Continuously Tunable Photonic Fractional Temporal Differentiator Based on a Tilted Fiber Bragg Grating

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***Abstract*—A tunable temporal photonic fractional differentiator implemented based on a tilted fiber Bragg grating is proposed and demonstrated. The phase response at a cladding mode resonant wavelength is strongly polarization-dependent and the fractional order of the photonic differentiator can be continuously tuned by changing the polarization state of the input light wave. A proof-of-concept experiment is carried out. The fractional differentiation of an optical Gaussian pulse with a bandwidth of 40 GHz is demonstrated, in which the fractional order is continuously tuned from 0.81 to 1.42.**

***Index Terms*—Polarization, tilted fiber Bragg grating (TFBG), tunable photonic fractional differentiator.**

I. Introduction

A TEMPORAL photonic differentiator is a basic operator that performs real-time differentiation of the field envelope of an optical signal in the optical domain, which can find applications in numerous fields such as ultrafast signal generation [1], [2], and pulse characterization [3]. Compared with a puely electronic temporal differentiator, a photonic temporal differentiator implemented in the optical domain would provide a much higher speed and wider bandwidth.

Numerous techniques have recently been proposed to implement a photonic temporal differentiator. In general, a photonic temporal differentiator can be realized using an optical device that has a transfer function with the form $[j(\omega-\omega_0)]^n$, where n is the differentiation order, ω is the optical frequency and ω_0 is the optical carrier frequency of the optical signal. A temporal differentiator can be realized based on cross-gain modulation in a semiconductor optical amplifier (SOA) [4]. A temporal differentiator can also be realized using a π phase shifted fiber Bragg grating (PS-FBG) [5], [6], a long period fiber grating (LPFG) [7], or a silicon micro-ring resonator [8]. However, the differentiation order in the above mentioned photonic differentiators is an integer and is hard to be changed once the optical device is fabricated.

When the differentiation order n is no longer kept as an integer, the operator is generalized to be a photonic temporal fractional differentiator. Recently, fractional differentiator has attracted great interests due to its potential applications in pulse shaping, signal processing and ultrafast optical signal coding [9], [10]. Cuadrado–Laborde and Andrés demonstrated a photonic fractional differentiator that was implemented based on an asymmetrical PS-FBG in reflection [9]. One limitation of this technique is that the fractional order cannot be adjusted once the asymmetrical PS-FBG is fabricated.

In this letter, we propose and demonstrate a photonic fractional temporal differentiator that has a continuously tunable fractional order implemented using a tilted fiber Bragg gratings (TFBG). TFBG has multiple cladding-mode resonances and the phase response within a cladding-mode resonance is highly polarization dependent. Therefore, the fractional order of the photonic differentiator can be continuously tuned by changing the polarization state of the input optical signal. An experiment is performed. A differentiator with a fractional order tunable from 0.81 to 1.42 is achieved. The implementation of the differentiator to achieve temporal differentiation of an optical Gaussian pulse with a bandwidth of 40 GHz is demonstrated.

II. Principle

For an input signal $x(t)$, the Fourier transform of an nth order differentiation $dx^n(t)/dt^n$ can be expressed as $[j(\omega-\omega_0)]^n X(\omega-\omega_0)$. The transfer function of the differentiator is then given by

$$H(\omega)=\begin{cases} j^n(\omega-\omega_0)^n \\ -j^n[-(\omega-\omega_0)^n] \end{cases} = \begin{cases} e^{jn\left(\frac{\pi}{2}\right)}(\omega-\omega_0)^n \\ e^{jn\left(-\frac{\pi}{2}\right)}[-(\omega-\omega_0)^n] \end{cases} \quad (1)$$

When the fractional order is 1, a first-order differentiator with a transfer function of $j(\omega-\omega_0)$ is implemented if the TFBG has a π phase shift at the resonance wavelength. The operation of a continuously tunable photonic fractional differentiator using a TFBG is illustrated in Fig. 1. The transmission spectrum of a TFBG has the longest wavelength resonance corresponding to the reflection of the core mode, and the shorter wavelength resonances corresponding to the contra-propagating coupling between the core mode and the cladding modes. With a linearly polarized input light, the TFBG can couple to two distinct groups of cladding modes depending on the orientation of the light polarization that is parallel or perpendicular to the tilt plane [11]. Tuning the polarization state between these extremes yields pairs of neighboring resonances with adjustable relative amplitudes. When a signal with an optical carrier located in one of the loss peak, the pulse will be differentiated by the TFBG with a fractional order determined by the input polarization state.

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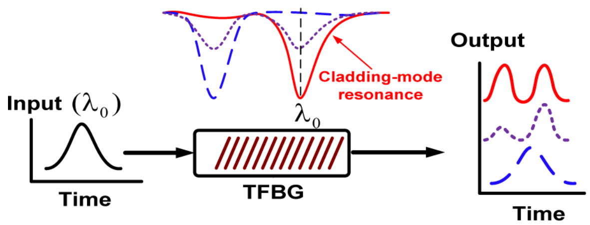

Fig. 1. Operation of a continuously tunable temporal photonic fractional differentiator based on a TFBG. When the incident light wave is adjusted with three different polarization states, a differentiated output with three different differentiation orders is achieved. TFBG: tilted fiber Bragg grating.

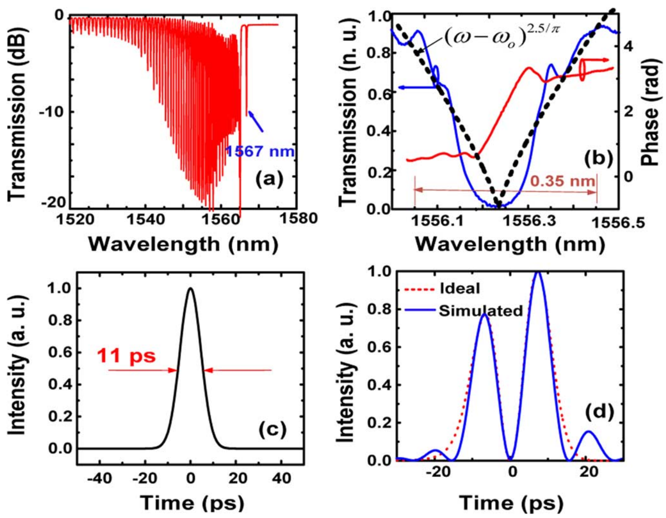

Fig. 2. (a) Transmission spectrum of a 4°-TFBG. (b) Magnitude and phase responses of one channel of the transmission spectrum. The dotted line is a polynomial fitting curve of the magnitude response with a polynomial order of $2.54/\pi$. (c) An input Gaussian pulse with a pulsewidth of 11 ps. (d) Simulated output pulse from the 4°-TFBG and an ideal photonic differentiator with a fractional order of $2.54/\pi$.

Fig. 2(a) shows the transmission spectrum of a TFBG with a tilt angle of 4°. The Bragg wavelength is 1567.6 nm and the total length of the TFBG is 10 mm. The TFBG is inscribed in hydrogen-loaded Corning SMF-28 fiber using a pulsed KrF excimer laser and a uniform phase mask [12]. Fig. 2(b) shows the magnitude and phase response of one channel of the transmission spectrum. It can be seen that a phase shift of about 2.54 rad is generated at the resonance frequency. The magnitude response is close to a polynomial fitting curve with a polynomial order of $2.54/\pi$, as shown in Fig. 2(b). Therefore, the transfer function $H(\omega)$ can be expressed as [9]

$$H(\omega) = \begin{cases} e^{j\left(\frac{2.54}{2}\right)}(\omega-\omega_0)^{\frac{2.54}{\pi}} \\ e^{j\left(-\frac{2.54}{2}\right)}\left[-(\omega-\omega_0)^{\frac{2.54}{\pi}}\right] \end{cases} \tag{2}$$

A phase shift of 2.54 rad at $\omega=\omega_0$ is generated and the fractionally order is $n = 2.54/\pi$. An optical signal will be fractional differentiated by the TFBG when the optical carrier of the signal is located at the resonance frequency. The operation bandwidth of the photonic fractional differentiator is 0.35 nm, or equivalently 43.75 GHz.

It is worth noting that the magnitude response of the TFBG and the ideal magnitude response are not perfectly matched, as shown in Fig. 2(b). To investigate the influence of the mismatch on the fractional differentiation performance, we numerically simulate the propagation of a Gaussian pulse with a pulsewidth of 11 ps through the TFBG, as shown in Fig. 2(c). Here, we utilize the measured phase response and compare the output pulses obtained using the measured (solid) and the ideal (dashed) magnitude responses. As can be seen from Fig. 2(d) a good agreement between the simulated and the ideal output pulses is achieved. The root mean square (RMS) error is about 4.27%. Thus, the influence of the mismatch between the ideal and obtained magnitude responses is small and can be ignored.

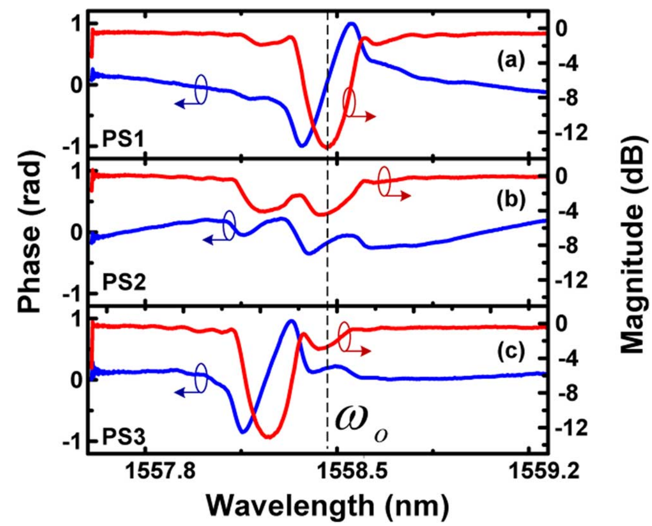

Fig. 3. Polarization dependence of the magnitude and phase responses of a 12° TFBG. (a) Polarization state 1, (b) polarization state 2, and (c) polarization state 3. PS: polarization state.

To evaluate the polarization dependence, a measurement of the magnitude and phase responses for an input light wave with different polarization state is performed. This is done using a 12° TFBG. Fig. 3 shows the magnitude and phase responses of the transmission spectrum when the input light is polarized at three different polarization states. When the light is polarized at state 1, as shown in Fig. 3(a), the notch depth of the long wavelength loss peak is about 13 dB while the short wavelength loss peak almost disappears. A maximum phase shift of about 1.92 rad is generated at the resonance wavelength. When the polarization state of the light is switched to state 3 (90° with respect to polarization state 1), the short wavelength loss peak is fully excited while the long wavelength loss peak almost disappears, as shown in Fig. 3(c). Fig. 3(b) shows that an identical notch depth of the short- and long-wavelength loss peaks is achieved when the light is polarized at state 2 (45° with respect to polarization state 1). It is also verified that each cladding mode resonance in the TFBG is formed by two cladding modes with orthogonal mode field distributions.

III. Experiment

The setup for a proof-of-concept experiment is shown in Fig. 4. A mode-locked laser (MLL) is used to generate an ultrashort pulse train with a repetition rate of 48.6 MHz, a pulsewidth of 550 fs. The pulse has a full-width at half-maximum (FWHM) of 8 nm with a central wavelength of 1558.6 nm. Since the optical pulse from the MLL is too narrow in the time domain to be differentiated by the TFBG, an optical bandpass filter (OBPF) is used to filter the optical spectrum of the MLL to broaden the optical pulse. The OBPF is central-wavelength tunable with a 3-dB bandwidth of 0.33 nm. As shown in Fig. 5(a),

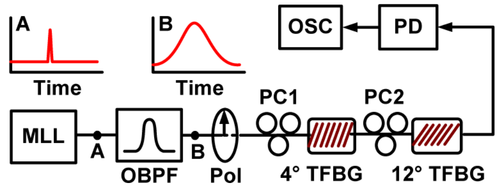

Fig. 4. Experimental setup. MLL: mode-locked laser; OBPF: optical bandpass filter; Pol: polarizer; TFBG: tilted fiber Bragg grating; PC: polarization controller; PD: photodetector; OSC: oscilloscope.

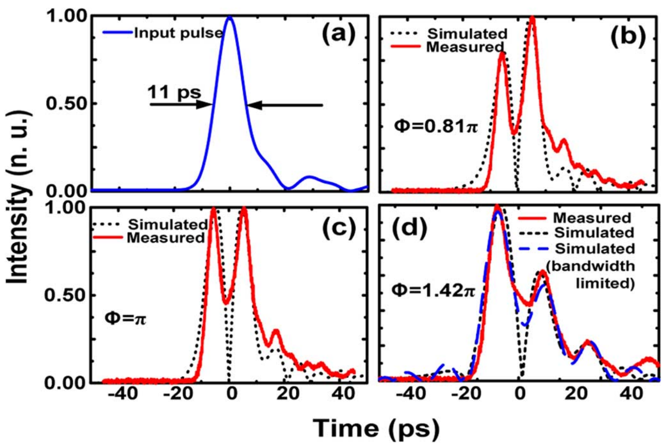

Fig. 5. Experimental results. (a) The Gaussian pulse after the optical filter, and the simulated (dotted line) and measured (solid line) output pulses from the photonic fractional differentiator with a fractional order of (b) 0.81, (c) 1.0, and (d) 1.42. The simulated output pulse (dashed line) with consideration of the limited bandwidth of the PD is also shown in (d).

the pulsewidth of the Gaussian pulse after the optical filter is about 11 ps when the central wavelength of the optical filter is tuned to 1558.44 nm. Since the time-bandwidth product of a Gaussian-shaped pulse is ~ 0.44, the bandwidth of the filtered pulse is about 40 GHz. The optical Gaussian pulse is then polarized by an in-line polarizer. Since a π phase shift cannot be achieved in the cladding mode resonance by only using a single TFBG, in the experiment the 4° TFBG is cascaded with the 12° TFBG to obtain a larger fractional order tunable range. Two polarization controllers (PCs) are connected before the two TFBGs to control the polarization states of the optical pulse. Note that the central wavelength of one channel of the 4° TFBG is 1558.44 nm. The 12° TFBG is mechanically stretched to ensure one of its cladding mode resonances spectrally match with that of the 4° TFBG. The differentiated optical pulse is detected by a 53-GHz photodetector (PD) with its waveform observed by a sampling oscilloscope (OSC, Agilent 86116A).

By tuning PC2, the phase shift located in the wavelength resonance of the 12° TFBG can be changed from 0 to 1.92 rad, as shown in Fig. 3. In addition, a phase shift of about 2.54 rad is obtained at the same resonance of the 4° TFBG, as shown in Fig. 2(b). Therefore, a fractional photonic differentiator with a fractional order tunable from 0.81 to 1.42 can be realized using the two cascaded TFBGs. Fig. 5(b) shows the measured output pulse from the fractional photonic differentiator with a fractional order of 0.81. A simulated pulse is also shown in Fig. 5(b). As can be seen, the simulated and experimental results agree well. The RMS error is calculated to be about 11.50%. Note that, the peak power of the measured differentiated pulse is about 1.97 mW, which can be further improved by using an optical amplifier. The photonic differentiator with a fractional order of 1 and 1.42 are also evaluated. The simulated and experimental results are shown in Fig. 5(c) and (d). Again, a good agreement is reached. The RMS errors of the differentiator with a fractional order of 1 and 1.42 are about 14.12% and 10.19%, respectively. Fig. 5(d) also shows a simulated output pulse, which is obtained assuming it is detected by a PD with a bandwidth of 53 GHz. The simulated output pulse after detection with the 53-GHz PD is much closer to the measured pulse. The RMS error between the simulated and the measured pulse is 3.87%. Thus, the measured processing error of the photonic fractional differentiator is partly caused due to the limited bandwidth of the PD.

IV. Conclusion

A technique using a TFBG to implement a temporal photonic fractional differentiator with a tunable fractional order was proposed for the first time. The photonic differentiation was implemented at one of the cladding mode resonances of the TFBG in transmission. The key feature of this technique is that the fractional order of the photonic differentiator is continuously tunable. The proposed technique was verified by an experiment, in which a 4° TFBG cascaded with a 12° TFBG was employed to provide a tunable fractional order from 0.81 to 1.42. The differentiation of an optical Gaussian pulse with a bandwidth of 40 GHz at different fractional orders was demonstrated. The temporal photonic differentiator has a potential to change the fractional order at a speed up to tens of GHz by using a high speed polarization modulator.

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