

User Oriented Resource Management with Virtualization: A Hierarchical Game Approach

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Abstract—The explosive advancements in mobile Internet and Internet of Things challenge the network capacity and architecture. The ossification of wireless networks hinders the further evolution towards the fifth generation of mobile communication systems. Ultra-dense small cell networks are considered as a feasible way to meet high-capacity demands. Meanwhile, ultra-dense small cell network virtualization also exploits an insightful perspective for the evolution because of its superiority, such as diversity, flexibility, low cost, and scalability. In this paper, we specify the necessity of resource management in virtualized ultra-dense small cell networks through a mapping and management architecture, and consider the problem of user-oriented virtual resource management. Then, we model the virtual resource management problem as a hierarchical game and obtain the closed-form solutions for spectrum, power, and price, respectively. Furthermore, we propose a customer-first (CF) algorithm that characterizes the user-oriented service of virtualization, and analyze its convergence. Simulation results present the effectiveness of the proposed CF algorithm.

Index Terms—Hierarchical game, low-complexity, user-oriented, ultra-dense small cell networks, virtualization.

I. INTRODUCTION

The rapid development of mobile Internet and Internet of Things and the emergence of various services have led to high-capacity demands, which challenge the network capacity and architecture. The fifth-generation of mobile communication systems aims at breaking the capacity bottleneck by densification [1]. The basic idea is to form ultra-dense small cell networks to promote network capacity by increasing the deployment density of low-power small base stations [2], [3]. However, network densification introduces even more complex challenges. For example, the capacity limitation exacerbated

by scarce wireless resources and the network load imbalance in ultra-dense small cell networks are among the challenges. Therefore, when scarce resources encounter explosive demands, it is urgent to approach the perfect matching of resources and demands by effective resource management methods to boost network capacity and balance network load.

Moreover, the ossification of ultra-dense small cell networks grows steadily. On the one hand, the operating expenses and capital expenses are high because of the dense deployment of small base stations. Hence, it is necessary to convert the service mode from physical to virtual entities to reduce expenses [4]. On the other hand, it is hard to bring in new technologies or adjust the existing technologies because of the complex composition of infrastructure service providers. Consequently, the decoupling of services and infrastructures is necessary in order to meet more service demands.

Ultra-dense small cell network virtualization can solve the ossification problem and has been considered as a promising technology for the evolution of the fifth-generation of mobile communication systems [5]. Based on [6], we define ultra-dense small cell network virtualization as follows. By virtualization, the physical resources of small base stations can be completely abstracted, pooled, and integrated into many virtual resources, which then can be shared by multiple virtual entities [7], [8].

For one thing, the nature of virtualization is a resource sharing technology. Resource virtualization can provide the complete set of resources to serve users flexibly [9]. By integrating the resources of multiple small base stations and converting the service mode from physical to virtual entities, the resources can be shared among different demands. For another, by the decoupling of services and infrastructures, virtualization can provide a platform for the current architecture and implement new technologies, functionalities, and applications. The feasibility of abstracting all available resources of multiple small base stations is studied in [10].

In virtual networks, resource management plays an important role in boosting network capacity, improving resource utilization, promoting quality of service, and balancing network load, especially in virtualized ultra-dense small cell networks. The cuboid filling model [11] describes the significance of effective resource management schemes intuitively. It means that effective resource management schemes can make the cuboid fully filled and leave resource holes as little as possible. Resource management in virtual networks aims at approaching the perfect matching between resources and demands.

One of the main aims of virtual resource management is to provide user-oriented customized service by abstracting

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resources and demands. Meanwhile, through forming virtualized ultra-dense small cell networks and converting the architecture from cell-centric to user-centric, user quality of service can also be enhanced [3]. Therefore, it is beneficial to bring virtualization into ultra-dense small cell networks to implement the goal of user-oriented resource management in the fifth-generation of mobile communication systems.

The remainder of the paper is organized as follows. Section II surveys the different works related to virtual resource management and summarizes the contributions of the paper. In section III, we introduce the system model and assumptions, present the proposed two-layer architecture, and specify the necessity of resource management in virtual networks. In section IV, we formulate the resource management problem for different entities. In section V, we provide the closed-form solutions for spectrum, power, price, and analyze the existence and uniqueness of the equilibrium solution. Furthermore, we propose a distributed virtual resource management algorithm. Simulation results are presented in section VI. Followed by the conclusions in section VII.

II. RELATED WORK AND CONTRIBUTIONS

In this section, we review works that relate to virtual resource management. We survey the virtual resource management schemes based on optimization theory, cooperative game, and non-cooperative game, and discuss in detail how our proposed scheme based on hierarchical game advances the state of the art. We then summarize and highlight the contributions of this paper.

A. Related Work Based on Optimization Theory

So far, there have been many types of research about virtual resource management in the academic community [12]–[16]. A survey about the challenges and techniques of resource management in virtualized networks is investigated in [12]. In [13], scheduling resource blocks among different service providers is studied, and the spectrum demands of providers are determined according to an evaluation mechanism. Moreover, the performance of different schemes with or without virtualization is compared, showing the advantage of virtualization in improving spectrum utilization. An uplink virtual spectrum allocation scheme is presented in [14], which defines the utilities of users, service providers, and mobile network operators by treating sub-carriers as commodities. Meanwhile, the spectrum demands of users are given, and the price is fixed for all users. Furthermore, the authors in [15] consider a spectrum waste that the demands are calculated at the peak and are fixed in the whole day, and propose an algorithm based on the opportunistic spectrum sharing allocation. Another spectrum waste is considered in [16], and the primary operators divide their spectrum resource and lease a certain percentage of the spectrum to a given mobile virtual network operator. Hence, resources are wasted if they are neither used by the primary operators nor leased to the mobile virtual network operators. Thus, by setting high percentages of leasing to the mobile virtual network operators, the spectrum resource utilization can be enhanced significantly.

Resource management with virtualization in small cell networks is investigated in [17] and [18]. The authors in [17] introduce a resource negotiation in virtualized LTE-A small cell networks, whose main idea is to reallocate the spectrum resources of two base stations to respond to traffic dynamics. Furthermore, in [18], a scalable resource negotiation is proposed in LTE-A heterogeneous ultra-dense small cell networks. The authors focus on enabling base stations to interconnect through the logical X2 interface and proceed with reallocating resources cooperatively based on the traffic. The extra signaling overhead is analyzed and calculated. Moreover, the scheme can be applied in multi-service and multi-operator scenarios.

Discussions: The distinctions between our paper and the above paper [13]–[18] are mainly in server points. First, different from [13], using a contract constraint to allocate bandwidth, our paper defines detailed utilities for virtual resource providers (VRPs) and virtual resource requesters (VRRs), and obtains the closed-form solutions for the demands of VRRs to perform on-demand allocation. Second, different from [14] in which the price is fixed for all users, the price for VRRs is adaptive based on the demands of VRRs in our paper. Next, both [15] and [16] study opportunistic spectrum sharing allocation, while we concentrate on the complete matching between resources and demands. Then, [17] and [18] focus on traffic-variational resource allocation and signaling overhead, and we investigate on-demand resource allocation and energy efficient. In addition, based on the above review, we can find out that many virtual resource management schemes are centralized, which are not suitable for large-scale ultra-dense small cell network scenarios because of high signaling overhead [13]–[16]. A few schemes are distributed but only consider spectrum allocation, and the transmission power per resource block is fixed and equal among small base stations [17], [18]. Moreover, when the scenarios become complex, and the optimal objectives increase, it is difficult to use these schemes to model and describe the complicated interactions. Therefore, a more effective method is necessary for modeling and solving the resource management problem in complex scenarios. Different from [13]–[18], we develop a distributed scheme to perform user-oriented virtual resource management in ultra-dense small cell networks based on game theory.

Game theory has been a promising method for studying resource management problem because of the following advantages.

- First, game theory comprises of abundant game models and is an effective modeling tool to depict and reflect different interactions and constraints among VRRs, VRPs, and both.
- Second, there exists an equilibrium state to achieve the optimal utility.
- Lastly, game theory is appropriate for designing distributed schemes.

B. Related Work Based on Game Theory

The cooperative game refers that players are cooperative and emphasizes on collective rationality. Spectrum management

with virtualization based on different cooperative game models in different networks is studied in [19], [21], and [23]. A bankruptcy game model is established in [19] to achieve resource block allocation based on the LTE virtualization architecture proposed in [20]. The cooperative game model is proposed according to a specific predefined contract. The main idea is that the resources are abstracted into a pool and can be shared among mobile virtual operators. A bargaining game model is modeled for bandwidth allocation in Infrastructure as a service (IaaS) data center networks in [21] to achieve fairness. Two main objectives are achieved, including basic bandwidth guarantee based on the bandwidth requirements of virtual machines and residual bandwidth reallocation based on the weights of virtual machines. Moreover, both offline algorithm and online algorithm are developed to optimize Nash bargaining solution. A resource allocation mechanism in IaaS cloud system based on coalition game and the uncertainty of game theory [22] is presented in [23]. The cloud is modeled as a multi-agent set, and different agents have different capabilities. Resource virtualization is done by forming coalitions among multiple agents, and then the virtual resources are allocated to different agents with different tasks. In addition, a power allocation scheme based on the cooperative game in wireless relay virtual networks is proposed in [24]. The authors indicate that combining cooperative communication into virtual networks can promote the network performance toward delay-sensitive multimedia services in mobile wireless networks. The proposed scheme can satisfy the diverse requirements for wireless multimedia services.

The non-cooperative game refers that players are non-cooperative and emphasizes on individual rationality. The bandwidth allocation problem is modeled as a non-cooperative game in [25], and Nash equilibrium is obtained in a simple physical topology scenario. In addition, a pricing-based power resource allocation optimization problem in wireless virtual networks is studied in [26] to maximize the energy efficiency of virtual network operators based on non-cooperative game. The authors in [27] develop two effective resource management approaches on account of both cooperative and non-cooperative game in cloud federation, which is an emerging technology to improve the utilization ratio of cloud resources. A trust index is created to filter out untrustworthy cloud service providers, and then the resource management approaches are applied to trusted cloud service providers.

Hierarchical game [28]–[30] is non-cooperative in nature which has different hierarchies with different utilities. Several resource management with virtualization works have been done using hierarchical game [31]–[34]. A bandwidth allocation approach between service providers and infrastructure providers based on two-layer non-cooperative game in cellular networks is proposed in [31]. The first layer is that each infrastructure provider decides whether to accept the request of the service provider. The second layer is that different service providers compete for bandwidth in a shared infrastructure provider. A three-stage Stackelberg model is used to formulate and model the spectrum and power management problem with full-duplex and virtualization in the cellular network [32]. There are four logical roles, including spectrum

providers, base station providers, relay providers, and service providers. A hierarchical game approach [33] in softwarized networks is explored to design a distributed network function virtualization system, which is beneficial to telecom operators. The servers' relationship is modeled as non-cooperation game, and the users' relationship is modeled as evolutionary game. A reinforcement learning method is developed to accelerate convergence. Resource allocation in fog computing networks is studied in [34]. Hierarchical game is used to handle the challenges in resource matching and information asymmetry, which takes data service subscribers, fog nodes, and data service operators into account.

Discussions: Several points in our paper are different from the above papers [31]–[34]. On the one hand, the idea in [31] is to allocate bandwidth between service providers and infrastructure providers, while our paper jointly considers user-oriented adaptive spectrum allocation, adaptive power allocation, and adaptive price allocation between VRPs and VRRs. Moreover, a distributed and nested scheme is developed in our paper, in which the inner nest aims to perform user-oriented virtual resource blocks (VRBs) and power allocation for given price to maximize each VRR' utility, and the outer nest aims to decide the price for each VRR to maximize the VRP' utility. Further, the interaction between the inner nest and the outer nest finally converges to equilibrium. On the other hand, the power optimization and the bandwidth optimization are separate in [32], whereas we formulate and analyze VRR utility as a joint optimization function in spectrum and power. In addition, the scenarios in [33] and [34] are softwarized networks and fog computing networks, while our focus is virtualized ultra-dense small cell networks.

C. Contributions

Although a number of research on resource management in virtualized networks based on game theory have been done, user-oriented virtual resource management based on hierarchical game in ultra-dense small cell networks is still at the beginning level. In this paper, we jointly consider the number of VRBs scheduled to VRRs, the power assigned to VRRs, and the price determined by VRPs. In this way, user quality of service and network performance can be improved. The contributions are summarized as follows.

- A two-layer architecture is presented to map the resource management problem from physical to virtual networks. We specify the necessity of resource management in virtual networks. Virtualized resource management in ultra-dense small cell networks can be decomposed into two entities, VRPs and VRRs. The decomposition enables flexibility in the process of network management.
- Hierarchical game is adapted to depict and reflect the matching interactions between VRPs and VRRs in the proposed two-layer architecture, and also to model the virtualized resource management problem and maximize each entity's revenue. By jointly considering the number of VRBs scheduled to VRRs and the power assigned to VRRs, energy-efficiency and on-demand allocation are taken into account.

- We exploit and explore the nature of virtualization in which resources and demands are aspectant. We further propose a low-complexity distributed customer-first (CF) scheme that characterizes the user-oriented service of virtualization to improve user quality of service, reduce network energy overhead, and promote network resource utilization ratio.
- The existence and uniqueness of sub-game perfect equilibrium (SPE) and hierarchical Nash equilibrium (HNE) are analyzed and proved for the proposed CF scheme.
- Simulation results confirm the effectiveness of the proposed scheme in improving network utility, system rate, network energy efficiency, network resource utilization ratio, and reducing user access-reject probability.

III. SYSTEM MODEL AND ARCHITECTURE

In this section, we illustrate the virtual network model and the two-layer architecture in a straightforward and insightful perspective.

A. Virtual Network Model

After virtualization, the services are decoupled and the resources are abstracted from infrastructures, respectively. As shown in Fig. 1, we present a virtual network model including infrastructure layer, pool layer, and service layer. The infrastructure layer has many small base stations, which own physical resource blocks (PRBs). The pool layer consists of a large number of VRBs, which are abstracted from the infrastructure layer and mapped from PRBs. A VRB has the same size as a PRB. VRB is distributed, which means the sub-carriers scheduled to one VRR are scattered on the whole bandwidth. Moreover, more than one VRB can be scheduled to one VRR in each transmission time interval. The service layer is composed of lots of VRRs. These VRRs are contained in different virtual networks based on geographic locations, channel conditions, and service types. There are three virtual networks in the service layer of Fig. 1. The solid lines depict virtual networks. The dotted lines represent VRRs, and the colored VRBs contained therein are scheduled to VRRs. Each virtual network serves a certain number of VRRs and different VRRs may have different demands to ensure communication.

By virtualization, some details in the infrastructure layer can be shielded, and the available resources are integrated to be managed uniformly by the pool layer. The service layer can get rid of the management and control of the infrastructure layer, carrying out self-operating and self-management mode to provide user-oriented service. VRRs in the service layer are serviced by leasing resources from the pool layer, and the pool layer obtains revenue by providing resources for the service layer. Therefore, the resource management problem in virtualized ultra-dense small cell networks is mainly to achieve the matching between pool layer and service layer as much as possible, and determine the transmission power. That is, resources can be fully used, and VRRs can be served with satisfaction, which promotes network throughput positively.

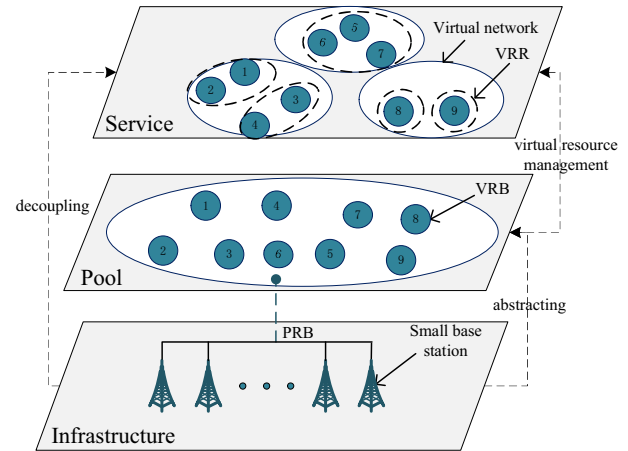


Fig. 1. Virtual network model.

B. Two-layer Architecture

Fig. 2 depicts the proposed two-layer architecture. Two essential elements, physical networks and virtual networks are included. In physical networks, the components related to resource management are demands and resources, and the resource management problem is the matching of the demands and resources of each small base station. In virtual networks, the components related to resource management are abstracted as VRRs and VRPs. The resource management problem is the matching of the demands and resources of all small base stations, which corresponds to the matching of the service layer and the pool layer in Fig. 1. We can observe the relationship between two elements in Fig. 2. The traffic in physical networks can be mapped as the demands of VRRs in virtual networks, which is equivalent to the decoupling of the infrastructure layer and the service layer in Fig. 1. The resources in physical networks can be virtualized and managed by VRPs in virtual networks, which is equivalent to the abstracting of the infrastructure layer and the pool layer in Fig. 1.

In virtual networks, VRPs own all available resources to provide customized service to VRRs. VRPs are providers, and VRRs are requesters. For example, if we treat VRBs as commodity, VRRs and VRPs play the role of suppliers and customers, respectively, and the related condition is the price provided by VRPs and the demands provided by VRRs. Meanwhile, the transmission power of VRRs should be determined, taking energy efficiency into account.

C. Channel Model

Suppose that there is one VRP providing service and M VRRs receiving service, $\forall m \in \mathcal{M}$. The VRP owns K VRBs. The transmission rate of VRR m can be denoted as

$$R_m = b_m \kappa \log_2(1 + \gamma_m), \quad (1)$$

where b_m is the number of VRBs scheduled to VRR m . κ is the size of each VRB. $b_m \kappa$ denotes the bandwidth occupied by VRR m . γ_m is the received signal-to-noise ratio (SNR) of

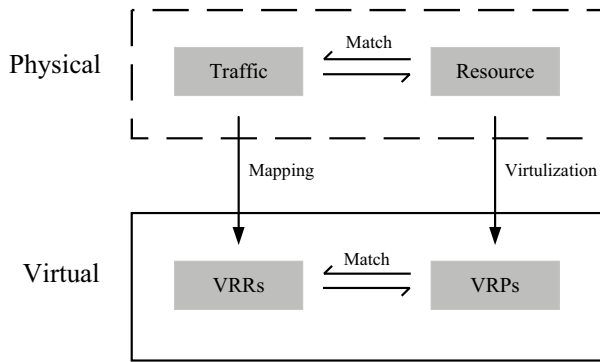


Fig. 2. Two-layer architecture.

VRR m . Consider an additive white Gaussian noise channel, γ_m can be denoted as

$$\gamma_m = \frac{p_m g_m}{N_0}, \quad (2)$$

where p_m is the power allocated to VRR m . g_m is the link gain of VRR m . N_0 denotes the background noise power.

IV. PROBLEM FORMULATION

In this section, we formulate the user-oriented resource management problem with virtualization as a hierarchical game model in ultra-dense small cell networks. The formulated model takes customized allocation into account.

Definition 1: The hierarchical game model is a two-layer game, namely VRP and VRR. In this game, the VRP firstly moves by deciding and broadcasting the price for all VRRs on the basis of VRRs' demands to maximize the VRP' utility, and then VRRs move subsequently by deciding the demands for VRBs based on the communication requirements and the price provided by the VRP, and determining the transmission power to maximize each VRR' utility.

A. Resource Management of VRR

In the VRR layer, VRRs evaluate the demands for VRBs and the power to maximize the utility. We only consider one VRP and focus on the number of VRBs. Inspired by [30], we

TABLE I
TABLE OF NOTATIONS

Symbol	Meaning
K	the number of VRBs
\mathcal{M}	the set of VRRs
m	the index of the VRR
U_m^{VRR}	the utility of VRR m
U^{VRP}	the utility of the VRP
b_m	the number of VRBs scheduled to VRR m
κ	the size of each VRB
γ_m	the received SNR of VRR m
p_m	the power allocated to VRR m
g_m	the link gain of VRR m
N_0	the background noise power
ℓ	the unit revenue in the transmission rate
ϖ_m	the price of unit bandwidth for VRR m
γ_{th}	the minimum required SNR for VRRs
p^{\max}	the maximum power

adopt the quadratic utility function to formulate the utility of VRR m :

$$U_m^{VRR} = \ell R_m - \varpi_m b_m \kappa - \frac{1}{2} (b_m \kappa)^2, \quad (3)$$

where ℓ is related to the revenue in the transmission rate by occupying VRBs. R_m is the transmission rate of VRR m . ϖ_m is the price of unit bandwidth for VRR m provided by the VRP. $b_m \kappa$ denotes the bandwidth occupied by VRR m .

The motivations for using (3) include the following aspects.

- The utility U_m^{VRR} is concave versus the demand b_m and the power p_m with boundary condition. Thus, it is workable to get the maximum utility of U_m^{VRR} .
- The demand function b_m is linear versus the price ϖ_m . It facilitates the analysis for the VRP.

The resource management problem in the VRR layer is to maximize the utility of VRR m while guaranteeing the related constraints as follows:

$$\max_{b_m, p_m} U_m^{VRR}(b_m, p_m | \varpi_m), \forall m \in \mathcal{M}, \quad (4)$$

s.t.

$$b_m = 1, \dots, K - 1, \forall m \in \mathcal{M}, \quad (5)$$

$$\sum_{m=1}^M b_m = 1, \dots, K - 1, \quad (6)$$

$$\gamma_m \geq \gamma_{th}, \forall m \in \mathcal{M}, \quad (7)$$

$$0 < \sum_{m=1}^M p_m \leq p^{\max}, \quad (8)$$

where γ_{th} is the minimum required SNR for VRRs. Constraint (5) means that each VRR accepts service and the required VRBs are no more than the total number of VRBs. Constraint (6) indicates that the required VRBs of all VRRs do not exceed the total number of VRBs. Constraint (7) sets the minimum threshold γ_{th} and represents that the received SNR of any VRR should be greater than the minimum required SNR. Constraint (8) means that the power of any VRR m should be greater than 0 and the total power of all VRRs is no more than the maximum power p^{\max} .

B. Pricing of VRP

In the VRP layer, the VRP gets revenue by leasing VRBs with appropriate price, which is related to the number of VRBs scheduled to VRRs. We define the utility function of the VRP as

$$U^{VRP} = \sum_{m=1}^M \varpi_m b_m \kappa. \quad (9)$$

In the same way, the resource management problem in the VRP layer is to maximize the utility of the VRP by the following optimization problem:

$$\max_{\varpi_m} U^{VRP}(\varpi_m | b_m, p_m), \quad (10)$$

s.t.

$$\varpi_m > 0, \forall m \in \mathcal{M}. \quad (11)$$

The VRP and VRRs are all autonomous decision-makers, who are rational and selfish. Each VRB is exclusively allocated to one VRR and one VRR can use more than one VRB in each transmission time interval.

In summary, we divide the resource management problem into two parts. For VRR m , the demand b_m and the power p_m are to be optimized. For the VRP, the price ϖ_m is to be optimized. Both the VRP and VRRs expect to maximize their utilities, respectively.

V. ANALYSIS OF HIERARCHICAL GAME

In this section, we analyze the formulated problem for VRRs and obtain the solution b_m^*, p_m^* in subsection A. We also analyze the formulated problem for the VRP and get the solution ϖ_m^* in subsection B. We prove the existence and uniqueness of hierarchical game in subsection C. Finally, the proposed CF algorithm is represented in subsection D.

Definition 2: The solution of the hierarchical game model is named HNE and can be obtained by backward induction method. HNE is composed of $\{b_m^*, p_m^*, \varpi_m^*, \forall m \in \mathcal{M}\}$, where b_m^* is the optimal number of VRBs scheduled to VRR m and p_m^* is the optimal power allocated to VRR m . Both are VRR m 's strategies to maximize the utility of VRR m . ϖ_m^* is the optimal price for VRR m , which is the VRP' strategy to maximize the utility of the VRP.

A. Analysis of VRR

The solutions of the formulated problem for VRR m are the demand b_m (integer) and the power p_m (continuous). The utility function is nonlinear. Hence, the problem is a mixed integer nonlinear programming, which is NP-hard in general. Thus, we transfer (4) into continuous space. The original problem is relaxed as follows:

$$\max_{b_m, p_m} U_m^{VRR}(b_m, p_m | \varpi_m), \forall m \in \mathcal{M}, \quad (12)$$

s.t.

$$b_m > 0, \forall m \in \mathcal{M}, \quad (13)$$

$$\sum_{m=1}^M b_m < K, \quad (14)$$

$$\gamma_m \geq \gamma_{th}, \forall m \in \mathcal{M}, \quad (15)$$

$$0 < \sum_{m=1}^M p_m \leq p^{\max}. \quad (16)$$

(12) is a multi-variable problem with continuous space. Given a feasible price for VRR m , we can get the solution of b_m and p_m based on (1)(2)(3) and (12). The solving steps are described as follows:

- By the property of Hessian Matrix, we ensure the concavity of U_m^{VRR} . Moreover, the constraint set is affine because it consists of linear constraints. Thus, the formulated problem is a convex problem.
- Based on decomposition theory, we convert the multi-variable problem into two dependent single-variable problems, i.e., the problem for spectrum adaptation (for b_m) and the problem for power adaptation (for p_m).

- The two single-variable problems can be solved by the Karush-Kuhn-Tucker (KKT) conditions.
- The solutions for the two problems are interactive and the unique SPE in the VRR layer can be found.

Theorem 1: The utility function U_m^{VRR} is concave if $\forall m \in \mathcal{M}$, b_m satisfy

$$b_m \geq \frac{\ell}{\kappa \ln 2}. \quad (17)$$

Proof: Hessian Matrix is adopted to guarantee the strict concavity of U_m^{VRR} . First, we make a preliminary definition including $A = \frac{\partial^2 U_m^{VRR}}{\partial b_m^2}$, $B = \frac{\partial^2 U_m^{VRR}}{\partial b_m \partial p_m}$, $C = \frac{\partial^2 U_m^{VRR}}{\partial p_m^2}$. Then, we derive the second-order derivative of U_m^{VRR} with respect to b_m and p_m , respectively.

$$\frac{\partial^2 U_m^{VRR}}{\partial b_m^2} = -\kappa^2, \quad (18)$$

$$\frac{\partial^2 U_m^{VRR}}{\partial b_m \partial p_m} = \frac{\ell \kappa g_m}{N_0 (1 + \gamma_m) \ln 2}, \quad (19)$$

$$\frac{\partial^2 U_m^{VRR}}{\partial p_m^2} = -\frac{\ell \kappa g_m^2 b_m}{N_0^2 (1 + \gamma_m)^2 \ln 2}. \quad (20)$$

We can observe $A < 0, C < 0$. We guarantee the non-negative definiteness of Hessian Matrix by deriving the sufficient condition of $AC - B^2 = \frac{\ell \kappa^2 g_m^2 (\kappa b_m \ln 2 - \ell)}{(N_0 + p_m g_m)^2 (\ln 2)^2} \geq 0$. Thus, when $b_m \geq \frac{\ell}{\kappa \ln 2}$, U_m^{VRR} is concave.

According to Theorem 1, the minimal b_m should satisfy (17) to ensure the concavity of U_m^{VRR} . It is also known that the affine constraint set is convex set. Notice that since we are facing a convex optimization problem, a local optimum of the problem is also globally optimal. In addition, duality gap is zero under the constraints, so the KKT conditions are necessary and sufficient. In the following, we first transform the objective problem into two problems based on decomposition theory, and then use the KKT conditions to solve the two problems, respectively [35].

1) *Spectrum adaptation:* Based on (12), for a given price ϖ_m and a feasible power p_m , VRR m can evaluate the demand for VRBs by solving the following problem:

$$\max_{b_m} U_m^{VRR}, \forall m \in \mathcal{M}, \quad (21)$$

s.t.

$$b_m \geq \frac{\ell}{\kappa \ln 2}, \forall m \in \mathcal{M}, \quad (22)$$

$$\sum_{m=1}^M b_m \leq K. \quad (23)$$

The Lagrangian function is as follows:

$$\begin{aligned} L(b_m, \nu_m, \lambda) &= \ell R_m - \varpi_m b_m \kappa - \frac{1}{2} (b_m \kappa)^2 - \nu_m \left(\frac{\ell}{\kappa \ln 2} - b_m \right) \\ &\quad - \lambda \left(\sum_{m=1}^M b_m - K \right). \end{aligned} \quad (24)$$

We know Lagrangian multipliers are nonnegative, i.e., $\nu_m \geq 0, \lambda \geq 0$. With the KKT conditions, we have the Lagrange

derivation equation with the complementary slackness conditions for VRR m as

$$\frac{\partial L(b_m, \nu_m, \lambda)}{\partial b_m} = 0, \quad (25)$$

$$\nu_m \left(\frac{\ell}{\kappa \ln 2} - b_m \right) = 0, \quad (26)$$

$$\lambda \left(\sum_{m=1}^M b_m - K \right) = 0. \quad (27)$$

We can see if $b_m \neq \frac{\ell}{\kappa \ln 2}$, there is $\nu_m = 0$. The same to (28), there is $\lambda = 0$ if $\sum_{m=1}^M b_m \neq K$. According to (25), we can obtain

$$\ell \kappa \log_2 \left(1 + \frac{p_m g_m}{N_0} \right) - \varpi_m \kappa - \kappa^2 b_m + \nu_m - \lambda = 0. \quad (28)$$

Thus, we have b_m^* as follows:

$$b_m^* = \left\{ \frac{1}{\kappa} \left[\ell \log_2 \left(1 + \gamma_m \right) - \varpi_m \right] + \frac{1}{\kappa^2} (\nu_m - \lambda) \right\}. \quad (29)$$

From (29), we can observe b_m^* is related with the power strategy p_m and the price strategy ϖ_m . Given a feasible power p_m and price ϖ_m , we can get the number of VRBs scheduled to VRR m .

Sub-gradient method [35] is used to update the Lagrangian multiplier ν_m and λ . Thus, we have

$$\nu_m^{t+1} = [\nu_m^t + \tau^t \left(\frac{\ell}{\kappa \ln 2} - b_m^* \right)]^+, \quad (30)$$

$$\lambda^{t+1} = [\lambda^t + \tau^t \left(\sum_{m=1}^M b_m^* - K \right)]^+, \quad (31)$$

where $[\cdot]^+$ denotes the projection onto the non-negative area. τ^t is an adjustable stepsize, which sensitively affects the convergence. Theoretically, τ^t can be constant or diminishing. However, the convergence cannot be guaranteed when using the constant stepsize $\tau^t = \tau^1$, $\tau^1 > 0$, which is because the iteration may repeatedly proceed near the closed-form optimal solution. On the contrary, by using the adjustable diminishing stepsize, where $\tau^1 > 0$, $\lim_{t \rightarrow \infty} \tau^t = 0$ and $\sum_{t=1}^{\infty} \tau^t = \infty$, the convergence of the optimal solution can be guaranteed. For example, in our implementation, we set $\tau^t = \frac{\tau^1}{t}$, $\tau^1 = 1$, $t = 1, 2, \dots$. The initialization are $\nu_m^1 = 4 * 10^3$ and $\lambda^1 = 10$.

Furthermore, without loss of generality, we set the iterative terminal conditions using

$$\|\nu_m^{t+1} - \nu_m^t\| \leq \varepsilon_1, \quad (32)$$

$$\|\lambda^{t+1} - \lambda^t\| \leq \varepsilon_2, \quad (33)$$

where ε_1 and ε_2 are any small positive.

We can learn that with the convergence settings of (32) and (33), b_m can converge to a steady solution after limited iterations. The detailed user-oriented spectrum adaptation scheme for all VRRs is shown in Algorithm 1, which can be implemented distributively. That is, each VRR updates its own multipliers ν_m and λ until convergence.

Algorithm 1 User-oriented spectrum adaptation

Input: M , $\{p_m\}$, $\{g_m\}$, $\{\varpi_m\}$, N_0 , κ , ℓ , K .

Output: $\{b_m\}$, $\{\nu_m\}$, λ .

```

1: Initialization:  $\nu_m \leftarrow 4 * 10^3$ ,  $\lambda \leftarrow 10$ ,  $\tau \leftarrow 1$ ,  $t \leftarrow 1$ 
2: while not converged do
3:   Update  $\lambda$  using (31).
4:   for all  $m \in M$  do
5:     Update  $\nu_m$  according to (30).
6:     Calculate  $b_m$  according to (43).
7:   end for
8:    $t \leftarrow t + 1$ .
9:    $\tau \leftarrow 1/t$ .
10: end while
11:  $b_m^* = \text{ceil}(b_m)$ .
```

Up to now, we have gotten the solution for b_m^* by solving (21). We should map the solution to the original problem. The common method is to round up to an integer. Thus, the solution is given by the following operation:

$$b_m^* = \lceil b_m^* \rceil. \quad (34)$$

2) *Power adaptation:* Based on (12), for a given price ϖ_m and the obtained demand b_m^* , the power of VRR m can be obtained by solving the following problem:

$$\max_{p_m} (U_m^{VRR} | b_m^*), \forall m \in \mathcal{M}, \quad (35)$$

s.t.

$$\gamma_m \geq \gamma_{th}, \forall m \in \mathcal{M}, \quad (36)$$

$$0 < \sum_{m=1}^M p_m \leq p^{\max}. \quad (37)$$

Similar to the problem for spectrum adaptation, we can form the Lagrangian function as follows:

$$\begin{aligned} \tilde{L}(p_m, \phi_m, \pi) &= \ell R_m - \varpi_m b_m \kappa - \frac{1}{2} (b_m \kappa)^2 - \phi_m (\gamma_{th} - \gamma_m) \\ &\quad - \pi \left(\sum_{m=1}^M p_m - p^{\max} \right), \end{aligned} \quad (38)$$

where ϕ_m and π are non-negative Lagrangian multipliers. For each given ϕ_m and π , the power of VRR m can be obtained by solving the following equation.

$$p_m^* = \arg \min_{p_m} \tilde{L}(p_m, \phi_m, \pi). \quad (39)$$

According to the KKT conditions, we have the following equations:

$$\frac{\partial \tilde{L}(p_m, \phi_m, \pi)}{\partial p_m} = 0, \quad (40)$$

$$\phi_m (\gamma_{th} - \gamma_m) = 0, \quad (41)$$

$$\pi \left(\sum_{m=1}^M p_m - p^{\max} \right) = 0. \quad (42)$$

Algorithm 2 User-oriented power adaptation**Input:** $M, \{b_m\}, \{g_m\}, N_0, \kappa, \iota, \gamma_{th}, p^{\max}$.**Output:** $\{p_m\}$.

```

1: Initialization:  $\phi_m \leftarrow 10^{-3}, \pi_i \leftarrow 10^5, \eta \leftarrow 1, t \leftarrow 1$ 
2: while not converged do .
3:   Update  $\pi$  using (46).
4:   for all  $m \in M$  do
5:     Update  $\phi_m$  according to (45).
6:     Calculate  $p_m$  according to (43).
7:   end for
8:    $t \leftarrow t + 1$ .
9:    $\eta \leftarrow 1/t$ .
10: end while

```

From (41), we can see if $\gamma_m \neq \gamma_{th}$, $\phi_m = 0$. Similar to (42), if $\sum_{m=1}^M p_m \neq p^{\max}$, $\pi = 0$. Based on (38), we can get p_m^* as

$$p_m^* = \left[\frac{b_m \kappa \ell}{\left(\pi - \frac{\phi_m g_m}{N_0}\right) \ln 2} - \frac{N_0}{g_m} \right]_{p_m^{\min}}^{p_m^{\max}}, \quad (43)$$

where \llbracket_x^y is the projection onto the area in $[x, y]$. Based on (2) and (7), p_m^{\min} can be obtained as

$$p_m^{\min} = \frac{\gamma_{th} N_0}{g_m}. \quad (44)$$

Moreover, p^{\max} should be configured more than $\sum_{m=1}^M p_m^{\min}$ according to (37). At the same time, $\pi - \frac{\phi_m g_m}{N_0} > 0$ should be guaranteed based on (43).

In the same way, sub-gradient projection method is used for the updating of Lagrangian multipliers:

$$\phi_m^{t+1} = \left[\phi_m^t + \eta^t (\gamma_{th} - \frac{p_m^* g_m}{N_0}) \right]^+, \quad (45)$$

$$\pi^{t+1} = \left[\pi^t + \eta^t \left(\sum_{m=1}^M p_m^* - p^{\max} \right) \right]^+, \quad (46)$$

where \llbracket^+ denotes the projection onto the non-negative area. η^t is an adjustable stepsize. The same as the analysis of spectrum management, in our implementation, we use the diminishing stepsize, and set $\eta^t = \eta^0 / t$, $\eta^0 = 1$, $t = 1, 2, \dots$. For the initial multipliers ϕ_m and π , we set $\phi_m^1 \leftarrow 10^{-3}$ and $\pi^1 \leftarrow 10^5$.

In the same way, we determine the iterative terminal conditions using

$$\|\phi_m^{t+1} - \phi_m^t\| \leq \varepsilon_3, \quad (47)$$

$$\|\pi^{t+1} - \pi^t\| \leq \varepsilon_4, \quad (48)$$

where ε_3 and ε_4 are any small positive.

With the iterative terminal conditions of (47) and (48), we can learn that p_m can converge to a steady solution after limited iterations. The detailed user-oriented power adaptation scheme for all VRRs is shown in Algorithm 2, which can be implemented distributively. That is, each VRR m updates its own multipliers ϕ_m and π until convergence.

According to Algorithm 2, the multipliers ϕ_m and π update accordingly until convergence. Then, the calculated power is

taken into account Algorithm 1, and then an updated demand is produced, which will be used in Algorithm 2 again. This procedure will be executed iteratively until convergence, as shown in Algorithm 3.

3) *The existence of SPE:* The existence of SPE in the VRR layer is ensured by the following theorem [36].

Theorem 2: There exists at least one SPE (b_m^*, p_m^*) in the VRR layer if $\forall m \in \mathcal{M}$: the set (b_m, p_m) is a nonempty compact set, and the utility U_m^{VRR} is continuous on (b_m, p_m) and concave on (b_m, p_m) . (b_m, p_m) satisfies the following equation.

$$(b_m^*, p_m^*) = \arg \max_{b_m^*, p_m^*} U_m^{VRR}. \quad (49)$$

Proof: As mentioned before, the set is a convex and compact set, and the utility is continuous on (b_m, p_m) . Furthermore, the joint concavity of U_m^{VRR} with respect to (b_m, p_m) is proved and guaranteed in Theorem 1. Therefore, at least one SPE exists.

4) *The uniqueness of SPE:* The uniqueness of SPE is ensured by potential game [37] in the following theorem.

Theorem 3: A game $[\mathcal{M}, s_m, U_m(\cdot)]$ is a potential game if there exists a potential function $G(\cdot)$ that satisfies $U_m(s_m', \mathbf{s}_{-m}) - U_m(s_m, \mathbf{s}_{-m}) = G(s_m', \mathbf{s}_{-m}) - G(s_m, \mathbf{s}_{-m})$, $\forall m \in \mathcal{M}$, where s_m and s_m' represent any two different strategies of VRR m .

A sufficient and essential condition to ensure the existence of $G(\cdot)$ is $\frac{\partial U_m}{\partial s_m} = \frac{\partial G}{\partial s_m}$, $\forall m \in \mathcal{M}$. When the condition is satisfied, the optimum solution of $G(\cdot)$ is the equilibrium solution.

The game in the VRR layer can be considered as a potential game if $\forall m \in \mathcal{M}$, $G(\cdot)$ is defined as

$$G = \ell \sum_{m=1}^M R_m - \kappa \sum_{m=1}^M \varpi_m b_m - \frac{\kappa^2}{2} \sum_{m=1}^M b_m^2, \quad (50)$$

we can prove the potential function $G(\cdot)$ satisfies $\frac{\partial U_m}{\partial b_m} = \frac{\partial G}{\partial b_m}$ and $\frac{\partial U_m}{\partial p_m} = \frac{\partial G}{\partial p_m}$.

Proof: It is known that the potential function possesses some convenient properties. If player m aims at maximizing $G(\cdot)$, keeping the other players' strategies invariable, the equilibrium point will not be changed [38]. In addition, the potential function $G(\cdot)$ is strictly concave and continuously differentiable in the non-negative area bounded by $\sum_{m=1}^M p_m = p^{\max}$ and $\sum_{m=1}^M b_m = K$. It can be concluded that there exists a unique solution because U_m^{VRR} is strictly concave with respect

Algorithm 3 SPE in the VRR layer**Input:** $M, \{\varpi_m\}$.**Output:** $\{b_m^*, p_m^*\}$.

```

1: Initialization.
2: while not converged do
3:   Perform Algorithm 1.
4:   Perform Algorithm 2.
5:    $t \leftarrow t + 1$ .
6: end while

```

to (b_m, p_m) , which is proved by (18)(19)(20). In consequence, the uniqueness of SPE in the VRR layer is ensured.

B. Analysis of VRP

To maximize the total revenue, the VRP needs to decide the optimal price for VRRs based on the scheduled VRBs. we substitute (29) into (9) and get

$$U^{VRP} = \sum_{m=1}^M \varpi_m \left[\frac{\ell \kappa \log_2 \left(1 + \frac{p_m g_m}{N_0} \right) - \kappa \varpi_m + \nu_m - \lambda}{\kappa} \right]. \quad (51)$$

Theorem 4: The utility U^{VRP} is strictly concave, and the VRP has the maximum revenue when ϖ_m^* satisfies

$$\varpi_m^* = \frac{\ell}{2} \log_2 \left(1 + \frac{p_m^* g_m}{N_0} \right) + \frac{\nu_m - \lambda}{2\kappa}. \quad (52)$$

Proof: We derive the second-order derivative of U^{VRP} versus ϖ_m and get $\frac{\partial^2 U^{VRP}}{\partial \varpi_m^2} = -\frac{2}{\kappa} < 0$. Thus, the utility is strictly concave. Given $\frac{\partial U^{VRP}}{\partial \varpi_m} = 0$, we can derive $\varpi_m^* = \frac{\ell}{2} \log_2 \left(1 + \frac{p_m^* g_m}{N_0} \right) + \frac{\nu_m - \lambda}{2\kappa}$, by which the VRP can get the maximum revenue.

It can be observed that the optimal price strategy of the VRP is related to the spectrum strategy and power strategy of VRRs.

Theorem 5: There exists only one SPE (ϖ_m^*) in the VRP layer if $\forall m \in \mathcal{M}$: the set (ϖ_m) is a nonempty compact set, and the utility function U^{VRP} is continuous on (ϖ_m) and strictly concave on (ϖ_m) . That is, (ϖ_m) satisfies $(\varpi_m^*) = \arg \max_{\varpi_m} U^{VRP}$.

Proof: The constraint set is affine as it consists of linear constrains. Thus, it is a convex and compact set. We can also know that the utility is continuous on (ϖ_m) . Furthermore, the strict concavity of U^{VRP} with respect to (ϖ_m) is proved and guaranteed in Theorem 4. Therefore, only one SPE exists.

C. The Existence and Uniqueness of HNE

Definition 3: $(b_m^*, p_m^*, \varpi_m^*, \forall m \in \mathcal{M})$ is the unique HNE for the proposed CF algorithm if and only if $(b_m^*, p_m^*, \forall m \in \mathcal{M})$ and $(\varpi_m^*, \forall m \in \mathcal{M})$ are unique SPEs for each layer, respectively.

Combining Theorem 2, Theorem 3, and Theorem 5, we know that $(b_m^*, p_m^*, \forall m \in \mathcal{M})$ and $(\varpi_m^*, \forall m \in \mathcal{M})$ are unique SPEs for each layer. Moreover, each SPE is Nash equilibrium [27], [32]. Therefore, there is a unique HNE $(b_m^*, p_m^*, \varpi_m^*, \forall m \in \mathcal{M})$.

D. CF Algorithm

The details of the proposed CF Algorithm is presented in Algorithm 4. Due to the leading role of the VRP, the Algorithm will converge to HNE if for $\forall m \in \mathcal{M}$, the price satisfies $\frac{\|\varpi_m^{t+1} - \varpi_m^t\|}{\|\varpi_m^t\|} \leq \varepsilon_5$. Meanwhile, we can observe the scheme is distributed not only in the VRP layer and the VRR layer but also within the VRR layer. This is because the VRP and VRRs are only for maximizing their own utilities. Moreover, the VRP and VRRs can termly update the CF scheme if the demand and price change. The convergence and effectiveness of the proposed CF Algorithm are verified in section VI.

Algorithm 4 CF based on hierarchical game

Input: M .

Output: $\{b_m^*, p_m^*, \varpi_m^*\}$.

- 1: Initialization.
- 2: **while** not converged **do**
- 3: Perform Algorithm 3.
- 4: Update prices using (52).
- 5: $t \leftarrow t + 1$.
- 6: **end while**

VI. SIMULATION RESULTS AND DISCUSSIONS

In this section, the convergence and effectiveness of the proposed CF Algorithm are demonstrated by simulation results. The simulation parameters are described as follows.

We consider a hexagon area which is covered by one VRP and some VRRs. The total number of VRBs is 100, which are abstracted from all small base stations. The size of each VRB is 180 kHz. VRRs are distributed randomly within the hexagon area. The maximal power of each small base station is set to 30 dBm. The background noise power N_0 is set to -100 dBm. The path loss model is denoted as $PL = 140.7 + 36.7 \log_{10}(R[km])$. The small-scale fading is modeled as Rayleigh random variables. The minimum required SNR is set to 5. ℓ is 100. The basic configurations are based on 3GPP TR36.814. Each VRR can be serviced by more than one VRB and each VRB can be used to serve only one VRR. Meanwhile, it is known that each small base station can only provide service for a certain number of VRRs if without virtualization, while the management and the service are more flexible and beneficial if with virtualization.

A. Convergence

For illustrative purposes, we consider a scenario with 4 VRRs. Fig. 3 verifies the convergence of the proposed CF scheme, including the convergence of user-oriented spectrum adaptive algorithm in Fig. 3(a), the convergence of user-oriented power adaptive algorithm in Fig. 3(b), and the convergence of price adaptive algorithm in Fig. 3(c). In Fig. 3(a), the X-axis is the number of iterations, and the Y-axis is demand for each VRR. Similarly to Fig. 3(b) and (c), the X-axis is the number of iterations, and the Y-axis is power and price for each VRR, respectively. We can see that demand, power, and price for each VRR converge quickly with the increasing number of iterations. It can be observed that the curves start to converge at the first iteration and reach steady state at the second iteration. In addition, we can also see that the demands are different for different VRRs and the corresponding price provided by the VRP is also different. Furthermore, it can be seen that when the demands of VRRs are equal (such as VRR 1, VRR 2, and VRR 4 in Fig. 3(a)), the prices are also equal (such as VRR 1, VRR 2, and VRR 4 in Fig. 3(c)). Moreover, the higher the demands are (such as VRR 3 and VRR 4 in Fig. 3(a)), the higher the corresponding prices are (such as VRR 3 and VRR 4 in Fig. 3(c)), which is due to the linear relation between the price and the demand.

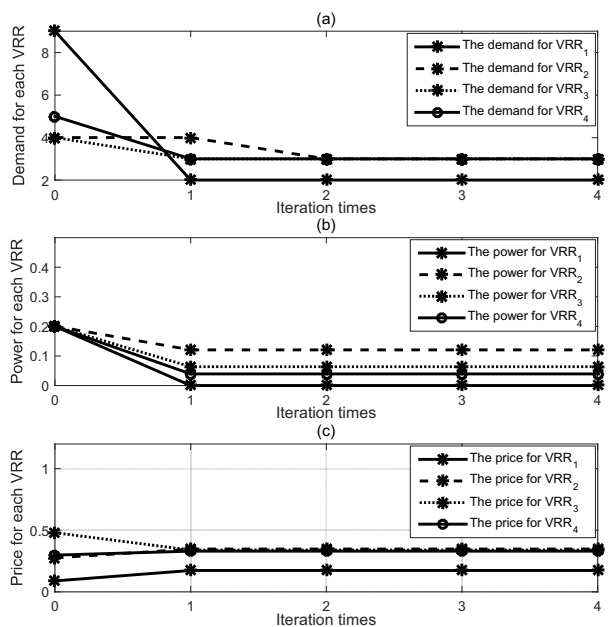


Fig. 3. Convergence of the proposed CF scheme based on hierarchical game in virtualized ultra-dense small cell networks. (a) Convergence of user-oriented spectrum adaptive algorithm. (b) Convergence of user-oriented power adaptive algorithm. (c) Convergence of price adaptive algorithm.

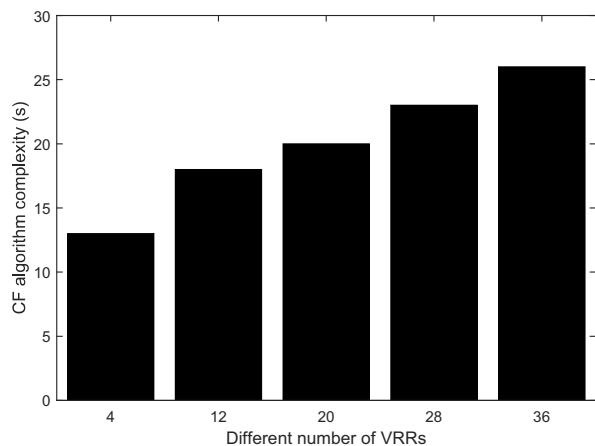


Fig. 4. CF algorithm complexity with respect to the different number of VRRs.

Further, the complexity of the proposed CF scheme is evaluated in Fig. 4. The X-axis is the different number of VRRs, and the Y-axis is the execution time of CF scheme. The unit of time is second (s). The complexity is counted under the condition that the iterations is 9 and the statistic times is 20. We can see that the rise is gently with the increasing number of VRRs. When the number of VRRs is 4, the algorithm complexity is 13s. When the number of VRRs is 36, the algorithm complexity is 26s. Thus, the number of VRRs increases 8-fold while the complexity only increases 1-fold. Therefore, the proposed CF scheme is low-complexity.

B. Performance Comparison

We show performance curves from the following aspects:

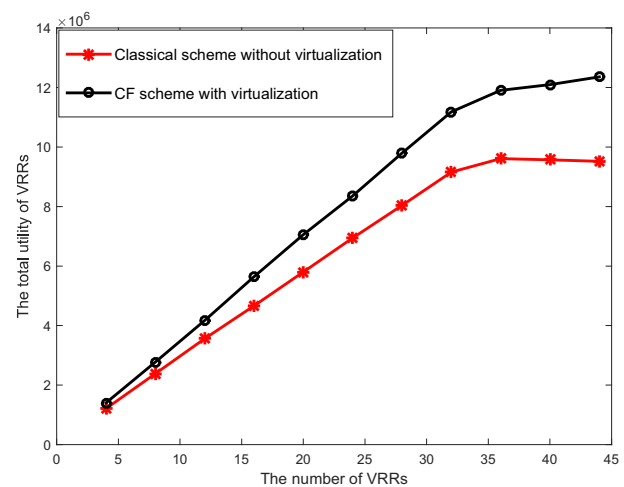


Fig. 5. Comparison of the total utility of VRRs for different schemes with the different number of VRRs.

- Comparison of the total utility of VRRs with and without virtualization in Fig. 5.
- Comparison of the utility of the VRP with and without virtualization in Fig. 6.
- Comparison of the average energy efficiency in the whole networks for different schemes in Fig. 7.
- Comparison of the system rate with and without virtualization in Fig. 8.
- Comparison of the resource utilization ratio with and without virtualization in Fig. 9.
- Comparison of the access-reject probability of VRRs with and without virtualization in Fig. 10.

The total utility of VRRs is illustrated in Fig. 5. We compare the performance of the CF scheme with the classical scheme without virtualization. The classical scheme is with fixed spectrum-allocation, average power-allocation, and average price-allocation. More specifically, each small base station has 50 VRBs, and the power is 30 dBm. Each VRR get 3 VRBs, and each small base station can serve 16 VRRs at most. The power is 0.02×3 , and the price adopts the statistical average of CF algorithm. It is easy to understand that the total utility of VRRs increases steadily with the growth of VRRs because more VRRs will bring more revenue. We can also observe that the CF scheme with virtualization always has an advantage over the classical scheme without virtualization. When the number of users is 44, the CF scheme with virtualization achieves the utility improvement of almost 22% than the classical scheme without virtualization. The reason is that VRRs can receive better quality of service and connect to a better condition of service with customized and on-demand resource allocation. That is, a higher degree of freedom of resource allocation is realized. Moreover, another reason is that the lower resource price is consumed for VRRs. More interestingly, when the network scale is extended through the number of VRRs, the utility of VRRs still increases with respect to the number of VRRs. Thus, the proposed CF scheme is especially suited to the network, which is large-scale or ultra-dense.

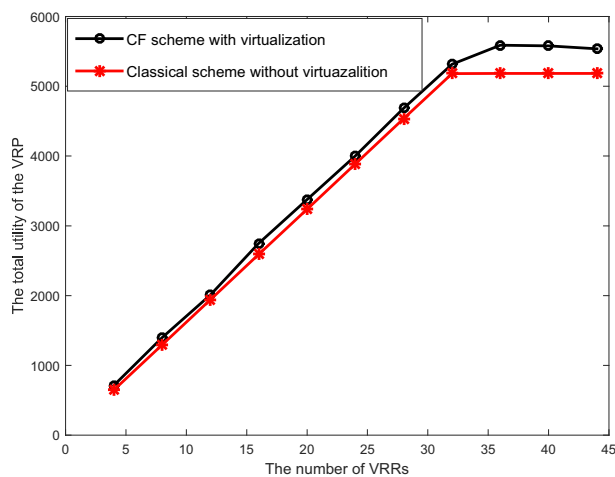


Fig. 6. Comparison of the total utility of the VRP for different schemes with the different number of VRRs.

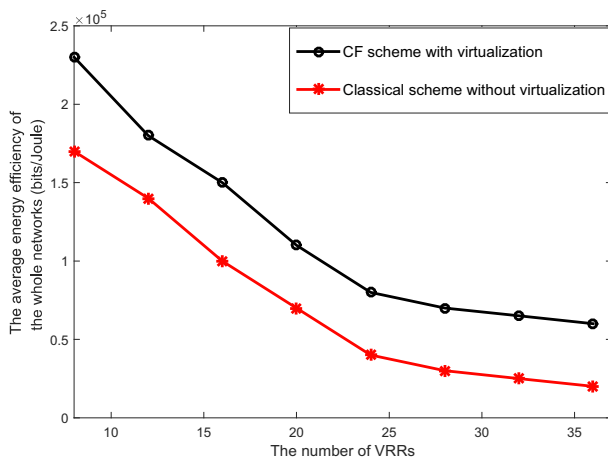


Fig. 7. Comparison of the average energy efficiency in the whole networks for different schemes with the different number of VRRs.

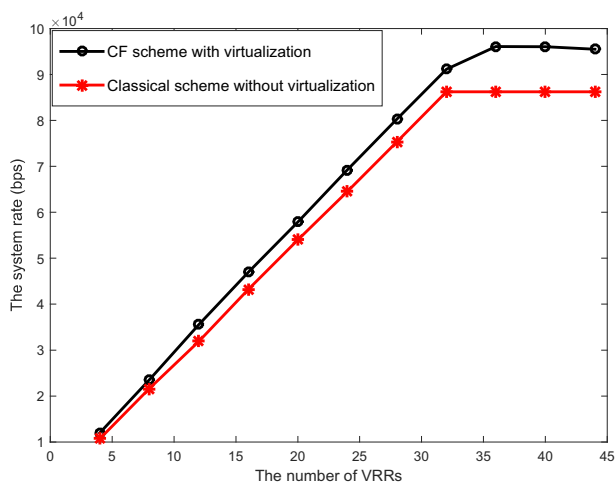


Fig. 8. Comparison of the system rate for different schemes with the different number of VRRs.

We also evaluate the total utility of the VRP versus the number of VRRs in Fig. 6. The classical scheme is same as the

one in Fig. 5. With the growth of VRRs, the utility of the VRP increases gradually because VRRs consume more resources. We can see that the utility of the CF scheme is always higher than the utility of the classical scheme. Meanwhile, when the number of VRRs is greater than 32, the superiority of the CF scheme is more obvious than the classical scheme. The reason is that more resources are leased in case of virtualization, thus, more revenues are obtained. Therefore, similarly to Fig. 5, the proposed CF scheme is applicable to the large-scale or ultra-dense networks.

We evaluate the performance of the CF scheme in terms of the average energy efficiency of the whole networks in Fig. 7. We can see that the CF scheme outperforms the classical scheme that is same as the one in Fig. 5. This is because the energy efficiency has been taken into account in the utility as an optimal objective. It can also be observed that the curve shows a descending tendency with the number of VRRs, which can be explained by the different growth rates in the average energy efficiency and the number of VRRs. Even so, when the number of users is 36, 3-fold energy efficiency improvement is achieved compared to the classical scheme.

We also evaluate the system rate with the different number of VRRs in Fig. 8. The classical scheme for fixed spectrum-allocation and average power-allocation is same as the one in Fig. 5. It shows that the curves rise with the growth of VRRs because of the multi-connectivity gain. Moreover, the CF scheme has higher rate than the classical scheme. The reason is that more VRRs are served with the same amount of resources by virtualization and higher quality of service are guaranteed by on-demand allocation. It can also be observed that the performance improvement with a large number of VRRs is higher than the one with a small amount of VRRs. When the number of users is 8, the system rate with the CF scheme is 2 Kbps higher than the one with the classical scheme. When the number of users is 36, the system rate with the CF scheme is 10 Kbps higher than the one with the classical scheme.

As shown in Fig. 9, the resource utilization ratio using different schemes with and without virtualization is compared. The classical scheme adopts on-demand spectrum-allocation. Each small base station has 50 VRBs, therefore the total demands of VRRs served by an small base station cannot exceed 50 if without virtualization. The resource utilization ratio increases with the growth of VRRs, which is because more resources are consumed. We can observe that the resource utilization ratio is equal when the number of users is smaller than 32. The reason is that the resources are enough to meet VRRs' demands. Furthermore, it can be observed that the resource utilization ratio using the CF scheme outperforms the one using the classical scheme when the number of VRRs is larger than 32. The reason is that the comparison scheme adopts the same spectrum allocation algorithm as the CF scheme but without using virtualization, which limits the flexibility of resource allocation and each small base station is only able to provide service for a certain number of VRRs. In other words, when the number of VRRs becomes large, the demands for resources become more, which may result in some VRRs not being served due to the insufficient

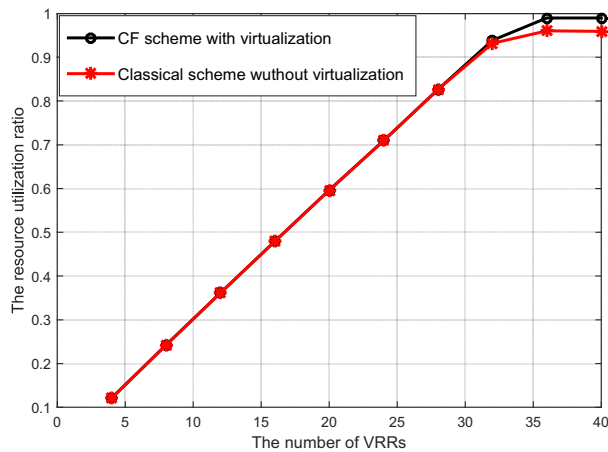


Fig. 9. Comparison of the resource utilization ratio with and without virtualization for the different number of VRRs.

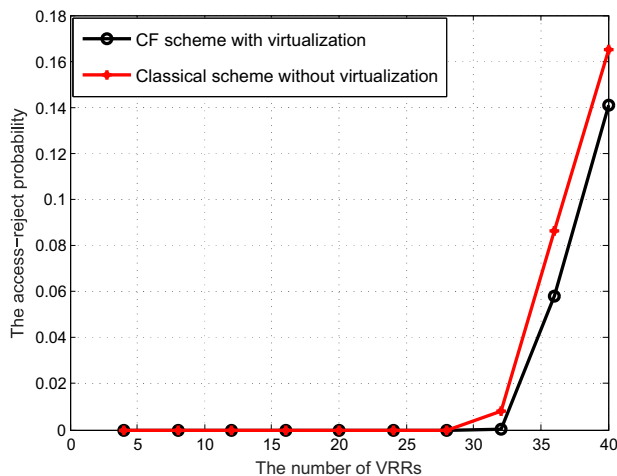


Fig. 10. Comparison of the access-reject probability with and without virtualization for the different number of VRRs.

resources if without virtualization. Thus, the residual resources are wasted and the resource utilization ratio is low. On the contrary, virtualization can integrate resources among multiple small base stations, which reduces the resource waste and improves the resource utilization ratio. We can calculate the promotion of the resource utilization ratio is about 4% when the number of VRRs is 40.

Further, we represent the access-reject probability with and without virtualization in Fig. 10. The classic scheme for spectrum-allocation is same as the scheme in Fig. 8. The access-reject probability refers that the ratio of VRRs that cannot be served due to the limited resources. We can see that for both CF scheme and classical scheme, the access-reject probability is zero when the number of VRRs is not larger than 28, which is because the resources are adequate to satisfy the demands. It can be seen that when the number of VRRs is 32, the access-reject probability begins to increase for the classical scheme while is still zero for the CF scheme, which is because the resources of some small base stations are inadequate to meet VRRs' demands if without virtualization. That is, by integrating resources with virtualization, more

VRRs can receive service at the same time. Moreover, we can see that when the number of VRRs is more than 32, the access-reject probability increases with the increasing VRRs for both CF scheme and classical scheme. With the growth of VRRs, the tendency is upward because of the inadequate resources and the added traffic. However, the access-reject probability with virtualization is always lower than the one without virtualization, and the superiority is even more obvious when the number of VRRs is large. This is because the integrating and sharing characteristics of virtualization, which means that the same amount of resources is able to meet more VRRs demands. It can be calculated that the access-reject probability of the CF scheme is 18% less than the one of the classical scheme when the number of VRRs is 40. That is, the same resources can serve 7 people more with the CF scheme. Therefore, combining with the Fig. 8, a conclusion can be obtained that the CF scheme can be applied in the ultra-dense large-connection scenario.

VII. CONCLUSIONS

In this paper, we investigated the user-oriented resource management problem with virtualization in ultra-dense small cell networks based on hierarchical game. We proposed a two-layer architecture specifying the necessity of resource management from physical networks to virtual networks. In virtual networks, there exist two components: VRPs and VRRs. We formulated the problem as a hierarchical game to maximize the utilities of both the VRP and VRRs. In the VRP layer, the VRP decides and broadcasts the price for VRRs based on demands. In the VRR layer, VRRs evaluate the demands for VRBs based on the communication requirements and the price provided by the VRP, and determine the transmission power. We formulated the utility of VRRs as a joint optimization function in spectrum and power. To solve the problem efficiently, we used decomposition method and standard convex optimization method to obtain the closed-form solutions for spectrum, power, and price, respectively. Most importantly, we proposed a low-complexity CF algorithm and HNE was found. Simulation results confirmed the convergence and effectiveness of the proposed CF algorithm.

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