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Better Understanding of Resonance through Modeling and Visualization

Daniel O. Ludwigsen
Kettering University

Cayla Jewett

Matthew Jusczcyk

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Daniel O. Ludwigsen[*], Cayla Jewett and Matthew Jusczcyk
Kettering University
*Corresponding author: 1700 W Third Ave, Flint, MI   48504, dludwigs@kettering.edu

Abstract: Students encounter cavity resonance and waveguide phenomena in acoustics courses and texts, where the study is usually limited to cases with simple geometries: parallelepipeds, cylinders, and spheres. Long-wavelength approximations help with situations of more complexity, as in the classic Helmholtz resonator. At Kettering University, we are beginning to employ finite element modeling in our acoustics classes to help undergraduates better understand the acoustic modes of actual structures. This approach to the time-independent wave equation (the Helmholtz equation) was first used in a research and measurements class to investigate two classic resonance problems. The first problem was a study of resonance in bottles of various shapes. The second problem, a standard application of the Helmholtz resonator, aimed to control noise in a duct at a single "problem frequency." Students employed swept-sine tests with their structures to determine acoustic mode frequencies. For some of the bottles, pressure mode shapes were also measured by moving a small microphone. The measurements were then compared to results from a time-harmonic finite element model, and when possible, to predictions based on simplified models (the Helmholtz resonator and cylinders). The dependence of the mode shape on varying cross-section enriched the understanding that the textbooks could deliver. In the noise control problem with a duct and resonator, the interaction of the resonator with standing waves of the duct was made clear through visualization. In particular, the model could simulate an infinite duct—not available in our lab!—to clarify the effect of the Helmholtz resonator. Measurements and models from student work will accompany discussion, and ideas for future implementation in courses will be mentioned.

Keywords: acoustic modes, cavity resonance, Helmholtz resonator

1. Introduction

The Applied Physics program at Kettering University currently offers three courses in acoustics to upper-level undergraduates, with a possible minor or concentration in the topic. The first is a survey of topics in sound, while the second course consists of topics in vibration. The third consists of a series of intensive laboratory exercises. The minor may be constructed with an appropriate additional course from either Mechanical or Electrical engineering. In lieu of the laboratory course, students occasionally elect to pursue an ongoing research topic through independent study, working individually with a faculty member. This opportunity gave rise to the present work combining fundamental concepts of resonant cavities with complex geometry through finite element modeling.

Projects involving two students are presented in this work. Both employed the acoustics application mode in FEMLAB 3.1. The time-independent wave equation, known as the Helmholtz equation, governs this application. Eigenvalue or eigenfrequency analysis was used to determine the mode frequencies and mode shapes of the pressure function, while the time-harmonic analysis provides a computational analog to the swept-sine test used in the laboratory. Boundary conditions were typically straightforward; most enclosures provided sufficiently rigid boundaries. Open ends were considered pressure release ($p = 0$) boundaries, neglecting the mass load that leads to end corrections. In one case the "radiation" boundary condition was used to simulate an infinite duct. All solid modeling was accomplished in FEMLAB, as well as post-processing.

2. Resonances in bottles

Several bottles of various geometries were studied through finite element models and in the laboratory. The classic shape of a glass Coca-Cola™ bottle provides an excellent example of these investigations. The profile of the cavity

was determined by filling the bottle to known depths, and measuring the mass (and thereby, the volume) of the water. This profile was rotated around the axis to create the 3-d interior of the bottle.

To explain the acoustic behavior of a bottle, the analytical model is typically either a cylinder or a Helmholtz resonator (a neck with inertia and a cavity with compliance).[1] The Helmholtz resonator has a single resonance as if it were a mass on a spring. The cylinder model is based on a closed/open pipe, with its series of harmonically-related modes.

2.1 Mode shapes and frequencies

With the interior geometry of the glass Coca-Cola™ bottle modeled in FEMLAB, an eigenvalue analysis provided insight into the possible modes of that shape. Parameters were set to typical room values (density 1.2 kg/m^3 and speed of sound 344 m/s), and the mesh was created with the default settings. Figure 1 gives selected modes with their frequencies.

The lowest of these modes (309 Hz) can be interpreted as a Helmholtz resonator mode. This lumped-element view requires wavelengths that are much great: the pressure in the cavity will be acceptably uniform, and the air in the neck will oscillate as a single mass. As the bulk of the bottle is red, there is fairly uniform pressure. The neck has gently sloping "shoulders", and is more difficult to distinguish. However, the only significant component of pressure gradient (related to particle velocity) occurs in the neck.

The modes at 1339 and 4468 Hz (the second and fifth for this bottle) represent a series of cylinder-type modes which match the boundary conditions. These have pressure variation on the bottle (x) axis, primarily, and nodal surfaces are nearly cross-section planes. Where the profile of the bottle is steeply sloped, those nodal surfaces curve to remain perpendicular to the wall. This may be seen in the 4468 Hz mode, near the sloping shoulders of the bottle. Also, where the cross-sectional area decreases, pressure amplitude increases compared to a cylinder.

The mode at 4850 Hz is a combination of the lowest cylinder-type mode and the lowest cross mode (at 4697 Hz). Cross modes feature a nodal plane along the bottle axis, as well as degeneracy in the y and z directions.
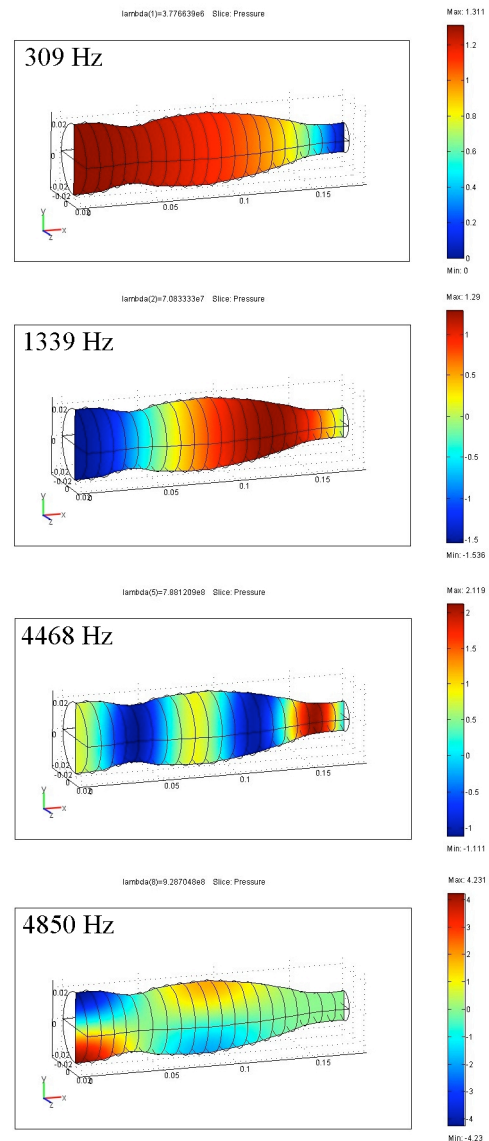
Figure 1. Selected modes of the bottle model. Note: color maps are different for each plot.

The results of this eigenvalue analysis confirm the typical analytical approaches. At the same time, there are subtleties revealed by the finite element approach.

2.2 Experimental verification

To measure the bottle's response in a swept-sine test, a speaker was placed near the bottle to drive its mouth. A reference microphone (a lavaliere electret mic, Radio Shack 33-3013) was

placed near the speaker, and a similar test microphone was placed inside the bottle, near the bottom. The driving frequency was swept through 100 to 6500 Hz, and the result was a clear series of peaks in the magnitude of the frequency response (the ratio of the test mic signal to the reference mic signal). These well-defined peaks at least 30 dB above the background lie below about 4500 Hz. Above that frequency, the results seemed very noisy or corrupted. This corruption occurred at different frequencies for bottles of different geometries.

The lower distinct peaks could be matched to eigenfrequencies. For example, the lowest was at 256 Hz (at a temperature of 71° F). This is well below the 309 Hz from the lowest mode of the eigenvalue analysis, but the discrepancy may be due to the missing end correction. The radius of the neck is a significant fraction of its length, so the end correction would also be significant. There may be additional effects related to the pressure variation discussed below.

The problems encountered in the laboratory, around 4500 Hz for the Coca-Cola™ bottle, are due to the appearance of cross modes in that frequency range. Driving the mouth with a uniform pressure will not readily excite these, and if the test microphone is near the axis of the bottle, it sits at a pressure node and will not give a worthwhile response. Different bottles will have the lowest cross modes at very different frequencies, explaining the variation in the result for different cross-section geometries.

For the axial modes, it is possible to compare axial pressure functions from FE results to measurements made with the test microphone fixed on the axis at various depth. Figure 2 presents is comparison for three of the modes in Figure 1. The vertical axis in each plot is self-normalized; only relative measurements were important. The frequencies used in the lab were those of the peak response magnitudes.

The match between the laboratory and FE results is worst for the lowest Helmholtz mode. The FE results show a nearly uniform pressure in the bottle until the neck contracts at about 15 cm. In contrast, the narrowed "waist" of the bottle seems to decrease rather than increase the pressure measured in the lab. This unexpected pressure variation as yet lacks a reasonable explanation. The Helmholtz resonator may not be an accurate model for this lowest mode of the familiar bottle shape.
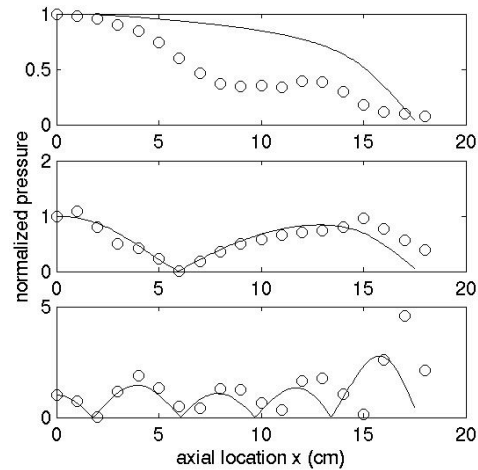
Figure 2. Comparison between laboratory measurements of on-axis pressure (circles) and FE pressure from post-processing (solid line). Data are individually normalized to the result at 0 cm. Data corresponds to lab/FE frequencies, respectively from the top, 256/309, 1274/1339, and 3980/4468 Hz.

3. Noise control application

The Helmholtz resonator combines an acoustic inertia and compliance which can be used to support oscillation at its resonance frequency, as in the bottle. Alternatively, when added as a branch off a duct, it can be used to control noise at that resonance frequency, given in Eq. (1).

$$f_0 = \frac{c}{2}\sqrt{\frac{S}{Vl}} \qquad (1)$$

Here c is the speed of sound, S the cross-sectional area of the neck, V is the volume of the cavity, and l' is the effective length of the neck (including end corrections). At that frequency, the resonator returns energy to the duct out of phase with the original wave traveling down the duct.[2] Through the visualization power of the post-processed finite element model, this interaction of the Helmholtz resonator and the duct can be understood more fully.

3.1 Verification: experiment and FE

A simple demonstration apparatus shows the narrow-band filtering of a Helmholtz resonator

attached to a duct. The laboratory measurements in this work come from a 2-m duct constructed of 1/2" medium density fiberboard (MDF). The square cross-section has sides of 9.8 cm, which limits the frequency range below about 3500 Hz to assure only plane waves propagate down the duct. At the driven end, a small enclosure containing a 4" speaker (Audax HP100M0) was coupled to the duct via a wider chamber, partially stuffed with polyester fiber batting. Silicone sealant was used to close seams.

The Helmholtz resonator, also constructed of MDF, consisted of a fixed-geometry neck and a cavity whose volume varied with a sliding piston. The square cross-section of the neck had an area of $S = 25.6$ cm^2 and an physical length of $l = 4.55$ cm. The cavity cross-sectional area was 59.3 cm^2, and allowed a maximum depth of 13.5 cm. From Eq. (1), the lowest possible Helmholtz resonance frequency is 322 Hz. The frequency of the Helmholtz resonator will thus fall in the midst of a series of peaks, a region where the transducers respond with reasonable linearity. The distance from the closed end of the duct to the speaker cone was 208 cm, so that the duct resonance frequencies are 83 Hz apart.

Swept sine tests recorded the frequency response of the duct/resonator combination, using a spectrum analyzer (Standard Research Systems 785) and two microphones. The first (PCB Piezotronics, Inc. U130D20) was mounted flush with the closed boundary on the far end of the tube. The reference signal was provided by a lavaliere microphone (Radio Shack 33-3013) placed at the entrance to the duct near the driver to remove the speaker's response characteristics.

The simulation created in the FEMLAB environment mirrored the lab apparatus. All boundaries of the gas volumes were assumed to be rigid, except the end of the duct nearest the speaker. A pressure amplitude of 1.0 Pa was specified across this plane (this corresponds to a sound pressure level of 94 dB re: 20 Pa). Density of the gas was 1.20 kg/m^3, and the speed of sound was specified to be 344 m/s.

The mesh for this model was built with the default settings; element size was appropriate to the geometry and the wavelengths for sound in our frequency range. The solver calculated the pressure at element vertices at each frequency specified. A transfer function was calculated between the closed and driven end in the post-

processing environment to replicate the frequency response measured in the lab.

Figure 3 displays transfer function magnitudes from both the laboratory apparatus (solid line) and the finite element simulation (dashed line). The effect of the filter is clearly seen in a notch at 530 Hz. The spacing between the five highest duct resonances is an average of 83.5 Hz, confirming that the series of peaks is due to standing waves in the duct. The lower peaks show the effects of the driven boundary condition; this is not the harmonic series for a tube closed at both ends, rather, the driven end 'shifts' the series. Also, as a practical note, the speaker is not effective at the lowest frequencies.

The difference in transfer function magnitude overall is a result of frequency resolution and, perhaps losses in the laboratory apparatus. Because the FE transfer function is sampled at 10 Hz intervals, some of the maxima are not represented in detail. The third peak, for example, lies quite close to the 210 Hz data point and its height is better represented. Also, the FE model includes no losses. The actual duct has walls with some roughness; the effect of this could be to broaden the peaks and thereby raise the minima.

Aside from issues of frequency resolution, the agreement between laboratory and finite element results was acceptable. Helmholtz resonator frequencies varied by an average of 3% between estimates of the lab and FE notch frequencies. For improvement, the model would be recalculated with smaller frequency steps.

3.2 Infinite "virtual" duct

The intent of the work was less focused on precision and more on understanding through visualization. Toward that goal, the rigid termination of the duct was replaced with a radiation boundary condition to avoid reflections to return to the Helmholtz resonator. The semi-infinite duct transfer functions near the resonance of the Helmholtz device are shown in Figure 4. The heavy solid curve corresponds to the type of measurement made with laboratory apparatus, with a microphone at the far end of the duct. The notch occurs at 514 Hz, in agreement with theory, and a peak at 516 Hz. To determine the nature of the peak, the duct was lengthened to 2.1 m. The light solid curve is the result from the longer semi-infinite duct. The
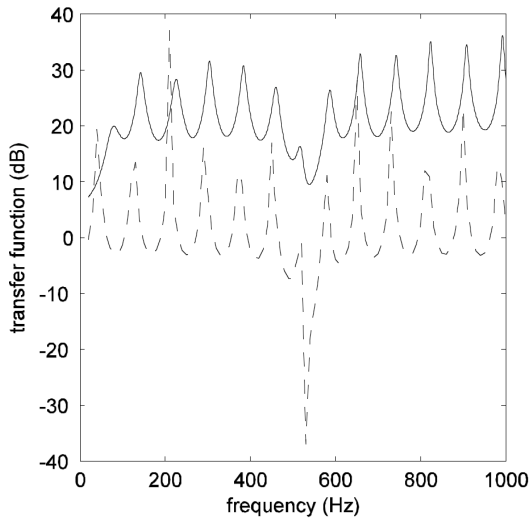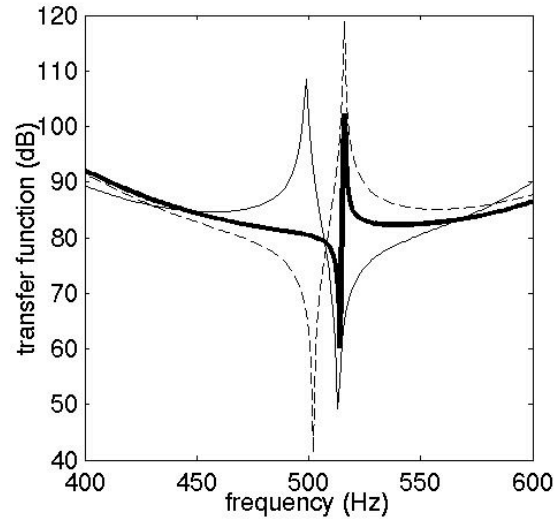
Figure 3. Comparison of the transfer functions for the duct with a Helmholtz resonator of moderate volume (375 cm3). The solid line is laboratory data (sampled 1 Hz) and the dashed line is from the FE model (10 Hz resolution).

Figure 4. Transfer functions for semi-infinite FE duct with a 6.33-cm Helmholtz resonator, with 1 Hz resolution.

peak frequency was reduced, while the notch remained in the same place. This peak is thus demonstrated to be a resonance of the duct itself.

The dashed curve of Figure 4 shows a transfer function involving the center of the duct exactly at the site of the Helmholtz resonator, rather than the far end. It is evident that for a "listener" at this location, the resonator reduces noise at a slightly different frequency; here, the notch is at 502 Hz. The discrepancy in frequency motivates a more informative view of the wave in the duct near the resonator.

The spatial pressure distribution of the semi-infinite duct and Helmholtz resonator provides some understanding of the difference in the notch frequency. As shown in Figure 5, the semi-infinite duct has no reflections from the terminal end. However, there is a standing wave set up between the source end and the Helmholtz resonator. The nodes of this standing wave appear as bands across the duct. Their spacing and location depend on the wavelength and thus the frequency of the wave in relation to the boundary conditions. At 514 Hz, the node nearest the Helmholtz resonator bends to incorporate the entire downstream portion of the duct.

This node does not happen to pass through the center of the duct. Instead, at 502 Hz with

slightly longer wavelength, the node in fact passes through the center. It does not extend to incorporate the rest of the duct, but ends at the far wall.

3.3 Noise control ramifications

The lumped-element approach is often used to determine Helmholtz resonator frequencies. With additional effort in creating a FE model, greater knowledge of the spatial can inform design for particular applications. The cancellation, for example, occurs at a slightly different frequency downstream than directly at the center of the duct. Another benefit from this kind of modeling effort can inform a better understanding of the standing wave created *upstream* of the Helmholtz resonator.

Because the Helmholtz resonator creates a side branch with its own impedance to wave propagation, the point to which it is attached becomes a point of reflection. The Helmholtz resonator becomes a boundary condition for the upstream side. Resulting standing waves at frequencies that match the boundary conditions are responsible for the nodes and antinodes seen in Figure 5. The maximum amplitudes, seen at the cavity of the Helmholtz resonator, vary with frequency. The peak at 516 Hz for the 2.0-m duct (1.0 m upstream) is associated with one of these
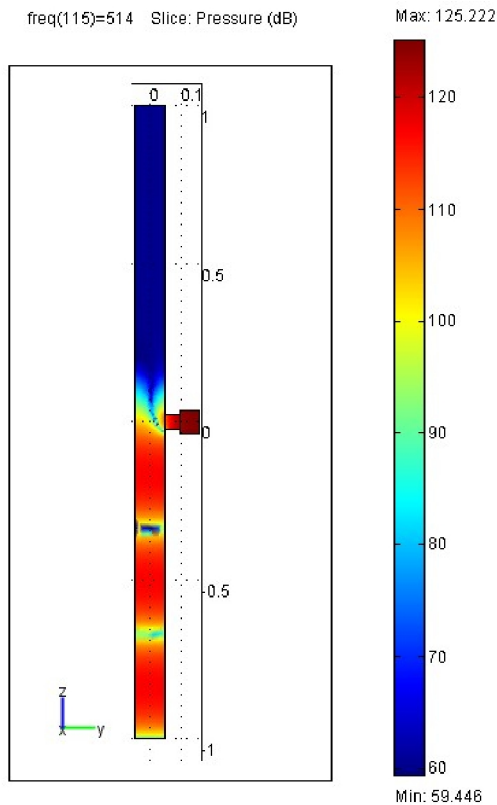
Figure 5. The pressure distribution (in dB) for the 2.0-m duct at 514 Hz.

standing waves that matches the boundary conditions best.

4. Conclusions

Courses in acoustics (and occasionally even introductory physics) for engineering and science majors often include the topic of resonance in simple structures such as Helmholtz resonators and pipes. Applying these models to real objects requires serious consideration of the necessary approximations. By using the accessible numerical environment offered by FEMLAB and now COMSOL Multiphysics, students can visualize the differences that arise when the model may not quite fit the geometry.

In the work reported here, student projects investigated how the Helmholtz resonator and pipe models fit two situations: the classic shape of a soda bottle, and a demonstration apparatus for noise control of sound propagating down a duct. Detailed results of the FE models provided

further insight into the approximations and assumptions inherent in the analytical models used in textbook cases. In turn, the students (as well as their instructor) became more familiar with the analytical models. Further work will investigate the variation of pressure in cylinders of varying cross-section, and attempt to propose and justify a simple correction to the volume of a Helmholtz resonator for greater accuracy in frequency.

5. References

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