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Quality Evaluation Methods - A Review

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Abstract: Quality is understood as a set of features that determine how a product fits to satisfy certain needs. Therefore, a set of measurements $X_1, X_2, \dots, X_n$ characterizes each product, and also serves as information in evaluation of product's quality. Individual measurements X_i are quantitative (numerical) or qualitative (non-numerical) variables expressing the value of the ith index of quality. Evaluation of the overall level of quality, comparison of similar products, measuring and reporting quality improvement and other related problems need construction of some integral quality index Q in which all individual measurements are taken into account. Several proposals for integral quality indices, such as differential approach, weighted means, mixed method, generalized index, algorithmic methods as KORTER, and PATTERN, and a multiplicative integral index are considered. A brief discussion of the properties of each approach is provided. Advantages and disadvantages are pointed out. Examples of how to find the product that exhibits the best fit to its multivariate quality index, and how to rate products according to their quality in a set of similar products are briefly shown.

1 Introduction

Product is any object, subject, concept or process aimed or used to satisfy some needs. Determination of the *Quality of Product* (QP) is a discussable problem. There are more than 40 different definitions of Quality of Product. British standard – BS4779 e.g. says: *QP is the collection of all feasible properties that determine the ability of a product to satisfy certain needs.*

For describing a product and to specify the QP, a set of characteristics is used (see e.g. Table 1). Characteristics of QP can be quantitative (numerical), and/or qualitative (non-numerical) variables. The i-th characteristic (or parameter, or index of the QP) is given by the pair (X^i, ε_i), which expresses the degree of possession of this i-th property and its actual direction, where

$$\varepsilon_i = \begin{cases} +1, & \text{if an increase in } X^i \text{ means better Quality;} \\ -1, & \text{if a decrease in } X^i \text{ means better Quality.} \end{cases}$$

Group of Indices of Q	Characterizing Properties
designation	basic functions the product is designed to fulfill
reliability	duration of use, repairability
ergonomic	man-machine relationship – convenience, useful
aesthetic	rationality of forms, composition, perfection
technology	manufacturing
portability	size and weight
standardization	level of unification and conformability
ecological	level of damaging effects
safety	safety of staff
economic	manufacturing, exploitation, and maintenance expenses
patent-rights	patent protection

Table 1. Indices involved in quality evaluation

Any product (P) with n characteristics involved in its description, is specified by the two vectors:

$$\vec{X} = (X^1, X^2, \ldots, X^n),$$

and

$$\vec{\varepsilon} = (\varepsilon^1, \varepsilon^2, \ldots, \varepsilon^n).$$

where the latter is constant for any item of product **P**.

Given m items of product **P**, the item j is presented by $\vec{\varepsilon}$, and the vector

$$\vec{X}_j = (X_j^1, X_j^2, \ldots, X_j^n),\ j = 1, 2, \ldots, m.$$

Here we discuss approaches to the following problems:

1. Given $\vec{\varepsilon}$ and $\vec{X}_j = (X_j^1, X_j^2, \ldots, X_j^n)$, $j = 1, \ldots, m$ for product P. Evaluate the overall QP for a particular item on the background of the items from that set.

2. Compare two or more items. Select the "best item".
3. Evaluate the possible improvement of the QP, if new characteristics are added, or some characteristics are disregarded.

A satisfactory solution can only be obtained by a real valued function (we name it *Overall Quality Level, OQL*) that incorporates the levels of all individual characteristics (X_j^1, X_j^2, ..., X_j^n), i.e.

$$OQL = f(X_j^1, X_j^2, \ldots, X_j^n).$$

In this paper we review some models for OQL, propose a new model, and discuss its properties.

1.1 Classical Approach

The main idea of the classical approach is to use a "basic analog"

$$\vec{X}_{ba} = (X_{ba}^1, X_{ba}^2, \ldots, X_{ba}^n),$$

where X_{ba}^i is the desired numerical value of the degree of possession of the ith characteristic. This almost drops the necessity of any statistical modeling. The quantity

$$q_i = \left(\frac{X^i}{X_{ba}^i}\right)^{\varepsilon_i}, \; I = 1, 2, \ldots, n$$

named *homonymous indicator*, shows the *relative index of quality* of the i-th characteristic.

Note: Some statistical approaches use as a basic analog the sample mean of a set of items under consideration.

1.2 Methods

1.2.1 Differential Method

Based on the basic analog the overall quality level OQL is defined by

$$OQL(\vec{X}, \vec{\varepsilon}) = \min(q_1, q_2, \ldots, q_n).$$

An item j is considered acceptable only if it satisfies

$$OQL(\vec{X}_j, \vec{\varepsilon}) \geq 1.$$

1.2.2 Method of The Weighted Average

Each characteristic X_i is weighted according to its importance with respect to the quality Q. Usually the weights w_i are determined by experts' opinion and satisfy

$$\vec{w} = (w_1, w_2, \ldots, w_n), \; w_i \geq 0.$$

Then

$$OQL(\vec{X}, \vec{\varepsilon}, \vec{w}) = \frac{w_1 q_1 + w_2 q_2 + \ldots + w_n q_n}{\sum_{i=1}^n w_i}.$$

1.2.3 Mixed Method

This method divides the index set of the quality characteristics $\{1, 2, \ldots, n\}$ into r disjoint subsets (or groups) $I_1, I_2, \ldots, I_r$, and introduces weights for each subset

$$\vec{W} = (W_1, W_2, \ldots, W_r).$$

The $OQL_k(\vec{X}, \vec{\varepsilon})$ for a characteristic of the group I_k is defined by

$$Q_k = \min_{i \in I_k} q_i,$$

and the OQL is given by

$$OQL(\vec{X}, \vec{\varepsilon}, \vec{W}\,) = W_1Q_1 + W_2Q_2 + \ldots + W_rQ_r.$$

2 Hierarchy of Overall Quality Indices

Assume that the n characteristics are divided into r groups with W_k - the weight of the k-th group, where $\sum_{k=1}^{r} W_k = 1$ Assume further that the weights w_i of any characteristic i within group k (again $\sum_{i \in G_k} w_i = 1.0$) is known, too.

Then the OQL for the group k is given by $Q_k = \sum_{i \in G_k} w_i q_i$, where q_i is the relative quality of the ith characteristic of the item.

Finally, the quantity

$$OQL = \sum_{k=1}^{r} W_k Q_k$$

is the hierarchical (aggregated) OQL of the particular item.

Example: Table 2 shows the groups of activities and its parameters used in a promotion procedure.

Group	Weight	Parameter	Weight
Teaching	50%	T. Load	40%
		T. Evaluation	60%
Research	20%	# publications	70%
		# citations	30%
Service	30%	# committees	35%
		Activity	65%

Table 2. Evaluation of overall achievements in promotional procedure

Let $x_{10}, x_{20}, \ldots, x_{60}$ be the required (control) numbers for the parameters given in Table 2. Assume that Professor X shows the value $\vec{X} = (X_1, X_2, \ldots, X_6)$ accumulated during the promotion period. His/her relative indices of Q would be then

$$q_i = X_i / x_{i0}.$$

For example, let

$$q_1 = 1.2;\ q_2 = 1.1;\ q_3 = 1;\ q_4 = 0;\ q_5 = .9;\ q_6 = .9.$$

Then the quality of the groups of characteristics is given by

$$\begin{array}{rcl} \text{Teaching:} & Q_1(\vec{X}) & = .4(1.2) + .6(1.1) = 1.14; \\ \text{Research:} & Q_2(\vec{X}) & = .7(1) + .3(0) = .7; \\ \text{Service:} & Q(\vec{X}) & = .35(.9) + .65(.9) = .9; \\ \text{Overall:} & OQL(X) & = .5(1.14) + .2(.7) + .3(.9) = .98. \end{array}$$

Despite the fact that there is a zero activity parameter, there will be relatively high overall evaluation, due to the additive form of this OQL.

Briefly said, the classic methods may have the following *Advantages:*

- easy to calculate;
- easy to use and explain;
- certainty, explicitness in interpretation and conclusions.

However, we point out also the following *Disadvantages:*

- low sensitivity X^i and w_i;
- give high OQL values even when some low individual characteristics are present;
- subjective views are incorporated in subjective decisions;
- the random nature of individual characteristics is somewhat ignored.

3 Contemporary Methods

Quality is related to a multidimensional problem as one has to operate with:

- large number of individual characteristics;
- random, uncertain behavior of characteristics and measures;

- evaluation of the OQL is restricted on a finite set (m) of similar items;
- possible incomplete information.

The initial information in quality evaluation comes from a data matrix X that has the form shown in Table 3.

Objects (items)	X^1	X^2	...	X^i	...	X^n
1	X_1^1	X_1^2	...	X_1^i	...	X_1^n
2	X_2^1	X_2^2	...	X_2^i	...	X_2^n
.	...	...	...	...	...	...
j	X_j^1	X_j^2	...	X_j^i	...	X_j^n
.	...	...	...	...	...	...
m	X_m^1	X_m^2	...	X_m^i	...	X_m^n
Weights	w_1	w_2	...	w_i	...	w_n
Directions	ϵ_1	ϵ_2	...	ϵ_i	...	ϵ_n

Table 3. Data Matrix X. Starting point in quality evaluation.

Generally, evaluation of OQL is limited within group of products where only chosen characteristics are involved. The latter are random variables. Therefore this is a problem of the *Multivariate Statistical Analysis.*

All contemporary methods of solution algorithms are based on a sequence of statistical (computational) manipulations with the data matrix X. Frequently it is assumed that the measured values of each i-th index among the present m products are i. i. d. random variables, that $X_1^i, X_2^i, \dots, X_m^i$ form a random sample. The units of measurement of each quality index X_k^i must be the same for all m items.

3.1 Method PATTERN: Planning Assistance Through Technical Evaluation of Relevant Numbers

THE ALGORITHM:

1. Calculate the column sums

$$X_{\sum}^i = \sum_{j=1}^{m} X_j^i \,,\; i = 1, \dots, n.$$

2. Transform each entry of X into fractional variables

$$\hat{X}_j^i = \frac{X_j^i}{X_{\sum}^i} \,,\; i = 1, \dots, n;\; j = 1, \dots, m \,,$$

where the quantity $(\hat{X}_j^i)100\%$ expresses how the i-th characteristic of item j participates in formation of corresponding total.

3. The number

$$R_j = \sum_{i=1}^{n} w_i \hat{X}_j^i$$

is the rank of j among the chosen items.

4. Arrange $R_1, R_2, \ldots, R_m$ to see how the items have to be arranged according to their overall quality indices.

3.2 An Alternative Based On PATTERN

Forbrig, Kuck, and Wolf (1984) proposed the following algorithm:

1a. Obtain "standardized" values for each particular characteristic i for the item of interest j

$$X_{j*}^i = \frac{X_j^i - \bar{X}^i}{S_i},$$

where

$$\bar{X}^i = \frac{1}{m}(X_1^i + X_2^i + \ldots + X_m^i)$$

is the sample mean of the i-th characteristic X^i, and

$$S_i = \frac{1}{m}(\mid X_1^i - \bar{X}^i \mid + \ldots + \mid X_m^i - \bar{X}^i \mid)$$

is the average of absolute deviation of the i-th characteristic from its sample mean.

2a. Compute rank of item j according to

$$R_j^* = w_1 X_{j*}^1 + w_2 X_{j*}^2 + \ldots + w_n X_{j*}^n.$$

3a. Use these ranks as in PATTERN to scale the set of items.

3.3 Other Methods

An alternative which uses more complicated statistical models, is discussed by Christozov (1997). Other alternatives that may be constructed on the base of statistical regression analysis, could be recognized in the work of Kordonsky and Gertsbakh (1996). An interesting approach to overall evaluation is proposed by Dimitrov (1996) in the field of education.

3.4 A New Method

This method uses the modified data matrix X:

$X = \{\epsilon_i X_j^i\},\ i = 1, \ldots, n;\ j = 1, \ldots, m.$

It is based on the concept that informative quality parameters exhibit high variability.

The algorithm consists of the following steps:

Step 1.
Calculate the weights w_i of each quality index.

1.1. Perform factor analysis for the set of indices $X^1, \ldots, X^n$ (see Harman, 1962). Obtain

Factors:	$F_1,$	$F_2,$	$\ldots,$	F_r
Indices involved:	$\{1, \ldots, n_1\},$	$\{n_1 + 1,$	$\ldots, n_1 + n_2\}, \ldots,$	$\{\ldots\}$
Factor weights	$\alpha_1,$	$\alpha_2,$	$\ldots,$	α_r

Reduce the set of indices from $\{1, 2, \ldots, n\}$ to $\{1, 2, \ldots, n_1 + \ldots + n_r\}$, which should be less than n.

1.2. Calculate Orthogonal Factors.

Find the groups of independent parameters

Group (set of indices)	Factor Weight
$I_1 = \{1, \ldots, n_1\},$	α_1
$I_2 = \{n_1 + 1, \ldots, n_1 + n_2\},$	α_2
...	...
$I_r = \{n_1 + \ldots + n_{r-1} + 1, \ldots, n_1 + \ldots + n_r\},$	α_r

Set $W_i = \alpha_i,\ i = 1, \ldots, r$ as common weight for the i-th group.

Set $w_i = W_k / n_k$ for weight of i-th parameter when $i \epsilon I_k$ (equally weighted parameters within a factor).

1.3. Calculate the Weight of each group G_k:

$$W_{G_k} = \sum_{i \epsilon G_k} w_i.$$

To get estimators of the Quality Level we need the following

Assumption: The observations $X_1^i, X_2^i, \ldots, X_m^i$ form a random sample from a population controlled by the marginal c. d. f.

$G_i(x) = P\{X_j^i \leq x\}.$

Step 2.
Estimate the particular quality level of each index (characteristic of each item):

A If $G_i(x)$ is known, set

$$q_j^i = G_i(X_j^i).$$

B If $G_i(x)$ is unknown, use a parametric or non-parametric estimator.

In case of a non-parametric approach, proceed in the following way:

2.1. Calculate

$$X_j^{*i} = \frac{X_j^i - \bar{X}^i}{\hat{S}},$$

where

$$\bar{X}^i = \frac{1}{m-1}\sum_{k\neq j} X_k^i; \; \hat{S}_i^2 = \frac{1}{m-2}\sum_{k\neq j}(X_k^i - \bar{X}^i)^2.$$

2.2. Find a spline estimator $\hat{H}_i(.)$ (e.g. the *ogive*) to the c. d. f. of variable X^{*i}, and calculate

$$\hat{q}_j^i = \hat{H}_i(X_j^{*i}).$$

In most cases of sufficiently large set of competing items (objects) the following statement would work:

2.3. If $m \to \infty$, then

$$\hat{q}_j^i \approx \Phi(X_j^{*i}).$$

where a value of $\hat{q}_j^i$ to 0 means bad quality, while a value of $\hat{q}_j^i$ close to 1 means good quality of the item j with respect to its i-th characteristic.

Note: bad $\leftarrow 0 \le \hat{q}_j^i \le 1 \rightarrow$ excellent.

Step 3.
Estimate the OQL.

For this step a suitable estimates transforming function has to be defined. Let the estimators q_j^i and the weights w_i be obtained. Introduce class of estimator functions

$$G = \{g(q, w)\ ,\ q\epsilon[0,1]\ ,\ w\epsilon[0,1]\}$$

by the following requirements, (here $\uparrow\downarrow$ mean monotonic convergence, or tendency):

1. $g(q, w) \uparrow 1$ when $w \downarrow 0$;

2. $g(q, w) \to q$ when $w \uparrow 1$;

3. $g(q, w) \downarrow 0$ when $q \downarrow 0$, and $w > 0$;

4. $g(q, w) \uparrow 1$ when $q \uparrow 1$, and $w > 0$;

5. $g(q, w)$ is increasing in q, and decreasing in w.

Example: The function $g(q, w) = q^w$ satisfies all requirements 1. – 5.

Definition: For an item $P_j = (X_j^1, X_j^2, \ldots, X_j^n)$ with particular quality levels $\vec{q}_j = (q_j^1, q_j^2, \ldots, q_j^n)$, and weights $\vec{w} = (w_1, \ldots, w_n)$, the quantity, computed according to the formula

$$OQL_n(j) = \{g(q_j^1, w_1) \times \ldots \times g(q_j^n, w_n)\}^{\frac{1}{n}}.$$

is called Overall Quality Level (OQL). •

The suggested rule of determination of OQL possesses all the naturally expected features.

F1. Perfect balance: $\min_k q_j^k \leq OQL_n(j) \leq \max_k q_j^k$;

F2. $OQL_n(j)$ is an increasing function of q_j^i and

$OQL_n(j) \downarrow 0$ if some $q_j^k \downarrow 0$ and $w_k > 0$;

F3. $OQL_n(j)$ "neglects" parameters with zero weight ($w_k = 0$), or with perfect quality ($q_j^k = 1$).

In addition, the following "balancing theorem" holds.

Theorem. (Sensitivity to the addition of new characteristics) The relationships shown in Table 4 between the OQL with $(n+1)$ characteristics, the OQL with n characteristics and the quality of the added new characteristic are true.

$(n+1)$ characteristics	n characteristics	$(n+1)^{st}$ characteristic	OQL with (n) characteristics
$Q_{n+1}(j)$	$> Q_n(j)$,	if $g(w_{n+1}, q^{n+1})$	$> Q_n$;
	$= Q_n(j)$,	if $g(w_{n+1}, q^{n+1})$	$= Q_n$;
	$< Q_n(j)$,	if $g(w_{n+1}, q^{n+1})$	$< Q_n$.

Table 4. Relationships between OQL's when including additional characteristic

The following example illustrates how the theorem works.

Example: Let the estimator function be $g(q, w) = q^w$. For $n = 3$ (and $n = 4$) let the quality levels and weights are:

$$\begin{array}{llll} q^1 = .7, & q^2 = .8, & q^3 = .9 & (q^4 = .88); \\ w^1 = .3, & w^2 = .3, & w^3 = .4 & (w^4 = 0); \\ w^1 = .25, & w^2 = .25, & w^3 = .35 & w^4 = .15. \end{array}$$

Then we have the overall quality level for 3 characteristics involved:

$$OQL_3(.) = \{(.7)^{.3}(.8)^{.3}(.9)^{.4}\}^{1/3} = .930503.$$

The quality level of the added $4 - th$ characteristic is

$$g(q^4, w^4) = (.9)^{15} = .98432 > OQL_3(.).$$

The ultimate OQL for 4 characteristics involved becomes

$$OQL_4(.) = \{(.7)^{.25}(.8)^{.25}(.9)^{.35}(.9)^{.15}\}^{1/4} = .943675.$$

Quality increases with inclusion of the fourth characteristic.

4 Conclusions

Evaluating the Overall Quality Level is an important exiting problem. It has many approaches, depending on the information at hand, the complexity of the problem, the rigorousness of the desired result. Classic approaches have limited applications. Stochastic approaches need enough data, assumptions, and sequences of statistical and complementary algebraic manipulations to be performed. The need of new approaches is evminent. Advantages and disadvantages of each approach should be known to be helpful in practice.

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