VARIATIONAL CONSTRAINTS FOR DA WITH ALADIN-NH DYNAMICS

CARLOS GEIJO and PAU ESCRIBÀ

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PRESENTATION LAYOUT

- MOTIVATION FOR THE DEVELOPMENT OF THE METHOD
- FORMALISM OF THE VARIATIONAL CONSTRAINTS (VC) METHOD
- TESTS WITH SYNTHETIC OBS AND IMPLEMENTATION IN HARMONIE-AROME CY40
- FIRST RESULTS WITH FIELD ALIGNMENT, 3D-VAR (CARLOS GEIJO) AND LETKF (PAU ESCRIBÀ)
- CONCLUSIONS AND OUTLOOK

INTRODUCTION, MOTIVATION

- Many variational techniques with constraints have been proposed in the past, also for mesoscale and convective scale DA (e.g. mass-field consistent wind fields)

- 3DVAR can be regarded as one such variational technique but it also shares important aspects wih statistical interpolation methods (B matrix of model error covariances)

- “The intermittency of mesoscale phenomena makes it difficult to derive meaningful covariances for use in statistical objective analysis procedures” (R . Daley, 1991)

- Algorithms like Field Alignment are intrinsically flow-dependent, but it introduces imbalances that have a detrimental impact on the NWP system performance

- This work aims at producing a numerical scheme that can effectively reduce these imbalances

Formulation of Balances for ALADIN-NH dynamics

- ALADIN-NH dynamics currently consists of a spectral formulation, with semi-lagrangian advection (SL) and semi-implicit time stepping (SI)

- The SI system consists of a set of five linear equations, with local and non-local ops, that can be used to give a precise definition of NH-balances

- Formulated in terms of rotational invariant scalars and a resting base state (no FlowDep !). In terrain-following coordinates, but with flat orography ($\pi_s^* = Cte$)

- Different formulations of the SI have been considered, they all with identical physical content and they all consistent within the linear approximations to the state equation and the gepotential equation

$$PD = \partial\Psi + T = \partial\phi + \partial\pi + T \quad ; \quad \phi(\xi) - \phi(\bar{\xi}) = \phi(\xi) = -G[\partial\phi] = -G[PD] + G[T] + \pi_s - \pi$$

- Two different numerical schemes (VFD and VFE) have been implemented for solving it

Formulation of Balances for ALADIN-NH dynamics

- Once the time discretization is performed, and *before vertical discretation*, the SI can be casted in different forms

- The GEO-GW formulation reads (with allowance for different scaling in the vertical and horizontal momentum eqs.):

$$D - K^2\left(T + (\partial + 1)\Psi\right) = D^\bullet \qquad K^2 = (kH)^2\omega_b^2 \quad \omega_b^2 = \frac{g\,\Delta t^2}{H} = \frac{g^2\,\Delta t^2}{RT^*} \quad ; \quad D = D'\,\Delta t$$

$$gw - \omega_e^2\left((\partial + 1)T + (\partial + 1)\partial\Psi\right) = gw^\bullet \qquad \omega_e^2 = \frac{g^2\,\Delta t^2}{RT_e^*} \qquad gw = gw'\,\Delta t \big/ RT_e^*$$

$$T + \frac{R}{c_v}\left(D - \chi\,\partial gw\right) = T^\bullet \qquad T = T' \big/ T^* \quad \chi = \frac{T_e^*}{T^*}$$

$$\pi_s + N[D] = \pi_s^\bullet \qquad \pi_S = \pi_S' \big/ \pi_S^*$$

$$\Psi - \chi\,gw + S[D] = \Psi^\bullet \qquad \Psi = (\Phi_S + \Phi') \big/ RT^* + \pi' \big/ \pi^*$$

Formulation of Balances for ALADIN-NH dynamics

- All these formulations lead to a Boundary Value problem with the same 2nd order differential operator

$$\left(-\lambda+\partial(\partial+1)\right)[\chi\, gw]=-\frac{\left(1+K^2\gamma\right)}{\omega_e^2\chi\,\gamma}\chi\, gw^\bullet+\left(\frac{R}{c_p}+\partial\right)D^\bullet-\frac{1}{\gamma}\left(\partial+1+K^2\right)T^\bullet-\left(\frac{1}{\gamma}\partial-\frac{R}{c_p}K^2\right)(\partial+1)\,\Psi^\bullet$$

$$\left(1+K^2\gamma\right)D-K^2(1+\gamma\,\partial)\,\chi\, gw=D^\bullet+K^2\left((\partial+1)\Psi^\bullet+T^\bullet\right)$$

$$T+\frac{R}{c_v}\left(D-\chi\,\partial gw\right)=T^\bullet \qquad\qquad \lambda=\frac{1+K^2\,\gamma\,(1+\omega_e^2\,\chi\,{R}/{c_p})}{\omega_e^2\,\chi\,\gamma}$$

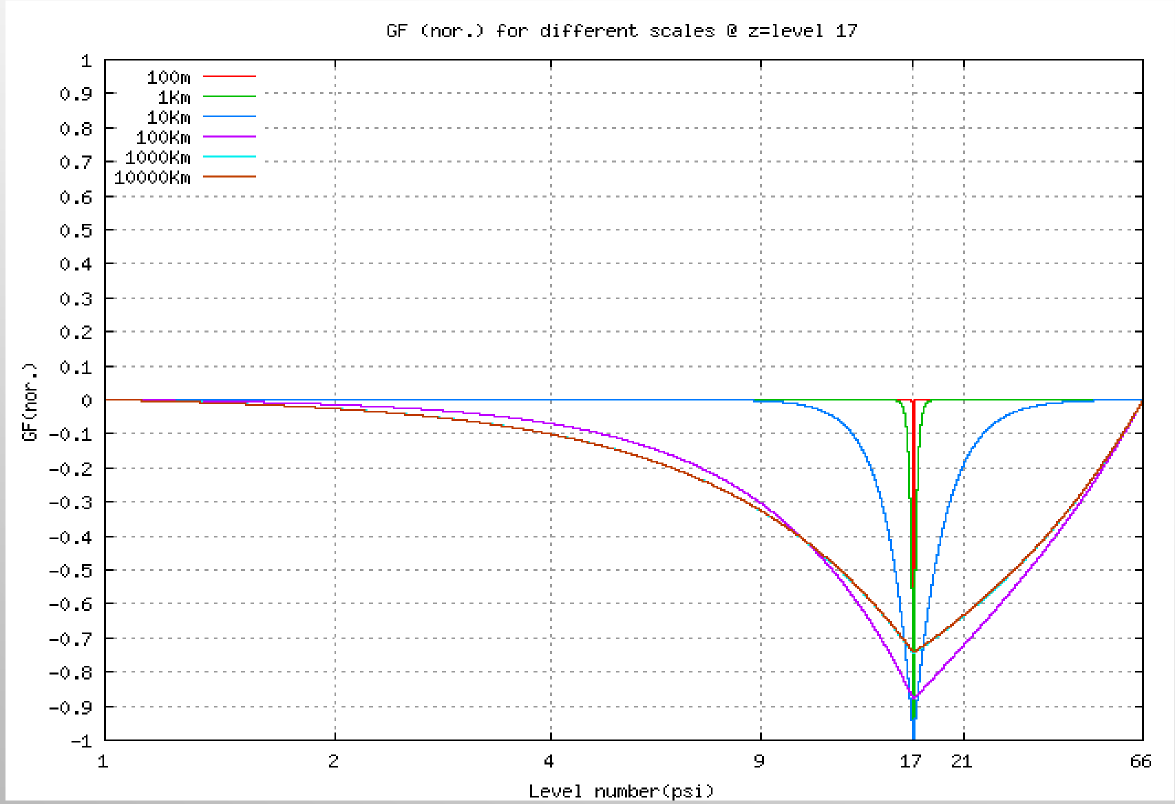
$$\pi_S+N\left[D\right]=\pi_S^\bullet$$

$$\Psi-\chi\, gw+S\left[D\right]=\Psi^\bullet \qquad\qquad (*)\quad \omega_e^2\chi=\omega_b^2$$

- Due to (*) the introduction of a different H scaling in the vertical momentum eq. (i.e. $\chi\neq 1$) has no impact on the free-mode spectrum of the system M[x]=0

Formulation of Balances for ALADIN-NH dynamics

- The GF for the operator $\lambda + \partial(\partial + 1)$ **with suitable conditions on gw and/or ∂gw at the upper and lower boundaries is determined**, and allows computation of the solution by means of quadratures on smooth interpolations of the RHS (e.g. cubic splines)



- **No need to discretize the vertical operators** : all algebraic constraints and conservation laws satisfied
- Incorporation of the upper BC on gw and/or ∂gw in the computation of the solution, enabling so **better nesting**
- **No need for staggering** in the vertical. $\vec{V}, \partial\vec{V}$ calculated at the same levels
- **Accurate** (quadratures on splines computed analytically) and **stable** (GF is a compact operator)

Formulation of Balances for ALADIN-NH dynamics

- **In the spirit of this GF approach** to the ALADIN-NH SI dynamics, let us consider the minimization of

$$2J(x^k) = \int_o^{\bar{\xi}} w_o^k \left\| x^k - x_o^k \right\|^2 + w_c^k \left\| Mx^k - x_\bullet^k \right\|^2$$

- This variational problem leads to another Boundary Value Elliptical Problem **also solvable by means of GFs**

- The k super-index in the x’s indicates fourier components, the double bars that these components are complex numbers. However, M is real and does not couple different wave-numbers, therefore these specifications can be dropped

- The “•” sub-index for $x_\bullet$ indicates that this symbol is in correspondence with the RHS dot-terms. Therefore it is computed from the model state at a different time step from the searched solution x (without dot). M is a time-step forward operator. The **constraint is then “non-holonomic” (constraints derivatives and not the variables themselves)**. This is a subtle difference with ordinary 3D-Var, where x_b takes the place of $x_\bullet$

- The “o” sub-index in x_o indicates that this quantity is given by observations. The w’s are relative weights and they are adjustable by try and check

- **The constraint on Ψ is treated as "strong"**
- **Incremental method:** solution in the vicinity of the background (which is, of course, balanced !)

$$x = x_b + \Delta x \quad ; \quad x - x_o = -d + \Delta x \quad ; \quad d = x_o - x_b \quad ; \quad Mx - x_\bullet = M\Delta x \quad ; \quad Mx_b = x_\bullet$$

$$2J(\Delta x) = \int_o^{\bar{\xi}} w\left(\Delta x - d\right)^2 + \left(M\ \Delta x\right)^2 = \int_0^{\bar{\xi}} w\left(\Delta x - d\right)^2 + C_1^2 + C_2^2 + C_3^2 + C_4^2$$

$$\Delta x^T = \left(\Delta gw, \Delta D, \Delta T, \Delta\pi_s\right) \quad ; d^T = \left(gw_o - gw_b, D_o - D_b, T_o - T_b, \pi_{s,o} - \pi_{s,b}\right) = \left(d_{gw}, d_D, d_T, d_{\pi_S}\right)$$

$$C_1 = \left(-\lambda + \partial\left(\partial + 1\right)\right)\Delta gw \equiv L\left[\Delta gw\right] \quad ; \qquad \Delta gw = \frac{(\Delta gw')\Delta t}{RT^*} \quad ; \quad \lambda = \frac{1 + \gamma K^2\left(1 + \frac{R}{c_p}\omega_b^2\right)}{\gamma\omega_b^2}$$

$$C_2 = -K^2\left(1 + \gamma\,\partial\right)\Delta gw + \left(1 + \gamma K^2\right)\Delta D \quad ; \qquad \Delta D = \left(\Delta D'\right)\Delta t$$

$$C_3 = \Delta T + \frac{R}{c_v}\left(\Delta D - \partial\Delta gw\right) \quad ; \qquad \Delta T = \frac{\Delta T'}{T^*}$$

$$C_4 = \Delta\pi_s + N\left[\Delta D\right] \quad ; \qquad \Delta\pi_s = \frac{\Delta\pi_s'}{\pi_s^*}$$

When **J is made stationary** the following system of equations is obtained

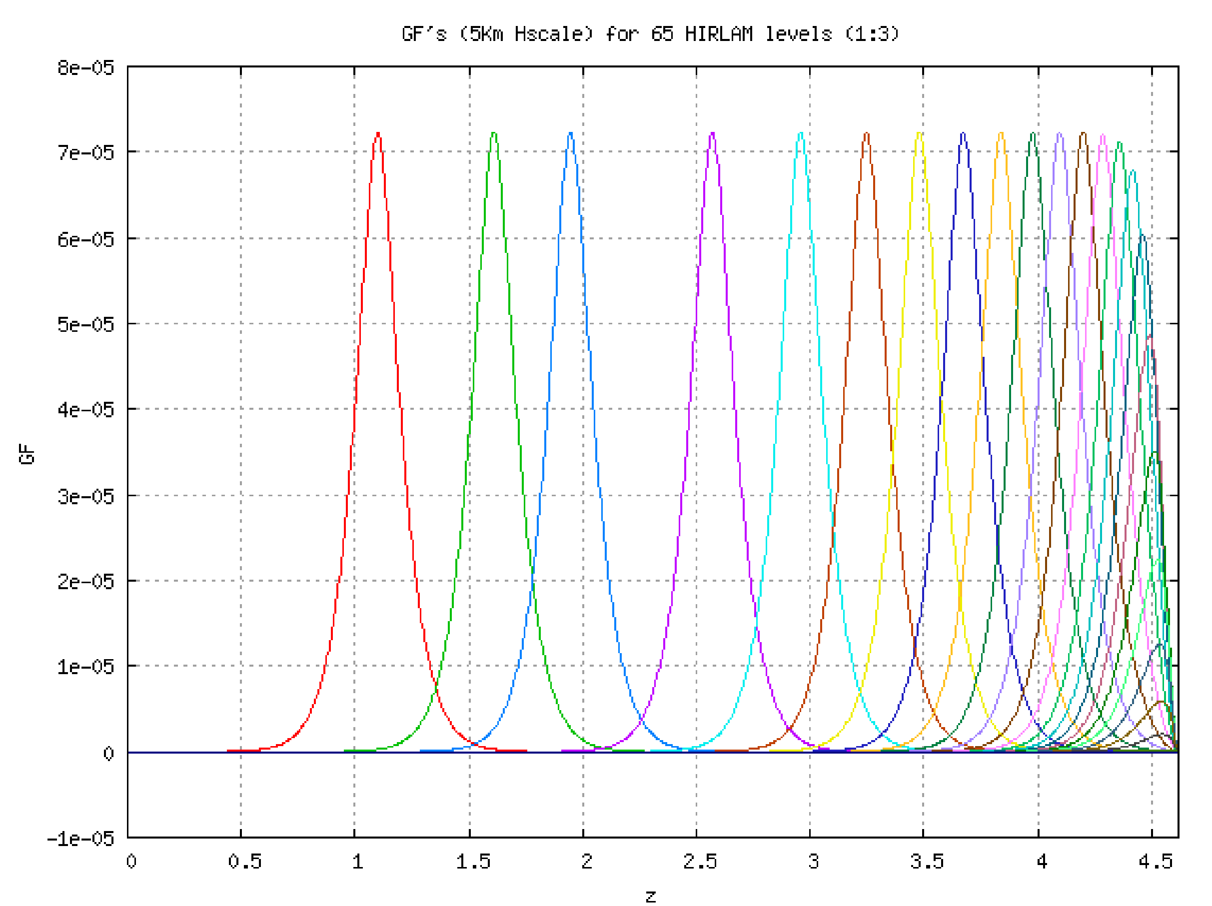
$$M^{+}M\,\Delta x + w\,\Delta x = w\,d; \quad w = \frac{w_o}{w_c};$$

$$M = \begin{bmatrix} L & 0 & 0 & 0 \\ -K^2\left(1+\gamma\,\partial\right) & \left(1+K^2\,\gamma\right) & 0 & 0 \\ -\frac{R}{c_v}\partial & \frac{R}{c_v} & 1 & 0 \\ 0 & N[\ ] & 0 & 1 \end{bmatrix} \quad M^{+} = \begin{bmatrix} L^{+} & -K^2\left(1-\gamma\,\partial\right) & \frac{R}{c_v}\partial & 0 \\ 0 & \left(1+K^2\,\gamma\right) & \frac{R}{c_v} & N^{+}[\ ] \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

which turns out to be another elliptical boundary value problem (of 4 th order) on Δgw

$$\Delta gw\left(0\right) = \Delta gw(\bar{\xi}) = \Delta\partial gw\left(0\right) = \Delta\partial gw\left(\bar{\xi}\right) = 0$$

Formulation of Balances for ALADIN-NH dynamics



- The Greens Function for the elliptical operator involved in the problem is positive definite and symmetric
- These GFs exhibit a clear broadening with larger horizontal scales
- Shift from “vorticity-implied” balances to “vertical velocity-implied” balances

The set of NH dynamical variables can be completed making use of :

$$\left.\begin{aligned} -\Delta gw + S\left[\Delta D\right] + \Delta\Psi = 0 \\ \Delta PD - S\left[\Delta D\right] + \gamma\left(\Delta D - \partial\Delta gw\right) = 0 \end{aligned}\right\} \qquad \Delta PD = \partial\Delta\Psi + \Delta T$$

It happens making use of the vertical momentum eq brings ΔPD values in better agreement with BG values

$$\Delta gw - \omega_e^2\left(\partial + 1\right)\Delta PD = 0 \quad ; \quad \omega_e^2 = \frac{g\Delta t}{RT_e^*}$$

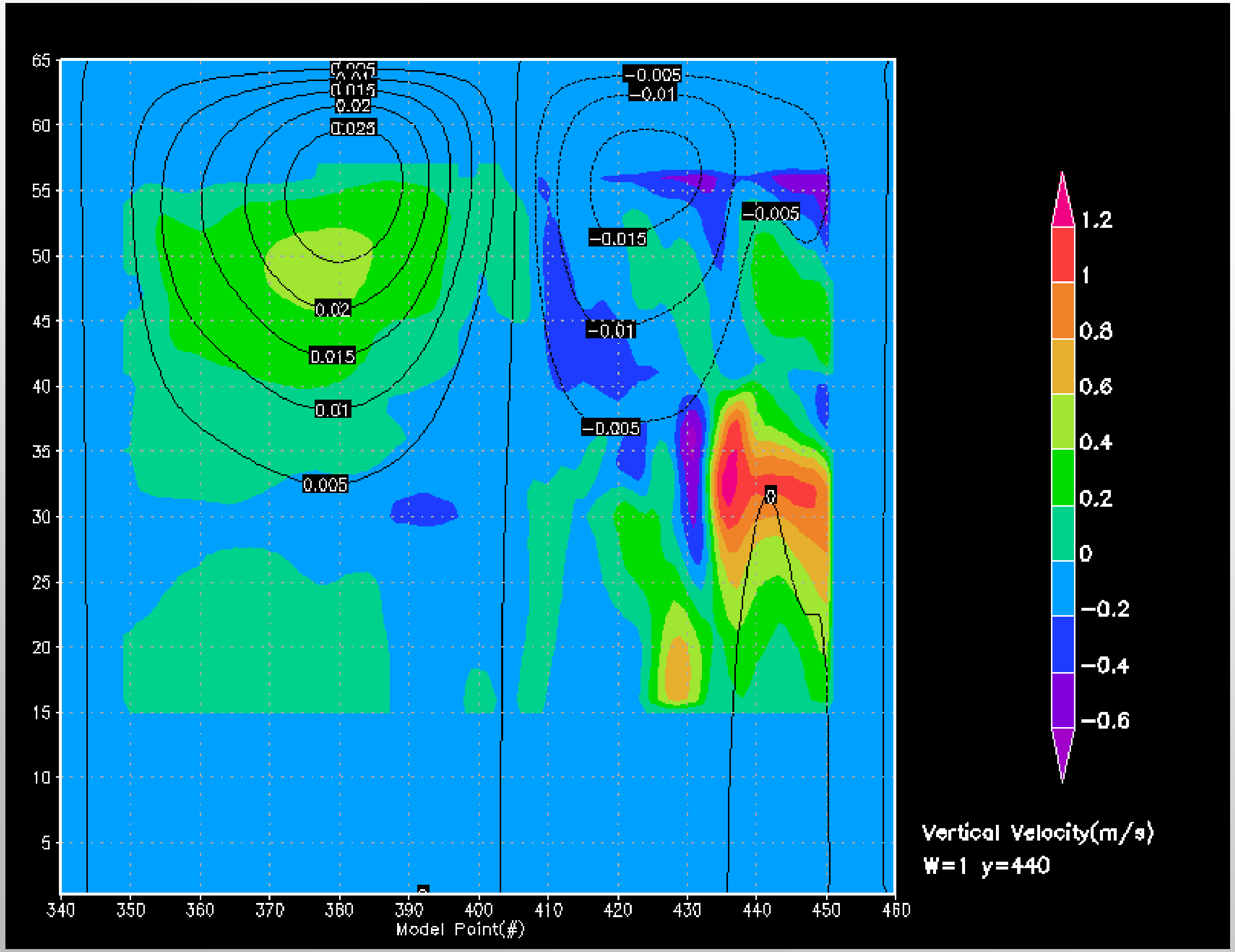
or, if we want the BC at top and bottom to be satisfied :

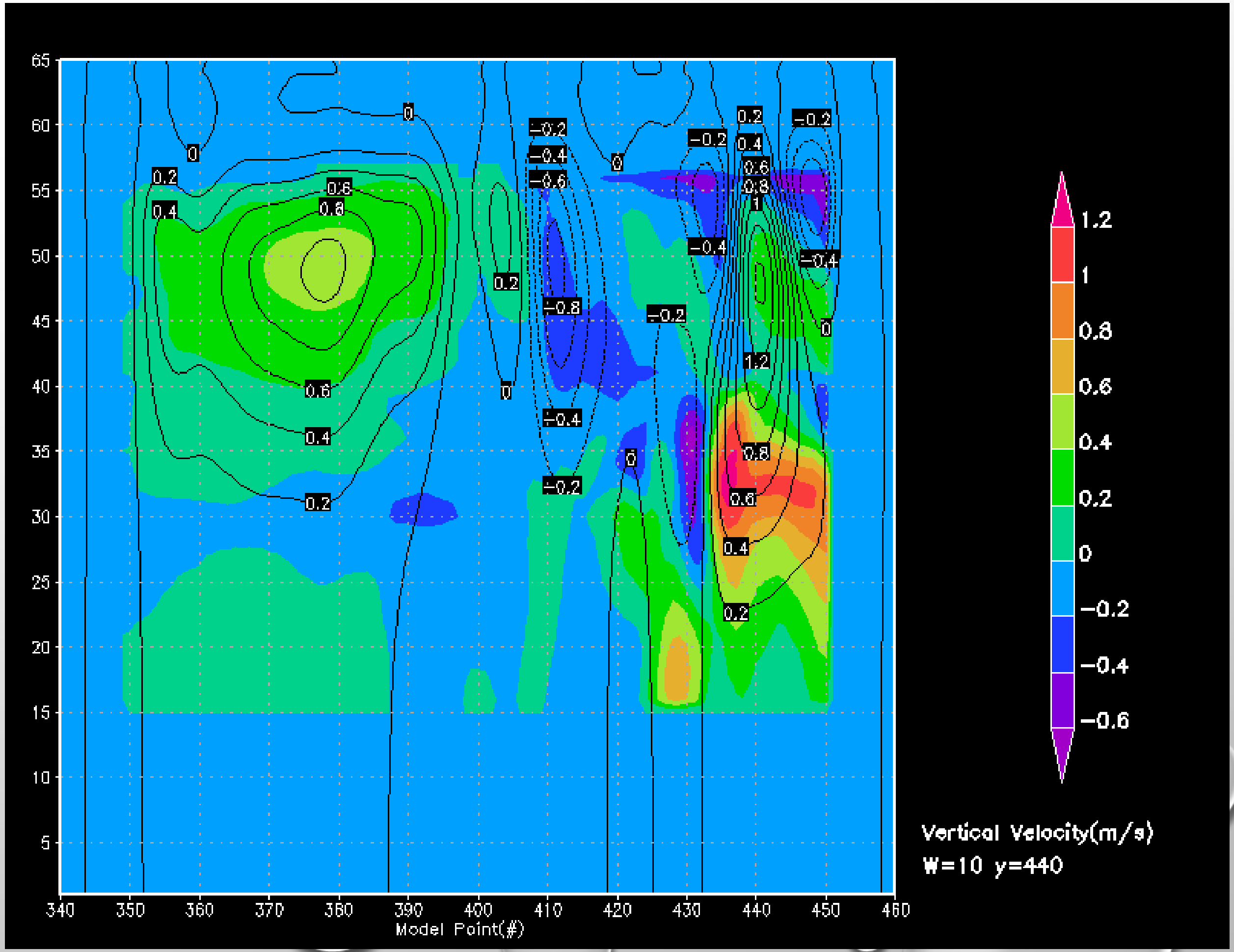
$$\partial\left(\Delta gw - \omega_e^2\left(\partial + 1\right)\Delta PD\right) = 0 \quad \begin{cases} \Delta PD(\xi = 0) = -\gamma\left(\Delta D - \partial\Delta gw\right)_{\xi=0} \\ \Delta PD(\xi = \bar{\xi}) = N\left[\Delta D\right] - \gamma\left(\Delta D - \partial\Delta gw\right)_{\xi=\bar{\xi}} \end{cases}$$

TESTS WITH SYNTHETIC OBS AND IMPLEMENTATION IN CY40

Analysis of Vertical Velocity from Vertical Wind Pseudo-Obs

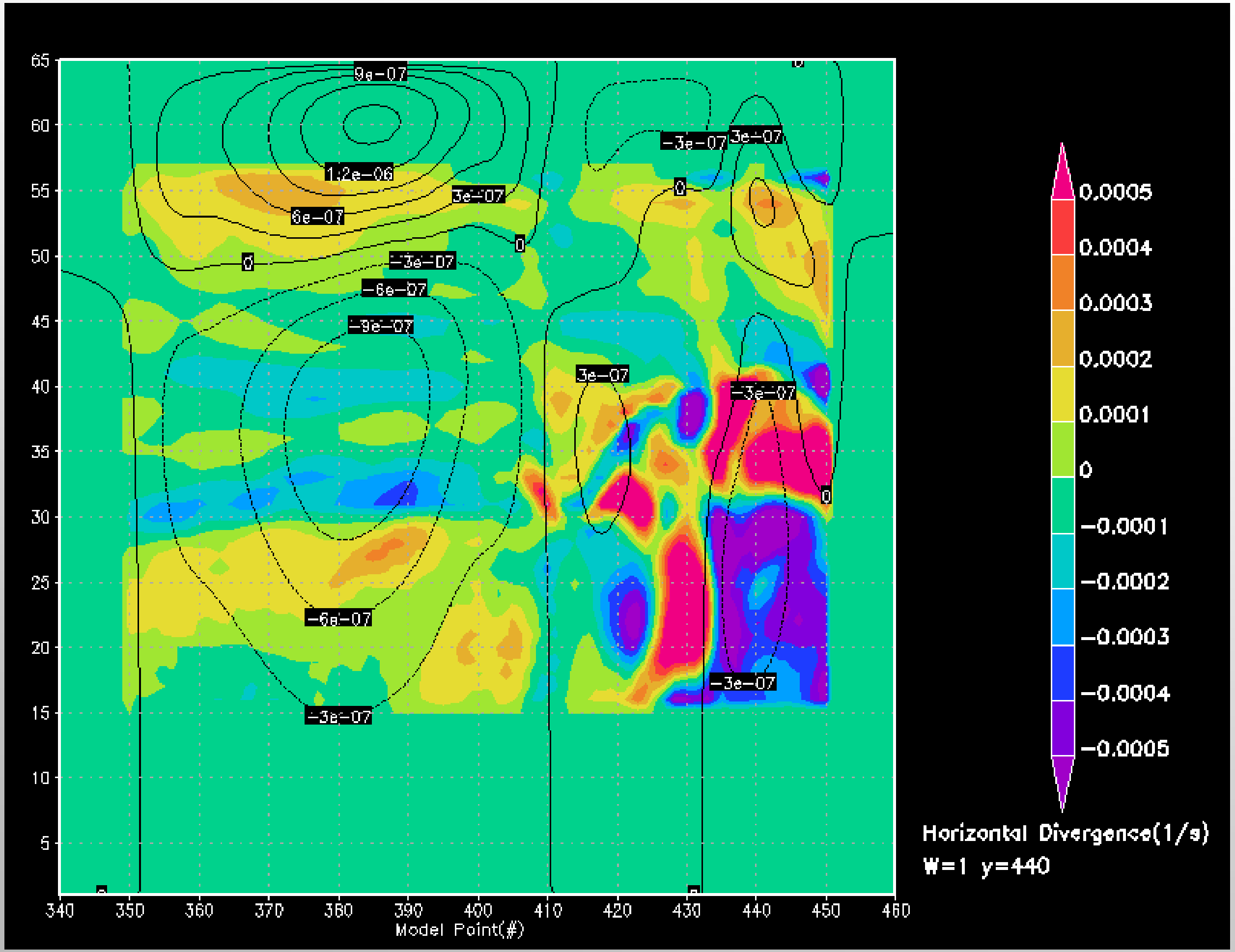
LEFT W=1 ; RIGHT W = 10 + scale factor ;   Contours are Analysed Fields, Shaded Field is the “Truth”

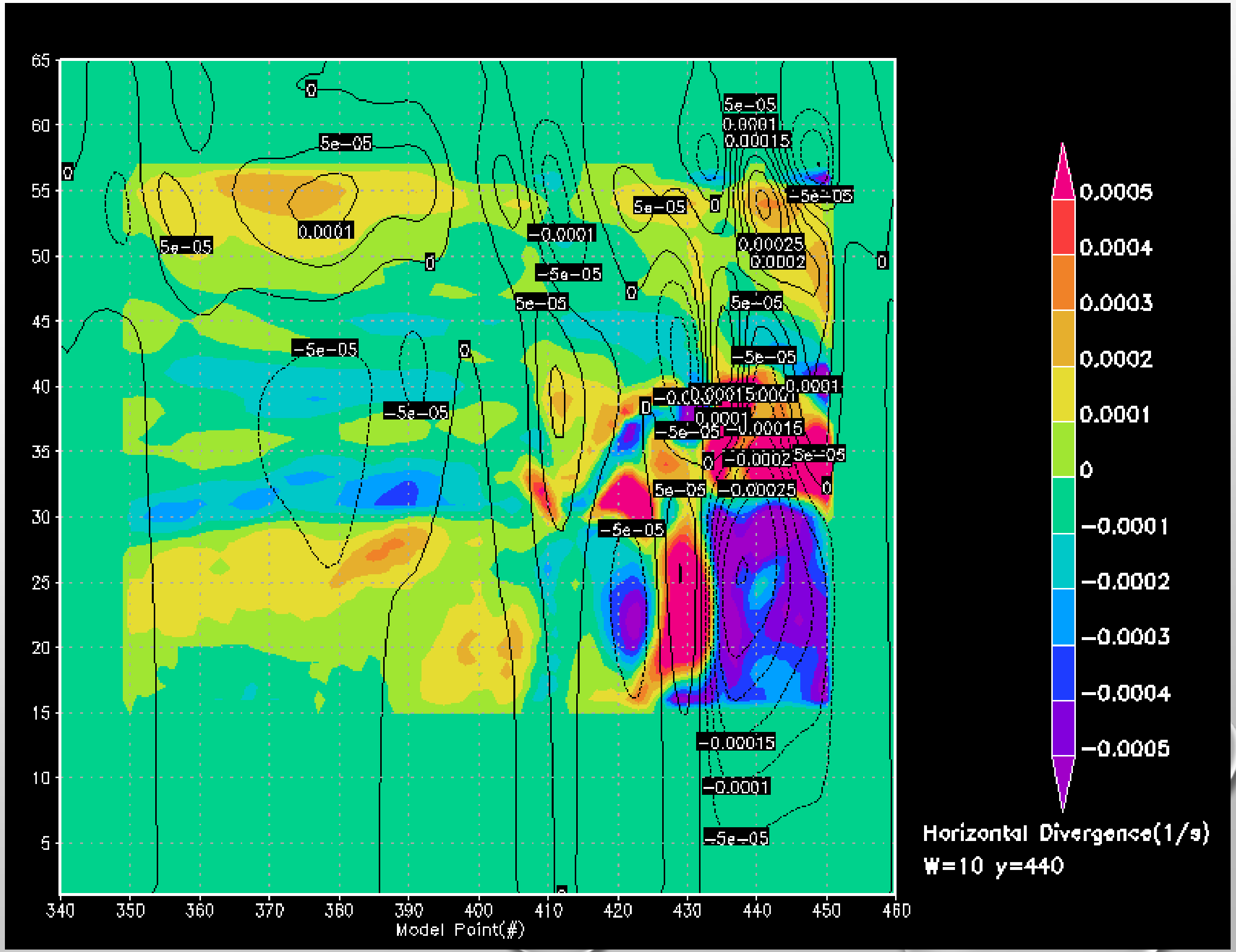




Analysis of Horizontal Divergence from Vertical Wind Pseudo-Obs

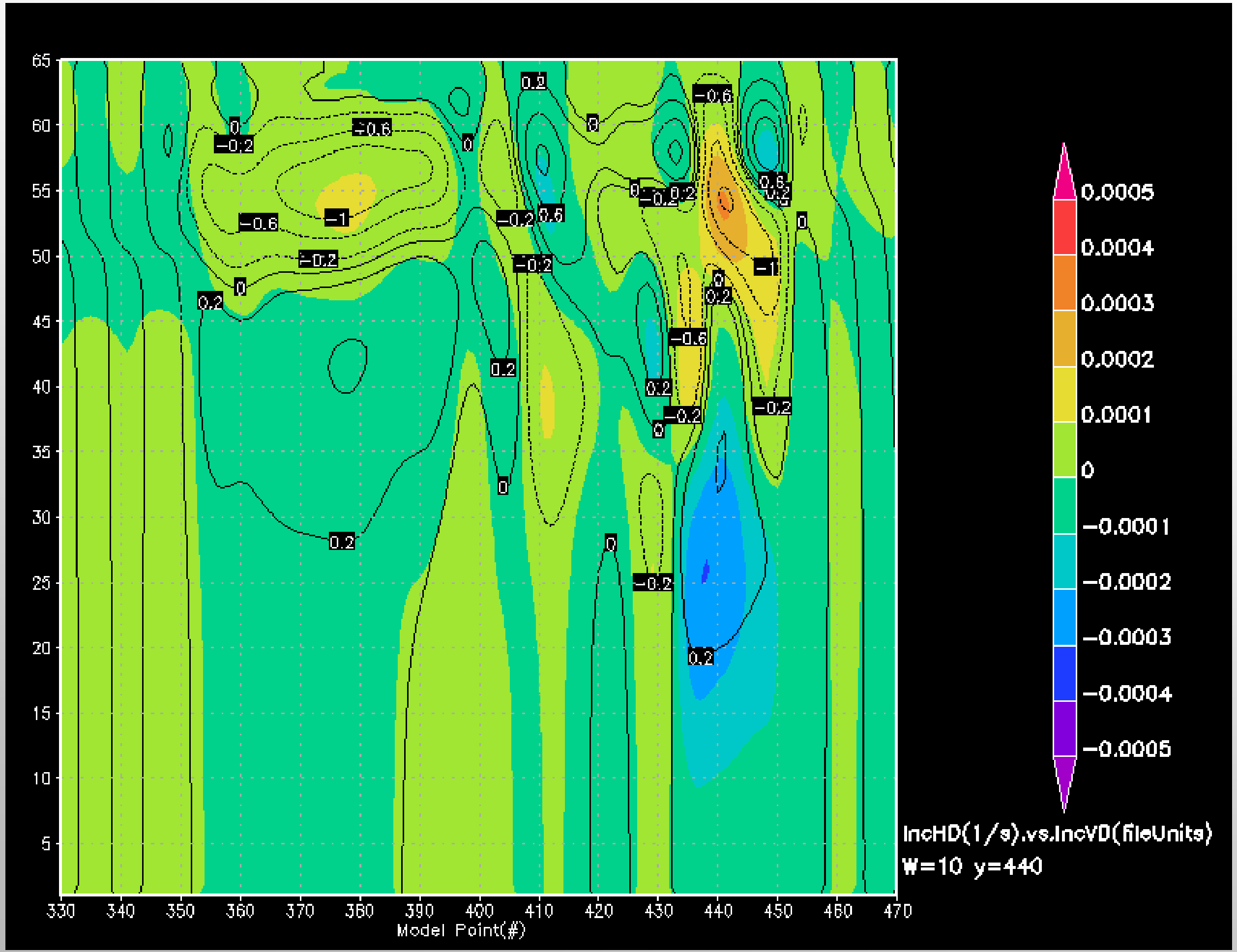
LEFT W=1 ; RIGHT W = 10 + scale factor ; Contours are Analysed Fields, Shaded Field is the “Truth”

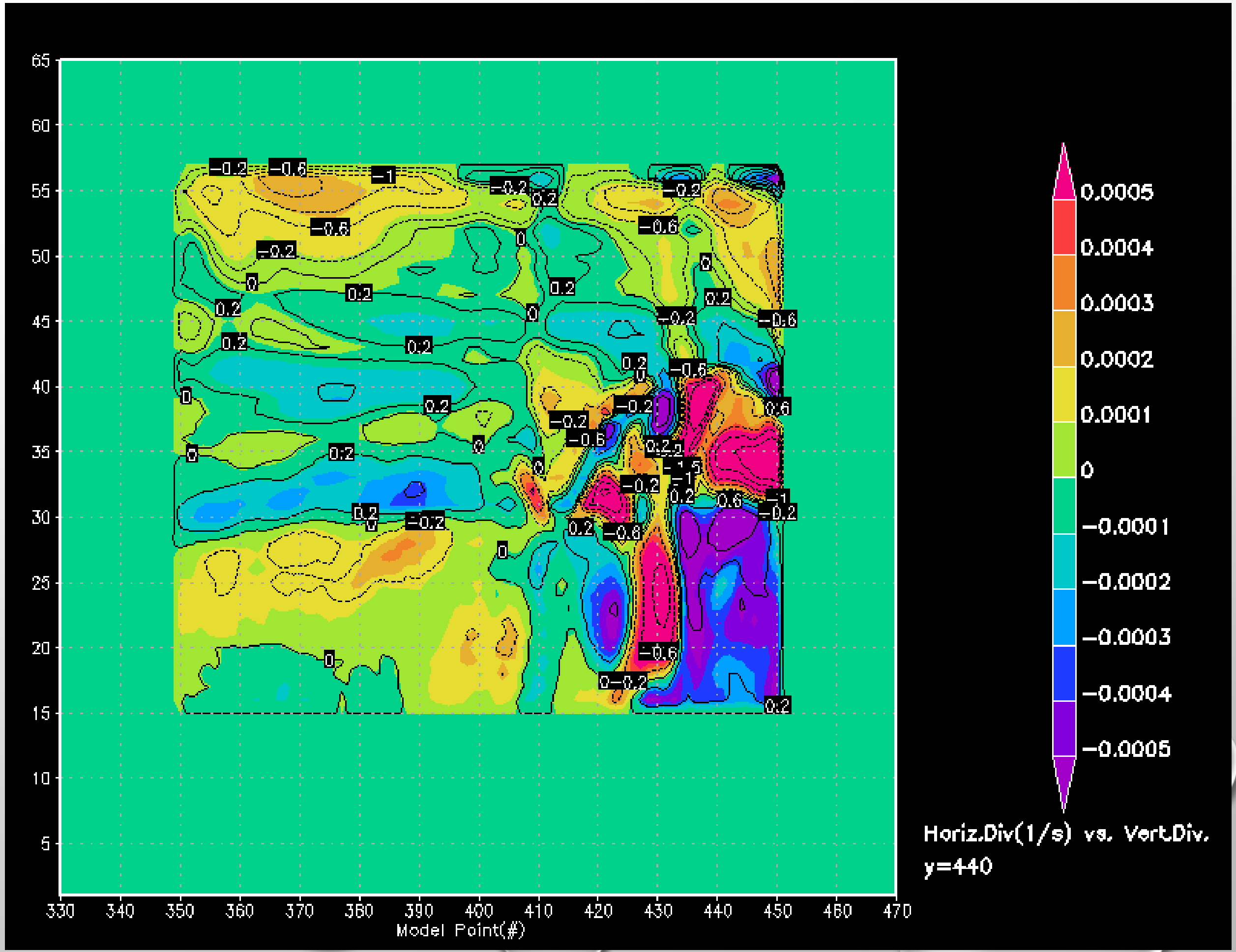




Balance between Horizontal and Vertical Divergence in Analysis from Vertical Wind Pseudo Observations

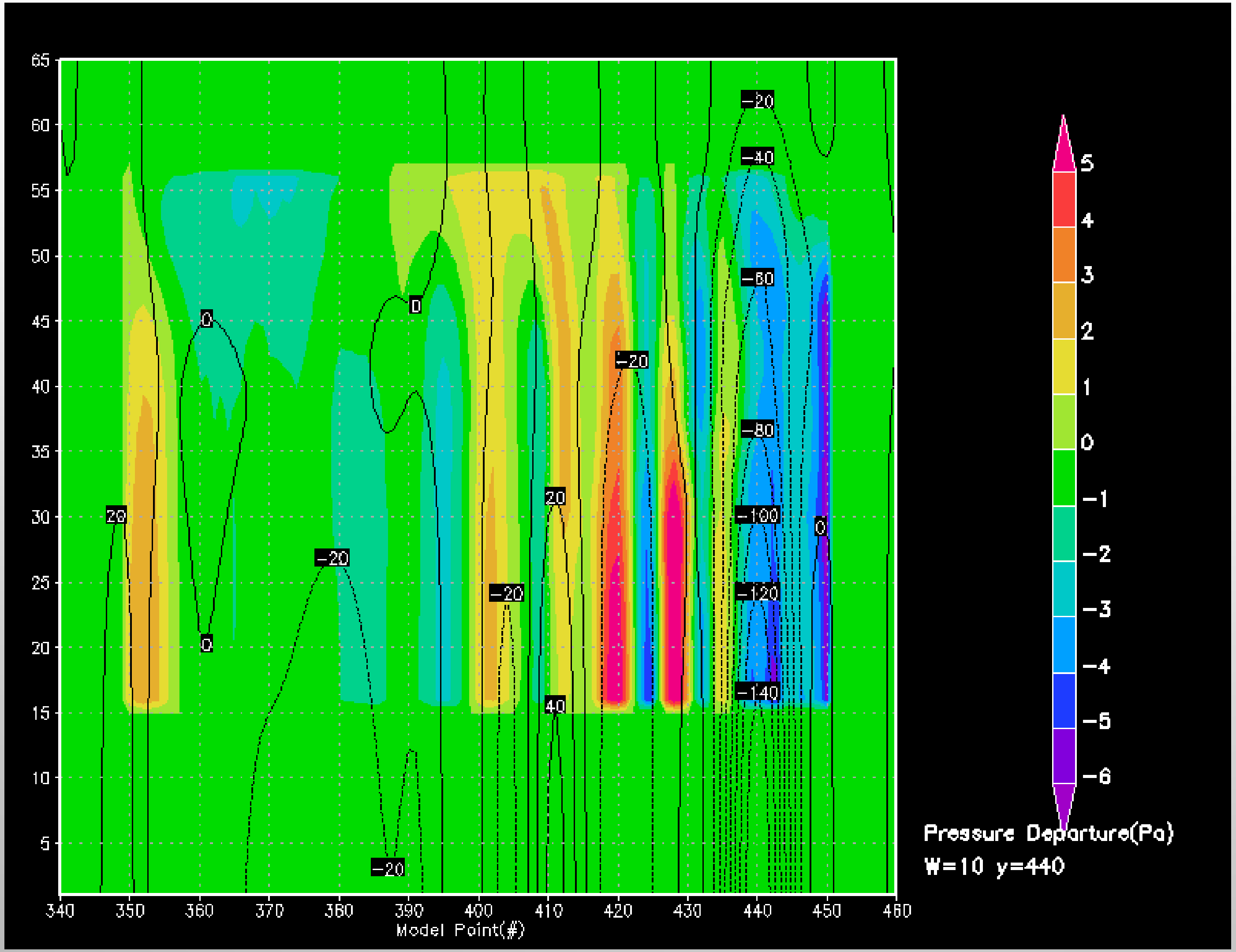
LEFT Analysed Fields ;   RIGHT Forecast Fields





Analysis of Pressure Departure from Vertical Wind Pseudo-Obs

Contours for the Pressure Departure Analysed Field, Shaded Field is the “Truth”

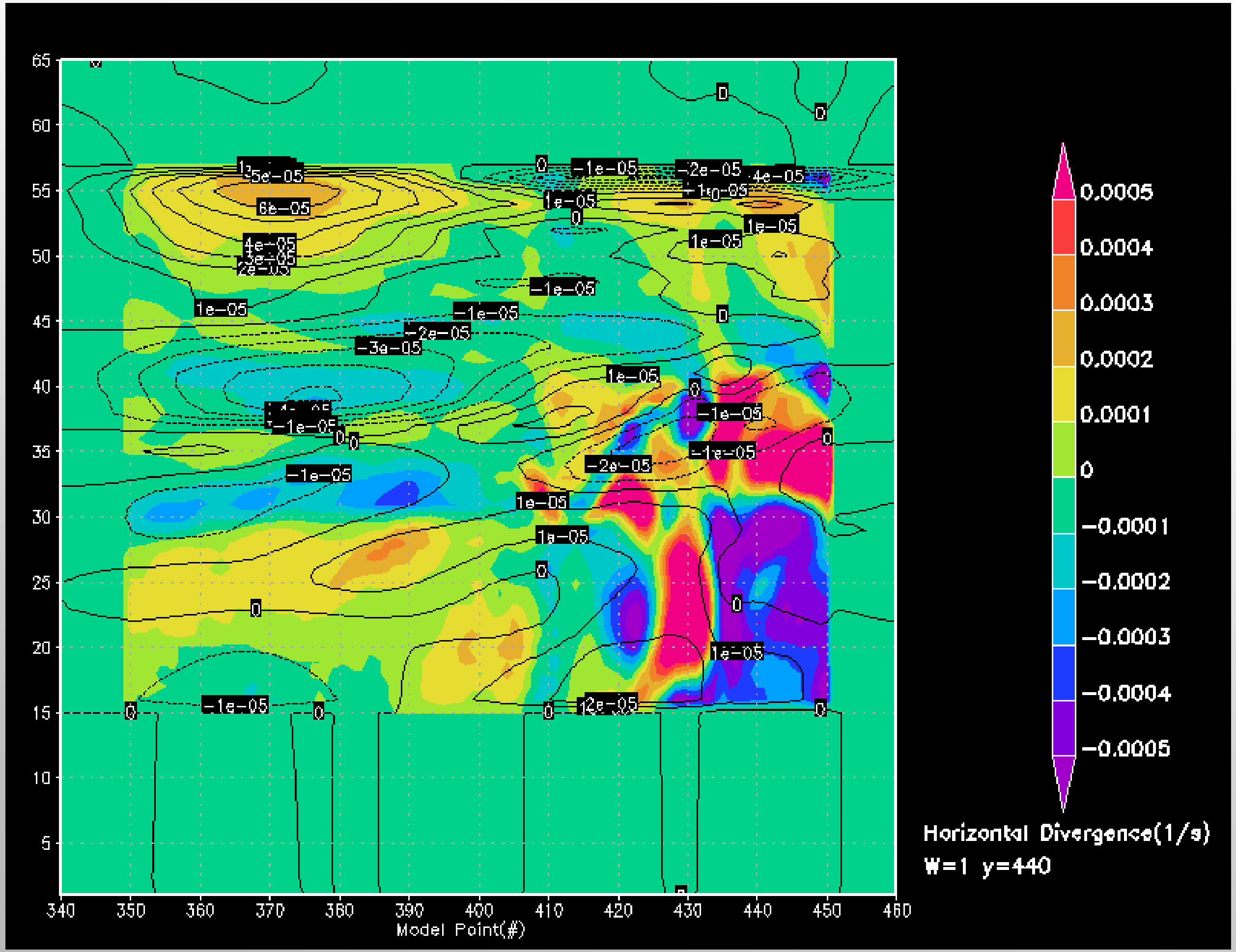


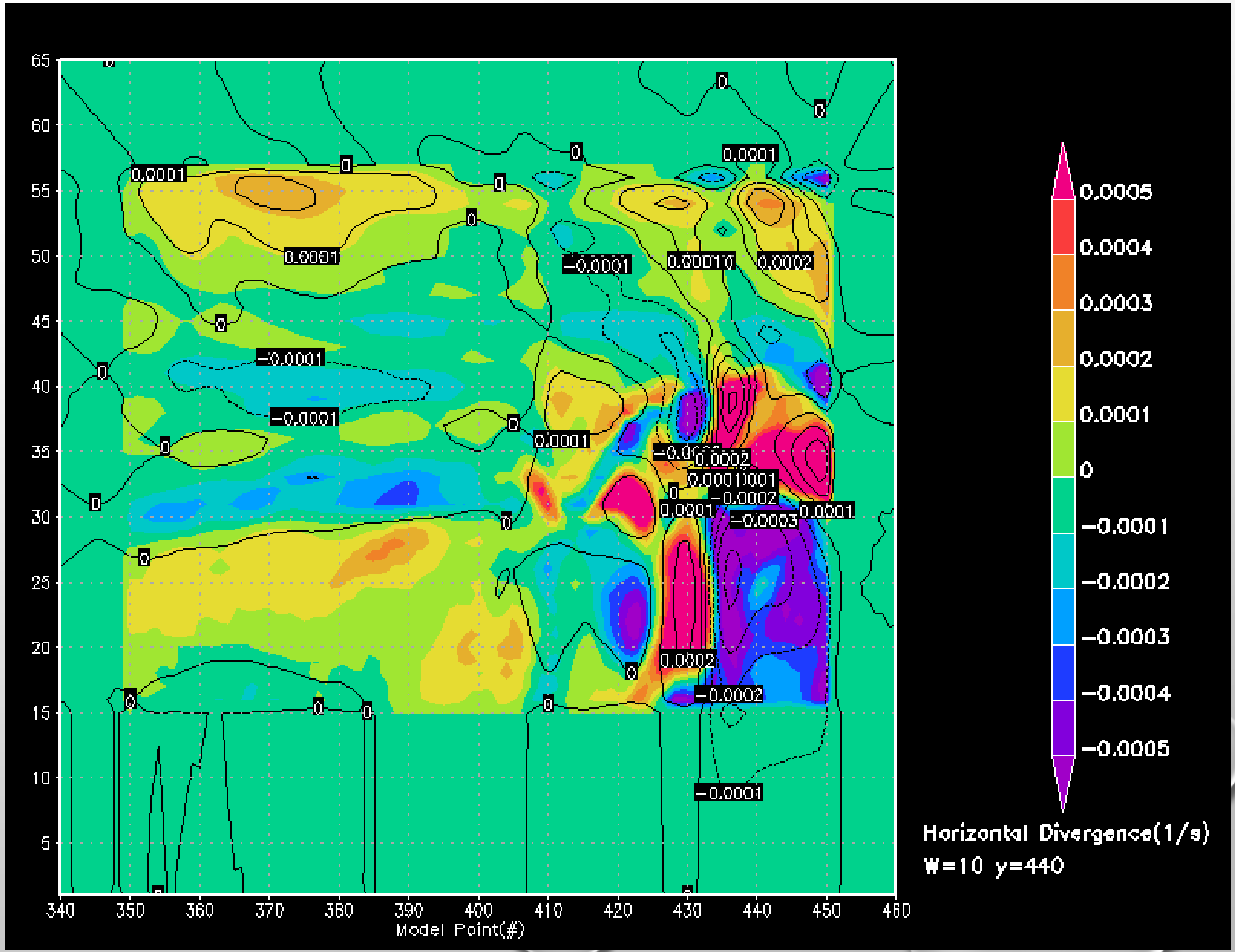
Analysis *from Vertical Wind (Pseudo)Observations*

- Analysis horizontal divergence fields is correct
- Surface pressure field is reasonably well analysed
- Analysis of pressure departure field is also acceptable
- Analysis of temperature is based on compressibility balance, which is second order as compared with diabatic processes
- Analysis of wind field contains only the divergent component

Analysis of Horizontal Divergence from Wind Pseudo-Obs

LEFT W=1 ; RIGHT W = 10 + scale factor ; Contours are Analysed Fields, Shaded Field is the “Truth”





Analysis of Vertical Velocity from Wind Pseudo-Obs

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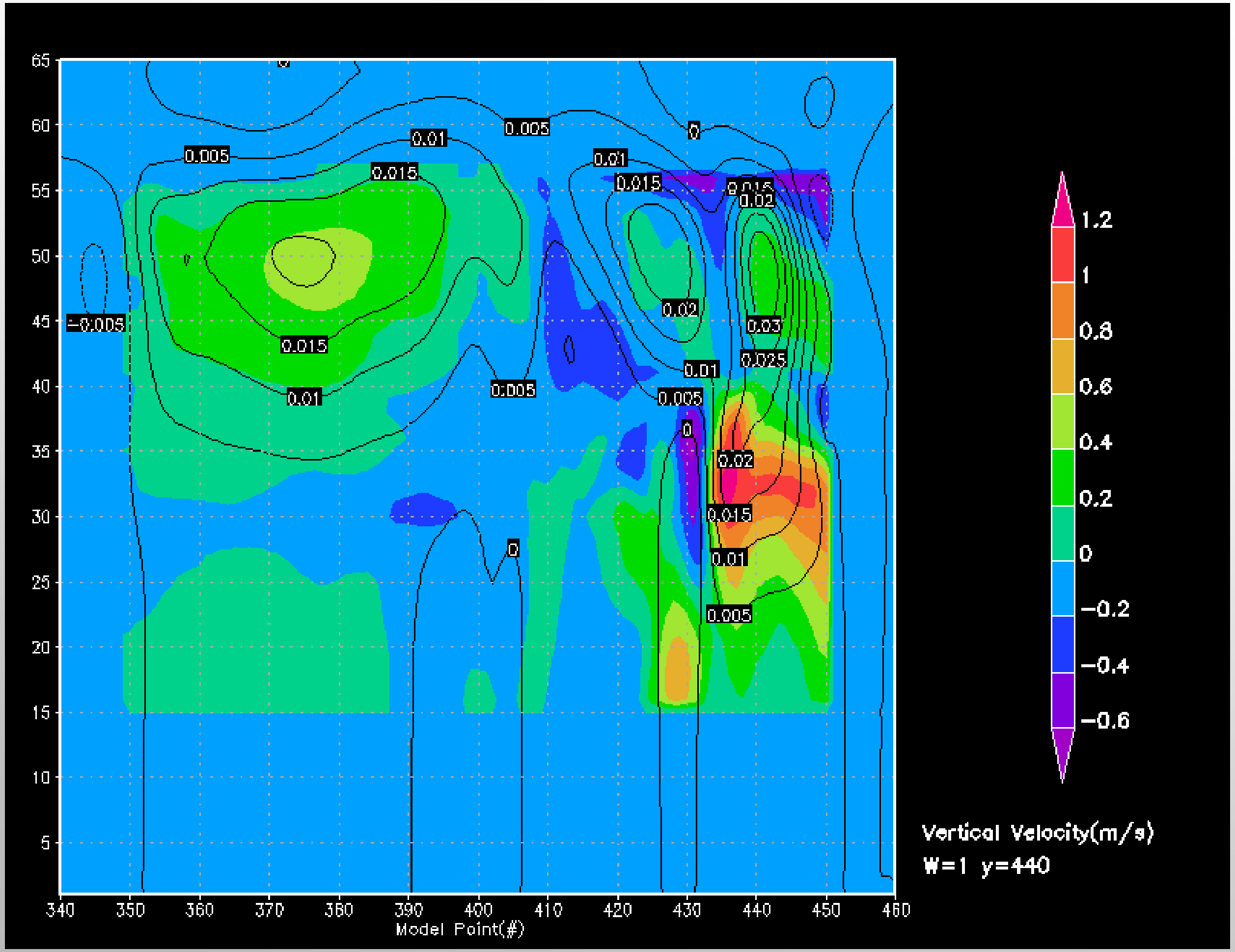
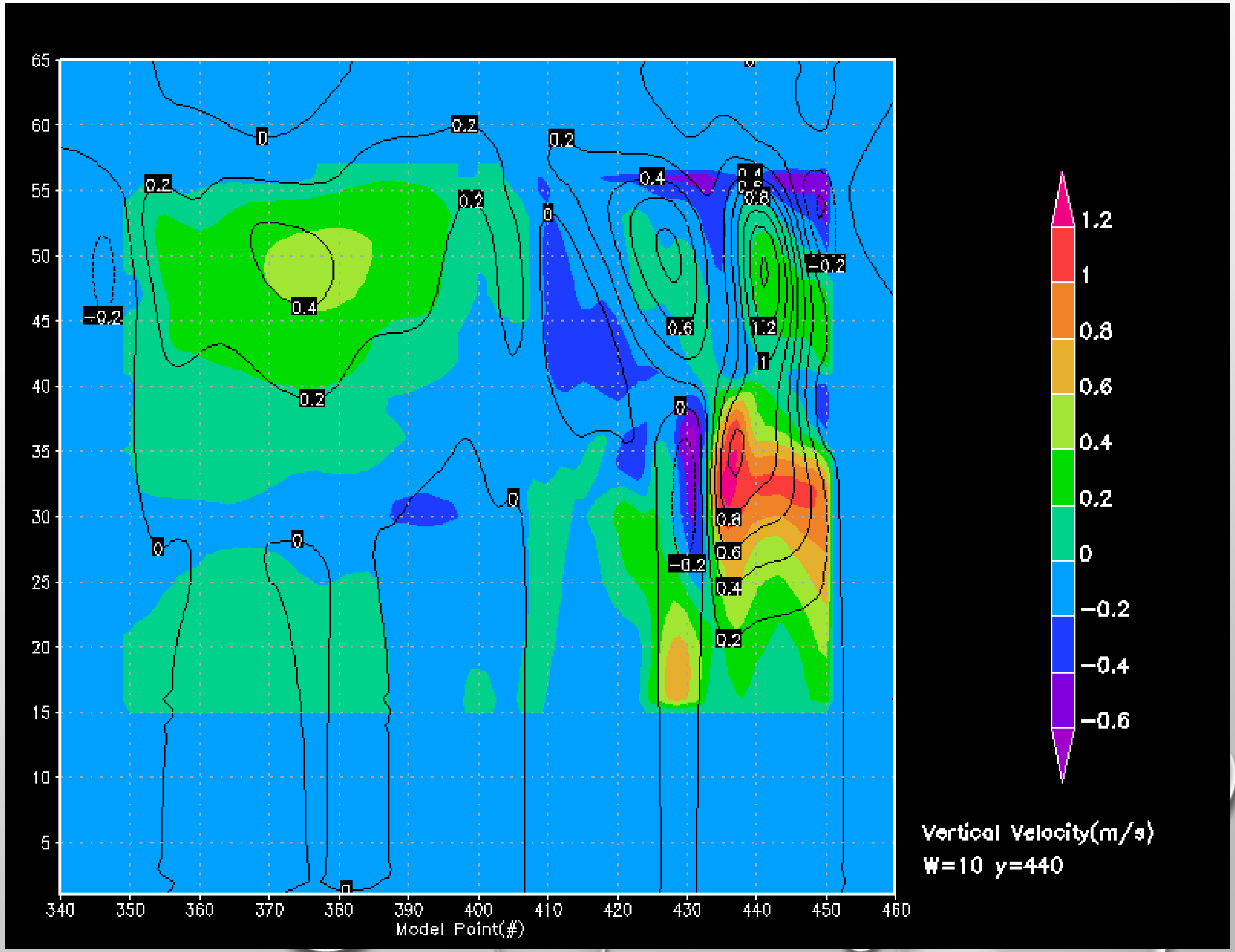
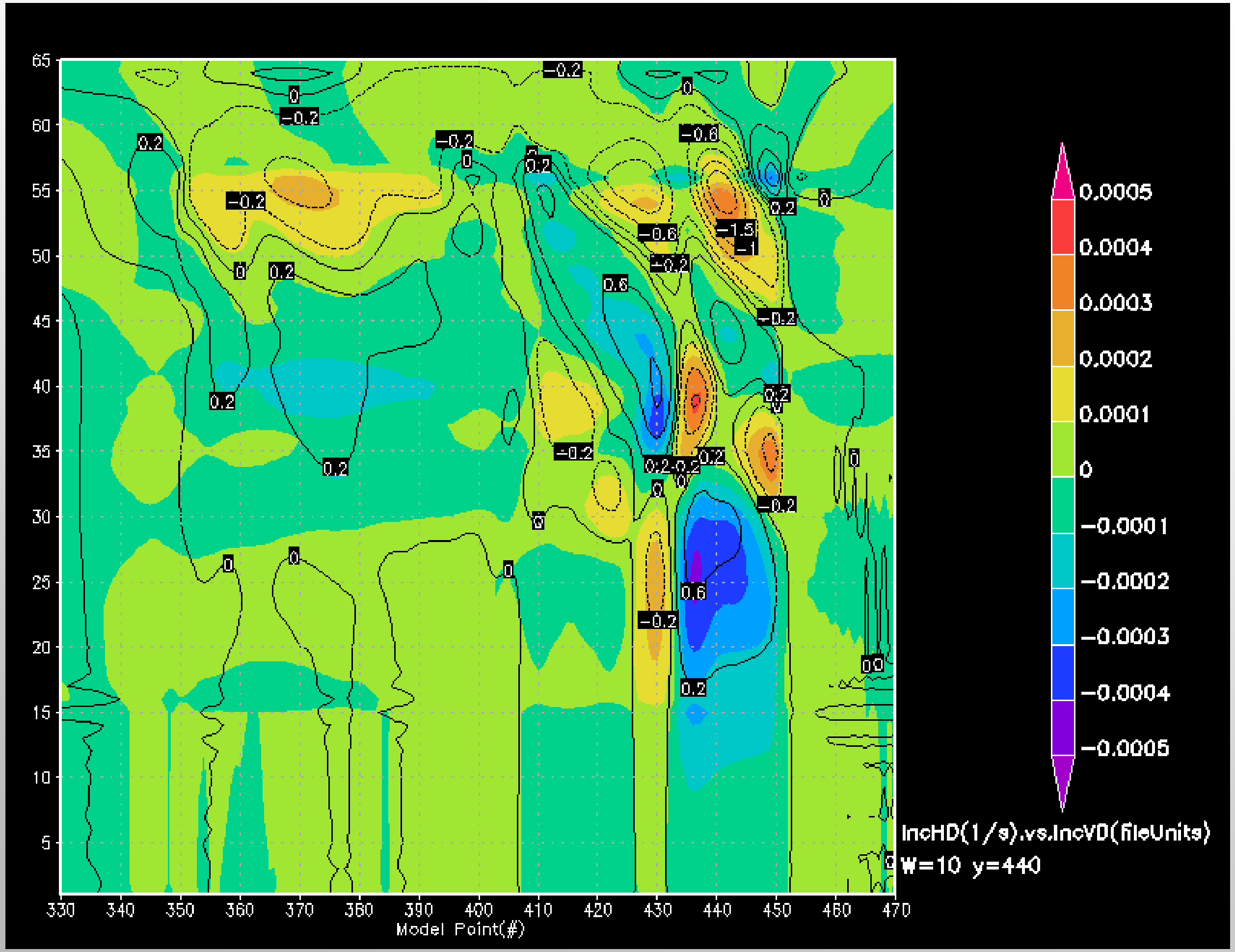
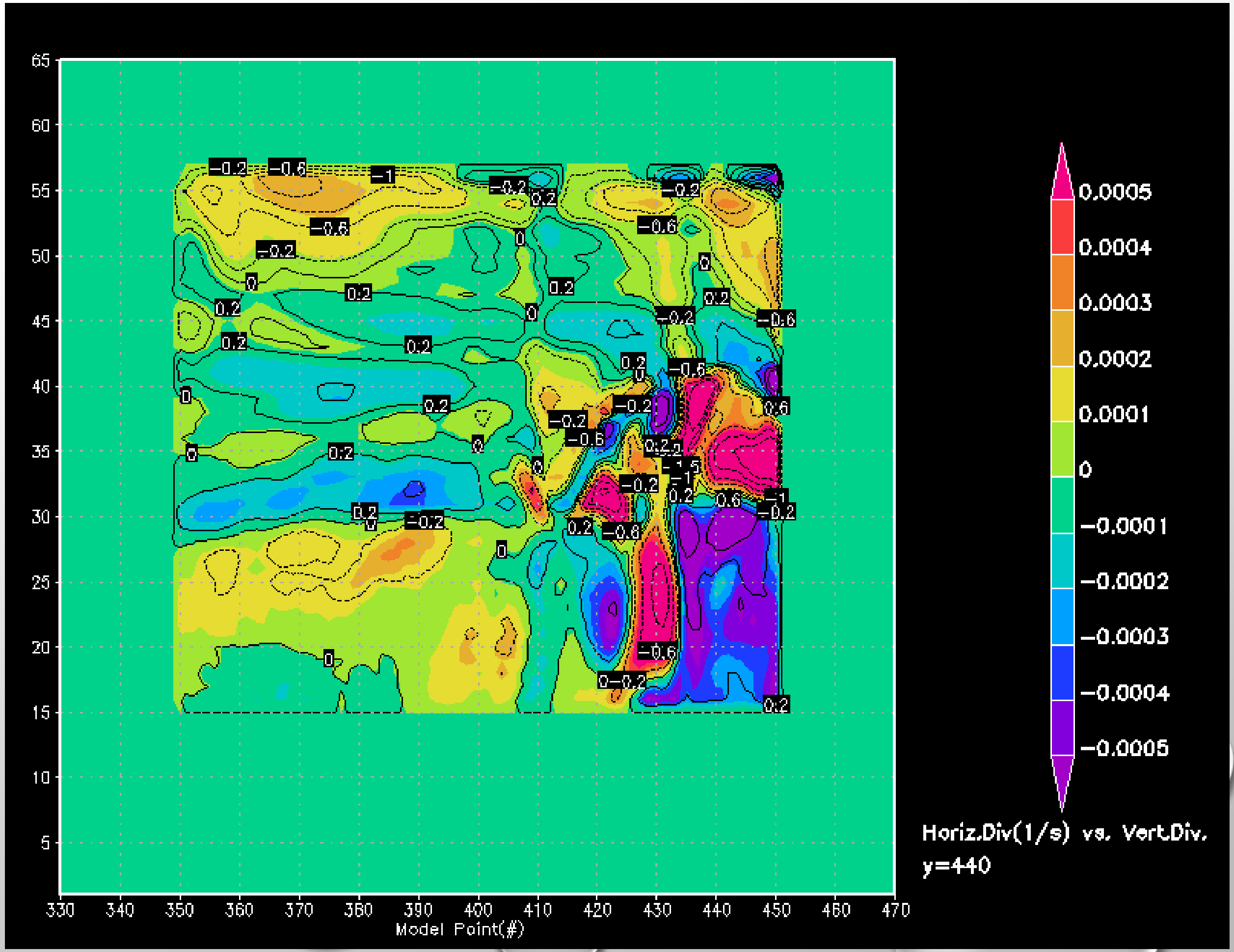




Balance between Horizontal and Vertical Divergence in Analysis from Wind Pseudo Observations

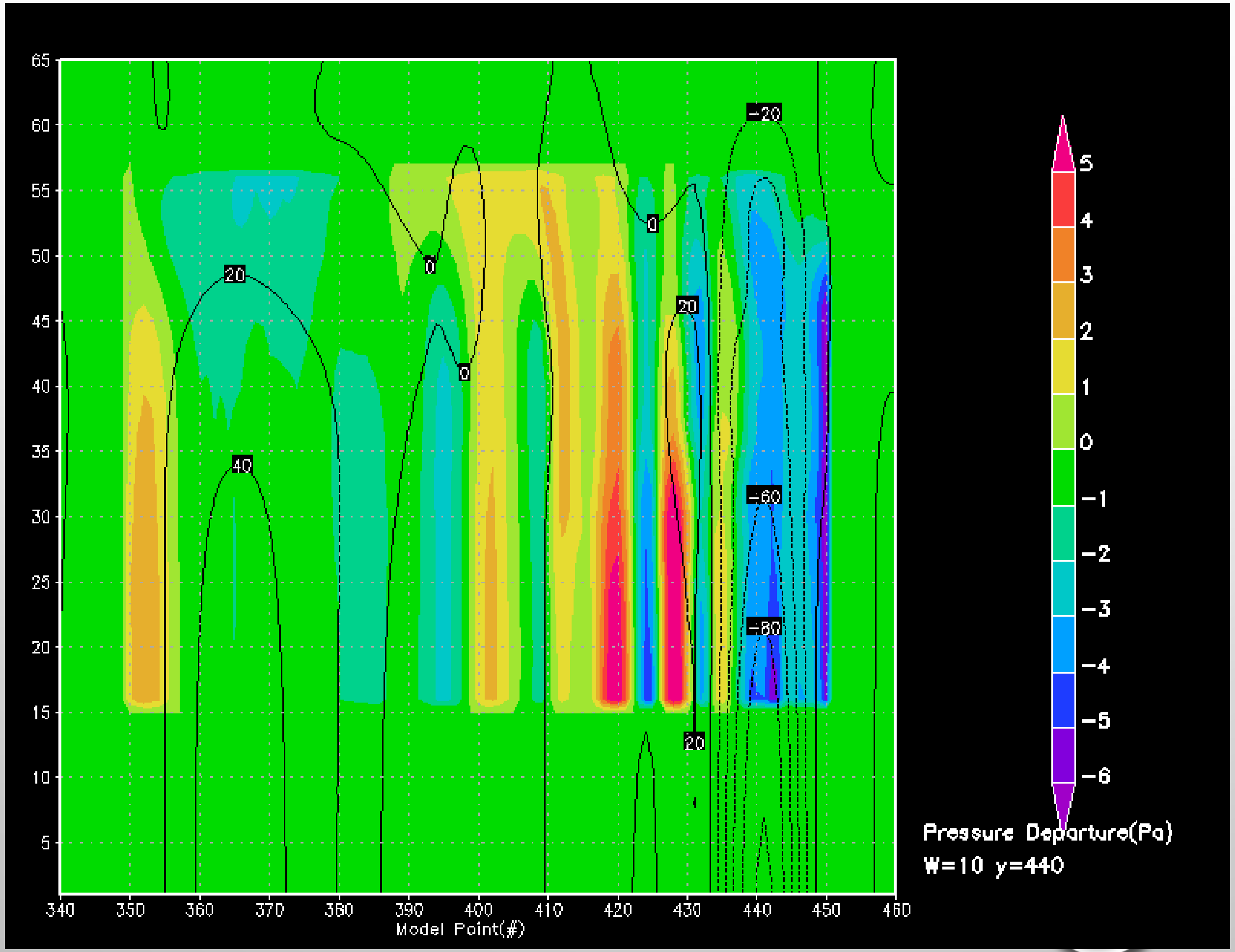
LEFT Analysed Fields ;   RIGHT Forecast Fields





Analysis of Pressure Departure from Wind Pseudo-Obs

Contours for the Pressure Departure Analysed Field, Shaded Field is the "Truth"



Analysis *from Wind (Pseudo)Observations*

- Analysis vertical wind fields is correct
- Surface pressure field is reasonably well analysed
- Analysis of pressure departure field is also acceptable
- Analysis of temperature is based on compressibility balance, which is second order as compared with diabatic processes
- Analysis of wind field is correct

IMPLEMENTATION ON CY40

- The VC functionality is **integrated in the FA software** and built by MakeUp as an utility

- The VC functionality runs as an **alternative to the DF (Digital Filter)** functionality

- In the light of tests with pseudo-obs, the following (**free**)**parameters** are included:

  ✓ Relative weight between obs-forcing and constraints
  ✓ Scale factor for the vertical velocity filtered increments
  ✓ Damping factor for pressure departure increments
  ✓ Switch to enable filtering of horizontal vorticity. The filter is designed from the filtering performed on the horizontal divergence

TESTS WITH LETKF ANALYSIS

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TEST WITH LETKF

- HARMONIE-AROME contains LETKF code to perform atmospheric analysis. It has been developed by Mats Hamrud (ECMWF) and adapted to LELAM and tunned by Pau Escribà and Jelena Bojorova from HIRLAM consortium

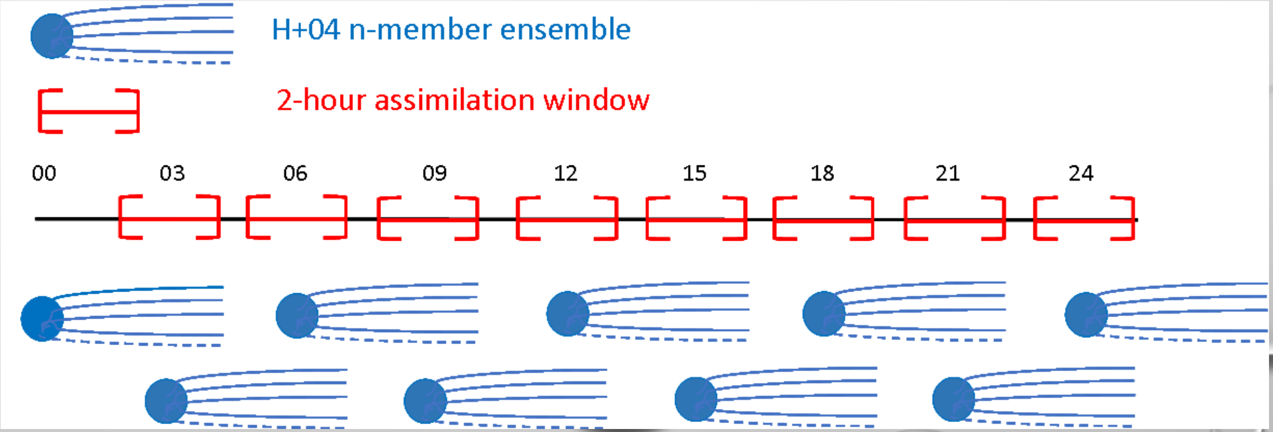
- Unbalances with initialization of LETKF analysis are present, in line with expectations (Hunt et al, 2006). Can this new VC functionality solve these problems ?

- Some bug-fixing and (partial) parameter tunning has been done as experiments have progressed (very preliminary results)

TEST WITH LETKF

•Basic characteristics of LETKF experiments are:

- Low frequency ass. cycle (3Hours)
- Only conventional observations (~$3*10^3$ /cycle) assimilated
- No surface analysis
- Small ensemble (10 members)
- Scaled Lagged Averaging Forecasts (SLAF) as LBCs
- Localization: horizontal (500 Km) and vertical log (p/p_s)~0.5 (both gaussian tapered)



TEST WITH LETKF CHKEV0 diagnostic tool

- Red line gives evolution of pressure tendency at Time-step resolution for an integration started from a 3H Forecast (“perfect balancing”)

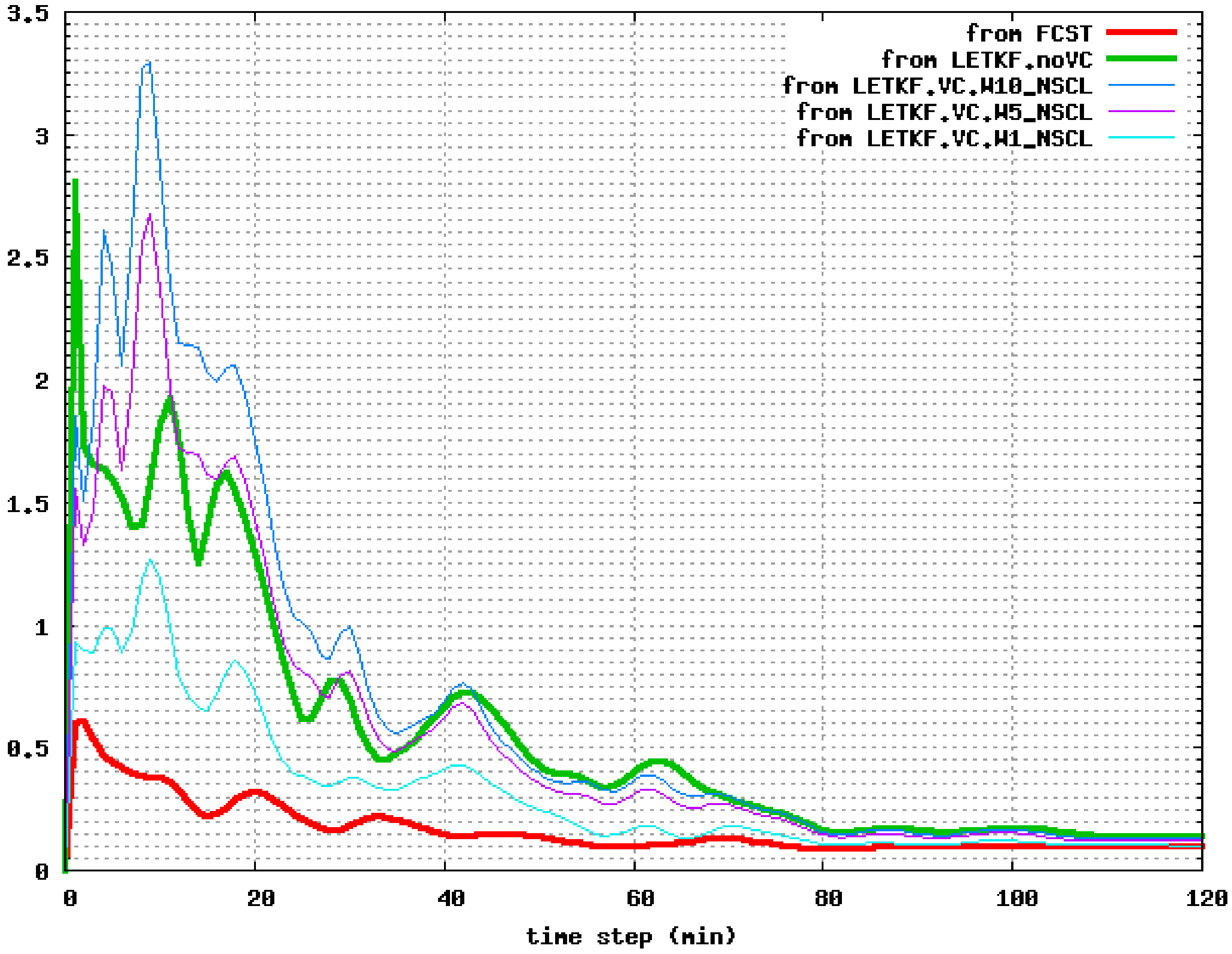
- Green line gives this evolution for an integration started from the LETKF analysis. Unbalances are apparent

- The other lines show the evolution for an integration started from initial conditions computed as :

FG + VCFilt [LETKF_analysis – FG]

where VCFilt [ ] denotes a filter constructed using a Variational Constraints technique. These constraints come from SI-NH dynamics

The three lines differ in the relative weight (W=10, 5, 1) between “raw increment forcing” (i.e. LETKF – FG) and SI constraints. The bigger is W, the smaller the relative weight given to these constraints



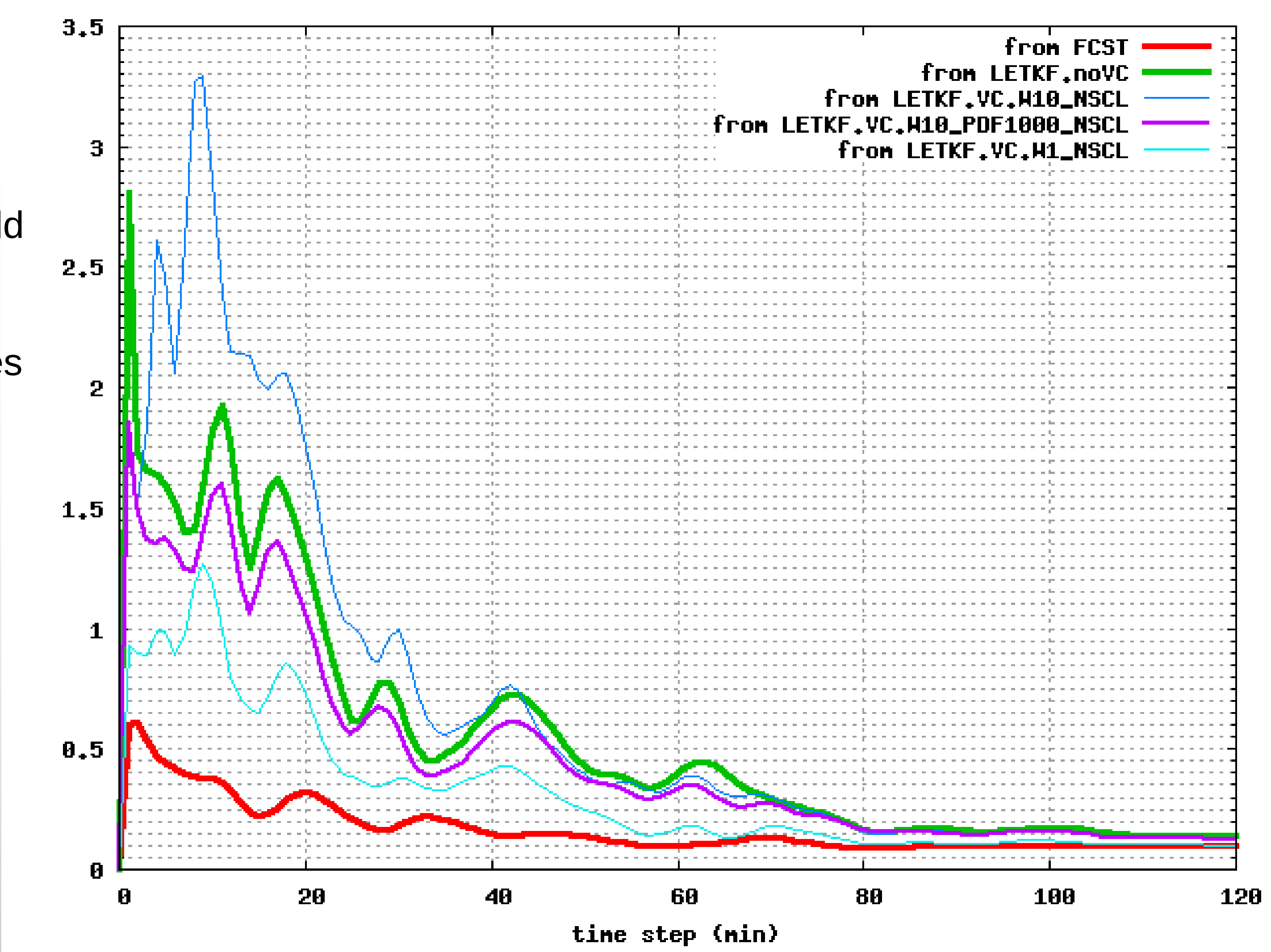
TEST WITH LETKF CHKEV0 diagnostic tool revisited

- When the PD increment field is damped out, the "spurious" oscillations are greatly attenuated, pointing out that the origin of the remaining unbalances in the VC analysis can be connected to the way this PD field is analysed

- The close match between the purple and green curves is surely due to the tight fit (W=10) to the raw LETKF analysis increments

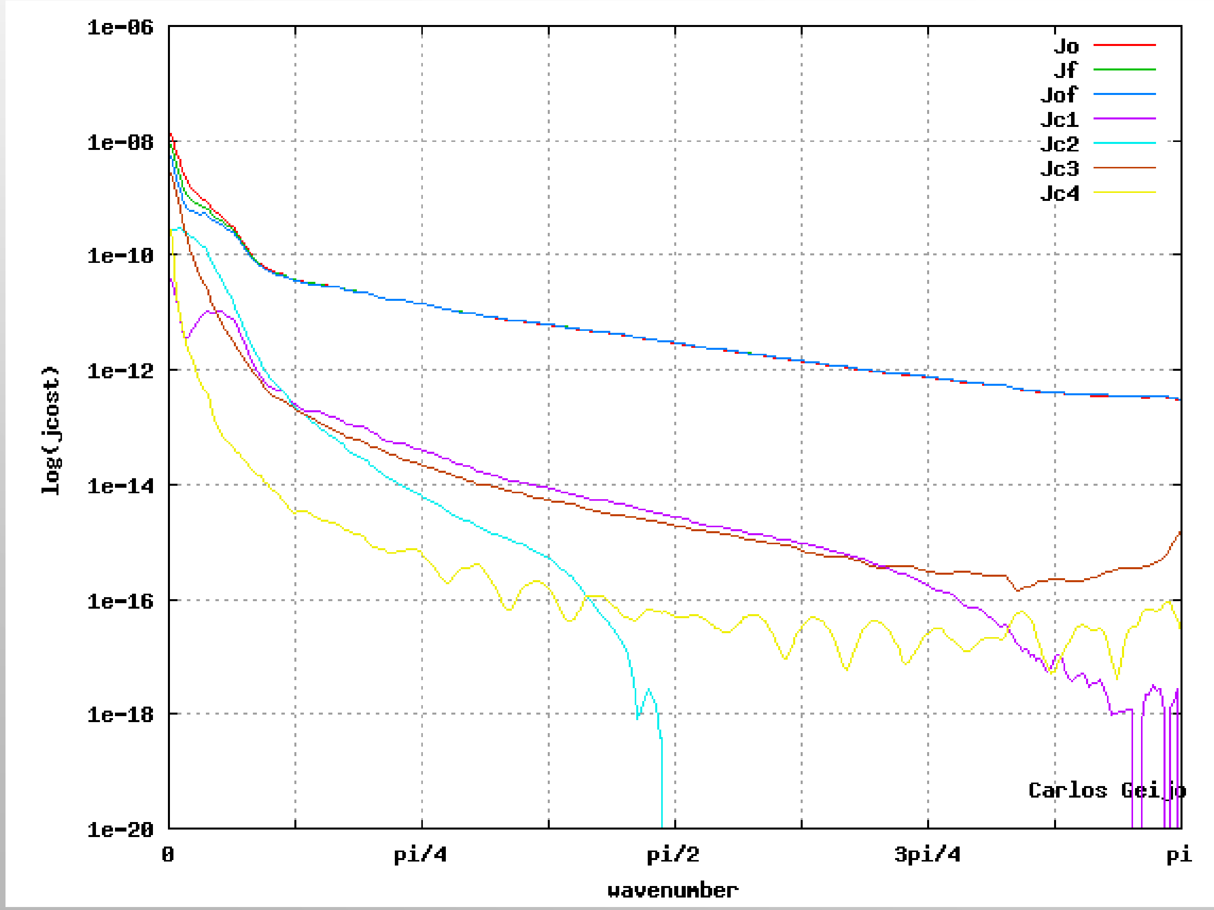
- It is an open question whether these "spurious" oscillations are in fact produced by differences in the numeric schemes used to run the model forward (Finite Differences) and that used to obtain the VC analyses (Green Functions)

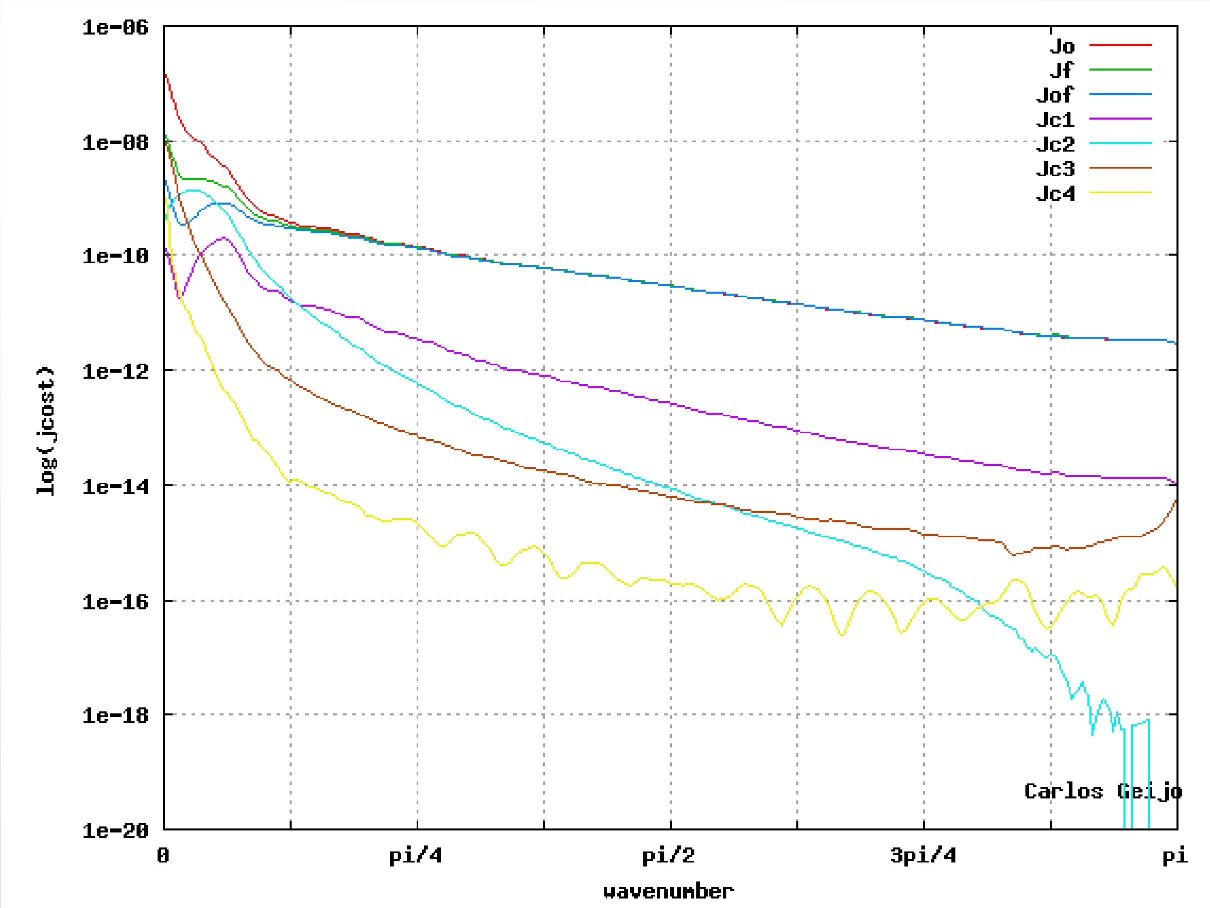
- It is also to be elucidated which VC analysis parameters yield better verification scores



TEST WITH LETKF COST FUNCTION minimization

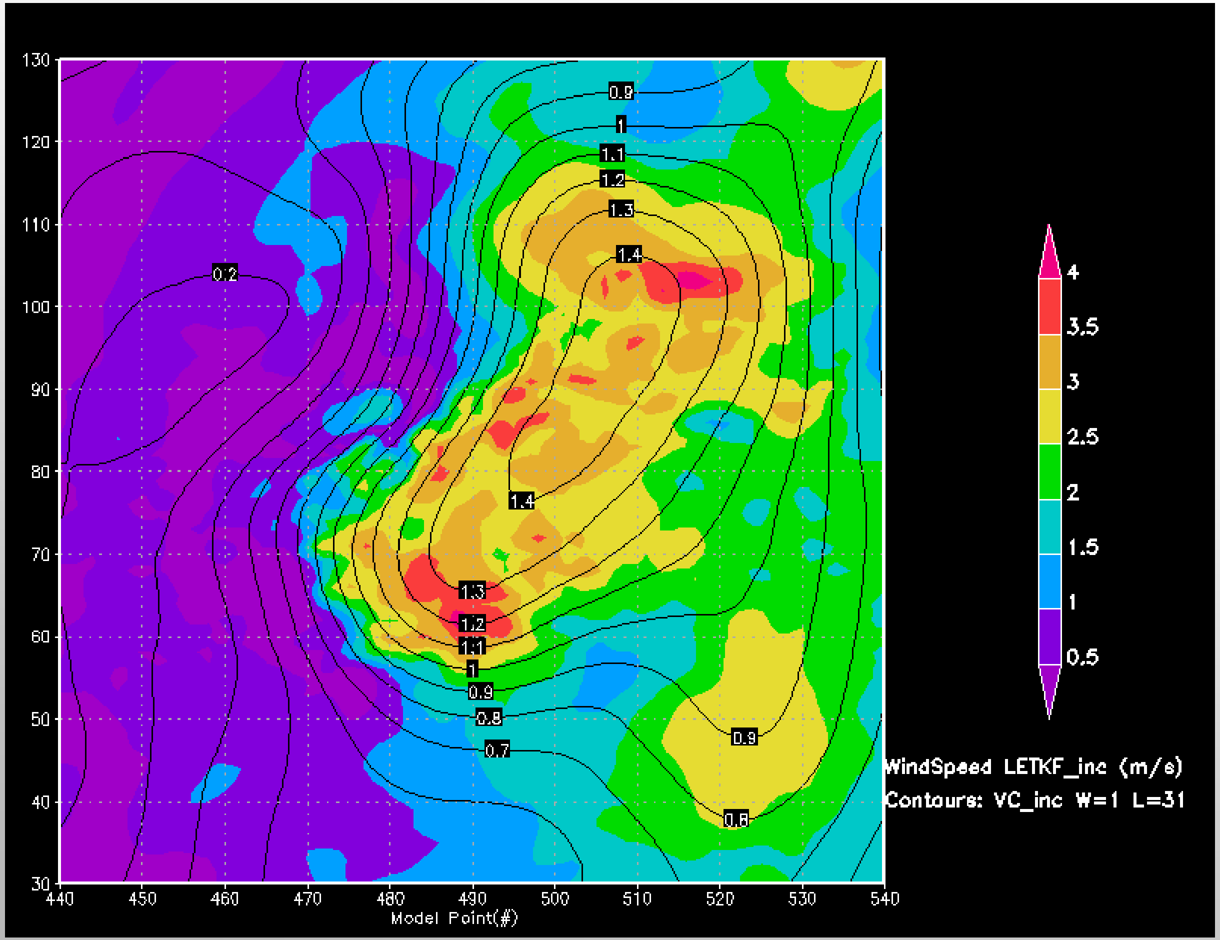
- These figures show the spectral distribution for the values of the different terms in the cost function to be minimized by the VC algorithm. Note the logarithmic scale
- Jo is the initial value for the LETKF increments to be filtered. Jof is the corresponding final value (w=1 left, W=10 right)
- Jc1 is for the vertical momentum constraint, Jc2 for the horizontal momentum constraint, Jc3 is the compressibility constraint and Jc4 is the surface pressure tendency constraint.

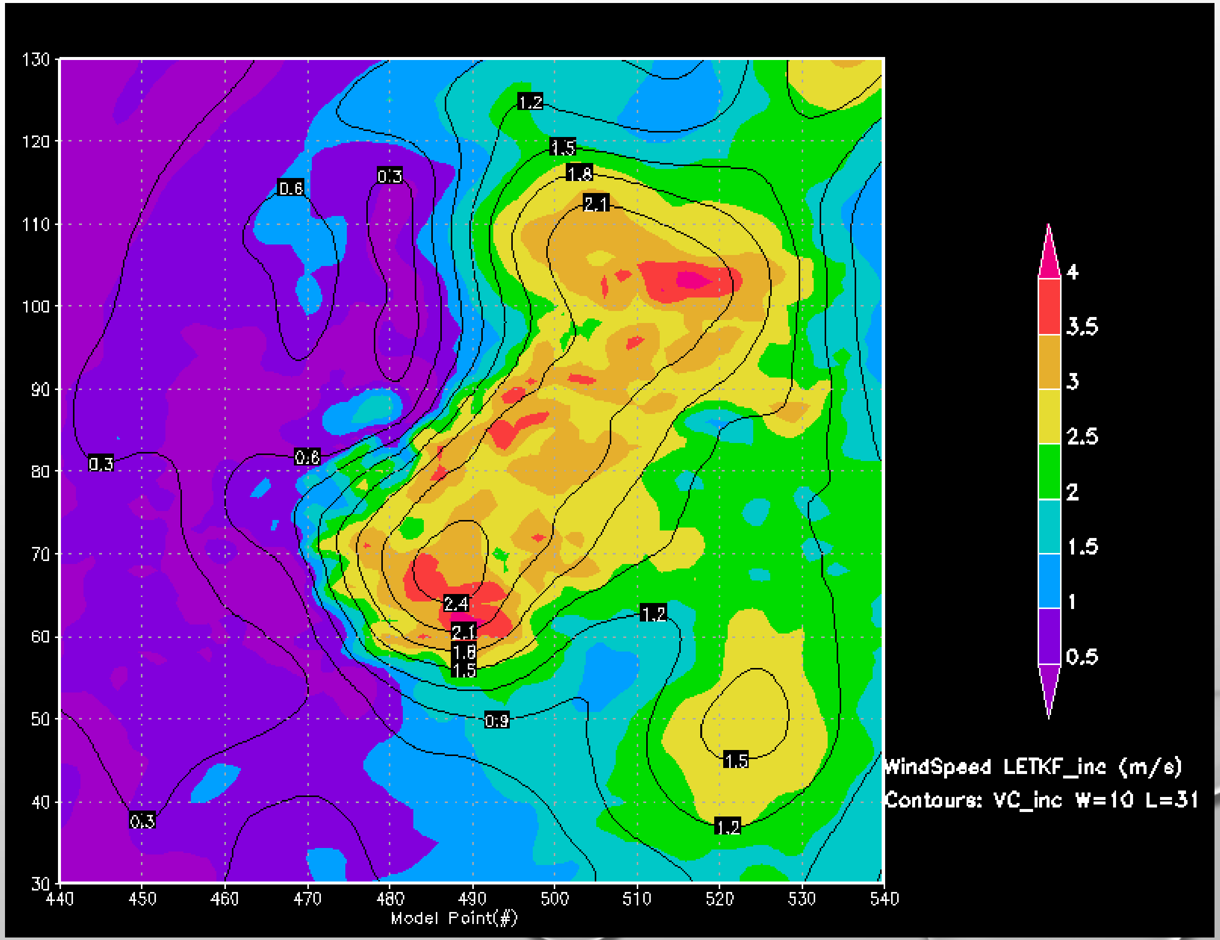




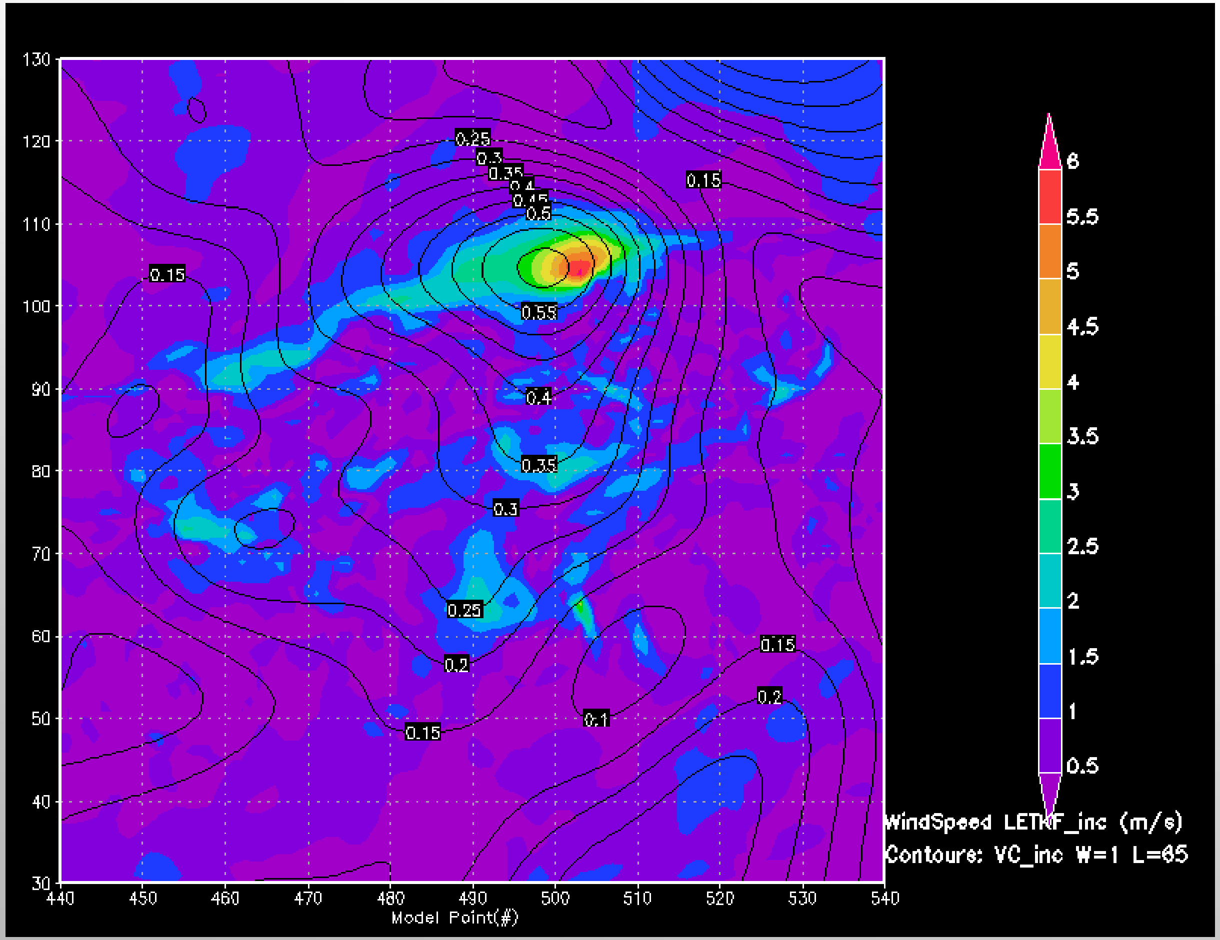
TEST WITH LETKF WIND FIELD Analysis Increments (level 31)

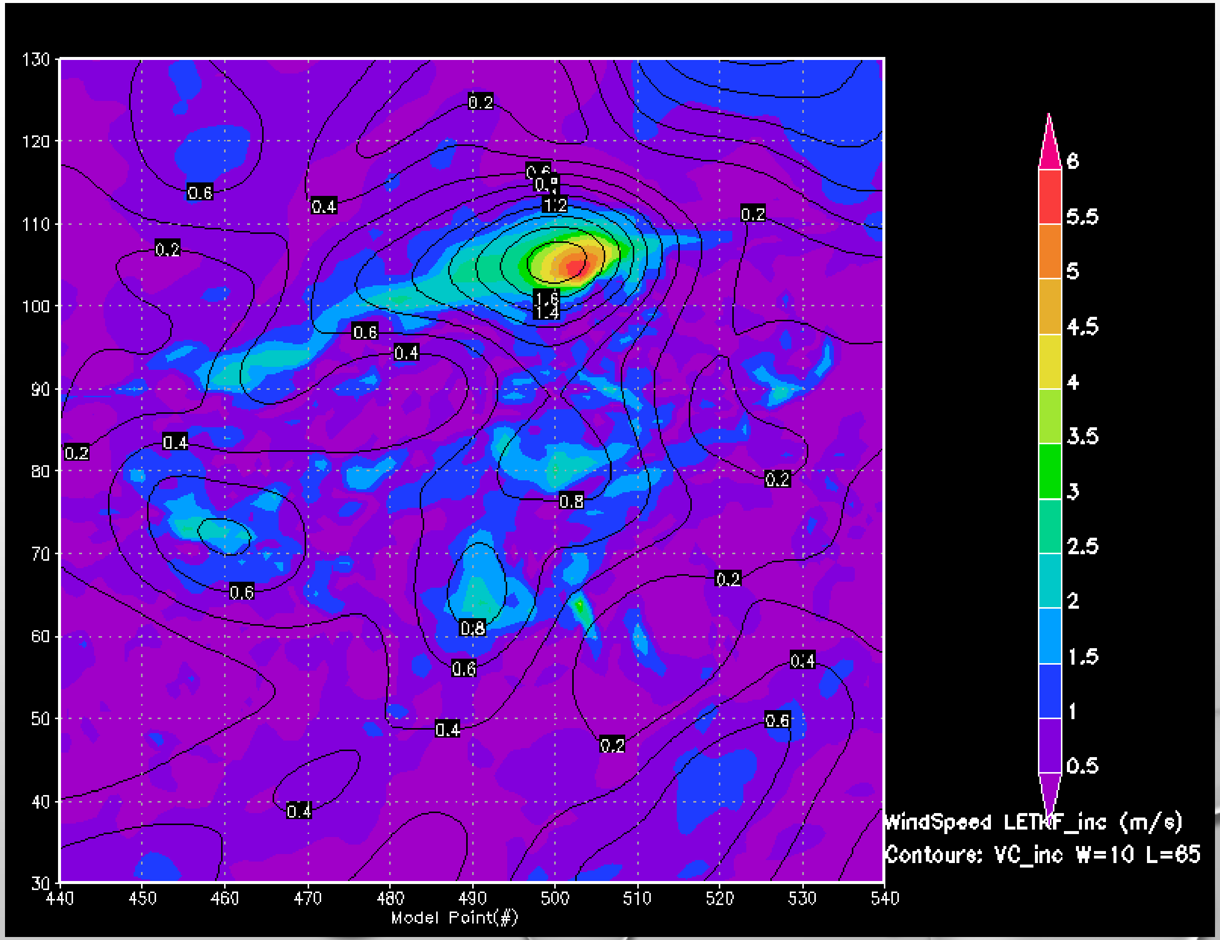
- The shaded field in the background corresponds to the "raw" LETKF wind vector difference analysis increments
- The contours in the foreground show the filtered wind vector difference increments for two values of the W parameter





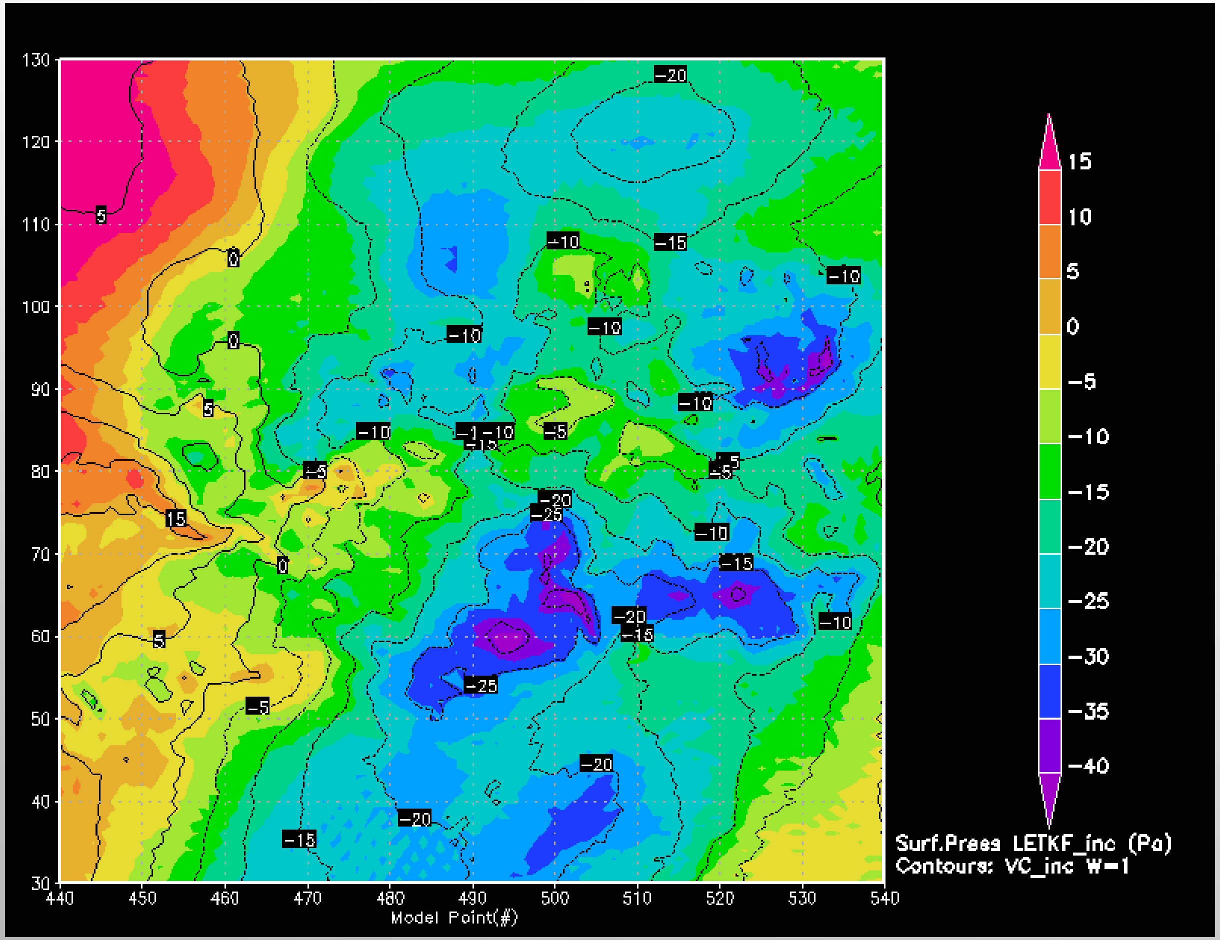
TEST WITH LETKF WIND FIELD Analysis Increments (level 65)

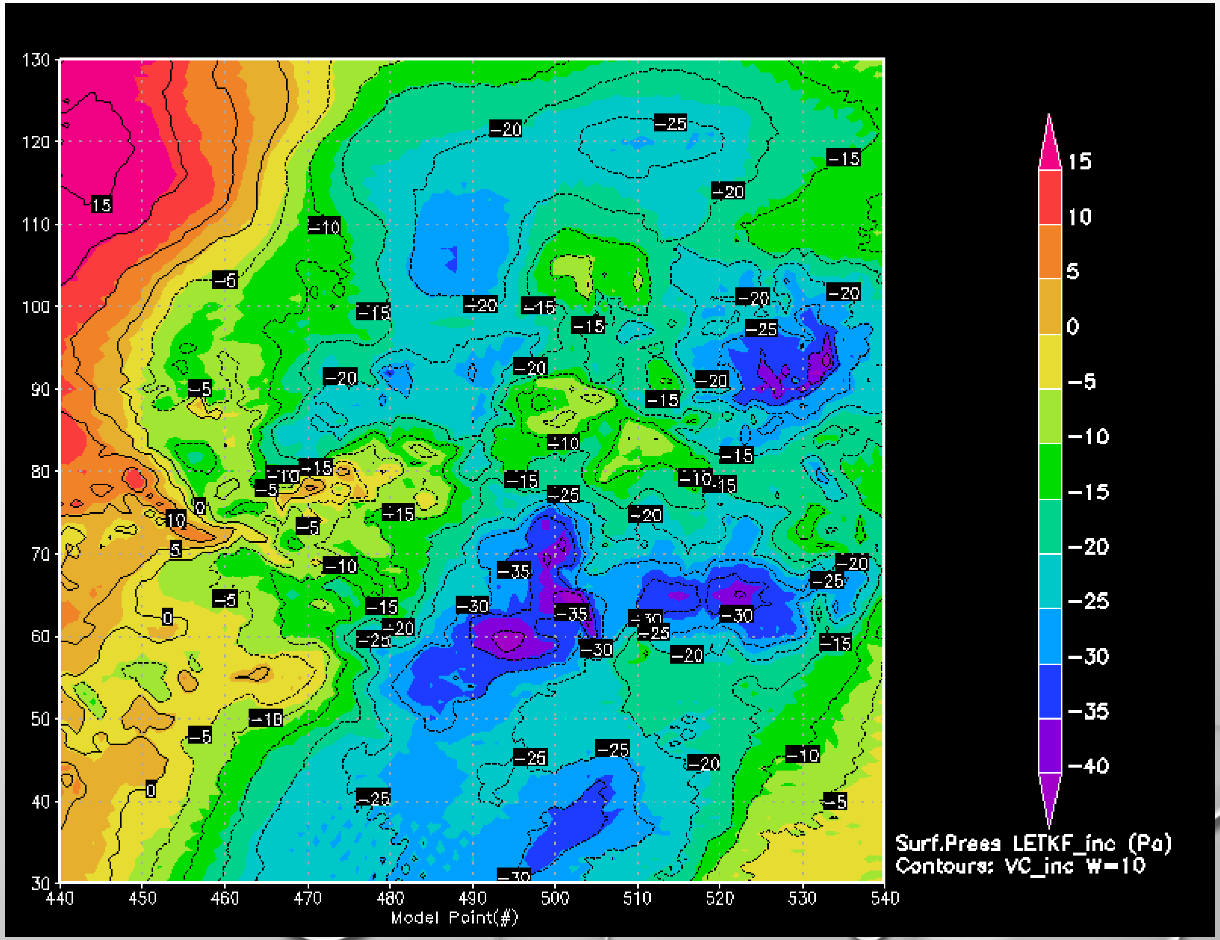




SURFACE PRESSURE Analysis Increments

- Ps enters in the SI-NH dynamics via the (linearized) tendency equation. For W=10 (right figure) the analysis closely follows the original LETKF increments. For W=1 (left) this dynamic constraint forces the Ps field to adjust to the value of net horizontal divergence in the air-column above, and this smooths out many small scale, noisy features in the LETKF increments

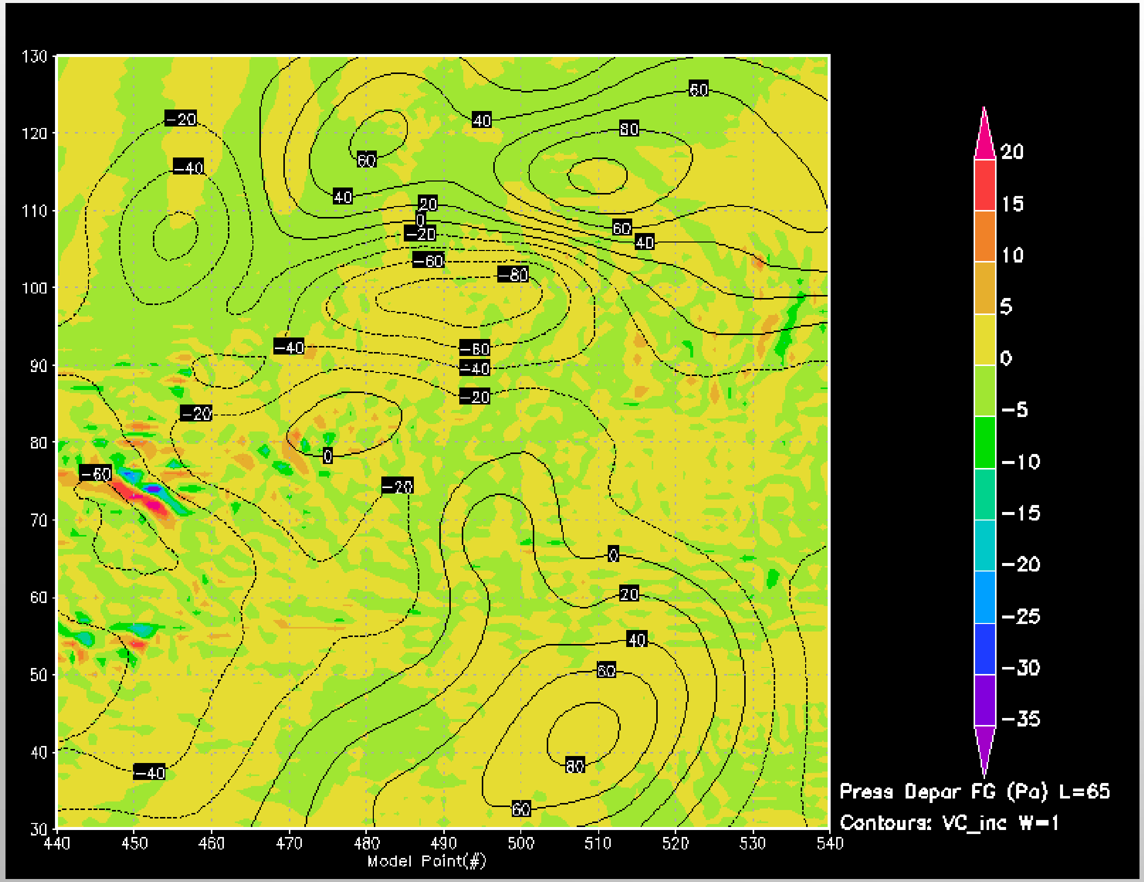


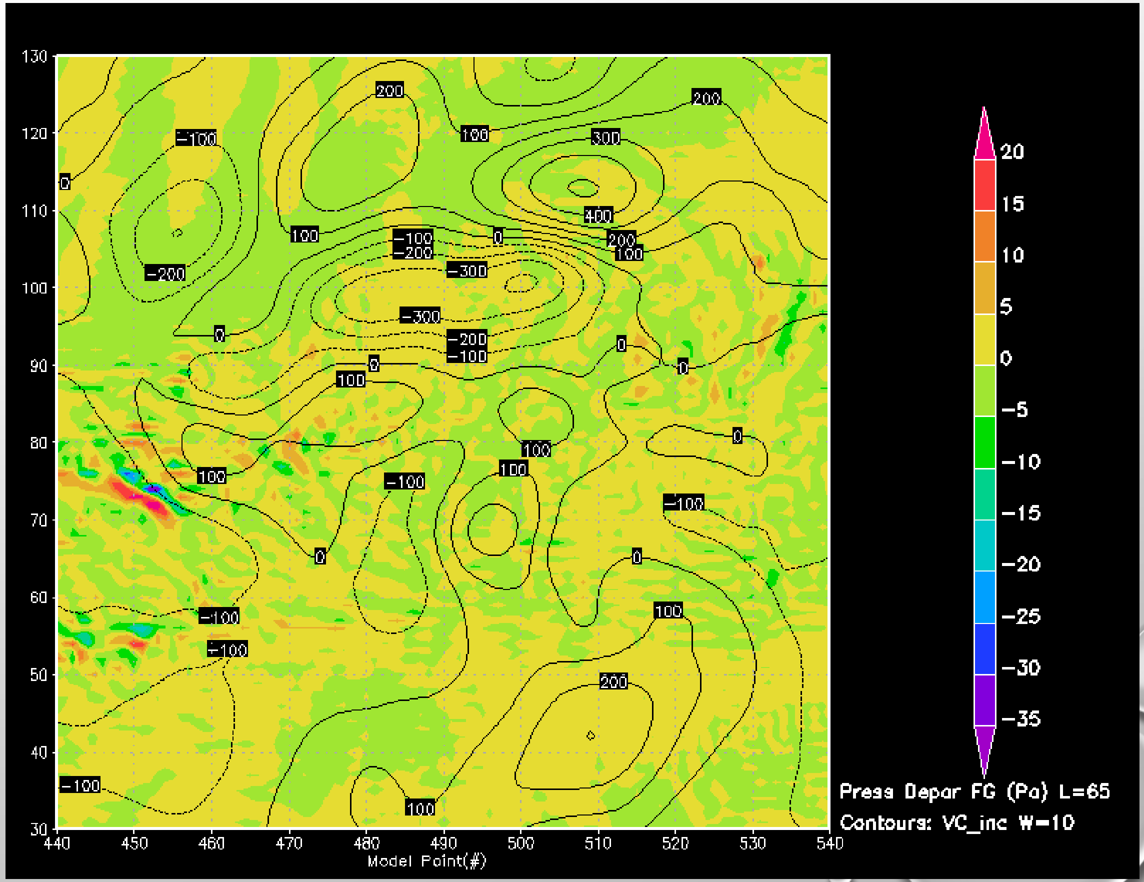


PRESSURE DEPARTURE Analysis Increments (level 65)

- The SI-NH dynamics contains the PD parameter, therefore, **the VC algorithm has the capacity to produce PD analyses**. The shaded background field in these figures does NOT correspond to LETKF increments (because this scheme does not analyse this field). The shaded background field displays now the FG field, and contours the VC increments

- Amplitude and spatial structure of PD FG fields and PD VC increments fields differ clearly. This demands further investigation.

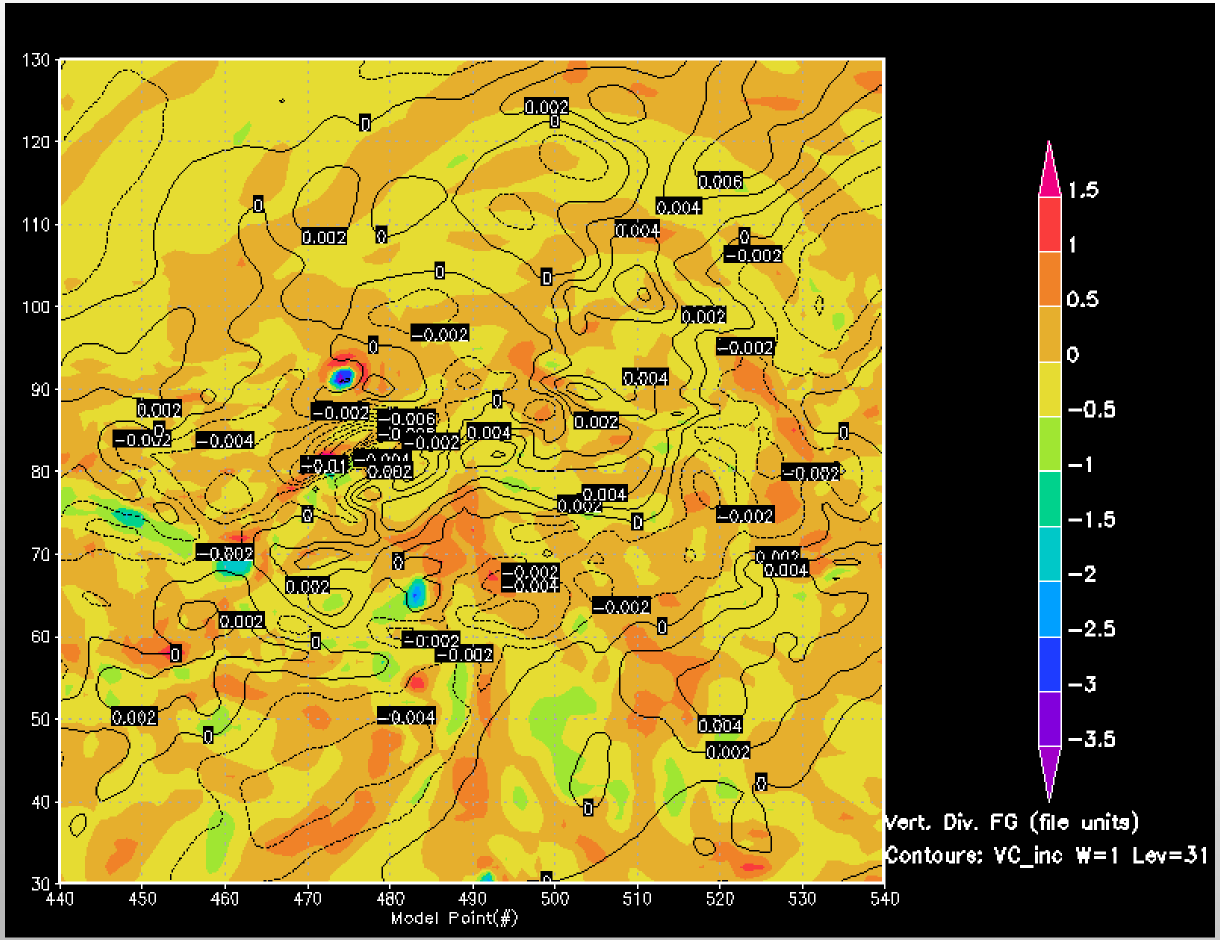


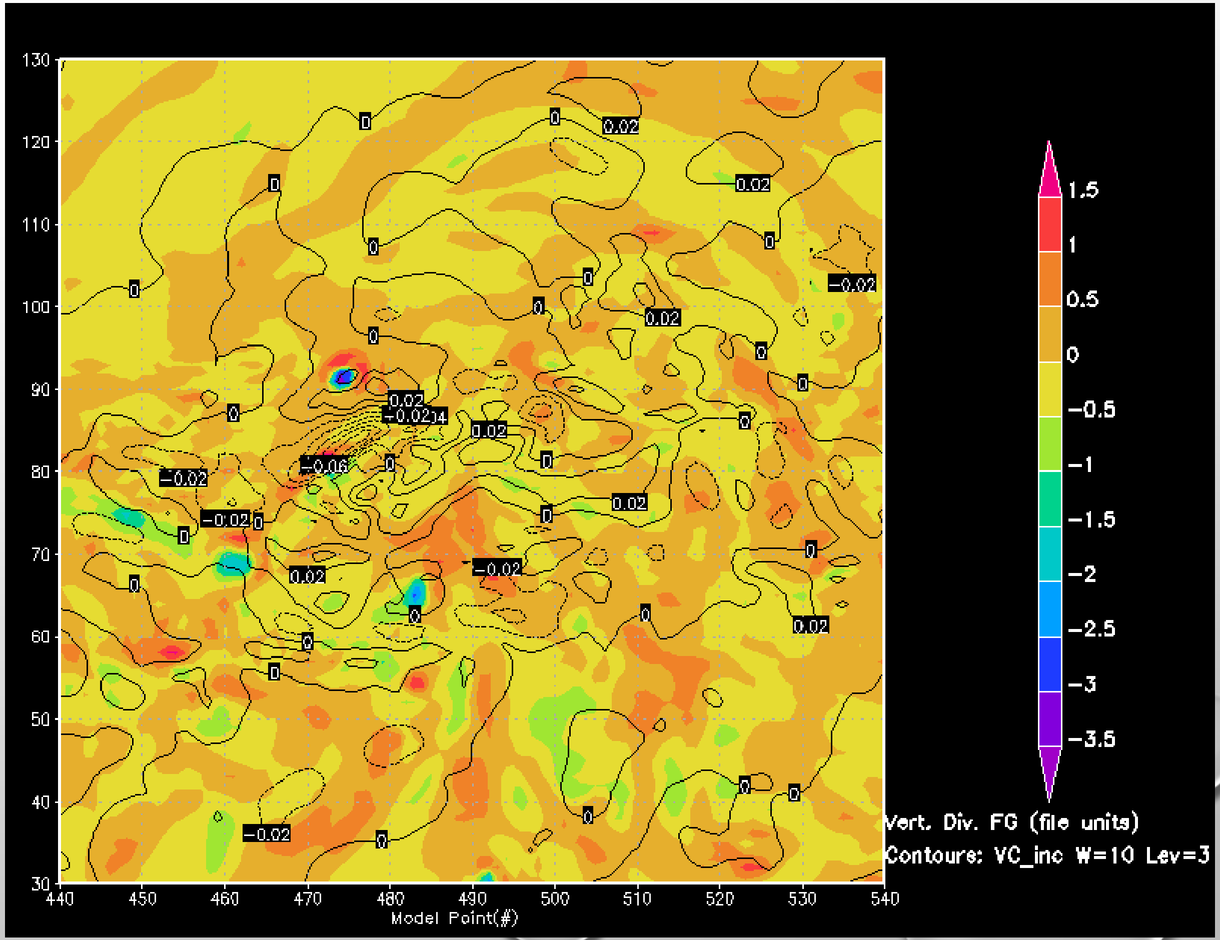


TEST WITH LETKF

VERTICAL DIVERGENCE Analysis Increments (level 31)

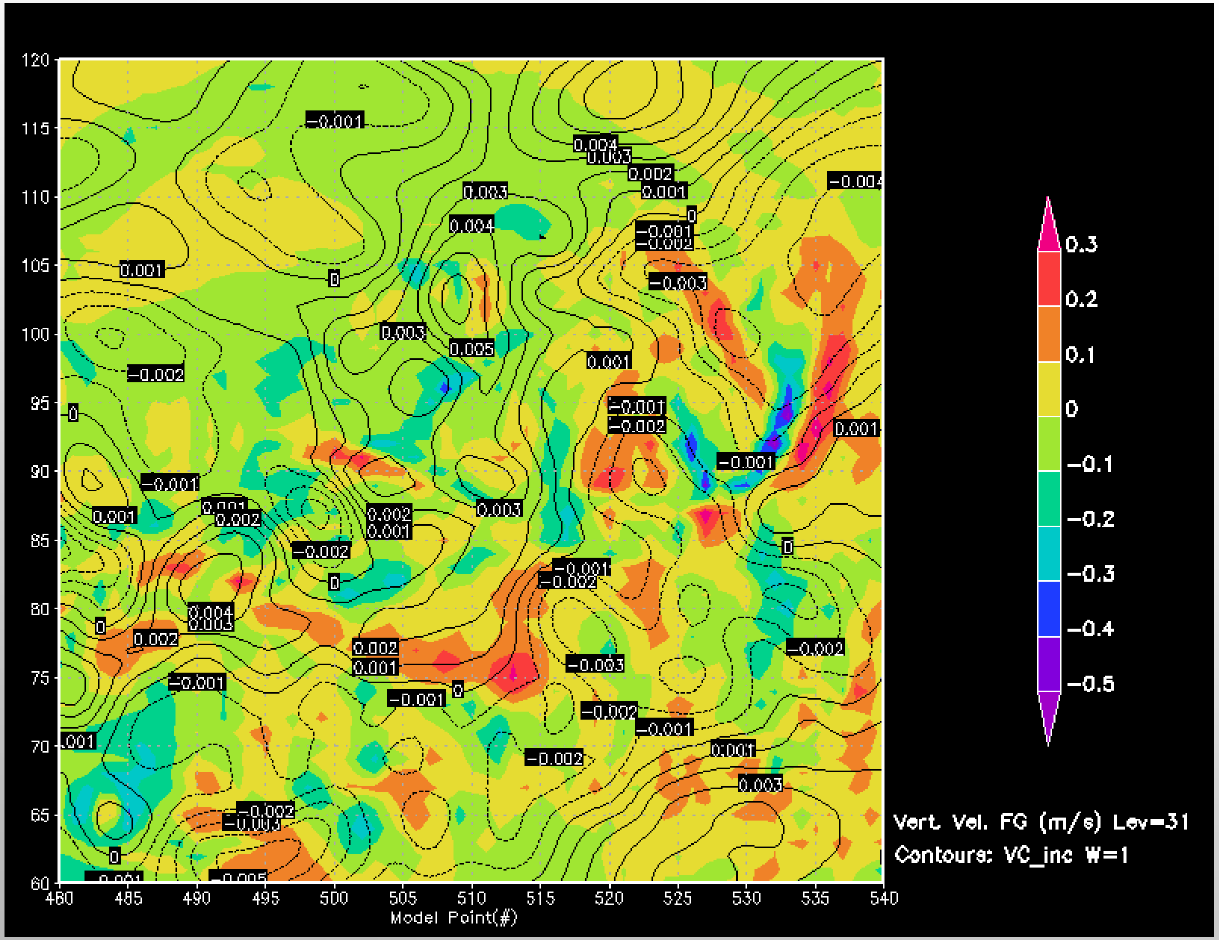
- **The VC algorithm also produces Vertical Wind and Vertical Divergence analyses**. The agreement in terms of scale and structure with the FG is now better than for the PD case. In fact, the VC algorithm preserves in an acceptable way the balance between horizontal and vertical divergence that is present in model states.

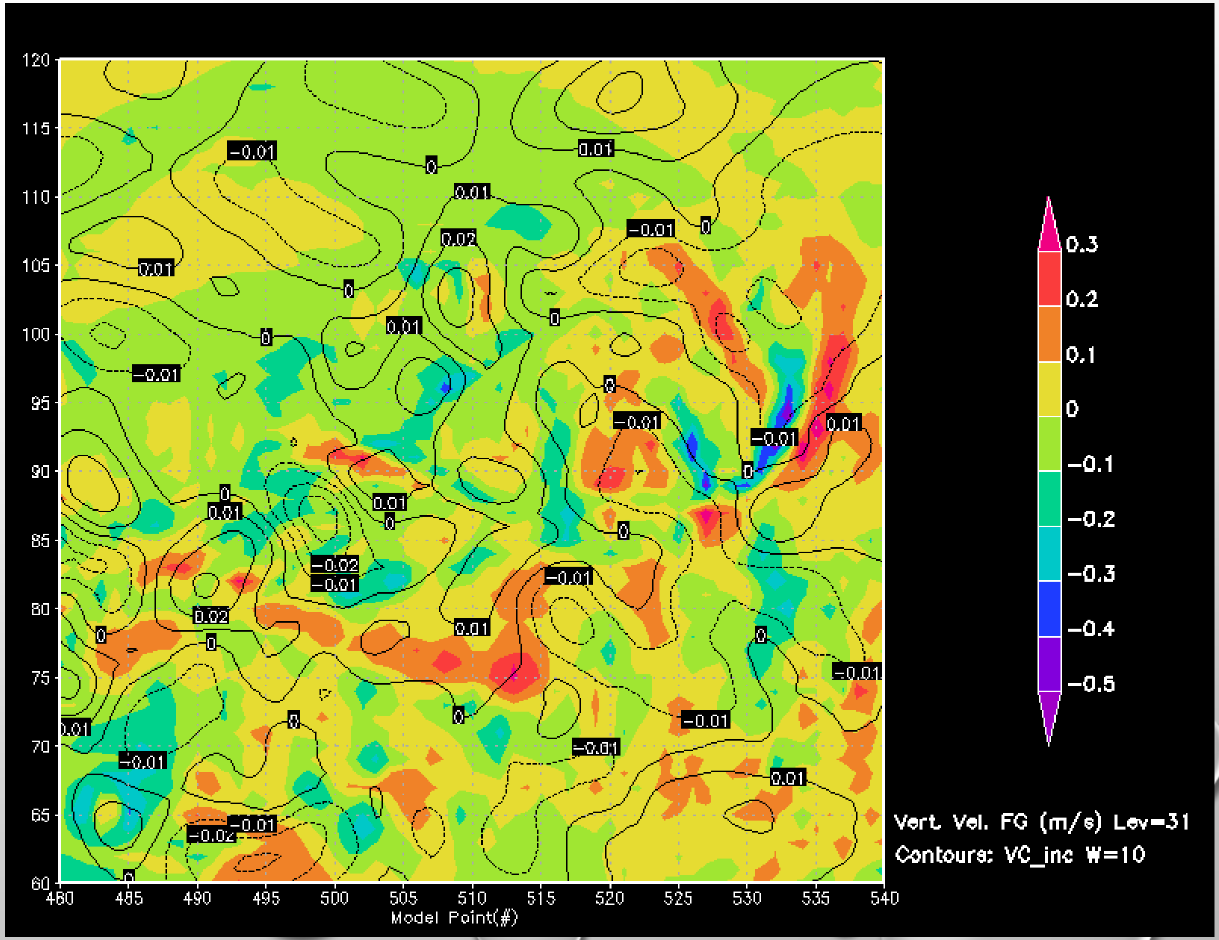




TEST WITH LETKF

VERTICAL VELOCITY Analysis Increments (level 31)

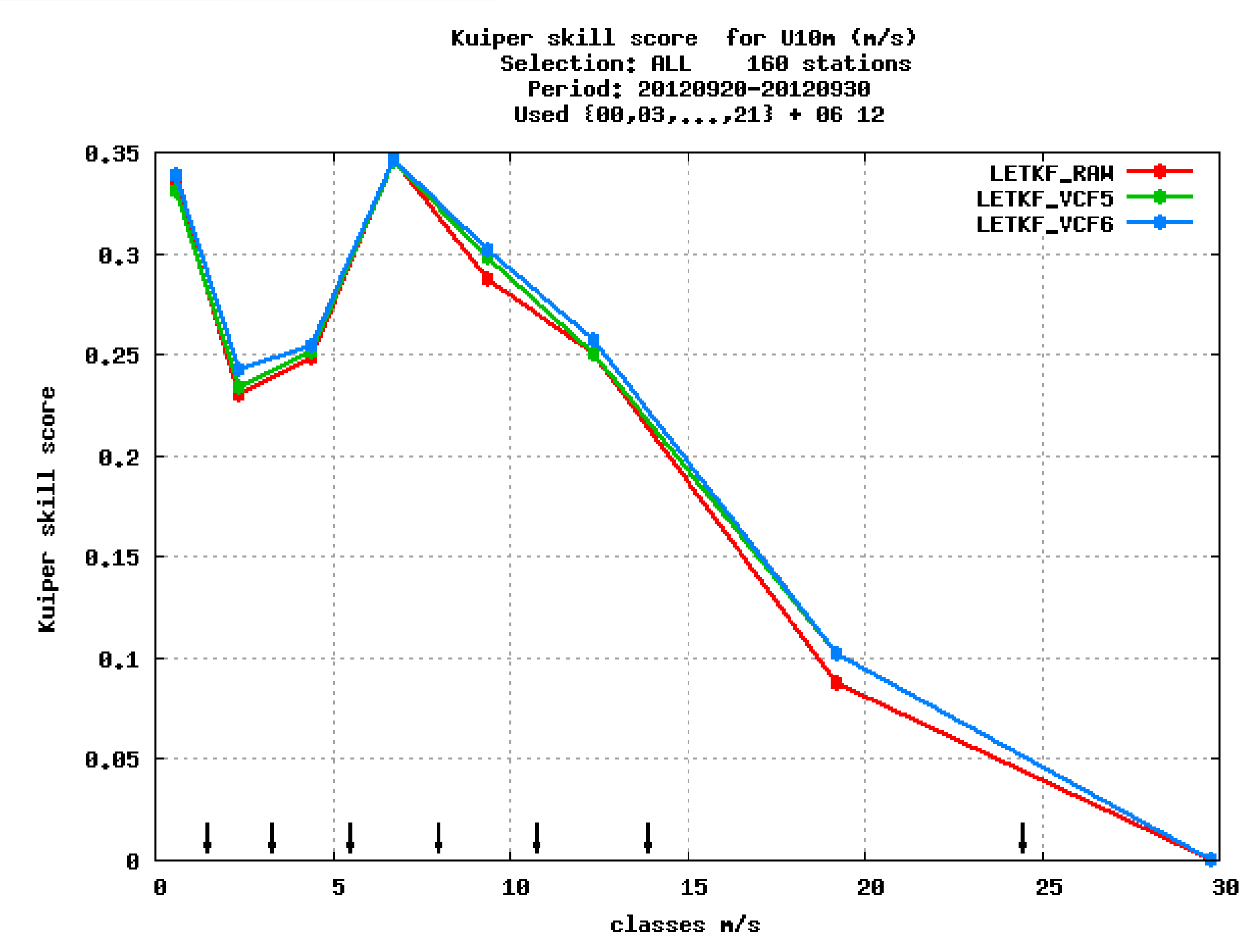


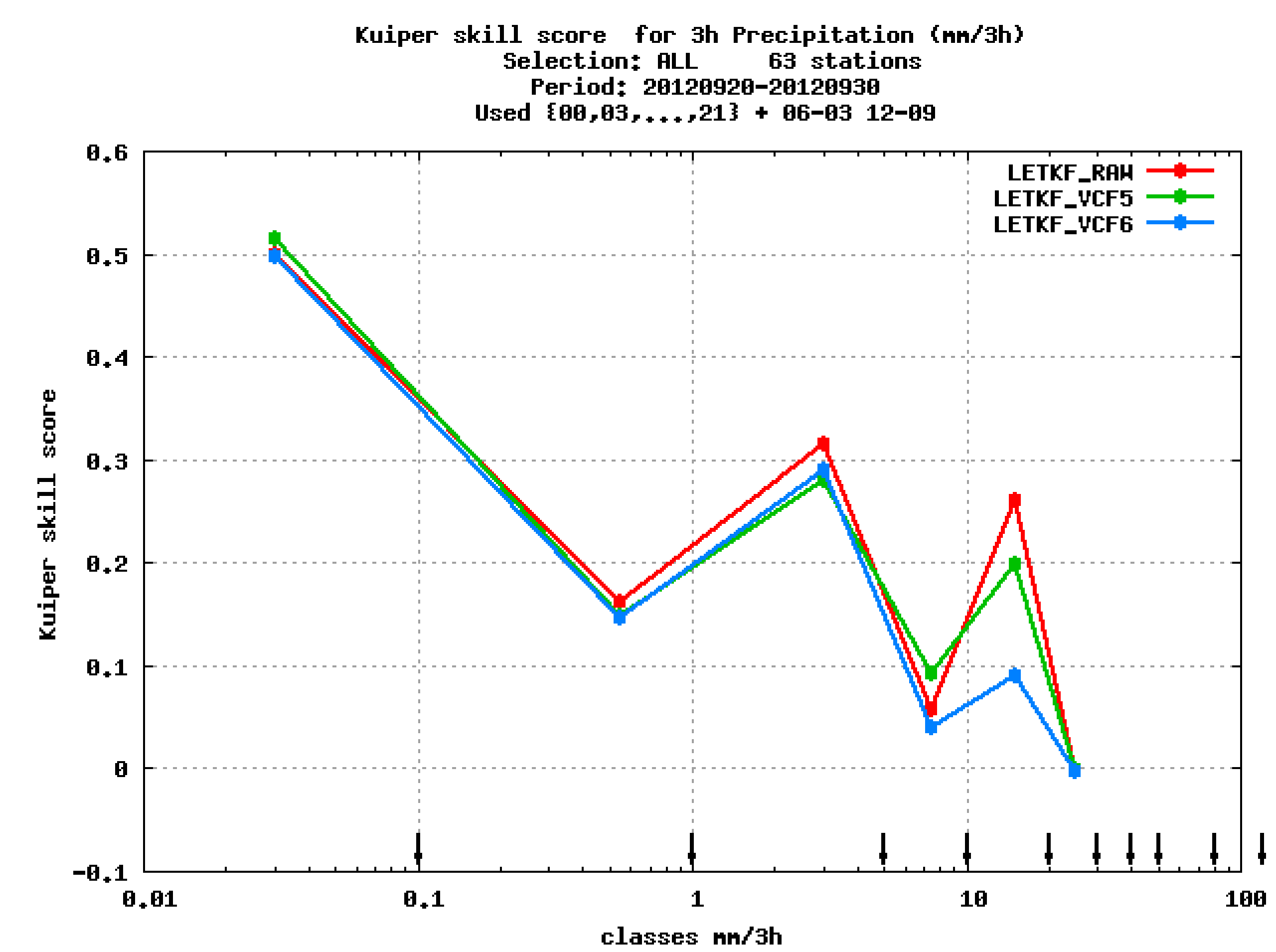


TEST WITH LETKF (Deterministic verification, only control member)

LETKF_VCF5: W=10 and no scale

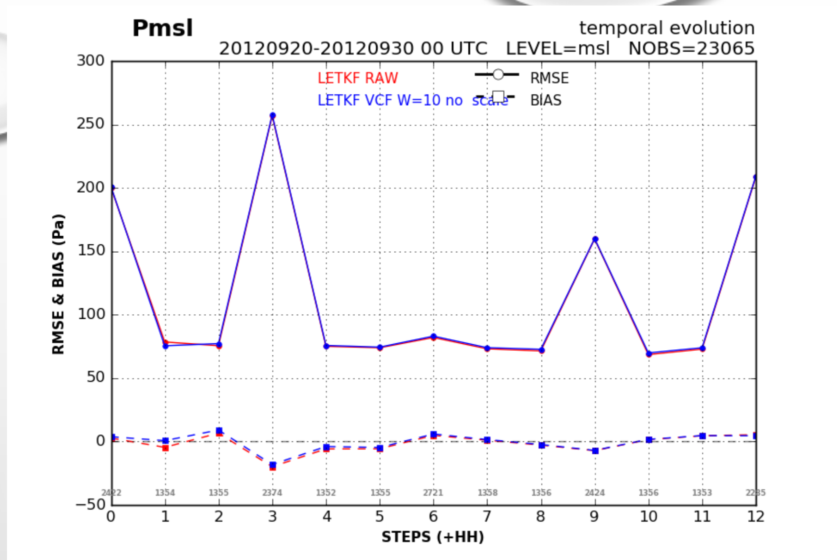
LETKF_VCF6: W=10 and yes scale

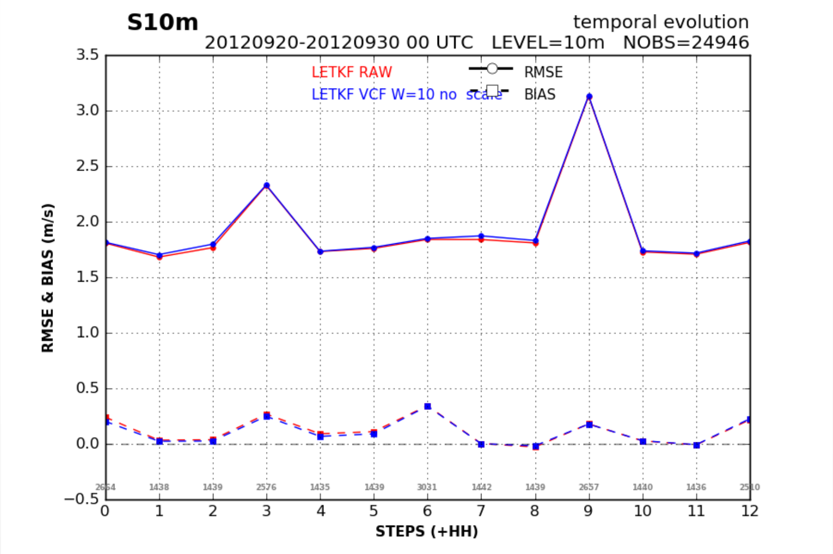


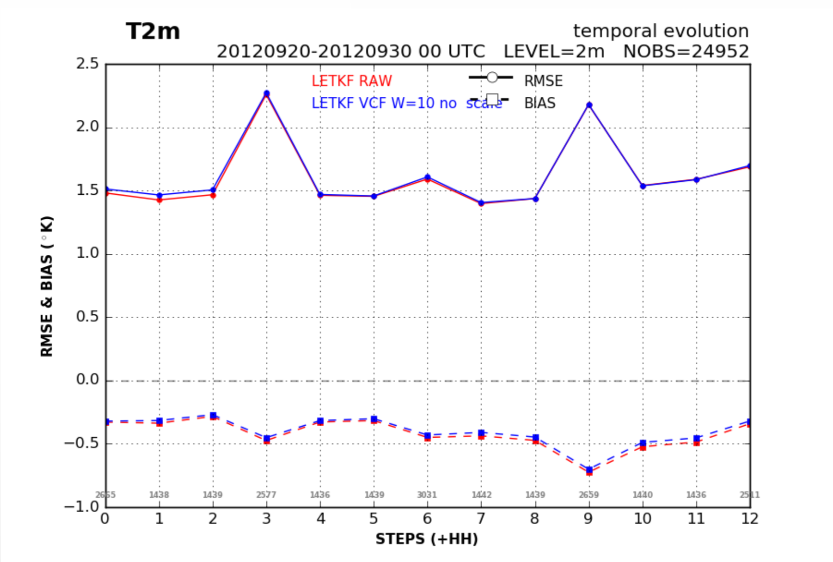


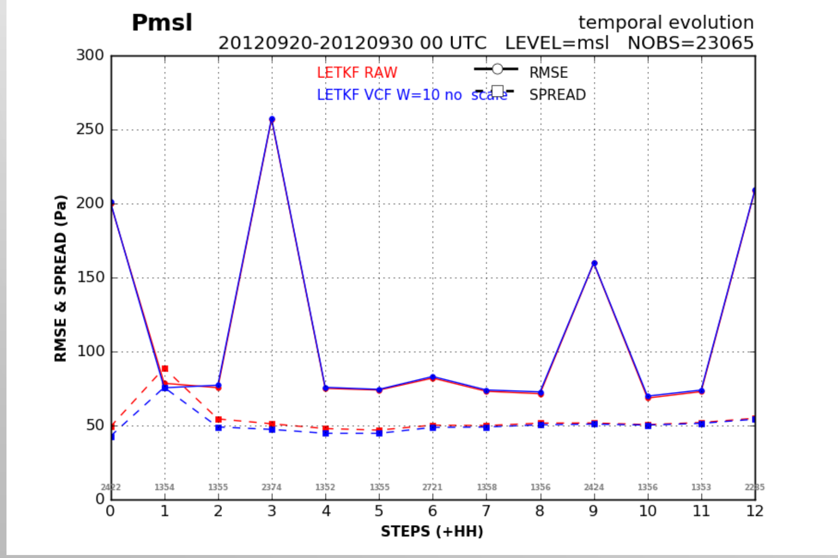
TEST WITH LETKF : Ensemble Verification

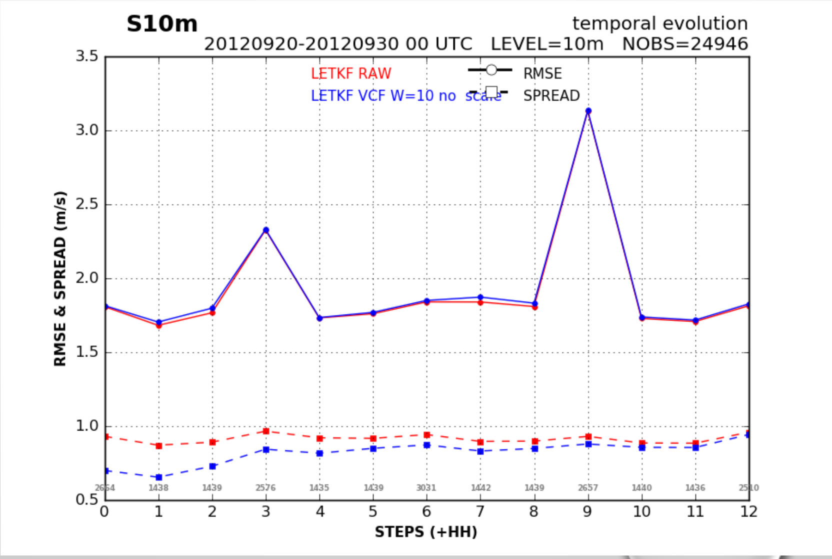
LETKF_VCF5: W=10 and no scale

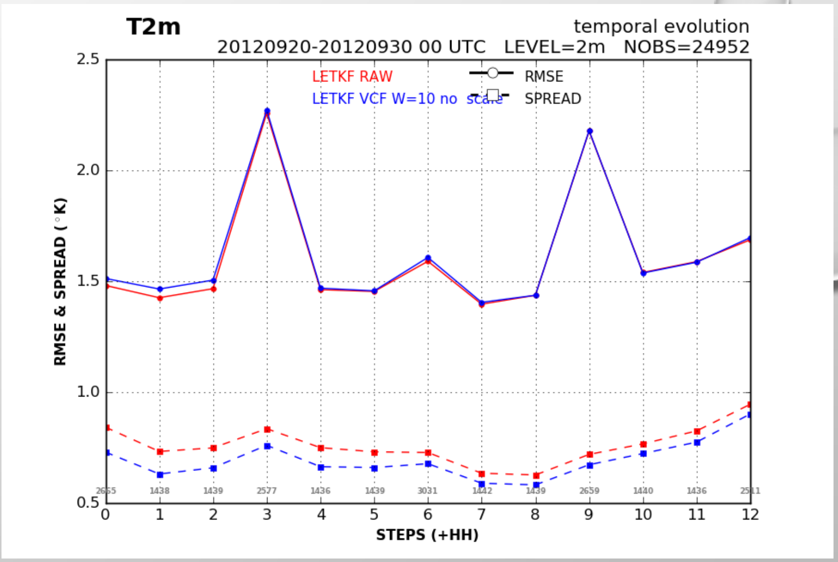












TEST WITH LETKF: COMMENTS ON SOME (PRELIMINARY) VERIFICATION PLOTS

- Positive impact on Ps tendency diagnostic. Expectations for improvements with shorter assimilation cycles, where initialization can make a difference
- Control run: slight positive impact in KSS for U10m and neutral to s.neg. for PE3h
- Ensemble-based verification: (Moderate) reduction in spread (“good spread” vs. “bad spread” (i.e. noise))
- No decrease in ensemble RMSE
- Possible issue: does the filter introduce bias ?
- Not all the territory of parameters explored. Scope for more (fine) tuning

TESTS WITH FIELD ALIGNMENT AND 3DVAR-ANALYSIS

TEST WITH FIELD ALIGMENT

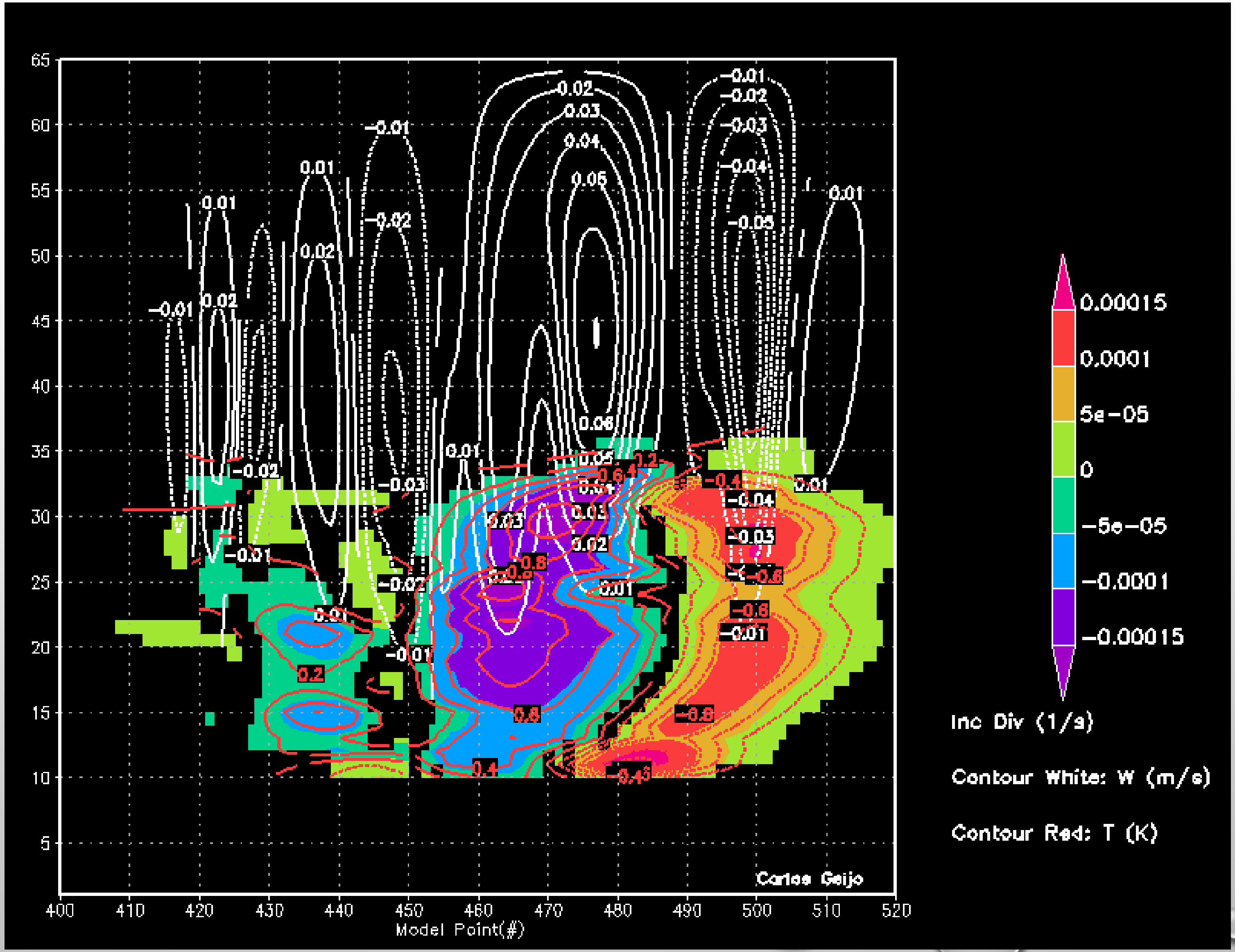
- “Twin Experiments” with **simulated Doppler Wind data** assimilated using the Field Alignment technique for 23 consecutive hourly cycles

- The aligned fields are balanced using this VC method prior to start of forecast (no 3DVar)

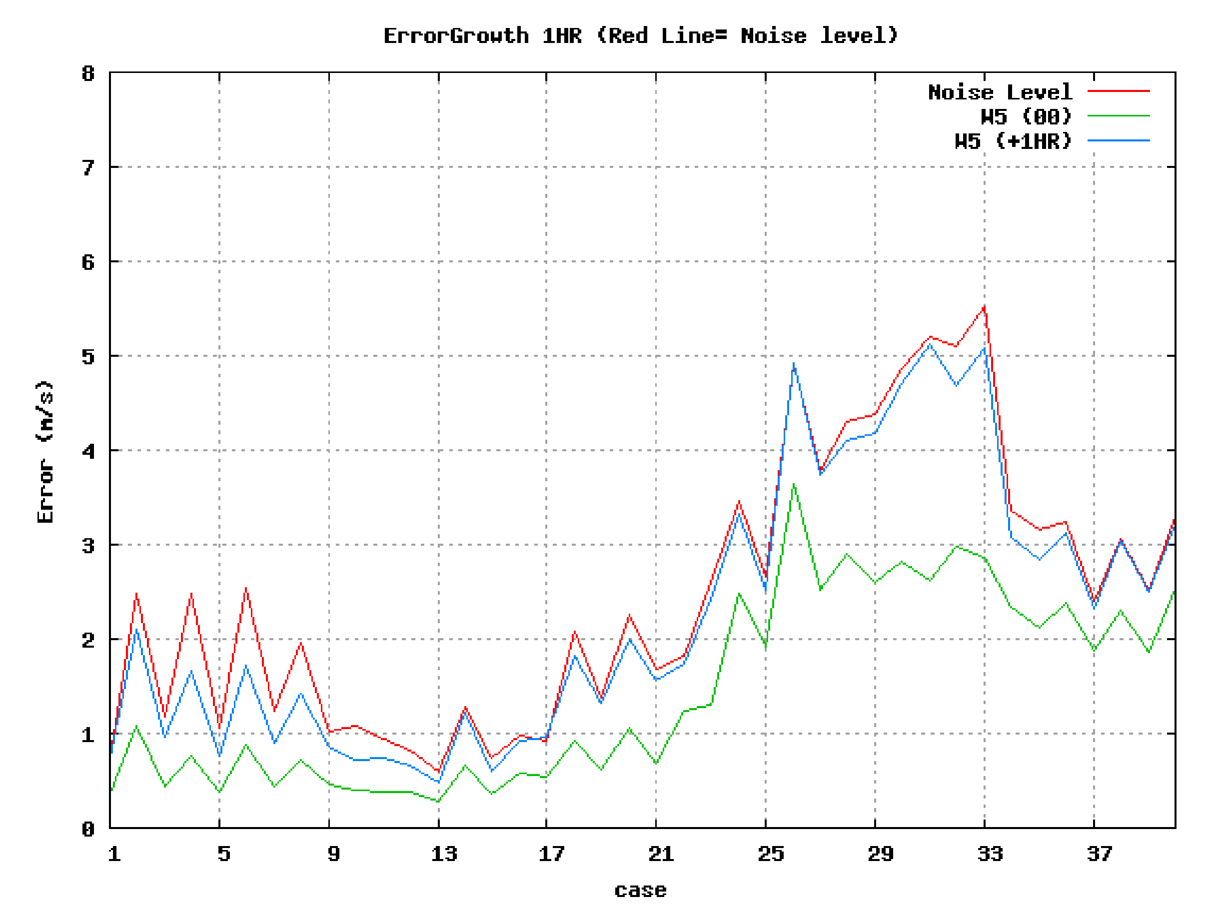
- The validation parameter is difference of Doppler Wind between forecast and simulated observations

TEST WITH FIELD ALIGMENT

Balanced Vertical Motions



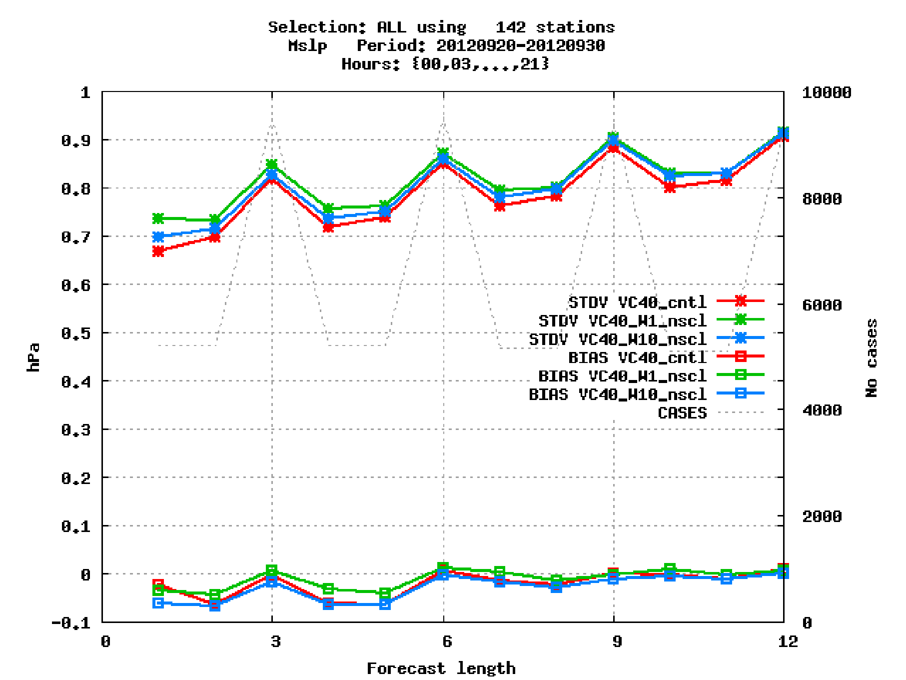
TEST WITH FIELD ALIGMENT: VALIDATION

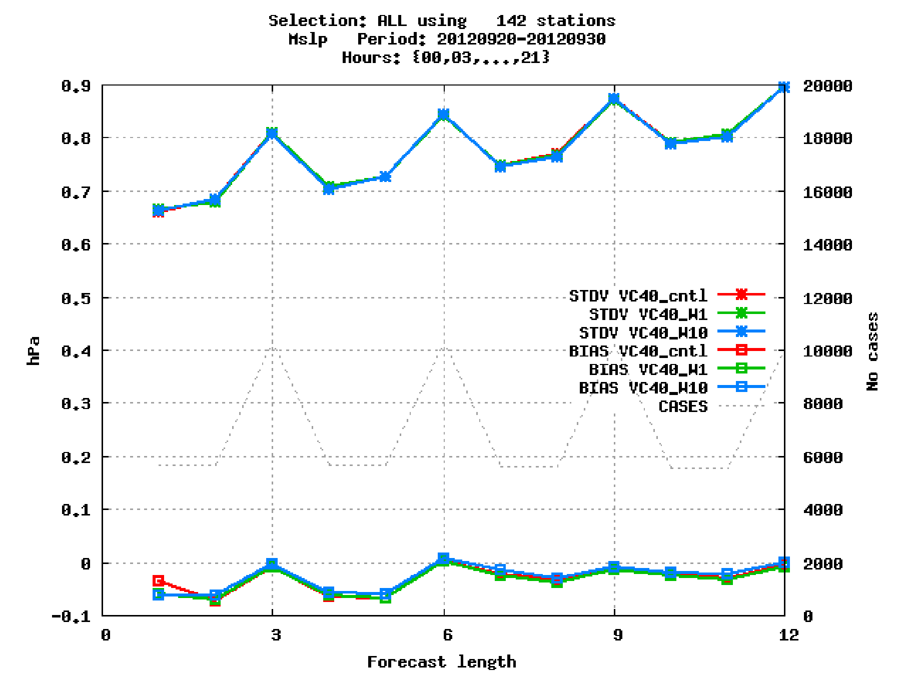


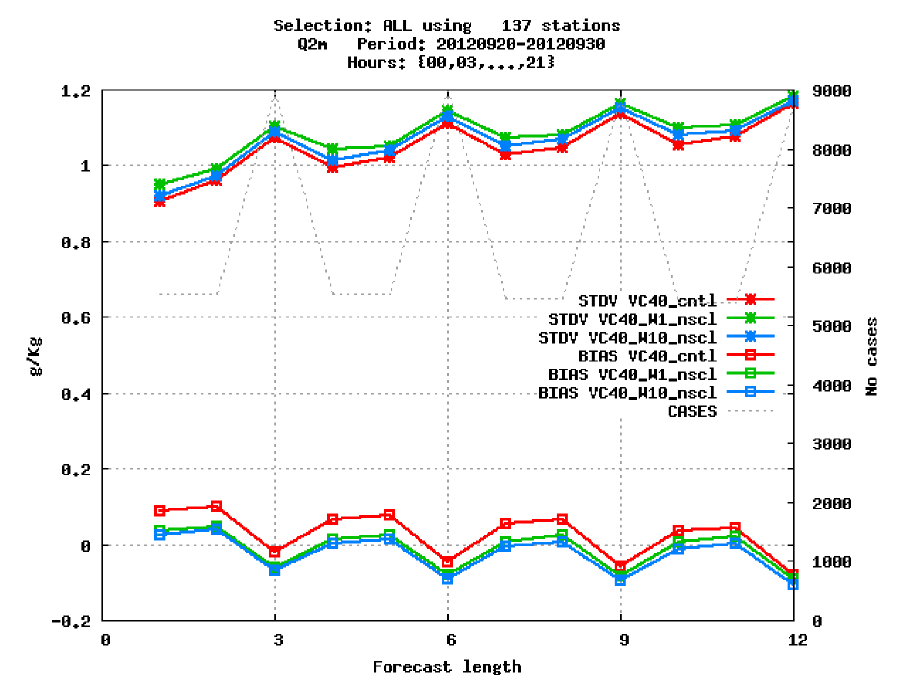
TEST WITH 3DVAR

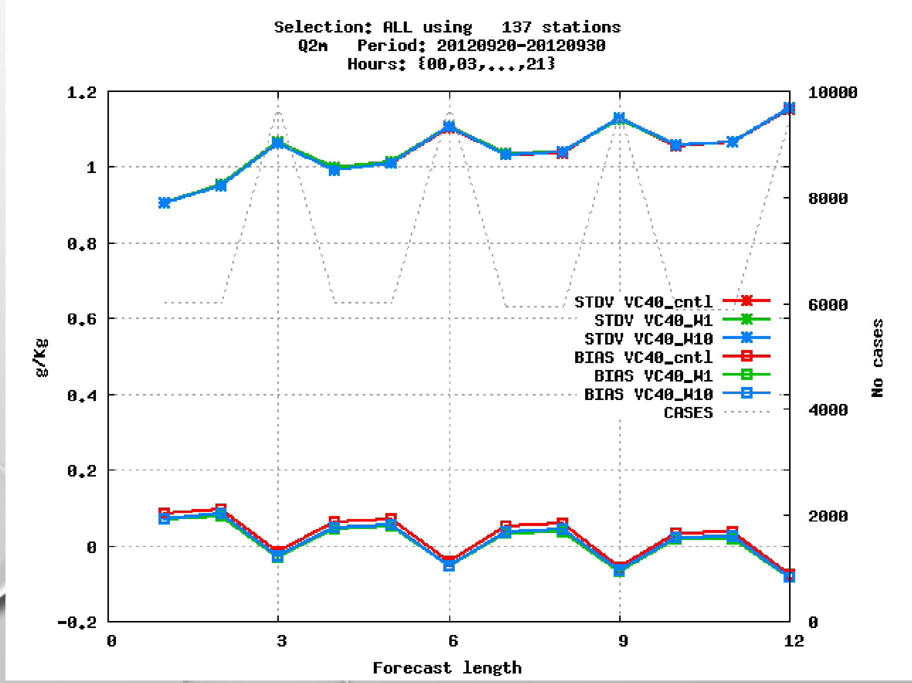
- 3H-DA cycles with conventional obs (~$3*10^3$ obs/cycle) . No surface analysis.
- **Control**: 3D-VAR
- **Experiments**: 3D-VAR with no statbal (LUNIVARIATE=.T.) and VC for balancing
- Verification using the standard HIRLAM “monitor” utility

TEST WITH 3DVAR

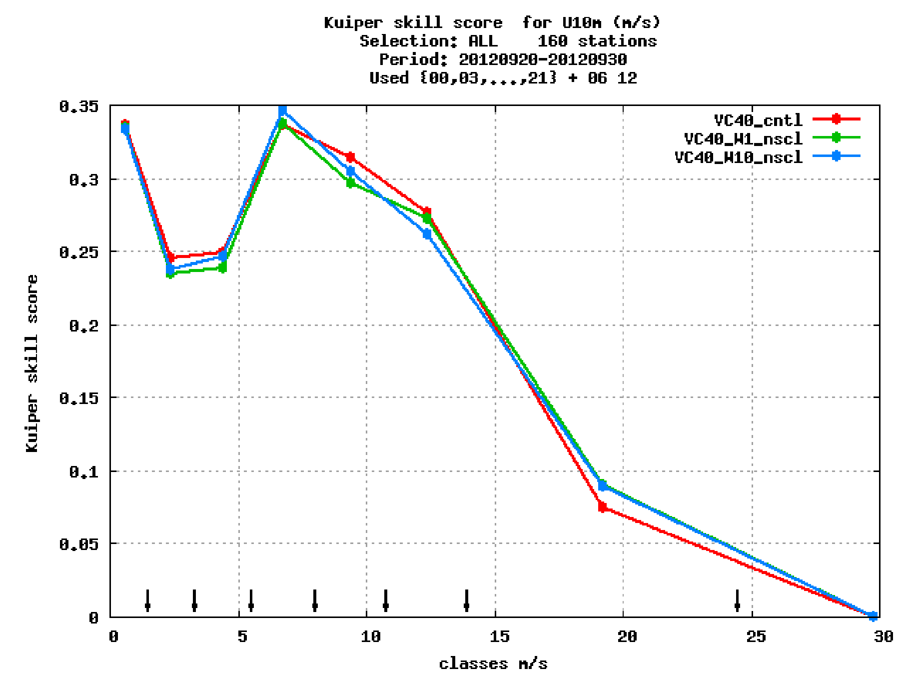


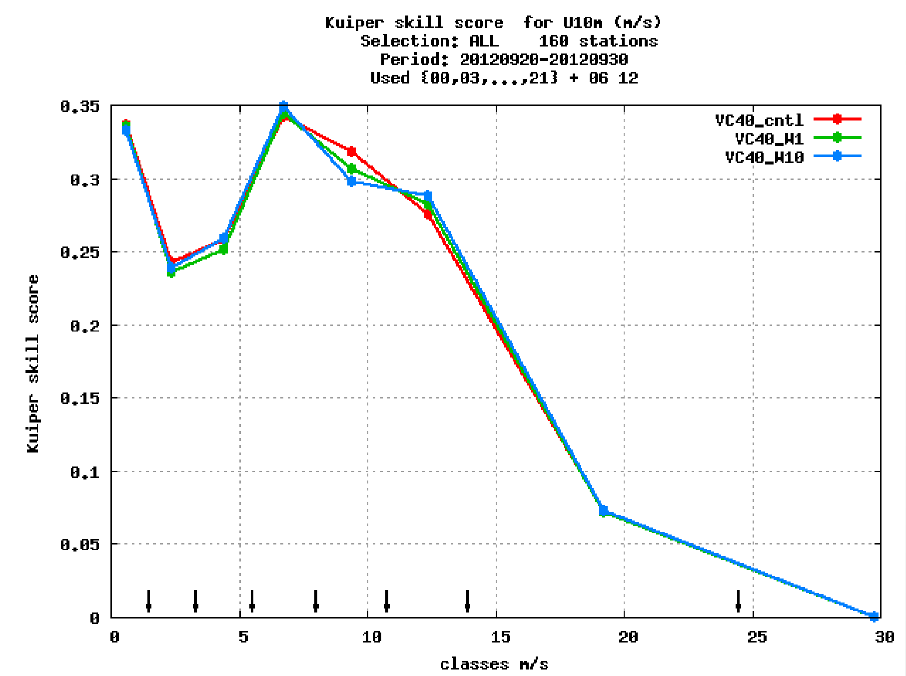


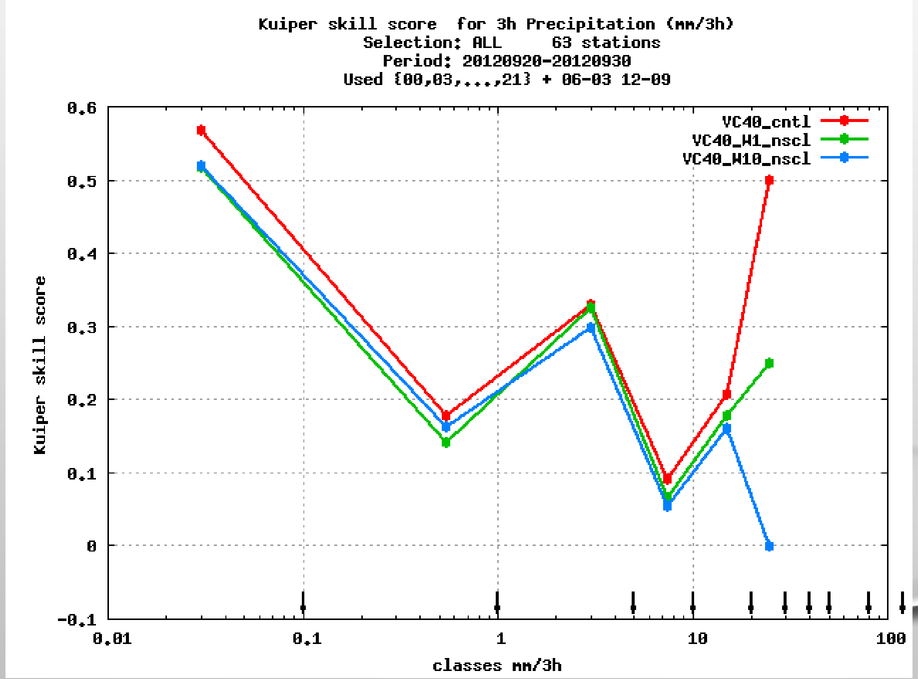


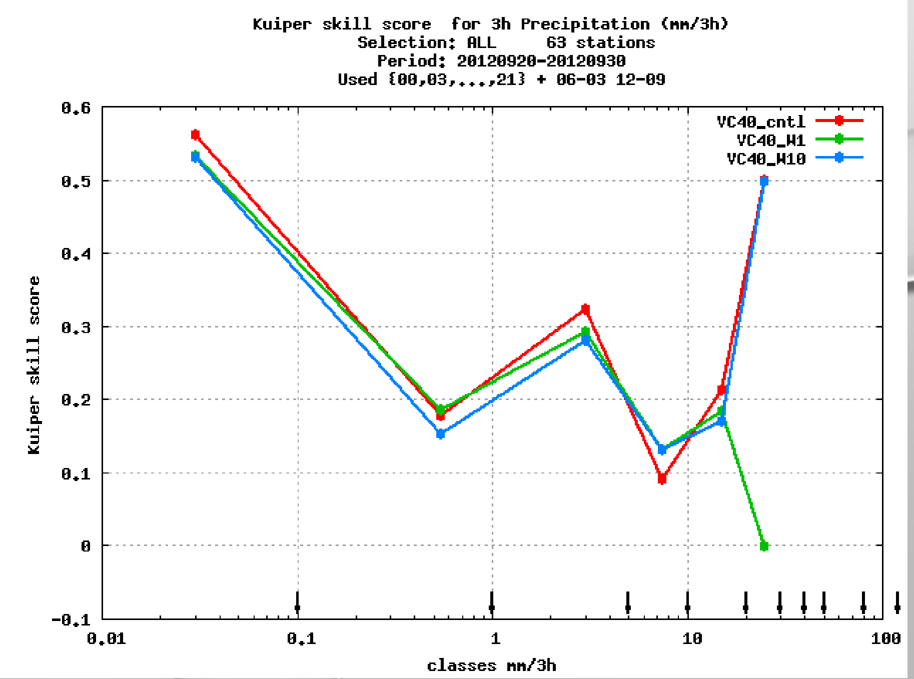


TEST WITH 3DVAR









CONCLUSIONS AND OUTLOOK

- The SI equations define dynamic relations among several variables that can be used in DA. The algorithm is given by the solution to a variational problem which can be obtained using GFs. The SI system is a time-step forward operator, and this property gives to this new algorithm a nudging-like functionality. It is therefore well suited for "continuous-in-time" DA. It also brings to the DA process the vertical velocity and PD fields.

- The implementation of the idea (GF numerics, as a external utility) is not optimal in terms of the integration in the system. Nonetheless it has demostrated its capacity to provide good equilibrium among horizontal and vertical momentum analysed fields and to filter out spurious oscillations in the surface pressure tendency field from LETKF analysis.

- First experiments with the LETKF DA algorithm (3h DA cycle, only conv. obs) show a clear reduction in ensemble spread. Although this result is to be expected, there is no simultaneous reduction in ensemble error and the verification scores in consequence are not favorable.

- The method has also demostrated its potential to improve balances of wind fields generated by FA with radar images. First results with 3D-VAR do not show big gains with respect to the current statistical balances method. The tests however, have been done with 3 hours DA cycles and small amount of observations, conditions where initialization issues do not manifest in the foreground. The introduction of some ad-hoc tunable parameters helps to improve the verification scores.

- In agreement with the HIRLAM RWP2018, many of these issues will be addressed in the coming months.