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The Ability of Pupils in a Selected High School to Estimate Quantitative Measurements

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THE ABILITY OF PUPILS IN A SELECTED HIGH SCHOOL TO ESTIMATE QUANTITATIVE MEASUREMENTS

By

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CHAPTER I

INTRODUCTION

Throughout the elementary and secondary school mathematics curriculum, emphasis is placed upon developing accuracy. Emphasis is also placed upon the processes of reasoning and comprehension, but accurate computation remains a primary objective of mathematics training. However, it is obvious that there are many occasions in daily life when it is advantageous to be able to quickly estimate quantitative measurements with approximate accuracy. It is the observation of the writer that, despite the obvious desirability of pupils acquiring competency in such estimating, the schools give little, if any, training in this factor.

I. THE PROBLEM

Statement of the problem. The purpose of this study is two-fold: (1) to ascertain with what degree of accuracy pupils of junior and senior level in a selected high school are able to estimate quantitative measurements; and (2) to determine whether or not the ability to make accurate estimates is related to such factors as mental ability, computational ability, or the amount of formal mathematics taken in high school.

Delimitation of the problem. This study is limited

to data secured from junior and senior pupils in Highland High School, Albuquerque, New Mexico. Juniors and seniors were selected because they were a group who had almost completed their formal public school education, including required and elective mathematics courses. This study is further limited to only those pupils for whom scores were obtained on each of the following three tests: (1) The Schorling-Clark-Potter Hundred Problem Arithmetic Test; (2) the Otis Self-Administering Test of Mental Ability, Form A; and (3) the Thirty-two Problem Estimating Test devised by the writer. Finally, the estimation phase of this study is limited to quantitative problems of measurement the answers to which can only be estimated, not computed.

Importance of the problem. Three factors frequently accentuate the need for ability to estimate quantitative measurements with approximate accuracy. (1) Time necessary to make or compute exact measurements may be limited. (2) Measurement instruments may not be available at the time of need. (3) Only approximate rather than exact measurements are frequently necessary. Because the resulting estimates made in such cases are often used as a basis for a person's subsequent actions and expenditures, it is important that such estimates

be made as accurate as possible under the circumstances. If this investigation should reveal that the high school pupils exhibit a deficiency in estimating ability, it could serve as an aid in planning future courses of study in mathematics.

Limitations of the study. The investigator is aware of a lack of evidence to substantiate the validity of the Estimating Ability Test which he devised. Since many of the estimation problems are concerned with the locale and objects within observation of the subject taking the test, validity would have to be established by administering the test to other groups at the same vantage point, in this instance, Highland High School, Albuquerque, New Mexico. Some groups which employ quantitative measurements, possibly engineers, consequently would have to assemble and take the test in order to obtain indices of validity. The writer wishes to stress again, however, that none of the thirty-two problems can be computed. Thus, performance depends entirely on one's ability to estimate the correct answers. In this sense one may say that the test measures what it purports to measure.

II. DEFINITION OF TERMS

Quantitative measurements. Quantitative measurements are those measurements found to exist when accurate measuring instruments are applied to any object. For example, an object might be an angle or arc containing a certain number of degrees; a stick a certain number of inches in length; or a pail containing some symbol of volume.

Estimation. Estimation is used as the ability to arrive at a value either by inspection without calculating the result or by a rough calculation.

III. ORGANIZATION OF THE REMAINDER OF THE STUDY

The remainder of this study is presented in four chapters. Chapter II presents a brief statement about related studies; the method of conducting the investigation is described in Chapter III; Chapter IV presents an analysis of the data; the conclusions and recommendations drawn from the study are presented in Chapter V. In the Appendix, the test instruments used in this study as well as certain organized data are exhibited.

CHAPTER II

SURVEY OF RELATED LITERATURE

It is apparent that very little investigation has been done in the area of measuring pupils' estimating ability in elementary or secondary school mathematics. The investigator was unable to find a single study directly related to study of the ability of pupils to estimate quantitative measurements.

Although considerable investigation has been made in the field of approximation, such studies have concerned one's ability to first approximate answers in order to check against actual computation. Consequently, such studies are not directly related to the problem of this investigation.

The most extensive work in approximate computation is contained in the Twelfth Yearbook of the National Council of Teachers of Mathematics.[1] This volume contains an analysis of absolute measurements in relation to pure mathematical procedure, and is therefore not related to the concept of estimating ability used in this study.

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[1] Aaron Bakst, The Twelfth Yearbook of the National Council of Teachers of Mathematics, (New York: Bureau of Publications, Teachers College, Columbia University, 1937).

Eddington,[2] in a study of numbers of absolute quantities, presents still another concept of approximation in his statement that data relative to measurements are always approximate. He lists three factors necessarily eliminating "absolute" measurement: (1) there is no measuring instrument however precise that can measure exactly; (2) the units of measure themselves are only approximately known; and (3) the errors that arise from the process of measurement are unavoidable due to the individual characteristics of those who perform the measurements. The present study is not concerned with this concept of approximation.

A third aspect of approximation as it relates to computation concerns the theory of error in abstract measurement. The term approximate as used in this sense is further related to certain quantities of precision beyond human control. These brief statements concerning approximation as found in the literature have been presented to show the reader that such studies as have been carried on in this field have very little relationship to the concept used in this study and consequently are not related.

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[2] Sir Arthur S. Eddington, The Nature of the Physical World, (Cambridge: Cambridge University Press, 1932), p. 23.

CHAPTER III

METHOD OF CONDUCTING THE INVESTIGATION

The scores made on three tests by junior and senior pupils at Highland High School, as well as school marks which they attained, constitute the sources of data for this study. Subject matter units and semester marks were obtained from the files in the office of the school principal. The test results were obtained from pupils in five different homerooms. Only junior and senior pupils were tested, as previously stated, because they had practically completed their formal school mathematics experiences.

The tests were administered to 164 pupils, but due to absences and other factors, only the results for 102 pupils who had taken the complete test battery were used for analysis. The three tests administered to the pupils were: (1) The Schorling-Clark-Potter Hundred Problem Arithmetic Test; (2) Otis Self-Administering Test of Mental Ability, Form A; and (3) The Thirty-two Problem Estimating Test devised by the writer.

In order that the reader may better understand certain groupings of raw data composed of school marks and the test scores, a brief description of them is presented.

High-School marks. In passing, the writer wishes to emphasize the unreliability of teachers' marks. The view of Heston[3] that marks are "psychological nonsense" is popular and may sometimes be well-founded. It is true that the variability and the subjectivity of marks cannot be ignored, yet it must be remembered that, inaccurate and unfair as marks may sometimes be, they are virtually the only medium used to evaluate success in the American system of formal education. Consequently, mathematics marks as well as marks obtained in all subjects, were recorded for each of the 102 pupils as a criterion of achievement. All marks were arbitrarily converted into numerical scores so that they could be compared and used in statistical analysis. This was done by assigning the following values: A, four points; B, three points; C, two points; D, one point: F, zero points. These grade points were then totaled to find the grade point average. The grade point average was computed by dividing the number of units of high school credit into the total number of grade points attained.

Mathematics units. The number of units of math-

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[3] Joseph C. Heston, "The Graduate Record Examination vs. Other Measures of Aptitude and Achievement," Educational and Psychological Measurement, 7:618-30, No. 3, Autumn, 1947.

ematics studied by each of the 102 pupils was ascertained by examination of the office files of the school. Commercial arithmetic and formal types of mathematics such as algebra and geometry composed the mathematics unit classification. A semester of work in a given subject was counted as a half unit.

Hundred Problem Arithmetic Test. The Schorling-Clark-Potter Hundred Problem Arithmetic Test was used in order to measure computational ability. The test is divided into five sections. Section I is a measure of ability to add columns of figures, fractions, decimals, and dollars and cents. Section II is a measure of ability to subtract figures, fractions, decimals, and dollars and cents. Section III is a measure of ability to multiply fractions, plain numbers, decimals, and dollars and cents. Section IV is a measure of ability to divide fractions and decimals. Section V is a measure of ability to translate figures into percentages, decimal numbers and fractions, in addition to general problems involving the same mathematical operations.

Otis Self-Administering Test of Mental Ability, Form A. The Otis Self-Administering Test of Mental Ability, Form A, is a test containing some seventy-five thought and recognition questions from which one may determine

intelligence quotients. Form A is designed for high school and college level students, and was included in the test battery in order to compare pupils' mental ability with their ability to make accurate estimates.

Thirty-two Problem Estimating Test. The Thirty-two Problem Estimating Test, is a test containing thirty-two problems involving estimation, none of the answers to which can be computed. It is made up of two types of problems: (1) estimation of something about objects that can be observed while making an estimate; and (2) estimation of something about objects that cannot be observed while making an estimate. Five alternate answers, one of which is correct, are presented in either ascending or descending order for each problem.

Scoring of the Thirty-two Problem Estimating Test. The Thirty-two Problem Estimating Test was scored twice, first for the correct answers and secondly for nearly correct answers. On this test there are five possible choices for each question designated by (a), (b), (c), (d), and (e). Since the possible choices are arranged in either ascending or descending order, on the second scoring, choices just above or below the correct answer given were scored as correct. For example, if (b) was the correct answer, (a) and (c), just above and below,

were also counted correct. The investigator felt that the pupils' ability to estimate the correct answers was important. However, the pupils' ability to estimate nearly correct answers seemed to give an even better indication of this estimating ability.

Administration of the Tests. The Hundred Problem Arithmetic Test was administered first on December 12, 1949, and the other two tests subsequently followed at uniform intervals of two school weeks. The tests were administered on all occasions by the writer and four trained graduate students in order to obtain uniformity of administration.

CHAPTER IV

ANALYSIS OF THE DATA

I. ABILITIES OF THE PUPILS

The means and standard deviations of the scores made on the Otis and the Hundred Problem Arithmetic Test were computed in order to compare the scores of pupils in this study with the norms obtained on standardization of the tests.

Table I presents the mean score and the standard deviation, made by the pupils in this study compared with the norms for the same measures on the Otis test. The mean score for the pupils in this study was found to be 51.73 with a standard deviation of 6.16 as compared with the norm mean score of 45.15 and a standard deviation of 9.5. From these results one may conclude: (1) the superior mean score of the pupils in this study indicates a level of greater mental ability. This tends to substantiate a cursory observation of the writer that the pupils at Highland High School tend to come from homes of higher socio-economic status than the average of the nation. Thus, some evidence is available that the pupils whose estimating ability the investigator attempts to measure have above average mental ability as a basis for making

TABLE I

COMPARISON OF THE MEAN AND STANDARD DEVIATION NORMS ON THE OTIS SELF-ADMINISTERING TEST OF MENTAL ABILITY, FORM A, WITH SCORES MADE BY PUPILS IN THIS STUDY

Group	Mean	Standard Deviation
Test norms	45.15	9.5
Pupil scores in this study	51.73	6.16

estimates; (2) because of less dispersion in the standard deviations, the pupils in this study were a more homogeneous group mentally than the pupils whose scores furnished a basis for standardization.

Table II presents the mean and the standard deviation of scores made by the pupils in this study compared with the norms for the same measures on the Hundred Problem Arithmetic Test.

The mean score for the pupils in this study was 72.1 with a standard deviation of 16.10 as compared with the norm mean score of 68.60 and a standard deviation of 20.20. One may thus conclude that above average performance was achieved by the pupils in this study as compared to those whose scores furnished a basis for the test norms. This conclusion is similar to that derived previously from analysis of Table I.

II. RELATIONSHIP BETWEEN FACTORS OF ABILITY

Coefficients of correlation were computed between the scores of fifteen different combinations of variables as well as the respective probable errors of these coefficients. These computations were made to determine the extent of relationship which exists between scores for such variables as intelligence, computational ability, mathematics marks, total grade marks, and estimating

TABLE II

COMPARISON OF THE MEAN AND STANDARD DEVIATION NORMS ON THE SCHORLING-CLARK-POTTER HUNDRED PROBLEM ARITHMETIC TEST WITH SCORES MADE BY PUPILS IN THIS STUDY

Group	Mean	Standard Deviation
Test norms	68.6	20.2
Pupil scores in this study	72.06	16.10

ability.

In order to facilitate clear understanding, this study will employ the following arbitrarily determined descriptive and quantitative values to interpret coefficients as recommended by Darley:[4]

.80 and up equals very high correlation
.50 to .80 equals substantial correlation
.30 to .50 equals some correlation
.20 to .30 equals slight correlation
.00 to .20 equals practically no correlation

Table III presents a complete list of the coefficients of correlation computed and the probable errors for these coefficients. These coefficients are arranged in descending order of magnitude.

The reader will note that the first six highest correlations presented are between measures of achievement, that is between such variables as total grade point averages, mathematics marks, mental ability scores, and computational ability. In none of these does the variable of estimating ability appear. By themselves these coefficients of correlation are not too significant, but when they are compared to coefficients between measures of achievement and estimating ability, they are by contrast quite significant. The highest coefficient of

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[4] John G. Darley, Testing and Counseling in the High School Guidance Program, (Chicago: Science Research Associates, 1943), p. 71.

TABLE III

COEFFICIENTS OF CORRELATION BETWEEN THE SCORES MADE ON VARIOUS MEASURES USED IN THE STUDY

Coefficient of Correlation	PE	Variables Correlated
.65	.0399	Mathematics marks and total grade point averages.
.62	.0141	Mathematics marks and Hundred Problem Arithmetic Test.
.56	.0463	Mathematics marks and Otis Self-Administering Mental Ability Test.
.51	.0502	Otis Self-Administering Mental Ability Test and Hundred Problem Arithmetic Test.
.45	.0540	Otis Self-Administering Mental Ability Test and Hundred Problem Arithmetic Test.
.43	.0550	Total grade point average and Hundred Problem Arithmetic Test.
.35	.0592	Estimating Test and Hundred Problem Arithmetic Test.
.33	.0600	Otis Self-Administering Mental Ability Test and Estimating Test.
.32	.0597	Estimating Test and allowances for Near Estimating Test.

TABLE XII

COEFFICIENTS OF CORRELATION BETWEEN APE SCORES
MADE ON VARIOUS MEASURES USED IN THE STUDY

Coefficients of Correlation	PE	Variables Correlated
.65	.0399	Mathematics marks and total grade point average.
.62	.0414	Mathematics marks and Hundred Problem Arithmetic Tests.
.76	.0403	Mathematics marks and Otis Self-Administering Mental Ability Test.
.51	.0502	Otis Self-Administering Mental Ability Test and Hundred Problem Arithmetic Test.
.45	.0540	Otis Self-Administering Mental Ability Tests and Hundred Problem Arithmetic Test.
.43	.0550	Total grade point average and Hundred Problem Arithmetic Test.
.35	.0592	Mathematics Test and Hundred Problem Arithmetic Test.
.33	.0600	Otis Self-Administering Mental Ability Test and Mathematics Test.
.32	.0597	Mathematics Test and Language Test.

TABLE III (continued)

COEFFICIENTS OF CORRELATION BETWEEN THE SCORES MADE ON VARIOUS MEASURES USED IN THE STUDY

Coefficient of Correlation	PE	Variables Correlated
.26	.0624	Otis Self-Administering Mental Ability Test and allowance for Near Estimating Test.
.25	.0633	Hundred Problem Arithmetic Test and allowance for Near Estimating Test.
.22	.0645	Mathematics marks and allowance for Near Estimating Test.
.07	.0666	Mathematics marks and Estimating Test
.05	.0667	Allowance for Near Estimating Test and total grade point averages.
.03	.0668	Total grade point averages and Estimating Test.

correlation between measures of achievement, namely, mathematics marks and total grade point averages, is .65, as compared to .35 for the highest correlation between a measure of achievement and estimating ability.

The previously mentioned coefficient of .65 according to the criteria used in this study for magnitude is considered a substantial correlation. Even the lowest coefficient in this highest group of six is .43 which represents some correlation.

In the lowest group of correlations or the last nine presented, coefficients are between measures of achievement and estimating ability. The highest coefficient of correlation in this group is .35 between the scores made on the Hundred Problem Arithmetic Test and the Estimating Test. In comparing this group with the higher correlations between measure of achievement variables, the absence of correlation is especially noticeable. According to the criteria used for magnitude of correlations, only the highest three manifest some correlation. In fact, the lowest three coefficients of correlation in this group manifest practically no correlation according to the same criteria for magnitude. The same data are presented in the usual inter-correlation form in Table IV, but actually present no new information.

TABLE IV

INTERCORRELATIONS BETWEEN THE SCORES MADE BY THE PUPILS ON VARIOUS MEASURES USED IN THIS STUDY

	Otis Mental Ability	Hundred Problem Arithmetic Test	Thirty-two Problem Estimating Test	Allowance for Near Estimates	Mathematics Marks	Total Grade Point Average
Otis Mental Ability		.51	.33	.26	.56	.45
Hundred Problem Arithmetic Test			.35	.25	.62	.43
Thirty-two Problem Estimating Test				.32	.07	.03
Allowance for Near Estimates					.22	.05
Mathematics Marks						.65
Total Grade Point Average						

From this total group of coefficients of correlation one may draw two conclusions: (1) there is some to substantial correlation between the variables which measure achievement; (2) there is slight to practically no correlation between measures of achievement and estimating ability. Specifically, one may conclude from Tables III and IV that the ability of pupils to estimate with competency is not related to their mental abilities, computational abilities, or marks received in mathematics classes.

III. ESTIMATING ABILITY AND QUANTITY OF MATHEMATICS UNITS COMPLETED

The pupils were classified according to the number of semester units of high school mathematics they had completed. The objective was to determine whether the amount of mathematics the pupils had completed might possibly improve their ability to estimate quantitative measurements. The students were classified as follows:

Group A - pupils with zero to one and one-half (0 - 1½) units
Group B - pupils with two units
Group C - pupils with more than two units

Table V presents the means and standard deviations of the scores made on the Estimating Test for the three groups.

TABLE V

MEAN SCORES MADE BY THREE GROUPS OF PUPILS WITH VARYING NUMBER OF MATHEMATICS UNITS ON THE THIRTY-TWO PROBLEM ESTIMATING TEST

Group	Number of Pupils	Range	Mean	PE Mean	Standard Deviation
Group A (0-1½ units)	12	9-22	16.50	.72	3.69
Group B (2 units)	48	7-22	12.16	.48	4.90
Group C (over 2 units)	42	7-24	13.83	.36	3.59
Totals	102	7-24	13.28	.23	3.49

The reader will note that the mean of 13.3 for the total group indicates, even without norms with which to compare, not too high a degree of competency in estimating accurately. Since the highest possible score on the test is thirty-two, a mean of 13.3 represents considerably less than 50 per cent correct responses. Also, when one recalls that a mean score is an average score, approximately 50 per cent of the pupils made lower scores than 13.3. Actual evidence of this is apparent in the distribution of scores for each group. No evidence is apparent in the table indicating greater competency in estimating as the number of mathematics units completed increases. In fact, Group A, representing pupils with the lowest number of units of mathematics, has a mean almost three points greater than the highest mean of the remaining two groups. Even discounting the size of the probable error due to the small number of cases in Group A, the same conclusion prevails.

One may conclude from these data that: (1) the degree of accuracy with which the pupils were able to make estimates is low; (2) the number of units of mathematics completed seems to have very little relation to the pupils' ability to accurately estimate quantitative measurements. In fact, a limited sampling would appear to indicate a slight tendency for an inverse

relationship to exist.

Table VI presents the means and standard deviations of the scores made on the Estimating Test for the three groups when allowances for near estimates were made. The mean scores as might be expected are considerably greater than in the previous tables when allowances for near estimates were not considered. This would indicate that the pupils were fairly competent in making rough or approximate estimates as evidenced by the mean of 24.60 for the total group. Although the possible choices for each problem on the test were arranged in either ascending or descending order, the difference from the true values in many instances represented considerable quantity. Although the competency of the pupils to make rough estimates was found to be considerably better than their ability to make correct estimates, there still remained ample room for improvement in accuracy. Finally, the evidence does not seem any more conclusive than was previously given to indicate improved estimating ability as the number of units of mathematics completed increases. However, more specific evidence is presented in regard to this factor in the next two tables.

Table VII presents the comparison of the means attained by pupils in mathematical Groups B and C with Group A, on the Thirty-two Problem Estimating Test as

TABLE VI

MEAN SCORES MADE BY THREE GROUPS OF PUPILS WITH VARYING NUMBER OF MATHEMATICS UNITS ON THE THIRTY-TWO PROBLEM ESTIMATING TEST WHEN ALLOWANCES FOR NEAR ESTIMATES WERE MADE

Group	Number of Pupils	Range	Mean	PE Mean	Standard Deviation
Group A (0-1½ units)	12	19-31	25.34	.70	3.59
Group B (2 units)	48	17-30	23.90	.30	3.05
Group C (over 2 units)	42	19-29	25.69	.24	2.34
Totals	102	17-32	24.64	.28	4.22

TABLE VII

COMPARISON OF THE MEANS ATTAINED BY PUPILS IN MATHEMATICS GROUPS B AND C WITH BROUP A ON THE THIRTY-TWO PROBLEM ESTIMATING TEST AS INDICATED BY THE CRITICAL RATIOS

Group	Number of Pupils	Mean	Difference of Mean from Group A	PE_d of the Means	Critical Ratios
A (0-1½ units)	12	16.5			
B (2 units)	48	12.16	-4.3	.25	3.88
C (over 2 units)	42	13.8	-2.7	.35	3.30

indicated by the critical ratios.

There is no certainty that the true difference between the means for Groups A and B is greater than zero since the critical ratio is 3.88. Consequently, the difference found in this study between means of the two stated groups is due to chance of the sampling, since the critical ratio would need to be 4.0 in order to insure a true difference. A similar conclusion can be drawn for Group C in comparison with Group A, because the critical ratio of 3.30 indicates the difference is due to chance of the sampling.

Table VIII presents the comparison of the mean scores attained by pupils in mathematics Groups B and C with Group A, on the Thirty-two Problem Estimating Test with allowances for near estimates as indicated by the critical ratios.

With the following obtained critical ratios: 1.88 for Group B in comparison with Group A, .43 for Group C in comparison with Group A, a similar conclusion can be drawn for these groupings as was drawn for the same groups on the Estimating Test, namely, the differences found in this study between the means of Near Estimating scores made by the various groups is due to chance of the sampling.

In conclusion, one may say that there is no evidence in the data to indicate that the number of mathemat-

TABLE VIII

COMPARISON OF THE MEANS ATTAINED BY PUPILS IN MATHEMATICS GROUPS B AND C WITH GROUP A ON THE THIRTY-TWO PROBLEM ESTIMATING TEST WITH ALLOWANCES FOR NEAR ESTIMATES AS INDICATED BY THE CRITICAL RATIOS

Group	Number of Pupils	Mean	Difference of Mean from Group A	PE_d of the Means	Critical Ratios
A (0-1½ units)	12	25.34			
B (2 units)	48	23.90	-1.44	.40	1.88
C (over 2 units)	42	25.	.35	.46	.43

ics units completed in high school by the pupils is related to their ability to make accurate estimates.

IV. ANALYSIS OF CERTAIN ITEMS ON THE THIRTY-TWO PROBLEM ESTIMATING TEST

In an effort to determine whether any difference could be ascertained between the pupils' ability to estimate something about objects which they could see or observe in contrast with those they could not observe, twenty items were selected from the Estimating Test and divided as follows: (1) ten items in which the objects could be observed while making an estimate; and (2) ten items in which the objects could not be observed while making an estimate.

As presented in Table IX, the mean score attained by the pupils for the observable group of items is 4.52 and 3.51 for the unobservable group. It will also be observed that the critical ratio is .67 from which one can conclude that there is no true difference between the mean scores made by the pupils for the two types of test objects. Consequently, any apparent evidence that the pupils made more accurate estimates when the objects were observable than when unobservable, is due to chance of the sampling and is not statistically significant.

Table X presents question number three on the

TABLE IX

THE MEANS ATTAINED ON TEN SELECTED OBSERVABLE AND UNOBSERVABLES ITEMS OF THE THIRTY-TWO PROBLEM ESTIMATING TEST

Type of Objects	Number of Pupils	Test Selected	Mean	PE_M	Critical Ratios
Observed	102	10	4.52	.012	.67
Unobserved	102	10	3.51	.010	

TABLE X

PUPIL RESPONSES MADE ON QUESTION THREE OF THE THIRTY-TWO PROBLEM ESTIMATING TEST

Question	Answers	Number of Pupils	Per Cent	Percentage of incorrect responses after allowance for near estimates
How high is the ceiling in this room (in feet)?	25	7	6.85	20.55
	20	14	13.70	
	15	47	46.08	
	12*	30	29.20	
	9	4	3.92	

* Correct answer 12

Estimating Test. This was designed to show the reader the range of pupil performance on representative items of the test. The reader will observe that only 29.2 per cent of the pupils chose the correct answer. When one considers that this is only a little more than 25 per cent of the pupils participating in the study, he may feel that this represents very low achievement. But the reader will also observe that when this same question was rescored on the basis of being nearly correct, (that is, one alternative choice above or below the correct answer), almost 80 per cent of the pupils chose an approximately correct answer.

Tables XI through XVI present selected questions on the Estimating Test and it will be noted that the same trend is observable in these questions as was seen in question three, mainly: (1) when scored on the basis of the correct answer, usually less than half of the pupils made the correct choice; and (2) when scored on a nearly correct basis, in most cases at least 80 per cent chose the correct answer. Because of this similarity in results obtained, interpretation of each table will not be presented.

From the results shown one may conclude that the pupils are poor estimators of exact measurements, but when nearly correct or approximate answers are taken into consideration, their performance is much more satisfactory.

TABLE XI

PUPIL RESPONSES MADE ON QUESTION FOUR OF THE THIRTY-TWO PROBLEM ESTIMATING TEST

Question	Answers	Number of Pupils	Per Cent	Percentage of incorrect responses after allowance for near estimates
How wide is the door in this classroom (in feet?)	2½	8	7.83	
	3	27	26.52	
	3½*	54	52.91	10.77
	5	10	9.80	
	6	3	2.94	

* Correct answer 3½

TABLE XII

PUPIL RESPONSES MADE ON QUESTION FIFTEEN OF THE THIRTY-TWO PROBLEM ESTIMATING TEST

Question	Answers	Number of Pupils	Per Cent	Percentage of incorrect responses after allowance for near estimates
Estimate the length of line AB (in inches)	2	1	0.98	
	3	9	8.83	
	4*	56	54.84	10.78
	5	26	25.55	
	6	10	9.80	

A B

* Correct Answer 4

TABLE XIII

PUPIL RESPONSES MADE ON QUESTION SIXTEEN OF THE THIRTY-TWO PROBLEM ESTIMATING TEST

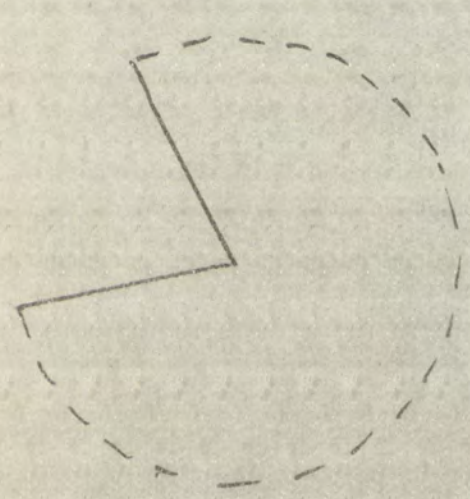
Question	Answers	Number of Pupils	Per Cent	Percentage of incorrect responses after allowance for near estimates
Estimate the length (in inches) of a line drawn diagonally across this $8\frac{1}{2}$ x 14 inch page	13	10	9.80	18.63
	16	33	32.40	
	18*	37	36.30	
	20	13	12.70	
	22	9	8.80	

* Correct answer 18

TABLE XIV

PUPIL RESPONSES MADE ON QUESTION TWENTY OF THE THIRTY-TWO PROBLEM ESTIMATING TEST

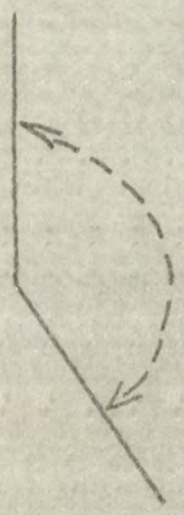
Question	Answers	Number of Pupils	Per Cent	Percentage of incorrect responses after allowance for near estimates
A reasonable estimate of the number of degrees in the angle below is:	65	17	16.71	
	75*	40	39.20	
	85	35	34.30	10.79
	95	7	6.85	
	105	3	2.94	



* Correct answer 75

TABLE XV

PUPIL RESPONSES MADE ON QUESTION TWENTY-TWO OF THE THIRTY-TWO PROBLEM ESTIMATING TEST

Question	Answers	Number of Pupils	Per Cent	Percentage of incorrect responses after allowance for near estimates
A reasonable estimate in degrees of the angle below is: 	105	5	4.80	
	120	9	8.83	
	135	16	15.74	13.73
	150*	38	37.21	
	165	34	33.42	

* Correct answer 150

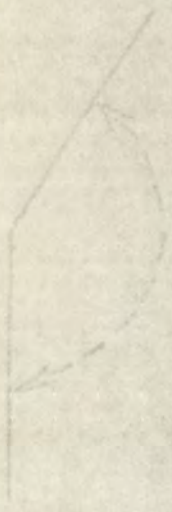


TABLE XVI

PUPIL RESPONSES MADE ON QUESTION TWENTY-SEVEN OF THE THIRTY-TWO PROBLEM ESTIMATING TEST

Question	Answers	Number of Pupils	Per Cent	Percentage of incorrect responses after allowance for near estimates
What is the length of the piece of wood held by the examiner?	24	12	11.81	
	32*	71	69.52	
	40	16	15.73	2.94
	48	2	1.96	
	54	1	0.98	

* Correct answer 32

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

I. CONCLUSIONS

In light of the data presented in this study, the following conclusions are in order:

1. The mental ability of a pupil, as measured by the Otis Self-Administering Mental Ability Test, Form A, bears no statistically significant relationship to the pupil's ability to estimate quantitative measurements.

2. Computational ability, as measured by the Schorling-Clark-Potter Hundred Problem Arithmetic Test, bears no statistically significant relationship to a pupil's ability to accurately estimate quantitative measurements. The same relationship exists between the factor of estimating ability and marks received by the pupils in high school mathematics courses.

3. The number of units of formal mathematics completed by high school pupils is no indication of their ability to estimate quantitative measurements.

4. The pupils were unable to estimate quantitative measurements about objects which they could observe any better than they could estimate objects which could not be observed. Any slight difference found in favor

of estimating objects which could be observed was not statistically significant and due to chance of sampling.

5. The pupils exhibited a low degree of accuracy in estimating exact measurements, but the ability to estimate rough or approximate measurements was quite satisfactory for at least 80 per cent of the pupils.

II. RECOMMENDATIONS

1. It is recommended that more extensive training in estimating quantitative measurements should be included in the elementary and secondary school mathematics courses. As an objective, this should rank more prominently along with such other formal mathematics objectives as accuracy, reasoning, and comprehension.

2. It is recommended that the Thirty-two Problem Estimating Test be standardized so that it can be used for further investigation in the field of estimating.

3. It is recommended that further and more extensive studies be conducted in the field of estimating quantitative measurements. The field presents numerous opportunities as yet unexplored.

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APPENDIX

OTIS SELF-ADMINISTERING TESTS OF MENTAL ABILITY

43

By ARTHUR S. OTIS

Formerly Development Specialist with Advisory Board, General Staff, United States War Department

HIGHER EXAMINATION: FORM A

20

For High Schools and Colleges

Score.....

Read this page. Do what it tells you to do.

Do not open this paper, or turn it over, until you are told to do so. Fill these blanks, giving your name, age, birthday, etc. Write plainly.

Name.....Age last birthday.....years
First name, initial, and last name

Birthday.....Class.....Date.....19....
Month Day

School or College.....City.....

This is a test to see how well you can think. It contains questions of different kinds. Here is a sample question already answered correctly. Notice how the question is answered:

Which one of the five words below tells what an apple is?

1 flower, 2 tree, 3 vegetable, 4 fruit, 5 animal.....(4)

The right answer, of course, is "fruit"; so the word "fruit" is underlined. And the word "fruit" is No. 4; so a figure 4 is placed in the parentheses at the end of the dotted line. This is the way you are to answer the questions.

Try this sample question yourself. Do not write the answer; just draw a line under it and then put its number in the parentheses:

Which one of the five words below means the opposite of north?

1 pole, 2 equator, 3 south, 4 east, 5 west.....( )

The answer, of course, is "south"; so you should have drawn a line under the word "south" and put a figure 3 in the parentheses. Try this one:

A foot is to a man and a paw is to a cat the same as a hoof is to a — what?

1 dog, 2 horse, 3 shoe, 4 blacksmith, 5 saddle.....( )

The answer, of course, is "horse"; so you should have drawn a line under the word "horse" and put a figure 2 in the parentheses. Try this one:

At four cents each, how many cents will 6 pencils cost?.....( )

The answer, of course, is 24, and there is nothing to underline; so just put the 24 in the parentheses. If the answer to any question is a number or a letter, put the number or letter in the parentheses without underlining anything. Make all letters like printed capitals.

The test contains 75 questions. You are not expected to be able to answer all of them, but do the best you can. You will be allowed half an hour after the examiner tells you to begin. Try to get as many right as possible. Be careful not to go so fast that you make mistakes. Do not spend too much time on any one question. No questions about the test will be answered by the examiner after the test begins. Lay your pencil down.

Do not turn this page until you are told to begin.

PRINTED IN U.S.A.

Published by World Book Company, Yonkers-on-Hudson, New York, and 2126 Prairie Avenue, Chicago

EXAMINATION BEGINS HERE:

1. The opposite of hate is (?)
   1 enemy, 2 fear, 3 love, 4 friend, 5 joy ..... ( )
2. If 3 pencils cost 5 cents, how many pencils can be bought for 50 cents? ..... ( )
3. A bird does not always have (?)
   1 wings, 2 eyes, 3 feet, 4 a nest, 5 a bill ..... ( )
4. The opposite of honor is (?)
   1 glory, 2 disgrace, 3 cowardice, 4 fear, 5 defeat ..... ( )
5. A fox most resembles a (?)
   1 wolf, 2 goat, 3 pig, 4 tiger, 5 cat ..... ( )
6. Quiet is related to sound in the same way that darkness is related to (?)
   1 a cellar, 2 sunlight, 3 noise, 4 stillness, 5 loud ..... ( )
7. A party consisted of a man and his wife, his two sons and their wives, and four children in each son's family. How many were there in the party? ..... ( )
8. A tree always has (?)
   1 leaves, 2 fruit, 3 buds, 4 roots, 5 a shadow ..... ( )
9. The opposite of economical is (?)
   1 cheap, 2 stingy, 3 extravagant, 4 value, 5 rich ..... ( )
10. Silver is more costly than iron because it is (?)
    1 heavier, 2 scarcer, 3 whiter, 4 harder, 5 prettier ..... ( )
11. Which one of the six statements below tells the meaning of the following proverb? "The early bird catches the worm." ..... ( )

    1. Don't do the impossible.
    2. Weeping is bad for the eyes.
    3. Don't worry over troubles before they come.
    4. Early birds like worms best.
    5. Prompt persons often secure advantages over tardy ones.
    6. It is foolish to fret about things we can't help.

12. Which statement above tells the meaning of this proverb? "Don't cry over spilt milk." .... ( )
13. Which statement above explains this proverb? "Don't cross a bridge till you get to it." .... ( )
14. An electric light is related to a candle as an automobile is to (?)
    1 a carriage, 2 electricity, 3 a tire, 4 speed, 5 glow ..... ( )
15. If a boy can run at the rate of 6 feet in $\frac{1}{4}$ of a second, how many feet can he run in 10 seconds? ( )
16. A meal always involves (?)
    1 a table, 2 dishes, 3 hunger, 4 food, 5 water ..... ( )
17. Of the five words below, four are alike in a certain way. Which is the one not like these four?
    1 bend, 2 shave, 3 chop, 4 whittle, 5 shear ..... ( )
18. The opposite of never is (?)
    1 often, 2 sometimes, 3 occasionally, 4 always, 5 frequently ..... ( )
19. A clock is related to time as a thermometer is to (?)
    1 a watch, 2 warm, 3 a bulb, 4 mercury, 5 temperature ..... ( )
20. Which word makes the truest sentence? Men are (?) shorter than their wives.
    1 always, 2 usually, 3 much, 4 rarely, 5 never ..... ( )
21. One number is wrong in the following series. What should that number be?
    1 4 2 5 3 6 4 7 5 9 6 9 ..... ( )
22. If the first two statements following are true, the third is (?) All members of this club are Republicans. Smith is not a Republican. Smith is a member of this club.
    1 true, 2 false, 3 not certain ..... ( )
23. A contest always has (?)
    1 an umpire, 2 opponents, 3 spectators, 4 applause, 5 victory ..... ( )
24. Which number in this series appears a second time nearest the beginning?
    6 4 5 3 7 8 0 9 5 9 8 8 6 5 4 7 3 0 8 9 1 ..... ( )
25. The moon is related to the earth as the earth is to (?)
    1 Mars, 2 the sun, 3 clouds, 4 stars, 5 the universe ..... ( )
26. Which word makes the truest sentence? Fathers are (?) wiser than their sons.
    1 always, 2 usually, 3 much, 4 rarely, 5 never ..... ( )

27. The opposite of awkward is (?)
1 strong, 2 pretty, 3 short, 4 graceful, 5 swift ..... ( )

28. A mother is always (?) than her daughter.
1 wiser, 2 taller, 3 stouter, 4 older, 5 more wrinkled ..... ( )

29. Which one of the six statements below tells the meaning of the following proverb? "The burnt child dreads the fire." ..... ( )

1. Frivolity flourishes when authority is absent.
2. Unhappy experiences teach us to be careful.
3. A thing must be tried before we know its value.
4. A meal is judged by the dessert.
5. Small animals never play in the presence of large ones.
6. Children suffer more from heat than grown people.

30. Which statement above explains this proverb? "When the cat is away, the mice will play." ( )

31. Which statement above explains this proverb? "The proof of the pudding is in the eating." ( )

32. If the settlement of a difference is made by mutual concession, it is called a (?)
1 promise, 2 compromise, 3 injunction, 4 coercion, 5 restoration ..... ( )

33. What is related to disease as carefulness is to accident?
1 doctor, 2 surgery, 3 medicine, 4 hospital, 5 sanitation ..... ( )

34. Of the five things below, four are alike in a certain way. Which is the one not like these four?
1 smuggle, 2 steal, 3 bribe, 4 cheat, 5 sell ..... ( )

35. If 10 boxes full of apples weigh 400 pounds, and each box when empty weighs 4 pounds, how many pounds do all the apples weigh? ..... ( )

36. The opposite of hope is (?)
1 faith, 2 misery, 3 sorrow, 4 despair, 5 hate ..... ( )

37. If all the odd-numbered letters in the alphabet were crossed out, what would be the tenth letter not crossed out? Print it. *Do not mark the alphabet.*
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z ..... ( )

38. What letter in the word SUPERFLUOUS is the same number in the word (counting from the beginning) as it is in the alphabet? Print it ..... ( )

39. What people say about a person constitutes his (?)
1 character, 2 gossip, 3 reputation, 4 disposition, 5 personality ..... ( )

40. If $2\frac{1}{2}$ yards of cloth cost 30 cents, how many cents will 10 yards cost? ..... ( )

41. If the words below were arranged to make a good sentence, with what letter would the second word of the sentence begin? Make it like a printed capital.
same means big large the as ..... ( )

42. If the first two statements following are true, the third is (?) George is older than Frank. James is older than George. Frank is younger than James.
1 true, 2 false, 3 not certain ..... ( )

43. Suppose the first and second letters in the word CONSTITUTIONAL were interchanged, also the third and fourth letters, the fifth and sixth, etc. Print the letter that would then be the twelfth letter counting to the right ..... ( )

44. One number is wrong in the following series. What should that number be?
0 1 3 6 10 15 21 28 34 ..... ( )

45. If $4\frac{1}{2}$ yards of cloth cost 90 cents, how many cents will $2\frac{1}{2}$ yards cost? ..... ( )

46. A man's influence in a community should depend upon his (?)
1 wealth, 2 dignity, 3 wisdom, 4 ambition, 5 political power ..... ( )

47. What is related to few as ordinary is to exceptional?
1 none, 2 some, 3 many, 4 less, 5 more ..... ( )

48. The opposite of treacherous is (?)
1 friendly, 2 brave, 3 wise, 4 cowardly, 5 loyal ..... ( )

49. Which one of the five words below is most unlike the other four?
1 good, 2 large, 3 red, 4 walk, 5 thick ..... ( )

50. If the first two statements following are true, the third is (?) Some of Brown's friends are Baptists. Some of Brown's friends are dentists. Some of Brown's friends are Baptist dentists.
1 true, 2 false, 3 not certain ..... ( )

51. How many of the following words can be made from the letters in the word LARGEST, using any letter any number of times?
great, stagger, grasses, trestle, struggle, rattle, garage, strangle ..... ( )

52. The statement that the moon is made of green cheese is (?)
1 absurd, 2 misleading, 3 improbable, 4 unfair, 5 wicked ..... ( )

 Do not stop. Go on with the next page.

53. Of the five things following, four are alike in a certain way. Which is the one not like these four?
1 tar, 2 snow, 3 soot, 4 ebony, 5 coal..... ( )

54. What is related to a cube in the same way in which a circle is related to a square?
1 circumference, 2 sphere, 3 corners, 4 solid, 5 thickness..... ( )

55. If the following words were seen on a wall by looking in a mirror on an opposite wall, which word would appear exactly the same as if seen directly?
1 OHIO, 2 SAW, 3 NOON, 4 MOTOR, 5 OTTO..... ( )

56. If a strip of cloth 24 inches long will shrink to 22 inches when washed, how many inches long will a 36-inch strip be after shrinking?..... ( )

57. Which of the following is a trait of character?
1 personality, 2 esteem, 3 love, 4 generosity, 5 health..... ( )

58. Find the two letters in the word DOING which have just as many letters between them in the word as in the alphabet. Print the one of these letters that comes first in the alphabet.
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z..... ( )

59. Revolution is related to evolution as flying is to (?)
1 birds, 2 whirling, 3 walking, 4 wings, 5 standing..... ( )

60. One number is wrong in the following series. What should that number be?
1 3 9 27 81 108..... ( )

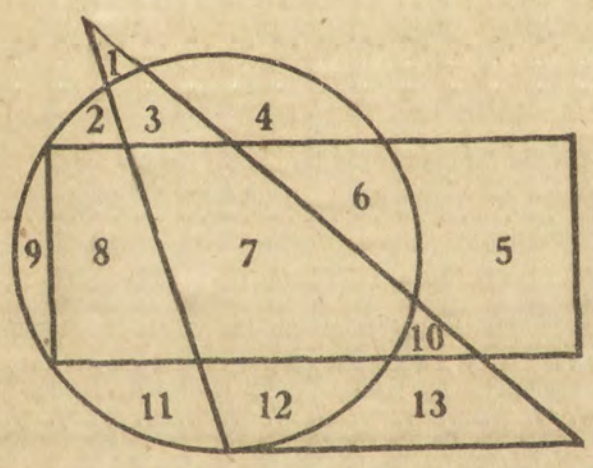
61. If Frank can ride a bicycle 30 feet while George runs 20 feet, how many feet can Frank ride while George runs 30 feet?..... ( )

62. Count each N in this series that is followed by an O next to it if the O is not followed by a T next to it. Tell how many N's you count.
N O N T Q M N O T M O N O O N Q M N N O Q N O T O N A M O N O M..... ( )

63. A man who is averse to change and progress is said to be (?)
1 democratic, 2 radical, 3 conservative, 4 anarchistic, 5 liberal..... ( )

64. Print the letter which is the fourth letter to the left of the letter which is midway between O and S in the alphabet..... ( )

65. What number is in the space which is in the rectangle and in the triangle but not in the circle? ( )



66. What number is in the same geometrical figure or figures as the number 8?..... ( )

67. How many spaces are there that are in any two but only two geometrical figures?..... ( )

68. A surface is related to a line as a line is to (?)
1 solid, 2 plane, 3 curve, 4 point, 5 string..... ( )

69. If the first two statements following are true, the third is (?) One cannot become a good violinist without much practice. Charles practices much on the violin. Charles will become a good violinist.
1 true, 2 false, 3 not certain..... ( )

70. If the words below were arranged to make the best sentence, with what letter would the last word of the sentence end? Print the letter as a capital.
sincerity traits courtesy character of desirable and are..... ( )

71. A man who is influenced in making a decision by preconceived opinions is said to be (?)
1 influential, 2 prejudiced, 3 hypocritical, 4 decisive, 5 impartial..... ( )

72. A hotel serves a mixture of 2 parts cream and 3 parts milk. How many pints of cream will it take to make 15 pints of the mixture?..... ( )

73. What is related to blood as physics is to motion?
1 temperature, 2 veins, 3 body, 4 physiology, 5 geography..... ( )

74. A statement the meaning of which is not definite is said to be (?)
1 erroneous, 2 doubtful, 3 ambiguous, 4 distorted, 5 hypothetical..... ( )

75. If a wire 20 inches long is to be cut so that one piece is $\frac{2}{3}$ as long as the other piece, how many inches long must the shorter piece be?..... ( )

45

HUNDRED-PROBLEM ARITHMETIC TEST

WHOLE NUMBERS—COMMON FRACTIONS—DECIMAL FRACTIONS—PER CENTS

By Raleigh Schorling
Head of Department of Mathematics, the University High School,
and Professor of Education, University of Michigan
John R. Clark
The Lincoln School, Teachers College, Columbia University
and Mary A. Potter
Supervisor of Mathematics, Public Schools, Racine, Wisconsin

V

Total Number Right	
%-ile Rank	

TEST: FORM V

Name..... Date..... Grade.....

Age.....years and.....months. Teacher.....

School..... City..... State.....

Directions

Do not turn this page until you are told to do so. Read the following directions.

This test contains several groups of arithmetic examples. When you finish one group, go right on to the next. If you come to an example that you cannot do, skip it and try it again later if you have time. Begin at the top of each column and work down the page.

You are not expected to finish every example, but work steadily and do the best you can.

You may do your figuring on the test paper or on the blank paper that has been given you. But you must be sure to **write the answer to each example in the box near the example.**

Do not turn the page until I say the word *Begin.*

Parts	Number Correct	+ Number Wrong	+ Number Omitted	= Total Number
I. Addition		+	+	= 10
II. Subtraction		+	+	= 10
III. Multiplication		+	+	= 15
IV. Division		+	+	= 15
V. Fractions, Decimals, and Per Cents		+	+	= 50
VI. Total		+	+	= 100

[This test is a revision of the *Schorling-Clark-Potter Arithmetic Test,* Form A (1928).]

Published by World Book Company, Yonkers-on-Hudson, New York, and Chicago, Illinois
 HPAT: V-14

PRINTED IN U.S.A.

a

I. ADDITION

Add:

1. 8 / 7 / 3 / 6 / 9 / 5 / 8 = ☐ (1)

2. 463 / 877 / 539 / 198 = ☐ (2)

3. $386.85 / 96.66 / 6.57 / .98 / 100.00 / 5.94 / 60.00 = ☐ (3)

4. $\frac{7}{10}$ / $\frac{3}{5}$ = ☐ (4)

5. $\frac{7}{8} + \frac{3}{16} =$ ☐ (5)

6. $17\frac{5}{6}$ / $5\frac{1}{3}$ = ☐ (6)

7. $\frac{3}{5} + \frac{1}{2} + \frac{7}{10} =$ ☐ (7)

8. $9\frac{3}{4}$ / $27\frac{7}{8}$ / $8\frac{9}{16}$ = ☐ (8)

9. $.07 + 5.23 + 8.29 + 1.40 =$ ☐ (9)

10. $\$2.25 + \$14.70 =$ ☐ (10)

When you finish this part, go right on with the next.

II. SUBTRACTION

Subtract:

11. 1124 − 742 = ☐ (11)

12. 880.75 − 785.78 = ☐ (12)

13. $\frac{11}{12} - \frac{1}{6} =$ ☐ (13)

14. $8\frac{3}{8}$ − $5\frac{3}{4}$ = ☐ (14)

15. $2\frac{3}{4} - \frac{2}{3} =$ ☐ (15)

16. $\$5.04 - 18¢ =$ ☐ (16)

17. $9.752 - 6.007 =$ ☐ (17)

18. $\$32 - \$6.58 =$ ☐ (18)

19. $9.25 - 2.20 =$ ☐ (19)

20. 9006 − 4039 = ☐ (20)

When you finish this part, go right on with the next.

III. MULTIPLICATION

Multiply:

Do your work here.
95
82

21. 95 × 82 = ☐ (21)

Write your answer in the box.

Do your work here.
609
40

22. 609 × 40 = ☐ (22)

Write your answer in the box.

Do your work here.
769
708

23. 769 × 708 = ☐ (23)

Write your answer in the box.

24. $\frac{3}{4} \times 60 =$ ☐ (24)

25. $\frac{5}{4} \times \frac{3}{2} =$ ☐ (25)

26. $\frac{5}{8} \times \frac{12}{10} =$ ☐ (26)

27. $45 \times \frac{2}{5} =$ ☐ (27)

28. $20\frac{3}{5} \times 12 =$ ☐ (28)

29. $1\frac{1}{2} \times 2\frac{1}{4} \times \frac{3}{4} =$ ☐ (29)

Do your work here.
4.928
3.2

30. 4.928 × 3.2 = ☐ (30)

Write your answer in the box.

(Part III is continued on the next page.)

III. MULTIPLICATION

(*Continued*)

e answers in the following examples ve not been "pointed off." Put the deci- l point in each answer where it belongs.

31. $20 \times .20 =$ 400 (31)

32. $1.6 \times 0.3 =$ 48 (32)

33. $0.5 \times 5 =$ 25 (33)

34. $0.2\,4\,5 \times 2 =$ 490 (34)

5. Does 1.2×0.5 equal 6.0 or .60 or .060 or 60? ☐ (35)

hen you finish this part, go right on h the next.

IV. DIVISION

vide:

36. $36 \div 3 =$ ☐ (36)

37. $636 \div 6 =$ ☐ (37)

38. $948 \div 9 =$ ☐ (38)

39. $.0\,0\,4\overline{).0\,2\,8\,4}$ ☐ (39)

40. $.3\,4\overline{)1\,0\,5.4}$ ☐ (40)

The answers in the following examples have not been "pointed off." Place the decimal point in each answer where it belongs, adding zeros when necessary.

41. $.1\,2\,3\overline{)5\,6\,0.8\,8}$ 456 (41)

42. $1.2\,3\overline{).5\,6\,0\,8\,8}$ 456 (42)

43. $1\,2.3\overline{)5\,6.0\,8\,8}$ 456 (43)

44. Does $4786 \div 10$ equal 4.786 or 47.86 or 478.6 or 4786? ☐ (44)

45. $2\frac{1}{2} \div 4\frac{1}{2} =$ ☐ (45)

46. $3\frac{3}{4} \div \frac{3}{4} =$ ☐ (46)

47. $\frac{3}{8} \div 4 =$ ☐ (47)

48. $4\frac{1}{2} \div 8 =$ ☐ (48)

49. $2\,1\overline{)8\,8\,2}$ ☐ (49)

50. $8\,3\overline{)1\,1\,3\,7\,1}$ ☐ (50)

When you finish this part, go right on with the next.

V. FRACTIONS, DECIMALS, AND PER CENTS

Write each of the following as per cent:

SAMPLE $\frac{1}{5} =$ (Your answer should read $\frac{1}{5} = 20\%$.)

51. $\frac{3}{100} =$ % (51)

52. $\frac{3}{5} =$ % (52)

53. $\frac{5}{8} =$ % (53)

54. $.75 =$ % (54)

55. $.075 =$ % (55)

56. $\frac{4}{5} =$ % (56)

57. $\frac{1}{3} =$ % (57)

58. $\frac{3}{8} =$ % (58)

59. $.2 =$ % (59)

60. $0.875 =$ % (60)

Write each of the following as a decimal fraction:

61. $\frac{3}{10} =$ ☐ (61)

62. $\frac{1}{4} =$ ☐ (62)

63. $\frac{2}{5} =$ ☐ (63)

64. $60\% =$ ☐ (64)

65. $7\frac{1}{2}\% =$ ☐ (65)

66. $\frac{7}{100} =$ ☐ (66)

67. $\frac{3}{5} =$ ☐ (67)

68. $\frac{1}{8} =$ ☐ (68)

(Part V is continued on the next page.)

PART V. (*Continued*)

Write each of the following as a decimal fraction:

69. $12\frac{1}{2}\% =$ [ ] (69)

70. $37\frac{1}{2}\% =$ [ ] (70)

Write each of the following as a common fraction:

71. $20\% =$ [ ] (71)

72. $9\% =$ [ ] (72)

73. $25\% =$ [ ] (73)

74. $12\frac{1}{2}\% =$ [ ] (74)

75. $33\frac{1}{3}\% =$ [ ] (75)

Complete the following:

76. 25% of $120 =$ [ ] (76)

77. 2.3% of $40 =$ [ ] (77)

78. 120% of $20 =$ [ ] (78)

79. $\frac{2}{3}\%$ of $3000 =$ [ ] (79)

80. [ ] % of $24 = 8$.

81. [ ] % of $60 = 6$.

82. [ ] % of $20 = 25$.

83. $4 =$ [ ] % of 20.

84. $9 =$ [ ] % of 18.

85. $8 =$ [ ] % of 80.

Write these decimals as per cents:

86. $.355 =$ [ ] % (86)

87. $.123 =$ [ ] % (87)

88. $.1825 =$ [ ] % (88)

Rewrite the following decimals, arranging them in the order of their size, the largest first and the smallest last:

89. .93 .15 .94 [ ] (89)

90. .40 2.5 .875 [ ] (90)

Write these as decimal fractions; carry the answer to three places and round off to two places:

91. $\frac{7}{16} =$ [ ] (91)

92. $\frac{5}{16} =$ [ ] (92)

93. Mary bought an \$8 dress at a 15% discount. What did she pay for the dress? [ ] (93)

94. What is the interest for a year on \$175 at 6%? [ ] (94)

95. Mr. Brown found that $22\frac{1}{2}\%$ of his peaches were not good enough to sell. Out of 80 bushels he could sell ? bushels. [ ] (95)

96. Carl earned \$32 during his summer vacation. He spent $14\frac{1}{2}\%$ of this money for schoolbooks. How much did his books cost? [ ] (96)

97. What do you pay for goods marked \$13.50 with a discount of 2%? [ ] (97)

98. What per cent of your investment do you make if you invest \$125 and gain \$5? [ ] % (98)

99. What is the interest for a year on \$300 at $4\frac{1}{2}\%$? [ ] (99)

100. There are 2150 pupils in one junior high school of this city. The principal of this school expects an increase of 6% in the number of pupils next semester. How many pupils does he plan to have next semester? [ ] (100)

When you finish this part, go back and make sure that your work is correct.

APPENDIX C

32 Problems of Quantity Estimation

Name _____ Date ___ Grade _____
Age ___ Teacher _____ School _____

This exercise contains estimation questions which cannot be actually computed. For each question you have a choice of 5 answers, one of which is correct. When you select the answer which you think is correct, place the letter of that answer in the space designated at the right side of the page. Here is an example:

1. How many eggs in a dozen?
   (a) 10
   (b) 8
   (c) 7
   (d) 6
   (e) 12

   1. e

This exercise aims to measure only your ability to estimate. It is very important that you try to do your best. When you complete one problem go right on to the next.

1. How far is it from here (Highland High) to the Hilton Hotel (in miles)?
   (a) 1
   (b) $2\frac{1}{2}$
   (c) $3\frac{1}{2}$
   (d) $4\frac{1}{2}$
   (e) 6

   1. _____

2. How wide are the hallways in this building (in feet)?
   (a) 27
   (b) 20
   (c) 15
   (d) 12
   (e) 8

   2. _____

3. How high is the ceiling in this room (in feet)?
   (a) 25
   (b) 20
   (c) 15
   (d) 12
   (e) 9

   3. _____

4. How wide is the door in this classroom (in feet)?
   (a) $2\frac{1}{2}$
   (b) 3
   (c) $3\frac{1}{2}$
   (d) 5
   (e) 6

4. ____

5. How long is this room (in feet)?
   (a) 12
   (b) 14
   (c) 16
   (d) 18
   (e) 24

5. ____

6. How wide is this room (in feet)?
   (a) 12
   (b) 15
   (c) 20
   (d) 24
   (e) 32

6. ____

7. How far is it from Highland High School to the nearest base of the Sandia Mountains (in miles)?
   (a) 2
   (b) 3
   (c) 5
   (d) 7
   (e) 10

7. ____

8. How many steps will an average man take in walking a mile (5280 feet in a mile)?
   (a) 210
   (b) 2100
   (c) 3500
   (d) 21,000
   (e) 210,000

8. ____

9. How many words are there in an average line of type in your school books?
   (a) 8
   (b) 10
   (c) 12
   (d) 14
   (e) 16

9. ____

EFFICIENCY

ERASE BOARD

RAG CONSIST

10. How many single dips of ice cream can you get out of a gallon?
(a) 70
(b) 60
(c) 50
(d) 40
(e) 30

10. ____

11. How many cubic feet of space are there in your school locker? (A cubic foot is one foot long, one foot wide, and one foot high).
(a) $2\frac{1}{2}$
(b) $3\frac{1}{2}$
(c) 5
(d) $6\frac{1}{2}$
(e) 8

11. ____

12. How high is the door in this classroom (in feet)?
(a) $5\frac{1}{2}$
(b) 7
(c) $8\frac{1}{2}$
(d) 10
(e) 11

12. ____

13. How many steps are there in the flight of stairs between the first and second floors of this school?
(a) 18
(b) 24
(c) 30
(d) 36
(e) 42

13. ____

14. Estimate the length of line AB (in inches).

A _____ _____ B

(a) 2
(b) 3
(c) 4
(d) 5
(e) 6

14. ____

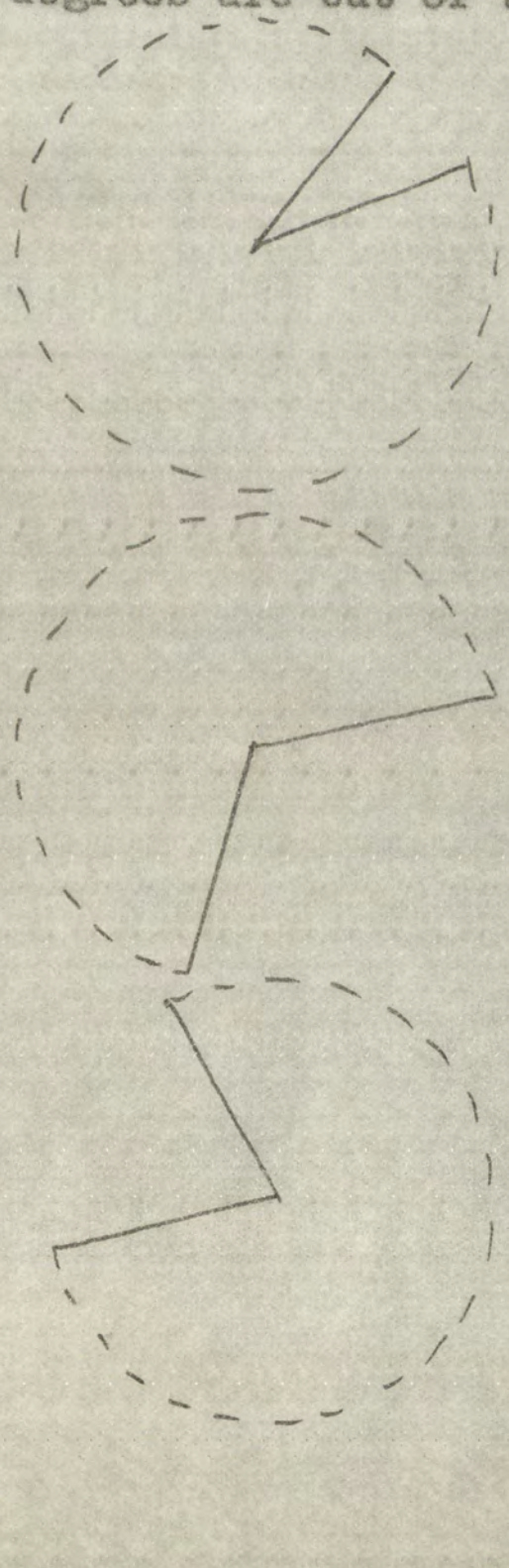
15. Estimate the length (in inches) of a line drawn diagonally across this 8½" by 14" page.
(a) 13
(b) 16
(c) 18
(d) 20
(e) 22

15. ____

16. How many level tablespoons of coffee are there in a pound of coffee?
(a) 75
(b) 65
(c) 55
(d) 45
(e) 35

16. ____

17. How many degrees are cut of the following circle?
(a) 10
(b) 20
(c) 30
(d) 40
(e) 50



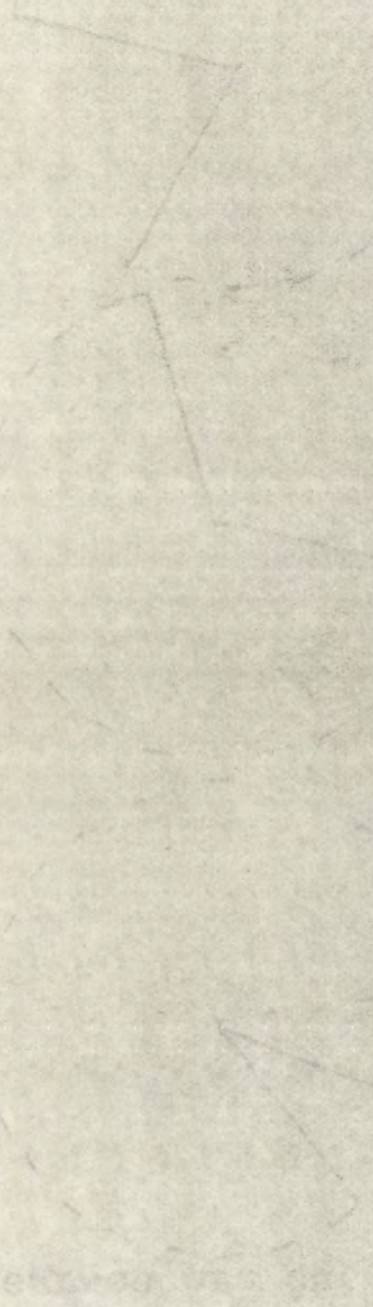
17. ____

18. (a) 80
(b) 100
(c) 120
(d) 140
(e) 160

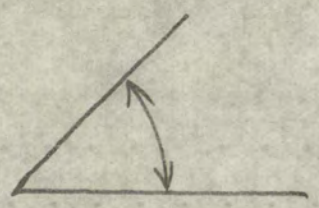
18. ____

19. (a) 65
(b) 75
(c) 85
(d) 95
(e) 105

19. ____

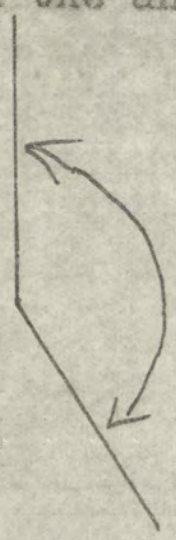


20. A reasonable estimate of the number of degrees in the angle below is:
(a) 15
(b) 25
(c) 35
(d) 45
(e) 55



20. _____

21. A reasonable estimate of the number of degrees in the angle below is:
(a) 105
(b) 120
(c) 135
(d) 150
(e) 165



21. _____

22. What is the approximate number of square feet in this room?
(a) 500
(b) 600
(c) 700
(d) 800
(e) 900

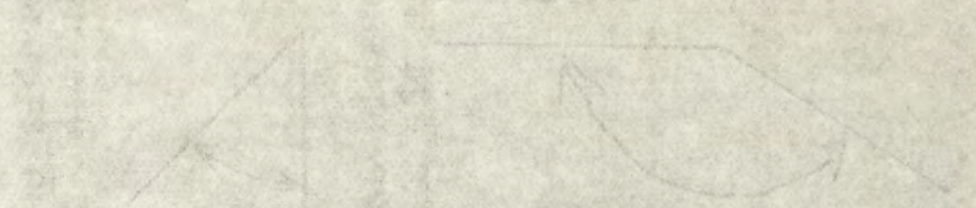
22. _____

23. What is the verticle dimension of the blackboard slate in this room (in feet)?
(a) 2
(b) 3
(c) 4
(d) 5
(e) 6

23. _____

24. How many square inches are there in the triangle held by the examiner?
(a) 24
(b) 32
(c) 36
(d) 40
(e) 50

24. _____



25. How many square inches are there in the rectangle held by the examiner?
(a) 98
(b) 108
(c) 118
(d) 128
(e) 138

25. _____

26. What is the length of the piece of wood held by the examiner (in inches)?
(a) 24
(b) 32
(c) 40
(d) 48
(e) 54

26. _____

27. How many slices of bread are there in the loaf held by the examiner? (counting the crusts).
(a) 35
(b) 30
(c) 25
(d) 20
(e) 15

27. _____

28. How many square inches are there in the circle held by the examiner?
(a) 60
(b) 70
(c) 80
(d) 90
(e) 100

28. _____

29. There are 981 pages in this book; estimate the number of pages he holds between his fingers.
(a) 70
(b) 80
(c) 90
(d) 100
(e) 150

29. _____

30. How many cups of water will this can hold?
(a) 7
(b) 8
(c) 9
(d) 10
(e) 12

30. _____

EFFICIENCY

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31. What is the approximate height of the person giving the test?
(a) 5'2"
(b) 5'7"
(c) 5'9"
(d) 5'11"
(e) 6'2"

31. ____

32. What is the approximate weight of the person giving the test?
(a) 140 lbs
(b) 160 lbs
(c) 180 lbs
(d) 200 lbs
(e) 220 lbs

32. ____

D.T.

IMPORTANT!

Special care should be taken to prevent loss or damage of this volume. If lost or damaged, it must be paid for at the current rate of typing.