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CP violation in $B \to \phi K_S$ in a model III two Higgs doublet model

Chao-Shang Huang*
Institute of Theoretical Physics, Academia Sinica, Beijing 100080, China

Shou-Hua Zhu†
Ottawa-Carleton Institute for Physics, Department of Physics, Carleton University, Ottawa, Canada K1S 5B6

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The mixing induced time dependent *CP* asymmetry, direct *CP* asymmetry, and branching ratio in $B \to \phi K_S$ in a model III two Higgs doublet model are calculated, in particular, neutral Higgs boson contributions are included. It is shown that satisfying all the relevant experimental constraints, for time dependent *CP* asymmetry $S_{\phi K}$ model III can agree with the present data, $S_{\phi k} = -0.39 \pm 0.41$, within a 1σ error, and the direct *CP* asymmetry which is zero in the SM can be about -8% to -20% in the reasonable regions of the parameters.

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I. INTRODUCTION

The recently reported measurements of time dependent *CP* asymmetries in $B \to \phi K_S$ decays[1] by BaBar [3]

$$\sin[2\beta(\phi K_S)]_{BaBar} = -0.19^{+0.52}_{-0.50} \pm 0.09 \tag{1}$$

and Belle [4]

$$\sin[2\beta(\phi K_S)]_{Belle} = -0.73 \pm 0.64 \pm 0.18 \tag{2}$$

result in the error weighted average

$$\sin[2\beta(\phi K_S)]_{ave} = -0.39 \pm 0.41 \tag{3}$$

with errors added in quadrature. The value in Eq. (2) corresponds to the coefficient of the sine term in time dependent *CP* asymmetry [6]; see Sec. IV. Belle also quotes a value for the direct *CP* asymmetry $A_{CP} = -C_{\phi K}$, i.e., the cosine term, $C_{\phi K} = -0.19 \pm 0.30$ [4,5]. Although there are at present large theoretical uncertainties in calculating strong phases, we still examine direct *CP* asymmetry in the paper in order to obtain qualitatively feeling for the effects of new physics on *CP* violation.

In the SM the above asymmetry is related to that in $B \to J/\psi K_S$ [7–10] by

$$\sin[2\beta(\phi K)] = \sin[2\beta(J/\psi K)] + O(\lambda^2), \tag{4}$$

where $\lambda \simeq 0.2$ appears in Wolfenstein's parametrization of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and $\sin[2\beta(J/\psi K_{S,L})]_{worldave} = +0.734 \pm 0.054$. Therefore, Eq. (3) violates the SM at the 2.7 σ deviation. The impact of these experimental results on the validity of CKM and SM is currently statistics limited. Future prospects at the *B*-factories are that the statistical error $\sigma_{\phi K_S}(\text{stat})$ can be expected to improve roughly by a factor of three with an increase of integrated luminosity from $0.1 ab^{-1}$ to $1 ab^{-1}$ [11] and it will take some time before we know the deviation with sufficient precision to draw final conclusions.

However, the possibility of a would-be measurement of $\sin[2\beta(\phi K_S)] = -0.39$ or a similar value which departs drastically from the SM expectation of Eq. (4) has attracted much interest to search for new physics, in particular, supersymmetry, two Higgs doublet model (2HDM), and model-independent way [12,13]. In the paper we consider the decay $B \to \phi K_S$ in a model III 2HDM. It is well known that in the model III 2HDM the couplings involving Higgs bosons and fermions can have complex phases, which can induce *CP* violation effects, even in the simplest case in which all tree-level flavor charging neutral current (FCNC) couplings are negligible. The effect of the color dipole operator on the phase from the decay amplitudes, $\Delta\Phi \equiv \arg(\bar{A}/A)$, in $b \to s\bar{s}s$ in the model III 2HDM has been studied in the second paper of Ref. [12] by Hiller and the result is $\Delta\Phi \leqslant 0.2$ which is far from explaining the deviation. We would like to see if it is possible to explain the deviation in the model III 2HDM under all known experimental constraints by extending to include the neutral Higgs boson (NHB) contributions and calculate hadronic matrix elements to the α_s order. Some relevant Wilson coefficients at the leading order (LO) in the model III 2HDM have been given [14]. Because the hadronic matrix elements of relevant operators have been calculated to the α_s order [15], we can obtain the amplitude of the process to the α_s order if we know the relevant Wilson coefficients at the next to leading order (NLO). In the paper we calculate them at NLO in the model III 2HDM. Furthermore, as pointed out in Ref. [13] the NHB penguin induced operators contribute sizably to both the branching ratio (Br) and time dependent *CP* asymmetry $S_{\phi k}$ in supersymmetrical models. In the paper we calculate the Wilson coefficients of NHB penguin induced operators in the model III 2HDM. Our results show that in the model III 2HDM, the *CP* asymmetry $S_{\phi K}$ can agree with the present data, $S_{\phi k} = -0.39 \pm 0.41$, within the 1σ error. Even if the $S_{\phi k}$ is measured to a level of

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*Email address: csh@itp.ac.cn
†Email address: huald@physics.carleton.ca
[1]The 2003 new results are $S_{\phi k} = -0.96 \pm 0.50^{+0.09}_{-0.11}$ by Belle [1] and $+0.45 \pm 0.43 \pm 0.07$ by BaBar [2].

-0.4 ± 0.1 in the future, the model III can still agree with the data at the 2σ level. And the direct *CP* asymmetry can reach about -20%.

The paper is organized as follows. In Sec. II we describe the model III 2HDM briefly. In Sec. III we give the effective Hamiltonian responsible for $B\to\phi K_s$ in the model. In particular, we give the Wilson coefficients at NLO for the operators which exist in SM and at LO for the new operators which are induced by NHB penguins, respectively. We present the decay amplitude and the *CP* asymmetry $S_{\phi K}$ in $B\to\phi K_s$ in Sec. IV. Section V is devoted to numerical results. In Sec. VI we draw our conclusions and present some discussions.

II. MODEL III TWO-HIGGS-DOUBLET MODEL (2HDM)

In model III 2HDM, both the doublets can couple to the up-type and down-type quarks; the details of the model can be found in Ref. [16]. The Yukawa Lagrangian relevant to our discussion in this paper is

$$\begin{aligned}\mathcal{L}_Y=&-\frac{g}{2M_W}(H^0\cos\alpha-h^0\sin\alpha)(\bar{U}M_UU+\bar{D}M_DD)\\&-\frac{H^0\sin\alpha+h^0\cos\alpha}{\sqrt{2}}\left[\bar{U}\left(\hat{\xi}^U\frac{1}{2}(1+\gamma^5)+\hat{\xi}^{U\dagger}\frac{1}{2}(1-\gamma^5)\right)U+\bar{D}\left(\hat{\xi}^D\frac{1}{2}(1+\gamma^5)+\hat{\xi}^{D\dagger}\frac{1}{2}(1-\gamma^5)\right)D\right]\\&+\frac{iA^0}{\sqrt{2}}\left[\bar{U}\left(\hat{\xi}^U\frac{1}{2}(1+\gamma^5)-\hat{\xi}^{U\dagger}\frac{1}{2}(1-\gamma^5)\right)U-\bar{D}\left(\hat{\xi}^D\frac{1}{2}(1+\gamma^5)-\hat{\xi}^{D\dagger}\frac{1}{2}(1-\gamma^5)\right)D\right]\\&-H^+\bar{U}\left[V_{\rm CKM}\hat{\xi}^D\frac{1}{2}(1+\gamma^5)-\hat{\xi}^{U\dagger}V_{\rm CKM}\frac{1}{2}(1-\gamma^5)\right]D-H^-\bar{D}\left[\hat{\xi}^{D\dagger}V^\dagger_{\rm CKM}\frac{1}{2}(1-\gamma^5)-V^\dagger_{\rm CKM}\hat{\xi}^U\frac{1}{2}(1+\gamma^5)\right]U,\end{aligned}\tag{5}$$

where U represents the mass eigenstates of u,c,t quarks and D represents the mass eigenstates of d,s,b quarks, $V_{\rm CKM}$ is the Cabibbo-Kobayashi-Maskawa matrix and the FCNC couplings are contained in the matrices $\hat{\xi}^{U,D}$. The Cheng-Sher ansatz for $\hat{\xi}^{U,D}$ is [16]

$$\hat{\xi}^{U,D}_{ij}=\lambda_{ij}\frac{g\sqrt{m_im_j}}{\sqrt{2}M_W}\tag{6}$$

by which the quark-mass hierarchy ensures that the FCNC within the first two generations are naturally suppressed by the small quark masses, while a larger freedom is allowed for the FCNC involving the third generations.[2] In the ansatz the residual degree of arbitrariness of the FC couplings is expressed through the λ_{ij} parameters which are of order one and need to be constrained by the available experiments. In the paper we choose $\xi^{U,D}$ to be diagonal and set the u and d quark masses to be zero for the sake of simplicity so that besides Higgs boson masses only $\lambda_{ii}, i=s,c,b,t$, are the new parameters and will enter into the Wilson coefficients relevant to the process.

III. EFFECTIVE HAMILTONIAN

The effective Hamiltonian for charmless B decays with $\Delta B=1$ is given by [13,20]

$$\begin{aligned}\mathcal{H}_{\rm eff}=&\frac{G_F}{\sqrt{2}}\sum_{p=u,c}V_{pb}V^*_{ps}\Bigg(C_1Q^p_1+C_2Q^p_2+\sum_{i=3,\dots,16}[C_iQ_i\\&+C'_iQ'_i]+C_{7\gamma}Q_{7\gamma}+C_{8g}Q_{8g}+C'_{7\gamma}Q'_{7\gamma}+C'_{8g}Q'_{8g}\Bigg)\\&+\text{H.c.}\end{aligned}\tag{7}$$

Here Q_i are quark and gluon operators and are given by

$$Q^p_1=(\bar{s}_\alpha p_\beta)_{V-A}(\bar{p}_\beta b_\alpha)_{V-A},$$

$$Q^p_2=(\bar{s}_\alpha p_\alpha)_{V-A}(\bar{p}_\beta b_\beta)_{V-A},$$

$$Q_{3(5)}=(\bar{s}_\alpha b_\alpha)_{V-A}\sum_q(\bar{q}_\beta q_\beta)_{V-(+)A},$$

$$Q_{4(6)}=(\bar{s}_\alpha b_\beta)_{V-A}\sum_q(\bar{q}_\beta q_\alpha)_{V-(+)A},$$

$$Q_{7(9)}=\frac{3}{2}(\bar{s}_\alpha b_\alpha)_{V-A}\sum_q e_q(\bar{q}_\beta q_\beta)_{V+(-)A},$$

$$Q_{8(10)}=\frac{3}{2}(\bar{s}_\alpha b_\beta)_{V-A}\sum_q e_q(\bar{q}_\beta q_\alpha)_{V+(-)A},$$

$$Q_{11(13)}=(\overline{sb})_{S+P}\sum_q\frac{m_q\lambda^*_{qq}(\lambda_{qq})}{m_b}(\bar{q}q)_{S-(+)P},$$

[2]Model III 2HDM has a remarkably stable FCNC suppression when one evolves the FCNC Yukawa coupling parameters by the RGE's to higher energies [17].

$$Q_{12(14)}=(\bar{s}_i b_j)_{S+P}\sum_q \frac{m_q\lambda^*_{qq}(\lambda_{qq})}{m_b}(\bar{q}_j q_i)_{S-(+)P},$$

$$Q_{15}=\overline{s\sigma^{\mu\nu}}(1+\gamma_5)b\sum_q \frac{m_q\lambda_{qq}}{m_b}\bar{q}\sigma_{\mu\nu}(1+\gamma_5)q,$$

$$Q_{16}=\bar{s}_i\sigma^{\mu\nu}(1+\gamma_5)b_j\sum_q \frac{m_q\lambda_{qq}}{m_b}\bar{q}_j\sigma_{\mu\nu}(1+\gamma_5)q_i,$$

$$Q_{7\gamma}=\frac{e}{8\pi^2}m_b\bar{s}_\alpha\sigma^{\mu\nu}F_{\mu\nu}(1+\gamma_5)b_\beta,$$

$$Q_{8g}=\frac{g_s}{8\pi^2}m_b\bar{s}_\alpha\sigma^{\mu\nu}G^a_{\mu\nu}\frac{\lambda_a^{\alpha\beta}}{2}(1+\gamma_5)b_\beta, \tag{8}$$

where $(V\pm A)(V\pm A)=\gamma^\mu(1\pm\gamma_5)\gamma_\mu(1\pm\gamma_5)$, $(\bar{q}_1 q_2)_{S\pm P}=\bar{q}_1(1\pm\gamma_5)q_2$, $q=u,d,s,c,b$, e_q is the electric charge number of q quark, λ_a is the color SU(3) Gell-Mann matrix, α and β are color indices, and $F_{\mu\nu}$ $[G_{\mu\nu}]$ are the photon [gluon] field strengths. The operators Q_i's are obtained from the unprimed operators Q_is by exchanging $L\leftrightarrow R$. In Eq. (7) operators Q_i, $i=11,\ldots,16$, are induced by neutral Higgs boson mediations [13].

The Wilson coefficients C_i, $i=1,\ldots,10$, have been calculated at LO [20,14]. We calculate them at NLO in the NDR scheme and results are as follows:

$$C_1(M_W)=\frac{11}{2}\frac{\alpha_s(M_W)}{4\pi}, \tag{9}$$

$$C_2(M_W)=1-\frac{11}{6}\frac{\alpha_s(M_W)}{4\pi}-\frac{35}{18}\frac{\alpha}{4\pi}, \tag{10}$$

$$C_3(M_W)=-\frac{\alpha_s(M_W)}{24\pi}\{\tilde{E}_0(x_t)+E_0^{III}(y)\}+\frac{\alpha}{6\pi}\frac{1}{\sin^2\theta_W}[2B_0(x_t)+C_0(x_t)], \tag{11}$$

$$C_4(M_W)=\frac{\alpha_s(M_W)}{8\pi}\{\tilde{E}_0(x_t)+E_0^{III}(y)\}, \tag{12}$$

$$C_5(M_W)=-\frac{\alpha_s(M_W)}{24\pi}\{\tilde{E}_0(x_t)+E_0^{III}(y)\}, \tag{13}$$

$$C_6(M_W)=\frac{\alpha_s(M_W)}{8\pi}\{\tilde{E}_0(x_t)+E_0^{III}(y)\}, \tag{14}$$

$$C_7(M_W)=\frac{\alpha}{6\pi}[4C_0(x_t)+\tilde{D}_0(x_t)], \tag{15}$$

$$C_8(M_W)=0, \tag{16}$$

$$C_9(M_W)=\frac{\alpha}{6\pi}\left[4C_0(x_t)+\tilde{D}_0(x_t)+\frac{1}{\sin^2\theta_W}[10B_0(x_t)-4C_0(x_t)]\right], \tag{17}$$

$$C_{10}(M_W)=0, \tag{18}$$

$$C_{7\gamma}(M_W)=-\frac{A(x_t)}{2}-\frac{A(y)}{6}|\lambda_{tt}|^2+B(y)\lambda_{tt}\lambda_{bb}e^{i\theta}, \tag{19}$$

$$C_{8G}(M_W)=-\frac{D(x_t)}{2}-\frac{D(y)}{6}|\lambda_{tt}|^2+E(y)\lambda_{tt}\lambda_{bb}e^{i\theta}, \tag{20}$$

where $x_t=m_t^2/M_W^2$, and $y=m_t^2/M_{H^\pm}^2$. Here the Wilson coefficients $C_{7\gamma}$ and C_{8g} at LO which are given in Ref. [14] have also been written. The Wilson coefficients $C_{7\gamma}$ and C_{8g} at NLO in SM have been given but they at NLO in model III 2HDM have not been calculated yet. Because we calculate the decay amplitude only to the α_s order it is enough to know them at LO. Here

$$A(x)=x\left[\frac{8x^2+5x-7}{12(x-1)^3}-\frac{(3x^2-2x)\ln x}{2(x-1)^4}\right], \tag{21}$$

$$B(y)=y\left[\frac{5y-3}{12(y-1)^2}-\frac{(3y-2)\ln y}{6(y-1)^3}\right], \tag{22}$$

$$D(x)=x\left[\frac{x^2-5x-2}{4(x-1)^3}+\frac{3x\ln x}{2(x-1)^4}\right], \tag{23}$$

$$E(y)=y\left[\frac{y-3}{4(y-1)^2}+\frac{\ln y}{2(y-1)^3}\right], \tag{24}$$

$$B_0(x_t)=\frac{1}{4}\left[\frac{x_t}{1-x_t}+\frac{x_t\ln x_t}{(x_t-1)^2}\right], \tag{25}$$

$$C_0(x_t)=\frac{x_t}{8}\left[\frac{x_t-6}{x_t-1}+\frac{3x_t+2}{(x_t-1)^2}\ln x_t\right], \tag{26}$$

$$D_0(x_t)=-\frac{4}{9}\ln x_t+\frac{-19x_t^3+25x_t^2}{36(x_t-1)^3}+\frac{x_t^2(5x_t^2-2x_t-6)}{18(x_t-1)^4}\ln x_t, \tag{27}$$

$$\tilde{D}_0(x_t)=D_0(x_t)-\frac{4}{9} \tag{28}$$

and

$$E_0(x_t)=-\frac{2}{3}\ln x_t+\frac{x_t(18-11x_t-x_t^2)}{12(1-x_t)^3}+\frac{x_t^2(15-16x_t+4x_t^2)}{6(1-x_t)^4}\ln x_t, \quad (29)$$

$$\tilde{E}_0(x_t)=E_0(x_t)-\frac{2}{3}, \quad (30)$$

$$E_0^{III}(y)=|\lambda_{tt}|^2\left\{\frac{16y-29y^2+7y^3}{36(1-y)^3}+\frac{2y-3y^2}{6(1-y)^4}\ln y\right\}. \quad (31)$$

The Wilson coefficients C_i, $i=11,\ldots,16$, at the leading order can be obtained from C_{Q1} and C_{Q2} in Ref. [19]. The nonvanishing coefficients at m_W are

$$C_{11}(M_W)=\frac{\alpha}{4\pi}\frac{m_b}{m_\tau\lambda^*_{\tau\tau}}(C_{Q1}-C_{Q2}),$$

$$C_{13}(M_W)=\frac{\alpha}{4\pi}\frac{m_b}{m_\tau\lambda_{\tau\tau}}(C_{Q1}+C_{Q2}). \quad (32)$$

We shall omit the contributions of the primed operators in numerical calculations for they are suppressed by m_s/m_b in model III 2HDM.

For the process we are interested in for this paper, the Wilson coefficients should run to the scale of $O(m_b)$. C_1-C_{10} are expanded to $O(\alpha_s)$ and NLO RGEs should be used. However for the C_{8g} and $C_{7\gamma}$, LO results should be sufficient. The details of the running of these Wilson coefficients can be found in Ref. [20]. The one loop anomalous dimension matrices of the NHB induced operators can be divided into two distangled groups [23]

$\gamma^{(RL)}=$ (33)

	O_{11}	O_{12}
O_{11}	−16	0
O_{12}	−6	2

and

$\gamma^{(RR)}=$ (34)

	O_{13}	O_{14}	O_{15}	O_{16}
O_{13}	−16	0	1/3	−1
O_{14}	−6	2	−1/2	−7/6
O_{15}	16	−48	16/3	0
O_{16}	−24	−56	6	−38/3

For Q_i' operators we have

$$\gamma^{(LR)}=\gamma^{(RL)} \quad \text{and} \quad \gamma^{(LL)}=\gamma^{(RR)}. \quad (35)$$

Because at present no NLO Wilson coefficients $C_i^{(\prime)}$, $i=11,\ldots,16$, are available we use the LO running of them in the paper.

IV. THE DECAY AMPLITUDE AND *CP* ASYMMETRY IN $B_d^0\to\phi K_S$

We use the BBNS approach [18] to calculate the hadronic matrix elements of operators. In the approach the hadronic matrix element of a operator in the heavy quark limit can be written as

$$\langle\phi K|Q|B\rangle=\langle\phi K|Q|B\rangle_f\left[1+\sum r_n\alpha_s^n\right], \quad (36)$$

where $\langle\phi K|Q|B\rangle_f$ indicates the naive factorization result. The second term in the square bracket indicates higher order α_s corrections to the matrix elements [18]. We calculate hadronic matrix elements to the α_s order in the paper. In order to see explicitly the effects of new operators in the model III 2HDM we divide the decay amplitude into two parts. One has the same form as that in SM, the other is new. That is, we can write the decay amplitude, to the α_s order, for $B\to\phi K$ in the heavy quark limit as [15,21,13]

$$A(B\to\phi K)=\frac{G_F}{\sqrt{2}}A\langle\phi|\bar{s}\gamma_\mu s|0\rangle\langle K|\bar{s}\gamma^\mu b|B\rangle,$$

$$A=A^o+A^n, \quad (37)$$

$$A^o=V_{ub}V_{us}^*\left[a_3^u+a_4^u+a_5^u-\frac{1}{2}(a_7^u+a_9^u+a_{10}^u)+a_{10a}^u\right]+V_{cb}V_{cs}^*\left[a_3^c+a_4^c+a_5^c-\frac{1}{2}(a_7^c+a_9^c+a_{10}^c)+a_{10a}^c\right], \quad (38)$$

$$A^n=-V_{tb}V_{ts}^*\left(a_4^{neu}+\frac{m_s}{m_b}\left[-\frac{1}{2}\lambda_{ss}^*(a_{12}+a_{12}')+\lambda_{ss}\frac{4m_s}{m_b}(a_{16}+a_{16}')\right]\right). \quad (39)$$

For the hadronic matrix elements of the vector current, we can write $\langle\phi|\bar{s}\gamma_\mu b|0\rangle=m_\phi f_\phi\epsilon_\mu^\phi$ and $\langle K|\bar{s}\gamma^\mu b|B\rangle=F_1^{B\to K}(q^2)(p_B^\mu+p_K^\mu)+[F_0^{B\to K}(q^2)-F_1^{B\to K}(q^2)](m_B^2-m_K^2)\times q^\mu/q^2$. Here, the coefficients $a_i^{u,c}$ in Eq. (38) are given by[3]

$$a_3^u=a_3^c=c_3+\frac{c_4}{N}+\frac{\alpha_s}{4\pi}\frac{C_F}{N}c_4F_\phi,$$

[3]The explicit expressions of the coefficients $a_i^{u,c}$ have been given in Ref. [21]. Because there are minor errors in the expressions in the paper and in order to make this paper self-contained we reproduce them here, correcting the minor errors.

$$a_4^p=c_4+\frac{c_3}{N}+\frac{\alpha_s}{4\pi}\frac{C_F}{N}\bigg[c_3[F_\phi+\hat{G}_\phi(s_s)+\hat{G}_\phi(s_b)]+c_2\hat{G}_\phi(s_p)+(c_4+c_6)\sum_{f=u}^{b}\tilde{G}_\phi(s_f)+c_{8g}G_{\phi,8g}\bigg],$$

$$a_5^u=a_5^c=c_5+\frac{c_6}{N}+\frac{\alpha_s}{4\pi}\frac{C_F}{N}c_6(-F_\phi-12),$$

$$a_7^u=a_7^c=c_7+\frac{c_8}{N}+\frac{\alpha_s}{4\pi}\frac{C_F}{N}c_8(-F_\phi-12),$$

$$a_9^u=a_9^c=c_9+\frac{c_{10}}{N}+\frac{\alpha_s}{4\pi}\frac{C_F}{N}c_{10}F_\phi,$$

$$a_{10}^u=a_{10}^c=c_{10}+\frac{c_9}{N}+\frac{\alpha_s}{4\pi}\frac{C_F}{N}c_9F_\phi,$$

$$a_{10a}^p=\frac{\alpha_s}{4\pi}\frac{C_F}{N}\bigg[(c_8+c_{10})\frac{3}{2}\sum_{f=u}^{b}e_f\hat{G}_\phi(s_f)+c_9\frac{3}{2}[e_s\hat{G}_\phi(s_s)+e_b\hat{G}_\phi(s_b)]\bigg], \tag{40}$$

where p takes the values u and c, $N=3$, $C_F=(N^2-1)/2N$, and $s_f=m_f^2/m_b^2$,

$$\hat{G}_\phi(s)=\frac{2}{3}+\frac{4}{3}\ln\frac{m_b}{\mu}-G_\phi(s),$$

$$G_\phi(s)=-4\int_0^1 dx\Phi_\phi(x)\int_0^1 du u(1-u)\times\ln[s-u(1-u)(1-x)],$$

$$\tilde{G}_\phi(s)=\hat{G}_\phi(s)-\frac{2}{3},$$

$$G_{\phi,8g}=-2G_\phi^0,\quad G_\phi^0=\int_0^1\frac{dx}{\bar{x}}\Phi_\phi(x),$$

$$F_\phi=-12\ln\frac{\mu}{m_b}-18+f_\phi^I+f_\phi^{II},$$

$$f_\phi^I=\int_0^1 dx g(x)\phi_\phi(x),$$

$$g(x)=3\frac{1-2x}{1-x}\ln x-3i\pi,$$

$$f_\phi^{II}=\frac{4\pi^2}{N}\frac{f_Kf_B}{F_1^{B\to K}(0)m_B^2}\int_0^1 dz\frac{\phi_B(z)}{z}\times\int_0^1 dx\frac{\phi_K(x)}{x}\int_0^1 dy\frac{\phi_\phi(y)}{y}, \tag{41}$$

where $\phi_i(x)$ are meson wave functions,

$$\phi_B(x)=N_Bx^2(1-x)^2\exp\bigg[-\frac{m_B^2x^2}{2\omega_B^2}\bigg],$$

$$\phi_{K,\phi}(x)=6x(1-x), \tag{42}$$

with normalization factor N_B satisfying $\int_0^1 dx\phi_B(x)=1$. Fitting various B decay data, ω_B is determined to be 0.4 GeV [22]. In Eq. (42) the asymptotic limit of the leading-twist distribution amplitudes for ϕ and K has been assumed.

The coefficients a_i in Eq. (39) are

$$a_4^{neu}=\frac{C_F\alpha_s}{4\pi}\frac{P_{\phi,2}^{neu}}{N_c},$$

$$a_{12}=C_{12}+\frac{C_{11}}{N_c}\bigg[1+\frac{C_F\alpha_s}{4\pi}(-V'-f_\phi^{II})\bigg],$$

$$a_{16}=C_{16}+\frac{C_{15}}{N_c}, \tag{43}$$

where

$$V'=-12\ln\frac{\mu}{m_b}-6+\int_0^1 dx g(\bar{x})\phi_\phi(x), \tag{44}$$

and

$$\begin{aligned}P_{\phi,2}^{neu}=&-\frac{1}{2}(C_{11}+C_{11}')\bigg[\frac{m_s\lambda_{ss}^*}{m_b}\bigg(\frac{4}{3}\ln\frac{m_b}{\mu}-G_\phi(0)\bigg)+\lambda_{bb}^*\bigg(\frac{4}{3}\ln\frac{m_b}{\mu}-G_\phi(1)\bigg)\bigg]+(C_{13}+C_{13}')\lambda_{bb}\bigg[-2\ln\frac{m_b}{\mu}G_\phi^0-GF_\phi(1)\bigg]\\&-4(C_{15}+C_{15}')\lambda_{bb}\bigg[\bigg(-\frac{1}{2}-2\ln\frac{m_b}{\mu}\bigg)G_\phi^0-GF_\phi(1)\bigg]\\&-8(C_{16}+C_{16}')\bigg[\lambda_{bb}\bigg(-2\ln\frac{m_b}{\mu}G_\phi^0-GF_\phi(1)\bigg)+\lambda_{cc}\bigg(\frac{m_c}{m_b}\bigg)^2\bigg(-2\ln\frac{m_b}{\mu}G_\phi^0-GF_\phi(s_c)\bigg)\bigg].\end{aligned} \tag{45}$$

In Eq. (45)

$$GF_{\phi}(s)=\int_0^1 dx\frac{\Phi_{\phi}(x)}{\bar{x}}GF(s-i\epsilon,\bar{x}),$$

$$GF(s,x)=\int_0^1 dt\,\ln[s-xt\bar{t}], \tag{46}$$

with $\bar{x}=1-x$. In calculations we have set $m_{u,d}=0$ and neglected the terms which are proportional to m_s^2/m_b^2 in Eq. (45). We have included only the leading twist contributions in Eq. (43). In Eq. (39) a_i' is obtained from a_i by substituting the Wilson coefficients C_j's for C_js. In numerical calculations a_i' is set to be zero because we have neglected C_j's. We see from Eq. (45) that the new contributions to the decay amplitude can be large if the coupling λ_{bb} is large due to the large contributions to the hadronic elements of the NHB induced operators at the α_s order arising from penguin contractions with b quark in the loop.

The decay rate can be obtained [21]

$$\Gamma(B\to\phi K)=\frac{G_F^2}{32\pi}|A|^2 f_\phi^2 F_1^{B\to K}(m_\phi^2)^2 m_B^3 P_{K\phi}^{3/2}, \tag{47}$$

where $P_{ij}=(1-m_i^2/m_B^2-m_j^2/m_B^2)^2-4m_i^2m_j^2/m_B^4$.

The time-dependent *CP*-asymmetry $S_{\phi K}$ is given by

$$a_{\phi K}(t)=-C_{\phi K}\cos(\Delta M_{B_d^0}t)+S_{\phi K}\sin(\Delta M_{B_d^0}t), \tag{48}$$

where

$$C_{\phi K}=\frac{1-|\lambda_{\phi k}|^2}{1+|\lambda_{\phi k}|^2},\quad S_{\phi K}=\frac{2\,\mathrm{Im}\,\lambda_{\phi k}}{1+|\lambda_{\phi k}|^2}. \tag{49}$$

Here $\lambda_{\phi k}$ is defined as

$$\lambda_{\phi k}=\left(\frac{q}{p}\right)_B\frac{\mathcal{A}(\bar{B}\to\phi K_S)}{\mathcal{A}(B\to\phi K_S)}. \tag{50}$$

The ratio $(q/p)_B$ is nearly a pure phase. In SM $\lambda_{\phi k}=e^{i2\beta}+O(\lambda^2)$. As pointed out in Introduction, the model III can give a phase to the decay which we call ϕ^{III}. Then we have

$$\lambda=e^{i(2\beta+\phi^{\mathrm{III}})}\frac{|\bar{\mathcal{A}}|}{|\mathcal{A}|}\Rightarrow S_{\phi K}=\sin(2\beta+\phi^{\mathrm{III}}) \tag{51}$$

if the ratio $|\bar{\mathcal{A}}|/|\mathcal{A}|=1$. In general the ratio in the model III is not equal to one and consequently it has an effect on the value of $S_{\phi K}$, as can be seen from Eq. (49). Thus the presence of the phases in the Yukawa couplings of the charged and neutral Higgs bosons can alter the value of $S_{\phi K}$ from the standard model prediction of $S_{\phi K}=\sin 2\beta_{J/\psi K}\sim 0.7$.

V. NUMERICAL ANALYSIS

A. Parameters input

In our numerical calculations we will use the following values for the relevant parameters: $m_b=4.8$ GeV, $m_c=1.5$ GeV, $m_t=175$ GeV, $\Lambda^{(5)}=225$ MeV, $2\times10^{-4}<\mathrm{Br}(B\to X_s\gamma)<4.5\times10^{-4}$, $d_n<10^{-25}$ e cm, $f_\phi=0.233$ GeV, $f_K=0.158$ GeV, $f_B=0.18$ GeV, and $F_1^{B\to K}(m_\phi)=0.3$. The parameters for CKM are $s_{12}=0.2229$, $s_{13}=0.0036$, $s_{23}=0.0412$, and $\delta_{13}=1.02$.

B. Constraints from $B\to X_s\gamma$ and neutron electric dipole moment (NEDM)

It is shown in Ref. [14] that the most strict constraints come from $B\to X_s\gamma$ and neutron electric dipole moment (NEDM). For completeness, we write the formulas as follows [24]:

$$\frac{\mathrm{Br}(B\to X_s\gamma)}{\mathrm{Br}(B\to X_c e\bar{\nu}_e)}\frac{|V_{ts}^*V_{tb}|^2}{|V_{cb}|^2}\frac{6\alpha_{\mathrm{em}}}{\pi f(m_c/m_b)}|C_{7\gamma}(m_b)|^2, \tag{52}$$

where $f(z)=1-8z^2+8z^6-z^8-24z^4\ln z$ and $\mathrm{Br}(b\to ce^-\bar{\nu})=10.45\%$.

The NEDM can be expressed as

$$d_n^g=10^{-25}\ e\ \mathrm{cm}\ \mathrm{Im}(\lambda_{tt}\lambda_{bb})\left(\frac{\alpha(m_n)}{\alpha(\mu)}\right)^{1/2}\left(\frac{\xi_g}{0.1}\right)H\left(\frac{m_t^2}{M_{H^\pm}^2}\right), \tag{53}$$

with

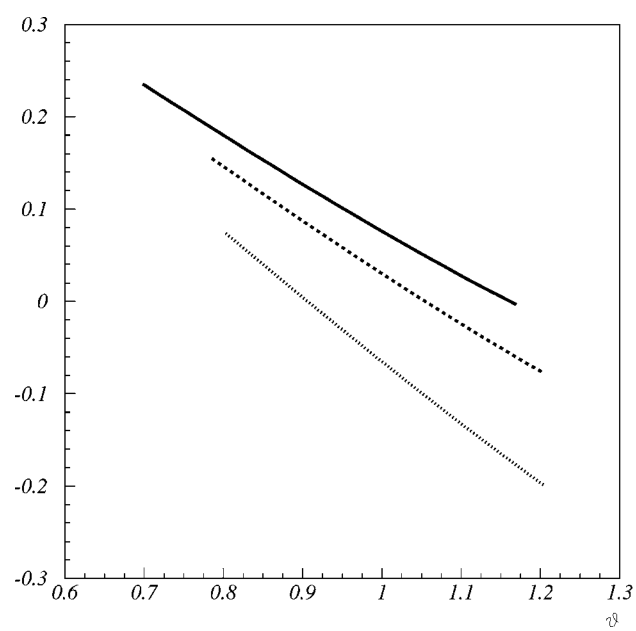

FIG. 1. $S_{\phi K}$ as a function of $\theta\equiv\theta_{bb}+\theta_{tt}$ with $\mu=2m_b$ (solid), m_b (dashed) and $m_b/2$ (dotted), where $m_{H^\pm}=200$ GeV, $|\lambda_{tt}|=0.03$, $|\lambda_{bb}|=100$, $\lambda_{ss}=\lambda_{cc}=100e^{-i\pi/2}$. The parameter ξ_g in neutron EDM expression is 0.03 [14,25].

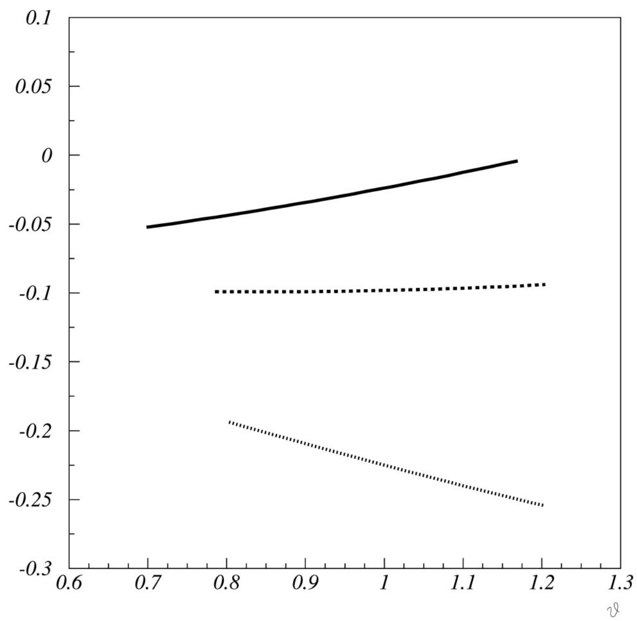

FIG. 2. ΔS (defined as the difference between $S_{\phi K}$ with and without NHB contributions) as a function of θ with $\mu = 2m_b$ (solid), m_b (dashed) and $m_b/2$ (dotted). Other parameters are the same as in Fig. 1.

$$H(y) = \frac{3}{2}\frac{y}{(1-y)^2}\left(y - 3 - \frac{2\log y}{1-y}\right). \tag{54}$$

C. Numerical results for $B\rightarrow K_s\phi$

We have scanned the parameter space in model III; in the following we will show the results for several specific parameters.

Figures 1–4 are devoted to the case in which neutral Higgs boson masses are set to be $m_{h^0} = 115$ GeV, $m_{A^0} = 120$ GeV, $m_{H^0} = 160$ GeV, which are the same with Ref. [19], and consequently $C_{11}(m_W) \gg C_{13}(m_W)$. Figures 1 and 2 show the $S_{\phi K}$ and ΔS, defined as the $S_{\phi K}$ difference with and without NHB contributions, as a function of $\theta \equiv \theta_{bb} + \theta_{tt}$ with $m_{H^\pm} = 200$ GeV. Note that there is another allowed region of θ, about -1.2 to -0.7, in which $S_{\phi K}$ is about 1. Therefore, we do not present the results in the figures. From the figures we can see that in model III, the charged and neutral Higgs boson contributions can decrease the value of $S_{\phi K}$ down to -0.2, in the allowed parameter space. It should be emphasized that the NHB contributions are sizable. In Figs. 3 and 4, we show the direct CP violation variable $C_{\phi K}$ and $\Delta C_{\phi K}$, defined as $C_{\phi K}$ difference with and without NHB contributions, as a function of θ. It is obvious that $C_{\phi K}$ can be 8–20 %, i.e., it can be in agreement with the data within 1σ deviation, while it is zero in the SM. At the same time, the NHB contributions can only change $C_{\phi K}$ by less than 3%.

Figures 5–8 (and also in Figs. 9–11) are plotted for the case in which the masses of NHBs have large splitting, $m_{A^0} = m_{H^0} = 1$ TeV$\gg m_{h^0} = 115$ GeV, and consequently $C_{11}(m_W)$ is the same order of magnitude, compared to

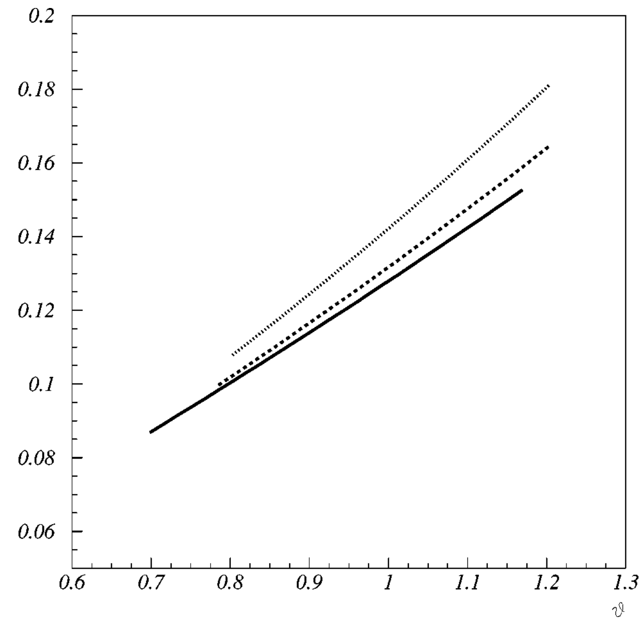

FIG. 3. $C_{\phi K}$ as a function of θ. Other parameters and conventions are the same as in Fig. 1.

$C_{13}(m_W)$. Now the NHB contributions are as important as those of the charged Higgs boson and $S_{\phi K}$ can reach about -0.6, as expected.

In order to demonstrate the NHB contributions, in Figs. 9–11, we show $S_{\phi K}$ as functions of the phases of λ_{bb} and λ_{ss}, θ_{bb} and θ_{ss}, and the correlation between $S_{\phi K}$ and $C_{\phi K}$, respectively. It is clear that $S_{\phi K}$ is sensitive to the phases. At the same time, in the range $[-\pi,\pi]$ of θ_{bb} and θ_{ss} $C_{\phi K}$ changes only several percents. There is a strong correlation between $S_{\phi K}$ and $C_{\phi K}$ and $C_{\phi K}$ is always positive regardless of the sign of $S_{\phi K}$, which is opposite to that of the central

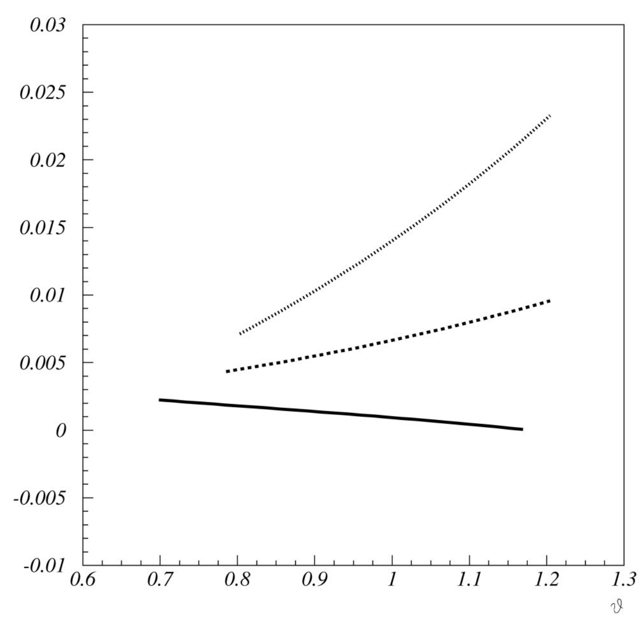

FIG. 4. ΔC (defined as the difference between $C_{\phi K}$ with and without NHB contributions) as a function of θ. Other parameters and conventions are the same as in Fig. 3.

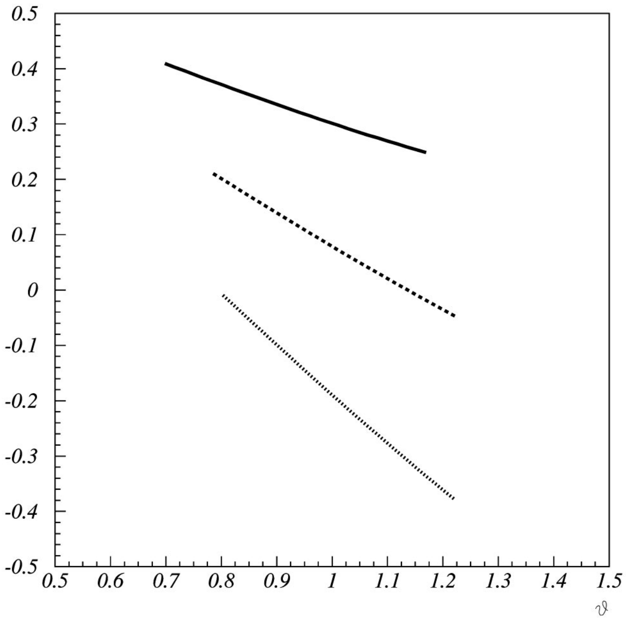

FIG. 5. $S_{\phi K}$ as a function of θ with $\mu=2m_b$ (solid), m_b (dashed) and $m_b/2$ (dotted), where $m_{H^\pm}=200$ GeV, $|\lambda_{tt}|=0.03$, $|\lambda_{bb}|=100$, $\lambda_{ss}=\lambda_{cc}=100e^{-i\pi/2}$. Note that the masses of NHB (in Figs. 5–11) are different than those in Figs. 1–4.

value of measurements. Therefore, if the minus $C_{\phi K}$ is confirmed in coming experiments the model III 2HDM could be excluded.

VI. CONCLUSIONS AND DISCUSSIONS

In summary we have calculated the Wilson coefficients at NLO for the operators in the SM (except for $Q_{7\gamma}$ and Q_{8g}), and at LO for the new operators which are induced by NHB penguins in the model III 2HDM. Using the Wilson coefficients obtained, we have calculated the mixing induced time-dependent *CP* asymmetry $S_{\phi k}$, branching ratio and direct *CP* asymmetry $C_{\phi K}$ for the decay $B\rightarrow\phi K_s$. It is shown that in the reasonable region of parameters where the constraints from B-$\overline{B}$ mixing, $\Gamma(b\rightarrow s\gamma)$, $\Gamma(b\rightarrow c\tau\overline{\nu_\tau})$, $\rho_0, R_b, B\rightarrow\mu^+\mu^-$, and electric dipole moments (EDMs) of the electron and neutron are satisfied, the branching ratio of the decay can reach 10×10^{-6}, $C_{\phi K}$ can reach 18% and $S_{\phi k}$ can be negative in quite a large region of parameters and as low as -0.6 in some regions of parameters.

Let us separately discuss the two cases: (1) only the charged Higgs contributions and (2) only the NHB contributions, in addition to the SM ones. Without NHB contributions, i.e., in the first case, the charged Higgs contributions can only decrease $S_{\phi k}$ to around 0. That is, the model III can agree with the present data, $S_{\phi k}=-0.39\pm0.41$, within 1σ error.

For the second case, our results show that the effects of NHB induced operators can be sizable even significant, depending on the characteristic scale μ of the process. Due to the large contributions to the hadronic elements of the operators at the α_s order arising from penguin contractions with b quark in the loop, both the Br and $S_{\phi K}$ are sizable or significantly different from those in SM.

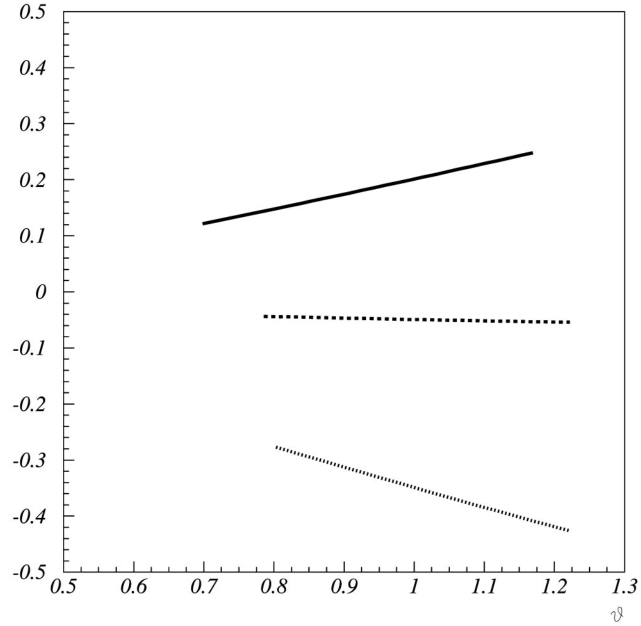

FIG. 6. ΔS as a function of θ with $\mu=2m_b$ (solid), m_b (dashed) and $m_b/2$ (dotted). Other parameters are the same as in Fig. 5.

Putting all the contributions together, we conclude that the model III can agree with the present data, $S_{\phi k}=-0.39\pm0.41$, within the 1σ error. Even if the $S_{\phi k}$ is measured to a level of -0.4 ± 0.1 in the future, the model III can still agree with the data at the 2σ level in quite a large regions of parameters and at the 1σ level in some regions of parameters. As for $C_{\phi K}$, our result is that it is positive, which is opposite to that of the measured central value. Considering

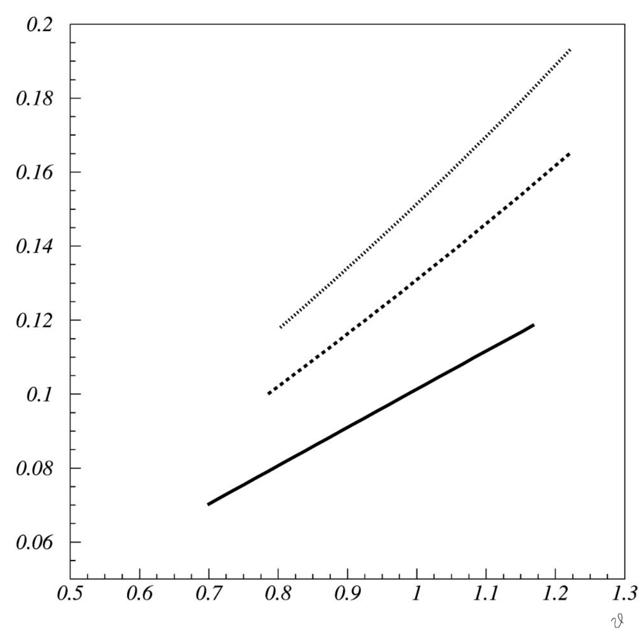

FIG. 7. $C_{\phi K}$ as a function of θ. Other parameters and conventions are the same as in Fig. 5.

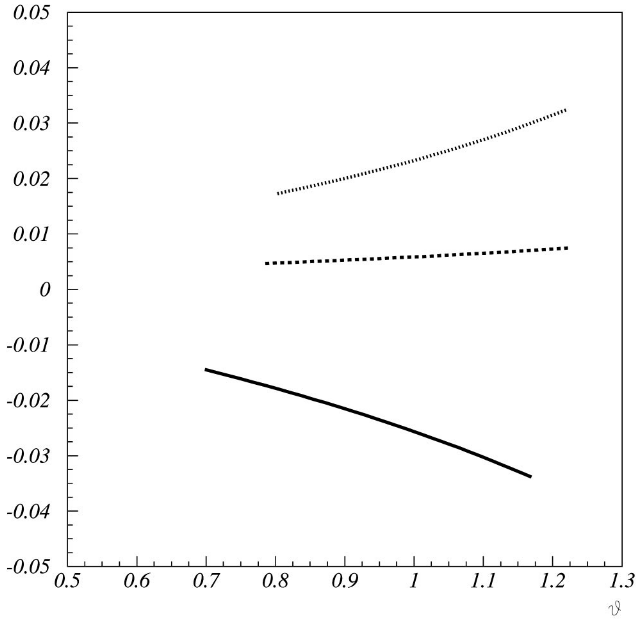

FIG. 8. ΔC as a function of θ. Other parameters and conventions are the same as in Fig. 7.

the large uncertainties both theoretically and experimentally at present, we should not take it seriously.

Our results show that both the Br and $S_{\phi k}$ (as well as $C_{\phi K}$) of $B\to\phi K_S$ are sensitive to the characteristic scale μ of the process, as can be seen from Eq. (45) and the SM amplitude. The significant scale dependence comes mainly from the $O(\alpha_s)$ corrections of hadronic matrix elements of the operators Q_i, $i=11,\ldots,16$ and also from leading order

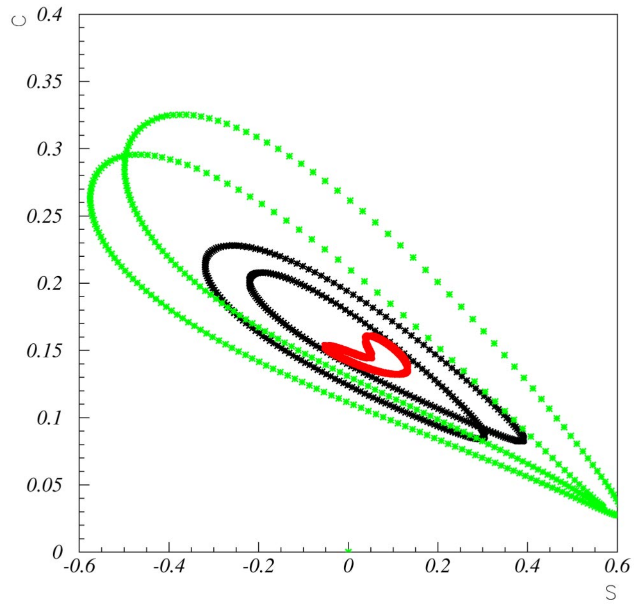

FIG. 10. Correlation between $C_{\phi K}$ and $S_{\phi K}$; other parameters are the same as in Fig. 9. The outmost two curves correspond to $\mu=m_b/2$, the curve in kernel is for $\mu=m_b$ and the other two curves are for $\mu=2m_b$.

Wilson coefficients C_i, $i=8g,11,\ldots,16$. However, despite there is the scale dependence, the conclusion that the model III can agree with the present data, $S_{\phi k}=-0.39\pm0.41$, within the 1σ error can still be drawn definitely.

Note added. We noticed Ref. [26] while completing this work. In Ref. [26] the mixing induced *CP* asymmetry $S_{\phi K}$ in the model III 2HDM is investigated. Comparing with the

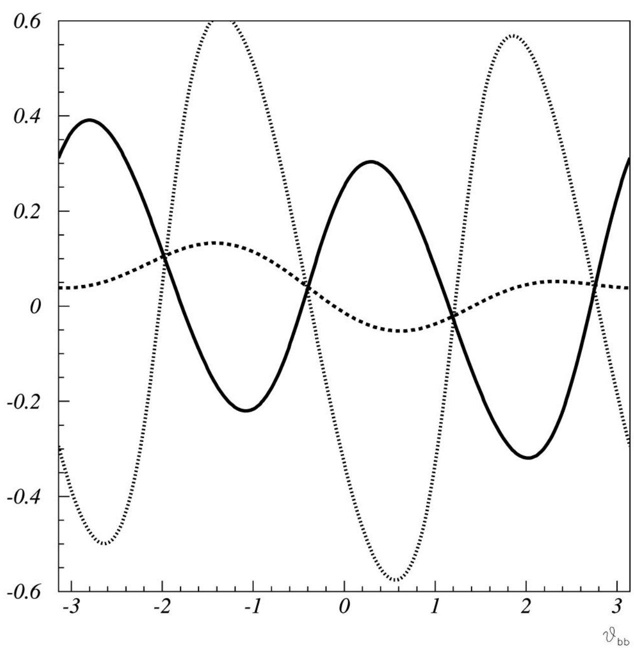

FIG. 9. $S_{\phi K}$ as a function of θ_{bb} with $\mu=2m_b$ (solid), m_b (dashed) and $m_b/2$ (dotted), where $m_{H^\pm}=200$ GeV, $|\lambda_{tt}|=0.03$, $|\lambda_{bb}|=100$, $\theta=1.15$ and $\lambda_{cc}=\lambda_{ss}=100e^{i\pi/4}$.

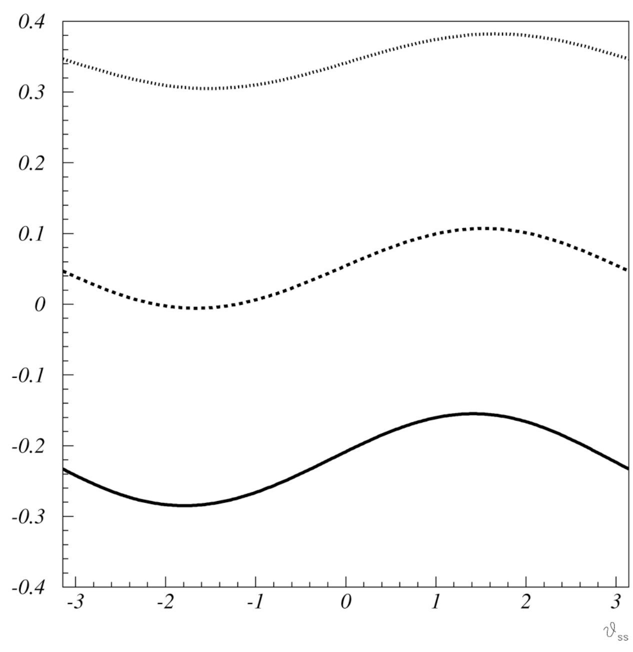

FIG. 11. $S_{\phi K}$ as a function of θ_{ss} with $\mu=2m_b$ (solid), m_b (dashed) and $m_b/2$ (dotted), where $m_{H^\pm}=200$ GeV, $|\lambda_{tt}|=0.03$, $|\lambda_{ss}|=100$, $\theta=1.15$, $\lambda_{bb}=100e^{-i\pi/4}$ and $\lambda_{cc}=100e^{i\pi/4}$.

paper, our results on the Wilson coefficients of the operators which exist in SM at NLO are in agreement. We differ significantly from the paper in the neutral Higgs boson contributions included. Furthermore, we calculate hadronic matrix elements of operators to the α_s order by BBNS's approach while the paper uses the naive factorization, i.e., at the tree level. Therefore, our numerical results and consequently conclusions are significantly different from those in the paper. Even without including the NHB contributions our results are also different from theirs due to the different precisions of calculating hadronic matrix elements, to which $S_{\phi k}$ is sensitive.

During the publication processing we became aware of Ref. [27] in which the LO anomalous dimensions for the mixing of $Q_{11,12}$ onto $Q_{3,\ldots,6}$ and Q_9 are given and those for the mixing of $Q_{13,\ldots,16}$ onto $Q_{7\gamma,8g}$ given in Ref. [28] are confirmed. In this paper these mixings are not taken into account. If we included them, the numerical results would change but the qualitative features of the results would be the same. We shall include them in a forthcoming paper on *CP* asymmetries in $B\to\eta' K_S$ and ϕK_s in a model III 2HDM.

ACKNOWLEDGMENTS

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