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On the Role of the Input Power Constraint in the Beamforming Optimality Range in TIMO Channels

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Abstract— **In this paper, the effect of the input power constraint on the beamforming optimality range in Gaussian two-input multiple-output (TIMO) channels is explored. The obtained results, using standard Lagrangian formulation, determine explicitly the range of the input SNR for which rank-1 signaling (beamforming) is optimal in TIMO channels for both the common power constraint and the individual power constraint cases. Moreover, the obtained results are extended to random TIMO channels, with channel state information at the receiver only, using the Jensen's upper bound on the mutual information.**

Keywords; MIMO channels, mutual information, input covariance matrix, beamforming, common power constraint, individual power constraints.

I. Introduction

In the literature on the design of the optimal transmission schemes for Gaussian multiple-input multiple-output (MIMO) channels, [1, 2] and references therein, the range of optimality of beamforming is of relevance since scalar coding can be used to achieve the channel capacity. Moreover, the introduction of distributed MIMO systems with non-uniform individual power constraints [3, 4] and OFDM-MIMO systems with uniform power constraints [5] has motivated the research on the design of the optimal input covariance for such systems. In this paper, the use of the standard Lagrangian formulation, to characterize the optimal input covariance matrix in Gaussian two-input multiple-output (TIMO) channels, has led to expressing the optimal input correlations (among the entries of the zero-mean complex Gaussian input vector) and consequently the range of optimality of beamforming (rank-1 signaling) in terms of the input SNR and the correlation between the channel columns for a TIMO channel with a common power constraint. Moreover, similar results are obtained for MIMO channels with individual power constraints. Finally, these results are extended to random channels using Jensen's upper bound on the mutual information.

In the paper, uppercase letters denote deterministic matrices and bold-faced uppercase letters denote random matrices. For vectors, bold-faced lowercase letters are used for both deterministic and random vectors where the distinction is assumed to be clear context-wise. For a matrix A, $tr(A)$ denotes the trace of a matrix, $[A]^H$ denotes the Hermitian transpose of a matrix. For a complex number z, the conjugate of z, the real and imaginary parts of z are denoted by z^*, $\Re e(z)$ and $\Im m(z)$, respectively

II. Maximaization of the Mutual Information in TIMO Channels

A. Deterministic channel matrix

The maximum mutual information, channel capacity, of a MIMO channel with L transmit and M receive elements for a deterministic channel matrix $H \in C^{M\times L}$ can be expressed as [6, 7]

$$\begin{aligned} C &= \sup_{tr(Q)\le P} \log\det\left[I_M + \frac{1}{\sigma_n{}^2} HQH^H\right] \\ &= \sup_{tr(Q)\le P} \log\det\left[I_L + \frac{1}{\sigma_n{}^2} QH^H H\right] \end{aligned}, \tag{1}$$

where P is the total power constraint and $Q \in C^{L\times L}$ is the positive semi-definite covariance matrix of the proper complex Gaussian input vector (Theorem 2 in [8]). In (1), I_M denotes the $M\times M$ identity matrix and $\sigma_n{}^2$ is the variance of the complex Gaussian noise at each receive element. In subsequent derivations the noise variance $\sigma_n{}^2$ is set to unity for notational simplicity.

Proposition 1:

(i) The range of the normalized input SNR for which rank-1 signaling is strictly optimal (capacity-achieving) for TIMO channels with a common power constraint is

$$0 < P \le \frac{2\left|\boldsymbol{h}_1{}^H \boldsymbol{h}_2\right|}{\left[\left|\boldsymbol{h}_1\right|^2\left|\boldsymbol{h}_2\right|^2 - \left|\boldsymbol{h}_1{}^H \boldsymbol{h}_2\right|^2\right]} \quad, \boldsymbol{h}_1 \ne a\boldsymbol{h}_2, M > 1 \quad, \qquad (2)$$

(ii) the range of the normalized input SNR for which rank-1 signaling is strictly optimal (capacity-achieving) for TIMO channels with individual power constraints is

$$0 < P \le 2\sqrt{\mathrm{K}^2 + \Delta^2} \quad , \quad 0 \le \Delta < \frac{P}{2}, \qquad (3\text{-a})$$

(iii) the range of the normalized input SNR for which rank-1 signaling is strictly sub-optimal for TIMO channels with individual power constraints is

$$P > 2\sqrt{\mathrm{K}^2 + \Delta^2} \quad , \quad 0 \le \Delta < \frac{P}{2}, \qquad (3\text{-b})$$

where a is a constant, Δ is the deviation from equal power allocation due to the individual power constraints, and $\mathrm{K} = \frac{\left|\boldsymbol{h}_1{}^H \boldsymbol{h}_2\right|}{\left[\left|\boldsymbol{h}_1\right|^2\left|\boldsymbol{h}_2\right|^2 - \left|\boldsymbol{h}_1{}^H \boldsymbol{h}_2\right|^2\right]}, \ \boldsymbol{h}_1 \ne a\boldsymbol{h}_2, M > 1$.

Proof: In the case of L=2 and M >1[1], the optimal values of the input covariance matrix entries, the correlation coefficient ρ_{12}, and the allocated input powers σ_1 and σ_2 can be determined by solving the following optimization problem:

$$\begin{aligned} &\max_{\sigma_1, \sigma_2, \rho_{12}} \ \det\left[I_2 + QH^H H\right] , \\ &s.t. \quad \sigma_1{}^2 + \sigma_2{}^2 = P \quad and \quad \left|\rho_{12}\right|^2 \le 1 \end{aligned} . \qquad (4)$$

It is straightforward, as shown in the Appendix I, to derive the followings

$$\rho_{12} = \frac{\boldsymbol{h}_1{}^H \boldsymbol{h}_2}{\sigma_1\sigma_2\left[\left|\boldsymbol{h}_1\right|^2\left|\boldsymbol{h}_2\right|^2 - \left|\boldsymbol{h}_1{}^H \boldsymbol{h}_2\right|^2\right] + \mu} \quad , \qquad (5\text{-a})$$

$$where \begin{cases} \mu = 0, \ \left|\rho_{12}\right|^2 < 1 \\ \mu > 0, \ \left|\rho_{12}\right|^2 = 1, \end{cases}$$

$$\rho_{21} = \rho_{12}{}^* \ , \qquad (5\text{-b})$$

$$\begin{aligned} &\sigma_1\left[\left|\boldsymbol{h}_1\right|^2 + \sigma_2{}^2\left|\boldsymbol{h}_1\right|^2\left|\boldsymbol{h}_2\right|^2 - \left|\boldsymbol{h}_1{}^H \boldsymbol{h}_2\right|^2\left[1 - \rho_{12}\rho_{21}\right] - \lambda\right], \\ &= -\sigma_2\Re e\left[\rho_{12}\boldsymbol{h}_2{}^H \boldsymbol{h}_1\right] \end{aligned} \qquad (5\text{-c})$$

$$\begin{aligned} &\sigma_2\left[\left|\boldsymbol{h}_2\right|^2 + \sigma_1{}^2\left|\boldsymbol{h}_1\right|^2\left|\boldsymbol{h}_2\right|^2 - \left|\boldsymbol{h}_1{}^H \boldsymbol{h}_2\right|^2\left[1 - \rho_{12}\rho_{21}\right] - \lambda\right]. \\ &= -\sigma_1\Re e\left[\rho_{12}\boldsymbol{h}_2{}^H \boldsymbol{h}_1\right] \end{aligned} \qquad (5\text{-d})$$

In (5-a) to (5-d), $\boldsymbol{h}_l$ denotes the lth column of the channel matrix and μ is the Lagrangian multiplier associated with the inequality constraint. In (5-c) and (5-d), λ is the Lagrangian multiplier associated with the input power constraint. The allocated powers σ_1 and σ_2, in (6-a) to (6-d) and subsequent analysis, are normalized by $\sigma_n{}^2 = 1$. The positive semi-definiteness condition is satisfied as far as $\left|\rho_{12}\right|^2 \le 1$ which is a consequence of the fact that the determinants of the principal minors of Q have to be non-negative as a sufficient condition for Q to be positive semi-definite. In (5-a), an analytical insight can be seen by considering the case when the magnitude of ρ_{12} is unity, this result in

$$\sigma_1\sigma_2\left[\left|\boldsymbol{h}_1\right|^2\left|\boldsymbol{h}_2\right|^2 - \left|\boldsymbol{h}_1{}^H \boldsymbol{h}_2\right|^2\right] + \mu = \left|\boldsymbol{h}_1{}^H \boldsymbol{h}_2\right| \ . \qquad (6)$$

But since μ has to be non-negative, then

$$\sigma_1\sigma_2 \le \frac{\left|\boldsymbol{h}_1{}^H \boldsymbol{h}_2\right|}{\left[\left|\boldsymbol{h}_1\right|^2\left|\boldsymbol{h}_2\right|^2 - \left|\boldsymbol{h}_1{}^H \boldsymbol{h}_2\right|^2\right]} \ , \boldsymbol{h}_1 \ne a\boldsymbol{h}_2, M > 1 \ . \qquad (7)$$

However, equal power allocation maximizes the product $\sigma_1\sigma_2$ and we may write

$$P \le \frac{2\left|\boldsymbol{h}_1{}^H \boldsymbol{h}_2\right|}{\left[\left|\boldsymbol{h}_1\right|^2\left|\boldsymbol{h}_2\right|^2 - \left|\boldsymbol{h}_1{}^H \boldsymbol{h}_2\right|^2\right]} \quad , \quad \boldsymbol{h}_1 \ne a\boldsymbol{h}_2, M > 1 \ . \qquad (8)$$

Hence the input power in (2) represents the input normalized SNR below which rank-1 signaling becomes strictly optimal since any other power allocation, in (7), will result in higher input normalized SNR condition for the optimality of beamforming.

In the SVD approach presented in [7], this corresponds to the case where the allocation of all the power to the largest channel matrix eigenvalue and the corresponding eigenvector, becomes optimal resulting in the so-called beamforming capacity [1]. However, the SVD approach does not provide such explicit expressions.

For MIMO channels with individual power constraints, the allocated powers do not relate to the channel matrix parameters and they either satisfy the condition in (7) or they do not. Assuming that the deviation from the equal power allocation is quantified by Δ as $\sigma_1 = \sqrt{\frac{P}{2} \pm \Delta}$ and $\sigma_2 = \sqrt{\frac{P}{2} \mp \Delta}$. Substituting in (7) will result in

[1] When M=1, the optimality of rank-1 signaling for a common power constraint, is straightforward [2].

$$\sqrt{\frac{P^2}{4}-\Delta^2} \le \frac{\left|\boldsymbol{h}_1{}^H \boldsymbol{h}_2\right|}{\left[\left|\boldsymbol{h}_1\right|^2\left|\boldsymbol{h}_2\right|^2-\left|\boldsymbol{h}_1{}^H \boldsymbol{h}_2\right|^2\right]}\ ,\quad .$$

$$\boldsymbol{h}_1 \ne a\boldsymbol{h}_2\,,\ M>1\,,\ 0\le\Delta<\frac{P}{2}$$

Eqn. (3) follows directly and the last part of the proposition (part (iii)) is simply the complement of part (ii). □

The results in (3-a) and (3-b) show that the range of beamforming optimality in distributed MIMO channels is affected not only by the correlations among the channel matrix columns (disparity of channel matrix eigenvalues [2]) but also by the allocated powers disparity resulting from the individual power constraints. MIMO channels with uniform individual power constraints tend to have the smallest range since equal power allocation is always adopted.

Remark: For L=3, the optimization problem can be expressed in a similar form to (4). However, the condition for Q to be semi-positive definite is more involved for L=3. Using Sylvester's criterion for positive semi-definiteness, the conditions for the first and second principal minors, are the same as for L=2, for the third principal minor, the condition can be expressed as

$$1+2\Re e\left[\rho_{12}\rho_{23}\rho_{31}\right]-\left|\rho_{13}\right|^2-\left|\rho_{12}\right|^2-\left|\rho_{23}\right|^2\ge 0\ . \tag{9}$$

The expression of the determinant for L=3 is too lengthy and is not included due to space limitation. Using a similar procedure to the one in Appendix I, the optimal correlation coefficients can be derived as

$$\rho_{12}= \frac{\left[\boldsymbol{h}_1{}^H\boldsymbol{h}_2+\sigma_1\sigma_3\rho_{13}B_1+\sigma_2\sigma_3\rho_{32}B_2+\sigma_3{}^2B_3+\sigma_1\sigma_2\sigma_3{}^2\rho_{13}\rho_{32}A_7\right]}{\sigma_1\sigma_2\left[\left|\boldsymbol{h}_1\right|^2\left|\boldsymbol{h}_2\right|^2-\left|\boldsymbol{h}_1{}^H\boldsymbol{h}_2\right|^2-\sigma_3{}^2A_5\right]} \tag{10}$$

where

$$B_1=\left[\boldsymbol{h}_3{}^H\boldsymbol{h}_1\boldsymbol{h}_1{}^H\boldsymbol{h}_2-\boldsymbol{h}_3{}^H\boldsymbol{h}_2\left|\boldsymbol{h}_1\right|^2\right],$$

$$B_2=\left[\boldsymbol{h}_1{}^H\boldsymbol{h}_2\boldsymbol{h}_2{}^H\boldsymbol{h}_3-\boldsymbol{h}_1{}^H\boldsymbol{h}_3\left|\boldsymbol{h}_2\right|^2\right],$$

$$B_3=\left[\boldsymbol{h}_1{}^H\boldsymbol{h}_2\left|\boldsymbol{h}_3\right|^2-\boldsymbol{h}_1{}^H\boldsymbol{h}_3\boldsymbol{h}_3{}^H\boldsymbol{h}_2\right].$$

Similarly,

$$\rho_{13}= \frac{\left[\boldsymbol{h}_1{}^H\boldsymbol{h}_3+\sigma_1\sigma_2\rho_{12}B_4+\sigma_2\sigma_3\rho_{23}B_5+\sigma_2{}^2B_6+\sigma_1\sigma_2{}^2\sigma_3\rho_{12}\rho_{23}A_6\right]}{\sigma_1\sigma_3\left[\left|\boldsymbol{h}_1\right|^2\left|\boldsymbol{h}_3\right|^2-\left|\boldsymbol{h}_1{}^H\boldsymbol{h}_3\right|^2-\sigma_2{}^2A_8\right]} \tag{11}$$

where

$$B_4=\left[\boldsymbol{h}_2{}^H\boldsymbol{h}_1\boldsymbol{h}_1{}^H\boldsymbol{h}_3-\boldsymbol{h}_2{}^H\boldsymbol{h}_3\left|\boldsymbol{h}_1\right|^2\right],$$

$$B_5=\left[\boldsymbol{h}_3{}^H\boldsymbol{h}_2\boldsymbol{h}_1{}^H\boldsymbol{h}_3-\boldsymbol{h}_1{}^H\boldsymbol{h}_2\left|\boldsymbol{h}_3\right|^2\right],$$

and

$$B_6=\left[\boldsymbol{h}_1{}^H\boldsymbol{h}_3\left|\boldsymbol{h}_2\right|^2-\boldsymbol{h}_2{}^H\boldsymbol{h}_3\boldsymbol{h}_1{}^H\boldsymbol{h}_2\right].$$

Finally,

$$\rho_{32}= \frac{\left[\boldsymbol{h}_3{}^H\boldsymbol{h}_2+\sigma_1\sigma_2\rho_{12}B_7+\sigma_1\sigma_3\rho_{31}B_8+\sigma_1{}^2B_9+\sigma_1{}^2\sigma_2\sigma_3\rho_{12}\rho_{31}A_6\right]}{\sigma_2\sigma_3\left[\left|\boldsymbol{h}_2\right|^2\left|\boldsymbol{h}_3\right|^2-\left|\boldsymbol{h}_2{}^H\boldsymbol{h}_3\right|^2-\sigma_1{}^2A_4\right]}\ , \tag{12}$$

where

$$B_7=\left[\boldsymbol{h}_2{}^H\boldsymbol{h}_1\boldsymbol{h}_3{}^H\boldsymbol{h}_2-\boldsymbol{h}_3{}^H\boldsymbol{h}_1\left|\boldsymbol{h}_2\right|^2\right],$$

$$B_8=\left[\boldsymbol{h}_3{}^H\boldsymbol{h}_2\boldsymbol{h}_1{}^H\boldsymbol{h}_3-\boldsymbol{h}_1{}^H\boldsymbol{h}_2\left|\boldsymbol{h}_3\right|^2\right],$$

and

$$B_9=\left[\boldsymbol{h}_3{}^H\boldsymbol{h}_2\left|\boldsymbol{h}_1\right|^2-\boldsymbol{h}_1{}^H\boldsymbol{h}_2\boldsymbol{h}_3{}^H\boldsymbol{h}_1\right].$$

Checking the positive definiteness condition, as given in (9), is difficult for MIMO channels with a common power constraint since both the optimal allocated powers and the correlation coefficients that satisfy that condition need to be computed. For MIMO channels with individual power constraints, the optimal correlation coefficients can be computed numerically by solving (10-12) since the allocated powers are known and the positive definiteness condition can be checked, and the rank of the input covariance matrix can be determined. However, the range of beamforming optimality is difficult to express analytically. So, for MIMO channels with individual power constraints, global solutions (for L>2) seem to be only numerical and the well-known SVD plus water-filling has to be for MIMO channels with a common power constraint.

The expressions in (10) to (12) show that uniform individual power allocation results in the smallest correlation coefficient between the corresponding input signals. Moreover, the uncorrelated transmission can be considered as a practical transmission scheme whenever the optimality condition for beam forming is not satisfied.

B. Random channel matrix

A well-known result for a random MIMO channel matrix is that the ergodic capacity with perfect channel state information at the transmitter (CSIT) and at the receiver (CSIR) is the average of capacities achieved for each "deterministic" channel realization [1]

$$C = E_{\boldsymbol{H}}\left[\sup_{Q;\, tr(Q)\le P} \log\det\left[I_L + \boldsymbol{Q}\boldsymbol{H}^H\boldsymbol{H}\right]\right], \qquad (13)$$

where E is the expectation operator. It is known that the capacity of a flat fading channel with perfect CSIT and CSIR is the average of the maximum mutual information for each channel realization [1, 2]; hence the results in (2) and (3) should apply for each fading state.

When perfect CSIR only is assumed, the ergodic capacity, is given as [7]

$$C = \sup_{Q;\, tr(Q)\le P} E_{\boldsymbol{H}}\left[\log\det\left[I_L + \boldsymbol{Q}\boldsymbol{H}^H\boldsymbol{H}\right]\right]\ . \qquad (14)$$

The optimal Q is dependent on the stationary distribution of the channel process and has been solved only for some special cases [2]. One way to approximate the capacity is to optimize for the Jensen's upper-bound on the mutual information obtained by using Jensen's inequality [9] as

$$\begin{aligned} &E_{\boldsymbol{H}}\left[\log\det\left[I_L + \boldsymbol{Q}\boldsymbol{H}^H\boldsymbol{H}\right]\right] \\ &\le \log\det\left[I_L + QE_{\boldsymbol{H}}\left[\boldsymbol{H}^H\boldsymbol{H}\right]\right] \end{aligned}. \qquad (15)$$

Then similar to the obtained result in (5-a), we may express the normalized input SNR for L=2 as

$$P \le \frac{2E\left[\left|\boldsymbol{h}_1^H\boldsymbol{h}_2\right|\right]}{\left[E\left|\boldsymbol{h}_1\right|^2 E\left|\boldsymbol{h}_2\right|^2 - E\left|\boldsymbol{h}_1^H\boldsymbol{h}_2\right|^2\right]}\ ,\ E\left|\boldsymbol{h}_1\right|^2 E\left|\boldsymbol{h}_2\right|^2 \ne E\left|\boldsymbol{h}_1^H\boldsymbol{h}_2\right|^2, \qquad (16)$$

and an analogous form of Proposition 1 will follow.

III. CONCLUSIONS

In this paper, the beamforming optimality range in Gaussian TIMO channels is considered. Carrying out the maximization of the mutual information analytically, without resorting to the eigen-decomposition approach, has led to an explicit expression for the range of input SNR for which rank-1 signaling is optimal in TIMO channels with both a common and individual power constraints. The results are extended to random channels using Jensen's upper bound on mutual information. The obtained results can be used to switch between the two common signaling schemes of beamforming and uncorrelated transmission.

Appendix I

To solve the optimization problem in (4), we use the fact that the $\log\det$ function can be maximized by maximizing its argument "the determinant term" [6, Chap 10]. So, expanding the expression in (4)

$$\det\left[I_2 + QH^H H\right] =$$
$$\det\begin{bmatrix} 1+\sigma_1^2\left|\boldsymbol{h}_1\right|^2 + \rho_{12}\sigma_1\sigma_2\boldsymbol{h}_2^H\boldsymbol{h}_1 & \sigma_1^2\boldsymbol{h}_1^H\boldsymbol{h}_2 + \rho_{12}\sigma_1\sigma_2\left|\boldsymbol{h}_2\right|^2 \\ \rho_{21}\sigma_1\sigma_2\left|\boldsymbol{h}_1\right|^2 + \sigma_2^2\boldsymbol{h}_2^H\boldsymbol{h}_1 & 1+\rho_{21}\sigma_1\sigma_2\boldsymbol{h}_1^H\boldsymbol{h}_2 + \sigma_2^2\left|\boldsymbol{h}_2\right|^2 \end{bmatrix} \qquad \text{(A-1)}$$

The expression in (A-1) can be further expanded and the Lagrangian equation, considering the fact that the logdet function is concave on the set of positive matrices Q and using [10, Theorem 18.6], can written as

$$\begin{aligned} &L(\sigma_1, \sigma_2, \rho_{12}, \rho_{21}, \lambda, \mu) \\ &= 1 + \sigma_1^2\left|\boldsymbol{h}_1\right|^2 + \sigma_2^2\left|\boldsymbol{h}_2\right|^2 + \sigma_1\sigma_2\left[\rho_{12}\boldsymbol{h}_2^H\boldsymbol{h}_1 + \rho_{21}\boldsymbol{h}_1^H\boldsymbol{h}_2\right] \\ &+ \sigma_1^2\sigma_2^2\left[\left|\boldsymbol{h}_1\right|^2\left|\boldsymbol{h}_2\right|^2 - \left|\boldsymbol{h}_1^H\boldsymbol{h}_2\right|^2\right]\left[1-\rho_{12}\rho_{21}\right] \\ &+ \lambda\left(\sigma_1^2 + \sigma_2^2 - P\right) + \mu\left(\left|\rho_{12}\right|^2 - 1\right), \\ &\textit{where } \mu = 0 \textit{ for } \left|\rho_{12}\right|^2 < 1 \textit{ and } \mu > 0 \textit{ for } \left|\rho_{12}\right|^2 = 1. \end{aligned}$$

Equating the derivatives with respect to the variables ρ_{12}, ρ_{21}, σ_1 and σ_2 to zeros and solving the corresponding equations will result in the expressions in (5-a) to (5-d).

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