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THE USE OF MODELS IN LITIGATION: CONCISE OR CONTRIVED?

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A universal difficulty faced by attorneys during the litigation of a case is the effective presentation of the facts which gave rise to the lawsuit. Evidence is sought to be presented in a manner which permits the trier of fact to reconstruct and evaluate the relevant events. Because the multiple factors that influence situations which provoke litigation are often complex, it is difficult to effectively portray them through verbal or pictorial representation. Such portrayals, although superficially adequate, are frequently incomplete and inadequate because they fail to demonstrate in a discernible fashion the interrelationships among the innumerable elements involved in the case. As a result, jurors and judges who are not familiar with the subject of the litigation must make judgments based on conflicting presentations of individual factors of a given case rather than upon a cohesive representation of the entire situation.

The use of modeling is one method by which presentation of complex and multifaceted elements of a given factual situation is simplified. Modeling is a product of the scientific environment and is the application of scientific attitudes and associated techniques to the study of operations.[1] It is an attempt to present the best representation of a total situation, rather than its individual segments, thereby considering as many factors as possible, whether complementary or conflicting. In addition, those factors are evaluated against the environment within which the process takes place.

For the lawyer, the use of the model in litigation enhances his ability to describe and comprehend facts more effectively than through the use of verbal description alone. By so doing, he often can uncover relationships not apparent in the verbal description. The model also provides an overview of a factually complex case, allowing the attorney to consider all relevant evidence simultaneously in his preparation.

The combination of the quantifying techniques involved in modeling and the capacity of the computer provides a powerful tool for improving our system of justice by producing more meaningful evidence. However, it also presents the real danger of introducing erroneous and misleading evidence.

The purpose of this article is to provide an understanding of those factors which influence a model's meaningfulness in order to allow the courts

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1. A. SCHUCHMAN, SCIENTIFIC DECISION MAKING IN BUSINESS 24 (1963) [hereinafter cited as SCHUCHMAN].

and practitioners to both effectively use modeling techniques as evidence where appropriate and to prevent the admission of evidence employing modeling techniques when they have been applied inappropriately. This article first establishes a basic understanding of models by explaining the role of the model and the methods and techniques involved in developing models. Based on this understanding, the article then suggests the practical considerations of using modeling techniques as evidence.

A Basic Understanding of Models

The Role of the Model

Modeling applies the method of research used in the natural sciences to the study of business problems. An important ingredient in this method is the formulation of an hypothesis or theory regarding the nature of the mechanism underlying a phenomenon. This theory is then tested against observed facts and modified in the light of test results. The modified theory is then tested and modified and the process is continued until the scientist is satisfied that his theory accounts for the observed facts with sufficient accuracy for his purposes.[2]

Since scientists and businessmen can rarely study or manipulate the phenomenon directly, it is more common to test the theory by constructing a replica of the cause and effect relationships which can be analyzed.[3] These replicas, known as models, embody in physical, graphical or mathematical form the scientist's theory of the origin or nature of the phenomenon.[4] The type of model most frequently used is the mathemetical model.

Mathematical models can be divided into two categories. The deterministic model assumes all relevant information concerning the problem is completely and surely known and the analysis is to search among all feasible alternatives and find the actual alternative which will provide the optimum solution.[5] The probabilistic or statistical model assumes information concerning the problem is not completely known, but can be specified by probabilities and the analysis is to search for the strategy which will optimize the expected value of the outcomes.[6] In either case, complicated mathematical techniques are often employed and are usually most effectively implemented on the computer. Both categories of models can be appropriately applied to evaluating specific types of situations in litigation.

2. C.W. Churchman, R. Ackoff, E.L. Arnoff, Introduction to Operations Research 61 (1957) [hereinafter cited as Churchman].
3. *Id.*
4. *Id.*
5. K. Chu, Quantitative Methods for Business and Economic Analysis xi (1969) [hereinafter cited as Chu].
6. *Id.*

Models may be thought of as serving, though imperfectly, four functions. One is *organizing* by ordering and relating disjointed data and showing similarities and connections between them which had previously remained unperceived. A second function is *heuristic*, providing devices which lead to the discovery of new facts and new methods through the transference of familiar patterns to an unfamiliar environment. A third function is *predictive*, the ability to predict results ranging from simple yes-or-no predictions to completely quantitative predictions. A fourth, and final, function is that of *measurement*, the ability to measure operations to which the model is connected by processes clearly understood.[7] All four functions of models have the potential for application in the courtroom with the measurement function having a particularly significant potential for measuring results which have already occurred.

The Development of the Model

The development of the model involves a methodical scientific set of steps.[8] The process begins with a formulation of the problem and the construction of a mathematical model which reflects it. An optimum solution is then sought to be derived from the model, followed by testing both it and the solution to establish control over the solution. This is then followed by the model's implementation.

To identify and formulate the problem, its frame of reference must first be determined. The model used as evidence should help identify the source or location of the problem. It should determine whether the situation was the result of an operational problem within an existing process or the result of an invalid process.[9] Therefore, when using models in litigation, the process represented by the model will be valid. The results of a valid process can then be measured against the results represented by the situation being litigated.

Second, the operational characteristics of the situation should be defined. It must be determined what the purpose of the activity was, the elements which were necessary in contributing to the results, the degree of accuracy required in the results and the extent to which the situation can be measured. Through this information, it is possible to determine the kind of data necessary and the extent to which mathematical tools can be employed.[10]

Finally, while defining the problem, there must be a continual reassessment of the definition as additional knowledge is gained. As newly acquired

7. SCHUCHMAN, *supra* note 1, at 79-80.
8. See CHURCHMAN, *supra* note 2, at 13 for the major phases.
9. SCHUCHMAN, *supra* note 1, at 43-44.
10. *Id.* at 44.

information is fed back, the definition of the problem will possibly be modified.[11]

The construction of the mathematical model then follows. First, units of measurement must be determined. Since the model is quantitative and the purpose of the model has been determined, the appropriate unit of measurement must be selected. This unit will reflect the closest measurement of the degree of accomplishment or movement toward the accomplishment of the ultimate purpose.[12]

Second, the model must be built. In other words, the situation must be defined in a form which allows logical and cogent development of the range of practicable alternative solutions.[13] At this point, the decision must be made as to what factors are to be selected from real life and incorporated into the model. This is a human decision and has ultimate impact on the validity of the model. Thirdly, the model must be converted to symbols and the appropriate mathematical techniques applied. The result is a symbolically represented problem which will evaluate quantitatively represented data and yield quantitative results.[14]

Finally, for most situations, it is necessary to convert the mathematical model to a form which can be processed by the computer. This is not always a requirement of model development, but is often essential because it is the only viable method for manipulating large volumes of data and complex algorithms in a limited time frame.

Following the construction of the mathematical model, the optimum solution for the model will be determined. It may be derived from the model in two ways: analytical or numerical. During an analytical process, mathematical deduction is applied through the application of various types of mathematical tools, and solutions are obtained in abstract. Numbers are substituted for the symbols after the solution is obtained.[15] The numerical process involves trying various values in the variables of the model and comparing the results. This process is called iterative because of the successive attempts to approach an optimum solution. For a numerical process, there must be a set of rules which identifies the optimum solution once it is obtained. When models cannot be evaluated numerically because of mathematical or practical considerations, sampling techniques are used to obtain approximate evaluations.[16]

Since the model will never be more than a partial representation of reality,[17] its adequacy must be evaluated. Since the model is being used to

11. *Id.*
12. *Id.* at 45.
13. *Id.*
14. *Id.* at 47.
15. CHURCHMAN, *supra* note 2, at 14.
16. *Id.*
17. *Id.*

evaluate real world predictions, it must be tested through judicious collection of data and the measurement of the model's effectiveness in the manipulation of that data.[18] Specifically, by analyzing the model in light of data from known situations, the assumptions of the model can be evaluated by analyzing certain characteristics of the model. First, it must be determined whether the model was able to describe correctly, and more clearly, known facts and situations. Second, it must be determined if the model was able to describe causes of known effects on the basis of the relationships among factors represented by it. Third, the model must be able to substantiate general relationships described with specific events. Fourth, the testing process should identify the limitations of the model by varying the values of its principal factors to test the consistency of the answers. Finally, the validity of the model itself must be evaluated by varying the principal factors to test the plausibility of the answers.[19]

Since the collection of complete and accurate data is often impractical, if not impossible, the model may be tested by a second statistical model. This model would deal with the measurement data and would determine the degree of inaccuracy which would be considered normal for a specific type of data. If the mathematical model then yields results which exceed the limits established by the statistical model, the data and/or model must be re-evaluated.[20] These processes of testing the model with historical data (either complete or sampled) must be used for all models, including those used to measure a specific result.

When the model is used on an on-going basis, it is important to regularly assess it to ensure that it continues to accurately reflect the real world it represents. If one or more factors or relationships represented in the model change significantly, the solution itself loses its validity. It is, therefore, necessary to develop tools for determining when changes occur.[21]

The models discussed in this article are not intended to be used on an on-going basis. This aspect does, however, become important if the use of a modeling technique is submitted as evidence to substantiate the basis for an original decision or series of decisions.

Ordinarily, once the solution to a model is determined, it must be translated into a set of operating procedures capable of being understood and applied.[22] It must be capable of being implemented in a manner which represents the intent of the solution and, on some occasions, it may be necessary to re-evaluate the model and its solution if implementation as

18. SCHUCHMAN, *supra* note 1, at 71.
19. *Id.* at 47.
20. *Id.* at 75.
21. CHURCHMAN, *supra* note 2, at 14-15.
22. *Id.* at 15.

originally intended is not feasible. Although a model designed for the purpose of measuring a decision after the fact is not likely to be implemented, it must be evaluated for the viability of its implementation. If the model yields solutions which cannot be implemented, there must be serious questions raised as to the practicality of the information learned from that particular model.

Techniques Used in Modeling

Model development applies a number of mathematical and scientific techniques, among them, linear programming, game theory, classical statistics and probability theory. This section will briefly describe them and, wherever possible, provide indications of applications which might become involved in litigation.

Linear programming is a mathematical optimizing technique used when available quantities of resources or factors of production are limited and when there are only a finite number of production processes to choose from. The objectives and constraints of the problem must be expressed by linear functions.[23] The objectives of the organization are represented by a linear function and the constraints—such as capital, labor and other resources—within which the organization operates are represented by several linear inequalities. The solution of a linear programming problem may thus be considered as the optimum use of the available scarce resources to achieve the objectives.[24]

Examples of applications of linear programming are the assignment of personnel or equipment to various tasks to complete a job at least cost, the determination of the route of a traveling salesman to allow him to cover several cities at least cost, and the assignment of transportation equipment to carry cargo from several initial points to destinations at minimum cost.[25] In all cases, the use of linear programming is directly applicable to decisions concerning the use of resources. This, of course, has direct implications concerning the profitability of an organization.

Game theory is a mathematical theory which has been developed to describe certain cases of conflicting interest. It is often used to represent the competitive process and provide the method for competitors to choose the optimum strategy in order not to lose. The *game* establishes the rules for play. It serves as the model. A *play* is a single course of action chosen by a player from the list of courses available to him. The *payoff* is the agreement about payment among players at the end of the game. The player's objective is assumed to be maximizing gain or minimizing loss.[26]

23. CHU, *supra* note 5, at 9.
24. *Id.* at 62.
25. *Id.* at 27-29.
26. *Id.* at 99-100.

Since game theory is used to determine the outcome of a combination of "plays" chosen by competitors according to their selected strategies, it would be suitable to simulate the results of various alternative decisions available to a businessman in response to the different moves of competitors. It could, therefore, be used to determine whether the businessman selected the best approach to a problem (yielding the maximum payoff) from among the available alternatives.

Classical statistics provides the methods for arranging or classifying information in order to describe the universe from which the data were collected.[27] It provides standard definitions for compiling and measuring data that are applied uniformly. As a result, through the use of classical statistics certain qualities of the universe can be communicated. Classical or "descriptive" statistics provide the characteristics of a population through measures of location (e.g., mean, median, mode), measures of variation (e.g., range, mean deviation, variance and standard deviation) and frequency distribution (the frequency with which individual elements fall into a specific location.)[28]

In addition, when it is not possible to observe an entire population, classical statistics through random sampling leads to the theories of estimation, testing hypotheses and analysis of variance. Because classical statistics is based on samples and cannot ensure complete confidence, answers are given with specified error probabilities or confidence levels.[29]

Since classical statistics is specifically a measurement technique, and provides rules for defining error probabilities, it can serve as a valuable tool in evaluating the likelihood, and predictability, of specific events which impacted the results of a decision, particularly if those events were beyond the control of the decision maker. In other words, if a businessman made a decision based on the assumption that a specific event would occur, classical statistics can be used to evaluate historical data to determine whether it was appropriate (the error probabilities were reasonable) to rely upon the occurence of that particular event.

Classical statistics can also be used to measure the performance of a particular enterprise relative to other similar entities or a total environment. Specific applications include evaluating the organization for potential violations such as employment discrimination and anti-trust practices including price-fixing and conspiring to refuse to deal.

When making a decision, it is often impossible to predict the outcome. When choosing among alternatives, the businessman must often estimate consequences based on partial information. Statistical decision theory provides a framework for choosing decision strategy by assigning probabilities to

27. *Id.* at 119-20.
28. *Id.* at 187.
29. *Id.*

the possible outcomes of each course of action.[30] The probabilities may be based on historical data or, if there is no data, assigned equally to all possible outcomes or assigned subjectively to reflect expectations about the future. Bayes' theorem further states that, if it is possible to obtain sample observations, the probabilities of specific outcomes can be further refined by combining the results obtained from assigning the original (a priori) probabilities to the outcomes with the results of the sampling process according to a specific mathematical formula.[31] This process may be repeated several times as new samples are obtained, with each additional sample observation further refining the probability of an outcome. The formal application of probability theory to decision making, given the same set of initial facts and data from subsequent events, could potentially provide a meaningful basis for reconstructing a business situation and evaluating the reasonableness of results derived from a less formal process.

Probability theory, particularly Bayesian theory, may also be used to reconstruct a prior situation for which no detailed history is available. An example of such a situation is the determination of the value of an inventory base when converting to LIFO accounting techniques. If no detailed usage records are available, operating management's perspective of use can be expressed in terms of probabilities. The probabilities can continue to be refined as additional input is gathered from different members of operating management. The resulting usage records can then be applied against the detail provided by purchase invoices and the LIFO inventory base determined. Although this technique is still an estimate, it has scientifically applied all data that are available at the time.

An extension of probability theory is a technique known as the Markov Chain Process, which is the assignment of probabilities to a series of events, the outcome of each event depending on the outcome of the preceding one. This dependence upon the outcome of the prior event is the same for all stages of events.[32] An example of a use for the Markov Chain is an investment decision model based on the probabilities of certain changes in interest rates based on the current interest rate.

There are many other techniques used in developing models. One is the queuing model which establishes a balance between the required work load or service and the available personnel and facilities[33] in an attempt to minimize waiting time. Another is breakeven analysis which determines the breakeven point, the volume or level of operation at which total revenue and total costs are exactly equal.[34] This serves as the basis for determining a profitable level of production.

30. *Id.* at 202.
31. R. Levin & C.A. Kirkpatrick, Quantitative Approaches to Management 75 (1965) [hereinafter cited as Levin & Kirkpatrick].
32. Chu, *supra* note 5, at 207.
33. *Id.* at 225.
34. Levin & Kirkpatrick, *supra* note 31, at 18.

Finally, there are other special purpose models including inventory models for the optimization of operational policy to reduce total inventory costs, including carrying costs, setup or ordering costs, and shortage costs;[35] critical path for controlling the progress of a project by describing the project as a network of events and activities;[36] and non-linear programming for applying mathematical techniques when linear programming cannot be applied.

The Practical Considerations of Using Models as Evidence

There are a number of practical considerations which must be addressed both when using the results of a model as evidence or when confronting evidence produced by a model. When models are properly applied, their use has some distinct advantages. It must be recognized, however, that when models are submitted as evidence, there are difficult questions which must be addressed.

The General Advantages of Models

Models can provide advantages to the litigator whether introduced by him or his opponent. The advantages of the model include its ability to describe and comprehend the facts of the situation better than any verbal description can hope to do. It can uncover relationships between the various aspects of the problem which are not apparent in the verbal description. It also can indicate what data should be collected to deal with the problem quantitatively, establish measures of effectiveness and explain situations that have been left unexplained in the past by giving cause and effect relationships. A mathematical model makes it possible to deal with the problem in its entirety and allow a consideration of all the major variables of the problem simultaneously. It provides for the capability of being enlarged step by step to a more comprehensive model to include factors that are neglected in verbal descriptions. It also uses mathematical techniques that might otherwise appear to have no applicability to the problem. In addition, a mathematical model frequently leads to a solution that can be adequately described and justified on the basis of verbal descriptions. Finally, it is often the case that the factors entering into the problem are so many that only elaborate data processing procedures can yield significant answers. In such a case, a mathematical model forms an immediate bridge to the use of large-scale electronic data processors.[37]

Questions to Ask Concerning the Acceptability of the Model

There are a number of important questions which must be answered concerning the acceptability of a model submitted as evidence. The issues

35. Chu, *supra* note 5, at 246.
36. *Id.* at 317-19.
37. Schuchman, *supra* note 1, at 65, 94-95.

to be addressed are associated both with the model development process and the techniques applied.

When a model and its results have been submitted as evidence in a case, the first determination which must be made is whether the model is relevant to the problem. There must first be adequate analysis to ensure that the model is relevant as formualted and deals with the appropriate issues for measuring the reasonableness of a particular situation. Second, since the model should represent a valid process in order to determine the source of failure (an invalid process or an operational failure within a valid process), its validity must be attested to. If there is more than one potentially valid process, the alternative processes must also be presented and evaluated to determine the relative appropriateness of each. If the intent is to isolate an operational failure within the process, all acceptable alternative processes must be evaluated to ensure that it was, in fact, an operational failure within any of the processes.

Third, it must be determined that the application of mathematical tools is appropriate. If mathematical tools are applied inappropriately, the results are meaningless regardless of how quantified they are. Finally, consideration must be given to the data available at the time of the original decision as opposed to the data available when the measuring model is being developed. Since hindsight inevitably provides a more astute perception than when looking forward, it is essential that the information reflected by the model be no more accurate than was possible at the time of the decision.

When evaluating the construction of a particular model, it is important to determine that the units of measurement selected are most representative of the situation. For example, in an antitrust case, if market share is an issue, it must be determined whether market share would be measured in units or dollars since each has its own implications, particularly, when dealing with a generic class of products, with each product at a different unit price.

Another important issue is the factors selected to represent real life within the model. Since few models can totally replicate a real life situation, the factors which are reflected within the model must be relevant and inclusive of all important aspects of the process. The inappropriate representation of less relevant factors and omission of significant factors can invalidate the model's ability to simulate real life.

The mathematical techniques selected for constructing the model must be appropriate, otherwise the model will not have actually performed the function it was intended to perform. Since the determination of the appropriateness of specific techniques is a mathematical issue, assessment will have to be made by an individual trained in mathematical techniques.

Finally, the construction of the model includes the translation of the general problem to mathematical symbols. And, if the model is processed on a computer, a further translation to computer language must take place. Both of these translation processes must be valid and accurate. Once again, this must be determined by individuals familiar with mathematical techniques and, if necessary, computer processing.

One or more solutions will be derived from the model. If the model is an analytical model, the proper numbers must be substituted in the optimum solution to obtain a meaningful result. Regardless of the validity of the solution, the substitution of improper numbers would yield invalid results. If the model represents a numerical process, the rules for identifying the optimum solution must be valid also. These must therefore be confirmed.

Specific techniques have inherent problems which must be addressed. Notably, if statistical sampling is performed, the error probabilities or confidence levels of the solution must be acceptable. If probability theory is applied, the assignment of probabilities to represent subjective judgments must be careful to eliminate knowledge which was gained subsequent to the time of original decision so as to avoid making invalid comparisons. Finally, in applying the computer to the solution of the problem, all of the concerns about the integrity of data and computer systems mentioned in *Data Processing Evidence—Is It Different?*[38] must be addressed.

Since all models must be tested to confirm their validity, proof of the test procedure should be provided. In addition, the testing techniques should be evaluated. A specific example of the concern over validity of testing techniques arises when sampling of data is used to validate the model. In such instances, the sampling process itself must be valid.

The solution derived from the model should be evaluated to determine the viability of its implementation at the time the original decision was made. If the model solution is not practical enough to have been implemented, there are grounds for questioning the applicability of the model. As apparently strong as the concepts may be, they are of little use if they cannot be applied to the problem.

CONCLUSION

As can be seen from the above, models tend to provide distinct absolutes concerning elements of a problem and the relationships between those elements. They, therefore, provide a clear, concise method of presenting an argument and, conversely, an equally clear target for refuting the argument. Therefore, modeling techniques have the potential to play a substantial role in litigation. On the other hand, all models must be carefully analyzed by those bearing the proper skills before being accepted as evidence.

38. DeHetre, *Data Processing Evidence—Is It Different?*, *supra* at 567.