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Dynamic Controllability Checking for Conditional Simple Temporal Networks with Uncertainty: New Sound-and-Complete Algorithms based on Constraint Propagation

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Abstract

A Conditional Simple Temporal Network with Uncertainty (CSTNU) is a data structure for representing and reasoning about time. CSTNUs incorporate *observation time-points* from Conditional Simple Temporal Networks (CSTNs) and *contingent links* from Simple Temporal Networks with Uncertainty (STNUs). A CSTNU is *dynamically controllable* (DC) if there exists a strategy for executing its time-points that guarantees the satisfaction of all relevant constraints no matter how the uncertainty associated with its observation time-points and contingent links is resolved in real time. This paper presents the first sound-and-complete DC-checking algorithms for CSTNUs based on the propagation of labeled constraints and demonstrates their practicality.

1 Introduction

A Conditional Simple Temporal Network with Uncertainty (CSTNU) is a data structure for representing and reasoning about time in domains where some constraints may apply only in certain scenarios and some events may have uncontrollable, but bounded durations [1]. They were defined to represent important features of, for example, (1) *workflow systems* used to automate medical-treatment processes [2],[3], and (2) *planning systems* when uncertain durations are present [4]. A CSTNU may include *observation time-points* and *contingent links.* An observation time-point represents a test action whose execution generates a truth value for a corresponding propositional letter. A contingent link represents an action with an uncertain, but bounded duration. Observation time-points and contingent links both involve uncertainty *and* uncontrollability, since the outcomes of tests and contingent durations are not known in advance and are not controlled by the scheduling agent; they are only *observed* during execution.

CSTNUs generalize Conditional Simple Temporal Networks (CSTNs) [5] and Simple Temporal Networks with Uncertainty (STNUs) [6]. The *dynamic controllability* (DC) property for CSTNUs generalizes the corresponding properties for CSTNs and STNUs. In brief, a CSTNU is DC if there exists a strategy for executing its time-points that guarantees the satisfaction of all relevant constraints no matter how the uncertainty associated with its observation time-points and contingent links is resolved during execution. The *DC-checking problem* for CSTNUs is that of determining whether arbitrary CSTNUs are DC.

Combi et al. [7, 8] presented sound constraint-propagation rules for CSTNUs, but did not address completeness. Following their approach, but focusing on CSTNs, Hunsberger et al. [9] presented the first practical sound-and-complete DC-checking algorithm for CSTNs. Recently, they presented a faster version of their algorithm, called the π-DC-checking algorithm [10], that will be used in this paper. In other approaches, Hunsberger and Posenato [11] presented an algorithm that views the DC-checking problem for CSTNs as a two-player game, searching an abstract game tree to find a "winning" strategy, guided by Monte-Carlo Tree Search and Limited Discrepancy Search; and Cimatti et al. [12] reduced the DC-checking problem for CSTNUs (and a broader class of networks) to a controller-synthesis problem for timed game automata, but have not shown whether that approach can be made practical for CSTNUs.

Contribution. This paper presents the first *practical sound-and-complete* DC-checking algorithms for CSTNUs. The first algorithm reduces the DC-checking problem for CSTNUs to the DC-checking problem for CSTNs; the second propagates constraints directly in the input CSTNU. The paper proves that both algorithms are correct and empirically evaluates their performance.

2 Conditional STNs with Uncertainty (CSTNUs)

This section recalls the definition of a well-defined CSTNU, which allows contingent links (as in an STNU) and observation time-points (as in a CSTN). The presentation combines and extends definitions from earlier work [13, 1, 8, 10]. The notion of a *streamlined* temporal network from recent work on CSTNs [14] is then applied to CSTNUs.

Definition 1 (P-Labels)**.** For a set $\mathcal{P}$ of propositional letters, a *p-label* is a (possibly empty) conjunction of (positive or negative) literals from $\mathcal{P}$. The *empty p-label* is notated $\boxdot$. For any p-label ℓ, and any $p \in \mathcal{P}$, if $\ell \models p$ or $\ell \models \neg p$, we say that p *appears* in ℓ. For p-labels, ℓ_1 and ℓ_2, if $\ell_1 \models \ell_2$, we say that ℓ_1 *entails* ℓ_2. If $\ell_1 \wedge \ell_2$ is satisfiable, we say that ℓ_1 and ℓ_2 are *consistent.* $\mathcal{P}^*$ denotes the set of all *satisfiable* p-labels with literals from $\mathcal{P}$.

Definition 2 (CSTNU)**.** A *Conditional Simple Temporal Network with Uncertainty* is a tuple, $\langle \mathcal{T}, \mathcal{P}, L, \mathcal{C}, \mathcal{OT}, \mathcal{O}, \mathcal{L} \rangle$, where:

- $\mathcal{T}$ is a finite set of real-valued time-points (i.e., variables with continuous domain);
- $\mathcal{P}$ is a finite set of propositional letters;
- $L \colon \mathcal{T} \to \mathcal{P}^*$ assigns p-labels to time-points;
- $\mathcal{C}$ is a set of *labeled* constraints, each of the form, $(Y - X \leq \delta,\ \ell)$, where $X, Y \in \mathcal{T}$, $\delta \in \mathbb{R}$, and $\ell \in \mathcal{P}^*$;
- $\mathcal{OT} \subseteq \mathcal{T}$ is a set of *observation* time-points;
- $\mathcal{O} \colon \mathcal{P} \to \mathcal{OT}$ is a bijection from propositional letters to observation time-points;
- $\mathcal{L}$ is a set of *contingent links* each of the form (A, x, y, C), where: $A \in \mathcal{T}, C \in \mathcal{T} \backslash \mathcal{OT}$, $A \not\equiv C$ are called the *activation* and *contingent* time-points, respectively; $L(A) = L(C)$; $0 < x < y < \infty$; and distinct contingent links have distinct contingent time-points.

By convention, for each $p \in \mathcal{P}$, $\mathcal{O}(p)$ (i.e., the observation time-point whose execution determines the truth value for p) may be denoted by $P?$.

Hunsberger *et al.* [9] called a CSTN *well-defined* if the p-labels on its time-points and constraints satisfied certain properties. CSTNs that are not well defined turn out to be useless. Definition 3 extends well-definedness to CSTNUs.

Definition 3 (Well-defined CSTNU)**.** A CSTNU is *well defined* if:

1. for each $(Y - X \leq \delta, \ell) \in \mathcal{C}$, $\ell \models L(X) \wedge L(Y)$;
2. for each $T \in \mathcal{T}$, and each p appearing in $L(T)$:
   (a) $\ell \models L(P?)$; and
   (b) $(P? - T \leq -\epsilon, L(T)) \in \mathcal{C}$, for some $\epsilon > 0$;
3. for each $(Y - X \leq \delta, \ell) \in \mathcal{C}$, and each p appearing in ℓ, $\ell \models L(P?)$; and
4. for each $(A, x, y, C, \ell) \in \mathcal{L}$, $L(A) = L(C) = \ell$. $\Leftarrow$ *This property is new for CSTNUs.*

Cairo *et al.* [14] showed that if $\mathcal{S}$ is a well-defined CSTN (i.e., satisfies properties 1–3 above), then there is a CSTN $\mathcal{S}_{\boxdot}$ that

1. does *not* have any labels on its time-points, and
2. is DC if and only if $\mathcal{S}$ is DC.

$\mathcal{S}_{\boxdot}$ is called a *streamlined* CSTN. We say that $\mathcal{S}_{\boxdot}$ is *DC-equivalent* to $\mathcal{S}$. This result extends easily to CSTNUs.

Definition 4 (Streamlined CSTNU)**.** Given a well-defined CSTNU $\mathcal{S}$, its *streamlined* version $\mathcal{S}_{\boxdot}$ is the same as $\mathcal{S}$ except that its time-points have no p-labels.

It is straightforward to show that if $\mathcal{S}$ is a *well-defined* CSTNU, then $\mathcal{S}$ is DC if and only if its streamlined version $\mathcal{S}_{\boxdot}$ is DC. For this reason, this paper restricts attention to streamlined CSTNUs, which simplifies definitions and proofs of the main results with no loss of generality.

Each CSTNU has a corresponding graph whose nodes represent time-points, and whose edges represent various kinds of temporal constraints. To accommodate the propositional labels (p-labels) from CSTN graphs and the alphabetic labels (a-labels) from STNU graphs, each edge in a CSTNU graph may be annotated by both p-labels *and* a-labels. This paper treats a-labels more rigorously than in prior work in anticipation of how they are conjoined and modified during constraint propagation. In particular, an a-label is liberally defined to allow *conjunctions of upper-case letters* that may arise during constraint propagation.

Definition 5 (A-Letters, A-Labels)**.** If $C_1, \ldots, C_k$ are the contingent time-points for a CSTNU $\mathcal{S}$, then $\mathcal{A} = \{c_1, \ldots, c_k, C_1, \ldots, C_k\}$ is the set of *alphabetic letters (a-letters)* for $\mathcal{S}$; $c_1, \ldots, c_k$ are the *lower-case* (LC) a-letters; and $C_1, \ldots, C_k$ are the *upper-case* (UC) a-letters. An *a-label*, $\aleph$, is a set of a-letters that: (1) is empty, notated as $\diamond$; (2) contains exactly one LC a-letter, notated as c_i; or (3) contains *one or more* UC a-letters, notated as $C_{i_1} \ldots C_{i_m}$. The set of all a-labels with letters from $\mathcal{A}$ is denoted by $\mathcal{A}^*$. $\mathcal{A}^*_u$ denotes the set of *UC a-labels* (that is, a-labels that contain zero or more UC a-letters). For any $\aleph, \aleph' \in \mathcal{A}^*_u$, their *conjunction* is given by their union (i.e., $\aleph\aleph' = \aleph \cup \aleph'$).

Each edge in a CSTNU graph is annotated by a triple, called a *labeled value,* that generalizes: (1) the numerical weights that appear on edges in STN graphs; (2) the *lower-case* and *upper-case* a-letters that appear on edges in STNU graphs; and (3) the p-labels that appear on edges in CSTN graphs. Since any pair of time-points, X and Y, may participate in multiple constraints, an edge from X to Y may have multiple labeled values, each of the form, $\langle \delta_i, \aleph_i, \ell_i \rangle$.

Definition 6 (Labeled values)**.** A *labeled value* is a triple, $\langle \delta, \aleph, \ell \rangle$, where: $\delta \in \mathbb{R}$, $\aleph \in \mathcal{A}^*$, and $\ell \in \mathcal{P}^*$.

In a CSTNU graph, an *ordinary* STN constraint, $Y - X \leq \delta$, is represented by an edge from X to Y annotated by $\langle \delta, \diamond, \boxdot \rangle$; and a conditional constraint, $(Y - X \leq \delta, \ell)$, is represented by an edge from X to Y annotated by $\langle \delta, \diamond, \ell \rangle$. Edges associated with contingent links require further introduction.

In an *STNU* graph, each contingent link (A, x, y, C) gives rise to two edges: a *lower-case* edge from A to C labeled by $c : x$ that represents the possibility that the duration $C - A$ might take on its minimum value x; and an *upper-case* edge from C to A labeled by $C : -y$ that represents that $C - A$ might take on its maximum value y. For a contingent link (A, x, y, C) in a CSTNU graph, the lower-case edge is annotated by $\langle x, c, \boxdot \rangle$, and the upper-case edge by $\langle -y, C, \boxdot \rangle$.

Figure 1 shows the graph for a sample CSTNU. Z is a time-point whose value is fixed at 0. Each time-point X is implicitly constrained to occur at or after Z. The dashed edges represent constraints obtained by Algorithm 2, described in section 6. It determines that this CSTNU is not DC.

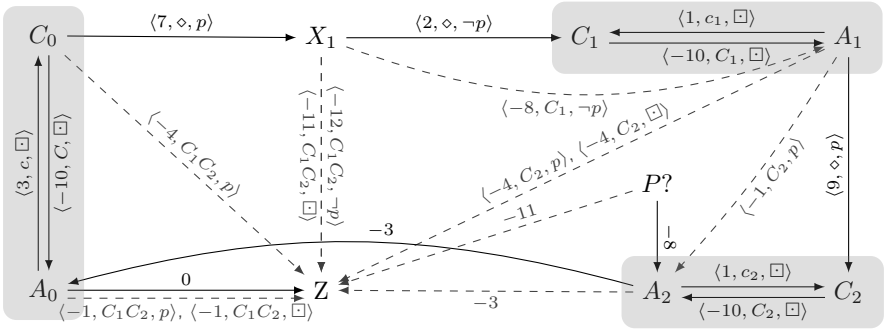


Figure 1: A CSTNU graph with 3 contingent links and one observation tp $P?$

3 Dynamic Controllability for CSTNUs

The truth values of propositions in a CSTNU are not known in advance; they are incrementally revealed as observation time-points are executed. Similarly, the durations of contingent links are only observed as the contingent time-points happen to execute. However, a *dynamic strategy* for executing the time-points in a CSTNU can *react* to observations and contingent executions in real time. A *viable* strategy is one that guarantees that all relevant constraints will be satisfied no matter which truth values and durations are revealed over time. A CSTNU with a dynamic and viable strategy is *dynamically controllable* (DC).

Like much recent work on STNUs and CSTNs [13, 9, 15], this paper defines the DC property for CSTNUs to allow execution strategies to react *instantaneously* to observations, instead of requiring arbitrarily small delays. It generalizes the DC semantics for STNUs [13] and the π-DC semantics for CSTNs [15].

This paper focuses on CSTNUs whose sets of contingent and observation time-points are distinct—with no loss of generality because any contingent observation time-point could be represented by two time-points, C and $P?$, constrained to occur simultaneously. An instantaneously reactive strategy could wait for C to execute and then execute $P?$ at the same time.

Preliminaries. A *scenario* s specifies a truth value for each proposition, and a *situation* ω specifies a duration for each contingent link. A *drama* is then a scenario-situation pair (s, ω). The projection of a CSTNU onto a drama (s, ω) is the *STN* obtained by restricting attention to the constraints whose labels are true under s and assigning each contingent duration to the value specified by ω.

Definition 7 (Scenario/Situation/Drama/Projection)**.** A *scenario* is a function, $s\colon \mathcal{P} \to \{true, false\}$, that assigns a truth value to each $p \in \mathcal{P}$. A scenario also determines the truth value, $s(\ell)$, for any p-label $\ell \in \mathcal{P}^*$. The set of all scenarios over $\mathcal{P}$ is denoted by $\mathcal{I}$. If $(A_1, x_1, y_1, C_1), \ldots, (A_k, x_k, y_k, C_k)$ are the contingent links for a CSTNU $\mathcal{S}$, then $\Omega = [x_1, y_1] \times \ldots \times [x_k, y_k]$ is called the *space of situations* for $\mathcal{S}$, and any $\omega = (\omega_1, \ldots, \omega_k) \in \Omega$ is called a *situation.* A *drama* is any pair $(s, \omega) \in \mathcal{I} \times \Omega$, where $s \in \mathcal{I}$ is a scenario, and $\omega \in \Omega$ is a situation. Let $\mathcal{S} = \langle \mathcal{T}, \mathcal{P}, \mathcal{C}, \mathcal{OT}, \mathcal{O}, \mathcal{L}\rangle$ be any CSTNU, and (s, ω) any drama for $\mathcal{S}$, where $\omega = (\omega_1, \ldots, \omega_k)$. The *projection* of $\mathcal{S}$ onto (s, ω)—denoted by $Prj(\mathcal{S}, s, \omega)$—is

the STN, $(\mathcal{T}, \mathcal{C}_s)$, where:[1]

$$\mathcal{C}_s = \{(Y - X \leq \delta) \mid \exists \ell, (Y - X \leq \delta, \ell) \in \mathcal{C} \text{ and } s(\ell) = \mathit{true}\} \\ \cup \{(C_i - A_i = \omega_i) \mid (A_i, x_i, y_i, C_i) \in \mathcal{L}\}$$

3.1 Execution strategies.

Cairo et al. [15] introduced the π-DC semantics for CSTNs that, unlike prior versions [9, 16], does not permit a kind of circular dependency among simultaneous observations. A π-dynamic strategy must specify, for each scenario, both a *schedule* for the time-points, and an *order of dependency* among the observation time-points. *This section extends the π-DC semantics to cover CSTNUs.*

Definition 8 (Schedule)**.** A *schedule* for a set $\mathcal{T}$ of time-points is a *complete* mapping, $\psi : \mathcal{T} \to \mathbb{R}$. For any $X \in \mathcal{T}$, and any schedule ψ, the execution time for X in ψ is denoted by $[\psi]_X$. The set of schedules for $\mathcal{T}$ is denoted by Ψ.

Definition 9 (Order of Dependency)**.** Let $\mathcal{OT} = \{P_1?, \ldots, P_n?\}$ be a set of observation time-points. Any permutation π over $(1, 2, \ldots, n)$ effectively specifies an order for those observation time-points. For any $P? \in \mathcal{OT}$, let $\pi(P?) \in \{1, 2, ..., n\}$ denote the (integer) position of $P?$ in the order determined by π; let Π_n denote the set of all permutations over $(1, 2, \ldots, n)$; and let $\Pi = \cup_{n \geq 0} \Pi_n$.

Definition 10 (π-Execution Strategy)**.** A *π-execution strategy* for a CSTNU $\mathcal{S}$ is a mapping, $\sigma : (\mathcal{I} \times \Omega) \to (\Psi \times \Pi)$, where for each drama $r = (s, \omega) \in \mathcal{I} \times \Omega$, $\sigma(s, \omega)$ is a pair (ψ_r, π_r) such that $\psi_r : \mathcal{T} \to \mathbb{R}$ is a schedule, and $\pi_r \in \Pi_{|\mathcal{OT}|}$ determines an order of dependency among the observation time-points. For convenience, π is extended such that $\pi(C) = 0$ for each contingent time-point C, and $\pi(X) = \infty$ for each non-contingent, non-observation time-point X.

The strategy σ is *viable* if for each drama $r = (s, \omega)$, the schedule ψ_r is a solution to the projection $Prj(\mathcal{S}, s, \omega)$. And σ is *coherent* if for each drama $r = (s, \omega)$, and any $P?$ and $Q?$ in $\mathcal{OT}$, $[\psi_r]_{P?} < [\psi_r]_{Q?}$ implies $\pi_r(P?) < \pi_r(Q?)$ (i.e., if ψ_r *schedules* $P?$ before $Q?$, then π_r *orders* $P?$ before $Q?$).

Definition 11 (π-History)**.** Let $\mathcal{S} = \langle \mathcal{T}, \mathcal{P}, \mathcal{C}, \mathcal{OT}, \mathcal{O}, \mathcal{L} \rangle$ be a CSTNU; σ a π-execution strategy for $\mathcal{S}$; $r = (s, \omega)$ a drama; $(\psi_r, \pi_r) = \sigma(s, \omega)$; $t \in \mathbb{R}$; and $d \in \{1, 2, \ldots, |\mathcal{OT}|; \infty\}$. Then $\mathcal{H}(t, d, s, \omega, \sigma) = (\mathcal{H}_s, \mathcal{H}_\omega)$ is the *π-history* of (t, d) for the drama (s, ω) and strategy σ, where:

$$\mathcal{H}_s = \{(p, s(p)) \mid P? \in \mathcal{OT},\ [\psi_r]_{P?} \leq t, \text{ and } \pi_r(P?) < d\}\,; \\ \mathcal{H}_\omega = \{(A, C, [\psi_r]_C - [\psi_r]_A) \mid \exists x, y \text{ such that } (A, x, y, C) \in \mathcal{L}, \text{ and } [\psi_r]_C \leq t\}\,.$$

$\mathcal{H}_s$ specifies the truth values of all propositions p observed *before* time t in the schedule ψ_r, as well as those observed *at* time t if $P?$ is ordered *before* position d by the permutation π_r. Similarly, $\mathcal{H}_\omega$ specifies the durations of all contingent links that completed *at or before* time t in the schedule ψ_r.

Definition 12 (π-Dynamic Execution Strategy)**.** A π-execution strategy, σ, for a CSTNU $\mathcal{S}$, is called *π-dynamic* if for every pair of dramas, (s_1, ω_1) and (s_2, ω_2), and every *non-contingent* (but possibly observation) time-point X:

[1] $C_i - A_i = \omega_i$ abbreviates the pair of constraints, $C_i - A_i \leq \omega_i$ and $A_i - C_i \leq -\omega_i$.

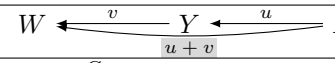
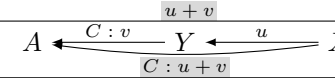
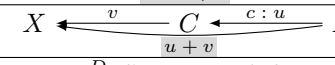
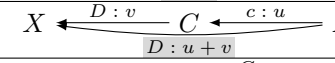
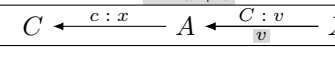
Rule	Conditions	Pre-existing and generated edges
No Case (NC):		
Upper Case (UC):		
Lower Case (LC):	$(v < 0)$	
Cross Case (CC):	$(D \not\equiv C$ and $v < 0)$	
Label Removal (LR):	$(v \geq -x)$	

Table 1: Morris-Muscettola rules for DC-checking STNUs

let:	$(\psi_1, \pi_1) = \sigma(s_1, \omega_1)$ and $(\psi_2, \pi_2) = \sigma(s_2, \omega_2)$,
let:	$t = [\psi_1]_X$, and $d = \pi_1(X) \in \{1, 2, \ldots, \lvert\mathcal{OT}\rvert; \infty\}$.
if:	$\mathcal{H}(t, d, s_1, \omega_1, \sigma) = \mathcal{H}(t, d, s_2, \omega_2, \sigma)$
then:	$[\psi_2]_X = t$ and $\pi_2(X) = d$.

Thus, if, in the drama (s_1, ω_1), σ executes X at time t and position d, and the relevant histories are the same, then σ must also execute X at t and d in (s_2, ω_2).

Definition 13 (π-DC)**.** A CSTNU, $\mathcal{S}$, is *π-dynamically controllable* (π-DC) if there exists a π-execution strategy for $\mathcal{S}$ that is both viable and π-dynamic.

4 DC-Checking for STNUs and CSTNs

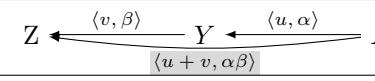
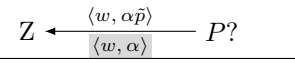
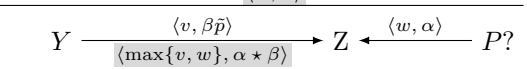
This section summarizes the DC-checking algorithms for STNUs and CSTNs, due to Morris and Muscettola [17] and Hunsberger and Posenato [10], respectively, that play important roles in our *new* CSTNU DC-checking algorithms.

4.1 DC checking for STNUs.

Table 1 lists the five constraint-propagation rules for STNUs due to Morris and Muscettola [17].[2] The *No Case* rule captures standard constraint propagation for STNs. The *Upper Case* rule generates a *conditional* constraint that guards against the possibility that the contingent duration $C - A$ might take on its maximum value. It can be glossed as: "While C remains unexecuted, X must *wait* at least $-u - v$ after the execution of A."[3] The *Lower Case* rule generates a constraint that guards against $C - A$ taking on its minimum value. The *Cross Case* rule generates a conditional constraint that guards against one contingent duration $C - A$ taking on its minimum value, while another $D - X$ takes on its maximum value. The *Label Removal* rule specifies when a conditional constraint can have the force of an unconditional constraint.

The Morris-Muscettola DC-checking algorithm applies the rules from Table 1 in at most $O(N^2)$ rounds, at a cost of $O(N^3)$ per round. Afterward, it computes the *AllMax* STN, which is the *STN* projection in which each contingent link is set to its maximum duration. The *AllMax* STN is computed from the *fully propagated* STNU by: (1) removing all lower-case edges; and (2) removing the

[2]Later STNU algorithms [13, 18] use techniques that do not readily transfer to CSTNUs.
[3]Wait constraints are only relevant if the wait time, $-u - v$, is positive (i.e., if $u + v < 0$).

Rule	Conditions	Pre-existing and Generated Edges
LP:	$u + v < 0$, $\alpha\beta \in \mathcal{P}^*$	
$\mathbf{qR}_0$:	$w < 0$, $\alpha \in \mathcal{Q}^*$	
$\mathbf{qR}_3^*$:	$w < 0$, $\alpha, \beta \in \mathcal{Q}^*$	

In each rule, $X, Y \in \mathcal{T}$; $P? \in \mathcal{OT}$; and $\mathrm{Z} = 0$. In qR_0 and qR_3^*, $\tilde{p} \in \{p, \neg p, ?p\}$; and p does not appear in α or β (in any form).

Table 2: Constraint-propagation rules for π-DC-checking CSTNs

upper-case letters from all (original or generated) upper-case edges. If the *AllMax* STN is consistent, then the STNU is declared to be DC.

4.2 π-DC checking for CSTNs.

Hunsberger et al. [9] presented a 6-rule IR-DC-checking algorithm for CSTNs ("IR" for "instantaneous reaction") that is based on the propagation of labeled constraints. Hunsberger and Posenato [10] subsequently introduced a faster, 3-rule version of their algorithm, called the π-DC-checking algorithm, which is used in this paper. The π-DC-checking algorithm generates constraints whose labels may include *q-literals,* such as $?p$, that indicate that a constraint need only hold as long as the value of p is unknown.

Definition 14 (Q-literals, q-labels)**.** If $p \in \mathcal{P}$, then $?p$ is a *q-literal.* A *q-label* is a (possibly empty) conjunction of literals and/or q-literals. $\mathcal{Q}^*$ denotes the set of all q-labels.
(For example, $p(?q)\neg r$ and $(?q)(?r)t\neg u$ are both q-labels.)

The $\star$ operator extends ordinary conjunction to q-labels. Intuitively, if constraint C_1 is labeled by p, and constraint C_2 is labeled by $\neg p$, then *both* C_1 and C_2 must hold as long as p is unknown, which is represented by $p \star \neg p = ?p$.

Definition 15 ($\star$)**.** The operator, $\star : \mathcal{Q}^* \times \mathcal{Q}^* \to \mathcal{Q}^\star$, is defined thusly. First, for any $p \in \mathcal{P}$, $p \star p = p$ and $\neg p \star \neg p = \neg p$; otherwise, for any $p_1, p_2 \in \{p, \neg p, ?p\}$, $p_1 \star p_2 = ?p$. Next, for any $\ell_1, \ell_2 \in \mathcal{Q}^*$, $\ell_1 \star \ell_2 \in \mathcal{Q}^*$ denotes the conjunction obtained by applying $\star$ in pairwise fashion to matching literals from ℓ_1 and ℓ_2, and conjoining any unmatched literals.
(For example: $(p\neg q(?r)t) \star (qr\neg s) = p(?q)(?r)\neg st$.)

Table 2 lists the sound-and-complete propagation rules for the π-DC-checking algorithm for CSTNs. The LP rule implements ordinary STN constraint propagation except that the labels, α and β, from the parent edges are conjoined in the generated edge. The qR_0 rule stipulates that a lower-bound constraint on $P?$ cannot depend on the value of p determined by executing $P?$. The qR_3^* rule specifies when an occurrence of $p, \neg p$ or $?p$ can be removed from a propositional label. The qR_3^* rule can generate edges whose labels are q-labels.

The π-DC-checking algorithm applies the rules from Table 2 until either: *(Non-DC)* a negative self-loop with a consistent label is found; or *(DC)* no new edges can be generated. The completeness proof for the π-DC-checking algorithm shows how, in positive instances, to construct the *earliest-first strategy,*

whose execution decisions are based on tracking the *current partial scenario* and computing *effective lower bounds* for unexecuted time-points. The *spreading lemma* ensures that lower-bound execution constraints are already present in the fully propagated network, courtesy of the qR_0 and qR_3^* rules. The proof also shows that upper-bound execution constraints cannot generate negative loops in the relevant STN projection. Termination is guaranteed by inserting a global upper bound (or *horizon*), whose value is $h = nM$, where $n = |\mathcal{T}|$ and M is the maximum absolute value of any negative edge. The horizon constraints do not affect the DC property, assuming that all edge weights are rational [14].

An upper bound for the computational complexity of the π-DC-checking algorithm is $O(M|\mathcal{T}|(|\mathcal{P}||\mathcal{T}|3^{|\mathcal{P}|} + |\mathcal{T}|^2 2^{|\mathcal{P}|})) = O(M|\mathcal{T}|^4 3^{|\mathcal{P}|})$. Although exponential in the worst case, it has been shown to be practical across a variety of networks.

5 Algorithm 1: Reducing CSTNU-DC to CSTN-DC

This section introduces a novel DC-checking algorithm for CSTNUs that first transforms its input CSTNU $\mathcal{S}$ into a *DC-equivalent* CSTN $\mathcal{S}'$, and then applies the π-DC-checking algorithm for CSTNs to $\mathcal{S}'$. The transformation for contingent links is illustrated below. For each contingent link, (A, x, y, C), a new observation time-point $P_c?$ is introduced that is constrained to occur exactly x after A. Executing $P_c?$ generates a value for p_c that determines whether the duration $C - A$ shall be x or y.[4] If $p_c = \mathit{true}$, then C must co-occur with $P_c?$ (i.e., x after A); otherwise, C must execute $y - x$ after $P_c?$ (i.e., y after A). Because the CSTNU has been transformed into a CSTN, the horizon value $h = nM$ can be applied to that CSTN without affecting the DC property. The computational cost of this CSTNU-to-CSTN transformation is $O(|\mathcal{L}|)$ (i.e., linear).

$$A \underset{\langle -x, \boxdot \rangle}{\overset{\langle x, \boxdot \rangle}{\longleftrightarrow}} P_c? \underset{\langle 0, \boxdot \rangle,\ \langle x - y, \neg p_c \rangle}{\overset{\langle 0, p_c \rangle,\ \langle y - x, \boxdot \rangle}{\longleftrightarrow}} C$$

6 Algorithm 2: Propagating in the CSTNU

This section introduces a novel DC-checking algorithm for CSTNUs that propagates constraints in the CSTNU using the rules in Table 3. The names of the rules reflect the STNU and CSTN rules from Tables 1 and 2 that they generalize, except that z! is a new kind of rule that *forward propagates* upper-case a-labels. The z! rule is not needed for DC-checking STNUs, but is needed to ensure completeness for CSTNU DC-checking. Note that z! and zqR_3^* can generate *conjunctions* of upper-case a-labels, which can be handled by all of the other rules.

To ensure termination, the algorithm inserts the same horizon constraints seen earlier, then it exhaustively applies the rules from Table 3. It outputs "not

[4] Cairo and Rizzi [19] proved that restricting contingent durations to be either the minimum or maximum value, but nothing in between, does not affect the DC property.

Rule	Conditions	Pre-existing and Generated Edges
(zLp/Nc/Uc)	$u+v<0$, $\alpha\beta\in\mathcal{P}^*$	$\mathrm{Z} \xleftarrow{\langle v,\aleph,\beta\rangle} Y \xleftarrow{\langle u,\diamond,\alpha\rangle} X$; $\mathrm{Z} \xleftarrow{\langle u+v,\aleph,\alpha\beta\rangle} X$
(zLc/Cc)	$x+v<0$, $C\notin\aleph$, $\beta\in\mathcal{P}^*$	$\mathrm{Z} \xleftarrow{\langle v,\aleph,\beta\rangle} C \xleftarrow{\langle x,c,\boxdot\rangle} A$; $\mathrm{Z} \xleftarrow{\langle x+v,\aleph,\beta\rangle} A$
(z!)	$-y+v<0$, $\beta\in\mathcal{P}^*$	$\mathrm{Z} \xleftarrow{\langle v,\aleph,\beta\rangle} A \xleftarrow{\langle -y,C,\boxdot\rangle} C$; $\mathrm{Z} \xleftarrow{\langle -y+v,C\aleph,\beta\rangle} C$
(zLr)	$m=\max\{v,w-x\}$, $C\notin\aleph\aleph_1$, $\beta,\gamma\in\mathcal{Q}^*$	$Y \xrightarrow{\langle v,C\aleph,\beta\rangle} \mathrm{Z} \xleftarrow{\langle w,\aleph_1,\gamma\rangle} A \xrightarrow{\langle x,c,\boxdot\rangle} C$; $Y \xrightarrow{\langle m,\aleph\aleph_1,\beta\star\gamma\rangle} \mathrm{Z}$
(zqR_0)	$w<0$, $\beta\in\mathcal{Q}^*$, $\tilde{p}\in\{p,\neg p,?p\}$	$\mathrm{Z} \xleftarrow{\langle w,\aleph,\beta\tilde{p}\rangle} P?$; $\mathrm{Z} \xleftarrow{\langle w,\aleph,\beta\rangle} P?$
(zqR_3^*)	$w<0$, $\beta,\gamma\in\mathcal{Q}^*$, $\tilde{p}\in\{p,\neg p,?p\}$	$Y \xrightarrow{\langle v,\aleph,\beta\tilde{p}\rangle} \mathrm{Z} \xleftarrow{\langle w,\aleph_1,\gamma\rangle} P?$; $Y \xrightarrow{\langle \max\{v,w\},\aleph\aleph_1,\beta\star\gamma\rangle} \mathrm{Z}$

$\mathrm{Z}=0$; $A,C,X,Y\in\mathcal{T}$; C is contingent; $P?\in\mathcal{OT}$; $\aleph,\aleph_1\in\mathcal{A}_u^*$.

Table 3: Constraint-propagation rules for CSTNU Algorithm 2

Algorithm 2: CSTNU-DC-CH(G)

Input: $G=\langle\mathcal{T},\mathcal{P},\mathcal{C},\mathcal{OT},\mathcal{O},\mathcal{L}\rangle$: a CSTNU instance
Output: the dynamic controllability status of G.
G' = distance graph of G;
$h=M|\mathcal{T}|$, where M is the maximum absolute value of any negative edge;
foreach $X\in\mathcal{T}$ **do** Add the edges, $\mathrm{Z}\xrightarrow{\langle h,\diamond,\boxdot\rangle}X$ and $X\xrightarrow{\langle 0,\diamond,\boxdot\rangle}\mathrm{Z}$, to G'. ;
do
  $G'=\mathrm{zqR}_0(G')$; // Label Modification
  $G'=\mathrm{zqR}_3^*(G')$;
  $G'=zLp/Nc/Uc(G')$; // Edge Generation
  $G'=z!(G')$;
  $G'=zLc/Cc(G')$;
  $G'=zLr(G')$;
  if *(any negative self-loop with a p-label has been found)* **then return** *not DC*;
while *(rules continue to generate new edges)*;
return *DC*

DC" if a negative self-loop with a consistent p-label is found; otherwise, "DC". The pseudo-code for the algorithm is given in Algorithm 2.

We begin with relevant definitions, then prove soundness and completeness.

Definition 16 (Precedes)**.** Let σ be a π-dynamic strategy, (s,ω) any drama, and $(\psi,\pi)=\sigma(s,\omega)$. For any $X,Y\in\mathcal{T}$, if either $[\psi]_X<[\psi]_Y$ or ($[\psi]_X=[\psi]_Y$ and $\pi(X)<\pi(Y)$) then we say that X *precedes* Y in (ψ,π), notated $X\prec_\psi^\pi Y$.

If $X\prec_\psi^\pi P?$, then the decision to execute X cannot depend on the observation of p. In addition, if X and Y are distinct time-points, and at least one of them is an observation time-point, then $X\prec_\psi^\pi Y$ if and only if $\neg(Y\prec_\psi^\pi X)$.

Definition 17 (Satisfy a Labeled Constraint)**.** A π-execution strategy σ *satisfies the labeled constraint* $(Y-X\le\delta,\ell)$, where $\ell\in\mathcal{P}^*$, if, for each drama (s,ω), either $s(\ell)=$ *false* or $[\psi]_Y-[\psi]_X\le\delta$, where $(\psi,\pi)=\sigma(s,\omega)$.

Definition 18 (Satisfy a Contingent Link)**.** A π-execution strategy σ *satisfies the contingent link* (A_i, x_i, y_i, C_i) if for each drama (s, ω), $[\psi]_{C_i} - [\psi]_{A_i} = \omega_i$. In such a case, we also say that σ satisfies the lower-case and upper-case edges associated with that contingent link.

From Defns. 7 and 12, it follows that a viable π-execution strategy σ must satisfy all of the (original) labeled constraints in $\mathcal{S}$ (before any constraint propagation) and all of the (original) lower- and upper-case edges in $\mathcal{S}$.

A constraint-propagation rule is sound if whenever a viable and dynamic σ satisfies the pre-existing edge(s) in that rule, σ must also satisfy the edge generated by that rule. Now, the rules in Table 3 only generate edges pointing at Z, which represent lower-bound constraints; however, the generated edges may have a-labels with multiple UC letters and q-labels (e.g., see rules z!, zLr and zqR_3^*); and many of the rules can propagate such labeled values. Therefore, the semantics of satisfying a lower-bound edge must accommodate such a-labels and q-labels.

Definition 19 (Satisfy a Lower-Bound Constraint)**.** A π-execution strategy σ *satisfies the lower-bound constraint* $(Y \geq \delta, \langle \aleph, \beta \rangle)$ represented by the edge from Y to Z labeled by $\langle -\delta, \aleph, \beta \rangle$, where $\beta \in \mathcal{Q}^*$, and $\aleph \in \mathcal{A}_u^*$, if for each drama (s, ω), *any* of the following hold, where $(\psi, \pi) = \sigma(s, \omega)$:

(1) $[\psi]_Y \geq \delta$;

(2) for some $C_i \in \aleph$, where $(A_i, x_i, y_i, C_i) \in \mathcal{L}$, $\omega_i < y_i$
(i.e., the contingent link does *not* take on its maximum duration);

(3) for some $p \in \beta$, $s(p) = \mathit{false}$;

(4) for some $\neg p \in \beta$, $s(p) = \mathit{true}$; or

(5) for some $?p \in \beta$, $P? \prec_{\psi}^{\pi} Y$.

If $(Z - Y \leq -\delta, \beta)$ (i.e., $(Y \geq \delta, \beta)$) is a labeled constraint in a CSTNU (prior to any propagation), then $\beta \in \mathcal{P}^*$, and the corresponding edge in the graph has the labeled value $\langle -\delta, \diamond, \beta \rangle$. For this edge, clauses (2) and (5) in Defn. 19 are vacuous, whence satisfaction reduces to: $[\psi]_Y \geq \delta$ or $s(\beta) = \mathit{false}$. Thus, Defn. 19 reduces to Defn. 17 for original labeled edges that happen be lower-bound edges.

More generally, it will be useful to note that for any lower-bound edge labeled by $\langle -\delta, \aleph, \beta \rangle$, where $\beta \in \mathcal{P}^*$, satisfaction (i.e., Defn. 19) reduces to:

(i) $[\psi]_Y \geq \delta$;

(ii) for some $C_i \in \aleph$, where $(A_i, x_i, y_i, C_i) \in \mathcal{L}$, $\omega_i < y_i$; or

(iii) $s(\beta) = \mathit{false}$. (‡)

Definition 20 (Soundness)**.** A constraint-propagation rule is *sound* if whenever a *viable* and π-*dynamic* execution strategy σ satisfies the rule's pre-existing (parent) edges, it also satisfies the rule's generated (child) edge.

Note. In each of the soundness proofs below, σ is assumed to be a viable and π-dynamic strategy that satisfies the parent edges in the rule under consideration. Note, too, that soundness proofs for the (zqR_0), (zqR_3^*) and (zLp/Nc/Uc) rules are skipped to save space.

Lemma 1. *The (zLc/Cc) rule from Table 3 is sound.*

Proof. Suppose σ does *not* satisfy the generated edge from A to Z in rule (zLc/Cc). Since the p-label $\alpha\beta$ on the generated edge is consistent, it follows from Defn. 19 that there is some drama (s, ω) such that *all* of the following hold:

($\neg$ i) $[\psi]_A < -u - v$;

($\neg$ ii) for each $C_i \in \aleph$, where $(A_i, x_i, y_i, C_i) \in \mathcal{L}$, $\omega_i = y_i$; and

($\neg$ iii) $s(\alpha\beta) = \mathit{true}$.

First, ($\neg$ iii) implies that $s(\alpha) = \mathit{true}$ and $s(\beta) = \mathit{true}$. Therefore, since σ satisfies the edge from C to Z, ($\neg$ ii) implies that $[\psi]_C \geq -v$. Next, let ω' be the same as ω except that the contingent link AC takes on its *minimum* value u; and let $(\psi', \pi') = \sigma(s, \omega')$. Since σ is viable, $[\psi']_C - [\psi']_A = u$. However, since $C \notin \aleph$, ($\neg$ ii) also holds for ω'; thus, $[\psi']_C \geq -v$ must hold. And, since the only difference between (s, ω) and (s, ω') is the duration of the contingent link AC, the first difference between ψ and ψ' must occur when C executes, which happens *after* A executes. Thus, $[\psi]_A = [\psi']_A = [\psi']_C - u \geq -v - u$, contradicting ($\neg$ i). □

Lemma 2. *The (z!) rule from Table 3 is sound.*

Proof. Let (s, ω) be any drama for which all $C_i \in C\aleph$ take on their maximum durations (i.e., $\omega_i = y_i$), and such that $s(\alpha\beta) = \mathit{true}$; and let $(\psi, \pi) = \sigma(s, \omega)$. Then $s(\beta) = \mathit{true}$ and all $C_i \in \aleph$ take on their maximum durations. Therefore, since σ satisfies the parent edge from A to Z, it follows that $[\psi]_A \geq -v$. Next, since $s(\alpha) = \mathit{true}$, and C takes on its maximum duration, then $[\psi]_C = [\psi]_A - u \geq -v - u$. Thus, σ satisfies the generated edge from C to Z. □

Lemma 3. *The (zLr) rule from Table 3 is sound.*

Proof. Suppose that σ does *not* satisfy the generated edge in rule (zLr). Then, by Defn. 19, there is a drama (s, ω) for which *all* of the following hold:

($1^\dagger$) $[\psi]_Y < -m$;

($2^\dagger$) for each $C_i \in \aleph\aleph_1$, where $(A_i, x_i, y_i, C_i) \in \mathcal{L}$, $[\psi]_{C_i} - [\psi]_{A_i} = y_i$;

($3^\dagger$) for each $p \in \beta \star \gamma$, $s(p) = \mathit{true}$;

($4^\dagger$) for each $\neg p \in \beta \star \gamma$, $s(p) = \mathit{false}$; and

($5^\dagger$) for each $?p \in \beta \star \gamma$, $\neg(P? \prec^{\pi}_{\psi} Y)$.

where $(\psi, \pi) = \sigma(s, \omega)$.

Since σ is valid and satisfies the parent edge from Y to Z, one of the following must hold, by ($\ddagger$), above.

(1) $[\psi]_Y \geq -v$;

(2) for some $C' \in C\aleph$, where $(A', x', y', C') \in \mathcal{L}$, $[\psi]_{C'} - [\psi]_{A'} < y'$;

(3) for some $p \in \beta$, $s(p) = \mathit{false}$;

(4) for some $\neg p \in \beta$, $s(p) = \mathit{true}$; or

(5) for some $?p \in \beta$, $P? \prec^{\pi}_{\psi} Y$.

Now, (1) contradicts ($1^{\dagger}$), since $-v \geq -m$. (2) holding for some $C' \in \aleph$ would contradict ($2^{\dagger}$). And (5) contradicts ($5^{\dagger}$), since $?p \in \beta$ implies $p \in \beta \star \gamma$. Therefore, either (2) holds for $C' = C$ (i.e., $[\psi]_C - [\psi]_A < y$) or some instance(s) of (3) or (4) hold.

Suppose some instance(s) of (3) or (4) hold. For (3), if $p \in \beta$ and $s(p) = \mathit{false}$, then to avoid contradicting ($3^{\dagger}$), we must have $?p \in \beta \star \gamma$, which, by ($5^{\dagger}$), implies that $\neg(P? \prec^{\pi}_{\psi} Y)$. We can assume Y and $P?$ are distinct because any occurrence of p in β could be removed by (zqR_0). Therefore, $Y \prec^{\pi}_{\psi} P?$ must hold. A similar argument applies to any occurrence of $\neg p \in \beta$ that makes (4) hold. Thus, Y must precede any $P?$ for which p or $\neg p$ makes (3) or (4) hold, respectively.

Let s' equal s, except that: if $p \in \beta$ makes (3) hold, then $s'(p) = \mathit{true}$; and if $\neg p \in \beta$ makes (4) hold, then $s'(p) = \mathit{false}$. Since σ satisfies the parent edge from Y to Z, one or more clauses from Defn. 19 must hold for $(\psi', \pi') = \sigma(s', \omega)$. By construction, (3) and (4) do *not* hold for s'; thus, one of the following must hold:

($1'$) $[\psi']_Y \geq -v$;

($2'$) for some $C' \in C\aleph, [\psi']_{C'} - [\psi']_{A'} < y'$; or

($5'$) for some $?p \in \beta$, $P? \prec^{\pi'}_{\psi'} Y$.

Now (ψ, π) and (ψ', π') each determine a sequence of events that can be ordered, first by execution time and, second, for simultaneous events, by order of dependence. Let t' be the *earliest* time at which the two sequences differ. By construction, it must be where some $[\psi]_{R?} = [\psi']_{R?} = t'$, but $s(r) \neq s'(r)$. Furthermore, $Y \prec^{\pi}_{\psi} R?$ and, thus, by the definition of t', $[\psi']_Y = [\psi]_Y$. But then ($1^{\dagger}$) implies that $[\psi']_Y = [\psi]_Y < -m \leq -v$, whence ($1'$) is false. As for ($5'$), if $?q \in \beta$ (and hence $?q \in \beta \star \gamma$) and $Q? \prec^{\pi'}_{\psi'} Y$, then $[\psi']_{Q?} \leq t'$, whence $[\psi']_{Q?} = [\psi]_{Q?}$ and, thus, $Q? \prec^{\pi}_{\psi} Y$, which contradicts ($5^{\dagger}$). Thus, ($2'$) must hold for some $C' \in C\aleph$. Since σ is valid, and the contingent durations in ω did not change from (s, ω) to (s', ω), ($2'$) and (2) are equivalent. Thus, the only possibility is that (2) holds for $C' = C$ (i.e., $[\psi]_C - [\psi]_A < y$).

Next, suppose that $[\psi]_Y < [\psi]_C$. Let ω^+ be the same as ω except that $C - A = y$. It is not hard to check that in the drama (s', ω^+), conditions ($1^{\dagger}$)–($5^{\dagger}$) all hold, but that *none* of the conditions ($1'$)–($5'$) can hold. (Changing to the situation ω^+ removed the last possibility (i.e., that $C - A < y$).) But that contradicts that σ satisfies the parent edge from Y to Z. Thus, $[\psi]_Y \geq [\psi]_C$.
Next, since σ satisfies the edge from A to Z, by Defn. 19, one of these must hold:

(1_A) $[\psi]_A \geq -w$;

(2_A) $\exists C' \in \aleph_1$, $[\psi]_{C'} - [\psi]_{A'} < y'$;

(3_A) for some $p \in \gamma$, $s(p) = \mathit{false}$;

(4_A) for some $\neg p \in \gamma$, $s(p) = \mathit{true}$; or

(5_A) for some $?p \in \gamma$, $P? \prec^{\pi}_{\psi} A$.

Now, ($2^{\dagger}$) contradicts (2_A). And ($5^{\dagger}$) contradicts (5_A). (We can assume that A and $P?$ are distinct since, otherwise, rule (zqR_0) could have been used to remove

any occurrence of $P?$ from γ.) And $[\psi]_A \leq [\psi]_C - x \leq [\psi]_Y - x < -m - x \leq -w$ implies (1_A) is false. (The inequalities follow from σ being viable, from $[\psi]_C \leq [\psi]_Y$, from $(1^\dagger)$, and $m = \max\{v, w - x\}$.) Finally, any $?p$ making (5_A) true would contradict $(5^\dagger)$, since $[\psi]_A < [\psi]_C \leq [\psi]_Y$. Thus, (3_A) or (4_A) must hold.

Now, suppose that some p makes both (3) and (3_A) true. Then $p \in \beta \star \gamma$ and $s(p) = \mathit{false}$, contradicting $(3^\dagger)$. Thus, the letters that make (3) true, if any, must be distinct from the letters that make (3_A) true, if any. Similarly, the letters that make (4) true must be distinct from those that make (4_A) true. In addition, any p that makes (3) true requires $s(p) = \mathit{false}$, which implies that p cannot simultaneously make (4_A) true; and any p that makes (4) true cannot simultaneously make (3_A) true. In short, the letters that make (3) or (4) true are *distinct* from those that make (3_A) or (4_A) true. And, to avoid contradicting $(3^\dagger)$ or $(4^\dagger)$, for any $\pm p$ that makes (3), (4), (3_A) or (4_A) true, $?p$ must be in $\beta \star \gamma$, whence $(5^\dagger)$ yields that Y must precede $P?$ (i.e., $Y \prec^{\pi}_{\psi} P?$).

So, let s'' be the same as s' except that if p makes either (3_A) or (4_A) true, then $s''(p) \neq s'(p) = s(p)$. (s and s' only differ on letters that make (3) or (4) true. Since those letters are distinct from the letters making (3_A) or (4_A) true, s and s' must agree on all letters that make (3_A) or (4_A) true.) Let $(\psi'', \pi'') = \sigma(s'', \omega)$. Let t'' be the first time when the events in (ψ'', π'') and (ψ, π) differ. Then for some $U?$, $[\psi'']_{U?} = [\psi]_{U?} = t''$ and $s''(u) \neq s(u)$; and Y precedes $U?$ in (ψ'', π'') and (ψ, π). Therefore, $[\psi'']_Y = [\psi']_Y = [\psi]_Y \leq t'' \leq t'$.

Since σ satisfies the edge from A to Z, one or more clauses from Defn. 19 must hold for that edge. By construction, the only candidates are:

$(1''_A)$ $[\psi'']_A \geq -w$;

$(2''_A)$ for some $C' \in \aleph_1$, $[\psi'']_A \geq [\psi'']_{C'}$; or

$(5''_A)$ for some $?p \in \gamma$, $P? \prec^{\pi''}_{\psi''} A$.

Since all events occurring before time t'' are executed identically by (ψ, π) and (ψ'', π''), (1_A) being false implies that $(1''_A)$ must also be false, since $[\psi]_A < [\psi]_Y \leq t''$. Similarly, (2_A) being false implies that $(2''_A)$ must also be false. Finally, if $?g \in \gamma$ makes $(5''_A)$ true, that contradicts $(5^\dagger)$, since $?g \in \beta \star \gamma$. Therefore, Case 1 invariably yields a contradiction! □

Theorem 1. *The rules from Table 3 are complete.*

Proof. Let $\mathcal{S}$ be any CSTNU; let $\mathcal{S}^*$ be the CSTNU obtained by fully propagating $\mathcal{S}$ using the rules from Table 3; and suppose that no negative loop with a consistent p-label was found and, thus, the DC-checking algorithm returned *DC.* Let $\mathcal{S}^*_x$ be the *AllMax* CSTN obtained by deleting all LC edges from $\mathcal{S}^*$ and removing all UC a-labels from labeled values in $\mathcal{S}^*$. By construction, the *AllMax* CSTN must already be fully propagated. To see this, note that ignoring the a-labels in the (zLp/Nc/Uc), zqR_0 and zqR^*_3 rules for CSTNUs from Table 3 reduces them to the LP, qR_0 and qR^*_3 rules for CSTNs from Table 2, respectively. Since no negative loop with consistent p-label was found by the CSTNU DC-checking algorithm, none exist in the *AllMax* CSTN; hence it too must be DC.

Construct the *earliest-first strategy* σ for $\mathcal{S}$, as follows. Let α be the *current partial scenario* (CPS), initially $\boxdot$; and let $\mathcal{T}_u$ be the unexecuted time-points, initially $\mathcal{T} \setminus \{Z\}$. For each $X \in \mathcal{T}_u$, compute its *effective lower bound:* $\text{ELB}(X) =$

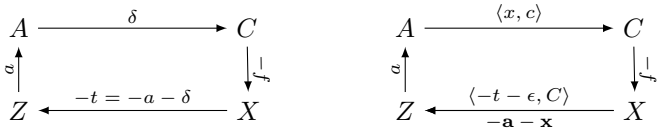


Figure 2: A negative loop in the modified (left) and original (right) CSTNU $\mathcal{S}^*$

$\max\{\delta \mid \exists(X \geq \delta, \ell) \in \mathcal{S}_x^*, appl(\ell, \alpha)\}$.[5] Let (Λ, χ) be the first execution decision: "if nothing happens before time Λ, then execute the time-points in χ", where $\Lambda = \min\{\text{ELB}(X) \mid X \in \mathcal{T}_u\}$; and $\chi = \{X \in \mathcal{T}_u \mid \text{ELB}(X) = \Lambda\}$ [20].

Case 1: No contingent time-point executes before time Λ.
For each active contingent link, (A_i, x_i, y_i, C_i), raise its lower bound to $\Lambda - a_i$, where $a_i = [\sigma(s)]_{A_i}$. This cannot introduce any new constraints into $\mathcal{S}^*$ or $\mathcal{S}_x^*$; thus, both are still DC. And, since no ELB values have changed, (Λ, χ) is the earliest-first decision for the CSTN $\mathcal{S}_x^*$. Thus, inserting the relevant execution constraints cannot introduce any negative loops into any relevant STN projection [10]. Remove any executed time-points from $\mathcal{T}_u$; update the CPS α to include any new observations; and delete any labeled values that are inconsistent with those observations.

Case 2: A contingent time-point C executes at some time $t \leq \Lambda$.
Update $\mathcal{S}^*$, as follows. First, replace the labeled value $\langle -y, C, \boxdot\rangle$ on the original UC edge from C to A with $\langle -\delta, \diamond, \boxdot\rangle$, where $\delta = t - [\sigma(s)]_A$ is the observed duration for the link (A, x, y, C); and replace the labeled value $\langle x, c, \boxdot\rangle$ on the original lower-case edge from A to C by $\langle \delta, \diamond, \boxdot\rangle$. The execution semantics ensures that $\delta \in [x, y]$. Second, *remove* any labeled value $\langle w, \aleph, \beta\rangle$ from $\mathcal{S}^*$ for which $C \in \aleph$. Third, for each $X \in \mathcal{T}_u$, insert a lower-bound constraint, $(X \geq t, \alpha)$. (Although $ELB(X, \alpha) \geq \Lambda \geq t$, the ELB value could have been due to a C-labeled edge which has since been removed.) Finally, fully propagate the modified $\mathcal{S}^*$ CSTNU.

Suppose a negative loop with consistent p-label is discovered in the modified $\mathcal{S}^*$. Any such loop must include the edge from A to C since, otherwise, nothing could prevent the corresponding loop in the original $\mathcal{S}^*$ from being generated, which would be even more negative. First, consider the graphs shown in Fig. 2, where irrelevant details (e.g., p-labels) have been omitted to improve clarity. The lower-bound $X \geq t = a + \delta$ in the modified $\mathcal{S}^*$ generates a negative loop only if $-f < 0$. But that lower bound on X can only be new/relevant if X's original ELB value arose from a C-labeled edge as illustrated on the righthand side, where $-t - \epsilon < -t$. But then the (zLr) rule would have generated the labeled value shown in blue, whence propagating backward from X to C to A to Z, courtesy of the (zLp/Nc/Uc), (z!) and (zLc/Cc) rules, would have generated a loop of length $(-a - x) + (-f) + x + a = -f < 0$ in the original, a contradiction. The only other possibility is if the path from C to Z in Fig. 2 included an occurrence of the (formerly UC) edge from C to A. This case would require a negative path from C to C, which would have made the original $\mathcal{S}^*$ non-DC, a contradiction. Thus, the modified $\mathcal{S}^*$ is necessarily DC. Compute ELB values based on the updated and propagated $\mathcal{S}_x^*$ graph and continue recursively.

The construction of the strategy will be complete (and the network still consistent) once all time-points have been executed. □

[5] $appl(\ell, \alpha)$ holds if ℓ is *applicable* given the CPS α [9]. Formally, $appl(\ell, \alpha)$ holds if each p that appears in *both* ℓ and α appears as p in both or as $\neg p$ in both.

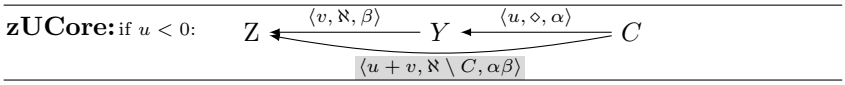


Table 4: Constraint-propagation rule zUCore for CSTNU Algorithm 2

6.1 Computational complexity.

An upper bound for the computational complexity of Algorithm 2 can be derived by adapting the original CSTN DC-checking algorithm complexity, while also considering the presence of a-labels. The three CSTN rules may have to consider all possible combinations of a-labels and q-labels. It is a matter of combinatoric operation to show that the complexity of such rules dominates the final upper bound, $O(M|\mathcal{T}|^4 3^{|\mathcal{P}|} 2^{|\mathcal{L}|})$, where M is the maximum absolute value of any weight in the graph.

7 Empirical Evaluation

This section presents an empirical comparison of the performance of the two DC-checking algorithms introduced in this paper. `Alg1-DC-Ch` is our implementation of Algorithm 1 (cf. Sect. 5), which converts CSTNUs to CSTNs; `Alg2-DC-Ch` is our implementation of Algorithm 2 (cf. Sect. 6) which directly propagates CSTNU constraints.

In `Alg2-DC-Ch` we added also a further rule, zUCore only for improving the practical performance of the algorithm. Rule zUCore optimizes zLp/Nc/Uc when some conditions are satisfied simplifying the a-label $\aleph$ of the new generated labeled value. In general, reducing the size of new a-label allows the algorithm to propagate less labeled values and, therefore, to converge faster. In more details, if a labeled value $\langle v, \aleph, \beta \rangle$ present in an edge heading Z has to be back propagated to a contingent time-point C adding a negative labeled value (as done by zLp/Nc/Uc), then the new labeled value for the edge $C \to$ Z can be simplified removing the C letter in its a-label $\aleph$.

Theorem 2 (Rule `zUCore`)**.** *Rule* ***`zUCore`*** *is sound.*

Proof. Let σ be a viable and π-dynamic strategy that satisfies the parent edges in the (zUCore) rule, (s, ω) any drama and $(\psi, \pi) = \sigma(s, \omega)$ the determined pair of scheduler ψ and observation time-point order π.

Let us assume that to apply zLp/Nc/Uc to the parent edges presented in Table 4 and that letter C is present in $\aleph$. The resulting labeled value $\langle u + v, \aleph, \alpha\beta \rangle$ has the a-label $\aleph$ containing C. Soundness of zLp/Nc/Uc guarantees that σ satisfies the constraint associated to such labeled value.

Without loss of generality, let us assume that $\aleph$ contains only C and that α and β are true. In this case, σ satisfies $Z \xleftarrow{\langle v, C, \beta \rangle} Y$ because either (1) $[\psi]_Y \geq -v$ or (2) $[\psi]_Y \geq [\psi]_C$ and it satisfies $Y \xleftarrow{\langle u, \diamond, \alpha \rangle} C$ because (3) $[\psi]_C \geq [\psi]_Y - u$. Note that since u is negative by hypothesis, C must be schedule strictly after Y.

Therefore, the constraint $Z \xleftarrow{\langle u+v, C, \alpha\beta \rangle} C$ is satisfied either conditions (1) and (3) hold or (1) and (2) hold. The combination of conditions (1) and (3) determines that $[\psi]_C \geq -(u + v)$. The combination (2) and (3) is impossible because it determines that $[\psi]_Y \geq [\psi]_C [\psi]_C \geq [\psi]_Y - u$.

Therefore, the generated constraint is satisfied only because $[\psi]_C \geq -(u+v)$. The a-label C is useless and it can be removed as done by zUCore. □

Both implementations were made in Java and executed on a JVM 8 in a Linux machine with an Intel(R) Xeon(R) E5-2637@3.5 GHz and 128GB of RAM. The source code is freely available [21].

For testing, we built benchmarks with a structure similar to those proposed by Hunsberger and Posenato [22]. They proposed four benchmarks, each having DC CSTNs and non-DC CSTNs, obtained from random workflow schemata generated by the ATAPIS toolset [23]. Each benchmark is generated from random workflows fixing the number of activities, N, and varying the number of observations, $|\mathcal{P}|$. Here, we created three benchmarks, called B3, B4, and B5, each having 250 DC CSTNUs and 250 non-DC CSTNUs, obtained from random workflow schemata with (1) $N = 10$, (2) $|\mathcal{P}|$ equal to 3, 4 and 5, for B3, B4, and B5, respectively, and converting activities as contingents links. It is possible to show that given a workflow instances having N tasks, k XOR connectors (which determines the number of observations), and j AND connectors, the corresponding CSTNU instance has $5 + 2N + 6k + 6j$ nodes, N contingent links, and k observations. In order to evaluate the experimental execution time on CSTNU instances of the same order (=number of nodes), we decided to divide each benchmark, B3, B4, and B5, in five sub-benchmarks, Bj_i with $j = 3, 4, 5$ and $i = 0, 1, 2, 3, 4$, each having 50 instances, generated by fixing also the number of AND connectors to value 0, 1, 2, 3, and 4, respectively. In this way it is possible also to evaluate the impact of the parallel components (AND connectors) on the DC execution time.

Fig. 3 displays the average execution times of the two algorithms over all five sub-benchmarks of B3, B4, and B5 considering only DC instances.

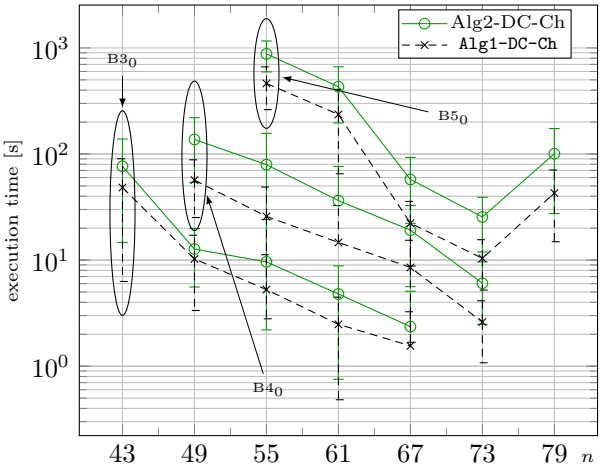


Figure 3: Benchmark B3, B4, and B5

Each data point value is the sample average $\overline{X}_{50} = \frac{\sum_{i=1}^{50} X_i}{50}$ of average execution times obtained considering the fifty instances of the relative sub benchmark. Indeed, each X_i is the average execution time obtained executing 3 times the algorithm on instance having index i in the considered sub benchmark[6]. The error

[6]The determination of all values required to execute the DC checking for 24 000 times,

bar of each data point represents 2.010 times the standard error of the mean, $\frac{S_{50}}{\sqrt{50}}$, where S_{50} is the corrected sample standard deviation, $S_{50} = \sqrt{\frac{\sum_{i=1}^{50}(X_i - \overline{X}_{50})^2}{49}}$. Value 2.010 is the Student's t distribution value with 49 degrees of freedom. Therefore, the error bar represents a 95% confidence interval for the average execution time of the algorithm on instances of the considered benchmark.

From the data in Fig. 3, it emerges that the most difficult instances are related to workflow schemata having no parallel connectors (i.e., instances belonging to the first sub-benchmark of each main benchmark). Moreover, in benchmarks B3 and B4 the two algorithms performs better as the number of AND connector increases. Such behavior is not confirmed in B5, where instances of the last sub benchmark $B5_4$ result to be no the easiest instances as in $B3_4$ and $B4_4$. After a thorough instance structure comparison of all instances in sub benchmarks Bi_4, $i = 3, 4, 5$, we discovered that the setting of parameters for ATAPIS random generator determined that the generator built instances giving priority to the layout of AND connectors before the layout of XOR ones. In this way, in all sub benchmarks but Bi_0, $i = 3, 4, 5$ and $B5_4$, the majority of instances have observation time points that are almost not in sequence. In $B5_4$ this behavior didn't occur and the benchmark contains many instances having three-four observation time points over five in sequence determining a greater number of possible scenarios and, hence, a greater execution time for the checking. In any case, benchmarks Bi_0, $i = 3, 4, 5$ represent always the worst case for the two algorithms.

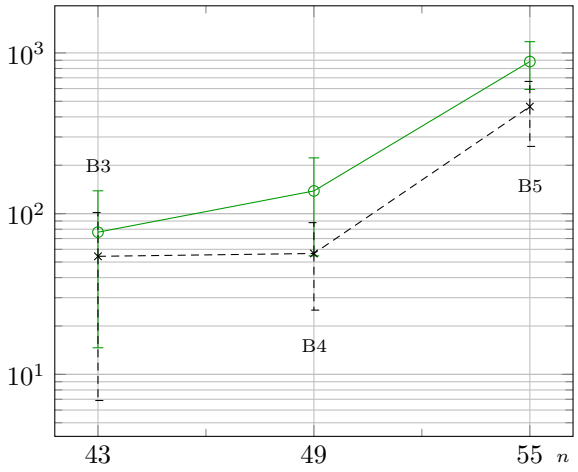


Figure 4: Worst Case in B3, B4, and B5 of DC instances

Fig. 4 reports only the average execution times obtained from instances of sub-benchmark $B3_0$, $B4_0$, and $B5_0$, respectively. Such sub benchmarks represent the worst cases for both the algorithms. Since the y-scale is logarithmic, the diagram shows that the increment of number of propositions determines a significant increment of execution time of the two algorithms.

Many of data points depicted in Figure 3 and Figure 4 have an error bar of relevant size. Therefore, we studied the distribution of execution time of the two algorithms in all benchmarks. We observed that in all benchmarks the

83.18 hours.

distribution is similar to the one depicted in Fig. 5 relative to `Alg1-DC-Ch` in the benchmark B5. The distribution is described in terms of quartiles. Each box has the lower edge equal to the first quartile, Q_1, while the upper edge equal to the third one, Q_3. In this way, each box "contains" 50% of execution times. The line inside each box represents the median of the sample. Horizontal lines outside a box represent the whiskers. The lower whisker value is the smallest data value which is larger than $Q_1 - 1.5IQR$, where IQR is the inter-quartile-range, i.e., $Q_3 - Q_1$. The upper whisker is the largest data value which is smaller than $Q_3 + 1.5IQR$. Diamond represents the average value of the benchmark. Dots above the upper whisker represent the data value outliers. The dot in the highest position in each set of data represents the worst case value of the benchmark. For each sub benchmark, outliers are around 10% of sub benchmark size (50 instances), but their value are very high with respect to the median/mean value (the scale of execution time axis is logarithmic).

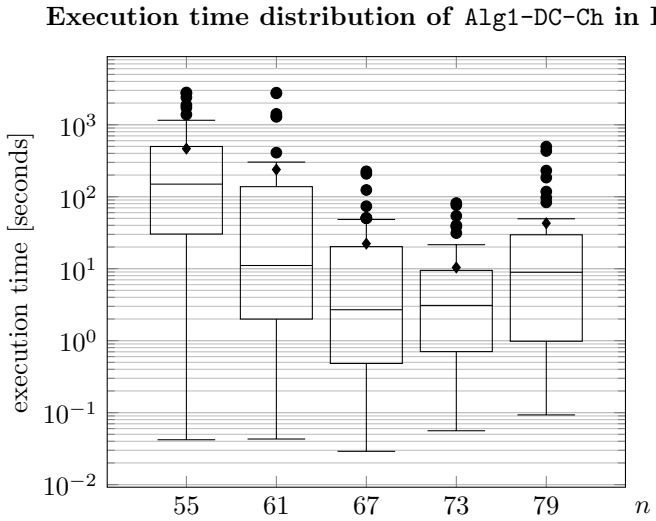


Figure 5: Execution time distribution `Alg1-DC-Ch` in B4 of DC instances

Fig. 6 displays the average execution times of the two algorithms over all five sub-benchmarks of B3, B4, and B5 considering only NOT DC instances.

The behavior of the two algorithms with respect to the sub-benchmarks in checking NON-DC instances is similar to the one in checking DC instances. However, for NON-DC instances the average execution time of each sub benchmark is higher till one order of magnitude than the average execution time of the corresponding DC-instance sub benchmark, cf. Figure 3 vs. Figure 6. As an example, `Alg1-DC-Ch` in sub benchmark $B5_0$ (benchmark with the worst average execution time) has an average execution time of 463 ± 708 seconds when instances are all DC and 1290 ± 1203 seconds when instances are all NON-DC.

In summary, we can state that experimentally Algorithm `Alg1-DC-Ch` performs better than `Alg2-DC-Ch`, and the improvement increases considerably as the size of the instances increases.

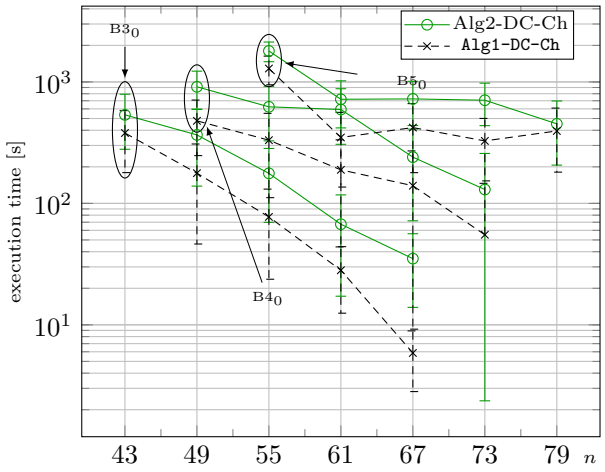


Figure 6: Benchmark B3, B4, and B5 of NON-DC instances

8 Conclusions

This paper presented the first *practical, sound-and-complete* DC-checking algorithms for CSTNUs. The first algorithm converts an input CSTNU into a DC-equivalent CSTN, then runs an existing CSTN algorithm. The second directly propagates CSTNU constraints, using new rules that ensure completeness. An empirical evaluation demonstrated their practicality across a variety of benchmarks. Future work aims to determine whether adding new rules can speed up the algorithms.

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