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ASYMMETRIC TECHNOLOGICAL CHANGE IN THE MELITZ MODEL: ARE FOREIGN TECHNOLOGICAL IMPROVEMENTS HARMFUL?

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Abstract

Foreign technological advance unambiguously reduces home welfare in a popular variant of the Melitz (2003) model that assumes the presence of a costlessly traded homogeneous good (Demidova, 2008). The present paper shows that this result is sensitive to the presence of such a good and is reversed in its absence. Indeed, in a generalized version of the Melitz model that adds a nontraded good and nests the original version as a special case, we show that foreign technological advance always improves home welfare. We derive relations that require information on only a few parameters to calibrate the model to data. These relations are used to calibrate an international trade model for the United States for quantitative analysis of the welfare effects. US is found to gain much less from foreign technological improvements than its trading partners from US improvements.

Key Words: Heterogeneous Firms, Technological Change, Gains from Trade, Nontraded Goods

JEL Codes: F10, F12

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1. Introduction

What role does international trade play in transmitting the welfare effects of technological improvements in a country to its trading partner? In traditional models of international trade (based on Ricardian or Heckscher-Ohlin frameworks), the terms of trade adjustment allows the trading partner to share the benefits of the country's technological advance. Recent models with monopolistic competition and heterogeneous firms have additional adjustment mechanisms that can lead to different welfare implications. Indeed, using a popular variant of the Melitz (2003) model that assumes the presence of a freely traded homogeneous (outside) good, Demidova (2008) has demonstrated that foreign technological advance unambiguously reduces home welfare. However, it is not clear if this result is a robust implication of the Melitz model and will hold for other versions of the model without the outside good.

The outside-good assumption has been used widely to simplify the solution of the Melitz model by tying the home wage to foreign wage.[1] The welfare implications of the model, however, can be very sensitive to the presence of the outside good. For example, Demidova and Rodriguez-Clare (2013) show that unilateral trade liberalization always improves the welfare of the liberalizing country in the absence of an outside good.[2] This result contrasts sharply with the welfare implications of unilateral trade liberalization in models with an outside good such as Demidova (2008) and Melitz and Ottaviano (2008) that the liberalizing country can suffer a welfare loss. The role of the outside good in

[1] This assumption goes back to Helpman, Melitz and Yeaple (2004), and has been used by a number of recent studies, for example, Baldwin and Forslid (2010), Baldwin and Okubo (2009) and Chor (2009).
[2] Also see Felbermayr and Jung (2012), who generalize the Demidova- Rodriguez-Clare analysis to a two country version with fully endogenous wage rates, and show that the welfare effect of a tariff reduction differs from that of a decrease in (iceberg) import cost.

determining the welfare effect of an asymmetric change in technology, however, has not been examined so far and is explored in the present paper.

We use a variant of the Melitz model that excludes the outside good but adds a homogeneous nontraded good. This version is appealing as it allows for nontraded goods that represent a large and a growing sector in most economies and are assigned an important role in many macroeconomic and international trade models.[3] The presence of the nontraded good also introduces an additional adjustment mechanism operating via the relative price of nontraded to traded goods. This mechanism is highlighted in the Balassa-Samuelson theory of how productivity improvement in the traded good sector affects the real exchange rate.[4] The share of the nontraded good is an important parameter in our model, but we can let it equal zero and thus nest the basic Melitz version as a special case of our model and explore how results differ with and without the nontraded good.

The solution of the model with the nontraded good is more complex, but we derive three relations that determine the home and foreign real wage (representing the welfare of the representative households in the home and foreign countries), and the real exchange rate.[5] We consider technological advance arising from improvements in labor efficiency as well as in the productivity distribution (in the form of either higher dispersion or support of Pareto distribution) of the differentiated good. For each of these improvements, we show that a small foreign technological improvement (around an

[3] Our setup, moreover, is analytically equivalent to a widely used model, in which the traded differentiated product is an intermediate input used in a nontraded and homogeneous final good.

[4] Also see Ghironi and Melitz (2005) who examine the effect of an improvement in aggregate productivity on the real exchchange rate in a DSGE version of the Melitz model.

[5] The home and foreign real wage is defined, respectively, in units of home and foreign aggregate consumption (including both the traded and nontraded goods) and the real exchange rate is the price of foreign in terms of home consumption.

initial symmetric state) would always improve home welfare. This result holds regardless of whether the nontraded good is present or not, although its presence makes an important difference to the transmission process and the magnitude of the welfare effect. Thus the effect of foreign technological change on home welfare in the model without the outside good is opposite to that with the outside good.

The paper also examines the welfare implications of an asymmetric change in the size of labor force. We show that a larger foreign size improves home welfare and is similar to foreign technical progress in this respect. We also identify conditions under which foreign technological improvements decrease the real exchange rate (depreciate the real value of the foreign currency) contrary to the predictions of the conventional Balassa-Samuelson theory. The conditions for the reversal of the Balassa-Samuelson effect have been examined for the monopolistic competition model with homogeneous firms (e.g., see Choudhri and Schembri, 2010), but have not been explored for the Melitz model with heterogeneous firms.

Our three-equation representation of the model facilitates the calibration of the model to a real economy. We develop a procedure that requires knowledge of only a subset of model parameters to calibrate the model to data. We use this procedure to develop a quantitative international trade model for the United States. As income per capita and the shares of traded goods differ significantly between the US and its trading partners, our numerical analysis allows for initial asymmetries. We are also able to examine the effect of large changes. The numerical analysis supports the analytical results based on small changes around initial symmetry. We find, however, that the increase in the welfare of US trading partners resulting from US technological improvements is much stronger than the increase in US welfare caused by non-US technological advance. The US is generally viewed as the technological leader, and one concern is that catch up by less advanced countries may hurt the US.[6] The results of our calibrated model suggest that narrowing of the technological gap would confer positive, albeit small, benefits to US.

The model is briefly described in Section 2. Section 3 discusses key analytical results. Numerical analysis based on the calibration of the model to US economy is presented in Section 4. Section 5 concludes the paper.

2. Model

This section describes a setup that introduces a nontraded good in a model with heterogeneous firms and two asymmetric countries. The consumption basket in each country includes a nontraded homogeneous good and a bundle of domestic and foreign varieties of a traded differentiated good. There is only one primary factor, labor, which is used in the production of both goods. In describing the model below, we focus on the relations for the home economy. Analogous relations hold for the foreign economy, where an asterisk is used to denote foreign variables and parameters.

The utility level depends on aggregate consumption, which is determined by the following Cobb-Douglas function:

$$C = N^{1-\alpha} Z^{\alpha} / [\alpha^{\alpha} (1-\alpha)^{1-\alpha}], \tag{1}$$

where N is the amount of the nontraded good and Z represents an index of the traded-good bundle. The nontraded good is produced under perfect competition, and one unit of

[6] For example, Krugman's (1986) model of technological gap based on the Ricardian framework implies that technical progress in less advanced countries may be harmful for more advanced countries, although less advanced countries always benefit from technological improvements in more advanced countries.

labor is needed to produce one unit of this good. Let L_N denote the amount of labor employed in the production of this good. Also, let w and p denote the real wage rate and the real price of the traded good bundle in units of aggregate consumption, C. Then, noting that $L_N = N$ and the real wage equals the real price of the nontraded good, we obtain the following demand functions from (1):

$$L_N = (1-\alpha)C / w, \tag{2}$$

$$Z = \alpha C / p. \tag{3}$$

Moreover, since the real price of the numeraire C equals unity, (1) implies that

$$1 = w^{1-\alpha} p^{\alpha}. \tag{4}$$

The above setup is analytically equivalent to a widely–used framework where inputs of labor and an intermediate-good bundle of traded varieties are used to produce a nontraded final good, C.[7]

The model for the differentiated good is based on Melitz (2003), but allows labor efficiency and productivity distribution for firms to differ between the home and foreign countries. The production of one unit of a home variety requires $1/\varepsilon\theta$ units of home labor. Parameter ε is a measure of home labor's efficiency while θ represents a firm-specific productivity index, which is drawn from a distribution, $g(\theta)$, after incurring a sunk entry cost, ψ / ε, in units of home labor. Each Firms also faces (in units of domestic labor) a fixed cost per period, ϕ / ε, to produce a variety, and an additional fixed cost per period, ϕ_E / ε, to export the variety. The corresponding costs for foreign firms are

[7] For example if instead of (1), we assume that C is produced according to the production function, $C = L_N^{1-\alpha} Z^{\alpha} / [\alpha^{\alpha}(1-\alpha)^{1-\alpha}]$, we would still get relations (2)-(4).

$\psi^* / \varepsilon^*, \phi^* / \varepsilon^*$ and ϕ_E^* / ε^* in units of foreign labor. The production requirements of a unit of a foreign variety are $1/\varepsilon^*\theta^*$ units of foreign labor with θ^* drawn from a distribution, $g^*(\theta^*)$. Technological differences between the home and foreign countries can arise because of differences in labor efficiency ($\varepsilon \neq \varepsilon^*$) or productivity distribution $[g(.) \neq g^*(.)]$. Exports of a variety by a home or foreign firm is also assumed to be subject to a melting iceberg trading cost such that $\tau > 1$ units of the variety have to be shipped for one unit to arrive.

The traded-good bundle includes a continuum of home and foreign varieties and is defined as

$$Z = \left[Z_H^{(\sigma-1)/\sigma} + Z_{FE}^{(\sigma-1)/\sigma} \right]^{\frac{\sigma}{\sigma-1}}, \tag{5}$$

where $Z_H = \left[\int_{\theta \in \Omega_H} Z_H\left(\theta\right)^{(\sigma-1)/\sigma} d\theta \right]^{\frac{\sigma}{\sigma-1}}$, $Z_{FE} \equiv \left[\int_{\theta^* \in \Omega_{FE}} Z_{FE}\left(\theta^*\right)^{(\sigma-1)/\sigma} d\theta^* \right]^{\frac{\sigma}{\sigma-1}}$, are the aggregates of varieties supplied by home firms and foreign exporters; $Z_H(\theta)$ and $Z_{FE}(\theta^*)$ are the amounts produced by firms indexed by θ and θ^*, respectively; Ω_H is the set of home firms while Ω_{FE} is the subsets of foreign firms representing exporters; and σ is the elasticity of substitution between varieties (assumed, for simplicity, to be the same between any pair of varieties regardless of where they are produced).

The demand for each type of variety is given by

$$\begin{gathered} Z_H(\theta) = Z_H \left[\frac{p_H(\theta)}{p_H} \right]^{-\sigma}, Z_{FE}(\theta^*) = Z_{FE} \left[\frac{p_{FE}(\theta^*)}{p_{FE}} \right]^{-\sigma}, \\ Z_H = Z\left(\frac{p_H}{p} \right)^{-\sigma}, Z_{FE} = Z\left(\frac{p_{FE}}{p} \right)^{-\sigma}, \end{gathered} \tag{6}$$

where $p_H(\theta)$, and $p_{FE}(\theta^*)$ are real prices in units of C. The real price index of the aggregate bundle (also in units of C) equals:

$$p = \left(p_H^{1-\sigma} + p_{FE}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \quad (7)$$

where $p_H = \left[\int_{\theta \in \Omega_H} p_H(\theta)^{1-\sigma} \, d\theta \right]^{\frac{1}{1-\sigma}}$, and $p_{FE} = \left[\int_{\theta^* \in \Omega_{FE}} p_{FE}(\theta^*)^{1-\sigma} \, d\theta^* \right]^{\frac{1}{1-\sigma}}$.

The optimal real prices for home and foreign varieties in the home market are set as

$$p_H(\theta) = \left(\frac{\sigma}{\sigma - 1} \right) \left(\frac{w}{\varepsilon \theta} \right), \; p_{FE}(\theta^*) = \left(\frac{\sigma}{\sigma - 1} \right) \left(\frac{\tau q w^*}{\varepsilon^* \theta^*} \right), \quad (8)$$

where w^* is the real wage in units of C^*, and q denotes the real exchange rate as the relative price of C^* in terms of C (the real value of the foreign currency).

The minimum levels of θ required for a home firm to produce a variety for the home and foreign markets are readily derived from (6), (8) and their foreign counterparts. Use these relations to express real profits of home firms from domestic sales (π_H) and exports (π_{HE}) as

$$\pi_H(\theta) = D w^{1-\sigma} (\varepsilon \theta)^{\sigma - 1} - (\phi / \varepsilon) w, \; \pi_{HE}(\theta) = D^* q^\sigma (\tau w)^{1-\sigma} (\varepsilon \theta)^{\sigma - 1} - (\phi_E / \varepsilon) w, \quad (9)$$

where $D \equiv (p^\sigma Z / \sigma)[\sigma / (\sigma - 1)]^{1-\sigma}$ and $D^* \equiv (p^{*\sigma} Z^* / \sigma)[\sigma / (\sigma - 1)]^{1-\sigma}$ are indexes of demand levels at home and abroad. Real profits from each operation increase in θ. Let $\bar{\theta}_H$ and $\bar{\theta}_{HE}$ denote the cutoff productivity levels for local producers (supplying only domestic market) and exporters (supplying both domestic and foreign markets), i.e., the minimum values of θ that imply $\pi_H = 0$ and $\pi_{HE} = 0$. From (9), these levels are determined as

$$\bar{\theta}_H = \left(\frac{\phi w^{\sigma}}{\varepsilon^{\sigma} D} \right)^{1/(\sigma-1)}, \bar{\theta}_{HE} = \left(\frac{\phi_E \tau^{\sigma-1} w^{\sigma}}{\varepsilon^{\sigma} D^* q^{\sigma}} \right)^{1/(\sigma-1)}. \quad (10)$$

The cutoff values for foreign firms, $\bar{\theta}_F$, and $\bar{\theta}_{FE}$ are defined analogously. We assume that $\phi_E \tau^{\sigma-1} D > \phi D^* q^{\sigma}$ (and $\phi_E^* \tau^{\sigma-1} D^* q^{\sigma} > \phi^* D$) in equilibrium, so that $\bar{\theta}_H < \bar{\theta}_{HE}$ ($\bar{\theta}_F < \bar{\theta}_{FE}$) and the sorting pattern (suggested by empirical evidence) that non-exporting firms are less productive than exporting firms is satisfied.

Following Melitz (2003), define the average productivity for all firms and exporters (by aggregating over the relevant productivity range) as

$$\tilde{\theta}_H \equiv \left[\int_{\bar{\theta}_H}^{\infty} \frac{\theta^{\sigma-1} g(\theta)}{1-G(\bar{\theta}_H)} d\theta \right]^{1/(\sigma-1)} \text{ and } \tilde{\theta}_{HE} \equiv \left[\int_{\bar{\theta}_{HE}}^{\infty} \frac{\theta^{\sigma-1} g(\theta)}{1-G(\bar{\theta}_{HE})} d\theta \right]^{1/(\sigma-1)},$$

where $G(\theta)$ is the cumulative distribution function. We assume that $g(\theta)$ follows a Pareto distribution as in Helpman, Melitz and Yeaple (2004), with shape parameter k ($>\sigma-1$) on support $\theta \in [b, \infty]$ for some $b > 0$. In this case, $G(\theta) = 1-(b/\theta)^k$, $g(\theta) = kb^k / \theta^{k+1}$, and thus

$$\tilde{\theta}_H == \left[\frac{k}{k-(\sigma-1)} \right]^{1/(\sigma-1)} \bar{\theta}_H, \tilde{\theta}_{HE} = \left[\frac{k}{k-(\sigma-1)} \right]^{1/(\sigma-1)} \bar{\theta}_{HE}. \quad (11)$$

The foreign distribution, $g^*(\theta^*)$, also assumed to be Pareto, could have different shape parameter and support (i.e., $g^*(\theta^*) = k^* b^{*k*} / \theta^{*k*+1}$).

Denote the probability that a home firm will survive after drawing its productivity level by $\nu_H\, [=1-G(\bar{\theta}_H)]$, and the conditional probabilities that a survivor will export by $\nu_{HE}\, [=1-G(\bar{\theta}_{HE})]/[1-G(\bar{\theta}_H)]$. These probabilities are determined according to the Pareto distribution as

$$\nu_H = (b / \bar{\theta}_H)^k, \quad \nu_{HE} = (\bar{\theta}_H / \bar{\theta}_{HE})^k . \tag{12}$$

The price indexes in (7) can be related to average productivity indexes as

$$p_H = n_H^{1/(1-\sigma)} \left[\frac{\sigma w}{(\sigma-1)\varepsilon\tilde{\theta}_H} \right], \; p_{FE} = n_{FE}^{1/(1-\sigma)} \left[\frac{\sigma \tau q w^*}{(\sigma-1)\varepsilon^* \tilde{\theta}_{FE}^*} \right], \tag{13}$$

where n_H is the mass of firms in the set Ω_H, and n_{FE} represents the mass of firms in the set Ω_{FE}. Also, using (9) and (11), we can relate average profits from different operations of surviving home firms to average productivity indexes as

$$\tilde{\pi}_H = Dw^{1-\sigma}(\varepsilon\tilde{\theta}_H)^{\sigma-1} - (\phi / \varepsilon)w, \tilde{\pi}_{HE} = D^* q^\sigma (\tau w)^{1-\sigma} (\varepsilon\tilde{\theta}_{HE})^{\sigma-1} - (\phi_E / \varepsilon) w . \tag{14}$$

Assume that after entry, a firm in each period faces an exogenous probability, δ, that it will exit (die). In steady-state equilibrium, the mass of firms is constant, so that the number of new firms entering in each period equals the number exiting in the period. Free entry implies that the present discounted value of ex-ante average profits equals the entry cost. Assuming, for simplicity that the discount rate equals zero,[8] this condition can be expressed as

$$w\psi / \varepsilon = \nu_H \tilde{\pi} / \delta , \tag{15}$$

where $\tilde{\pi} = \tilde{\pi}_H + \nu_{HE}\tilde{\pi}_{HE}$ represents average ex-post profits for all firms. Since ν_{HE} represents the relative frequency of home firms undertaking exports in steady state, we also have

$$n_{HE} = \nu_{HE} n_H . \tag{16}$$

Let L_Z denote total labor demand by home producers of the differentiated good. The condition for labor-market equilibrium in steady state is

[8] This is a standard simplification that has been used by Melitz (2003) among others. See Baldwin (2005) for further discussion of this simplifying assumption.

$$L = L_N + L_Z + (\psi/\varepsilon)(\delta n_H/\nu_H), \tag{17}$$

where L is home labor supply, and $\psi\delta n_H/\varepsilon\nu_H$ represents labor requirements of $\delta n_H/\nu_H$ firms in the entry process (a proportion ν_H of this number of firms successfully enters the industry and replaces δn_H exiting firms in steady-state equilibrium).

Moreover, trade balance in steady state implies that

$$p_{HE}Z^*_{HE} = p_{FE}Z_{FE}. \tag{18}$$

Finally, since the free entry condition implies that total entry costs ($n_H w\psi\delta/\varepsilon\nu_H$) equal total profits [$n_H\tilde{\pi}$], it follows that expenditure equals labor income: [9]

$$C = wL. \tag{19}$$

3. Analytical Results

The home welfare level (for a representative agent) is measured by C/L, which equals w according to (19). Thus the model needs to be solved for w to determine the welfare level. To obtain the solution for w, we proceed as follows. First, we use the free-entry condition and the relation for profits to show that w is a function of the ratio of cutoff levels of non-exporters to exporters ($\bar{\theta}_H/\bar{\theta}_{HE}$). Next, we show that this ratio depends on w/w^* and q. We then derive two relations, which are used to determine these variables.

3.1 Key Relations

[9] To derive (19), note first that $C = wL_N + pZ$ from (2) and (3). Next, use (6), (7), (13), (14) and their foreign counterparts, and (18) to obtain: $C = w(L_N + L_Z) + n_H\tilde{\pi}_H + n_{HE}\tilde{\pi}_{HE}$. The right hand side of this equation equals wL since $wL - w(L_N + L_Z) = n_H w\psi\delta/\varepsilon\nu_H$ from (17) and $n_H w\psi\delta/\varepsilon\nu_H = n_H\tilde{\pi}_H + n_{HE}\tilde{\pi}_{HE}$ from (15).

We start by relating D (the demand level index) to w. Use the definition of D in (9) and use (3), (4) and (19) to substitute for p and Z in it to obtain

$$D = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \frac{\alpha}{\sigma} L w^{1-(\sigma-1)(1-\alpha)/\alpha} . \tag{20}$$

Note that the elasticity of D with respect to w equals $1-(\sigma-1)(1-\alpha)/\alpha$, and its sign depends on the relative strength of the positive income effect via C and the negative substitution effect via p. The substitution effect operates via the adjustment in the relative price of nontraded to traded goods and is absent in the model without the nontraded good. Making use of (10), (12) and (21), we can express the probability that a new entrant will survive as

$$\nu_H = \Gamma \varepsilon^{k\sigma/(\sigma-1)} L^{\frac{k}{\sigma-1}} \left[w\right]^{-k/\alpha} , \tag{21}$$

where $\Gamma \equiv \left[b(\sigma-1)/\sigma\right]^k \left[\alpha/(\phi\sigma)\right]^{k/(\sigma-1)}$. The negative relation between ν_H and w arises because an increase in w leads (though its direct plus indirect effect via D) to an increase in the cutoff level, $\bar{\theta}_H$, which lowers ν_H.

Letting $\Pi \equiv \tilde{\pi}/w$ denote the ratio of average ex-post profits to the wage rate (i.e., average profits in units of labor), substituting the expression for ν_H in (21) into the free-entry condition (15), and solving for w, we obtain

$$w = \left(\Pi \Gamma L^{k/(\sigma-1)} \varepsilon^{1+k\sigma/(\sigma-1)} / \psi\delta\right)^{\alpha/k} . \tag{22}$$

The positive association between w and Π in (22) arises because an increase in Π must be offset by a decrease in ν_H to satisfy the free entry condition. The decrease in ν_H, in turn, requires (for reasons discussed above) an increase in w.

We next express the profit-wage ratio as an increasing function of the ratio of cutoff levels. Use (10), (11), (12), and (14) to obtain the following relation (see Appendix A for the derivation):

$$\Pi = \Lambda \left[1 + \frac{\phi_E}{\phi} \left(\frac{\bar{\theta}_H}{\bar{\theta}_{HE}} \right)^k \right], \tag{23}$$

where $\Lambda \equiv \frac{\phi}{\varepsilon} \left[\frac{k}{k-(\sigma-1)} - 1 \right]$. According to (10), the cutoff ratio equals

$\frac{\bar{\theta}_H}{\bar{\theta}_{HE}} = \tau^{-1} \left(\frac{\phi q^\sigma D^*}{\phi_E D} \right)^{1/(\sigma-1)}$. We can use (20) and its foreign counterpart to express the ratio as[10]

$$\frac{\bar{\theta}_H}{\bar{\theta}_{HE}} = \tau^{-1} \left(\frac{\phi L^*}{\phi_E L} \right)^{1/(\sigma-1)} q^{\sigma/(\sigma-1)} \left(\frac{w}{w^*} \right)^\beta, \tag{24}$$

where $\beta \equiv \frac{1-\alpha}{\alpha} - \frac{1}{\sigma-1}$, and its sign is positive (negative) if the substitution effect of w on D is stronger (weaker) than the income effect. As (24) indicates, the traded good share plays an important role in determining the effect of the relative wage on the cutoff ratio. In the special case of $\alpha = 1$ (no nontraded good), β is negative. As α decreases, β increases and turns positive at a sufficiently low value. Finally, using (24) to substitute for $\bar{\theta}_H / \bar{\theta}_{HE}$ in (23) and then using the resulting expression to eliminate Π in (22), we derive the following relation, in which w is a function of w/w^* and q:

$$w = \left[\Theta \left\{ L^{k/(\sigma-1)} + \Phi^{-1} L^{*k/(\sigma-1)} \left(w/w^* \right)^{k\beta} q^{k\sigma/(\sigma-1)} \right\} \right]^{\alpha/k}, \tag{25}$$

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[10] For simplicity, we assume that $\alpha^* = \alpha$. This assumption is relaxed in the numerical analysis below.

where $\Theta = \frac{\Gamma \Lambda \varepsilon^{1+k\sigma/(\sigma-1)}}{\psi \delta}$ and $\Phi = \tau^k \left(\phi_E / \phi\right)^{\frac{k}{\sigma-1}-1}$.

Although (25) is derived to facilitate the solution of this paper's model, the derivation utilizes only the relations for the differentiated good. Thus (25) also holds for the original Melitz (2003) version with one differentiated good as well as the Helpman, Melitz and Yeaple (2004) variant that adds an outside good. If the home and foreign countries are symmetric, $q = 1$ and $w / w^* = 1$, and (25) is sufficient to determine home welfare.[11] If countries are not homogeneous, further analysis is needed to determine w / w^* and q. In the setup with an outside good, $w = qw^*$.[12] Under this condition, (25) implies that w is, a decreasing function of w^* and thus an increase in foreign wage brought about by a foreign technological improvement would lower home welfare as in Demidova (2008).[13]

We derive two further relations to solve the model with the nontraded good. Using foreign counterparts of (20)-(24), we first obtain the following relation for the foreign wage:

$$w^* = \left[\Theta^* \left\{ L^{*k^*/(\sigma-1)} + \Phi^{*-1} L^{k^*/(\sigma-1)} \left(w^* / w\right)^{k^*\beta} q^{-k^*\sigma/(\sigma-1)} \right\} \right]^{\alpha^*/k^*}, \qquad (26)$$

[11] Choudhri and Marasco (2013) use this approach to determine the gains from trade and FDI for symmetric countries in the presence of a nontraded good.
[12] This equality holds because the price of the homogeneous traded good is the same in the two countries and equals the wage rate in each country (by normalization). Without symmetry, however, $q \neq 1$ and $w \neq w^*$.
[13] Note that with $q = w / w^*$, the expression $\left(\frac{w^*}{w}\right)^{k\beta} q^{\frac{k\sigma}{\sigma-1}}$ in (25) equals $= \left(\frac{w^*}{w}\right)^{-k/\alpha}$, and (25) simplifies to $w = \left[\Theta \left\{ L^{k/(\sigma-1)} + \Phi^{-1} L^{*k/(\sigma-1)} \left(w / w^*\right)^{k/\alpha} \right\} \right]^{\alpha/k}$.

where $\Theta^* = \frac{\Gamma^* \Lambda^* \varepsilon^{*1+k^* \sigma/(\sigma-1)}}{\psi^* \delta^*}$ and $\Phi^* = \tau^{k^*} \left(\phi_E^* / \phi^*\right)^{\frac{k^*}{\sigma-1}-1}$. The second relation is based on the equations for the home and foreign prices of the traded good and the conditions for balanced trade, and takes the following form:

$$\frac{w}{w^*} = \frac{\Phi + \left(\frac{L^*}{L}\right)^{k/(\sigma-1)} q^{\sigma k/(\sigma-1)} \left(\frac{w}{w^*}\right)^{k\beta}}{\left(\frac{L^*}{L}\right)^{\frac{k}{\sigma-1}-1} q^{\frac{\sigma k}{\sigma-1}-1} \left(\frac{w}{w^*}\right)^{k\beta} \left[\Phi^* \left(\frac{L^*}{L}\right)^{\frac{k^*}{\sigma-1}} q^{\frac{\sigma k^*}{\sigma-1}} \left(\frac{w}{w^*}\right)^{k^*\beta} + 1\right]}. \tag{27}$$

This relation is derived in the Appendix A.

3.2 Model Solution

The system consisting of (25), (26) and (27) can be solved for w, w^* and q. To simplify the solution of the model, we take a first-order log-linear approximation of these relations around an initial state where the two countries are symmetric. Let a bar denote the initial value of a variable in the symmetric state and a hat the log deviation around this value (i.e., $\hat{q} = (q - \bar{q}) / \bar{q}$), and express (25), (26) and (27) as

$$\hat{w} = \frac{1}{1+\Phi}\left[\alpha\beta(\hat{w} - \hat{w}^*) + \frac{\alpha\sigma}{\sigma-1}\hat{q}\right], \tag{28}$$

$$\hat{w}^* = \frac{-1}{1+\Phi}\left[\alpha\beta(\hat{w} - \hat{w}^*) + \frac{\alpha\sigma}{\sigma-1}\hat{q}\right], \tag{29}$$

$$\hat{w} - \hat{w}^* = -\frac{2\Phi}{1+\Phi}\beta k\left(\hat{w} - \hat{w}^*\right) + \left[1 - \frac{2\Phi}{1+\Phi}\frac{\sigma k}{\sigma-1}\right]\hat{q}\,. \tag{30}$$

The log-linear model can be solved recursively. First, it can be reduced to a system of two equations in two variables, the change in relative welfare $\hat{w} - \hat{w}^*$, and the change in real exchange rate $\hat{q}$. Subtracting (29) from (28), and noting that

$\alpha\beta = 1 - \frac{\alpha\sigma}{\sigma-1}$, we obtain

$$c_{11}(\hat{w} - \hat{w}^*) + c_{12}\hat{q} = 0, \tag{31}$$

where $c_{11} = \left[1 - \frac{2}{1+\Phi}\left(1 - \frac{\alpha\sigma}{\sigma-1}\right)\right]$ and $c_{12} = -\frac{2}{1+\bar{\Phi}}\left(\frac{\alpha\sigma}{\sigma-1}\right)$. Express (30) as

$$c_{21}(\hat{w} - \hat{w}^*) + c_{22}\hat{q} = 0, \tag{32}$$

where $c_{21} = 1 + \frac{2\Phi}{1+\Phi}\beta k$ and $c_{22} = \frac{2\Phi}{1+\Phi}\left(\frac{\sigma k}{\sigma-1}\right) - 1$. We have $c_{12} < 0$. Also, we have

$c_{11} > 0$ and $c_{22} > 0$ since $\Phi = \tau^k \left(\frac{\phi_E}{\phi}\right)^{\frac{k}{\sigma-1}-1} > 1$ and $1 - \frac{\alpha\sigma}{\sigma-1} < 1$. For $\alpha = 1$,

$c_{21} = 1 - \frac{2\Phi}{1+\Phi}\frac{k}{\sigma-1} < 0$ as $\Phi > 1$ and $k > \sigma - 1$. For $\alpha < 1$, however, the sign of c_{21} is

given by the following result:

Result 1. There exists a value of σ, $\sigma_1 \in \left(1, \frac{1}{1-\alpha}\right)$ *such that*

$$c_{21}\begin{cases} < 0 & if\ \sigma < \sigma_1 \\ > 0 & if\ \sigma > \sigma_1 \end{cases}.$$

The result is proved in the Appendix A. Note that β is positive or negative as σ is

greater or smaller than $\frac{1}{1-\alpha}$. For $\sigma > \frac{1}{1-\alpha}$, c_{21} is clearly positive. Result 1 shows that

c_{21} is also positive for a range of values of σ below $\frac{1}{1-\alpha}$.

The solution of the model is illustrated in Figure 1 for the case of $\sigma > \frac{1}{1-\alpha}$,

which implies $\beta > 0$ and $\sigma > \sigma_1$. In this figure, FE represents an upward-sloping curve between $\hat{w}-\hat{w}^*$ and $\hat{q}$ implied by (31) based on the free entry condition, with its slope equal to $-c_{12}/c_{11}$. In the illustrated case, PT represents a downward-sloping curve between the two variables derived from (32) based on the price relations for traded goods and the trade balance condition, with its slope equal to $-c_{22}/c_{21}$. The initial equilibrium is at the symmetric state with $\hat{w}-\hat{w}^*=0$ and $\hat{q}=0$.

For later use, the figure also includes a $\bar{U}$ curve that shows the values of the two variables that would maintain the initial level of home welfare. This relation is obtained by setting the left hand side of (28) equal to zero and is given by

$$\beta(\hat{w}-\hat{w}^*)+\frac{\sigma}{\sigma-1}\hat{q}=0. \qquad (33)$$

The sign of β determines the slope of the $\bar{U}$ curve. In the illustrated case with $\beta>0$, the $\bar{U}$ curve slopes downwards and is steeper than the PT curve.[14] In this case, it is easily verified that at any point above the $\bar{U}$ curve, home welfare is higher than the initial level.

The terms of trade (the relative price of the export to import bundle) can also be related to w/w^* and q. Noting that $p_{HE}/p_{FE}=Z_{FE}/Z_{HE}^*$ from (18), we can use (6), (20) and their foreign counterparts to express the terms of trade as

$$p_{HE}/p_{FE}=\left(Zp^{\sigma}/Z^*p^{*\sigma}q^{\sigma}\right)^{1/(1-\sigma)}=\left(D/D^*q^{\sigma}\right)^{1/(1-\sigma)}=(L^*/L)^{1/(\sigma-1)}(w/w^*)^{\beta}q^{\sigma/(\sigma-1)}.$$

Holding the terms of trade constant, log-linearization of this relation around a symmetric initial state also yields (33). Thus along the $\bar{U}$ curve, both the home welfare and the terms of trade are unchanged. Any point above the $\bar{U}$ curve represents an improvement in the terms of trade over the initial level.

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[14] The slope of the $\bar{U}$ curve equals $-\sigma/\beta(\sigma-1)$, and its absolute value is greater than the absolute value of the slope of the PT curve.

3.3 Foreign Changes

In the present model, improvements in foreign technology can result from increase in foreign labor efficiency (larger ε^*) or a better (Pareto) productivity distribution in the form of an increase in the minimum productivity level (higher b^*) or productivity dispersion (lower k^*).[15] The effects of each of these foreign technological improvements on the real exchange rate and both relative and absolute home welfare are summarized in Proposition 1.

Proposition 1. Given a symmetric initial steady state; (i) a small increase in either ε^* or b^* causes an increase in w, a decrease in w/w^*, and an increase or a decrease in q as σ is greater or smaller than σ_1; and, (ii) a small decrease in k^* causes an increase in w, and an ambiguous effect on w/w^* and q.

Proposition 1 is proved in Appendix A. We can explain the effects in this proposition with the help of Figure 1, which assumes $\sigma > \frac{1}{1-\alpha}$. An increase in either ε^* or b^* would shift FE curve rightwards, but would not affect the PT curve. In the new equilibrium, shown in Figure 1 by the point where FE' and PT curves intersect, w/w^* is lower while q is higher. Also, w increases since the new equilibrium point is above the $\bar{U}$ curve (which does not shift), As the terms of trade also increase at this point, higher

[15] Notice that both an increase in the support b^* and a decrease in the shape parameter k^* ensure that the foreign distribution first order stochastically dominates (FSD) the home distribution of productivities, while only with a lower k^* will the foreign distribution satisfy the stronger condition of hazard rate stochastic dominance (HRSD) over the domestic distribution. This is because the hazard rate function for a Pareto distribution with support *b* and shape parameter k is $\frac{g(\theta)}{1-G(\theta)} = \frac{kb^k/\theta^{k+1}}{(b/\theta)^k} = \frac{k}{\theta}$ which is not a function of the support *b*.

home welfare can be attributed to an improvement in the terms of trade. A decrease in k^* would lead to rightward shifts in both FE and PT curves, but would not shift the $\bar{U}$ curve. Thus q and w both increase, but the effect on w/w^* depends on the relative magnitude of the shifts of the two curves. Figure 1 illustrates an equilibrium for this case (shown by the intersection of FE' and PT') where the shift in the FE curve is larger and w/w^* decreases.

Note that if $\sigma < \frac{1}{1-\alpha}$, the $\bar{U}$ would slope upwards, but the results would not be affected.[16] If $\sigma < \sigma_1$, moreover, the PT curve also slopes upwards and can be shown to be steeper than both $\bar{U}$ and FE curves. In this case, an increase in ε^* (or b^*), which shifts only the FE curve right, would lead to a decrease in q (as well as w/w^*). There would still be an increase in w. The qualitative effects are the same if $\alpha = 1$.[17] Thus, in the model without the nontraded good, higher foreign productivity improves home welfare, but contrary to the Balassa-Samuelson model, depreciates the real value of the foreign currency.[18] A decrease in k^* would shift both FE and PT curves, and the effect on q is uncertain even in the case of $\sigma < \sigma_1$ (or $\alpha = 1$).

Larger foreign size can also have important welfare implications. Key effects of an increase in foreign labor force are described by Proposition 2.

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[16] In this case, points below the $\bar{U}$ curve represent an improvement in welfare.

[17] Since, $\beta < 0$ and $c_{21} < 0$ for $\alpha = 1$, this case also implies that both $\bar{U}$ and PT curves slope upwards and PT is steeper than $\bar{U}$.

[18] Note, however, that the real exchange rate in our model is defined in terms of welfare-based price indexes, and as shown by Ghironi and Melitz (2005) , its behavior differs from the real exchange rate defined in terms of price indexes representing average prices of varieties.

Proposition 2. Given a symmetric initial steady state, a small increase in L^* *causes an increase in* w *, a decrease in* w/w^**, and an ambiguous effect on* q*.*

The proof of Propositions 2 is provided in Appendix A. Figure 1 can also be used to understand the results in this proposition. A larger foreign labor force would shift FE curve rightwards and PT curve leftwards (shifts associated with Proposition 2 are not shown to keep the figure simple). Thus w/w^* would decrease, but the effect on q is uncertain. The effect on w is not clear in the figure as $\bar{U}$ curve would also shifts towards the left. However, the result in Appendix A that w increases unambiguously suggests that the new $\bar{U}$ curve lies below (above) the new equilibrium point if the $\bar{U}$ curve slopes downwards (upwards). Welfare implications of a larger foreign size are thus similar to those of foreign technological advance.

4. Numerical Analysis

To complement our analytical results, this section undertakes numerical analysis to examine welfare effects for a real economy. The model is calibrated to US economy, which is treated as home economy. Foreign economy is defined as trade-weighted aggregate of major US trading partners. Derivation of analytical results was simplified by the assumption that home and foreign economies are symmetric initially. However, as the US economy differs significantly from its trading partners, the numerical analysis allows us to explore whether Propositions 1 and 2 hold in the presence of initial international differences. These differences could also introduce asymmetries in the welfare effects of home and foreign technological improvements and we also explore such asymmetries. Finally, the numerical analysis also enables us to examine the effect of large technological changes in each economy.

4.1 Calibration

Quantitative welfare analysis in our model is based on the system of three nonlinear equations (25)-(27), modified to allow the share of traded goods in consumption, α, to differ from its foreign counterpart, α^*. We develop a procedure (explained in Appendix B) that calibrates all parameters needed to solve the system. This procedure requires estimates of foreign relative labor supply (L^* / L), initial steady-state values of the relative foreign real wage $(w^* q/w)$, the share of imports in consumption $(p_{FE} Z_{FE} / C)$, and a subset of parameter, $S = [\tau, \sigma, k, k^*, \alpha, \alpha^*]$.

To determine initial steady state values, we use long-period averages of data for the US and its trading partners (Appendix B provides further details). Letting a bar over a variable denote the initial steady-state value, we normalize $\bar{q} = \bar{w} = L = 1$. Given this normalization, data on income per capita (in constant US dollars) and the population of US trading partners relative to US levels is used to estimate $\bar{w}^* (= \bar{w}^* \bar{q} / \bar{w})$ and $L^* (= L^* / L)$. The import share estimate utilizes US data on the share of trade flows (average of imports and exports) in GDP.

Parameters in the subset S have received considerable attention and we choose their values based on the estimates suggested in the literature. Variable trade costs consist of transportations costs, tariffs and nontariff barriers and have been measured directly as well as indirectly. Direct estimates of different components of variable trade costs suggest that a conservative estimate of the variable cost parameter, τ, equals 1.2 (i.e., proportional trade costs are 20%).[19] Indirect measures indicate that τ is not much higher

[19] Anderson and Wincoop (2004) survey the measurement of trade costs. Their representative estimate of policy barriers (tariffs and nontariff barriers) is 8% for industrialized countries. The estimate of directly

than this value.[20] We let τ equal 1.2 in the baseline model, and explore higher values in the sensitivity analysis.

Traditional classification identifies traded goods with non-service industries representing largely the products of Agriculture, Mining and Manufacturing sectors. It is generally recognized, however, that significant international trade also occurs in certain service industries (especially, industries producing financial, business and communication services). Data limitation, however, make it difficult to obtain good estimates of the share of service industries open to international trade. Reasonable assumptions about the shares of traded service industries for US and its trading partners suggest that α is between 0.38 and 0.42 while α^* is between 0.46 and 0.49 (see Appendix B).

To estimate the Pareto shape parameter, it is useful to express it as a sum of two components: $k = \gamma + (\sigma - 1)$. Under the assumption of Pareto distribution, γ would equal the inverse of the standard deviation of the log of firm sales. Helpman, Melitz and Yeaple's (2004) estimates of the mean value of this standard deviation based on plant data for the US and firm data for Europe suggest a range of from 0.6 to 0.8 for γ. We use a value of 0.7 for γ to determine k conditional on the value of σ. US trading partners are likely to have, on average, lower productivity dispersion ($k^* > k$) and thus we assume a large value of 1.1 for $\gamma^* (= k^* - \sigma + 1)$.

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measured freight costs (based on US data) is 12%. Transportation costs would be higher if the cost of the time value of goods in transit is added. They also find that additional border costs are substantial (some of these costs may be included in fixed export costs in our model).

[20] For example, Obstfeld and Rogoff (2001) suggest a value of 1.3 for this parameter to explain certain empirical puzzles in open economy macroeconomics.

Estimates of the elasticity of substitution between home and foreign goods vary over a wide range depending upon the estimating procedure and the type of data used. [21] A number of calibrated models (e.g., Obstfeld and Rogoff, 2007) assume a value for σ equal to 2 or 3. However, the assumed sorting pattern that exporters are more productive than local producers in each country significantly constraints the feasible range of values for σ for our calibrated economy. These constraints are explained in Appendix B. In our asymmetric case (with much lower foreign real wage), one important restriction is that there is an upper bound on σ, which is a function of τ, α and α^*. Assuming that $\tau = 1.2$, the upper limit is just below 2.05 for the low values of the shares of traded goods $(\alpha = 0.38, \alpha^* = 0.46)$ and slightly below 2.30 for the high values ($\alpha = 0.42, \alpha^* = 0.49$). We assume a baseline value of 2.0 for σ, which is below the upper bound implied by the sorting pattern.[22]

4.2 Results

This section discusses the basic results of the numerical analysis on the welfare implications of changes, first in the foreign and then in the home economy.[23] Recall that in our calibration exercise, the home economy is the US while the foreign economy is the

[21] Estimates of the average value of the substitution elasticity estimated from disaggregated trade data tend to be large even if sectoral elasticities are assumed to be homogeneous (see Imbs and Mejean, 2011). These estimates are based on the preference structure typically assumed in trade models, in which the elasticity of substitution between a pair of varieties is the same regardless of where they are produced. Note, however, that the estimates of the elasticity of substitution between domestic and foreign bundles are generally much lower in an alternative specification where this elasticity is assumed to differ from the elasticity of substitution between varieties in each bundle (see Feenstra et al., 2012),

[22] It is also above the critical value, σ_1, and thus rules out the reversal of the Balassa-Samuelson effect. Note that σ_1, is less than $1/(1-\alpha)$ for the case $\alpha = \alpha^*$. In our numerical analysis, we allow for $\alpha \neq \alpha^*$. Our baseline value of σ exceeds both $1/(1-\alpha)$ and $1/(1-\alpha^*)$ for the entire range of values of α and α^*.

[23] The basic results are based on the baseline values of τ, γ, σ and the two sets of values for α and α^*. We also did sensitivity analysis to variations in τ (in the 2.0-2.5 range), γ (within a range of 0.6-0.8) and σ (within a range consistent with the sorting pattern), and found that the results did not change much.

combined economy of its trading partners. Table 1 shows the effects of changes in foreign technology and size on absolute and relative home welfare as well as the real exchange rate. We consider three sources of an improvement in foreign technology (an increase in ε^*, an increase in b^* or a decrease in k^*) and one source of larger foreign size (an increase in L^*). To facilitate comparisons between different types of foreign changes, we assume the magnitude of the change in each case to be such that it leads to a 10 % increase in foreign welfare (w^*). Under this assumption, a change in ε^* or b^* has the same effect. The effect of a change in k^*, however, differs from that of ε^* or b^*. Determination of the effect of a decrease in k^*, moreover, requires estimates of additional parameters. As explained in Appendix B, we obtain these estimates by using suitable normalizations and assuming a reasonable value for the survivor probability for a firm (ν_F).

The results in the table show that Propositions 1 and 2 hold even for big changes (large enough to cause a 10 % increase in foreign welfare). The numerical analysis also provides results on the magnitude of the effects on home welfare in response to foreign changes. Foreign technological improvements lead to a very small increase in absolute home welfare, and thus cause a large decline in relative home welfare. For example, assuming high values for the home and foreign shares of traded goods, the increase in ε^* or b^* improves home welfare only by 0.16% and relative home welfare deteriorates by 9.84%. The most favorable case for home welfare is represented by the decrease in k^* (under high traded goods shares), but even in this case, home welfare increases by mere 0.43 % (relative welfare decreases by 9.57 %).

Larger foreign size causes more substantial improvements in home welfare. The foreign increase in L^* brings about a 1.9 % increase home welfare in the case of high traded goods shares, and an increase of 2.12 % in the case of low shares. The sign of the effect of larger foreign size on real exchange rate could not be determined in the theoretical analysis (see Proposition 2). The numerical analysis, however, indicates that the increase in L^* significantly lowers the real exchange rate (by about 4 %).

Table 2 examines the effects of technological and size changes in the home economy on foreign welfare. The table shows the effects of an increase in ε or b, a decrease in k, and an increase in L. Again, to facilitate comparison between different changes, the magnitude of each of these changes is assumed to be such that it leads to a 10 % increase in home welfare (w). The signs of the welfare effects in Table 2 are the same as in Table 1, but there are important asymmetries in the magnitudes of these effects. The effects of home changes on foreign welfare turn out to be much stronger than the effects of foreign changes on home welfare. For example, an improvement in home labor efficiency that generates a 10 % increase in the home real wage leads to 1.7 % increase in the foreign real wage (in the case of high traded goods shares) while an analogous foreign technological improvement causes a 0.16 % increase in the home real wage. Other home changes also produce a much larger effect on the foreign real wage than foreign changes on the home real wage.

Asymmetries in the magnitudes of welfare effects can be attributed mostly to differences in the shares of traded goods and initial wages (income per capita) between the US and its trading partners. For example, if the US traded goods share is raised to the non-US level of 0.49 (in the high share case), the effect of the foreign improvement in labor efficiency on home welfare would increase from 0.16 % to 0.44 % while the effect of home improvement on foreign welfare would decrease from 1.7 % to 0.86 %. The remaining difference between the effects on home and foreign welfare is accounted for largely by initial wage differences. These results suggest that a country with a higher share of traded goods and lower income per capita than its trading partner can expect a larger benefit from foreign technological advance.

5. Conclusions

Recently, there has been much interest in international trade models based on Melitz (2003), which highlight trade in differentiated goods produced by hetereogeneous firms under monopolistic competition. In such models, one concern is that foreign technological progress harms a country as home firms are displaced by more productive foreign firms. A justification for this concern is provided by Demidova (2008) in a variant of the Melitz model that includes a competitive sector producing a costlessly traded homogeneous good.

This paper argues that a country should not fear foreign technological progress but rather welcome it in the absence of a freely traded homogeneous good. In this case, the real exchange rate adjusts to maintain balanced trade in the differentiated good. The exchange rate adjustment in response to improved foreign productivity allows home firms to compete with more poductive foreign firms. We show that foreign technological improvements (in a variety of forms) always improve home welfare even if the Melitz model is genralized to include a nontraded good.

The paper also examines the effect of foreign technological improvements on the real exchange rate. For an improvement in foreign labor productivity in the traded good, we derive the condition under which the real value of foreign currency depreciates, contrary to the predictions of the Balassa-Samuelson model. Interestingly, the reversal of the Balassa-Samuelson effect always occurs in the model without the nontraded good.

A model with trade mainly in differentiated goods and a large nontraded goods sector seems relevant for many countries. We calibrate our model to data for the US and its trading partners. This case provides an interesting example of the welfare implications of technological change in trading countries that are significantly different. The numerical analysis reveals important asymmetries in the transmission of welfare effects: US gains much less from foreign technological improvements than its trading partners from US improvements. We attribute these asymmetries mainly to a smaller share of traded goods and a higher income per capita in the US than in its trading partners.

Appendix A

Derivation of (23)

Noting that $\Pi = [\tilde{\pi}_H + \nu_{HE}\tilde{\pi}_{HE}]/w$, use (14) to express it as

$$\Pi = Dw^{-\sigma}(\varepsilon\tilde{\theta}_H)^{\sigma-1}[1+\nu_{HE}(D^*/D)q^{\sigma}(\tau)^{1-\sigma}(\tilde{\theta}_{HE}^{\sigma-1}/\tilde{\theta}_H^{\sigma-1})]-(\phi/\varepsilon)[1+\nu_{HE}(\phi_E/\phi)]. \quad \text{(A1)}$$

Also, (10) and (11) imply that $Dw^{-\sigma}(\varepsilon\tilde{\theta}_H)^{\sigma-1} = \dfrac{k\phi/\varepsilon}{k-\sigma+1}$ while (11) and (12) and their foreign counterparts imply that $\nu_{HE}(\tilde{\theta}_{HE}^{\sigma-1}/\tilde{\theta}_H^{\sigma-1}) = (\bar{\theta}_H/\bar{\theta}_{HE})^{k-(\sigma-1)}$ and $\nu_{FE}(\tilde{\theta}_{FE}^{\sigma-1}/\tilde{\theta}_F^{\sigma-1}) = (\bar{\theta}_F/\bar{\theta}_{FE})^{k-(\sigma-1)}$. Use these expressions in (A1) and simplify the resulting equation to obtain (23)

Derivation of (27)

Use (7) and (11)-(13) and the corresponding foreign equations to obtain

$$p^{1-\sigma} = n_H\left(\frac{(\sigma-1)\varepsilon\tilde{\theta}_H}{\sigma w}\right)^{\sigma-1}\left[1+\frac{n_F(\varepsilon^*\bar{\theta}_F^*)^{\sigma-1}}{n_H(\varepsilon\bar{\theta}_H)^{\sigma-1}}\left(\frac{\tau q w^*}{w}\right)^{1-\sigma}\left(\frac{\bar{\theta}_F^*}{\bar{\theta}_{FE}^*}\right)^{k*-(\sigma-1)}\right], \quad \text{(A2)}$$

$$p^{*1-\sigma} = n_H\left(\frac{(\sigma-1)\varepsilon\tilde{\theta}_H}{\sigma w^*}\right)^{\sigma-1}\left[\frac{n_F(\varepsilon^*\bar{\theta}_F^*)^{\sigma-1}}{n_H(\varepsilon\bar{\theta}_H)^{\sigma-1}}+\left(\frac{\tau w}{q w^*}\right)^{1-\sigma}\left(\frac{\bar{\theta}_H}{\bar{\theta}_{HE}}\right)^{k-(\sigma-1)}\right]. \quad \text{(A3)}$$

Next, use (6), (11), (13), (16) and their foreign counterparts to express

$p_{HE}Z_{HE}^* = n_H(\varepsilon\tilde{\theta}_H)^{\sigma-1}\left[q^{\sigma}D^*(\bar{\theta}_H/\bar{\theta}_{HE})^{k-(\sigma-1)}\sigma(\tau w)^{1-\sigma}\right]$, and

$p_{FE}Z_{FE} = n_F(\varepsilon^*\tilde{\theta}_F^*)^{\sigma-1}\left[D(\bar{\theta}_F^*/\bar{\theta}_{FE}^*)^{k*-(\sigma-1)}\sigma(\tau q w^*)^{1-\sigma}\right]$. Use these expressions in (18) to get

$$\frac{n_F(\varepsilon^*\tilde{\theta}_F^*)^{\sigma-1}}{n_H(\varepsilon\tilde{\theta}_H)^{\sigma-1}} = \left(\frac{q^{\sigma}D^*}{D}\right)\left(\frac{w}{q w^*}\right)^{1-\sigma}\frac{(\bar{\theta}_H/\bar{\theta}_{HE})^{k-(\sigma-1)}}{(\bar{\theta}_F^*/\bar{\theta}_{FE}^*)^{k*-(\sigma-1)}}. \quad \text{(A4)}$$

Divide (A2) by (A3), and in the resulting expression, use (A4) to substitute for

$\frac{n_F(\varepsilon^*\tilde{\theta}_F^*)^{\sigma-1}}{n_H(\varepsilon\tilde{\theta}_H)^{\sigma-1}}$ and (10) and its foreign counterpart to substitute for $\frac{\bar{\theta}_H}{\bar{\theta}_{HE}}$ and $\frac{\bar{\theta}_F^*}{\bar{\theta}_{FE}^*}$ to derive

$$\left(\frac{p}{p^*}\right)^{1-\sigma} = \left(\frac{w}{w^*}\right)^{1-\sigma}\left[\frac{1+\Phi^{-1}(D^*q^\sigma/D)^{k/(\sigma-1)}}{\Phi^{-1}(qw^*/w)^{\sigma-1}\left\{\Phi^*(D^*q^\sigma/D)^{(k+k^*)/(\sigma-1)-1}+(D^*q^\sigma/D)^{k/(\sigma-1)-1}\right\}}\right]. \text{(A5)}$$

Finally, note that (4) and its foreign counterpart imply that $\left(\frac{p}{p^*}\right)^{1-\sigma} = \left(\frac{w}{w^*}\right)^{-(1-\alpha)(1-\sigma)/\alpha}$

while (20) and its foreign counterpart imply that $\frac{D^*}{D} = \frac{L^* w^{*1-(\sigma-1)(1-\alpha)/\alpha}}{Lw^{1-(\sigma-1)(1-\alpha)/\alpha}}$. Making these substitutions in (A5) and simplifying this equation, we obtain (27).

Proof of Result 1

For $\sigma \in \left(\frac{1}{1-\alpha},\infty\right)$, $c_{21} = 1+\frac{2\Phi}{1+\Phi}\beta k > 0$ since $\beta = \left(\frac{1-\alpha}{\alpha}-\frac{1}{\sigma-1}\right) \geq 0$.
For $\sigma \in \left(1,\frac{1}{1-\alpha}\right)$, $\lim_{\sigma\to 1} c_{21} = -\infty$, $\lim_{\sigma\to 1/(1-\alpha)} c_{21} = 1$, and $\partial c_{21}/\partial\sigma > 0$, since $\lim_{\sigma\to 1}\beta = -\infty$, $\lim_{\sigma\to 1/(1-\alpha)}\beta = 0$, and $\partial\beta/\partial\sigma > 0$. Thus there exists a value of σ in this interval, σ_1, for which $c_{21} = 0$, and c_{21} is greater (less) than zero as σ is greater (less) than σ_1.

Proofs of Propositions

Express the nonlinear relations (25), (26) and (27) as $w = f(\frac{w}{w^*},q,S)$, $w^* = f^*(\frac{w}{w^*},q,S)$ and $\frac{w}{w^*} = g(\frac{w}{w^*},q,S)$, where S is the set of model parameters. In the initial state, assume that the home and foreign countries are homogeneous (the home and foreign parameters are symmetric), and normalize $\varepsilon = L = w = 1$. To examine the effect of a change in parameter $x(\in S)$, take the log-linear approximation of the relations $\frac{w}{w^*}=\frac{f(.)}{f^*(.)}$ and $\frac{w}{w^*}=g(.)$ around a symmetric initial state to obtain the following two-equation system:

$$c_{11}(\hat{w}-\hat{w}^*)+c_{12}\hat{q}=d_1\hat{x}, \quad c_{21}(\hat{w}-\hat{w}^*)+c_{22}\hat{q}=d_2\hat{x}, \tag{A6}$$

where $d_1=\bar{x}\left(\frac{\partial \ln f}{\partial x}-\frac{\partial \ln f^*}{\partial x}\right)$ and $d_2=\bar{x}\frac{\partial \ln g}{\partial x}$; $c_{11}, c_{12}, c_{21},$ and c_{22} are defined (in terms of initial values of parameters) as in (31) and (32); and $c_{11}>0, c_{12}<0, c_{22}>0$ while the sign of c_{21} is given by Result 1 . The system can be solved for $\hat{w}-\hat{w}^*$ and $\hat{q}$ as

$$\hat{w}-\hat{w}^*=\frac{d_1c_{22}-d_2c_{12}}{\Delta}\hat{x}, \quad \hat{q}=\frac{d_2c_{11}-d_1c_{21}}{\Delta}\hat{x}, , \tag{A7}$$

where $\Delta=\begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix}=\left(\frac{2\Phi}{1+\Phi}\right)\left(\frac{\sigma k}{\sigma-1}\right)-1+\frac{2}{1+\Phi}>0$, since $\frac{\sigma k}{\sigma-1}>\sigma$ and $\Phi>1$.

Substitute these values in the log-linear approximation of the relation, $w=f(.)$, and simplify to get

$$\hat{w}=\frac{1}{\Delta(1+\Phi)}\left\{-d_1+d_2\frac{\alpha\sigma}{\sigma-1}\right\}\hat{x}+\frac{\bar{x}}{\bar{f}}\frac{\partial f}{\partial x}\hat{x}. \tag{A8}$$

To prove Proposition 1, part (i), first let $x=\varepsilon^*$. Note that (27) and (29) imply that $\bar{\varepsilon}^*\frac{\partial \ln f}{\partial \varepsilon^*}=0, \ \bar{\varepsilon}^*\frac{\partial \ln g}{\partial \varepsilon^*}=0$. Use (28) to derive $\bar{\varepsilon}^*\frac{\partial \ln f^*}{\partial \varepsilon^*}=\frac{\alpha\sigma}{\sigma-1}$. Thus, $d_1=-\frac{\alpha\sigma}{\sigma-1}$, and $d_2=0$. Substituting these values in (A7) and (A8) and using Result 1, it can be shown that for $\hat{\varepsilon}^*>0$, $\hat{w}>0, \hat{w}-\hat{w}^*<0$ and $\hat{q}$ is positive or negative as σ is greater or less than σ_1 . Next, let $x=b^*$. Note that $\bar{b}^*\frac{\partial \ln f}{\partial b^*}=0, \ \bar{b}^*\frac{\partial \ln g}{\partial b^*}=0$ according to (27) and (29), and use (28) to derive $\bar{b}^* \dfrac{\partial \ln f^*}{\partial b^*} = \alpha$. Thus, $d_1 = -\alpha,$ and $d_2 = 0.$ Again,

substituting these values in (A7) and (A8) and using Result 1, we can derive the results

for $\hat{b}^* > 0$.

To prove Proposition 1, part (ii), let $x = k^*$. Use (28) to derive

$$\bar{k}^* \frac{\partial \ln f^*}{\partial k^*} = \alpha \left[\frac{1}{\bar{k}^*} \ln \bar{\Gamma}^* - \frac{1}{\bar{k}^* - (\sigma - 1)} - \frac{1}{1+\bar{\Phi}} \ln \left(\frac{\phi_E \tau^{\sigma-1}}{\phi} \right)^{\frac{1}{\sigma-1}} \right] < 0,$$

since $\dfrac{\phi_E \tau^{\sigma-1}}{\phi} > 1$, and $\dfrac{1}{k^*} \ln \Gamma^* < 0$ (because (21) implies that in the initial state with

normalization, $\bar{\Gamma}^* = \bar{v}_H < 1$). Also, use (29) to derive

$\bar{k}^* \dfrac{\partial \ln g}{\partial k^*} = -\bar{k}^* \dfrac{\bar{\Phi}}{1+\bar{\Phi}} \ln \left(\dfrac{\phi_E \tau^{\sigma-1}}{\phi} \right)^{\frac{1}{\sigma-1}} < 0$. Thus, noting that $\bar{k}^* \dfrac{\partial \ln f}{\partial k^*} = 0$ from (27), we

have $d_1 > 0$ and $d_2 < 0$. Given these signs, (A7) and (A8) imply that $\hat{w} > 0$ for $\hat{k}^* < 0$

(higher dispersion), but the signs of $\hat{w} - \hat{w}^*$ and $\hat{q}$ are indeterminate as $d_1 c_{22}$ is positive

and $d_2 c_{12}$ can be positive or negative.

To prove Proposition 2, let $x = L^*$. Use (27)-(29 to obtain

$\bar{L}^* \dfrac{\partial \ln f}{\partial L^*} = \left(\dfrac{\alpha}{\sigma - 1} \right) \dfrac{1}{1+\Phi}$, $\bar{L}^* \dfrac{\partial \ln f^*}{\partial L^*} = -\left(\dfrac{\alpha}{\sigma - 1} \right) \dfrac{\Phi}{1+\Phi}$, and $\bar{L}^* \dfrac{\partial \ln g}{\partial L^*} = 1 - \dfrac{k}{\sigma - 1} (\dfrac{2\Phi}{1+\Phi})$.

Thus $d_1 = \left(\dfrac{\alpha}{\sigma - 1} \right) \left(\dfrac{1-\Phi}{1+\Phi} \right) < 0$ and $d_2 < 0$. The signs of $\hat{w} - \hat{w}^*$ and $\hat{q}$, are indeterminate

as the sign of the expression $d_2 c_{11} - d_1 c_{21}$ in (A7) is ambiguous. To prove the effect on w,

first note that substituting for the value of $\bar{L}^* \dfrac{\partial \ln f}{\partial L^*}$ in (A8), we can express

$\hat{w} = \frac{\alpha}{\Delta(1+\bar{\Phi})(\sigma-1)}\left\{-d_1\frac{\sigma-1}{\alpha}+d_2\sigma+\Delta\right\}\hat{L}^*$. Recalling that

$\Delta = \left(\frac{2\Phi}{1+\Phi}\right)\left(\frac{\sigma k}{\sigma-1}\right)-1+\frac{2}{1+\Phi}$ and using the values of d_1, and d_2, the expression in the curly bracket is simplified as $\frac{\Phi-1}{1+\Phi}+\sigma-1+\frac{2}{1+\Phi}$, which is positive since $\Phi > 1$. Thus $\hat{w} > 0$ for $\hat{L}^* > 0$.

Appendix B

Data-based parameters

Steady-state values of the relative foreign real wage ($\bar{w}^* / \bar{w}$) and relative foreign labor supply (L^* / L) are measured, respectively, as weighted averages of GDP per capita (in US dollars, averaged over 2001-2010) and population (also averaged over 2001-2010) of major US trading partners relative to US levels. The trading partners group includes 15 leading US trade partners accounting for 73.4 % of US imports and 71.5 % of US exports.[24] The weights used to compute averages for this group are based on the share of trade (average of imports and exports) with each partner in the total trade with the group. The source of the data for GDP per capita and population is the World Development Indicators 2011 (from the World Bank) and for trade flows is the US Census Bureau.

To measure home and foreign shares of traded goods (α, α^*), we need estimates of the shares of non-service industries and service industries with trade for both the US and its trading partners. For the share of non-service industries, we use Word Bank data,

[24] The countries included in the US trading partner group are: Canada, Mexico, China, Japan, Germany, U.K., South Korea, France, Taiwan, Brazil, Netherlands, Singapore, Venezuela, Saudi Arabia, and India.

which are available for US as well as nearly all countries in the US trading partner group. Other sources of sectoral data (e.g. STAN data base) provide more disaggregated sector-level data, but are not available for some important US trading partners. The share of non-service industries in GDP [defined as one minus the share of service industries (ISIC 51-99), and averaged over 2001-2010] is 0.23 for the US and 0.37 for the trading partner group (based on trade-weighted averages).[25] The share of service industries open to international trade is difficult to estimate as the data on trade in services by industry are not available in sufficient detail. We assume that the share of traded service industries in all service industries is between 0.20 and 0.25 for the US and between 0.15 and 0.20 for its trading partners.[26] Calculating the share of traded services industries in GDP under this assumption and adding it to the share of non-service industries, we obtain a range of 0.38-0.42 for α and 0.46-0.49 for α^*.

Calibration procedure

To allow for $\alpha \neq \alpha^*$, express the model consisting of (25)-(27) as

$$w = \left[\Theta \left\{ L^{k/(\sigma-1)} + \Phi^{-1}(\alpha^* / \alpha) L^{*k/(\sigma-1)} \left(w^* / w \right)^{-k\beta} q^{k\sigma/(\sigma-1)} \right\} \right]^{\alpha/k}, \qquad \text{(B1)}$$

$$w^* = \left[\Theta^* \left\{ L^{*k*/(\sigma-1)} + \Phi^{*-1}(\alpha / \alpha^*) L^{k*/(\sigma-1)} \left(w^* / w \right)^{k*\beta*} q^{-k*\sigma/(\sigma-1)} \right\} \right]^{\alpha*/k*}, \qquad \text{(B2)}$$

[25] For this calculation, Taiwan is excluded because the share data for this country were not available from the World Bank source.

[26] International trade in services is likely to occur largely in industries producing information, finance and insurance, and professional and business services. Based on the US (Bureau of Economic Analysis) data, the average share of these sectors (over 2001-2010) in all service sectors is 0.29. We assume that a major portion of these sectors represents traded service industries. Shares of these industries are not available for all US trading partners, but STAN data suggest that the share of similar industries tends to be smaller in non-US countries.

$$\frac{w}{w^*} = \left[\frac{\Phi + \left(\frac{\alpha^* L^*}{\alpha L}\right)^{k/(\sigma-1)} q^{\sigma k/(\sigma-1)} \frac{w^{k\beta}}{w^{*k\beta^*}}}{\left(\frac{\alpha^* L^*}{\alpha L}\right)^{\frac{k}{\sigma-1}-1} q^{\frac{\sigma k}{\sigma-1}-1} \frac{w^{k\beta}}{w^{*k\beta^*}} \left[\Phi^* \left(\frac{\alpha^* L^*}{\alpha L}\right)^{\frac{k^*}{\sigma-1}} q^{\frac{\sigma k^*}{\sigma-1}} \frac{w^{k^*\beta}}{w^{*k^*\beta^*}} + 1\right]} \right]. \tag{B3}$$

To calibrate Φ, Φ^*, Θ and Θ^*, we derive an additional relation that determines the share of imports in consumption defined as $impsh \equiv \frac{p_{FE} Z_{FE}}{C}$ as follows. First, use (3), (6) and (7) to write the import share as

$impsh = \alpha \left(\frac{p_{FE}}{p}\right)^{1-\sigma} = \frac{1}{\left[(p_H / p_{HE})(p_{HE} / p_{FE}\right]^{1-\sigma} + 1}$. Next, use (11), (12), (13) and its foreign counterpart, (16) and (24) modified to let $\alpha \neq \alpha^*$, to obtain

$\left(p_H / p_{HE}\right)^{1-\sigma} = \Phi q^{\sigma - \sigma k/(\sigma-1)} \left(\frac{\alpha^* L^*}{\alpha L}\right)^{1-k/(\sigma-1)} w^{*-\beta^*(\sigma-1-k)} w^{\beta(\sigma-1-k)}$. Also noting that $p_{HE} / p_{FE} = Z_{FE} / Z^*_{HE}$ from (18), and using (6), (20) and their foreign counterparts modified to allow $\alpha \neq \alpha^*$, we get

$\left(p_{HE} / p_{FE}\right)^{1-\sigma} = (q^\sigma D^* / D)^{-1} = q^{-\sigma} \left(\frac{\alpha^* L^*}{\alpha L} w^{*-\beta^*(\sigma-1)} w^{\beta(\sigma-1)}\right)^{-1}$. Using these expressions in the relation for the import share, we have

$$impsh = \frac{\alpha}{\Phi q^{-\sigma k/(\sigma-1)} \left(\frac{\alpha^* L^*}{\alpha L}\right)^{-k/(\sigma-1)} w^{*\beta^* k} w^{-\beta k} + 1}. \tag{B4}$$

Given $L^*, \bar{w}^*$, the values of parameters in the subset $S = [\sigma, \tau, k, k^*, \alpha, \alpha^*]$, and our normalization $(\bar{q} = \bar{w} = L = 1)$, (B4) can be used to determine Φ as

$$\Phi = \frac{(\alpha / \overline{impsh}) - 1}{(\frac{\alpha^* L^*}{\alpha})^{-k/(\sigma-1)} \overline{w}^{*\beta^* k}}. \tag{B5}$$

Next given Φ, Φ^* can be determined from (B3) as

$$\Phi^* = \left\{ \frac{\Phi + \left(\frac{\alpha^* L^*}{\alpha} \right)^{k/(\sigma-1)} \frac{1}{\overline{w}^{*k\beta^*}}}{\left(\frac{\alpha^* L^*}{\alpha} \right)^{\frac{k}{\sigma-1}-1} \frac{1}{\overline{w}^{*1+k\beta^*}}} - 1 \right\} \div \left[\left(\frac{\alpha^* L^*}{\alpha} \right)^{\frac{k^*}{\sigma-1}} \frac{1}{\overline{w}^{*k^*\beta^*}} \right]. \tag{B6}$$

Finally, we can determine Θ and Θ^* from (B1) and (B2) as follows

,

$$\Theta = 1 / \left\{ 1 + \Phi^{-1} (\alpha^* / \alpha) L^{*k/(\sigma-1)} \left(w^* \right)^{-k\beta} \right\}, \tag{B7}$$

$$\Theta^* = w^{*k^*/\alpha^*} / \left\{ L^{*k^*/(\sigma-1)} + \Phi^{*-1} (\alpha / \alpha^*) \left(w^* \right)^{k^*\beta^*} \right\}. \tag{B8}$$

Constraints on parameters in S

The sorting pattern for home and foreign exporters and local producers requires that $\phi_E \tau^{\sigma-1} D > \phi D^* q^\sigma$ and $\phi_E^* \tau^{\sigma-1} D^* q^\sigma > \phi^* D$. Using (20) and its foreign counterpart, these constraints under our normalization can be expressed as

$$\frac{\phi_E}{\phi} > \frac{1}{\tau^{\sigma-1}} \frac{\alpha^*}{\alpha} L^* w^{*1-(\sigma-1)(1-\alpha^*)/\alpha^*}, \tag{B9}$$

$$\frac{\phi_E^*}{\phi^*} > \frac{1}{\tau^{*\sigma-1}} \frac{\alpha}{\alpha^*} \frac{1}{L^* w^{*1-(\sigma-1)(1-\alpha^*)/\alpha^*}}. \tag{B10}$$

Note that $\frac{\phi_E}{\phi} = \left(\Phi \tau^{-k} \right)^{\frac{\sigma-1}{k-(\sigma-1)}}$ and $\frac{\phi_E^*}{\phi^*} = \left(\Phi^* \tau^{*-k^*} \right)^{\frac{\sigma-1}{k^*-(\sigma-1)}}$. The values assumed for parameters in S must satisfy (B9) and (B10) and are thus constrained.

Changes in ε^*, b^* and k^*

The effect of a change in ε^* or b^* operates via Θ^*. Noting that

$\Gamma^* \equiv \left[b^*(\sigma-1)/\sigma\right]^{k*}\left[\alpha^*/(\phi^*\sigma)\right]^{k*/(\sigma-1)}$ and $\Lambda^* \equiv \frac{\phi^*}{\varepsilon^*}\left[\frac{k^*}{k^*-(\sigma-1)}-1\right]$, we have

$$\Theta^* = \frac{b^{*k*}\phi^{*1-k*/(\sigma-1)}\varepsilon^{*k^*\sigma/(\sigma-1)}[(\sigma-1)/\sigma]^{k*}(\alpha^*/\sigma)^{k*/(\sigma-1)}[k^*/(k^*-(\sigma-1))]}{\psi^*\delta^*}. \quad \text{(B11)}$$

Θ^* increases in both ε^* and b^*, and its elasticity with respect to ε^* and b^* is $k*\sigma/(\sigma-1)$ and k^*, respectively. The effect of changes in ε^* or b^* in Table 1 is calculated by determining the change in Θ^* that would cause a 10% increase in w^*.

A change in k^* would affect both Θ^* and Φ^*. As (B11) indicates, values of b^*, ϕ^* and ε^* would be needed to determine the elasticity of Θ^* with respect to k^*. Our strategy is to normalize $b^*=1$, $\varepsilon^*=1$ and make a reasonable assumption about ν_F to derive ϕ^* as follows. First, letting $\bar{k}^*$ denote the initial value of k^* and assuming that $\nu_F = 0.5$, we use the foreign counterpart of (21), with $\varepsilon^*=1$, to determine Γ^* as

$\Gamma^* = \frac{\nu_F}{L^{*\frac{\bar{k}*}{\sigma-1}}w^{*-\frac{\bar{k}*}{\alpha*}}}$. Next, with $b^*=1$, we determine ϕ^* as

$\phi^* = \left(\frac{\left[(\sigma-1)/\sigma\right]^{\bar{k}*}\left[\alpha^*/\sigma\right]^{\bar{k}*/(\sigma-1)}}{\Gamma^*}\right)^{\frac{\sigma-1}{\bar{k}*}}$. Given ϕ^* and $\varepsilon^*=1$, Λ^* is determined, which

allows us to pin down $\psi^*\delta^* = \frac{\Gamma^*\Lambda^*}{\Theta^*}$. These values are used to calculate how Θ^* would

change in response to a change in k^*. Since $\frac{\phi_E^*}{\phi^*} = \left(\Phi^*\tau^{*-\bar{k}*}\right)^{\frac{\sigma-1}{\bar{k}*-(\sigma-1)}}$, we can readily

determine the effect of a change in k^* on Φ^*. A similar approach is used to determine the effect of changes in ε or b and k in Table 2.

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Figure 1. Diagrammatic Analysis of the Model

Note: *FE* is the free entry relation (eq. 31); *PT* is the price trade balance relation (eq. 32); and $\bar{U}$ is the relation for costant home welfare (eq. 33). *FE'* represents a shift in FE line due to an increase in $\varepsilon*$ or $b*$ or to a decrease in $k*$ while *PT'* represents a shift in *PT l*ine due to a decrease in $k*$.

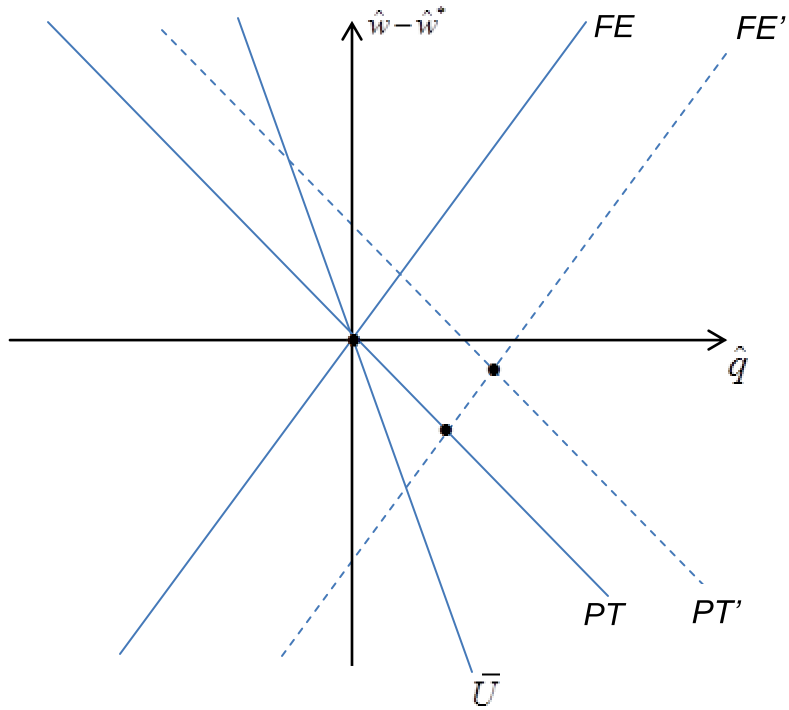

Table 1. Effects of Changes in Foreign Technology and Size

	Percentage Change in		
	Absolute Home Welfare (w)	Relative Home Welfare (w/w^*)	Real Exchange (q)
Low Shares ($\alpha = 0.38, \alpha^* = 0.46$)			
An increase in ε^* or b^*	0.10	-9.90	3.43
A decrease in k^*	0.21	-9.79	3.81
An increase in L^*	2.12	-7.88	-3.94
High Shares ($\alpha = 0.42, \alpha^* = 0.49$)			
An increase in ε^* or b^*	0.16	-9.84	2.49
A decrease in k^*	0.43	-9.57	3.57
An increase in L^*	1.90	-8.10	-4.09

Note: In each case, $\tau = 1.2$ and $\sigma = 2.0$. Magnitude of each change is chosen to ensure that w^* increases by 10 %.

Table 2. Effects of Changes in Home Technology and Size

	Percentage Change in		
	Absolute Foreign Welfare (w^*)	Relative Foreign Welfare (w^* / w)	Real Exchange (q)
Low Shares ($\alpha = 0.38, \alpha^* = 0.46$)			
An increase in ε or b	2.08	-7.92	-5.32
A decrease in k^*	4.79	-5.21	-10.18
An increase in L	5.83	-4.17	3.32
High Shares ($\alpha = 0.42, \alpha^* = 0.49$)			
An increase in ε or b	1.70	-8.31	-4.04
A decrease in k^*	4.11	-5.89	-8.67
An increase in L	4.86	-5.14	3.32

Note: In each case, $\tau = 1.2$ and $\sigma = 2.0$. Magnitude of each change is chosen to ensure that w increases by 10 %.