

Finite-time trajectory tracking control for rigid 3-DOF manipulators with disturbances

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ABSTRACT This paper addresses the robust finite time trajectory tracking control problem for a rigid three-degrees-of-freedom (3-DOF) manipulator system, which is subject to model uncertainty and external disturbances. The disturbances are assumed to be upper bounded. The proposed method incorporates a finite time velocity observer and disturbance observer into the proposed nonlinear control scheme and the conditions of global finite time stability are established. The global finite-time stability of the controlled system is proved by using the finite-time Lyapunov stability theory. Simulation results of a 3-DOF manipulator show that the trajectory tracking was achieved with a finite period of settling time.

INDEX TERMS Finite-time tracking, global finite-time stability, disturbance observer, velocity observer.

I. INTRODUCTION

THE problem of high-precision, fast-response trajectory tracking control of robotic manipulators has long been a research hotspot [1]–[4]. A number of strategies have been proposed for this purpose, including computed torque control [5]–[8] and inverse-dynamics control [6], [9], to guarantee the asymptotic stability of robotic manipulators system [10]. However, the actual trajectories generated using these control methods usually cannot converge to the desired ones within a finite period of settling time. To address this issue, finite-time stabilization strategies have been developed, enabling robotic manipulators to converge fast with high precision within a finite period of time [11]–[18].

Most finite-time control methods need joint position and velocity measurements [15]–[20], which not only require extra sensors, but also add to the cost, weight, size, and noise to the servo systems of manipulators. A number of nonlinear velocity observers have been proposed [21]–[23], however, most of these velocity observers provide estimation in the sense of infinite time. In order to provide faster velocity estimation and higher estimation accuracy, finite time velocity observers have been proposed [22].

In recent years, the disturbance observer (DO) has been introduced for robotic manipulator control [24], [25] in order to deal with the robust problem associated with uncertainties, external disturbances, and unmodeled friction forces in robot

manipulator systems [3], [8], [9], [26]. In general, the main purpose of using DO is to further deduce external unknown or uncertain disturbance torques without the use of an additional sensor [27], [28]. In particular, finite-time DO has been proposed to ensure the finite-time convergence of disturbance estimation [15].

Recently, Bouakrif [23] proposes a trajectory tracking strategy with a nonlinear disturbance observer and velocity observer for robot manipulators. Using position measurements only, it demonstrated that the actual trajectories rapidly tracked the desired trajectories under the conditions of model uncertainties and external disturbances, however, the trajectory tracking was achieved asymptotically.

This paper introduces a finite-time nonlinear control scheme for rigid 3-DOF manipulators with external disturbance and model uncertainty. The proposed method incorporates a finite time velocity observer and disturbance observer into the proposed nonlinear control scheme and the conditions of global finite time stability are established. The global finite-time stability of the controlled system is proved by using the finite-time Lyapunov stability theory. Simulation results of a 3-DOF manipulator shows that trajectory tracking was achieved with a finite period of settling time.

The rest of this paper is organized as follows. The dynamics of a rigid 3-DOF manipulator is introduced in Section 2. Section 3 describes the design of a finite-time disturbance

observer, and the input torque is generated by incorporating the disturbance observer with a nonlinear controller. Then, the finite-time velocity observer is introduced. In Section 4, global finite-time stabilization of a 3-DOF manipulators system is proved by using Lyapunov stability theory. In Section 5, numerical simulation are provided to verify the correctness of the theoretical derivation. Section 6 draws a conclusion and gives some suggestions for further research.

II. THE DYNAMICS OF A RIGID 3-DOF MANIPULATOR

In this section, a rigid 3-DOF manipulator system, which suffers external disturbance and model uncertainty is presented. Later, according to the manipulator model, the state equation of the system is derived and the tracking error dynamics of the manipulator is obtained.

Consider a rigid 3-DOF manipulator system given by the following model:

$$\tau = M(\theta)\ddot{\theta} + B(\theta)\left[\dot{\theta} \cdot \dot{\theta}\right] + C(\theta)\left[\dot{\theta}^2\right] + G(\theta) + \tau_d \quad (1)$$

where $\theta = [\theta_1, \theta_2, \theta_3]^T$, $\dot{\theta}, \ddot{\theta} \in R^{3 \times 1}$ are the position, velocity, and acceleration vectors of manipulator joints, respectively. $M(\theta) \in R^{3 \times 3}$ is the positive definite inertia matrix. $B(\theta) \in R^{3 \times 3}$ is the vector of Coriolis torques. $C(\theta) \in R^{3 \times 3}$ represents the vector of centripetal forces. $[\dot{\theta} \cdot \dot{\theta}] \in R^{3 \times 1}$ and $[\dot{\theta}^2] \in R^{3 \times 1}$ are defined as $[\dot{\theta} \cdot \dot{\theta}] = [\dot{\theta}_1 \dot{\theta}_2, \dot{\theta}_1 \dot{\theta}_3, \dot{\theta}_2 \dot{\theta}_3]^T$ and $[\dot{\theta}^2] = [\dot{\theta}_1^2, \dot{\theta}_2^2, \dot{\theta}_3^2]^T$. $G(\theta) \in R^{3 \times 1}$ is the vector of gravitational torques. $\tau = [\tau_1, \tau_2, \tau_3]^T$ represents the vector of input torques. τ_d , which is (by assumption) absolutely continuous, represents the external disturbance and model uncertainty, and its derivative $\dot{\tau}_d$ is assumed to be a locally bounded Lebesgue measurable to ensure the existence of control τ . In consequence, $\|\tau_d\|$ and $\|\dot{\tau}_d\|$ are assumed to be upper estimated as follows:

$$\|\tau_d\| \leq \beta_0(t), \|\dot{\tau}_d\| \leq \beta_1(t) \quad (2)$$

where $\beta_0(t)$ and $\beta_1(t)$ are known non-negative functions.

The rigid 3-DOF manipulator model is shown in Fig. 1.

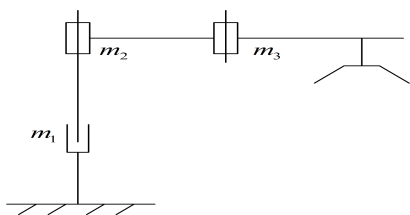


FIGURE 1. Structure of 3-DOF manipulator

As seen from this figure, the first joint is the translational joint, which can move up and down. Both the second and the third joints can rotate freely. The weights of the three joints are $m_1 = 1kg$, $m_2 = 1kg$, and $m_3 = 1kg$, respectively. The movement range d of the first link, satisfies $d = 0 - 0.21m$. The lengths of the other two links are $l_2 = 0.2m$ and $l_3 = 0.2m$, respectively.

Therefore, according to the 3-DOF manipulator system in (1), matrices M , B , C , and G of this 3-DOF manipulator are given as follows:

$$\begin{aligned} M(\theta) &= \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}, B(\theta) = \begin{bmatrix} 0 & 0 \\ 0 & -l_2 l_3 m_3 \sin \theta \\ 0 & 0 \end{bmatrix}, \\ C(\theta) &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} l_2 l_3 m_3 \sin \theta \\ 0 & -\frac{1}{2} l_2 l_3 m_3 \sin \theta & 0 \end{bmatrix}, \\ G(\theta) &= \begin{bmatrix} (m_1 + m_2 + m_3)g \\ 0 \\ 0 \end{bmatrix}. \end{aligned} \quad (3)$$

The components of M are given by

$$\begin{cases} m_{11} = m_1 + m_2 + m_3 \\ m_{12} = m_{13} = m_{21} = m_{31} = 0 \\ m_{22} = \frac{1}{3} m_2 l_2^2 + \frac{1}{3} m_3 l_3^2 + m_3 l_2^2 + l_2 l_3 m_3 \cos \theta_3 \\ m_{23} = \frac{1}{2} l_2 l_3 m_3 \cos \theta_3 + \frac{1}{3} m_3 l_3^2 \\ m_{32} = \frac{1}{2} l_2 l_3 m_3 \cos \theta_3 + \frac{1}{3} m_3 l_3^2 \\ m_{33} = \frac{1}{3} m_3 l_3^2. \end{cases} \quad (4)$$

The state variables x and v represent positions and velocities, respectively:

$$\begin{cases} x = \theta \\ v = \dot{x} \end{cases} \quad (5)$$

where $x = [x_1, x_2, x_3]^T$ and $v = [v_1, v_2, v_3]^T$. Thus, the state equation of the robot manipulator can be described by

$$\begin{cases} \dot{x} = v \\ \dot{v} = f(x, v) + \phi(x, v, t) + p(x)(\tau + \Delta u) \end{cases} \quad (6)$$

where $p(x) = M^{-1}(x)$, $f = -p(x)(B(x)\langle v, v \rangle + C(x)v^2) - p(x)G(x)$, $\phi = -p(x)\tau_d$, and $\Delta u = u - \tau$. u is the control vector of the robot system.

Considering the saturation constraint of the joints' actuators, we design the saturation function to limit the control vector u :

$$u = \begin{cases} u_{upper} & \text{if } \tau > u_{upper} \\ \tau & \text{if } -u_{lower} \leq \tau \leq u_{upper} \\ -u_{lower} & \text{if } \tau < -u_{lower} \end{cases} \quad (7)$$

where u_{upper} and u_{lower} are the known values of the saturation function. Note that $u_{upper} \neq u_{lower}$ is always satisfied.

Despite the uncertainties, external disturbances, unmodeled friction forces, and actuators' saturation, all trajectories of manipulator joints can reach the desired trajectories in finite time for suitable values of the input torque τ .

$x_d = [x_{d1}, x_{d2}, x_{d3}]^T$ represents the desired trajectories and $v_d = [v_{d1}, v_{d2}, v_{d3}]^T$ gives the derivatives of the desired trajectories; thus, tracking errors can be described by $e_x = x - x_d$ and $e_v = v - v_d$. Based on the state space model in (6) the tracking error dynamics of the manipulator are given as follows:

$$\begin{cases} \dot{e}_x = e_v \\ \dot{e}_v = f(x, v) + \phi(x, v, t) + p(x)(\tau + \Delta u) - \dot{v}_d \end{cases} \quad (8)$$

III. DESIGN OF THE CLOSED-LOOP CONTROL SYSTEM

In this section, we describe the design of the closed-loop control system satisfying the conditions of global finite time stability based on the dynamics of the 3-DOF manipulator given in the last section. Firstly we propose a disturbance observer to estimate the disturbance within finite time. Second, by incorporating the nonlinear sliding mode controller with the disturbance observer, input torque is determined to guarantee that the trajectory error converges to zero within finite settle time. In the following subsection, a finite time velocity observer is designed to replace the velocity in the disturbance observer and input torque with its estimation. Finally, the control input is acquired in virtue of modified input torque and the saturation function, which indicates that the closed-loop control system is established.

A. DESIGN OF THE DISTURBANCE OBSERVER AND INPUT TORQUE

In order to estimate the disturbance within finite settle time, the disturbance observer is designed as follows:

$$\begin{cases} \dot{\hat{\phi}} = -K\omega - f(x, v) - \gamma \text{sig}^c(\omega) \\ \quad - \beta_0(t)\lambda_{\max}(M^{-1})\text{sgn}(\omega) - |f(x, v)|\text{sgn}(\omega) \\ \dot{z} = -K\omega + pu - \dot{v}_d - \beta_0(t)\lambda_{\max}(M^{-1})\text{sgn}(\omega) \\ \quad - \gamma \text{sig}^c(\omega) - |f(x, v)|\text{sgn}(\omega) \\ \hat{\tau}_d = p^{-1}\hat{\phi}, \omega = z - e_v \end{cases} \quad (9)$$

where $\hat{\phi}$ and $\hat{\tau}_d$ are the estimates of ϕ and τ_d , respectively. $\omega = [\omega_1, \omega_2, \omega_3]^T$ represents the subsidiary vector. $K\omega = [K_1\omega_1, K_2\omega_2, K_3\omega_3]^T$, $\gamma = [\gamma_1, \gamma_2, \gamma_3]^T$, and $\text{sig}^c(\omega) = [|\omega_1|^c \text{sgn}(\omega_1), |\omega_2|^c \text{sgn}(\omega_2), |\omega_3|^c \text{sgn}(\omega_3)]^T$, where $k_j > 0$, $\gamma_j > 0$ and $0 < c < 1$ are arbitrary coefficients, and $\text{sgn}(\omega)$ denotes the signum function of vector ω . $\lambda_{\max}(M^{-1})$ is the maximum eigenvalue of M^{-1} . Note that variables x and v are assumed to be directly measurable here.

According to the disturbance observer, $\hat{\tau}_d$ can exactly converge to τ_d after finite time T_a , satisfying the following inequality:

$$T_a \leq \frac{\ln \left(\alpha \sqrt{\left(\sum_{j=1}^3 s_j^2(0) + \omega_j^2(0) \right)^{1-c} + \delta} \right) - \ln \delta}{\alpha(1-c)} \quad (10)$$

where parameters α and δ satisfy $\alpha = \min(\min \varepsilon_j, \min k_j)$ and $\delta = \min(\min \gamma_j, \min c_j)$ in which ε_j and c_j are arbitrary coefficient.

Then, for the objective of designing the input torque, a nonlinear sliding mode controller is also introduced. In this paper, we choose the sliding surface $s = [s_1, s_2, s_3]^T$ as follows:

$$s = \omega + \dot{e}_v + \text{sig}^{\varpi_x}(e_v) + \text{sig}^{\varpi_v}(\psi(e_x, e_v)). \quad (11)$$

In (11), parameters $\varpi_x = [\varpi_{x1}, \varpi_{x2}, \varpi_{x3}]^T$ and $\varpi_v = [\varpi_{v1}, \varpi_{v2}, \varpi_{v3}]^T$ are defined, where $0 < \varpi_{xj} < 1$ is an arbitrary coefficient and $\varpi_{vj} = \varpi_{xj}(2 - \varpi_{xj})^{-1}$ for $j = 1, 2, 3$. Also, parameter $\psi(e_x, e_v)$ is given by $\psi(e_x, e_v) = [\psi_j(e_{x1}, e_{v1}), \psi_j(e_{x2}, e_{v2}), \psi_j(e_{x3}, e_{v3})]^T$, where $\psi_j(e_{xj}, e_{vj}) = (2 - \varpi_{xj})^{-1} |e_{vj}|^{2 - \varpi_{xj}} \text{sgn}(e_{vj}) + e_{xj}$ for $j = 1, 2, 3$. By incorporating the controller with a disturbance observer, the input torque τ can be chosen such that the control system achieves global finite-time stability. For this purpose, the control law of τ is proposed as follows:

$$\begin{cases} \tau = p^{-1}(-f(x, v) - \text{sig}^{\varpi_x}(e_v) - \text{sig}^{\varpi_v}(\psi(e_x, e_v)) + \dot{v}_d + \tau_s) - \Delta u \\ \dot{\tau}_s = -\varepsilon s - \varsigma \text{sig}^c(s) - \hat{\phi} - (\beta_0(t)\lambda_{\max}(M^{-1}) + \beta_2(t))\text{sgn}(s) \end{cases} \quad (12)$$

where function $\beta_2(t) = \beta_0(t)\|p(x)\| + \beta_1(t)\lambda_{\max}(M^{-1})$, $\varepsilon s = [\varepsilon_1 s_1, \varepsilon_2 s_2, \varepsilon_3 s_3]^T$, and $\varsigma = [\varsigma_1, \varsigma_2, \varsigma_3]^T$ are as defined. When the input torque τ is applied to the error dynamics of the manipulator in (8), the tracking error is able to reach the sliding surface $s = 0$ in finite time T_a , and then converges to zero after finite time T_b . In other words, all tracking errors reach zero within finite time $T_{tol} = T_a + T_b$.

B. DESIGN OF THE VELOCITY OBSERVER AND CONTROL INPUT

Owing to the drawbacks of direct measurement of joint velocity, the method used in this work only measures position x directly, while velocity v is estimated by velocity observer in adjustable finite time. Assume that velocity v is bounded as follows:

$$\|v\| \leq \kappa \quad (13)$$

where κ is a known constant.

Hence, the velocity observer is designed as follows:

$$\begin{cases} \dot{\hat{x}} = \hat{v} - (\sigma_0 + \sigma_3) \text{sig}(\hat{x} - x) \\ \quad - h(x, \hat{v}) \text{sgn}(\hat{x} - x) - \sigma_2 \text{sig}^\xi(\hat{x} - x) \\ \dot{\hat{v}} = -\sigma_1 \hat{v} - \sigma_3 \|\hat{x} - x\| + f(x, \hat{v}) + p(x)u \\ h(x, \hat{v}) = (\|\hat{v}\| + \kappa)L_1(x, \hat{v}) + L_2(x, \hat{v}) \\ \quad + \sigma_1 \kappa + \sigma_2 (\|\hat{v}\| + \kappa)^\xi + \lambda_{\max}(M^{-1})\beta_0(t) \end{cases} \quad (14)$$

where $\hat{x} = [\hat{x}_1, \hat{x}_2, \hat{x}_3]^T$ and $\hat{v} = [\hat{v}_1, \hat{v}_2, \hat{v}_3]^T$ are estimates of x and v , respectively. $\sigma_0, \sigma_2, \sigma_3 > 0$, and $0 < \xi < 1$ are arbitrary parameters which can be set by users. Define $h(x, \hat{v}) = B(x)\langle \hat{v}, \hat{v} \rangle + C(x)\hat{v}^2$, $L_1(x, \hat{v}) = \lambda_{\max}(M^{-1})(\|B\| + \|C\|)(\|\hat{v}\| + \kappa)$, $L_2(x, \hat{v}) = 2\lambda_{\max}(M^{-1})(\|B\| + \|C\|)\|\hat{v}\|^2$, and $\sigma_1 = 1 + \sigma_0$. Clearly, through this velocity observer, the obtained estimate of velocity \hat{v} can converge to v within finite time T_c , which satisfies the following inequality:

$$T_c \leq \frac{\ln \left(\sigma_0 (\|\bar{e}_x(0)\| + \|\bar{e}_v(0)\|)^{1-\xi} + \sigma_2 \right) - \ln \sigma_2}{\sigma_0(1-\xi)} \quad (15)$$

where $\bar{e}_x = \hat{x} - x$, $\bar{e}_v = \hat{v} - v$.

Remark 1: By substituting \hat{v} for v , the disturbance observer in (9) and input torque in (12) are modified. Since the design of the finite-time velocity observer is finished, the global finite-time is changed as $T_{tol}' = T_{tol} + T_c = T_a + T_b + T_c$.

In order to guarantee that the system achieves global stability in finite time T_{tol}' , the modified input torque $\tilde{\tau}$ is proposed as follows:

$$\tilde{\tau} = \begin{cases} c\tau & 0 \leq t < T_c \\ \tau & t \leq T_c \end{cases} \quad (16)$$

where $c\tau \in R^{3 \times 1}$ is an arbitrary continuous function.

Hence, based on the saturation function in (7), the control input u is given by

$$u = \begin{cases} u_{upper} & \text{if } \tilde{\tau} > u_{upper} \\ \tilde{\tau} & \text{if } -u_{lower} \leq \tilde{\tau} \leq u_{upper} \\ -u_{lower} & \text{if } \tilde{\tau} < -u_{lower} \end{cases} \quad (17)$$

Therefore, the closed-loop control system of the 3-DOF manipulator, which is composed of the disturbance observer, velocity observer, and the nonlinear controller, is established to guarantee the trajectory tracking in finite time T_{tol}' .

IV. GLOBAL FINITE-TIME STABILITY

By using Lyapunov theory, this section proves the global finite-time stability of the closed-loop control system. That is to say, the finite-time tracking toward the desired trajectory can be testified.

Two lemmas are introduced to contribute to the demonstration of global finite-time stability.

Lemma 1 [11]: Consider a nonlinear system $\dot{x}(t) = f(t)$, $x \in R^n$, $x(0) = x_0$ with the equilibrium point $x = 0$. Assume that there exist three real numbers $\rho_1 > 0$, $\rho_2 > 0$, $0 < \beta < 1$ and a continuously differentiable positive function $V(x) : R^n \rightarrow R$ such that the inequality $\dot{V}(x) + \rho_1 V(x) + \rho_2 V^\beta(x) \leq 0$ is always fulfilled for any solution $x(t, x_0)$ of the system. Then, the equilibrium point $x = 0$ is globally finite-time stable and the finite convergence time T , which is called the finite settling time, satisfies the inequality $T(x_0) \leq (\rho_1(1-\beta))^{-1} (\ln(\rho_1 V^{1-\beta}(x_0) + \rho_2) - \ln \rho_2)$.

Lemma 2 [29]: Consider a double integrator system is expressed by differential equations $\dot{x} = v$, $\dot{v} = u$, where u is the control input, and x and v are the state variables. This system can be made globally finite-time stable by implementing control input (18).

$$\begin{aligned} u &= -|v|^\varpi \text{sgn}(v) - |\psi(x, v)|^{\varpi(2-\varpi)-1} \text{sgn}(\psi(x, v)) \\ \psi(x, v) &= x + (2-\varpi)^{-1} |v|^{2-\varpi} \text{sgn}(v). \end{aligned} \quad (18)$$

where tuning coefficient $0 < \varpi < 1$ can be used to adjust the finite convergence time. Furthermore, by using control input (18), state variables x and v exactly converge to zero in the finite convergence time specified as

$$\begin{aligned} T(x(0), v(0)) &\leq \frac{V(x(0), v(0))^F}{Fl}, \\ F &= (3-\varpi)^{-1} (1-\varpi) \end{aligned} \quad (19)$$

where positive function $V(x, v)$ and coefficient l are defined such that arbitrary parameters ζ and Φ fulfill constraints $0 < \zeta < 1$ and $\Phi > 1$, respectively [29].

$$\begin{aligned} V &= \frac{1}{J} |\psi(x, v)|^J + \zeta v \psi(x, v) + \frac{\Phi}{3-\varpi} |v|^{3-\varpi} \\ l &= -\max_{(x,v) \in D} \dot{V}(x, v), D = \{(x, v) : V(x, v) = 1\} \end{aligned} \quad (20)$$

where $J = (2-\varpi)^{-1} (3-\varpi)$.

Theorem 1 demonstrates that the tracking error dynamics expressed by (8) reach sliding surface $s = 0$ in finite time T_a . Theorem 2 shows that the estimation of disturbances $\hat{\tau}_d$ converges to actual disturbances $s = 0$ in finite time T_a . Theorem 3 proves that the tracking errors, which had reached $s = 0$, arrive at zero in finite time T_b . Moreover, Theorem 4 shows that the estimations of velocity converge to actual velocity in finite time T_c . Finally, Theorem 5 shows that trajectory tracking is achieved within finite time $T_{tol}' = T_a + T_b + T_c$.

Theorem 1: Consider control torque (12) and sliding mode (11). Then, the tracking errors expressed by (8) reach sliding surface $s = 0$ within adjustable finite time T_a given by (10).

Proof: Consider candidate Lyapunov function $V = 0.5 (s^T s + \omega^T \omega)$. By substituting for τ from (12) into (8), one can obtain the following equation:

$$\dot{e}_v = \phi - \text{sig}^{\varpi x}(e_v) - \text{sig}^{\varpi v}(\psi(e_x, e_v)) + \tau_s \quad (21)$$

Then, by substituting \dot{e}_v from (21) into (11), sliding surface s is derived as $s = \phi + \tau_s + \omega$ so that $\dot{s} = \dot{\phi} + \dot{\tau}_s + \dot{\omega}$. By taking into account $\dot{\omega}$ and $\dot{\tau}_s$ expressed by (9) and (12), this yields

$$\begin{aligned} \dot{s} &= \dot{\phi} - \phi - \varepsilon s - \zeta \text{sig}^c(s) \\ &\quad - (\beta_0(t) \lambda_{\max}(M^{-1}) + \beta_2(t)) \text{sgn}(s) \end{aligned} \quad (22)$$

By employing inequalities $\|s\| \leq \sum_{j=1}^3 |s_j|$, $s^T \dot{\phi} \leq \|s\| \|\dot{\phi}\|$, and $-s^T \phi \leq \|s\| \|\phi\|$, it follows that

$$\begin{aligned} s^T \dot{s} &\leq \|s\| \|\dot{\phi}\| + \|s\| \|\phi\| - \sum_{j=1}^3 \zeta_j |s_j|^{c+1} \\ &\quad - \sum_{j=1}^3 \varepsilon_j s_j^2 - (\beta_0(t) \lambda_{\max}(M^{-1}) + \beta_2(t)) \|s\| \end{aligned} \quad (23)$$

According to the definition $\phi = -p(x) \tau_d$, inequalities (2), and inequality $M^{-1} \leq \lambda_{\max}(M^{-1})$ [14], the following inequalities are obtained:

$$\|\dot{\phi}\| \leq \beta_2(t), \|\phi\| \leq \beta_0(t) \lambda_{\max}(M^{-1}) \quad (24)$$

where function $\beta_2(t)$ is defined as $\beta_2(t) = \beta_0(t) \|p(x)\| + \beta_1(t) \lambda_{\max}(M^{-1})$.

Applying (24) to (23) results in

$$s^T \dot{s} \leq -\varepsilon_{\min} \sum_{j=1}^3 s_j^2 - \zeta_{\min} \sum_{j=1}^3 |s_j|^{c+1} \quad (25)$$

where $\varepsilon_{\min} = \min \varepsilon_j$ and $\zeta_{\min} = \min \zeta_j$. Based on (9), the derivation of ω is obtained as follows:

$$\begin{aligned} \dot{\omega} &= \dot{z} - \dot{e}_v = -K\omega + pu - \dot{v}_d - \zeta \text{sig}^c(\omega) - \dot{e}_v \\ &\quad - \beta_0(t) \lambda_{\max}(M^{-1}) \text{sgn}(\omega) - |f(x, v)| \text{sgn}(\omega) \end{aligned} \quad (26)$$

Applying (8) to (26), this yields

$$\dot{\omega} = -K\omega - \beta_0(t) \lambda_{\max}(M^{-1}) \operatorname{sgn}(\omega) - \gamma \operatorname{sig}^c(\omega) - |f(x, v)| \operatorname{sgn}(\omega) - f(x, v) - \phi. \quad (27)$$

Thus,

$$\omega^T \dot{\omega} = -\omega^T (f(x, v) + \phi + \gamma \operatorname{sig}^c(\omega) + K\omega) - \beta_0(t) \lambda_{\max}(M^{-1}) \left(\sum_{j=1}^3 |\omega_j| \right) - |\omega|^T |f(x, v)|. \quad (28)$$

Consider that $-|\omega|^T |f(x, v)| - \omega^T f(x, v) < 0$, $-\omega^T \phi \leq \|\omega\| \|\phi\|$ and $\|\omega\| \leq \sum_{j=1}^3 |\omega_j|$ are always fulfilled. Thus, $\omega^T \dot{\omega}$ satisfies the following inequality:

$$\omega^T \dot{\omega} \leq -\sum_{j=1}^3 k_j \omega_j^2 - \sum_{j=1}^3 \gamma_j |\omega_j|^{c+1} + \|\omega\| (\|\phi\| - \beta_0(t) \lambda_{\max}(M^{-1})) \quad (29)$$

By employing $\|\phi\| \leq \beta_0(t) \lambda_{\max}(M^{-1})$, (29) can be simplified to

$$\omega^T \dot{\omega} \leq -k_{\min} \sum_{j=1}^3 \omega_j^2 - \gamma_{\min} \sum_{j=1}^3 |\omega_j|^{c+1} \quad (30)$$

Taking into account (30) and (25), and applying definitions $\alpha = \min(\min \varepsilon_j, \min k_j)$ and $\delta = \min(\min \gamma_j, \min \zeta_j)$, derivation \dot{V} can be written as

$$\begin{aligned} \dot{V} &= 0.5 (s^T \dot{s} + \omega^T \dot{\omega}) \\ &\leq -\alpha \sum_{j=1}^3 (\omega_j^2 + s_j^2) - \delta \sum_{j=1}^3 (|\omega_j|^{c+1} + |s_j|^{c+1}) \end{aligned} \quad (31)$$

Introducing inequalities $\sum_{i=1}^3 |a_i|^\beta \geq \left(\sum_{i=1}^3 |a_i| \right)^\beta$ and $\sum_{i=1}^3 |a_i|^{1+\beta} \geq \sqrt{\left(\sum_{i=1}^3 |a_i|^2 \right)^{1+\beta}}$ for $0 < \beta < 1$ results in

$$\begin{aligned} \dot{V} &\leq -2\alpha V - \delta \left(\sqrt{\left(\sum_{j=1}^3 (s_j^2 + \omega_j^2) \right)^{c+1}} \right) \\ &= -2\alpha V - \delta \sqrt{2^{c+1}} \sqrt{V^{c+1}} \end{aligned} \quad (32)$$

Suppose $\rho_1 = 2\alpha$, $\rho_2 = \delta \sqrt{2^{c+1}}$, and $\beta = 0.5(c+1)$. Thus, $\dot{V}(x) + \rho_1 V(x) + \rho_2 V^\beta(x) \leq 0$. According to Lemma 1, the equilibrium point $s = 0$ is stable and $\omega = 0$ is attainable after finite time T_a . This completes the proof.

Theorem 2: Consider disturbance observer (9). Then, disturbance errors converge to zero within finite time T_a given by (10).

Proof: From Theorem 1, it is known that $\omega=0$ is attainable for $t \geq T_a$ such that $\dot{\omega} = 0$. By setting $\bar{\phi} = \hat{\phi} - \phi$, this yields

$$\begin{aligned} \dot{\bar{\phi}} &= -\dot{\phi} - K\omega - f(x, v) - \beta_0(t) \lambda_{\max}(M^{-1}) \operatorname{sgn}(\omega) \\ &\quad - \gamma \operatorname{sig}^c(\omega) - |f(x, v)| \operatorname{sgn}(\omega) = -\dot{\phi} - K\omega + \phi \\ &\quad - \dot{e}_v + p(x)u - \dot{v}_d - \beta_0(t) \lambda_{\max}(M^{-1}) \operatorname{sgn}(\omega) \\ &\quad - \gamma \operatorname{sig}^c(\omega) - |f(x, v)| \operatorname{sgn}(\omega) = \dot{z} - \dot{e}_v = \dot{\omega} \end{aligned} \quad (33)$$

Clearly, for $t \geq T_a$, $\bar{\phi} = 0$, as a result, $\hat{\phi} = \phi$. Thus, after finite time T_a , $\hat{\tau}_d = p^{-1} \hat{\phi}$ converges to $\tau_d = p^{-1} \phi$. This completes the proof.

Theorem 3: Consider tracking error dynamics (8) and sliding mode (11). Then, trajectory tracking errors, which had arrived at $s = 0$, converge to zero within finite time T_b specified as

$$T_b \leq \max_j \left\{ \frac{1}{l_j F_j} (V_j(e_{xj}(T_a), e_{vj}(T_a)))^{F_j} \right\} \quad (34)$$

with $F_j = (3 - \varpi_{xj})^{-1} (1 - \varpi_{xj})$

where positive function $V_j(e_{xj}, e_{vj})$ and l_j are defined so that parameters ζ_j and Φ_j follow $0 < \zeta_j < 1$ and $\Phi_j > 1$, respectively.

$$\begin{aligned} V_j &= \frac{|\psi_j(e_{xj}, e_{vj})|^{J_j}}{J_j} + \zeta_j e_{vj} \psi_j(e_{xj}, e_{vj}) + \frac{\Phi_j |e_{vj}|^{3-\varpi_{xj}}}{3 - \varpi_{xj}}, \\ l_j &= -\max_{(e_{xj}, e_{vj}) \in D_j} \dot{V}_j(e_{xj}, e_{vj}), \\ D_j &= \{(e_{xj}, e_{vj}) : V_j(e_{xj}, e_{vj}) = 1\}. \end{aligned} \quad (35)$$

where $J_j = (2 - \varpi_{xj})^{-1} (3 - \varpi_{xj})$.

Proof: According to Theorem 1, $s = \omega = 0$ for $t \geq T_a$. So, from (11) one can obtain

$$\dot{e}_v = -\operatorname{sig}^{\varpi_x}(e_v) - \operatorname{sig}^{\varpi_v}(\psi(e_x, e_v)). \quad (36)$$

Therefore, (8) can be rewritten as

$$\begin{aligned} \dot{e}_x &= e_v \\ \dot{e}_v &= -\operatorname{sig}^{\varpi_x}(e_v) - \operatorname{sig}^{\varpi_v}(\psi(e_x, e_v)) \end{aligned} \quad (37)$$

In consequence, three independent double integrator subsystems that belong to system (37) are described as below for $j = 1, 2, 3$:

$$\begin{aligned} \dot{e}_{xj} &= e_{vj} \\ \dot{e}_{vj} &= -|e_{vj}|^{\varpi_{xj}} \operatorname{sgn}(e_{vj}) - |\psi_j(e_{xj}, e_{vj})|^{\varpi_{vj}} \operatorname{sgn}(\psi_j(e_{xj}, e_{vj})) \end{aligned} \quad (38)$$

Based on Lemma 2 and the conclusion of [29], the following inequality can be written as

$$\begin{aligned} \dot{V}_j(e_{xj}, e_{vj}) &\leq l_j (V_j(e_{xj}, e_{vj}))^{\frac{2}{3 - \varpi_{xj}}}, \\ &\text{for all } [e_{xj}, e_{vj}]^T \in R^2. \end{aligned} \quad (39)$$

where $l_j = -\max_{(e_{xj}, e_{vj}) \in D_j} \dot{V}_j(e_{xj}, e_{vj})$. As a result, tracking errors in each subsystem converge to zero after finite time ${}_j T_b$ specified as

$${}_j T_b \leq \frac{1}{l_j F_j} (V_j(e_{xj}(T_a), e_{vj}(T_a)))^{F_j} \quad (40)$$

Obviously, for system (37) all tracking errors that had arrived at $s = 0$ converge to zero after finite time T_b . In other words, tracking errors converge to zero within finite time $T_{tot} = T_a + T_b$.

Proposition 1: $f(x, v) \in R^3$, which is defined in (6), satisfies the following relation:

$$\|f(x, \hat{v}) - f(x, v)\| \leq L_1(x, \hat{v}) \|\hat{v} - v\| + L_2(x, \hat{v}) \quad (41)$$

Proof: From (6), this yields

$$\begin{aligned} \|f(x, \hat{v}) - f(x, v)\| &= \|M^{-1}(B(\langle v, v \rangle - \langle \hat{v}, \hat{v} \rangle) + C(v^2 - \hat{v}^2))\| \\ &\leq \|M^{-1}B(\langle v, v \rangle - \langle \hat{v}, \hat{v} \rangle)\| + \|M^{-1}C(v^2 - \hat{v}^2)\| \\ &\leq \lambda_{\max}(M^{-1})(\|B\|(\|v\| + \|\hat{v}\|) + \|C\|(\|v\|^2 + \|\hat{v}\|^2)) \end{aligned} \quad (42)$$

Based on the definitions of $\langle v, v \rangle$ and v^2 , the following inequalities hold:

$$\|\langle v, v \rangle\| \leq \|v\|^2, \quad \|v^2\| \leq \|v\|^2 \quad (43)$$

So, (42) can be rewritten as

$$\begin{aligned} &\|f(x, \hat{v}) - f(x, v)\| \\ &\leq \lambda_{\max}(M^{-1})(\|B\|(\|v\|^2 + \|\hat{v}\|^2) + \|C\|(\|v\|^2 + \|\hat{v}\|^2)) \\ &\leq \lambda_{\max}(M^{-1})(2(\|B\| + \|C\|)\|\hat{v}\|^2) \\ &\quad + \lambda_{\max}(M^{-1})(\|B\| + \|C\|)(\|v\| - \|\hat{v}\|)(\|v\| + \|\hat{v}\|) \end{aligned} \quad (44)$$

Then, in view of (13), one can obtain

$$\begin{aligned} &\|f(x, \hat{v}) - f(x, v)\| \\ &\leq \lambda_{\max}(M^{-1})(2(\|B\| + \|C\|)\|\hat{v}\|^2 + (\|B\| + \|C\|)(\|\hat{v} - v\|)(\|v\| + \|\hat{v}\|)) \\ &= L_1(x, \hat{v})\|\hat{v} - v\| + L_2(x, \hat{v}). \end{aligned} \quad (45)$$

This completes the proof.

Theorem 4: Consider velocity observer (14) and assume that estimation errors should not have a finite escape time. Then, estimates \hat{x} and \hat{v} converge to x and v exactly within finite time T_c given by (15).

Proof: Based on (14) and the non-existence of a finite escape time in estimation errors, the error dynamics can be written as follows:

$$\begin{aligned} \dot{\bar{e}}_x &= \bar{e}_v - (\sigma_0 + \sigma_3) \text{sig}(\bar{e}_x) - h(x, \hat{v}) \text{sgn}(\bar{e}_x) - \sigma_2 \text{sig}^\xi(\bar{e}_x) \\ \dot{\bar{e}}_v &= -\sigma_1 \hat{v} - \sigma_3 |\bar{e}_x| + f(x, \hat{v}) - f(x, v) - \phi(x, v, t) \end{aligned} \quad (46)$$

Here, the candidate Lyapunov function is chosen as $V = \|\bar{e}_x\| + \|\bar{e}_v\|$, such that its time derivative $\dot{V} = \|\dot{\bar{e}}_x\| - \|\dot{\bar{e}}_v\|$ where $(\bar{e}_x)_j$ is the j^{th} element of \bar{e}_x . Introducing inequalities $\sum_{i=1}^3 |a_i|^\beta \geq \left(\sum_{i=1}^3 |a_i|\right)^\beta$ and

$$\sum_{i=1}^3 |a_i|^{1+\beta} \geq \sqrt{\left(\sum_{i=1}^3 |a_i|^2\right)^{1+\beta}} \quad \text{for } 0 < \beta < 1 \text{ results in}$$

$$\sum_{j=1}^3 |(\bar{e}_x)_j| \geq \|\bar{e}_x\|, \quad \sum_{j=1}^3 |(\bar{e}_x)_j|^{\xi+1} \geq \|\bar{e}_x\|^{\xi+1} \quad (47)$$

By substituting for $\dot{\bar{e}}_x$ and $\dot{\bar{e}}_v$ from (46) into $\dot{V} = \|\dot{\bar{e}}_x\| - \|\dot{\bar{e}}_v\|$, and employing (47), one can obtain

$$\begin{aligned} \dot{V} &\leq \|\bar{e}_x\|^{-1} \bar{e}_x^T \bar{e}_v - (\sigma_0 + \sigma_3) \|\bar{e}_x\| - h(x, \hat{v}) - \sigma_1 \|\bar{e}_v\| \\ &\quad - \sigma_2 \|\bar{e}_x\|^\xi - \sigma_1 \|\bar{e}_v\|^{-1} \bar{e}_v^T v - \sigma_3 \|\bar{e}_v\|^{-1} \bar{e}_v^T |\bar{e}_x| \\ &\quad - \|\bar{e}_v\|^{-1} \bar{e}_v^T \phi + \|\bar{e}_v\|^{-1} \bar{e}_v^T (f(x, \hat{v}) - f(x, v)) \end{aligned} \quad (48)$$

Applying inequalities $|\bar{e}_v^T (f(x, \hat{v}) - f(x, v))| \leq \|\bar{e}_v^T\| \|f(x, \hat{v}) - f(x, v)\|$, $|\bar{e}_v^T v| \leq \|\bar{e}_v\| \|v\|$, $|\bar{e}_v^T \phi| \leq \|\bar{e}_v\| \|\phi\|$ results in

$$\begin{aligned} \dot{V} &\leq \|\bar{e}_v\| - (\sigma_0 + \sigma_3) \|\bar{e}_x\| - h(x, \hat{v}) - \sigma_1 \|\bar{e}_v\| + \|\phi\| \\ &\quad + \sigma_1 \|v\| - \sigma_2 \|\bar{e}_x\|^\xi + \sigma_3 \|\bar{e}_x\| + \|f(x, \hat{v}) - f(x, v)\| \end{aligned} \quad (49)$$

By employing inequalities (13) and (24), equality $\sigma_1 = 1 + \sigma_0$ and proposition 1, and substituting for $h(x, \hat{v})$, (48) can be rewritten as

$$\begin{aligned} \dot{V} &\leq -\sigma_0 \|\bar{e}_v\| - \sigma_0 \|\bar{e}_x\| - \sigma_2 \|\bar{e}_x\|^\xi \\ &\quad - \sigma_2 (\|\hat{v}\| + \kappa)^\xi - L_2(\|\hat{v}\| + \kappa) + L_2 \|\bar{e}_v\| \end{aligned} \quad (50)$$

Then, by incorporating with inequalities $\|\bar{e}_v\| \leq (\|\hat{v}\| + \kappa)$ and $-(\|\hat{v}\| + \kappa)^\xi \leq -\|\bar{e}_v\|^\xi$, this yields

$$\dot{V} \leq -\sigma_0 (\|\bar{e}_x\| + \|\bar{e}_v\|) - \sigma_2 (\|\bar{e}_x\|^\xi + \|\bar{e}_v\|^\xi) \quad (51)$$

According to inequality $\sum_{i=1}^3 |a_i|^\beta \geq \left(\sum_{i=1}^3 |a_i|\right)^\beta$, $\|\bar{e}_x\|^\xi + \|\bar{e}_v\|^\xi \geq (\|\bar{e}_x\| + \|\bar{e}_v\|)^\xi$ is obtained.

Moreover, in view of function $V = \|\bar{e}_x\| + \|\bar{e}_v\|$, (51) can be rewritten as $\dot{V} + \sigma_0 V + \sigma_2 V^\xi \leq 0$. By setting $\rho_1 = \sigma_0$, $\rho_2 = \sigma_2$, $\beta = \xi$, inequality $\dot{V}(\bar{e}_x, \bar{e}_v) + \rho_1 V(\bar{e}_x, \bar{e}_v) + \rho_2 V^\beta(\bar{e}_x, \bar{e}_v) \leq 0$ is satisfied. Considering Lemma 1, errors \bar{e}_x and \bar{e}_v reach zero for $t \geq T_c$. This completes the proof.

Theorem 5: Consider a closed loop system of a 3-DOF manipulator, which only measures positions of joints. Then, joint positions x converge to the desired trajectories x_d within finite time $T'_{tol} = T_a + T_b + T_c$, where T_c satisfies inequality (15), and T_b and T_a are determined by

$$T_b \leq \max_j \left\{ \frac{V_j(e_{xj}(T_a + T_c), e_{vj}(T_a + T_c))^{F_j}}{l_j F_j} \right\} \quad (52)$$

$$T_a \leq \frac{\ln \left(\alpha \sqrt{\left(\sum_{j=1}^3 s_j^2(T_c) + \omega_j^2(T_c) \right)^{1-c}} + \delta \right) - \ln \delta}{\alpha(1-c)} \quad (53)$$

Proof: Based on Theorem 4, estimate \hat{v} converges to v after finite time T_c using the velocity observer. Note that position x is applied to observer (14) as input to estimate v , and for $t \geq T_c$ improved torque $\tilde{\tau}$ becomes τ , and \hat{v} becomes v . Consequently, by considering Theorem 1, 2, 3 and 4, it is clear that x converges to x_d within finite time $T'_{tol} = T_a + T_b + T_c$, where T_c satisfies inequality (15). In addition, it is clear that initial condition $t = 0$ in (10) could be substituted

by $t = T_c$, and initial condition $t = T_a$ in (33) could be replaced by $t = T_a + T_c$. This completes the proof.

Remark 2: The constraint of input torque indicated in (17), which is employed to design control signal for eliminating the effect of actuator saturation, can enhance the performance of the control system of manipulator.

Remark 3: The parameters of nonlinear observers given in (9) and (14) as well as those of control torque provided in (12), which are in proper design, can decrease global finite time and improve control precision.

V. NUMERICAL SIMULATION

In this section, finite-time trajectory tracking is verified by numerical simulation for the 3-DOF manipulator described in Section 2.

A schematic block diagram of the control system of the closed-loop 3-DOF manipulator is given as Fig. 2.

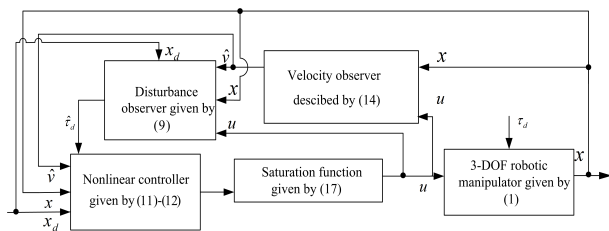


FIGURE 2. Closed-loop control system of 3-DOF manipulator.

Consider the model of the 3-DOF manipulator in (1) and the matrix parameters in (3). The desired trajectory was chosen as $x_d = [\sin(8t), \cos(8t), 0.5 \cos(8t)]^T$. The initial position and velocity were selected as $x(0) = [-\frac{\pi}{3}, -\pi, \frac{\pi}{3}]^T$ and $v(0) = [0, 0, 0]^T$, respectively. The initial values of the velocity observer were considered to be $\hat{x}(0) = [0, 0, 0]^T$ and $\hat{v}(0) = [5, -6, -3]^T$. The disturbance matrix was chosen as $\tau_d = (1 + \sin(t)) [1, 1, 1]^T + 0.2x(t)$. Based on (17), values of $u_{upper1} = u_{upper2} = u_{upper3} = 80$ and $u_{lower1} = u_{lower2} = u_{lower3} = 60$ were selected. The upper bound of velocity was chosen as $\kappa = 25$. The parameters of the sliding mode controller were taken to be $\varpi_{x1} = \varpi_{x2} = \varpi_{x3} = 0.5$, $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 5$ and $\varsigma_1 = \varsigma_2 = \varsigma_3 = 2$. The parameters of the velocity observer were assumed to be $\sigma_0 = \sigma_2 = \sigma_3 = 1.5$, and $\xi = 0.8$, and the disturbance parameters were chosen as $c = 0.2$, $\gamma_1 = \gamma_2 = \gamma_3 = 15$, and $k_1 = k_2 = k_3 = 10$.

From Fig. 3, it is clear that the estimated velocities $\hat{v}_1(t)$, $\hat{v}_2(t)$, and $\hat{v}_3(t)$ converge to actual velocities $v_1(t)$, $v_2(t)$, and $v_3(t)$ in finite time $T_c \approx 4.2(s)$. In Fig. 4, the estimated disturbances $\hat{\tau}_{d1}$, $\hat{\tau}_{d2}$, and $\hat{\tau}_{d3}$ converge to actual disturbances τ_{d1} , τ_{d2} , and τ_{d3} within finite time $T_a \approx 1.0(s)$. Fig. 5 illustrates how the actual positions x_1 , x_2 , and x_3 track exactly the desired positions x_{d1} , x_{d2} , and x_{d3} in total finite time $T'_{tol} \approx 8.5(s)$.

As shown in Fig. 3, 4, and 5, global finite-time stability in a closed-loop system of a 3-DOF manipulator is attainable, consistent with the results of the theoretical analysis.

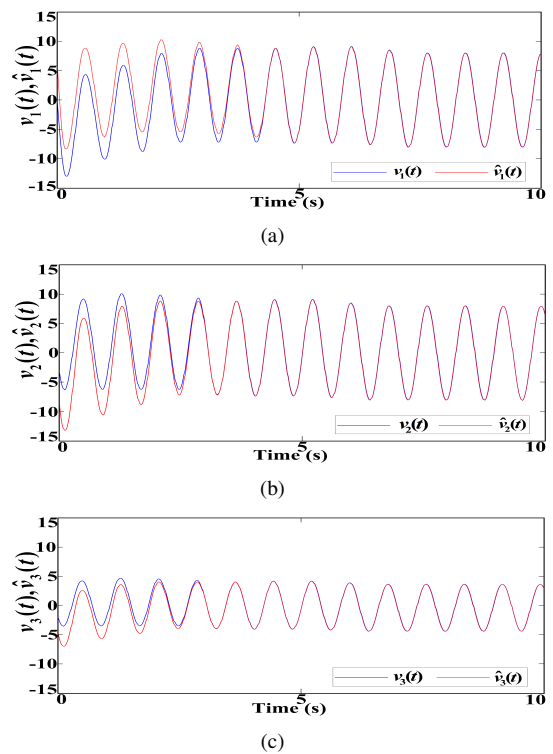


FIGURE 3. The estimates of joints velocities. a: Time responses of \hat{v}_1 and v_1 . b: Time responses of \hat{v}_2 and v_2 . c: Time responses of \hat{v}_3 and v_3 .

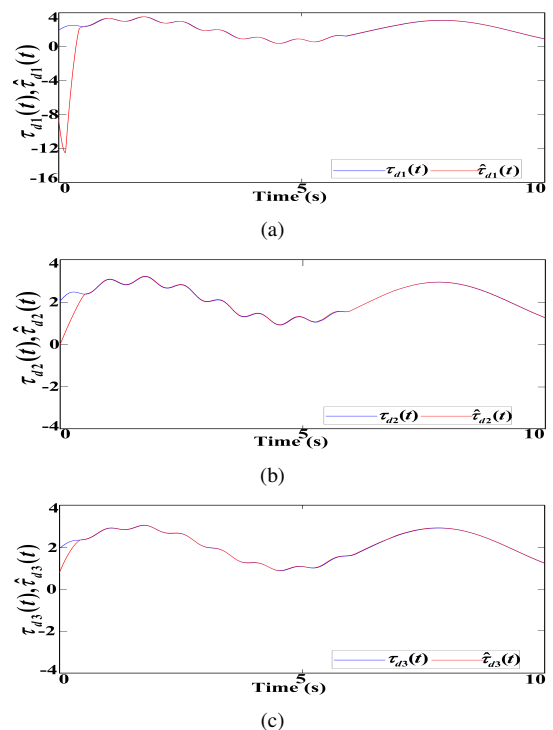


FIGURE 4. The estimates of external disturbances. a: Time responses of $\hat{\tau}_{d1}$ and τ_{d1} . b: Time responses of $\hat{\tau}_{d2}$ and τ_{d2} . c: Time responses of $\hat{\tau}_{d3}$ and τ_{d3} .

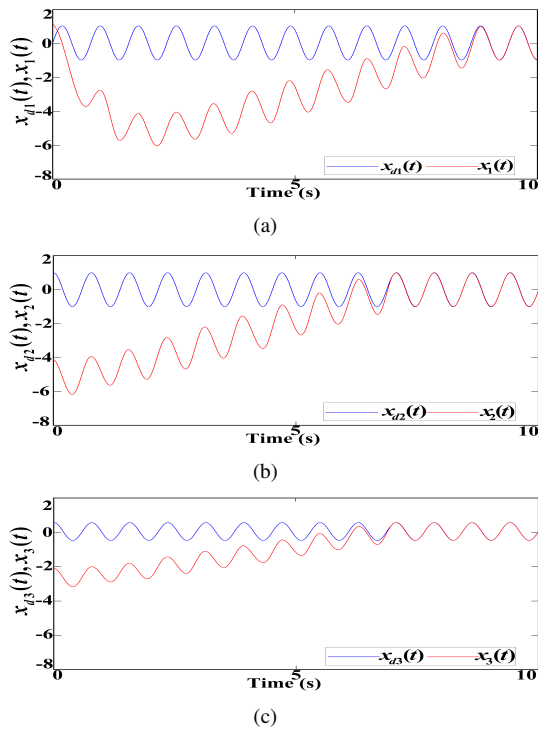


FIGURE 5. Finite time trajectory tracking. a: Time responses of x_{d1} and x_1 . b: Time responses of x_{d2} and x_2 . c: Time responses of x_{d3} and x_3 .

VI. CONCLUSION

This paper addresses the finite-time trajectory tracking control problem for the rigid 3-DOF manipulator system in the presence of system external disturbance. The finite-time velocity observer is designed to estimate velocity from direct measurement of joint position. To improve the robustness of the manipulator system against the disturbances, a disturbance observer is also designed. By applying the disturbance observer and velocity observer with a nonlinear controller, the conditions of global finite-time stability is established. Based on Lyapunov stability theory, it is proved that the finite-time trajectory tracking is achieved for the closed-loop manipulator system. Simulation is performed and the results demonstrate the effectiveness of the proposed control method. In the future, the finite-time trajectory tracking control based on the observers will be further investigated for other nonlinear systems. In addition, the finite-time convergence rate for the robot system will be studied by virtue of new control schemes.

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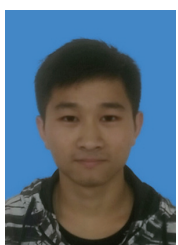
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