Economic Analysis of Liability Apportionment Among Multiple Tortfeasors: A Survey, and Perspectives in Large-Scale Risks Management

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ECONOMIC ANALYSIS OF LIABILITY APPORTIONMENT AMONG MULTIPLE TORTFEASORS: A SURVEY, AND PERSPECTIVES IN LARGE-SCALE RISKS MANAGEMENT

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I. INTRODUCTION

Many human activities, especially industrial processes, can cause damage to the environment: chemicals, energy supply, even the agro-food industry involves health risks. Because it can be difficult and/or costly for potential injury victims to avoid these risks, public regulation has been put in place in order to: 1) incite industries to engage in efforts to reduce the level of accident risk; and 2) compensate the victims in case of damage.

For industrial activities, one of the main regulatory tools in place is civil liability.[1] Civil liability responds to the two goals of public regulation: it compels the injurers to compensate the victims in case of damage (via a damages payment), and this threat of payment *ex post* provides incentives to engage in effort *ex ante* to reduce the probability of an accident occurring.

However, most industrial activities require the involvement of several decision makers, who all have different and autonomous legal entities. When each of these decision makers has an impact on the overall level of risk, the efficiency of civil liability can be found wanting. Indeed, the total debt (to compensate victims) has to be shared among the different parties who have contributed to the occurrence of the damage. However, since the work of A.C. Pigou in *The Economics of Welfare*,[2] economic theory has

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1. Civil liability is not the only regulatory tool in place. *Command and control* systems are also in place, such as approvals (authorizations to operate, which are granted only if the firm is able to demonstrate its ability to adequately control the level of risk). Approvals can be combined with random *in situ* inspections once the firm is operating. *See, e.g.*, Chemical Accident Prevention Provisions, 40 C.F.R. § 68 (1996).

2. A.C. PIGOU, THE ECONOMICS OF WELFARE pt. II, ch. IX (4th ed. 1932).

taught us that, in order to provide an agent with the incentive to optimally control the level of "nuisance" he produces (e.g., pollution, risk of accident, etc.), this agent must take into account the whole nuisance[3] he causes. But when several agents contribute to a common damage, how can the debt of liability be shared efficiently?

To illustrate, consider the case of energy. Taking the example of gas power plants, the level of accident risk depends on the level of care provided by the operator (e.g. the frequency at which the gas pressure is controlled). In a French illustration, the French national electricity manufacturer, Électricité de France (EDF), is the decision maker. But for a given level of care provided by the operator, the level of risk also depends on the reliability of the production process (e.g., keeping with France, the reliability of the turbine provided by the French manufacturer Alstom). We can also use the agro-food industry as another example. The level of health risk depends on the level of care taken by the operator, such as the quality of the sterilization process, for example. However, for a given level of care, the level of health risk also depends on the quality of inputs provided by suppliers, for example the food containers (i.e. their ability not to oxidize in contact with food).[4]

In this article we seek to address the optimal way of sharing a debt in liability for a common (and high) damage in which the different contributors have an impact on the likelihood of the damage occurring. We focus our analysis on the question of the *efficiency* of the sharing, i.e. sharing liability in order to provide each decision maker with efficient incentives to undertake sufficient efforts to optimally control the level of risk. We set aside other important questions, such as fairness issues (e.g. a sufficient level of compensation for the victims and fairness in the allocation of the debt between the different contributors).

The following analysis is developed more generally in Julien Jacob and Bruno Lovat's article titled, *Multiple Tortfeasors in High Risk Indus-*

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3.   To be precise, the agent has to take into account the *marginal* (expected) damage he causes: for instance, if he expects to increase his level of activity, he has to (financially) take into account the increase in damage resulting from the increase in activity (e.g. he has to pay a tax corresponding to the damage caused by the increase in CO2 emissions).

4.   We also could think of the controversy surrounding the use of Bisphenol A, which is suspected to be an endocrine disruptor. The suspected risks, related to exposure to endocrine disruptors, are long-term health risks. In this case, applying civil liability could face difficulties because of the difficulty gathering sufficient evidence to prove the causal link between the exposure and the damage suffered. A first economic analysis of the incentives provided by civil liability in cases of long-term latent hazards is provided by Al H. Ringleb & Steven N. Wiggins, *Liability and Large Scale, Long-Term Hazards*, 98 J. POL. & ECON. 98, 574–95 (1990). A practical example reflecting the difficulty in enforcing civil liability in case of latent hazards is given by the Diethylstilbestrol (DES) case. *See* Caroline Politi, *Procès du Distilbène: deux laboratoires condamnés en appel*, L'EXPRESS, Oct. 26, 2012.

tries: How to Share Liability?.[5] First, we present the originality of our contribution with respect to the literature on law and economics. We then present our assumptions and the main results of our analysis, and show how our sharing rule could apply using a numerical example.

II. THE LITERATURE AND OUR CONTRIBUTION

Despite the wide range of applications, the economic literature on the optimal apportionment of liability is relatively scarce, and mainly North American.

In the United States, there are two main ways of sharing liability: using a joint-and-several liability rule or a non-joint liability rule. In each case, several injurers are liable for a common damage. In each case, each injurer has to pay a share of the common debt (this share is determined by the judge).[6] The main difference between these two rules is the fact that liability is *joint* in the first case and *non-joint* in the second. In a case of non-joint liability, each injurer has to pay its share of liability to the limit of its level of assets.[7] In cases of joint and several liability, the different contributors are jointly liable: if one contributor is financially unable to pay for a share of liability, the remaining debt has to be paid by the other (solvent) contributors. As a consequence, joint and several liability allows the victims to sue only one contributor and to claim the entire damage from this sole contributor. This contributor then has to sue the other contributors for their shares of the liability. By pursuing only one (highly solvent) contributor, joint and several liability leads to a decrease in litigation costs for the victims, who can more easily exercise their right to redress. Hence, joint and several liability is the default apportionment rule in the United States in cases of damage with multiple defendants.[8]

However, in the 1980s, a tort reform movement developed with the aim of restricting the application of joint and several liability to economic damage only (e.g. loss of gross revenue). Nowadays, in the United States, most states apply the rule of non-joint liability for non-economic damage,

5.   Julien Jacob & Bruno Lovat, Multiple Tortfeasors in High Risk Industries: How to Share Liability? (2015) (unpublished manuscript) (on file with the Bureau d'Economie Théorique et Appliquée).

6.   *See infra* section III.B.

7.   We assume that each injurer benefits from limited liability; they cannot pay more than their level of wealth/assets. As a consequence, if the amount of debt exceeds the level of wealth, a part of the debt remains unpaid.

8.   Han-Duck Lee et al., *How Does Joint and Several Tort Reform Affect the Rate of Tort Filings? Evidence from the State Courts*, 61 J. RISK & INS. 295, 316 (1994).

especially environmental damage.[9] This change was introduced as a response to the Liability Insurance Crisis which affected certain "hazardous" sectors (e.g. chemicals) in the United States during the 1980s; the application of joint and several liability, combined with the increasing use of strict liability[10] and the difficulty of assessing (and forecasting) environmental damage led to an unexpected increase in debts for environmental damages.[11] As a consequence, certain partners of these sectors decided to limit their exposure. Liability insurers excluded environmental damages[12] and banks[13] became more reluctant to grant loans.[14] Nevertheless, the debate is still open in the United States; not all states enacted such tort reform, and some federal laws, like CERCLA, still use joint and several liability.[15]

In this debate, the law and economic literature has developed comparative analyses between joint-and-several and non-joint liability, and/or has developed normative analyses aiming to find an optimal sharing rule (as mentioned before, we restrict our attention to the problem of providing optimal incentives to control the risk). A study of the literature on law and economics shows that the optimal way to share liability between multiple defendants is closely related to the characteristics of the situation to be regulated. Considering several contributors to a common damage (instead of only one injurer) leads to a multiplicity of possible situations to regulate. The situations can be differentiated according to three criteria: (i) the type of actions that may be undertaken by the decision makers; (ii) the chronol-

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9.   *See* Yiling Deng & George Zanjani, *What Drives Tort Reform Legislation? Economics and Politics of the State Decisions to Restrict Liability Torts* (Ga. State Univ. Dep't of Risk & Ins. Working Paper, 2014); Lee et al., *supra* note 8, at 298.

10.   Negligence (i.e. liability is subject to a deviation from a standard of due care) still remains the default rule of liability. But for the case of environmental damages, strict liability (no need to demonstrate a deviation from a standard to establish liability) is increasingly used. *See, e.g.*, Comprehensive Environmental Response, Compensation and Liability Act (CERCLA), 42 U.S.C. §§ 9601-9675 (2011).

11.   Strict liability is also increasingly used in cases of work exposures. Ringleb & Wiggins, *supra* note 4, at 574–95 (showing the underlying problems for the sectors which expose their workers to risks during the production process and how these industries react in order to reduce their own exposure to liability claims); *see also* Martin T. Katzman, *Pollution Liability Insurance and Catastrophic Environmental Risk*, 55 J. RISK & INS. 75 (1998).

12.   *See* Dan R. Anderson, *Development of Environmental Liability Risk Management and Insurance in the United States: Lessons and Opportunities*, 2 RISK MGMT. & INS. REV. 1 (1998).

13.   Two reasons justify the reluctance of the banking sector. First, if liability insurers do not cover certain risks, the expected liability debt for the firms is higher. As a consequence, the probability of default increases (especially environmental damages cases, which can be considered as senior debts). Second, some U.S. environmental laws, such as CERCLA, introduce an extension of liability to the financial partners; if the firm is unable to pay for the liability, the remaining damages can be claimed from the banks.

14.   *See* Rohan Pitchford, *How Liable Should a Lender be? The Case of Judgment-Proof Firms and Environmental Risk*, 85 AM. ECON. REV. 1171 (1995).

15.   42 U.S.C. § 9607.

ogy of actions; and (iii) the way in which the different actions combine to lead to the occurrence of the damage.

In *Joint Liability in Torts: Marginal and Inframarginal Efficiency*, Thomas Miceli and Kathleen Segerson distinguish two types of actions: binary actions (to engage or not engage in an activity),[16] or the choice of a degree of effort within a continuum of possibilities (e.g., which degree of care to exert).[17] The two types of actions do not lead to the same method of optimally sharing a debt. In *Multiple Tortfeasors: An Economic Analysis*, Robert Young et al. analyze how the way in which the actions combine (to lead to the damage) alters the optimal apportionment of liability. They distinguish actions *in series* from actions *in parallel*.[18] Actions in series need the involvement of all contributors, simultaneously, to provoke the damage; if one contributor does not act, the damage cannot occur. Actions in parallel are actions that can lead to the damage independently of each other; each contributor can cause the damage alone through its own action. Our analysis adopts the following position.[19]

As indicated above, we restrict our attention to the efficiency of incentives. We are seeking to find a sharing rule that can provide optimal incentives to several decision makers who, by their actions, have the possibility of controlling the probability of a given damage occurring.

We choose to analyze a common situation whereby a provider of products/technologies is in a relationship with an industrial operator that uses these technologies within its production process. Although only the industrial operator's activity can "directly" cause the damage, the upstream technical provider has an impact on the probability of the damage occurring because of its effort in the quality/reliability of the technology it provides (irrespective of the degree of prevention adopted by the regulator). In this sense, the technical provider can be considered an "indirect" contributor to the damage and has to receive optimal incentives to provide a technology of "good" quality.

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16.   Thomas J. Miceli & Kathleen Segerson, *Joint Liability in Torts: Marginal and Inframarginal Efficiency*, 11 INT'L REV. L. & ECON. 235 (1991); *see also* STEVEN SHAVELL, ECONOMIC ANALYSIS OF ACCIDENT LAW ch. 7 (1987); Steven Shavell, *Uncertainty over Causation and the Determinants of Civil Liability*, 28 J. L. & ECON. 587 (1985); Robert Young et al., *Multiple Tortfeasors: An Economic Analysis*, 3 REV. L. & ECON. 111 (2007) [hereinafter Young, *Multiple Tortfeasors*]; Robert Young et al., *Causality and Causation in Tort Law*, 24 INT'L REV. L. & ECON. 507 (2004).

17.   *See* Lewis A. Kornhauser & Richard L. Revesz, *Sharing Damages Among Multiple Tortfeasors*, 98 YALE L.J. 831 (1989).

18.   Young, *Multiple Tortfeasors*, *supra* note 16.

19.   Jacob & Lovat, *supra* note 5.

Considering the classification introduced by Miceli and Segerson and Young et al.,[20] we observe that, to the best of our knowledge, this kind of situation has not been considered before. Our aim is to regulate levels of care (the agents are already engaged in their activities, which are supposed to be socially desirable). But in regard to the way their actions combine, the situation we consider is a new one: the industrial operator is both necessary and sufficient to cause the damage. Without this operator, no damage can occur (necessity), and its mere presence is sufficient to cause the damage, irrespective of the presence or not of the technical provider.[21] The technical provider (hereinafter the "innovator") is neither necessary nor sufficient to cause the damage; it cannot cause the damage alone, and the operator can cause the damage without it.

Another original aspect of our work is that it takes into account both the capacity of each agent to be financially unable to pay for damage caused (insolvency), and the market relationship which links the operator to the innovator. Since Steven Shavell's, *The Judgment Proof Problem*, it has been well-established in the law and economic literature that the possibility of being insolvent can reduce the incentives to control the level of risk. When the amount of damages exceeds the financial capacity of the firm, the principle of limitation of liability prevents the firm from taking into account the whole damage it causes in its economic calculus.[22] Hence its care effort will be insufficient.[23] Insolvency in the case of multiple defendants is studied by Lewis Kornhauser and Richard Revesz, but in a different framework. In their work, the decision-makers have an impact on the magnitude of the damage (which occurs with certainty).[24] Instead, we consider decision-makers that have the ability to reduce the probability of the damage occurring.

Our contribution is also original in that it takes into account the market relationship between the two injurers: the operator buys an input product, or a productive technology, from the innovator. The price is a conse-

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20.   Miceli & Segerson, *supra* note 16; Young, *Multiple Tortfeasors*, *supra* note 16, at 121.

21.   This means that the industrial operator is able to operate without the technology provided by the provider. For instance, the operator owns a "basic technology," and the technical provider offers an alternative technology or an upgrade of the basic technology. This assumption can be removed and the qualitative results remain the same: the operator would be only necessary, but no longer sufficient.

22.   Steven Shavell, *The Judgment Proof Problem*, 6 INT'L REV. L. & ECON. 45 (1986).

23.   To illustrate, consider a firm with a financial capacity of $1 million. It can cause damage of $10 million. Because of limited liability, it only has to pay $1 million in case of an accident. From this firm's point of view, this is *as if* it faced a damage of $1 million (and not $10 million). If the firm were endowed with a higher level of wealth, it could face a higher loss in case of an accident. This would provide incentives to make more effort to reduce the likelihood of this bad event.

24.   Lewis A. Kornhauser & Richard L. Revesz, *Apportioning Damages Among Potentially Insolvent Actors*, 19 J. LEGAL STUD. 617 (1990).

quence of bargaining between the two actors. Depending on the intensity of competition on the research and development ("R&D") market, bargaining power would tilt in favor of one actor or the other. If the innovator benefits from a monopoly position on the R&D market, it would be able to fix a high selling price (because it is the only existing provider). Conversely, if the innovator faces a large number of competitors on the R&D market, it would have to moderate its selling price to conclude the transaction (otherwise, the operator could turn to other competitors). The innovator's ability (or inability) to determine its selling price has an impact on the incentives to make efforts to improve the quality of the technology, and this degree of quality has an impact on the likelihood of damage occurring. So the sharing rule, which aims to provide all agents with optimal incentives to control the level of risk, has to take the characteristics of the R&D market into account.

Finally, our analysis is explicitly a normative one. We do not compare different existing rules of apportionment. We aim to define an optimal sharing rule, irrespective of any existing rule, but by taking important legal constraints into account, such as the limitation of liability. Our only objective is to define an optimal rule that provides each agent with optimal incentives to make optimal efforts to control the level of risk. Despite its novelty, the sharing rule we propose is intended to be easily applied, without any drastic modification in the prevailing legal corpus.

III. ANALYTICAL FRAMEWORK

As a first step, we introduce the basic assumptions of our analysis before highlighting the two main originalities of our contribution: the possibility for each injurer to be insolvent and the market relationship that links them.

A. Basic Assumptions

We consider the case of two firms. The first one is an industrial operator, denoted by O. The operator is engaged in an industrial activity that can cause damage to third parties and/or the environment. The magnitude of the potential damage is given by HH. The operator is endowed with a basic productive technology, but it has the possibility of buying a new one from an innovator, denoted by II. This new technology is more reliable and reduces the probability of an accident occurring.

More precisely, the probability of a damage occurring is denoted $p(x, e)$ which depends on a level of care, x, which is adopted by the opera-

tor, and an effort in R&D, e, which is adopted by the innovator and which determines the degree of technical performance of the technology. The higher x, the lower probability $p(x, e)$: $\frac{\delta p(x,e)}{\delta x} < 0$. The same property holds with R&D: $\frac{\delta p(x,e)}{\delta e} < 0$. But the efficiency of x and e in reducing the probability is decreasing: $\frac{\delta^2 p(x,e)}{\delta x^2} > 0$, $\frac{\delta^2 p(x,e)}{\delta e^2} > 0$. The following specification satisfies these properties: $(x, e) = \frac{\exp(-\alpha x) + \exp(-\beta e)}{\gamma}$, with $\alpha > 0, \beta > 0$ and $\gamma \geq 2$. We use this specification for our theoretical analysis and to calibrate the numerical calculus we provide later. The operator is initially endowed with a technology with a degree 0 of technical performance ($e = 0$), but it has the possibility of buying from the innovator a more advanced technology ($e > 0$). To exercise a care effort, as well as an effort in R&D, is costly: the total cost of applying a given level of care x is $c.x$, and the total cost of applying a given level of R&D e is $k.e$.

Each firm is endowed with a level of wealth, W_O and W_I, respectively for the operator and the innovator, from which the damages will be financed in case of accident. We suppose that entering in activity allows a firm perceiving a revenue R_i, $i = I, O$, from which it can finance its R&D activities (firm I), its prevention activities or the purchase of a new technology (firm O). We suppose that no firm is able, alone, to pay for the total damage: $W_O < H$, $W_I < H$. Each firm is subject to an insolvency constraint. However, we assume that, taken together, both firms have sufficient wealth to pay for the overall damage[25]: $W_O + W_I > H$. So the relevant question is how to share H in order to provide both firms with optimal incentives to exercise care and R&D in order to optimally control the level of risk.

Finally, we assume that strict liability holds. This allows us to simplify the analysis and thus to focus our attention on finding the optimal sharing rule. We set aside the issue of fairness in the apportionment, which could take place with a negligence rule (e.g., by taking into account the relative degree of negligence of each injurer). Moreover, with regards to damage to the environment and/or the presence of "high risks," strict liability tends to become the default liability rule.[26]

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25.   We assume that the two firms are sufficiently wealthy to repair the total damage H (when taken together): victims are fully compensated. This choice is an arbitrary one (it serves to lighten the calculus). However, the sum of the total damages to be paid could be different from H, and the qualitative results should not be affected.

26.   In the United States, strict liability holds under CERCLA. Shavell illustrates the increasing use of strict liability in the case of "abnormally dangerous" damage and in the case of "ultrahazardous" activities. STEVEN SHAVELL, FOUNDATIONS OF ECONOMIC ANALYSIS OF LAW 204–05 (2004) (citing

Now we introduce our particular scheme of liability sharing.

B. Apportioning Liability Under Insolvency

Because of individual insolvency constraints ($W_O < H, W_I < H$), there are constraints on the liability sharing: each agent cannot pay more than its financial capacity. As a consequence, if we denote as D_O and D_I the amount in damages to be paid respectively by the operator and the innovator, we have to take into account the following constraints: $D_O < H, D_I < H$.

The liability scheme can be illustrated by the following figure.

Figure 1: apportionment under individual insolvencies
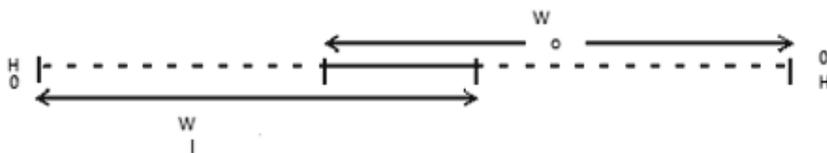
The top of the figure highlights damages to be paid by the operator, with a reading from right to left. Damages to be paid by the innovator are represented at the bottom, from left to right. For a good understanding of how this sharing rule works, we consider the following illustration.

Because of the limited liability constraint, the maximum amount in damages that can be claimed from the innovator is W_I. As a consequence, the minimum amount in damages that will be claimed from the operator is the complement $H - W_I$ (highlighted by the dotted line on the right). Applying similar reasoning, we can say that the minimum amount in damages that will be claimed from the innovator is $H - W_O$ (dotted line on the left). Because of individual constraints of limited liability, these two amounts are incompressible. As a consequence, the amount of debt to be shared is:

$$H - (H - W_I) - (H - W_O) = W_I + W_O - H$$

Put differently, when considering the total amount of damages to be paid, but abstracting from the minimum (and incompressible) amounts in damages, the "sharing zone" is restricted to $W_I + W_O - H$, which corre-

JOHN FLEMING, THE LAW OF TORTS 15–18, 97 (9th ed. 1998)); *see also* W.P. KEETON ET AL., PROSSER AND KEETON ON TORTS 536–38 (5th ed. 1984).

sponds to the total wealth over the amount of damage to be remedied. So it is *only* to this sharing zone that our sharing rule will apply.

As a consequence, the amounts in damages to be paid by each agent are:

$$D_O = D_O(\theta) = W_O - \theta.[W_I + W_o - H]$$
$$D_I = D_I(\theta) = W_I - (1 - \theta).[W_I + W_o - H]$$

With θ the share of "sharing zone" which is attributed to the innovator, and $(1 - \theta)$ the share attributed to the operator, θ takes values between 0 and 1. It is important to keep in mind that the apportionment only applies to the sharing zone: hence, $\theta = \frac{1}{2}$ does not mean that each firm pays for one half of the total damage H, but that they each pay for one half of the "sharing zone". Below highlights the originality of our sharing rule with respect to existing rules of apportionment.

The different rules currently enforced often define the apportionment relative to the contribution of each agent to the overall damage. In our case, the magnitude of H is given and is independent from the actions decided by the agents (recall that the agent can alter the probability of the damage occurring, not its magnitude). So we cannot define a sharing rule based on the individual relative contributions to the damage. Moreover, θ is defined in a specific manner, taking into account individual insolvency constraints; these constraints are *ex ante* explicitly taken into account. This is very different from the current functioning of the liability system where the agents are *a priori* liable for a given share of the damage (according to criteria based on the relative contribution on the damage) but, *ex post*, they can escape from paying their share of liability thanks to insolvency (what is referred to as "judgment-proofness" in the literature). Here, such a mechanism is excluded: *a priori* and *a posteriori* payments are known and identical.[27]

Now that our rule of apportionment is presented, we have to determine the optimal value of θ, so as to provide optimal incentives for prevention (x) and innovation (e) in order to "properly" control the level of the risk of

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27.   In case of joint and several liability it is possible for an agent to be financially unable to pay for its *a priori* share of liability. In that case, the remaining damages are passed on to another solvent agent. In our system of liability, an agent with a low level of solvency will pay for a low share of the common debt. But there is no "one for one" relationship between the (in)solvency of one agent, and the remaining debt attributed to the other agent. Technically, a change in W_I or in W_o W_O leads to the definition of a new value of θ θ. There is not necessarily a transfer of "one for one" between the two agents (as it is the case under joint and several liability).

damage. For this, we first have to define the optimal situation, and then to set θ in such a way as to reach this situation.

C. First Best Situation

In economic analyses, it is usually recognized that optimality—or, equivalently, a "first-best" situation—consists of reaching the maximum level of social welfare. Social welfare is defined as the sum of all individual welfares. In our case, this corresponds to the sum of the individual profits of the operator and the innovator, minus the level of the risk of damage borne by third parties. So the definition of optimality responds to the following problem:

$$Max_{x,e}\ SW(x,e) = W_I + W_O + R_I + R_O - cx - ke - p(x,e)H$$

with $p(x,e) = \frac{\exp(-\alpha x) + \exp(-\beta e)}{\gamma}$ in our illustration.

We have to find the values of x and e which maximize this social welfare function.

So, optimality comes from a hypothetical situation where all interests and costs are taken into account by an "omniscient, omnipotent and benevolent dictator," who aims to maximize the global welfare of society. This "ideal" situation is used as a benchmark toward which we should strive.

The socially optimal values for x and e (which we respectively denote x^{**} and e^{**}) are:[28]

$$x^{**} = \frac{1}{\alpha} ln\left(\frac{\alpha H}{\gamma c}\right)$$

$$e^{**} = \frac{1}{\beta} ln\left(\frac{\beta H}{\gamma k}\right)$$

We verify that the higher the level of damage H, the higher the optimal values of prevention and innovation. The higher the cost c of prevention (respectively, the cost k of innovation), the lower the optimal value of prevention (respectively, innovation).

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28. These values are derived by using the classical method of finding the first-order conditions of x and e (i.e. by solving $\frac{\delta SW(x,e)}{\delta x} = 0$ and $\frac{\delta SW(x,e)}{\delta e} = 0$ respectively).

At the optimum, the probability of causing an accident should be:

$$p(x^{**}, e^{**}) = \frac{c\beta + \alpha k}{\alpha\beta H}$$

The higher the level H of damage, the lower the probability should be. The higher the cost of making efforts on care and/or in innovation, the lower the social values for prevention and/or innovation and, as a consequence, the higher the socially optimal value of the probability of an accident occurring.
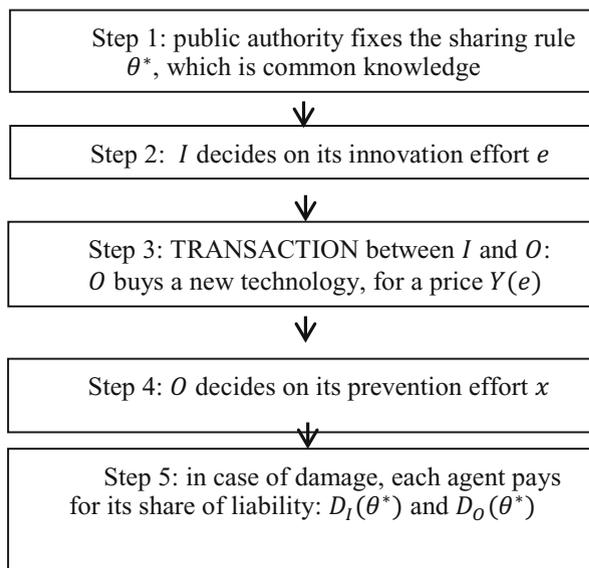
In reality, of course, no such a dictator exists. Reality involves several private agents, facing their own constraints and wanting to maximize their own objectives. The aim of the public regulator is thus to enforce a policy in such a way as to come as close as possible to the optimal situation, but taking into account the private behaviors of firms.

To determine the optimal value of the sharing rule θ, we have to properly define all the private interests of both agents, I and O. So we have to define the market relationship that links them to each other.

D. Two Defendants Linked by a Market Relationship

We know that the operator has the possibility of buying a new production technology from the innovator. More precisely, the relationship between the two agents can be summarized in the following manner.

Figure 2. Schedule of individual decisions

Step 1: public authority fixes the sharing rule θ^*, which is common knowledge

↓

Step 2: I decides on its innovation effort e

↓

Step 3: TRANSACTION between I and O: O buys a new technology, for a price $Y(e)$

↓

Step 4: O decides on its prevention effort x

↓

Step 5: in case of damage, each agent pays for its share of liability: $D_I(\theta^*)$ and $D_O(\theta^*)$

The optimal value of the sharing rule θ, which is denoted by θ^*, has to be known before any private decision-making. The two private actors, I and O, will make their private decisions (in order to maximize their own profit) in light of the rule θ^*.

Moreover, the transaction between the two agents (step 3, figure 2) also has an influence on their decision-making: the selling price of the new technology alters the incentives of the innovator to design a more or less reliable technology. The selling price is the result of a bargain between I and O. The outcome of the bargain depends on the degree of competition on the R&D market (the market on which the firm I is located). Two extreme cases can be considered:

(i) The R&D market is a perfectly competitive one. In that case, firm I faces a multitude of competitors, who can also supply firm O. Firm I has no freedom in fixing its selling price; it has to be in line with the most efficient competitor (otherwise, it cannot "win the contract"). Suppose that the selling price offered by the most efficient competitor is Y_0.

(ii) The R&D market is a monopolistic one. Firm I is the only one to offer new production technology, and it has the possibility of fixing its own selling price. The only constraint it faces is firm O having an interest in buying its new technology. So, firm I will fix the maximum selling price, to which firm O is indifferent on whether to buy the new technology or not—the selling price is equal to the benefit firm O can enjoy from using the new technology (in terms of decreasing the probability of an accident occurring and paying $D_O(\theta^*)$):

$$Y_I(e) = D_O(\theta^*) \cdot \big(p(x,0) - p(x,e^*)\big)$$

with $p(x,0) - p(x,e^*)$ being the decrease in the probability of an accident when using the new technology (with a degree of technical advancement of e^*) instead of the basic technology (with a degree of technical advancement of 0).

These two examples are the two extreme cases that can be encountered on a market. But between them, there is a continuum of intermediate degrees of competition. As a result, denoting as λ the degree of competition on the R&D market ($\lambda = 0$ means no competition (i.e., a monopoly); $\lambda = 1$ means perfect competition), the price of the new technology is:

$$Y(e) = \lambda Y_0 + (1 - \lambda)Y_I(e)$$

This price, which depends on the degree of competition (which determines the relative bargaining power of each firm), reduces the profit of the operator and increases the profit of the innovator. We remark that in all cases except perfect competition, price $Y(e)$ increases with e: the higher the technical advancement of the new technology, the higher its selling price.[29] So this selling price has an impact on the incentives to provide an R&D effort. Moreover, the lower the degree of competition on the R&D market (lower λ), the higher the sensitivity of $Y(e)$ to the level of e. Thus, through the bargaining which takes place between the two firms to fix $Y(e)$, the degree of competition has an impact on the incentives to innovate.

Now that our framework and the social benchmark is outlined, we will define the optimal sharing rule and discuss the possible policy implications.

IV. RESULTS

In this section, we first present our theoretical results. Then we provide a numerical example in order to illustrate how the sharing rule could apply.

A. Theoretical Predictions

To determine the optimal value of θ, i.e. the value which maximizes social welfare given the fact that each firm pursues its own private interest, we first have to determine the private decisions made by the firms. To do so, we use the methodology of backward induction: we determine the last decision of the schedule (other decisions and parameters being given), and then roll back the schedule to determine the upstream decisions (the downstream decisions being known and given).

As a first step, we have to determine how the operator chooses its level of effort, x, for a given production technology (with a given degree of advancement e). The operator has to find the value of x which maximizes its private profit:

$$Max_x \ \pi_O(x,e) = \ W_O + R_O - Y(e) - c.x - p(x,e).D_O(\theta)$$

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29. $Y_I(e) = D_O(\theta^*).\left(p(x,0) - p(x,e^*)\right)$ increases with the level of e, because $p(x,e)$ is decreasing in e. The higher the degree of technical advancement, the more reliable the new technology (the lower the probability of causing a damage). So, the expected cost of having to pay D_O decreases and this can be valued for the agent O.

with, in our example, $p(x,e) = \frac{\exp(-\alpha x)+\exp(-\beta e)}{\gamma}$ and $D_O(\theta) = W_O - \theta.[W_I + W_O - H]$. The value of x which maximizes the operator's private profit $\pi_O(x,e)$ is:

$$x^* = \frac{1}{\alpha} ln\left(\frac{\alpha D_O(\theta)}{\gamma c}\right)$$

We can easily check that: $x^* < x^{**}$ because $D_O(\theta) < H$. As a consequence, whatever the sharing rule θ, insufficient incentives for care will be provided. This problem is well known in law and economics, especially since the work of Shavell.[30] Because of limited liability, the operator does not take into account the entire damage its activity causes to society. As a consequence, it does not perceive the whole social benefit of care in terms of reducing the level of risk of accident. Nevertheless, because $D_O(\theta)$ decreases in θ—recall that θ is the portion of the "sharing zone" which is attributed to the innovator $((1 - \theta)$ is attributed to the operator)—the lower the value of θ, the higher the value of x^*.

In a second step, x^* being defined and given, we have to analyze how the innovator determines its effort e^* in terms of innovation. Its effort is determined in such a way as to maximize its private profit:

$$Max_e \; \pi_I(x,e) = \; W_I + R_I + Y(e) - k.e - p(x^*,e).D_I(\theta)$$

with, in our example, $p(x,e) = \frac{\exp(-\alpha x)+\exp(-\beta e)}{\gamma}$, $D_I(\theta) = W_I - (1-\theta).[W_I + W_O - H]$ and $Y(e) = \lambda Y_0 + (1-\lambda)\big[D_O(\theta).\big(p(x^*,0) - p(x^*,e)\big)\big]$.

The value of e which maximizes the operator's private profit $\pi_I(x,e)$ is:

$$e^* = \frac{1}{\beta} ln\left(\frac{\beta(H - \lambda D_O(\theta))}{\gamma k}\right)$$

30. *See* Shavell, *supra* note 22.

Two remarks can be made. First, for a given degree of competition λ different from 0 (i.e., except the case of a monopoly), it is easy to check that we have $e^* < e^{**}$ (because of $(H - \lambda D_O(\theta) < H)$. We also observe that the higher θ, the higher the effort provided in innovation. The intuition is very simple: a higher θ means a higher degree of liability in case of accident, so the incentives to provide efforts aiming to reduce the likelihood of damage occurring are strengthened. We can also remark that in a case of a monopoly ($\lambda = 0$), the private innovation effort equals the socially optimal one, whatever the sharing rule θ: $e^* = e^{**}$. The intuition is as follows: in a case of monopoly, the innovator can fix the maximum selling price, $Y_I(e) = D_O(\theta) \cdot (p(x^*, 0) - p(x^*, e))$, which is equal to the (whole) benefit the innovation provides to the operator in terms of improving the efficiency of prevention measures, x, to reduce the likelihood of damage occurring.

All these results can be summarized in the following proposition:

Proposition 1: The Incentives Proposition

Consider two firms, both having an impact on the probability of a common damage occurring.

    I. *Sharing liability between the two firms does not lead to an optimal level of effort from the operator (the agent downstream)*;

    II. *When there is a strictly positive degree of competition on the innovation market (where the upstream agent works), sharing liability does not lead to an optimal level of innovation*;

    III. *The higher the degree of liability for an agent, the higher the effort it provides*; and

    IV. *In the case of a monopoly on the innovation market, an optimal level of innovation is provided, whatever the apportionment of liability.*

We have determined how the private efforts are chosen. Now, with these elements in mind, we are able to determine the optimal value of θ. As mentioned above, we focus our analysis on the issue of providing optimal incentives to "properly" control the level of risk. So we need to find the value of θ that maximizes social welfare, taking into account the way in which the private agents choose their level of effort. So the optimal value of θ responds to the following objective:

$$Max_\theta \, SW(x, e, \theta) = W_I + W_O + R_I + R_O - cx^* - ke^* - p(x^*, e^*)H$$

With:

$$p(x,e) = \frac{\exp(-\alpha x) + \exp(-\beta e)}{\gamma}$$

$$x^* = \frac{1}{\alpha} \ln\left(\frac{\alpha D_O(\theta)}{\gamma c}\right)$$

$$e^* = \frac{1}{\beta} \ln\left(\frac{\beta(H - \lambda D_O(\theta))}{\gamma k}\right)$$

$$D_O(\theta) = W_O - \theta.[W_I + W_o - H]$$

We find the following results below.

Proposition 2: The Optimal Apportionment Proposition

The optimal apportionment of liability between the two firms depends on the degree of competition on the R&D market.

    I.   *If the innovator benefits from a monopoly position ($\lambda = 0$), it has to assume the minimum share of liability (i.e. $\theta^* = 0$, $D_I(\theta) = H - W_O$). A maximum share of liability is assigned to the operator: $D_O(\theta) = W_O$.*

    II. *For all other degrees of competition (i.e. $\lambda \neq 0$), there is an optimal apportionment $\theta \in]0,1[$ if the following conditions are met:*

        a. $\dfrac{\alpha/c}{\beta/k}$ *is higher than* $\dfrac{1}{3}\dfrac{(\lambda-1)^2}{2\lambda+1}$

        b. *The level of individual wealth W_O and W_I are sufficiently high.*[31]

Point (I) of Proposition 2 can easily be deducted from Proposition 1: because the power of the monopoly allows the innovator to perceive the whole social benefit from its innovation (so that it receives optimal incentives to invest in R&D), the only problem is the presence of suboptimal incentives for the operator for risk prevention. As a consequence, maximum

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31. For proof of Proposition 2, see the Appendix attached hereto.

incentives have to be provided to the operator, via the setting of the maxi-
mum degree of liability: $\theta^* = 0$, so as to obtain $D_O(\theta) = W_O$.

Point (ii) of Proposition 2 teaches us that for an interior solution to ex-
ist (i.e. a value of θ which is different from 0 and 1), it is necessary for the
efficiency-cost ratio of prevention (i.e. α/c) to be sufficiently higher than
that of innovation (i.e. β/k). Given that λ takes a value in [0,1], we know
that $\frac{1}{3}\frac{(\lambda-1)^2}{2\lambda+1}$ takes a value in [0, 1/3]. As a consequence, if the efficiency-
cost ratio of prevention is equal to the efficiency-cost ratio of innovation,
this necessary condition is satisfied. Point (ii) also teaches us that an interi-
or solution requires conditions regarding the firms' wealth to be satisfied.
The intuition is as follows: first, it is necessary for the wealth to be suffi-
ciently high to exceed the overall damage. The higher the sum of the indi-
vidual wealth, the higher the "sharing zone" defined in section III.B *su-
pra*.[32] Second, having an optimal value of θ different from 0 and 1 means
that the situation requires "balanced incentives" to be provided; there is no
need to provide maximum incentives to one agent (and minimum incen-
tives to the other one). In other words, the lack of incentives is similar for
the two agents. All else being equal, incentives are provided through the
size of the wealth; the more an agent can lose in case of accident, the high-
er the incentives to avoid the accident. So, all else being equal, the more
similar the agents' wealth, the more balanced (between the two agents) the
incentives to be provided.

Finally, it is worth developing a numerical illustration. It allows us to
test for the application of such a policy (within different situations), and al-
so to see how the performance of the regulation evolves when the degree of
competition on the R&D market varies.

B. Numerical Illustration

For a given set of parameters, we calculate the private efforts in care
and R&D (and the corresponding level of social welfare), depending on the
sharing rule θ. Thereby we deduce the optimal value of θ (which induces
the maximum level of social welfare). Then, we seek how the optimal shar-
ing rule varies when the degree of competition on the R&D market chang-
es.

The basic case we study is the following:

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32. If the sum of the two sets of wealth W_O and W_I is just equal to the amount of damage H, the
sharing zone equals 0: each firm is liquidated to pay for the liability, and the amount to be paid in dam-
ages is not a decision variable.

$$H = 1400 \qquad R_O = 100 \qquad c = 2 \qquad \alpha = 0,5$$

$$W_O = 1300 \quad R_I = 100 \qquad k = 1,98 \quad \beta = 0,55$$

$$W_I = 1000 \qquad\qquad\qquad\quad \lambda = 1 \qquad \gamma = 2$$

For that specific case, the optimal apportionment of liability is: $\theta^* = 0,653$. This means that it is optimal to allocate 65.3% of the sharing zone to the innovator, and 34.7% to the operator.

Given these different elements, the minimum amount in damages the operator has to pay is: $H - W_I = 1400 - 1000 = 400$, the innovator has to pay: $H - W_O = 1400 - 1300 = 100$, and the sharing zone is: $W_I + W_O - H = 1000 + 1300 - 1400 = 900$.

It follows that the optimal amount in damages the operator has to pay is: $400 + 0.347*900 = 712.3$, i.e. 50.88% of the total damage H, and the optimal amount in damages the innovator has to pay is: $100 + 0.653*900 = 687.7$, i.e. 49.12% of the total damage H.

We then calculate how the optimal apportionment θ^* reacts when the degree of competition on the R&D market changes.

We find that the lower (respectively higher) the degree of competition on the R&D market, the lower (respectively higher) the degree of liability to assign to the innovator. This result is a consequence of Point (I) of Proposition 2: in case of a monopoly on the R&D market, the innovator has optimal incentives to innovate. Starting from this point, a higher degree of competition on the R&D market prevents the innovator from fixing a high selling price for its technology and, above all, from making the price dependent on the innovation effort. This leads to lower incentives to innovate, which have to be compensated by a higher degree of liability for the innovator. This point is illustrated, in our numerical example, by Figure 3 below.

Figure 3. Optimal sharing rule θ and share of global liability $(\frac{D_O}{H})$ depending on the degree of competition λ
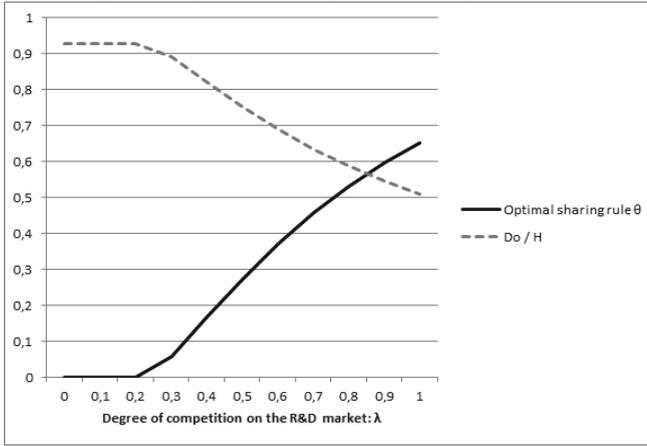
A higher degree of competition on the R&D market (a higher value of λ, on the x-axis) is associated with a higher value of θ^* (the black solid line), meaning that the share of liability assigned on the operator decreases ($\frac{D_O}{H}$, the grey dashed line).

V. POLICY IMPLICATIONS AND CONCLUDING REMARKS

Our analysis aims to propose a rule of apportionment of liability in order to regulate frequent situations in which two actors, with low levels of solvency, have an impact on the likelihood of a common damage occurring. Each actor, via its decision, alters the probability of an accident occurring. Moreover, the two actors are linked by a market relationship, as one actor provides a production technology to the other (and the reliability of this technology has an impact on the ability to "properly" control the level of risk).

The originality of our work is twofold. First, to the best of our knowledge, this is the first study to analyze this kind of situation. Second, our originality also lies in our adoption of a normative perspective. We do not propose a comparison between existing rules of apportionment (such as joint and several liability and non-joint liability), but instead propose a new rule, regardless of the existing ones, which aims to provide optimal incentives for controlling the risk of damage. Nevertheless, in order to ensure a potential application, we take into account important practical "constraints," such as the principle of limitation of liability.

The rule of apportionment we suggest has no equivalent in practice. We highlight the fact that the nature of the market relationship which links the two agents is a key factor in the definition of the optimal sharing rule. We specifically show that in the case of a monopoly on the R&D market (upstream agent / provider), a minimum liability has to be assigned to the innovator since its market power provides sufficient incentives to provide a "high-quality" technology (via the ability to fix a high selling price). When a higher degree of competition holds on this market, the optimal apportionment depends on the ratio of efficiency/cost of prevention and R&D, and on individual relative wealth. Nevertheless, all else being equal, a higher degree of competition on the R&D market calls for a higher degree of liability for the innovator.

This result leads to at least two remarks. We show that on a monopolistic R&D market, efficiency calls for a minimum liability for the innovator. This result could be used in combination with the rationale of the patent system, which gives an innovator a monopolistic position, in such a way as to foster the incentives provided by this system. The importance of the degree of competition in the definition of the optimal apportionment of liability leads to another policy implication. Our result suggests that, all else being equal, a variation in the degree of competition on the R&D market leads to a variation in the optimal apportionment of liability. Thus, having knowledge about the degree of competition on this market is a necessary condition to properly configure the liability system. Our result thus calls for collaboration between the competition authorities, which have sufficient expertise to assess the degree of competition, and the legislatures and courts. Here, the fundamental idea is to build a "liability formula" which gives the amount to be paid for liability by each defendant. This formula depends on different parameters such as the relative efficiency/cost ratios (of care and R&D), the amounts of assets, and the overall damage and the degree of competition on the market. This formula, which should be common knowledge (in order to provide *ex ante* incentives), would be used *ex post* by the courts to establish liability. The competition authority's expertise should be necessary *ex ante* and *ex post*. *Ex ante*, its expertise is needed to determine this formula (i.e., the way in which competition affects the incentives). *Ex post*, in case of damage, its expertise is needed to assess the degree of competition on the relevant market (thus allowing a calculation of the amounts in damages to be claimed). This necessity for collaboration is also extended to regulation agencies, since the optimal apportionment also depends on characteristics of the production technologies.

Our analysis, like any exploratory study, is built on simple and strong assumptions that should be removed in further analyses. Our results have to be interpreted in the light of these assumptions. Assuming strict liability allows us to set aside the issues related to fairness in the apportionment of debt. Such an assumption simplifies the analysis but, from a practical point of view, could be justified by the fact that strict liability is increasingly used in environmental damage cases and for controlling highly hazardous activities—which are the activities we had in mind when developing our analysis. However, other assumptions should be removed and thus call for further studies. For instance, we greatly simplify the analysis when we assume that R&D activities always (and rapidly) succeed in improving the technology. We know that R&D activities are highly uncertain, and the possible benefits can be earned within a medium/long-term perspective. Conversely, prevention activities have an impact in the short term, and their efficiency is less uncertain. Nevertheless, R&D activities have an impact on the technological trajectory, i.e. on the long-run ability of preventive measures to reduce the level of risk. So their overall long-run impact may be higher than the benefit of providing a higher level of prevention, but with a "less advanced" technology. As a consequence, the trade-off between prevention and R&D calls for a broader analysis of certain short-term benefits versus more uncertain (but potentially higher) long-run benefits.

APPENDIX

Proof of Proposition 2

Point (II) can be demonstrated in the following manner:

To lighten the calculations, we directly search for an optimal value of D_O, the amount in damages to be paid by the operator. We just have to keep in mind that D_O only takes values in $[H - W_I, W_O]$. Then, with the value of D_O, it is easy to find the corresponding value of θ: $\theta = \frac{W_O - D_O}{W_I + W_O - H}$

The optimal value of D_O maximizes social welfare, taking into account the private levels of efforts.

$$Max_\theta \, SW(x, e, D_O) = W_I + W_O + R_I + R_O - cx^* - ke^* - p(x^*, e^*)H$$

With:

$$p(x, e) = \frac{\exp(-\alpha x) + \exp(-\beta e)}{\gamma}$$

$$x^* = \frac{1}{\alpha} \ln\left(\frac{\alpha D_O}{\gamma c}\right)$$

$$e^* = \frac{1}{\beta} \ln\left(\frac{\beta(H - \lambda D_O)}{\gamma k}\right)$$

The first order condition, $\frac{\delta SW(x, e, D_O)}{\delta D_O} = 0$, is:

$$\frac{-c}{\alpha D_O} + \frac{cH}{\alpha(D_O)^2} - \frac{\lambda k H}{\beta(H - \lambda D_O)^2} + \frac{\lambda k}{\beta(H - \lambda D_O)} = 0$$

$$\Rightarrow \frac{-(D_O)^3 \lambda^2(c\beta + k\alpha) + (D_O)^2 c\beta H(2\lambda + \lambda^2) - D_O c\beta H^2(2\lambda + 1) + c\beta H^3}{\alpha\beta(D_O)^2(H - \lambda D_O)^2} = 0$$

So, the optimal value of D_O satisfies:

$$(D_O)^3 \lambda^2(c\beta + k\alpha) - (D_O)^2 c\beta H(2\lambda + \lambda^2) + D_O c\beta H^2(2\lambda + 1) - c\beta H^3 = 0$$

Dividing by $c\beta$, the condition can be rewritten:

$$F(D_O) = (D_O)^3 \lambda^2 (1 + \tau) - (D_O)^2 H(2\lambda + \lambda^2) + D_O H^2 (2\lambda + 1) - H^3 = 0, \text{with } \tau = \frac{\alpha/c}{\beta/k}$$

This first order condition is a third-degree equation in D_O, which we denote $F(D_O)$.

Differentiating $F(D_O)$ leads to a second-degree polynomial:

$$F'(D_O) = 3(D_O)^2 \lambda^2 (1 + \tau) - 2D_O H(2\lambda + \lambda^2) + H^2 (2\lambda + 1)$$

whose discriminant is: $\Delta = 4H^2 [(2\lambda + \lambda^2)^2 - 3\lambda^2 (\tau + 1)(2\lambda + 1)]$. The discriminant Δ is of the sign of $(2\lambda + \lambda^2)^2 - 3\lambda^2 (\tau + 1)(2\lambda + 1)$.

We can deduce that for $\tau > \frac{1}{3} \frac{(\lambda-1)^2}{2\lambda+1}$, $F(D_O)$ is always non negative, and so $F(D_O)$ is an increasing function with a unique root. So before D_O the function SW increases, and decreases after D_O: D_O is the unique maximize of SW.

Now we have to take into account the constraints on the value of D_O. We know that D_O takes only values in $[H - W_I, W_O]$. As a consequence:

If the optimal value of D_O is lower than $H - W_I$, this means that a minimum liability has to be assigned to the operator and we obtain: $D_O = H - W_I$. This situation is not optimal, but because of the limited liability principle, we can only obtain this "second best" choice.

If the optimal value of D_O is higher than W_O, this means that a maximum liability has to be assigned to the operator and we obtain: $D_O = W_O$. This situation is not optimal, but because of the limited liability principle, we can only obtain this "second best" choice.

So, we look at the conditions which ensure an interior solution, i.e. an optimal value of D_O which lies in $[H - W_I, W_O]$. For this, let us define:

$t_O = \frac{W_o}{H}$, i.e. the maximum payment in damages the operator can bear (its wealth, W_O), expressed as a percentage of the overall damage H. This allows us to rewrite:

$$\max D_O = W_O = t_O H$$

$t_I = \frac{H-W_I}{H}$, i.e. the remaining amount of damage (which remains to be remedied) when a maximum liability is assigned to the innovator ($H - W_I$), expressed as a percentage of the overall damage H. This allows us to rewrite:

$$\min D_O = H - W_I = t_I H$$

To obtain $D_O \in]H - W_I, W_O[$ requires the following condition to be satisfied:

$$F(t_I H) \le 0 \le F(t_O H)$$

which is equivalent to:

$$\frac{(t_O)^3(-\lambda^2) + (t_O)^2(\lambda^2 + 2\lambda) + t_O(-2\lambda - 1) + 1}{(t_O)^3 \lambda^2} \le \tau$$
$$\le \frac{(t_I)^3(-\lambda^2) + (t_I)^2(\lambda^2 + 2\lambda) + t_I(-2\lambda - 1) + 1}{(t_I)^3 \lambda^2}$$

We observe that:

$\frac{(t_O)^3(-\lambda^2)+(t_O)^2(\lambda^2+2\lambda)+t_O(-2\lambda-1)+1}{(t_O)^3 \lambda^2}$ is decreasing in W_O, and $\frac{(t_I)^3(-\lambda^2)+(t_I)^2(\lambda^2+2\lambda)+t_I(-2\lambda-1)+1}{(t_I)^3 \lambda^2}$ is increasing in W_I. So the condition is more easily satisfied when both W_O and W_I are sufficiently high.