eta G + \rho G^2) + \beta(G)E$.

Appendix C

This appendix describes the logic behind Figure 7. Let us first prove the proposition that the competitive equilibrium occurs at the maximum feasible level of employment on the L curve only when $d\beta/dG = 0$. By solving Eq. 9a for E and computing $dE/dG = 0$ to select the maximum employment point, we obtain

$$(\eta + 2\rho G - \lambda) - E(d\beta/dG) = 0. \quad (C1)$$

Now, the competitive equilibrium point on the L curve is now obtained by combining Equations 9a and 9b to obtain

$$(\eta + 2\rho G - \lambda) - 0.5E(d\beta/dG) = 0. \quad (C2)$$

Clearly, Equations C1 and C2 are equivalent only when $d\beta/dG = 0$; therefore, the competitive equilibrium employment level is the maximum employment level only when $d\beta/dG = 0$.

We now demonstrate the locations of points A , B , and C in Figure 7. Rearranging Eq. 9b, we obtain

$$(\eta + 2\rho G - \lambda) = 0.5E(d\beta/dG) = 0. \quad (C3)$$

Assuming that $d^2\beta/dG^2 = 0$, an increase in G increases the left-hand side of Eq. C3; as a result, the right-hand side must also increase. If $d\beta/dG > 0$, the right-hand side will increase only when E rises. This result implies that, in this case, we are at point B in Figure 7. Similarly, if $d\beta/dG < 0$, E must decrease when G increases; in this case, we must be at point C in Figure 7.