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Modelling of earthquake rupturing as a stochastic process and estimation of its distribution function from earthquake observations

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SUMMARY

The effect on earthquake rupturing of heterogeneities in tectonic stress and in material strength along a large fault zone is incorporated in the potential dynamic stress drop, defined as the difference between the tectonic shear stress and the dynamic frictional strength according to a slip-weakening model. The distribution of the potential dynamic stress drop $\Delta\tau_d(x)$ along the strike of the fault plane is modelled as a 1-D stochastic process. Using a simple dynamic fracture criterion, a relation is established between earthquake rupturing and potential dynamic stress drop, by which any earthquake rupture process can be regarded as a segment of a realization of the process $\Delta\tau_d(x)$ where $\Delta\tau_d(x) > 0$. Since dynamic slip varies approximately linearly with dynamic stress drop, it has the same distribution function as $\Delta\tau_d(x)$, provided that $\Delta\tau_d(x)$ is a Gaussian process. Three independent earthquake observations, i.e. the average stress drop, the Gutenberg–Richter relation and the surface slip along earthquake faults, are used to estimate the distribution function of $\Delta\tau_d(x)$. An analytical solution is derived for the distribution function of $\Delta\tau_d(x)$, which shows that, among all known distribution models, only the fractional Brownian motion with index $H \to 0$ (fractal dimension $D = 2$ in the 1-D case) can give rise to the observed approximately constant stress drop independent of earthquake size. The probability distribution of the size of zerosets of the fractional Brownian motion shows a power-law relation with frequency, which resembles the frequency–seismic-moment relation. Using an average b value of 1.0 for small earthquakes, an index $H \to 0$ of the fractional Brownian motion is obtained. The model predicts that the b value for large earthquakes is smaller than that for small earthquakes along the same fault zone, which is in agreement with observations. The surface slip data of two strike-slip-dominated earthquake faults with rupture lengths larger than 100 km are inverted using power spectral analysis. Both data sets display a power-law relation between the sample power spectrum and the spatial frequency, which implies a fractional Brownian distribution. The estimated index H is close to zero for both earthquake faults. Stress drops, b values, and surface slips all independently suggest that the earthquake rupturing process can be modelled stochastically as a fractional Brownian motion with index $H \to 0$.

Key words: *b* values, earthquake rupture, stochastic process, stress drops, surface slips.

INTRODUCTION

Earthquake sequences are complex dynamic processes associated with brittle faulting. The spatial and temporal distribution of earthquakes has a significant random component. This is reflected in the fact that many physical properties of earthquakes have been formulated through statistical approaches, such as the frequency–magnitude relation, the average stress drop–earthquake size relation and the recurrence times of large events (Wyss 1973; Kanamori & Anderson 1975; Caputo 1977; Hanks 1977; Wesnousky, Scholz & Shimazaki 1983; Singh, Rodriguez & Esteva 1983; Hanks & Boore 1984). It has also been recognized that the heterogeneities in tectonic stress and in rock strength along active fault zones may play an important role in earthquake mechanics (Das & Aki 1977; Aki 1979; Lay & Kanamori 1981; Lay, Kanamori & Ruff 1982; Aki 1984; Lomnitz-Adler & Lemus-Diaz 1989; Aki 1992; Dmowska &

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Lovison 1992; Ruff 1992). Consequently, understanding the distribution characteristics of stress–strength heterogeneities may clarify the earthquake mechanism.

Two empirical statistical relations seem to hold universally. One is the Gutenberg–Richter (1954) frequency–magnitude relation. The other is the constancy of average stress drop (e.g. Aki 1972; Thatcher & Hanks 1973; Kanamori & Anderson 1975; Hanks 1977). As the logarithm of seismic moment has statistically a linear relation with the magnitude, assuming a constant stress drop, earthquake frequency can be related to seismic moment by a power law (Brune & King 1967; Wyss & Brune 1968; Kanamori & Anderson 1975). This power-law relation has led to the conclusion that the distribution of earthquakes is fractal (Hanks 1979; von Seggern 1980; Andrews 1980; Aki 1981; Huang & Turcotte 1988). Using different b values, different fractal dimensions have been obtained; they nearly cover the permissible range from 2 to 3 in the 2-D case (Hanks 1979; von Seggern 1980; Andrews 1980; Huang & Turcotte 1988). Thus, the variation in b value is thought to be a consequence of the spatial and temporal variation of the fractal dimension D (von Seggern 1980; Huang & Turcotte 1988). Investigations have also been carried out into the spectral properties of stress–strength heterogeneities based on the constant average stress drop (Hanks 1979; Andrews 1980). In spite of the same assumptions that (1) average stress drop is constant and (2) stress–strength heterogeneities follow a fractal distribution [specifically, a power-law dependence of the spectrum of potential stress drop on wavenumber by Hanks (1979), and a power-law dependence of the frequency of ruptures on size by Andrews (1980)], Hanks (1979) infers the fractal dimension $D=2$, while Andrews (1980) obtains $D=3$ (for the 2-D case). Theoretically, the physical properties of the fractal distribution of stress–strength heterogeneities are quite different for different Ds. It appears that the physical significance of the b value and the constancy of average stress drop are still imperfectly understood.

The measured surface slips of large earthquake ruptures show heterogeneous features (Sharp *et al.* 1982; Crone & Machette 1984; Deng *et al.* 1986; Zhang *et al.* 1987; Nakata *et al.* 1990; Yoshida & Abe 1992). The variation of slip along the strike of an earthquake fault is very irregular. This irregularity cannot be interpreted solely in terms of variation of surface lithology and fault geometry, but it must reflect complex physical processes along the fault. Since slip is directly related to stress drop, measured surface slips are useful to infer the distribution of stress–strength heterogeneities.

In this paper, the effects of heterogeneity in stress and in strength on earthquake rupturing are incorporated in the potential dynamic stress drop, defined as the difference between the tectonic shear stress and the dynamic frictional strength according to a slip-weakening model (Scholz & Aviles 1986). The potential dynamic stress drop allows both stress and frictional strength to vary along the fault, i.e. it poses no restrictions on the residual frictional stress. The potential dynamic stress drop is modelled as a stochastic process. Our analysis differs from previous ones mainly in three aspects:

(1) the basis of our model is the potential dynamic stress drop, which has a clear physical meaning;

(2) the analysis proceeds in the stochastic domain and follows the fundamentals of stochastic processes;

(3) no *a priori* assumption is made on the distribution of stress–strength heterogeneities, whereas a fractal (fractional Brownian motion) distribution of the heterogeneities is assumed in almost all previous models.

Three independent earthquake observations, i.e. the constancy of average stress drops, the frequency–magnitude relation, and the surface slips of earthquake faults, are used to explore the characteristics of the potential dynamic stress drop distribution.

EARTHQUAKE RUPTURING AS A STOCHASTIC PROCESS

Assume a planar fault with area A_{max} that can generate earthquakes of all sizes; the maximum magnitude corresponds to rupture of the entire area. All earthquakes nucleate at some point on the fault plane, and the earthquake size (magnitude or seismic moment) depends on the ultimate rupture area (Hanks 1979; Andrews 1980; von Seggern 1980; Huang & Turcotte 1988). When an earthquake nucleates, the stress at the nucleation point must satisfy the Coulomb–Navier static fracture criterion:

$$\tau_s(x_0, y_0) - \tau_f(x_0, y_0) \geq 0, \tag{1}$$

where $\tau_s(x_0, y_0)$, $\tau_f(x_0, y_0)$ are static shear stress and static frictional strength respectively, and (x_0, y_0) are the coordinates of the nucleation point in a Cartesian system on the fault plane with the x-axis and y-axis parallel to the strike and the dip direction respectively. Once the rupture has nucleated, dynamic fracture criteria govern the propagation of rupture. A simple dynamic criterion can be expressed as

$$a\tau_s(x, y) - \tau_f(x, y) \geq 0 \tag{2}$$

(Richards 1976; von Seggern 1980), where $a \geq 1$ is a factor that brings the static stress up to a dynamic stress due to the stress concentration at the crack tips. The rupture will propagate if the dynamic criterion is satisfied in the area around the nucleation point. The magnitude of stress concentration at the crack tips is approximately equal to the dynamic stress drop of the adjacent ruptured area (Nur 1978). Therefore, inequality (2) can be written also as

$$\tau_s(x, y) + \Delta\tau_d(x-\Delta x, y-\Delta y) - \tau_f(x, y) \geq 0, \tag{3}$$

where $\Delta\tau_d(x-\Delta x, y-\Delta y)$ is the dynamic stress drop of the adjacent ruptured area at position $(x-\Delta x, y-\Delta y)$. Here, we adopt a slip-weakening model as proposed by Scholz & Aviles (1986) in which $\Delta\tau_d(x-\Delta x, y-\Delta y)$ is related to the static shear stress τ_s and the dynamic frictional strength τ_d as

$$\Delta\tau_d(x-\Delta x, y-\Delta y) = \tau_s(x-\Delta x, y-\Delta y) - \tau_d(x-\Delta x, y-\Delta y). \tag{4}$$

We term $\Delta\tau_d(x, y)$ the potential dynamic stress drop, since before any earthquake occurs $\Delta\tau_d(x, y)$ exists potentially on the fault surface. $\Delta\tau_d(x, y)$ incorporates the effects of heterogeneity, both in stress and in strength, on rupture propagation.

If either $\tau_s(x, y)$ or $\tau_d(x, y)$ (or both) is a stochastic process, $\Delta\tau_d(x, y)$ will be a 2-D stochastic process. A detailed discussion of the physical genesis of random distributions of $\tau_s(x, y)$ and $\tau_d(x, y)$ is beyond the scope of this paper. It is apparent, however, that $\tau_s(x, y)$ and $\tau_d(x, y)$ are not only functions of position, but also of time, since earthquake sequences involve the time-scale. Several arguments favour the stochastic nature of $\tau_s(x, y)$ and $\tau_d(x, y)$:

(1) spatial and temporal variations in principal stress directions and magnitudes may result in a random distribution in both shear stress and strength on the fault plane (Lana & Correig 1987);

(2) the frictional coefficient shows a time-dependent behaviour (Dieterich 1972, 1978; Dieterich & Conrad 1984);

(3) the occurrence of small earthquakes causes random redistributions of the shear stress along the fault plane (Andrews 1980);

(4) irregular variations of pore pressure and stress corrosion give rise to randomness in the time dependence of the strength (e.g. Scholz 1990, pp. 29–35).

In this paper, we consider only the variation of $\Delta\tau_d(x, y)$ along the strike direction and model $\Delta\tau_d(x)$ by a 1-D stochastic process, as a profile of the 2-D process $\Delta\tau_d(x, y)$. When an earthquake nucleates at a point, the inequality $\tau_s(x, y) - \tau_f(x, y) < 0$ must hold everywhere except at the nucleation point. The rupture terminates at points where the potential stress drop $\Delta\tau_d(x) \le 0$, according to inequality (3). However, $\Delta\tau_d(x) \le 0$ is a sufficient but not necessary condition for the termination of the rupture. The rupture can also stop at points where $\Delta\tau_d(x)$ is small and the state of stress does not satisfy the dynamic criterion (inequality 3). Consequently, under this simple dynamic criterion, earthquake rupturing is connected with the potential stress drop. Any earthquake faulting process can be regarded as a segment of a realization of the stochastic process $\Delta\tau_d(x)$, that is, a segment where $\Delta\tau_d(x) > 0$. Different distribution functions (or spectra) of $\Delta\tau_d(x)$ will give rise to different patterns of earthquake distributions.

Many physical processes in nature can be modelled by Gaussian (normal) processes (Priestley 1981). If both $\tau_s(x)$ and $\tau_d(x)$ are Gaussian processes with a constant mean, $\Delta\tau_d(x)$ must also be a Gaussian process with a constant mean $E[\Delta\tau_d(x)] = E[\tau_s(x)] - E[\tau_d(x)]$ because $\tau_s(x)$ and $\tau_d(x)$ are independent. Earthquake observation suggests that during a seismic cycle the regional tectonic stress increases with time, and rock strength decreases with time, due to stress corrosion and wearing-out of asperities. Both effects will cause an increase of the mean of $\Delta\tau_d(x)$ with time. Because a larger segment of $\Delta\tau_d(x)$ will be shifted above zero due to the increase of the mean, the probability of large earthquakes along a given fault increases with time. Seismic cycles have been interpreted in terms of variation of the spectrum of stress and strength from roughness to smoothness (von Seggern 1980; Huang & Turcotte 1988). They could also result from a stationary process (usually with rough paths) coupled with a time-varying (increasing) mean of the potential dynamic stress drop. Throughout this paper, we focus our attention on the spatial variation of the potential dynamic stress drop. The effect of the time factor on the occurrence of earthquakes is discussed only briefly. Further investigation into the time variation of the potential dynamic stress drop and its effect on earthquake occurrence is currently in progress.

We use three independent earthquake observations, i.e. the constancy of average stress drops, the Gutenberg–Richter relation, and the surface slips of earthquake faults, to infer the distribution function of $\Delta\tau_d(x)$.

AVERAGE STRESS DROP

One of the most important features of the earthquake process is that average stress drops are statistically constant. The average stress drops on seismic faults are almost always found to lie within the range of 1–10 MPa, with a mean value of about 5 MPa, independent of source dimension and of geographic location over twelve orders of magnitude in seismic moment (e.g. Aki 1972; Thatcher & Hanks 1973; Kanamori & Anderson 1975; Hanks 1977). Some investigations have been carried out into the spectral properties of stress–strength heterogeneities based on this observation [see the discussion of the work by Hanks (1979) and Andrews (1980) above].

An earthquake rupture process can be regarded as a segment of a realization of the stochastic process $\Delta\tau_d(x, y)$, where $\Delta\tau_d(x, y) > 0$. Thus the average stress drop is the integral of $\Delta\tau_d(x, y)$ over the rupture area, divided by the area. In the 1-D case, the average stress drop can be expressed accordingly as

$$\overline{\Delta\tau_d}(L) = \frac{1}{L}\int_L \Delta\tau_d(x)\,dx, \qquad \Delta\tau_d(x) \ge 0. \tag{5}$$

For the average stress drop to be constant, the mean value of $\overline{\Delta\tau_d}(L)$ must be independent of L, where L is the rupture length. Here, we examine how constancy of average stress drop may arise from a stochastic process.

First we consider a special case, i.e. assume a fractional Brownian distribution for $\Delta\tau_d(x)$. Fractional Brownian motion with index $H(0 < H < 1)$ is defined as follows (Mandelbrot 1983, pp. 350–352; Feder 1988, p. 170; Falconer 1990, p. 246; Turcotte 1992, p. 75): (1) $Z(x)$ is continuous and $Z(0) = 0$ with probability $P = 1$; (2) for any $x \ge 0$ and $s > 0$, the increment $Z(x + s) - Z(x)$ follows the normal distribution with zero mean and variance $s_0 s^{2H}$, that is

$$P\{Z(x+s) - Z(x) \le z\} = \frac{1}{\sqrt{2\pi s_0 s^H}}\int_{-\infty}^{z} \exp\left(-\frac{u^2}{2 s_0 s^{2H}}\right) du, \tag{6}$$

where s_0 is a factor identical to the variance of the increment $Z(x + s) - Z(x)$ when $s = 1$. This definition can be extended to processes with constant mean (m) by changing the definition (1) from $Z(0) = 0$ to $Z(0) = m$. The fractional Brownian motion comprises a family of random functions described by the index $H(0 < H < 1)$. When $H = 1/2$, it reduces to classical Brownian motion. Fig. 1 shows examples of fractional Brownian motion with zero mean and different indexes H, simulated using the method proposed by Feder (1988, pp. 172–174). The spectral properties of the processes are quite different for different Hs. The smaller H is, the more conspicuous the high-frequency component of the process, and the process tends to be stationary for $H \to 0$. The path (graph) of a fractional Brownian motion has an approximate fractal (Hausdorff) dimension of $D \approx 2 - H$ for the 1-D case and $D \approx 3 - H$ for the 2-D case (Falconer 1990, pp. 246–247).

The paths (graphs) of the fractional Brownian motion are self-affine fractals, i.e. they look the same upon changing scales $X \to pX$ and $\Delta\tau_d(x) \to p^H \Delta\tau_d(x)$, where p is a constant and H is the index (Wong & Lin 1988). However, this scale-invariant property of fractional Brownian motion is only valid for the statistical average of ensembles. For any given finite realization, the two records can be quite different and the vertical scale will not be changed by the expected factor p^H (Feder 1988, p. 167). This means that the spatial position of $\Delta\tau_d(px)$ is not equal to that of $p^H \Delta\tau_d(x)$ for any finite records, but their mean values are identical. Using this scale invariance and recalling eq. (5), the mean value of average stress drop over a rupture

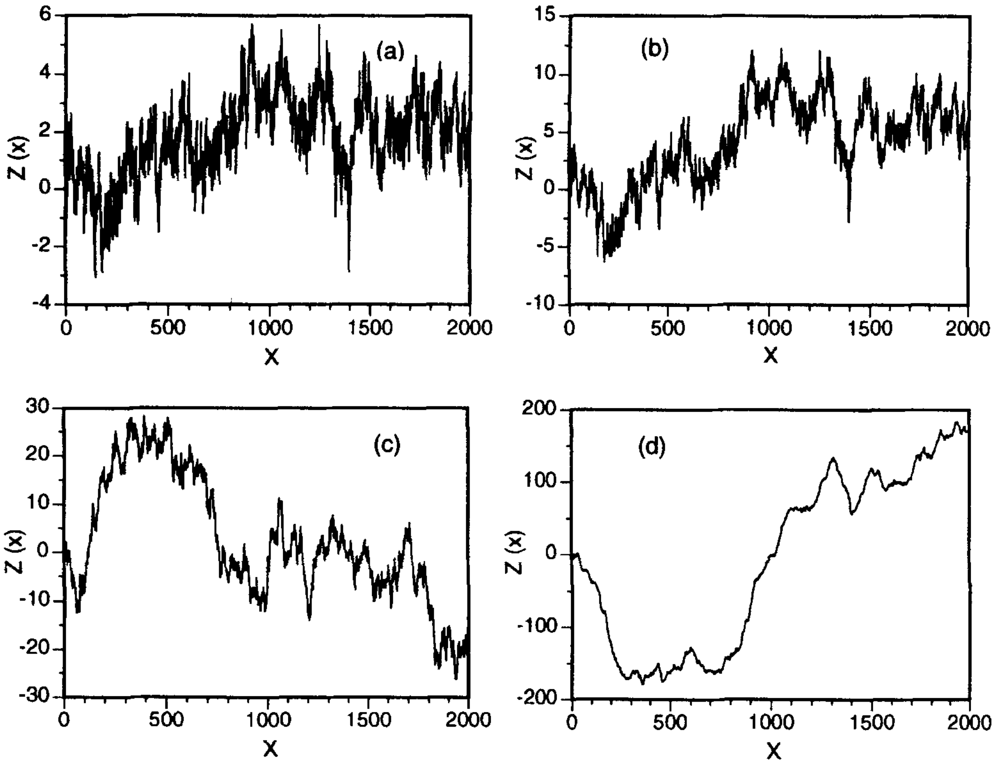

Figure 1. Simulated 1-D fractional Brownian processes for different indexes H, with mean zero and variance equal to x^{2H}. (a) $H=0.01$; (b) $H=0.2$; (c) $H=0.5$ and (d) $H=0.8$.

length pL is

$$E[\overline{\Delta\tau_d}(pL)] = \frac{1}{pL}\int_{pL} E[\Delta\tau_d(px)]\,d(px) = p^H E[\overline{\Delta\tau_d}(L)], \qquad \Delta\tau_d(px)\geq 0\,. \qquad (7)$$

Eq. (7) shows that if $\Delta\tau_d(x)$ follows a fractional Brownian distribution, ruptures with different lengths (or earthquakes with different magnitudes) can have a constant average stress drop only if $H\to 0$; otherwise, the mean value of average stress drops would increase systematically with rupture size. Furthermore, eq. (7) also agrees with the observation that average earthquake stress drops, although having a constant mean value, vary within a finite dispersion (Hanks 1977).

Now we take the general case, i.e. without any assumption about the probability distribution of $\Delta\tau_d(x)$. As $\Delta\tau_d(x)$ is a continuous stochastic process, its sample mean is

$$\hat{m}_\tau = \frac{1}{x_1-x_0}\int_{x_0}^{x_1}\Delta\tau_d(x)\,dx\,. \qquad (8)$$

Note that $\hat{m}_\tau$ is different from $\overline{\Delta\tau_d}(L)$ given in eq. (5). The former is the sample mean of a segment of a realization of the process $\Delta\tau_d(x)$, which can take either a positive or a negative value. The latter is the sample mean of a portion of a realization where $\Delta\tau_d(x)\geq 0$. Since $\Delta\tau_d(x)$ is a random variable for any fixed x, it follows that $\hat{m}_\tau$ is also a random variable. Its mean can be derived from eq. (8) as

$$E[\hat{m}_\tau] = \frac{1}{x_1-x_0}\int_{x_0}^{x_1}E[\Delta\tau_d(x)]\,dx\,, \qquad (9)$$

where $E[\Delta\tau_d(x)]$ is the mean of $\Delta\tau_d(x)$ at a point x, and its variance is

$$V(\hat{m}_\tau) = E[(\hat{m}_\tau - E[\hat{m}_\tau])^2] = E\left[\left(\frac{1}{x_1-x_0}\int_{x_0}^{x_1}\{\Delta\tau_d(x)-E[\Delta\tau_d(x)]\}\,dx\right)^2\right]. \qquad (10)$$

Consequently, any $\overline{\Delta\tau_d}(L)$ (the average earthquake stress drop) can be considered as a sample of all positive values taken by the random variable $\hat{m}_\tau$.

Taking a Gaussian process as an example, $\hat{m}_\tau$ follows the normal distribution with mean and variance given by eqs (9) and (10) respectively (Priestley 1981, p. 91). Because $\overline{\Delta\tau_d}(L)$ takes only positive values, its mean value is always larger than the mean of $\hat{m}_\tau$, and it also increases as the mean of $\hat{m}_\tau$ increases. Therefore, for the average stress drop to have a constant mean, the mean of $\hat{m}_\tau$ must be constant. Eq. (9) shows that this condition is satisfied only if $\Delta\tau_d(x)$ has a constant mean $E[\Delta\tau_d(x)] = m_\tau$, independent of position. Eq. (9) is thus reduced to

$$E[\hat{m}_\tau] = m_\tau\,, \qquad (11)$$

where m_τ is the mean of the process $\Delta\tau_d(x)$. However, the constant mean is a necessary but not a sufficient condition for $\Delta\tau_d(x)$ to give rise to the constancy of average stress drops. This point can be seen clearly by considering the variance of $\hat{m}_\tau$.

Fig. 2 shows two distributions of $\hat{m}_\tau$ with an identical mean and different variances. The mean value of the average stress drop increases with the variance of the sample mean. Consequently, for the average stress drop to maintain a constant mean value, $\hat{m}_\tau$ must have not only a constant mean, but also a constant variance. Substituting eq. (11) into

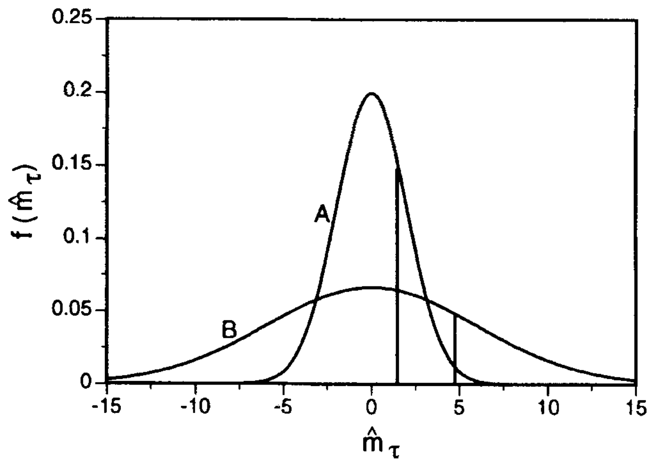

Figure 2. Schematic diagram of two normal distributions of $\hat{m}_\tau$, with mean zero and variance (A) $V=4$ and (B) $V=36$. The two vertical lines show the positions of the mean of $\overline{\Delta\tau_d}(L)$ for the two distributions.

eq. (10) yields

$$V(\hat{m}_\tau) = E[(\hat{m}_\tau - m_\tau)^2]$$

$$= E\left[\frac{1}{(x_1-x_0)^2}\int_{x_0}^{x_1}\{\Delta\tau_d(x)-m_\tau\}\,dx\int_{x_0}^{x_1}\{\Delta\tau_d(t)-m_\tau\}\,dt\right]. \quad (12)$$

Changing the integral variables from x, t to x, $s=t-x$, we thus obtain

$$V(\hat{m}_\tau) = \frac{1}{(x_1-x_0)^2}\int_{-(x_1-x_0)}^{x_1-x_0} ds \int_{x_0}^{x_1-s} \mathrm{cov}[\Delta\tau_d(x), \Delta\tau_d(x+s)]\,dx, \qquad s\geq 0$$

$$V(\hat{m}_\tau) = \frac{1}{(x_1-x_0)^2}\int_{-(x_1-x_0)}^{x_1-x_0} ds \int_{x_0+|s|}^{x_1} \mathrm{cov}[\Delta\tau_d(x), \Delta\tau_d(x+s)]\,dx, \qquad s< 0 \quad (13)$$

where $\mathrm{cov}[\Delta\tau_d(x), \Delta\tau_d(x+s)]$ is the autocovariance function of $\Delta\tau_d(x)$. Eq. (13) shows that a necessary condition for $V(\hat{m}_\tau)$ to be constant is that the autocovariance function be independent of position. This result demonstrates that $\Delta\tau_d(x)$ must be a stationary process. If the fractional Brownian motion is used to model the process $\Delta\tau_d(x)$, only one member in this family can become a candidate, that is, the one with index $H\to 0$, because the fractional Brownian motions with other indexes are non-stationary. This conclusion is consistent with that based on scale invariance.

Stationarity is not a sufficient condition for $\Delta\tau_d(x)$ to produce a constant earthquake stress drop. By taking the autocovariance function, which is now a function of s only, out of the second integral, eq. (13) reduces to

$$V(\hat{m}_\tau) = \frac{1}{x_1-x_0}\int_{-(x_1-x_0)}^{x_1-x_0} \mathrm{cov}[\Delta\tau_d(x), \Delta\tau_d(x+s)]\left(1-\frac{|s|}{x_1-x_0}\right)ds. \quad (14)$$

For most stationary processes, such as autoregressive, moving average, and general linear processes, since $\mathrm{cov}[\Delta\tau_d(x), \Delta\tau_d(x+s)]$ is an even and decreasing function of $|s|$, $V(\hat{m}_\tau)$ is also a decreasing function of the sample size, and usually $V(\hat{m}_\tau)\to 0$ as the sample size (x_1-x_0) goes to infinity (Priestley 1981, p. 320). Because the mean value of $\overline{\Delta\tau_d}(L)$ increases slowly with increasing $V(\hat{m}_\tau)$ (see Fig. 2), small earthquakes should have a mean value of average stress drop slightly larger than that of large earthquakes. Besides, the dispersion of average stress drops should decrease as earthquake size increases. Statistical results on earthquake stress drop show that neither of these predictions is fulfilled (Hanks 1977). This excludes those stationary processes whose autocovariance function is a decreasing function of $|s|$. However, a stationary process exists that can lead both to a constant mean and a constant variance of $\hat{m}_\tau$, and consequently give rise to a constant mean value of earthquake stress drops, independent of earthquake size. This is the fractional Brownian motion with index $H\to 0$. Recalling the definition of fractional Brownian motion, the autocovariance function is

$$\mathrm{cov}[\Delta\tau_d(x), \Delta\tau_d(x+s)] = \frac{s_0}{2}[x^{2H}+(x+s)^{2H}-s^{2H}], \qquad s\geq 0$$

$$\mathrm{cov}[\Delta\tau_d(x), \Delta\tau_d(x+s)] = \frac{s_0}{2}[(x-|s|)^{2H}+x^{2H}-|s|^{2H}], \qquad s<0. \quad (15)$$

When $H\to 0$, the autocovariance function is approximately independent of both x and s, and s_0 becomes the variance of the process $\Delta\tau_d(x)$. Thus, $V(\hat{m}_\tau)$ can be obtained from eqs (14) and (15) as

$$V(\hat{m}_\tau) \simeq \frac{s_0}{2(x_1-x_0)}\int_{-(x_1-x_0)}^{x_1-x_0}\left(1-\frac{|s|}{x_1-x_0}\right)ds \simeq \frac{s_0}{2}. \quad (16)$$

The variance of $\hat{m}_\tau$ (sample mean), which is approximately equal to half the variance of the process, is constant and independent of sample size. Consequently, the mean value of $\overline{\Delta\tau_d}(L)$ (average earthquake stress drop) is also constant and independent of rupture length.

In summary, the observed constancy of average stress drop requires that $\Delta\tau_d(x)$ be a stationary process, which is implied by the fact that the statistical properties of the earthquake process do not change over position. Physically, this can be explained by postulating that the overall levels of both the static shear stress and the dynamic frictional strength remain constant on average and do not fluctuate too far from their mean values along the fault. In other words, the fault (and by extension the brittle crust) is in a metastable state, everywhere near failure (see also Sornette, Davy & Sornette 1990a). Consequently, the mean and variance of the dynamic stress drop can be expected to be constant and not to increase or decrease systematically with position. However, the constancy of average stress drop implies rather more than this. Many stationary processes whose autocovariance function is characterized by a decreasing dependence on $|s|$ cannot give rise to a constant mean value of the average stress drop independent of rupture size. Of the known models, only the fractional Brownian motion with $H\to 0$ can reproduce the observations. $H\to 0$ corresponds to $D\approx 3$ and shows a very rough profile of $\Delta\tau_d(x)$ (see Fig. 1a), which contrasts with the very low value of D measured for the roughness of the fault surface (Aviles & Scholz 1987). This implies that the distribution of $\Delta\tau_d(x)$ is affected not only by fault geometry, but also by many other factors, such as occurrences of small earthquakes, local fluctuations in lithology and fluid pressure. Main (1988) presented a model of recursive fault bends with low topological dimensions which nevertheless gave rise to a very rough stress profile.

The fractional Brownian motion with $H \to 0$, which is often termed 'flicker noise', is an especially important stationary process. It has found many applications in geophysical modelling (see Jensen *et al.* 1991 for general discussion). The flicker noise differs from other stationary processes mainly in its correlation function. It has a long-run correlation between positions of $\Delta\tau_d(x)$ in the 'past' and in the 'future' along the x-axis, whereas other stationary processes are either 'memoryless', such as the white noise, or of 'poor memory'. Although the increments of flicker noise are antipersistent (i.e. have a negative correlation), the process itself has a positive (persistent) correlation. One can derive from eq. (15) that the autocorrelation function for the flicker noise ($H \to 0$) is approximately constant and equal to 1/2. A detailed discussion of the physical genesis of the long-distance positive correlation of $\Delta\tau_d(x)$ is beyond the scope of this paper. However, it is not difficult to find seismological and geological phenomena conforming to this property. For instance, the tectonic stress is persistently low along a fault segment where a recent large earthquake occurred and is continuously high along another segment where there has been no large earthquake for a long time. The large-scale variation in lithology may give rise to long-distance positive correlation in fault frictional strength.

With respect to the effect of the time factor on the occurrence of earthquakes, the mean of the process $\Delta\tau_d(x)$ is likely to increase during an earthquake cycle. When the slip-rate is high along a fault zone, the mean of $\Delta\tau_d(x)$ increases more quickly with time, and the occurrence of large earthquakes is more frequent along the fault zone, i.e. the recurrence times of large earthquakes are short. The recurrence times of large earthquakes depend on both the time-rate of the mean and the distribution of $\Delta\tau_d(x)$. The former, which depends on the slip-rate of plate motion, is likely to be deterministic, and the latter is stochastic. Therefore, the recurrence times of large earthquakes have a random component. We can determine only the average recurrence time of large earthquakes, which is related to the slip-rate of plate motion.

THE GUTENBERG–RICHTER RELATION

Besides the average stress drop, another important statistical property of earthquakes is the Gutenberg–Richter frequency–magnitude relation. Statistics of seismicity show that the exponential relation between the cumulative number of earthquakes and the magnitude holds universally over a broad range of magnitudes, although the parameters may vary from one area to another (e.g. von Seggern 1980; Main & Burton 1984; Rydelek & Sacks 1989; Main 1992; Pacheco, Scholz & Sykes 1992). Because the logarithm of seismic moment has a statistically linear relation with magnitude, a constant stress drop results in a power-law relation between the cumulative number of earthquakes and the seismic moment (Brune & King 1967; Wyss & Brune 1968; Kanamori & Anderson 1975):

$$N(M_0) = c_1 M_0^{-(b/c)}, \tag{17}$$

where c_1 is a constant, and b and c are the slopes of the logarithmic magnitude–frequency relation and of the logarithmic seismic moment–magnitude relation respectively. This power-law scaling relation has led to the proposal that earthquakes are manifestations of a fractal physical process (Hanks 1979; von Seggern 1980; Andrews 1980; Aki 1981; Huang & Turcotte 1988).

Investigations into the statistical behaviour of seismicity have followed two different directions. In the first approach, the Gutenberg–Richter relation is interpreted in terms of the random spatial (fractal) distribution of active fault sizes (Turcotte 1986; Hirata 1989; Sornette, Davy & Sornette 1990b; Sornette & Davy 1991). In the second approach, the Gutenberg–Richter relation is ascribed to the random distribution of inhomogeneous stress and strength along a single fault (Hanks 1979; von Seggern 1980; Andrews 1980; Huang & Turcotte 1988). More recently, a few models have been proposed that combine these two interpretations (Sornette, Vanneste & Sornette 1991; Lomnitz-Adler 1992). Here, our analysis is restricted to the physical genesis of seismicity along a single fault.

Although the frequency–magnitude relation is usually derived from statistics of regional or global seismicity, studies of seismicity along a single fault zone suggest that it is also applicable in this case (Wesnousky *et al.* 1983; Singh *et al.* 1983; Schwartz & Coppersmith 1984; Davison & Scholz 1985). For small earthquakes with rupture radius $r \le h_0/2 \sin\theta$, where h_0 is the thickness of the seismogenic layer and θ is the dip of the fault plane, with an average c-value equal to 1.5, eq. (17) can be expressed in terms of the rupture area:

$$N(A) = c_2 A^{-b}, \tag{18}$$

where c_2 is a constant (von Seggern 1980; Scholz 1982; Huang & Turcotte 1988). Assuming $b \approx 1.0$, von Seggern (1980) obtained $D = 3$ (fractal dimension for the graph of the 2-D fractional Brownian motion). Huang & Turcotte (1988) obtained $b \approx 0.77$–1.11 by numerical simulation of the 2-D fractional Brownian motion with fractal dimension $D \approx 2.2$–2.4. In both cases, the spatial and temporal variations in the b value were considered as a reflection of the variation in the fractal dimension.

In our model, the propagation of earthquake rupture in principle stops at positions where $\Delta\tau_d(x, y) = 0$. Therefore, the study of the earthquake size distribution is approximately equivalent to the study of the size distribution of zerosets of the stochastic process $\Delta\tau_d(x, y)$. The zerosets are defined as sets of intersections between $\Delta\tau_d(x, y)$ and the plane $\Delta\tau_d(x, y) = 0$, and their size is the area within the intersection line. For 2-D fractional Brownian motion, the number of zerosets with size larger than A is given as (Adler 1981, p. 215; Mandelbrot 1983, p. 260 and p. 272)

$$N(A) = c_3 A^{-(2-H)/2}, \tag{19}$$

where c_3 is a constant and H is the index. $D_z = 2 - H$ is often called the fractal (or Hausdorff) dimension of the zerosets; the relation between the fractal dimension of the zerosets and the fractal dimension of the path of fractional Brownian motion is $D_z = D - 1$. Eq. (19) is derived under the condition that the process has a mean of zero, whereas the mean of $\Delta\tau_d(x, y)$ is likely to increase during a seismic cycle and may not be zero. It has been proven, however, that eq. (19) is still valid for any level sets defined as sets of intersections between $\Delta\tau_d(x, y)$ and the plane $\Delta\tau_d(x, y) = B$, where B is any given value, as long as the level sets are not empty (Adler 1981, p. 251). This implies that eq. (19) can be applied to $\Delta\tau_d(x, y)$ with non-zero mean as well. An exactly solvable model for eq. (19), applicable to the magnitude–frequency distribution, has been provided by Hemmer & Hansen (1992) and Sornette (1992).

Eq. (19) shows that the fractional Brownian motion gives rise to a power-law relation between the cumulative number and the size of zerosets similar to the frequency–rupture area relation given in eq. (18). Comparison of the two equations yields

$$H = 2 - 2b. \tag{20}$$

Based on eq. (20), the spatial and temporal variation in b value can be interpreted in terms of the variation of fractal dimension D of $\Delta\tau_d(x, y)$ (von Seggern 1980; Huang & Turcotte 1988). However, since the cumulative number of zerosets with size larger than A is a random variable, eq. (19) refers to its mean value (Adler 1981, p. 215; Karatzas 1988, pp. 400–417). Consequently, for a given H or D, two records of any given finite realization of $\Delta\tau_d(x, y)$ do not give rise to the same number–size relation, but the mean value of the cumulative number of zerosets for all realizations is related to the size by a power-law relation (eq. 19). Since each realization of the process $\Delta\tau_d(x, y)$ corresponds to an instant on the time-axis, the mean value of the cumulative number of zerosets is equivalent to the cumulative number of earthquakes during a long period of time. Therefore, the variation in b values can arise from a dispersion from the mean value due to the randomness of zerosets, rather than from a one-to-one relation between the b value and the fractal dimension D. This inference is supported by the observation that the average b value over long time periods is stable and approximately equal to 1.0, although over short times it varies from 0.5 to 1.5 (Shi & Bolt 1982; Scholz 1990, pp. 188–189). Taking on the average $b \approx 1.0$, it follows that $H \to 0$. This result is consistent with that based on the observation of constant average stress drop.

For larger earthquakes, i.e. earthquakes with rupture length $L \geq h_0/\sin\theta$, where h_0 and θ have the same meanings as in eq. (18), a model has been proposed by Scholz (1982, 1990, pp. 180–189). This model (termed the L-model), assuming an average c-value equal to 1.5 and a constant fault width, expresses the cumulative number of large earthquakes with rupture length $\geq L$ as

$$N(L) = c_4 L^{-(4b/3)}, \tag{21}$$

where c_4 is a constant.

The ultimate rupture sizes of large intraplate earthquakes depend predominantly on how far the rupture can propagate along the strike direction, since the rupture width is constrained by the thickness of the seismogenic layer. Therefore, their frequency–size relation is equivalent to the distribution of zerosets of the 1-D process $\Delta\tau_d(x)$. The mean value of the cumulative number of zerosets with length larger than L is given by (Adler 1981, p. 215; Mandelbrot 1983, p. 354)

$$N(L) = c_5 L^{-(1-H)}, \tag{22}$$

where c_5 is a constant and H is the index ($D_z = 1 - H$ is the fractal or Hausdorff dimension of the zerosets of the 1-D fractional Brownian motion). Comparing eq. (22) with eq. (21), we obtain

$$H = 1 - \frac{4b}{3}. \tag{23}$$

The average b value for large earthquakes along a single fault zone is not available because the historical record is not sufficient to do meaningful statistics for a single fault. Using $H \to 0$, we predict that on average $b \approx 0.75$, i.e. the b value for large earthquakes along a single fault zone is smaller than that for small earthquakes. A few case studies have suggested that the maximum earthquake size is greatly underestimated by the extrapolation of the size distribution of small earthquakes for the same fault, which implies a smaller b value for large earthquakes (Wesnousky *et al.* 1983; Singh *et al.* 1983; Schwartz & Coppersmith 1984; Davison & Scholz 1985). This result is in agreement with our theoretical prediction.

The Gutenberg–Richter relation describes the number of earthquakes of given magnitude (or moment) per unit time. The time-rate of occurrence of earthquakes has been considered in other models (Lomnitz-Adler 1988; Rundle 1989). In this work, we assume that the mean of the process $\Delta\tau_d(x)$ increases with time during an earthquake circle. Along the time-axis, different points correspond to different means of the process $\Delta\tau_d(x)$. The fractional Brownian motion with any mean yields a power-law relation between the number and the size of the zerosets (eq. 19). Therefore, the fractional Brownian distribution of $\Delta\tau_d(x)$ can generate the Gutenberg–Richter relation per unit time.

SURFACE SLIPS OF EARTHQUAKE FAULTS

Investigation of large earthquake ruptures has shown that the variation of surface slips along the strike of faults is very irregular (Sharp *et al.* 1982; Crone & Machette 1984; Deng *et al.* 1986; Zhang *et al.* 1987; Nakata *et al.* 1990; Yoshida & Abe 1992). This irregularity cannot be interpreted in terms of the variation of surface lithology and fault geometry alone, but it must reflect a complex physical process along the fault.

On the basis of the circular rupture model and/or the rectangular L-model, a linear relation is obtained between the average stress drop and the mean slip (Scholz 1990, p. 181):

$$\overline{\Delta\tau_d} = c_6 \overline{\Delta u_d}, \tag{24}$$

where c_6, for a given rupture, is a constant that depends on the rupture model and material parameters. We rewrite eq. (24) as

$$\Delta\tau_d(x) = c_6 \Delta u_d(x), \qquad \Delta\tau_d(x) \geq 0, \tag{25}$$

in which both quantities are expressed as a function of position. For a given rupture, integration of eq. (25) yields eq. (24). Since $\Delta\tau_d(x)$ is a stochastic process, the dynamic slip $\Delta u_d(x)$ is also a stochastic process. Its mean is

$$E[\Delta u_d(x)] = c_7 E[\Delta\tau_d(x)], \qquad \Delta\tau_d(x) \geq 0, \tag{26}$$

where $c_7 = 1/c_6$ is a constant for a given earthquake. Its autocovariance function is

$$\operatorname{cov}[\Delta u_d(x), \Delta u_d(x+s)] = c_7^2 \operatorname{cov}[\Delta\tau_d(x), \Delta\tau_d(x+s)], \qquad \Delta\tau_d(x) \geq 0. \tag{27}$$

Eq. (27) shows that the process $\Delta u_d(x)$ has the same distribution function as $\Delta\tau_d(x)$, but with a different mean and variance, provided that $\Delta\tau_d(x)$ is a Gaussian process. This can be seen clearly from their autocorrelation functions.

Eq. (27) gives

$$\rho_u(s) = \frac{\mathrm{cov}[\Delta u_d(x), \Delta u_d(x+s)]}{\{V[\Delta u_d(x)]V[\Delta u_d(x+s)]\}^{1/2}} = \rho_\tau(s), \tag{28}$$

where $\rho_u(s)$ and $\rho_\tau(s)$ are the autocorrelation functions of the two processes. Since their autocorrelation functions are identical, $\Delta u_d(x)$ and $\Delta\tau_d(x)$ have the same normalized power spectrum. Therefore, the distribution function of the potential dynamic stress drop $\Delta\tau_d(x)$ can be estimated by power spectral analysis of surface slip data.

Spectral analysis is a common method to estimate the distribution function of stationary stochastic processes (see Priestley 1981, pp. 432–449). Application to non-stationary fractional Brownian motion yields an approximate power-law dependence of the power spectral density on the wavenumber κ, provided that κ is relatively large:

$$\hat{h}(\kappa) \simeq c_s \kappa^{-(1+2H)} \tag{29}$$

where c_s is a constant and H is the index (Mandelbrot 1983, p. 388; Falconer 1990, pp. 155–158). Consequently, spectral analysis can be applied to infer whether a given record originates in a particular stationary process or in one of the fractional Brownian family by comparing the sample power spectral density function with theoretical ones.

In order to make the estimation rigorous, slip data sets should satisfy the following conditions: (1) the earthquake fault is large; (2) there exist no or little surface unconsolidated sediments; and (3) the size of the data set is large, i.e. there are many measurements of slip along the earthquake fault. Although there are many earthquake ruptures whose slips have been measured, data sets that satisfy the above three conditions are rare. We analyse two earthquake ruptures reasonably suitable for this purpose. One is the 1920 Haiyuan earthquake ($M = 8.7$) in north-western China (36.7°N, 105.3°E) (Deng *et al.* 1986; Zhang *et al.* 1987). The other is the 1990 Luzon earthquake ($M_s = 7.8$) in the Philippines (15.7°N, 121.2°E) (Nakata *et al.* 1990; Yoshida & Abe 1992). The Haiyuan earthquake caused a surface breakage about 220 km long. The fault is predominantly strike-slip with a relatively small dip-slip component. Although the earthquake occurred several decades ago, the displacements are still visible in most places along the fault trace. Deng *et al.* (1986) and Zhang *et al.* (1987) took detailed measurements of the horizontal component of displacement associated with this earthquake, for a total of 168 slip measurements along the 220 km long surface fault (the maximum measured horizontal displacement is about 12 m). The surface fault associated with the Luzon, Philippines earthquake is about 120 km long, with a predominant strike-slip component. The surface ruptures are impressive, offsetting roads, foot-paths, streams, paddy dykes and so on (Nakata *et al.* 1990; Yoshida & Abe 1992). The displacements were measured in detail along the fault (Nakata *et al.* 1990; Yoshida & Abe 1992). There are in total 111 separate measurements, and the measured maximum amount of slip is about 6 m. [See Deng *et al.* (1986) and Yoshida & Abe (1992) for detailed information about the surface displacement measurements of the two seismic faults.]

The surface displacements are affected by many factors, such as the inertial force, surface lithology, and measurement errors. We assume that the effects of surface lithology and measurement errors are relatively small. The inertial force or 'overshooting' causes the final static slip to be on average about 30 per cent larger than the dynamic slip (Scholz 1990, p. 174) and therefore its effect on the dynamic slip can be approximated by the surface displacements divided by a constant factor. Since a Gaussian process times a constant factor is still a Gaussian process, the use of the original data does not affect the estimation. We have applied power spectral analysis to the measured surface slips of the two earthquake faults to estimate the distribution function of $\Delta\tau_d(x)$. The estimated power spectra of surface slips for the two earthquake faults are shown in Fig. 3. The estimation is not robust due to the relatively poor data sets (small sample size and irregular sampling), and the curves of power spectral density fluctuate more than the theoretical ones. However, the sample power spectral density shows clearly a power-law dependence on the spatial frequency f (wavelength is $1/f$) for both data sets. These results confirm that the process $\Delta\tau_d(x)$ follows the fractional Brownian distribution. The estimated index, calculated using the method of least squares, is $\hat{H} = 0.13$ for the Haiyuan fault and $\hat{H} = 0.27$ for the Luzon fault; both values are small. Consequently, this result does not contradict the conclusion $H \to 0$ obtained from the observation of constant average stress drop and the Gutenberg–Richter relation. It also shows that, even just before the two large earthquakes occurred, the spatial series of the potential dynamic stress drop were very rough, contrary to the hypothesis (von Seggern 1979; Huang & Turcotte 1988) of

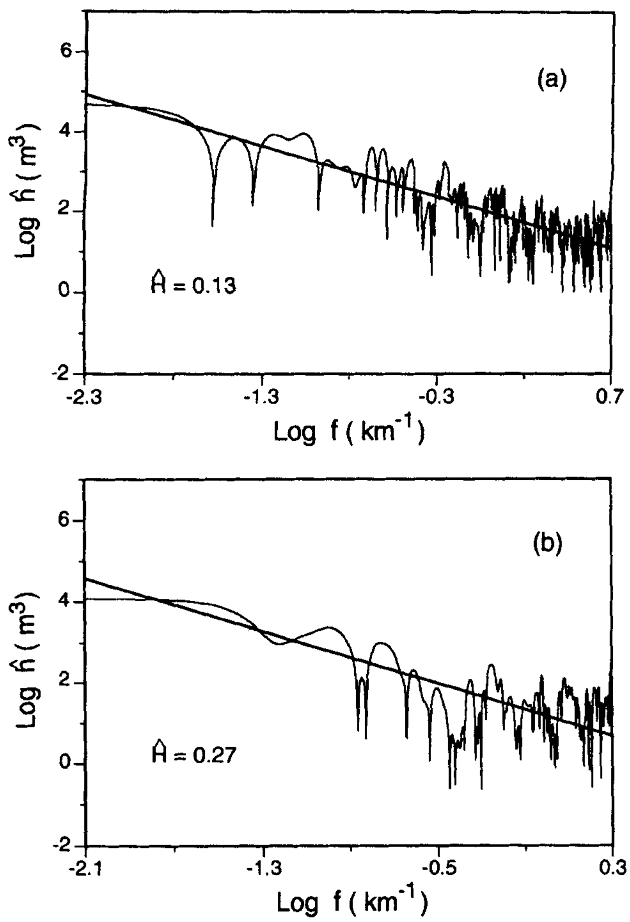

Figure 3. Logarithmic power spectral density of surface slip. (a) Haiyuan fault; (b) Luzon fault. The linear least-squares best fits are denoted by the straight lines. $\hat{H}$ is the estimated index.

a gradual change from roughness to smoothness of the stress–strength difference during the earthquake cycle.

CONCLUSIONS

Nearly all previous stochastic models of the earthquake process assume *a priori* a fractal distribution of stress and strength heterogeneities along the seismic fault. This assumption is dropped in this paper. Assuming a stick-slip weakening model, a relation is established between heterogeneities in stress–strength and the potential dynamic stress drop (and, consequently, fault slip). The variation of the potential dynamic stress drop $\Delta\tau_d(x)$ along the strike of the fault plane is modelled as a 1-D stochastic process. Thus, any seismic rupture can be regarded as a segment of a realization of $\Delta\tau_d(x)$ where $\Delta\tau_d(x) > 0$.

Three sets of independent observations (observed stress drop, Gutenberg–Richter magnitude–frequency relation, and measured surface slip along a seismic fault) are used to infer the properties of $\Delta\tau_d(x)$. The constancy of the observed average stress drop places important constraints on the distribution function of $\Delta\tau_d(x)$. The property of scale invariance shows that, if the distribution of $\Delta\tau_d(x)$ is of fractional Brownian type, only the member of the Brownian family with index $H \rightarrow 0$ can give rise to a constant mean value of average stress drop. The result for the general case, i.e. without the assumption of fractal distribution, is the same as that based on the property of scale invariance. Among several possible stochastic models, it appears that only the fractional Brownian motion with $H \rightarrow 0$ (fractal dimension $D = 2$ in the 1-D case) can produce an approximately constant mean value of seismic stress drop, and so fit the observations.

The distribution of zerosets in the fractional Brownian motion has the same power-law dependence on frequency as the earthquake rupture size, and therefore the fractal dimension D can be related to the b value in the magnitude–frequency law. However, the spatial and temporal variations of the b value can be interpreted to be a consequence of the randomness of the zerosets, rather than a consequence of variations of D. Using an average value of $b \approx 1.0$, the result $H \approx 0$ is obtained for the case of small earthquakes, which is in agreement with that based on the constancy of average stress drop. We also predict that the b value for large earthquakes is somewhat lower than that for small earthquakes along a common fault. This inference seems to be confirmed by observation and is potentially of importance for the estimation of the maximum magnitude.

Using the relation between the average stress drop and the average slip, the dynamic slip can be expressed as a linear function of the dynamic stress drop, both of which are a function of position along the seismic fault. The dynamic slip follows the same distribution as the dynamic stress drop, provided that the latter is a Gaussian process. Consequently, the distribution function of the potential dynamic stress drop can be inferred using earthquake fault slip data. The slip data of two large predominantly strike-slip earthquake faults have been analysed by power spectral analysis. Both faults show clearly a power-law relation between the power spectrum and the spatial frequency, which confirms that the process $\Delta\tau_d(x)$ follows the fractional Brownian distribution. The result of estimation, although not robust due to relatively small sample sizes and irregular sampling, does not contradict the conclusion $H \rightarrow 0$ obtained from the observation of constant average stress drop and from the Gutenberg–Richter relation.

Further work is required to confirm the fractional Brownian distribution (with $H \rightarrow 0$) of the earthquake process, using other arguments, based for instance on records of strong ground motion. Our analysis suggests that many earthquake parameters, such as the b value and the average stress drop, are random variables. The dispersions as well as the central values of these variables are likely to pose additional constraints on the distribution function of $\Delta\tau_d(x)$. In addition, the model presented in this paper is purely stochastic and does not consider the deterministic properties of the seismic occurrence. Models combining stochastic and deterministic analyses will improve our understanding of the earthquake process.

With respect to earthquake prediction, the model is pessimistic about the relevance of the variation of b values to the likelihood of large earthquakes, because the b value is a random variable and is unstable in short time periods. The model also suggests that understanding the variation of $\Delta\tau_d(x)$ along a single large fault zone at depth is of primary importance for earthquake prediction. Any workable approach to earthquake prediction should incorporate the stochastic analysis of seismicity, the distribution pattern of small earthquakes, and space–time correlations between earthquakes along a single fault.

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