

COMMENT ON CHAMLEY'S (1986) OPTIMAL TAXATION OF CAPITAL

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A central result in Chamley's (1986)¹ influential article on optimal taxation of capital is Theorem 2, which characterizes the trajectory of the optimal tax rate. The present comment identifies, and then fills, a serious gap in the proof of this theorem.

In the last paragraph of the proof, Chamley considers the case in which $v = 0 < \dot{v}$ at time t_1 , and shows that this case leads to a contradiction. To complete the proof, it is also necessary to rule out the following alternative case, which he ignores: $v = \dot{v} = 0$ at t_1 , but $v > 0$ and $\dot{v} > 0$ immediately after t_1 (since $\bar{r} = 0$ whenever $t_1 < t < t_2$).

In this alternative case, (32) implies that $Z = 0$ at t_1 , and (33) then implies that the following two conditions hold immediately after t_1 . First, $\dot{Z} > 0$ because, by continuity, Z is close to 0. (Recall that $\lambda - \mu > 0$ in a second-best solution.) Consequently, $Z > 0$ as the second condition. Under these two conditions, (32) and (33) imply that for *all* $t > t_1$, $v > 0$ and thus $\bar{r} = 0$. This implication, however, is absurd (as Chamley explains). Therefore, the alternative case is also impossible.

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¹ Christophe Chamley, "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives", *Econometrica*, Vol. 54, May 1986, pp. 607-622.