The Role of Mortality Rate in the Transition from Stagnation to Growth

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Contents

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1 Introduction

In literature there is no agreement on which are the main determinants of the extraordinary development in the last five centuries of Western economies and of the related phenomenon denoted Great Divergence. Many scholars (see, e.g., Glaeser et al., 2004) following a literature starting in 1960' (see, e.g., Cipolla (1962)) argue that the (differences in the) accumulation of human capital is the main source of long-run growth, and therefore the root(s) of this development (and divergence) should be searched in the factors affecting its accumulation.

However, other scholars point to the quality of institutions as the main determinant of the long-run growth of a country. In one of the most important contributions Acemoglu et al. (2001) argument that a lower settler mortality, favoring a better quality of institutions, explains the differences in income between North American and Center and Southern American countries.

These two explanations can be also viewed as complementary, but the prevalence of one or the other has crucial policy implications. For example, if the quality of institutions is the key factor of development then the adoption of Western institutions (e.g. democracy) is the main policy recommendation to poor countries; differently, the attention should be on all the factors benefiting the accumulation of human capital (e.g. public expenditure in education).

The purpose of this paper is twofold: i) to discuss how changes in mortality, affecting the accumulation of human (and physical) capital, provides a plausible additional determinant of the long-run growth of countries; and ii) to reconcile the empirical evidence presented in Acemoglu et al. (2001) with the explanation that human capital accumulation, and not the quality of institutions, is the main determinant of long-run growth of countries.

Our theoretical framework explicitly refers to the Unified Growth Theory proposed by Galor and Weil (2000) and Galor (2005), where long-run growth is the result of accumulation of human capital and the pattern of development presents three regimes; however, here the transition from the first regime of stagnation to the third regime of modern growth is jointly driven by technological progress *and* by the mortality rate of individuals.

Importantly, following Preston (1975) and Easterlin (2004), changes in mortality rate are assumed to be exogenous as technological progress; indeed both authors convincingly argue that, at least for the period we are interested in here (i.e. the period from the 18th to the 20th century), health improvements are the result of the so-called "Mortality Revolution", which it is counterpart of Industrial Revolution for the improvements in productivity. Moreover, according to Easterlin (2004), both revolutions have the same source in the Scientific Revolution of the 17th century, with the mortality revolution displaying its main effects one century later with respect to Industrial Revolution. In Easterlin (2004)'s words: "the Industrial and Mortality

Revolution are two of a kind. Both mark the onset of accelerated technological change in their respective fields. Both reflect the cumulation of empirically tested knowledge dating from the seventeenth century onward. ... In seeking an explanation of both the Industrial and Mortality Revolution, one must ask what is new on the scene. The answer suggested here is the emergence and growth of modern science ... "(see Easterlin, 2004, pp. 99-100).[1]

The model proceeds along the lines of the theoretical framework presented in Galor and Moav (2004), where changes in the type of technologies and factors used in the economy characterized the three different regimes. In particular, economy can stay in i) an agricultural regime, where output is produced with an agricultural technology, whose factors are unskilled labour and land; ii) in a pre-modern-growth regime, where output is produced using only physical capital and unskilled labour in an industrial sector; and iii) in a modern-growth regime where both physical and human capital are used in the industrial sector. The transition from a regime to another depends on the change in the relative productivity of agricultural and modern technology and on the stock of physical and human capital in the economy.

The accumulation of physical and human capital is the result of choice of agents living potentially for two periods: childhood and adulthood. Every adult has just one child, but the size of population inversely varies with the mortality rate.[2] Every agent devotes the first period of her life to the acquisition of human capital (if any) and in the second period she employs her human capital and inherited wealth, i.e. physical capital, in production and allocate the resulting income between consumption and a bequest for her offspring (this bequest is positive only other a given threshold of income).

Parents choose to allocate this bequest between the finance of offspring's education and a transfer devoted to finance the future wealth of offspring, in order to maximize the future income of children.

We assume that people potentially live two periods: childhood and adulthood. Childhood is a certainty and the risk of mortality occurs in adulthood [3] .

The optimal investment in human capital is always decreasing in the mortality rate, since a higher mortality rate decreases the return on investing in education (the agent has less time to recover from her investment). However, a decrease in mortality rate has two opposite effects on the amount of inherited wealth. On one hand, it raises the consumption of parent reducing the

[1]The idea that Industrial Revolution is mainly the result of a cultural revolution caused by the emergence of the new scientific method elaborated in the 17th century (which particularly permeated the English society in the 18th and 19 centuries) has strong advocates; see for a discussion Mokyr (1999).

[2]It is straightforward to extend the analysis to the case of a growth rate of population dependent of mortality rate by assuming more than one child for every adult.

[3]The possibility to die also in the childhood is out of the scope of the paper.

amount of transfer to offspring; on the other hand, it also increases labour income and hence the transfer. We show that at low level of income (i.e. in the agricultural and pre-modern regimes) the first effect can dominate the second and induces a reduction in the bequest to offspring and therefore in the capital available in the economy. This result is in line with the empirical evidence discussed in Cervellati and Sunde, 2011 on nonlinear relationship between life expectancy and growth rate of per capita income

An economy with a low productivity in the industrial sector (with respect to agricultural sector) and a high mortality rate is deemed to stay in the agricultural regime, where the very low income allows for a limited bequest entirely concentrated in a direct transfer given the low return of the investment in education and where such transfer is not invested in the industrial technology because not sufficiently productive with respect to agricultural technology; a raise in the productivity of industrial sector can push the economy in the pre-modern-growth regime, where wage (for unskilled labour) in the industrial sector are expected higher than in the agricultural sector even though a low stock of capital. As discussed above, the impact of a possible fall in mortality rate on this dynamic is ambiguous : at low levels of income, it could decrease inherited wealth and therefore act as a counterbalancing force with respect to the increase of productivity of the industrial sector. In particular, we find that under plausible assumptions the rise in life expectancy, at low levels of income, has a positive effect on intergenerational transfers only if remains within a certain threshold, then it can have a favorable effect and push an economy towards a pre-modern regime. On the other hand, if longevity exceeds this threshold, then it may have a negative effect on economic growth and the economy can be pushed back to an agricultural regime.

Finally, at high levels of income, the rise in life expectancy always allows the transition from a pre-modern-growth regime to a modern-growth regime. Thus, at high levels of income both the increase in the productivity of industrial technology and the decrease in mortality rate increases the size of bequest and the incentive to invest in human capital by raising the wage of skilled workers. This is caused both by the increases in the stock of capital and by the longer period agent can have to recover from her investment. Therefore, in this regime the positive effect on country's income by the Industrial and by the Mortality revolutions should reinforce one another.

Our work is related to a large body of recent literature that analyzes the interaction between human capital formation and the rise in life expectancy in the process of development. Cervellati and Sunde (2005), Boucekkine et al. (2003) focus on the reinforcing interaction between life expectancy and human capital accumulation in the transition from stagnation to growth. Hazan and Zoabi (2006) show that the complementarity between health and education in the production of human capital is crucial to generate the quantity - quality trade off and therefore the transition from stagnation to growth. Others theoretical contributions suggest that the decline in mortality rates can increase the return to investment in human capital via: (a) a prolongation of life that rising the period in which individuals may receive returns on their investment promotes investment in human capital (Soares, 2005; De la Croix and Licandro, 1999), (b) a high population density which rises the efficiency in human capital production through an increase in the spread of knowledge (Lagerlof, 2003), (b) the increase in population growth and in the advancement of skill-biased technologies (Weisdorf, 2004).

The interaction between mortality rates and economic development is also the focus of many empirical studies. Lorentzen et al. (2008), Bloom et al. (2004), Bloom and Canning (2005) among others, find a positive effect of life expectancy on economic growth. In the other hand, Acemoglu and Johnson (2007) shows that the rise in life expectancy leading to a significant increase in population which is not sufficiently compensated by the reduction in the birth rates, has a negative effect on income per capita.

The paper proceeds as follows; Section 2 presents the model; Section 3 analyses the development process; and Section 4 concludes.

2 The Model

To measure the effect of mortality reductions on growth we extend the framework presented in Galor and Moav (2004). We consider an economy populated by an overlapping generations of people who potentially live for two periods: childhood and adulthood. They live in childhood for sure but are subjected to a mortality risk $(1-p) \in (0,1)$ during the adulthood. More specifically, p defines the probability for an individual belonging to cohort t of reaching the end of period $t+1$. In every period t labour force consists of pL_t workers and each worker receives a labour income which rises with p.

2.1 Production

In every period, the economy produces a single material good, the price of which is normalized to 1. Production may take place with two different methods: an agricultural technology that employs unskilled labour and land, and an industrial technology that employs physical capital and skilled labour.

We assume that in the early stage of development the industrial technology is too inefficient to be used and production is conducted using the agricultural technology. However, in the process of development the productivity of the industrial technology grows faster than that of the agricultural technology and at some point becomes profitable to employ the industrial technology. Thus, the industrial technology will replace the agricultural technology and the process of modernization begins.

The agricultural production function is given by:

$$Y_t^a = A^a (pL_t)^{1-\lambda} (T)^{\lambda}, \tag{1}$$

where A^a is a productivity parameter, pL_t is the supply of unskilled labour and T is the quantity of land which is normalized to 1 for simplicity.

The industrial production function is given by:

$$Y_t^m = A(pH_t)^{1-\alpha} K_t^{\alpha} = Ap^{1-\alpha} H_t k_t^{\alpha}, \tag{2}$$

where $k_t = K_t/H_t$, $\alpha \in (0,1)$ and $A > 0$ is a technological parameter. Note that pH_t with $H_t = h_t L_t$ is the aggregate level of human capital given by the individual level of human capital h_t and labour force pL_t. As established below human capital increases with the resources invested in education and when these resources are zero $H_t = L_t$.

The problem of producers is to maximize profits subject to the production function. When agricultural technology is operating, in absence of property rights to land, workers receive their average product :

$$w_t^a = A^a (pL_t)^{-\lambda}. \tag{3}$$

When industrial technology is operating, producers choose the level of physical capital K_t and the efficiency units of labour H_t such that $\{K_t, H_t\} = \arg\max\, [Ap^{1-\alpha} H_t k_t^{\alpha} - wpH_t - r_t K_t]$. Thus, the rate of return to capital r_t and the wage rate per efficiency unit of labor w_t are given by:

$$r_t = \alpha A p^{1-\alpha} k_t^{\alpha-1}; \tag{4}$$

$$w_t^m = (1-\alpha)\, Ap^{-\alpha} k_t^{\alpha}. \tag{5}$$

We assume that in the early stage of development production is conducted using the old technology since the productivity of the new technology A is lower than the productivity of the old technology A^a (see Galor and Mountford (2008)). The economy will start to employ the new technology when the value of the marginal product of unskilled workers using the new technology, $(1-\alpha)\, Ap^{-\alpha} k_t^{\alpha}$ is at least high as that of unskilled workers using the agricultural technology $(1-\lambda) A^a p^{-\lambda} (L_t)^{-\lambda}$. Therefore production will be conducted using the new technology when:

$$k_t \geq k^o,$$

where:

$$k^o = \left[\left(\frac{1-\lambda}{1-\alpha} \right) \frac{A^a}{A} p^{\alpha-\lambda} (L_t)^{-\lambda} \right]^{1/\alpha}. \tag{6}$$

Notice that the effect of higher life expectancy on the threshold level k^o is ambiguous and depends on the relationship between α and λ. In order to reduce the number of possible scenarios we assume that

Assumption 1

$$\alpha > \lambda.$$

2.2 Consumption and Total Transfers

We consider an overlapping generations economy where agents live in childhood and as adults. As established above, adults have a probability $p \in (0, 1)$ of surviving during the second period. Each individual has a single parent and a single child and adult population in period t is normalized to 1. They care about consumption c_{t+1} and a transfer to the offspring b_{t+1}. The expected utility function is therefore[4]:

$$U = p[(1-\beta)\log(c_{t+1}) + \beta \log(b_{t+1} + \theta)], \tag{8}$$

where $\beta \in (0, 1)$ is the discount factor and $\theta > 0$ implies that children receive a positive transfer only when parent's income is sufficiently high (see Eq. (14) below).

Agents devote the first period of their lives to the acquisition of human capital. In particular, human capital of children is an increasing function of expected level of education that parents give to the offspring, i.e pe_t, that is:

$$h_{t+1} = (1 + pe_t)^\gamma \tag{9}$$

where $h(0) = 1$, $h'(0) = \gamma$ and $\lim_{e_t \to \infty} h'(pe_t) = 0$ (Galor and Moav, 2004, 2006).

In the second period of life agents work and allocate their income between consumption and a bequest for their children. In particular, income of each agent is given by labor income ω_{t+1} and an inheritance x_{t+1} that she received from parents. The expected labour income rises with

[4]Following Rosen (1988) we assume the expected utility in the second period is given by the utility of state "life "given by the utility from consumption and the bequest to the children and the utility of state "death "given by M which is assumed to be equal to zero for simplicity:

$$U = p[(1-\beta)\log(c_{t+1}) + \beta \log(b_{t+1} + \theta)] + (1-p)M, \tag{7}$$

life expectancy and is given by pw^a_{t+1} when agricultural technology is employed and $pw^m_{t+1}h_{t+1}$ when industrial technology is employed. Therefore, individuals' second period income y_{t+1} is:

$$y_{t+1} = \omega_{t+1} + x_{t+1}, \tag{10}$$

Parents allocate this income between consumption pc_{t+1} and a transfer to the offspring pb_{t+1}:

$$y_{t+1} = pc_{t+1} + pb_{t+1}, \tag{11}$$

where $b_{t+1} \geq 0$. As established below parents allocate the expected transfer pb_{t+1} between the spending in children's education pe_{t+1} and an amount ps_{t+1} which they save for the future wealth of children . Thus the inheritance x_{t+1} which agents receive in the adult age is given by return on parents' saving:

$$x_{t+1} = ps_t R_{t+1} = p(b_t - e_t)R_{t+1}, \tag{12}$$

where due to complete capital depreciation, i.e. $\delta = 1$, $R_{t+1} = 1 + r_{t+1} - \delta = r_{t+1}$. Thus when agricultural technology is operating $R_{t+1} = 0$.

Parents choose the level of consumption and the level of transfer to the offspring so as to maximize their expected utility subject to the budget constraint (11) :

$$c_{t+1} = \begin{cases} \frac{y_{t+1}}{p} & \text{if } y_{t+1} \leq \mu p\theta \\ \frac{\mu(y_{t+1}+p\theta)}{p(1+\mu)} & \text{if } y_{t+1} > \mu p\theta \end{cases}, \tag{13}$$

$$b_{t+1} = \begin{cases} 0 & \text{if } y_{t+1} \leq \mu p\theta \\ \frac{y_{t+1}-\mu p\theta}{p(1+\mu)} & \text{if } y_{t+1} > \mu p\theta \end{cases}, \tag{14}$$

where $\mu \equiv (1-\beta)/\beta$. Note that there is an interior solution for the optimal transfer only if parents' income is sufficiently high, i.e. $y_{t+1} > \mu p\theta$, otherwise parents devote their income totally to the consumption.

Parents choose the allocation of optimal transfer to the spending in children's education in order to maximize the future income of children i.e. y_{t+1}. Thus from equations (10) and (12) it follows that:

$$e^*_t = \underset{e_t \in [0,b_t]}{\arg\max}[pw^m_{t+1}h(pe_t) + p(b_t - e_t)R_{t+1}],$$

which using Equations (4), (5) and (9) yields the following optimal level of education:

$$e_t = \begin{cases} 0 & \text{if } 0 < k_{t+1} \leq \tilde{k}; \\ \frac{1}{p}\left[\left(\frac{k_{t+1}}{\tilde{k}}\right)^{\frac{1}{1-\gamma}} - 1\right] & \text{if } k_{t+1} > \tilde{k}, \end{cases} \tag{15}$$

where:

$$\tilde{k} = \frac{\alpha}{(1-\alpha)\gamma}. \tag{16}$$

Hence, there is an interior solution for the optimal education choice only if the capital labour ratio in period $t+1$ is sufficiently high, that is $k_{t+1} > \tilde{k}$.
Given that in the early stages of development, i.e. $k_t < k^o$, there is no demand for skilled individuals and the proportion of skilled labour in the labour force is zero, it seems reasonable to assume that:

Assumption 2

$$k^o < \tilde{k}, \tag{17}$$

which holds if:

$$A > A^{MIN} = \left[\frac{(1-\alpha)\,\gamma}{\alpha}\right]^{\alpha} \left(\frac{1-\lambda}{1-\alpha}\right) A^o p^{\alpha-\lambda} (L_t)^{-\beta}. \tag{18}$$

2.3 Equilibrium

The amount of transfer b_{t+1} and his allocation between education spending and the saving for the future wealth of children determine the aggregate level of physical capital K_{t+1} and human capital $\bar{H}_{t+1}$:

$$\begin{aligned} K_{t+1} &= ps_t = p(b_t - e_t), \\ H_{t+1} &= h(pe_t), \end{aligned} \tag{19}$$

where Thus the capital-labour ratio k_{t+1} is given:

$$k_{t+1} = \frac{p(b_t - e_t)}{h(pe_t)}. \tag{20}$$

Substituting equation (20) into equation (15) we get the following relationship between the expected education spending, i.e. $\bar{e_t} = pe_t$ and the expected intergenerational transfer, i.e. $\bar{b_t} = pb_t$

$$\bar{e_t} = \begin{cases} 0 & \text{if } 0 < \bar{b_t} \leq \tilde{k}; \\ \frac{\bar{b_t} - \tilde{k}}{1+\tilde{k}} & \text{if } \bar{b_t} > \tilde{k}, \end{cases} \tag{21}$$

Notice that a higher adult longevity, lowers the threshold level $\tilde{k}$ and hence rises the optimal amount of education.

Thus, parents choose to invest in children education only if the expected intergenerational transfer $\bar{b_t}$ is sufficiently high otherwise the optimal choice for education $\bar{e_t}$ is zero. This implies that the expected amount which parents save for the future wealth of children, i.e. $\bar{s}_t = ps_t$, is given:

$$\bar{s}_t = \begin{cases} \bar{b_t} & \text{if } 0 < \bar{b_t} \leq \tilde{k}; \\ \frac{\tilde{k}(1+\bar{b_t})}{1+\tilde{k}} & \text{if } \bar{b_t} > \tilde{k}. \end{cases} \tag{22}$$

Therefore, when bequest is $0 < b_t \leq \tilde{k}$ the optimal choice for education is zero and total transfer is devoted to finance future wealth of children. When bequest is $b_t > \tilde{k}$, both e_t and s_t increase with respect to b_t.

Equations (20) and (22) imply that the capital-labor ratio in period t is determined by the amount of transfer in period t, that is:

$$k_{t+1} = k(b_t), \tag{23}$$

thus we can define $k^o = b^o$ and $\tilde{k} = \tilde{b}$.

Using equations Equations (3)-(5), (14) and (21) we can characterize the dynamic of expected transfers for each child in period $t+1$, i.e. $\bar{b}_{t+1}$, as function of intergenerational transfers in the preceding period :

$$\bar{b}_{t+1} = \begin{cases} \max[(A^a p^{1-\lambda} - \mu p\theta)/(1+\mu), 0] & \text{if } \bar{b_t} \in [0, b^o); \\ \\ (Ap^{1-\alpha}\bar{b_t}^{\alpha} - \mu\theta p)/(1+\mu) & \text{if } \bar{b_t} \in [b^o, \tilde{b}); \\ \\ \left[Ap^{1-\alpha}\tilde{b}^{\alpha}\left(\frac{1+\bar{b_t}}{1+\tilde{b}}\right)^{\gamma(1-\alpha)+\alpha} - \mu\theta p\right]/(1+\mu) & \text{if } \bar{b}_t \in [\tilde{b}, +\infty). \end{cases} \tag{24}$$

The three ranges of $\bar{b_t}$ identify the three distinct regimes: the agricultural regime, i.e. $\bar{b_t} \in [0, b^o)$, where production is conducted using agricultural technology, whose factors are unskilled labour and land; a pre-modern-growth regime $\bar{b_t} \in [0, b^o)$, where output is the result of using physical capital and unskilled labour in an industrial sector; and a modern-growth regime, i.e. $\bar{b_t} \in [0, b^o)$ where both physical and human capital are used in the industrial sector.

3 Dynamics Within and Between Regimes

The dynamics of economics within each regime and the transition through the three regimes depend on the different combinations of technology and survival probability (see figure 1). The following proposition characterizes the dynamic of total transfer within each regime.

Proposition 1 *Under Assumptions (1), (2) and assuming* $\frac{A^a}{(1-\lambda)\mu} < \theta < \frac{(1+\mu)}{\gamma\mu}$*:*

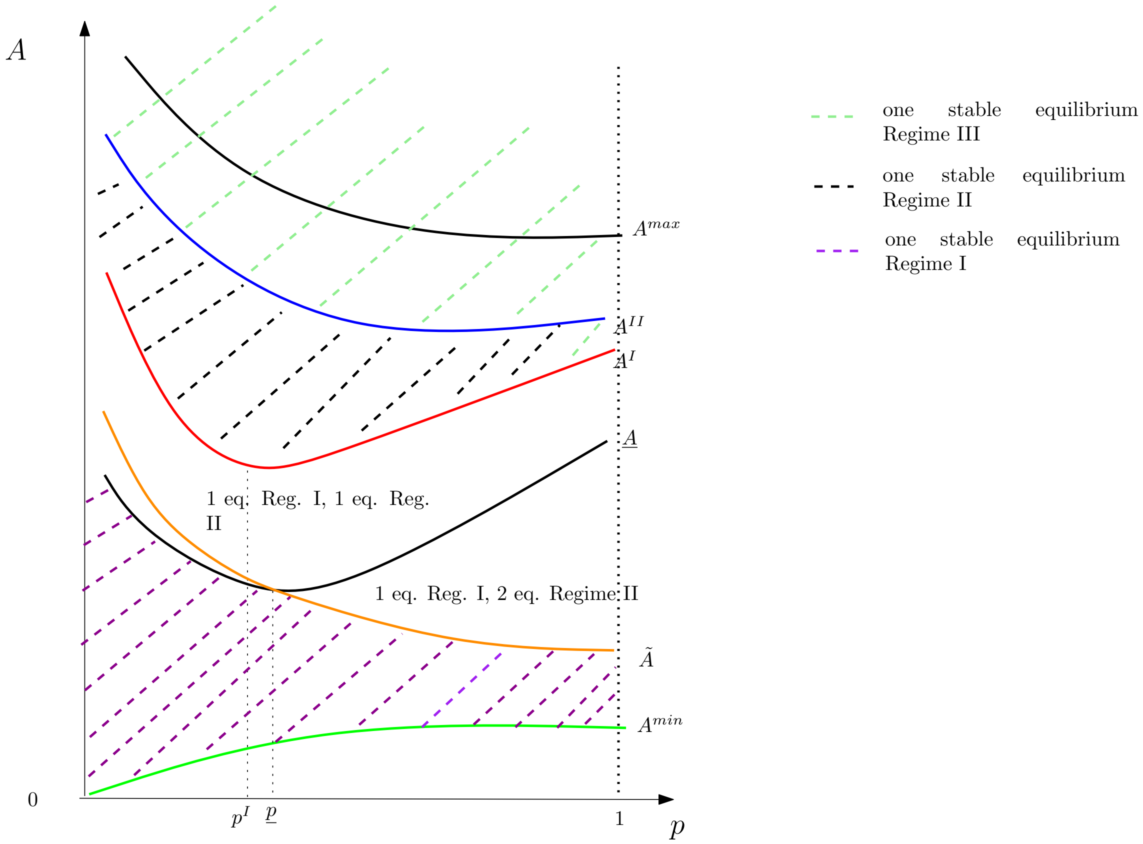

Figure 1: Dynamics between and within regimes

- *An economy shows only one stable equilibrium in regime I if* $A < \underline{A}$ *for* $0 \leq p < \underline{p}$ *and* $A < \tilde{A}$ *for* $\underline{p} < p \leq 1$.
- *An economy shows only one stable equilibrium in regime II if* $A^I < A < A^{II}$.
- *An economy shows only one stable equilibrium in regime III if* $A > A^{II}$.

Proof. see Appendix A ■

At early stages of development, in an agricultural regimes, when technology is relatively low, the industrial technology is latent and the economy stagnates. The low level of income results in a zero or a very low bequest which is not invested in industrial technology because it is not sufficiently productive compared to agricultural technology. Therefore, the economy converges to a stable equilibrium, i.e. E^L in which there is neither investment in physical capital nor in human capital (see figure 2).

The rise in productivity of the industrial sector, can lead to three phases. As long as it is relatively low, i.e. $A < A^I$, the dynamic of total transfers show multiple equilibria: if the

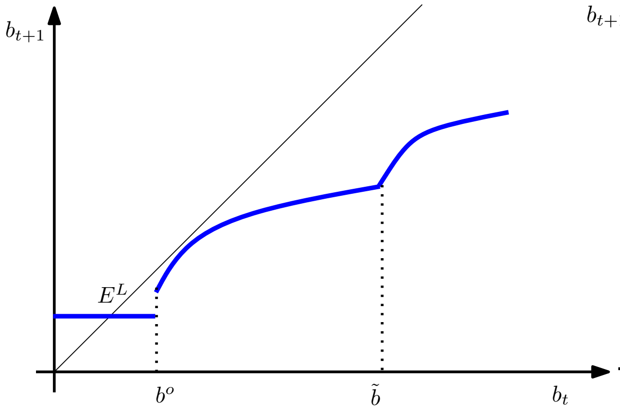

Figure 2: One Stable Equilibrium Regime I

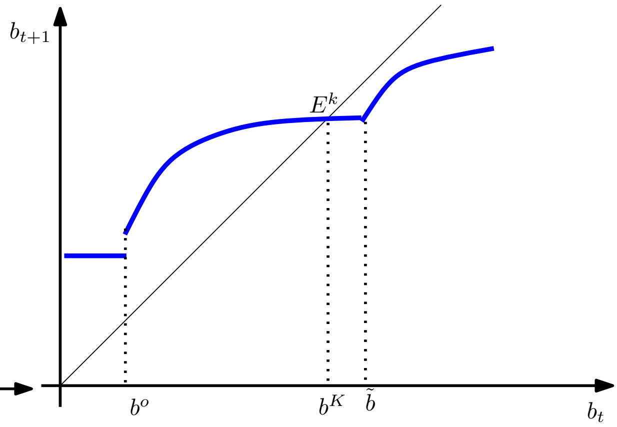

Figure 3: One Stable Equilibrium Regime II

initial level of b_t is low, i.e. $b_t < b^L$, it will not be profitable to use industrial technology, therefore the intergenerational transfers contract over time and the system converges to the stable equilibrium E^L. If $b_t > b^L$ then the economy begins to accumulate physical capital and converges to the stable equilibrium i.e. E^K where the evolution of output is driven by physical capital accumulation (see figure 5). As the productivity of the industrial sector continues to grow, the unstable equilibrium vanishes (see figure 6). When industrial technology becomes sufficiently productive, i.e. $A > A^I$, regardless of the initial level of bequest, it will become profitable to use industrial technology, and the economy converges to a stable equilibrium, i.e. E^K, where the evolution of output is driven by physical capital accumulation (see figure 3).

Finally, the further increase in the productivity of industrial technology i.e. $A > A^{II}$, allows the economy to converge to a globally steady state equilibrium E^H where the evolution of output is driven by the accumulation of human capital as well as physical capital (see Figure 4).

The reduction of mortality can have important effects on this dynamic. In particular, a decrease in mortality has two opposing effects on intergenerational transfers. On the one hand, higher longevity increases consumption by parents, thus reducing transfer to their offspring; on the other hand, parents who live longer, work for a longer period, thus increasing labor income and raising transfers to their children. When the initial level of income is sufficiently high, the second effect always prevails whereas at low levels of income, if longevity increases above a certain threshold, i.e. $p > \underline{p}$ the first effect can prevail. If however, longevity remains within a certain threshold, then it can have a positive effect on the intergenerational transfers thus leading to physical capital accumulation and to a pre-modern regime. The basic motivation underlying this result is that income increases at decreasing rates with respect to

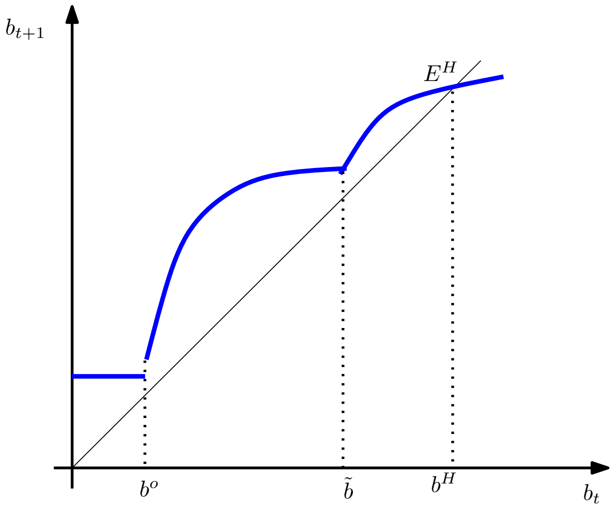

Figure 4: One Stable Equilibrium Regime III

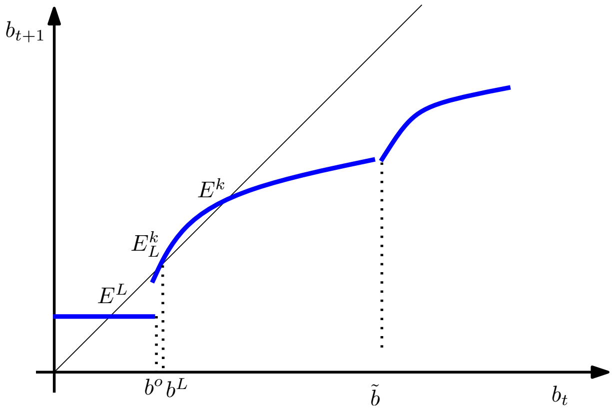

Figure 5: Multiple Equilibria (a)

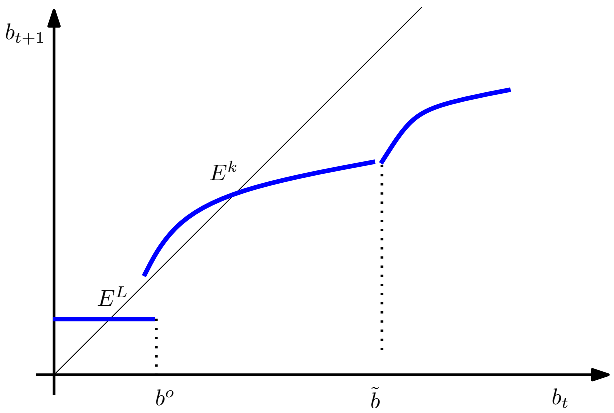

Figure 6: Multiple Equilibria (b)

longevity. This implies that when longevity increases above a certain threshold, at low levels of income, the rise in income is insufficient to compensate the rise in consumption. Reduction in intergeneration transfer, in turn reduces physical capital accumulation, pushing the economy towards an agricultural regime. On the other hand, at high levels of income the economy can accumulate human capital and therefore the rise in longevity always allows a level of income sufficiently high to compensate for the rise in consumption, thus leading to a modern regime (see figure 7). Indeed, if income is sufficiently high, the rise in longevity increases the return on investment in education and therefore high income perpetuates.

The effect of mortality reductions on the intergenerational transfer is summarized in the following proposition.

Proposition 2 *Under Assumptions (1), (2) and assuming* $\frac{A^a}{(1-\lambda)\mu} < \theta < \frac{(1+\mu)}{\gamma\mu}$, *an increase in longevity always has a positive effect on intergenerational transfer at high levels of income and a non-linear effect at low levels of income: the effect is positive if longevity remains below a certain threshold, i.e.* $\partial b_{t+1}/\partial p > 0$ *if* $p < \underline{p}$, *and is negative, if longevity exceeds that threshold, i.e.* $\partial b_{t+1}/\partial p < 0$ *if* $p > \underline{p}$.

Proof. see Appendix B ■

Thus mortality reduction has important implications for the transition through the three regimes (see figure 1). Consider an economy which shows a unique stable equilibrium in Regime II. If initial income is sufficiently high, the rise in life expectancy triggers a rapid transition toward the modern-growth regime. The rise in longevity, indeed, ensuring higher returns in human capital, makes investing in education more profitable. This leads to a higher income per capita, increasing the intergenerational transfer.

On the other hand, at low levels of income, as long as life expectancy remains within a certain threshold, i.e. $p < \underline{p}$, then it can have a favorable effect and push an economy towards a pre-modern regime. However, if life expectancy rises above a certain threshold, the low equilibrium E^L may emerge. Thus the economy may be pushed back to an agricultural regime.

These results are in line with the empirical evidence discussed in Cervellati and Sunde, 2011 which show a non linear relationship between life expectancy and economic growth. In particular, they show that this relationship is negative before the onset of the demographic transition and strongly positive after its onset. The basic idea behind this results is that increased life expectancy might have a negative effect on growth in income per capita if it accelerates population growth.

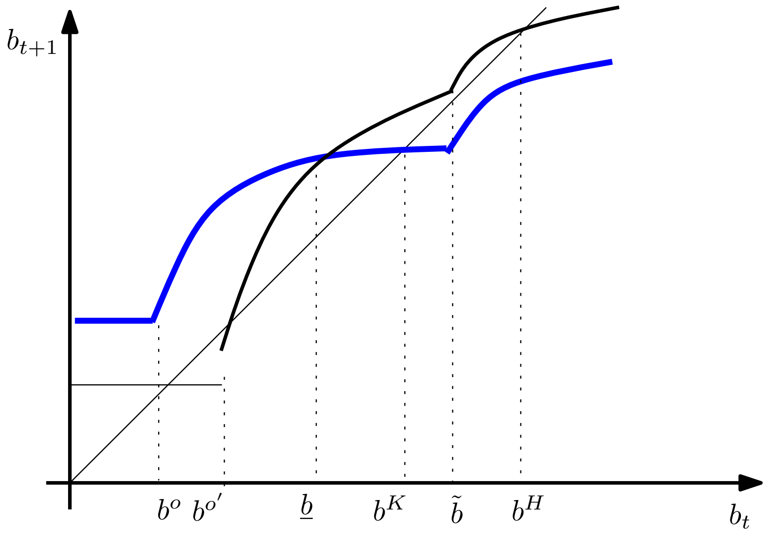

Figure 7: The rise in life expectancy

4 Conclusions

This paper contributes to the literature on the role of mortality reductions for economic growth by accounting for the differential effects of life expectancy during the different phases of economic development. According to the existing literature, we find that the rise in technological progress always allows the transition from stagnation to growth. The rise in longevity can have important effects on this dynamic. It has a positive effect on intergenerational transfer at high levels of income and a non-linear effect at low levels of income: this effect is positive if longevity remains within a certain threshold and becomes negative if longevity exceeds that threshold. The basic motivation underlying this result is that income increases at decreasing rates with respect to longevity. Thus if longevity increases above a certain threshold, at low levels of income, the rise in income is insufficient to compensate the rise in consumption. Reduction in intergeneration transfer, in turn reduces physical capital accumulation, pushing the economy towards an agricultural regime. On the other hand, at high income levels, rising longevity doesn't has the same opposite effects. The rise in longevity, indeed, increasing the return on investment in education, stimulates investment in human capital and increases labour income. Thus the rise in income is sufficiently high to compensate the rise in consumption due to higher longevity, leading to higher intergenerational transfers.

Thus we show that the rise in life expectancy may have direct effects on economic growth, although they appear to be non-monotonic and depend in particular, on the level of development. When income is sufficiently high, improvements in life expectancy always increase the probability of transition towards the modern growth regime. However, at low levels of income, if the rise of longevity exceeds a certain threshold, the economy can be pushed back to a stagnation regime.

Two important caveats to our analysis is that we ignore the potential role of endogenous fertility and endogenous mortality. We choose to not consider endogenous fertility in order to highlight the central role of mortality decline in the explanation of the observed patterns of development of the most of Western countries. An extension of the model to include an endogenous fertility rate should be advised but we argue that it should not substantially affect the main results of the paper.

With respect the second argument, we argue that the introduction of endogenous mortality should not affect the qualitative results of the paper but just adding a possible self-reinforcing mechanism to the transition from a regime to the other.

References

Acemoglu, D. and S. Johnson (2007). "Disease and Development: the Effect of Life Expectancy on Economic Growth". *Journal of Political Economy 115*(61), 925985.

Acemoglu, D., S. Johnson, and J. A. Robinson (2001). "The Colonial Origins of Comparative Development: An Empirical Investigation". *American Economic Review 91*(5), 1369–1401.

Bloom, D. and D. Canning (2005). "Health and Economic Growth: Reconciling the Micro and Macro Evidence". *Working Paper no. 42 (February),*, Center Democracy, Development, and Rule of Law, Stanford Inst. Internat. Studies, Stanford, CA.

Bloom, D., D. Canning, and J. Sevilla (2004). "The Effect of Health on Economic Growth: A Production Function Approach". *World Development 32*(1), 1–13.

Boucekkine, R., D. D. la Croix D., and O. Licandro (2003). "Early Mortality Declines at the dawn of modern growth". *Scandinavian Journal of Economics 105*(3), 401–418.

Cervellati, M. and U. Sunde (2005). "Human Capital Formation, Life Expectancy and the Process of Development". *American Economic Review 95*, 1653–1672.

Cervellati, M. and U. Sunde (2011). "Life Expectancy and economic growth: the role of the demographic transition". *Journal of economic growth 16*, 99–133.

Cipolla, C. (1962). *The Economic History of World Population.* Harmondsworth, Penguin Books.

De la Croix, D. and O. Licandro (1999). "Life Expectancy and Endogenous Growth". *Economic Letters* (65), 225–263.

Easterlin, R. A. (2004). *The Reluctant Economist Perspectives on Economics, Economic History, and Demography.* Cambridge University Press.

Galor, O. (2005). "From Stagnation to Growth: Unified Growth Theory ". *in The Handbook of Economic Growth (A 'Classical' Prespective) 1*(Edited by Philippe Aghion and Steven Durlauf), 171–293.

Galor, O. and O. Moav (2004). "From Physical to Human Capital Accumulation: Inequality and the Process of Development". *Review of Economic Studies 71*, 1001–1026.

Galor, O. and O. Moav (2006). "Das Human Kapital: A Theory of the Demise of Class Structure". *Review of Economic Studies 73*, 85–117.

Galor, O. and A. Mountford (2008). " Trading Population for Productivity : Theory and Evidence". *Review of Economic Studies 75*, 1143–1179.

Galor, O. and D. Weil (2000). "Population, Technology and Growth: From the Malthusian Regime to the Demographic Transition and Beyond". *American Economic Review) 90*, 806–828.

Glaeser, E. L., R. L. Porta, F. L. de Silanes, and A. Shleifer (2004). "Do Institutions Cause Growth? ". *Journal of Economic Growth 9*(3), 271–303.

Hazan, M. and H. Zoabi (2006). "Does longevity cause growth? a theoretical critique ". *Journal of Economic Growth 11*(4), 363–376.

Lagerlof, N. P. (2003). "From Malthus to Modern Growth: Can Epidemics Explain the three Regimes". *International Economic Review 44*, 755–777.

Lorentzen, P., J. McMillan, and R. Wacziarg (2008). "Death and development". *Journal of Economic Growth 13*(2), 81–124.

Preston, S. (1975). "The Changing Relation between Mortality and Level of Economic Development". *Population Studies 29*(2), 231–248.

Rosen, S. (1988). "The Value of Changes in Life Expectancy". *Journal of Risk and Uncertainty 1*(3), 285–304.

Soares, R. (2005). "Mortality Reductions, Educational Attainment and Fertility Choice". *American Economic Review 95*(3), 580–601.

Weisdorf, J. L. (2004). "From stagnation to Growth: Revisiting three historical regimes". *Journal of Population Economics 17*, 455472.

A Proof of Proposition 1

From equation (14), when $\bar{b}_t \in [0, b^0]$, parents leave a positive bequest to their children if:

$$A^a > \mu\theta p^\lambda \tag{25}$$

Under assumption 1, simple calculations show that if $\bar{b}_{t+1} > 0$ for $\bar{b}_t \in [0, b^o]$, then it also holds that $\bar{b}_{t+1} > 0$ for each $\bar{b}_t > b^o$. Otherwise if condition (25) doesn't hold, then for each $\bar{b}_t > b^o$, the intergenerational transfer will be positive if b_t is sufficiently high, that is, $b_t > \hat{b}$:

$$\hat{b} = \frac{\mu\theta p^\alpha}{A} \tag{26}$$

where $\hat{b} < b^o$ if the following condition holds:

$$A^a > \mu\theta p^\lambda \left(\frac{1-\alpha}{1-\lambda}\right) \tag{27}$$

- An economy shows only one stable equilibrium in regime I if technological level is: $A < \underline{A}$ when $0 < p < \underline{p}$ and $A < \tilde{A}$ when $\underline{p} < p < 1$ (see figure 8).
- An economy shows only one stable equilibrium in regime II if $A^I < A < A^{II}$.
- An economy shows only one stable equilibrium in regime III if $A > A^{II}$.

Proof:
An economy shows only **one stable equilibrium** in the range $b_t \in [0, b^o)$ if:

$$\lim_{b_t \to b^{o-}} b_{t+1} \leq b^o,$$

$$\lim_{b_t \to b^{o+}} b_{t+1} \leq b^o,$$

$$\left.\frac{\partial b_{t+1}}{\partial b_t}\right|_{b_t = b^o} < 1,$$

$$\lim_{b_t \to \tilde{b}^-} b_{t+1} \leq \tilde{b},$$

$$\lim_{b_t \to \tilde{b}^+} b_{t+1} \leq \tilde{b},$$

$$\left.\frac{\partial b_{t+1}}{\partial b_t}\right|_{b_t = \tilde{b}} < 1,$$

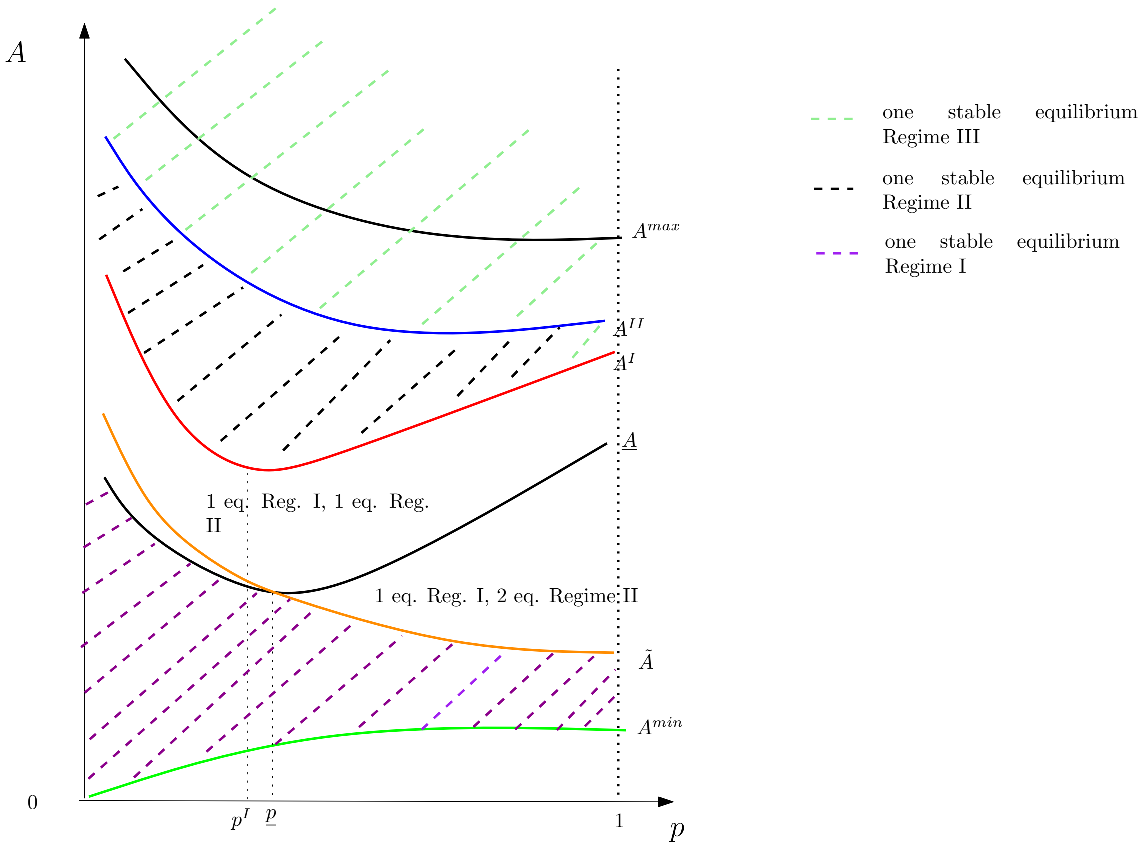

Figure 8: Dynamics between and within regimes

The first condition holds if $A < A^I$

$$A^I = \left(\frac{1-\lambda}{1-\alpha}\right) A^a \left(\frac{1+\mu}{A^a p^{\frac{\lambda(1-\alpha)}{\alpha}} - \mu\theta p^{\frac{\lambda}{\alpha}}}\right)^{\alpha}. \tag{28}$$

where $\lim_{p\to 0} A^I = \infty$ and $\partial A^I/\partial p > 0$ if:

$$p > p^I = \left[\frac{A^a(1-\alpha)}{\mu\theta}\right]^{1/\lambda}. \tag{29}$$

The second condition holds if:

$$A < \underline{A} = \left(\frac{1-\lambda}{1-\alpha}\right) A^a \left(\frac{1+\mu}{A^a \left(\frac{1-\lambda}{1-\alpha}\right) p^{\frac{\lambda(1-\alpha)}{\alpha}} - \mu\theta p^{\frac{\lambda}{\alpha}}}\right)^{\alpha}, \tag{30}$$

where $\lim_{p\to 0} \underline{A} = \infty$ and $\partial \underline{A}/\partial p > 0$ if:

$$p > \underline{p} = \left[\frac{A^a(1-\lambda)}{\mu\theta}\right]^{1/\lambda}, \tag{31}$$

which we assume < 1, that is:

$$A^a < \frac{\mu\theta}{1-\lambda} \tag{32}$$

Simple calculations show that, if assumption 1 holds then $A^I > \underline{A}$ and $p^I < \underline{p}$.
The third condition holds if:

$$A < \tilde{A} = \left(\frac{1-\lambda}{1-\alpha}\right) A^a \left[\frac{1+\mu}{\alpha A^a p^{\frac{\lambda(1-\alpha)}{\alpha}} \left(\frac{1-\lambda}{1-\alpha}\right)}\right]^{\alpha} \tag{33}$$

The fourth and fifth conditions hold if:

$$A < A^{II} = \frac{\tilde{b}(1+\mu) + \mu\theta p}{\tilde{b}^{\alpha} p^{1-\alpha}} \tag{34}$$

where $\lim_{p\to 0} A^{II} = \infty$ and $\partial A^{II}/\partial p < 0$ if:

$$\gamma < \frac{1+\mu}{\mu\theta} \tag{35}$$

Finally, the sixth condition holds if:

$$A < A^{max} = \frac{1+\mu}{\gamma(1-\alpha)p^{1-\alpha}\tilde{b}^{\alpha}}. \tag{36}$$

We find that $\tilde{A} > \underline{A}$ if $p < \underline{p}$ while $\tilde{A} < \underline{A}$ if $p > \underline{p}$.
Moreover, $\tilde{A} > A_{min}$ if conditions (32) and (35) hold.

Simple calculations show that $A^{max} > A^{II}$ if condition (35) holds (that is $A^{max} > A^{II}$ if $p < (1+\mu)/\gamma\mu\theta$ which is higher than one if (35) holds).

An economy shows only **one stable equilibrium** in the range $b_t \in [b^o, \tilde{b})$ if:

$$\lim_{b_t \to b^{o-}} b_{t+1} \geq b^o,$$

$$\lim_{b_t \to b^{o+}} b_{t+1} \geq b^o,$$

$$\lim_{b_t \to \tilde{b}^-} b_{t+1} \leq \tilde{b},$$

$$\lim_{b_t \to \tilde{b}^+} b_{t+1} \leq \tilde{b},$$

$$\left.\frac{\partial b_{t+1}}{\partial b_t}\right|_{b_t=\tilde{b}} < 1,$$

The first and the second conditions hold if $A > A^I$. The third condition holds if $A < A^{II}$. Finally the fourth condition holds if $A < A^{max}$.

An economy shows **one stable equilibrium** in the range $b_t \in [\tilde{b}, \infty)$ if:

$$\lim_{b_t \to b^{o-}} b_{t+1} \geq b^o,$$

$$\lim_{b_t \to b^{o+}} b_{t+1} \geq b^o,$$

$$\lim_{b_t \to \tilde{b}^-} b_{t+1} \geq \tilde{b},$$

$$\lim_{b_t \to \tilde{b}^+} b_{t+1} \geq \tilde{b},$$

The first and second conditions hold if $A > A^I$ and the third and the fourth conditions hold if $A > A^{II}$.

Finally, when $\underline{A} < A < A^I$ the economy shows a stable equilibrium in Regime I and a stable equilibrium in Regime II, and for each $\underline{p} < p < 1$ and $\tilde{A} < A < \underline{A}$ the economy shows a stable equilibrium in Regime I and two equilibria (one stable and one unstable) in Regime II:

B Proof of Proposition 2

- In the first regime $\bar{b}_t \in [0, b^0]$, $\partial b_{t+1}/\partial p > 0$ if $p < \underline{p}$.

- In the second regime, that is for $b_t \in [b^o, \tilde{b})$, $\partial b_{t+1}/\partial p > 0$ if:

$$b_t > \underline{b} = \left[\frac{\mu\theta}{A(1-\alpha)}\right]^{1/\alpha} p \tag{37}$$

where $\underline{b} > b^o$ if $p > \underline{p}$.

We have that $\underline{b} < b^k$ if:

$$b_{t+1}(\underline{b}) - \underline{b} > 0 \tag{38}$$

which holds if:

$$A > \hat{A} = \frac{\mu\theta}{(1-\alpha)}\left[\frac{(1+\mu)(1-\alpha)}{\mu\theta\alpha}\right]^{\alpha} \tag{39}$$

Simple calculations show that, for each $p > \underline{p}$, $\hat{A} < \tilde{A}$. This would imply that, for each $p > \underline{p}$ then $\underline{b} < b^k$.

- In the third regime, i.e. $b_t \in [\tilde{b}, +\infty)$, $\partial b_{t+1}/\partial p > 0$ if:

$$\gamma < \frac{1+\mu}{\mu\theta} \tag{40}$$

Proof: In particular $\partial b_{t+1}/\partial p > 0$ for $b_t \in [\tilde{b}, +\infty)$ if:

$$b > \hat{b}, \tag{41}$$

where :

$$\hat{b} = \frac{\mu\theta p^{\alpha}}{A(1-\alpha)\tilde{b}^{\alpha}}^{\frac{1}{\gamma(1-\alpha)+\alpha}}(1+\tilde{b}) - 1, \tag{42}$$

where $\tilde{b} > \hat{b}$ if:

$$A > A^* = \frac{\theta\mu p^{\alpha}}{\tilde{b}^{\alpha}(1-\alpha)}, \tag{43}$$

where $A^{II} > A^*$ if:

$$\gamma < \frac{1+\mu}{\mu\theta} \tag{44}$$