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Capacity Planning and Resource Acquisition Decisions Using Robust Optimization

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by

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To my parents Inta Bērziņa and Dr. Oļǧerts Jakubovskis,

to my wife Marika, and my sons Eric and Arthur

ABSTRACT

This dissertation studies strategic capacity planning and resource acquisition decisions, including the facility location problem and the technology choice problem. These decisions are modeled in an integrative manner, and the main purpose of the proposed models and numerical experiments is to examine the effects of economies of scale, economies of scope, and the combined effects of scale and scope under uncertain demand realizations using robust optimization. The type of capacities, or technology alternatives, that a firm can acquire can be classified on two basic dimensions. The first dimension relates to the effects of scale via distinction between labor-intensive (less automated) technologies and capital-intensive (more automated) technologies. The second dimension relates to the effects of scope via distinction between product-dedicated and flexible technologies. Moreover, each of the product-dedicated and flexible technologies can have different levels of labor or capital-intensiveness, leading to the joint effects of economies of scale and economies of scope. Each of the technology alternatives possesses certain cost structures. Labor-intensive technologies are characterized by low fixed costs and high variable costs, whereas capital-intensive technologies are characterized by just the opposite cost structure, i.e., high fixed costs and low variable costs. Flexible technologies cost more than product-dedicated technologies, both in terms of fixed and variable costs. Robust optimization methodology is used to investigate how different levels of robustness, and facility and technology costs affect the quantities, types and allocation of technologies to facilities. Results show that specific technology choice patterns emerge depending on various cost structures and different levels of model robustness specified to accommodate uncertain demand realizations. The results obtained by the two-stage robust optimization approach are compared to the results obtained by a non-robust approach and a stochastic programming approach.

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Chapter 1

Introduction

1.1 Research Motivation

According to popular managerial publications, many industries such as automotive, steel, and semiconductor, suffer from chronic or cyclical overcapacity that threatens the profitability and even the survival of companies in these sectors. The reasons for this overcapacity are complex, industry-specific, and related to overall global economic trends. Yet what is common in all industries is that the capacity levels are determined based on future demand forecasts, which are inherently uncertain. The main motivating factor for our research is to provide insights and approaches to deal with uncertain market conditions when making high-level, or strategic, infrastructure investments.

Capacity investment decisions, as strategic level decisions, are characterized not only by their long-term impact, high fixed costs, and irreversible consequences, but also by their relatively high reliance on qualitative judgement-based approaches, as opposed to data-driven approaches. Data-driven approaches for capacity investment decisions are well suited for operational or tactical level decisions that can be modeled using traditional stochastic programming methodology. Therefore, we believe, the application of robust optimization, which requires little distributional information, is an appropriate tool for capacity planning and resource acquisition decisions. In this case, the chosen methodology fits the subject matter – reliable quantitative results can be obtained based on scarce historical data or very limited knowledge about the potential future realizations of demand. Demand uncertainty is probably the most recognized, but not the only source of uncertainty; for example, other sources of uncertainty include supplier reliability, costs, productivity, and many others. We intend to develop an approach that can be extended to address the uncertainty in these other parameters. It can be noted that the uncertain parameters can be grouped into three categories depending on their place in optimization models, i.e., they can be either uncertain objective function vectors, uncertain right hand side vectors, or uncertain left hand side matrices.

Optimization in a stochastic environment presents three challenges: first, the limitations of computational capabilities in solving real-world large size problems, second, the limited availability of efficient algorithms and solution procedures, and third, the appropriateness of applying a particular approach in capturing the randomness of data. While the first two of these issues are being addressed by the rapid advances in information technology and operations research, the third issue to a large extent depends on our understanding of uncertainty, which is one of the fundamental epistemological questions in general. Thus, it is our objective to contribute to the field of robust optimization, which has the potential for both providing computationally tractable problem formulations and incorporating random data with limited information about the nature of uncertainty. Also, the direction of our work is consistent with the increase in the number of research publications that explicitly incorporate stochastic features in model formulations. We believe that our modeling approach has a prototypic value and that it could be adapted to a variety of industry applications. For example, although we specifically address demand uncertainty ("downstream randomness”), one could easily adopt this approach to supply uncertainty (“upstream randomness”).

1.2 Research Questions

The unifying theme of this work is the issue of how a firm can position its strategic resources considering the trade-off between capacity shortage and capacity excess under uncertain demand conditions. However, the quantitative assessment of exogenous demand uncertainty is not sufficient to establish the appropriate levels of capacity. A firm must also decide to what extent it is willing to satisfy excessive demand realizations, i.e., how *robust* a firm’s overall capacity should be to accommodate demand randomness. This dissertation provides answers to the following key questions:

1. How do different levels of robustness, facility, and technology costs affect the quantities, types and allocation of technologies to facilities using robust optimization?
2. How do robust optimization solutions differ from non-robust solutions, with respect to the quantities, types and allocation of technologies to facilities for varying levels of robustness?
3. How do robust optimization solutions differ from stochastic programming solutions, with respect to total costs and quantities, types and allocation of technologies to facilities for varying levels of robustness.

All three questions are addressed by using two versions of the facility location, capacity acquisition, and technology choice model. The first version, the single product model is a step towards a more general model, the multi-product model. While the main purpose of the single product model is to examine the effects of economies of scale, the main purpose of the multi-product model is to examine the combined effects of economies of scale and economies of scope.

1.3 Contributions

Our implementation of the integrated facility location, capacity acquisition, and technology choice model in a two-stage robust optimization setting builds upon a solid and extensive theoretical foundation that encompasses a rather broad range of research areas. However, despite the variety of concepts utilized in our work, we have maintained a clear focus and a distinct unifying theme throughout this research. A firm, to be able to fully satisfy customer requirements and maintain or increase its competitiveness, must wisely acquire means of production that include various resources and capacities. These acquisition decisions include spatial aspects as well as temporal aspects, which means that a firm must decide where to *locate* these production capacities by taking into account the *time* lag between capacity investment decisions ("here-and-now" decisions) and demand realizations ("wait-and-see" decisions). We offer a novel view on the capacity planning process with the consideration of these spatial and temporal aspects using the two-stage robust optimization methodology. In particular, we contribute to the literature on strategic capacity planning and resource acquisition decisions as follows.

We propose a framework according to which a firm must find an optimal mix of *dedicated* vs. *flexible* (capable of producing multiple product types) technologies on one hand, and *labor-intensive* vs. *capital-intensive* technologies on the other hand. For convenience we will refer to a technology with low fixed and high variable costs as a "labor-intensive" one, whereas we refer to a technology with high fixed and low variable costs as a "capital-intensive" one. In this case, "labor-intensive" does not mean a manual low-productivity type of activity, it just means that this type of technology (or process) possesses this particular cost structure. We show that there exist specific relationships between these four types of capacities (labor-intensive and capital-intensive, and dedicated and flexible) that form the basis for the scale effects, scope effects, and the joint scale and scope effects under uncertain demand conditions. We also show that with the increase of uncertainty (or increase in robustness level, using robust optimization terminology), these four technology types exhibit characteristic trends.

Within the context of our robust models, we contribute to the understanding of capacity optimization as a trade-off between the requirement for acquiring larger amounts of capacity to increase the chances of meeting demand vs. additional cost for unused capacity. The *level of robustness* in this setting means a *weight* assigned by the decision-maker who determines an appropriate balance between the probability that the model remains feasible under uncertain demand realizations by acquiring larger amounts of capacity, and the degree of deterioration in the objective value (*"price of robustness"*) associated with his additional capacity. We emphasize the distinction between the *largest* (in unit terms) demand realizations vs. the *costliest* demand realizations. The robust optimization model ensures (within a specified "budget of robustness") that the solution remains feasible under uncertain demand realizations in unit terms, and that the solution does not become suboptimal under uncertain demand realizations in cost terms. This two-sided requirement leads to a bilinear robust model formulation. We illustrate the bilinear nature of the robust recourse subproblem, and address the computational challenges of these types of bilinear problems.

We compare the solutions obtained using the robust optimization methodology to those obtained using a non-robust approach as well as to those obtained using traditional stochastic programming. We argue that in order to compare robust optimization to stochastic programming some implicit assumptions are required to make the comparison of the "worst-case" approach of robust optimization and the "average" approach of stochastic programming possible. More specifically, we are interested in comparing the performance of the *first stage* solutions, as these first stage decisions (facility locations, capacity amounts, and technology types) constitute the prime focus of our research.

1.4 Scope of Research

In Chapter 6 some of the future research directions are discussed. Here, however, some comments are provided to delineate the scope of this dissertation. In the operations management literature the concept of capacity is very broad and includes several distinct sub-fields related to different contexts in which the terms *capacity* and *resources* are used. In our work, capacity is understood as *strategic* capacity that includes the major infrastructure components of a firm, such as plant facilities, production lines, capital equipment, etc., that determine a certain level of potential aggregate output to satisfy market demand. We have chosen to implement our models as *static* (one period) as opposed to dynamic (multi-period) ones. This choice is supported by the background literature presented in Chapter 2 and can be explained by the strategic nature of the problem and by computational considerations. We make an assumption that strategic capacity investments are typically done in large chunks over a long time horizon, and not in small increments over multiple “time buckets.” Also, as noted in the literature, the multi-period models in a stochastic setting are much more computationally demanding compared to single period models, yet provide relatively little additional insights. In this work it is assumed that only the market demands are random, all other parameters have deterministic values. Likewise, only capacity investment costs have a concave non-decreasing cost structure; the production and transportation costs are assumed to be linear in the amount produced and the shipping distance. All of the above mentioned restrictions are recognized by taking into account the trade-off between maximum potential insight gained and the computational burden. Finally, while recognizing that outsourcing is an important consideration when making capacity decisions, these considerations are beyond the scope of this work; here it is assumed that all capacity is acquired by a firm.

1.5 Organization of the Dissertation

The reminder of this dissertation is structured as follows. In Chapter 2 we present a review of the literature that includes both background on the subject matter as well as on methodological approaches for dealing with uncertainty, including robust optimization. We establish the foundation for our research by analyzing the relevant issues in the capacity planning and resource acquisition literature, and identify the opportunities for combining an integrative perspective with a stochastic environment. In Chapter 3 the deterministic (or *nominal*) models are presented, including the single product version and the multi-product version. This distinction between single product and multi-product settings is maintained throughout this work as this distinction allows study of the effects of economies of scale and the economies of scope under uncertain demand conditions. Chapter 4 is dedicated to a detailed presentation of robust optimization methodology, including reformulation of the nominal models, and solution algorithm for the two-stage robust counterpart problem. In Chapter 5 extensive numerical studies are presented, including the comparative analysis of robust optimization and stochastic programming based results. Finally, in Chapter 6 we present conclusions and discuss areas of future research. In addition, complete sets of experimental outputs are included in the Appendix.

Chapter 2

Review of the Relevant Literature

The review of literature is structured taking into consideration the various aspects of capacity acquisition as a strategic level decision. First, we discuss the characteristics of strategic level decisions, and distinguish them from tactical and operational level decisions, Second, we review capacity acquisition issues in conjunction with two other common strategic level decisions, namely facility location and technology choice decisions, and emphasize the integrative nature of these. Finally, we address decision making in a stochastic environment in general, as well as in the context of production and distribution network design, and supply chain management. The unifying theme, thus, for the review of relevant literature is the integration of strategic level decisions, including capacity acquisition, in a setting characterized by uncertain parameter realizations.

2.1 Strategic Decision Level

The distinction between the strategic, tactical and operational decision-making levels is widely recognized in the logistics and operations management literature, although the specific contents of these levels as well as the basis of classification may differ among researchers. A comprehensive classification and analysis of the three decision levels, along with a literature review, is provided in Schmidt and Wilhelm (2000) [48]. According to [48], the strategic level decisions are concerned with prescribing facility locations, production technologies and plant capacities, whereas the tactical level decisions are concerned with the material flow management policy, including production levels at all plants, assembly policy, inventory levels and lot sizes. The operational level decisions include schedule coordination and customer service objectives. Higher decision levels establish the constraints for lower decision levels, and each decision level "addresses a particular time frame." In Schmidt and Wilhelm (2000), the modeling issues are discussed, and prototypic formulations are presented for each of the decision levels. In Santoso et al. (2005) [47], a strategic level supply chain network design problem is presented, where the strategic components include the number, location, capacity, and technology of the facilities. The tactical level planning, according to [47], includes deciding the aggregate quantities and material flows for purchasing, processing, and distribution of products, and the efficiency of the tactical operations relies heavily on the supply chain configuration at the strategic level. The importance of strategic level decisions are characterized by their long-term impact and substantial capital requirements (Baron et al., 2011 [4]). In addition, strategic decisions, such as capital investment, are usually deemed irreversible (Van Mieghem, 2003 [53]).

Martinez-Costa et al. (2014) [40] provide a literature survey along with a conceptual framework for strategic capacity planning in manufacturing, and explain the essential difference between strategic and tactical decision levels. In their opinion, this difference should not be primarily based on the time horizon, as sometimes suggested in the literature, but on the consideration of assets that are the object of decisions. Tactical decisions, according to Martinez-Costa et al. (2014), involve production and inventory decisions, i.e., what is usually referred to as aggregate planning, and can include the modification of workforce size or work hours, but not the decisions that involve facilities or equipment. The other factor that potentially diminishes the importance of the temporal factors in the classification of decision levels is the pace of changes in technology and demand in various industries; for example, in the semiconductor industry the time horizon for strategic decisions may be less than a year.

Another aspect of the strategic decisions (especially from a modeling perspective), in addition to their long-term impact, high capital expenditures, and irreversibility, is the level of uncertainty about the future states of the world. Snyder (2006) [50], for example, make a distinction between the strategic phase and the tactical phase based on the two-stage nature of decision-making under uncertainty. In the two-stage framework, a common approach to modeling decision-making in stochastic environments, the strategic phase involves making capital investments under uncertainty, whereas the tactical phase involves actions after the uncertainty is resolved. In tactical (and operational) level models, it is assumed that the strategic level decisions are fixed (see, for example, [4], [48]).

2.2 Capacity Planning Issues

Capacity planning encompasses several distinct fields of study, including capacity expansion, plant location, technology management, new product development, production or aggregate planning, inventory and supply chain management (Van Mieghem, 2003 [53]). Van Mieghem defines capacity as follows:

> Capacity is a measure of processing abilities and limitations that stem from the scarcity of various processing resources and is represented as a vector of stocks of various processing resources. ... While *capacity* refers to stocks of various resources, *investment* refers to the change of that stock over time. Investment thus involves the monetary flow ... .

Martinez-Costa et al. (2014) [40] write that capacity is not the total volume of output in a given period, as sometimes understood, but, because the production output depends on a product mix, what defines capacity is not generally the volume of outputs that the system can generate in a given time, but the availability of various types of productive resources. In Martinez-Costa et al. the terms *resource* and *capacity type* have the same meaning and can refer to, for example, a machine, a process, or an assembly line.

Many of the dynamic (multi-period) capacity planning, or *capacity expansion* issues and problems are discussed and analyzed in the seminal work by Luss (1982) [38]. According to Luss (1982), capacity expansion planning consists primarily of determining future expansion times, sizes, and locations, as well as the types of production facilities. Single period (static) capacity planning is not addressed in [38] – the major reasons for the exclusion of the static models from this capacity planning survey is the author's view that they do not adequately capture the issues of economies of scale and the time value of money. Furthermore, Luss states that comparing facility location problems to capacity expansion problems, it appears that most of the location literature is devoted to static problems. Regardless of the appropriateness of static vs. dynamic approaches for a specific research goal, Luss (1982) identifies several important research questions related to capacity planning in general. The capacity size decisions are by necessity linked with the facility location decisions, as the transportation costs of the products to the demand locations can not be neglected, and thus the location issue becomes an important part of the capacity planning process. Luss (1982) also recognizes that capacities of different types, i.e., different technologies, represent alternative cost structures, and can greatly influence the optimal investment policies. Capacity expansion costs are usually concave, exhibiting economies of scale. The most popular capacity cost functions are either the power cost function, or the fixed charge cost function.

Most of the literature that deals with capacity planning or capacity acquisition clearly identifies this problem as a dynamic one, in line with the earlier works analyzed in Luss (1982) [38]. Verter and Dincer (1992) [56] state that the capacity expansion problem can be formulated over either an infinite time period, or a discrete period finite time horizon, and provide a summary of sub-categories of problems within the capacity expansion problem, listing the following: planning horizon and discount rate, the set of feasible expansion sizes, demand pattern, capacity acquisition costs and other cost factors, number of facilities, and number of products involved. They also note that, with some exceptions, the capacity expansion models are one-directional, i.e., they do not allow for capacity contractions. Li and Tirupati (1994) [34] offer a heuristic algorithm for a dynamic capacity expansion problem. The problem formulation is similar to the ones presented in previous works, but with some generalizations with regards to demand patterns and cost functions, and a special focus on the multi-product aspect of the problem. Aghezzaf (2005) [1] also considers a multi-period capacity planning environment, but with some extensions compared to the previous literature. He considers the plant capacity planning decisions and warehouse location decisions as some of the most important strategic decisions a firm can make, and offers a model formulation where these two problems are solved jointly. In Aghezzaf's model the traditional capacity expansion model is embedded into a two-echelon supply chain model, with a warehouse stage between plants and customer markets.

Olhager et al. (2001) [45] provide a conceptual model that combines two perspectives in manufacturing strategy: a perspective that deals with long-term decisions involving capacity levels, facilities, production processes, and vertical integration, and a second perspective the deals with such decisions as sales and operations planning (S&OP). The aggregate capacity levels are based on long term sales forecasts; however, capacity typically can be added (or reduced) only in large discrete steps, necessitating a firm to select an appropriate manufacturing strategy that includes either lag, lead, or tracking options. From the S&OP perspective there are three options – level, chase, and mix. Thus, while the focus from the strategic perspective is on the timing of capacity changes, the focus from the S&OP perspective is the rate of production relative to sales. According to the Olhager et al. (2001) conceptual framework, while the lead capacity strategy is compatible with the chase S&OP approach, allowing for resource availability and flexibility, the lag capacity strategy is compatible with the level S&OP approach, allowing for maximum resource utilization. The combination of lead capacity strategy with the level S&OP is neither conflicting, nor supportive; however, the combination of lag capacity strategy with the chase S&OP can lead to negative consequences.

Van Mieghem (2003) [53] describes general capacity investment issues, as well as the optimal capacity investment policies for three settings: stationary, dynamic, and risk-averse. In optimization models, capacity is often the upper bounds on some processing resources. The tactical level models assume the capacity to be fixed, and the outcomes of these models depend, in part, on the amount of available capacity. In a stochastic setting, i.e., in recourse problems, the capacity investment decisions can be based on the newsvendor principle, where capacity shortages or excesses can be dealt with by tactical countermeasures. Another important capacity planning issue, according to Van Miegham is the nature of "capacity adjustment costs." Typically, the changes in capacity levels are not gradual or incremental – investments in capacity are usually "lumpy" because of either indivisibility of capital assets, non-linear capacity investment costs (i.e., economies of scale), or the fact that capacity investment decisions are often irreversible. Van Mieghem suggests that many capacity models simply ignore tactical flows, and discusses the appropriateness of such an approach, citing the need to strike a balance between complexity and realism. The justification for the separation of strategic capacity decisions and tactical decisions can be based on the "time-scale separation", which means that capacity changes are infrequent relative to tactical decisions, and therefore in tactical flow models the capacity levels are taken as fixed.

Even though the capacity planning literature is dominated by multi-period models, some authors argue that in some instances the capacity investment decisions can be reduced to single period models. Van Mieghem (2003) discusses the theoretical results that indicate that under independent and identically distributed random variable structure, stationary environment, and independent periods, a multi-period capacity planning problem can be reduced to a single period one. The author suggests that while being reformulated as static, these models, while losing their time dimension, become less complex and are able therefore to include more details regarding the problem specifics, and better express the nature of uncertainty.

Ahmed and Garcia (2003) [2] specifically consider a two stage stochastic capacity planning model, and argue that although the model considers multiple discrete periods over a long time horizon, the capacity expansion decisions are strategic in nature and should be made at the beginning of the planning period in stage one. The operational, or recourse, decisions can be made when more information becomes available. Ahmed and Garcia (2003) state that the multi-period (two-stage) capacity decisions could be in principle converted into a multi-stage stochastic integer program; however, at a disadvantage of becoming computationally almost impossible to solve. Moreover, they argue that the two-stage approach is a good enough approximation of the multi-stage problem. Santoso et al. (2005) [47] present a single period two-stage stochastic supply chain model, and discuss the computational challenges associated with modeling the joint realization of uncertainties even for a relatively small problem instance.

2.3 Capacity Acquisition and Other Strategic Level Decisions

Although strategic capacity decisions are typically made in conjunction with several other major decisions, two of them – facility location and technology choice – have received more consideration in the literature on strategic capacity planning in comparison to other related decisions. For example, Vidal and Goetschalckx (1997) [58] mention supplier selection and transportation mode choice as additional strategic decisions within the scope of a firm's global supply chain design, while the decisions concerning location, capacity, and type of manufacturing plants are placed on the top on the list of strategic decisions. Similarly, Verter and Dincer (1992) [56] include product mix, time-phasing of investments, and financial planning as the components of overall manufacturing strategy, yet location, capacity, and technology decisions are of paramount importance within the context of developing effective global manufacturing strategies.

2.3.1 Facility Location

Facility location problems have been extensively studied in the literature, and there exist various typologies for these problems that depend on the underlying modeling assumptions, solution approaches, and other factors. The most notable distinction is between the continuous and discrete models. The continuous facility location models are less utilized in practice. However, they can offer intuitive and insightful solutions, compared to solutions obtained by means of discrete mathematical progamming models. The latter are good at incorporating many details and specifics, but, according to Dasci and Laporte (2005) [18] fail to explain *why* an optimal solution is what it is. Regardless of the relative scarcity of continuous location models, they can be particularly useful in addressing strategic level problems. For example, Dasci and

Verter (2001) [19], in addition to the fixed facility and transportation costs, include capacity acquisition and operating costs (fixed and variable, linear as well as non-linear) that incorporate the effects of scale economies. Dasci and Laporte (2005) [18] extend the traditional market area model by assuming that the demand is uncertain, and show that the optimal solution depends not only on the trade-off between the fixed facility and transportation costs, but also on the ratio of unit variable capacity costs to unit shortage costs, taking into account probability distribution.

The body of literature devoted to discrete facility location problems is very extensive. Owen and Daskin (1998) [46] review facility location problems that explicitly address the strategic nature of the problem, by considering either dynamic, or stochastic characteristics, as opposed to the static and deterministic models. According to [46], facility location is a critical aspect of strategic planning, and the extension of facility location models to dynamic or stochastic settings can better capture the real-world complexities and uncertainties. Thus, the incorporation of temporal and stochastic aspects proactively make the models more reliable, as opposed to the analyses of solution sensitivity in a reactive manner.

Klose and Drexl (2005) [33] view facility location problems as a core component of a firm's distribution system design, itself being a strategic issue. Klose and Drexl provide a classification of facility location models that range from simple deterministic single-period, single-product models to non-linear and probabilistic models, and discuss the common solution approaches for various classes of location models. There exist hierarchical relationships between facility location models for distribution system design, and the multi-product, multi-period, or multi-echelon models are essentially the extensions of either *uncapacitated* or *capacitated* facility location problems (UFLP or CFLP), which themselves are NP-hard. While addressing dynamic location models, Klose and Drexl (2005) question the practical relevance of the multi-period models for several reasons, including the issues of selecting the appropriate time horizon, the amount and accuracy of data requirements, and the solution difficulties associated with increased complexity.

Snyder (2006) [50] provides a comprehensive review and analysis of facility location problems under uncertainty. Snyder mentions that facility location decisions share the characteristics of strategic decisions, namely that they are costly, have a long-term impact, and are difficult to reverse. It is reiterated that traditionally facility location problems under uncertainty have been modeled in a two stage framework – capital investments are made during the first, strategic phase, followed by the tactical phase, after uncertainties are resolved. The basis for Snyder's classification is the distinction between three decision-making environments – certainty, risk, and uncertainty – and the facility location models are grouped into three broad categories that correspond to the three environments – deterministic, stochastic, and robust, respectively.

Melo et al. (2009) [43] provide a review of facility location models in the context of supply chain management, and more specifically, the role of these models in the supply chain *network design*. It is suggested that facility location models should be extended to include four features to be useful for supply chain models – multiple echelons, multiple commodities, multiple periods, and stochastic parameters. When considering the types of decisions modeled, in addition to location-allocation decisions, capacity planning, inventory management, and production decisions are the most common, based on the reviewed literature. The capacity planning literature often considers also the choice of technology. It is suggested that, in fact, the technology determines the capacity, not vice versa. Melo et al. (2009) conclude that as the supply chain modeling efforts should seek increased integration between strategic and tactical/operational levels to avoid sub-optimality, the facility location models, being part of supply chain network design, should avoid simplifications and include more features relevant to real-life supply chain management problems.

2.3.2 Technology Choice, and Economies of Scale and Scope

It has been recognized in the early literature that industrial facilities exhibit economies of scale. Some of these works are referenced in Verter and Dincer (1995) [57]. Verter and Dincer present an integrated approach for the simultaneous optimization of facility location and capacity acquisition decisions. It is assumed that the capacity cost function is a monotone increasing concave function, i.e., power function in this case, and a linear approximation technique is used to solve the problem. Although in [57] the resulting piecewise linear segments are not explicitly associated with a particular *capacity type*, in a later work (Verter, 2002 [54]) the different monotone increasing cost functions (power, or piecewise linear) are clearly identified as *technology alternatives*. A view that each segment, or range of the piecewise concave cost function represents a single technology is also shared by other researchers, e.g., Luss (1982) [38], Li and Tirupati (1994) [34], Ahmed and Sahinidis (2008) [3].

Verter and Dincer (1992) [56] argue that the integrated facility location, capacity acquisition, and technology selection decisions are the building blocks for a firm's global production and distribution network. They note that the technology selection problem can be traced to the historical trend of labor-intensive processes being replaced by capital-intensive production processes, where the labor-intensive processes are characterized by low fixed and high variable costs, and the capital-intensive processes are characterized by high fixed and low variable cost structures. Summarizing the results from the literature survey, Verter and Dincer (1992) state that the benefits of automated capital-intensive technologies go beyond the effects of scale economies, however, and include improved quality, higher responsiveness to market needs, and increased productivity. In addition, optimal technology selection decisions would be those that choose to invest in more capital-intensive technologies with ever non-increasing per unit production costs.

Another dimension in capacity typology, in addition to the level of capital intensity, is the distinction between the *dedicated* and *flexible* technologies. While the concept of capital intensity is usually presented as a single product issue in the context of scale economies, the notion of flexibility of capacity is treated as a multi-product (often two product) issue in the context of scope economies. The conceptual links between dedicated technologies and economies of scale, and between flexible technologies and economies of scope were established with the emergence of modern manufacturing capabilities. Goldhar and Jelinek (1983) [26], and Goldhar and Jelinek (1985) [27] address the need to shift the strategic management approaches from the traditional scale perspective, which “means unlearning a host of familiar scale-based assumptions”, to an economies of scope perspective that is characterized by product variety, customization, and responsiveness.

Although the concept of flexibility is rather broad and can encompass various meanings, in the literature it is most often associated with an ability to produce more than one kind of product, i.e., *product flexibility*. The concepts, approaches, and results by Fine and Freund (1990) [22] have motivated and influenced a steady stream of literature devoted to the issue of optimal amounts of dedicated vs. flexible technology that a firm must acquire to maximize profits. Fine and Freund present a two-stage stochastic model, where in the first stage the dedicated capacity and flexible capacity investment decisions are made, and in the second stage, after uncertain demand is observed, production decisions are made. Capacity investment costs are linear, and flexible capacity costs are higher than those for any type of dedicated technology. Production costs are also linear, and an assumption is made that they are technology independent, i.e., they are the same for dedicated and flexible technologies. In a multi-product setting the effects of demand correlation have substantial impact on the optimal combination of dedicated and flexible capacities. Fine and Freund showed that with the increased uncertainty in demand, flexible capacity becomes more valuable in the case of negatively correlated demand, and has no value in the case of perfectly positively correlated demands. Van Mieghem (1998) [52] showed that flexible technology can be valuable even in the case of perfectly positively correlated demands, as the company has the ability to exploit price differentials, and produce more profitable products using flexible technology at the expense of less profitable products. Extensive research follows the works by Fine and Freund (1990) and Van Mieghem (1998) to analyze the intricate dynamics between product-flexible capacities and optimal profitability conditions though responsive pricing (price postponement) approaches, and product substitutability (cross-price) effects (see, for example, Bish and Wang (2004) [14], Chod and Rudi (2005) [16], Biller et al. (2006) [12], Lus and Muriel (2009) [37], Goyal and Netessine (2011) [28]).

According to Li and Tirupati (1994) [34], an optimal capacity strategy typically includes certain proportions of dedicated and flexible capacities that depend on such factors as the demand patterns, the relative investment costs of flexible technology, and economies of scale. Investments in flexible technologies are economically justified, even at higher investment costs. Moreover, Li and Tirupati (1994) suggest, based on experimental results, that there is no inherent incompatibility between economies of scale and economies of scope. The results in Chen et al. (2002) [15] indicate that flexibility is more useful in the case of individual demand variability, in comparison with total demand variability, and that the optimal amount of flexible capacity depends on a particular problem, not on a general rule of thumb. Ahmed and Sahinidis (2008) [3] present a solution approach for a multi-product and multi-period capacity planning problem, while suggesting that the dynamic demand environment and short product life cycles have placed the technology adoption decisions among the key strategic decisions for a firm.

In their review of the strategic supply chain network design literature, Melo et al. (2009) [43] indicate that capacity decisions are dominated by the choice of equipment and/or technology decisions. In turn, the primary consideration related to the choice of technology, according to Verter (2002) [54] is their acquisition and operation cost structures, and economies of scale, which depend on the level of automation or productivity.

Graves and Tomlin (2003) [30] develop a quantifiable flexibility measure using the concept of product-plant links that is "based on the excess capacity available to any subset of products, relative to an equal-share allocation of the capacity." Graves and Tomlin also emphasize the importance of configuration of the product-plant links, not just the number of these links. In particular, they note that "closed" configurations outperform configurations that consist of numerous distinct product-plant based chains. Furthermore, Graves and Tomlin (2003) examine the impact of random demand realizations on different capacity flexibility policies, and extend the flexibility measure to multi-stage supply chains. In [30] the analytically derived flexibility measure is validated experimentally using simulation.

2.4 Integrative Approach to Strategic Decisions

In his survey of capacity related literature, Luss (1982) [38] writes that in the early literature (e.g., Manne, 1967 [39]) capacity expansion decisions are explicitly considered in connection with the optimal location decisions. Similarly, the assumption of different capacity types, or production facility types, with different cost structures means that they can be considered alternative technologies. The integrative nature of these strategic problems has manifested itself through numerous works, both of theoretical as well as practical orientation.

Verter and Dincer (1992) [56] provide a literature review specifically dedicated to an integrative evaluation of facility location, capacity acquisition, and technology selection. They identify these three factors as the building blocks for a firm's global manufacturing strategies, and claim this integration is even more important than for domestic production-distribution strategies. Verter and Dincer conclude that each of the three factors – location, capacity, and technology – is a complex area of research by itself, and that there exist potential for a theoretical synthesis of these areas.

Verter (2002) [54], and Verter and Dasci (2002) [55] present formal models that build upon the conceptual considerations in Verter and Dincer (1992), and explicitly include in an integrated manner the facility location, capacity acquisition, and technology choice variables with the purpose of studying the effects between these three decisions. Both models are static, deterministic, mixed integer non-linear optimization problems, solved using a piecewise approximation algorithm. The non-linearity in these models is caused by the capacity acquisition and operating costs modeled as a power function, or more generally, as any monotone increasing concave function (fixed charge linear, and piecewise linear functions belong to this category, and they are also used in [54] and [55]). In Verter (2002) [54], a single product model is offered that includes alternative technologies, which represent economies of scale that affect the number and size of facilities. In Verter and Dasci (2002) [55], the facility location, capacity acquisition, and technology selection model is extended to include multiple products, additional capacity types, and dedicated and flexible technologies to capture the economies of scope in facility location and sizing decisions. The authors suggest that a firm's manufacturing strategy can be designed as being positioned between the market-focus and product-focus ends of the spectrum. Under a pure market-focus strategy, a firm would manufacture all the products needed for the particular markets in a plant assigned to these markets. Under a pure product-focus strategy, a firm would concentrate the production in plants with dedicated technologies to take advantage on the scale economies. An optimal solution of the model would prescribe a hybrid strategy along the market/product focus spectrum.

Lim and Kim (1999) [35] propose a deterministic multi-period integrated plant location and capacity acquisition (or disposal) problem, where the types of capacities include dedicated and flexible *facilities* (i.e., technologies). In [35], a slightly different terminology is used – *plant* in this case means a collection of facilities that are capable of producing different types of products. Integrating location, capacity, and technology decisions in a dynamic setting is especially hard to solve, and a heuristic algorithm based on Lagrangian relaxation, decomposition, and a cut-and-branch procedure is presented along with computational experiments. Lim and Kim (1999) attempt to incorporate many simultaneous decisions related to plant opening, acquisition of dedicated and flexible capacities, and capacity allocation to operations. The model also incorporates the investment budget over the planning horizon and the discount rate for costs. The Lim and Kim model allows for multiple types of flexible technologies, which makes the model more realistic, yet also more complex. The authors suggest that their approach may be well suited for global manufacturing companies in industries that are characterized by rapid changes in capacity and product requirements, for example, in automotive or electronics industries where the strategic level decisions have to be made on a more frequent basis.

From a more practical perspective, Eppen et al. (1989) [20] present an integrated multi-product, multi-period, multi-plant capacity planning model under risk. A major decision management needs to make is the right trade-off between profit and risk when considering capacity investments. Eppen et al. state that the fundamental issue is to "determine the appropriate type and level of production capacity at each of several locations." The problem is presented in the context of the automotive industry, and some of the top managerial concerns are addressed, such as chronic excess capacity. Although the Eppen et al. model was developed for General Motors, the concepts and the dynamics between various strategic planning factors can be extended to other industries and other settings. For example, the product mix, plant allocation, and capacity flexibility options are relevant for any complex production system. Karabuk and Wu (2003) [31] provide another example from the semi-conductor industry where the decisions about capacity levels and the decisions about the technology mix are inseparable as strategic capacity planning is an iterative process with the two main components – capacity expansion and capacity configuration. Another applied strategic capacity planning case from the automotive industry is presented in Fleischmann et al. (2006) [23]. As one would expect, in a more applied planning environment, the number of factors that have to be considered in the model increases. Fleischmann et al. model the BMW global production network as a strategic problem that includes decisions to allocate multiple products to multiple plants in a multi-year dynamic environment taking into account the potential uncertainty in demand and corporate policies on capacity reserves. In addition to these considerations, the model has to account for real-world restrictions such as the maximum number of sites a product can be allocated to, local content requirements, and taxation systems of different countries. In the BMW case, the global production and capacity planning is done in conjunction with the investment and cash flow planning. It appears from the above mentioned examples that the more integrative approaches dominate in practical industry cases, and are primarily driven by the necessity to accommodate the needs of real-world strategic planning efforts.

2.5 Decisions Under Uncertainty

Discussion and classification of the different types of uncertainty in a business environment is provided by Klibi et al. (2010) [32]. Some authors have adopted a distinction between certainty, risk, and uncertainty. However, as noted by Klibi et al. (2010), this classification of uncertainty is not shared by other authors, who associate the concept of risk not only with the probability of an occurrence of an event, but also with the magnitude of the value lost or gained. An uncertain event, according to this view, is value-neutral, and the decisions are made under certainty when perfect information exists, or under uncertainty when only partial information is available. A probabilistic interpretation of uncertainty is the prevailing interpretation of randomness in management science, although it is not the only way to formalize uncertainty. These alternative formalisms include, for example, the set-based approach, which constitutes the methodological foundation of the uncertainty set based robust optimization.

When classifying the issues of the strategic capacity planning problem, Martinez-Costa et al. (2014) [40] make a distinction between the *decisions* addressed in the problem (e.g., capacity size, capacity location, allocation, capacity configuration and technology selection) and the external *factors* included in the problem statement, such as uncertainty. Martinez-Costa et al. (2014) offer a taxonomy of capacity models, based on three criteria: the nature of the problem (deterministic or stochastic), the type of capacity decision, and the number of locations involved in the capacity decisions (single-site or multi-site). Regardless of the model classification schemes proposed by different authors, the stochastic vs. deterministic approach is often clearly identified (e.g., Vidal and Goetschalckx (1997) [58], Melo et al. (2009) [43], Farahani et al. (2014) [21]).

The deterministic approach has been the dominant approach for many decades in the production, distribution, and supply chain design models, as reviewed and classified, for example, in Meixell and Gargeya (2005) [41]. The majority of the proposed models do not explicitly consider the impact of uncertainty on the optimal solution. In the Melo et al. (2009) [43] review, the deterministic models dominate as well, especially in the multi-product category. In addition, there is a scarcity of models that consider stochasticity beyond one or two echelons in the supply chain network.

Demand (individual product and product mix) uncertainty is probably the most recognized type of uncertainty. But it is not the only source of uncertainty; for example, other sources of uncertainty include supplier reliability, costs, productivity, and many others. Other factors in addition to uncertainty complicate the capacity planning and resource acquisition decisions: constant changes in technologies and short product life cycles force companies not only to determine the right amount of capacity, but also to ensure that the capacity is flexible and adaptable to the new technologies and new products. The difficulties associated with modeling of uncertainty and incorporating uncertainty into capacity planning models are recognized by many researchers. Uncertainty can be modeled in a variety of ways. However, regardless of the approach, the addition of uncertainty to the underlying already difficult-to-solve deterministic models can make the resulting models intractable. Addressing specifically the existing literature on production planning, Graves (2008) writes [29]:

> This literature is largely oblivious to uncertainty. Much like research on the economic-order-quantity (EOQ) model, the contention is that the value of these models is in optimizing critical cost tradeoffs, often in the context of tight constraints. The research perspective is that dealing with uncertainty is of secondary importance to getting the tradeoffs right; furthermore, there is the assumption that the uncertainties can be handled by other measures, which are independent of the determination of the production plan. Nevertheless, there is also the recognition that the deterministic assumptions are a shortcoming of this research, but were necessary in order to keep the models tractable.

It appears that the general trend in modeling production-distribution networks is increasing attention to explicitly incorporate uncertainty in model formulation. For example, in a more recent survey, Farahani et al. (2014) [21], the share of models with stochastic features is quite substantial, although the majority of models addressed still consider demand uncertainty as the sole source of randomness.

2.6 Summary of Literature Review and Research Implications

Each of the topics, discussed in the previous sections – facility location, capacity acquisition, and technology choice – is a vast field of research by itself. Our intent is not so much to provide an exhaustive review of each of them as to show that these topics are naturally linked as a part of complete production-distribution, or supply chain networks. Indeed, some of the earlier works, mentioned in our review, have explicitly recognized that a firm's decision to locate a facility cannot be separated from the decision about its capacity and the type of this capacity. In this dissertation we examine an integrated model that considers simultaneously all three decisions, and our approach is to some extent a response to researchers' suggestions for more integrated and holistic view on these strategic level decisions. One of the purposes of our review is to show that an integrative view is a logical extension of the previous research, and at the same time to establish a theoretical foundation for our modeling approach. Another purpose of this literature review is to examine the extent of application of methodologies that deal with uncertainty to the facility location, capacity acquisition, and technology choice models. Although historically deterministic models have been dominant in the logistics and supply chain management literature, a more recent trend indicate that a growing number of publications consider modeling of stochastic parameters an essential part of state-of-the-art research. The recognition of the inherent uncertainty in the logistics and supply chain models is similar to the recognition of the integrative nature of these models in that in both instances the complexity of these models not only prove to be computationally challenging, but also present difficulties in deriving general insights from specific problem instances. Our work is dedicated to the application of robust optimization methodology to the facility location, capacity acquisition, and technology choice model, i.e, we believe that this way we can simultaneously address both the integrative nature of the strategic decision-making process, and the stochastic environment in which these decisions are made. We have identified a number of publications related to the application of robust optimization methodology to the field of logistics and supply chain management. At the same time these robust optimization applications appear to be "disconnected": they don't exhibit the same methodological unity and standardization that deterministic optimization or stochastic programming applications do. With these considerations we believe that there exist research directions that would address some of the gaps in the literature, and more specifically, the application of robust optimization approach to an integrated capacity planning and resource acquisition problem.

Chapter 3

Nominal Model Formulations

3.1 Problem Description

In Chapter 3, the nominal base models are described, including the single product model and the multi-product model, presented in Sections 3.2 and 3.3, respectively. In this initial Section 3.1, however, some common features are discussed. The individual components of the proposed models can be found in previous works, described in Chapter 2, and they reflect common modeling approaches. The strategic level production-distribution network design models can be either cost minimization, or profit maximization models. In some instances, the objective function can include multiple objectives that can lead to a goal programming approach. We have chosen to use the cost minimization objective, in part because in non-deterministic settings, variable price and variable demand lead to non-linear objective functions. Some of the model features include such common characteristics as the selection of facility locations, and capacity types (technology alternatives) and sizes, as well as the determination of production quantities and the optimal allocation of products to customer zones. All the costs, except capacity investment costs, are either fixed charge or linear. Capacity investment costs have a fixed charge piecewise linear structure. The closest to our formulation of the nominal models are the models in Verter (2002) [54] and Verter and Dasci (2002) [55]. However, their approach is strictly deterministic, and their focus, besides demonstrating the integrative nature of strategic decisions, is algorithmic development. Among other models that share similarities with our approach is the model described in Baron et al. (2011) [4] in the context of robust optimization, even though they do not make a distinction between capacity types, which is one of the essential features of the models presented here.

Both the single and multi-product versions of our models are static (one period), two-tier (production facilities and customer zones) models, and there are no upper or lower limits placed on facility capacities. These characteristics indicate that the proposed models are closely related to the *uncapacitated facility location problems* (UFLP). However, the concave capacity investment cost structure makes the technology choice decisions and facility location decisions interdependent, and, thus, our formulations, just as the ones presented in Verter (2002) [54] and Verter and Dasci (2002) [55], cannot be reduced to UFLP. Similarly, in the multi-product setting, the dedicated and flexible capacity investment decisions and facility locations decisions are interdependent. In Chapter 4 this mutual dependency is explored under the conditions of demand uncertainty.

The nominal[1] formulation presented in this chapter makes no distinction between decision stages. Therefore, the deterministic facility opening, technology selection, capacity investment level, production, and transportation decisions are optimized as a single monolithic problem[2]. To maintain the integrity of the piecewise linear structure of capacity investment costs, a restriction is placed on the number of capacity types, or technologies, that can be established at a site, i.e., at most one. This re-

[1]To follow a terminological convention in robust optimization we use term *nominal* instead of *deterministic* to distinguish a deterministic problem from its *robust counterpart*, which is also a deterministic problem.

[2]The two-stage equivalents of both the single and multiple models will be introduced in Chapter 4.

striction applies to both single and multi-product models (see constraints (3.6) and (3.15)–(3.16)). There is no such restriction, however, enforced on simultaneous placement of dedicated and flexible technologies at the same site, or two different dedicated technologies for two products at the same site. We are implementing our models with minimum restrictions related to simultaneous placement of different types of technologies at a single site to be able to observe more "natural" unconstrained outcomes resulting from interdependencies between model components. We could, in addition to the above mentioned restrictions (3.6) and (3.15)–(3.16), include various other logical constraints. For example, we could restrict the production plants to either dedicated, or flexible technologies only, or enforce other restrictions. However, to gain maximum insight from our numerical studies, we prefer to use limited number of artificially imposed conditions, although such conditions may be of great importance in practical industrial applications.

3.2 Single Product Model

In this section, the notation and a formal problem description is presented for the single product model.

3.2.1 Notation and Assumptions for the Single Product Model

Table 3.1 provides a detailed description of sets, parameters, and variables used in the single product model formulation.

Parameters	
$i \in I$	set of production sites
$j \in J$	set of customer zones
$l \in L$	set of technologies
f_i	fixed production facility investment cost at site i

e_{il}	fixed capacity investment cost for technology l at site i
g_{il}	unit capacity investment cost for technology l at site i
c_{ijl}	unit production cost using technology l at site i, including transportation cost to customer zone j
d_j	demand of customer zone j
Variables	
z_{il}	units of capacity established using technology l at site i
x_{ijl}	units produced using technology l at site i and transported to customer zone j
y_i	1 if production facility at site i opened, 0 otherwise
v_{il}	1 if capacity using technology l at site i established, 0 otherwise

Table 3.1: Notation for the nominal single product model.

The single product model includes two sets of binary variables – y_i and v_{il}, and two sets of continuous variables – z_{il} and x_{ijl}.

3.2.2 Single Product Model Formulation

The nominal single product model is formulated as a mixed integer program:

$$\min_{y,v,z,x} \quad \sum_i f_i y_i + \sum_i \sum_l e_{il} v_{il} + \sum_i \sum_l g_{il} z_{il} + \sum_i \sum_j \sum_l c_{ijl} x_{ijl} \tag{3.1}$$

$$\text{s.t.} \quad \sum_i \sum_l x_{ijl} \geq d_j, \quad \forall j \tag{3.2}$$

$$\sum_j x_{ijl} \leq z_{il}, \quad \forall i, l \tag{3.3}$$

$$z_{il} \leq M v_{il}, \quad \forall i, l \tag{3.4}$$

$$v_{il} \leq y_i, \quad \forall i, l \tag{3.5}$$

$$\sum_l v_{il} \leq 1, \quad \forall i \tag{3.6}$$

$$y_i, \ v_{il} \in \{0, 1\}; \ z_{il}, \ x_{ijl} \geq 0, \quad \forall i, j, l,$$

where M is a sufficiently large constant, representing the bounds on the z_{il} variables, for example, $M = \sum_j d_j$. The objective function (3.1) minimizes the sum of fixed production facility investment costs, the sum of fixed capacity investment costs for all technologies, the sum of unit capacity investment costs for all technologies, and the sum of production and transportation costs. Constraint (3.2) stipulates that the demands must be satisfied for each customer zone j. Constraint (3.3) states that the total number of units produced using technology l at site i and transported to customer zone j cannot exceed the number units of capacity established using technology l at site i. Constraint (3.4) states that no amount of capacity is established without corresponding fixed charges. Constraint (3.5) states that capacity using any technology l is established only at an open production site i. Constraint (3.6) allows at most one type of technology l per production site i.

3.3 Multi-product Model

In this section, the notation and a formal problem description is presented for the multi-product model.

3.3.1 Notation and Assumptions for the Multi-product Model

Table 3.2 provides a detailed description of sets, parameters, and variables used in the multi-product model formulation.

Parameters

$i \in I$	set of production sites
$j \in J$	set of customer zones
$k \in K$	set of products
$l \in L$	set of technologies
f_i	fixed production facility investment cost at site i

e^D_{ikl}	fixed capacity investment cost for dedicated technology l for product k at site i
e^F_{il}	fixed capacity investment cost for flexible technology l at site i
g^D_{ikl}	unit capacity investment cost for dedicated technology l for product k at site i
g^F_{il}	unit capacity investment cost for flexible technology l at site i
h_{ikl}	units of capacity of flexible technology l required to produce one unit of product k at site i
c^D_{ijkl}	unit production cost for product k using dedicated technology l at site i, including transportation cost to customer zone j
c^F_{ijkl}	unit production cost for product k using flexible technology l at site i, including transportation cost to customer zone j
d_{jk}	demand of customer zone j for product k
Variables	
z^D_{ikl}	units of capacity established using dedicated technology l for product k at site i
z^F_{il}	units of capacity established using flexible technology l at site i
x^D_{ijkl}	units of product k produced using dedicated technology l at site i and transported to customer zone j
x^F_{ijkl}	units of product k produced using flexible technology l at site i and transported to customer zone j
y_i	1 if production facility at site i opened, 0 otherwise
v^D_{ikl}	1 if capacity using dedicated technology l for product k at site i established, 0 otherwise
v^F_{il}	1 if capacity using flexible technology l at site i established, 0 otherwise

Table 3.2: Notation for the nominal multi-product model.

The multi product model includes three sets of binary variables – y_i, v^D_{ikl}, and v^F_{il}, and four sets of continuous variables – z^D_{ikl}, z^F_{il}, x^D_{ijkl}, and x^F_{ijkl}.

3.3.2 Multi-product Model Formulation

The nominal multi-product model is formulated as a mixed integer program:

$$\min_{y,v,z,x} \quad \sum_i f_i y_i + \sum_i \sum_l \left(\sum_k e^D_{ikl} v^D_{ikl} + e^F_{il} v^F_{il} \right) + \sum_i \sum_l \left(\sum_k g^D_{ikl} z^D_{ikl} + g^F_{il} z^F_{il} \right) +$$

$$\sum_i \sum_j \sum_k \sum_l \left(c^D_{ijkl} x^D_{ijkl} + c^F_{ijkl} x^F_{ijkl} \right) \tag{3.7}$$

$$\text{s.t.} \quad \sum_i \sum_l \left(x^D_{ijkl} + x^F_{ijkl} \right) \geq d_{jk}, \quad \forall j,k \tag{3.8}$$

$$\sum_j x^D_{ijkl} \leq z^D_{ikl}, \quad \forall i,k,l \tag{3.9}$$

$$\sum_j \sum_k h_{ikl} x^F_{ijkl} \leq z^F_{il}, \quad \forall i,l \tag{3.10}$$

$$z^D_{ikl} \leq M v^D_{ikl}, \quad \forall i,k,l \tag{3.11}$$

$$z^F_{il} \leq M v^F_{il}, \quad \forall i,l \tag{3.12}$$

$$v^D_{ikl} \leq y_i, \quad \forall i,k,l \tag{3.13}$$

$$v^F_{il} \leq y_i, \quad \forall i,l \tag{3.14}$$

$$\sum_l v^D_{ikl} \leq 1, \quad \forall i,k \tag{3.15}$$

$$\sum_l v^F_{il} \leq 1, \quad \forall i \tag{3.16}$$

$$y_i,\ v^D_{ikl},\ v^F_{il} \in \{0,1\};\ z^D_{ikl},\ z^F_{il},\ x^D_{ijkl},\ x^F_{ijkl} \geq 0, \quad \forall i,j,k,l,$$

where M is a sufficiently large constant, representing the bounds on the z^D_{ikl} and z^F_{il} variables. The objective function (3.7) minimizes the sum of fixed production facility investment costs, the sum of fixed capacity investment costs for all dedicated and flexible technologies, the sum of unit capacity investment costs for all dedicated and flexible technologies, and the sum of production and transportation costs. Constraint (3.8) stipulates that the demands must be satisfied for each customer zone j for product k, produced using either a dedicated or flexible technology l. Constraints (3.9) and (3.10) state, for dedicated and flexible technologies, respectively, that the total number of units of product k produced using technology l at site i and transported to customer zone j cannot exceed the number units of capacity established using technology l at site i. Constraints (3.11) and (3.12) state, for dedicated and flexible technologies, respectively, that no amount of capacity is established without corresponding fixed charges. Constraints (3.13) and (3.14) state, for dedicated and flexible technologies, respectively, that capacity using any technology l is established only at an open production site i. Constraints (3.15) and (3.16) allow, for dedicated and flexible technologies, respectively, at most one type of technology l per production site i.

Chapter 4

Robust Reformulations and Solution Methods

4.1 Overview of the Robust Optimization Paradigm

In a general sense, robust optimization is a collection of different approaches that allow the decision-maker to pro-actively consider the impact of random parameters on the optimal solution. The unifying aspect of these different approaches is that the uncertainty is analyzed from the *worst-case* perspective, as opposed to the *expected value* perspective. Gabrel et al. (2014) [25] provide an overview of the theoretical results and applications in robust optimization, and emphasize that the main question within the robust optimization paradigm is the issue of conservatism, i.e., it is the question of the right trade-off between the performance of the model and the level of protection, or immunization, against the adverse effects of randomness. The issue of conservativeness is related to the choice of appropriate requirements for the worst-case solution that the decision-maker can specify in advance. There lies an important distinction between robust optimization and *stochastic programming*. Within the stochastic programming framework, the random behavior of data is independent from the decision-making process, even when we have incomplete or poor information about the probability distribution. When we make assumptions or guesses about probability distributions, our desire is to describe the behavior of data "as close to reality as possible", i.e., this behavior, more or less accurately captured, is *exogenous* to our preferences. On the contrary, within the robust optimization paradigm our preferences as a decision-maker are incorporated in the model solution by making the solution *insensitive* to the randomness of the outside reality. Moreover, it is possible to numerically specify the desired level of robustness. Ben-Tal et al. (2009) [5] compare this situation to engineering design process when safety-related parameters are increased by a factor to account for material quality, environmental hazards, etc.

Another traditional methodology, in addition to stochastic programming, for dealing with uncertain data is *sensitivity analysis*. It is a "local" *post-optimization* tool, when by changing parameters within ranges that represent the potential random realizations of these parameters, the impact on the objective value is observed. However, with sensitivity analysis usually there is no systematic way in which the source of the greatest impact on the objective is determined. One can vary different parameters one (or several) at a time, but there are no guarantees that the chosen changes have the greatest impact, or are the most sensitive, to the solution value. Mulvey et al. (1995) [44] refer to sensitivity analysis as a *reactive* approach. With the robust optimization methodology the process of finding a model solution that is "insensitive" to random data realizations is *proactive*: it is part of the optimization process when the greatest possible deterioration of the objective (i.e., the "worst-case") is found. The robust solution guaranties that no other parameter change, or multiple parameter changes, will give a "worse" solution value; of course, provided that the allowable ranges within which a parameter can change, as well as the overall level of allowable simultaneous changes are specified.

Robust optimization is a *deterministic* (Bertsimas et al., 2011 [9]) optimization method, although its purpose is to address random data perturbations. In robust optimization data are not "modeled" in the stochastic programming sense. Instead, the robust problems are constructed in a way that will ensure that the model will remain feasible (and not sub-optimal) in a random environment. To avoid terminological confusion, in the robust optimization literature the deterministic "version" of a robust problem is called the *nominal* problem, and the robust "version" of a deterministic problem is called the *robust counterpart* problem.

Within set-induced robust optimization, the *uncertainty sets* play an important role. There exist various approaches to specify uncertainty sets: they can be specified as a convex hull of a finite set of scenarios, defined as a vector norm, or constructed in some other way. It appears that there is no unified axiomatic interpretation of the uncertainty sets. We follow a vector norm based interpretation of uncertainty sets, with its underlying assumptions; this interpretation can be considered the dominant one in the robust optimization literature, although alternative approaches exist.

There are two issues related to the construction of the norm-related uncertainty sets: the magnitude of parameter deviation from a central value (the deviation interval), and the overall limit of joint parameter deviation (the robustness budget). To combine the requirements for maximum allowed *individual* deviations with the requirement for maximum allowed *joint* deviations, the robust uncertainty sets are constructed as intersections of primitive sets. Thus, two commonly used uncertainty sets can be obtained: the *ellipsoidal* uncertainty set and the *polyhedral* uncertainty set, both constructed as intersections with the *box* uncertainty set. The role of the box (L_∞ norm) is to control the individual deviations, and the role of the ellipsoid (L_2 norm), or the polyhedron (L_1 norm) is to control the joint deviations. In the n-dimensional case, the ratio of "sizes" between the ellipsoid and the box, or the polyhedron and the box is termed the *robustness parameter*. This parameter of robustness, or "budget of uncertainty", as it is often referred to in the robust optimization literature, is a parameter that allows control of the trade-off between the system performance and robustness (i.e., insensitivity) against random data perturbations.

Considering a robust optimization problem, a distinction is made between the *constraint-wise* uncertainty that is associated with the overly conservative box uncertainty set, and the *row-wise* uncertainty that leads to less conservative solutions based on ellipsoidal or polyhedral uncertainty sets. Another important topic related to the choice of uncertainty sets is the problem of computational tractability. Extensive description and analysis of the different uncertainty sets in robust optimization is given, for example, in Ben-Tal and Nemirovski (1998 and 1999) [7] [8], Bertsimas and Sim (2004) [11], Bertsimas et al. (2011) [9].

The solution of the robust counterpart can be neither sub-optimal, nor infeasible for any realizations of the random data within the specified set. The issue of the trade-off between optimality and feasibility is formalized in Mulvey et al. (1995) [44] using a robust optimization approach that combines the scenario based approach of stochastic programming with a goal programming approach that includes a weight parameter, which controls the trade-off between "solution robustness" (robustness with respect to the objective value) and "model robustness" (robustness with respect to feasibility of the model under uncertain data realizations). This weight parameter can be viewed as an analog to the robustness parameters for the uncertainty set-based approach. The Mulvey et al. (1995) model allows for "soft" constraints and is, therefore, inconsistent with the robust counterpart approach (in the uncertainty set-based robust optimization, the constraints are assumed to be "hard"). Ben-Tal and Nemirovski (1998) [7] note that if these constraints are made "obligatory", the Mulvey et al. (1995) approach would be a particular case of robust optimization approach, where the uncertainty set is constructed as a convex hull of scenarios. Mulvey et al. (1995), although published several years before the emergence of the widely accepted set-based robust optimization, can help shed light on the intuitive meaning of the commonly used robustness parameters (i.e., Ω and Γ) in the context of goal programming, where the decision-maker sets the desired trade-off between conflicting goals; that is, in this case between the protection level against random parameter realizations, and the acceptable level of deterioration in the objective function. Bertsimas and Sim (2004) [11] quantify this trade-off by theoretically deriving probability bounds of constraint violation, and show that by allowing a relatively small deterioration in the objective value ("price of robustness"), the robust solution remains feasible with high probability. In [11], several versions of the probability bounds are derived. An important characteristic of these bounds is that they are independent of problem solution, and they are the functions of just the robustness parameter and the dimensionality of the uncertainty set. Bertsimas and Sim (2004) provide simulation results for robust solutions under different robustness parameters and illustrate that these results are consistent with theoretically derived probabilistic guarantees.

4.2 Uncertainty Sets

The uncertainty set based approach has become common in robust optimization. The concept of interval/box uncertainty goes back to Soyster (1973) [51], who termed his approach as "inexact linear programming." The information available on the uncertain vector $\mathbf{d}$ is that each j^{th} element of $\mathbf{d}$, d_j is a symmetric and bounded random variable and takes values in the interval $[\bar{d}_j - \hat{d}_j,\ \bar{d}_j + \hat{d}_j]$, where $\bar{d}_j$ is the *nominal* value of d_j, and $\hat{d}_j$ is its *maximum deviation*. The *scaled deviation* of parameter d_j from its nominal value is defined as $w_j = (d_j - \bar{d}_j)/\hat{d}_j$, which takes values in $[-1, 1]$.

Although the simplest of the uncertainty sets, the box, based on simple interval uncertainty, does not present by itself much interest from a practical perspective because of its ultra-conservative results, it is used to create more advanced uncertainty sets, namely ellipsoidal and polyhedral ones, constructed as their intersections with the box uncertainty set. The polyhedral uncertainty set here has the same meaning as the "budgeted", or "cardinality constrained" uncertainty set, as sometimes referred to in the literature. The following are the definitions of these sets for the scaled deviation variable, using the *robustness parameters* Ω and Γ that control the trade-off between robustness and optimality:

$$\textit{Box:} \quad \mathcal{U}_{box} := \Big\{ \mathbf{w} \;\Big|\; |w_j| \leq 1,\ \forall j \Big\} \tag{4.1}$$

$$\textit{Ellipsoidal:} \quad \mathcal{U}_{\Omega} := \Big\{ \mathbf{w} \;\Big|\; \sqrt{\sum_j w_j^2} \leq \Omega;\ |w_j| \leq 1,\ \forall j \Big\} \tag{4.2}$$

$$\textit{Polyhedral:} \quad \mathcal{U}_{\Gamma} := \Big\{ \mathbf{w} \;\Big|\; \sum_j |w_j| \leq \Gamma;\ |w_j| \leq 1,\ \forall j \Big\}, \tag{4.3}$$

where w_j is the j^{th} element of $\mathbf{w}$. With appropriately selected robustness parameters Ω and Γ, $\mathcal{U}_{\Gamma}$ is a linear approximation of $\mathcal{U}_{\Omega}$ (see Figure 4.1), and as such, yield more conservative solutions. However, the polyhedral uncertainty set possesses a very valuable feature – its robust counterpart is a linear optimization program. Bertsimas and Sim (2004) [11] and Ben-Tal et al. (2009) [5] provide the analysis and discussion regarding the relative degree of conservativeness of the solutions depending on the choice of uncertainty sets. Also, the fundamental relationship between the robustness parameters $\Omega = \Gamma/\sqrt{\text{card}(J)}$ allows comparison of the robust objective values resulting from using either ellipsoidal or polyhedral uncertainty sets. The budget of robustness Γ for the polyhedral uncertainty set can take the values in the interval $\Gamma \in \big[0,\ \text{card}(J)\big]$. Assuming the absolute worst-case robustness level (equaling the box uncertainty), gives $\Gamma = \text{card}(J)$ and $\Omega = \sqrt{\text{card}(J)}$. When $\Gamma = 0$, and $\Omega = 0$, the problem reduces to the nominal one.

The definition for the demand uncertainty set that can be readily included in the

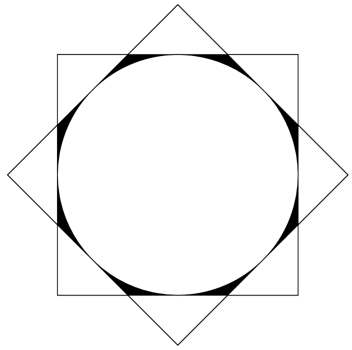


Figure 4.1: Unit box, ellipsoidal, and polyhedral uncertainty sets: $\Omega = 1$ and $\Gamma = \sqrt{2}$.

reformulated robust model (single product version) is

$$\mathcal{D}(j) := \Big\{ \mathbf{d} \;\Big|\; d_j = \bar{d}_j + \hat{d}_j w_j,\ \forall j;\ \mathbf{w} \in \mathcal{U} \Big\}. \tag{4.4}$$

Uncertainty sets for the multi-product case (in terms of scaled deviation) are defined as follows[1]:

$$\textit{Box:}\quad \mathcal{U}_{box} := \Big\{ \mathbf{w} \;\Big|\; |w_{jk}| \le 1,\ \forall j,k \Big\} \tag{4.5}$$

$$\textit{Ellipsoidal:}\quad \mathcal{U}_{\Omega} := \Big\{ \mathbf{w} \;\Big|\; \sqrt{\sum_j \sum_k w_{jk}^2} \le \Omega;\ |w_{jk}| \le 1,\ \forall j,k \Big\} \tag{4.6}$$

$$\textit{Polyhedral:}\quad \mathcal{U}_{\Gamma} := \Big\{ \mathbf{w} \;\Big|\; \sum_j \sum_k |w_{jk}| \le \Gamma;\ |w_{jk}| \le 1,\ \forall j,k \Big\}. \tag{4.7}$$

The budget of robustness Γ for the polyhedral uncertainty set (multi-product case) can take the values in the interval $\Gamma \in \big[0,\ \mathrm{card}(J) \times \mathrm{card}(K)\big]$. This implies that, considering the relationship between Γ and Ω, the corresponding budget of robustness for the ellipsoidal uncertainty set $\Omega = \Gamma / \sqrt{\mathrm{card}(J) \times \mathrm{card}(K))}$. The demand

[1]In the multi-product case, the uncertainty set can be developed applying vectorization operator to the location/product demand matrix: $\mathrm{vec}(\mathbf{D}) = [d_{11}, \ldots, d_{j1}, d_{12}, \ldots, d_{j2}, \ldots, d_{1k}, \ldots, d_{jk}]^\top$.

uncertainty set for the multi-product case is as follows:

$$\mathcal{D}(jk) := \Big\{ \mathbf{d} \;\Big|\; d_{jk} = \bar{d}_{jk} + \hat{d}_{jk} w_{jk},\ \forall j, k;\ \mathbf{w} \in \mathcal{U} \Big\}. \tag{4.8}$$

The maximum demand deviation $\hat{\mathbf{d}}$ does not have to be the same specific percentage of $\bar{\mathbf{d}}$ for all locations and/or products. However, in the literature a constant ratio $\delta = \hat{\mathbf{d}}/\bar{\mathbf{d}}$ between the values of maximum deviations and the values of nominal demands is sometimes assumed (see, for example, Ben-Tal et al., 2005 [6], Baron et al., 2011 [4]).

4.3 Two-stage Decision Framework

The two-stage optimization approach has been widely implemented within the stochastic programming context since its inception. A comprehensive theoretical treatment of this subject can be found in, for example, Birge and Louveaux (2011) [13]. This approach has been extended to robust optimization implementations. Zeng and Zhao (2013) [59] propose a *column-and-constraint generation* (or *primal cut*) algorithm to solve the two-stage robust optimization problem, and demonstrate its superior performance compared to more generic cutting plane algorithms. They formulate the two-stage robust model as a *minimax* problem with uncertain right hand side parameters (i.e., demands). A similar model, but using dual cutting plane solution approach, is proposed in Gabrel et al. (2014) [24].

Applying the two-stage principles to the nominal model formulations in Chapter 3, a two-stage robust counterpart formulation is obtained. The *first stage* decisions include facility location decisions $\mathbf{y}$, and capacity investment decisions $\mathbf{v}$ and $\mathbf{z}$; the *second stage* decision variables, or *recourse* variables, are production and transportation quantities $\mathbf{x}$:

$$\min_{\mathbf{y},\mathbf{v},\mathbf{z}} \quad \mathbf{f}^\top\mathbf{y} + \mathbf{e}^\top\mathbf{v} + \mathbf{g}^\top\mathbf{z} + \max_{\mathbf{d}\in\mathcal{D}} \min_{\mathbf{x}} \quad \mathbf{c}^\top\mathbf{x} \tag{4.9}$$

$$\text{s.t.} \quad \mathbf{x} \geq \mathbf{d} \qquad (\boldsymbol{\lambda}) \tag{4.10}$$

$$\mathbf{x} \leq \mathbf{z} \qquad (\boldsymbol{\pi}) \tag{4.11}$$

$$\mathbf{z} \leq M\mathbf{v} \tag{4.12}$$

$$\mathbf{v} \leq \mathbf{y} \tag{4.13}$$

$$\mathbf{y},\ \mathbf{v} \in \{0,1\};\ \mathbf{z},\ \mathbf{x} \geq \mathbf{0},$$

where $\boldsymbol{\lambda}$ and $\boldsymbol{\pi}$ denote the dual variables for constraints (4.10) and (4.11), respectively. Formulation (4.9)–(4.13) represents in a general form the robust reformulation of (3.1)–(3.6) and (3.7)–(3.16) for the single product and multi-product models, respectively. In this reformulation, $\mathbf{d}$ is a random variable that belongs to the above defined uncertainty set $\mathcal{D}$, and the problem has a *min-max-min* structure. To obtain the second stage recourse problem, the inner $min_{\{\mathbf{x}\}}$ is converted to $max_{\{\boldsymbol{\lambda},\boldsymbol{\pi}\}}$ and combined with $max_{\{\mathbf{d}\in\mathcal{D}\}}$, yielding the following subproblem for a fixed $\mathbf{z}^*$:

$$\mathcal{Q}(\mathbf{z}^*) = \max_{\mathbf{d}\in\mathcal{D},\boldsymbol{\lambda},\boldsymbol{\pi}} Q(\mathbf{z}^*,\mathbf{d}) = \max_{\mathbf{d}\in\mathcal{D},\boldsymbol{\lambda},\boldsymbol{\pi}} \quad \mathbf{d}^\top\boldsymbol{\lambda} - \mathbf{z}^{*\top}\boldsymbol{\pi} \tag{4.14}$$

$$\text{s.t.} \quad \boldsymbol{\lambda} - \boldsymbol{\pi} \leq \mathbf{c} \tag{4.15}$$

$$\boldsymbol{\lambda},\ \boldsymbol{\pi} \geq \mathbf{0}.$$

The relaxed master problem for the two-stage robust formulation is described on page 50 as a part of a *minimax* decomposition algorithm.

4.4 Robust Recourse Problems

The following four formulations are obtained by combining the definitions of uncertainty sets (4.4) and (4.8) with the dual of the subproblem (4.14)–(4.15).

Single product, ellipsoidal uncertainty set:

$$\max_{w,\lambda,\pi} \quad \sum_j \left(\bar{d}_j + \hat{d}_j w_j\right)\lambda_j - \sum_i \sum_l z^*_{il}\pi_{il} \tag{4.16}$$

$$\text{s.t.} \quad \lambda_j - \pi_{il} \le c_{ijl}, \quad \forall i,j,l \tag{4.17}$$

$$\sqrt{\sum_j w_j^2} \le \Omega \tag{4.18}$$

$$w_j \le 1, \quad \forall j \tag{4.19}$$

$$w_j,\ \lambda_j,\ \pi_{il} \ge 0, \quad \forall i,j,l.$$

Single product, polyhedral uncertainty set:

$$\max_{w,\lambda,\pi} \quad \sum_j \left(\bar{d}_j + \hat{d}_j w_j\right)\lambda_j - \sum_i \sum_l z^*_{il}\pi_{il} \tag{4.20}$$

$$\text{s.t.} \quad \lambda_j - \pi_{il} \le c_{ijl}, \quad \forall i,j,l \tag{4.21}$$

$$\sum_j w_j \le \Gamma \tag{4.22}$$

$$w_j \le 1, \quad \forall j \tag{4.23}$$

$$w_j,\ \lambda_j,\ \pi_{il} \ge 0, \quad \forall i,j,l.$$

Multi-product, ellipsoidal uncertainty set:

$$\max_{w,\lambda,\pi} \quad \sum_j \sum_k \left(\bar{d}_{jk} + \hat{d}_{jk} w_{jk}\right)\lambda_{jk} - \sum_i \sum_l \left(\sum_k z^{*D}_{ikl}\pi^D_{ikl} + z^{*F}_{il}\pi^F_{il}\right) \tag{4.24}$$

$$
\begin{aligned}
\text{s.t.} \quad & \lambda_{jk} - \pi^D_{ikl} \le c^D_{ijkl}, \quad \forall i,j,k,l && (4.25)\\
& \lambda_{jk} - h_{ikl}\pi^F_{il} \le c^F_{ijkl}, \quad \forall i,j,k,l && (4.26)\\
& \sqrt{\sum_j \sum_k w^2_{jk}} \le \Omega && (4.27)\\
& w_{jk} \le 1, \quad \forall j,k && (4.28)\\
& w_{jk},\ \lambda_{jk},\ \pi^D_{ikl},\ \pi^F_{il} \ge 0, \quad \forall i,j,k,l.
\end{aligned}
$$

Multi-product, polyhedral uncertainty set:

$$
\begin{aligned}
\max_{w,\lambda,\pi} \quad & \sum_j \sum_k \left(\bar{d}_{jk} + \hat{d}_{jk} w_{jk}\right)\lambda_{jk} - \sum_i \sum_l \left(\sum_k z^{*D}_{ikl}\pi^D_{ikl} + z^{*F}_{il}\pi^F_{il}\right) && (4.29)\\
\text{s.t.} \quad & \lambda_{jk} - \pi^D_{ikl} \le c^D_{ijkl}, \quad \forall i,j,k,l && (4.30)\\
& \lambda_{jk} - h_{ikl}\pi^F_{il} \le c^F_{ijkl}, \quad \forall i,j,k,l && (4.31)\\
& \sum_j \sum_k w_{jk} \le \Gamma && (4.32)\\
& w_{jk} \le 1, \quad \forall j,k && (4.33)\\
& w_{jk},\ \lambda_{jk},\ \pi^D_{ikl},\ \pi^F_{il} \ge 0, \quad \forall i,j,k,l.
\end{aligned}
$$

The above formulations are for the *optimality* subproblems. To obtain the *feasibility* subproblems, the following modifications are introduced: c_{ijl}, or c^D_{ijkl} and c^F_{ijkl} are set to 0, and the *normalization* constraints $\lambda_j \le 1$ and $\pi_{il} \le 1$ are added to the single product subproblems, or $\lambda_{jk} \le 1$, $\pi^D_{ikl} \le 1$, and $\pi^F_{il} \le 1$ are added to the multi-product subproblems.

Problems (4.16)–(4.19), (4.20)–(4.23), (4.24)–(4.28), and (4.29)–(4.33) are difficult to solve bilinear optimization problems because of the product $\mathbf{w}\boldsymbol{\lambda}$. While it is possible to apply the standard linearization techniques to (4.20)–(4.23) and (4.29)–(4.33), i.e., to the formulations for the polyhedral uncertainty set, and solve as mixed integer programs, efficient solution procedures for (4.16)–(4.19) and (4.24)–(4.28) appear to be elusive. Therefore, the formulations involving ellipsoidal uncertainty sets can be solved for only small sizes, using global solvers for non-linear non-convex problems[2].

Bilinear subproblems for the polyhedral uncertainty sets (4.20)–(4.23) and (4.29)–(4.33) can be converted to mixed integer linear problems as, for example, in Gabrel et al. (2014) [24], where the continuous $\mathbf{w} \in [0, 1]$ is replaced by a binary variable, and the product $\mathbf{w}\boldsymbol{\lambda}$ replaced by variable $\boldsymbol{\lambda'}$. However, this conversion places restrictions on Γ values, i.e., they can only be integers. This outcome is contrary to the original meaning of Γ, as explained in Bertsimas and Sim (2004) [11], where the robustness parameter Γ, *not necessarily integer*, can take values in the interval $\big[0,\ \mathrm{card}(J)\big]$. To address the issue, we introduce *fractional deviations* $\hat{\mathbf{d}}' = \hat{\mathbf{d}}\big(\Gamma - \lfloor\Gamma\rfloor\big)$, and propose a modified formulation that converts the bilinear problem to a mixed integer linear problem, and at the same time preserves the original meaning of Γ. For expositional clarity, only the single product model is presented:

$$\max_{w,\lambda,\lambda',\lambda'',\pi} \quad \sum_j \bar{d}_j\lambda_j + \sum_j \hat{d}_j\lambda'_j + \sum_j \hat{d}'_j\lambda''_j - \sum_i\sum_l z^*_{il}\pi_{il} \tag{4.34}$$

$$\text{s.t.} \quad \lambda_j - \pi_{il} \le c_{ijl}, \quad \forall i,j,l \tag{4.35}$$

$$\sum_j w_j \le \Gamma \tag{4.36}$$

$$\lambda'_j \le \lambda_j, \quad \forall j \tag{4.37}$$

$$\lambda'_j \le M w_j, \quad \forall j \tag{4.38}$$

$$\sum_j w'_j \le 1 \tag{4.39}$$

$$w_j + w'_j \le 1, \quad \forall j \tag{4.40}$$

$$\lambda''_j \le \lambda_j, \quad \forall j \tag{4.41}$$

$$\lambda''_j \le M w'_j, \quad \forall j \tag{4.42}$$

[2] For example, LINDO Global, or BARON, available through GAMS IDE (integrated development environment).

$$w_j,\ w'_j \in \{0,1\};\ \lambda_j,\ \lambda'_j,\ \lambda''_j,\ \pi_{il} \geq 0, \quad \forall i,j,l,$$

where $\boldsymbol{\lambda''} = \boldsymbol{\lambda}\mathbf{w'}$, just as $\boldsymbol{\lambda'} = \boldsymbol{\lambda}\mathbf{w}$. Parameter M is a sufficiently large constant, representing the bounds on the $\boldsymbol{\lambda}$ and $\boldsymbol{\lambda''}$ variables. The extended formulation (4.34)–(4.42), for the first time proposed in this dissertation, can be beneficial for uncertainty sets with fewer elements, e.g., fewer than 20, or in situations when fractional Γ values are needed to achieve greater precision for analysis and comparative purposes.

4.5 Solution Algorithm

The solution approach used here to solve robust optimization problems is adapted from Zeng and Zhao (2013) [59]. They make a distinction between a type of a *Benders-dual cutting plane algorithm* and the *primal cut algorithm*, also termed the *column-and-constraint generation* (C&CG) procedure. Under Benders-dual method, the objective value is gradually constructed using the (dual) cut coefficients, obtained from solving the second stage recourse problem. However, under the primal cut, or C&CG, approach, the dual information is not used to generate cuts. Instead, the primal cut procedure generates constraints along with the copies of *primal* recourse decision variables, using the information from the worst-case solution of the second stage. Thus, at each iteration a new column of primary recourse variables as well as a set of "scenarios" for the uncertain demand, i.e., constraints, are created.

In the two-stage robust optimization setting, the primal cut algorithm exhibits superior performance in terms of both the number of iterations and solution time. For example, Zeng and Zhao (2013) show that the number of iterations is at least an order of magnitude fewer using the primal cut approach, compared to Benders-dual approach. Although the dimensionality of the primal recourse variables increases at each iteration, thus increasing the computational complexity, this increased complexity is more than offset by stronger cuts generated by the primal cut algorithm.

In addition, the primal cut algorithm provides a unified approach to the optimality and feasibility, i.e., there are no two separate sets of cut coefficients – one for optimality cuts and one for feasibility cuts, as in Benders-dual approach.

Primal Cut Algorithm

---

1. Set $LB = -\infty$, $UB = +\infty$, the iteration counter $t = 1$, and the optimality cut-set $\mathcal{O} = \varnothing$; select convergence tolerance parameter ε.
2. Solve the restricted master problem:

$$\min_{\mathbf{y},\mathbf{v},\mathbf{z},\eta,\mathbf{x}} \quad \mathbf{f}^\top\mathbf{y} + \mathbf{e}^\top\mathbf{v} + \mathbf{g}^\top\mathbf{z} + \eta \tag{4.43}$$

$$\text{s.t.} \quad \eta \geq \mathbf{c}^\top\mathbf{x}^s, \quad \forall s \in \mathcal{O} \tag{4.44}$$

$$\mathbf{x}^s \geq \mathbf{d}^s, \quad \forall s \leq t-1 \tag{4.45}$$

$$\mathbf{x}^s \leq \mathbf{z}, \quad \forall s \leq t-1 \tag{4.46}$$

$$\mathbf{z} \leq M\mathbf{v} \tag{4.47}$$

$$\mathbf{v} \leq \mathbf{y} \tag{4.48}$$

$$\mathbf{y},\ \mathbf{v} \in \{0,1\};\ \mathbf{z},\ \eta,\ \mathbf{x}^s \geq \mathbf{0}.$$

   Obtain optimal solution $(\mathbf{y}^*, \mathbf{v}^*, \mathbf{z}^*, \eta^*, \mathbf{x}^{s*})$ and update $LB = \mathbf{f}^\top\mathbf{y}^* + \mathbf{e}^\top\mathbf{v}^* + \mathbf{g}^\top\mathbf{z}^* + \eta^*$.
3. Solve subproblem $\mathcal{Q}(\mathbf{z}^*)$.

   (a) If $\mathcal{Q}(\mathbf{z}^*) < +\infty$, solve

$$\mathcal{Q}(\mathbf{z}^*) = \max_{\mathbf{w}\in\mathcal{U},\boldsymbol{\lambda}\geq\mathbf{0},\boldsymbol{\pi}\geq\mathbf{0}} \left\{ \left(\bar{\mathbf{d}} + \hat{\mathbf{d}}\mathbf{w}\right)^\top \boldsymbol{\lambda} - \mathbf{z}^{*\top}\boldsymbol{\pi} \ \text{ s.t. } \ \boldsymbol{\lambda} - \boldsymbol{\pi} \leq \mathbf{c} \right\},$$

   and update $UB = \min\left(UB,\ \mathbf{f}^\top\mathbf{y}^* + \mathbf{e}^\top\mathbf{v}^* + \mathbf{g}^\top\mathbf{z}^* + \mathcal{Q}(\mathbf{z}^*)\right)$.

   (b) If $UB - LB < \varepsilon$, stop; otherwise assign cut parameter $\mathbf{d}^t = \bar{\mathbf{d}} + \hat{\mathbf{d}}\mathbf{w}^*$, create variables $\mathbf{x}^t$, and add constraints $\eta \geq \mathbf{c}^\top\mathbf{x}^t$, $\mathbf{x}^t \geq \mathbf{d}^t$, and $\mathbf{x}^t \leq \mathbf{z}$ to the master problem. Update $t = t+1$, $\mathcal{O} = \mathcal{O} \cup \{t\}$, and go to Step 2.

(c) If $\mathcal{Q}(\mathbf{z}^*) = +\infty$, solve

$$\mathcal{Q}(\mathbf{z}^*) = \max_{\mathbf{w}\in\mathcal{U},\boldsymbol{\lambda}\geq\mathbf{0},\boldsymbol{\pi}\geq\mathbf{0}} \left\{ \left(\bar{\mathbf{d}} + \hat{\mathbf{d}}\mathbf{w}\right)^\top \boldsymbol{\lambda} - \mathbf{z}^{*\top}\boldsymbol{\pi} \text{ s.t. } \boldsymbol{\lambda} - \boldsymbol{\pi} \leq \mathbf{0},\ \boldsymbol{\lambda} \leq \mathbf{1},\ \boldsymbol{\pi} \leq \mathbf{1} \right\}.$$

Assign cut parameter $\mathbf{d}^t = \bar{\mathbf{d}} + \hat{\mathbf{d}}\mathbf{w}^*$, create variables $\mathbf{x}^t$, and add constraints $\mathbf{x}^t \geq \mathbf{d}^t$, and $\mathbf{x}^t \leq \mathbf{z}$ to the master problem. Update $t = t + 1$, and go to Step 2.

The general structure of the primal cut algorithm is as follows. The first step is to initialize the lower and upper bounds, to set the iteration counter, and to select a convergence tolerance parameter. The second step is to solve the restricted master problem and to update the lower bound. The third step is to solve the subproblem: if the subproblem is not unbounded, the optimality subproblem is solved and the upper bound is updated; if the subproblem is unbounded, the feasibility subproblem is solved. When the difference between the upper and lower bounds is less than the tolerance parameter, the algorithm converges. At each iteration either optimality or feasibility cuts are added, and a new dimension of primary recourse variables is created. According to the primal cut algorithm, the set of feasibility cuts is a subset of optimality cuts, i.e, when the optimality subproblem is solved, cuts (4.44), (4.45), and (4.46) are added to the master problem, and when the feasibility subproblem is solved, only cuts (4.45) and (4.46) are added to the master problem.

Robust Optimization Problem: An Illustrative Example

The following example illustrates the bilinear formulation of the subproblem that leads to a robust solution, which ensures that a sufficient amount of capacity is acquired to meet the *largest* (in unit terms) demand deviations within the specified uncertainty set, and that the objective value represents the *costliest* demand deviations within the specified uncertainty set. We also provide a numerical interpretation of the primal cut algorithm, using the single product model and the polyhedral uncertainty set. To simplify our exposition, we assume only one technology type and no fixed capacity investment costs.

The objective function of the subproblem

$$\max_{\mathbf{w},\boldsymbol{\lambda},\boldsymbol{\pi}} \quad \left(\bar{\mathbf{d}} + \hat{\mathbf{d}}\mathbf{w}\right)^{\top}\boldsymbol{\lambda} - \mathbf{z}^{*\top}\boldsymbol{\pi}$$

is the same for both optimality and feasibility sub-problems. By solving the *optimality* subproblem, the *costliest* realizations of the uncertain demand $\left(\bar{\mathbf{d}} + \hat{\mathbf{d}}\mathbf{w}^{*}\right)^{\top}\boldsymbol{\lambda}$ within a specified set are obtained. By solving the *feasibility* subproblem, the *largest* (in unit terms) realizations of the uncertain demand $\bar{\mathbf{d}} + \hat{\mathbf{d}}\mathbf{w}^{*}$ within a specified set are obtained (as $\boldsymbol{\lambda}^{*} = \mathbf{1}$, due to the normalization constraints $\boldsymbol{\lambda} \leq \mathbf{1}$ and $\boldsymbol{\pi} \leq \mathbf{1}$).

The data set for the sample problem is chosen as follows:

- $f_1 = 50,\ f_2 = 55,\ g_1 = 1,\ g_2 = 1$
- $c_{11} = 2,\ c_{12} = 3,\ c_{21} = 1,\ c_{22} = 6$
- $\bar{d}_1 = 60,\ \bar{d}_2 = 30,\ \hat{d}_1 = 30,\ \hat{d}_2 = 15,\ \Gamma = 1.$

Using the primal cut algorithm presented on page 50, this small problem converges in four iterations, which are described below in detail (the integrality and non-negativity constraints are omitted for more concise description).

Iteration 1 Solve the master problem (the sets of cuts are empty and there are no primal recourse variables x created yet)

$$\begin{aligned} \tau = \min_{y,z,\eta} \quad & 50y_1 + 55y_2 + 1z_1 + 1z_2 + \eta \\ \text{s.t.} \quad & z_1 \leq My_1,\ z_2 \leq My_2, \end{aligned}$$

which gives a solution of $\tau^* = 0,\ y_1^* = 0,\ y_2^* = 0,\ z_1^* = 0,\ z_2^* = 0$. Update $LB = 0$. Because the subproblem is unbounded, solve the modified feasibility subproblem

$$\begin{aligned} \theta = \max_{w,\lambda,\pi} \quad & (60 + 30w_1)\lambda_1 + (30 + 15w_2)\lambda_2 - 0\pi_1 - 0\pi_2 \\ \text{s.t.} \quad & \lambda_1 - \pi_1 \leq 0,\ \lambda_1 - \pi_2 \leq 0,\ \lambda_2 - \pi_1 \leq 0,\ \lambda_2 - \pi_2 \leq 0 \end{aligned}$$

$$
\begin{aligned}
& w_1 + w_2 \le 1 \\
& w_1 \le 1,\ w_2 \le 1 \\
& \lambda_1 \le 1,\ \lambda_2 \le 1 \\
& \pi_1 \le 1,\ \pi_2 \le 1,
\end{aligned}
$$

which gives a solution of $w_1^* = 1$, $w_2^* = 0$. The upper bound remains $UB = +\infty$. Create new variables x_{11}^1, x_{12}^1, x_{21}^1, x_{22}^1, and add the following cuts to the master problem:

$$
\begin{aligned}
& x_{11}^1 + x_{21}^1 \ge 90,\ x_{12}^1 + x_{22}^1 \ge 30 \\
& x_{11}^1 + x_{12}^1 \le z_1,\ x_{21}^1 + x_{22}^1 \le z_2.
\end{aligned}
$$

Iteration 2 Solve the master problem

$$
\begin{aligned}
\tau = \min_{y,z,\eta,x} \quad & 50y_1 + 55y_2 + 1z_1 + 1z_2 + \eta \\
\text{s.t.} \quad & x_{11}^1 + x_{21}^1 \ge 90,\ x_{12}^1 + x_{22}^1 \ge 30 \\
& x_{11}^1 + x_{12}^1 \le z_1,\ x_{21}^1 + x_{22}^1 \le z_2 \\
& z_1 \le My_1,\ z_2 \le My_2,
\end{aligned}
$$

which gives a solution[3] of $\tau^* = 170$, $y_1^* = 1$, $y_2^* = 0$, $z_1^* = 120$, $z_2^* = 0$. Update $LB = 170$. Solve the subproblem

$$
\begin{aligned}
\theta = \max_{w,\lambda,\pi} \quad & (60 + 30w_1)\lambda_1 + (30 + 15w_2)\lambda_2 - 120\pi_1 - 0\pi_2 \\
\text{s.t.} \quad & \lambda_1 - \pi_1 \le 2,\ \lambda_1 - \pi_2 \le 3,\ \lambda_2 - \pi_1 \le 1,\ \lambda_2 - \pi_2 \le 6 \\
& w_1 + w_2 \le 1 \\
& w_1 \le 1,\ w_2 \le 1,
\end{aligned}
$$

which gives a solution of $\theta^* = 270$, $w_1^* = 1$, $w_2^* = 0$. Update $UB = \min(UB,\ 50 + 120 + 270) = 440$. Create new variables x_{11}^2, x_{12}^2, x_{21}^2, x_{22}^2, and add the following cuts to the master problem:

$$
\begin{aligned}
& \eta \ge 2x_{11}^2 + 3x_{12}^2 + 1x_{21}^2 + 6x_{22}^2 \\
& x_{11}^2 + x_{21}^2 \ge 90,\ x_{12}^2 + x_{22}^2 \ge 30 \\
& x_{11}^2 + x_{12}^2 \le z_1,\ x_{21}^2 + x_{22}^2 \le z_2.
\end{aligned}
$$

[3]Solutions for x's and η are omitted from the description.

Iteration 3 Solve the master problem

$$\begin{aligned}
\tau = \min_{y,z,\eta,x} \quad & 50y_1 + 55y_2 + 1z_1 + 1z_2 + \eta \\
\text{s.t.} \quad & \eta \geq 2x_{11}^2 + 3x_{12}^2 + 1x_{21}^2 + 6x_{22}^2 \\
& x_{11}^1 + x_{21}^1 \geq 90,\ x_{12}^1 + x_{22}^1 \geq 30 \\
& x_{11}^2 + x_{21}^2 \geq 90,\ x_{12}^2 + x_{22}^2 \geq 30 \\
& x_{11}^1 + x_{12}^1 \leq z_1,\ x_{21}^1 + x_{22}^1 \leq z_2 \\
& x_{11}^2 + x_{12}^2 \leq z_1,\ x_{21}^2 + x_{22}^2 \leq z_2 \\
& z_1 \leq My_1,\ z_2 \leq My_2,
\end{aligned}$$

which gives a solution of $\tau^* = 405$, $y_1^* = 1$, $y_2^* = 1$, $z_1^* = 30$, $z_2^* = 90$. Update $LB = 405$. Solve the subproblem

$$\begin{aligned}
\theta = \max_{w,\lambda,\pi} \quad & (60 + 30w_1)\lambda_1 + (30 + 15w_2)\lambda_2 - 30\pi_1 - 90\pi_2 \\
\text{s.t.} \quad & \lambda_1 - \pi_1 \leq 2,\ \lambda_1 - \pi_2 \leq 3,\ \lambda_2 - \pi_1 \leq 1,\ \lambda_2 - \pi_2 \leq 6 \\
& w_1 + w_2 \leq 1 \\
& w_1 \leq 1,\ w_2 \leq 1,
\end{aligned}$$

which gives a solution of $\theta^* = 240$, $w_1^* = 0$, $w_2^* = 1$. Update $UB = \min(UB,\ 105 + 120 + 240) = 440$. Create new variables x_{11}^3, x_{12}^3, x_{21}^3, x_{22}^3, and add the following cuts to the master problem:

$$\begin{aligned}
& \eta \geq 2x_{11}^3 + 3x_{12}^3 + 1x_{21}^3 + 6x_{22}^3 \\
& x_{11}^3 + x_{21}^3 \geq 60,\ x_{12}^3 + x_{22}^3 \geq 45 \\
& x_{11}^3 + x_{12}^3 \leq z_1,\ x_{21}^3 + x_{22}^3 \leq z_2.
\end{aligned}$$

Iteration 4 Solve the master problem

$$\begin{aligned}
\tau = \min_{y,z,\eta,x} \quad & 50y_1 + 55y_2 + 1z_1 + 1z_2 + \eta \\
\text{s.t.} \quad & \eta \geq 2x_{11}^2 + 3x_{12}^2 + 1x_{21}^2 + 6x_{22}^2 \\
& \eta \geq 2x_{11}^3 + 3x_{12}^3 + 1x_{21}^3 + 6x_{22}^3 \\
& x_{11}^1 + x_{21}^1 \geq 90,\ x_{12}^1 + x_{22}^1 \geq 30 \\
& x_{11}^2 + x_{21}^2 \geq 90,\ x_{12}^2 + x_{22}^2 \geq 30 \\
& x_{11}^3 + x_{21}^3 \geq 60,\ x_{12}^3 + x_{22}^3 \geq 45
\end{aligned}$$

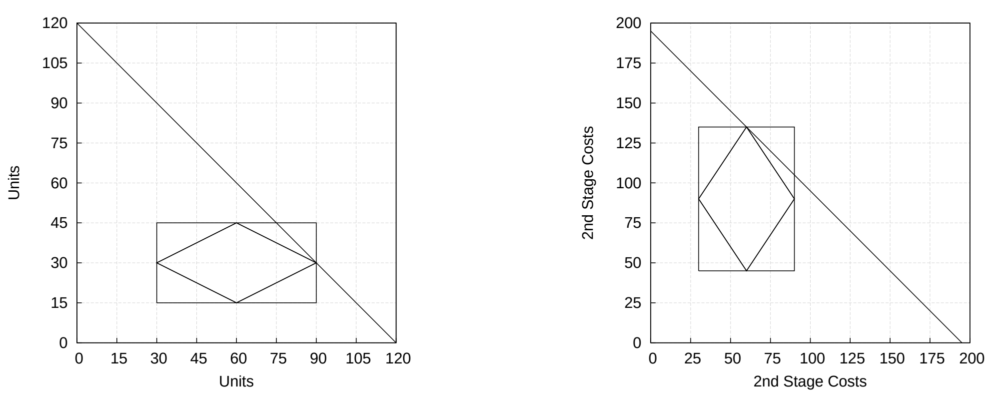
$$x_{11}^1 + x_{12}^1 \le z_1,\ x_{21}^1 + x_{22}^1 \le z_2$$
$$x_{11}^2 + x_{12}^2 \le z_1,\ x_{21}^2 + x_{22}^2 \le z_2$$
$$x_{11}^3 + x_{12}^3 \le z_1,\ x_{21}^3 + x_{22}^3 \le z_2$$
$$z_1 \le My_1,\ z_2 \le My_2,$$

which gives a solution of $\tau^* = 420,\ y_1^* = 1,\ y_2^* = 1,\ z_1^* = 45,\ z_2^* = 75$. Update $LB = 420$. Solve the subproblem

$$\begin{aligned} \theta = \max_{w,\lambda,\pi} \quad & (60 + 30w_1)\lambda_1 + (30 + 15w_2)\lambda_2 - 45\pi_1 - 75\pi_2 \\ \text{s.t.} \quad & \lambda_1 - \pi_1 \le 2,\ \lambda_1 - \pi_2 \le 3,\ \lambda_2 - \pi_1 \le 1,\ \lambda_2 - \pi_2 \le 6 \\ & w_1 + w_2 \le 1 \\ & w_1 \le 1,\ w_2 \le 1, \end{aligned}$$

which gives a solution of $\theta^* = 195,\ w_1^* = 0,\ w_2^* = 1$. Update $UB = \min(UB,\ 105 + 120 + 195) = 420$, which is equal to LB.

According to the solution, the first stage costs are 225 and the second stage costs are 195. Figure 4.2 shows the solution of the sample problem from two perspectives – from the capacity perspective and from the cost perspective. The solution must be feasible for *any* one out of two deviations, either $w_1 = 1$ and $w_2 = 0$, or $w_1 = 0$ and $w_2 = 1$. This is why a total of 120 units of capacity ($z_1^* = 45$ and $z_2^* = 75$) is needed



(a) Largest deviation: $w_1 = 1$, $w_2 = 0$.

(b) Costliest deviation: $w_1 = 0$, $w_2 = 1$.

Figure 4.2: Total demand for a robust solution.

to ensure feasibility, but it happens in this instance that only a total of 105 units of demand ($d_1 = 60 + 30w_1^* = 60$ and $d_2 = 30 + 15w_2^* = 45$) has to be satisfied to ensure optimality. A *slack* of 15 units is created by the robust solution to make the solution *less sensitive* to random demand realizations. As a matter of managerial interest, in this case the safety capacity is *optimally* distributed between the two facilities, while to total amount of this safety capacity is set according to a pre-specified robustness level. This example, as well our numerical experiments in Chapter 5 indicate that under uncertain demand realizations the capacity may not be fully utilized. This insight is consistent with analytical results of Van Mieghem (2003) [53], who states that a key feature of this safety capacity is that it is unbalanced, meaning that regardless how the uncertain demand is realized one will typically not utilize all capacities.

Chapter 5

Numerical Studies

The overall goals of the numerical studies are three-fold. First, the proposed robust optimization models are implemented and solved to provide insights related to finding optimal capacity types and quantities under different facility and technology costs, and for varying levels of robustness (Section 5.2.1). Second, the solutions obtained by robust optimization are compared to non-robust model solutions (Section 5.2.2). Third, the performance of robust optimization solutions is compared to that of stochastic programming solutions, including the effects of demand correlations (Section 5.2.3). All experiments are conducted for both the single product and multi-product model versions, corresponding to their formulations in Chapters 3 and 4.

Computational experiments are designed and implemented using recognized practices from the literature. For example, Baron et al. (2011) [4] offer a multi-period facility location model under demand uncertainty. They use robust optimization methodology in conjunction with simulation to show that the topology of the solution, the optimal facility sizes, and operational profits depend on the decision-maker's assumptions about the nature of uncertainty. This work is related to our study in that it uses a joint capacity location and capacity sizing model using uncertainty set-based robust optimization. However, the Baron et al. model considers only a single product and a single capacity type with linear unit capacity acquisition costs (as opposed to concave piecewise linear costs in our work). The other characteristics that differentiate their work from ours is that we formulate our models as two-stage models with recourse, and solve them for a whole spectrum of different robustness parameter Ω and Γ values. Regardless of the differences between the two studies, we adapt from Baron et al. (2011) [4] the idea to generate samples of location coordinates and customer demands, and to solve samples of problem instances using robust optimization, albeit in a different setting. Another example of a computational study relevant to our experiments, an application of stochastic programming approach to supply chain design, is Santoso et al. (2005) [47]. As our intent is to use stochastic programming only for benchmarking purposes, our adaptation of the Sample Average Approximation scheme, a stochastic programming method, from Santoso et al. (2005) [47] and related works, is straightforward.

5.1 Experimental Design

The general experimental approach is similar between the single product and the multi-product model; however, the data as well as the test instances are described separately. In addition, while the main purpose of the single product model is to investigate the effects of economies of scale in the context of an integrated facility location, capacity acquisition, and technology choice model, the main purpose of the multi-product model is to investigate the effects of economies of scope, as well as the combined effects of scale and scope. All computational experiments are conducted with randomly generated data that reflect a relatively wide range of possible problem parameters in order to obtain more general insights. Most of the experiments, with the exception of instances when robust optimization solutions are compared to stochastic programming solutions, are conducted using the polyhedral uncertainty set, due to the solution efficiency that greatly exceeds the efficiency of the ellipsoidal version of the problem.

5.1.1 Generation of Problem Samples

A similar sample generation approach is used for both the single product and multi-product models. These same samples are used for the robust model as well as for a "non-robust" model, described in Section 5.1.2. The first step is to identify the number of test instances that represent different combinations of facility and technology type costs, as well as the cost ratios between flexible and dedicated technologies (for the multi-product model). Each of the facility and technology type costs ($\mathbf{f}$, $\mathbf{e}$, and $\mathbf{g}$, defined in Chapter 3) is scaled to either "low" or "high" values, as described below. This way, a wide range of possible outcomes can be observed, and at the same time the number of these possible outcomes is contained to a manageable number of combinations. The second step is to generate two samples – one for the single product model and one for the multi-product model – and to scale the corresponding facility and technology costs.

It is assumed that the facility and demand locations (x and y -coordinates) are uniformly generated in a 100×100 square, transportation costs are set proportional to the Euclidean distances between the locations, and the customer zone demands are drawn from a uniform distribution (see, for example, Cornuejols et al., 1991 [17], Lim and Kim, 1999 [35], Melkote and Daskin, 2001 [42], Baron et al., 2011 [4]). The fixed production facility investment costs, fixed technology investment costs, and unit technology investment costs are assumed to be equal for all facilities, i.e., $f_i = f$, $e_{il} = e_l$, and $g_{il} = g_l$. In addition, we assume that the dedicated capacity investment costs are the same for both products – this assumption may be too restrictive for real world problems; however, in support of our approach it is not uncommon in the analytical literature on dedicated vs. flexible capacities to assume that the unit capacity investment costs for both products are the same (see, for example, Fine and Freund (1990) [22], Lus and Muriel (2009) [37]). Thus, $e^D_{ikl} = e^D_l$ and $g^D_{ikl} = g^D_l$.

The sampling of e_l and g_l is described as follows. We assume that there exist three different technology types ($l1$, $l2$, and $l3$) that represent a piecewise linear non-decreasing cost structure. We also assume that $l1$ represents the most "labor-intensive" technology, while $l3$ represents the most "capital-intensive", or the most automated technology ($l2$ is an intermediate alternative between $l1$ and $l3$). Thus, $e_1 < e_2 < e_3$, and $g_1 > g_2 > g_3$. The fixed and unit technology costs, e_l and g_l, are generated from $U(0, e_{max})$ and $U(0, g_{max})$, respectively, and sorted to satisfy the following conditions:

$$0 < e_1 < e_2 < e_3 < e_{max}$$

$$g_{max} > g_1 > g_2 > g_3 > 0.$$

The fixed facility costs are sampled from $U(0, f_{max})$. Scalars f_{max}, e_{max}, and g_{max} represent the upper limit the corresponding costs f, e_l, and g_l can be drawn from. Each of the *max* parameters is scaled to "low" or "high" to control the relative magnitude of f vs. e_l, e_l vs. g_l, and f vs. g_l. It is assumed that the second stage production costs are technology independent[1], and are sampled from $U(1, 10)$.

The sample size[2] for the single-product model is 150, and for the multi-product model 100. Each of the individual problems is solved for varying levels of robustness, i.e., for 21 levels of Γ, where $\Gamma \in [0, 20]$.

[1] The technology-based differences in unit costs are already reflected in g_l.

[2] The sample sizes are based on two factors: the magnitude of standard error and computational burden.

Single Product Model Dataset

Table 5.1 summarizes the test instances for the single product model. Each individual random single product problem is replicated $R = 150$ times, and the robust problem (using the polyhedral uncertainty set) is solved, yielding a total of 25,200 instances of individual *min-max* robust problems ($2f_{max} \times 2e_{max} \times 2g_{max} \times 21\Gamma \times 150R$).

Data	**Values**	**Description**
I	10	number of production facilities
J	20	number of customer zones
L	3	number of technology types
f_{max}	5,000 or 20,000	maximum facility costs
e_{max}	10,000 or 15,000	maximum fixed technology costs
g_{max}	7.5 or 10.0	maximum variable technology costs
c_{ijl}	$U(1, 10) + 1\times$dist.	production and transportation costs
$\bar{d}_j$	$U(50, 450)$	nominal demands
$\hat{d}_j$	$0.5\bar{d}_j$	maximum deviation of demands
Γ	$0, 1, \ldots, 20$	robustness levels
R	150	number of replications

Table 5.1: Data for the single product model.

Multi-product Model Dataset

Table 5.2 summarizes the test instances for the multi-product model. Each individual random multi-product problem is replicated $R = 100$ times, and the robust problem (using the polyhedral uncertainty set) is solved, yielding a total of 12,600 instances of individual *min-max* robust problems ($6\Delta \times 21\Gamma \times 100R$). It can be emphasized that in the multi-product model each of the dedicated technologies and the flexible technology has two sub-types (labor-intensive and capital-intensive), resulting in 6 technologies total in the model; for example, the variables for units of capacity established are z^D_{i11}, z^D_{i21}, z^D_{i12}, z^D_{i22}, z^F_{i1}, and z^F_{i2}.

Data	**Values**	**Description**
I	10	number of production facilities
J	10	number of customer zones
K	2	number of products
L	2	number of technology types
f_{max}	5,000	maximum facility costs
e^D_{max}	15,000	maximum fixed dedicated technology costs
g^D_{max}	10.0	maximum variable dedicated technology costs
Δ	1, 1.25, 1.5, 1.75, 1.9, 2	flexible to dedicated technology cost ratio
c_{ijkl}	$U(1, 10) + 1\times$dist.	production and transportation costs
$\bar{d}_{j1}$	$U(50, 450)$	product 1 nominal demands
$\bar{d}_{j2}$	$U(200, 400)$	product 2 nominal demands
$\hat{d}_{jk}$	$0.5\bar{d}_{jk}$	maximum deviation of demands
Γ	$0, 1, \ldots, 20$	robustness levels
R	100	number of replications $r \in R$

Table 5.2: Data for the multi-product model.

The flexible technology costs are established as follows. As in the case for the single product model, the technology costs are the same for all facility locations. To obtain the flexible capacity investment costs we use the *flexible to dedicated technology cost ratio* Δ (see Table 5.2). As a result, $e^F_l = \Delta e^D_l$ and $g^F_l = \Delta g^D_l$. We have selected six different values for this ratio, including extreme cases of $\Delta = 1$ and $\Delta = 2$ that represent a range of possible values for Δ in a two-product setting.

5.1.2 Non-robust Model Assumptions

The non-robust model used for comparison purposes to assess the quality of solutions obtained using robust optimization is constructed as follows. We use a deterministic[3] "box-robustness" model that is solved for different, gradually increasing deviation intervals. In Section 4.2 we defined the box and polyhedral uncertainty sets as follows:

$$\textit{Box:}\quad \mathcal{U}_{box} := \Big\{ \mathbf{w} \;\Big|\; |w_j| \leq 1,\ \forall j \Big\}$$

$$\textit{Polyhedral:}\quad \mathcal{U}_{\Gamma} := \Big\{ \mathbf{w} \;\Big|\; \sum_j |w_j| \leq \Gamma;\ |w_j| \leq 1,\ \forall j \Big\}.$$

According to the box uncertainty set definition, the random variable w_j, $j \in J$ is bounded by a J-dimensional unit hypercube, and in robust solutions all w_j's will take values of 1. Therefore, to restrict the worst-case solution when all w_j's equal to 1, the robustness parameter Γ in the definition of the polyhedral uncertainty set controls the number dimensions that can deviate from 0. The importance of how the polyhedral uncertainty set is defined is that it considers *joint* deviations. The box uncertainty set allows all *individual* deviations. The non-robust model, therefore, is constructed for gradually increasing intervals from 0 to 1, i.e., it is a box robustness model for different interval sizes. For example, for card(J) = 20, the polyhedral robust model with $\Gamma = 10$ is compared to the non-robust box model with $|w_j| \leq 0.5$, i.e., instead of allowing *any* 10 out of 20 w_j's to deviate within the interval $[0, 1]$, the box model allows *all* 20 to deviate within the interval $[0, 0.5]$.

5.1.3 Assumptions for Stochastic Programming Implementation

The data selection process when comparing the robust optimization solutions to stochastic programming solutions is similar to the process described in Section 5.1.1; however, except for using samples of problem instances, we randomly select one in-

[3]We call this model *non-robust* instead of *deterministic* because robust optimization is a deterministic methodology itself.

stance for the single product model, and one instance for the multi-product model. These two instances are solved using both ellipsoidal and polyhedral uncertainty set-based robust optimization. We also solve four instances using stochastic programming (one for the single product model and three for the multi-product model). We then simulate the performance of the robust solution using the same batch of samples that is used to estimate the statistical upper bound in the stochastic solution. For the multi-product case we investigate the effects of demand correlation: we use uncorrelated, -0.95, and 0.95 correlation levels between the two products.

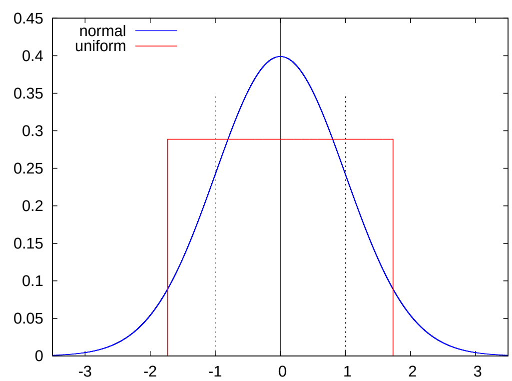


Figure 5.1: Standard deviations of normal and uniform set equal.

One of the main questions that needs to be addressed when comparing the results obtained using robust optimization and those obtained using stochastic programming is the relationship between the robust interval and a variability measure for the distribution type used in stochastic programming. One approach is to make an assumption that the half-length of the robust interval is equal to three standard deviations with the probability of 99.7% (in the case of normal distribution). An alternative approach, used in our study, is to assume that the robust interval corresponds to a symmetric uniform distribution with its half-support equal to the robust interval[4]. Then, provided that such assumption is justified, we set equal the standard deviation for the

[4]Here we are including an additional assumption about the distribution of the random parameter within the robust interval, i.e., we are assuming that it follows a uniform distribution, whereas the original definition of the random parameter in robust optimization includes no such assumption.

uniform distribution and the standard deviation for the normal distribution, in which case the standard deviation for demands is $\sigma = \hat{d}/\sqrt{3}$ (see Figure 5.1). Using the two-stage decision framework (see Section 4.3), the stochastic program for our models can be formulated as follows:

$$\min_{\mathbf{y},\mathbf{v},\mathbf{z}} \quad \mathbf{f}^\top \mathbf{y} + \mathbf{e}^\top \mathbf{v} + \mathbf{g}^\top \mathbf{z} + \mathbb{E}_{\mathbf{d}}\big[Q\big(\mathbf{z}, \mathbf{d}(\omega)\big)\big]. \tag{5.1}$$

We use the *Sample Average Approximation* method in our stochastic programming implementation (notation in Table 5.3). Our implementation is based on works by Santoso et al. (2005) [47], Linderoth et al. (2006) [36], and Shapiro et al. (2009) [49]. In SAA, the continuous expectation function in (5.1) is approximated using Monte Carlo sampling:

$$\min_{\mathbf{y},\mathbf{v},\mathbf{z}} \quad \mathbf{f}^\top \mathbf{y} + \mathbf{e}^\top \mathbf{v} + \mathbf{g}^\top \mathbf{z} + \frac{1}{N}\sum_{n=1}^{N} Q(\mathbf{z}, \mathbf{d}^n). \tag{5.2}$$

N	scenarios in the sampled problem ($n = 1, \ldots, N$)
M	number of replications of the SAA problem ($m = 1, \ldots, M$)
N'	sample size to estimate the objective function value ($n' = 1, \ldots, N'$)
u	decision variables of the first stage
$\phi(u)$	objective function of the two-stage stochastic problem
ϑ	optimal value of the true problem
$\hat{\phi}_N(u)$	objective function of the two-stage SAA problem
$\hat{\vartheta}_N^m$	optimal objective value of the SAA problem
$\hat{u}_N^m$	decision variables in the optimal solution of the SAA problem
$\bar{\vartheta}_{N,M}$	statistical lower bound for ϑ
$\hat{\sigma}^2_{N,M}$	estimate of the variance of $\bar{\vartheta}_{N,M}$
$L_{N,M}$	$(1-\alpha)$ confidence lower bound
u^*	feasible solution of the true problem
$\hat{\phi}_{N'}(u^*)$	statistical upper bound for ϑ
$\hat{\sigma}^2_{N'}(u^*)$	estimate of the variance of $\hat{\phi}_{N'}(u^*)$
$U_{N'}(u^*)$	$(1-\alpha)$ confidence upper bound

$\text{gap}(u^*)$	estimate of the optimality gap
$\sigma^2_{\text{gap}(u^*)}$	estimate of the variance of $\text{gap}(u^*)$

Table 5.3: Notation used for SAA.

We use the following sample sizes: $M = 20$ batches of $N = 50$ and $N' = 1{,}000$, which is consistent with the stochastic programming literature (e.g., Santoso et al. (2005) [47]). The Sample Average Approximation method is summarized as follows, using notation provided in Table 5.3.

SAA Algorithm

1. For $m = 1, \ldots, M$ repeat the following steps.

   (a) Generate an i.i.d. random sample $\mathbf{d}^1, \ldots, \mathbf{d}^N$.

   (b) For $\mathbf{d}^1, \ldots, \mathbf{d}^N$, solve the SAA problem, and let $\hat{\vartheta}_N^m$ and $\hat{u}_N^m$ be the optimal objective value and the optimal solution, respectively.

   (c) Generate an i.i.d. random sample $\mathbf{d}^1, \ldots, \mathbf{d}^{N'}$, independent from sample $\mathbf{d}^1, \ldots, \mathbf{d}^N$ generated in Step 1a.

   (d) Select a feasible solution u^* to the true problem (5.1), i.e., the optimal solution of the SAA problem $\hat{u}_N^m$, and estimate the true objective function value $\phi(u^*)$ (a statistical upper bound), the variance of this estimate, and a $(1-\alpha)$ confidence upper bound as follows:

$$\hat{\phi}_{N'}(u^*) := \mathbf{f}^\top \mathbf{y}^* + \mathbf{e}^\top \mathbf{v}^* + \mathbf{g}^\top \mathbf{z}^* + \frac{1}{N'} \sum_{n'=1}^{N'} Q\big(\mathbf{z}^*, \mathbf{d}^{n'}\big), \tag{5.3}$$

$$\hat{\sigma}^2_{N'}(u^*) := \frac{1}{N'(N'-1)} \sum_{n'=1}^{N'} \Big(\mathbf{f}^\top \mathbf{y}^* + \mathbf{e}^\top \mathbf{v}^* + \mathbf{g}^\top \mathbf{z}^* + Q\big(\mathbf{z}^*, \mathbf{d}^{n'}\big) - \hat{\phi}_{N'}(u^*)\Big)^2, \tag{5.4}$$

$$U_{N'}(u^*) := \hat{\phi}_{N'}(u^*) + z_{(\alpha)} \hat{\sigma}_{N'}(u^*). \tag{5.5}$$

2. Estimate a statistical lower bound to ϑ, its variance, and a $(1-\alpha)$ confidence lower

bound as follows:

$$\bar{\vartheta}_{N,M} := \frac{1}{M}\sum_{m=1}^{M}\hat{\vartheta}_N^m, \tag{5.6}$$

$$\hat{\sigma}^2_{N,M} := \frac{1}{M(M-1)}\sum_{m=1}^{M}\left(\hat{\vartheta}_N^m - \bar{\vartheta}_{N,M}\right)^2, \tag{5.7}$$

$$L_{N,M} := \bar{\vartheta}_{N,M} - t_{(\alpha,M-1)}\hat{\sigma}_{N,M}. \tag{5.8}$$

3. For each solution $\hat{u}_N^m$, $m = 1, \ldots, M$, estimate the optimality gap, and its variance:

$$\text{gap}(u^*) := \hat{\phi}_{N'}(u^*) - \bar{\vartheta}_{N,M} \tag{5.9}$$

$$\sigma^2_{\text{gap}(u^*)} := \hat{\sigma}^2_{N'}(u^*) + \hat{\sigma}^2_{N,M} \tag{5.10}$$

The most difficult part of SAA is the Step 1b. For smaller problem instances the SAA problem can be solved directly by CPLEX; however, larger problems require application of a decomposition approach.

5.2 Results and Analysis

The results in this section are presented in the order that corresponds to the order of the research questions (RQ1, RQ2, and RQ3) formulated in Chapter 1. In the subsequent subsections, related to the research questions, the findings are presented first for the single product model and then for the multi-product model. In general, while the primary purpose of the single product model is to examine the effects of economies of scale, the purpose of the multi-product model is to examine the joint effects of scale and scope. Therefore, the single product model can be viewed as a "building block" for a more general multi-product model.

5.2.1 Findings Related to Research Question 1

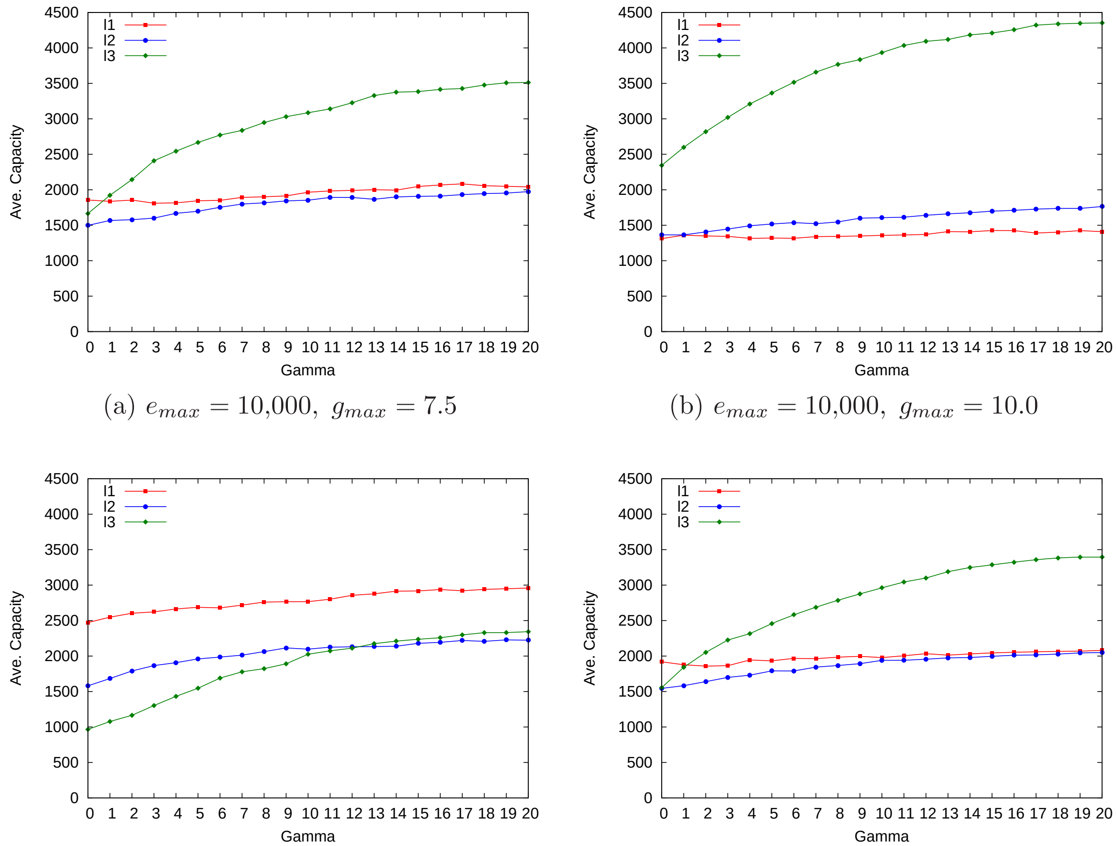
In numerical studies related to RQ 1 we examine how different levels of robustness, facility, and technology costs affect the quantities, types and allocation of technologies to facilities.

Single Product Model

The single product experiments consist of eight instances that correspond to the number of combinations formed by two f_{max} values (i.e., "low" and "high"), two e_{max} values, and two g_{max} values. Detailed results for these eight problem instances are presented in Appendix Tables A1–A8[5]. It can be noted that the low vs. high values of these parameters were obtained by *scaling* them within the same sample, and not by generating different random values. This way we are able to look at strictly the effects of cost magnitude alone without any additional "random noise." Each of the eight cost combinations was solved for varying levels of robustness, represented by Γ. For the purposes of expositional clarity, here, as well as in the following sections, we graphically present only select figures that help facilitate the analysis and discussion.

As noted in Table 5.1, in the single product model experiments we use three technologies of different capital-intensity, $l1$, $l2$, and $l3$. Figure 5.2 shows the detailed results for the same level of fixed facility costs (f_{max} = 5,000). The top row of Figure 5.2 (5.2a and 5.2b) represents low e_{max}, while the bottom row (5.2c and 5.2d) represents high e_{max}. Likewise, the left column of Figure 5.2 (5.2a and 5.2c) represents low g_{max}, while the right column (5.2b and 5.2d) represents high g_{max}. Figure 5.2 leads to two observations with regards to the level of robustness and the relative magnitude of fixed and unit technology costs. First, as the level of robustness increases, for low-to-medium Γ values, the quantity of the most capital-intensive technology $l3$ increases at a rate that substantially exceeds the rates of increase for technologies $l1$ and $l2$.

[5] Abbreviations used in these and other tables are explained on page 95.



(a) $e_{max} = 10{,}000,\ g_{max} = 7.5$

(b) $e_{max} = 10{,}000,\ g_{max} = 10.0$

(c) $e_{max} = 15{,}000,\ g_{max} = 7.5$

(d) $e_{max} = 15{,}000,\ g_{max} = 10.0$

Figure 5.2: Average capacity levels with $f_{max} = 5{,}000$.

In other words, most of additional capacity that is acquired due to increased levels of robustness, can be attributed to the most capital-intensive technology. Second, the relative magnitude of fixed and unit technology costs has the following impact on the quantity of capacity:

- *higher* fixed costs e lead to *lower* levels of capital-intensive technology $l3$,
- *higher* fixed costs e lead to *higher* levels of labor-intensive technology $l1$,
- *higher* unit costs g lead to *higher* levels of capital-intensive technology $l3$,
- *higher* unit costs g lead to *lower* levels of labor-intensive technology $l1$,
- neither fixed nor unit costs have substantial impact on the levels of intermediate technology $l2$ for the parameter values considered.

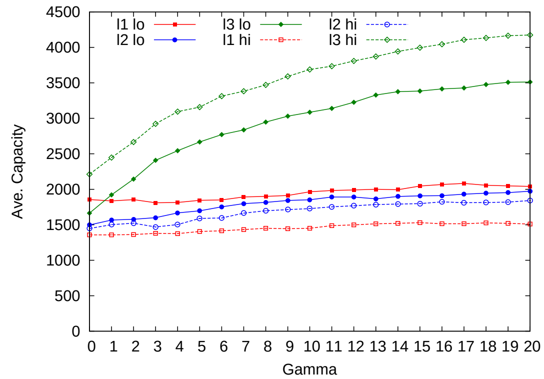
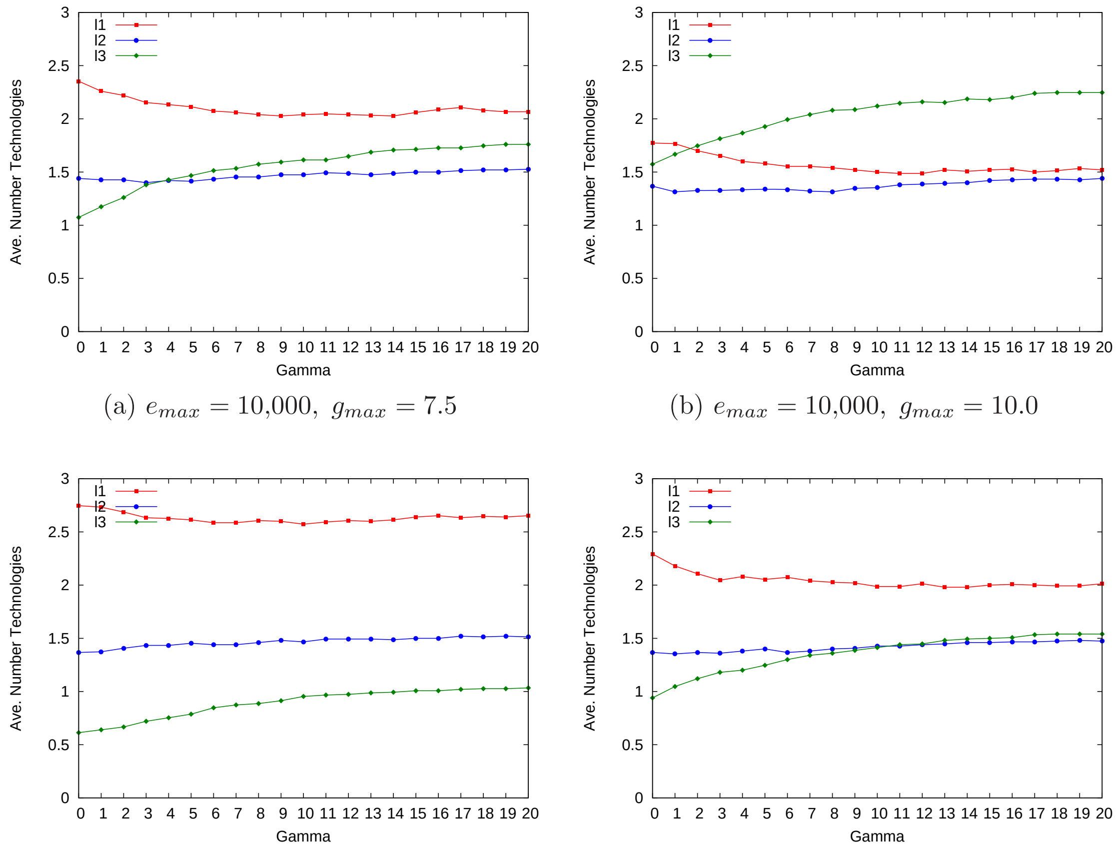


Figure 5.3: The impact on the average capacity levels of "high" (f_{max} = 20,000) vs. "low" (f_{max} = 5,000) fixed facility costs for e_{max} = 10,000 and g_{max} = 7.5.

The outcomes for high fixed facility costs f_{max} = 20,000 show very similar impact (see Figure 5.3) for all four combinations of technology costs. The average quantity of $l3$ technology is approximately 15-30% *higher* over the range of Γ for f_{max} = 20,000 vs. f_{max} = 5,000, the average quantity of $l1$ technology is approximately 35% *lower* for f_{max} = 20,000 vs. f_{max} = 5,000, and the average quantity of $l2$ remains about the same[6]. We can conclude that higher fixed facility costs lead to higher utilization of more capital intensive technologies and lower utilization of labor-intensive technologies, because for f_{max} = 20,000 there are fewer facilities on average, which in turn favors high fixed cost and low unit cost technology $l3$.

Figure 5.4 shows the average number of installations of technologies $l1$, $l2$, and $l3$ per facility with f_{max} = 5,000. Due to constraint $\sum_l v_{il} \leq 1$ (see page 32), the sum of the number of average technology installations is equal to the average number of facilities open. Just like in the case with regards to the quantities of technology types, the average number of technology installations are affected by the level of robustness and the relative magnitude of fixed and unit technology costs. The average number of high capital intensity installations $l3$ *increase* with the increase of robustness level Γ, the average number of low capital intensity installations $l1$ *decrease* with the increase

[6]Because of high similarity of the impact of fixed facility costs, we include comparative results only for the first instance (Figure 5.2a).



(a) $e_{max} = 10{,}000,\ g_{max} = 7.5$

(b) $e_{max} = 10{,}000,\ g_{max} = 10.0$

(c) $e_{max} = 15{,}000,\ g_{max} = 7.5$

(d) $e_{max} = 15{,}000,\ g_{max} = 10.0$

Figure 5.4: Average number of technology installations with with $f_{max} = 5{,}000$.

of Γ, while the average number of medium capital intensity installations $l2$ remain the same. The impact of the first stage costs (the fixed facility costs, and fixed and unit technology costs) on the average number of technology installations are summarized as follows:

- *higher* fixed costs e lead to *fewer* $l3$ installations on average,
- *higher* fixed costs e lead to *more* $l1$ installations on average,
- *higher* unit costs g lead to *more* $l3$ installations on average,
- *higher* unit costs g lead to *fewer* $l1$ installations on average,
- neither fixed nor unit costs have substantial impact on the number of $l2$ installations on average for the parameter values considered.

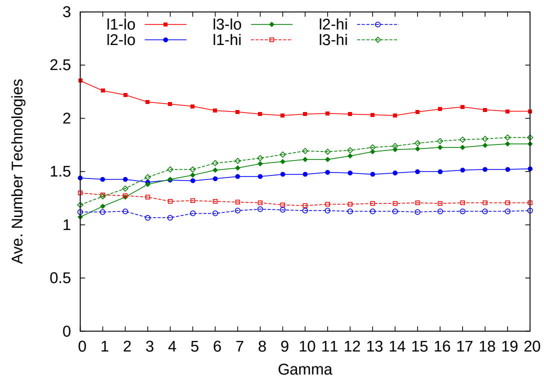


Figure 5.5: The impact on the average number of technology installations of "high" (f_{max} = 20,000) vs. "low" (f_{max} = 5,000) fixed facility costs for e_{max} = 10,000 and g_{max} = 7.5.

Figure 5.5 shows the impact of high vs. low fixed facility costs on the average number of technology installations[7]. With the increase in fixed facility costs the number of $l1$ and $l2$ installation decreases substantially (approximately 40% for $l1$ and 25% for $l2$), while the number of $l3$ installations show only a slight increase. Especially sensitive to the fixed facility costs is the average number of established capacities of type $l1$, as the low-fixed-cost benefits of $l1$ diminish with the increases in fixed facility costs. The increase in fixed facility costs lead to fewer, but larger capacity installations represented by technology $l3$.

The average capacity sizes per technology installations, as expected, are larger for $l3$, and smaller for $l1$. The impact of different e_{max} and g_{max} values on the installation sizes appear to be insignificant. However, the average sizes depend on the fixed facility costs f_{max} and the different levels of robustness.

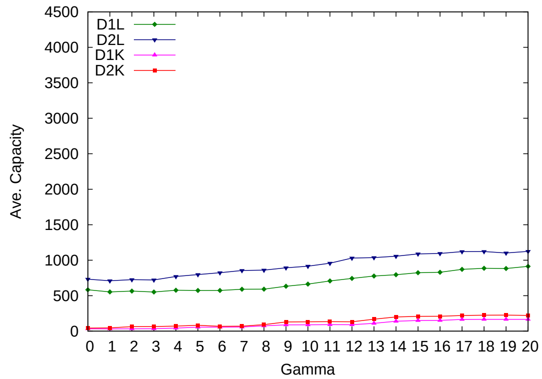
Multi-product Model

The multi-product experiments consist of six instances that represent the different levels of flexible to dedicated cost ratio. Detailed results for these six problem instances are presented in Appendix Tables A9–A20. The insights obtained from these test instances are related not only to the relative amounts of capacity established,

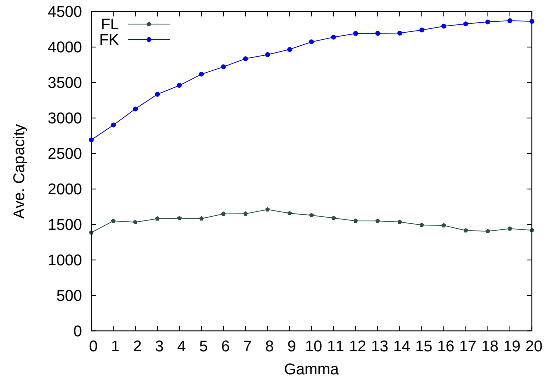
[7]The effects of high fixed facility costs are shown for Figure 5.4a

represented by use of dedicated vs. flexible technologies under different technology costs between the two, but also to the interdependence between the labor-intensive and capital-intensive technologies on the one hand, and the dedicated and flexible technologies on the other hand. The levels of robustness, just as in the single product case, is varied from 0 to 20. The multi-product version of the problem is solved for two technologies, *labor-intensive* and *capital-intensive*, hereafter denoted L and K, respectively[8]. The combination of L and K technologies with D (dedicated) and F (flexible) produce the following technology types in a two-product setting: D_1L, D_2L, D_1K, D_2K, FL, and FK. Symbol D without an index means the combined amount of dedicated capacities of the two products, i.e., $D = D_1 + D_2$. It is important to note that in the multi-product version of the model we are not varying the fixed facility costs, and the *dedicated* fixed and variable unit capacity investment costs – they are $f_{max} = 5{,}000$, $e^D_{max} = 15{,}000$, and $g^D_{max} = 10.0$, respectively. What is changing, however, is the ratio of flexible capacity investment costs to the dedicated capacity investment costs, denoted Δ. The instances with $\Delta = 1$ and $\Delta = 2$ represent extreme cases, when the flexible capacity either costs the same as each dedicated capacity (assuming both dedicated capacities have the same costs), or the flexible capacity costs the same as the sum of costs of two dedicated capacities. These two extreme cases are included for illustration purposes only to show that when $\Delta = 1$, a firm would never invest in dedicated capacities, and when $\Delta = 2$, a firm would never invest in flexible capacity. Thus, in the interest of analyzing more interesting instances, we will focus four alternatives with $\Delta = 1.25$, $\Delta = 1.5$, $\Delta = 1.75$, and $\Delta = 1.9$. Figure 5.6 reflects the dynamics between D, F, L and K with changing Δ. The left column of Figure 5.6 shows the changes in dedicated technology levels with increasing Δ, whereas the right column shows the changes in flexible technology levels. In the case of *dedicated* technologies D, the labor-intensive technology L dominates, in

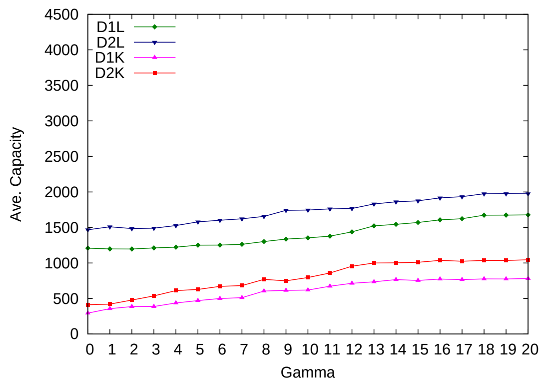
[8]For notational simplicity we are using L and K, instead of $l1$, $l2$, etc., as we are considering only two levels of capital-intensity in the multi-product case.



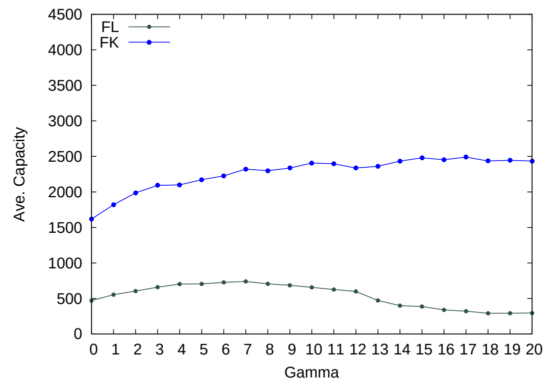
(a) D capacities, $\Delta = 1.25$



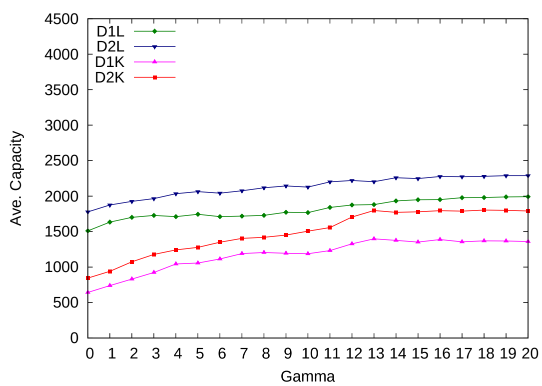
(b) F capacities, $\Delta = 1.25$



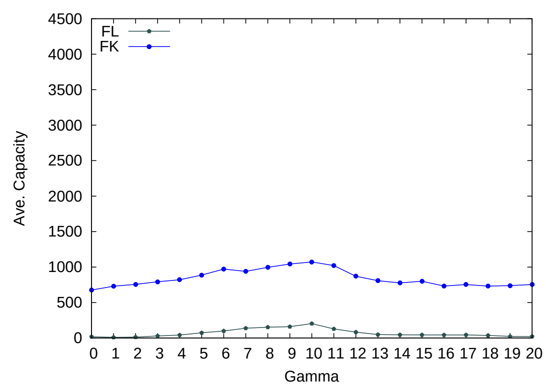
(c) D capacities, $\Delta = 1.5$



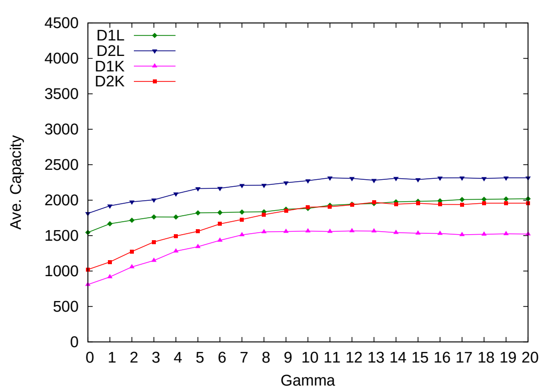
(d) F capacities, $\Delta = 1.5$



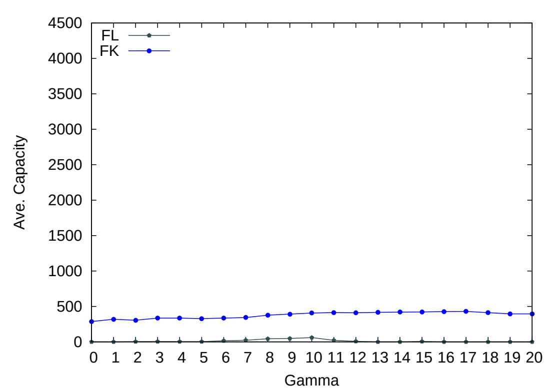
(e) D capacities, $\Delta = 1.75$



(f) F capacities, $\Delta = 1.75$



(g) D capacities, $\Delta = 1.9$



(h) F capacities, $\Delta = 1.9$

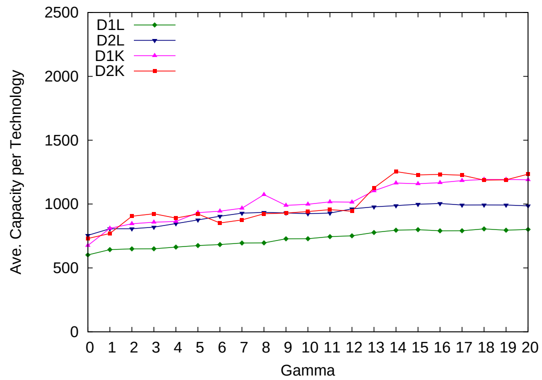
Figure 5.6: Average capacity levels for various flexible to dedicated technology cost ratio Δ values.

terms of quantities, over capital-intensive technology K, i.e., DL dominates over DK. The opposite situation can be observed for *flexible* technologies, that is, the capital-intensive flexible technology FK has higher capacity levels than the labor-intensive flexible technology FL. With the increase of the flexible capacity costs relative to dedicated capacity costs, the diminishing amounts of flexible capacity are replaced by the *capital-intensive* dedicated technologies DK, while the levels of labor-intensive dedicated technologies DL show only smaller increases. Another observation pertains to the impact of the level of robustness on the relative amounts of technology types. It appears that higher levels of Γ "favor" more capital-intensive technologies: for low Δ values (1.25), FK clearly dominates over other types; for high Δ values (1.9 and 2), DK almost doubles the established capacity. Considering labor-intensive flexible capacity FL, the results show that its use becomes insignificant for higher Δ values (1.75 and 1.9); however, for lower Δ values (1.25 and 1.5) FL exhibits significant presence. Moreover, it appears that FL has a tendency to decrease with increasing robustness levels.

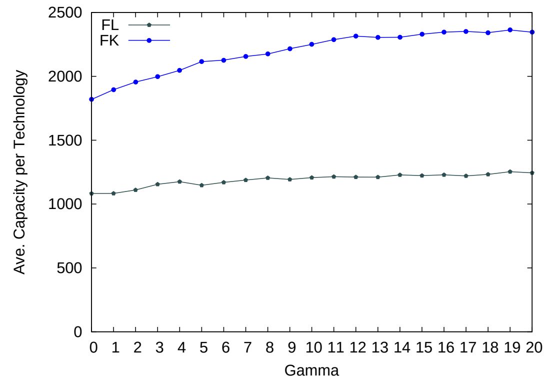
Figure 5.7 shows the average sizes per technology installation for different Δ values (1.25, 1.5, and 1.75). As expected, capital-intensive technologies have larger sizes per installation, both for dedicated and flexible technologies. This result for the multi-product case is consistent with the result obtained in the single product case, and it confirms the effects of economies of scale in a multi-product setting.

In the multi-product case there is no restriction placed on how many dedicated or flexible technologies can be established at a single facility. Therefore, one can consider a question about the average number of technology installations per facility, taking into account varying robustness levels. Our findings indicate that the number of technology installations per facility depends on the flexible to dedicated cost ratio Δ alone, and not on the level of robustness.

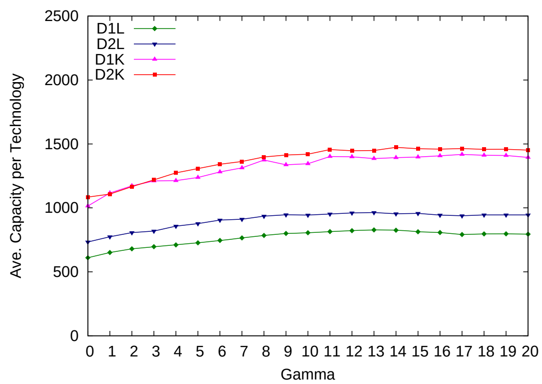
In this section we focused on four outcomes that emerge from different combi-



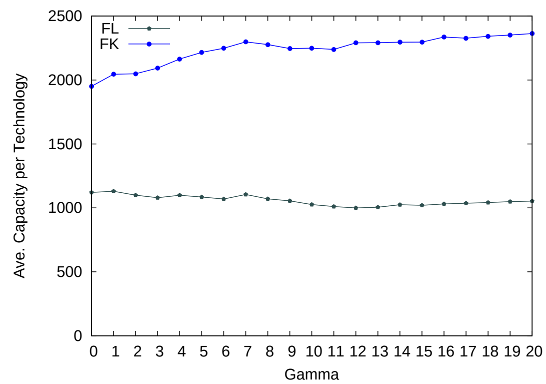
(a) D capacities, $\Delta = 1.25$



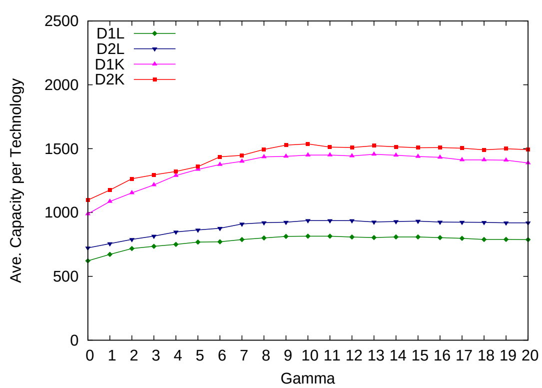
(b) F capacities, $\Delta = 1.25$



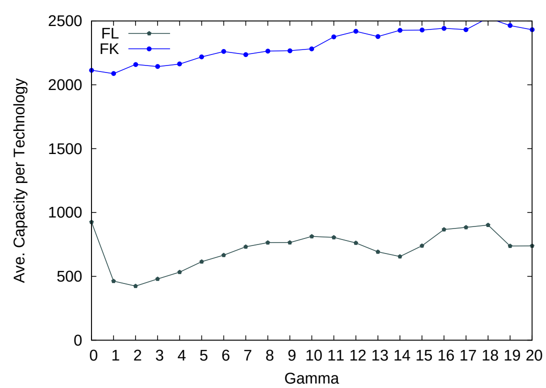
(c) D capacities, $\Delta = 1.5$



(d) F capacities, $\Delta = 1.5$



(e) D capacities, $\Delta = 1.75$



(f) F capacities, $\Delta = 1.75$

Figure 5.7: Average capacity levels per technology installation for various flexible to dedicated technology cost ratio Δ values.

nations of first stage costs and varying levels of robustness: the number of facilities open, the relative quantities of different technologies installed, the average sizes of technology installations, and the number of technologies established per facility. Our findings related to the first research question show that both the single product and multi-product model solutions exhibit behavior that is consistent with the effects of economies of scale and scope.

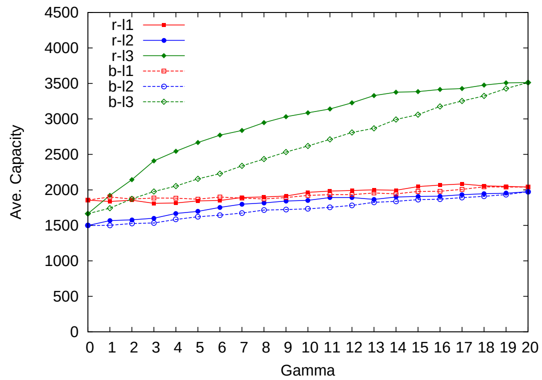
5.2.2 Findings Related to Research Question 2

In this section, we look at how robust optimization solutions differ from *non-robust solutions*, with respect to the quantities, types and allocation of technologies to facilities for varying levels of robustness. A method for developing a non-robust (or a "box") model was described in Section 5.1.2. As previously, we will consider the single product case first, followed by a more general multi-product version of the problem. The issue of comparing a robust solution to a non-robust solution is an important one as it allows to separate the "effects of robustness" from the effects of scale and scope. We will address the question of why non-robust solutions are inferior to robust ones in Section 5.2.3. These robustness effects manifest themselves in manner that can be considered similar to the "risk pooling" behavior.

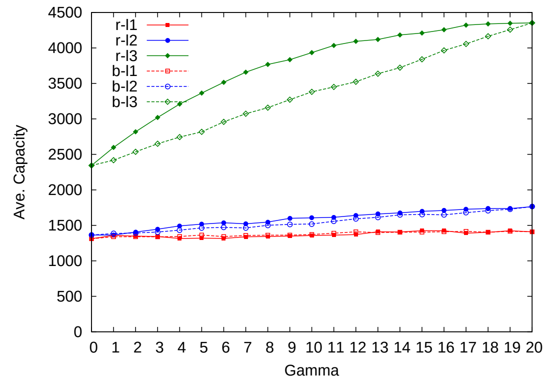
Single Product Model

In the single product case the comparison between robust and non-robust solutions is done for the three technologies of different capital intensity, $l1$, $l2$, and $l3$. The robust problem and solution is described in detail in Section 5.2.1. Figure 5.8 illustrates the differences between the robust and non-robust ("box") solutions, considering different first stage costs[9]. The solutions for both models are the same when $\Gamma = 0$ and when $\Gamma = \text{card}(J)$ by construction, representing the nominal and worst-case (as defined in robust optimization) instances, respectively. Figure 5.8 considers a case when f_{max} = 5,000; similar results are obtained with f_{max} = 20,000. Accord-

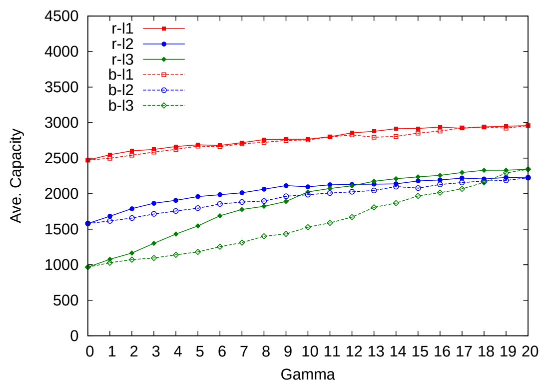
[9] The solid lines and letter r denote robust solutions, while the dashed lines and letter b denote non-robust ("box") solutions.



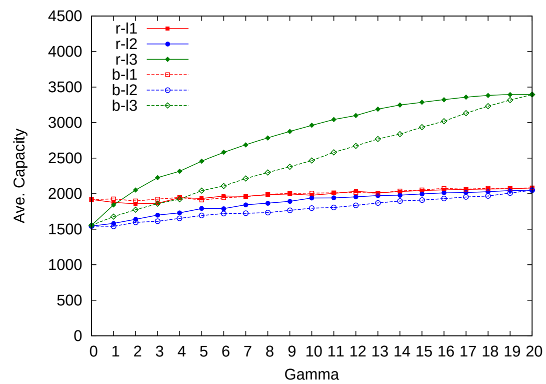
(a) $e_{max} = 10{,}000,\ g_{max} = 7.5$



(b) $e_{max} = 10{,}000,\ g_{max} = 10.0$



(c) $e_{max} = 15{,}000,\ g_{max} = 7.5$



(d) $e_{max} = 15{,}000,\ g_{max} = 10.0$

Figure 5.8: Average capacity levels: robust vs. non-robust ("box") solutions.

ing to the solved instances, for all cost combinations there is a substantial difference between capacity levels for high capital-intensity technology $l3$ according to the robust vs. non-robust solutions. The capacities for low capital-intensity technology $l1$ are essentially the same for both robust and non-robust solutions. The capacities for medium capital-intensity technology $l2$ are just slightly higher for the robust solutions. In terms of the number of total facilities and the number of facilities with certain technologies[10], the robust vs. non-robust solution differs as follows: while the total number of facilities is the same for both robust and non-robust solutions, the number of $l3$ technologies is higher and the number of $l1$ technologies is lower for the

[10]There is only one technology per facility in the single product case (see constraint 3.6 on page 32).

robust solutions compared to non-robust ones.

These results suggest that the robust solution prescribes larger facilities, represented by more capital-intensive technology $l3$, in which case the production-distribution network can better respond to *joint* demand deviations. The non-robust model is optimized only for *individual* demand deviations. Thus, in the single product case, the robust solution can be considered more "volume flexible"[11], which means greater cost-effectiveness in responding to spatially distributed uncertain demand realizations.

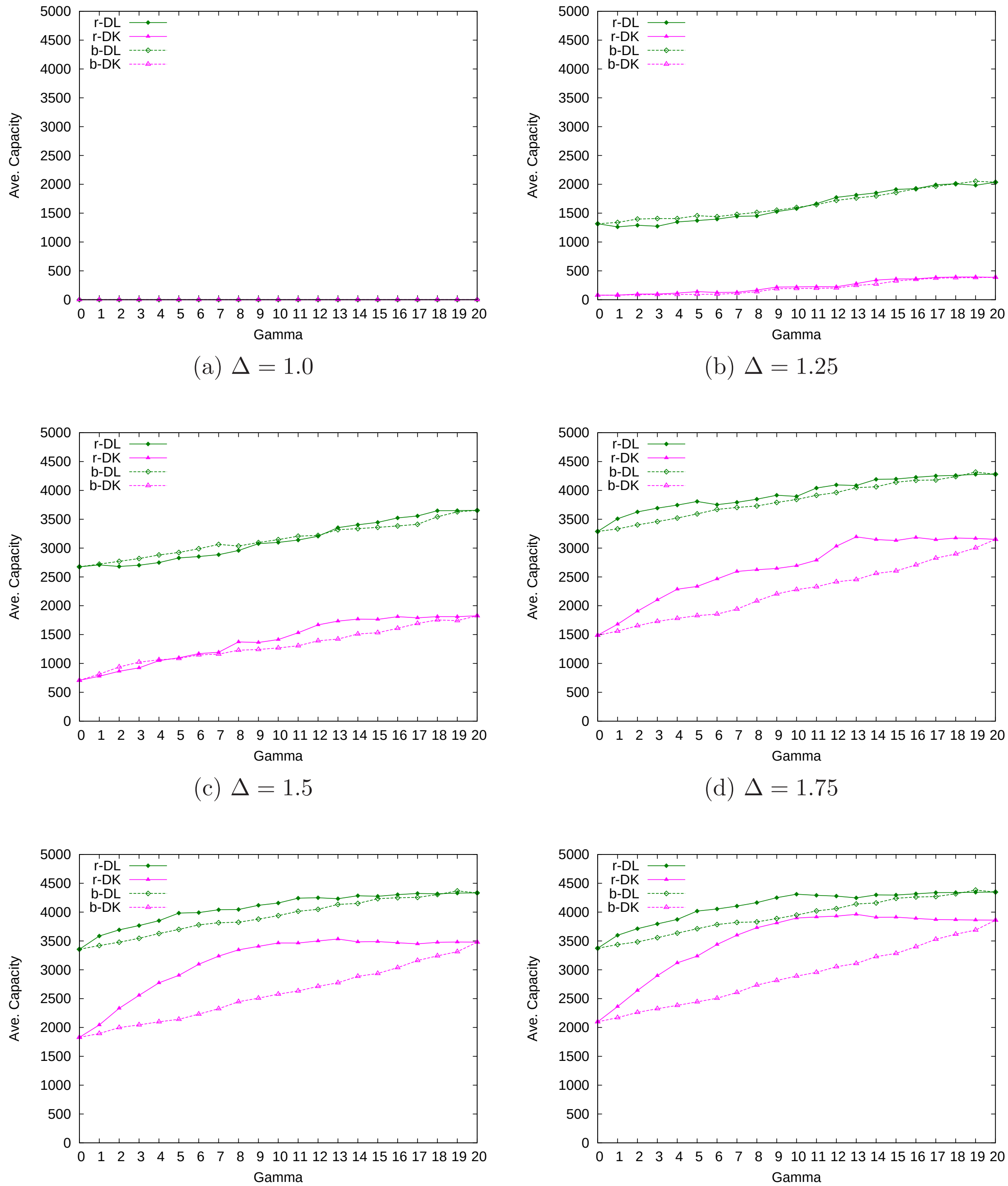
Multi-product Model

In the multi-product case the comparison between robust and non-robust solutions is done for the dedicated as well as flexible capacities. In this section we consider the total amount of dedicated capacity for each of the two levels of capital-intensity. Figure 5.9 shows the robust solutions compared to non-robust solutions as the flexible capacity becomes more expensive compared to dedicated capacity[12], expressed in the value of Δ. Figure 5.9a indicates that in the case of $\Delta = 1$ dedicated capacity is not used; the inclusion of this case is for illustration purposes only to show that when $\Delta = 1$, a firm would never invest in dedicated capacities. According to Figure 5.9 the quantities of both the labor-intensive technology DL and the capital-intensive technology DK increase with the increase in Δ. However, while the robust and non-robust solutions do not differ much for the labor-intensive technology, the robust solutions for the capital-intensive technology show notable differences. This observation supports the claim that when the levels of robustness increase, and the flexible capacity becomes more expensive, a firm would rely more on dedicated capital-intensive technology DK.

Figure 5.10 reflects the changes in the quantities of flexible technologies FL and

[11] Not to be confused with the flexibility concept as used in this work and applied to multi-product technology flexibility.

[12] The solid lines and letter r denote robust solutions, while the dashed lines and letter b denote non-robust ("box") solutions.



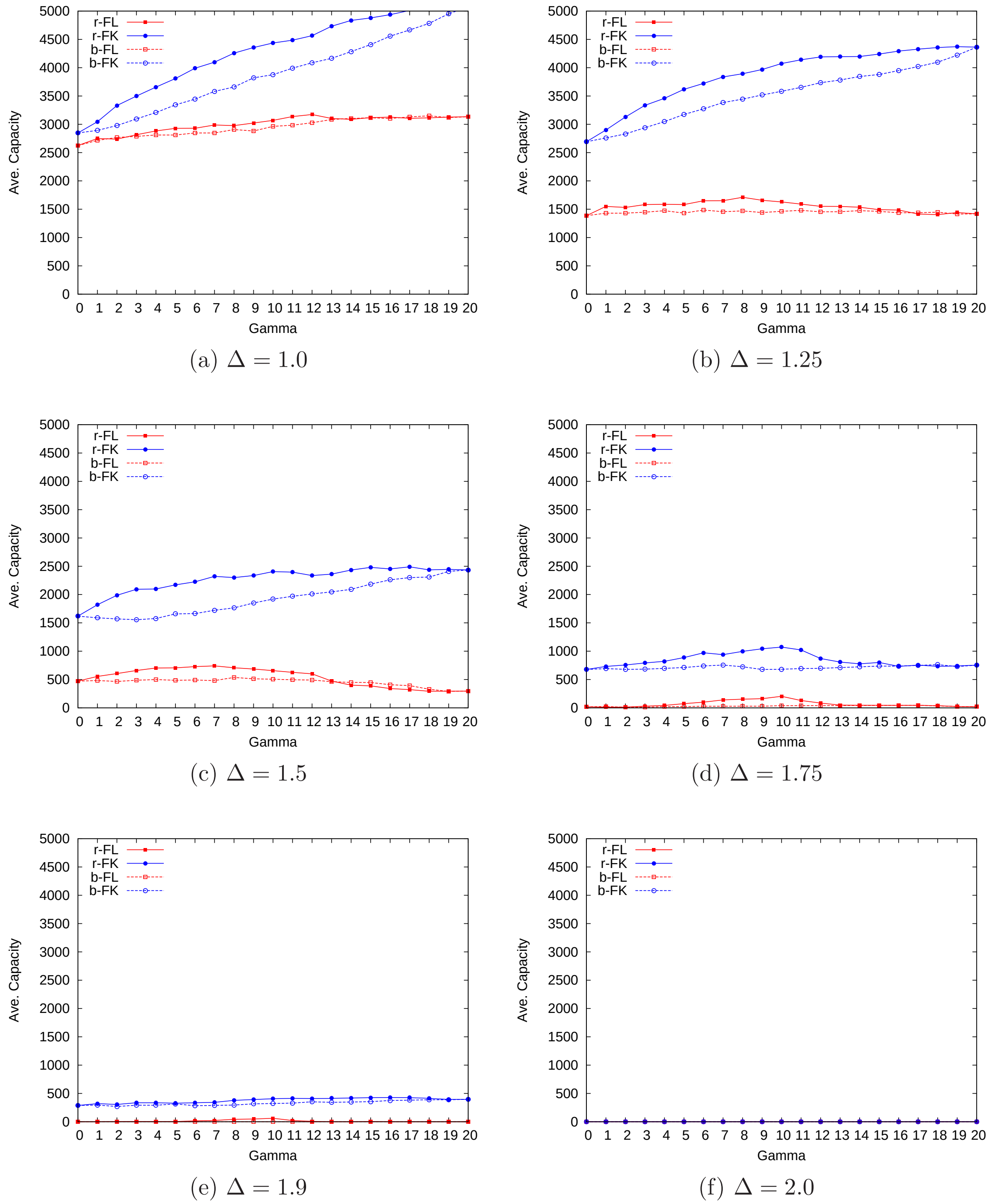
(a) $\Delta = 1.0$ (b) $\Delta = 1.25$

(c) $\Delta = 1.5$ (d) $\Delta = 1.75$

(e) $\Delta = 1.9$ (f) $\Delta = 2.0$

Figure 5.9: Comparison of robust to non-robust ("box") solutions for DL and DK technologies for different Δ values.

FD as the Δ increases, i.e., the flexible technology becomes more expensive relative to dedicated technology. Figure 5.10f represents an instance when $\Delta = 2$, in which case a firm is not investing in flexible capacity. Just like for the dedicated capacity, this case



(a) $\Delta = 1.0$ (b) $\Delta = 1.25$

(c) $\Delta = 1.5$ (d) $\Delta = 1.75$

(e) $\Delta = 1.9$ (f) $\Delta = 2.0$

Figure 5.10: Comparison of robust to non-robust ("box") solutions for FL and FK technologies for different Δ values.

is included for illustration purposes. As flexible capacity becomes more expensive, its overall amount is decreasing. However, there exist notable differences between the robust and non-robust solutions. For Δ values 1.0, 1.25, and 1.5 the robust solutions indicate higher levels for the capital-intensive technology FK than the non-robust solutions. For the flexible labor-intensive technology FL, the respective amounts used between robust and non-robust solutions are similar, although robust solutions indicate slightly higher levels of FL for $\Delta = 1.0$ to $\Delta = 1.75$. The behavior of the flexible technologies indicate a similar pattern to dedicated technologies, namely, the differences between robust and non-robust solutions can be attributed mainly to the capital-intensive technology FK. Our experiments comparing robust vs. non-robust solutions also revealed that in cases when Δ is not too high (i.e., 1.0, 1.25, and 1.5) the robust solutions prescribe more flexible capacity compared to non-robust solutions. That is, the combined quantity $FL + FK$ is higher for the robust than non-robust solutions. The overall pattern of behavior of the four technologies (DL, DK, FL, and FK) under increasing level of robustness indicate that while the flexible capacity has relatively low cost, a firm invests in capital-intensive flexible technology to better cope with demand uncertainty. With the increased cost of flexible technology, a firm shifts its increased amounts of capacity due to increased robustness to capital-intensive dedicated technologies.

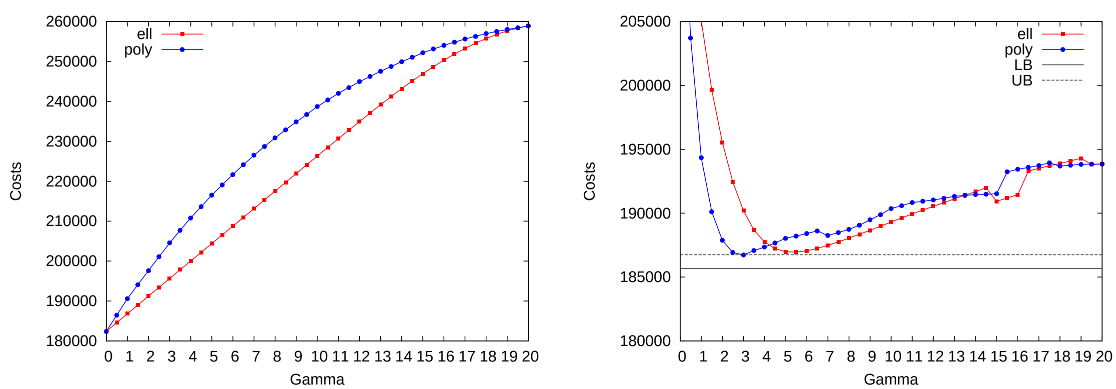
Our numerical studies related to the second research question indicated that there exist differences between robust and non-robust solutions with regards to how increasing levels of robustness affect the optimal quantities of different capacity types. Robust solutions show a more rapid rate of increase in capital intensive as well as flexible technologies, compared to the non-robust solutions. These increases can not be attributed just to the effects of economies of scale and scope.

5.2.3 Findings Related to Research Question 3

This section presents results related to the comparative performance of solution obtained by robust optimization compared to those obtained by stochastic programming. Detailed results are presented in Appendix Tables A21–A28. As previously, we first review results related to the single product model, and then turn to analysis of the multi-product model. The stochastic programming results are obtained using the Sample Average Approximation method as described in Section 5.1.3. The results for both the single product and multi-product model are compared in two aspects: first, in terms of costs, and second, in terms of capacities of different technologies.

Single Product Model

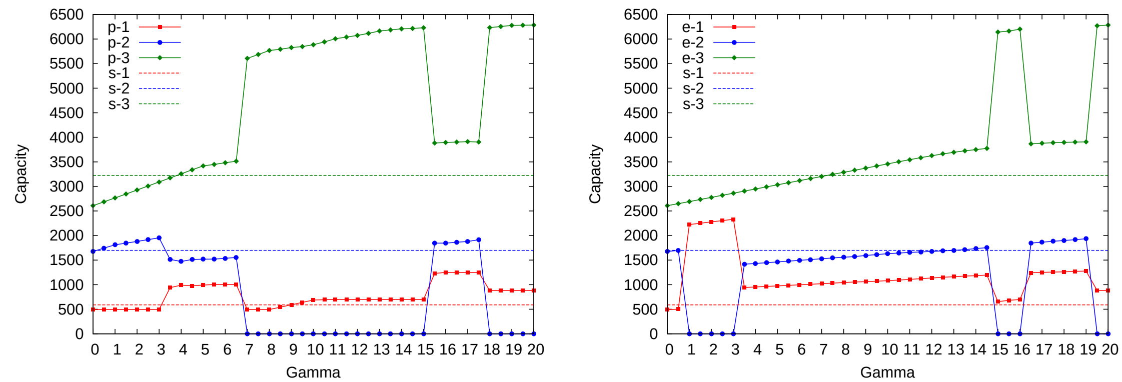
Figure 5.11 illustrates the comparison of results obtained by robust optimization to those obtained by stochastic programming. In Figure 5.11a the total robust solutions for the ellipsoidal and polyhedral uncertainty sets are presented. In Figure 5.11b



(a) Robust solution. (b) Stochastic solution.

Figure 5.11: Single product model costs.

the performance of the first stage robust solution is compared to the stochastic solution. It shows that by appropriately selecting the robustness parameters Γ or Ω, the performance in terms of costs of the first stage solution obtained via robust optimization is of comparable quality to the stochastic first stage solution. Specifically, the best performance in the case of polyhedral uncertainty set is obtained with $\Gamma = 3.0$ (with value 186,730.9), and in the case of ellipsoidal uncertainty set with $\Gamma = 5.5$, which is equivalent to $\Omega = 1.23$ (with value 186,953.9). In both instances, these values are within the ranges of standard errors for the estimates of robust solutions and the stochastic upper bound ($UB = 186{,}747.4$). These results are consistent with the theoretically recommended robustness budget Γ that scales with $\sqrt{\mathrm{card}(J)}$ (e.g., Bertsimas et al., 2013 [10]), and are based, according to [10], on implications of the probability laws, specifically, the central limit theorem. Thus, as a general guideline for selecting an appropriate level of robustness, in the case when solving the model for all Γ is not practical, we would chose $\Gamma = \sqrt{\mathrm{card}(J)}$ (or in this case, $\sqrt{20} \approx 4.47$, which is equivalent to $\Omega = 1$).



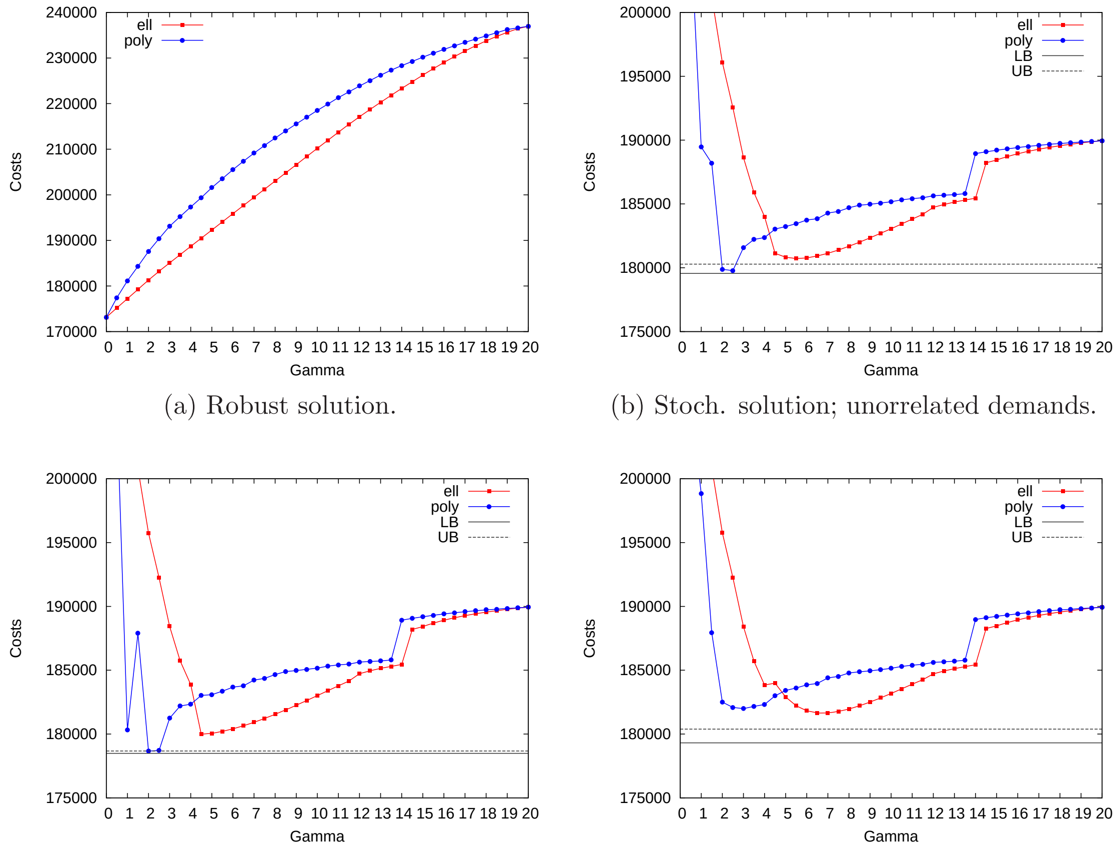
(a) Polyhedral set. (b) Ellipsoidal set.

Figure 5.12: Capacity levels for the single product solutions (robust vs. stochastic).

Taking into account the above considerations with regards to the appropriate values of the robustness parameters, we can address the issue of the best capacity configuration for the single product model with three technology alternatives $l1$, $l2$, and $l3$. According to Figure 5.12, we can observe that the robust capacity solutions for the three technologies are relatively close to the horizontal dashed lines, representing the stochastic capacity solutions, at $\Gamma = 4.47$ (Figure 5.12a) and $\Omega = 1$ (Figure 5.12b.) It is a characteristic for the objective values of robust solutions (e.g., red and blue solid lines in Figure 5.11a) to exhibit "smooth" increases along horizontal axis. However, the capacity graphs (in Figure 5.12) exhibit "erratic behavior" that represents qualitative shifts from one technology type to another. This outcome can be explained, as suggested in Chapter 4, by the differences between the *largest* and the *costliest* demand deviations in the "optimal robust" solutions. By gradually increasing Γ, the objective value reflect the gradually increasing *costs* of these deviations, thus the resulting "smoothness" of the objective value graph in Figure 5.11a. However, when we consider gradually increasing Γ, the resulting capacity size increases or transitions from one technology to another do not have to be "gradual" or "smooth." In other words, because the *costliest* demand deviations are not the same as the *largest* demand deviations, the *rates of cost increases* and the *rates of capacity increases* with the increase in Γ can be very different. Another factor that contribute to the qualitative shifts between technologies is that we do not impose lower and upper limits on the amounts that can be produced using particular technology types, which means that slight cost differences can lead to radically different outcomes with respect to technology types.

Multi-product Model

The cost results for the robust vs. stochastic multi-product model are presented in Figure 5.13, specifically, Figure 5.13a shows the robust solutions for both the ellipsoidal and polyhedral uncertainty sets, and Figures 5.13b – 5.13d show the stochastic solutions as well as simulated first stage robust solutions for different levels of correlation. In the case of the multi-product version of the model, for both polyhedral and ellipsoidal uncertainty sets, the estimate of the cost performance of the first stage solution is slightly worse at the respective lowest points (see Figure 5.13), and is outside the range of the standard errors (with the exception for the polyhedral uncertainty set with uncorrelated and negatively correlated demands). However, this difference is roughly 0.5% of the total costs, which is about the same as the gap between stochastic upper bound and lower bound. Based on the extensive simulation results we can not conclude that the performance of the first stage robust solution is substantially worse than the performance offered by stochastic solution, provided that the values of ro-



(a) Robust solution.

(b) Stoch. solution; unorrelated demands.

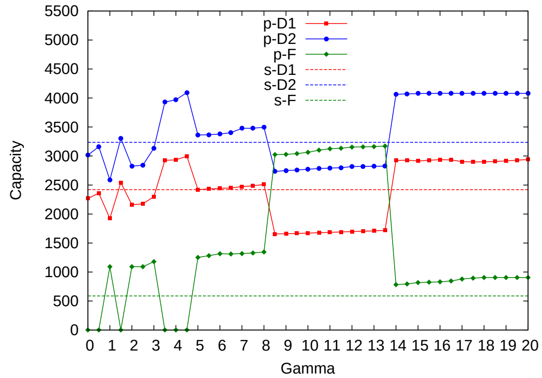
(c) Stoch. solution, −0.95 correlated demands.

(d) Stoch. solution, 0.95 correlated demands.

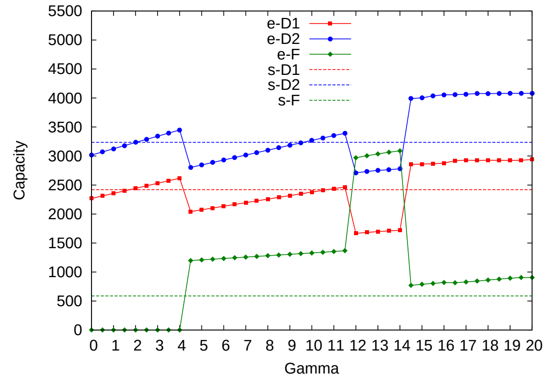
Figure 5.13: Multi-product model costs.

bustness parameters are chosen appropriately. Quite the opposite: we can argue that the robust solution performs very well taking into account the absence of demand correlation information, or any distributional information explicitly included in the robust optimization model.

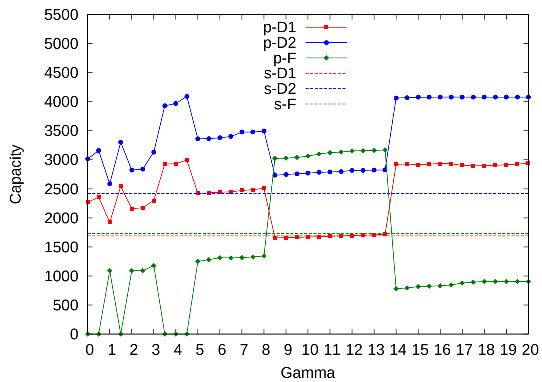
When considering the capacity levels of different technology types (Figure 5.14), we can observe that the robust solutions are relatively close to stochastic solutions in the *uncorrelated* case at $\Gamma = 4.47$ and $\Omega = 1$, which is similar to our observations for the single product model. However, for both the negatively and positively correlated cases, the proportions of flexible vs. dedicated capacities given by the robust solution are different from the proportions given by the stochastic solution. According to the stochastic solution, the levels of flexible capacity are much higher in the case of



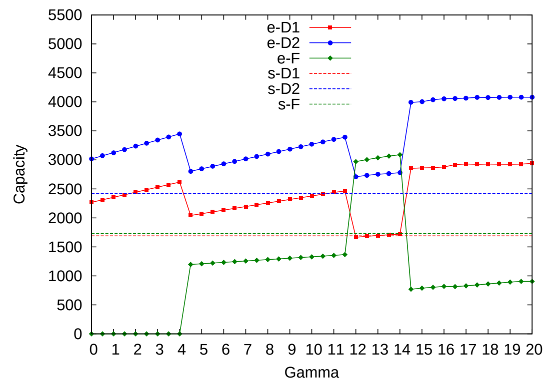
(a) Polyhedral set, uncorrelated.



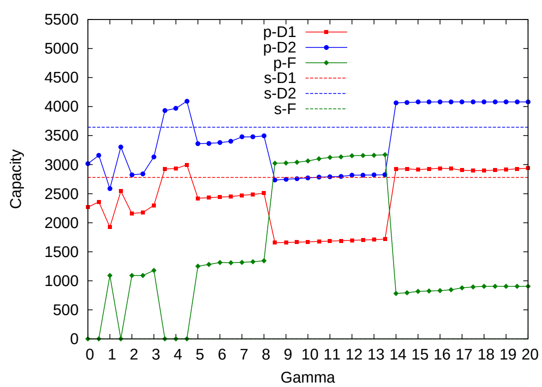
(b) Ellipsoidal set, uncorrelated.



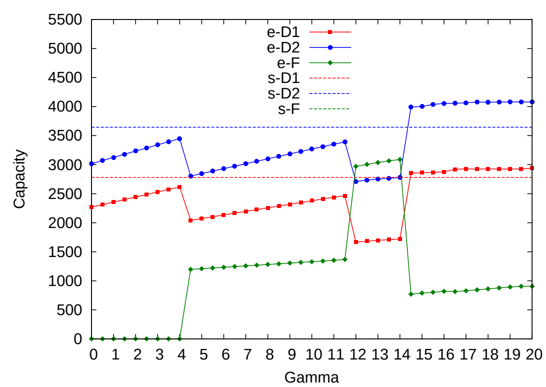
(c) Polyhedral set, −0.95 correlation.



(d) Ellipsoidal set, −0.95 correlation.



(e) Polyhedral set, 0.95 correlation.



(f) Ellipsoidal set, 0.95 correlation.

Figure 5.14: Capacity levels for the multi-product solutions (robust vs. stochastic).

negatively correlated demands, and there is no need for flexible capacity in the case of positively correlated demands – this result is consistent with results from the flexible capacity literature.

To conclude our discussion with regards to experimental results in this section, we need to be aware about the implicit assumptions we make when we compare the results obtained by using robust optimization methodology vs. the results obtained by using stochastic programming. Without making these assumptions we would not be able to "compare apples to oranges." Therefore, we need to use caution when making conclusive statements about the relative performance of these two approaches. In our presentation of results we discussed the *performance of the first stage solution*, not the performance in terms of *total costs*, which include both the first stage and the recourse costs. Of course, the robust optimization method will give much higher total costs, because it is the *worst case* approach vs. the *expected value* approach. However, from the perspective of strategic capacity acquisition perspective we are, in fact, interested primarily in these first stage solutions.

Chapter 6

Conclusions and Future Research

In this dissertation we provided theoretical background on capacity planning and resource acquisition decisions, applied the robust optimization methodology to the integrated facility location, capacity acquisition, and technology choice problem, and conducted rigorous computational studies. We have obtained insights for making strategic level capacity investment decisions and insights regarding the uncertain market environment in which these decisions are made. The importance of determining appropriate locations and technology types, and a correct assessment of changing market dynamics becomes even more pronounced for firms with global operations. We believe that the simultaneous consideration of various strategic decision level issues is a step towards a more integrative approach, as often recommended in the literature, in production-distribution network and supply chain modeling. As a result, we obtained insights for further research as well as for managerial considerations. These insights can be summarized as follows.

- Robust optimization provides solutions that are both feasible and optimal for all random demand realizations within a specific uncertainty set and level of robustness. However, due to the differences between largest vs. costliest demand realizations, the total amount of capacity established will be larger than the total aggregate demand under stochastic demand conditions. The difference between the total installed capacity and total realized demand constitute the safety capacity. In an optimization model context, this safety capacity is represented as slacks in supply constraints. The robust solution not only determines the appropriate amount of this safety capacity, but it also places these slacks in those supply constraints that will provide maximum robustness at minimum cost. In this sense robust optimization can be interpreted as "optimization of slacks."

- For varying levels of robustness, we showed that the fixed facility costs, the fixed and unit technology costs, and the flexible-to-dedicated cost ratio affect the number of facilities open, the relative amounts of different technologies installed, the average sizes of technology installations, and the number of technologies established per facility. At the same time our findings indicate that the choice of a particular technology type is highly volatile with respect to increasing levels of robustness, even though the total capacity amounts show more gradual increases. We provided an explanation regarding these apparent shifts, and we attributed this phenomenon to the uncertainty set based robust optimization approach that leads to such outcome, which is absent in both deterministic optimization and stochastic programming.
- Our numerical studies in both single product and multi-product settings showed a consistent pattern of behavior of robust solutions with regards to capital-intensive and flexible technologies. We showed that this behavior, i.e., a risk-pooling behavior, can not be attributed just to the effects of economies of scale and the effects of economies of scope alone by comparing robust solutions to non-robust solutions. In the single-product setting these "robustness effects" mean that with the increase of robustness levels, the additional safety capacity will be allocated to the most capital-intensive technology. In the multi-product

setting, the "robustness effects" depend on the relative cost of flexible vs. dedicated technologies. When the flexible technology is not too expensive relative to the dedicated technology costs, the safety capacity will be allocated to a more capital-intensive flexible technology. When the flexible technology costs become expensive relative to the dedicated technology costs, the safety capacity will be allocated to capital-intensive dedicated technologies. We also analyzed the behavior of labor-intensive technologies. Labor-intensive dedicated technologies indicate higher amounts of capacity than capital-intensive dedicated technologies; however, the opposite can be observed for flexible technologies: the amounts of labor-intensive flexible technologies used are always lower than the amounts of capital-intensive flexible technologies for our setting.

- We conducted experimental studies with the purpose of comparing the results obtained using robust optimization, using both the ellipsoidal and polyhedral uncertainty sets, to the results obtained using stochastic programming. An insight from the comparison is that our numerical studies confirmed the theoretical guidelines related to the "best" choice of the robustness parameter within the context of traditional stochastic programming and the probabilistic paradigm. Also, we provided an interpretation of the concept of value-of-information and determined that there exists a level of robustness under which the robust solutions demonstrate comparable performance to stochastic solutions. In addition to comparing the robust optimization solutions to the stochastic programming solutions we compared the results of robust optimization based on different uncertainty sets, i.e., the ellipsoidal uncertainty set and the polyhedral uncertainty set. As expected, the polyhedral version of the robust problem gives more conservative results. This comparison contributed not only to understanding of the magnitude of differences between the two robust solutions, but also to understanding of computational challenges associated with solving a non-convex bilinear ellipsoidal uncertainty set based robust problem.

Our facility location, capacity acquisition, and technology choice model has certain practical implications. First, a number of conditions have to be present for this model to be applicable to practical industry settings. These conditions stipulate that there exist spatially distributed random demands and that the transportation costs are not negligible. Furthermore, there exist production capacities of different technology types that lead to economies of scale and economies of scope. These are relatively restrictive conditions; therefore, it is more likely that this model is applicable to firms with global operations that face spatially distributed markets, substantial production and transportation costs, and a range of technologies and product families. One example of a potential industrial application would be agricultural supply chains, where transportation of raw agricultural output is cost-prohibitive, and therefore the processing facilities need to be located near the areas of varying density and varying scope of agricultural output.

There are some limitations to our work. One of the limitations is related to the type of study we conduct. Any research findings that are based on computational experiments can only be generalized using a great deal of caution. These findings can either offer insights by confirming theoretical conclusions, or lead to more extensive studies of questions that require additional research. Our experimental design was developed with an intent to minimize the possibility of outcomes that are based on specific problem parameters, rather than on relationships of more fundamental nature. Another limitation is related to the fact we have not provided a detailed industrial application using the facility location, capacity acquisition and technology choice model using robust optimization methodology.

Future research may focus on additional questions that were identified in the course of working on this dissertation. Some of the potential research directions include the extension of the uncertainty set formulations to other problem parameters (e.g., costs), the examination of the model in a multi-period setting, and the inclusion of additional supply chain echelons. But perhaps one of the more interesting extensions of the model would be to formulate the production and transportation costs in a way that would lead to similar effects of economies of scale and scope for the second stage recourse problem. This formulation would be possible due to the ability of the primal cut algorithm to solve mixed integer subproblems, as binary variables would be needed to formulate production and transportation costs as piecewise linear concave costs. Another research direction is the focus on a large-scale implementation of this model. As a part of this implementation, a method for finding tight big-M values needs to be developed. Overall, we believe that the modeling approach presented in this dissertation is applicable to a wide range of problems, and that this work using robust optimization methodology has a "prototypic value" beyond specific problem context.

APPENDIX

Abbreviations for Appendix Tables A1–A20:

Y	$\sum_r \sum_i y_i(r)/R$
Z1	$\sum_r \sum_i z_{i1}(r)/R$
Z2	$\sum_r \sum_i z_{i2}(r)/R$
Z3	$\sum_r \sum_i z_{i3}(r)/R$
V1	$\sum_r \sum_i v_{i1}(r)/R$
V2	$\sum_r \sum_i v_{i2}(r)/R$
V3	$\sum_r \sum_i v_{i3}(r)/R$
ZD1	$\sum_r \sum_i \sum_l z^D_{i1l}(r)/R$
ZD2	$\sum_r \sum_i \sum_l z^D_{i2l}(r)/R$
ZF	$\sum_r \sum_i \sum_l z^F_{il}(r)/R$
VD1	$\sum_r \sum_i \sum_l v^D_{i1l}(r)/R$
VD2	$\sum_r \sum_i \sum_l v^D_{i2l}(r)/R$
VF	$\sum_r \sum_i \sum_l v^F_{il}(r)/R$
ZD11	$\sum_r \sum_i z^D_{i11}(r)/R$
ZD12	$\sum_r \sum_i z^D_{i12}(r)/R$
ZD21	$\sum_r \sum_i z^D_{i21}(r)/R$
ZD22	$\sum_r \sum_i z^D_{i22}(r)/R$
ZF1	$\sum_r \sum_i z^F_{i1}(r)/R$
ZF2	$\sum_r \sum_i z^F_{i2}(r)/R$
err	standard error

Abbreviations for Appendix Tables A21–A28:

(p)	polyhedral uncertainty set
(e)	ellipsoidal uncertainty set
Rob(*)	objective values of robust solutions
Est(*)	estimates of the first stage robust solutions
LB	lower statistical bound of stochastic solutions
UB	upper statistical bound of stochastic solutions
Z1(*)	robust solution for $\sum_i z_{i1}$
Z2(*)	robust solution for $\sum_i z_{i2}$
Z3(*)	robust solution for $\sum_i z_{i3}$
ZD1(*)	robust solution for $\sum_{il} z^D_{i1l}$
ZD2(*)	robust solution for $\sum_{il} z^D_{i2l}$
ZF(*)	robust solution for $\sum_{il} z^F_{il}$
Z1(stoch)	stochastic solution for $\sum_i z_{i1}$
Z2(stoch)	stochastic solution for $\sum_i z_{i2}$
Z3(stoch)	stochastic solution for $\sum_i z_{i3}$
ZD1(stoch)	stochastic solution for $\sum_{il} z^D_{i1l}$
ZD2(stoch)	stochastic solution for $\sum_{il} z^D_{i2l}$
ZF(stoch)	stochastic solution for $\sum_{il} z^F_{il}$
err	standard error

	Objective		Solutions									
Γ	*ave. value*	*err*	*Z1*	*errZ1*	*Z2*	*errZ2*	*Z3*	*errZ3*	*Y*	*V1*	*V2*	*V3*
0	167,992.7	2,258.5	1,856.1	168.8	1,497.4	166.9	1,663.8	161.6	4.87	2.35	1.44	1.07
1	176,588.7	2,345.1	1,836.6	172.1	1,567.2	173.3	1,921.3	174.1	4.86	2.26	1.43	1.17
2	183,587.5	2,415.2	1,854.7	178.2	1,577.5	179.8	2,143.7	185.8	4.91	2.22	1.43	1.26
3	189,695.9	2,493.1	1,809.8	177.9	1,599.8	185.9	2,408.0	193.9	4.93	2.15	1.40	1.38
4	195,156.9	2,569.9	1,814.3	182.7	1,667.5	194.4	2,543.9	202.2	4.98	2.13	1.42	1.43
5	200,078.7	2,646.6	1,844.6	188.4	1,696.3	198.4	2,666.6	208.3	4.99	2.11	1.41	1.47
6	204,555.9	2,717.3	1,849.7	193.8	1,752.8	202.1	2,770.5	215.6	5.02	2.07	1.43	1.51
7	208,655.5	2,781.0	1,892.7	200.7	1,798.8	207.1	2,836.8	222.9	5.05	2.06	1.45	1.53
8	212,403.7	2,840.9	1,900.1	203.4	1,815.3	211.2	2,947.5	227.5	5.07	2.04	1.45	1.57
9	215,840.8	2,896.1	1,912.7	207.0	1,842.1	212.9	3,030.9	231.6	5.09	2.03	1.47	1.59
10	218,995.5	2,948.6	1,964.9	212.6	1,852.1	216.0	3,085.9	237.4	5.13	2.04	1.47	1.61
11	221,866.2	2,999.5	1,982.8	214.7	1,891.3	219.0	3,139.6	241.3	5.15	2.05	1.49	1.61
12	224,487.9	3,044.2	1,991.4	214.5	1,891.2	220.9	3,226.1	243.4	5.17	2.04	1.49	1.65
13	226,878.7	3,083.8	2,001.1	217.1	1,865.7	220.7	3,327.9	246.2	5.19	2.03	1.47	1.69
14	229,033.7	3,120.9	1,994.5	218.7	1,898.4	224.0	3,375.9	249.3	5.22	2.03	1.49	1.71
15	230,951.9	3,154.1	2,045.8	223.5	1,907.6	225.7	3,384.2	252.6	5.27	2.06	1.50	1.71
16	232,643.7	3,182.5	2,067.3	226.1	1,911.3	226.2	3,415.2	254.7	5.31	2.09	1.50	1.73
17	234,129.7	3,205.9	2,082.5	228.0	1,931.8	227.4	3,428.1	255.9	5.35	2.11	1.51	1.73
18	235,387.8	3,224.7	2,056.7	225.3	1,945.3	228.8	3,476.8	255.2	5.35	2.08	1.52	1.75
19	236,395.1	3,238.7	2,048.5	225.9	1,953.3	229.7	3,507.5	257.1	5.35	2.07	1.52	1.76
20	237,114.6	3,250.2	2,040.7	223.9	1,973.4	229.8	3,512.0	257.4	5.35	2.07	1.53	1.76

Table A1: Single product solution: $f_{max} = 5{,}000,\ e_{max} = 10{,}000,\ g_{max} = 7.5$.

	Objective		Solutions									
Γ	*ave. value*	*err*	*Z1*	*errZ1*	*Z2*	*errZ2*	*Z3*	*errZ3*	*Y*	*V1*	*V2*	*V3*
0	173,239.0	2,320.3	1,310.9	150.0	1,363.0	159.6	2,343.5	175.6	4.71	1.77	1.37	1.57
1	181,979.4	2,411.8	1,358.9	156.5	1,363.6	164.5	2,598.1	187.5	4.75	1.77	1.31	1.67
2	189,105.9	2,487.4	1,348.2	161.4	1,405.9	170.9	2,818.7	196.3	4.77	1.70	1.33	1.75
3	195,354.9	2,569.4	1,342.7	164.5	1,446.0	177.5	3,019.9	205.8	4.79	1.65	1.33	1.81
4	200,936.9	2,649.3	1,314.7	164.9	1,491.7	185.7	3,208.7	210.3	4.80	1.60	1.33	1.87
5	205,962.3	2,728.5	1,318.9	169.3	1,517.1	188.6	3,364.4	215.8	4.85	1.58	1.34	1.93
6	210,536.6	2,801.6	1,315.9	171.0	1,536.5	192.2	3,515.7	220.8	4.88	1.55	1.33	1.99
7	214,728.6	2,869.1	1,338.3	175.7	1,521.0	193.2	3,658.0	226.8	4.91	1.55	1.32	2.04
8	218,573.0	2,932.3	1,342.6	179.5	1,546.3	197.1	3,767.1	231.5	4.93	1.54	1.31	2.08
9	222,098.0	2,990.3	1,348.2	180.8	1,599.4	201.5	3,834.4	234.4	4.95	1.52	1.35	2.09
10	225,333.6	3,044.8	1,356.4	183.6	1,605.9	205.4	3,934.3	237.6	4.97	1.50	1.35	2.12
11	228,284.9	3,097.5	1,361.8	186.9	1,613.7	206.1	4,034.7	241.8	5.01	1.49	1.38	2.15
12	230,980.2	3,144.9	1,369.6	188.4	1,640.8	207.8	4,093.3	245.3	5.03	1.49	1.39	2.16
13	233,428.1	3,186.3	1,410.0	190.9	1,659.4	210.3	4,119.5	247.8	5.07	1.52	1.39	2.15
14	235,626.8	3,224.4	1,406.5	193.1	1,676.1	212.4	4,183.5	251.4	5.09	1.51	1.40	2.19
15	237,601.9	3,259.7	1,424.6	195.0	1,698.0	214.0	4,210.0	253.0	5.12	1.52	1.42	2.18
16	239,342.2	3,289.8	1,424.0	195.8	1,709.2	216.2	4,256.9	255.2	5.15	1.53	1.43	2.20
17	240,865.8	3,315.1	1,389.5	195.3	1,727.0	217.6	4,321.8	258.1	5.17	1.50	1.43	2.24
18	242,157.2	3,334.4	1,402.6	196.4	1,736.4	219.0	4,338.9	259.5	5.19	1.51	1.43	2.25
19	243,193.7	3,348.6	1,424.1	197.1	1,737.1	220.0	4,347.2	259.8	5.21	1.53	1.43	2.25
20	243,932.1	3,360.0	1,409.0	197.4	1,763.7	222.6	4,353.4	262.4	5.21	1.52	1.44	2.25

Table A2: Single product solution: $f_{max} = 5{,}000,\ e_{max} = 10{,}000,\ g_{max} = 10.0$.

	Objective		Solutions									
Γ	*ave. value*	*err*	*Z1*	*errZ1*	*Z2*	*errZ2*	*Z3*	*errZ3*	*Y*	*V1*	*V2*	*V3*
0	174,787.7	2,349.3	2,472.2	187.3	1,579.5	178.5	965.6	148.2	4.73	2.75	1.37	0.61
1	183,629.9	2,431.6	2,547.9	195.4	1,682.9	189.3	1,076.1	158.7	4.75	2.73	1.37	0.64
2	190,852.2	2,502.6	2,603.3	204.0	1,789.7	200.3	1,164.4	168.0	4.76	2.69	1.41	0.67
3	197,146.2	2,574.7	2,622.2	209.1	1,865.8	209.3	1,302.2	176.5	4.79	2.63	1.43	0.72
4	202,776.1	2,648.2	2,663.5	213.2	1,906.8	214.8	1,431.5	182.3	4.81	2.63	1.43	0.75
5	207,874.9	2,723.2	2,687.1	218.2	1,959.6	219.6	1,545.7	189.2	4.85	2.61	1.45	0.79
6	212,504.6	2,794.7	2,678.7	221.9	1,985.6	222.4	1,689.0	195.7	4.87	2.59	1.44	0.85
7	216,721.5	2,860.0	2,717.6	228.1	2,012.8	225.7	1,777.9	201.8	4.90	2.59	1.44	0.87
8	220,583.5	2,920.9	2,761.2	232.6	2,062.5	230.4	1,822.0	205.7	4.95	2.61	1.46	0.89
9	224,133.6	2,976.7	2,765.2	234.2	2,114.0	235.2	1,890.1	211.3	4.99	2.60	1.48	0.91
10	227,374.9	3,028.7	2,766.2	234.8	2,096.5	235.4	2,025.6	215.2	4.99	2.57	1.47	0.95
11	230,334.6	3,080.2	2,800.9	237.0	2,124.5	237.6	2,072.8	219.8	5.05	2.59	1.49	0.97
12	233,022.3	3,125.7	2,857.0	240.8	2,130.2	238.7	2,110.8	220.8	5.07	2.61	1.49	0.97
13	235,473.8	3,166.6	2,876.7	243.7	2,133.3	239.4	2,174.4	224.4	5.08	2.60	1.49	0.99
14	237,687.5	3,205.0	2,915.1	248.5	2,137.9	240.3	2,210.5	228.7	5.09	2.61	1.49	0.99
15	239,663.5	3,239.1	2,918.4	251.2	2,178.6	242.4	2,236.0	231.6	5.15	2.64	1.50	1.01
16	241,406.9	3,268.8	2,937.9	251.9	2,193.7	244.1	2,258.2	232.4	5.16	2.65	1.50	1.01
17	242,946.1	3,294.8	2,920.1	253.0	2,219.7	247.0	2,298.1	233.8	5.17	2.63	1.52	1.02
18	244,247.8	3,314.6	2,939.9	254.2	2,207.2	247.3	2,329.0	236.0	5.19	2.65	1.51	1.03
19	245,286.9	3,328.7	2,949.7	255.5	2,227.4	249.4	2,330.3	236.8	5.19	2.64	1.52	1.03
20	246,026.6	3,339.3	2,958.6	255.7	2,223.5	248.8	2,344.0	237.7	5.20	2.65	1.51	1.03

Table A3: Single product solution: $f_{max} = 5{,}000,\ e_{max} = 15{,}000,\ g_{max} = 7.5$.

	Objective		Solutions									
Γ	*ave. value*	*err*	*Z1*	*errZ1*	*Z2*	*errZ2*	*Z3*	*errZ3*	*Y*	*V1*	*V2*	*V3*
0	181,106.0	2,413.2	1,918.3	168.0	1,543.9	171.3	1,555.1	159.0	4.60	2.29	1.37	0.94
1	190,132.8	2,497.6	1,877.7	174.7	1,582.3	176.4	1,841.3	173.3	4.58	2.18	1.35	1.05
2	197,486.4	2,567.1	1,858.1	177.0	1,639.0	181.9	2,052.0	182.9	4.59	2.11	1.37	1.12
3	203,932.7	2,642.3	1,864.0	181.3	1,698.7	188.9	2,226.1	194.7	4.59	2.05	1.36	1.18
4	209,708.7	2,717.5	1,944.2	188.9	1,730.1	195.7	2,315.5	199.8	4.66	2.08	1.38	1.20
5	214,935.7	2,795.1	1,932.6	191.6	1,790.3	202.4	2,457.7	207.2	4.70	2.05	1.40	1.25
6	219,674.6	2,869.5	1,966.8	196.8	1,789.5	206.3	2,584.2	215.1	4.74	2.07	1.37	1.30
7	224,001.6	2,936.7	1,963.2	200.3	1,844.0	211.6	2,686.6	221.1	4.76	2.04	1.38	1.34
8	227,956.8	2,999.8	1,984.7	205.3	1,866.8	214.5	2,784.4	224.5	4.79	2.03	1.40	1.36
9	231,599.6	3,058.5	1,996.6	209.0	1,892.3	218.5	2,875.5	231.1	4.81	2.02	1.41	1.39
10	234,929.4	3,113.1	1,978.2	207.1	1,938.8	221.5	2,963.1	233.5	4.83	1.99	1.43	1.41
11	237,966.0	3,167.1	2,004.4	211.5	1,939.8	223.1	3,043.6	238.6	4.85	1.99	1.43	1.44
12	240,726.9	3,214.8	2,032.8	214.4	1,954.7	224.6	3,100.4	241.5	4.90	2.01	1.44	1.45
13	243,237.7	3,257.2	2,012.6	216.9	1,974.0	228.0	3,189.9	247.1	4.91	1.98	1.45	1.48
14	245,509.5	3,296.3	2,029.6	217.6	1,979.3	228.4	3,248.5	248.0	4.93	1.98	1.46	1.49
15	247,550.7	3,333.4	2,043.7	218.5	1,996.4	230.3	3,287.0	249.3	4.96	2.00	1.46	1.50
16	249,352.8	3,365.1	2,053.6	219.7	2,011.1	232.0	3,321.5	251.1	4.98	2.01	1.47	1.51
17	250,941.8	3,392.4	2,060.2	222.6	2,016.3	233.1	3,358.6	253.5	5.00	2.00	1.47	1.53
18	252,279.8	3,413.3	2,065.5	223.9	2,027.6	234.3	3,382.5	255.3	5.01	1.99	1.47	1.54
19	253,352.2	3,428.0	2,068.6	224.6	2,043.6	235.1	3,394.6	256.6	5.01	1.99	1.48	1.54
20	254,110.9	3,439.2	2,080.8	225.0	2,049.8	235.7	3,395.5	256.7	5.03	2.01	1.47	1.54

Table A4: Single product solution: $f_{max} = 5{,}000,\ e_{max} = 15{,}000,\ g_{max} = 10.0$.

Γ	Objective		Solutions									
	ave. value	*err*	*Z1*	*errZ1*	*Z2*	*errZ2*	*Z3*	*errZ3*	*Y*	*V1*	*V2*	*V3*
0	196,063.0	2,778.7	1,357.5	160.1	1,448.0	167.2	2,211.9	178.6	3.61	1.30	1.12	1.19
1	204,888.1	2,866.7	1,358.1	164.4	1,501.6	175.0	2,447.9	189.1	3.67	1.28	1.12	1.27
2	212,216.5	2,938.9	1,361.8	167.7	1,522.3	181.2	2,665.7	197.9	3.74	1.27	1.13	1.34
3	218,617.6	3,010.3	1,380.0	169.3	1,469.5	181.7	2,922.8	202.5	3.77	1.26	1.07	1.45
4	224,394.8	3,078.5	1,374.9	173.1	1,503.2	186.4	3,094.7	210.4	3.81	1.22	1.07	1.52
5	229,616.1	3,147.4	1,405.1	175.3	1,589.6	194.8	3,158.7	215.9	3.85	1.23	1.11	1.52
6	234,398.6	3,213.1	1,415.2	178.8	1,596.3	196.4	3,312.9	222.1	3.91	1.22	1.11	1.58
7	238,741.8	3,273.8	1,432.9	185.1	1,663.7	202.8	3,382.8	228.9	3.95	1.21	1.13	1.60
8	242,697.1	3,330.6	1,450.5	189.5	1,697.7	206.3	3,471.5	233.0	3.98	1.21	1.15	1.63
9	246,332.0	3,383.1	1,444.7	192.0	1,713.4	210.0	3,591.5	238.8	3.99	1.19	1.14	1.66
10	249,665.8	3,433.8	1,451.3	194.9	1,727.5	213.6	3,688.2	244.5	4.01	1.18	1.13	1.69
11	252,721.7	3,481.8	1,487.6	198.1	1,752.5	216.8	3,734.9	248.1	4.01	1.19	1.13	1.69
12	255,516.0	3,525.6	1,498.1	199.1	1,767.9	219.1	3,808.7	251.5	4.02	1.19	1.13	1.70
13	258,056.2	3,565.2	1,513.3	202.4	1,785.8	221.9	3,871.9	255.7	4.05	1.20	1.13	1.73
14	260,335.4	3,601.2	1,520.3	203.9	1,791.4	224.6	3,942.5	259.1	4.07	1.20	1.13	1.74
15	262,387.0	3,634.8	1,530.3	205.8	1,798.5	226.2	3,995.3	261.8	4.09	1.21	1.12	1.77
16	264,203.0	3,664.0	1,514.2	208.1	1,823.1	229.9	4,044.4	267.4	4.11	1.20	1.13	1.79
17	265,788.4	3,689.1	1,514.9	209.2	1,811.4	228.7	4,107.2	267.9	4.13	1.21	1.13	1.80
18	267,139.1	3,709.9	1,526.4	210.5	1,813.5	230.1	4,133.4	270.0	4.14	1.21	1.13	1.81
19	268,222.2	3,724.8	1,519.9	210.7	1,820.2	230.9	4,165.0	271.5	4.15	1.21	1.13	1.82
20	268,999.7	3,736.2	1,510.3	208.4	1,841.2	231.2	4,174.6	272.1	4.16	1.21	1.13	1.82

Table A5: Single product solution: $f_{max} = 20{,}000$, $e_{max} = 10{,}000$, $g_{max} = 7.5$.

	Objective		Solutions									
Γ	*ave. value*	*err*	*Z1*	*errZ1*	*Z2*	*errZ2*	*Z3*	*errZ3*	*Y*	*V1*	*V2*	*V3*
0	200,651.8	2,819.0	928.5	137.7	1,158.3	152.0	2,930.6	181.4	3.54	0.95	0.95	1.64
1	209,644.3	2,912.0	954.2	144.0	1,223.2	162.3	3,126.3	193.7	3.60	0.94	0.97	1.69
2	217,102.8	2,989.2	962.7	149.5	1,204.0	164.8	3,382.6	201.7	3.65	0.93	0.93	1.79
3	223,634.6	3,064.3	955.8	155.0	1,230.1	170.5	3,581.7	209.5	3.67	0.89	0.92	1.87
4	229,527.9	3,136.2	967.4	156.3	1,281.3	178.2	3,718.5	215.9	3.70	0.89	0.93	1.89
5	234,866.6	3,208.7	995.8	160.4	1,330.7	183.7	3,822.6	221.4	3.75	0.89	0.95	1.92
6	239,761.2	3,276.7	994.3	159.7	1,345.7	184.6	3,979.5	223.6	3.79	0.88	0.95	1.96
7	244,192.3	3,339.8	1,001.7	164.0	1,365.0	189.2	4,107.0	229.0	3.83	0.87	0.95	2.01
8	248,240.1	3,399.0	1,008.2	167.8	1,385.3	193.5	4,219.5	234.6	3.87	0.87	0.96	2.05
9	251,954.4	3,453.1	978.4	165.7	1,379.5	192.6	4,385.9	234.7	3.89	0.85	0.95	2.09
10	255,362.8	3,505.0	994.8	171.7	1,379.3	195.3	4,486.4	238.6	3.91	0.85	0.95	2.11
11	258,492.5	3,554.2	971.9	173.7	1,430.1	198.4	4,569.7	243.9	3.93	0.82	0.97	2.13
12	261,359.6	3,599.9	942.8	170.6	1,428.6	201.0	4,700.1	245.5	3.92	0.79	0.97	2.16
13	263,955.1	3,641.2	993.0	173.8	1,447.1	203.5	4,726.1	248.4	3.95	0.82	0.97	2.16
14	266,289.7	3,679.1	999.7	176.1	1,461.6	205.5	4,786.5	251.7	3.98	0.83	0.97	2.18
15	268,392.1	3,714.2	982.6	174.1	1,493.5	209.7	4,841.2	252.9	3.99	0.81	0.99	2.19
16	270,253.9	3,744.7	993.5	175.6	1,497.6	211.4	4,887.5	255.5	4.01	0.82	0.99	2.20
17	271,867.4	3,770.2	992.8	176.5	1,506.6	210.2	4,930.1	257.1	4.03	0.82	0.99	2.22
18	273,245.7	3,791.3	1,046.5	187.2	1,515.3	211.4	4,909.5	262.3	4.04	0.84	0.99	2.21
19	274,358.5	3,806.8	1,054.8	188.0	1,517.7	212.2	4,932.1	263.6	4.05	0.85	0.99	2.21
20	275,156.0	3,818.7	1,094.2	194.4	1,513.8	212.2	4,918.1	265.9	4.07	0.88	0.99	2.21

Table A6: Single product solution: $f_{max} = 20{,}000,\ e_{max} = 10{,}000,\ g_{max} = 10.0$.

Γ	Objective		Solutions									
	ave. value	*err*	*Z1*	*errZ1*	*Z2*	*errZ2*	*Z3*	*errZ3*	*Y*	*V1*	*V2*	*V3*
0	202,152.4	2,805.3	1,978.7	178.2	1,636.3	181.0	1,402.4	162.9	3.52	1.69	1.15	0.68
1	211,275.1	2,896.7	2,017.6	182.0	1,710.4	189.9	1,566.3	171.9	3.56	1.67	1.16	0.73
2	218,823.7	2,970.4	2,028.3	186.6	1,808.7	198.5	1,695.2	179.4	3.59	1.65	1.19	0.76
3	225,405.4	3,044.5	2,066.7	194.1	1,875.4	204.6	1,810.0	189.1	3.66	1.64	1.21	0.81
4	231,339.4	3,112.8	2,090.6	202.1	1,955.6	212.4	1,909.5	197.6	3.71	1.64	1.22	0.85
5	236,720.4	3,182.6	2,096.2	207.9	2,038.3	220.7	2,001.6	206.7	3.73	1.61	1.25	0.87
6	241,645.8	3,249.1	2,120.9	213.3	2,064.4	224.7	2,126.0	214.3	3.79	1.61	1.27	0.92
7	246,098.5	3,310.8	2,175.4	217.3	2,120.8	228.4	2,167.1	216.9	3.83	1.62	1.27	0.93
8	250,179.5	3,369.4	2,203.5	221.9	2,133.7	231.7	2,266.4	223.3	3.87	1.63	1.28	0.96
9	253,915.1	3,422.6	2,171.5	222.6	2,156.1	235.8	2,405.4	230.6	3.89	1.59	1.27	1.02
10	257,337.1	3,471.9	2,183.0	224.7	2,139.2	234.6	2,526.8	233.1	3.89	1.59	1.25	1.05
11	260,478.7	3,519.9	2,205.4	228.8	2,139.1	236.7	2,617.9	239.5	3.90	1.59	1.24	1.07
12	263,347.9	3,564.4	2,216.8	232.1	2,159.3	239.2	2,689.0	243.7	3.92	1.59	1.25	1.09
13	265,955.7	3,605.4	2,238.8	234.3	2,184.9	242.1	2,738.3	245.9	3.95	1.61	1.25	1.10
14	268,285.8	3,643.0	2,224.3	233.3	2,182.6	242.0	2,840.3	246.6	3.99	1.61	1.25	1.13
15	270,388.4	3,677.8	2,222.4	235.6	2,211.2	244.0	2,886.3	249.2	4.00	1.60	1.25	1.15
16	272,238.8	3,708.2	2,242.1	238.4	2,229.0	246.0	2,907.3	252.2	4.01	1.61	1.25	1.15
17	273,866.3	3,734.6	2,241.0	241.7	2,256.1	249.1	2,931.3	255.1	4.02	1.60	1.27	1.15
18	275,247.8	3,756.4	2,267.1	243.0	2,237.2	249.6	2,967.4	256.2	4.03	1.61	1.25	1.17
19	276,360.7	3,772.2	2,269.2	244.8	2,257.5	252.2	2,978.1	257.2	4.04	1.61	1.27	1.17
20	277,162.5	3,783.3	2,276.6	245.5	2,265.1	252.9	2,984.4	257.8	4.04	1.61	1.27	1.17

Table A7: Single product solution: $f_{max} = 20{,}000$, $e_{max} = 15{,}000$, $g_{max} = 7.5$.

	Objective		Solutions										
Γ	*ave. value*	*err*		*Z1*	*errZ1*	*Z2*	*errZ2*	*Z3*	*errZ3*	*Y*	*V1*	*V2*	*V3*
0	207,781.5	2,843.7		1,485.6	164.5	1,522.8	172.1	2,008.9	180.3	3.47	1.38	1.09	1.00
1	217,097.0	2,935.7		1,490.2	171.3	1,574.6	177.9	2,224.5	188.0	3.53	1.33	1.11	1.08
2	224,776.0	3,010.0		1,513.4	176.1	1,591.6	181.5	2,424.3	195.4	3.57	1.32	1.11	1.14
3	231,485.5	3,085.2		1,468.6	177.1	1,632.3	189.7	2,648.3	204.8	3.61	1.27	1.11	1.24
4	237,553.2	3,156.8		1,495.5	182.6	1,611.5	191.5	2,839.3	211.7	3.63	1.26	1.07	1.30
5	243,054.3	3,230.1		1,503.7	184.5	1,660.6	200.3	2,964.7	219.5	3.66	1.25	1.07	1.34
6	248,084.2	3,299.5		1,508.3	188.5	1,679.5	201.5	3,109.9	223.9	3.71	1.23	1.09	1.39
7	252,624.7	3,364.6		1,522.1	191.8	1,723.9	205.8	3,206.7	228.2	3.73	1.21	1.10	1.42
8	256,793.0	3,427.0		1,568.0	196.1	1,731.8	208.6	3,294.9	231.9	3.77	1.23	1.11	1.43
9	260,623.3	3,484.1		1,610.5	201.6	1,741.6	210.9	3,374.3	236.6	3.79	1.25	1.10	1.45
10	264,123.8	3,537.3		1,559.8	200.4	1,733.0	211.8	3,552.7	241.5	3.81	1.20	1.09	1.52
11	267,323.1	3,587.1		1,573.9	203.0	1,791.9	218.4	3,592.2	246.8	3.82	1.20	1.10	1.52
12	270,244.2	3,633.0		1,592.2	202.8	1,783.6	219.3	3,687.8	250.5	3.85	1.21	1.09	1.54
13	272,912.0	3,676.1		1,583.6	205.4	1,794.8	221.1	3,780.0	255.8	3.86	1.20	1.10	1.56
14	275,299.6	3,715.1		1,594.6	207.9	1,831.6	226.3	3,816.3	259.9	3.88	1.19	1.11	1.57
15	277,461.6	3,751.1		1,583.5	209.7	1,854.2	229.6	3,874.9	263.0	3.89	1.19	1.11	1.59
16	279,370.0	3,783.3		1,589.6	211.4	1,888.6	231.6	3,895.7	264.7	3.90	1.19	1.13	1.59
17	281,036.1	3,810.0		1,590.7	213.3	1,883.8	232.4	3,950.2	267.5	3.90	1.17	1.11	1.61
18	282,459.6	3,832.8		1,601.7	214.4	1,894.7	233.8	3,972.5	268.9	3.91	1.18	1.11	1.62
19	283,601.9	3,848.8		1,616.4	215.4	1,903.3	234.8	3,983.7	269.8	3.93	1.19	1.12	1.62
20	284,418.9	3,860.4		1,624.9	216.0	1,909.4	235.4	3,991.8	270.4	3.94	1.20	1.12	1.62

Table A8: Single product solution: $f_{max} = 20{,}000,\ e_{max} = 15{,}000,\ g_{max} = 10.0$.

	Objective		Solutions									
Γ	*ave. value*	*err*	*ZD1*	*errZD1*	*ZD2*	*errZD2*	*ZF*	*errZF*	*Y*	*VD1*	*VD2*	*VF*
0	187,315.7	3,295.6	0.0	0.0	0.0	0.0	5,471.3	40.3	4.05	0.00	0.00	4.05
1	195,926.0	3,410.5	0.0	0.0	0.0	0.0	5,794.4	42.3	4.13	0.00	0.00	4.13
2	203,114.6	3,526.2	0.0	0.0	0.0	0.0	6,069.6	45.8	4.14	0.00	0.00	4.14
3	209,487.8	3,634.5	0.0	0.0	0.0	0.0	6,313.9	48.2	4.16	0.00	0.00	4.16
4	215,293.6	3,737.8	0.0	0.0	0.0	0.0	6,539.2	50.4	4.24	0.00	0.00	4.24
5	220,623.7	3,832.4	0.0	0.0	0.0	0.0	6,736.1	52.1	4.25	0.00	0.00	4.25
6	225,566.4	3,924.7	0.0	0.0	0.0	0.0	6,919.6	53.9	4.28	0.00	0.00	4.28
7	230,158.9	4,014.7	0.0	0.0	0.0	0.0	7,084.7	55.5	4.29	0.00	0.00	4.29
8	234,364.2	4,093.4	0.0	0.0	0.0	0.0	7,234.0	56.4	4.29	0.00	0.00	4.29
9	238,293.2	4,171.1	0.0	0.0	0.0	0.0	7,377.3	56.8	4.31	0.00	0.00	4.31
10	241,924.8	4,246.9	0.0	0.0	0.0	0.0	7,505.4	57.1	4.32	0.00	0.00	4.32
11	245,303.8	4,317.6	0.0	0.0	0.0	0.0	7,621.4	57.9	4.34	0.00	0.00	4.34
12	248,403.1	4,381.9	0.0	0.0	0.0	0.0	7,737.2	57.6	4.38	0.00	0.00	4.38
13	251,239.2	4,440.5	0.0	0.0	0.0	0.0	7,835.2	58.0	4.38	0.00	0.00	4.38
14	253,830.5	4,495.8	0.0	0.0	0.0	0.0	7,921.8	58.7	4.39	0.00	0.00	4.39
15	256,165.8	4,545.1	0.0	0.0	0.0	0.0	7,995.7	58.7	4.41	0.00	0.00	4.41
16	258,258.9	4,592.4	0.0	0.0	0.0	0.0	8,062.7	59.3	4.43	0.00	0.00	4.43
17	260,086.1	4,633.0	0.0	0.0	0.0	0.0	8,115.3	59.2	4.44	0.00	0.00	4.44
18	261,605.2	4,669.8	0.0	0.0	0.0	0.0	8,157.2	60.0	4.45	0.00	0.00	4.45
19	262,810.6	4,700.6	0.0	0.0	0.0	0.0	8,188.3	60.2	4.45	0.00	0.00	4.45
20	263,624.6	4,715.8	0.0	0.0	0.0	0.0	8,206.9	60.5	4.45	0.00	0.00	4.45

Table A9: Multi-product solution: $\Delta = 1$.

Γ	Objective		Solutions									
	ave. value	*err*	*ZD1*	*errZD1*	*ZD2*	*errZD2*	*ZF*	*errZF*	*Y*	*VD1*	*VD2*	*VF*
0	197,322.4	3,497.2	617.3	94.7	776.4	117.2	4,077.6	218.0	3.80	1.02	1.03	2.76
1	206,450.3	3,610.4	585.5	91.8	755.8	112.2	4,449.3	211.7	3.87	0.90	0.94	2.96
2	213,974.5	3,726.8	598.4	93.0	789.3	115.1	4,660.9	217.0	3.90	0.91	0.97	2.98
3	220,636.2	3,839.8	586.6	92.8	785.9	116.7	4,917.3	220.9	3.94	0.89	0.95	3.04
4	226,704.1	3,947.5	620.1	97.3	842.1	122.9	5,046.1	231.0	3.98	0.92	0.99	3.04
5	232,257.1	4,045.5	629.9	98.2	880.3	127.2	5,199.8	235.8	4.02	0.91	1.00	3.09
6	237,402.2	4,141.1	630.2	97.9	891.5	130.8	5,371.1	239.2	4.09	0.90	0.99	3.16
7	242,135.2	4,231.4	648.6	100.7	924.5	134.3	5,487.3	245.1	4.13	0.91	1.00	3.17
8	246,501.4	4,311.7	666.7	101.4	952.1	135.2	5,603.6	246.9	4.18	0.92	1.02	3.21
9	250,554.7	4,391.5	722.5	107.4	1,023.1	143.5	5,625.4	259.1	4.21	0.96	1.10	3.18
10	254,301.6	4,468.5	752.8	109.8	1,046.8	148.0	5,702.1	265.9	4.21	1.00	1.13	3.16
11	257,775.3	4,540.6	799.3	115.1	1,091.2	150.9	5,730.6	273.4	4.21	1.04	1.17	3.12
12	260,944.9	4,605.2	835.2	117.8	1,161.7	154.5	5,740.6	278.8	4.24	1.08	1.21	3.09
13	263,840.3	4,664.7	887.6	123.2	1,205.6	160.5	5,743.8	289.0	4.27	1.10	1.21	3.10
14	266,494.7	4,721.2	935.0	127.3	1,257.1	164.2	5,731.8	295.8	4.26	1.12	1.23	3.07
15	268,905.9	4,772.8	973.9	132.2	1,296.8	167.9	5,731.7	305.7	4.27	1.16	1.26	3.04
16	271,040.9	4,819.8	981.5	134.0	1,304.1	170.1	5,779.7	309.6	4.27	1.18	1.26	3.04
17	272,884.0	4,860.1	1,036.2	141.1	1,341.8	176.0	5,742.2	322.2	4.28	1.24	1.31	3.00
18	274,416.2	4,896.2	1,053.0	143.4	1,347.9	178.6	5,760.1	327.9	4.30	1.24	1.32	3.00
19	275,630.1	4,925.7	1,049.5	144.8	1,327.4	180.8	5,813.4	332.0	4.30	1.25	1.30	3.00
20	276,453.1	4,939.9	1,080.0	149.9	1,345.8	183.0	5,781.1	339.5	4.32	1.28	1.32	3.00

Table A10: Multi-product solution: $\Delta = 1.25$.

	Objective		Solutions									
Γ	*ave. value*	*err*	*ZD1*	*errZD1*	*ZD2*	*errZD2*	*ZF*	*errZF*	*Y*	*VD1*	*VD2*	*VF*
0	203,130.3	3,672.1	1,502.6	110.5	1,878.8	129.5	2,089.8	242.8	3.69	2.27	2.38	1.25
1	212,803.4	3,781.3	1,555.4	117.6	1,930.3	135.3	2,373.7	251.8	3.73	2.16	2.33	1.38
2	220,676.6	3,891.6	1,583.5	121.8	1,962.6	137.1	2,591.6	252.6	3.79	2.09	2.25	1.52
3	227,503.2	3,998.2	1,599.2	119.6	2,026.7	135.0	2,751.3	249.9	3.85	2.06	2.26	1.61
4	233,698.3	4,104.7	1,659.0	122.6	2,137.7	137.2	2,801.9	254.1	3.88	2.08	2.26	1.61
5	239,385.2	4,202.3	1,720.7	122.0	2,206.3	137.8	2,876.4	254.1	3.92	2.10	2.28	1.63
6	244,599.1	4,295.1	1,751.7	122.1	2,271.4	139.8	2,952.7	257.4	3.94	2.07	2.27	1.67
7	249,445.4	4,388.6	1,775.1	124.6	2,303.6	143.1	3,061.1	264.1	3.96	2.04	2.28	1.68
8	253,944.9	4,479.9	1,906.7	130.6	2,424.5	151.6	3,006.4	274.0	4.01	2.10	2.32	1.67
9	258,065.0	4,561.1	1,951.5	129.9	2,489.4	152.5	3,021.6	274.6	4.06	2.13	2.37	1.69
10	261,870.9	4,638.8	1,972.0	129.2	2,540.5	154.2	3,062.8	276.9	4.10	2.14	2.41	1.71
11	265,299.5	4,711.7	2,049.6	133.9	2,621.1	155.5	3,022.5	281.6	4.11	2.17	2.44	1.69
12	268,426.1	4,773.4	2,151.8	138.1	2,722.1	158.9	2,935.7	288.7	4.11	2.26	2.50	1.62
13	271,281.1	4,833.1	2,257.1	145.9	2,830.6	167.1	2,833.3	303.0	4.11	2.37	2.59	1.50
14	273,864.3	4,885.5	2,309.5	148.4	2,862.5	173.7	2,833.3	313.9	4.10	2.42	2.63	1.45
15	276,177.9	4,933.2	2,324.6	152.8	2,884.5	180.1	2,867.4	328.0	4.14	2.47	2.65	1.46
16	278,212.1	4,977.7	2,380.5	155.6	2,951.8	181.6	2,792.8	332.5	4.15	2.54	2.74	1.38
17	279,974.8	5,016.7	2,388.7	156.5	2,957.8	183.9	2,810.7	337.6	4.15	2.59	2.76	1.38
18	281,457.7	5,049.2	2,448.8	159.5	3,009.7	186.0	2,727.1	342.7	4.16	2.65	2.80	1.32
19	282,636.3	5,077.6	2,449.4	159.4	3,010.8	186.0	2,739.2	344.1	4.16	2.65	2.80	1.32
20	283,450.1	5,092.4	2,458.0	160.2	3,020.1	186.0	2,728.8	345.2	4.16	2.67	2.81	1.31

Table A11: Multi-product solution: $\Delta = 1.5$.

	Objective		Solutions									
Γ	*ave. value*	*err*	*ZD1*	*errZD1*	*ZD2*	*errZD2*	*ZF*	*errZF*	*Y*	*VD1*	*VD2*	*VF*
0	205,191.2	3,732.8	2,153.3	84.0	2,622.9	91.1	695.1	167.9	3.66	3.08	3.23	0.34
1	214,984.2	3,835.5	2,372.1	92.7	2,815.6	100.0	739.7	178.6	3.73	3.11	3.28	0.37
2	223,092.4	3,947.8	2,531.0	98.1	3,000.7	105.1	768.5	184.0	3.76	3.09	3.29	0.38
3	230,129.5	4,056.2	2,651.7	102.4	3,143.3	110.3	821.5	191.2	3.82	3.11	3.32	0.43
4	236,442.3	4,162.6	2,755.3	106.7	3,276.0	114.6	864.2	197.3	3.88	3.09	3.34	0.46
5	242,238.6	4,261.8	2,801.1	109.7	3,340.9	117.2	961.1	199.3	3.91	3.06	3.33	0.52
6	247,584.0	4,354.8	2,825.0	113.5	3,391.4	122.4	1,071.7	208.8	3.90	3.03	3.27	0.58
7	252,521.1	4,448.6	2,908.8	111.6	3,478.8	120.7	1,078.3	202.4	3.93	3.03	3.25	0.61
8	257,057.3	4,537.3	2,935.2	114.7	3,536.6	122.8	1,148.9	206.3	3.95	3.00	3.25	0.64
9	261,232.7	4,617.7	2,966.8	118.0	3,594.7	129.0	1,203.2	215.9	3.99	3.01	3.27	0.67
10	265,090.0	4,693.2	2,956.5	117.9	3,633.3	129.0	1,275.4	217.1	4.02	2.99	3.25	0.72
11	268,309.9	4,760.1	3,072.4	118.4	3,758.0	131.6	1,150.1	223.3	4.03	3.11	3.38	0.59
12	271,215.5	4,824.8	3,201.9	117.8	3,925.0	126.9	954.3	219.9	4.03	3.24	3.50	0.47
13	273,874.3	4,881.0	3,277.7	116.1	4,000.0	124.8	856.7	219.6	4.04	3.30	3.56	0.41
14	276,301.9	4,930.4	3,307.9	116.3	4,030.2	124.3	822.3	221.3	4.05	3.34	3.60	0.39
15	278,510.9	4,975.0	3,301.3	116.8	4,026.1	124.1	845.5	223.3	4.06	3.35	3.59	0.39
16	280,469.2	5,016.8	3,339.5	112.1	4,071.3	120.0	776.0	217.3	4.09	3.40	3.65	0.35
17	282,178.0	5,054.2	3,333.1	112.1	4,062.3	118.9	798.0	217.9	4.10	3.44	3.65	0.36
18	283,634.2	5,082.8	3,350.0	111.3	4,082.1	118.8	767.8	218.4	4.10	3.48	3.68	0.33
19	284,836.6	5,107.6	3,356.1	111.5	4,087.1	118.7	761.2	219.5	4.11	3.49	3.69	0.33
20	285,653.9	5,120.6	3,352.0	111.7	4,079.1	119.0	775.8	220.7	4.11	3.51	3.69	0.34

Table A12: Multi-product solution: $\Delta = 1.75$.

	Objective		Solutions									
Γ	*ave. value*	*err*	*ZD1*	*errZD1*	*ZD2*	*errZD2*	*ZF*	*errZF*	*Y*	*VD1*	*VD2*	*VF*
0	205,531.4	3,733.7	2,351.8	62.8	2,832.6	63.4	286.9	111.5	3.66	3.29	3.44	0.13
1	215,355.3	3,835.5	2,582.6	69.2	3,046.4	69.0	319.5	118.9	3.74	3.32	3.49	0.14
2	223,473.0	3,946.7	2,773.1	75.2	3,250.7	73.0	307.9	121.2	3.78	3.31	3.53	0.14
3	230,524.5	4,054.6	2,910.0	82.7	3,413.9	79.2	339.7	131.9	3.82	3.33	3.57	0.15
4	236,855.0	4,160.0	3,041.4	84.3	3,582.1	79.1	338.9	132.6	3.90	3.35	3.63	0.16
5	242,676.7	4,259.8	3,163.0	85.7	3,723.3	80.5	331.2	134.9	3.92	3.36	3.67	0.16
6	248,071.8	4,355.9	3,257.1	89.4	3,831.0	83.0	351.0	138.1	3.90	3.39	3.65	0.18
7	253,047.8	4,450.4	3,341.2	91.8	3,935.4	85.4	365.4	141.2	3.94	3.41	3.68	0.19
8	257,615.8	4,539.0	3,386.4	95.0	4,005.7	90.7	419.8	151.2	3.96	3.36	3.64	0.24
9	261,808.3	4,619.5	3,428.6	96.9	4,094.7	93.1	439.2	153.5	4.00	3.37	3.66	0.26
10	265,710.8	4,696.6	3,446.2	98.4	4,175.4	95.1	467.6	156.1	4.00	3.37	3.64	0.28
11	268,813.9	4,761.2	3,484.8	99.2	4,220.1	95.0	434.9	159.0	4.04	3.43	3.74	0.22
12	271,660.7	4,823.7	3,507.9	98.8	4,240.3	94.5	417.6	161.0	4.05	3.48	3.77	0.19
13	274,279.2	4,878.0	3,516.0	98.7	4,248.4	94.6	416.1	163.1	4.06	3.51	3.78	0.17
14	276,689.8	4,926.8	3,517.3	98.7	4,249.8	94.4	419.5	164.5	4.07	3.53	3.80	0.17
15	278,894.9	4,971.7	3,515.8	98.3	4,246.6	94.3	427.8	165.9	4.08	3.55	3.79	0.18
16	280,840.4	5,013.7	3,518.8	98.6	4,252.1	94.1	426.9	167.6	4.11	3.58	3.84	0.17
17	282,542.2	5,050.9	3,519.8	98.6	4,252.6	94.0	429.1	168.5	4.12	3.63	3.85	0.17
18	283,995.2	5,079.4	3,529.3	98.9	4,263.1	93.6	412.1	168.5	4.12	3.65	3.86	0.16
19	285,195.8	5,104.2	3,540.3	97.8	4,271.2	92.8	394.4	168.2	4.13	3.67	3.88	0.15
20	286,010.9	5,117.4	3,540.3	97.8	4,271.2	92.8	395.4	168.7	4.12	3.69	3.88	0.15

Table A13: Multi-product solution: $\Delta = 1.9$.

	Objective		Solutions									
Γ	*ave. value*	*err*	*ZD1*	*errZD1*	*ZD2*	*errZD2*	*ZF*	*errZF*	*Y*	*VD1*	*VD2*	*VF*
0	205,601.7	3,733.8	2,485.6	39.5	2,985.7	18.1	0.0	0.0	3.66	3.42	3.57	0.00
1	215,427.9	3,835.3	2,738.6	44.1	3,221.7	21.2	0.0	0.0	3.73	3.45	3.63	0.00
2	223,545.8	3,946.4	2,927.3	46.5	3,426.0	23.8	0.0	0.0	3.78	3.44	3.66	0.00
3	230,600.3	4,054.3	3,085.2	48.8	3,607.3	26.1	0.0	0.0	3.82	3.47	3.71	0.00
4	236,941.6	4,159.7	3,218.9	49.9	3,772.5	27.5	0.0	0.0	3.91	3.50	3.78	0.00
5	242,767.3	4,259.4	3,341.0	51.5	3,912.6	27.9	0.0	0.0	3.92	3.51	3.82	0.00
6	248,168.7	4,355.0	3,453.9	53.4	4,038.2	27.7	0.0	0.0	3.91	3.54	3.81	0.00
7	253,146.8	4,449.7	3,548.7	55.1	4,154.5	27.4	0.0	0.0	3.94	3.57	3.84	0.00
8	257,724.8	4,538.3	3,628.5	57.0	4,265.7	27.1	0.0	0.0	3.96	3.58	3.86	0.00
9	261,927.9	4,619.1	3,687.5	58.4	4,374.2	27.1	0.0	0.0	4.00	3.62	3.90	0.00
10	265,835.1	4,695.6	3,728.4	59.2	4,478.5	27.2	0.0	0.0	4.00	3.62	3.90	0.00
11	268,920.4	4,759.7	3,728.4	59.2	4,478.5	27.2	0.0	0.0	4.03	3.62	3.92	0.00
12	271,758.8	4,822.5	3,728.4	59.2	4,478.5	27.2	0.0	0.0	4.05	3.66	3.94	0.00
13	274,375.0	4,876.8	3,728.4	59.2	4,478.5	27.2	0.0	0.0	4.06	3.68	3.95	0.00
14	276,783.1	4,925.7	3,728.4	59.2	4,478.5	27.2	0.0	0.0	4.07	3.70	3.97	0.00
15	278,987.7	4,970.7	3,728.4	59.2	4,478.5	27.2	0.0	0.0	4.08	3.73	3.97	0.00
16	280,928.7	5,012.5	3,728.4	59.2	4,478.5	27.2	0.0	0.0	4.11	3.75	4.01	0.00
17	282,629.0	5,049.8	3,728.4	59.2	4,478.5	27.2	0.0	0.0	4.12	3.80	4.02	0.00
18	284,080.8	5,078.3	3,728.4	59.2	4,478.5	27.2	0.0	0.0	4.12	3.81	4.02	0.00
19	285,279.8	5,103.3	3,728.4	59.2	4,478.5	27.2	0.0	0.0	4.13	3.82	4.03	0.00
20	286,094.7	5,116.4	3,728.4	59.2	4,478.5	27.2	0.0	0.0	4.12	3.84	4.03	0.00

Table A14: Multi-product solution: $\Delta = 2.0$.

Γ	*ZD11*	*ZD12*	*ZD21*	*ZD22*	*ZF1*	*ZF2*
0	0.0	0.0	0.0	0.0	2,624.1	2,847.2
1	0.0	0.0	0.0	0.0	2,749.3	3,045.1
2	0.0	0.0	0.0	0.0	2,738.0	3,331.6
3	0.0	0.0	0.0	0.0	2,816.0	3,497.9
4	0.0	0.0	0.0	0.0	2,882.3	3,656.9
5	0.0	0.0	0.0	0.0	2,928.0	3,808.0
6	0.0	0.0	0.0	0.0	2,929.9	3,989.6
7	0.0	0.0	0.0	0.0	2,987.7	4,097.0
8	0.0	0.0	0.0	0.0	2,977.1	4,256.9
9	0.0	0.0	0.0	0.0	3,022.3	4,355.0
10	0.0	0.0	0.0	0.0	3,069.3	4,436.1
11	0.0	0.0	0.0	0.0	3,136.2	4,485.2
12	0.0	0.0	0.0	0.0	3,172.8	4,564.4
13	0.0	0.0	0.0	0.0	3,103.4	4,731.8
14	0.0	0.0	0.0	0.0	3,087.5	4,834.4
15	0.0	0.0	0.0	0.0	3,117.5	4,878.2
16	0.0	0.0	0.0	0.0	3,126.6	4,936.1
17	0.0	0.0	0.0	0.0	3,105.2	5,010.1
18	0.0	0.0	0.0	0.0	3,113.7	5,043.6
19	0.0	0.0	0.0	0.0	3,126.6	5,061.8
20	0.0	0.0	0.0	0.0	3,134.6	5,072.3

Table A15: Multi-product capacity details: $\Delta = 1.0$.

Γ	*ZD11*	*ZD12*	*ZD21*	*ZD22*	*ZF1*	*ZF2*
0	583.5	33.8	732.6	43.8	1,385.9	2,691.6
1	553.1	32.4	709.7	46.2	1,549.2	2,900.1
2	564.6	33.8	726.0	63.3	1,532.1	3,128.8
3	552.2	34.3	721.2	64.7	1,582.3	3,334.9
4	577.0	43.2	770.9	71.2	1,586.8	3,459.3
5	573.9	55.9	797.2	83.0	1,582.5	3,617.3
6	573.5	56.7	823.4	68.1	1,649.3	3,721.8
7	590.6	58.0	854.5	70.1	1,650.9	3,836.4
8	591.6	75.1	859.8	92.4	1,710.6	3,892.9
9	633.4	89.1	893.1	130.1	1,658.0	3,967.4
10	662.9	89.8	915.0	131.8	1,629.5	4,072.5
11	707.7	91.6	957.3	133.9	1,590.8	4,139.8
12	743.8	91.3	1,029.6	132.1	1,550.6	4,190.0
13	777.2	110.4	1,036.7	169.0	1,549.6	4,194.2
14	795.4	139.7	1,056.5	200.6	1,535.2	4,196.6
15	823.3	150.7	1,088.0	208.8	1,491.8	4,239.9
16	829.8	151.8	1,094.7	209.4	1,486.5	4,293.2
17	870.5	165.7	1,121.2	220.6	1,415.2	4,327.0
18	886.1	167.0	1,122.3	225.6	1,404.6	4,355.5
19	882.7	166.9	1,101.4	226.0	1,441.5	4,371.8
20	913.3	166.7	1,123.7	222.1	1,418.4	4,362.7

Table A16: Multi-product capacity details: $\Delta = 1.25$.

Γ	*ZD11*	*ZD12*	*ZD21*	*ZD22*	*ZF1*	*ZF2*
0	1,208.3	294.3	1,466.7	412.1	470.8	1,619.0
1	1,198.2	357.2	1,509.4	420.9	553.7	1,820.0
2	1,196.7	386.8	1,484.5	478.1	604.6	1,987.0
3	1,211.8	387.5	1,490.4	536.3	658.4	2,092.9
4	1,222.4	436.6	1,526.2	611.5	703.1	2,098.8
5	1,250.4	470.4	1,578.9	627.4	705.2	2,171.2
6	1,252.1	499.6	1,601.0	670.4	727.2	2,225.5
7	1,263.1	512.1	1,622.3	681.3	740.1	2,321.0
8	1,302.0	604.7	1,655.6	768.8	706.7	2,299.7
9	1,336.6	614.9	1,740.4	748.9	685.8	2,335.8
10	1,353.2	618.8	1,745.4	795.0	656.6	2,406.2
11	1,376.9	672.8	1,762.0	859.1	626.4	2,396.1
12	1,438.0	713.8	1,767.3	954.8	599.9	2,335.8
13	1,522.7	734.4	1,831.2	999.4	472.5	2,360.8
14	1,543.5	766.0	1,860.5	1,002.0	399.8	2,433.6
15	1,570.0	754.6	1,875.5	1,009.0	387.6	2,479.8
16	1,606.5	773.9	1,915.7	1,036.1	340.3	2,452.5
17	1,623.1	765.6	1,933.1	1,024.7	321.2	2,489.5
18	1,672.5	776.3	1,974.6	1,035.1	291.7	2,435.4
19	1,674.2	775.2	1,975.2	1,035.6	293.7	2,445.5
20	1,677.1	781.0	1,974.7	1,045.3	294.9	2,434.0

Table A17: Multi-product capacity details: $\Delta = 1.5$.

Γ	*ZD11*	*ZD12*	*ZD21*	*ZD22*	*ZF1*	*ZF2*
0	1,510.6	642.7	1,777.8	845.2	18.5	676.6
1	1,633.5	738.6	1,874.8	940.8	9.3	730.4
2	1,700.5	830.6	1,926.3	1,074.4	12.7	755.8
3	1,727.5	924.2	1,964.8	1,178.4	28.8	792.7
4	1,710.9	1,044.4	2,034.1	1,241.9	42.6	821.6
5	1,744.3	1,056.7	2,063.2	1,277.7	73.8	887.3
6	1,710.9	1,114.1	2,041.4	1,350.0	99.9	971.8
7	1,718.2	1,190.6	2,074.8	1,403.9	139.0	939.3
8	1,729.5	1,205.7	2,117.3	1,419.3	152.8	996.1
9	1,772.1	1,194.7	2,143.9	1,450.8	160.6	1,042.6
10	1,768.0	1,188.5	2,127.3	1,506.0	203.1	1,072.3
11	1,840.1	1,232.3	2,200.7	1,557.2	128.8	1,021.3
12	1,874.8	1,327.1	2,219.7	1,705.2	83.8	870.5
13	1,880.1	1,397.6	2,203.3	1,796.7	48.4	808.3
14	1,932.0	1,375.9	2,258.7	1,771.5	45.9	776.5
15	1,949.0	1,352.3	2,247.6	1,778.5	44.3	801.1
16	1,950.8	1,388.7	2,276.4	1,794.9	43.3	732.7
17	1,977.5	1,355.5	2,273.4	1,788.9	44.2	753.8
18	1,980.2	1,369.8	2,278.6	1,803.5	36.1	731.7
19	1,988.7	1,367.4	2,288.3	1,798.9	22.1	739.1
20	1,991.7	1,360.3	2,288.5	1,790.6	22.2	753.6

Table A18: Multi-product capacity details: $\Delta = 1.75$.

Γ	*ZD11*	*ZD12*	*ZD21*	*ZD22*	*ZF1*	*ZF2*
0	1,543.7	808.1	1,811.9	1,020.7	0.0	286.9
1	1,666.0	916.6	1,919.2	1,127.2	0.0	319.5
2	1,716.3	1,056.7	1,975.7	1,275.0	3.1	304.8
3	1,762.3	1,147.8	2,004.1	1,409.8	4.4	335.3
4	1,761.0	1,280.5	2,089.5	1,492.7	3.1	335.7
5	1,819.8	1,343.2	2,163.4	1,559.9	3.7	327.5
6	1,824.7	1,432.4	2,167.1	1,663.9	15.4	335.6
7	1,831.7	1,509.5	2,208.5	1,726.9	22.5	342.9
8	1,834.9	1,551.5	2,211.0	1,794.6	42.8	377.0
9	1,871.9	1,556.7	2,245.4	1,849.3	47.5	391.7
10	1,882.3	1,563.8	2,274.6	1,900.8	61.1	406.6
11	1,927.4	1,557.4	2,314.0	1,906.1	21.4	413.5
12	1,942.1	1,565.8	2,306.5	1,933.8	7.3	410.2
13	1,952.1	1,563.9	2,279.4	1,969.0	0.0	416.1
14	1,975.6	1,541.7	2,307.9	1,941.9	0.0	419.5
15	1,982.9	1,532.9	2,290.4	1,956.2	5.3	422.5
16	1,989.5	1,529.3	2,313.1	1,939.0	0.0	426.9
17	2,008.4	1,511.4	2,314.8	1,937.8	0.0	429.1
18	2,011.1	1,518.1	2,305.8	1,957.3	0.0	412.1
19	2,015.6	1,524.7	2,314.0	1,957.3	0.0	394.4
20	2,018.5	1,521.8	2,314.1	1,957.1	0.0	395.4

Table A19: Multi-product capacity details: $\Delta = 1.9$

Γ	*ZD11*	*ZD12*	*ZD21*	*ZD22*	*ZF1*	*ZF2*
0	1,556.2	929.4	1,816.7	1,169.0	0.0	0.0
1	1,674.2	1,064.4	1,923.5	1,298.2	0.0	0.0
2	1,727.7	1,199.7	1,984.3	1,441.7	0.0	0.0
3	1,774.9	1,310.3	2,020.5	1,586.8	0.0	0.0
4	1,772.1	1,446.8	2,100.0	1,672.5	0.0	0.0
5	1,836.2	1,504.8	2,181.5	1,731.1	0.0	0.0
6	1,853.2	1,600.7	2,200.8	1,837.4	0.0	0.0
7	1,857.9	1,690.8	2,245.9	1,908.6	0.0	0.0
8	1,884.0	1,744.5	2,280.7	1,985.0	0.0	0.0
9	1,929.3	1,758.1	2,320.8	2,053.4	0.0	0.0
10	1,948.2	1,780.2	2,362.3	2,116.3	0.0	0.0
11	1,951.3	1,777.1	2,339.4	2,139.1	0.0	0.0
12	1,956.5	1,771.9	2,321.0	2,157.5	0.0	0.0
13	1,959.2	1,769.2	2,286.6	2,192.0	0.0	0.0
14	1,982.7	1,745.7	2,315.0	2,163.5	0.0	0.0
15	1,993.3	1,735.1	2,301.7	2,176.8	0.0	0.0
16	1,996.6	1,731.8	2,320.2	2,158.3	0.0	0.0
17	2,015.5	1,712.9	2,321.9	2,156.6	0.0	0.0
18	2,018.3	1,710.0	2,320.7	2,157.8	0.0	0.0
19	2,022.7	1,705.7	2,321.1	2,157.4	0.0	0.0
20	2,025.6	1,702.8	2,321.2	2,157.3	0.0	0.0

Table A20: Multi-product capacity details: $\Delta = 2.0$.

Γ	Ω	*Rob(p)*	*Rob(e)*	*Z1(p)*	*Z2(p)*	*Z3(p)*	*Z1(e)*	*Z2(e)*	*Z3(e)*
0.00	0.00	182,376.8	182,376.8	494.0	1,676.0	2,608.0	494.0	1,676.0	2,608.0
0.50	0.11	186,476.4	184,602.8	494.0	1,743.4	2,687.3	499.9	1,697.3	2,649.8
1.00	0.22	190,581.9	186,817.0	494.0	1,811.7	2,766.5	2,223.7	0.0	2,691.5
1.50	0.34	194,094.8	189,022.5	494.0	1,846.0	2,846.5	2,250.8	0.0	2,734.9
2.00	0.45	197,607.6	191,223.8	494.0	1,880.4	2,926.6	2,278.0	0.0	2,776.4
2.50	0.56	201,075.4	193,423.3	494.0	1,917.0	3,007.2	2,304.4	0.0	2,819.5
3.00	0.67	204,543.1	195,629.1	494.0	1,953.6	3,087.8	2,331.9	0.0	2,861.8
3.50	0.78	207,688.1	197,824.0	939.8	1,514.7	3,173.5	941.8	1,417.0	2,905.6
4.00	0.89	210,778.7	200,013.0	991.2	1,472.7	3,256.3	953.4	1,432.0	2,948.2
4.50	1.01	213,636.7	202,202.9	976.7	1,513.2	3,338.7	964.8	1,447.4	2,990.7
5.00	1.12	216,537.4	204,395.6	992.1	1,519.0	3,418.4	975.5	1,464.2	3,032.9
5.50	1.23	219,083.2	206,585.9	1,003.5	1,520.5	3,447.1	984.8	1,481.8	3,075.9
6.00	1.34	221,657.0	208,776.6	1,003.5	1,535.7	3,482.4	997.5	1,496.1	3,117.9
6.50	1.45	224,106.8	210,963.6	1,003.5	1,554.6	3,513.2	1,012.6	1,507.2	3,159.9
7.00	1.57	226,550.3	213,156.4	494.0	0.0	5,605.7	1,018.6	1,528.9	3,202.8
7.50	1.68	228,710.3	215,348.1	494.0	0.0	5,685.8	1,029.6	1,544.9	3,245.7
8.00	1.79	230,871.6	217,537.7	494.0	0.0	5,766.3	1,041.4	1,559.9	3,288.1
8.50	1.90	232,848.8	219,725.5	542.8	0.0	5,792.2	1,054.9	1,573.0	3,330.0
9.00	2.01	234,851.4	221,919.2	591.5	0.0	5,826.2	1,061.6	1,593.8	3,373.2
9.50	2.12	236,753.0	224,121.4	639.9	0.0	5,847.0	1,070.6	1,613.5	3,416.2
10.00	2.24	238,706.2	226,312.9	688.3	0.0	5,884.4	1,081.4	1,630.5	3,457.0
10.50	2.35	240,356.6	228,507.4	693.2	0.0	5,941.9	1,093.0	1,645.4	3,501.7
11.00	2.46	242,028.9	230,683.7	698.0	0.0	6,006.5	1,109.1	1,654.7	3,541.9
11.50	2.57	243,479.7	232,847.5	698.0	0.0	6,039.9	1,123.0	1,665.5	3,582.9
12.00	2.68	244,930.5	235,004.4	698.0	0.0	6,073.3	1,134.2	1,678.8	3,627.0
12.50	2.80	246,234.6	237,116.9	698.0	0.0	6,115.1	1,147.3	1,689.5	3,664.4
13.00	2.91	247,561.1	239,159.8	698.0	0.0	6,164.0	1,165.2	1,693.9	3,693.9
13.50	3.02	248,746.2	241,209.7	698.0	0.0	6,185.9	1,176.0	1,713.4	3,724.0
14.00	3.13	249,931.3	243,175.3	698.0	0.0	6,207.9	1,183.6	1,734.2	3,749.6
14.50	3.24	251,042.8	245,080.1	698.0	0.0	6,216.1	1,195.0	1,753.6	3,773.9
15.00	3.35	252,170.8	246,900.7	698.0	0.0	6,229.5	654.8	0.0	6,141.6
15.50	3.47	253,147.6	248,671.8	1,229.0	1,846.5	3,884.0	681.9	0.0	6,161.9
16.00	3.58	254,020.6	250,338.6	1,250.5	1,846.5	3,894.5	698.5	0.0	6,200.9
16.50	3.69	254,826.5	251,898.1	1,250.5	1,863.5	3,903.3	1,242.0	1,847.4	3,866.1
17.00	3.80	255,632.4	253,302.5	1,250.5	1,880.5	3,912.0	1,250.2	1,865.1	3,878.5
17.50	3.91	256,314.0	254,581.1	1,250.5	1,915.3	3,903.9	1,255.8	1,882.9	3,889.6
18.00	4.02	257,023.8	255,734.4	880.0	0.0	6,232.6	1,261.7	1,898.7	3,895.6
18.50	4.14	257,524.0	256,774.1	881.0	0.0	6,255.1	1,270.5	1,916.1	3,901.8
19.00	4.25	258,024.1	257,670.0	882.0	0.0	6,277.6	1,277.4	1,935.1	3,907.4
19.50	4.36	258,461.2	258,374.9	882.0	0.0	6,281.3	881.0	0.0	6,270.6
20.00	4.47	258,898.3	258,898.3	882.0	0.0	6,285.0	882.0	0.0	6,285.0

Table A21: Single product robust solution.

Γ	Ω	*E(p)*	*errE(p)*	*E(e)*	*errE(e)*
0.00	0.00	218,732.2	365.4	218,732.2	365.4
0.50	0.11	203,700.9	294.8	210,829.7	333.8
1.00	0.22	194,345.4	220.5	205,066.6	301.4
1.50	0.34	190,107.8	164.2	199,638.5	265.8
2.00	0.45	187,868.5	120.5	195,534.1	229.2
2.50	0.56	186,916.9	91.3	192,430.5	193.2
3.00	0.67	186,730.9	72.8	190,192.3	160.9
3.50	0.78	187,069.7	62.9	188,673.6	131.8
4.00	0.89	187,343.0	58.4	187,741.4	110.4
4.50	1.01	187,664.3	54.5	187,219.5	93.3
5.00	1.12	188,036.3	52.8	186,980.4	80.4
5.50	1.23	188,212.5	52.3	186,953.9	70.4
6.00	1.34	188,407.3	51.9	187,057.7	63.7
6.50	1.45	188,613.6	51.7	187,237.0	59.3
7.00	1.57	188,249.0	53.7	187,484.2	56.6
7.50	1.68	188,487.5	53.2	187,758.1	54.2
8.00	1.79	188,732.5	53.1	188,046.8	52.8
8.50	1.90	189,061.4	52.8	188,344.6	52.0
9.00	2.01	189,476.9	52.7	188,659.4	51.6
9.50	2.12	189,887.4	52.5	188,986.4	51.2
10.00	2.24	190,368.1	52.4	189,305.2	50.7
10.50	2.35	190,585.4	52.3	189,629.5	50.4
11.00	2.46	190,824.8	52.3	189,935.3	50.3
11.50	2.57	190,929.7	52.3	190,238.8	50.2
12.00	2.68	191,034.6	52.3	190,551.7	50.1
12.50	2.80	191,165.7	52.3	190,839.1	50.1
13.00	2.91	191,319.2	52.3	191,095.3	50.0
13.50	3.02	191,388.1	52.3	191,406.3	49.9
14.00	3.13	191,457.1	52.3	191,687.9	49.9
14.50	3.24	191,482.7	52.3	191,985.9	49.9
15.00	3.35	191,525.0	52.3	190,921.9	52.4
15.50	3.47	193,238.0	49.9	191,190.1	52.3
16.00	3.58	193,436.0	49.9	191,439.1	52.3
16.50	3.69	193,581.7	49.9	193,286.0	49.9
17.00	3.80	193,727.5	49.9	193,510.5	49.9
17.50	3.91	193,944.1	49.9	193,712.4	49.9
18.00	4.02	193,682.4	51.4	193,886.0	49.9
18.50	4.14	193,755.1	51.4	194,094.2	49.9
19.00	4.25	193,828.0	51.3	194,296.2	49.9
19.50	4.36	193,839.7	51.3	193,803.9	51.4
20.00	4.47	193,851.3	51.3	193,851.3	51.3

Table A22: Estimates of the single product robust solution.

Γ	Ω	*Rob(p)*	*Rob(e)*	*ZD1(p)*	*ZD2(p)*	*ZF(p)*	*ZD1(e)*	*DZ2(e)*	*ZF(e)*
0.00	0.00	173,151.9	173,151.9	2,268.0	3,016.0	0.0	2,268.0	3,016.0	0.0
0.50	0.11	177,399.7	175,183.4	2,359.8	3,159.6	0.0	2,311.5	3,070.1	0.0
1.00	0.22	181,137.1	177,214.8	1,924.4	2,586.8	1,091.0	2,355.0	3,124.2	0.0
1.50	0.34	184,301.3	179,246.3	2,543.1	3,303.0	0.0	2,398.5	3,178.3	0.0
2.00	0.45	187,593.8	181,277.0	2,158.9	2,824.3	1,091.0	2,442.0	3,238.4	0.0
2.50	0.56	190,381.1	183,241.9	2,175.5	2,842.0	1,091.0	2,485.5	3,286.5	0.0
3.00	0.67	193,102.7	185,060.5	2,299.3	3,135.4	1,177.6	2,530.2	3,340.9	0.0
3.50	0.78	195,223.6	186,873.5	2,926.2	3,929.7	0.0	2,573.9	3,395.0	0.0
4.00	0.89	197,327.9	188,686.4	2,933.5	3,970.6	0.0	2,617.6	3,449.2	0.0
4.50	1.01	199,364.2	190,479.7	2,997.3	4,094.7	0.0	2,042.3	2,803.8	1,196.5
5.00	1.12	201,573.7	192,270.9	2,421.8	3,361.5	1,250.4	2,072.4	2,846.4	1,209.2
5.50	1.23	203,532.0	194,064.6	2,432.0	3,367.4	1,281.1	2,103.1	2,888.6	1,220.9
6.00	1.34	205,535.5	195,860.1	2,442.2	3,379.0	1,314.3	2,133.8	2,931.8	1,232.9
6.50	1.45	207,331.6	197,648.9	2,449.1	3,403.1	1,310.0	2,164.9	2,974.7	1,245.7
7.00	1.57	209,174.6	199,442.2	2,472.6	3,477.4	1,317.3	2,194.9	3,015.5	1,256.7
7.50	1.68	210,781.4	201,236.4	2,485.9	3,478.8	1,327.4	2,225.9	3,058.4	1,268.6
8.00	1.79	212,466.6	203,028.2	2,510.2	3,495.3	1,344.8	2,256.4	3,101.1	1,280.8
8.50	1.90	214,027.4	204,819.3	1,655.5	2,734.7	3,023.8	2,287.0	3,142.7	1,292.4
9.00	2.01	215,544.6	206,612.1	1,659.7	2,747.6	3,028.0	2,317.7	3,185.5	1,304.8
9.50	2.12	217,027.3	208,402.0	1,664.6	2,760.4	3,042.0	2,348.3	3,227.6	1,316.7
10.00	2.24	218,513.9	210,190.3	1,669.5	2,771.2	3,063.8	2,378.8	3,269.5	1,327.6
10.50	2.35	219,902.0	211,965.6	1,679.2	2,783.9	3,101.3	2,409.2	3,311.3	1,339.6
11.00	2.46	221,316.1	213,724.8	1,684.6	2,791.8	3,124.3	2,438.0	3,352.3	1,352.6
11.50	2.57	222,583.6	215,457.6	1,688.8	2,798.4	3,134.3	2,463.9	3,391.3	1,367.2
12.00	2.68	223,868.7	217,131.2	1,696.6	2,818.5	3,154.5	1,665.9	2,710.0	2,970.7
12.50	2.80	225,060.6	218,730.2	1,702.8	2,820.3	3,157.9	1,682.0	2,733.3	3,004.6
13.00	2.91	226,252.1	220,294.5	1,709.0	2,822.1	3,161.3	1,696.5	2,751.4	3,036.6
13.50	3.02	227,345.2	221,816.9	1,718.2	2,827.3	3,170.8	1,709.1	2,763.8	3,065.0
14.00	3.13	228,344.4	223,337.4	2,922.7	4,063.5	782.0	1,719.4	2,779.8	3,088.6
14.50	3.24	229,257.7	224,802.4	2,928.7	4,071.3	793.3	2,853.7	3,990.9	769.4
15.00	3.35	230,185.9	226,256.7	2,914.5	4,079.1	817.4	2,861.4	4,003.4	787.7
15.50	3.47	231,053.3	227,674.4	2,926.1	4,080.3	823.8	2,865.2	4,037.7	802.6
16.00	3.58	231,920.9	229,032.6	2,937.5	4,081.5	830.9	2,877.0	4,055.5	818.3
16.50	3.69	232,671.3	230,321.3	2,932.8	4,081.5	845.2	2,918.6	4,057.6	814.7
17.00	3.80	233,439.4	231,552.8	2,904.4	4,081.5	877.8	2,928.9	4,065.1	828.1
17.50	3.91	234,153.5	232,697.8	2,897.2	4,081.5	893.5	2,922.8	4,077.7	844.3
18.00	4.02	234,873.1	233,763.9	2,898.1	4,081.5	904.5	2,928.2	4,075.5	861.3
18.50	4.14	235,557.6	234,745.1	2,905.5	4,081.5	904.5	2,925.9	4,078.9	877.0
19.00	4.25	236,258.7	235,640.5	2,916.4	4,081.5	904.5	2,924.2	4,081.4	893.2
19.50	4.36	236,615.0	236,447.2	2,928.2	4,081.5	904.5	2,926.5	4,081.5	904.2
20.00	4.47	236,971.3	236,971.3	2,940.0	4,081.5	904.5	2,940.0	4,081.5	904.5

Table A23: Multi-product robust solution.

Γ	Ω	$E(p)$	$errE(p)$	$E(e)$	$errE(e)$
0.00	0.00	230,838.6	517.3	230,838.6	517.3
0.50	0.11	206,098.2	361.0	219,115.2	448.2
1.00	0.22	189,455.0	274.3	209,230.5	382.3
1.50	0.34	188,187.0	186.7	201,233.7	321.1
2.00	0.45	179,876.1	84.3	196,091.0	265.9
2.50	0.56	179,776.8	82.4	192,548.3	219.7
3.00	0.67	181,579.0	67.8	188,633.9	174.3
3.50	0.78	182,218.4	66.2	185,888.3	141.1
4.00	0.89	182,362.2	65.6	184,005.7	122.0
4.50	1.01	183,032.0	64.0	181,129.0	83.1
5.00	1.12	183,228.3	64.2	180,822.8	75.7
5.50	1.23	183,453.3	63.7	180,721.0	71.3
6.00	1.34	183,737.5	63.2	180,772.0	68.9
6.50	1.45	183,843.2	63.2	180,928.7	67.4
7.00	1.57	184,284.8	62.7	181,131.4	65.8
7.50	1.68	184,411.8	62.6	181,391.2	64.9
8.00	1.79	184,702.7	62.5	181,685.7	64.3
8.50	1.90	184,900.0	62.5	182,000.6	63.8
9.00	2.01	184,973.1	62.8	182,345.6	63.3
9.50	2.12	185,069.4	62.7	182,699.3	63.0
10.00	2.24	185,170.5	62.6	183,057.3	62.7
10.50	2.35	185,322.9	62.0	183,431.6	62.5
11.00	2.46	185,418.1	61.8	183,810.4	62.3
11.50	2.57	185,481.3	61.8	184,187.0	62.2
12.00	2.68	185,640.9	61.6	184,733.0	62.2
12.50	2.80	185,684.5	61.6	184,956.1	62.0
13.00	2.91	185,728.3	61.6	185,147.8	61.9
13.50	3.02	185,804.4	61.6	185,302.2	61.8
14.00	3.13	188,946.0	59.4	185,454.1	61.9
14.50	3.24	189,081.6	59.3	188,226.3	59.6
15.00	3.35	189,212.9	59.3	188,441.8	59.5
15.50	3.47	189,314.0	59.3	188,704.0	59.4
16.00	3.58	189,419.6	59.3	188,940.1	59.4
16.50	3.69	189,496.0	59.3	189,114.4	59.4
17.00	3.80	189,593.1	59.3	189,286.7	59.3
17.50	3.91	189,670.1	59.2	189,427.1	59.3
18.00	4.02	189,751.6	59.2	189,558.5	59.3
18.50	4.14	189,785.1	59.2	189,672.6	59.3
19.00	4.25	189,834.9	59.2	189,790.2	59.3
19.50	4.36	189,888.9	59.2	189,879.2	59.2
20.00	4.47	189,943.2	59.2	189,943.2	59.2

Table A24: Estimates of the multi-product robust solution (uncorrelated demands).

Γ	Ω	$E(p)$	$errE(p)$	$E(e)$	$errE(e)$
0.00	0.00	230,202.3	294.0	230,202.3	294.0
0.50	0.11	205,615.4	242.9	218,508.9	276.2
1.00	0.22	180,320.3	57.8	208,693.5	250.6
1.50	0.34	187,898.0	138.6	200,777.2	220.9
2.00	0.45	178,667.4	25.5	195,744.4	187.7
2.50	0.56	178,710.0	24.1	192,254.6	156.1
3.00	0.67	181,237.2	16.3	188,435.4	127.1
3.50	0.78	182,193.8	16.7	185,735.2	103.1
4.00	0.89	182,341.5	16.0	183,873.4	85.6
4.50	1.01	183,024.2	14.6	179,987.1	26.9
5.00	1.12	183,086.4	14.2	180,057.6	24.6
5.50	1.23	183,354.5	14.1	180,198.3	22.9
6.00	1.34	183,673.6	14.0	180,397.1	21.1
6.50	1.45	183,776.4	13.8	180,651.5	19.6
7.00	1.57	184,225.8	13.5	180,918.3	18.5
7.50	1.68	184,360.5	13.5	181,226.0	17.7
8.00	1.79	184,662.6	13.5	181,556.0	16.8
8.50	1.90	184,896.1	13.6	181,895.6	16.1
9.00	2.01	184,969.5	13.5	182,261.2	15.6
9.50	2.12	185,065.9	13.5	182,630.3	15.1
10.00	2.24	185,166.9	13.5	182,999.9	14.6
10.50	2.35	185,319.3	13.5	183,384.7	14.3
11.00	2.46	185,414.5	13.5	183,772.1	14.0
11.50	2.57	185,477.7	13.5	184,156.2	13.8
12.00	2.68	185,636.9	13.5	184,728.8	13.7
12.50	2.80	185,680.5	13.5	184,951.9	13.6
13.00	2.91	185,724.3	13.5	185,143.7	13.5
13.50	3.02	185,800.6	13.4	185,298.2	13.4
14.00	3.13	188,922.3	12.6	185,450.3	13.3
14.50	3.24	189,061.6	12.6	188,196.1	12.7
15.00	3.35	189,198.6	12.6	188,419.0	12.7
15.50	3.47	189,300.9	12.6	188,686.0	12.7
16.00	3.58	189,407.5	12.6	188,925.8	12.7
16.50	3.69	189,485.5	12.6	189,099.5	12.6
17.00	3.80	189,584.8	12.6	189,274.1	12.6
17.50	3.91	189,662.4	12.7	189,416.4	12.6
18.00	4.02	189,744.1	12.7	189,549.3	12.6
18.50	4.14	189,777.6	12.7	189,664.2	12.6
19.00	4.25	189,827.4	12.7	189,782.4	12.6
19.50	4.36	189,881.5	12.7	189,871.7	12.7
20.00	4.47	189,935.8	12.7	189,935.8	12.7

Table A25: Estimates of the multi-product robust solution (-0.95 correlated demands).

Γ	Ω	*E(p)*	*errE(p)*	*E(e)*	*errE(e)*
0.00	0.00	230,270.1	730.3	230,270.1	730.3
0.50	0.11	205,702.1	511.7	218,555.5	637.8
1.00	0.22	198,830.7	437.1	208,766.6	546.1
1.50	0.34	187,941.7	251.9	200,865.1	457.6
2.00	0.45	182,501.5	169.1	195,769.9	367.0
2.50	0.56	182,085.9	158.8	192,262.4	296.9
3.00	0.67	181,987.9	86.2	188,415.5	238.3
3.50	0.78	182,167.2	83.4	185,706.7	199.2
4.00	0.89	182,316.1	83.1	183,842.8	170.8
4.50	1.01	182,997.6	82.1	183,987.1	176.6
5.00	1.12	183,413.5	84.3	182,902.5	150.3
5.50	1.23	183,606.3	83.6	182,214.5	128.4
6.00	1.34	183,855.4	82.8	181,823.2	110.1
6.50	1.45	183,962.9	82.9	181,661.3	95.2
7.00	1.57	184,395.9	82.6	181,656.1	87.8
7.50	1.68	184,512.8	82.4	181,769.4	84.6
8.00	1.79	184,787.3	82.1	181,965.5	83.2
8.50	1.90	184,877.2	81.4	182,219.5	82.3
9.00	2.01	184,950.8	81.2	182,518.5	82.5
9.50	2.12	185,046.9	81.0	182,839.0	82.8
10.00	2.24	185,147.8	80.9	183,175.2	82.6
10.50	2.35	185,299.6	80.9	183,532.0	82.3
11.00	2.46	185,394.4	80.8	183,896.0	82.1
11.50	2.57	185,457.5	80.8	184,258.1	81.9
12.00	2.68	185,616.8	80.8	184,712.6	81.2
12.50	2.80	185,660.4	80.8	184,934.1	81.1
13.00	2.91	185,704.2	80.9	185,124.8	81.0
13.50	3.02	185,780.3	80.8	185,278.6	80.9
14.00	3.13	188,977.5	76.1	185,430.6	80.8
14.50	3.24	189,106.2	76.0	188,267.0	76.2
15.00	3.35	189,225.1	75.9	188,470.4	76.2
15.50	3.47	189,323.3	75.8	188,723.9	76.0
16.00	3.58	189,425.8	75.8	188,952.2	75.8
16.50	3.69	189,497.0	75.7	189,128.0	75.9
17.00	3.80	189,585.0	75.4	189,294.2	75.8
17.50	3.91	189,658.8	75.3	189,428.4	75.7
18.00	4.02	189,738.2	75.2	189,554.6	75.6
18.50	4.14	189,771.8	75.2	189,664.6	75.5
19.00	4.25	189,821.5	75.2	189,778.8	75.3
19.50	4.36	189,875.6	75.2	189,865.9	75.2
20.00	4.47	189,929.8	75.2	189,929.8	75.2

Table A26: Estimates of the multi-product robust solution (0.95 correlated demands).

LB	*errLB*	*UB*	*errUB*	*gap*	*% gap*	*Z1*	*errZ1*	*Z2*	*errZ2*	*Z3*	*errZ3*
185,663.6	397.5	186,747.4	129.4	1,083.8	0.58	589.0	3.4	1,698.7	7.8	3,222.1	35.6

Table A27: Single product stochastic solution.

Corr.	*LB*	*errLB*	*UB*	*errUB*	*gap*	*% gap*	*ZD1*	*errZD1*	*ZD2*	*errZD2*	*ZF*	*errZF*
0.0	179,558.7	350.8	180,285.2	215.9	726.5	0.4	2,420.0	70.0	3,236.3	89.6	586.8	134.7
-0.95	178,484.5	123.9	178,673.1	76.2	188.6	0.1	1,690.5	91.7	2,419.7	84.1	1,731.7	153.8
0.95	179,309.7	483.3	180,388.8	209.5	1,079.1	0.6	2,781.2	26.5	3,644.9	27.3	0.0	0.0

Table A28: Multi-product stochastic solutions.

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