# Tunable Fractional Order Temporal Differentiator by Optically Pumping a Tilted Fiber Bragg Grating

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***Abstract*— We propose and demonstrate an optically tunable photonic fractional temporal differentiator using a tilted fiber Bragg grating written in an erbium/ytterbium (Er-Yb) co-doped fiber. Thanks to the high absorption of the Er-Yb co-doped fiber, when it is pumped the refractive index is changed, and thus the phase of a cladding mode resonant wavelength is changed continuously by continuous tuning of the pumping power. By locating the wavelength of the input light wave at the location of a cladding mode resonant wavelength, a temporal differentiator with a tunable fractional order is achieved. The proposed technique is experimentally evaluated. A temporal differentiator with a tunable fractional order is demonstrated. The use of the fractional differentiator to implement temporal differentiation of a Gaussian pulse with a bandwidth of 28 and 75 GHz is also demonstrated.**

***Index Terms*— Optical signal processing, tilted fiber Bragg grating (TFBG), tunable ultrafast fractional differentiator.**

## I. Introduction

With the rapid development of photonic technologies, the implementation of basic signal processing functions in the optical domain has been considered an effective solution for ultra-wideband signal processing. A differentiator is one of these essential signal processing elements which provides $n$-th order time derivative of the complex envelope of an arbitrary input optical pulse. In addition to signal processing purposes [1], a temporal differentiator can be used for ultra-fast signal generation [2], [3], and ultra-high-speed coding [4], [5].

Numerous techniques have been proposed recently to implement all-optical temporal differentiators. In [6], a temporal differentiator is implemented based on cross-gain modulation in a semiconductor optical amplifier (SOA). A temporal differentiator can also be achieved using a long period grating (LPG) [7], a pi-phase shifted fiber Bragg grating (PS-FBG) [8], [9], or a micro-ring resonator [10]. In addition to the implementation of a regular first-order differentiator, a temporal differentiator with a fractional order can also be implemented. In [11], a photonic fractional differentiator implemented based on an asymmetrical PS-FBG in reflection was demonstrated. The limitation of this technique is the absence of the differentiation order tunability.

Recently, we demonstrated the implementation of a fractional differentiator using a titled fiber Bragg grating (TFBG) [12]. In a TFBG, the phase at a cladding mode resonant wavelength is strongly polarization dependent, by continuously tuning the polarization state of the input light wave, the fractional order is tuned. However, the change of the polarization state usually involves mechanical movement, making the system complicated with low accuracy.

In this letter, we propose and demonstrate a continuously tunable fractional differentiator using a TFBG written in an erbium/ytterbium (Er/Yb) co-doped fiber. By optically pumping the TFBG, the phase of a cladding mode resonant wavelength is changed. By locating the wavelength of the input light wave at the location of the cladding mode resonant wavelength, a temporal differentiator with a tunable fractional order is achieved. An experiment is performed. A differentiator with a tunable fractional order is demonstrated. The use of the fractional differentiator to implement temporal differentiation of a Gaussian pulse with a bandwidth of 28 GHz and 75 GHz is also demonstrated.

## II. Principle

For a signal $x(t)$, the Fourier transform of its $n$th order differentiation, $dx^n(t)/dt^n$, is expressed as $[j(\omega-\omega_0)]^n X(\omega-\omega_0)$, where $\omega$ is the optical frequency, $\omega_0$ is the carrier frequency, and $X(\omega)$ is the Fourier transform of $x(t)$. Therefore, the differentiator can be considered as an optical filter with a frequency response given by

$$H_n(\omega) = [j(\omega-\omega_0)]^n = \begin{cases} e^{jn\left(\frac{\pi}{2}\right)}\,|(\omega-\omega_0)|^n & \omega > \omega_0 \\ e^{jn\left(-\frac{\pi}{2}\right)}\,|(\omega-\omega_0)|^n & \omega < \omega_0 \end{cases} \quad (1)$$

An optical filter with a frequency response given by (1) can be implemented using a TFBG. It is different from a regular FBG, a TFBG is fabricated with the variation of the refractive index at an angle to the optical axis of the optical fiber. In a TFBG, two different coupling, between the forward core mode and the backward core mode and between the core mode and the different cladding modes, would generate different resonance wavelengths. The resonance wavelength corresponding to the self-coupling of the core mode is given

$$\lambda_{Bragg} = \frac{2n_{eff,core}\Lambda_g}{\cos\theta} \quad (2)$$

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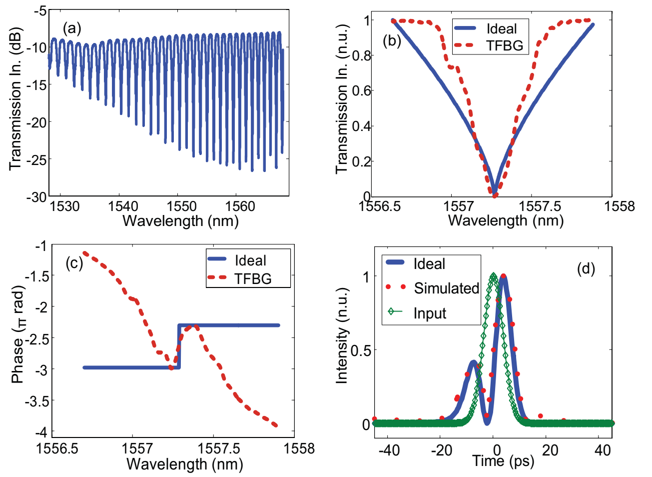


Fig. 1. (a) Transmission spectrum of a TFBG with a tilt angle of 10°. (b) Magnitude response. (c) Phase response of one channel of the TFBG. The solid line shows the magnitude and phase response of an ideal differentiator. (d) Simulated output pulse from the TFBG. The dotted line shows the output pulse from an ideal differentiator. The fractional order is 0.675. In.: Intensity.

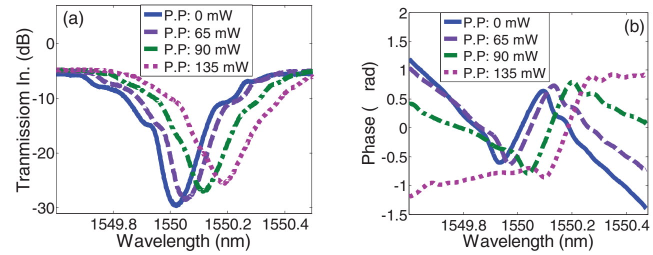


Fig. 2. (a) Magnitude, and (b) phase responses of a cladding mode resonance with a pumping power from 0 to 135-mW. P.P: pumping power. In.: Intensity.

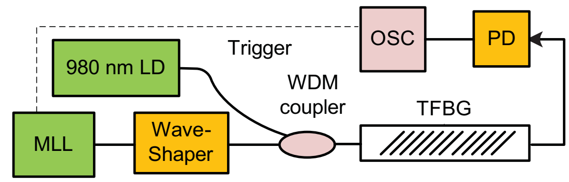


Fig. 3. Experimental setup. MLL: mode-locked laser, OSC: sampling oscilloscope, LD: laser diode, and PD: photodetector.

and the resonances corresponding to contra-propagating cladding modes are given by

$$\lambda_{coupling} = (n_{eff,cladding} + n_{eff,core})\frac{\Lambda_g}{\cos\theta} \tag{3}$$

where $\theta$ is the tilt angle of the TFBG, $\Lambda_g$ is the nominal grating period, $n_{eff,core}$ and $n_{eff,cladding}$ are the effective refractive indices of the core mode ($LP_{01}$) and a particular cladding mode, respectively. In [13], we have shown that by pumping an Er/Yb co-doped fiber with a 980-nm laser diode (LD), due to the high absorption of the Er/Yb co-doped fiber, the refractive index is changed. Based on (2) and (3), the resonance wavelengths are tuned. In addition, the coupling coefficient of each coupling mode also depends on $n_{eff,core}$ [14], thus different coupling coefficients are achieved, leading to the tuning of the phase of the resonance wavelengths. By locating the wavelength of the input light wave at the location of a specific cladding mode resonant wavelength, a temporal differentiator with a tunable fractional order is achieved.

Fig. 1(a) shows the transmission intensity spectrum of a 1-cm long TFBG with a tilt angle of 10°, and a Bragg wavelength of 1600 nm. Fig. 1(b) and (c) shows the magnitude and phase responses of one of the cladding-mode resonances with the resonance wavelength of 1557.27 nm measured by a LUNA optical vector analyzer. As can be seen the phase jump at this resonance wavelength is $0.675\pi$ rad, thus a differentiator with a fractional order of 0.675 can be achieved by introducing an input signal with the carrier wavelength of 1557.27 nm to this TFBG. The bandwidth of this differentiator is 0.8 nm or equivalently 100 GHz. Fig 1(a) and (b) also shows the magnitude and phase responses of an ideal fractional differentiator with an order of 0.675. As can be seen the magnitude and phase responses of the TFBG are close to those of an ideal fractional differentiator. The use of the differentiator to perform wideband differentiation of a Gaussian pulse is then simulated. The temporal full width at half maximum (FWHM) of the input pulse is 8 ps. The input pulse and output results are also shown in Fig. 1(d). A good agreement between the output pulse of the TFBG-based differentiator and that of an ideal differentiator is reached. The root mean square error (RMSE) is also calculated, which is 8.1%.

Then, the tunability is investigated. To do so, the TFBG is pumped by a 980-nm LD. By changing the pumping power, the magnitude and phase responses of the TFBG at different cladding mode resonances are changed. Fig. 2(a) and (b) shows the change of the magnitude and phase responses for a cladding mode resonance at 1550.3 nm of a TFBG with a tilt angle of 6° and a Bragg wavelength of 1570 nm. As can be seen in Fig. 2(b), by changing the pumping power from 0 to 135 mW, the resonance wavelength is shifted to longer wavelengths, and the introduced phase jump at the resonance wavelength is changed from $1.25\pi$ to $1.72\pi$, thus the order of the differentiator can be tuned from 1.25 to 1.72.

## III. Experiment

Fig. 3 shows the experimental setup. A mode-lock laser (MLL) is used to generate a short pulse with a temporal width of 550 fs. A 1-cm long TFBG with a tilt angle of 6° and a Bragg wavelength of 1570 nm is used as a differentiator. The TFBG is fabricated by using an excimer laser with a uniform phase mask. The tilt angle is introduced by using a focal lens. A 980-nm LD is used to pump the TFBG via a WDM coupler. The differentiated optical pulse is detected by a 53-GHz photodetector and its waveform is observed by a sampling oscilloscope.

First, we show the tuning of the fractional order and the differentiation of a Gaussian pulse at different differentiation order. In the experiment, a Gaussian pulse with a bandwidth of 28 GHz obtained by filtering an ultra-short optical pulse from the MLL using a WaveShaper is applied to the TFBG. A cladding mode resonance at 1550.04 nm is selected. As can be seen by increasing the pumping power from 0 to 135 mW (or the injection current to the LD from

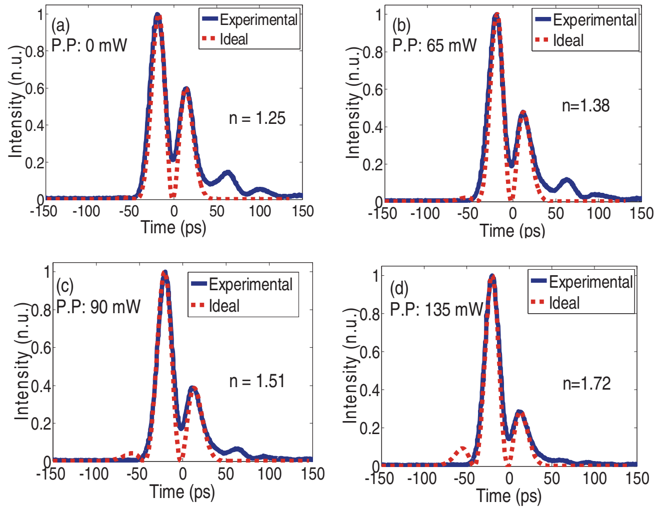


Fig. 4. Differentiated pulses at the output of the TFBG pumped with a pumping power of (a) 0 mW, (b) 65 mW, (c) 90 mW, and (d) 135 mW.

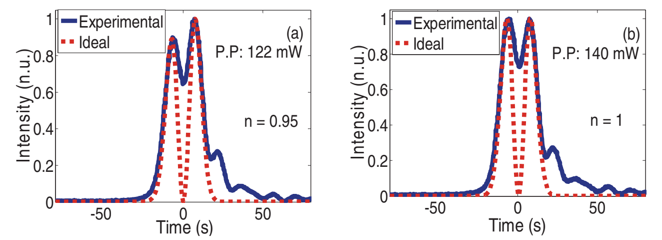


Fig. 5. Differentiated pulses at the output of the TFBG pumped with a power at (a) 122 mW and (b) 140 mW. The carrier wavelength is tuned at 1559 nm.

0 to 175 mA), the differentiation order is changed from 1.25 to 1.72. It should be mentioned that by increasing the pumping power from 0 to 135 mW, the resonance wavelength is shifted from 1550.04 to 1550.24 nm. Thus the carrier wavelength of the Gaussian pulse is adjustment by changing the central wavelength of the WaveShaper. Fig. 4 shows the differentiated pulses at different fractional orders. The pulses by an ideal differentiator are also shown in Fig. 4 for comparison. As can be seen a good agreement between the ideal output pulses and the experimentally differentiated pulses is reached. The RMSE for $n = 1.25$ is 8%, which is the largest error.

Then, a Gaussian pulse with a bandwidth of 75 GHz is generated by reconfiguring the WaveShaper and is applied to the TFBG. Since the bandwidth of the differentiator is 100 GHz, the input Gaussian pulse can be effectively differentiated. A different cladding mode resonance at 1559 nm is selected. The pumping power is tuned at 122 mW and 140 mW corresponding to a fractional order of 0.95 and 1, respectively. The generated waveforms are shown in Fig. 5(a) and (b). The errors compared with the ideal waveforms are larger; especially at the notch. The large errors are caused mainly due to the limited bandwidth of the photodetector (53 GHz). Again, the RMSE is also calculated, which is 16%.

## IV. Conclusion

Note that when the TFBG was pumped, both the magnitude and phase responses were changed. The magnitude response of an ideal fractional differentiator is given by $|(\omega - \omega_0)|^n$ which was not exactly satisfied when using a TFBG, as can be seen from Fig. 1(a). However, for the implementation of a differentiator, the phase response plays a much more important role [15]. This explains the generated waveforms were close to the waveforms based on an ideal differentiator with small errors.

The key significance of the approach is that a TFBG written in an Er/Yb co-doped fiber was employed. When pumped, the phase response of a cladding mode resonance was tuned, which led to the tuning of the fractional order. Compared with our earlier approach [12] where the tuning was done by mechanical tuning the incident light polarization, the proposed technique was simpler with higher accuracy. The proposed approach was evaluated by an experiment. A differentiator with a continuously tunable fractional order was demonstrated. The differentiation of a wideband Gaussian pulse with a bandwidth of 28 and 75 GHz was also performed.

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