# Estimación del error al medir Vapor de Agua Precipitable con radiosondeos

**Julio A. Castro-Almazán**
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**and**
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**Casiana Muñoz Tuñón**

**Sky Quality Team**
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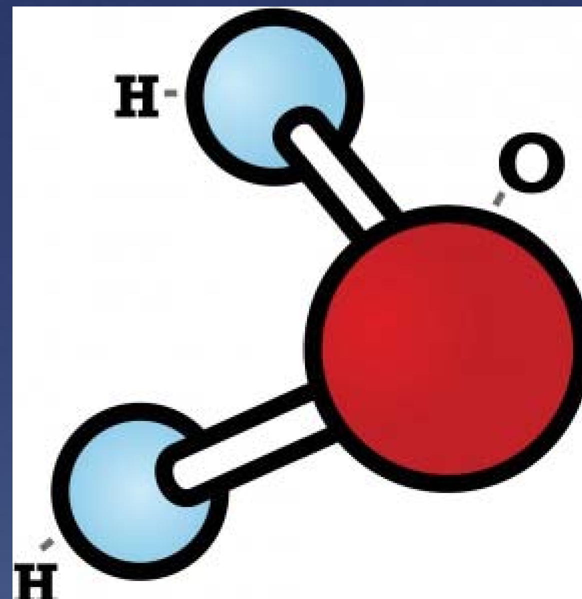
photo:Daniel López

# WATER VAPOUR

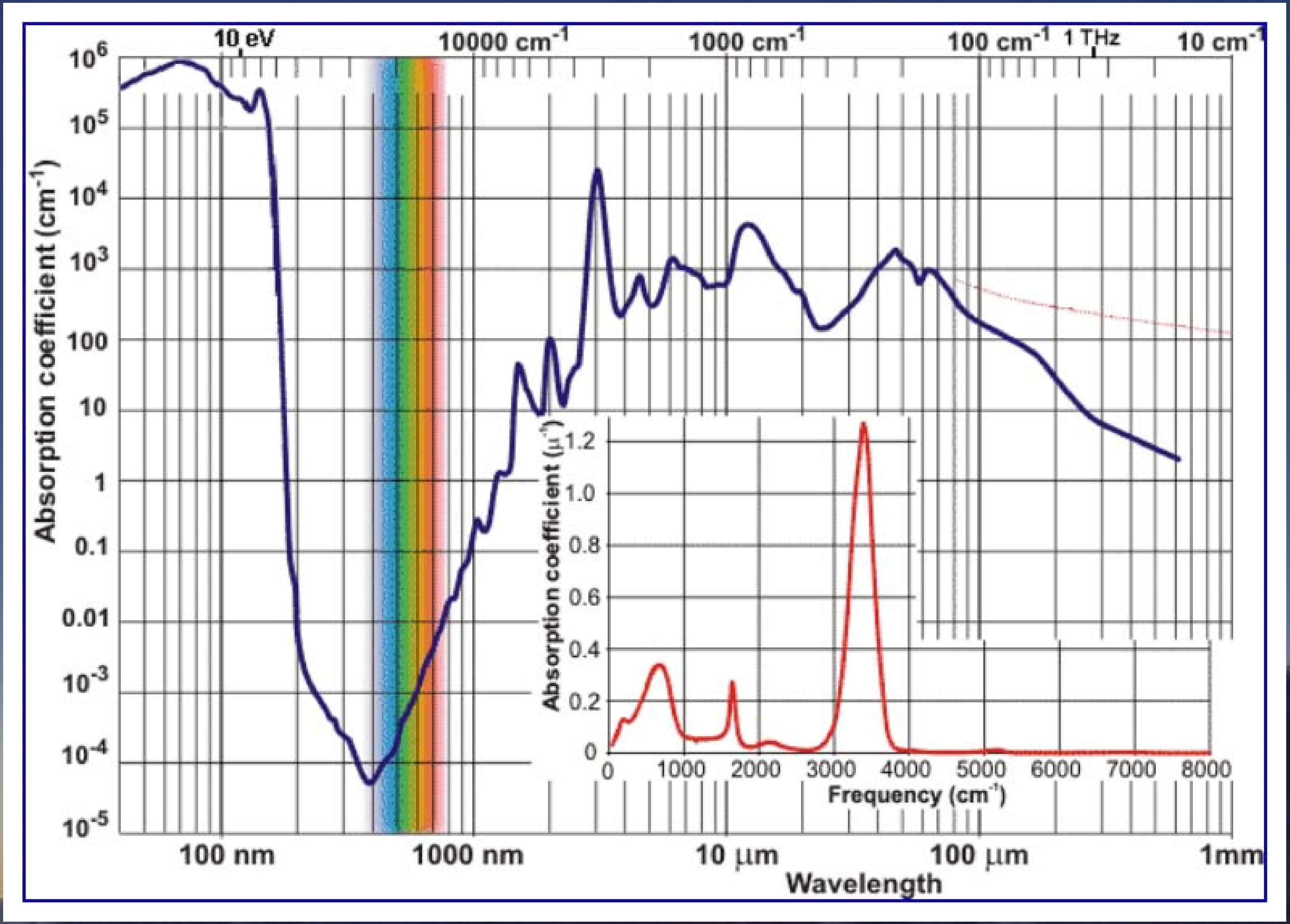


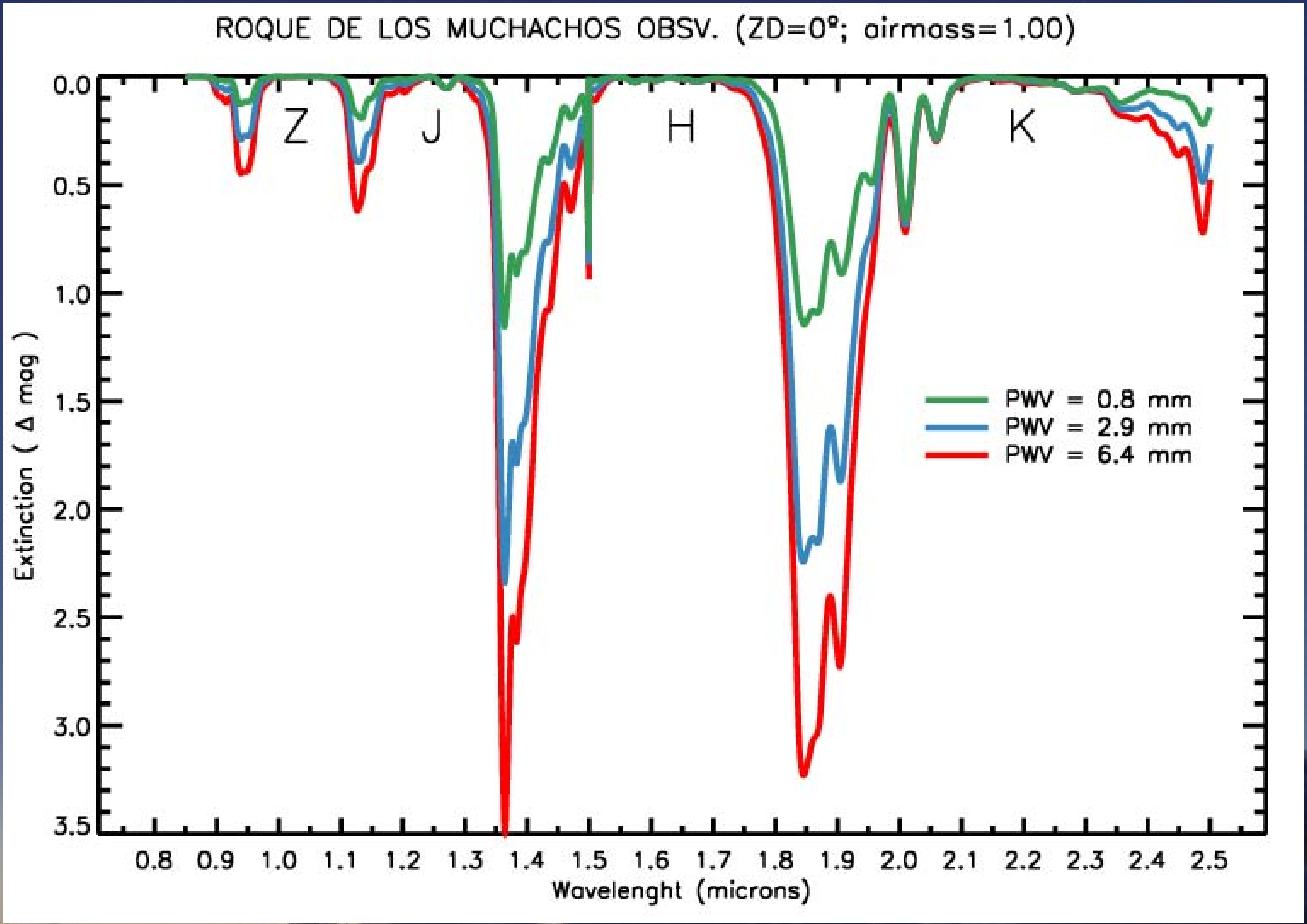

**<4 % of all atmospheric molecules**

**but...**

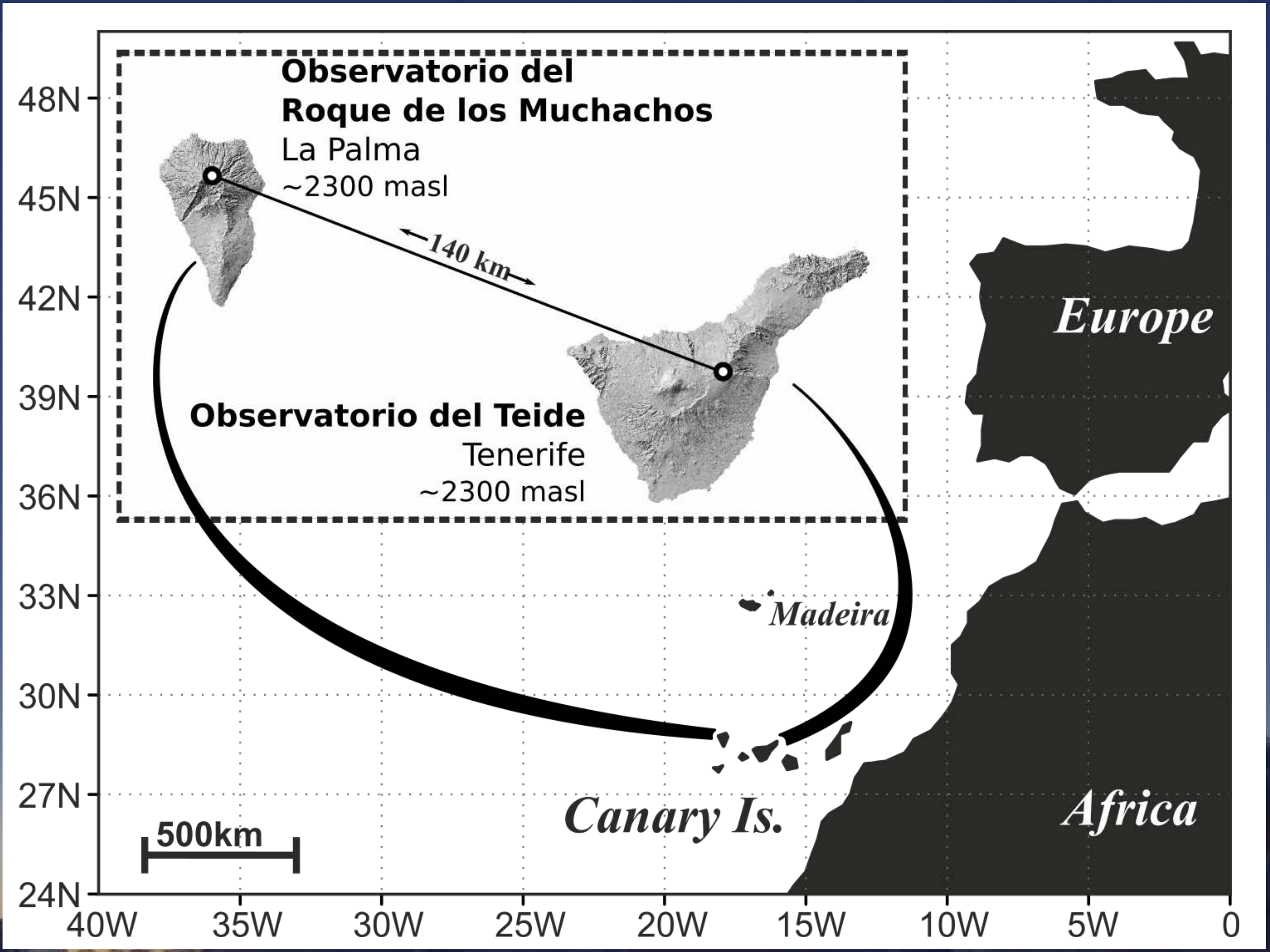
**is a powerful Greenhouse Gas, participates in processes affecting the global climate and ...**

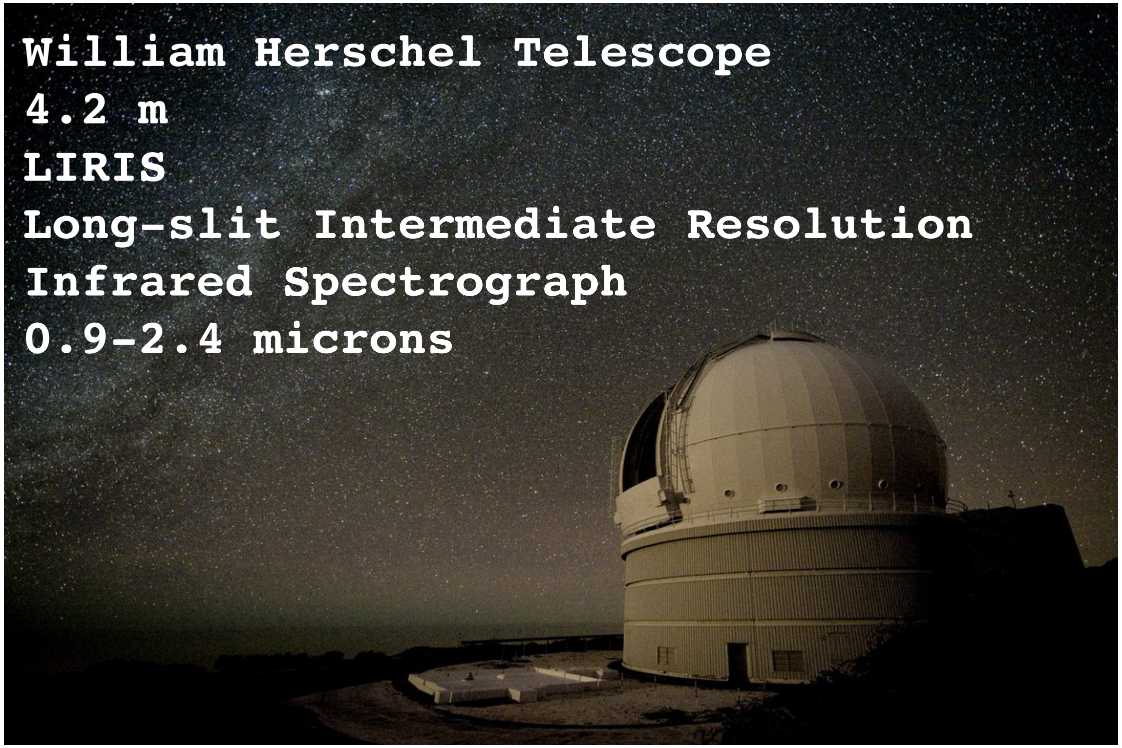
## ...and
## is the principal responsible for the atmospheric extinction in IR, submillimeter and microwave astronomical observations

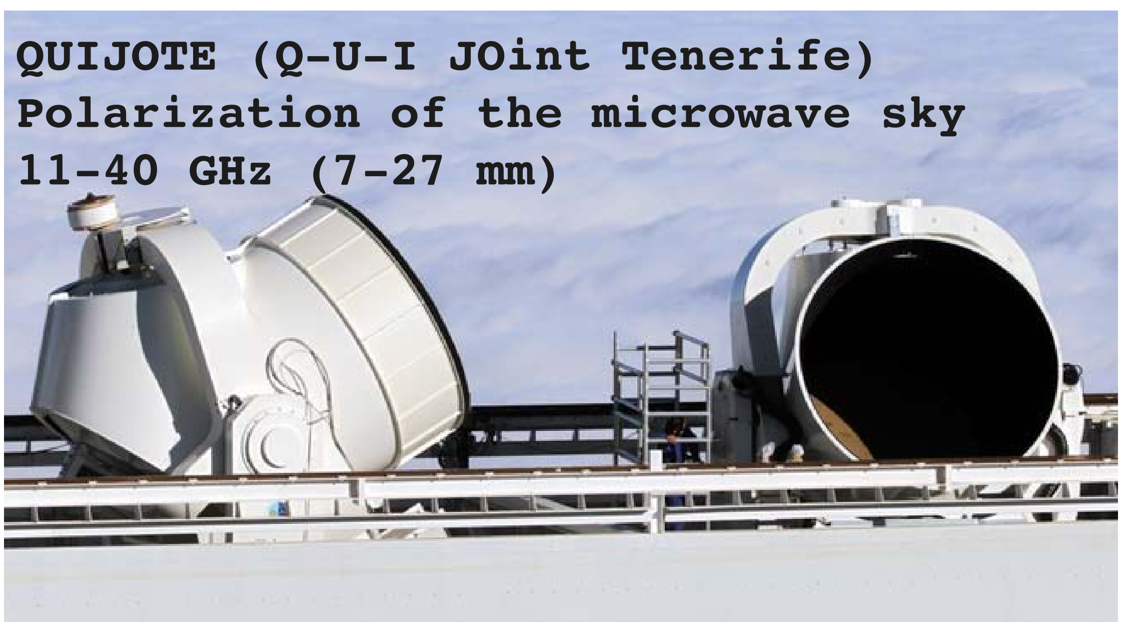


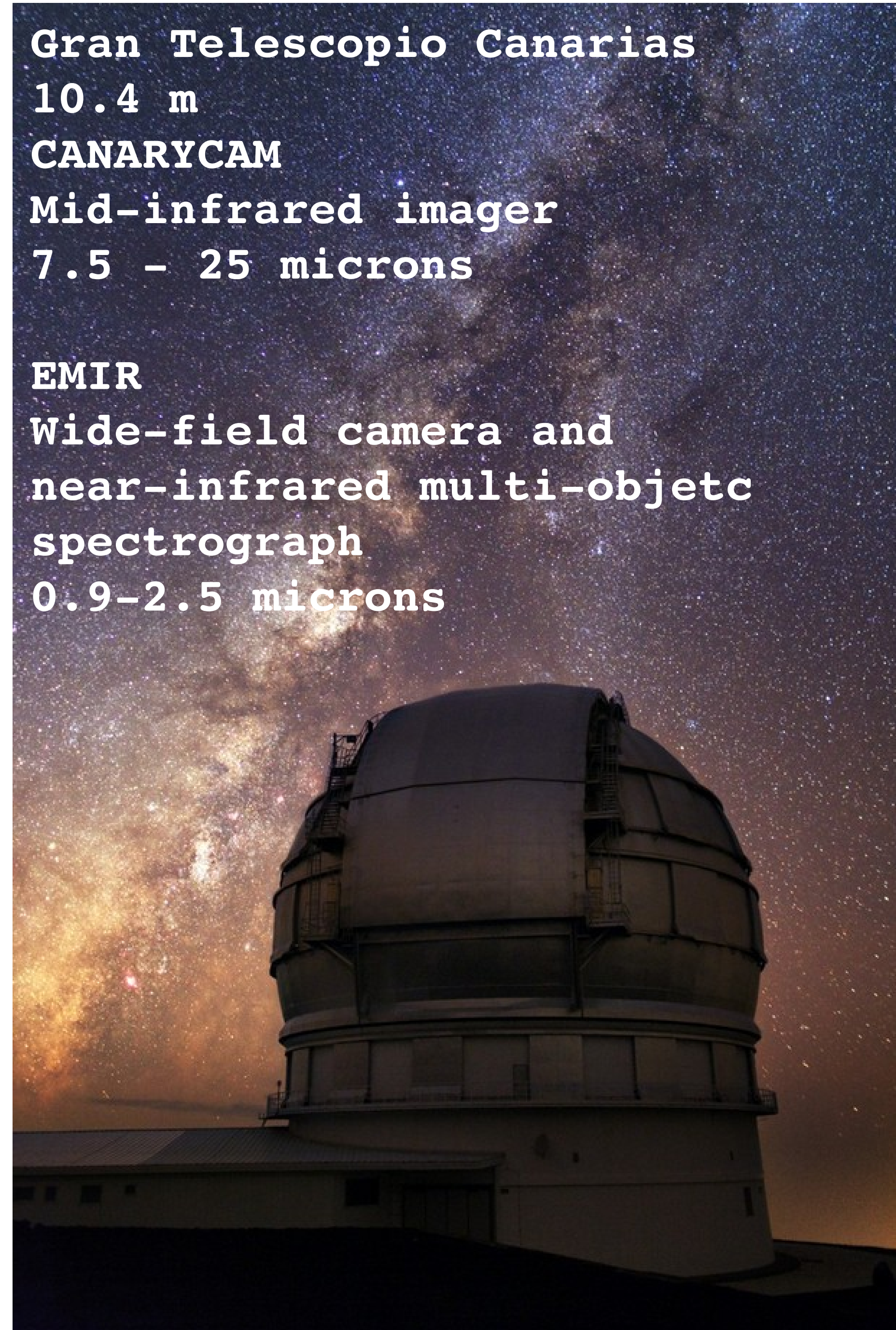



data from ATRAN (SOFIA project) synthetic model


Observatorio del
Roque de los Muchachos
La Palma
~2300 masl
←140 km→
Observatorio del Teide
Tenerife
~2300 masl
Europe
Madeira
Canary Is.
Africa
500km
48N
45N
42N
39N
36N
33N
30N
27N
24N
40W
35W
30W
25W
20W
15W
10W
5W
0


William Herschel Telescope
4.2 m
LIRIS
Long-slit Intermediate Resolution
Infrared Spectrograph
0.9-2.4 microns


QUIJOTE (Q-U-I JOint Tenerife)
Polarization of the microwave sky
11-40 GHz (7-27 mm)


Gran Telescopio Canarias
10.4 m
CANARYCAM
Mid-infrared imager
7.5 - 25 microns
EMIR
Wide-field camera and
near-infrared multi-objetc
spectrograph
0.9-2.5 microns

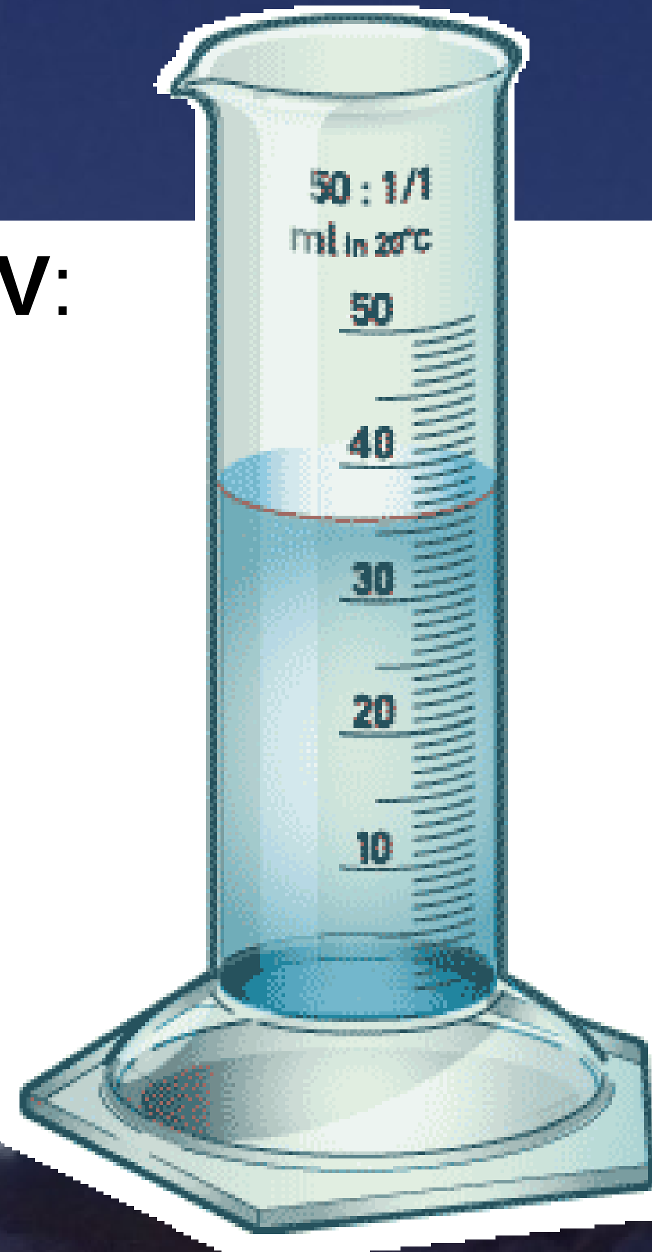
# Precipitable Water Vapour **PWV**:

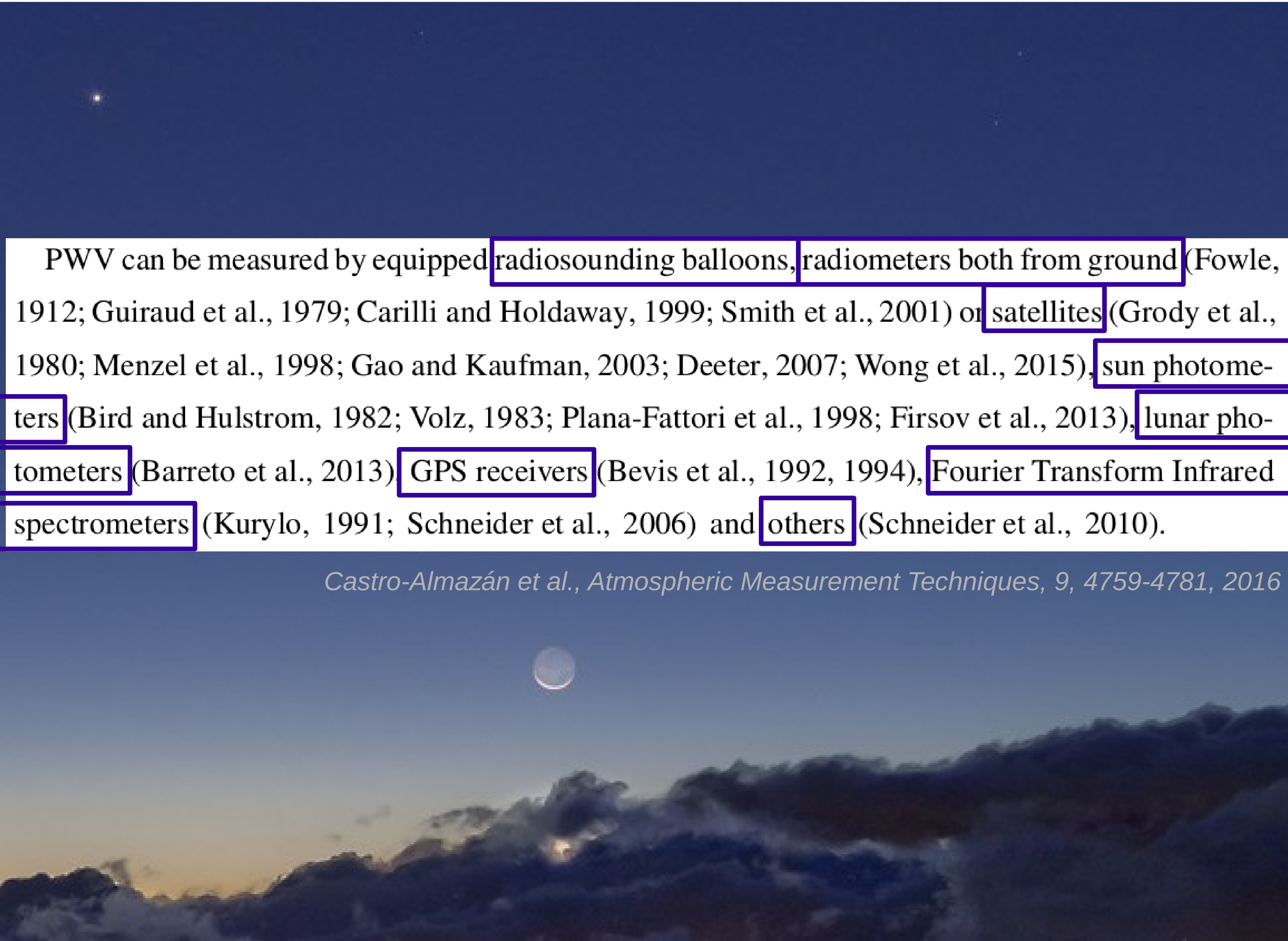
“Total water column height if integrated from the surface to the top of the atmosphere with unit cross-sectional”

*(glossary of the American Meteorological Society)*

Commonly expressed in **mm:**

the height to which that water would stand if completely condensed and collected in a vessel with a cross section of 1 $m^2$





PWV can be measured by equipped radiosounding balloons, radiometers both from ground (Fowle, 1912; Guiraud et al., 1979; Carilli and Holdaway, 1999; Smith et al., 2001) or satellites (Grody et al., 1980; Menzel et al., 1998; Gao and Kaufman, 2003; Deeter, 2007; Wong et al., 2015), sun photometers (Bird and Hulstrom, 1982; Volz, 1983; Plana-Fattori et al., 1998; Firsov et al., 2013), lunar photometers (Barreto et al., 2013), GPS receivers (Bevis et al., 1992, 1994), Fourier Transform Infrared spectrometers (Kurylo, 1991; Schneider et al., 2006) and others (Schneider et al., 2010).

*Castro-Almazán et al., Atmospheric Measurement Techniques, 9, 4759-4781, 2016*

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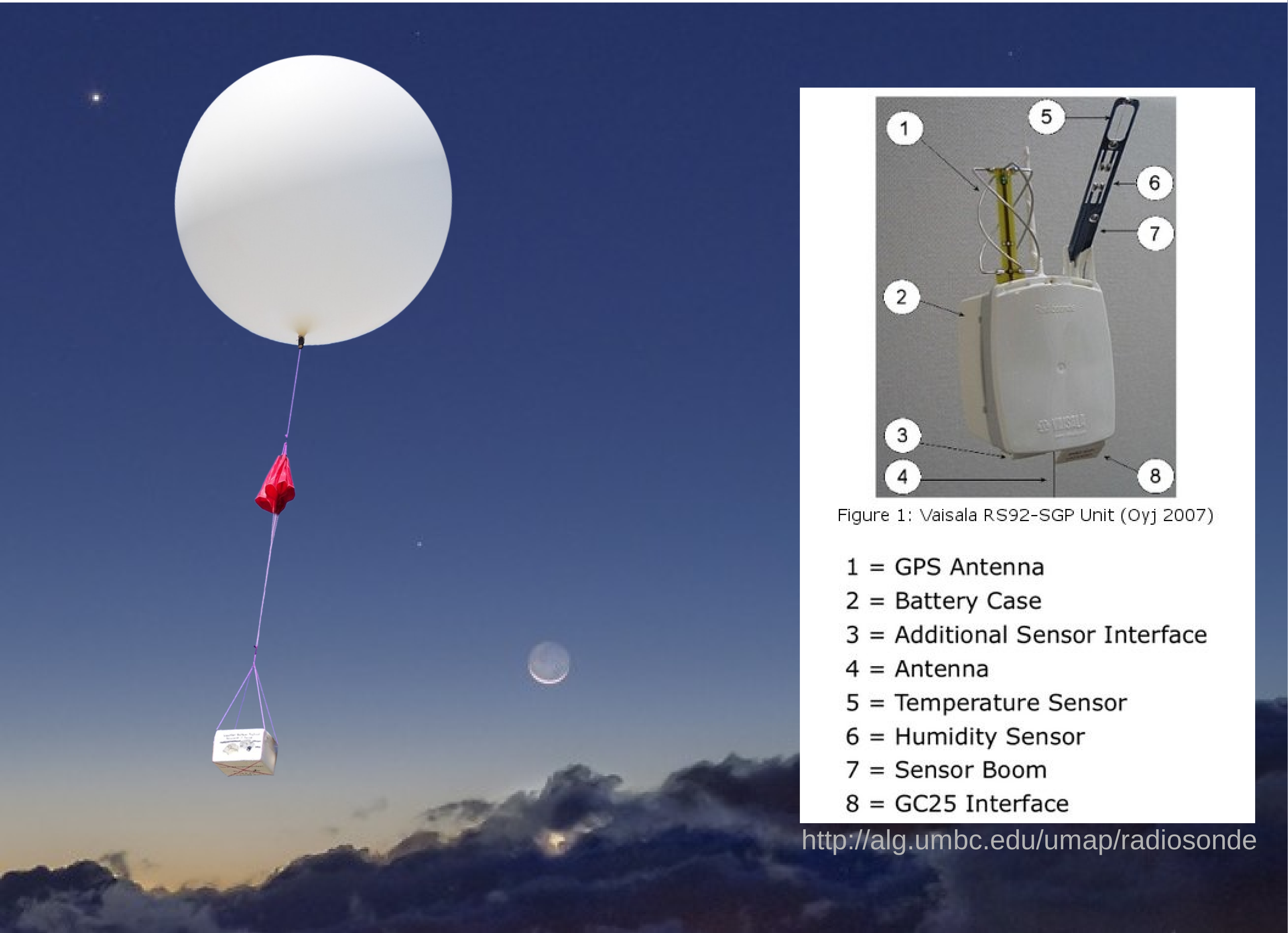


Figure 1: Vaisala RS92-SGP Unit (Oyj 2007)

1 = GPS Antenna
2 = Battery Case
3 = Additional Sensor Interface
4 = Antenna
5 = Temperature Sensor
6 = Humidity Sensor
7 = Sensor Boom
8 = GC25 Interface

http://alg.umbc.edu/umap/radiosonde

atmospheric radiosoundings are a direct in situ measurement and one of the most accurate methods of retrieving the PWV. Radiosoundings are also one of the current standards for atmospheric research and are widely used as a valid reference for comparison and calibration.

Accurate error estimation of PWV from radiosoundings is essential for regression analyses in comparison or calibration studies. In this sense, special care is needed when radiosoundings from different sources and with differing characteristics are being discussed, or when working in a particularly dry atmosphere.

*Castro-Almazán et al., AMT, 9, 4759-4781, 2016*

Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY

MNRAS **452**, 1992–2003 (2015) doi:10.1093/mnra

# Forecasting the precipitable water vapour content: validation for astronomical observatories using radiosoundings

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[4] *Department Astrophysique, Université de Nice–Sophia Antipolis, Nice, France*

# Precipitable Water Vapour at the Canarian Observatories (Teide and Roque de los Muchachos) from routine GPS

Julio A. Castro-Almazán[a,b], Casiana Muñoz-Tuñón[a,b], Begoña García-Lorenzo[a,b], Gabriel Pérez-Jordán[a*], Antonia M. Varela[a,b], and Ignacio Romero[c,d]

Observatory Operations: Strategies, Processes, and Systems VI, edited by Alison B. Peck, Robert L. Seaman, Chris R. Benn, Proc. of SPIE Vol. 9910, 99100P · © 2016 SPIE · CCC code: 0277-786X/16/$18 · doi: 10.1117/12.2232646

Proc. of SPIE Vol. 9910 99100P-1

Atmos. Meas. Tech., 9, 4759–4781, 2016
www.atmos-meas-tech.net/9/4759/2016/
doi:10.5194/amt-9-4759-2016


Atmospheric Measurement Techniques
Open Access

# A semiempirical error estimation technique for PWV derived from atmospheric radiosonde data

**Julio A. Castro-Almazán[1,2], Gabriel Pérez-Jordán[1], and Casiana Muñoz-Tuñón[1,2]**

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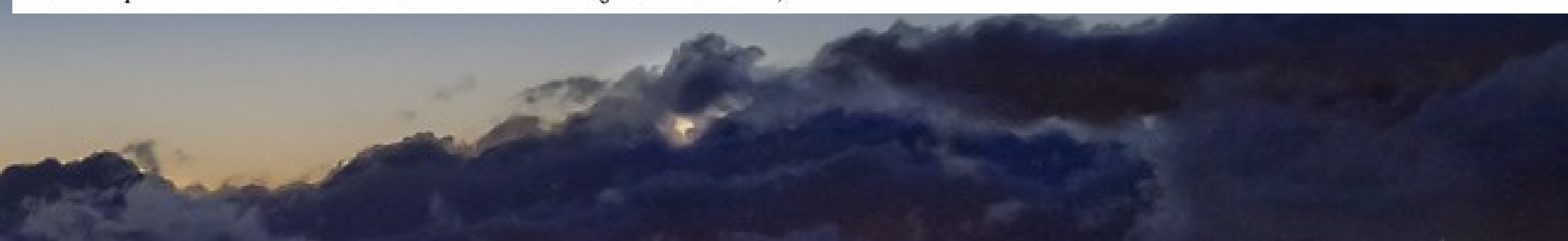
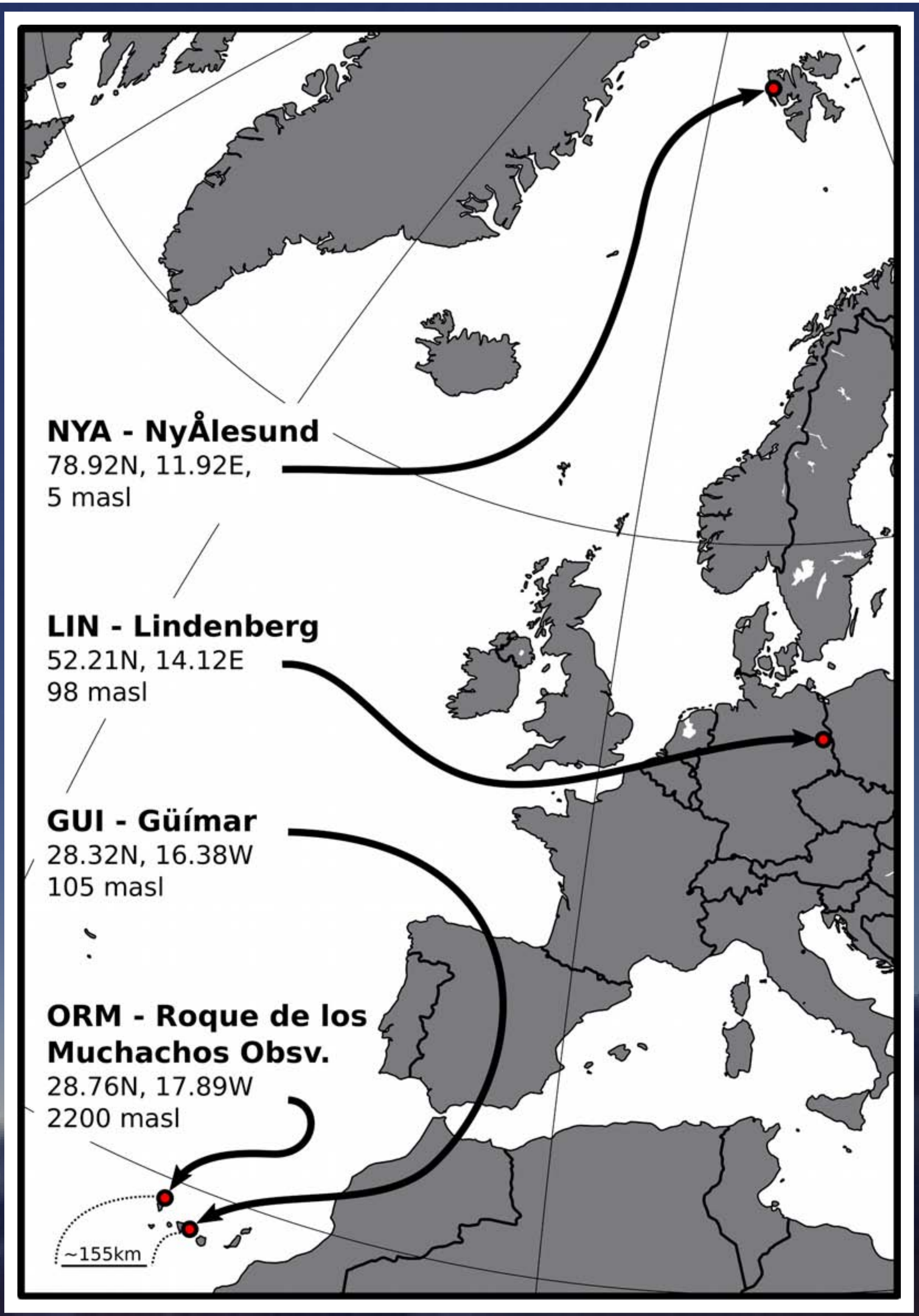
*Correspondence to:* Julio A. Castro-Almazán (jcastro@iac.es)




NYA - NyÅlesund
78.92N, 11.92E,
5 masl
LIN - Lindenberg
52.21N, 14.12E
98 masl
GUI - Güímar
28.32N, 16.38W
105 masl
ORM - Roque de los
Muchachos Obsv.
28.76N, 17.89W
2200 masl
~155km

**RS92 High Res.** *(>2500 lev)*

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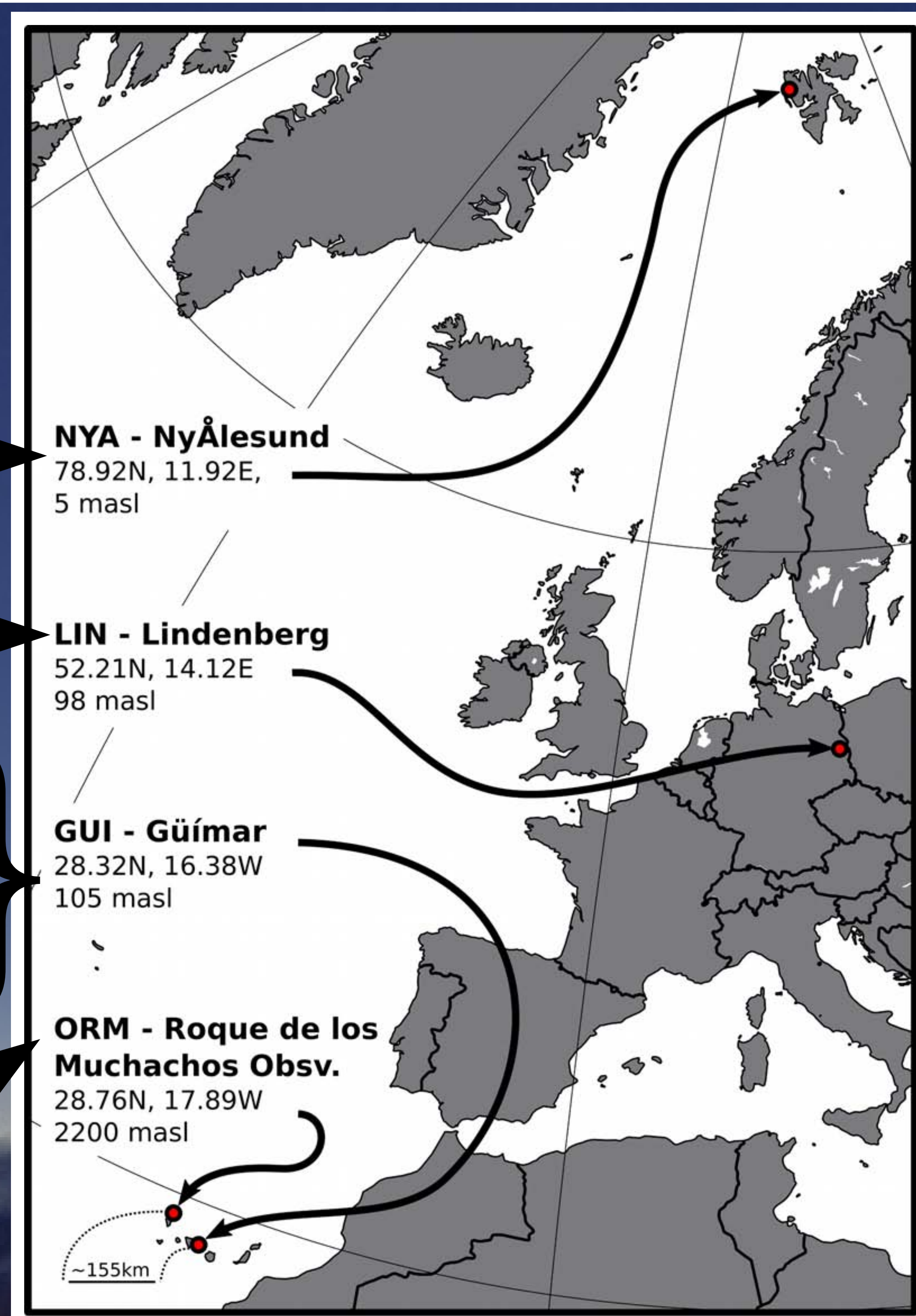
**STD Standard lev. (~15 lev)**
**SIG Significant lev. (~45 lev)**
**WYO Wyoming codif. (~100 lev)**

**RS80 High Res.** *(>2500 lev)*



## RS92 Vaisala

***data ~ 1 year***
***(2013-2014)***

***Quality Control:***
***AEMet (GUI)***
***GRUAN (LIN, NYA)***

## RS80 Vaisala

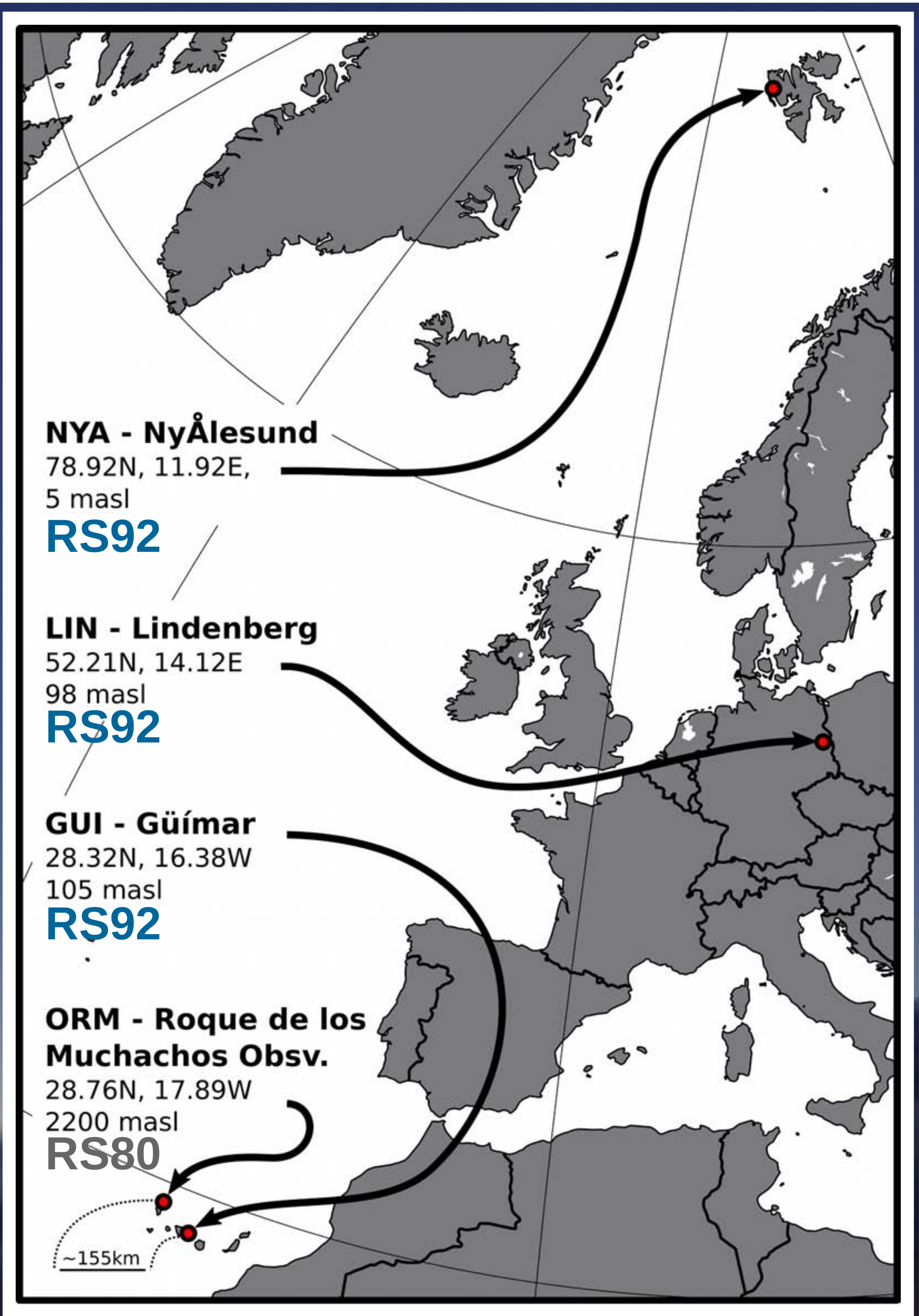
***data ~ intensive runs***
***(1990-APR-JUL)***
***(1995-NOV)***

***Quality Control:***
***Météo-France (ORM)***

## BOTH

***Uncertainties:***

$\pm 0.5\,^\circ\mathrm{C},\ \pm 5\,\%$ and $\pm 1\,\mathrm{hPa}$



# PWV from radiosondes

The PWV is obtained from the temperature $T$ (°C), atmospheric pressure $p$ (hPa) and relative humidity RH (%)

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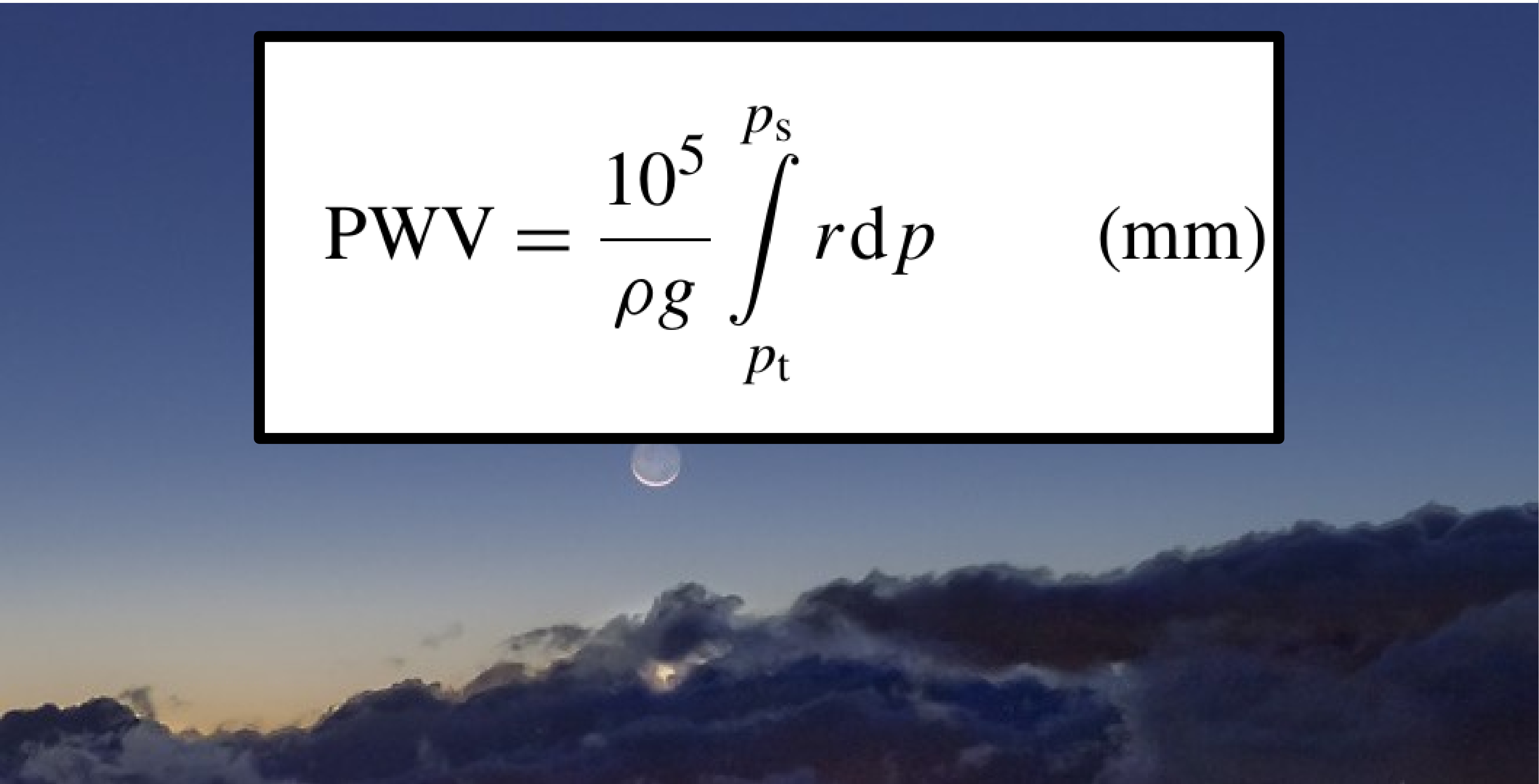
$$\mathrm{PWV} = \frac{10^5}{\rho g} \int_{p_\mathrm{t}}^{p_\mathrm{s}} r \, \mathrm{d} p \qquad \text{(mm)}$$



# PWV from radiosondes

The PWV is obtained from the temperature $T$ (°C), atmospheric pressure $p$ (hPa) and relative humidity RH (%)

$$\mathrm{PWV} = \frac{10^5}{\rho g} \int_{p_t}^{p_s} r \, \mathrm{d}p \qquad \text{(mm)}$$

$$e = e_{\mathrm{sat}} \cdot \frac{\mathrm{RH}}{100}$$

$$r = 0.622 \left( \frac{e}{p - e} \right)$$

$$e_{sat} = a_0 + T(a_1 + T(a_2 + T(a_3 + T(a_4 + T(a_5 + T a_6))))) \qquad \text{(hPa)}$$

# PWV from radiosondes

The PWV is obtained from the temperature $T$ (°C), atmospheric pressure $p$ (hPa) and relative humidity RH (%)

$$\mathrm{PWV} = \frac{10^5}{\rho g} \int_{p_\mathrm{t}}^{p_\mathrm{s}} r \, \mathrm{d}p \qquad \text{(mm)}$$

TRAPEZOIDAL SUMMATION

$$\mathrm{PWV} \approx \frac{1}{2} \sum_{i=0}^{N-1} \left( r_{i+1} + r_i \right) \cdot \left( p_{i+1} - p_i \right)$$

# Error budget for PWV from radiosonde data

$$\epsilon_{\mathrm{f}}^2 = \sigma^2 + \epsilon_{\mathrm{s}}^2$$

*uncertainty*
*sampling error*

# Error budget for PWV from radiosonde data

$$\epsilon_{\mathrm{f}}^2 = \underbrace{\sigma^2}_{} + \epsilon_{\mathrm{s}}^2 = \underbrace{\mathrm{var}_{\mathrm{f}} + 2 \cdot \mathrm{cov}_{\mathrm{f}}}_{} + \epsilon_{\mathrm{s}}^2$$

*uncertainty*
*sampling error*

# Error budget for PWV from radiosonde data

$$\epsilon_f^2 = \mathrm{var}_f + 2 \cdot \mathrm{cov}_f + \epsilon_s^2$$

*variance*

*covariance*

*sampling error*

# Error budget for PWV from radiosonde data

$$\epsilon_f^2 = \mathrm{var_f} + 2 \cdot \mathrm{cov_f} + \epsilon_s^2$$

*variance* → **analytical**

*covariance* → **empirical model**

*sampling error* → **empirical model**

# Error budget for PWV from radiosonde data

$$\epsilon_f^2 = \boxed{\mathrm{var_f}} + 2 \cdot \mathrm{cov_f} + \epsilon_s^2$$

*variance*

$$\mathrm{var_f} = \sum_{i=0}^{N-1} \sigma_i^2$$

**analytically:**
error propagation in the trapezoidal rule over all the sample levels

**note:**
In the trapezoidal rule,
each value, excluding the extremes,
is shared by adjacent bins; therefore,
the derivation of the variance must
avoid the double summation of error.

# Error budget for PWV from radiosonde data

$$\epsilon_f^2 = \boxed{\mathrm{var_f}} + 2 \cdot \mathrm{cov_f} + \epsilon_s^2$$

*variance*

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**note:**

in the trapezoidal rule, each value, excluding the extremes, is shared by adjacent bins; therefore, the derivation of the variance must avoid the double summation of error.

HUGE UGLY EQUATIONS (HERE) CENSORED!

(see the paper...)

# Error budget for PWV from radiosonde data

$$\epsilon_f^2 = \mathrm{var}_f + 2 \cdot \boxed{\mathrm{cov}_f} + \epsilon_s^2$$

*covariance*

**empirical model**

$$\mathrm{cov}_f = \sum_{i,j;\; j>i} \sigma_i \sigma_j \rho_{ij}$$

# Error budget for PWV from radiosonde data

$$\epsilon_f^2 = \mathrm{var}_f + 2 \cdot \mathrm{cov}_f + \epsilon_s^2$$

*covariance*

**empirical model:**

vertical profiles

⇩

'time' (space) series

⇩

semi-Markov process

(the probability of the next levels depend on the amount of time elapsed τ)

with τ exponentially distributed.

⇩

the covariance as a simplified pure exponential ρ ⇨

$$\mathrm{cov}_f = \sum_{i,j;\, j>i} \sigma_i \sigma_j \rho_{ij}$$

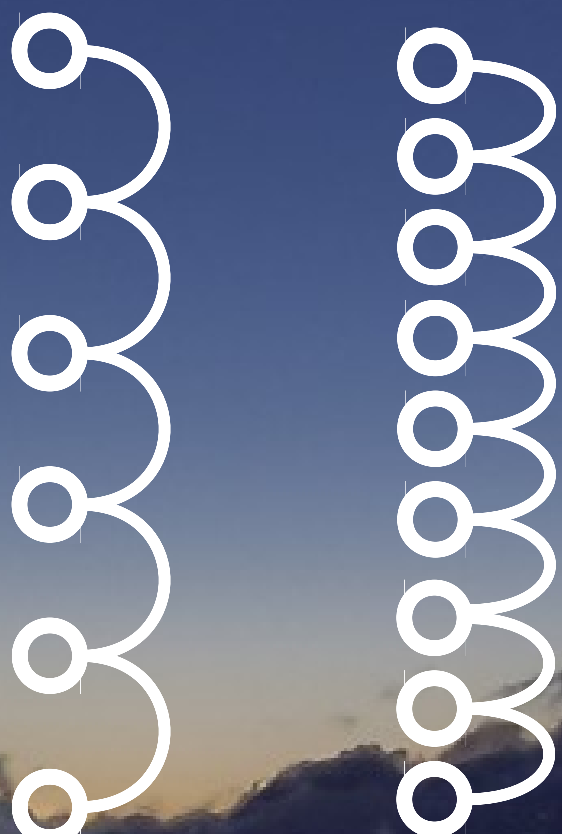
$$\mathrm{cov}_f = \sum_{i,j;\, j>i} \sigma_i \sigma_j e^{-(p_i - p_j)/\tau_\rho}$$

# Error budget for PWV from radiosonde data

$$\epsilon_f^2 = \mathrm{var_f} + 2 \cdot \mathrm{cov_f} + \epsilon_s^2$$

*sampling error*

**empirical model**



# Error budget for PWV from radiosonde data

$$\epsilon_f^2 = \mathrm{var_f} + 2 \cdot \mathrm{cov_f} + \epsilon_s^2$$

*sampling error*

High resolution data (>2500)

↓

negligible sampling error

↓

**empirical model**

$$\epsilon_s = \epsilon_s(N)$$

↓

recursively sub-sampling each profile (uniformly)
and
analysing the residuals:

$$\mathrm{res}_N = I - \tilde{I}_N$$

# Error budget for PWV from radiosonde data

$$\epsilon_f^2 = \text{var}_f + 2 \cdot \text{cov}_f + \epsilon_s^2$$

*sampling error*

High resolution data (>2500)

negligible sampling error

**empirical model**

$$\epsilon_s = \epsilon_s(N)$$

recursively sub-sampling each profile (uniformly)
and
analysing the residuals:

$$\text{res}_N = I - \widetilde{I}_N$$

**note:**
the **whole** atmospheric column is assumed to be **uniformly sampled**.

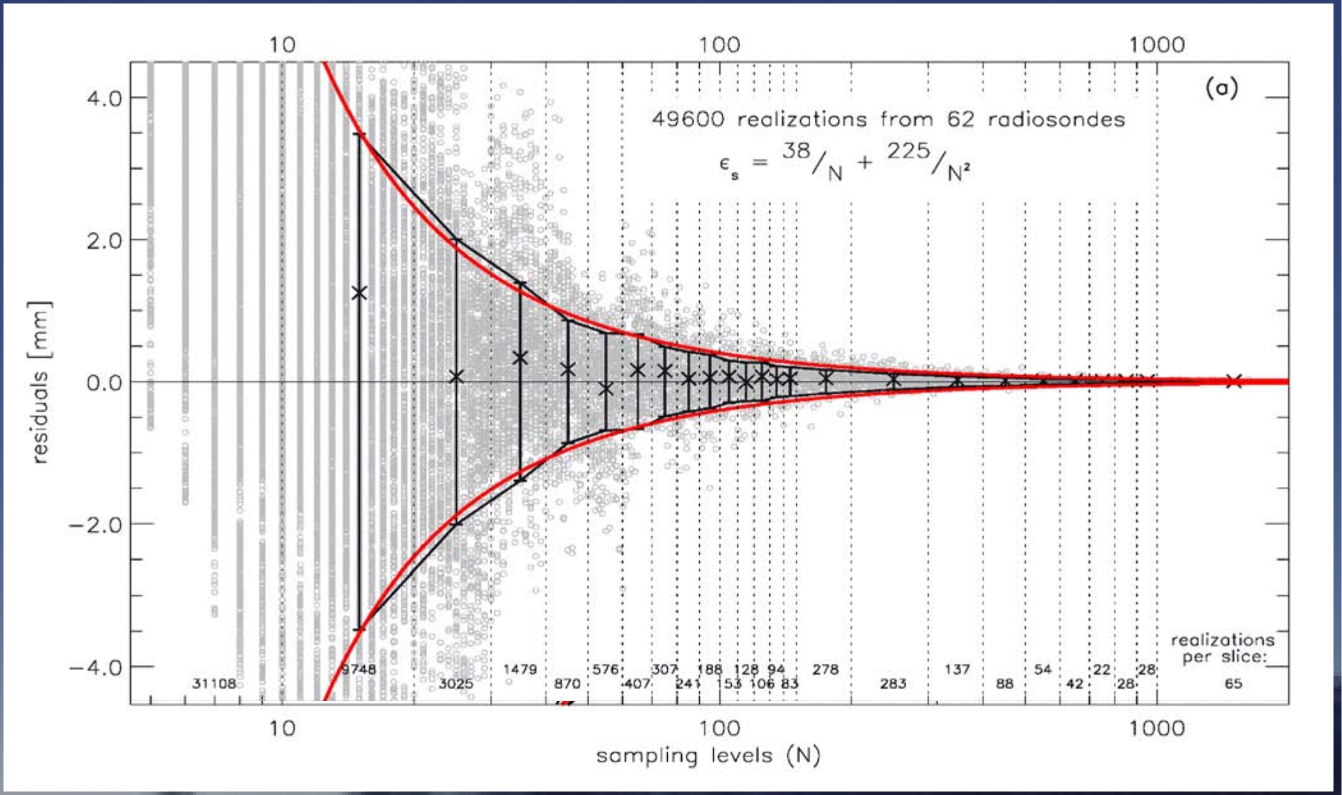
# Error budget for PWV from radiosonde data

$$\epsilon_f^2 = \mathrm{var}_f + 2 \cdot \mathrm{cov}_f + \epsilon_s^2$$

*sampling error*

$$\mathrm{res}_N = I - \widetilde{I}_N$$

**grouped in slices to fit a model...**



# Error budget for PWV from radiosonde data

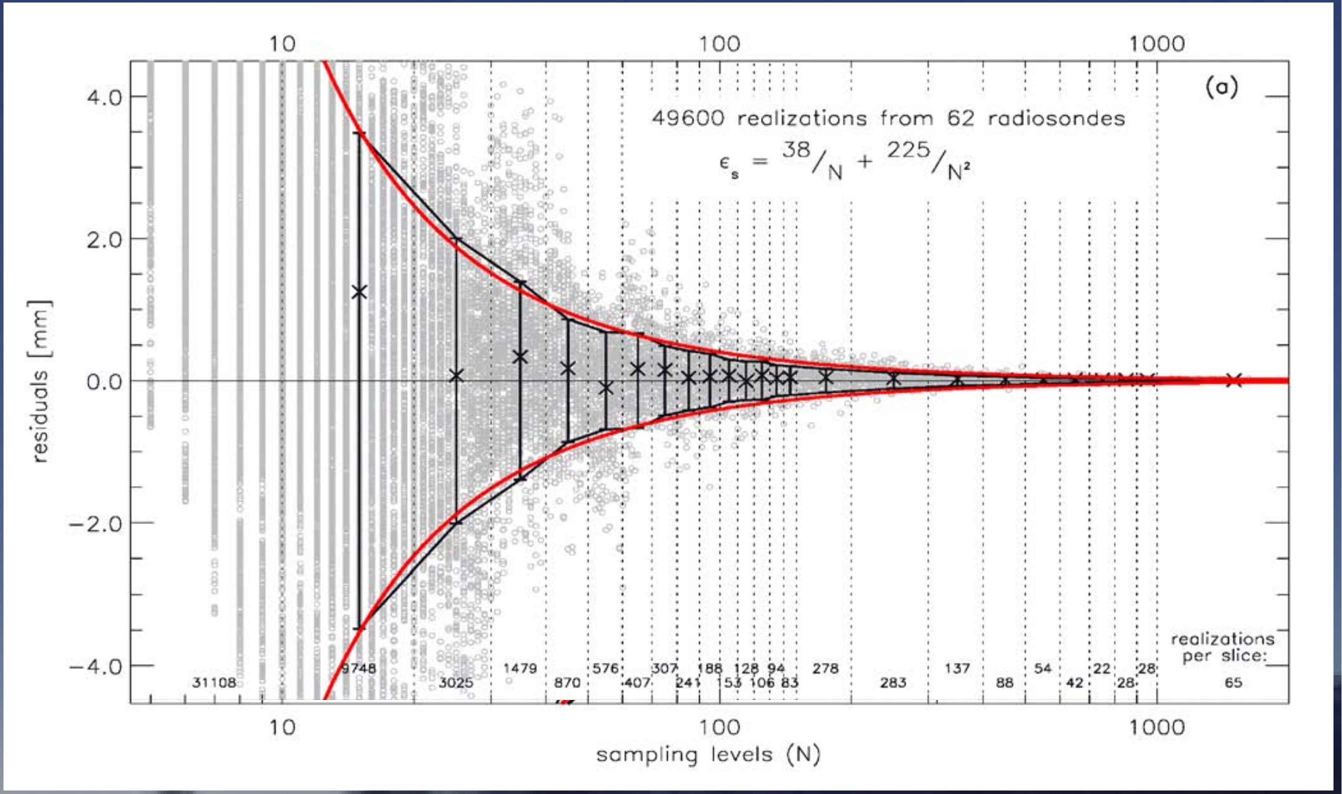
$$\epsilon_f^2 = \mathrm{var_f} + 2 \cdot \mathrm{cov_f} + \epsilon_s^2$$

*sampling error*

$$\mathrm{res}_N = I - \widetilde{I}_N$$

**dependence on N:**

$$\widetilde{I}_N = \widetilde{\mathrm{PWV}}(N^{-1}) + E(N^{-2})$$

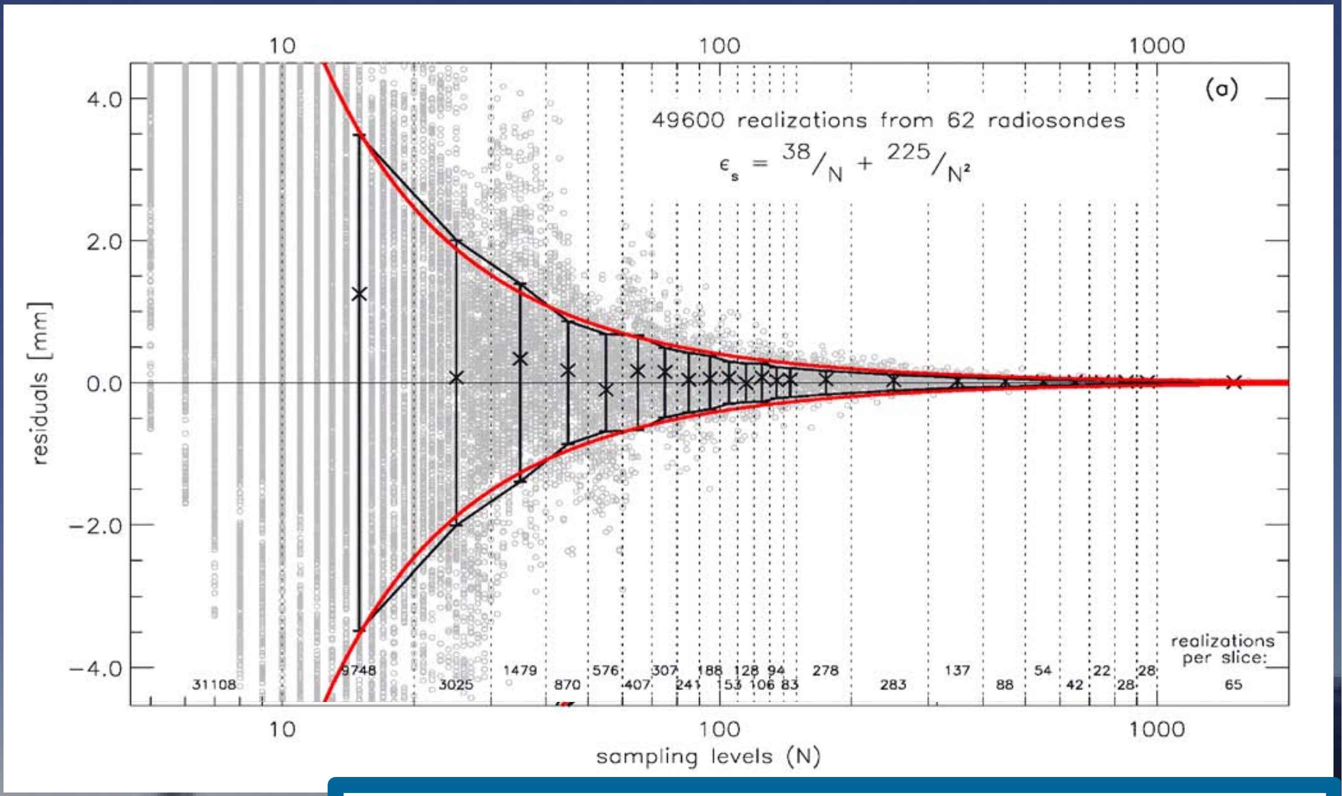


# Error budget for PWV from radiosonde data

$$\epsilon_{\mathrm{f}}^2 = \mathrm{var}_{\mathrm{f}} + 2 \cdot \mathrm{cov}_{\mathrm{f}} + \epsilon_{\mathrm{s}}^2$$

*sampling error*

$$\mathrm{res}_N = I - \tilde{I}_N$$




**dependence on N:**

$$\tilde{I}_N = \widetilde{\mathrm{PWV}}(N^{-1}) + E(N^{-2})$$

**fit a function:**

$$A/N + B/N^2 + C$$

$C = 0$, as

$$\lim_{(N\to\infty)} \mathrm{res}_N = 0,$$

$$\epsilon_{\mathrm{s}} = \frac{s_0}{N} + \frac{s_1}{N^2}; \qquad (N \gtrsim 10)$$

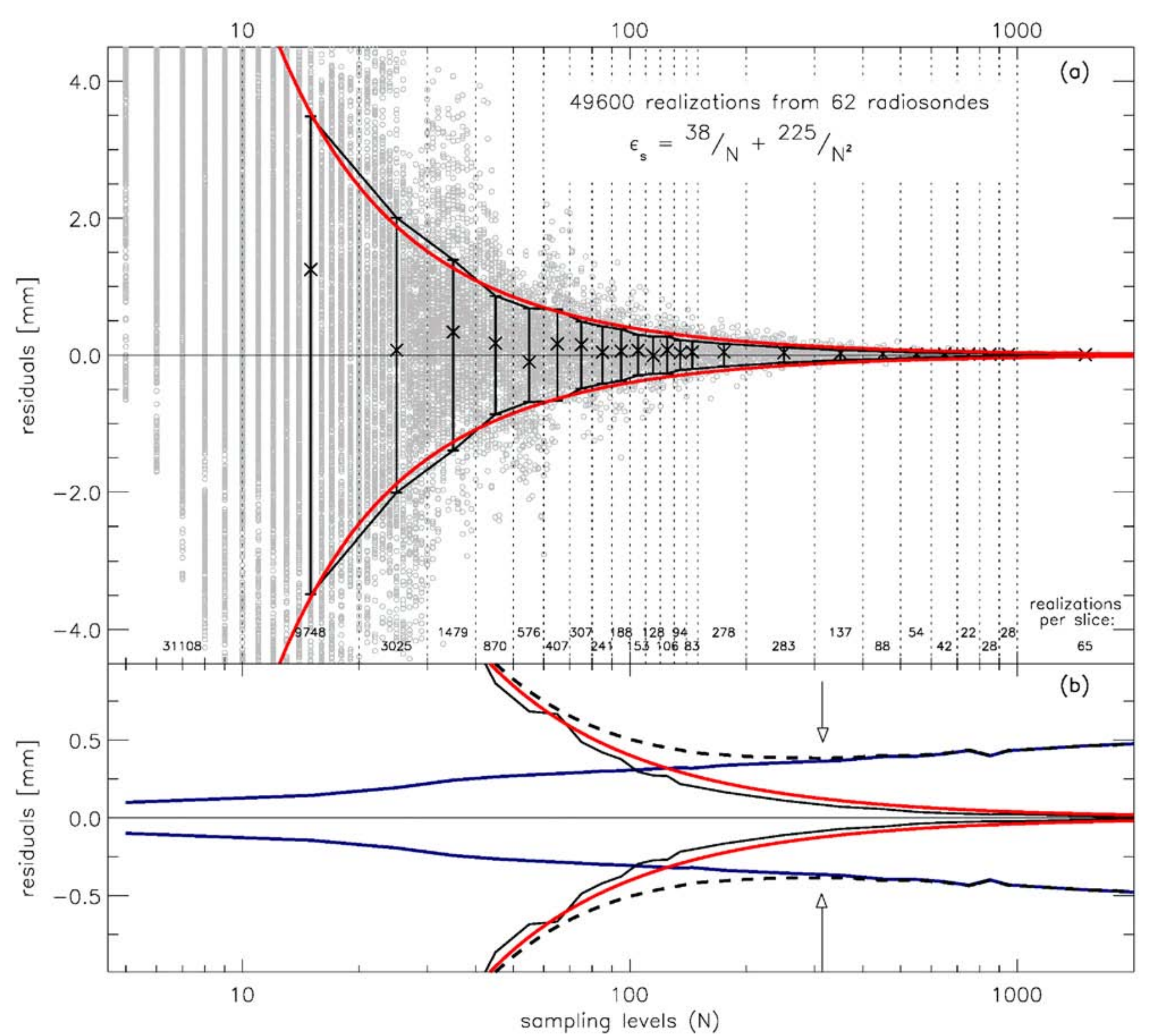
# Optimized error

$$\epsilon_f^2 = \sigma^2 + \epsilon_s^2$$

$$\epsilon_s^2$$

$$\sigma^2$$

$$\begin{aligned}\epsilon^2 &= \min[\epsilon_f^2(N, \mathrm{PWV})] \\ &= \min[\sigma^2(N, \mathrm{PWV}) + \epsilon_s^2(N)],\end{aligned}$$



# Optimized error

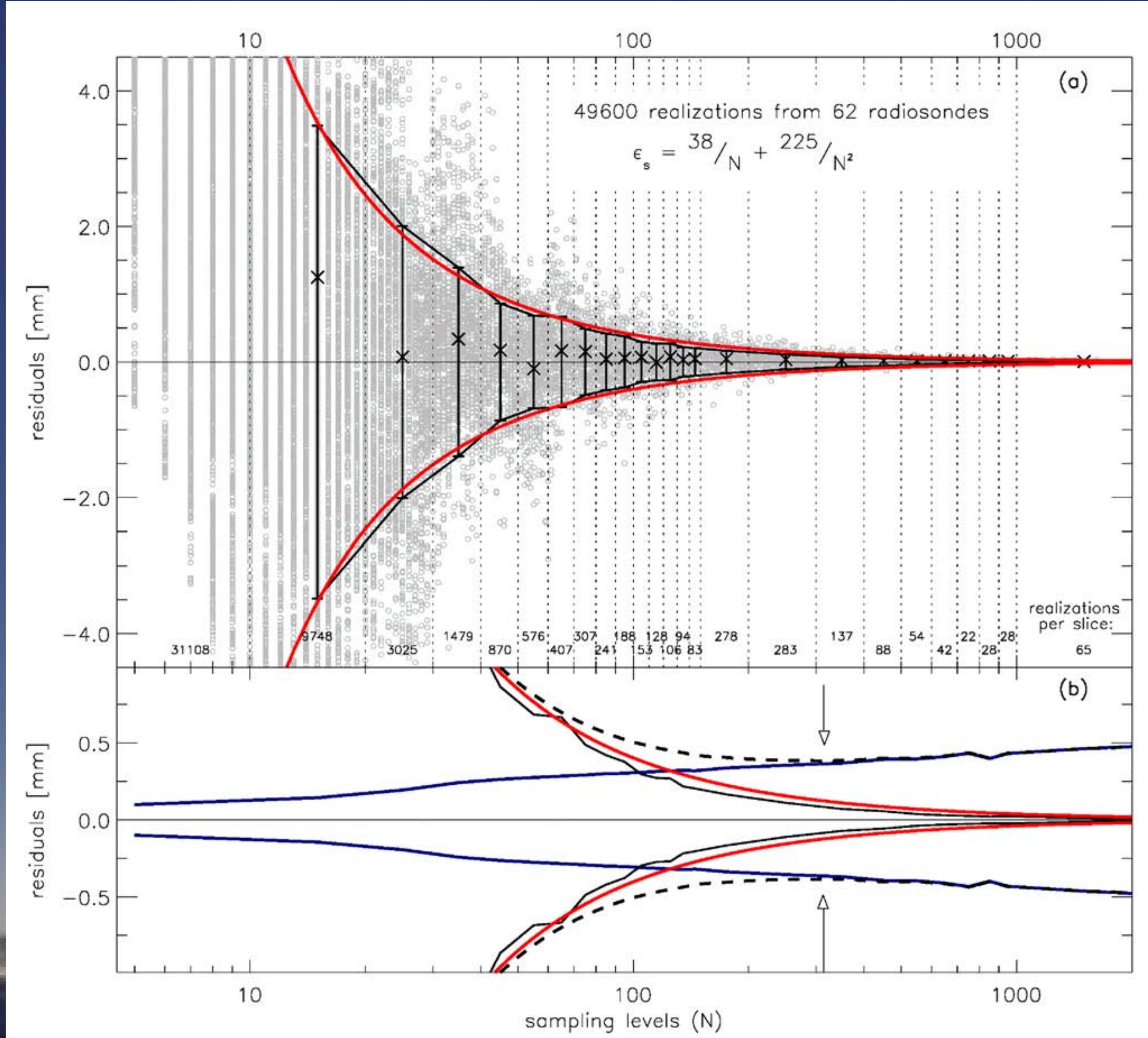
$$\epsilon_f^2 = \sigma^2 + \epsilon_s^2$$

$$N_0 = \arg\min[\epsilon_f]$$

$$\sigma_0 = \sigma(N_0, \mathrm{PWV}),$$

$$\epsilon_{s_0} = \epsilon_s(N_0).$$

$$\begin{aligned} \epsilon^2 &= \min[\epsilon_f^2(N, \mathrm{PWV})] \\ &= \min[\sigma^2(N, \mathrm{PWV}) + \epsilon_s^2(N)], \end{aligned}$$

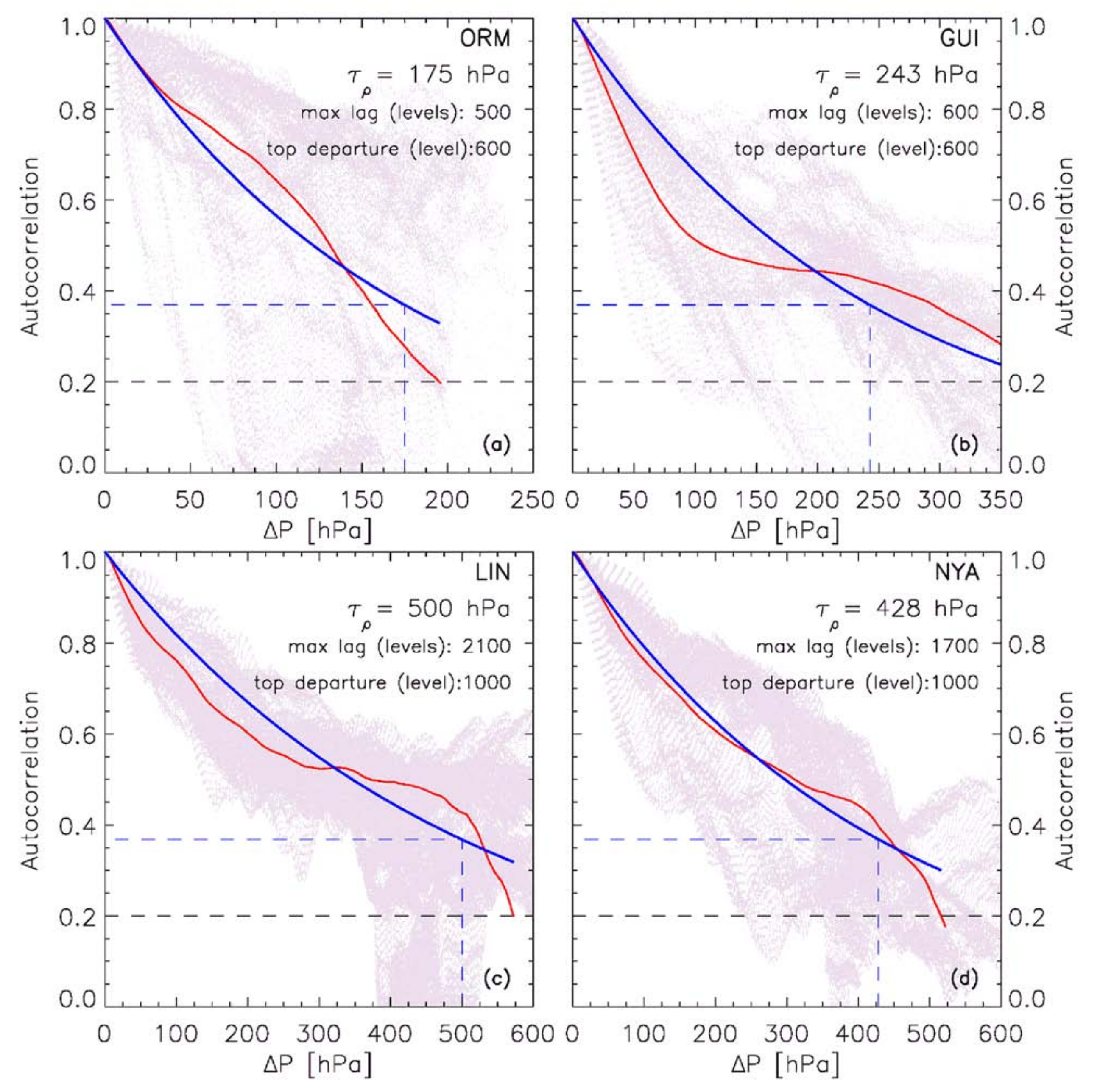


# Results

$$\mathrm{cov_f} = \sum_{i,j;\, j>i} \sigma_i \sigma_j e^{-(p_i - p_j)/\tau_\rho}$$

*covariance*

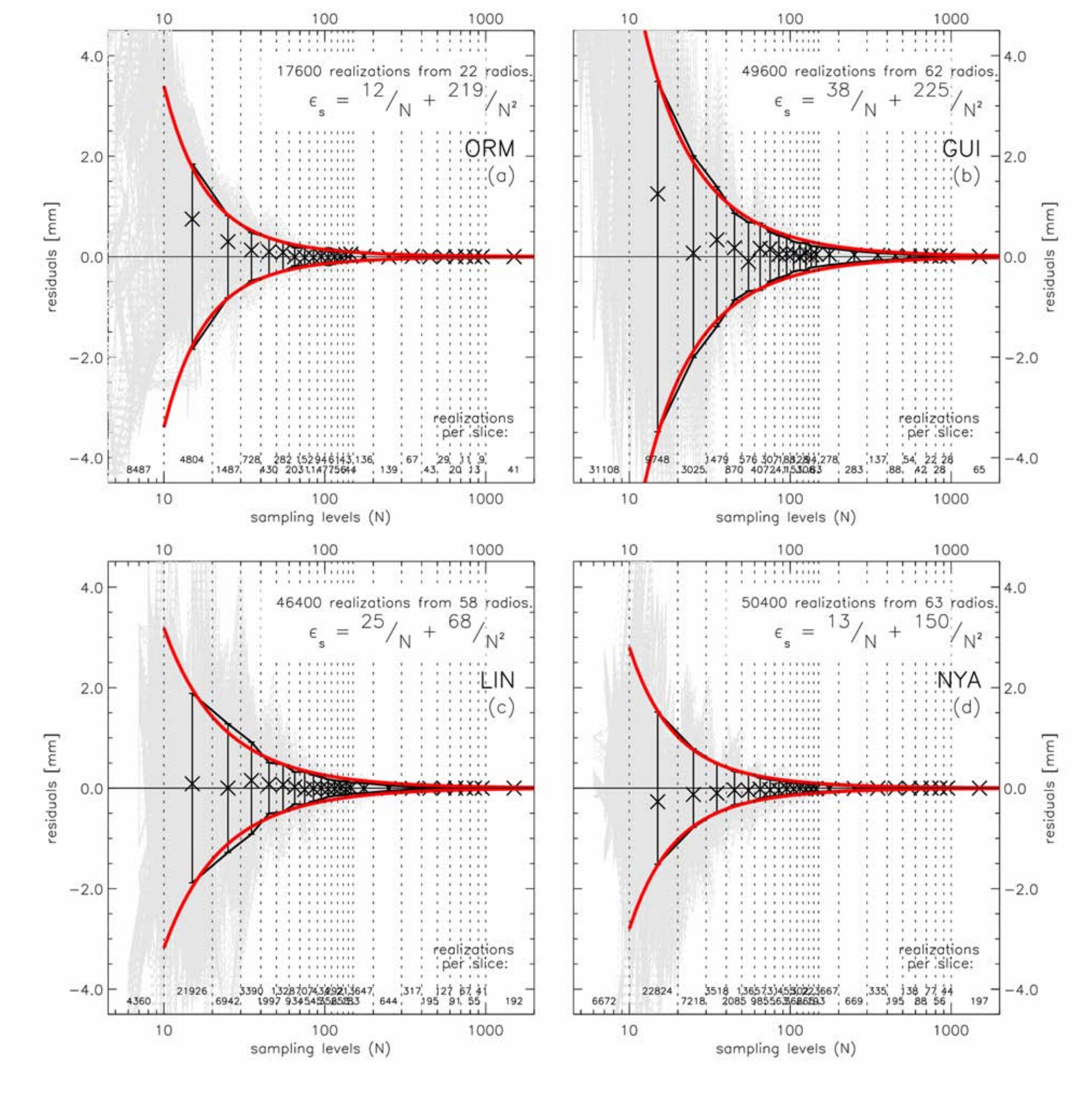
*exponential autocorrelation lags*



# Results

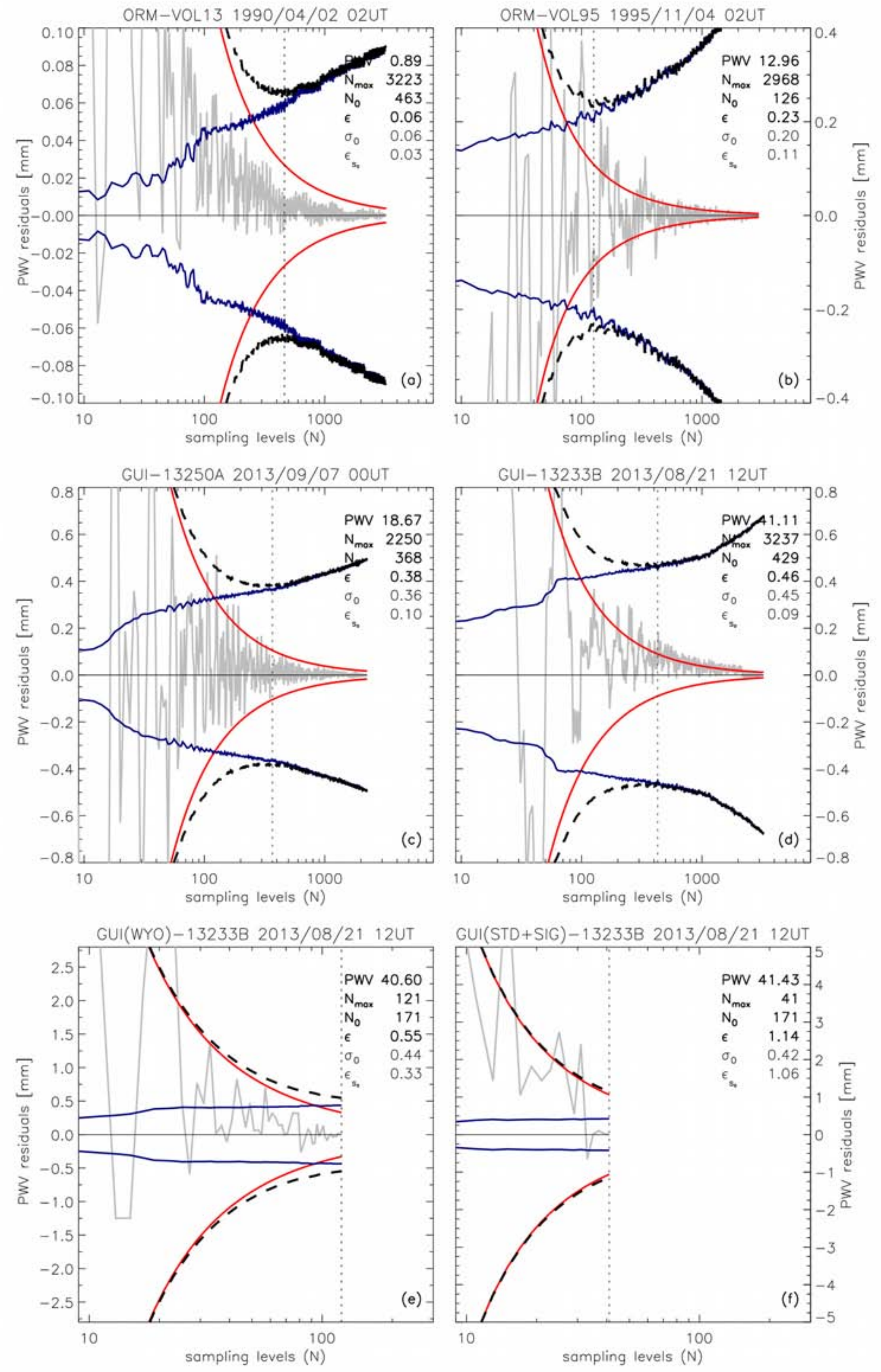
$$\epsilon_s = \frac{s_0}{N} + \frac{s_1}{N^2}; \qquad (N \gtrsim 10)$$

*sampling errors*

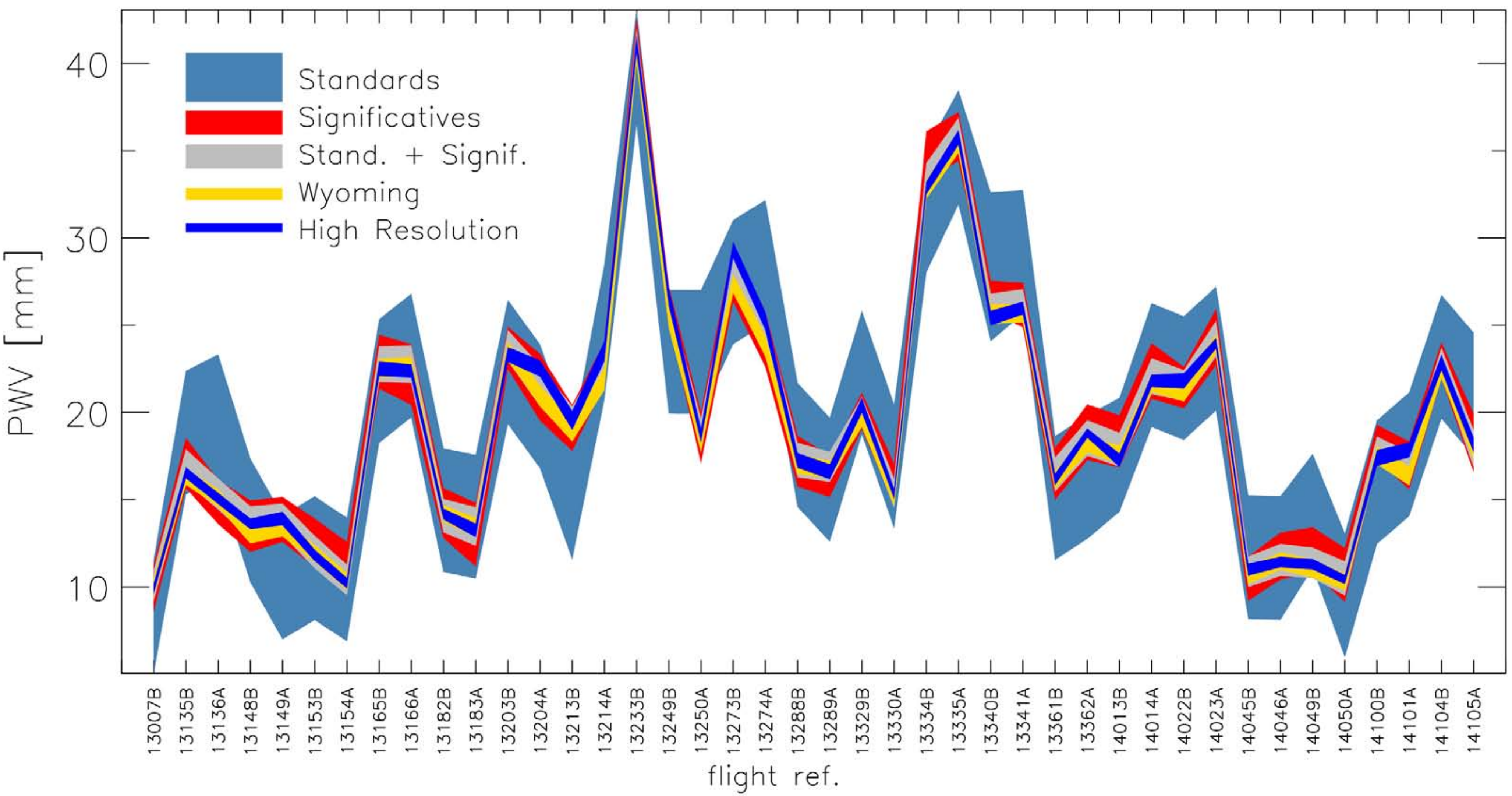


# Results

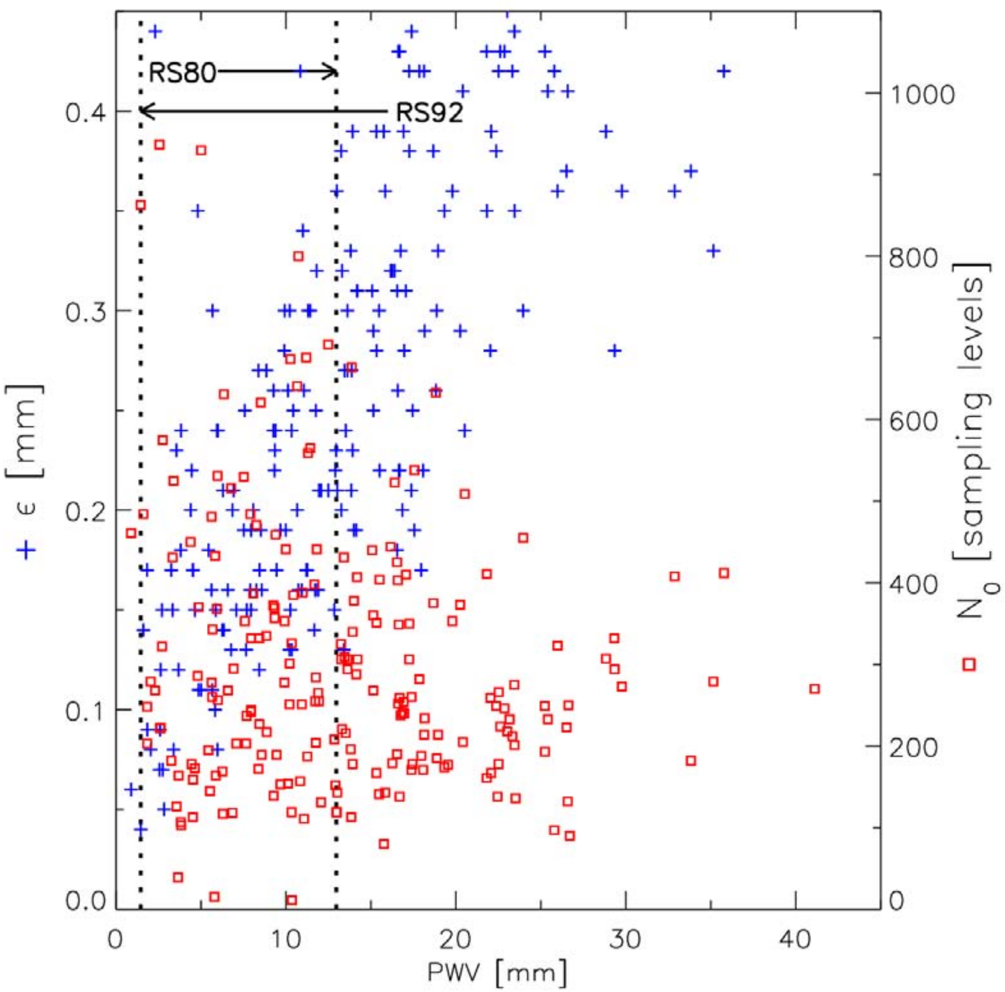
*examples*

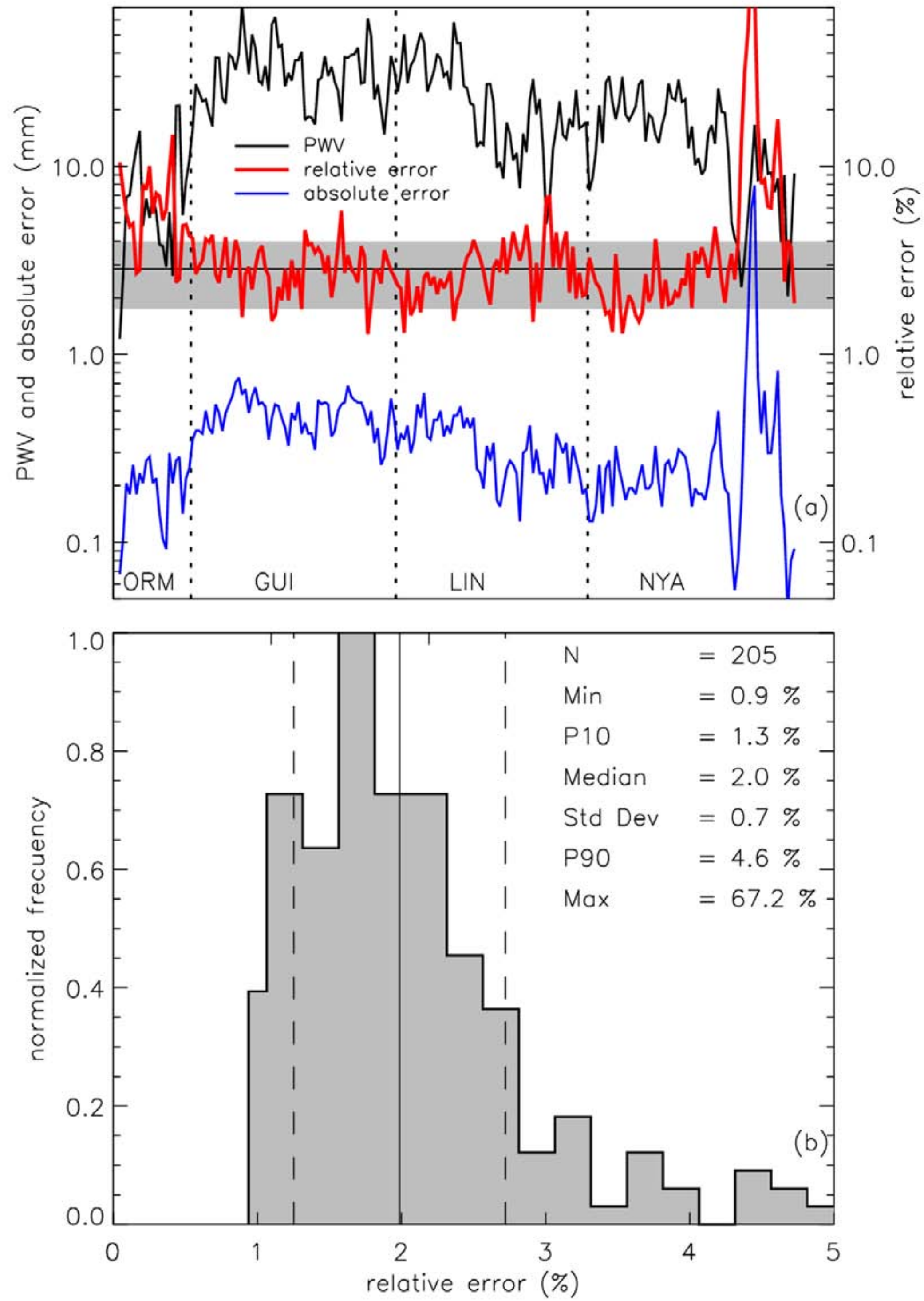


# Results


Standards
Significatives
Stand. + Signif.
Wyoming
High Resolution
PWV [mm]
10
20
30
40
13007B
13135B
13136A
13148B
13149A
13153B
13154A
13165B
13166A
13182B
13183A
13203B
13204A
13213B
13214A
13233B
13249B
13250A
13273B
13274A
13288B
13289A
13329B
13330A
13334B
13335A
13340B
13341A
13361B
13362A
14013B
14014A
14022B
14023A
14045B
14046A
14049B
14050A
14100B
14101A
14104B
14105A
flight ref.

# Results


RS80
RS92
ε [mm]
$N_0$ [sampling levels]
PWV [mm]
0.0
0.1
0.2
0.3
0.4
0
10
20
30
40
0
200
400
600
800
1000


PWV
relative error
absolute error
PWV and absolute error (mm)
relative error (%)
10.0
1.0
0.1
ORM
GUI
LIN
NYA
(a)
N = 205
Min = 0.9 %
P10 = 1.3 %
Median = 2.0 %
Std Dev = 0.7 %
P90 = 4.6 %
Max = 67.2 %
normalized frecuency
relative error (%)
(b)
0.0
0.2
0.4
0.6
0.8
1.0
0
1
2
3
4
5

# practical applications

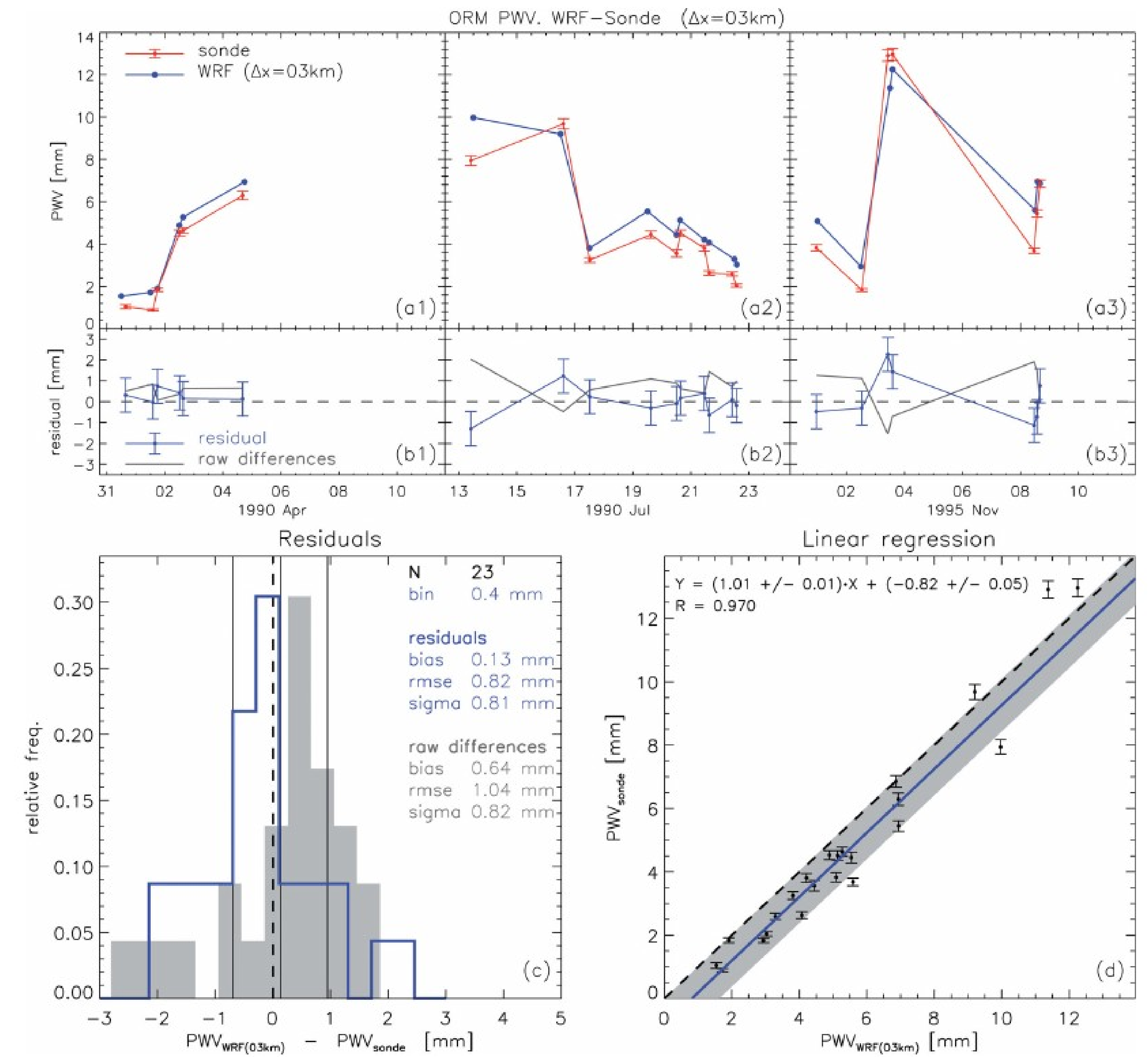



**Figure 8.** (a1–a3) Data series of PWV measured (red) and WRF forecast (blue) with horizontal resolution $\Delta x = 3$ km, (b1–b3) time series of residuals and raw differences, (c) distribution of residuals and (d) regression analysis at ORM. The indices 1, 2 and 3 refer to the 3 separated campaigns in 1990 April, 1990 July and 1995 November. The residuals are calculated after applying the linear fit model, while the raw differences are calculated directly as $PWV_{WRF} - PWV_{sonde}$. The shading in the linear fit is the estimation of the error as the squared sum of the RMSE of the residuals and uncertainties of the coefficients given in the equation. [A colour version of this figure is available in the online version.]

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**Forecasting the precipitable water vapour content: validation for astronomical observatories using radiosoundings**

G. Pérez-Jordán,[1★†] J. A. Castro-Almazán,[1,2†] C. Muñoz-Tuñón,[1,2†] B. Codina[3] and J. Vernin[4]

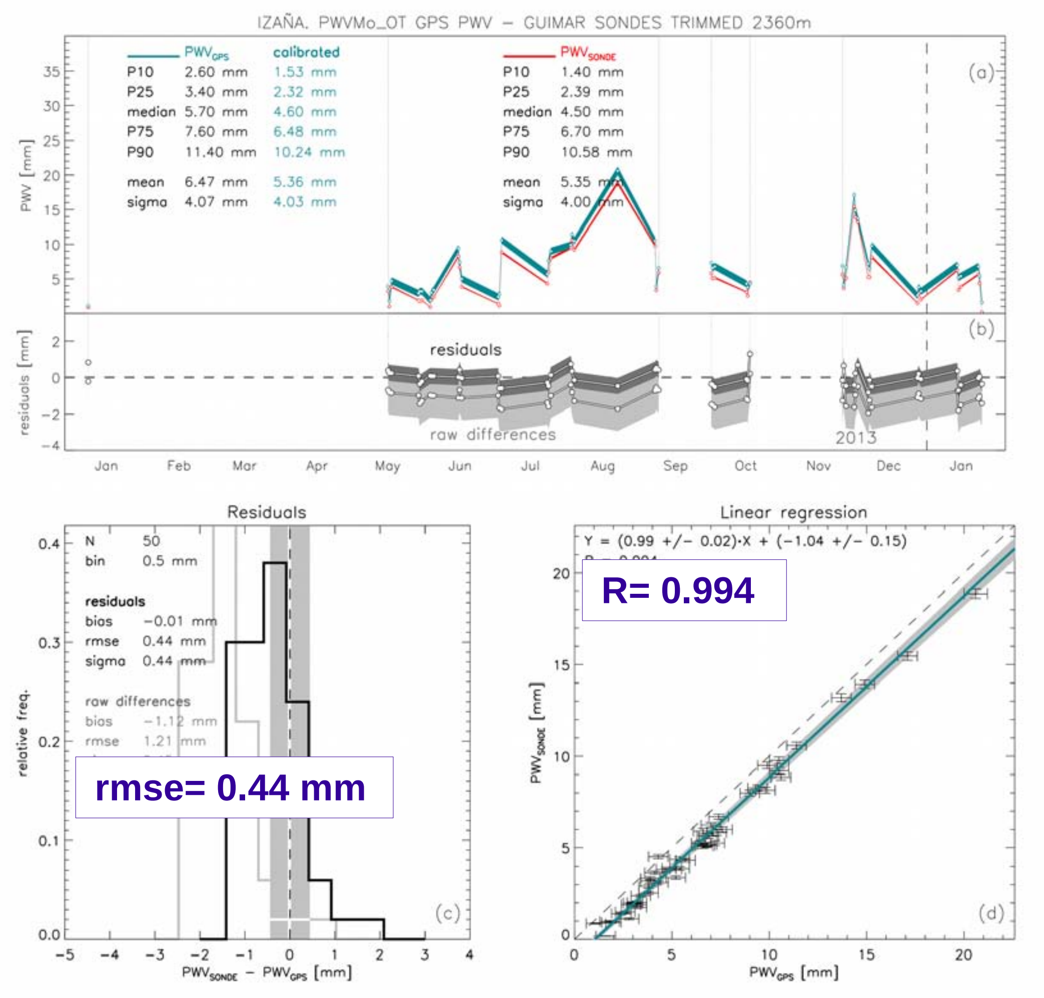
# practical applications

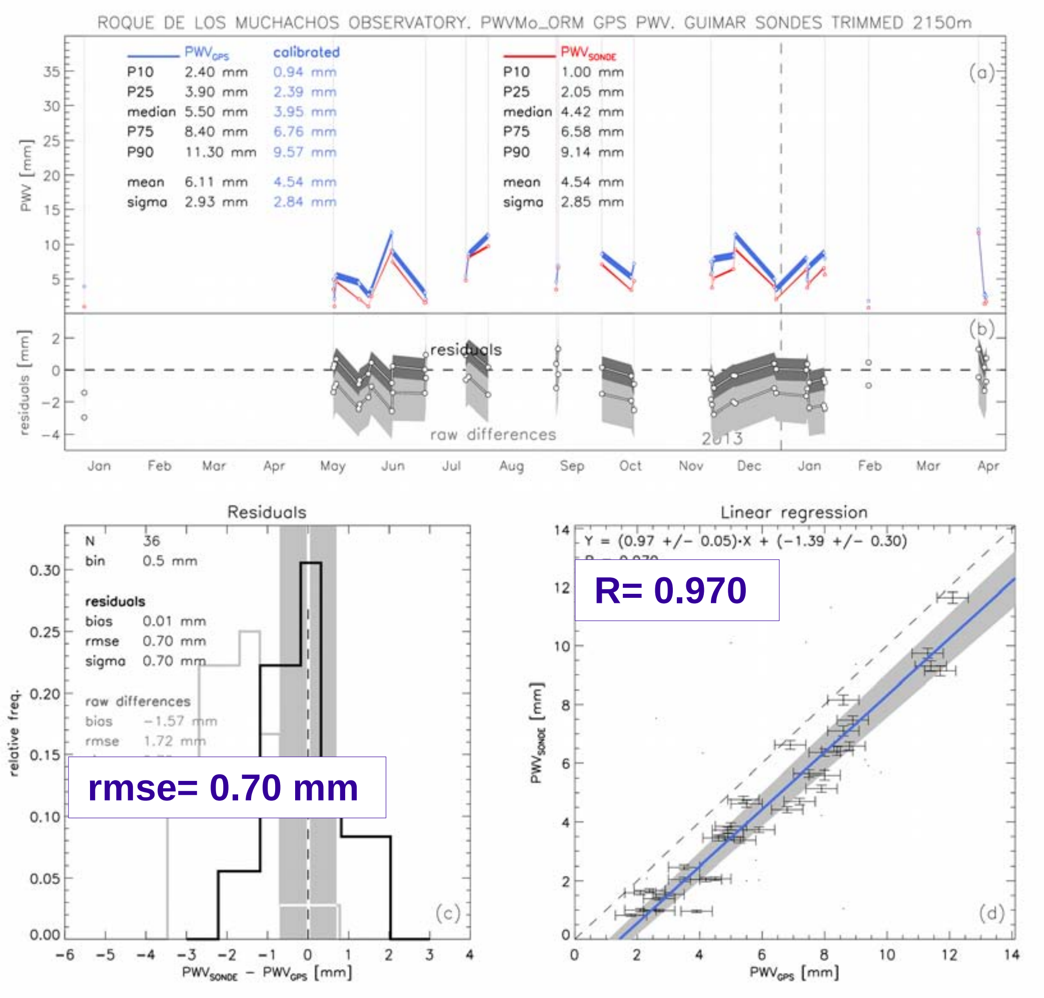
## Precipitable Water Vapour at the Canarian Observatories (Teide and Roque de los Muchachos) from routine GPS

Julio A. Castro-Almazán[a,b], Casiana Muñoz-Tuñón[a,b], Begoña García-Lorenzo[a,b], Gabriel Pérez-Jordán[a*], Antonia M. Varela[a,b], and Ignacio Romero[c,d]

Observatory Operations: Strategies, Processes, and Systems VI, edited by Alison B. Peck, Robert L. Seaman, Chris R. Benn, Proc. of SPIE Vol. 9910, 99100P · © 2016 SPIE · CCC code: 0277-786X/16/$18 · doi: 10.1117/12.2232646

Proc. of SPIE Vol. 9910 99100P-1



# CONCLUSIONS

- PWV strongly impacts the astronomical observations (IR, μ-w, …)
- A semiempirical method for estimating the **error** and **optimum number of sampled levels ($N_0$)** in **PWV** determinations from **radiosoundings** is proposed.
- The method was tested in **4 different locations** (different conditions, latitudes, PWV content,...).
- The error is roughly proportional to PWV whereas $N_0$ is the reverse.
- The value of **$N_0$ is less than 400** for 77 % of the profiles and the absolute **errors are always < 0.6 mm**.
  Median **relative error is 2.0 ± 0.7 %**
- These results reduce by more than a half the uncertainties previously reported in the literature.

# CONCLUSIONS

Therefore, not only the uncertainties define the error in PWV estimations from radiosoundings but also the autocorrelation between levels and the sampling. Here we have proposed that it is possible to optimize the number of sampled levels to minimize the error within the instrumental uncertainty. Whereas a radiosounding samples at least $N_0$ uniform vertical levels, depending on the WV content and distribution of the atmosphere, the error in the PWV estimate is likely to stay below $\approx 3\,\%$ (median + dispersion $= 2.7\,\%$) even for dry conditions.


thank you!
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