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Proportional signs in the works of Heinrich Schutz

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PROPORTIONAL SIGNS IN THE WORKS OF HEINRICH SCHÜTZ

A Dissertation Submitted in Partial Fulfillment
of the Requirements for the Degree of
Doctor of Arts

Jang Woo Park

College of Performing and Visual Arts
School of Music
Music History and Literature
May, 2010

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ABSTRACT

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    Some time signatures used in the *Neue Schütz Ausgabe* (Bärenreither, 1955–
2008) differ from both modern signatures and contemporary mensuration signs,
obscuring Schütz's original intentions. A review of the history of proportion signs from
the late 14th century to the 17th century shows that the four basic mensuration signs of the
late 14th century were the foundation of the proportion system throughout the period, and
that the proportion signs of the 16th and 17th century were adaptations of *modus cum
tempore* signs and fractions. Although confusion was created through misunderstandings
of the meanings of the signs and by attempts to reform the system, the original meanings
of the mensuration-proportion signs were retained throughout the period.

    A study of the proportion signs used in the *Psalmen Davids* (1619) and in the
Symphoniae Sacrae III (1650), as well as several signs found in a few of his other works,
shows that Schütz's notation is within the conventional practice of mensuration-

proportion notation. Some of Schütz's signs are open to more than one interpretation,

requiring an explanation of possible interpretations of the signs and some suggestions for

modern performance.

TABLE OF CONTENTS

LIST OF MUSIC EXAMPLES

vi

LIST OF TABLES

CHAPTER I

INTRODUCTION

At first glance, some of the meter signs used in the volumes of the Heinrich

Schütz *New Compelete Works* (*Neue Ausgabe sämtlicher Werke)* may seem strange when

viewed from the standpoint of modern notation. In fact, the signs in the complete works

volumes are not modern time signatures at all, but modified versions of the original

mensuration-proportion signs used by Schütz. The editors who transcribed the original

notation into modern modified them to convey to modern readers, scholars, and

performers the proportional relationships inherent in the original signs. Although the

introduction to the complete works provides common principles used in the transcriptions,

each editor has applied slightly different methods to the interpretation of the mensuration-

proportion signs. Some of these interpretations have gone rather far from Schütz's

intentions, even sometimes obscuring the meanings of the original signs, thus leading

performers to misunderstand the relative tempi of sections of music within a piece. In

order to solve this problem, it is necessary to recognize that Schütz followed the

traditional mensuration-proportion practice, which is strongly based on the concept of the

invariable uniform tactus and the original meanings of the mensuration signs.

Although great confusion on the interpretation of proportion signs existed

throughout the era of the proportional practice—from the early fourteenth century to the

early eighteenth century—and a strong transitional tendency from proportional practice

toward the modern notational system was a mainstream throughout the seventeenth

century, the discussions of the proportional signs presented in the writings of some of

Schütz's predecessors and contemporaries support his use of the signs within the tradition

of mensuration-proportion practice, in which the original meanings of the mensuration

signs are clearly retained: In particular the two treatises of Adriano Banchieri,

Conclusioni nel suono dell'Organo (1609) and *Cartella musicale* (1614), strongly

advocate the mensuration-proportional practice.

In this tradition, the mensuration-proportion signs are derived from the earlier

mensuration signs which are basically associated with the concept of tactus, although the

appearance of the term "tactus" in the written sources occurs much later than the

beginning of the use of the mensuration signs (around the 1250s). Giorgio Anselmi is the

earliest theorist who discusses the tactus concept in 1434 in his *De musica*. The tactus is

the regular beating of time, used from the very beginning of mensural notation; the

invariable uniform tactus concept was confirmed mainly for church vocal polyphonic

music in the late fifteenth century. Throughout the practice of the tactus, its speed was

generally considered to be equal to the pulse rate of a healthy person breathing under

normal conditions—that is, about 60 beats per minute. There are basically two kinds of

tactus: the equal and the unequal. In the equal tactus, the length of time for the down

motion and the up motion of a tactus is the same, resulting in a duple subdivision. The

unequal tactus uses the same two motions, down and up, but the down motion lasts twice

as long as the up motion, thus a triple subdivision.

The four basic mensuration signs used from around 1340— ☉, ☉, ☾, and ☾

—were replaced by the signs ☉, O, ☾, and C in the late fourteenth century; a detailed

explanation of these is given in chapter II. In the late fourteenth and the early fifteenth

centuries, only eight proportions, possibly indicated by the mensuration signs alone, were

used: *dupla* (2/1), *tripla* (3/1), *quadrupla* (4/1), *sesquialtera* (3/2), *sesquitertia* (4/3),

sesquioctava (9/8), *dupla sesquiquarta* (9/4), and *dupla superbipartiens tertia* (8/3).

Numeral fractions, which began to be used as proportion signs in the early 15th century,

gave composers a chance to try other proportions than the eight common ones. Other

types of signs, such as O2, C3, O22, C23, ☉2, or ☾3, called *modus cum tempore*,

also began to be used as proportion signs from around 1420. They indicated *modus* (the

mensuration of a longa into either perfect or imperfect subdivision) and *tempus* (the

mensuration of a breve into either perfect or imperfect subdivision); a more detailed

explanation of these signs is given in chapter II.

By about 1430, numbers and numerically or graphically modified mensuration

signs, such as 2, 3, ₵2, Φ3, ₵3, or ☉3, were added to the pre-existing

proportion signs. Around this time, the O and C signs were commonly used to

designate the *integer valor*, a standard note value to which proportions are compared,

with a semibreve tactus. Other signs were compared to them to create proportional

relationships: for example, in the proportion indicated by the C sign (*integer valor*)

followed by the C2 sign, two semibreves under C2 sign are sung in the time duration

of a semibreve under the *integer valor*, the C sign, creating the *dupla* (2/1) proportional

relationship. The addition of a stroke through any sign (called a cut sign), such as the ₵

sign or Φ sign, represents *diminutio dupla*, resulting in a doubling of the number of

basic unit notes within a tactus (see Ex.1.1), effectively doubling the tempo.

<u>Breve-tactus</u>   <u>Semibreve-tactus</u>   <u>Minim-tactus</u>
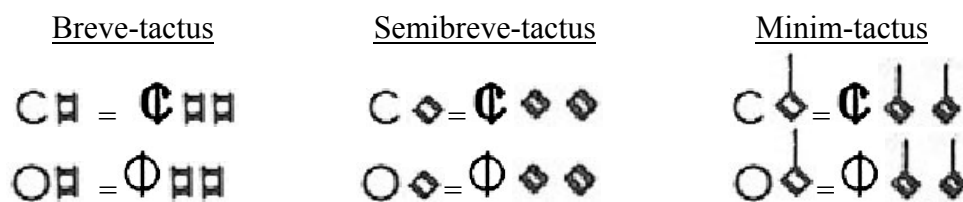
Ex.1.1 The Cut Signs

Theorists in the late fifteenth century, influenced by the rebirth of the

mathematics of Boethius, theorized complex proportions that were not adopted into

actual proportional practice. During the same period, reformers of the notational system,

represented by Tinctoris and Gaffurius, tried to change the conventionally non-

cumulative successive-proportional relationship, in which a succession of more than one

proportional relationship within a piece was interpreted as non-cumulative, to a

cumulative practice, in which each successive proportion is cumulative. This follows the

principle of the Hindu-Arabic fraction, which had become a commonly recognized

feature of mathematics. Although some followers of the reformers practiced the

cumulative relationship of successive proportions, others continued using the

conventional non-cumulative practice, creating an element of uncertainty in how to

interpret proportion signs for any given piece of music.

In addition to the pre-existing confusion of the use of proportion signs created by

individual interpretation without uniform understanding, the occurrence of the minim

tactus by the mid-sixteenth century creates even more uncertainty; notation under the

C and Φ signs became identical when using the minim tactus. Confusion about the

interpretations of proportional relationships when used when the semibreve is divided

into three (called major prolation, represented by the signs $\mathbb{C}$ and $\odot$) or into two

(called minor prolation, represented by the signs, C and O), which began in the early

fifteenth century, continued into the sixteenth and seventeenth centuries. However, the

interpretation of these relationships based on the original mensuration meanings of the

signs was never entirely given up by some composers.

By the mid-sixteenth century, the five simplest proportions—*dupla* (2/1), *tripla*

(3/1), *quadrupla* (4/1), *sesquialtera* (3/2), and *sesquitertia* (4/3)—were used most

commonly, with the more complex proportions avoided, due to their difficulty in singing.

By the late sixteenth century, the O sign was no longer used as the initiating sign

(*integer valor*), leaving the C and ₵ signs as the only initiating signs. In the early

seventeenth century, Adriano Banchieri pointed out the incorrect use of proportion signs

and provided a correct and systematic interpretation of proportion signs based on the

conventional mensuration-proportion practice, in which the original mensural meanings

of the mensuration signs, used as proportion signs, were clearly retained:

$C \to C\frac{3}{2}$, $C \to C\frac{3}{1}$, $₵ \to ₵\frac{3}{2}$, $C \to C6/4$, $C \to ₵\frac{3}{2}$, $C \to \phi\frac{3}{2}$, $₵ \to$

$\phi\frac{3}{2}$, $C \to \phi\frac{3}{1}$, $C \to ₵\frac{3}{2}$, and $₵ \to C\frac{3}{2}$. These are discussed in more detail in

Chapter II, section 3.

The proportion sign, a combination of a mensuration sign and a fraction, such as

$C\frac{3}{2}$, or just a fraction, such as $\frac{3}{2}$, or even just a number alone, such as 2, signals a

change of the number of unit notes within the tactus, with the lower number of the

fraction representing the number of unit notes in the tactus under the *integer valor*, and

the upper number representing the number of unit notes in the tactus under the proportion.

Throughout the entire era of proportional notation, six proportional indications,

$C \rightarrow C\,\frac{3}{2}$, $C \rightarrow C\,\frac{2}{1}$, $C \rightarrow C\,\frac{3}{1}$, $O \rightarrow \mathreferenced{C}$, $C \rightarrow \mathreferenced{C}\,\frac{3}{2}$, and $C \rightarrow \Phi\,\frac{3}{1}$, were

most commonly used within the conventional mensuration-proportion practice.

Nevertheless, theories about the proportional signs developed differently in

different regions; sometimes theorists even within a same region developed different

theories. The diversity of theories about proportion signs, and the fact that Schütz never

discussed his own practice, creates some difficulty in interpreting his use of the signs.

The goal of my research is to demonstrate that Schütz's use of the proportion signs

followed the tradition of the older mensuration-proportion practice inevitably associated

with the concept of tactus and the original meaning of mensuration signs.

To do this, it will be important to discuss the predominating conventions of

proportional signs with which Schütz would have been familiar. Afterwards, through the

study of exemplars of the original publications of *Psalmen Davids* (1619) and

Symphoniae Sacrae Tertia Pars (1650), I will carefully examine Schütz's use of

proportional signs and their meanings. Because these two works represent compositions

from earlier and later points in Schütz's career, it should be possible to determine if any changes in his use occurred.

This study will help me develop other alternative ways to represent proportion signs than those used by the editors involved with the Heinrich Schütz New Complete Works (*Neue Ausgabe sämtlicher Werke*), who took a middle ground between retaining the original proportion signs and using modern time signatures. In terms of both scholarly and practical use, this middle ground leaves some degree of discrepancy in the distribution of barlines, in the decision of tempo, and in the choice of modern time signatures for the original proportion signs. Two other approaches to transcription can be used that show more clearly the intended proportional relationship between tempos: The first is to transcribe the original proportion signs completely into modern time signatures, but include an indication of the tempo relationships between the different time signatures; the second is to keep the original proportion signs and note values and apply regular barlines only to the unit of a tactus. The latter method requires a thorough explanation of the original proportion system and would be more appropriate for scholarly use than for performance; nevertheless, it would be an excellent aid to musicians who are interested in historical performance and eager to perform the works of Schütz.

CHAPTER II

THE PROPORTIONAL PRACTICE AS INHERITED BY SCHÜTZ

In this chapter, I will describe the proportion practices from the beginning to the end, from the fourteenth century to the seventeenth century, as chronologically as possible to show how the practice developed and changed. Nevertheless, the primary purpose of this chapter is to trace the long convention of mensuration-proportion practice, which reached to Heinrich Schütz.

Mensuration Signs in the Fourteenth Century

The notation of the French *Ars nova* (the second half of the fourteenth century) uses four basic note values, maxima, longa, breve, and semibreve, each of which can be either perfect (triple subdivision) or imperfect (duple subdivision). The four basic mensuration signs represent the combination of divisions of the breve and semibreve into triple or duple. The earliest treatise to introduce the signs is Johannes de Muris's *Libellus cantus mensurabilis* (c. 1340): ☉ for *tempus perfectum cum prolatio maior*, ☉ for *tempus perfectum cum prolatio minor*, ☾ for *tempus imperfectum cum prolatio maior*,

and ℭ for *tempus imperfectum cum prolatio minor,*[1] which were replaced by the ⊙ , O ,

ℭ , and C signs beginning in the late 14th century and finalized in the fifteenth century.[2]

The mensuration of the breve was called *tempus* and that of the semibreve *prolatio*. Both

tempus and *prolatio* are classified into two kinds, perfect and imperfect. The perfect

tempus (*tempus perfectum*) divides a breve into three semibreves; the imperfect *tempus*

(*tempus imperfectum*) divides a breve into two. The perfect *prolatio* (*prolatio maior*)

divides a semibreve into three minims, and the imperfect *prolatio* (*prolatio minor*)

divides a semibreve into two. *Tempus perfectum cum prolatio maior* is indicated by

the ⊙ sign and *tempus imperfectum cum prolatio maior* by the ℭ sign. *Tempus*

perfectum cum prolatio minor is indicated by the O sign and *tempus imperfectum cum*

prolatio minor by the C sign (see Ex.2.1).[3]

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[1] Anna Maria Busse Berger, "The Evolution of Rhythmic Notation", in *The Cambridge History of Western Music Theory*, ed. Thomas Christensen (Cambridge: Cambridge University Press, 2002), 636-37.
[2] Willi Apel, *The Notation of Polyphonic Music 900-1600* (Cambridge: The Mediaeval Academy of America, 1953), 145.
[3] Apel, *The Notation of Polyphonic Music*, 96.

Tempus perfectum
cum prolatio maior

Tempus imperfectum
cum prolatio maior

Tempus perfectum
cum prolatio minor

Tempus imperfectum
cum prolatio minor
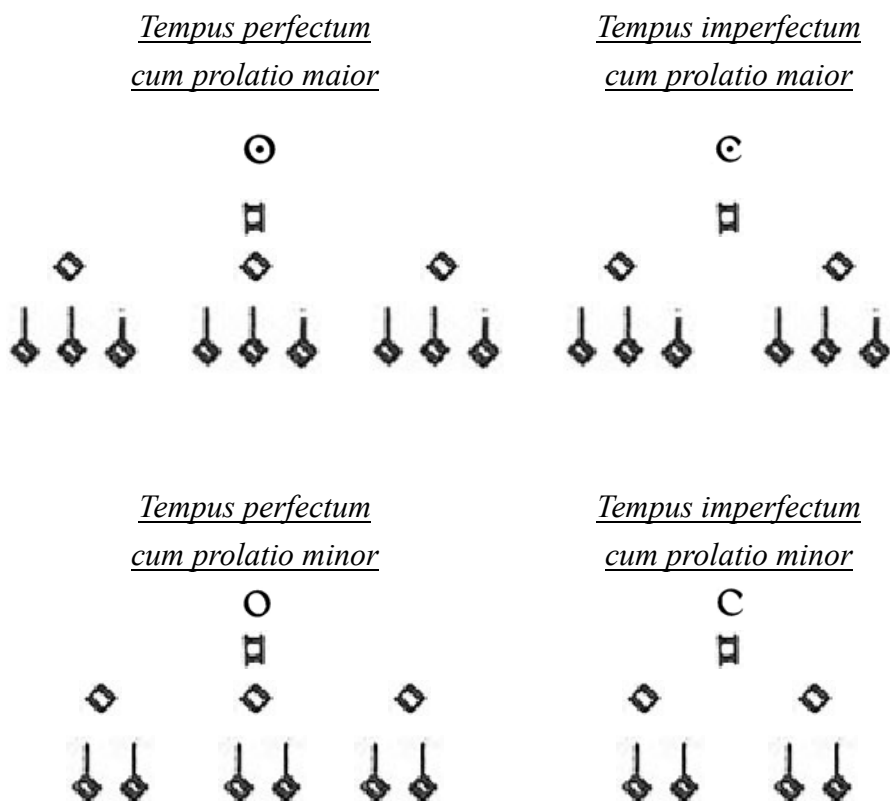
Ex 2.1 The basic four mensuration signs and their mensurations

The semibreve and the minim were understood as divisions of the breve, and the

longa as a multiplication of the breve. Thus the breve operates as the central note value.

Over the course of the fourteenth century, theorists began to speculate on the proportional

relationships between note values.[4] The earliest discussions of the *proportiones* take

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[4] Anna Maria Busse Berger, *Mensuration and Proportion Signs: Origins and Evolution* (Oxford: Clarendon Press, 1993), 33-34.

place in two different works: the treatise of Johannes de Muris, *Ars discantus secundum*

(c.1350), and in the anonymous treatise *De proportionibus* (c. 1350).[5]

The concept of a central breve is different from that of the equal breve. The

central breve is the central or standard note value of the mensuration, so larger note

values are indicated by a multiplication of it and the smaller by division. The time

duration of the central breve is variable depending on its mensuration (perfect or

imperfect). The equal breve is a kind of invariable time-length, regardless of its

mensuration.[6] However, the equal breve is not the same as the invariable uniform tactus

theorized since the late fifteenth century. The invariable time length of the equal breve is

confined within a specific proportional application. So the time duration of the equal

breve might change in different proportional applications.

The central breve practice actually began in the late thirteenth century. Following

this tradition, Johannes de Muris divided the breve into two to nine equal semibreves.

Development in innovative French Ars Nova notation shifted the central note value from

the breve to the minim.[7] In later fourteenth-century notation after Muris, the comparison

of different mensuration (perfect with imperfect) is based on the minim equality rather

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[5] J.A.Bank, *Tactus, Tempo and Notation in Mensural Music from the 13th to the 17th Century* (Amsterdam: Annie Bank, 1972), 43.
[6] Busse Berger, *Mensuration and Proportion Signs*, 81-82.
[7] Ibid., 80-81.

than breve or semibreve.[8] Some of the musical works of the *ars sublitior* style between

c.1380 and c.1420 show the earliest examples of the use of notation using mensuration

signs to indicate proportions rather than simple divisions. Mensuration signs used as

proportion signs overrode the minim-equality mensuration practice that predominated

during that period.[9] The central breve became the equal breve for proportional notation,

in which the time duration of the breve is the same and the smaller note values, such as

semibreve and minim, were compared proportionally.[10] An example of the proportional

use of mensuration signs at the end of the fourteenth century is the *sesquitertia* proportion

(4/3) on the minim level under breve equality, indicated by the ℭ sign (*integer valor*)

followed by the ℭ sign (= ¢ sign) (proportion), in which four minims in the proportion

are sung in the time duration of three minims in the *integer valor*: i.e., the initial

mensuration sign. The mensuration of the ℭ sign, *tempus imperfectum cum maior*

prolatio, divides a breve into two semibreve and successively subdivides each of the

semibreves into three minims resulting in six minims per breve. The mensuration of the

ℭ sign, a diminution by half of the *tempus imperfectum cum minor prolatio*, divides a

breve into two semibreves and successively subdivides each of the semibreves into two

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[8] Busse Berger, "The Evolution of Rhythmic Notation", 645.
[9] Roger Bowers, "Proportional Notation," *The New Grove Dictionary of Music and Musicians*, 2nd ed., ed. Stanley Sadie (London: Macmillan, 2001), vol. 20, 428.
[10] Busse Berger, "The Evolution of Rhythmic Notation", 642.

minims, after which each note value is multiplied by two, resulting in eight minims per breve. Under the equal-breve principle, the eight minims under the ↄ sign (proportion) are sung in the time duration of the six minims under the ₵ sign (*integer valor*) creating a *sesquitertia* (4/3) proportional relationship (see Ex.2.2). The *integer valor* or *integer* is the "full value" or standard value to which the note values in the proportion are compared.[11] Proportional notation then creates diversity of rhythmic expression within the mensuration system.[12]
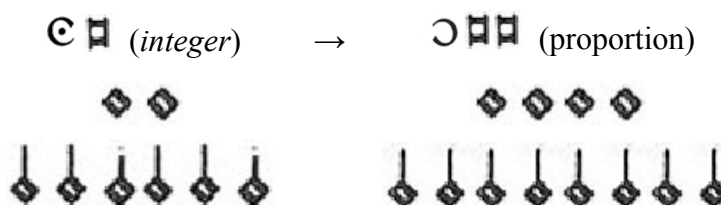
Ex.2.2 *Sesquitertia* proportion (4/3) by ₵ → ↄ on minim level in breve equality

Breve equality generally dominates throughout the fifteenth and the early sixteenth centuries until c. 1510, when the equal minim comes to the fore for the comparison of the different mensurations (perfect and imperfect), due to the success of the late fifteenth-century reform of proportional notation by Tinctoris and Gaffurius.

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[11]Bank, *Tactus, Tempo and Notation in Mensural Music from the 13th to the 17th Century*, 67-69.
[12] Margaret Bent, "Notation: Polyphonic Mensural Notation", *The New Grove Dictionary of Music and Musicians*, 2nd ed., ed. Stanley Sadie (London: Macmillan, 2001), vol. 18, 136-37.

Nevertheless, minim equality as advocated by the reformers was not newly invented, but

taken from the past—specifically speaking, from the practice of the *ars sublitior*.[13]

Fourteenth- and Fifteenth-century Practice

<u>The Tactus in the Fifteenth Century</u>

By 1430, theorists began to discuss the concept of *mensura*, *battuta*, or *tactus* as

the unit for beating time. In his treatise *De musica* (1434), Giorgio Anselmi addresses

tactus as a regular beating of time by the front of the foot, by clapping of the hands, or by

the hand on the back of the student. According to Anselmi, the tempo of the tactus is

moderate and not yet fixed as is the tactus theorized from the late 15th century, which falls

on the breve. Theorists after Anselmi advocated the tactus falling on the semibreve.[14] A

tactus, as explained in most of the 16th-century treatises, consists of two motions, usually

down-and-up or up-and-down hand motions.[15]

By the end of the fifteenth century, the concept of the invariable uniform tactus

was invented in the context of church polyphony with the actual speed of the notes

related to the tactus.[16] The speed of the tactus is equal to the pulse rate of a healthy

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[13] Busse Berger, *Mensuration and Proportion Signs*, 82-83.
[14] Ibid., 78.
[15] Bank, *Tactus, Tempo and Notation in Mensural Music from the 13th to the 17th Century*, 231.
[16] Bowers, "Proportional Notation," 429.

person breathing under normal conditions, about 60 beats per minute, and remains

constant throughout a section of a piece, no matter how the meter changes.[17]

The concept of the tactus began with the beginning of mensural music. Since the

late thirteenth century, after the time of Franco, the central breve had been the norm, with

the central breve functioning as the standard time value to which the other note values are

compared. In this way, it functioned similarly to the invariable breve tactus addressed in

treatises after the late fifteenth century. However, the central breve was actually flexible

in duration depending on the mensuration. After the time of Vitry and Marchettus, both

the central breve and the central semibreve were commonly used. The central note value

is named *mensura temporis* (time measure) in most of the treatises of the fifteenth century.

Treatises between 1450 and 1530 discuss the two or three note values that relate to the

tactus: the breve, the semibreve, and the minim. Before the invariable uniform tactus

became the norm in the 16th century, the flexible tactus was commonly practiced.[18]

Proportion Signs in the Late Fourteenth and the Early Fifteenth Centuries

The proportions commonly used from the late fourteenth to the early fifteenth

centuries were *dupla* (2/1), *tripla* (3/1), *quadrupla* (4/1), *sesquialtera* (3/2), *sesquitertia*

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[17] Mary Cyr, *Performing Baroque Music* (Portland: Amadeus Press, 1992), 30.
[18] Bank, *Tactus, Tempo and Notation in Mensural Music from the 13th to the 17th Century*, 115.

(4/3), *sesquioctava* (9/8), *dupla sesquiquarta* (9/4), and *dupla superbipartiens tertia* (8/3), which can be indicated by the mensuration signs alone. The use of mensuration signs as proportion signs during this period is generally based on breve equality to override the predominating rule of the minim equality used for mensuration. The *dupla* (2/1) proportion was indicated by C (*integer*) → Ɔ (proportion) on the level of breve, semibreve, or minim. In this proportion, a breve, two semibreves, and four minims in the *integer* correspond to two breves, four semibreves, and eight minims in the proportion respectively (see Ex.2.3). In the examples, the down and up arrows indicate the down and up motions of the tactus.
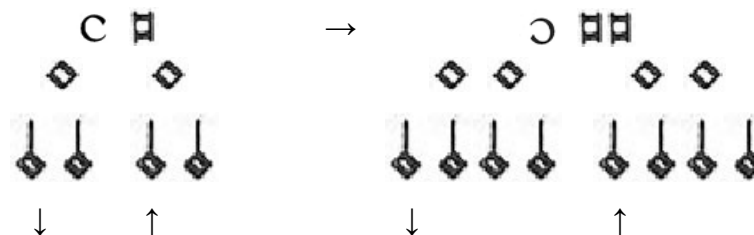
Ex.2.3 *Dupla* proportion (2/1) by C → Ɔ

The *tripla* proportion (3/1) was indicated by C (*integer*) → Φ (proportion) on the semibreve level, in which three semibreves under the proportion correspond to a semibreve under the *integer* (see Ex.2.4).

C ♮   →   Φ ♫♫
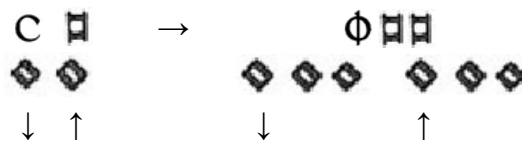
Ex.2.4 The *tripla* proportion (3/1) by   C→Φ

The *quadrupla* proportion (4/1) was indicated by   C (*integer*) → Φ (proportion) on the

semibreve level (see Ex.2.5).
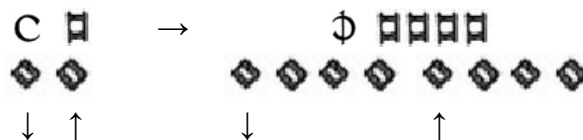
Ex.2.5 *Quadrupla* proportion (4/1) by   C→Φ

The *sesquialtera* proportion (3/2) was presented either by   C (*integer*) → O

(proportion) on the semibreve level or   C (*integer*) → ¢ (proportion) on the minim level

(see Ex.2.6).
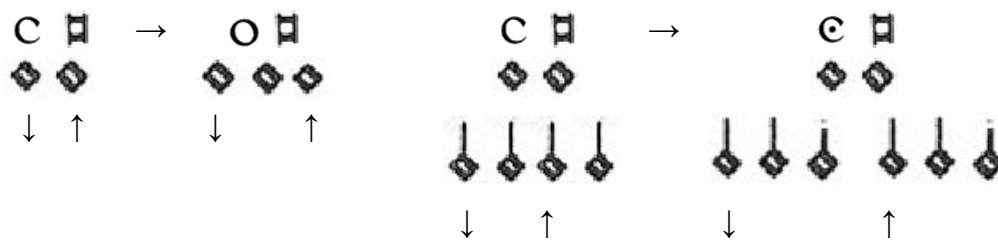
Ex.2.6 *Sesquialtera* proportion (3/2) by   C→O   and   C→¢

The *sesquitertia* proportion (4/3) was presented either by ◯(*integer*) → ☊ (proportion) on the semibreve level or by ☾ (*integer*) → ☊ (proportion) on the minim level (see Ex.2.7).
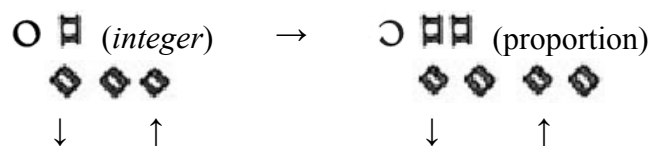
Ex.2.7 *Sesquitertia* proportion (4/3) by ◯→ ☊

The *sesquioctava* proportion (9/8) was indicated by ☊ (*integer*) → ☉ (proportion) on the minim level (see Ex.2.8).
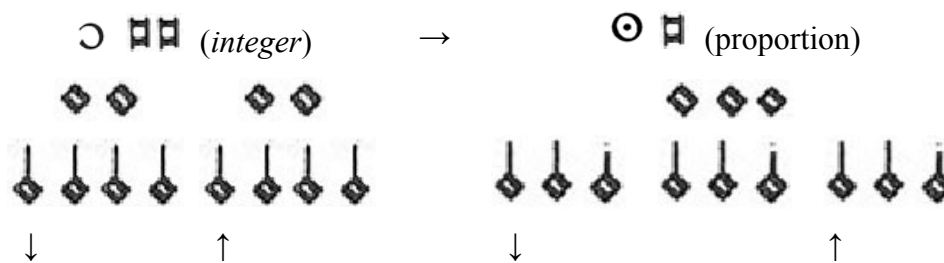
Ex.2.8 *Sesquioctava* proportion (9/8) by ☊→ ☉

The *dupla sesquiquarta* proportion (9/4) was represented by ☾ (*integer*) → ☉

(proportion) on the minim level (see Ex.2.9).
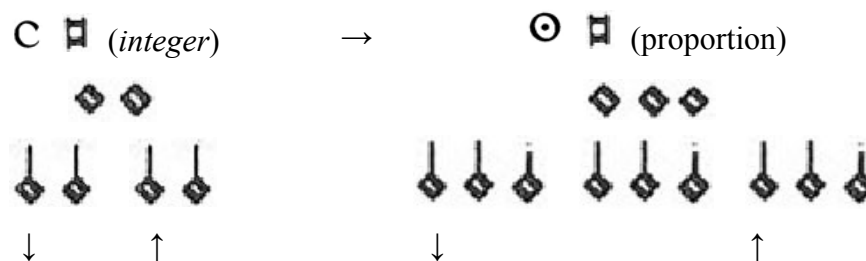
Ex.2.9 *Dupla sesquiquarta* proportion (9/4) by C → ☉

The *dupla superbipartiens tertia* proportion (8/3) was indicated by O(*integer*)

→ Φ(proportion) on semibreve level (see Ex.2.10).[19]
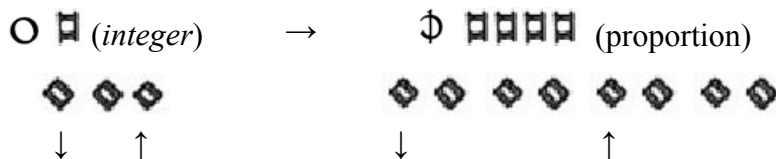
Ex.2.10 *Dupla superbipartiens tertia* proportion (8/3) O→ Φ

The Fraction

Around 1400, fractions were increasingly used as proportion signs, such as 2/1,

3/1, 3/2, 4/3, etc.[20] The fraction, which was introduced into the notation system as a

proportion sign in the early fifteenth century, functioned as a useful tool for composers to

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[19] Busse Berger, *Mensuration and Proportion Signs: Origins and Evolution*, 54-56, 166-75.
[20] Bank, *Tactus, Tempo and Notation in Mensural Music from the 13th to the 17th Century*, 65.

expand proportional possibilities beyond the basic eight proportions designated by the

mensuration signs mentioned above.[21] The earliest treatise addressing rhythmic

proportions using fractions is Prosdocimus de Beldemandis's *Tractatus practice de*

cantus mensurabilis (1408),[22] in which he discusses the *dupla* (2/1)*, tertia* (3/1)*,*

sesquialtera (3/2)*, sesquitertia* (4/3), *and dupla sesquiquarta* (9/4) proportions. [23]

Before the reform of Tinctoris in the late fifteenth century, successive proportions,

which are indicated by mensuration signs or numerical signs (single figures or fractions),

were not cumulative, but were always compared to the initiating sign (*integer valor*). For

example, with successive C , ₵ , and O signs, the ₵ sign compared to the *integer*

C sign creates *dupla* proportion (2/1) and the O sign compared to the *integer* C sign

independently creates *sesquialtera* proportion (3/2), both on the semibreve level (see

Ex.2.11).[24]
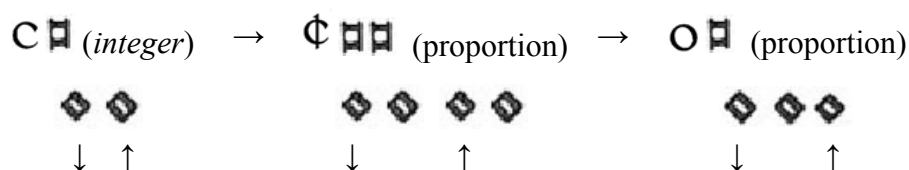
Ex.2.11 Non-cumulative proportional relationship

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[21] Busse Berger, *Mensuration and Proportion Signs: Origins and Evolution*, 166-178.
[22] Ibid., 164.
[23] Apel, *The Notation of Polyphonic Music 900-1600*, 145.
[24] Busse Berger, "The Evolution of Rhythmic Notation", 649.

This began to change in the early sixteenth century in the works of composers following

the reform.

Modus Cum Tempore Signs

The use of the *modus cum tempore* signs began from around 1420 and lasted

until the 1530s. These are combinations of a mensuration sign, such as the O, C, ℂ,

or ⊙ sign, and one or two figure, such as 2, 3, 22 or 23: O2, C3, O22, C23,

⊙2, or ℂ3, etc. There are three ways to interpret them. The usual interpretation of the

signs, supported by the largest group of theorists of the period, including John Hothby, is

that in a combination of a mensuration sign and a figure the mensuration sign represents

the *modus* (the mensuration of the longa or division into either two or three breves) and

the figure represents the *tempus* (the mensuration of the breve). For example, in the O2

sign, the mensuration sign O represents the perfect mensuration of the longa and the

figure 2 the imperfect mensuration of the breve. When a mensuration sign is combined

with two figures, the mensuration sign represents the mensuration of the maxima, the first

figure that of the longa, and the second figure that of the breve. For example, in the

C23 sign, the C indicates imperfect mensuration of the *maxima*, the first figure 2 the

imperfect mensuration of the *longa* (*modus*), and the second figure 3 the perfect

mensuration of the *breve* (*tempus*). When the ⊙ or ℭ sign is combined with a figure,

the circle or the semicircle represents the mensuration of the *longa* (*modus*), the figure

that of the *breve* (*tempus*), and the dot that of the *semibreve* (*prolatio*). For example, in

the ⊙2 sign, the Ο indicates the perfect mensuration of the *longa* (*modus*), the figure

2 the imperfect mensuration of the *breve* (*tempus*), and the dot the perfect mensuration of

the *semibreve* (*prolatio*).[25]

The second interpretation of the *modus cum tempore* signs, represented by a

small group of theorists in the late fifteenth and early sixteenth centuries, is that the

mensuration sign, the Ο or ℂ sign, always represents the mensuration of the breve

(*tempus*), the first figure the longa (*modus*), and the second figure the *maxima*. For

example, in the Ο32 sign, the Ο indicates the perfect mensuration of the breve, the

first figure 3 the perfect mensuration of the longa, and the second figure 2 the imperfect

mensuration of the maxima.[26]

In the third interpretation, represented by some theorists of the era, including

Sebald Heyden, the *modus cum tempore* signs are understood as follows: The

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[25] Busse Berger, *Mensuration and Proportion Signs: Origins and Evolution*, 20-23.
[26] Ibid., 21.

mensuration sign, the O or C sign, indicates the mensuration of the breve (*tempus*), a figure 2 the *proportio dupla* (2/1), and a figure 3 the *proportio tripla* (3/1). For example, in the O2 sign, the O sign is for the perfect mensuration of the *breve* (*tempus*) and the figure 2 the *proportio dupla* (2/1), in which two compared notes under proportion correspond to one note of the same value under *integer*. In this interpretation, the original meaning as the *modus cum tempore* sign is lost and a proportional meaning is attached to the sign.[27]

The adaption of the *modus cum tempore* signs into proportion signs, beginning in the later fifteenth century, results in the replacement of proportion signs for the older signs consisting of a mensuration sign and a fraction: For example, the C2 sign can be used instead of the C$\frac{2}{1}$ sign and the C3 sign instead of the C$\frac{3}{1}$ or the C$\frac{3}{2}$ sign.[28] However, during the almost 100 years of their use, such signs were used as either mensuration signs or proportion signs;[29] i.e., some composers use the O3, O2, C3, C2 signs as *modus cum tempore* signs while others use them as proportion signs, in which the figure 3 designates either *proportio tripla* (3/1) or *sesquialtera* (3/2) and the figure 2 *proportio dupla* (2/1).[30]

27 Busse Berger, *Mensuration and Proportion Signs: Origins and Evolution*, 21.
28 Bowers, "Proportional Notation," 428.
29 Busse Berger, *Mensuration and Proportion Signs: Origins and Evolution*, 148.
30 Bank, *Tactus, Tempo and Notation in Mensural Music from the 13th to the 17th Century*, 167.

In part to the controversial nature of the interpretation of the *modus cum tempore*

signs, after the invention of the music printing, music publishers use the simpler signs,

such as the ¢ and *sesquialtera* proportion signs, 3/2, instead of the complex *modus*

cum tempore signs, with the practical reason to sell more books to the large group of

singers who were not well educated in the use of proportion signs. In his treatise *De arte*

canendi (1540), Sebald Heyden tried to transcribe pieces with the old complex signs

using only the ¢ sign, because most musicians no longer understood the old *modus cum*

tempore signs. The Φ and ¢ signs (the cut signs) were already introduced in the late

fourteenth century as proportion signs indicating diminution by half, i.e., *dupla*

proportion (2/1).[31] Since then the ¢ and the Ɔ signs were treated as the same.

Almost throughout the fifteenth century, the O2 and C 2 signs were treated the

same as the Φ and ¢ signs respectively.[32] In the late fifteenth and the early sixteenth

centuries, the cut signs, ¢ and Φ, were interpreted in three different ways, as

diminution by one-half, diminution by one-third, or just a slightly faster tempo.[33] That

the ¢ sign and Φ sign were used to indicate either quicker performance or the

substitution of a breve for a semibreve, a semibreve for a minim, and a longa for a breve

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[31] Busse Berger, *Mensuration and Proportion Signs: Origins and Evolution*, 162-63.
[32] Busse Berger, "The Evolution of Rhythmic Notation", 647-48.
[33] Ibid., 638.

(resulting in *diminutio dupla*), is addressed by Anonymous XII (c.1460).[34]

Proportion Signs in the Fifteenth Century

By c.1430, proportion signs were diversified through the use of numbers as well as numerically or graphically modified mensuration signs, such as 2, 3, ¢2, Φ3, ℭ3, ☉3, etc.[35] The breve tactus was commonly used for the signs O, Φ, ℭ, ¢, C3, ¢3, and O3. By 1450, the semibreve tactus becomes the normal invariable tactus under the C sign , following the principle of the human pulse rate.[36] Nevertheless, throughout the century, the breve tactus and the semibreve tactus, the two common *tactus-mensurae*, were used together side-by-side.[37]

The *sesquialtera* (3/2), *dupla* (2/1), and *tripla* (3/1) proportions were most frequently used before 1450. The *sesquialtera* proportion (3/2) is indicated by the C sign as *integer valor* and the C $\frac{3}{2}$ sign as proportion on the semibreve level with a breve tactus, in which three semibreves under the proportion are sung in the time duration of two semibreves under the *integer* (see Ex. 1.12).

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[34] Bank, *Tactus, Tempo and Notation in Mensural Music from the 13th to the 17th Century*, 97.
[35] Bent, "Notation: Polyphonic Mensural Notation", 136-37.
[36] Bank, *Tactus, Tempo and Notation in Mensural Music from the 13th to the 17th Century*, 155.
[37] Ibid., 161-63.

C ◆ ◆ *(integer)*   →   C $\frac{3}{2}$ ◆ ◆ ◆ *(proportion)*

↓   ↑   ↓   ↑

Ex.2.12   C → C $\frac{3}{2}$: *sesquialtera* (3/2); semibreve level, breve-tactus

The *dupla* proportion (2/1) is indicated by the   C   sign *(integer)* followed by C $\frac{2}{1}$ or

C 2 (proportion) on the semibreve level with a semibreve tactus (see Ex.2.13).

C ◆ *(integer)*   →   C $\frac{2}{1}$ ◆ ◆ *(proportion)*

↓↑   ↓ ↑

Ex.2.13   C → C $\frac{2}{1}$: *dupla* (2/1): semibreve level, semibreve-tactus

The *tripla* proportion (3/1) is indicated by the   C sign *(integer)* followed by the   C $\frac{3}{1}$

or   C 3 sign on the semibreve level with a semibreve tactus (see Ex.2.14).[38]

C ◆ *(integer)*   →   C $\frac{3}{1}$ ◆ ◆ ◆ *(proportion)*

↓↑   ↓   ↑

Ex.2.14   C → C $\frac{3}{1}$: *tripla* (3/1); semibreve level, semibreve tactus

By 1450, the *sesquitertia* proportion (4/3) was designated by   O *(integer)*

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[38] Bowers, "Proportional Notation," 428.

→ ¢ (proportion) on the semibreve level with a breve tactus (see Ex.2.15).[39]

O�◆ ◆◆ (*integer*) →   ¢ ◆ ◆ ◆ ◆ (proportion)

↓　↑　　　　　↓　↑

Ex.2.15 O→ ¢ : *sesquitertia* (4/3)**;** semibreve level**,** breve tactus
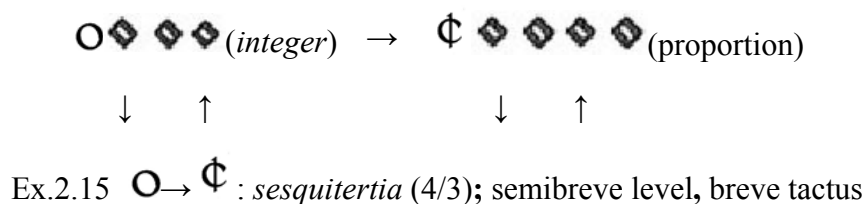
In the end of the 15th century, the *tripla* proportion (3/1) on the minim level with

a semibreve tactus was indicated by the symbol C followed by ¢ 𝟑/𝟐 (see Ex.2.16).[40]

C ◆ ◆ (*integer*) → ¢ 𝟑/𝟐 ◆ ◆ ◆ ◆ ◆ (proportion)

↓ ↑　　　　　↓　　　↑

Ex.2.16 C → ¢ 𝟑/𝟐: *tripla* proportion (3/1); minim level, semibreve tactus
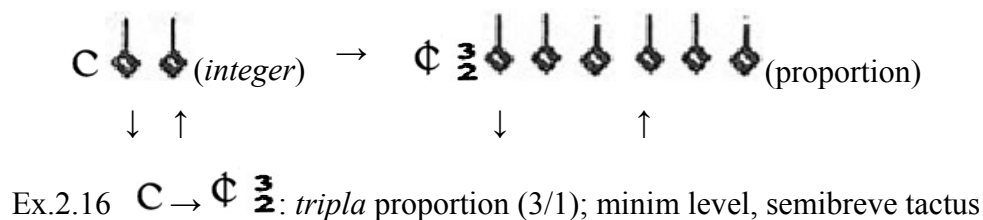
The Reform of Proportional Notation in the Late Fifteenth Century

Due to the rebirth of ancient Greek ideas, specifically the influence of the

mathematics of Boethius, late fifteenth-century treatises, including *De preceptis artis*

musice libellus of Guilelmus Monachus (c.1460), *Proportionale musices* of Johannes

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[39] Ibid.
[40] Bowers, "Proportional Notation," 429.

Tinctoris (1473), and *Practica musicae* of Franchinus Gaffurius (1497), categorize

proportions into five different species: *genus multiplex*, *genus superparticulare*, *genus*

superpartiens, *genus multiplex superparticulare*, and *genus multiplex superpartiens*. The

Genus multiplex consists of all the fractions whose denominator is 1: *proportio dupla*

(2/1), *tripla* (3/1), *quadrupla* (4/1), etc. The *genus superparticulare* consists of all the

fractions whose numerator is larger than the denominator by 1: *sesquialtera* (3/2),

sesquitertia (4/3), *sesquiquarta* (5/4), etc. The *genus superpartiens* contains the fractions

in which the numerator is larger than the denominator by two, three, etc.: *proportio*

superbipartiente tertias (5/3), *proportio supertripartiente quinta* (8/5), etc. The *genus*

multiplex superparticulare includes the fractions in which the numerator is one plus the

multiplication of the denominator and a given number: *proportio tripla sesquitertias*

((3x3+1)/3 = 10/3), *proportio quadrupla sesquiquinta* ((4x5+1)/5 = 21/5), etc. The *genus*

multiplex comprises fractions in which the numerator is the multiplication of the

denominator and a given number plus another given number: *proportio quadruple*

sesquiquinta ((4x2+3)/4 = 11/4), etc. Only simple proportions were used in actual

practice and complex proportions were retained in theory.[41]

The late fifteenth-century reformers of rhythmic notation, represented by

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[41] Apel, *The Notation of Polyphonic Music 900-1600*, 145-46.

Tinctoris and Gaffurius, tried to reform proportional notation by applying the arithmetic

rule of the Hindu-Arabic fraction against the old and contemporary proportional practice,

which was based on non-cumulative proportional relationship. Thus after Tinctoris's

reform of proportional notation, successive proportions are frequently interpreted

cumulatively following the arithmetic rule of the Hindu-Arabic fraction by followers of

the reformers. For example, with successive C , ₵ , and O signs, the *dupla* proportion

(2/1) created between the C and the ₵ signs is multiplied by the *sesquialtera*

proportion (3/2) between the C and the O signs. The result is a *tripla* proportion

(3/1) through the arithmetically cumulative process (2/1 x 3/2 =6/2=3/1) (see Ex.2.17).[42]

This contrasts to the earlier practice, where a new sign is always compared to the

initiating proportion sign, the *integer valor*.

C ☐ *(integer)* → ₵ ☐ ☐ *(2/1)* → O ☐ *(2/1 x 2/3 = 6/2)*

Ex.2.17 Cumulative proportional relationship

Because the Hindu-Arabic fraction was commonly taught and used, the

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[42] Busse Berger, "The Evolution of Rhythmic Notation", 649.

cumulative proportional relationship became a kind of trend in the early sixteenth century,

reflecting the great vogue of the use of the Hindu-Arabic fraction, and widely accepted

by composers. Nevertheless, the conventional mensuration-proportion practice based on

non-cumulative proportional relationship continued.

Sixteenth- and Seventeenth-century Practice

By the end of the fifteenth century, some composers arbitrarily used some

mensuration signs without proper knowledge of the original meanings of the signs and

used them interchangeably. Furthermore, the arbitrary use of proportion signs depending

on the personal choice of individual composers creates great confusion in understanding

their meaning. In the early sixteenth century, composers avoided a good deal of confusion

by generally using only simple proportion signs, leaving the more complex proportions

out of the actual practice. The growing tendency to use smaller note values resulted in the

need for the semibreve tactus and minim tactus as norms.[43]

The Tactus in the Sixteenth and Seventeenth Centuries

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[43] Bank, *Tactus, Tempo and Notation in Mensural Music from the 13th to the 17th Century*, 173.

By the early sixteenth century, three kinds of tactus are discussed most commonly in treatises: the unequal tactus, the breve tactus, and the semibreve tactus. The unequal tactus is the ternary tactus in which three notes fall on the down and up motions of the tactus beating: the first and second notes on the down motion and the last note on the up motion. The unequal tactus involves the *prolatio tactus* under the ₵ sign and the *tripla tactus* under the C 3 sign. The ₵ sign, following its original mensuration meaning, divides a breve into two semibreves and subdivides each of the two semibreves into three minims. When the ₵ sign as proportion follows the *integer* C sign, three minims under the ₵ sign correspond to two minims under the C sign, creating the *prolatio tactus* (see Ex.2.18). The *tripla tactus* occurs under the C 3 sign (proportion) in the proportional relationship to the *integer* C sign, in which three semibreves under the C 3 sign correspond to a semibreve under the *integer* C sign (see Ex.2.19).
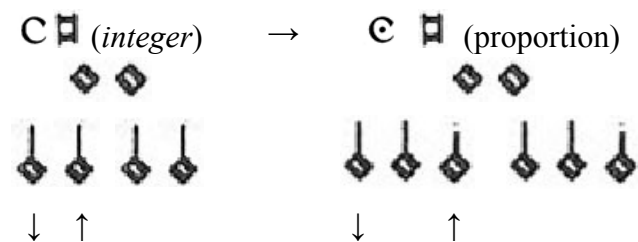
Ex.2.18 Unequal tactus, *prolatio tactus* in ₵ sign

C ◆ (*integer*)   →   C 3 ◆ ◆ ◆ (proportion)
   ↓↑   ↓   ↑
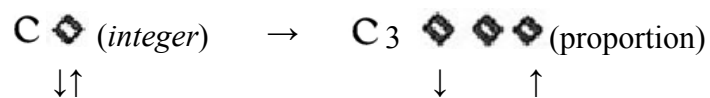
Ex.2.19 Unequal tactus, *tripla tactus* in C 3 sign (proportion)

The breve tactus occurs under the ¢ sign (proportion) in the proportional

relationship to the *integer* C sign in which two semibreve under the ¢ sign correspond to

a semibreve under the C sign (see Ex.2.20). The semibreve tactus occurs under the

integer C sign (see Ex.2.21); this is the most common tactus in both centuries.[44]

C ▯ (*integer*)   →   ¢ ▯ ▯ (proportion)

  ◆ ◆   ◆ ◆ ◆ ◆
   ↓↑   ↓ ↑
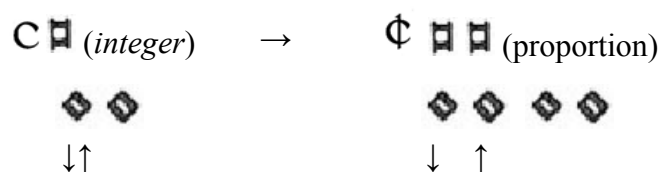
Ex.2.20 Breve tactus in ¢ sign

C ◆ (*integer*)

  ◆ ◆

  ↓ ↑
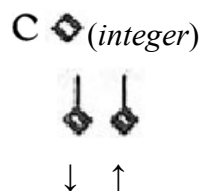
Ex.2.21 Semibreve tactus in C sign

In the first half of the sixteenth century, six sorts of tactus are also addressed by

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[44] Bank, *Tactus, Tempo and Notation in Mensural Music from the 13th to the 17th Century*, 121.

some theorists: *tactus generalis*, *sepcialis*, *semiditatis*, *augmentationis*, *diminutionis velocior*, and *diminutionis cum augmentione*. Out of the six, only the *tactus generalis* (general tactus) is used under *integer valor*, while the rest are used under proportion. The *tactus generalis* (general tactus) is the semibreve tactus in the O and C signs (see Ex.2.22). The *tactus specialis* (special tactus) is the breve tactus in the O2 and C2 signs when they are proportionally related to the O and C signs respectively; i.e., the O2 following O and the C2 following C (see Ex.2.23). The *tactus semiditatis* (half diminished tactus) is the breve tactus in the ₵ sign (see Ex.2.24). The *tactus augmentationis* (augmented tactus) is the minim tactus in the ₡ sign (see Ex.2.25). The *tactus diminutionis velocior* (diminished quicker tactus), represented by O → Φ, calls for a quickening of the tactus. The *tactus diminutionis cum augmentione* (diminution with augmentation) is the semibreve tactus in the Φ sign (see Ex.2.26).[45]

O ◻ (*integer*)   C ◻ (*integer*)
◇ ◇ ◇   ◇ ◇
↓↑   ↓↑

Ex.2.22 *tactus generalis* (*integer*), semibreve tactus in O and C signs

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[45] Bank, *Tactus, Tempo and Notation in Mensural Music from the 13th to the 17th Century*, 203-07.

O◇ (*integer*) → O2 ◈ ◇ (proportion) C ◇ (*integer*) → C 2 ◈ ◇ (proportion)

↓↑ ↓ ↑ ↓↑ ↓ ↑

Ex.2.23 *tactus specialis*, breve tactus in O2 and C 2 signs

C ◇ (*integer*) → ¢ ◈ ◇ (proportion)

↓↑ ↓ ↑

Ex.2.24 *tactus semiditatis*, breve tactus in ¢ sign

C▯ (*integer*) → ℂ▯ (proportion)

◈ ◇ ◈ ◇

↓↑ ♪ ♪ ♪

↓↑

Ex.2.25 *tactus augmentationis*: ℂ , minim tactus

C ◇ → ⊙ ♪ → φ ♪ ♪

↓↑ ↓↑ ↓ ↑

Ex.2.26 *tactus diminutionis cum augmentione*, semibreve tactus in Φ sign

In the mid-sixteenth century, Sebald Heyden addressed the idea of the invariable

semibreve tactus as a norm.[46] In his *Musicae Stoicheiosis* (1532), Heyden states that the

mensura or *tactus* consists of two equal movements to which the quantity of the note

values, both in the *integer valor* and the proportion, is arranged in such a way that the

proportioned note values in the proportion fall in the same time duration of the invariable

tactus falling on the semibreve in the *integer valor*.[47]

    In his *Dodekachordon* (1547), Heinrich Glareanus addresses the practice of a

flexible application of tactus. He explains that, to avoid the listener's weariness,

musicians quickened the tempo by adding a stroke to the sign, such as Φ or $\math␩{C}$, calling

them *diminutio*. However, in this context the term *diminutio* does not mean real

proportional diminution, but just a slightly quicker beating of the tactus.[48] An example

given by Glareanus is that sometimes the O, $\math␩{C}$, and Φ signs were used in the three

sections of Kyrie movement successively to speed up the tempi to avoid being tiresome

to the audience; the cut signs do not mean the exact duple diminution, but just a slight

quickening of the tempo.[49]

    Throughout the sixteenth and the seventeenth centuries, three basic tactus

concepts are used: *tactus maior*, *tactus minor*, and *tactus proportionatus*. The *tactus*

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[46] Bank, *Tactus, Tempo and Notation in Mensural Music from the 13th to the 17th Century*, 171.
[47] Bank, *Tactus, Tempo and Notation in Mensural Music from the 13th to the 17th Century*, 211.
[48] Ibid., 215.
[49] George Houle, *Meter in Music, 1600-1800* (Bloomington: Indiana University Press, 1987), 13.

maior (the greater tactus, the whole tactus, or the total tactus) is the true tactus, falling on

the semibreve under the *integer valor*, the full value before being proportioned: in other

words, the value to which the proportion is compared (see Ex.2.27).

C ◇

↓ ↑

Ex.2.27 *tactus maior*

The *tactus minor* or *semitactus* is twice as fast as the *tactus maior* and used by the

amateurs who cannot follow the *tactus maior* (see Ex.2.28). Both the *tactus maior* and

tactus minor are binary (equal tactus).

C ◇

↓↑ ↓↑

Ex.2.28 *tactus minor* or *semitactus*

The *tactus proportionatus* (the proportionate tactus) is the ternary tactus (unequal tactus),

used for *tripla* (3/1) or *sesquialtera* (3/2) proportions (see Ex.2.29).[50]

C ◇ (*integer*) → C $\frac{3}{1}$ ◇ ◇ ◇ (proportion)   C ◇ (*integer*) → C $\frac{3}{2}$ (proportion)

↓↑   ↓   ↑
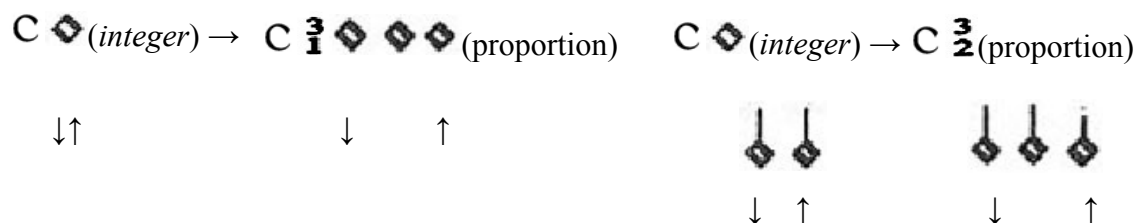
Ex.2.29 *tactus proportionatus* in the proportion

Again the tactus is invariable; in other words, the time duration of the tactus is fixed.

According to Dowland in late sixteenth century, the equal tactus features two

minims, four semiminims, eight eighth notes, or sixteen sixteenth notes within one

tactus.[51] According to Zarlino, the equal tactus is used in the O, C, Φ and ₵

signs, which designate the imperfect semibreve mensuration (*prolatio minor*). The

unequal tactus is used with the ⊙, ℭ, Φ, and ₵ signs, in which the dot designates

the perfect semibreve mensuration (*prolatio maior*).[52] One important point to make is

that most sixteenth-century theorists discuss the tactus in strict relation to mensural

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[50] Houle, *Meter in Music, 1600-1800*, 4.
[51] Houle, *Meter in Music, 1600-1800*, 4.
[52] Ibid.

notation: in other words, retaining the original meaning of the mensuration signs. This tendency continues in the seventeenth century, at least the first half of the century, as a mainstream idea.[53]

Sixteenth-century theorists remained in agreement about the speed of the tactus: As in earlier eras, the tempo of the *tactus maior* or the *tactus proportionatus* was MM 60-80, following the normal human pulse rate under normal conditions.[54] Nevertheless, according to Mersenne in the early seventeenth century, the tempo of the tactus varies according to the affection of the music, including the character, words, and emotion of the music.[55]

In sum, two different classifications of the tactus were used throughout the sixteenth and the seventeenth centuries. In the first classification, the tactus is divided into three kinds depending on the choice of the unit note-value: the breve-tactus, the semibreve-tactus, and the minim-tactus. In the breve tactus the tactus falls on the breve, in the semibreve tactus on the semibreve, and in the minim tactus on the minim. The time duration of the tactus is the same regardless of which kind of note it falls on; in other

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[53] Ibid.
[54] Houle, *Meter in Music, 1600-1800*, 5.
[55] Ibid.

words, the time duration of the breve tactus, the semibreve tactus, and the minim tactus is

the same (see Ex.2.30).[56]

Breve-tactus	Semibreve-tactus	Minim-tactus
¢ ☐ *(integer)*	C ◆ *(integer)*	ℂ ◆ *(integer)*
↓↑	↓↑	↓↑
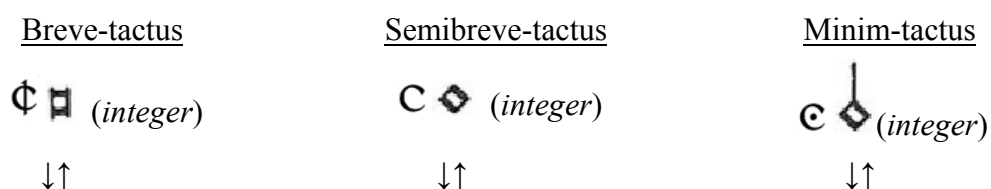
Ex.2.30 Tactus on different note values

Under the second classification, there are actually two kinds of tactus: the equal

or binary tactus and the unequal or ternary tactus (see Ex.2.31). In the equal tactus, the

time duration of the down motion is exactly as long as the up motion. For example, the

equal tactus falling on the semibreve is used in the imperfect mensuration of the

semibreve under *integer valor*; i.e., a *semibreve* is divided into two equal minims. Thus

the down hand motion falls on the first minim and the up hand motion on the second

minim. The unequal tactus uses the same two motions, down and up, but the down

motion lasts twice as long as the up motion. This is used in perfect mensuration. For

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[56] Bowers, "Proportional Notation," 428.

example, the unequal semibreve-tactus consists of 3 minims; the first and second minims

are indicated by the down motion of the hand and the third by the up motion.[57]

<div align="center">
Equal-tactus   Unequal-tactus
</div>

Ex.2.31 Equal-tactus and Unequal-tactus at the semibreve level

In the early seventeenth century, at the beginning of the Baroque era, innovations

in musical style and notation took place primarily in instrumental music and solo vocal

music, whereas the conservative stream of the old notational tradition based on

mensuration and proportion continued in the mainstream, especially for polyphonic vocal

ensemble works.[58]

Proportion Signs in the Sixteenth Century

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[57] Arnold Dolmetsch, *The Interpretation of The Music of the XVII and XVIII Centu*ries (London: Oxford University Press, 1946), 28-32.
[58] Houle, *Meter in Music, 1600-1800*, 8.

Many medieval proportion signs were used throughout the sixteenth century; however, individual applications of the signs beyond their original meanings created confusion.[59] By mid-sixteenth century, the occurrence of the minim tactus caused confusion in the use of proportional notation. For example, notation under the

C and ₵ signs is identical in the minim tactus (see Ex.2.32); the only way to distinguish between them is to use them in the *dupla* proportional relationship (see Ex.2.33).[60]

Ex.2.32 C and ₵ signs in the minim tactus

C ◇ (*integer*)   →   ₵ ◇ ◇ (proportion)

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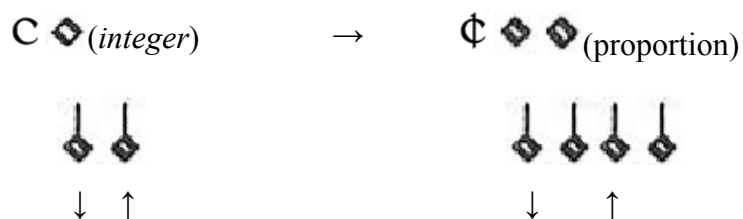
[59] Richard Rastall, *The Notation of Western Music*, 113.
[60] Richard Rastall, *The Notation of Western Music*, 188.

Ex.2.33 *dupla* proportional relationship of C → ¢

From the early fifteenth century, there was confusion about the relationships

between the major prolation or *prolatio perfecta* signs (ℭ and ☉), in which a

semibreve is divided into three minims, and the minor prolation or *prolatio imperfecta*

signs (C and O), in which a semibreve is divided into two minims, indicated by

C → ℭ and O → ☉ . A majority of theorists advocated the *sesquialtera* proportional

relationship on the minim level with a semibreve tactus, following the original meanings

of the mensuration signs (see Ex. 2.34 and Ex. 2.35).[61]
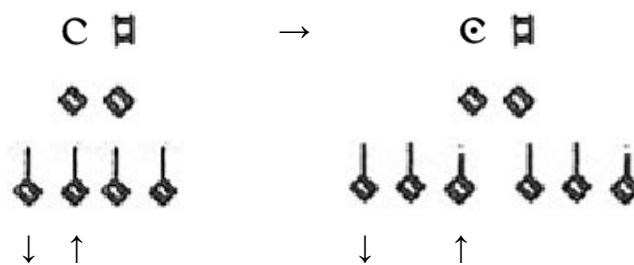
Ex.2.34 *Sesquialtera* proportional relationship by C → ℭ
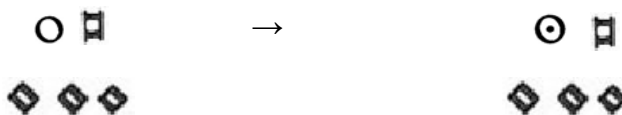
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[61] Busse Berger, *Mensuration and Proportion Signs: Origins and Evolution*, 101.
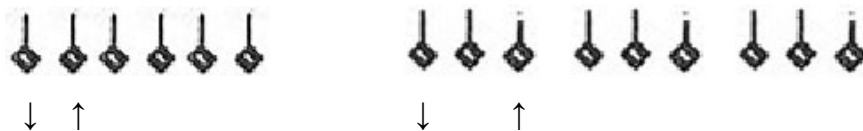
Ex.2.35 *Sesquialtera* proportional relationship by O→ ☉

On the other hand, there were five other possible interpretations of the

relationship between the major prolation and minor prolation signs, which do not retain

the original mensuration meaning of the signs. The first was based on minim equality,

thus a minim under the minor prolation signs (C or O) corresponded to a minim under

the major prolation signs (₵ or ☉) (see Ex.2.36).

Ex.2.36 Interpretation 1

The second interpretation was that a minim under the major prolation signs (₵ or ☉)

was equal to a breve under the cut minor prolation signs (₵ or Φ) (see Ex.2.37).

Ex.2.37 Interpretation 2

In the third interpretation, a minim under the major prolation signs (ℭ or ⊙)

corresponded to a semibreve under the cut minor prolation signs (¢ or Φ) (see Ex.2.38).

$$ \text{ℭ}_{\text{or}} \ ⊙ \ \downarrow \ = \ \text{¢}_{\text{or}} \ Φ \ ◇ $$

Ex.2.38 Interpretation 3

In the fourth interpretation, three minims under the major prolation signs (ℭ or ⊙)

corresponded to four minims under the minor prolation sign   C , creating *sesquitertia*

(see Ex.2.39).[62]

$$ \text{ℭ}_{\text{or}} \ ⊙ \ ◇ \quad \rightarrow \quad \text{ℭ} \ \square $$
$$ \downarrow\downarrow\downarrow \qquad \downarrow\downarrow\downarrow\downarrow $$

Ex.2.39 Interpretation 4

The fifth interpretation occurred when the major prolation signs (ℭ and ⊙) were used

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[62] Busse Berger, *Mensuration and Proportion Signs: Origins and Evolution*, 87-89.

for augmentation, in which the minim tactus under the major prolation signs was used

instead of the semibreve tactus under the minor prolation sign C ; in other words, a

minim under the major prolation signs (C and ⊙) was equal to a semibreve under the

 C sign (see Ex.2.40). With the *prolatio perfecta diminutionis cum augmentione* or

prolatio maior diminuta signs (¢ and Φ), a minim under the major prolation signs

(C and ⊙) was equal to a semibreve under the *prolatio maior diminuta* signs (¢ and

Φ) (see Ex.2.41).[63]

$$C \boxvoid \; \lozenge \; = \; C_{\text{ or}} \; \odot \; \downarrow$$

Ex.2.40 Interpretation 5: augmentation under C or ⊙

$$C_{\text{ or}} \; \odot \; \downarrow \; = \; ¢_{\text{ or}} \; Φ \; \lozenge$$

Ex.2.41 Augmentation under ¢ or Φ

In the sixteenth century, the cut signs, Φ and ¢ , were interpreted by some

musicians as either *diminutio semiditas* (half diminution) or *diminutio per tertiam partem*

(diminution by 1/3, leaving 2/3 of the original value), whereas others, such as Glareanus,

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[63] Bank, *Tactus, Tempo and Notation in Mensural Music from the 13th to the 17th Century*, 203-07.

interpreted them as indicating a slightly faster tempo instead of one exactly two times

faster. Regarding proportion signs derived from *modus cum tempore* signs, the O2 and

C2 signs were interpreted as either *diminutio per mediam partem* or *semiditas* like the

cut signs, while the O3 and C3 signs were interpreted as either *proportio tripla* (3/1)

or *sesquialtera*(3/2).[64] Practically, there was no uniform understanding or uniform

application of the proportion signs for the proportional relationships. The fact that the

choice of the proportion signs depended on the personal choice of the composers resulted

in different interpretations of them. In other words, composers arbitrarily selected

proportion signs that, in their opinion, fitted their compositions best.[65]

Fractions, modified mensuration signs, and a combination of a mensuration sign

and a figure were used even for the simplest proportions, *proportio dupla* and *proportio*

tripla: the ⁊, Ɔ, ₵, C2, Φ, and O2 signs for *proportio dupla*, and the ⁈, C3

and O3 for *proportio tripla*.[66] In his treatise *A Plain and Easy Introduction to Practical*

Music (1597), Thomas Morley mentions that five proportions – *dupla* (2/1), *tripla* (3/1),

quadruple (4/1), *sesquialtera* (3/2), and *sesquitertia* (4/3) – were used most commonly,

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[64] Bank, *Tactus, Tempo and Notation in Mensural Music from the 13th to the 17th Century*, 203-07.
[65] Bank, *Tactus, Tempo and Notation in Mensural Music from the 13th to the 17th Century*, 171-73.
[66] Apel, *The Notation of Polyphonic Music 900-1600*, 147.

while the more complex proportions were avoided due to their difficulty in singing.[67]

The *sesquitertia* proportion (4/3) is represented by ₵ as *integer valor* followed by ↄ

as a proportion: The *integer* ₵ sign retains the original mensuration meaning of the

tempus imperfectum cum prolatio maior, with three minims to the semibreve tactus, and

the proportion ↄ sign keeps its original mensuration meaning of the *alla breve* of the

tempus imperfectum cum prolatio minor, with four minims to the breve tactus. In other

words, the *sesquitertia* proportion (4/3) indicated by the ₵ (*integer*) → ↄ (proportion)

signs is on the minim level in proportional relationship in the semibreve tactus of the

integer. (see Ex.2.42).[68]
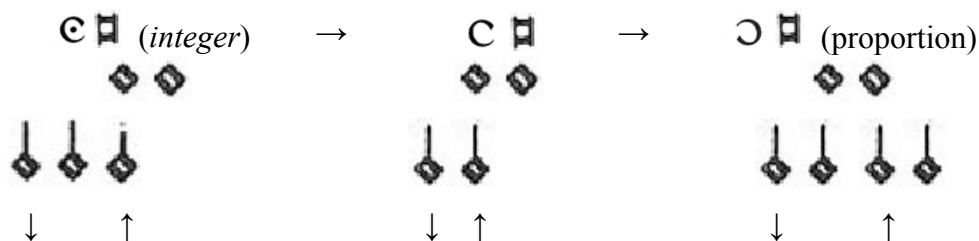
Ex.2.42 The *sesquitertia* (4/3) proportional relationship between ₵ (*tempus imperfectum cum prolatio maior* in semibreve tactus) and ↄ (*alla breve* of the *tempus imperfectum cum prolatio minor* in breve tactus)

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[67] Thomas Morley, *A Plain and Easy Introduction to Practical Music*, ed. R. Alec Harman (New York: W. W. Norton & Company, 1966), 47.
[68] Apel, *The Notation of Polyphonic Music 900-1600*, 151.

In the *tripla* (3/1) proportional relationship between the C sign and C 3 sign,

proportio tripla of *tempus imperfectum*, the C sign in the *integer valor* retains the

original mensuration of the *tempus imperfectum cum prolatio minor*, and the C in the

C 3 proportion sign preserves the original mensural meaning of the minor prolation

(imperfect semibreve). The mensuration of the breve in the proportion under the C 3

sign could be perfect or imperfect depending on the context. The *tripla* proportional

relationship occurs basically on the semibreve level in a semibreve tactus: i.e., one

semibreve in the *integer* in the time of three semibreves in the proportion (see Ex.2.43).[69]

C ◻ (*integer*)   →   C 3 (proportion)
◇ ◇   ◇ ◇ ◇
↓↑   ↓   ↑
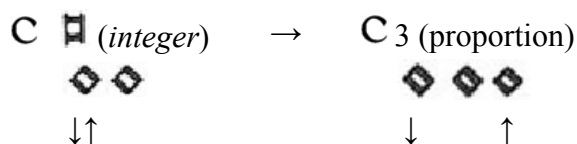
Ex.2.43 *Tripla* (3/1) proportional relationship between the C and C 3 signs

In the *tripla* (3/1) proportional relationship between the O and O3 signs

(*proportio tripla* of *tempus perfectum*) on the semibreve level with a semibreve tactus,

the O sign in the *integer valor* retains the original mensuration of the *tempus perfectum*

cum prolatio minor. The O sign in the O3 proportion sign keeps the original

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[69] Ibid., 155-56.

mensuration meaning of minor prolation. (see Ex.2.44).[70]

O ◻(*integer*)   →   O3 (proportion)
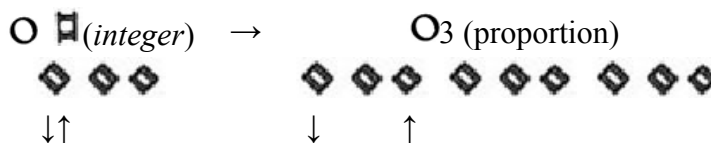
Ex.2.44 *Tripla* proportional relationship between O and O3 signs

In the *proportio quadrupla* (4/1), indicated by 4/1, 4, C 4/1, C 4, O4/1, O4,

¢ 2, Φ 2, Ɔ 2, Ɔ 2/1, Ɔ ¢ , or Φ signs in the proportion, the *integer valor* is

given a semibreve tactus, and the proportion, under one of the signs above, has four

semibreves per tactus. The mensuration of the breve is always imperfect and the tactus

falls on a *longa* in the proportion, so it is called *alla longa* (see Ex.2.45).[71]

C ◻ (*integer*)   →   C 4/1 ◻ ◻ (proportion)
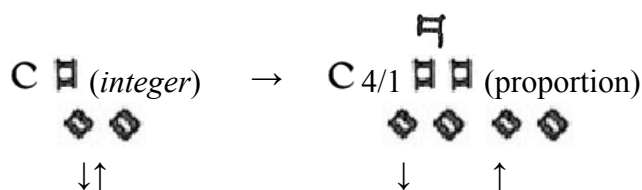
Ex.2.45 *Proportio quadrupla*

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[70] Apel, *The Notation of Polyphonic Music 900-1600*, 157.
[71] Apel, *The Notation of Polyphonic Music 900-1600*, 157-58.

The *proportio sesquialtera* (3:2), indicated by the $C\frac{3}{2}$, $O\frac{3}{2}$, $\frac{3}{2}$, or 3 signs in the proportion, can be presented in the breve tactus or the semibreve tactus. The *sesquialtera* proportion (3/2), indicated by the $C \rightarrow C\frac{3}{2}$ signs, takes place on the semibreve level in the breve tactus or on minim level with a semibreve tactus (see Ex.2.46).

$$C \ \square \quad \rightarrow \quad C\frac{3}{2} \ = \ \frac{3}{2} \ = \ 3$$
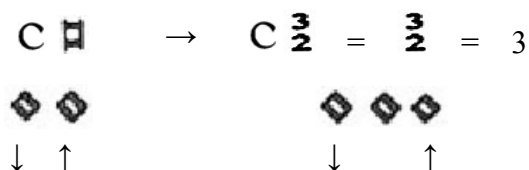
Ex.2.46 *Sesquilatera* (3/2) by $C \rightarrow C\frac{3}{2}$, semibreve level, breve tactus

The *sesquialtera* proportion (3/2), indicated by the $O \rightarrow O\frac{3}{2}$ signs, is on the minim level with a semibreve tactus (see Ex.2.47).

$$O \ \square \quad \rightarrow \quad O\frac{3}{2} \ = \ \frac{3}{2} \ = \ 3$$
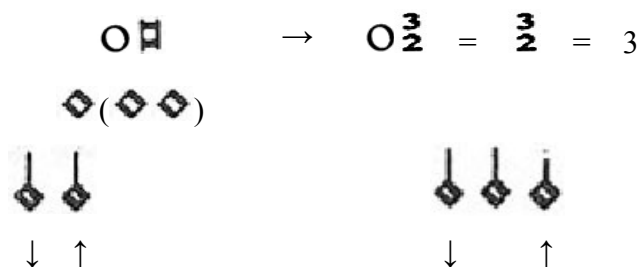
Ex.2.47 *Sesquilatera* (3/2) by $O \rightarrow O\frac{3}{2}$, minim level, semibreve tactus

The *sesquialtera diminuta* proportion (6/2), indicated by the $C \rightarrow \math01{\mathbb{C}} \; \frac{3}{2}$ signs, occurs on the semibreve level with a breve tactus, through the double operation of the two proportion signs in the proportion: the $\frac{3}{2}$ sign for *sesquialtera* (3/2) and the $\math01{\mathbb{C}}$ sign for *diminuta* (2/1), resulting in *sesquialtera diminuta* (6/2) (see Ex.2.48).[72]

$$C \; \square \; (integer) \quad \rightarrow \quad \frac{3}{2} \quad \rightarrow \quad \math01{\mathbb{C}} \quad = \quad \math01{\mathbb{C}} \; \frac{3}{2} \; (proportion)$$
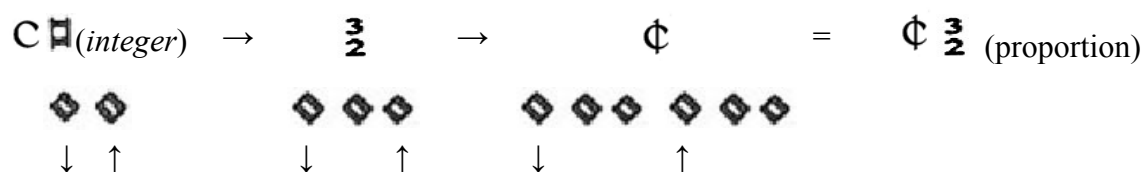
Ex.2.48 *Sesquialtera diminuta* (6/2) by $C \rightarrow \math01{\mathbb{C}} \; \frac{3}{2}$, semibreve level, breve tactus

The long tradition of using the mensuration-proportion signs within the original mensural meanings continued throughout the sixteenth century, together with the confused use of the signs, which was caused either by lack of proper knowledge of the mensuration-proportion convention or by attempts to establish a new standard of proportional practice or to theorize some complex proportional relationships for the sake of just theoretical. The convention of the mensuration-proportion practice continued into the seventeenth century.

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[72] Apel, *The Notation of Polyphonic Music 900-1600*, 158-59.

Proportion Signs in the Seventeenth Century

The transition from proportional notation to modern notation took place over the course of the seventeenth century. Nevertheless, the old conventions of the mensuration-proportion practice, based on the original mensural meanings and the invariable uniform tactus, continued at least until the mid-century. In the early seventeenth century, some composers used the minim tactus under the C or O signs, while others applied the minim tactus under the $\mathvarphi$ or ϕ signs.[73] Some composers maintained the exact proportional meaning of the C, $\mathvarphi$ and ϕ signs as 4:2:1 proportions, whereas others

interpreted them as relative tempo changes: the C sign represents a slow tempo; the $\mathvarphi$ sign a moderate tempo; and the ϕ sign a brisk tempo.[74]

In his treatise *Conclusioni nel suono dell'Organo* (1609), Adriano Banchieri presents a discussion of tactus and some proportions, in which he uses the term *tempo perfetto* for the tempo of the *tactus alla breve* (breve tactus) in $\mathvarphi$ sign, and *tempo imperfetto* for the tempo of the *tactus alla semibreve* (semibreve tactus) in C sign. Because his tactus for the proportions is the same invariable time duration that originated at the end of the fifteenth century, the time duration of both the *tempo perfetto* and the

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[73] Rastall, *The Notation of Western Music*, 188.
[74] Rastall, *The Notation of Western Music*, 188-89.

tempo imperfetto is actually the same. The *tactus equalis* is simply the binary tactus, and

the *tactus inequalis* the ternary.[75] Banchieri no longer uses the ◯ sign (*tempus*

perfectum) as the initiating sign (*integer valor*). Also at this time the C and ¢ signs,

when used as *integer valor*, are treated equally except for the tactus: the semibreve tactus

for the C sign and the breve tactus for the ¢ sign (see Ex.2.49).[76]
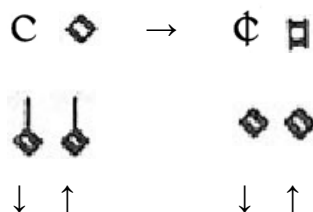
Ex.2.49 Semibreve tactus in C sign (left) and breve tactus in ¢ sign (right)

Banchieri indicates the *sesquialtera* proportion (3/2) either by ¢ → **3/2** on the semibreve

level in the breve tactus, or by C → **3/2** on the minim level in the semibreve tactus (see

Ex.2.50). The *tripla* proportion (3/1) is designated by C → **3/1** on the semibreve level in

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[75] Roger Bowers, "Proportioned Notation in Banchieri's Theory And Monteverdi's Music", *Performing Practice in Monteverdi's Music: The Historic – Philological Background*, ed. Raffaello Monterosso (Cremona: Fondazione Claudio Monteverdi, 1995), 56-57.

[76] Ibid., 56.

the semibreve tactus (see Ex.2.51).[77]
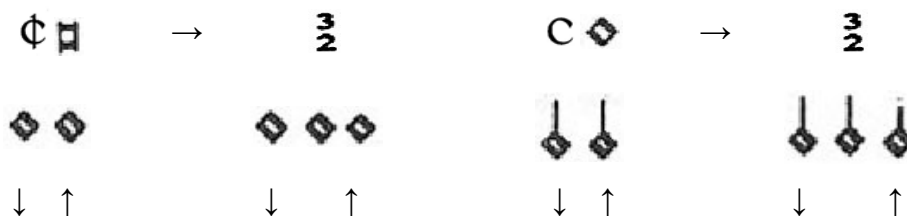
Ex.2.50 *Sesquialtera* (3/2) by $\Phi \to \frac{3}{2}$ in the breve tactus and by $C \to \frac{3}{2}$ in the semibreve tactus
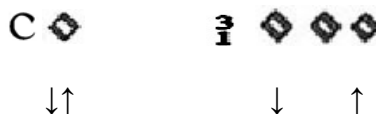
Ex.2.51 *Tripla* (3/1) by $C \to \frac{3}{1}$ in the semibreve tactus

In another treatise, *Cartella musicale* (1614), Banchieri tries to rectify other composers' incorrect usages of the proportional signs by describing their precise, rational, and conventional system. Banchieri believed that confusion in proportional notation was due to the arbitrary use of proportion signs by composers who had an incorrect understanding of the system of proportional notation.[78] Banchieri explained systematic proportion signs in the following way: The proportion signs are clearly and strictly related to the *integer valor*; when a mensuration sign has no proportional meaning, the

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[77] Roger Bowers, "Proportioned Notation in Banchieri's Theory And Monteverdi's Music," 59-60.
[78] Roger Bowers, "Proportioned Notation in Banchieri's Theory And Monteverdi's Music," 66.

mensuration is based on semibreve equality to what came before. For example,

the C sign followed by the O sign and the ₵ sign followed by the Φ sign do not have

proportional meaning, only mensurational. In the former (C→O) the C sign indicates

the *tempus imperfectum* mensuration, the duple division of the breve into the two

semibreves, and the O sign indicates the *tempus perfectum* mensuration, the triple

division of the breve into the three semibreves, in semibreve equality (see Ex.2.52).[79]

Each of the cut signs (₵ , Φ) indicates *diminutio dupla* to the former: Thus the ₵ and

Φ signs are *diminutio dupla* of the C and the O signs respectively.

C ▯   →   O ▯

◈ ◈   ◈ ◈ ◈

↓↑   ↓↑
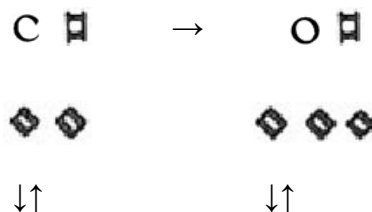
Ex.2.52 C→O, without proportional meaning in semibreve equality

On the other hand, when the ₵ sign is followed by the Φ $\frac{3}{2}$ sign, a

proportional relationship is indicated, with the two semibreves under the ₵ sign

(*integer valor*) counted in the breve tactus during the time of the three semibreves under

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[79] Ibid., 67.

the $\Phi\frac{3}{2}$ sign (see Ex.2.53). In this case only the $\frac{3}{2}$ sign in the $\Phi\frac{3}{2}$ sign indicates

the proportion, while the Φ sign only indicates the result of the *sesquialtera* proportion,

the triple division of the breve, but does not affect the proportional process.[80]

$$\mathbb{C}\,\square \quad \rightarrow \quad \Phi\,\frac{3}{2}\,\square$$
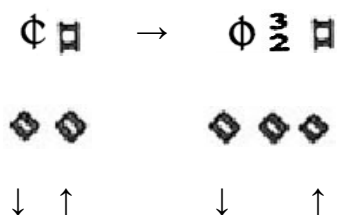
Ex.2.53 The single operation of the proportion signs in $\mathbb{C} \rightarrow \Phi\,\frac{3}{2}$ in breve tactus

Although the $\mathbb{C}$ and Φ signs can be equally treated as *integer valor*, when the Φ sign

follows $\mathbb{C}$, it designates the *proportio dupla* in the overall proportional context (see

Ex.2.54).[81]

$$\mathbb{C}\,\Diamond \quad \rightarrow \quad \mathbb{C}\,\square$$
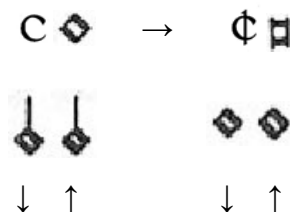
Ex.2.54 The *dupla* proportional relationship of $\mathbb{C} \rightarrow \Phi$ in overall proportional context

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[80] Roger Bowers, "Proportioned Notation in Banchieri's Theory And Monteverdi's Music," 67.
[81] Roger Bowers, "Proportioned Notation in Banchieri's Theory And Monteverdi's Music," 75.

All the diminution signs or cut signs affect the proportional relationship. For

example, when the C sign is followed by the $\Phi\,\frac{3}{2}$ sign, the cut sign, Φ, indicates the

dupla proportion (2/1) and the diminution sign, $\frac{3}{2}$, indicates the *sesquialtera* proportion

(3/2) resulting in the *tripla* proportion (3/1) by the double operation: 3/2 x 2/1=3/1 (see

Ex.2.55).[82]
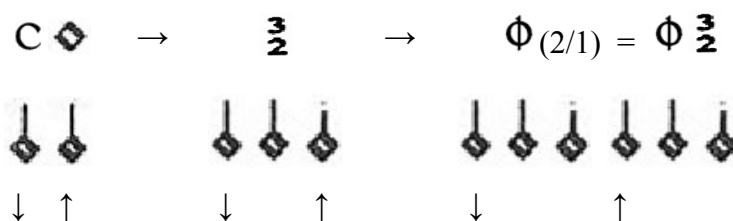
Ex.2.55 The double operation of the proportion signs in $C \rightarrow \Phi\,\frac{3}{2}$ in semibreve tactus

Based on the principles mentioned above, ten different proportional indications

are possible using proportion signs made up of the combination of a mensuration sign and

a fraction; these are shown in Table 2.1.

In the first four proportions in Table 1, the proportional relationships are

determined only by the numerical fractions, because the same mensuration signs, C and

$\mathrm{\mathcal{C}}$, are used in both the *integer* and the proportion. In such cases, the omission of the

mensuration sign of the proportion does not affect the result; for example, the first

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[82] Ibid.

proportion in Table 2.1, $C \rightarrow C\frac{3}{2}$ indicates the *sesquialtera* proportion (3/2) on the minim level in the semibreve tactus (see Ex.2.56). This proportion can also be written as $C \rightarrow \frac{3}{2}$ without any change in its meaning.[83]
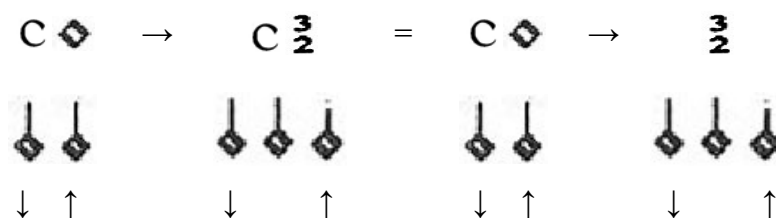
Ex.2.56 $C \rightarrow C\frac{3}{2}$: *sesquialtera* (3/2) on minim level in semibreve tactus

Table 2.1 Possible proportional relationships using signs made up of a mensural sign and a numerical fraction

1	$C \rightarrow C\frac{3}{2}$
2	$C \rightarrow C\frac{3}{1}$
3	$\mathitalic{\Cent} \rightarrow \Cent\frac{3}{2}$
4	$C \rightarrow C_{6/4}$
5	$C \rightarrow \Cent\frac{3}{2}$

---

[83] Bowers, "Proportioned Notation in Banchieri's Theory And Monteverdi's Music," 76.

6	$C \rightarrow \phi \frac{3}{2}$
7	$\mathsf{C} \rightarrow \phi \frac{3}{2}$
8	$C \rightarrow \phi \frac{3}{1}$
9	$C \rightarrow \mathsf{C} \frac{3}{2}$
10	$\mathsf{C} \rightarrow C \frac{3}{2}$

The second proportion in Table 2.1, the C sign followed by the $C\,\frac{3}{1}$ sign, indicates the

tripla proportion (3/1) on the semibreve level in the semibreve tactus (see Ex.2.57).

Ex.2.57 $C \rightarrow C\,\frac{3}{1}$: *tripla* (3/1) on semibreve level in semibreve tactus

The next proportion, $\mathsf{C} \rightarrow \mathsf{C}\,\frac{3}{2}$, indicates the *sesquialtera* proportion (3/2) on the

semibreve level in the breve tactus (see Ex.2.58).

Ex.2.58 $\mathsf{C} \rightarrow \mathsf{C}\,\frac{3}{2}$: *sesquialtera* (3/2) on semibreve level in breve tactus

The fourth proportion, the C sign followed by the C 6/4 sign, indicates the *sesquiquarta* proportion (6/4) on the semi-minim level in the semibreve tactus (see Ex.2.59).[84]
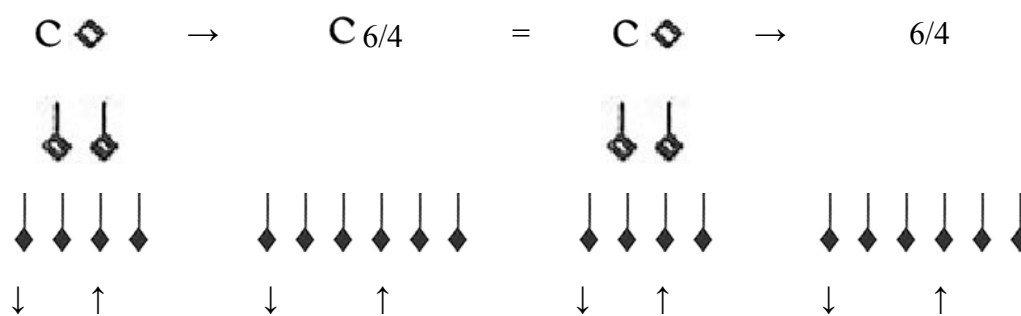
Ex.2.59 C → C 6/4: *sesquiquarta* (6/4) on semi-minim level in semibreve tactus

The next four proportional indications in Table 2.1 (numbers 5 through 8) use different mensuration signs in the *integer* and the proportion, providing a double proportional operation, represented by both the mensuration sign and the fraction. The C → ₵ $\frac{3}{2}$ signs indicate the *tripla* proportion (3/1) on the semibreve level in the semibreve tactus, with the 3/2 proportion multiplied by 2/1 proportion (see Ex.2.60).

C ◊ → $\frac{3}{2}$(1+1/2) ◊ → ₵ ◊ ◊ ◊ = ₵ $\frac{3}{2}$

[84] Bowers, "Proportioned Notation in Banchieri's Theory And Monteverdi's Music," 76.

Ex.2.60 $C \rightarrow \Phi \frac{3}{2}$: *tripla* (3/2 x 2/1 = 3/1) on semibreve level in semibreve tactus

The $C \rightarrow \Phi \frac{3}{2}$ signs also indicate the *tripla* proportion (3/2 x 2/1 = 3/1), but on the minim level in the semibreve tactus, with the 3/2 proportion multiplied by the 2/1 proportion (see Ex.2.61).
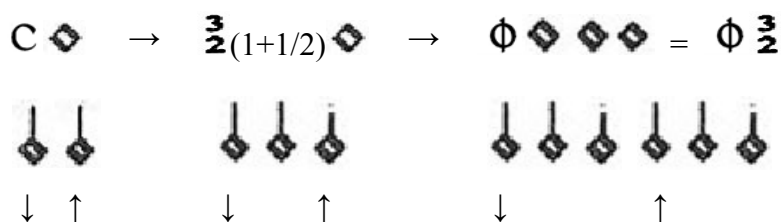
Ex.2.61 $C \rightarrow \Phi \frac{3}{2}$: *tripla* (3/2 x 2/1 = 3/1) on minim level in semibreve tactus

The $C \rightarrow \Phi \frac{3}{1}$ signs indicate the *sextupla* proportion (6/1) on the semibreve level in the semibreve tactus, with the 3/1 proportion multiplied by the 2/1 proportion (see Ex.2.62).[85]

This proportion was invented in the early 17th century.[86]

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85 Bowers, "Proportioned Notation," 76.
86 Bowers, "Proportional Notation," 429.

C ◇ → $\frac{3}{1}$ ◇ ◇◇ → Φ ◇ ◇◇ ◇ ◇◇ = Φ $\frac{3}{1}$

↓↑　　　　↓　　↑　　　　↓　　　↑
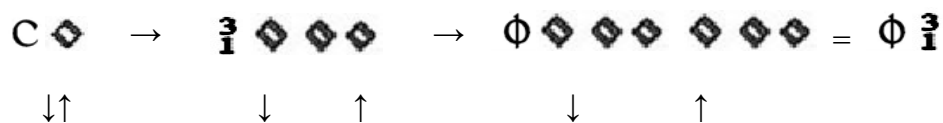
Ex.2.62 C → Φ $\frac{3}{1}$: *sextupla* (3/1 x 2/1 = 6/1) on semibreve level in semibreve tactus

The Ȼ → C $\frac{3}{2}$ signs indicate the *subsesquitertia* proportion (3/4) on the minim level in the breve tactus, with the 3/2 proportion multiplied by the 1/2 proportion (see Ex.2.63).[87]

Ȼ ▫ (*integer*)　→　C　→　C $\frac{3}{2}$ (*proportion*)

◇ ◇　　　　◇

↓ ◇ ↓◇　　↓◇ ↓　　　↓◇ ↓ ◇

↓　↑　　↓ ↑　　　↓　↑
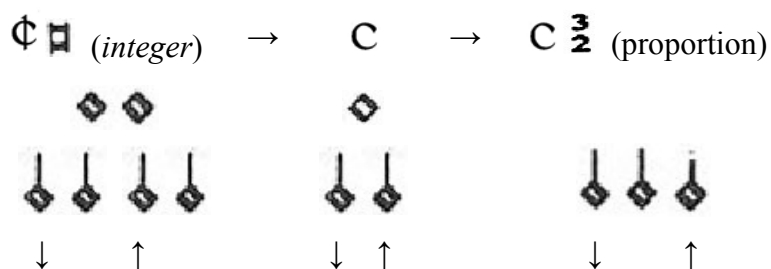
Ex.2.63　Ȼ → C $\frac{3}{2}$: *subsesquitertia* (3/2 x ½ = 3/4) on minim level in breve tactus

The last two proportional indications (numbers 9 and 10) also use different mensuration signs between the *integer valor* and the proportion. The different signs do not create a proportional relationship, but only show that the result retains the original mensuration meaning of the signs. Thus Ȼ → Φ $\frac{3}{2}$ signs indicate the *sesquialtera*

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[87] Bowers, "Proportioned Notation", 76.

proportion (3/2) on the semibreve level in the breve tactus (see Ex.2.64).

$$¢ \; \square \quad \rightarrow \quad \varphi \; \tfrac{3}{2}$$
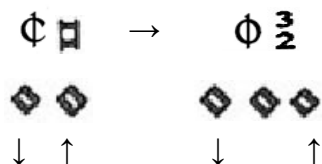
Ex.2.64 $¢ \rightarrow \varphi \tfrac{3}{2}$: *sesquialtera* (3/2) on the semibreve level in the breve tactus

Similarly, the $C \rightarrow \mathbb{C} \tfrac{3}{2}$ signs indicate the *sesquialtera* proportion (3/2) on minim level in the semibreve tactus (see Ex.2.65).[88]
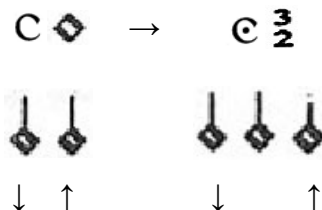
$$C \; \lozenge \quad \rightarrow \quad \mathbb{C} \; \tfrac{3}{2}$$

Ex.2.65 $C \rightarrow \mathbb{C} \tfrac{3}{2}$: *sesquialtera* (3/2) on minim level in the semibreve tactus

According to Roger Bowers, conventional proportional notation as seen in Table 2.1 was the mainstream practice in the early 17th century, especially during the first two or three decades, with the full understanding of the composers, who strictly applied the

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[88] Bowers, "Proportioned Notation", 76.

rules to their compositions. Bowers confirms that Claudio Monteverdi followed the

proportional practice as described by Banchieri.[89]

On the other hand, the use of Italian tempo words or mood words, such as *tarde*,

velociter, *adagio*, and *presto*, began in the early 17th century to indicate tempo inflections

that the proportional system was incapable of showing.[90] Over the course of the

seventeenth century, proportion signs began to be interpreted generally as tempo

modifications indicating a slower or quicker tempo. The fraction-proportion signs began

to be interpreted in the semi-minim equality; for example, the fraction 3/4, which used to

designate the proportional relationship of three semi-minims in the time duration of four

in the *integer valor*, came to be interpreted as three quarter notes in a measure.[91]

From the very beginning of mensuration practice in the fourteenth century,

composers expressed proportions inherent in the concept of mensuration based on the

central breve or the equal breve principle. Mensuration signs were useful tools for

composers to indicate proportional relationships within the breve equality, which

functioned as a ruler for regular *mensura* or *tactus*. Indeed, the regular beating practice

might have begun with the beginning of mensural notation. Most likely, from the very

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[89] Bowers, "Proportioned Notation", 77, 90.
[90] Houle, *Meter in Music, 1600-1800*, 2.
[91] Ibid.

beginning, the tempo of the tactus was flexible within the moderately controllable tempo of the human pulse rate under normal conditions of around M.M. 60. Although the invariable uniform tactus was discussed and strongly advocated for church polyphony by the majority of theorists from the late fifteenth century until allegedly the early seventeenth century, a flexible tempo range of the tactus was practiced according to the taste of the performers. However, the uniform tactus within a moderate range of flexibility functioned as the foundation of tempo, at least in vocal ensemble music of the late Medieval and Renaissance eras before the new Baroque musical styles, in which tempo or mood words began to be used to indicate tempos out of the normal tactus range, which initiated the dissolution of the invariable uniform tactus practice as well as proportional practice.

CHAPTER III

SCHÜTZ'S USE OF PROPORTIONAL SIGNS
IN HIS *PSALMEN DAVIDS* (1619) AND
SYMPHONIAE SACRAE TERTIA PARS (1650)

For the correct understanding of the proportional relationships used in the works

of Schütz, a precise knowledge of the note and rest signs of his time, which are a little bit

different form the modern signs, is required. During Schütz's time, composers retained

the note and rest signs used in the previous century; these are primarily the note values of

the breve, semibreve, minim, semi-minim, and fusa (see Table 3.1). When a note is dotted,

the dot means an addition of half of the note value (*punctis additionis*); for example, a

dotted breve designates the value of one and half breves and a dotted semibreve the value

of one and half semibreves. Under the ₵ 3 or ₵ 3/1 signs, the breve rest sign has the

same value as a dotted breve or three semibreves (see Ex. 3.7 and Ex. 3.15). Coloration –

that is, the use of blackened notes – principally indicates the *hemiola* rhythm (see Ex.

3.30 and Ex. 3.31).

Table 3.1 The primary signs for the notes and rests used during c. 1450 to 1600s

Note value	Note sign	Rest sign

Longa		
Breve		
Semibreve		
Minim		
Semi-minim		
Fusa		
Semi-fusa		

Proportional Signs Used in *Psalmen Davids* (1619)

The *Psalmen Davids* (1619) contains 26 pieces that include settings of 22 complete Psalm texts, a compiled Psalm text (verses from Ps. 96, 98, 148, 150 and the complete Ps. 117), two texts from other Biblical sources, and a motet text; Table 3.2 lists each piece, with the text it sets and its *Schütz Werke Verzeichnis* (SWV) catalog number.

Table 3.2 Titles of the 26 pieces in the *Psalmen Davids*

No.	Text	Title	SWV
1	Psalm 110	*Der Herr sprach zu meinem Herren*	22
2	Psalm 2	*Warum toten die Heiden*	23
3	Psalm 6	*Ach Herr straf mich nicht in deinem Zorn*	24
4	Psalm 130	*Aus der Tiefe ruf ich, Herr, zu dir*	25
5	Psalm 122	*Ich freu mich des, das mir geredt ist*	26
6	Psalm 8	*Herr unser Herrscher wie herrlich ist dein Nam*	27
7	Psalm 1	*Wohl dem, der nicht wandelt im Rat der Gottlosen*	28
8	Psalm 84	*Wie lieblich sind deine Wohnungen*	29
9	Psalm 128	*Wohl dem, der den Herren fürchtet*	30
10	Psalm 121	*Ich hebe meine Augen auf zu den Bergen*	31
11	Psalm 136	*Dancket dem Herren den der ist freundlich*	32
12	Psalm 23	*Der Herr ist mein Hirt*	33
13	Psalm 111	*Ich dancke dem Herrn von gantzem hertzen*	34
14	Psalm 98	*Singet dem Herr nein neues Lied*	35
15	Psalm 100	*Jauchtzet dem Herren alle Welt*	36
16	Psalm 137	*An den Wassern zu Babel fassen wir*	37
17	Psalm 150	*Alleluja. Lobet den Herren in seinem Heiligtum*	38
18	Psalm 103	*Lobe den Herren, meine Seele*	39
19	Jeremiah 31, 20	*Ist nicht Ephraim mein teuer Sohn*	40
20	Psalm 103	*Nun lob, mein Seel, den Herren*	41
21	Motet	*Die mit Tränen säen*	42
22	Psalm 115	*Nicht uns, Herr, sondern deinem Namen gib Ehre*	43
23	Psalm 128	*Wohl dem, der den Herren fürchtet*	44
24	Psalm 136	*Danket dem Herren, den er ist freundlich*	45
25	Isaiah 49: 14-16	*Zion spricht, der Herr hat mich verlassen*	46
26	Compiled Psalm texts	*Jauchzet dem Herren, alle Welt, singet, rühmet, lobet*	47

Schütz uses only three proportion signs in the *Psalmen Davids*: ¢ , 3, and ¢ 3.

Table 3.3 shows these proportional relationships and their possible interpretations, and

groups the 26 pieces according to which relationships are found in them.

Table 3.3 Categorized proportional indications in the *Psalmen Davids* (1619)

Category	Proportional indication	Possible interpretations	Nos.of the pieces
1	¢ → 3	1) ¢ → 3/1, breve tactus of the *integer* 2) ¢ → 3/2, breve tactus of the *integer* 3) ¢ → 3/1, semibreve tactus of the *integer* 4) ¢ → 3/2, semibreve tactus of the *integer*	1, 2, 11, 12, 20, 22, 24
2	¢ 3 → ¢	1) ¢ 3/1 → ¢ , breve tactus of the *integer* 2) ¢ 3/2 → ¢ , breve tactus of the *integer* 3) ¢ 3/1 → ¢ , semibreve tactus of the *integer* 4) ¢ 3/2 → ¢ , semibreve tactus of the *integer*	5, 7, 15, 17, 18, 26
3	¢	1) ¢ , breve tactus 2) ¢ , semibreve tactus	3, 4, 6, 8, 9, 10, 13, 14, 16, 19, 21, 23, 25

The first two possible interpretations in categories 1 and 2 are based on the breve tactus of the *integer valor*, while the third and fourth are on the semibreve tactus of the *integer valor*. In category 3, the first interpretation is based on the breve tactus and the second on the semibreve tactus. The tempo of the breve and the semibreve tactus is the same, around M.M. = 60.

An interesting feature of this publication, and others by Schütz, is that only the basso continuo parts show regular bar lines, corresponding to the *tactus maior*. Ex.3.1, the basso continuo part of *Lobe den Herren, meine Seele*, no.18 of the *Psalmen Davids*, shows an example of the printed regular barlines: In this case, barlines appear every six semibreves in the opening ¢ 3 section, and every two semibreves in the subsequent ¢ section, each corresponding to the *tactus maior* in that section.

Ex.3.1 Basso continuo part of *Lobe den Herren, meine Seele*, no.18 of the *Psalmen Davids*: printed regular barlines corresponding to the *tactus maior*

To clearly understand the possible interpretations of the proportional relationships, a short review of the possible tactus tempi in the time of Schütz is needed before a detailed explanation of the proportional interpretations. There are two possible tactus: the breve (▯) and the semibreve (◇). Strictly following the conventional mensuration-proportion practice, which retains the original meanings of the mensuration signs, the breve tactus is the *tactus maior* for all pieces in *Psalmen Davids* (1619). With the breve tactus as the *tactus maior*, the semibreve tactus becomes the *tactus minor*, having half of the time duration of the *tactus maior*, resulting in a tempo twice as fast as the *tactus maior* (see Ex.3.2).

Tactus maior   *Tactus minor*

¢ ♮   ¢ ♮

◆ ◆   ◆ ◆

↓ ↑   ↓ ↓ ↓ ↓

                                ↓ ↑
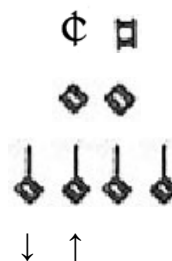
Ex.3.2 Breve tactus as *tactus maior* and semibreve tactus as *tactus minor*

When the *tactus maior* is used, a slight slowing down of the tempo might be needed sometimes in pieces having many semi-minims or fusas. Because the *tactus maior* was for professional musicians who could read the rhythm precisely, just a slight tempo adjustment would be good enough for the professional musicians to perform the works.

The semibreve tactus as *tactus minor* might have been used by semi-professional musicians when taken within the normal tempo range of the breve tactus (*tactus maior*), around M.M. = 60. This would require a change of tactus types between equal and unequal, which would help the musicians perform the rhythm of the pieces correctly. However, the overall flow of music could be slightly interrupted by switching between equal and unequal tactus.

Another possible way of choosing a tempo is to take the semibreve as the

invariable uniform tactus, whose tempo range is around M.M. = 60. This semibreve

tactus as the *tactus maior* or normal tactus might have been used for amateur musicians

who could not perform in the fast tempo with the breve tactus as the *tactus maior* or the

semibreve tactus as the *tactus minor* used by professional performers.

As discussed in Chapter II, the first category in Table 3.3, ₵ → 3, can be

interpreted as either the *tripla* (3/1) or *sesquialtera* proportion (3/2) within the

conventional mensuration-proportion practice. In the former interpretation, the *tripla*

proportion occurs on the semibreve level with a breve tactus of the *integer valor*,

resulting in three semibreves in the same time as the previous one semibreve. The *tactus*

minor beating, falling on the semibreve of the *integer*, creates a tactus-type switch

between equal and unequal tactus; instead of two equal down and up motions for a single

breve with the *tactus maior*, the tactus will have one longer down motion for two

semibreves and a shorter up motion for one semibreve under the proportion sign 3

(unequal tactus) following the equal tactus under the *integer* sign ₵ , in which each of

the down and up motions of equal time duration fall on a minim (see Ex.3.3). This

slightly interrupts the musical flow at these switching points.

¢ ◻ (*integer*)   →   3/1 (proportion)

◇   ◇   ◇ ◇◇   ◇ ◇◇

↓   ↑   ↓   ↑   : *tactus maior*

♦ ♦ ♦ ♦
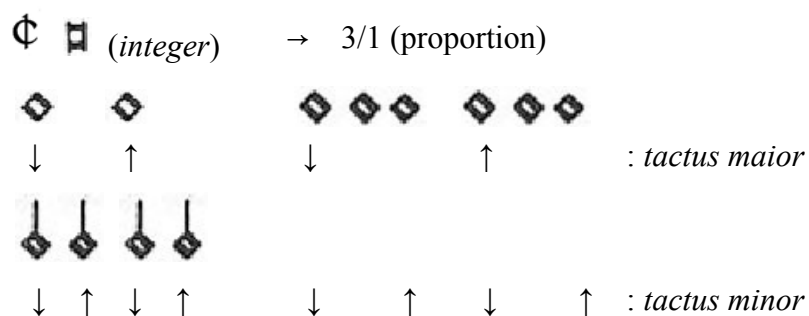
↓ ↑ ↓ ↑   ↓   ↑   ↓   ↑   : *tactus minor*

Ex.3.3 *tripla* (3/1) interpretation of ¢ → 3, *tactus maior* and *tactus minor*

Ex.3.4, showing this proportion, is from Schütz's *Der Herr ist mein Hirt*, no.12 of the

Psalmen Davids. The six semibreves under the proportion sign 3 are to be played during

the same time of the two semibreves or four minims or eight semi-minims under the

integer sign ¢ in the *tactus maior* (breve tactus), creating the *tripla* proportional

relationship. Ex.3.5 shows added vertical lines corresponding to the *tactus maior*. The

three longa rests and one semibreve rest at the beginning are equal to six and a half breve

rests, corresponding to six and a half bars in the *tactus maior*. Under the proportion sign 3,

a breve rest has the same value as three semibreves or a dotted breve, and a semibreve

rest has the same value as a semibreve. The *tactus maior* beating will be enough for

professional musicians, while the *tactus minor* beating would be helpful for semi-

professional musicians without slowing down the tempo (as shown in Ex.3.3).

Ex.3.4 *Der Herr ist mein Hirt*, no.12 of the *Psalmen Davids*: ₵ → 3

Ex.3.5 *Der Herr ist mein Hirt*, no.12: *tripla* (3/1) interpretation of ₵ → 3 with the breve tactus of the *integer*, with added vertical lines

The second interpretation of ₵ → 3 as ₵ → 3/2 as the *sesquialtera* proportion (3/2) occurs on the semibreve level with a breve tactus of the *integer* (*tactus maior*), in which three semibreves under the proportion sign 3 correspond to two semibreves under the *integer* sign ₵ (see Ex.3.6). The *tactus minor* beating under the proportion sign 3/2 is not supported by the actual notation, in which the consecutive three semibreves consist of the basic rhythmic construction with rarely used minims (see Exx.3.6 and 3.7). All the

actual triple-meter notation by Schütz under the ₵ 3 or 3 sign in the *Psalmen Davids* is

clearly on the semibreve level, not the minim level, with the predominating use of dotted

breves, pairs of a breve and a semibreve, and three consecutive semibreves as the units of

the triple rhythm, as seen in Ex.7, *Wohl dem, der nicht wandelt im Rat der Gottlosen*

(no.12) and *Der Herr ist mein Hirt* (no.12) from the *Psalmen Davids*. Non-consecutive

minims and semi-minims, shown in Ex.3.15, *Jauchtzet dem Herren alle Welt* (no.15 of

the *Psalmen Davids*), are used only rarely to create a dotted rhythm within the semibreve-

level rhythmic design. Thus, the *sesquialtera* proportion (3/2) with the *tactus minor* is

most likely not intended. Ex.3.8 shows the *sesquialtera* interpretation of no.12, *Der Herr*

ist mein Hirt, with the addition of vertical lines corresponding to the *tactus maior*, in

which three semibreves under the proportion sign 3 correspond to two semibreves under

the *integer* sign ₵.
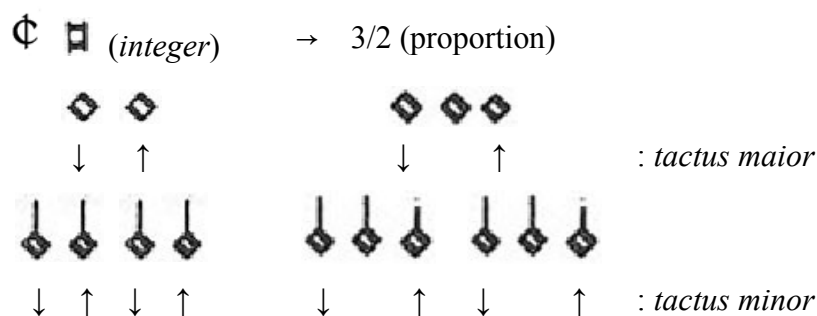
Ex.3.6 *sesquialtera* (3/2) interpretation of ₵ → 3, *tactus maior* and *tactus minor*

Ex.3.7 Actual notation under $\mathcal{C}$ 3 or 3 sign, nos. 7 and 12

Ex. 3.8 *Der Herr ist mein Hirt*, no.12: s*esquialtera* (3/2) interpretation of $\mathcal{C} \to$ 3 with the breve tactus of *integer*, on semibreve level, with added vertical lines

The third and fourth interpretations of the first category are based on the semibreve tactus of the *integer valor*, in which the tempo of the semibreve tactus is around M.M. = 60. These interpretations take a tempo twice as slow as the first and second, and are exactly the same interpretations as the first and second interpretations

except for the semibreve as the *tactus maior* instead of the breve.

The third interpretation of ₵ → 3 is as a *tripla*, with three semibreves under the

proportion sign 3 corresponding to one semibreve under the *integer* sign ₵ (see Ex.3.9).

As seen in the comparison of the first and third interpretations of the first category in

Ex.3.10, the only difference between these two is a tempo that is twice as slow in the

third interpretation than in the first. Ex. 3.11 shows this interpretation of no.12 in

Psalmen Davids, with the added vertical lines corresponding to the semibreve tactus of

the *integer valor*.

₵ ◇*(integer)* → 3 ◇ ◇ ◇ (proportion)

↓↑ ↓ ↑

Ex.3.9 *tripla* (3/1) interpretation of ₵ → 3 with the semibreve tactus of the *integer*

₵ ▯*(integer)* → 3/1 (proportion) : breve tactus of *integer*

◇ ◇ ◇ ◇◇ ◇ ◇◇

↓ ↑ ↓ ↑

₵ ◇*(integer)* → 3/1 ◇ ◇ ◇ (proportion) : semibreve tactus of *integer*

↓↑ ↓ ↑

Ex.3.10 comparison of interpretations 1) and 3) of category 1 in Table 3.3

Ex.3.11 *Der Herr ist mein Hirt*, no.12: *tripla* (3/1) interpretation of $\mathvarphi \to 3$ with the semibreve tactus of the *integer*, on semibreve level, with added vertical lines

The fourth interpretation of the first category in Table 3.3, *sesquialtera* (3/2) with the semibreve tactus of the *integer valor*, results in a tempo twice as slow as the second interpretation, *sesquialtera* (3/2) with the breve tactus of the *integer valor* (see Ex.3.12). As mentioned above, this fourth interpretation is not intended by Schütz because of his use of semibreves in triple meter. Thus, drawing barlines corresponding to the semibreve tactus of the *integer valor* is impossible under the proportion sign 3.

¢ ◻ *(integer)*   →   3/2 (proportion)   : breve tactus of *integer*

◇ ◇   ◇ ◇◇

↓ ↑   ↓   ↑

¢ ◇ *(integer)*   →   3/2 (proportion)   : semibreve tactus of *integer*

♦ ♦   ♦ ♦ ♦

↓ ↑   ↓   ↑
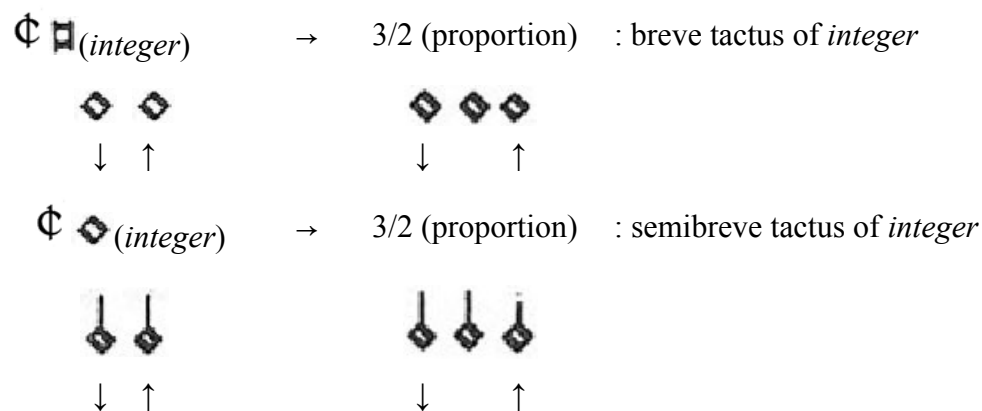
Ex.3.12 comparison of interpretations 2) and 4) of category 1 in Table 3.3

The second category in Table 3.3, ¢ 3 → ¢ , represents the exact same proportional relationship as the first category. The only difference is the use of ¢ 3 as the initial sign, which itself does not represent the *integer valor*. Only C , ¢ , O, and Φ signs function as *integer valor*, to which other proportion signs compare specific note values to create proportional relationships. Thus, in ¢ 3 → ¢ , the ¢ sign is the *integer valor*, as seen in Ex.3.13, which also shows the *tripla* (3/1) proportional relationship indicated by ¢ 3 → ¢ , in which three semibreves under the proportion sign ¢ 3 correspond to one semibreve under the *integer* sign ¢ . Ex.3.14 shows the *sesquialtera* (3/2) proportional interpretation of ¢ 3 → ¢ with the breve tactus of the *integer valor*, in which three semibreves under the proportion sign ¢ 3 correspond to

two semibreves under the *integer* sign ¢ . Ex.3.15, the original *Jauchtzet dem Herren*

alle Welt, no.15 in the *Psalmen Davids*, is an example of the ¢ 3 → ¢ proportion

indication, which can be interpreted as either *tripla* proportion (3/1) or *sesquialtera*

proportion (3/2). Ex 3.16 shows the *tripla* proportional interpretation using the breve

tactus of the *integer valor*, with added vertical lines corresponding to the breve tactus. Ex

3.17 shows the *sesquialtera* proportional interpretation with the added vertical lines

corresponding to the breve tactus.

¢ 3/1 (proportion)   →   ¢ ⊡ (*integer*)

◇ ◇◇ ◇ ◇◇   ◇   ◇

↓   ↑   ↓   ↑   : *tactus maior*

↓↓ ↓↓
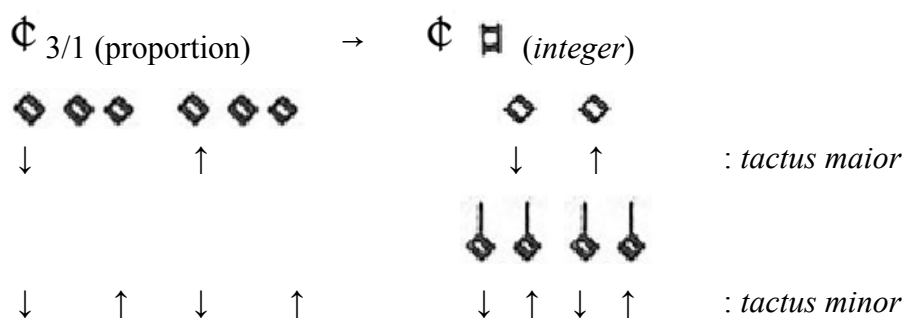
↓   ↑   ↓   ↑   ↓ ↑ ↓ ↑   : *tactus minor*

Ex.3.13 *tripla* (3/1) proportional interpretation of ¢ 3 → ¢ with the breve tactus of the
integer, *tactus maior* and *tactus minor*
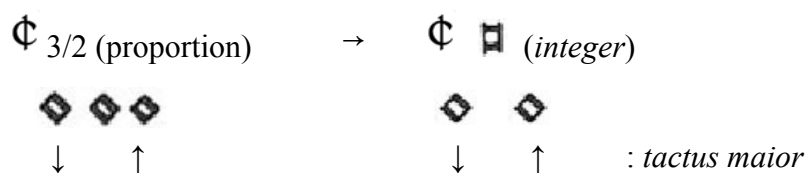
¢ 3/2 (proportion)   →   ¢ ⊡ (*integer*)

◇ ◇◇   ◇   ◇

↓   ↑   ↓   ↑   : *tactus maior*

Ex.3.14 *sesquialtera* (3/2) proportional interpretation of ¢ 3 → ¢ with the breve tactus
of the *integer*, *tactus maior*

Ex.3.15 *Jauchzet dem Herrn*, no.15: ¢ 3 → ¢

Ex 3.16 *Jauchzet dem Herrn*, no.15: *tripla* (3/1) proportional interpretation of ¢ 3 → ¢ with the breve tactus of the *integer*, with addition of vertical lines corresponding to the breve tactus

Ex.3 17 *Jauchzet dem Herrn*, no.15: *sesquialtera* (3/2) proportional interpretation of ¢ 3 → ¢ with the breve tactus of the *integer*, with addition of vertical lines corresponding to the breve tactus

The third and fourth interpretations of the second category of Table 3.3, the *tripla* (3/1) and *sesquialtera* (3/2) proportional interpretations with the semibreve tactus of the *integer valor,* are exactly the same as those of the first category.

For the third category in Table 3.3, which uses the ₵ sign alone, either *tactus maior* or *tactus minor* can be used. Following the convention of the *alla semibreve* sign C, in which the semibreve tactus functions as the *tactus maior*, the proportional relationship between the C sign and the ₵ sign (*alla breve*) is *dupla* (2/1), in which two semibreve under the proportion sign ₵ correspond to a semibreve under the *integer* sign C with a semibreve tactus of the *integer* (see Ex.3.18).

C (*integer*) → ₵ ◻ (proportion)
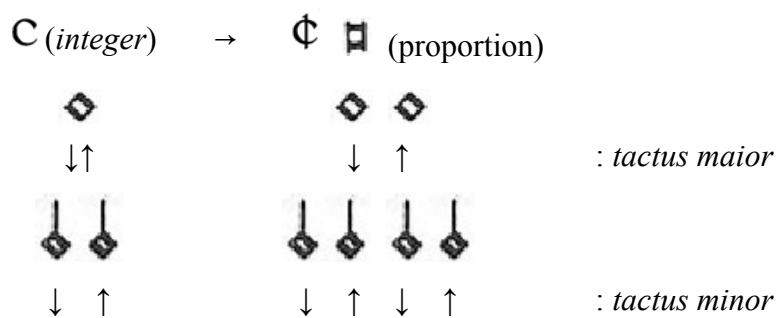
Ex.3.18 ₵ sign alone

Ex.3.19 shows the opening of *Aus der Tiefe ruf ich, Herr, zu dir*, no.4 in the *Psalmen Davids*, which uses only the ₵ sign throughout the piece. As explained above,

the breve tactus functions as the *tactus maior* under the *alla breve* sign $\mathrevC$, as shown in

Ex.3.20, in which the added vertical lines correspond to the breve tactus. Ex.3.21 shows

the interpretation of the semibreve tactus as the *tactus maior*, in which the added vertical

lines correspond to the semibreve tactus.

Ex.3.19 *Aus der Tieffe*, no.4: $\mathrevC$ sign alone

Ex.3.20 *Aus der Tieffe*, no.4; $\mathrevC$ sign alone with the breve tactus with added
vertical lines corresponding to the breve tactus

Ex.3.21 *Aus der Tieffe*, no.4: ¢ sign alone with the semibreve tactus with
added vertical lines corresponding to the semibreve tactus

Proportional Signs Used in
Symphoniae Sacrae Tertia Pars (1650)

The original *Symphoniae Sacrae Tertia Pars* was published in 1650 in twelve

partbooks: 1) Prima Vox, 2) Secunda Vox, 3) Tertia Vox, 4) Quarta & Ultima Vox, 5)

Violinum Primum, 6) Violinum Secundum, 7) Cantus Complementi, 8) Altus

Complementi, 9) Tenor Complementi, 10) Bassus Complementi, 11) Bassus Ad Organum,

and 12) Bassus Pro Violone.[92] Table 3.4 lists the pieces, their German titles, and SWV

numbers. Like the original prints of the *Psalmen Davids*, only the basso continuo parts

show regular barlines, corresponding to the *tactus maior*.

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[92] Werner Breig, Critical Notes to Volume 18, *Heinrich Schütz Neue Ausgabe sämtlicher Werke*, ed. Werner
Breig (New York: Bärenreiter, 1989), 143.

Table 3.4 The titles of the 21 pieces in *Symphoniae Sacrae III*

No.	Title	SWV no.
1	*Der Herr ist mein Hirt, mir wird nichts mangeln*	398
2	*Ich hebe meine Augen auf zu den Bergen*	399
3	*Wo der Herr nicht das Haus bauet*	400
4	*Mein Sohn, warum hast du uns das getan?*	401
5	*O Herr, hilf, O Herr, laß wohl gelingen*	402
6	*Siehe, es erschien der Engel des Herren Joseph im Traum*	403
7	*Feget den alten Sauerteig aus*	404
8	*O süßerJesu Christ, wer an dich recht gedenket*	405
9	*O jesu süß, wer dein gedenkt super Lilia convallium*	406
10	*Lasset uns doch den Herren, unsern Gott, loben*	407
11	*Es ging ein Sämann aus, zu säen seinen Samen*	408
12	*Seid barmherzig, wie auch euer Vater barmherzig ist*	409
13	*Siehe, dieser wird gesetzt zu einem Fall*	410
14	*Vater unser, der du bist im Himmel*	411
15	*Siehe, wie fein und lieblich*	412
16	*Hütet euch, daß eure Herzen nicht beschweret werden*	413
17	*Meister, wir wissen, daß du wahrhaftig bist*	414
18	*Saul, Saul, was verfolgst du mich*	415
19	*Herr, wie lange willst du mein so gar vergessen*	416
20	*Komm, heiliger Geist, Herre Gott*	417
21	*Nun danket alle Gott*	418

In Ex.3.22 , *Der Herr ist mein Hirt, mir wird nichts mangeln* (no.1 of the *Symphoniae Sacrae III*), it is hard to tell whether the initial sign and the sign in the fourth system are the ₵ sign or the ₵ sign. However, the ₵ sign of *Aus der Tiefe ruf ich,*

Herr, zu dir (no.4 in the *Psalmen Davids*), is crystal clear (see Ex.3.23). Nevertheless, a

close study of the original notations and their possible interpretations and a comparison to

those of the *Psalmen Davids* provide a clue to clarify the problem.

Ex.3.22 *Der Herr ist mein Hirt*, no.1: unclear C sign or ¢ signs

Ex.3.23 *Aus der Tiefe*, no.4 of the *Psalmen Davids*: clear $\mathrm{\mathsf{C}}\!\!\!\!|$ signs

Table 3.5 shows the series of proportion signs within each piece, as found in the

original 1650 publication.

Table 3.5 Proportional indications in *Symphoniae Sacrae III* (1650)

No.	Proportion signs as shown in a piece
1	¢ → **3/1** → ¢ → **3/1** → ¢ → **3/1** → ¢ → **3/1** → C
2	C → **3/1** → ¢ → **3/1**
3	C **3/1** → C → **3/1** → C → **3/1** → ¢ → **3/1** → ¢ → **3/1** → C
4	C → **3/1** → ¢
5	¢ → **3/1** → ¢
6	¢ → **3/1** → ¢ → **3/1** → ¢
7	¢ **3/1** → ¢
8	C → **3/1** → ¢ → **3/1**

Table 3.5 Continue

No.	Proportion signs as shown in a piece
9	C $\frac{3}{1}$→ ¢ →$\frac{3}{1}$→[C]→$\frac{3}{1}$→ ¢ →$\frac{3}{1}$
10	¢ →$\frac{3}{1}$→ ¢ → ¢
11	¢ →$\frac{3}{1}$→ ¢
12	¢
13	¢
14	C →$\frac{3}{1}$
15	¢ // ¢
16	¢ →$\frac{3}{1}$→ ¢ →$\frac{3}{1}$→ ¢
17	C // C →$\frac{3}{1}$→ ¢ →$\frac{3}{1}$→ ¢ →$\frac{3}{1}$
18	C $\frac{3}{1}$→ C
19	¢ // C →$\frac{3}{1}$→ ¢ →$\frac{3}{1}$→ ¢
20	¢ $\frac{3}{2}$
21	C → ¢ $\frac{3}{1}$→ C →$\frac{3}{1}$→ ¢ →$\frac{3}{1}$→ ¢ →$\frac{3}{1}$→ C

* [   ]: obviously missing sign in no.9

* //: double bars in nos. 15,17, and 19

Three different principles can be used to interpret the proportions in the 21 pieces

in the *Symphoniae Sacrae III*, based on the *tactus maior* as given by the *integer* sign of

each piece: a literal interpretation; an interpretation of the ₵ sign as representing a

slightly faster tempo than the C sign; and an interpretation of the C sign as a printing

error of the ₵ sign.

In the first of these interpretations, the proportion signs shown in the pieces are

accepted literally as the intended signs. For example, for the proportion signs in no.1 of

Table 3.5 (e.g. *Der Herr ist mein Hirt, mir wird nichts mangeln*, ₵ → **3̄1̄** → ₵ → **3̄1̄** →

₵ → **3̄1̄** → ₵ → **3̄1̄** → C), the *tactus maior* falls on two semibreves under ₵ , six

semibreves under **3̄1̄**, and one semibreve under C respectively (see Ex.3.24). In this

case, the C sign, not the initial ₵ sign, is the *integer valor*, to which other proportion

signs are compared to create proportional relationships, as explained in Chapter II; when

the signs C and ₵ are used together the proportional relationship is *dupla* (2/1),

indicated by C → ₵ , with the C as the integer and the ₵ as the proportion, in which

two semibreves and four minims under the ₵ sign correspond to one semibreve and two

minims under the *integer* sign C . The successive proportional relationship is non-

cumulative, with each proportion sign compared to the *integer* respectively, following the

conventional mensuration-proportion practice. Ex.3.25 shows the latter part of the

original print of no.1, while Ex.3.26 gives the literal interpretation of the part with added

vertical lines corresponding to the *tacti* according to the signs.

¢ ◊ ◊ → ³⁄₁ ◊ ◊ ◊ ◊ ◊ ◊ → C ◊ (*integer*)

↓ ↑   ↓   ↑   ↓↑   : *tactus maior*
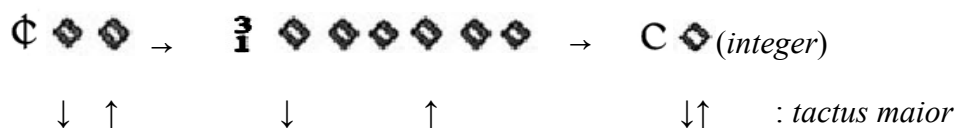
Ex.3.24 literal interpretation of no.1, *Der ist mein Hirt*.

Ex.3.25 *Der Herr ist mein Hirt*, latter part, no.1 in the *Symphoniae Sacrae III*

Ex.3.26 *Der Herr ist mein Hirt*, no.1 in the *Symphoniae Sacrae III*, with added vertical lines corresponding to the *tacti*

If one interprets the Ȼ sign as representing a slightly faster tempo than the C sign, as addressed in Chapter II as a tendency in the notational practice of the seventeenth century, the Ȼ → **3̶1̶** proportional relationship of the proportion signs of *Der Herr ist mein Hirt* (no.1 in Table 3.5), Ȼ → **3̶1̶** → Ȼ → **3̶1̶** → Ȼ → **3̶1̶** → Ȼ → **3̶1̶** → C, is *tripla* (3/1) on the semibreve level with a breve tactus as the *tactus maior* (see Ex.3.27). The tempo under the C sign at the end is intended to be slightly slower than that under Ȼ sign, creating a *ritardando* effect at the end of the piece.

¢ ◈ ◈ (*integer*)   →   𝟑/𝟏 ◈ ◈ ◈ ◈ ◈ ◈

↓ ↑   ↓   ↑   : *tactus maior*

Ex.3.27 interpretation 2 of no.1

On the other hand, if the C sign is regarded as a printing error of the ¢ sign, the proportion signs of no.1 in Table 3.5 become ¢ → 𝟑/𝟏 → ¢ → 𝟑/𝟏 → ¢ → 𝟑/𝟏 → ¢ → 𝟑/𝟏 → ¢ , creating only the *tripla* proportion (3/1), in which six semibreves under the 𝟑/𝟏 sign correspond to two semibreves under the ¢ sign (*integer*) in the *tactus maior* (breve tactus) (see Ex.3.27). Ex.3.28 shows the printing-error interpretation, with added vertical lines corresponding to the *tactus maior*.

Ex.3.28 *Der Herr ist mein Hirt,* no.1 in the *Symphoniae Sacrae III*: print error
interpretation with added vertical lines corresponding to the *tactus maior*

This last interpretation is most probably the correct one. Human error may have

been introduced when different compositors set the wrong type for the different

parkbooks for the printing. This possibility is addressed by Roger Bowers, who has

identified an inconsistency between the $\mathsf{\Phi}\,\frac{3}{2}$ and $\mathsf{\Phi}\,\frac{3}{1}$ signs in the last quarter of the

original partbooks of Monteverdi's *Selva Morale et Spirituale* (1641).[93] According to

Konrad Ameln, a similar printing error was caused by compositors when the number of

---

[93] Bowers, "Proportioned Notation in Banchieri's Theory And Monteverdi's Music", 77-79.

the printing type of a specific proportion sign was insufficient:

> The notes of the original were set in type and are not free of errors. Noteworthy is the frequent change of the mensural notation sign or meter: it is found as C or C without any apparent reason in the same voice part of one piece as well as in various parts of the same piece. Often, the vertical bar in the C is hardly visible. This is easy to explain: the typesetter did not have enough alla breve symbols in his case and resorted to the C. But it is clear that C was always intended because of the bar divisions in the thoroughbass, where the bar line is always placed after four half-note values (= 2 semibreves), even when C is placed in front.[94]

Regular printed barlines, in which each bar corresponds to the breve tactus of the *integer* C as *tactus maior,* are used in the basso continuo parts of all 21 pieces in *Symphoniae Sacrae III* with only a few exceptions. Ex.3.29 shows the regular barlines in nos.2 and 19; in both cases, the barlines correspond to the breve tactus, making it highly likely that the C sign was Schütz's original intention.

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[94] Konrad Ameln, Preface to Volume 7, *Heinrich Schütz Neue Ausgabe sämtlicher Werke,* ed. Konrad Ameln (New York: Bärenreiter, 1988), x.

Ex.3.29 Regular barlines corresponding to the breve tactus of the *integer* ₵ in *Ich hebe meine Augen auf* and *Herr, wie lange willst du*, nos.2 & 19 of *Symphoniae Sacrae III*

If all the C signs in *Symphoniae Sacrae III* are regarded as printing errors of the ₵ sign, only three types of proportion indications are found. These, and the pieces in which they occur, are shown in Table 3.6.

Table 3.6 Categorized proportional indications in *Symphoniae Sacrae III* (1650)

Category	Proportion signs	Nos.
1	₵ → $\frac{3}{1}$	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 14, 16, 17, 18, 19, 21
2	₵ alone	12, 13, 15
3	₵ $\frac{3}{2}$ alone	20

Category 1, indicated by Ȼ → **3̵/1̵**, clearly shows the *tripla* proportion (3/1) with

a breve tactus under the *integer* Ȼ sign as *tactus maior*. It would also be possible for the

tactus minor to be beaten for non-professional musicians (see Ex.3.30).

Ȼ ◻ (*integer*) → 3/1 (proportion)

◇ ◇   ◇ ◇◇   ◇ ◇◇
↓ ↑   ↓   ↑   : *tactus maior*
↓↑ ↓↑   ↓   ↑ ↓   ↑   : *tactus minor*
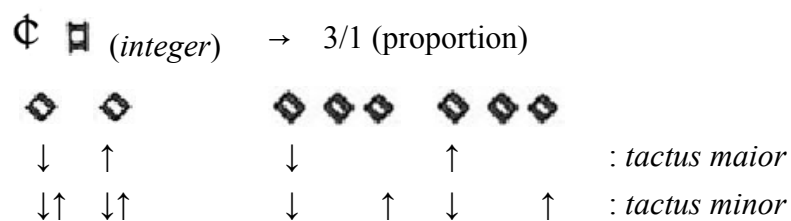
Ex.3.30 *tripla* (3/1) proportion indicated by Ȼ → 3/1, breve tactus

An example of this category is the piece previously discussed: *Der Herr ist mein*

Hirt, no.1 of *Symphoniae Sacrae III* (see Ex.3.31). It begins with Ȼ , changes to **3̵/1̵** in

the second system and returns to Ȼ in the fourth system. With the exception of the first

and third bars of the first system, and the second bar of the fourth system, all the bars

under the Ȼ sign represent the length of a breve. The barlines under the **3̵/1̵** sign are

completely regular for the *tactus maior*, with six semibreves per *tactus maior*. A

blackened semibreve followed by a blackened breve, shown in the last bar of the second

system, designates coloration, in which the rhythm changes without changing the note

values of the semibreve and breve. This coloration is different from the Renaissance

coloration, in which a colored note loses one thirds of its note value. The three

consecutive blackened breves at the beginning of the fourth system indicate a *hemiola*

rhythm without changing of the note value of the breve.

Ex.3.31 Der Herr ist mine Hirt, no.1 of *Symphoniae Sacrae III*: *tripla* (3/1) proportion
indicated by $\mathbb{C} \to \frac{3}{1}$, breve tactus

An example of the same category, but indicated by $\mathbb{C} \frac{3}{1} \to \mathbb{C}$ (with $\mathbb{C} \frac{3}{1}$ as the

initial sign), is no.18, *Saul, Saul was verfolgst du mich*, in which the regular barlines

correspond to the *tactus maior*, falling on the breve under the *integer* ₵ sign and on

six semibreves under the proportion sign ₵ $\frac{3}{1}$. The indication of a *hemiola* rhythm by

the use of three consecutive blackened breves is seen four times in the first and second

systems (see Ex.3.32).

Ex.3.32 *Saul, Saul was verfolgst du mich*, no 18 of *Symphoniae Sacrae III*: *tripla* (3/1)
proportion indicated by ₵ $\frac{3}{1}$ → ₵ , breve tactus

Category 2 of Table 3.6, indicated by ₵ alone, represents the breve tactus as

tactus maior, to which the barlines regularly correspond, as seen in no.13, *Siehe, dieser*

wird gesetzt (see Ex.3.33).

Ex.3.33 *Siehe, dieser wird gesetzt*, no 13 of *Symphoniae Sacrae III*: Category 2 indicated by ₵ alone, breve tactus

Category 3 of Table 3.6 is the use of ₵ 𝟑/𝟐 alone. The proportional relationship of the sign is compared to the *integer* ₵, even though it is not shown in the piece, creating the proportional relationship ₵ → ₵ 𝟑/𝟐, which represents the *sesquialtera* proportion (3/2) either on the semibreve level or on the minim level, with a breve tactus as the *tactus maior* under the *integer* (see Ex.3.34). Schütz uses the minim-level *sesquialtera* proportion in *Komm heiliger Geist*, no.20, with regular barlines corresponding to the *tactus maior*, six minims under the ₵ 𝟑/𝟐 sign (see Ex.3.35).
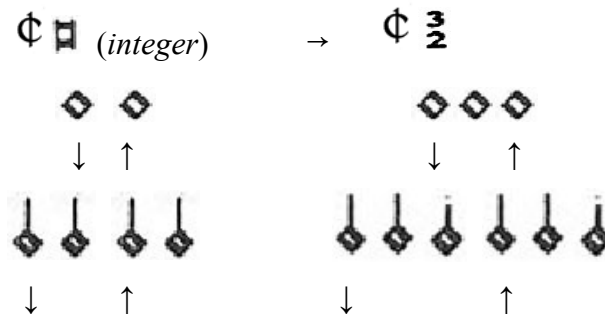
Ex.3.34 *Sesquialtera* proportion (3/2) by $\mathrm{C}\!\!\!| \rightarrow \mathrm{C}\!\!\!|\ \frac{3}{2}$, semibreve level or minim level

Ex.3.35 *Komm heiliger Geist*, no 20 in *Symphoniae Sacrae III*: Category 3 indicated by $\mathrm{C}\!\!\!|\ \frac{3}{2}$ alone, *tactus maior*

Comparison of the Usages of Proportional Signs in
Psalmen Davids and *Symphoniae Sacrae Tertia Par*s

Table 3.7 outlines the differences in notation and proportion signs between

Psalmen Davids (1619) and *Symphoniae Sacrae III* (1650).

Table 3.7 Comparison of notation between *Psalmen Davids* (1619) and *Symphoniae Sacrae* III (1650)

Category	*Psalmen Davids* (1619)	*Symphoniae Sacrae* III (1650)
1	¢ →3 or ¢ 3→ ¢	¢ → 3/1 or ¢ 3/1 → ¢
2	¢ alone	¢ alone
3		¢ 3/2 alone

In category 1, the proportion signs used in *Psalmen Davids*, ¢ →3 or ¢ 3→ ¢ ,

remain open to the two possible interpretations of *tripla* (3/1) and *sesquialtera* (3/2)

proportions. On the other hand, those used in *Symphoniae Sacrae III*, ¢ → 3/1   or

¢ 3/1 → ¢ , clarify the intended proportional relationship by using the figure 3/1 instead

of figure 3.

The proportion in category 2 of both the *Psalmen Davids* and the *Symphoniae*

Sacrae III , ¢ alone, does not create the same confusion as category 1; however, the

notation opens up two possibilities in the interpretation of the tactus: the breve tactus and

the sembreve tactus.

Category 3 shows Schütz's use of a new proportional indication in *Symphoniae*

Sacrae III, ¢ 3/2, which was not used in *Psalmen Davids*. By using regular barlines and

notation on the minim level, Schütz clearly shows his intention for the meaning of the proportional relationship indicated by the sign: *sesquialtera* (3/2) on the minim level in the breve tactus of the *integer*.

The comparison of the *Psalmen Davids* (1619) and the *Symphoniae Sacrare III* (1650) gives the strong impression that the earlier use of proportion signs in the *Psalmen Davids* was refined and clarified in the later use in the *Symphoniae Sacrae III*. The notation itself, in its use of note values and the printed barlines corresponding to the *tactus maior*, shows no difference between the two works. This is the case even when different proportion signs are used – the $\mathcal{C}$ 3 sign in the *Psalmen Davids* and the $\mathcal{C}$ 3/1 in *Symphoniae sacrae III*; the note values and the barlines used under the two signs are actually the same: Although the $\mathcal{C} \to \mathcal{C}$ 3 proportional indication in the *Psalmen Davids* provides the two possible interpretations, as *sesquialtera* (3/2) by $\mathcal{C} \to \mathcal{C}$ 3/2 and as *tripla* (3/1) by $\mathcal{C} \to \mathcal{C}$ 3/1, the comparison to the proportional indication $\mathcal{C} \to \mathcal{C}$ 3/1 in the *Symphoniae Sacrae III* reveals that the $\mathcal{C} \to \mathcal{C}$ 3 proportional indication in the *Psalmen Davids* is intended as the proportional indication $\mathcal{C} \to \mathcal{C}$ 3/1 shown in the *Symphoniae Sacrae III* because of the identical notation between the two, including the same barlines corresponding to the breve tactus of the *integer*.

Other Proportion Signs Used in Other Works
of Heinrich Schütz

In his other works, Schütz uses seven proportional relationships other than those

used in the *Psalmen Davids* and the *Symphoniae Sacrae III*. Table 3.8 shows the

proportions and examples of where they occur in Schütz's works.

Table 3.8 Other proportion signs than those used in the *Psalmen Davids* (1619) and the
Symphoniae Sacrae III (1650)

No.	Proportion signs	Example

1	C → $\frac{3}{1}$	*Symphoniae Sacrae I* (1629):
		    No.1 *Paratum cor meum* (SWV 257)
		    No.5 *Venite ad me omnes* (SWV 261)

2	C → C $\frac{3}{1}$ → C 6/4	*Symphoniae Sacrae II* (1647): No.10 *Lobet den Herrn in seinem Heiligtum* (SWV 50)
3	C → Φ $\frac{3}{2}$	*Cantiones Sacrae* (1625): No.15 *Dulcissime et benignissime Christie* (SWV 67)
4	Φ $\frac{3}{2}$ → ¢	*Syncharma musicum* (SWV 49)
5	C → $\frac{3}{2}$	*Beschluß, der Geburt unseres Herrn und Seligmachers Jesu Christi*
6	C 3 alone	*Gluck zu dem Helikon* (SWV 96)
7	C → 3	*Teutoniam dudum belli atra pericla molestant* (SWV 338)

The first proportion in Table 4.1, C → $\frac{3}{1}$, indicates the *tripla* proportion on the

semibreve level, in which three semibreves under the proportion sign $\frac{3}{1}$ correspond to

one semibreve under the *integer* sign C, as shown in Ex.3.36. Ex.3.37, *Paratum cor*

meum, no.1 in the *Symphoniae Sacrae I* (1629), and Ex.3.38, *Venite ad me omnes*, no.5 in

the *Symphoniae Sacrae I* (1629), are examples of this proportion. Ex.3.39 shows added

vertical lines corresponding to the semibreve tactus of the *integer* as the *tactus maior*.

$$C \diamond \quad (integer) \quad \rightarrow \quad \frac{3}{1} \diamond \quad \diamond\diamond$$

$$\downarrow\uparrow \qquad\qquad\qquad \downarrow \qquad \uparrow \quad : tactus\ maior$$
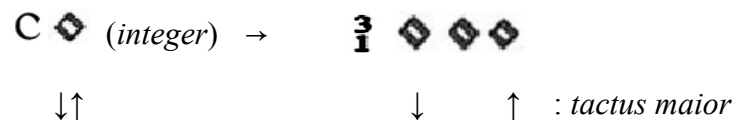
Ex.3.36 *tripla* proportion (3/1) by $C \rightarrow \frac{3}{1}$, semibreve level, semibreve tactus

Ex.3.37 *Paratum cor meum*, no.1 in the *Symphoniae Sacrae I* (1629): $C \rightarrow \frac{3}{1}$

Ex.3.38 *Venite ad me omnes*, no.5 in the *Symphoniae Sacrae I* (1629): $\mathbf{C} \rightarrow \frac{3}{1}$

Ex.3.39 *Paratum cor meum*, no.1 in the *Symphoniae Sacrae I* (1629): $\mathbf{C} \rightarrow \frac{3}{1}$, with added vertical lines corresponding to the semibreve tactus as *tactus maior*

In the second proportion in Table 3.8, $\mathbf{C} \rightarrow \mathbf{C}\,\frac{3}{1} \rightarrow \mathbf{C}\,6/4$, the proportion sign

$\mathbf{C}\,6/4$ is to be compared to the *integer* sign $\mathbf{C}$, following the non-cumulative feature

for successive proportions of the conventional mensuration-proportion practice. Thus the

first proportion, indicated by $\mathbf{C} \to \mathbf{C} \, \overset{\mathbf{3}}{\mathbf{1}}$, is exactly the same *tripla* proportion (3/1) as

the first proportion in Table 3.8. The other proportion, indicated by $\mathbf{C} \to \mathbf{C} \, 6/4$, is the

sesquialtera (3/2) proportional relationship on the semi-minim level with a semibreve

tactus as the *tactus maior*, in which six semi-minims under the proportion sign

$\mathbf{C} \to \mathbf{C} \, 6/4$ correspond to four minims under the *integer* sign $\mathbf{C}$ (see Ex.3.40). Ex.3.41,

from the original of *Lobet den Herrn in seinem Heiligtum*, no.10 in the *Symphoniae*

Sacrae II (1647), is an example of this proportional indication, but in this case the printed

barlines under the *integer* sign $\mathbf{C}$ are not regular. Those under the proportion sign

$\mathbf{C} \, \overset{\mathbf{3}}{\mathbf{1}}$ regularly correspond to two *tactus maiors* – six semibreves – except for in the first

system, while those under the $\mathbf{C} \, 6/4$ sign mostly correspond to the *tactus maior* – six

semi-minims – but sometimes to two or three *tactus maiors*, twelve or eighteen semi-

minims. Ex.3.42 is the same example as Ex.3.41, but with addition of vertical lines

corresponding to the *tactus maior*.
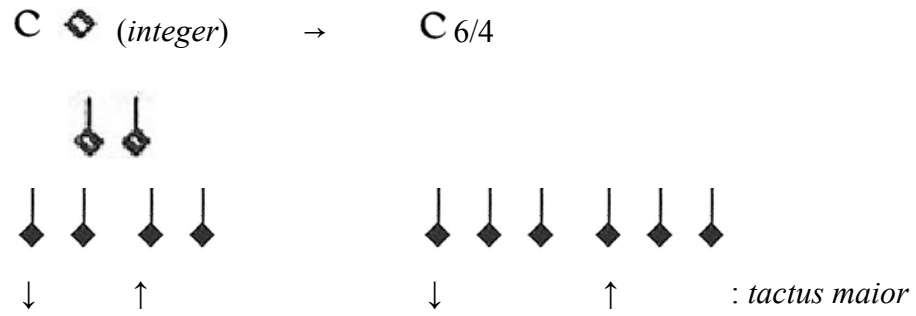
Ex.3.40 *sesquialtera* proportion (3/2) by C → C 6/4, semi-minim level, semibreve tactus of the *integer*

Ex.3.41 *Lobet den Herrn in seinem Heiligtum*, no.10 in the *Symphoniae Sacrae II* (1647):
$C \rightarrow C\,6/4$
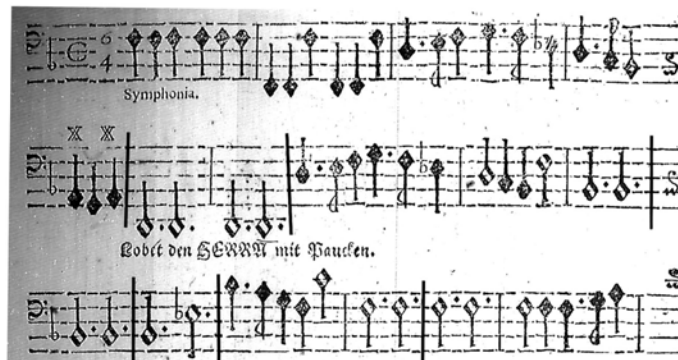
Ex.3.42 *Lobet den Herrn in seinem Heiligtum*, no.10 in the *Symphoniae Sacrae II* (1647):
$C \rightarrow C$ 6/4, with added vertical lines corresponding to the *tactus maior*

The third proportion in the Table 3.8, indicated by $C \rightarrow \Phi \frac{3}{2}$, is the tripla (3/1)

on the minim level with a semibreve tactus of the *integer* as the *tactus maior*, in which

three minims under the proportion sign $\Phi \frac{3}{2}$ correspond to one minim under the

integer sign C. The proportion indication involves a double operation of the proportion

sign $\Phi \frac{3}{2}$, in which $C \rightarrow \frac{3}{2}$ creates *sesquialtera* proportion (3/2) on the minim level

(three minims under the $\frac{3}{2}$ sign corresponding to two minims under the *integer*), while

the Φ sign operates as *dupla* diminution (2/1), resulting in the *tripla* proportion (3/1) on

the minim level (see Ex.3.43). An example of this proportional indication is *Dulcissime et*

benignissime Christe, no.15 in the *Cantiones Sacrae* (1625), in which the printed regular

barlines correspond exactly to two *tactus maiors* (see Ex.3.44). Ex. 3.45 shows the same

example with added vertical lines corresponding to the *tactus maior*.
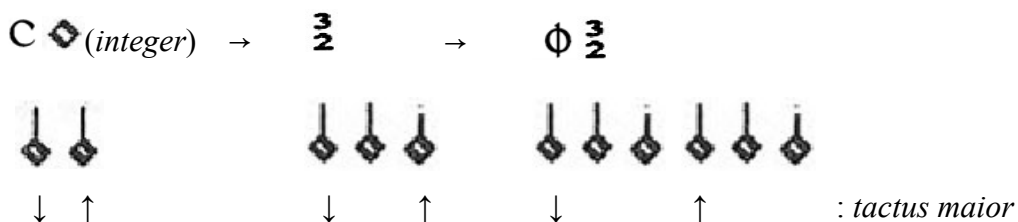
Ex.3.43 *tripla* proportion (3/1) by $C \rightarrow \Phi \frac{3}{2}$, minim level, semibreve tactus of the

integer

Ex.3.44 *Dulcissime et benignissime Christe*, no.15 in the *Cantiones Sacrae* (1625):
C → Φ $\frac{3}{2}$

Ex.3.45 *Dulcissime et benignissime Christe*, no.15 in the *Cantiones Sacrae* (1625), with added vertical lines corresponding to the *tactus maior*: C → Φ 𝟑/𝟐

The fourth proportion in Table 3.8, indicated by Φ 𝟑/𝟐 → ¢ or ¢ → Φ 𝟑/𝟐, is the

sesquialtera proportion (3/2) on the semibreve level in the breve tactus of the *integer* as

the *tactus maior*, in which three semibreves under the proportion sign $\phi\frac{3}{2}$ sign

correspond to two semibreves under the *integer* sign C (see Ex.3.46). An example of

this proportional indication is found in *Syncharma musicum*, in which the printed barlines

under the *integer* sign C correspond exactly to the *tactus maior* (one breve); those under

the proportion sign $\phi\frac{3}{2}$ correspond to two *tactus maiors*, except for the beginning of

the second system, where a *hemiola* occurs with a blackened breve followed by two

blackened semibreves, followed by a dotted semibreve and a minim (see Ex.3.47).

Ex.3.48 shows the same music but with added vertical lines corresponding to the *tactus*

maior, except for the semibreve right after the C sign at the beginning of the second

system, which was barred for a semibreve.
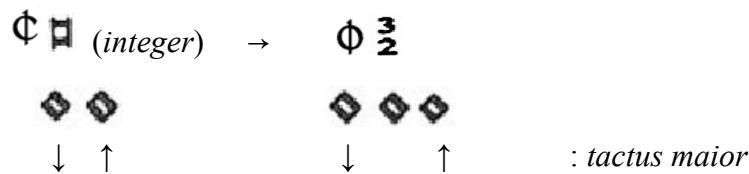
Ex.3.46 *sesquialtera* proportion (3/2) by $\mathrm{C} \rightarrow \phi\frac{3}{2}$, semibreve level, breve tactus of the *integer*

Ex.3.47 *Syncharma musicum* (SWV 49): Φ 𝟑/𝟐 → ¢

Ex.3.48 *Syncharma musicum* (SWV 49): Φ 𝟑/𝟐 → ¢ , with added vertical line
corresponding to the *tactus maior*

The fifth proportion in Table 3.8, indicated by C → 𝟑/𝟐 , is the *sesquialtera*

proportion (3/2) on the minim level with a semibreve tactus of the *integer* as the *tactus*

maior, in which three minims under the proportion sign $\mathbf{\frac{3}{2}}$ or $C\,\mathbf{\frac{3}{2}}$ correspond to two

minims under the integer sign C (see Ex.3.49). An example of this is found in the

Beschluß (conclusion) section of *der Geburt unseres Herrn und Seligmachers Jesu*

Christi.

$$C \lozenge \ (\textit{integer}) \quad \rightarrow \quad \mathbf{\frac{3}{2}}$$

    ↓ ↑   ↓   ↑   : *tactus maior*
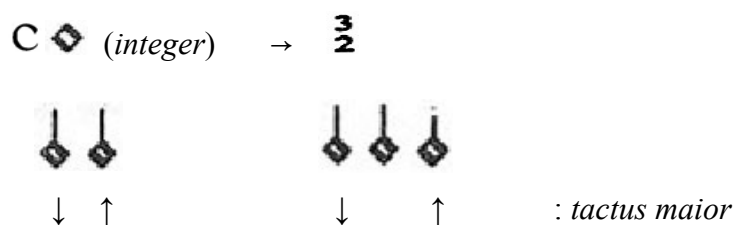
Ex.3.49 *sesquialtera* proportion (3/2) by $C \rightarrow \mathbf{\frac{3}{2}}$, minim level, semibreve tactus of the

integer

       The interpretation of the sixth proportion in Table 3.8, indicated by $C\,3$ alone,

is either as $C \rightarrow C\,\mathbf{\frac{3}{1}}$ or $C \rightarrow C\,\mathbf{\frac{3}{2}}$. The former is the *tripla* (3/1) proportional

relationship to the integer C on the semibreve level in the semibreve tactus of the

integer as the *tactus maior*, in which three semibreves under the proportion sign $C\,3$

correspond to one semibreve under the integer C, and exactly the same as the first

proportion in Table 3.8. The latter is the *sesquialtera* (3/2) proportion, which is exactly

the same as the fifth proportion in Table 3.8, except with the use of only the proportion

sign without the *integer*. An example of this proportional indication is *Gluck zu dem Helikon* (SWV 96).

The last proportional relationship in Table 3.8, ₵→3, is exactly the same as explanation of the sixth proportion indication, except for the use of both the *integer* sign ₵ and the proportion sign 3. An example of this is *Teutoniam dudum belli atra pericla molestant* (SWV 338).

As discussed above, the use of the proportional signs in all the works of Schütz follows the conventional mensuration-proportion practice, in which the original meanings of the mensuration signs are retained. The intended proportional relationships, indicated by the proportional signs used by Schütz, are clear based on the practice. The only practical problem to solve for performing the works is to decide the proper tempo, because Schütz does not differentiate in his use of barlines for pieces with primarily longer note values and others with primarily shorter note values. The former fits the normal tempo of the tactus, but the latter requires a slower tempo to perform the predominantly shorter notes. The tempo practice of the time of Schütz is based on the fixed universal tactus, around 60 M.M. per minute, but with a flexible application of the tactus depending on performance circumstances; mainly on the sort of performers

(professionals or amateurs), but also on other factors like the acoustics of the

performance hall.

CHAPTER IV

CONCLUSION

The editors of the Heinrich Schütz *New Complete Works* (*Neue Ausgabe*

sämtlicher Werke) were faced with the task of realizing Schütz's music into modern

notation, which has a very different set of conventions with regard to meter. Most of the

later editions clarify the metrical confusion created by the original proportion signs

through the use of modern time signatures, and distributing barlines according to a set

rule, whereby the barline corresponds to one semibreve of the *integer*. However, the

relationships of note values between the original proportion signs used in a piece, which

are based on the fixed tactus of the time of Schütz, are not clearly indicated, with the

exception of only a few editions, in which the editor includes an indication of the

relationship of note values between two time signatures, such as 𝆯 (one whole note) =

𝆯. (one dotted whole note).

The editorial principles of the later *New Complete Works* editions follow those of

the *Erbe deutscher Musik* (1967). The original note values under the triple metric

proportion signs, ₵ 3, ₵ 𝟯𝟭, and C 𝟯𝟭 are halved, while those under the duple

metric proportion signs, ₵ and C, are retained. The barlines correspond to the

semibreve of the *integer valor*, because the editors believed that the semibreve tactus

functions as the normal tactus (around M.M. = 60) in the time of Schütz.[95]

Table 4.1 is a comparison between the originally intended proportions, as

discussed in Chapter III, and their modern transcriptions in the Complete Works. In the

Table, the original signs come from the publications of Schütz's time; the work listed

serves as an example of the use of each original proportional indication; the signs under

the modern transcription are examples from the Complete Works; the interpretation under

the modern transcription shows the proportional relationship actually used for the

transcriptions; and the barlines show the note values to which the barlines correspond.

The table demonstrates which time signatures the editors used to replace the original

proportions signs, how differently the original proportion indications are interpreted in

the modern transcriptions in terms of the proportional indication, and whether the

barlines of the modern transcriptions correspond to the original barlines corresponding to

the *tactus maior*.

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[95] Werner Breig, Preface to Volume 20, *Heinrich Schütz Neue Ausgabe sämtlicher Werke*, ed. Werner Breig (London: Bärenreiter, 1996), xiv.

Table 4.1 Comparison between the interpretations in Chapter III and the modern
transcriptions in Heinrich Schütz Complete Works

No.	Original signs	Work	Modern transcription in the New Complete Works		
			Signs	Interpretation	Barlines
1	¢ → ¢ 3	Ps. D. no.15	¢ → ¢ 3	C → 3/1	Semibreve of *integer*
2	¢ alone	Ps. D. no.4	C	C	Semibreve of *integer*
3	¢ → 3/1	S.S.III no.1	C → 3/2	C → 3/1	Semibreve of *integer*
4	¢ 3/2 alone	S.S.III no.20	3/2	¢ 3/2	Semibreve of *integer*
5	C → 3/1	S.S.I no.1	C → 3/2	C → 3/1	Semibreve of *integer*
6	C → C 6/4	S.S.II no10	C → 6/4	C → 6/4	Semibreve of *integer*
7	C → φ 3/2	C.S. no.15	4/2 → 3/2	¢ → 6/4	4/2: 2 x *tactus maior* 3/2: 1 x *tactus maior*
8	¢ → φ 3/2	*Syncharma musicum* (SWV 49)	¢ → 3/2	C → 3/1	¢ : ½ x *tactus maior* 3/2: 1 x *tactus maior*

* Ps. D.: *Psalmen Davids*

* S.S.: *Symphoniae Sacrae*

* C.S.: *Cantiones Sacrae*

The originally intended proportion of no.1 of Table 4.1, ¢ → ¢ 3, is *tripla* (3/1)

on the semibreve level in the breve tactus of the *integer* as the *tactus maior*, in which six

semibreves under the proportion sign $\mathtext{¢}$ 3 correspond to two semibreves under the

integer sign $\mathtext{¢}$, as discussed in Chapter III. Ex.3.16, *Jauchzet dem Herrn*, no.15 of the

Psalmen Davids, shows the original proportional relationship with added barlines,

corresponding to the *tactus maior*; Ex.4.1, below, shows how the same passage was

transcribed in the *New Complete Works*: the original proportion signs were retained; the

note values under the ¢ 3 sign were halved; and the barlines correspond to the

semibreve of the *integer* (half of the *tactus maior*). In this case the proportional indication

of C → $\frac{3}{1}$, *tripla* proportion (3/1) on the semibreve level in the semibreve tactus of the

integer as the *tactus maior*, exactly fits the modern transcription, in which three

semibreves under the proportion sign $\frac{3}{1}$ correspond to one semibreve under the *integer*

sign C. Thus, the only difference between the original intention and the modern

transcription is the different tactus: the breve tactus in the original notation and the

semibreve tactus in the modern transcription (see Ex.4.2). Since the fixed universal tactus

of the time of Schütz is the breve (as discussed in Chapter III), with a tempo of around

M.M. = 60, the modern transcription is actually twice as slow as the original notation.

Although modern performers can easily perform the piece with the modern transcription

due to its implied slow tempo, the performance cannot properly express the affection of

the piece as originally intended.

Ex.4.1 *Jauchzet dem Herrn*, no.15 of the *Psalmen Davids*: Complete Works transcription of ₵ → ₵ 3

Ex.4.2 The modern interpretation of the original proportion indication of ₵ → ₵ 3

The original intention of no.2 of Table 4.1, ₵ alone, is the breve tactus as

discussed in Chapter III and shown in Ex.3.20, *Aus der Tiefe*, no.4 of the *Psalmen Davids*,

which shows the added barlines corresponding to the breve. Ex.4.3 is the same passage in

its modern transcription, in which the barlines now correspond to the semibreve, creating

a tactus tempo twice as slow as the original (see Ex.4.4). This modern transcription

creates the same performance problem as that of no.1 in Table 4.1.

Ex.4.3 *Aus der Tiefe*, no.4 of the *Psalmen Davids*: Complete Works transcription of
₵ alone

Ex.4.4 The modern interpretation of the original proportion indication of ₵ alone

The originally intended proportion of no.3 in Table 4.1, ₵ → **3̅1̅**, is *tripla* (3/1)

on the semibreve level in the breve tactus of the *integer* as the *tactus maior*. Ex.3.28, *Der*

Herr st mein Hirt, no.1 of the *Symphoniae Sacrae III*, is an example of this, in which a

few vertical lines were added to make the barlines exactly correspond to the *tactus maior*.

Ex.4.5 is its transcription from the *NSA*, in which the original ₵ and **3̅1̅** signs were

replaced by C and $\mathbf{\frac{3}{2}}$ and the barlines correspond to one semibreve under the C sign,

and three semibreves under the $\mathbf{\frac{3}{2}}$ sign, making the tactus tempo twice as slow as the

original. Moreover, according to the editorial principle, the note values under the original

$\mathbf{\frac{3}{1}}$ sign were halved (see Ex.4.5). This modern transcription creates the same performance

problem as that of no.1 in Table 4.1, while creating additional confusion about the

relationship of tempos between the duple and triple sections because of the halving of the

original note values under the $\mathbf{\frac{3}{1}}$ sign.

Ex.4.5 *Der Herr ist mein Hirt*, no.1 of the *Symphoniae Sacrae III*: Complete Works transcription of $\mathbf{\Phi} \rightarrow \mathbf{\frac{3}{1}}$

The originally intended tactus of no.4 in Table 4.1, $\mathbf{\Phi}\ \mathbf{\frac{3}{2}}$ alone, is six minims

per tactus, as shown in Ex.3.35, *Komm, heiliger Geist*, no.20 of the *Symphoniae Sacrae*

III. Ex.4.6 shows the same passage as transcribed in the *NSA*: a $\frac{3}{2}$ sign replaces the

original ₵ $\frac{3}{2}$ sign; the original note values are retained; and the barlines correspond to

three minims, which again creates a tactus tempo twice as slow as the original, resulting

in the same performance problem as that of no.1 in Table 4.1.

Ex.4.6 *Komm, Heiliger Geist*, no.20 of the *Symphoniae Sacrae III*: Complete Works
transcription of ₵ $\frac{3}{2}$ alone

The tempi in these four modern transcriptions—twice as slow as the original—

are caused by the interpretation of the normal tactus under the *integer* sign ₵ (*alla*

breve) as the semibreve instead of the breve. The editors of the *NSA* presupposed that the

semibreve tactus is the normal tactus in the time of Schütz, as represented in their

editorial principles. However, the discussions in Chapter III advocate for the breve tactus

as normal tactus in the proportions having the ₵ sign as *integer*. In the discussions in

Chapter III, the proportional indications retaining the *alla breve* sign ₵ as the

integer , ₵ →3, ₵ → $\frac{3}{1}$, and ₵ → Φ $\frac{3}{2}$, have the breve tactus under the *integer* as the

normal tactus, around M.M. = 60, and function as the *tactus maior*. And the

proportional indications retaining only one single sign with the *alla breve* sign ₵ in

them, ₵ alone and ₵ 𝟑/𝟐 alone, also have the breve tactus as the normal tactus.

     There are instances when the *tactus maior* is intended to be on the semibreve

level: When the integer sign is C. This is the case for no.5 in Table 4.1, C→𝟑/𝟏, where

the originally intended proportion is *tripla* (3/1) on the semibreve level in the semibreve

tactus of the *integer* as the *tactus maior*; three semibreves under the proportion sign 𝟑/𝟏

correspond to one semibreve under the integer sign C, as shown in Ex.3.39, *Partum cor*

meum, no.1 of the *Symphoniae Sacrae I*. Ex.4.7 is the modern transcription of the same

passage, in which the original proportion signs C and 𝟑/𝟏 are replaced by C and

𝟑/𝟐; the editor also halved the note values under the 𝟑/𝟐 sign, and inserted barlines

corresponding to the semibreve of the *integer*, resulting in the same tempo as the original.

Ex.4.7 *Paratum cor meum*, no.1 of the *Symphoniae Sacrae I*: Complete Works
transcription of C→𝟑/𝟏

The originally intended proportion of no.6 in Table 4.1, $\mathbf{C} \rightarrow \mathbf{C}$ 6/4, is

sesquialtera (3/2) on the semi-minim level in the semibreve tactus of the *integer* as the

tactus maior, in which six semi-minims under the proportion sign $\mathbf{C}$ 6/4 correspond to

four semi-minims under the integer sign $\mathbf{C}$ as shown Ex. 3.42, *Lobet den Herrn in*

seinem Heiligtum, no.10 of the *Symphoniae Sacrare II*, with barlines corresponding to the

semibreve tactus of the *integer*. In the modern transcription (see Ex.4.8), the original

proportion signs $\mathbf{C}$ and $\mathbf{C}$ 6/4 are replaced by $\mathbf{C}$ and 6/4. In this case the editor

retained the original note values under both signs, so the barlines correspond to the

semibreve tactus of the *integer* as in the original.

Ex.4.8 *Lobet den Herrn in seinem Heiligtum*, no.10 of the *Symphoniae Sacrae II*:
Complete Works transcription of $\mathbf{C} \rightarrow \mathbf{C}$ 6/4

In the transcriptions of nos. 5 and 6 in Table 4.1, the editorial principle of taking

the semibreve as the normal tactus unit coincides with the original proportional

indications, with the sign $\mathbf{C}$ (*alla semibreve*) as the *integer*; thus the tempi of these

transcriptions are the same as those intended by the original proportion indications. The

modern performer should therefore have little trouble with arriving at the correct tempi

through the transcription, and can properly express the affections of the pieces as

originally intended.

A different problem is encountered in transcriptions of pieces using the $\Phi \frac{3}{2}$

sign. The originally intended proportion of no.7 in Table 4.1, $\mathbf{C} \rightarrow \Phi \frac{3}{2}$, is tripla (3/1)

on the minim level in the semibreve tactus of the *integer* as the *tactus maior*, in which six

minims under the proportion sign $\Phi \frac{3}{2}$ correspond to two minims under the *integer*

sign $\mathbf{C}$. In Ex.3.45, *Dulcissime et benignissime Chirste*, no.15 of the *Cantiones Sacrae*,

the original barlines correspond to the semibreve tactus of the *integer*. Ex.4.9 shows the

modern transcription: Modern time signs, 4/2 and $\frac{3}{2}$, replace the original proportion

signs, $\mathbf{C}$ and $\Phi \frac{3}{2}$, respectively. With note values under the $\frac{3}{2}$ sign reduced by half,

and barlines under the 4/2 sign corresponding to twice the *tactus maior*, i.e., one breve,

the result is a tempo twice as fast as the original. On the other hand, the barlines under the

$\frac{3}{2}$ sign correspond to the *tactus maior* as in the original sign $\Phi \frac{3}{2}$. Thus different *tacti*

are applied for different signs. The modern transcription actually should have added

barlines corresponding to one whole note under the 4/2 sign to make the tempos balance

between the two signs, so that the originally intended affection can be properly expressed.

Ex.4.9 *Dulcissime et benignissime Chrite*, no.15 of the *Cantiones Sacrae*: Complete Works transcription of $C \rightarrow \Phi \, \frac{3}{2}$

The original intention of the last proportion in Table 4.1, $\mathsf{C} \hspace{-0.5em}\mid \rightarrow \Phi \, \frac{3}{2}$, is

sesquialtera (3/2) on the semibreve level in the breve tactus of the *integer* as the *tactus*

maior, in which three semibreves under the proportion sign $\Phi \, \frac{3}{2}$ correspond to two

semibreves under the *integer* sign $\mathsf{C} \hspace{-0.5em}\mid$, with barlines corresponding to the *tactus maior*,

as seen previously in Ex.3.48, *Syncharma musicum* (SWV 49). In the modern

transcription (Ex.4.10), the modern time signs, $\mathsf{C} \hspace{-0.5em}\mid$ and $\frac{3}{2}$, replace the original

proportion signs, $\mathsf{C} \hspace{-0.5em}\mid$ and $\Phi \, \frac{3}{2}$, respectively, while the note values under the $\frac{3}{2}$ sign

are halved. The barlines under the $\mathsf{C} \hspace{-0.5em}\mid$ sign correspond to half the *tactus maior*, i.e., the

semibreve, which results in a tempo twice as slow as the original, while the barlines

under the $\frac{3}{2}$ sign correspond to the *tactus maior* as in the original. Like the transcription

of no.7 in Table 4.1, this transcription applies different tactus unit to the different signs,

resulting in a totally different tempo relationship between the two signs. In order for the

transcription to retain the originally intended affection, every other barline under the

¢ sign has to be removed.

Ex.4.10 *Syncharma musicum* (SWV 49): Complete Works transcription of ¢ → Φ 𝟑𝟐

Except for the first transcription discussed in this chapter, which retains the

original signs with an editorial indication of the relationship of note values between the

two signs (𝗢 = 𝗢.), the rest of the transcriptions use modern time signatures with

their appropriate barlines. Only the transcriptions of nos. 5 and 6 of Table 4.1 match the

original intention in their distribution of barlines, because the original proportion

indications having the *alla semibreve* sign ₵ as their *integer* coincide with the editorial

principle of taking the semibreve tactus as the normal tactus, around M.M. = 60.

In the first four transcriptions in Table 4.1, which have the *alla breve* sign ₵ as

their *integer*, the originally intended breve tactus was disregarded and replaced, on the

presupposition of the editors, by the semibreve tactus as the normal tactus, resulting in a

tempo twice as slow as originally intended. If an historically-informed performance were

attempted with using these transcriptions, a tempo around M.M. = 120 per bar is needed.

In the last two transcriptions in Table 4.1, different tactus units were applied to the

different signs, resulting in totally different tempo relationships from the original

intention. To recover the originally intended tempo, the addition or subtraction of barlines

under one of the two signs in a piece would be required.

In general, the editors involved with the Heinrich Schütz *New Complete Works*

(*Neue Ausgabe sämtlicher Werke*), took a middle ground between retaining the original

proportion signs and using modern time signatures. However, this middle ground leaves

some degree of discrepancy in the distribution of barlines, in the decision of tempo, and

in the choice of modern time signatures to replace the original proportion signs.

It would be better in modern transcriptions, if barlines corresponded to the *tactus*

maior under both the *integer* and the proportion signs in the original notation, with an

indication of the relationships of the note values between different time signatures

provided, such as ⬭ = ⬭. . As discussed in Chapter III, the tempo decision in the

performance practice of the time of Schütz most likely depends on the capability of the

performers: Professional musicians could perform the pieces with the tempo

corresponding to the *tactus maior*, around M.M. = 60, without considerable slowing

down of the tempo, while amateur musicians would need to beat the *tactus minor* or slow

down the tempo considerably;   taking the semibreve as the *tactus maior* instead of the

breve, would result in a tempo twice as slow as the one indicated.

     To create a historically-informed performance from the *Neue Schütz Ausgabe*

transcription of, for example, *Syncharma musicum* (SWV 49) (Ex.4.10), the performer

would have to first refer to the preface to see if the original note values under the 3/2 sign

were halved or not, then know that the time length of the bars under the two modern time

signatures is equivalent (following the tactus practice in the time of Schütz), and finally

check in the preface or in the critical notes to see if the transcription's barlines correspond

to the *tactus maior* in the original proportion signs. Unfortunately, information on the

original proportion signs is not always provided in the *NSA*. Ideally, a new modern

transcription is needed to enable historically well informed performances. Such a

transcription would include barlines that correspond to the *tactus maior* under both the

integer sign and the proportion sign, and would clearly show the relationship of the note

values between the different time signatures. Ex.4.11, a portion of the continuo part of

Syncharma musicum (SWV 49), shows just such a transcription.

Ex.4.11 *Syncharma musicum* (SWV 49): a new transcription of ₵ → Φ 𝟑𝟐 showing the
original intention

Ideally, a new edition of Schütz's works should be made, using this transcription

process. The more precisely a modern transcription of the works of Heinrich Schütz can

reflect his intended proportional relationships, the more easily historically informed

performances of his music could be accomplished.

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