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Evaluation of the performance of a random coefficient regression model cumulative summation control chart under varying model conditions: with human services applications

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EVALUATION OF THE PERFORMANCE OF A RANDOM
COEFFCIENT REGRESSION MODEL CUMULATIVE
SUMMATION CONTROL CHART UNDER
VARYING MODEL CONDTIONS:
WITH HUMAN SERVICES
APPLICATIONS

A Dissertation Submitted in Partial Fulfillment
Of the Requirements for the Degree of
Doctor of Philosophy

Christopher John McKinney

College of Education and Behavioral Sciences
School of Educational Research, Leadership, and Technology
Program of Applied Statistics and Research Methods

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This Dissertation by: Christopher John McKinney

Entitled: *Evaluation of the Performance of a Random Coefficient Regression Model CUSUM Control Chart under Varying Model Conditions: With Human Services Applications*

has been approved as meeting the requirement for the Degree of Doctor of Philosophy in College of Education and Behavioral Science in School of Educational Research, Leadership, and Technology in Program of Applied Statistics and Research Methods

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ABSTRACT

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The use of quality control charts with metrics within the educational, behavioral, and other human services field has typically been considered very limited due to the complications imposed by nested structures, conditional relationships, and complex variance-covariance structures. The current study evaluated the performance of the random coefficient regression model control chart (RCRMCC) under conditions found commonly in the human services fields. Derived from the Regression Control Chart (RCC), the RCRMCC utilizes the residuals of the random coefficient regression model as inputs for the quality control charts. Using Monte Carlo simulations, the RCRMCC and RCC are compared, in regards to their average run lengths (ARL), under varying in-control and out-of-control population conditions, within the Cumulative Summation (CUSUM) Control Chart framework. It was found that the RCRMCC is advantageous under conditions similar to those found in the educational, behavioral, and other human services industries. Applications of the findings and future directions with the RCRMCC are discussed.

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CHAPTER I

INTRODUCTION

Background

Since the introduction of statistical process control (SPC) techniques in the early

1900's, multiple statistical tools have become available to ensure that products and

services meet prescribed specifications (Montgomery, 2005b). The most sophisticated of

the SPC tools is the control chart, introduced by Walter Shewhart (1926) in the 1920's.

The benefits of the Shewhart type control charts have been well established in the

manufacturing and service industries, where benefits have included improving

productivity, prevention of defects, reduction in unnecessary process adjustments,

providing process diagnostic information, and providing information about process

stability (Montgomery, 2005b). Further advantages of the statistical quality control

charts, focusing on the service industries, include: 1) assessing implementation of quality

improvement techniques, 2) providing feedback to employees, and 3) effectively

monitoring multiple processes occurring over time (Sulek, Marucheck, and Lind, 2006;

Palm, Rodriquez, Spiring, and Wheeler, 1997; Mehring, 1995; Sulek, Lind, and

Marucheck, 1995). Although advantages have been found in the use of quality control

charts, their implementation in the services industry has remained limited (Harvey, 1998).

 Reasons for this lack of implementation of process control techniques in the service

industries may be due to the wide variety and complexity of the service and quality data

collected, which some researchers believe imposes a limitation on the scope of SPC

techniques that can be utilized effectively with the data (Stewart, 2003). Within the educational, public service, and healthcare services industries, data may exist within complicated nested structures, where environmental and organismic variables at higher levels may affect the relationships between the outcome(s) and identified covariate(s) at lower levels of the data hierarchy (Raudenbush & Bryk, 2002; Longford, 1993). In order for the service industries to be able to adequately implement quality control techniques and process improvement, they need to overcome the issues of conditional relationships and nested structures. The solution to this issue evolves from the basic idea of a regression control chart (RCC) introduced by Mandel (1969) as a way to compensate for the existence of a covariate that affected the expected value of a process outcome.

Mandel (1969) introduced the problem of covariates in Shewhart-type control charts, with the example of monitoring the number of man hours required to sort incoming mail in a post office, where the volume of mail was the covariate. Since a post office required more man hours to sort larger volumes of mail, simply monitoring the average and variance of the man hours per post office would result in erroneous identification of out-of-control post offices. Mandel solved this problem by developing a regression model, predicting man hours as a function of mail volume, then establishing statistical limits about the regression line. In this fashion the man hours for a specified post office would be compared to the estimated average man hours, corrected for the post office's volume of mail. Though Mandel's (1969) work enhanced our ability to monitor manufacturing and service processes, where covariates exist for the characteristic of interest, the RCC may not be appropriate for many service processes.

Under situations where there exist conditional relationships and nested data structures, the typical regression model, and thus the regression control chart would be inappropriate (Raudenbush & Bryk, 2002). For example, in the mental health services field, the needs level at intake and the rate of recovery of a consumer may be dependent on factors such as diagnosis, level of drug abuse, and type of treatment. A consumer with a diagnosis of depression may start out with a lower level of initial level of service need and a lower rate of recovery than a consumer who has schizophrenia. Furthermore, a consumer with more resources, such as monetary savings, insurance, and close family/friends, typically would recover faster than a consumer without these resources. Given these differences in the initial level of service need and rate of recovery of the consumers, a single equation would not be appropriate, as some consumers may appear to be performing better than they really are, while others would appear to be doing poorly even though they are recovering as expected or better. In the situations where conditional relationships, such as differences in initial needs and rate of recovery, and nested regression models (rates of change within consumers) occur, the use of a conventional regression-adjusted control chart may result in increased erroneous out-of-control signals (false alarms), or the use of multiple control charts, needing corrections which adjust for the inflated type I error, both of which may be impractical. Though the typical regression control chart may not be appropriate in these situations, by extending the underlying regression model to a random coefficient regression model (RCRM) we are able to develop a set of control charts that account for nested data structures, while also allowing for conditional relationships among the outcomes and covariates.

The development and implementation of the RCRM (Longford, 1993), over the past three decades, has provided a solution to the issue of conditional relationships and nested structures common in the human service industries. Introduced in the social and behavioral sciences as hierarchical linear modeling (Raudenbush & Bryk, 2002), group/cluster randomized trials (Murray, 1998), and multilevel modeling (Goldstein, 1995), RCRM are a subset of the linear mixed effects models (LMM; Jiang, 2007; Raudenbush & Bryk). RCRMs allow for the partitioning of the within and between units/groups variance (Raudenbush & Bryk), similar to the typical split-plot design (Montgomery, 2005a). Furthermore, when covariates are used to adjust measures nested within units, RCRMs allow for the simultaneous estimation of conditional relationships based upon environmental and organismic characteristics of the units/groups at each level of the data hierarchy (Raudenbush & Bryk). Provided the previously described characteristics, using the RCRM requires only one equation to create regression-adjusted control charts, where the relationship between the covariates and characteristic of interest varies according to environmental or organismic factors affecting the units/groups.

Extending the regression control chart to encompass the RCRM would allow for more efficient monitoring of manufacturing and service processes. The partitioning of variance would allow for monitoring of the random error and the variability among the units/groups. Furthermore, the conditional regression coefficients would provide an overall model adaptive to the changing environment and organismic factors of the units/groups, increasing the likelihood of detecting a change in the process characteristic, while decreasing the rate of false alarms. All of the estimations would also occur simultaneously reducing the overall type I error, therefore further reducing the rate of

false alarms. The ability to monitor multiple variance components, adapt coefficients to changing environmental and organismic factors, and the decreased type I error provides for a more efficient regression control chart, for human services' processes.

Statement of the Problem

The existence of conditional relationships, in manufacturing and service process outcomes, can pose a problem with the implementation of a RCC and control charts in general. The extension of the RCC to encompass the RCRM would assist in alleviating the problem of conditional relationships. Though it is expected that a RCRM control chart (RCRMCC) would be beneficial as an SPC tool, there have been no studies that have established if a RCRMCC is beneficial, and if it is beneficial, under what circumstances the RCRMCC outperforms the typical RCC.

Several factors could affect the performance of RCRM and in turn the RCRMCC. These factors include the number of nested measurements per unit/group (m), the number of units/groups (J) upon which the measures are taken, the intra-class correlation coefficient (*ICC;* the ratio of the variance among the units in which the measures are nested (level 2 error) to the total variance in the model), and the coefficient of determination (R^2) for the regression coefficients within units/groups (Level 1; the percentage of the within unit/group variance accounted for by the level 1 covariate) and between units/groups (Level 2; the percentage of the variance of the level 2 coefficient accounted for by the level 2 covariate). It is expected that each of these factors will play a predominant role in the ability of the RCRMCC to detect a shift in the outcome of size delta (δ), as each is important in the calculation of power for the RCRM (Murray, 1998; Raudenbush & Bryk, 2002).

Purpose of the Study

The study examines:

1. Under which system circumstances the RCRM CUSUM control chart

   produces higher in-control average run-lengths (ARL). Factors evaluated are:

   a. *ICC*

   b. Levels 1 and 2 R^2

2. Under which system circumstances the RCRM CUSUM control chart

   produces lower out-of-control ARL for varying mean shifts (δ). Factors

   evaluated are:

   a. *ICC*

   b. Levels 1 and 2 R^2

Research Questions

Q1   Does the RCRMCC result in higher in-control average run lengths than
the RCC, under system conditions in which the *ICC*, Level 1 R^2 and Level
2 R^2 vary and no mean shift has occurred?

Q2   Does the RCRMCC result in lower out-of-control average run lengths than
the RCC, under system conditions in which the *ICC*, Level 1 R^2 and Level
2 R^2 vary and a mean shift has occurred?

Significance of the Study

Providing a more effective control chart for mean shifts in the presence of

conditional relationships and nested structures would be valuable to the human service

industries. Within these industries it is common for process outcomes to exist within a

nested structure and for relationships between outcomes and certain covariates to be

dependent on factors associated with the subject or group measured. By implementing a

control chart to correct for this issue, these service industries would be able to more effectively improve the overall quality of services.

<div align="center">Definitions</div>

In-Control Average Run Length – The average number of samples, evaluated from an in-control system, until the control chart produces a false alarm.

Out-of-Control Average Run Length – The average number of samples, evaluated from a system with a shift in the outcome (out-of-control), until the control chart produces an alarm.

CHAPTER II

REVIEW OF LITERATURE

This chapter reviews the literature pertinent to understanding the use of LMM in conjunction with the RCC, as applicable to the study. Provided the study is meant to be applicable primarily to the healthcare, behavioral health, and social (human) services sectors, the review of literature focuses on these areas. The chapter is broken down into five main parts: (1) a review of benefits and limitations of control charts to the behavioral and social services industries, (2) an overview of univariate statistical process control charts, (3) an overview of random coefficient regression models, (4) power of the RCRM and (5) the incorporation of mixed models into control charts.

Control Charts in the Services Industries

The use of SPC methods within the service industries is considered one of the most substantial advances in quality management of the last century (Wyckoff, 1984). Wyckoff argued that the use of quality control charts (QCC), a subset of SPC methods, can be effective in the evaluation of process quality by both managers and employees. Montgomery (2005b) also states that the QCC is an essential tool for the detection of unusual variation in the service's performance data. Multiple advantages associated with the utilization of the QCC has been demonstrated over the past several decades which include: 1) assessing implementation of quality improvement techniques, 2) providing feedback to employees, and 3) effectively monitoring of multiple processes occurring over time (Sulek, Marucheck, and Lind, 2006; Palm, Rodriquez, Spiring, and Wheeler,

1997; Mehring, 1995; Sulek, Lind, and Marucheck, 1995). Although advantages have been associated with the use of the QCC, implementation of the QCC in the service industries has remained limited (Harvey, 1998).

The limited use of control charts across the service industries is reflected within the field of mental healthcare, where the need for services is high, creating a great need for integration of quality control and efficiency techniques. Approximately one in four American adults, or almost 58 million people in the United States, suffers from a diagnosable mental illness in a given year (National Institute of Mental Health, 2008), where nearly 80% of American children and more than 67% of American adults who need mental health and addictions treatment services do not receive them (National Council for Community Behavioral Healthcare, 2010). As a result, mental disorders are the leading cause of disability in the United States and Canada for adults aged 18-44 (World Health Organization, 2004). Through the use of quality control techniques in the mental health services industry, more efficient use and allocation of resources can be obtained, allowing for a larger number of consumers to receive mental health treatment and promote greater and sustained mental health recovery, while mitigating costs to society as a whole.

Herbstman and Pincus (2009) noted that over the past decade a proliferation of mental health and/or substance abuse indicators has been developed in the United States for use in programs to assess mental healthcare service performance at the federal, state, and individual provider level, yet these indicators have not been successfully implemented into a coherent quality management program. Given the dynamic nature of mental health services outcomes and sometimes complicated nested structure of the

outcomes, implementing common QCCs may be difficult. Even provided this issue, in general, grantors, particularly those providing public funds, require high accountability and compliance with practices as mandated by payers (LaGanga and Lawrence, 2009); therefore, it is necessary to develop a quality control system that can help an organization adhere to the established standards, while also allowing for enhanced allocation of resources and expanded efficiency.

Similar to the mental healthcare services industry, other service industries have a high need to implement quality control practices in order to increase service quality, yet have similar nested data structures and conditional relationships related to environmental and unit/group factors/covariates. Within education, criminal justice, and other social services areas numerous examples of conditional and nested relationships exist (Raudenbush & Bryk, 2002). Another study demonstrated these structures can also be applied to political and opinion polling (Luke, 2004). The need for implementation of the QCC in the service industries, specifically social and human services, is great. In order to help further the implementation of the QCC, the issues due to the conditional and hierarchical nature of the process characteristic of interest must be addressed in a manner that is practical for implementation.

<div align="center">Overview of Univariate Statistical Process Control Charts</div>

This section reviews several common univariate control charts used in the manufacturing and services industries. The charts to be reviewed will be the Shewhart type charts, CUSUM charts, and the regression corrected control chart, as utilized with variable process characteristic data.

Within all processes that are functioning as specified, referred to as in-control processes, there exist random variations in the process characteristic of interest, also known as chance variation. When a systematic change occurs in the system, causing the system to be out-of-control, the mean and/or variance may shift. These shifts are referred to as assignable cause variation, where the main function of statistical process control is to identify and remove these assignable causes (Montgomery, 2005b).

One of the main tools utilized for SPC is the QCC. The QCC can be effective at monitoring variability of a process characteristic, thereby the quality of a process, helping to identify the assignable cause of an out-of-control process (Montgomery, 2005b). As the need to identify a variety of assignable causes under varying circumstances has grown, so has the number of QCC.

The most basic and utilized QCC are the Shewhart-type control charts. Introduced by Walter Shewhart (1926), these QCC monitor some specified process characteristic containing chance variation. Two phases are used to develop and implement the Shewhart-type control charts. Phase I involves the collection of a pre-determined number of unit samples from the in-control process of interest, where a process measurement is taken on each sampled unit. The measurements are then used to establish the mean and random variation of the process characteristic. These parameter estimates are then used in Phase II to establish the warning and control limits, along with the center line (CL), which is used for further monitoring of the process (Tracy, Young, & Mason, 1992). During Phase II the CL is used as the expected value of the process characteristic, where control limits are established around the CL. There is an upper control limit (UCL) and a lower control limit (LCL), along with upper and lower warning

limits. These limits are expressed in terms of the number of standard deviations (σ; sigma) above and below the CL (Montgomery, 2005b).

In order to establish when a process is out-of-control the Western Electric Handbook (1956) suggests a series of rules to detect a nonrandom pattern in the control chart. The Western Electric Handbook, as stated by Montgomery (2005b), assumes an out-of-control process if any of the following are found to be true regarding the control chart:

(1)   One point plots outside the three-sigma control limits

(2)   Two out of three consecutive points plot beyond the two-sigma warning limits

(3)   Four out of five consecutive points plot at a distance of one-sigma or beyond from the center line

or

(4)   Eight consecutive points plot on one side of the center line (pg. 166)

Interpretation of these rules is limited to one side, above or below the CL, at a time (Montgomery, 2005b).

The specific values of the CL, UCL, and LCL for an $\bar{x}$ chart can be computed using the range (R) or standard deviation (s) method, where the range and standard deviation can also be monitored through their respective R and s charts (Montgomery, 2005b). Assuming the process characteristic of interest is normally distributed and $x_1, x_2, x_3, \ldots, x_n$ is a sample of size n, then the mean of the sample is

$$\bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n} \qquad\qquad (2.01)$$

Now, let $\bar{x}_1, \bar{x}_2, \bar{x}_3, \ldots, \bar{x}_m$ be the average of each of m samples, then the best estimator

of the true process average, μ, is

$$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \ldots + \bar{x}_m}{m} \qquad (2.02)$$

where $\bar{\bar{x}}$, the grand average, is used as the value of the CL. The range for a single

sample is computed as

$$R = x_{max} - x_{min} \qquad (2.03)$$

where x_{max} is the largest value of $x_1, x_2, x_3, \ldots, x_n$ and x_{min} is the smallest value.

Using the R of each of m samples the average R is computed as

$$\bar{R} = \frac{R_1 + R_2 + \ldots + R_m}{m} \qquad (2.04)$$

The control limits of the $\bar{x}$ chart, based upon the range, is

$$UCL = \bar{\bar{x}} + A_2\bar{R} \qquad (2.05)$$

$$CL \quad = \bar{\bar{x}} \qquad (2.06)$$

$$LCL = \bar{\bar{x}} - A_2\bar{R} \qquad (2.07)$$

where A_2 is a constant for a specified sample size as defined by Montgomery. The

corresponding R chart for detecting shifts in the variability of the process characteristic is

established as

$$UCL = D_4\bar{R} \qquad (2.08)$$

$$CL \quad = \bar{R} \qquad (2.09)$$

$$LCL = D_3\bar{R} \qquad (2.10)$$

where D_3 and D_4 are constants for a specified sample size as defined by Montgomery.

The standard deviation (s) of a sample of size n is computed as

$$s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}} \qquad (2.11)$$

where the average of s for the m samples is

$$\bar{s} = \frac{1}{m}\sum_{i=1}^{m} s_i \qquad (2.12)$$

such that s_i is the standard deviation of sample i. Using the estimate of the average of s,

the parameters of the $\bar{x}$ chart become

$$UCL = \bar{\bar{x}} + A_3\bar{s} \qquad (2.13)$$

$$CL \quad = \bar{\bar{x}} \qquad (2.14)$$

$$LCL = \bar{\bar{x}} + A_3\bar{s} \qquad (2.15)$$

and the parameters for the corresponding s chart are

$$UCL = B_4\bar{s} \qquad (2.16)$$

$$CL \quad = \bar{s} \qquad (2.17)$$

$$LCL = B_3\bar{s} \qquad (2.18)$$

where A_3, B_3, and B_4 are constants dependent on the sample size, n, for each of the m

samples, as specified by Montgomery. Though the focus here is on mean values for each

sample, the Shewhart charts can also be reframed for individual observations. The

adjustment of the Shewhart charts for individual observations is accomplished by using

an average moving range, MR, comparing the present observation to the prior observation

such that

$$MR = |x_i - x_{i-1}| \qquad (2.19)$$

and

$$\overline{MR} = \frac{\sum_{i=1}^{m} MR_i}{m} \qquad (2.20)$$

Though the Shewhart $\bar{x}$, R, and s control charts can be effective at monitoring for

out-of-control processes, they tend to only detect large shifts in the process characteristic.

In order to detect small shifts in the process characteristic the Cumulative Summation

(CUSUM) and Exponentially Weighted Moving Average (EWMA) control charts are recommended. These charts are also recommended for Phase II control charts, versus continuing with a Shewhart-type chart. Both control charts utilize previous information in the series to determine if a process is out-of-control (Montgomery, 2005b).

The CUSUM can be represented using the tabular or the V-mask method (Montgomery, 2005b). This review focuses on only the tabular method. Under the tabular method a series of samples of size i, with mean $\bar{x}_i$, is assumed. When the process is in-control, the observations are assumed to have a normal distribution with a mean or target value, μ_0, and standard deviation, s. The tabular CUSUM works by summing the derivations above or below the target value, μ_0, using the upper (C_i^+) and lower (C_i^-) CUSUM statistics respectively. The statistics are computed as

$$C_i^+ = max[0, \bar{x}_i - (\mu_0 + K) + C_{i-1}^+] \qquad (2.21)$$

$$C_i^- = max[0, (\mu_0 - K) - \bar{x}_i + C_{i-1}^-] \qquad (2.22)$$

where C_i^+ and C_i^- both start at a value of 0, i.e. $C_0^+ = C_0^- = 0$, where each resets to zero when either becomes negative. K is generally called the allowance or slack value, which is usually chosen to be one-half of the magnitude of the out-of-control shift in the characteristic of interest (Montgomery, 2005b), such that

$$K = \frac{\delta}{2} s \qquad (2.23)$$

where δ is the magnitude of the shift in terms of number of standard deviations away from the target value. If either C_i^+ or C_i^- exceeds the decision interval, H, then the process is considered to be out-of-control. Though there are multiple ways to choose H, one generally accepted method is five (5) times the process standard deviation (Montgomery, 2005b). The CUSUM may also be extended to the monitoring of other

process parameters such as variability, by adjusting the target reference and standard deviations estimates to those appropriate for the process parameter. Furthermore, the CUSUM can also be used to monitor individual observations similar to the Shewhart charts, where the target value and standard deviation are adjusted for the estimate of the parameters for the series of individual observations (Montgomery, 2005b).

The RCC can build upon the Shewhart or CUSUM methods. Introduced by Mandel (1969), the RCC controls for variation due to a known covariate, to which the dependent variable is causally and linearly related. The Shewhart-type RCC is constructed through the previously described two phases. The first phase estimates the in-control coefficients for the identified covariates in regards to the process characteristic of interest. Using the residual of the regression model, upper and lower control limits are defined about the fitted regression line. In Phase II, all subsequently sampled units are then compared to the control limits about the regression line, where the value of the process characteristic of interest is compared to the expected value defined by the value of the covariate for the sampled unit. The RCC is interpreted in the same fashion as a Shewhart control chart, with the main difference being that the expected values and control limits follow a linear relationship with the covariate versus having a constant value for all samples. In order to use the RCC method with a CUSUM control chart the residuals from the expected value, based upon the estimated linear relationship, are used to determine the cumulative summation of the deviations from the target value (in this case the expected value due to the covariate).

The Random Coefficient Regression Model

The RCRM, also known as the hierarchical linear model (HLM), multilevel model (MLM), and covariance components model in the social and behavioral sciences (Raudenbush and Bryk, 2002; Singer, 1998, Longford, 1993), is based upon the linear mixed effects model (Jiang, 2007), where the effects are allowed to vary across the units. The RCRM can be extended from the general form of the regression model (Jiang; Raudenbush and Bryk). Assume an outcome (y_i) and predictor (x_i) are measured, such that n pairs of observations are taken, i.e. $i = 1,\ldots,n$. If a fixed intercept, β_0, and estimated relationship, β_1, for all of the y_i and x_i are assumed, then the general form of the regression model is

$$y_i = \beta_0 + \beta_1 x_i + r_i \qquad\qquad (2.28)$$

where r_i is the residual from the predicted value of y_i given x_i, and is distributed normally with a mean of 0 and variance σ^2.

Now assume that the n_j observations over time are nested within m individuals, and the intercept and estimated slopes are allowed to vary across the *m individuals.* This model now assumes varying coefficients for the intercepts and slopes across the *m* individuals, such that the regression model now becomes

$$y_{ij} = \beta_{0j} + \beta_{1j} x_{ij} + r_{ij} \qquad\qquad (2.29)$$

For each of the *m* individuals, an intercept, β_{0j}, and an estimated slope, β_{1j}, are calculated, such that $j = 1,\ldots,m$, based upon the n_j nested measurements. In this form, we assume *m* fixed intercepts and slopes, one for each individual. In order to move this form of the regression model to a RCRM, it is assumed the varying intercepts and slopes are random. Since the intercepts and slopes are random across the *m* individuals, another

set of regression models can be used to estimate the intercept and slope from the

characteristics of the individuals. Assuming W_j is the covariate of interest in estimating

the intercepts and slopes for the m individuals, the regression model for the random

intercepts is

$$\beta_{0j} = \gamma_{00} + \gamma_{01}W_j + u_{0j} \tag{2.30}$$

where γ_{00} is the average intercept across all m individuals, γ_{01} is the effect of W_j on the

intercept for unit j, and u_{0j} is the residual for the intercept of each of the m individuals

and is normally distributed with mean 0 and variance τ_{00}. In a similar fashion, we can

create a model for the random slopes across all of the individuals such that

$$\beta_{1j} = \gamma_{10} + \gamma_{11}W_j + u_{1j} \tag{2.31}$$

where γ_{10} is the mean slope across all m individuals, γ_{11} is the effect of W_j on the mean

slope for individual j, and u_{1j} is the residual for the slope of each of the m individuals

and is normally distributed with mean 0 and variance τ_{11}. Jointly, u_{0j} and u_{1j} are

assumed to be bivariate normally distributed with variance-covariance ($\boldsymbol{T}$) matrix:

$$Var \begin{bmatrix} \mu_{0j} \\ \mu_{1j} \end{bmatrix} = \begin{bmatrix} \tau_{00} & \tau_{01} \\ \tau_{10} & \tau_{11} \end{bmatrix} = \boldsymbol{T} \tag{2.32}$$

The correlation between the intercept and slope pairs $\rho(\beta_{0j}, \beta_{1j})$ is:

$$\rho(\beta_{0j}, \beta_{1j}) = \tau_{01} \Big/ \left(\sqrt{\tau_{00}}\sqrt{\tau_{11}}\right) \tag{2.33}$$

where τ_{01} is the covariance between the intercepts and slopes. Equations 2.30 and 2.31

allow us to model the random slopes and intercepts across the individuals, such that

variability in the intercepts and slopes can be explained using any number of W's.

The final step towards developing the RCRM involves replacing β_{0j} (intercepts) and β_{1j} (slopes) in equation 2.29 with equations 2.30 and 2.31 respectively. These combined equations provide the RCRM with form

$$y_{ij} = \gamma_{00} + \gamma_{01}W_j + \gamma_{10}x_{ij} + \gamma_{11}W_jx_{ij} +$$

$$u_{0j} + u_{1j}x_{ij} + r_{ij} \tag{2.34}$$

Thus, the final model given in equation 2.34 now contains three error terms, one each for the intercepts (u_{0j}), slopes ($u_{1j}x_{ij}$), and the residuals (r_{ij}), which partitions the variance among the nested levels. This model can be extended to have numerous x's for the nested measures, traditionally referred to as Level 1 covariates and W's for the individual or groups in which the measures are nested, traditionally referred to as Level 2 covariates.

Power and the Random Coefficient Regression Model

In general, power is defined as the ability of a statistical model to detect a significant difference, when a difference actually exists (Cohen, 1988). Since a QCC is developed to identify significant deviations from some target value, it is important to understand the components that are necessary to determine the power of a RCRM. Spybrook, Raudenbush, Congdon, and Martinez (2009) identify six general factors affecting the power of a RCRM, for any given α level; 1) size of the effect (δ), (2) the intra-class correlation (*ICC*), (3) the number of units/groups (*J*), (4) the number of measures/units (*n*) within each unit/group (*m*), (5) the amount of within unit/group variance accounted for by the Level 1 covariates (Level 1 R^2), and (6) the amount of between unit/group variance accounted for by the Level 2 covariates (Level 2 R^2). In regards to all of the factors except the *ICC*, an increase in the value of the factor results in

an increase in power, holding all other factors constant. On the other hand a decrease in

power is seen as the value of the *ICC* increases. The decrease in power is shown by

Murray (1998) to result from an increase in the variance estimate due to the larger *ICC*.

In order to effectively evaluate the derived or simulated power of a RCRM, all six of the

above listed factors should be taken into consideration.

<div align="center">
Random Coefficient Regression Model and the
Cumulative Summation Control Chart
</div>

Integrating the RCRM into the CUSUM control chart framework involves replacing

the references to the target values, μ_0, and raw values, x_i, of equations 2.21 and 2.22 with

the sum of the RCRM residuals, $u_{0j}, u_{1j}x_{ij}, r_{ij}$, for each of the *m* individuals, i.e.,

$$C_i^+ = \max[0, (\widehat{u_{0j}} + \widehat{u_{1j}}x_{ij} + \hat{r}_{ij}) - K + C_{i-1}^+] \qquad (2.35)$$

$$C_i^- = \max[0, |\widehat{u_{0j}} + \widehat{u_{1j}}x_{ij} + \hat{r}_{ij}| - K + C_{i-1}^-] \qquad (2.36)$$

In this fashion, the deviations of the raw scores from the target value are replaced with

the individual's deviation from the expected value provided by the RCRM. The

calculation of the sensitivity value, *K*, and of the decision interval, *H*, remain the same as

presented in equation 2.23. Reaching the decision interval indicates that the individual's

outcomes significantly differ from the RCRM estimated outcome, and thus review of the

individual is warranted.

CHAPTER III

METHODS

The study developed the foundation for a statistical QCC based upon the RCRM. By extending the basic process for a RCC as developed by Mandel (1969), the RCRMCC can account for nested data structures, where relationships between the characteristic of interest and explanatory covariates may vary across the higher level units/groups. The study utilized a Monte Carlo Simulation to compare the in-control and out-of-control ARL for the RCRMCC to those of the RCC within the CUSUM framework.

The first research question was answered through the evaluation of the in-control ARL under the RCRMCC and RCC, within the CUSUM framework. In order to identify under which system circumstances the RCRMCC CUSUM chart had a higher ARL than the RCC CUSUM chart, the following interacting factors, under an in-control system were evaluated. The in-control condition of a mean shift of zero (0) standard deviations in the intercept was utilized for this simulation.

1) Intra-class Correlations (*ICC*): The control condition of an *ICC* of 0.0001 was utilized (nearly no variability among groups/units in which measures are nested, an *ICC* of 0 could not be used due to issues with the variance-covariance matrix of level 2 parameters), along with *ICC's* of 0.05, 0.15, 0.25, 0.5. These *ICC* values were consistent with *ICC*'s found to be common within the social and behavioral sciences (Hedges and Hedberg, 2007;)

2) Coefficient of Determination (R^2): For level 1 and 2 equations (equations 2.30 and 2.31), the R^2 were varied among the values 0.01, 0.25, and 0.5 for R^2, where both of the level 2 equations had the same R^2 value. These values are consistent with R^2 values discussed in power studies with regression models (Cohen, 1988). The crossed R^2 values provide a total of 9 cells being evaluated.

The second research question was answered through the evaluation of the out-of-control ARL under the RCRMCC and RCC, within the CUSUM framework. In order to identify under which system circumstances the RCRMCC CUSUM chart had a lower out-of-control ARL than the RCC CUSUM chart, the previous interacting factors in Part 1 along with mean shift changes in the intercept (δ) were evaluated.

1) Mean Shifts (δ): The out-of-control conditions with mean shifts of 0.2, 0.5, and 0.8 standard deviations were evaluated. These values are consistent with another simulation study of HLM regression models carried out by Fang (2006). The shifts in the slope were not evaluated.

The total number of cells evaluated were 180 per control chart (RCRMCC versus RCC), given all system circumstances providing for an in-control and out-of-control system; five (5) ICC levels, nine (9) R^2 values, and four (4) mean shifts.

Apparatus/Instruments

All Monte Carlo Simulations were run under the R version 2.10.1 platform (R Development Core Team, 2009) utilizing the non-linear mixed effects (nlme; Pinheiro, Bates, Debroy, Sarkar, & The R Core Team, 2009) package to analyze the RCRM models. The multivariate normal function (mvtnorm; Genz, Bretz, Miwa, Mi, Leisch,

Scheipl, & Hothorn, 2010) was utilized to create the random data for the control charts.

Appendix A contains the R-scripts used to carry-out all simulations and analysis.

<p style="text-align: center">Basic Model Structure</p>

The model evaluated was a two-level nested model with a covariate at each level.

The two-level model provided a simple linear regression model at the first level with the

conditional relationships being specified by one covariate at the first level. A

corresponding simple linear regression model was utilized as the comparison to the two-

level RCRM. The RCRM was identical to the model specified by equation 2.34.

<p style="text-align: center">Simulation Procedures</p>

The following steps were utilized to produce the population values and

subsequent sample values used to evaluate the ARL of the RCRMCC and RCC under

varying system circumstances and out-of-control mean shifts as previously described.

All R-scripts used to perform the simulations are provided in Appendix A.

1. Under the in-control situation (no mean shift) a population of 1,000 groups was

   created where all the W_j's were randomly assigned to each group using a normal

   distribution with mean 0 and standard deviation determined as a function of the

   respective level 2 population parameters. These population values were saved for

   later simulations to ensure consistency in the Level 2 population parameters for

   each groups throughout the simulations. The next step randomly created 1,000

   paired x_{ij} and outcome values within each group, utilizing the randomly assigned

   values of the W_j's. The mean of the random data points varied according to the

   mean and standard deviation of the within group parameters for the 45 *ICC, Level*

1 R², and *Level2 R²* parameter crosses. The RCRM parameters utilized in the simulation model are shown in Table 1.

2. The parameter estimates for the RCRM and regression models were produced for each of the 45 sets of system circumstances using R. These parameter estimates were saved and used for all later residual calculations.

3. Under each of the sets of crossed system parameters, data points were simulated, which were then evaluated under the in-control RCRM and regression models produced in step 1. The residuals of the data points were evaluated under the CUSUM control chart, where data points were produced until the RCRMCC and RCC provided an out-of-control signal.

4. Step 3 was repeated 10,000 times, where the number of data points until an out-of-control signal were tabulated each time for both the RCC and RCRM CUSUM control charts.

5. The average and standard deviation of the number of data points for each of the 10,000 runs, under the RCRMCC and RCC, were calculated, for each crossed mean shift and sets of system circumstances.

Table 1

RCRM Simulation Parameters

δ	ICC	Level 1 R^2	Level 2 R^2	γ_{00}	γ_{01}	γ_{10}	γ_{11}	τ_{00}	τ_{11}	τ_{01}	σ^2
0.0	0.0001	0.01	0.01	0.00	0.1	0.1	0.1	0.00	0.00	0	0.99
0.0	0.0001	0.01	0.25	0.00	0.1	0.1	0.1	0.00	0.00	0	0.99
0.0	0.0001	0.01	0.50	0.00	0.1	0.1	0.1	0.00	0.00	0	0.99
0.0	0.0001	0.25	0.01	0.00	0.1	0.1	0.1	0.00	0.00	0	0.75
0.0	0.0001	0.25	0.25	0.00	0.1	0.1	0.1	0.00	0.00	0	0.75
0.0	0.0001	0.25	0.50	0.00	0.1	0.1	0.1	0.00	0.00	0	0.75
0.0	0.0001	0.50	0.01	0.00	0.1	0.1	0.1	0.00	0.00	0	0.50
0.0	0.0001	0.50	0.25	0.00	0.1	0.1	0.1	0.00	0.00	0	0.50
0.0	0.0001	0.50	0.50	0.00	0.1	0.1	0.1	0.00	0.00	0	0.50
0.0	0.05	0.01	0.01	0.00	0.1	0.1	0.1	0.02	0.02	0	0.94
0.0	0.05	0.01	0.25	0.00	0.1	0.1	0.1	0.02	0.02	0	0.94
0.0	0.05	0.01	0.50	0.00	0.1	0.1	0.1	0.01	0.01	0	0.94
0.0	0.05	0.25	0.01	0.00	0.1	0.1	0.1	0.02	0.02	0	0.71
0.0	0.05	0.25	0.25	0.00	0.1	0.1	0.1	0.02	0.02	0	0.71
0.0	0.05	0.25	0.50	0.00	0.1	0.1	0.1	0.01	0.01	0	0.71
0.0	0.05	0.50	0.01	0.00	0.1	0.1	0.1	0.02	0.02	0	0.48
0.0	0.05	0.50	0.25	0.00	0.1	0.1	0.1	0.02	0.02	0	0.48
0.0	0.05	0.50	0.50	0.00	0.1	0.1	0.1	0.01	0.01	0	0.48
0.0	0.15	0.01	0.01	0.00	0.1	0.1	0.1	0.07	0.07	0	0.84
0.0	0.15	0.01	0.25	0.00	0.1	0.1	0.1	0.06	0.06	0	0.84
0.0	0.15	0.01	0.50	0.00	0.1	0.1	0.1	0.04	0.04	0	0.84
0.0	0.15	0.25	0.01	0.00	0.1	0.1	0.1	0.07	0.07	0	0.64
0.0	0.15	0.25	0.25	0.00	0.1	0.1	0.1	0.06	0.06	0	0.64
0.0	0.15	0.25	0.50	0.00	0.1	0.1	0.1	0.04	0.04	0	0.64
0.0	0.15	0.50	0.01	0.00	0.1	0.1	0.1	0.07	0.07	0	0.43
0.0	0.15	0.50	0.25	0.00	0.1	0.1	0.1	0.06	0.06	0	0.43
0.0	0.15	0.50	0.50	0.00	0.1	0.1	0.1	0.04	0.04	0	0.43
0.0	0.25	0.01	0.01	0.00	0.1	0.1	0.1	0.12	0.12	0	0.74
0.0	0.25	0.01	0.25	0.00	0.1	0.1	0.1	0.09	0.09	0	0.74
0.0	0.25	0.01	0.50	0.00	0.1	0.1	0.1	0.06	0.06	0	0.74
0.0	0.25	0.25	0.01	0.00	0.1	0.1	0.1	0.12	0.12	0	0.56
0.0	0.25	0.25	0.25	0.00	0.1	0.1	0.1	0.09	0.09	0	0.56
0.0	0.25	0.25	0.50	0.00	0.1	0.1	0.1	0.06	0.06	0	0.56
0.0	0.25	0.50	0.01	0.00	0.1	0.1	0.1	0.12	0.12	0	0.38

Table 1, cont.

δ	ICC	Level 1 R^2	Level 2 R^2	γ_{00}	γ_{01}	γ_{10}	γ_{11}	τ_{00}	τ_{11}	τ_{01}	σ^2
0.0	0.25	0.50	0.25	0.00	0.1	0.1	0.1	0.09	0.09	0	0.38
0.0	0.25	0.50	0.50	0.00	0.1	0.1	0.1	0.06	0.06	0	0.38
0.0	0.50	0.01	0.01	0.00	0.1	0.1	0.1	0.25	0.25	0	0.50
0.0	0.50	0.01	0.25	0.00	0.1	0.1	0.1	0.19	0.19	0	0.50
0.0	0.50	0.01	0.50	0.00	0.1	0.1	0.1	0.13	0.13	0	0.50
0.0	0.50	0.25	0.01	0.00	0.1	0.1	0.1	0.25	0.25	0	0.38
0.0	0.50	0.25	0.25	0.00	0.1	0.1	0.1	0.19	0.19	0	0.38
0.0	0.50	0.25	0.50	0.00	0.1	0.1	0.1	0.13	0.13	0	0.38
0.0	0.50	0.50	0.01	0.00	0.1	0.1	0.1	0.25	0.25	0	0.25
0.0	0.50	0.50	0.25	0.00	0.1	0.1	0.1	0.19	0.19	0	0.25
0.0	0.50	0.50	0.50	0.00	0.1	0.1	0.1	0.13	0.13	0	0.25
0.2	0.0001	0.01	0.01	0.20	0.1	0.1	0.1	0.00	0.00	0	0.99
0.2	0.0001	0.01	0.25	0.20	0.1	0.1	0.1	0.00	0.00	0	0.99
0.2	0.0001	0.01	0.50	0.20	0.1	0.1	0.1	0.00	0.00	0	0.99
0.2	0.0001	0.25	0.01	0.20	0.1	0.1	0.1	0.00	0.00	0	0.75
0.2	0.0001	0.25	0.25	0.20	0.1	0.1	0.1	0.00	0.00	0	0.75
0.2	0.0001	0.25	0.50	0.20	0.1	0.1	0.1	0.00	0.00	0	0.75
0.2	0.0001	0.50	0.01	0.20	0.1	0.1	0.1	0.00	0.00	0	0.50
0.2	0.0001	0.50	0.25	0.20	0.1	0.1	0.1	0.00	0.00	0	0.50
0.2	0.0001	0.50	0.50	0.20	0.1	0.1	0.1	0.00	0.00	0	0.50
0.2	0.05	0.01	0.01	0.20	0.1	0.1	0.1	0.02	0.02	0	0.94
0.2	0.05	0.01	0.25	0.20	0.1	0.1	0.1	0.02	0.02	0	0.94
0.2	0.05	0.01	0.50	0.20	0.1	0.1	0.1	0.01	0.01	0	0.94
0.2	0.05	0.25	0.01	0.20	0.1	0.1	0.1	0.02	0.02	0	0.71
0.2	0.05	0.25	0.25	0.20	0.1	0.1	0.1	0.02	0.02	0	0.71
0.2	0.05	0.25	0.50	0.20	0.1	0.1	0.1	0.01	0.01	0	0.71
0.2	0.05	0.50	0.01	0.20	0.1	0.1	0.1	0.02	0.02	0	0.48
0.2	0.05	0.50	0.25	0.20	0.1	0.1	0.1	0.02	0.02	0	0.48
0.2	0.05	0.50	0.50	0.20	0.1	0.1	0.1	0.01	0.01	0	0.48
0.2	0.15	0.01	0.01	0.20	0.1	0.1	0.1	0.07	0.07	0	0.84
0.2	0.15	0.01	0.25	0.20	0.1	0.1	0.1	0.06	0.06	0	0.84
0.2	0.15	0.01	0.50	0.20	0.1	0.1	0.1	0.04	0.04	0	0.84
0.2	0.15	0.25	0.01	0.20	0.1	0.1	0.1	0.07	0.07	0	0.64
0.2	0.15	0.25	0.25	0.20	0.1	0.1	0.1	0.06	0.06	0	0.64
0.2	0.15	0.25	0.50	0.20	0.1	0.1	0.1	0.04	0.04	0	0.64

Table 1, cont.

δ	ICC	Level 1 R^2	Level 2 R^2	γ_{00}	γ_{01}	γ_{10}	γ_{11}	τ_{00}	τ_{11}	τ_{01}	σ^2
0.2	0.15	0.50	0.01	0.20	0.1	0.1	0.1	0.07	0.07	0	0.43
0.2	0.15	0.50	0.25	0.20	0.1	0.1	0.1	0.06	0.06	0	0.43
0.2	0.15	0.50	0.50	0.20	0.1	0.1	0.1	0.04	0.04	0	0.43
0.2	0.25	0.01	0.25	0.20	0.1	0.1	0.1	0.09	0.09	0	0.74
0.2	0.25	0.01	0.50	0.20	0.1	0.1	0.1	0.06	0.06	0	0.74
0.2	0.25	0.25	0.01	0.20	0.1	0.1	0.1	0.12	0.12	0	0.56
0.2	0.25	0.25	0.25	0.20	0.1	0.1	0.1	0.09	0.09	0	0.56
0.2	0.25	0.25	0.50	0.20	0.1	0.1	0.1	0.06	0.06	0	0.56
0.2	0.25	0.50	0.01	0.20	0.1	0.1	0.1	0.12	0.12	0	0.38
0.2	0.25	0.50	0.25	0.20	0.1	0.1	0.1	0.09	0.09	0	0.38
0.2	0.25	0.50	0.50	0.20	0.1	0.1	0.1	0.06	0.06	0	0.38
0.2	0.50	0.01	0.01	0.20	0.1	0.1	0.1	0.25	0.25	0	0.50
0.2	0.50	0.01	0.25	0.20	0.1	0.1	0.1	0.19	0.19	0	0.50
0.2	0.50	0.01	0.50	0.20	0.1	0.1	0.1	0.13	0.13	0	0.50
0.2	0.50	0.25	0.01	0.20	0.1	0.1	0.1	0.25	0.25	0	0.38
0.2	0.50	0.25	0.25	0.20	0.1	0.1	0.1	0.19	0.19	0	0.38
0.2	0.50	0.25	0.50	0.20	0.1	0.1	0.1	0.13	0.13	0	0.38
0.2	0.50	0.50	0.01	0.20	0.1	0.1	0.1	0.25	0.25	0	0.25
0.2	0.50	0.50	0.25	0.20	0.1	0.1	0.1	0.19	0.19	0	0.25
0.2	0.50	0.50	0.50	0.20	0.1	0.1	0.1	0.13	0.13	0	0.25
0.5	0.0001	0.01	0.01	0.50	0.1	0.1	0.1	0.00	0.00	0	0.99
0.5	0.0001	0.01	0.25	0.50	0.1	0.1	0.1	0.00	0.00	0	0.99
0.5	0.0001	0.01	0.50	0.50	0.1	0.1	0.1	0.00	0.00	0	0.99
0.5	0.0001	0.25	0.01	0.50	0.1	0.1	0.1	0.00	0.00	0	0.75
0.5	0.0001	0.25	0.25	0.50	0.1	0.1	0.1	0.00	0.00	0	0.75
0.5	0.0001	0.25	0.50	0.50	0.1	0.1	0.1	0.00	0.00	0	0.75
0.5	0.0001	0.50	0.01	0.50	0.1	0.1	0.1	0.00	0.00	0	0.50
0.5	0.0001	0.50	0.25	0.50	0.1	0.1	0.1	0.00	0.00	0	0.50
0.5	0.0001	0.50	0.50	0.50	0.1	0.1	0.1	0.00	0.00	0	0.50
0.5	0.05	0.01	0.01	0.50	0.1	0.1	0.1	0.02	0.02	0	0.94
0.5	0.05	0.01	0.25	0.50	0.1	0.1	0.1	0.02	0.02	0	0.94
0.5	0.05	0.01	0.50	0.50	0.1	0.1	0.1	0.01	0.01	0	0.94
0.5	0.05	0.25	0.01	0.50	0.1	0.1	0.1	0.02	0.02	0	0.71
0.5	0.05	0.25	0.25	0.50	0.1	0.1	0.1	0.02	0.02	0	0.71
0.5	0.05	0.25	0.50	0.50	0.1	0.1	0.1	0.01	0.01	0	0.71

Table 1, cont.

δ	ICC	Level 1 R^2	Level 2 R^2	γ_{00}	γ_{01}	γ_{10}	γ_{11}	τ_{00}	τ_{11}	τ_{01}	σ^2
0.5	0.05	0.50	0.01	0.50	0.1	0.1	0.1	0.02	0.02	0	0.48
0.5	0.05	0.50	0.25	0.50	0.1	0.1	0.1	0.02	0.02	0	0.48
0.5	0.05	0.50	0.50	0.50	0.1	0.1	0.1	0.01	0.01	0	0.48
0.5	0.15	0.01	0.01	0.50	0.1	0.1	0.1	0.07	0.07	0	0.84
0.5	0.15	0.01	0.25	0.50	0.1	0.1	0.1	0.06	0.06	0	0.84
0.5	0.15	0.01	0.50	0.50	0.1	0.1	0.1	0.04	0.04	0	0.84
0.5	0.15	0.25	0.01	0.50	0.1	0.1	0.1	0.07	0.07	0	0.64
0.5	0.15	0.25	0.25	0.50	0.1	0.1	0.1	0.06	0.06	0	0.64
0.5	0.15	0.25	0.50	0.50	0.1	0.1	0.1	0.04	0.04	0	0.64
0.5	0.15	0.50	0.01	0.50	0.1	0.1	0.1	0.07	0.07	0	0.43
0.5	0.15	0.50	0.25	0.50	0.1	0.1	0.1	0.06	0.06	0	0.43
0.5	0.15	0.50	0.50	0.50	0.1	0.1	0.1	0.04	0.04	0	0.43
0.5	0.25	0.01	0.01	0.50	0.1	0.1	0.1	0.12	0.12	0	0.74
0.5	0.25	0.01	0.25	0.50	0.1	0.1	0.1	0.09	0.09	0	0.74
0.5	0.25	0.01	0.50	0.50	0.1	0.1	0.1	0.06	0.06	0	0.74
0.5	0.25	0.25	0.01	0.50	0.1	0.1	0.1	0.12	0.12	0	0.56
0.5	0.25	0.25	0.25	0.50	0.1	0.1	0.1	0.09	0.09	0	0.56
0.5	0.25	0.25	0.50	0.50	0.1	0.1	0.1	0.06	0.06	0	0.56
0.5	0.25	0.50	0.01	0.50	0.1	0.1	0.1	0.12	0.12	0	0.38
0.5	0.25	0.50	0.25	0.50	0.1	0.1	0.1	0.09	0.09	0	0.38
0.5	0.25	0.50	0.50	0.50	0.1	0.1	0.1	0.06	0.06	0	0.38
0.5	0.50	0.01	0.01	0.50	0.1	0.1	0.1	0.25	0.25	0	0.50
0.5	0.50	0.01	0.25	0.50	0.1	0.1	0.1	0.19	0.19	0	0.50
0.5	0.50	0.01	0.50	0.50	0.1	0.1	0.1	0.13	0.13	0	0.50
0.5	0.50	0.25	0.01	0.50	0.1	0.1	0.1	0.25	0.25	0	0.38
0.5	0.50	0.25	0.25	0.50	0.1	0.1	0.1	0.19	0.19	0	0.38
0.5	0.50	0.25	0.50	0.50	0.1	0.1	0.1	0.13	0.13	0	0.38
0.5	0.50	0.50	0.01	0.50	0.1	0.1	0.1	0.25	0.25	0	0.25
0.5	0.50	0.50	0.25	0.50	0.1	0.1	0.1	0.19	0.19	0	0.25
0.5	0.50	0.50	0.50	0.50	0.1	0.1	0.1	0.13	0.13	0	0.25
0.8	0.0001	0.01	0.01	0.80	0.1	0.1	0.1	0.00	0.00	0	0.99
0.8	0.0001	0.01	0.25	0.80	0.1	0.1	0.1	0.00	0.00	0	0.99
0.8	0.0001	0.01	0.50	0.80	0.1	0.1	0.1	0.00	0.00	0	0.99
0.8	0.0001	0.25	0.01	0.80	0.1	0.1	0.1	0.00	0.00	0	0.75
0.8	0.0001	0.25	0.25	0.80	0.1	0.1	0.1	0.00	0.00	0	0.75

Table 1, cont.

δ	ICC	Level 1 R^2	Level 2 R^2	γ_{00}	γ_{01}	γ_{10}	γ_{11}	τ_{00}	τ_{11}	τ_{01}	σ^2
0.8	0.0001	0.25	0.50	0.80	0.1	0.1	0.1	0.00	0.00	0	0.75
0.8	0.0001	0.50	0.01	0.80	0.1	0.1	0.1	0.00	0.00	0	0.50
0.8	0.0001	0.50	0.25	0.80	0.1	0.1	0.1	0.00	0.00	0	0.50
0.8	0.0001	0.50	0.50	0.80	0.1	0.1	0.1	0.00	0.00	0	0.50
0.8	0.05	0.01	0.01	0.80	0.1	0.1	0.1	0.02	0.02	0	0.94
0.8	0.05	0.01	0.25	0.80	0.1	0.1	0.1	0.02	0.02	0	0.94
0.8	0.05	0.01	0.50	0.80	0.1	0.1	0.1	0.01	0.01	0	0.94
0.8	0.05	0.25	0.01	0.80	0.1	0.1	0.1	0.02	0.02	0	0.71
0.8	0.05	0.25	0.25	0.80	0.1	0.1	0.1	0.02	0.02	0	0.71
0.8	0.05	0.25	0.50	0.80	0.1	0.1	0.1	0.01	0.01	0	0.71
0.8	0.05	0.50	0.01	0.80	0.1	0.1	0.1	0.02	0.02	0	0.48
0.8	0.05	0.50	0.25	0.80	0.1	0.1	0.1	0.02	0.02	0	0.48
0.8	0.05	0.50	0.50	0.80	0.1	0.1	0.1	0.01	0.01	0	0.48
0.8	0.15	0.01	0.01	0.80	0.1	0.1	0.1	0.07	0.07	0	0.84
0.8	0.15	0.01	0.25	0.80	0.1	0.1	0.1	0.06	0.06	0	0.84
0.8	0.15	0.01	0.50	0.80	0.1	0.1	0.1	0.04	0.04	0	0.84
0.8	0.15	0.25	0.01	0.80	0.1	0.1	0.1	0.07	0.07	0	0.64
0.8	0.15	0.25	0.25	0.80	0.1	0.1	0.1	0.06	0.06	0	0.64
0.8	0.15	0.25	0.50	0.80	0.1	0.1	0.1	0.04	0.04	0	0.64
0.8	0.15	0.50	0.01	0.80	0.1	0.1	0.1	0.07	0.07	0	0.43
0.8	0.15	0.50	0.25	0.80	0.1	0.1	0.1	0.06	0.06	0	0.43
0.8	0.15	0.50	0.50	0.80	0.1	0.1	0.1	0.04	0.04	0	0.43
0.8	0.25	0.01	0.01	0.80	0.1	0.1	0.1	0.12	0.12	0	0.74
0.8	0.25	0.01	0.25	0.80	0.1	0.1	0.1	0.09	0.09	0	0.74
0.8	0.25	0.01	0.50	0.80	0.1	0.1	0.1	0.06	0.06	0	0.74
0.8	0.25	0.25	0.01	0.80	0.1	0.1	0.1	0.12	0.12	0	0.56
0.8	0.25	0.25	0.25	0.80	0.1	0.1	0.1	0.09	0.09	0	0.56
0.8	0.25	0.25	0.50	0.80	0.1	0.1	0.1	0.06	0.06	0	0.56
0.8	0.25	0.50	0.01	0.80	0.1	0.1	0.1	0.12	0.12	0	0.38
0.8	0.25	0.50	0.25	0.80	0.1	0.1	0.1	0.09	0.09	0	0.38
0.8	0.25	0.50	0.50	0.80	0.1	0.1	0.1	0.06	0.06	0	0.38
0.8	0.50	0.01	0.01	0.80	0.1	0.1	0.1	0.25	0.25	0	0.50
0.8	0.50	0.01	0.25	0.80	0.1	0.1	0.1	0.19	0.19	0	0.50
0.8	0.50	0.01	0.50	0.80	0.1	0.1	0.1	0.13	0.13	0	0.50
0.8	0.50	0.25	0.01	0.80	0.1	0.1	0.1	0.25	0.25	0	0.38

Table 1, cont.

δ	ICC	Level 1 R^2	Level 2 R^2	γ_{00}	γ_{01}	γ_{10}	γ_{11}	τ_{00}	τ_{11}	τ_{01}	σ^2
0.8	0.50	0.25	0.25	0.80	0.1	0.1	0.1	0.19	0.19	0	0.38
0.8	0.50	0.25	0.50	0.80	0.1	0.1	0.1	0.13	0.13	0	0.38
0.8	0.50	0.50	0.01	0.80	0.1	0.1	0.1	0.25	0.25	0	0.25
0.8	0.50	0.50	0.25	0.80	0.1	0.1	0.1	0.19	0.19	0	0.25
0.8	0.50	0.50	0.50	0.80	0.1	0.1	0.1	0.13	0.13	0	0.25

Analysis of the Average Run Lengths

Differences in the operation of the two QCC, RCRMCC and RCC, were evaluated through direct comparison of the RCRMCC and RCC ARL table cells. Further comparisons were made through graphical displays and descriptive summaries of the ARL table patterns, as appropriate.

CHAPTER IV

RESULTS

Parameter Estimates under the In-Control Population

The SLR model produced estimates of the slope (Min: 0.098, Max: 0.100) and intercept (Min: -0.001, Max: 0.003) that were consistent with the in-control population parameters, across all level 1 and level 2 R^2 values, under the *ICC* equal to 0.0001 condition. See Table 2 for the 95% confidence intervals for the parameter estimates. When the *ICC* is at the 0.05 level or above, the slope estimates of the SLR model are no longer consistent with the in-control population parameters, with slope estimates ranging from -0.00064 to 0.00116. Furthermore the range of estimates for the intercepts becomes much greater at -0.012 to 0.032. There appeared to be no effect of the level 1 and level 2 R^2 values on the intercept and slope estimates. Table 3 provides the intercept, slope and error estimates under all in-control population parameter crosses for the SLR models. The RCRM parameter estimates across all population conditions, though varied, were consistent with the in-control population parameters, see table 2 for the 95% confidence intervals for the parameter estimates. The main variables that affected the estimation of the RCRM parameters were the level 1 and 2 R^2 values, where greater deviation and variability of the parameter estimates is seen when either the level 1 or level 2 R^2 values are 0.01, the effect is greatest when both R^2 values are equal to 0.01. Furthermore, the estimates of the RCRM parameters are also more widely varied and deviated when the

ICC is equal to 0.0001. Table 4 provides the level 1 intercept and slope, level 2 covariate

effects, and variance estimates under all in-control population parameter crosses.

Table 2

95% Confidence Intervals for the In-Control SLR and RCRM Model Parameter Estimates

ICC	L1R2	L2R2	RCRM Model Parameter								OLS Model Parameters			
			Intercept		Xij		Wj		Wj::Xij		Intercept		Xij	
			Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
0.0001	0.01	0.01	0.00	0.00	0.10	0.10	-0.42	0.17	-0.16	0.40	0.00	0.00	0.10	0.10
		0.25	0.00	0.00	0.10	0.10	0.08	0.19	0.03	0.14	0.00	0.00	0.10	0.10
		0.50	0.00	0.00	0.10	0.10	0.09	0.17	0.09	0.17	0.00	0.00	0.10	0.10
	0.25	0.01	0.00	0.00	0.10	0.10	-0.22	0.28	0.04	0.19	0.00	0.00	0.10	0.10
		0.25	0.00	0.00	0.10	0.10	0.03	0.12	0.10	0.12	0.00	0.00	0.10	0.10
		0.50	0.00	0.00	0.10	0.10	0.04	0.11	0.09	0.11	0.00	0.00	0.10	0.10
	0.50	0.01	0.00	0.00	0.10	0.10	-0.10	0.32	0.01	0.14	0.00	0.00	0.10	0.10
		0.25	0.00	0.00	0.10	0.10	0.09	0.17	0.09	0.11	0.00	0.00	0.10	0.10
		0.50	0.00	0.00	0.10	0.10	0.05	0.11	0.09	0.11	0.00	0.00	0.10	0.10
0.05	0.01	0.01	-0.01	0.01	0.07	0.09	0.09	0.22	0.10	0.20	-0.01	0.00	0.00	0.00
		0.25	-0.01	0.01	0.07	0.08	0.08	0.10	0.07	0.09	0.00	0.00	0.00	0.00
		0.50	-0.01	0.01	0.09	0.10	0.09	0.10	0.09	0.10	0.00	0.01	0.00	0.00
	0.25	0.01	-0.01	0.01	0.09	0.11	0.06	0.19	0.02	0.14	0.00	0.01	0.00	0.00
		0.25	-0.01	0.01	0.09	0.11	0.09	0.11	0.09	0.11	-0.01	0.00	0.00	0.00
		0.50	-0.01	0.01	0.09	0.11	0.10	0.11	0.09	0.10	0.00	0.00	0.00	0.00
	0.50	0.01	-0.02	0.00	0.09	0.11	0.06	0.19	0.04	0.15	-0.01	0.00	0.00	0.00
		0.25	-0.01	0.01	0.09	0.11	0.09	0.11	0.09	0.11	0.00	0.00	0.00	0.00
		0.50	-0.01	0.01	0.09	0.10	0.10	0.11	0.09	0.10	-0.01	0.00	0.00	0.00
0.15	0.01	0.01	-0.01	0.02	0.08	0.11	0.03	0.16	-0.01	0.10	0.00	0.01	0.00	0.00
		0.25	-0.02	0.01	0.08	0.11	0.10	0.12	0.07	0.09	0.00	0.00	0.00	0.00
		0.50	-0.01	0.01	0.09	0.11	0.10	0.11	0.09	0.10	0.00	0.00	0.00	0.00

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Table 2, cont.

ICC	L1R2	L2R2	RCRM Model Parameter								OLS Model Parameters			
			Intercept		Xij		Wj		Wj::Xij		Intercept		Xij	
			Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
	0.25	0.01	-0.01	0.03	0.08	0.11	-0.02	0.11	0.03	0.15	0.01	0.01	0.00	0.00
		0.25	-0.01	0.02	0.09	0.12	0.08	0.11	0.09	0.11	0.02	0.02	0.00	0.00
		0.50	-0.01	0.02	0.08	0.11	0.09	0.10	0.10	0.11	-0.01	0.00	0.00	0.00
	0.50	0.01	-0.02	0.01	0.07	0.11	0.00	0.13	0.02	0.15	0.00	0.00	0.00	0.00
		0.25	0.00	0.02	0.09	0.12	0.09	0.11	0.09	0.11	0.01	0.01	0.00	0.00
		0.50	-0.01	0.01	0.09	0.11	0.09	0.11	0.09	0.11	0.00	0.01	0.00	0.00
0.25	0.01	0.01	-0.04	0.01	0.07	0.11	0.08	0.21	0.06	0.16	-0.01	-0.01	0.00	0.00
		0.25	-0.03	0.01	0.05	0.09	0.09	0.11	0.07	0.10	-0.01	-0.01	0.00	0.00
		0.50	-0.02	0.01	0.09	0.12	0.09	0.10	0.09	0.11	-0.01	-0.01	0.00	0.00
	0.25	0.01	-0.01	0.03	0.09	0.13	0.04	0.16	0.03	0.15	0.01	0.01	0.00	0.00
		0.25	-0.02	0.02	0.06	0.10	0.09	0.12	0.08	0.10	0.00	0.00	0.00	0.00
		0.50	-0.01	0.02	0.08	0.11	0.10	0.11	0.09	0.10	0.01	0.01	0.00	0.00
	0.50	0.01	-0.02	0.03	0.08	0.12	0.03	0.16	0.03	0.15	0.01	0.01	0.00	0.00
		0.25	0.00	0.04	0.09	0.13	0.09	0.11	0.09	0.12	0.03	0.03	0.00	0.00
		0.50	-0.02	0.01	0.09	0.12	0.09	0.11	0.09	0.11	-0.01	-0.01	0.00	0.00
0.50	0.01	0.01	-0.04	0.03	0.06	0.11	0.04	0.16	0.01	0.11	-0.01	0.00	0.00	0.00
		0.25	-0.04	0.01	0.06	0.11	0.08	0.11	0.07	0.09	-0.01	-0.01	0.00	0.00
		0.50	-0.03	0.02	0.08	0.12	0.10	0.11	0.09	0.10	0.01	0.01	0.00	0.00
	0.25	0.01	-0.02	0.04	0.09	0.16	0.05	0.18	0.07	0.19	0.00	0.01	0.00	0.00
		0.25	-0.01	0.05	0.06	0.12	0.09	0.11	0.08	0.10	0.00	0.00	0.00	0.00
		0.50	-0.01	0.03	0.07	0.11	0.09	0.11	0.09	0.11	0.02	0.03	0.00	0.00
	0.50	0.01	-0.02	0.04	0.09	0.15	0.05	0.18	0.13	0.25	0.01	0.01	0.00	0.00

Table 3

In-Control Population SLR Parameter Estimates
** $p < 0.05$ ^^ $p < 0.01$

ICC	L1R2	L2R2	Intercept (SE)		Xij (SE)		r
					Parameters		
0.0001	0.01	0.01	0.003	(0.00100)^^	0.100	(0.00099)^^	1.00
		0.25	0.001	(0.00099)	0.099	(0.00098)^^	0.99
		0.50	-0.001	(0.00100)	0.099	(0.00099)^^	1.00
	0.25	0.01	-0.001	(0.00087)	0.099	(0.00017)^^	0.87
		0.25	0.000	(0.00087)	0.098	(0.00017)^^	0.87
		0.50	-0.001	(0.00087)	0.099	(0.00017)^^	0.87
	0.50	0.01	0.000	(0.00071)	0.099	(0.00010)^^	0.71
		0.25	0.000	(0.00071)	0.099	(0.00010)^^	0.71
		0.50	0.000	(0.00071)	0.099	(0.00010)^^	0.71
0.05	0.01	0.01	-0.004	(0.00099)^^	0.000	(0.00008)^^	0.99
		0.25	0.002	(0.00099)	0.000	(0.00006)**	0.99
		0.50	0.006	(0.00099)^^	0.000	(0.00001)	0.99
	0.25	0.01	0.003	(0.00099)^^	0.000	(0.00001)^^	0.99
		0.25	-0.004	(0.00099)^^	0.000	(0.00001)^^	0.99
		0.50	-0.001	(0.00099)	0.000	(0.00002)^^	0.99
	0.50	0.01	-0.004	(0.00099)^^	0.000	(0.00000)^^	0.99
		0.25	0.000	(0.00099)	0.000	(0.00001)^^	0.99
		0.50	-0.005	(0.00099)^^	0.000	(0.00001)^^	0.99
0.15	0.01	0.01	0.003	(0.00096)^^	0.000	(0.00003)^^	0.96
		0.25	0.000	(0.00096)	0.000	(0.00001)^^	0.96
		0.50	-0.001	(0.00096)	0.000	(0.00010)^^	0.96
	0.25	0.01	0.009	(0.00096)^^	0.000	(0.00001)^^	0.96
		0.25	0.018	(0.00096)^^	0.000	(0.00000)^^	0.96
		0.50	-0.003	(0.00096)^^	0.000	(0.00003)^^	0.96
	0.50	0.01	-0.003	(0.00096)^^	0.000	(0.00001)^^	0.96
		0.25	0.008	(0.00096)^^	0.000	(0.00002)^^	0.96
		0.50	0.004	(0.00096)^^	0.000	(0.00000)^^	0.96
0.25	0.01	0.01	-0.011	(0.00094)^^	0.000	(0.00006)^^	0.94
		0.25	-0.012	(0.00093)^^	0.000	(0.00014)	0.93
		0.50	-0.008	(0.00094)^^	0.000	(0.00011)	0.94
	0.25	0.01	0.012	(0.00093)^^	0.000	(0.00002)^^	0.93
		0.25	-0.001	(0.00094)	0.000	(0.00004)^^	0.94
		0.50	0.007	(0.00094)^^	0.000	(0.00001)^^	0.94
	0.50	0.01	0.008	(0.00094)^^	0.000	(0.00002)**	0.94
		0.25	0.032	(0.00093)^^	0.000	(0.00003)^^	0.93
		0.50	-0.007	(0.00093)^^	0.000	(0.00000)^^	0.93

Table 3, cont.

ICC	L1R2	L2R2	Parameters				
			Intercept (SE)		Xij (SE)		r
0.50	0.01	0.01	-0.005	(0.00087)^^	0.001	(0.00037)**	0.87
		0.25	-0.007	(0.00086)^^	0.000	(0.00009)^^	0.86
		0.50	0.008	(0.00087)^^	0.000	(0.00014)	0.87
	0.25	0.01	0.006	(0.00087)^^	0.000	(0.00002)^^	0.87
		0.25	-0.002	(0.00087)	0.000	(0.00001)^^	0.87
		0.50	0.025	(0.00087)^^	0.000	(0.00002)^^	0.87
	0.50	0.01	0.013	(0.00087)^^	0.001	(0.00005)^^	0.87
		0.25	-0.003	(0.00087)^^	0.001	(0.00003)^^	0.87
		0.50	-0.010	(0.00086)^^	0.001	(0.00002)^^	0.86

Table 4

In-Control RCRM Population Parameter Estimates
** p < 0.05 ^^ p < 0.01

			Parameter Estimates (Standard Error)									
ICC	L1R2	L2R2	Intercept		Xij		Wj		Wj::Xij		u0j	u1j	rij	Total
0.0001	0.01	0.01	0.003	(0.01)^^	0.10	(0.001)^^	-0.12	(0.15)	0.12	(0.14)	0.00	0.00	0.99	0.99
		0.25	0.001	(0.01)	0.10	(0.001)^^	0.13	(0.03)^^	0.08	(0.03)^^	0.00	0.00	0.99	0.99
		0.50	-0.001	(0.01)	0.10	(0.001)^^	0.13	(0.02)^^	0.13	(0.02)^^	0.00	0.00	0.99	0.99
	0.25	0.01	-0.001	(0.01)	0.10	(0.001)^^	0.03	(0.13)	0.12	(0.04)^^	0.00	0.00	0.75	0.75
		0.25	0.000	(0.01)	0.10	(0.001)^^	0.07	(0.02)^^	0.11	(0.01)^^	0.00	0.00	0.75	0.75
		0.50	-0.001	(0.01)	0.10	(0.001)^^	0.07	(0.02)^^	0.10	(0.01)^^	0.00	0.00	0.75	0.75
	0.50	0.01	0.000	(0.01)	0.10	(0.001)^^	0.11	(0.11)	0.08	(0.03)^^	0.00	0.00	0.50	0.50
		0.25	0.000	(0.01)	0.10	(0.001)^^	0.13	(0.02)^^	0.10	(0.01)^^	0.00	0.00	0.50	0.50
		0.50	0.000	(0.01)	0.10	(0.001)^^	0.08	(0.02)^^	0.10	(0.01)^^	0.00	0.00	0.50	0.50
0.05	0.01	0.01	-0.004	(0.01)	0.08	(0.004)^^	0.15	(0.03)^^	0.15	(0.03)^^	0.02	0.02	0.94	0.98
		0.25	0.002	(0.01)	0.08	(0.004)^^	0.09	(0.01)^^	0.08	(0.01)^^	0.02	0.02	0.94	0.98
		0.50	-0.001	(0.01)	0.09	(0.004)^^	0.10	(0.01)^^	0.10	(0.01)^^	0.01	0.01	0.94	0.97
	0.25	0.01	0.003	(0.01)	0.10	(0.005)^^	0.12	(0.03)^^	0.08	(0.03)**	0.03	0.02	0.71	0.76
		0.25	-0.001	(0.01)	0.10	(0.004)^^	0.10	(0.01)^^	0.10	(0.01)^^	0.02	0.02	0.71	0.75
		0.50	-0.001	(0.01)	0.10	(0.004)^^	0.10	(0.01)^^	0.10	(0.01)^^	0.01	0.01	0.71	0.74
	0.50	0.01	-0.006	(0.01)	0.10	(0.005)^^	0.13	(0.03)^^	0.10	(0.03)^^	0.02	0.02	0.47	0.52
		0.25	0.003	(0.01)	0.10	(0.004)^^	0.10	(0.01)^^	0.10	(0.01)^^	0.02	0.02	0.47	0.51
		0.50	0.001	(0.01)	0.10	(0.003)^^	0.10	(0.01)^^	0.10	(0.01)^^	0.01	0.01	0.47	0.50
0.15	0.01	0.01	0.006	(0.01)	0.10	(0.008)^^	0.10	(0.03)^^	0.05	(0.03)	0.07	0.05	0.84	0.96
		0.25	-0.002	(0.01)	0.10	(0.007)^^	0.11	(0.01)^^	0.08	(0.01)^^	0.06	0.04	0.84	0.93
		0.50	-0.002	(0.01)	0.10	(0.006)^^	0.10	(0.01)^^	0.10	(0.01)^^	0.04	0.03	0.84	0.91

Table 4, cont.

ICC	L1R2	L2R2	Intercept		Xij		Wj		Wj::Xij		u0j	u1j	rij	Total
	0.25	0.01	0.010	(0.01)	0.10	(0.009)^^	0.05	(0.03)	0.09	(0.03)^^	0.07	0.07	0.64	0.78
		0.25	0.005	(0.01)	0.11	(0.008)^^	0.10	(0.01)^^	0.10	(0.01)^^	0.06	0.06	0.64	0.75
		0.50	0.004	(0.01)	0.10	(0.006)^^	0.10	(0.01)^^	0.10	(0.01)^^	0.04	0.04	0.64	0.71
	0.50	0.01	-0.002	(0.01)	0.10	(0.009)^^	0.06	(0.03)	0.08	(0.03)**	0.08	0.07	0.42	0.57
		0.25	0.010	(0.01)	0.10	(0.008)^^	0.10	(0.01)^^	0.10	(0.01)^^	0.06	0.06	0.42	0.54
		0.50	-0.001	(0.01)	0.10	(0.006)^^	0.10	(0.01)^^	0.10	(0.01)^^	0.04	0.04	0.43	0.51
0.25	0.01	0.01	-0.013	(0.01)	0.09	(0.010)^^	0.15	(0.03)^^	0.11	(0.03)^^	0.13	0.08	0.74	0.95
		0.25	-0.012	(0.01)	0.07	(0.010)^^	0.10	(0.01)^^	0.09	(0.01)^^	0.09	0.07	0.74	0.91
		0.50	-0.009	(0.01)	0.11	(0.008)^^	0.10	(0.01)^^	0.10	(0.01)^^	0.06	0.06	0.74	0.87
	0.25	0.01	0.012	(0.01)	0.11	(0.011)^^	0.10	(0.03)^^	0.09	(0.03)^^	0.12	0.12	0.56	0.80
		0.25	0.003	(0.01)	0.08	(0.010)^^	0.10	(0.01)^^	0.09	(0.01)^^	0.10	0.09	0.56	0.74
		0.50	0.005	(0.01)	0.09	(0.008)^^	0.10	(0.01)^^	0.10	(0.01)^^	0.06	0.06	0.56	0.68
	0.50	0.01	0.007	(0.01)	0.10	(0.011)^^	0.10	(0.03)^^	0.10	(0.03)^^	0.13	0.11	0.37	0.62
		0.25	0.020	(0.01)**	0.11	(0.011)^^	0.10	(0.01)^^	0.11	(0.01)^^	0.09	0.11	0.38	0.57
		0.50	-0.002	(0.01)	0.10	(0.008)^^	0.10	(0.01)^^	0.10	(0.01)^^	0.06	0.06	0.37	0.50
0.50	0.01	0.01	-0.006	(0.02)	0.08	(0.014)^^	0.10	(0.03)^^	0.06	(0.03)**	0.26	0.16	0.50	0.92
		0.25	-0.012	(0.01)	0.09	(0.013)^^	0.10	(0.01)^^	0.08	(0.01)^^	0.18	0.14	0.49	0.82
		0.50	-0.003	(0.01)	0.10	(0.011)^^	0.10	(0.01)^^	0.10	(0.01)^^	0.13	0.12	0.49	0.74
	0.25	0.01	0.008	(0.02)	0.13	(0.016)^^	0.12	(0.03)^^	0.13	(0.03)^^	0.26	0.25	0.37	0.88
		0.25	0.020	(0.01)	0.09	(0.014)^^	0.10	(0.01)^^	0.10	(0.01)^^	0.19	0.18	0.38	0.75
		0.50	0.010	(0.01)	0.09	(0.011)^^	0.10	(0.01)^^	0.10	(0.01)^^	0.13	0.13	0.38	0.63
	0.50	0.01	0.012	(0.02)	0.12	(0.016)^^	0.12	(0.03)^^	0.19	(0.03)^^	0.25	0.24	0.25	0.74
		0.25	-0.009	(0.01)	0.10	(0.014)^^	0.11	(0.01)^^	0.10	(0.01)^^	0.19	0.19	0.25	0.63
		0.50	-0.010	(0.01)	0.11	(0.011)^^	0.10	(0.01)^^	0.10	(0.01)^^	0.13	0.13	0.25	0.50

Parameter Estimates (Standard Error)

Comparison of Regression Control Chart and Random
Coefficient Regression Model Control Chart Average Run
Lengths under In-Control Population Parameters

Under the in-control population parameters the RCRMCC had higher or equal

ARL as compared to the RCC except under the *ICC* equal to 0.0001 crossed with the

level 1 R^2 equal to 0.01 conditions. Under the $ICC = 0.0001$ crossed with the level 1 $R^2 =$

0.01 conditions, the RCC performs 1% more efficiently than the RCRMCC. Under all

other conditions the RCRMCC has an equal or higher ARL, with a percentage difference

ranging from -1% to 58402%, where the percentage difference is the difference between

the RCRMCC and RCC ARL divided by the RCC ARL.   As the *ICC*, level 1 R^2, and

level 2 R^2 values increase the ARL differences also increase.   Tables 5 and 6 provide the

RCC and RCRMCC ARL under each of the in-control population conditions, with

figures 1 and 2 providing a graphical display of the ARL, respectively. Table 7 provides

the RCRMCC minus RCC ARL differences, with Table 8 providing the percentage of the

differences relative to the RCC ARL estimate, which are also graphed in Figure 3.

Table 5

In-Control CUSUM RCC ARL

		Intra-Class Correlation				
L1R2	L2R2	0.0001	0.05	0.15	0.25	0.50
0.01	0.01	73	75	86	93	143
0.01	0.25	72	75	85	98	159
0.01	0.50	72	75	85	96	152
0.25	0.01	150	75	85	97	144
0.25	0.25	149	74	86	98	147
0.25	0.50	148	75	86	100	151
0.50	0.01	649	76	85	93	153
0.50	0.25	646	75	84	95	139
0.50	0.50	640	76	86	101	148

Table 6

In-Control CUSUM RCRMCC ARL

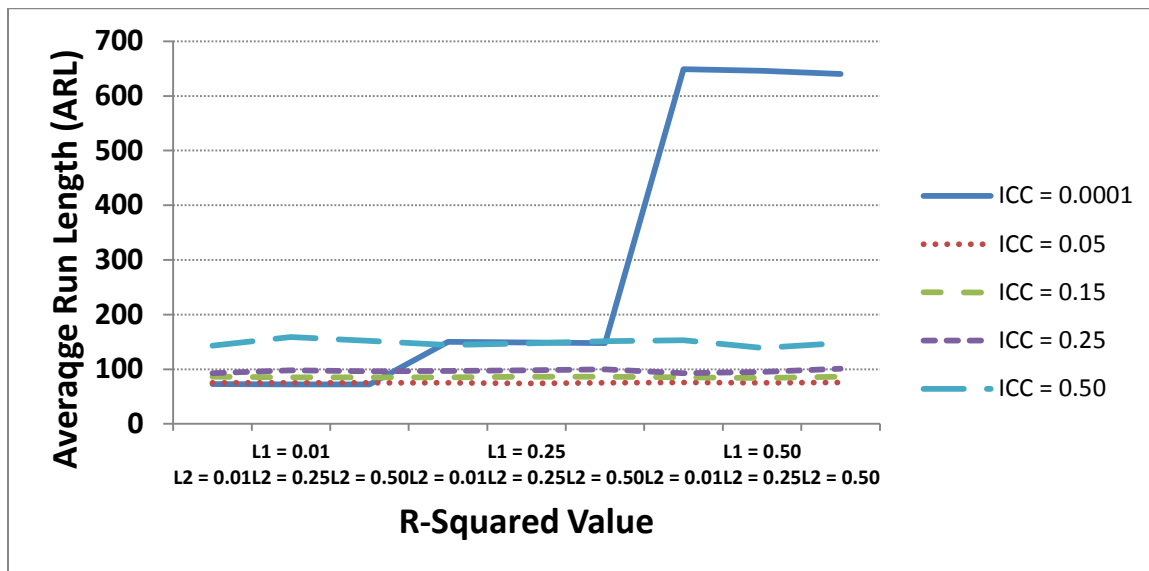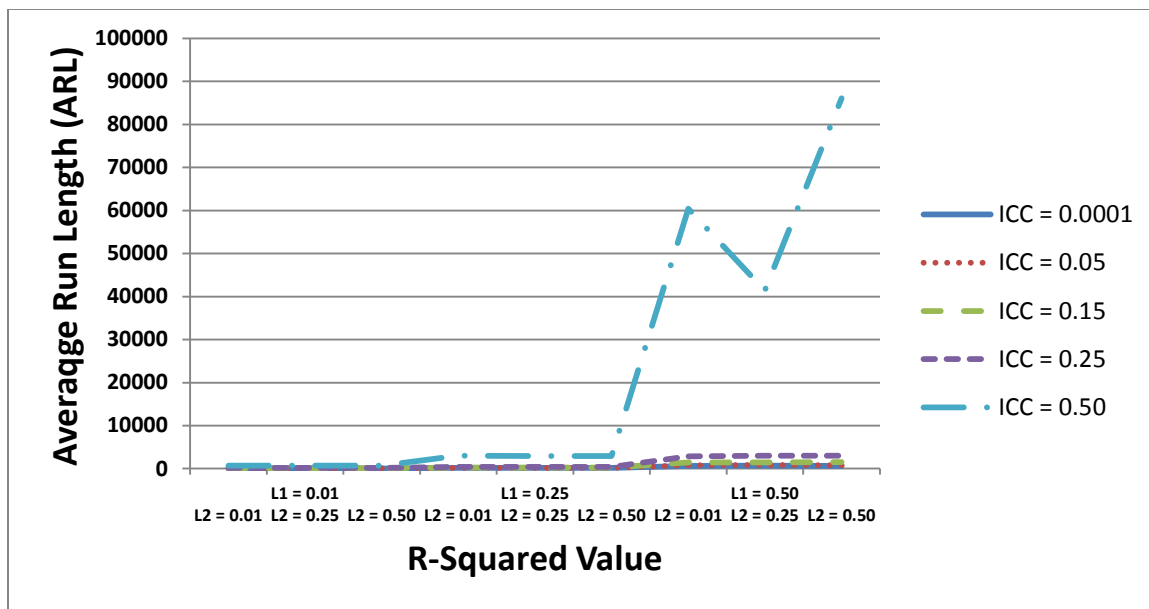|		Intra-Class Correlation				
L1R2	L2R2	0.0001	0.05	0.15	0.25	0.50
0.01	0.01	72	82	106	153	670
0.01	0.25	73	81	106	156	676
0.01	0.50	72	82	101	151	693
0.25	0.01	150	176	255	396	3035
0.25	0.25	151	177	248	400	2909
0.25	0.50	148	176	250	391	2905
0.50	0.01	664	811	1413	2860	60427
0.50	0.25	670	828	1459	3027	41405
0.50	0.50	655	815	1513	3011	86050

Figure 1.

In-Control CUSUM RCC ARL.

Figure 2.

In-Control CUSUM RCRMCC ARL.

Table 7

In-Control RCRMCC-RCC ARL Differences

L1R2	L2R2	Intra-Class Correlation				
		0.0001	0.05	0.15	0.25	0.50
0.01	0.01	-1	7	20	60	527
0.01	0.25	1	6	21	58	517
0.01	0.50	0	7	16	55	541
0.25	0.01	0	101	170	299	2891
0.25	0.25	2	103	162	302	2762
0.25	0.50	0	101	164	291	2754
0.50	0.01	15	735	1328	2767	60274
0.50	0.25	24	753	1375	2932	41266
0.50	0.50	15	739	1427	2910	85902

Table 8

In-Control RCRMCC-RCC ARL Percentage Difference in Comparison to RCC ARL

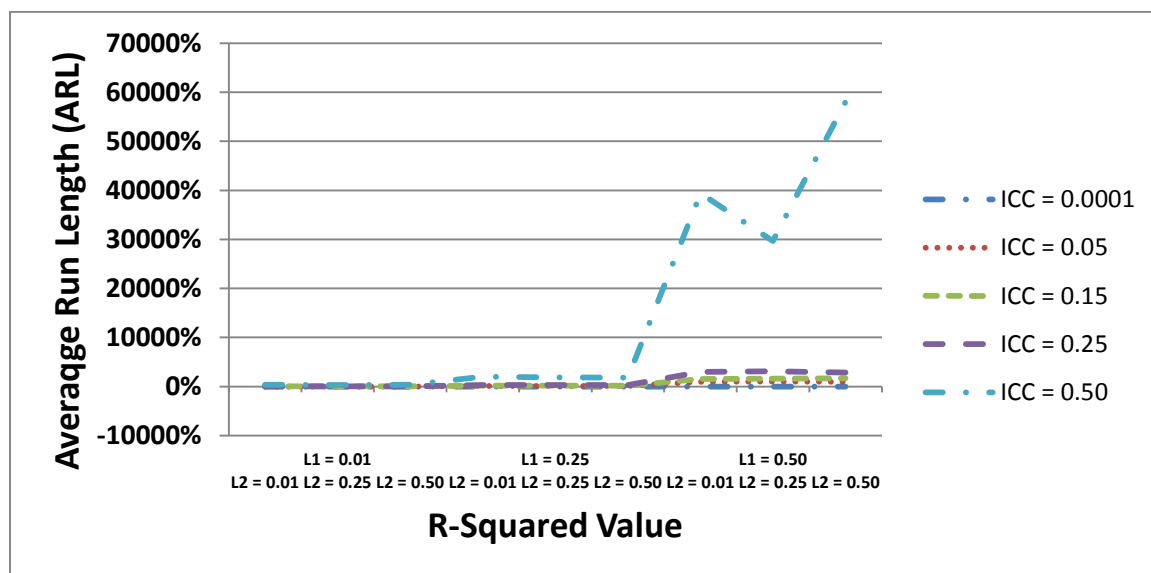|		Intra-Class Correlation				
Level 1 R2	Level 2 R2	0.0001	0.05	0.15	0.25	0.50
0.01	0.01	-1%	9%	23%	65%	369%
0.01	0.25	1%	8%	25%	59%	325%
0.01	0.50	0%	9%	19%	57%	356%
0.25	0.01	0%	135%	200%	308%	2008%
0.25	0.25	1%	139%	188%	308%	1879%
0.25	0.50	0%	135%	191%	291%	1824%
0.50	0.01	2%	967%	1562%	2975%	39395%
0.50	0.25	4%	1004%	1637%	3086%	29688%
0.50	0.50	2%	972%	1659%	2881%	58042%

Figure 3.

In-Control RCRMCC-RCC ARL percentage difference in comparison to RCC ARL.

Comparison of the Regression Control Chart and Random
Coefficient Regression Model Control Chart Average
Run Lengths under the Out-of-Control Population
Parameters

The out-of-control populations were composed of three mean shifts in the

intercept of the data values; 0.2, 0.5, and 0.8 standard deviations. Under the condition of

a mean shift of 0.2 standard deviations the RCRMCC had higher ARL than the RCC,

except in the case of the *ICC* being equal to 0.0001 where under some conditions the

RCRMCC had lower ARL. The differences in the ARL relative to the RCC ARL ranged

from -6% to 365%, with the percentage difference increasing with the *ICC*, level 1 R^2,

and level 2 R^2 values. Tables 9 and 10 provide the RCC and RCRMCC ARL under each

of the 0.2 standard deviation mean shift out-of-control population conditions, with figures

4 and 5 providing graphical displays of the ARL, respectively. Table 11 provides the

RCRMCC minus RCC ARL differences, with Table 12 providing the percentage of the

differences relative to the RCC ARL estimates and Figure 6 providing graphical displays

of the percentage differences. Under the condition of a mean shift of 0.5 standard

deviations, the RCRMCC had higher or equal ARL than the RCC. The differences in the

ARL, relative to the RCC ARL, ranged from 0% to 18%, with the percentage difference

increasing with the *ICC*, level 1 R^2, and level 2 R^2 values. Tables 13 and 14 provide the

RCC and RCRMCC ARL under each of the 0.5 standard deviations mean shift out-of-

control population conditions, with figures 7 and 8 providing graphical displays of the

ARL, respectively. Table 15 provides the RCRMCC minus RCC ARL difference, with

Table 16 providing the percentage of the difference relative to the RCC ARL estimate.

Figure 9 provides the graphical display of the percentage differences. Under the condition

of a mean shift of 0.8 standard deviations, the RCRMCC had ARL equal to the RCC.

Both the RCC and RCRMCC had an ARL of 10 for all of the crossed conditions. Tables

17 and 18 provide the RCC and RCRMCC ARL under each of the 0.8 standard deviation

mean shift out-of-control population conditions, with figures 10 and 11 providing

graphical displays of the ARL, respectively. Table 19 provides the RCRMCC minus

RCC ARL difference, with Table 20 providing the percentage of the difference relative to

the RCC ARL estimate and Figure 12 providing a graphical display of the percentage

differences.

Table 9

Out-of-Control CUSUM RCC ARL under Mean Shift of 0.2 Standard Deviations

		Intra-Class Correlation				
Level 1 R2	Level 1 R2	0.0001	0.05	0.15	0.25	0.50
0.01	0.01	45	46	48	51	60
0.01	0.25	45	45	48	52	63
0.01	0.50	44	45	48	51	61
0.25	0.01	62	45	48	51	61
0.25	0.25	61	45	48	50	62
0.25	0.50	63	45	48	52	61
0.50	0.01	103	46	49	51	63
0.50	0.25	103	46	48	52	61
0.50	0.50	102	45	48	52	63

Table 10

Out-of-Control CUSUM RCRMCC ARL under Mean Shift of 0.2 Standard Deviations

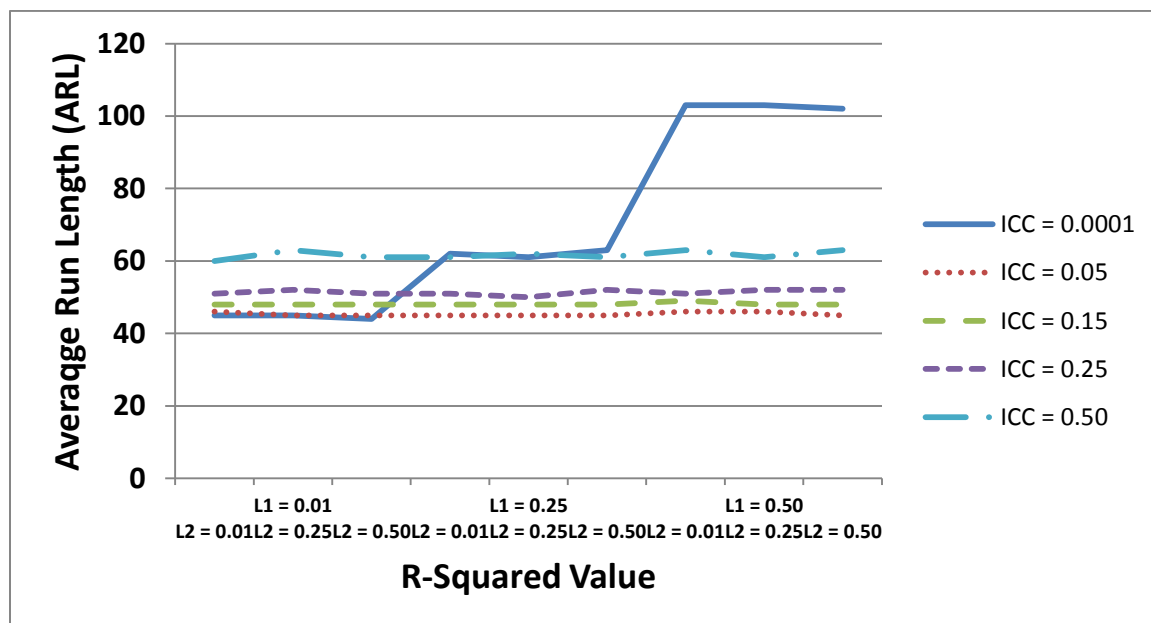| Level 1 R2	Level 2 R2	Intra-Class Correlation				
		0.0001	0.05	0.15	0.25	0.50
0.01	0.01	44	48	53	64	102
0.01	0.25	44	48	54	60	101
0.01	0.50	45	47	55	61	108
0.25	0.01	62	64	75	91	169
0.25	0.25	62	68	82	90	159
0.25	0.50	59	66	73	92	158
0.50	0.01	104	110	125	148	288
0.50	0.25	107	112	129	154	317
0.50	0.50	102	113	126	151	293

Figure 4.

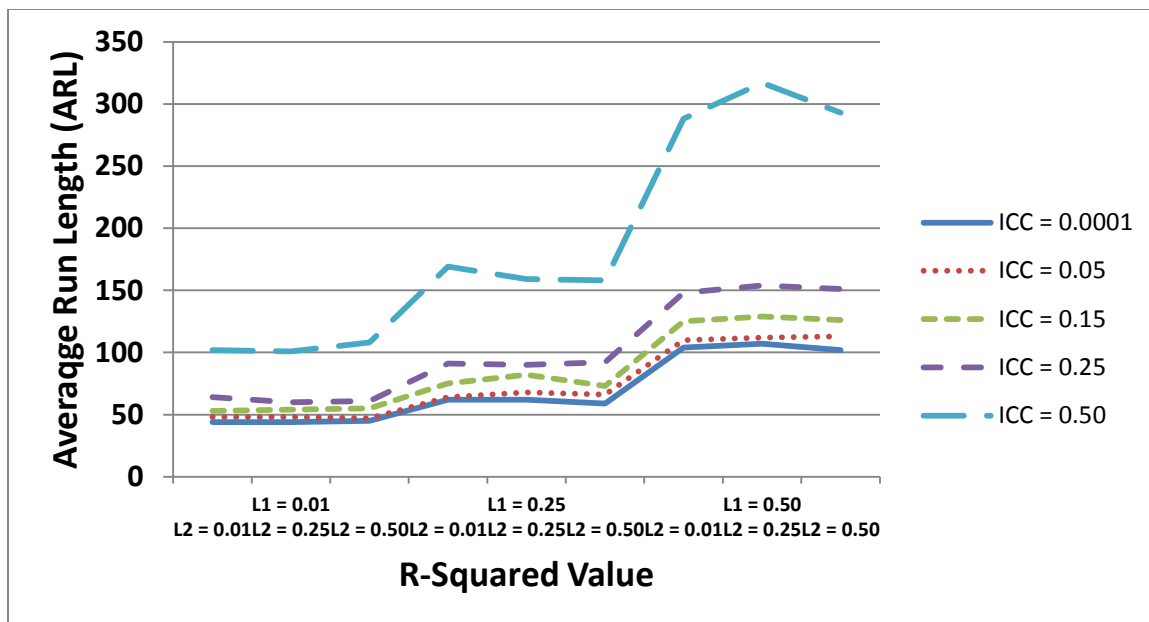Out-of-Control CUSUM RCC ARL under mean shift of 0.2 standard deviations.

Figure 5.

Out-of-Control CUSUM RCRMCC ARL under mean shift of 0.2 standard deviations.

Table 11

Out-of-Control RCRMCC-RCC ARL Differences under Mean Shift of 0.2 Standard Deviations

Level 1 R2	Level 2 R2	Intra-Class Correlation				
		0.0001	0.05	0.15	0.25	0.50
0.01	0.01	-1	2	5	13	42
0.01	0.25	-1	3	6	8	38
0.01	0.50	1	2	7	10	47
0.25	0.01	0	19	27	40	108
0.25	0.25	1	23	34	40	97
0.25	0.50	-4	21	25	40	97
0.50	0.01	1	64	76	97	225
0.50	0.25	4	66	81	102	256
0.50	0.50	0	68	78	99	230

Table 12

Out-of-Control RCRMCC-RCC ARL Percentage Difference in Comparison to RCC ARL under Mean Shift of 0.2 Standard Deviations

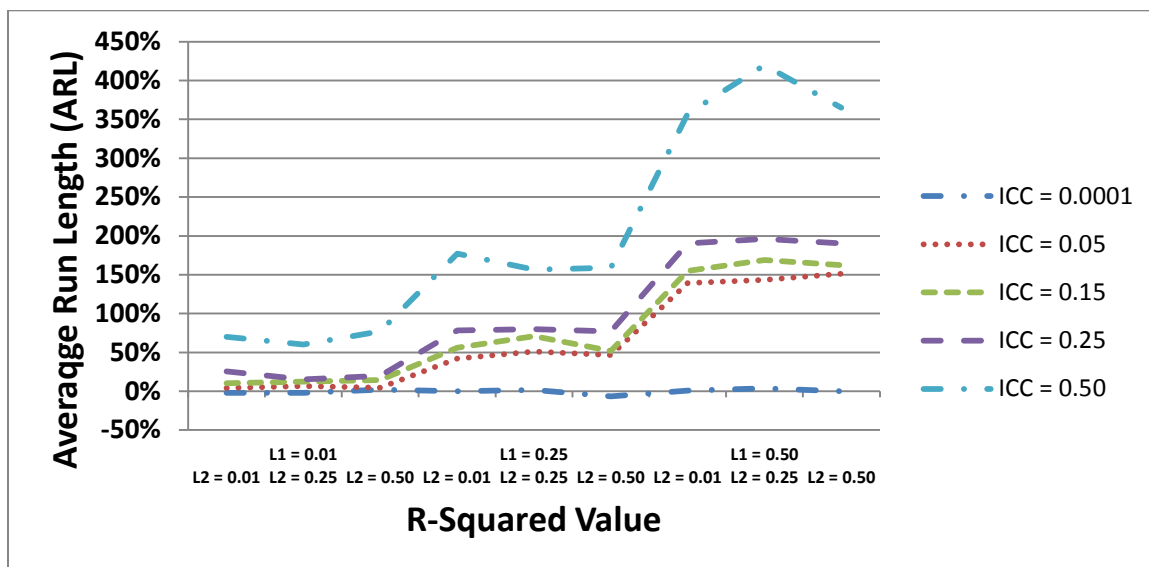|		Intra-Class Correlation				
Level 1 R2	Level 2 R2	0.0001	0.05	0.15	0.25	0.50
0.01	0.01	-2%	4%	10%	25%	70%
0.01	0.25	-2%	7%	13%	15%	60%
0.01	0.50	2%	4%	15%	20%	77%
0.25	0.01	0%	42%	56%	78%	177%
0.25	0.25	2%	51%	71%	80%	156%
0.25	0.50	-6%	47%	52%	77%	159%
0.50	0.01	1%	139%	155%	190%	357%
0.50	0.25	4%	143%	169%	196%	420%
0.50	0.50	0%	151%	163%	190%	365%

Figure 6.

Out-of-Control RCRMCC-RCC ARL percentage difference in comparison to RCC ARL under mean shift of 0.2 standard deviations.

Table 13

Out-of-Control CUSUM RCC ARL under Mean Shift of 0.5 Standard Deviations

Level 1 R2	Level 1 R2	Intra-Class Correlation				
		0.0001	0.05	0.15	0.25	0.50
0.01	0.01	17	17	17	18	18
0.01	0.25	17	17	17	18	18
0.01	0.50	17	17	17	18	18
0.25	0.01	18	17	17	18	18
0.25	0.25	18	17	17	18	18
0.25	0.50	18	17	17	17	18
0.50	0.01	19	17	17	18	18
0.50	0.25	19	17	17	18	18
0.50	0.50	19	17	17	18	18

Table 14

Out-of-Control CUSUM RCRMCC ARL under Mean Shift of 0.5 Standard Deviations

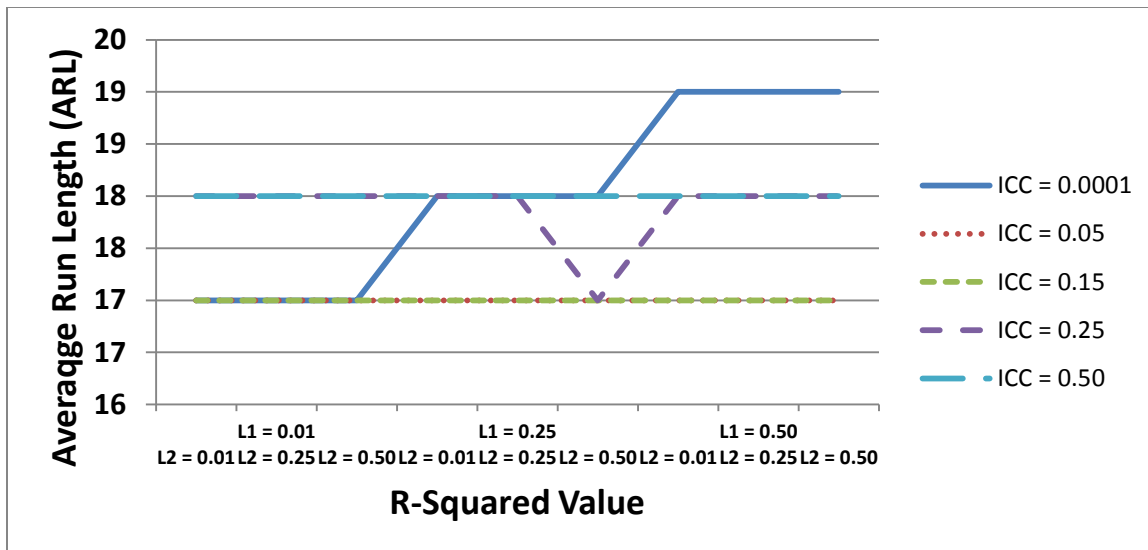| Level 1 R2	Level 2 R2	Intra-Class Correlation				
		0.0001	0.05	0.15	0.25	0.50
0.01	0.01	17	17	18	18	19
0.01	0.25	17	17	18	18	20
0.01	0.50	17	17	18	18	19
0.25	0.01	18	18	19	19	20
0.25	0.25	18	18	19	19	20
0.25	0.50	18	18	19	19	20
0.50	0.01	19	19	20	20	20
0.50	0.25	19	20	20	20	20
0.50	0.50	19	19	20	20	20

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Figure 7.

Out-of-Control CUSUM RCC ARL under mean shift of 0.5 standard deviations.
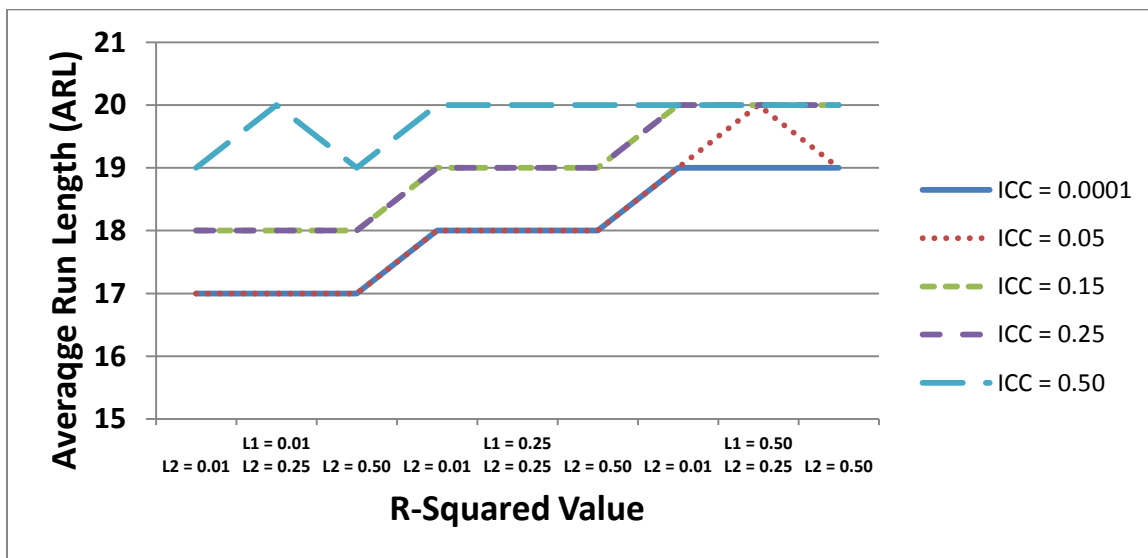

Figure 8.

Out-of-Control CUSUM RCRMCC ARL under mean shift of 0.5 standard deviations.

Table 15

Out-of-Control RCRMCC-RCC ARL Differences under Mean Shift of 0.5 Standard Deviations

		Intra-Class Correlation				
Level 1 R2	Level 2 R2	0.0001	0.05	0.15	0.25	0.50
0.01	0.01	0	0	1	0	1
0.01	0.25	0	0	1	0	2
0.01	0.50	0	0	1	0	1
0.25	0.01	0	1	2	1	2
0.25	0.25	0	1	2	1	2
0.25	0.50	0	1	2	2	2
0.50	0.01	0	2	3	2	2
0.50	0.25	0	3	3	2	2
0.50	0.50	0	2	3	2	2

Table 16

Out-of-Control RCRMCC-RCC ARL Percentage Difference in Comparison to RCC ARL under Mean Shift of 0.5 Standard Deviations

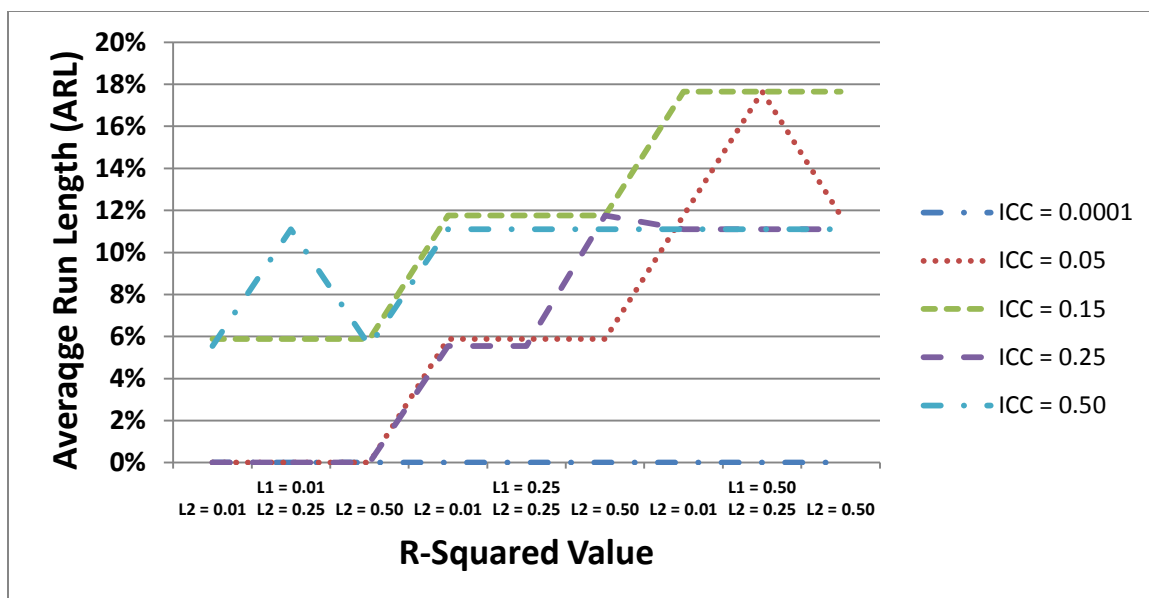|		Intra-Class Correlation				
Level 1 R2	Level 2 R2	0.0001	0.05	0.15	0.25	0.50
0.01	0.01	0%	0%	6%	0%	6%
0.01	0.25	0%	0%	6%	0%	11%
0.01	0.50	0%	0%	6%	0%	6%
0.25	0.01	0%	6%	12%	6%	11%
0.25	0.25	0%	6%	12%	6%	11%
0.25	0.50	0%	6%	12%	12%	11%
0.50	0.01	0%	12%	18%	11%	11%
0.50	0.25	0%	18%	18%	11%	11%
0.50	0.50	0%	12%	18%	11%	11%

Figure 9.

Out-of-Control RCRMCC-RCC ARL percentage difference in comparison to RCC ARL under mean shift of 0.5 standard deviations

Table 17

Out-of-Control CUSUM RCC ARL under Mean Shift of 0.8 Standard Deviations

		Intra-Class Correlation				
Level 1 R2	Level 1 R2	0.0001	0.05	0.15	0.25	0.50
0.01	0.01	10	10	10	10	10
0.01	0.25	10	10	10	10	10
0.01	0.50	10	10	10	10	10
0.25	0.01	10	10	10	10	10
0.25	0.25	10	10	10	10	10
0.25	0.50	10	10	10	10	10
0.50	0.01	10	10	10	10	10
0.50	0.25	10	10	10	10	10
0.50	0.50	10	10	10	10	10

Table 18

Out-of-Control CUSUM RCRMCC ARL under Mean Shift of 0.8 Standard Deviations

		Intra-Class Correlation				
Level 1 R2	Level 2 R2	0.0001	0.05	0.15	0.25	0.50
0.01	0.01	10	10	10	10	10
0.01	0.25	10	10	10	10	10
0.01	0.50	10	10	10	10	10
0.25	0.01	10	10	10	10	10
0.25	0.25	10	10	10	10	10
0.25	0.50	10	10	10	10	10
0.50	0.01	10	10	10	10	10
0.50	0.25	10	10	10	10	10
0.50	0.50	10	10	10	10	10

Figure 10.

Out-of-Control CUSUM RCC ARL under mean shift of 0.8 standard deviations.
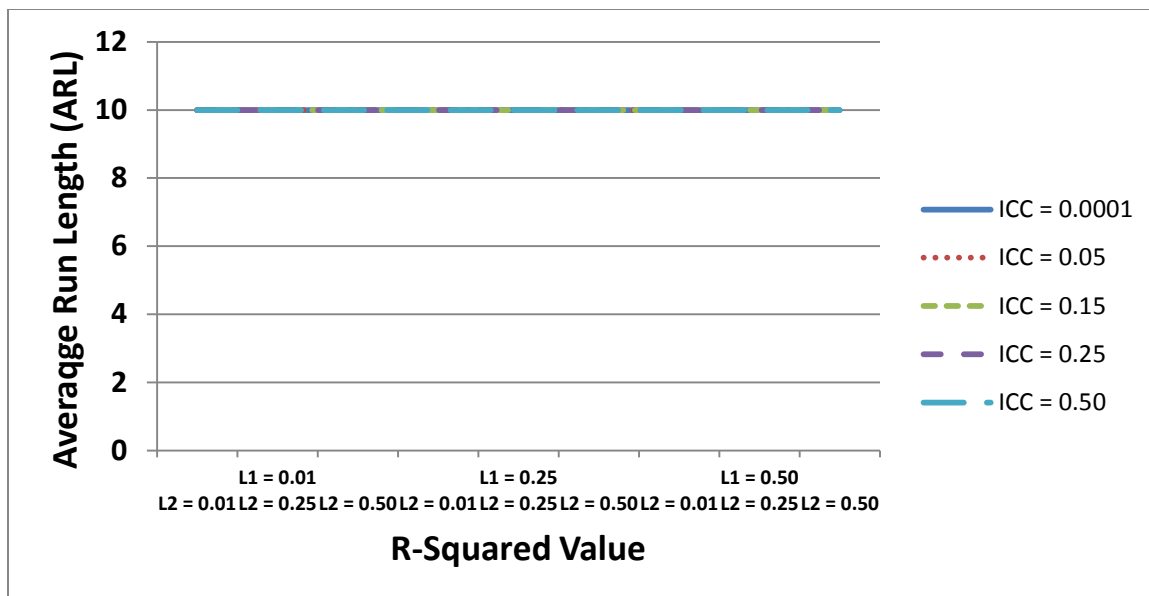
Figure 11.

Out-of-Control CUSUM RCRMCC ARL under mean shift of 0.8 standard deviations.

Table 19

Out-of-Control RCRMCC-RCC ARL Differences under Mean Shift of 0.8 Standard Deviations

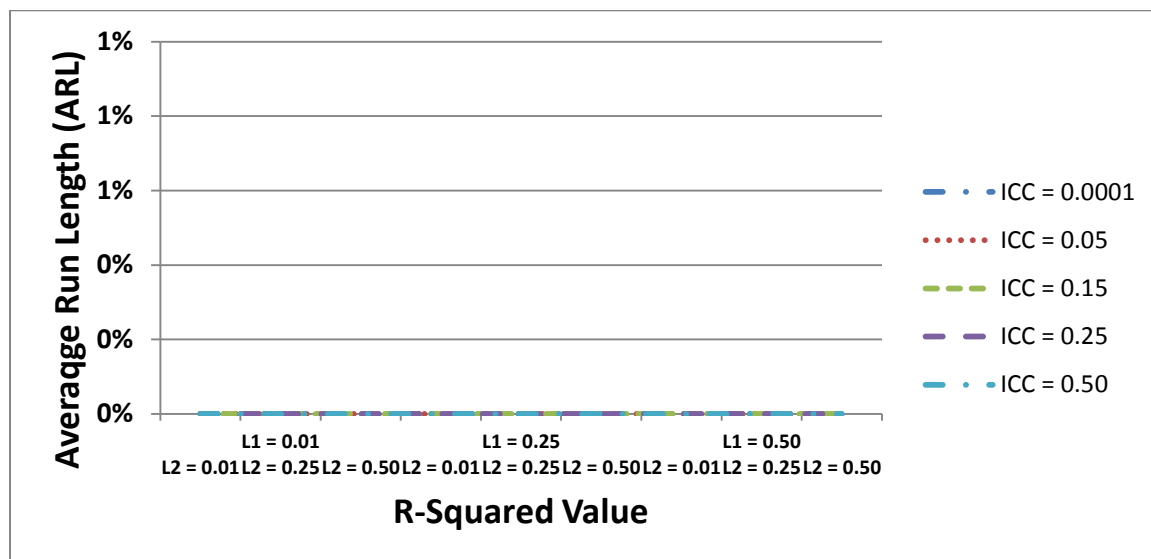| Level 1 R2	Level 2 R2	Intra-Class Correlation				
		0.0001	0.05	0.15	0.25	0.50
0.01	0.01	0	0	0	0	0
0.01	0.25	0	0	0	0	0
0.01	0.50	0	0	0	0	0
0.25	0.01	0	0	0	0	0
0.25	0.25	0	0	0	0	0
0.25	0.50	0	0	0	0	0
0.50	0.01	0	0	0	0	0
0.50	0.25	0	0	0	0	0
0.50	0.50	0	0	0	0	0

Table 20

Out-of-Control RCRMCC-RCC ARL Percentage Difference in Comparison to RCC ARL under Mean Shift of 0.8 Standard Deviations

		Intra-Class Correlation				
Level 1 R2	Level 2 R2	0.0001	0.05	0.15	0.25	0.50
0.01	0.01	0%	0%	0%	0%	0%
0.01	0.25	0%	0%	0%	0%	0%
0.01	0.50	0%	0%	0%	0%	0%
0.25	0.01	0%	0%	0%	0%	0%
0.25	0.25	0%	0%	0%	0%	0%
0.25	0.50	0%	0%	0%	0%	0%
0.50	0.01	0%	0%	0%	0%	0%
0.50	0.25	0%	0%	0%	0%	0%
0.50	0.50	0%	0%	0%	0%	0%

Figure 12.

Out-of-Control RCRMCC-RCC ARL percentage difference in comparison to RCC ARL under mean shift of 0.8 standard deviations

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Regarding the first research question, as expected, it was found that the RCRMCC has higher ARL than the RCC under the in-control population conditions, when the *ICC* is greater than or equal to 0.05. When the *ICC* is equal to 0.0001 the RCRMCC and RCC have equivalent ARL, where this would be expected given that the SLR parameter estimates are best under the *ICC* equal to 0.0001 conditions. Mixed results were found in regards to the second research question regarding the comparison of the RCRMCC and RCC under the out-of-control population conditions. The RCRMCC is found to be as efficient as the RCC under the out-of-control condition where the mean shift is equal to 0.8 standard deviations. Under the other out-of-control mean shifts the RCRMCC has a higher ARL than the RCC, but this is most likely due to the poor parameter estimates produced by the SLR for the RCC, resulting in larger residuals, thus producing the false appearance of having higher efficiency. Overall the results indicate that the RCRMCC is preferable under conditions where the *ICC* is greater than or equal to 0.05, where false alarms would be minimized under the RCRMCC, when the process is in-control.

Though the RCRMCC proved to be more advantageous than the RCC, a couple of abnormalities arose in the ARL obtained through the simulations. Under both the in-control and out-of-control conditions, the RCRMCC, under the *ICC* = 0.50 condition, and the RCC, under the *ICC* = 0.0001, produce very high ARL, under the level 1 R^2 equal to 0.5 condition, as compared to other ARL for the control charts. The increased

ARL are most likely due to the extremely low level of residuals produced by the RCRM and SLR models, see Tables 3 and 4, thereby producing much larger ARL since the small model residuals were less likely to increase the magnitude of the CUSUM statistics. Furthermore, I was unable to show that the RCRMCC was more efficient under the mean shifts of 0.2 and 0.5 standard deviations. In these cases the RCC consistently had lower ARL than the RCRMCC. This is most likely due to the poor estimation of the slope under the SLR model when the *ICC* is greater than or equal to 0.05. In these cases the SLR model estimated slopes around 0, which resulted in larger residuals, thus inflating the CUSUM statistics and leading to an out-of-control signal sooner than expected. Though both abnormalities exist, either the effect on the operation of the RCRMCC would be minimal or procedures could be implemented that would alleviate the issues. The latter is discussed more fully in the Future Studies subsection.

Overall when the *ICC* is greater than or equal to 0.05 the RCRMCC is more efficient (having a higher ARL) than the RCC under the in-control conditions and has equivalent efficiency (the same ARL) when a large mean shift has occurred in the system process. Thus, the RCRMCC would provide fewer false alarms under in-control system conditions and be able to detect large shifts in the system process under conditions common in the human services industry. Under the small and moderate mean shift out-of-control conditions, the RCRMCC appears to not detect the shift as quickly but other modifications to the CUSUM control chart parameters may allow for more equivalent rates of detection by the RCC.

Implication of Findings

In human services fields where the variability of the higher level units has an *ICC* level of 0.05 or greater the most appropriate control chart to be utilized would be the RCRMCC. Utilizing the RCRMCC would allow for fewer false alarms, simultaneous estimation of conditional relationships, and estimation of variability among units on which measures are taken. The RCRMCC thus would allow for better quality control charts within the human services field, opening up a new era of quality control within these industries.

Being able to utilize a more efficient quality control chart would allow for increased quality in the human services industry as based upon quantitative quality metrics for the field. By overcoming the obstacles associated with the conditional relationships and higher level variability, the RCRMCC allows for improved monitoring of human services process, identification of quality issues, and allowing for a minimal number of control charts to be utilized by the operator. Together this would further the implementation of more sophisticated quality control charts in the human services field, providing the ability to increase the quality of processes across many human services industries.

Limitations of Study

The current study focused on several key population characteristics that affect the estimation of parameters within the RCRM; *ICC,* level 1 R^2, and level 2 R^2 values. Though these parameters are integral to the estimation of the RCRM and thereby the performance of the RCRMCC, several other population characteristics would serve to further expand the application of the RCRMCC. The foremost limitation on the

simulations was that the RCRM was limited to one level of nesting, i.e. measures within units. Further levels of nesting can be accounted for in the RCRM, and thereby used to produce RCRMCCs with higher level conditional relationships. Another limitation was that the effect of the level 2 covariate on the intercept and slope was forced to be the same due to restrictions on the level 2 variance-covariance matrix. Since allowing for differing effects would have required differing variances of the effects, these variances were set to be fixed and equal to each other. This equivalence of the level 2 covariate effect on the intercept and slopes would not be a common event in practical application, and thus more likely scenarios may be considered in future simulations. A third limitation that should be considered is that both the RCC and RCRMCC CUSUM charts were set to detect a moderate shift in the mean, other settings may be of interest in future studies. Lastly, the effect of the covariate on the intercept and slope was fixed to have no correlation. In practical application this would not always be true, that the slopes and intercepts are completely independent of each other. Though none of these limitations take away from the end results of the study, they would be important consideration in the further exploration of the performance of the RCRMCC.

<div align="center">Human Services Applications</div>

The implementation of the RCRMCC in human services industries would utilize the same two phase process utilizes for other control charts. Phase I encompasses the identification of the quality metric of interest, nesting structure of the data, and covariates for each level of the RCRM. While the process is in an in-control state, a random sampling of units would be taken and all relevant quality measures and covariatestaken on each unit. Under the RCRM framework an appropriate and valid model would be

estimated from the randomly selected sample. The estimated in-control model would then be used to produce residuals on subsequently sample units in Phase II. Under Phase II a continuation of the sampling process would occur, where the residual from the in-control model for each unit would be computed. These residuals would then be fed into an appropriate QCC, i.e. CUSUM or Shewhart type, for monitoring of shifts in the process. In order to illustrate this process an example from the mental health services field is provided.

Over the past several years the Mental Health Center of Denver (MHCD; Denver, CO) has incorporated a RCRMCC, utilizing the CUSUM framework, into their overall quality control management system (McKinney, Olmos-Gallo, DeRoche, & LaGanga, 2009). In MHCD's system the level of nesting are measures over time (Level 1) within consumers (Level 2). The quality metric of interest was an internally developed recovery scale that incorporated items related to symptom management, engagement in services, housing status, and interest in education and employment. The level 1 covariate of interest was the number of months since treatment (rate of change in recovery), with level 2 covariates of diagnosis (mood, thought, or other diagnosis), treatment type (high intensity, moderate intensity, or outpatient treatment) and substance abuse (history of abuse or no history). During Phase I development, consumers were selected from programs known to be functioning at a high level of service fidelity. The recovery metric and covariates were measured at 2 month intervals from intake until discharge on all selected consumers. A RCRM was then developed and validated based upon the in-control measures. Under Phase II all consumer data were evaluated under the in-control RCRM model. The residuals obtained were then fed into the CUSUM QCC framework.

Using the electronic medical record system, clinical service staff could then retrieve a graph of the CUSUM chart to determine how the consumer was performing compared to the expected progress of consumers with the same diagnosis, treatment type, and substance abuse disorder. This helps to reduce the time the clinical service staff spend determining the progress of each of the consumers, while making it easier to detect consumers that may need further assistance or other services. Graphical examples of a consumer recovering at a faster rate than expected, as expected, and at a lower rate than expected under the RCRMCC implemented at MHCD are provided in Figures 13, 14, and 15 respectively.
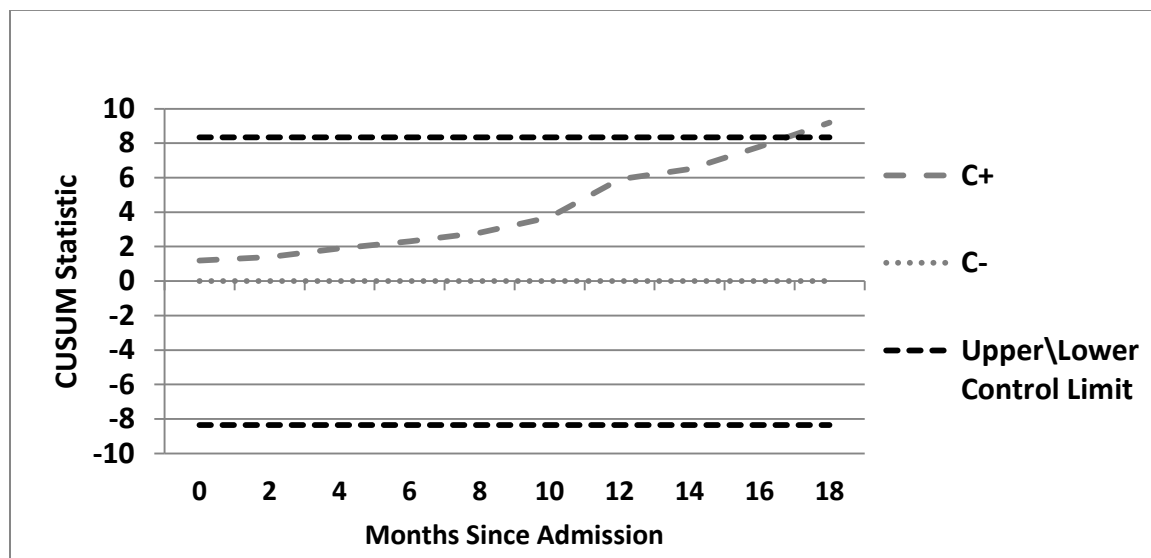

Figure 13.

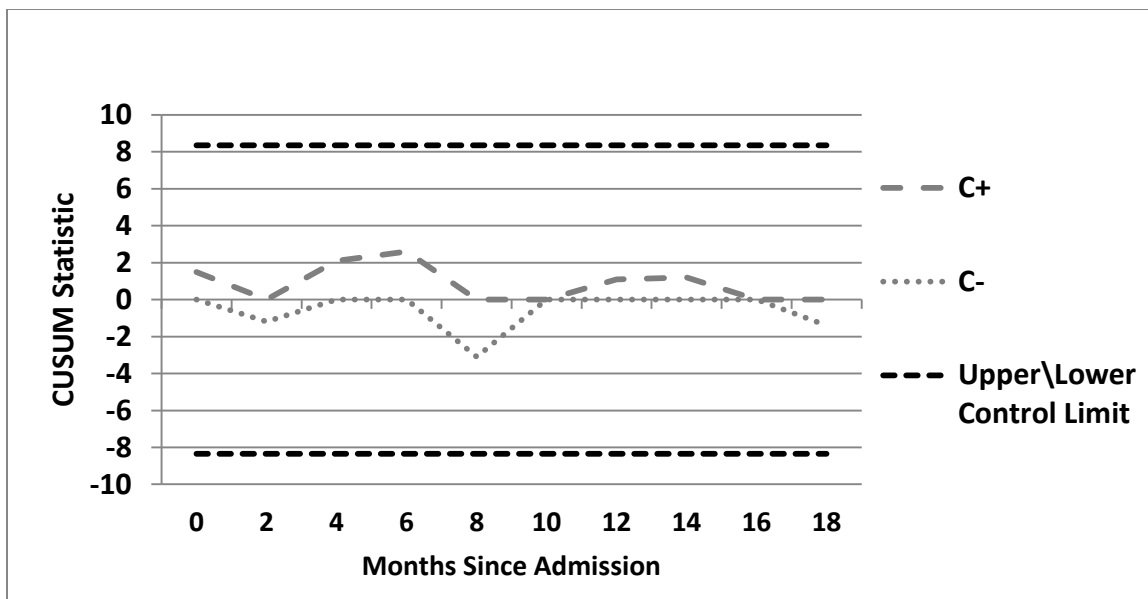A CUSUM RCRMCC example where the consumer is recovering faster than expected.

Figure 14.

A CUSUM RCRMCC example where the consumer is recovering as expected.
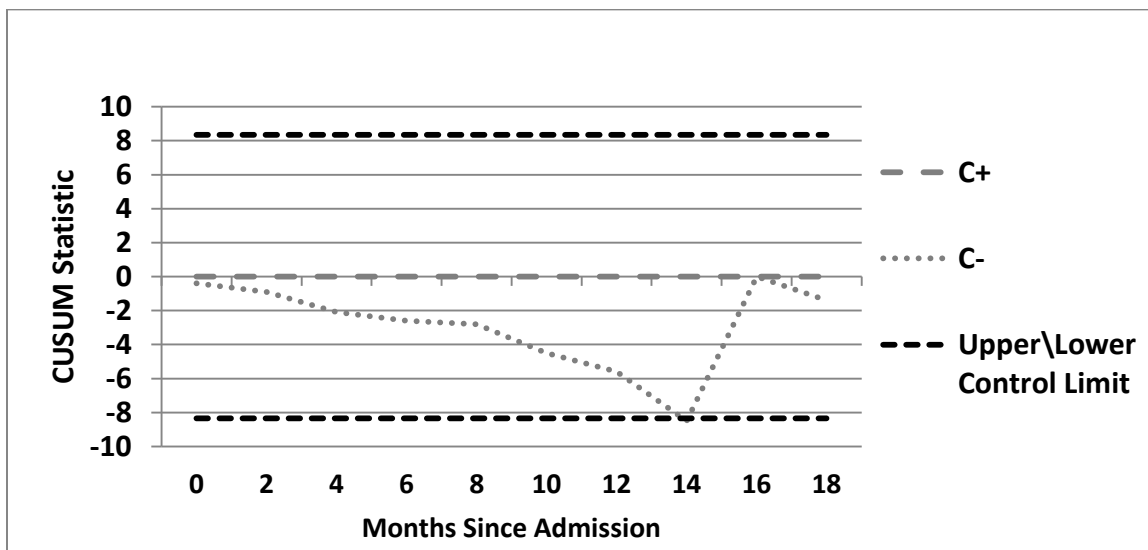

Figure 15.

A CUSUM RCRMCC example where the consumer is recovering at a lower rate than expected.

Limitations to the Application of the Random
Coefficient Regression Model Control
Chart

The RCRMCC has shown itself to be valuable under situations typical in the

human services industries, but the level of sophistication needed to implement and

monitor the RCRMCC may be beyond the training of most quality control persons within

these industries. Under Phase I of the implementation of the control chart the user must

be able to adequately develop a RCRM, considering all of the nuances that are associated

with the model. Furthermore, if the RCRMCC is not needed then many hours completing

complicated models would be wasted and non-beneficial to the operations. A more

commercially friendly version of this control chart should be developed to help alleviate

some of the issues associated with the use of these models and implementing them for

use in the RCRMCC.

Future Studies

Though the current study focused on many of the key population characteristics

related to the RCRMCC, future studies may want to focus on other areas that can help to

better understand the performance of the RCRMCC for varying applications. Due to the

RCRMCC having ARL that were higher under the 0.2 and 0.5 standard deviation mean

shifts, it would be pertinent to review the effect of Fast Initial Response (FIR;

Montgomery, 2005b) on the performance of the RCRMCC as compared to the unadjusted

RCC. FIR is intended to provide a faster response to a change in the system by starting

the RCRMCC with a predetermined accumulation of residuals, thus allowing the

CUSUM to signal a change more quickly if a system change has occurred. Other studies

may also want to look at control charts that monitor the higher level variances, where there are many control charts that have been established previously that look at changes in variance. Another item to consider in future studies would be the use of multivariate control charts to monitor the higher level effects and their relationship with the lower level covariates. Further study of these items would allow for a broader perspective of the applications of the RCRMCC.

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APPENDIX A

R SCRIPTS USED FOR SIMULATIONS

CODE for In-Control Population Creation
##45 tables for the in-control populations (no mean shifts) will be
##produced through the following procedures. The 45 populations will
##represent the crosses of the ICC values (0, .05, .15, .25, and .50),
##Level 1 R squared values of (0.01, 0.25, and 0.5), and Level 1 R
##squared values of (0.01, 0.25, and 0.5).

##Load multivariate distribution package
##Needed for production of random multivariate data
library(mvtnorm);
##Needed for linear mized effects model estimation
library(nlme);

##Set constant paramters for RCRM model, Note: G stands for gamma
##Set grand intercept over all units/pop
##Implies the mean of all vaues is 0
G00 <- 0.00;
##Set effect of Wj's on unit/group intercepts
##The unit increase for each value ofthe Wj's
G01 <- 0.1;
##Set grand slope over all units/pop
##Overall slope for all units
G10 <- 0.1;
##Set effect of Wj's on unit/group slopes
##Unit increase in slope for each unit increase in Wj's
G11 <- 0.1;
##Set Level 1 R quared values
L1R2 <- c(0.01, 0.25, 0.50);
##Set Level 2 R squared values
L2R2 <- c(0.01, 0.25, 0.50);
##Set ICC values
ICC <- c(0.5, 0.25, 0.0001, 0.05, 0.15);
##Set total Variance
Totvar <- 1;

for (ic in 1:5){
for (lr1 in 1:3){
for (lr2 in 1:3){

##Step 1, simulate the values for the 1000 pop/units within each
##population

##Set mean values for Wj's, u0j, and u1j
##Wj's will have a mean of zero, keeping the intercept at 0, where the u0j and u1j will

```
##have means of G00 and G10
distmean<-c(0,G00,G10);


##Set variance/co-variance matrix for Wj's, G00+u0j, and G10+u1j
##Set base matrix assuming variance of Wj is 1, u0j is 1, and u1j is 1
##Assumes the covariance between u0j and u1j is 0
distsigma<-matrix(c(1,1,1,1,1,0,1,0,1), ncol=3);

##Calculate true variance of u0j and u1j given system parameters
##Assumes 1/2 of the variance at level 2 is due to intercept and 1/2 due to slope
distsigma[2,2] <- distsigma[3,3] <- 1/2*ICC[ic]*Totvar;
##Var(Wj) = Cov(Xij, rij)/(0.1)
distsigma[1,1] <- (L2R2[lr2]* distsigma[2,2])/(G01*G01);
##Calculate covariance of Wjs and uoj/u1j base on L2R2
##Covariance formula is corr(x,y) * SDx * SDy
##Variance of Wj given by equation for beta1 (slope) of ols equation, beta1 = Cov(Wj,
u0j/u1j)/Var(Wj)
distsigma[1,2] <- distsigma[2,1] <- distsigma[1,3] <- distsigma[3,1] <- sqrt(L2R2[lr2])*
sqrt(distsigma[1,1]) * sqrt(distsigma[2,2]);

##Produce matrix of Wj, uoj, and u1j values for 1000 groups
grps <- rmvnorm(n=1000, mean=distmean, sigma=distsigma, method = "svd");
groups <- as.data.frame(grps);
remove(grps);
names(groups)<- c("Wj","u0j","u1j");


##Initalize population matrix for computing yij assign group numbers 1 through 1000
##The sequence of subtracting and adding 1 is to appriately calculate the pop numbers
group <- seq(1:1000000);
group <- group -1;
group <- group%/%1000;
group <-group + 1;
##Put data in a data frame for future calcs
pop <- data.frame(group);
##Removing extraneous data
remove(group);


##Loop that produces 1000 observations for each of 1000 groups
        for (g in 1:1000){

                ##Set mean values for Xij's, and rij
                ##Xij's and rijs will have a mean of zero and groups$u0j respectively,
keeping the intercept at 0
```

```
                distmean<-c(0,groups$u0j[g]);

                ##Set variance/co-variance matrix for Xij's and rij's base matrix
                ##Assumes the covariance  is 0
                distsigma<-matrix(c(1,0,0,1), ncol=2);

                ##Calculate true variance of Xij and rij given system parameters
                distsigma[2,2] <- Totvar * (1 - ICC[ic]);
                ##Var(Wj) = Cov(Xij, rij)/(u1j)
                distsigma[1,1] <- (L1R2[lr1]*
distsigma[2,2])/(groups$u1j[g]*groups$u1j[g]);
                ##Calculate covariance of Wjs and uoj/u1j base on L2R2
                ##Covariance formula is corr(x,y) * SDx * SDy
                ##Variance of Wj given by equation for beta1 (slope) of ols equation,
beta1 = Cov(Wj, u0j/u1j)/Var(Wj)
                distsigma[1,2] <- distsigma[2,1] <- (L1R2[lr1] * distsigma[2,2]) /
groups$u1j[g] ;


                ##Produce 1000 random numbers for group
                xrij <- as.data.frame(rmvnorm(n=1000, mean = distmean, sigma =
distsigma));
                names(xrij)<- c("Xij","Yij");

                ##Testing
                ##OLStest <- glm(Yij ~ Xij, data=xrij);
                ##summary(OLStest);
                start <-((g-1)*1000+1);
                end <- (g*1000)
                pop$yij[start:end] <- xrij$Yij;
                pop$xij[start:end] <- xrij$Xij;
                pop$Wj[start:end] <- groups$Wj[g];

                };

                filename <-
paste("C:/Dissertation/PoPData/Pop","_ICC_",ICC[ic],"_L1_",L1R2[lr1],"_L2_",L2R2[l
r2],".RData", sep="");
                save(pop, file = filename);
                filename <-
paste("C:/Dissertation/PoPData/Groups","_ICC_",ICC[ic],"_L1_",L1R2[lr1],"_L2_",L2R
2[lr2],".RData", sep="");
                save(groups, file = filename);
};
};
};
```

CODE for Estimation of Models from In-Control Populations
##45 OLS and 45 RCRM models for the in-control populations (no mean shifts) will be
##produced through the following procedures. The 45 models will
##represent the crosses of the ICC values (0, .05, .15, .25, and .50),
##Level 1 R squared values of (0.01, 0.25, and 0.5), and Level 1 R
##squared values of (0.01, 0.25, and 0.5).

##Needed for linear mized effects model estimation
library(nlme);

##Set Level 1 R quared values
L1R2 <- c(0.01, 0.25, 0.50);
##Set Level 2 R squared values
L2R2 <- c(0.01, 0.25, 0.50);
##Set ICC values
ICC <- c(0.0001, 0.05, 0.15, 0.25, 0.5);

##index for model parameters
ic <- 3;
lr1 <- 3;
lr2 <- 3;

##Due to memory management problems each model needed to be ran independently
##The above constants were used to run each model estimation procedure

##Load in-control population data

filenameData <-
paste("C:/Dissertation/PoPData/Pop","_ICC_",ICC[ic],"_L1_",L1R2[lr1],"_L2_",L2R2[l
r2],".RData", sep="");
load(filenameData);

memory.size(3000);

##Step 1, Obtain OLS and RCRM models for each in-control population as
##previously simulated

##RCRM Model, intercept and slopes vary within groups, where group is a random
factor
try(RCRMmodel <- lme(yij ~ 1*Wj + xij*Wj, random=~1+xij|group, data=pop,
control = lmeControl(niterEM = 500,msMaxIter = 1000, maxIter = 1000, returnObject =
TRUE)));
##OLS Model
try(OLSmodel <- lm(yij ~ 1 + xij, data=pop));

##Save Models

```
filenameO <-
paste("C:/Dissertation/ModelData/OLSmodel","_ICC_",ICC[ic],"_L1_",L1R2[lr1],"_L2_
",L2R2[lr2],".RModel", sep="");
filenameR <-
paste("C:/Dissertation/ModelData/RCRMmodel","_ICC_",ICC[ic],"_L1_",L1R2[lr1],"_L
2_",L2R2[lr2],".RModel", sep="");
save(RCRMmodel, file = filenameR);
save(OLSmodel, file = filenameO);

remove(RCRMmodel);
remove(OLSmodel);

}}};
##End repeated code

##Obtain summaries for each model
##This will be used to validate the population models
##and verify all models were estimated approriately before
##use in CUSUM control chart simulations

coeftable <- matrix(rnorm(1125,0,1), nrow=45);
count<-0;

for (ic in 1:5){
for (lr1 in 1:3){
for (lr2 in 1:3){

filenameO <-
paste("C:/Dissertation/ModelData/OLSmodel","_ICC_",ICC[ic],"_L1_",L1R2[lr1],"_L2_
",L2R2[lr2],".RModel", sep="");
load(filenameO);

filenameR <-
paste("C:/Dissertation/ModelData/RCRMmodel","_ICC_",ICC[ic],"_L1_",L1R2[lr1],"_L
2_",L2R2[lr2],".RModel", sep="");
load(filenameR);

count<-count+1;
coeftable[count,1]<-ICC[ic];
coeftable[count,2]<-L1R2[lr1];
coeftable[count,3]<-L2R2[lr2];
coeftable[count,4]<-summary(RCRMmodel)[[20]][[1]];
coeftable[count,5]<-summary(RCRMmodel)[[20]][[2]];
coeftable[count,6]<-summary(RCRMmodel)[[20]][[3]];
coeftable[count,7]<-summary(RCRMmodel)[[20]][[4]];
coeftable[count,8]<-summary(RCRMmodel)[[20]][[5]];
```

```
coeftable[count,9]<-summary(RCRMmodel)[[20]][[6]];
coeftable[count,10]<-summary(RCRMmodel)[[20]][[7]];
coeftable[count,11]<-summary(RCRMmodel)[[20]][[8]];
coeftable[count,12]<-summary(RCRMmodel)[[20]][[17]];
coeftable[count,13]<-summary(RCRMmodel)[[20]][[18]];
coeftable[count,14]<-summary(RCRMmodel)[[20]][[19]];
coeftable[count,15]<-summary(RCRMmodel)[[20]][[20]];
coeftable[count,16:18]<-VarCorr(RCRMmodel)[,1];
coeftable[count,19]<-summary(OLSmodel)[[4]][[1]];
coeftable[count,20]<-summary(OLSmodel)[[4]][[2]];
coeftable[count,21]<-summary(OLSmodel)[[4]][[3]];
coeftable[count,22]<-summary(OLSmodel)[[4]][[4]];
coeftable[count,23]<-summary(OLSmodel)[[4]][[7]];
coeftable[count,24]<-summary(OLSmodel)[[4]][[8]];
coeftable[count,25]<-summary(OLSmodel)[[6]];

remove(RCRMmodel);
remove(OLSmodel);

}}};

ctable <- paste("C:/Dissertation/Tables/Parameters.RTable", sep="");
write.table(coeftable, file = ctable);
```

CODE for simulation of RCC
##10,000 simulations of the in-control and three mean shift CUSUM charts
##will be performed for each of the OLS and RCRM models, with system paramters of
##produced through the following procedures. ICC values (0, .05, .15, .25, and .50),
##Level 1 R squared values of (0.01, 0.25, and 0.5), and Level 1 R
##squared values of (0.01, 0.25, and 0.5), being manipulated.

##Load multivariate distribution package
##Needed for production of random multivariate data
library(mvtnorm);
##Needed for linear mized effects model estimation
library(nlme);

##Set constant paramters for RCRM model, Note: G stands for gamma
##Set grand intercept over all units/pop
##Implies the mean of all vaues is 0
G00 <- 0.00;
##Set effect of Wj's on unit/group intercepts
##The unit increase for each value ofthe Wj's
G01 <- 0.1;
##Set grand slope over all units/pop
##Overall slope for all units
G10 <- 0.1;
##Set effect of Wj's on unit/group slopes
##Unit increase in slope for each unit increase in Wj's
G11 <- 0.1;
##Set Level 1 R quared values
L1R2 <- c(0.01, 0.25, 0.50);
##Set Level 2 R squared values
L2R2 <- c(0.01, 0.25, 0.50);
##Set ICC values
ICC <- c(0.0001, 0.05, 0.15, 0.25, 0.5);
##Set total Variance
Totvar <- 1;
##Set Mean Shifts
shift <- c(0.0, 0.2, 0.5, 0.8);

turn<-0;
avgTable<-1:45;
varTable<-1:45;

for (sh in 1:4){
        for (ic in 1:5){
                for (lr1 in 1:3){

```
for (lr2 in 1:3){

        memory.size(3000);
        ##Create the list of value to get residuals and feed into
CUSUM simulations
        ##Step 1, load the population paramters for each group
        filename <-
paste("C:/Dissertation/PoPData/Groups","_ICC_",ICC[ic],"_L1_",L1R2[lr1],"_L2_",L2R
2[lr2],".RData", sep="");
        load(filename);
        remove(filename);
        ##Step 2Change mean of intercept for each group by mean
shift.  Since overall variability is 1
        ##this results in an equivelant change in the G00
        groups$u0j<-groups$u0j+shift[sh];
        ##Step 3 load RCRM and OLS models
        filenameO <-
paste("C:/Dissertation/ModelData/OLSmodel","_ICC_",ICC[ic],"_L1_",L1R2[lr1],"_L2_
",L2R2[lr2],".RModel", sep="");
        load(filenameO);
        filenameR <-
paste("C:/Dissertation/ModelData/RCRMmodel","_ICC_",ICC[ic],"_L1_",L1R2[lr1],"_L
2_",L2R2[lr2],".RModel", sep="");
        load(filenameR);



        ##Produce values and run through CUSUM 10,000

        points<-1:10000;
        H<-5;
        for (cusum in 1:10000){
                Cp<-Cn<-0;
                count <- 0;
                while ((Cp < H) & (Cn < H)) {
                        count <- count +1;
                        ##Initalize population matrix for computing
yij assign group numbers 1 through 1000
                        ##The sequence of subtracting and adding 1
is to appriately calculate the pop numbers
                        memory.size(3000);
                        group <- trunc(runif(1)*999 + 1);
                        ##Put data in a data frame for future calcs
                        samp <- data.frame(group);
                        memory.size(3000);
```

```
                                    ##Set mean values for Xij's, and rij
                                    ##Xij's and rijs will have a mean of zero and
groups$u0j respectively, keeping the intercept at 0
                                    distmean<-c(0,groups$u0j[group]);

                                    ##Set variance/co-variance matrix for Xij's
and rij's base matrix
                                    ##Assumes the covariance  is 0
                                    distsigma<-matrix(c(1,0,0,1), ncol=2);

                                    ##Calculate true variance of Xij and rij
given system parameters
                                    distsigma[2,2] <- Totvar * (1 - ICC[ic]);
                                    ##Var(Wj) = Cov(Xij, rij)/(u1j)
                                    distsigma[1,1] <- (L1R2[lr1]*
distsigma[2,2])/(groups$u1j[group]*groups$u1j[group]);
                                    ##Calculate covariance of Wjs and uoj/u1j
base on L2R2
                                    ##Covariance formula is corr(x,y) * SDx *
SDy
                                    ##Variance of Wj given by equation for
beta1 (slope) of ols equation, beta1 = Cov(Wj, u0j/u1j)/Var(Wj)
                                    distsigma[1,2] <- distsigma[2,1] <-
(L1R2[lr1] * distsigma[2,2]) / groups$u1j[group] ;


                                    ##Produce random numbers for group
                                    xrij <- as.data.frame(rmvnorm(n=1, mean =
distmean, sigma = distsigma));

                                    names(xrij)<- c("Xij","Yij");

                                    samp$yij <- xrij$Yij;
                                    samp$xij <- xrij$Xij;
                                    samp$Wj <- groups$Wj[group];

                                    OLSpredict <- predict(OLSmodel,
newdata=samp);

                                    Cp <- max(0, samp$yij-
(OLSpredict[1]+(0.5/2)) + Cp);

                                    Cn <- max(0, (OLSpredict[1]-(0.5/2)) -
samp$yij + Cn);

                                    };
                              points[cusum]<-count;
                              };
                        avg<-mean(points);
```

```
                                vari<-var(points);
                                turn<-turn+1;
                                avgTable[turn]<-avg;
                                varTable[turn]<-vari;

                                filenameavg <-
paste("C:/Dissertation/ModelData/OLSAVGTable","_shift_",shift[sh],".RTable",
sep="");
                                save(avgTable, file = filenameavg);
                                filenamevar <-
paste("C:/Dissertation/ModelData/OLSVARTable","_shift_",shift[sh],".RTable",
sep="");
                                save(varTable, file = filenamevar);

                                };
                        };
                };
        };
```

CODE for the Simulation of RCRMCC
##10,000 simulations of the in-control and three mean shift CUSUM charts
##will be performed for each of the OLS and RCRM models, with system paramters of
##produced through the following procedures. ICC values (0, .05, .15, .25, and .50),
##Level 1 R squared values of (0.01, 0.25, and 0.5), and Level 1 R
##squared values of (0.01, 0.25, and 0.5), being manipulated.

##Load multivariate distribution package
##Needed for production of random multivariate data
library(mvtnorm);
##Needed for linear mized effects model estimation
library(nlme);

##Set constant paramters for RCRM model, Note: G stands for gamma
##Set grand intercept over all units/pop
##Implies the mean of all vaues is 0
G00 <- 0.00;
##Set effect of Wj's on unit/group intercepts
##The unit increase for each value ofthe Wj's
G01 <- 0.1;
##Set grand slope over all units/pop
##Overall slope for all units
G10 <- 0.1;
##Set effect of Wj's on unit/group slopes
##Unit increase in slope for each unit increase in Wj's
G11 <- 0.1;
##Set Level 1 R quared values
L1R2 <- c(0.01, 0.25, 0.50);
##Set Level 2 R squared values
L2R2 <- c(0.01, 0.25, 0.50);
##Set ICC values
ICC <- c(0.0001, 0.05, 0.15, 0.25, 0.5);
##Set total Variance
Totvar <- 1;
##Set Mean Shifts
shift <- c(0.0, 0.2, 0.5, 0.8);
turn<-0;
avgTable<-1:45;
varTable<-1:45;

for (sh in 1:4){

```
for (ic in 1:5){
        for (lr1 in 1:3){
                for (lr2 in 1:3){

                        memory.size(3000);
                        ##Create the list of value to get residuals and feed into
CUSUM simulations
                        ##Step 1, load the population paramters for each group
                        filename <-
paste("C:/Dissertation/PoPData/Groups","_ICC_",ICC[ic],"_L1_",L1R2[lr1],"_L2_",L2R
2[lr2],".RData", sep="");
                        load(filename);
                        remove(filename);
                        ##Step 2Change mean of intercept for each group by mean
shift.  Since overall variability is 1
                        ##this results in an equivelant change in the G00
                        groups$u0j<-groups$u0j+shift[sh];
                        ##Step 3 load RCRM and OLS models
                        filenameO <-
paste("C:/Dissertation/ModelData/OLSmodel","_ICC_",ICC[ic],"_L1_",L1R2[lr1],"_L2_
",L2R2[lr2],".RModel", sep="");
                        load(filenameO);
                        filenameR <-
paste("C:/Dissertation/ModelData/RCRMmodel","_ICC_",ICC[ic],"_L1_",L1R2[lr1],"_L
2_",L2R2[lr2],".RModel", sep="");
                        load(filenameR);




                        ##Produce values and run through CUSUM 10,000

                        points<-1:10000;
                        H<-5;
                        for (cusum in 1:10000){
                                Cp<-Cn<-0;
                                count <- 0;
                                while ((Cp < H) & (Cn < H)) {
                                        count <- count +1;
                                        ##Initalize population matrix for computing
yij assign group numbers 1 through 1000
                                        ##The sequence of subtracting and adding 1
is to appriately calculate the pop numbers
                                        memory.size(3000);
                                        group <- trunc(runif(1)*999 + 1);
                                        ##Put data in a data frame for future calcs
                                        samp <- data.frame(group);
```

```
memory.size(3000);

##Set mean values for Xij's, and rij
##Xij's and rijs will have a mean of zero and
groups$u0j respectively, keeping the intercept at 0
distmean<-c(0,groups$u0j[group]);

##Set variance/co-variance matrix for Xij's
and rij's base matrix

##Assumes the covariance  is 0
distsigma<-matrix(c(1,0,0,1), ncol=2);

##Calculate true variance of Xij and rij
given system parameters

distsigma[2,2] <- Totvar * (1 - ICC[ic]);
##Var(Wj) = Cov(Xij, rij)/(u1j)
distsigma[1,1] <- (L1R2[lr1]*
distsigma[2,2])/(groups$u1j[group]*groups$u1j[group]);
##Calculate covariance of Wjs and uoj/u1j
base on L2R2

##Covariance formula is corr(x,y) * SDx *
SDy
##Variance of Wj given by equation for
beta1 (slope) of ols equation, beta1 = Cov(Wj, u0j/u1j)/Var(Wj)
distsigma[1,2] <- distsigma[2,1] <-
(L1R2[lr1] * distsigma[2,2]) / groups$u1j[group] ;


##Produce random numbers for group
xrij <- as.data.frame(rmvnorm(n=1, mean =
distmean, sigma = distsigma));

names(xrij)<- c("Xij","Yij");

samp$yij <- xrij$Yij;
samp$xij <- xrij$Xij;
samp$Wj <- groups$Wj[group];

RCRMpredict <- predict(RCRMmodel,
newdata=samp);

Cp <- max(0, samp$yij-
(RCRMpredict[1]+(0.5/2)) + Cp);

Cn <- max(0, (RCRMpredict[1]-(0.5/2)) -
samp$yij + Cn);

};
points[cusum]<-count;
```

```
                    };
                    avg<-mean(points);
                    vari<-var(points);
                    turn<-turn+1;
                    avgTable[turn]<-avg;
                    varTable[turn]<-vari;

                    filenameavg <-
paste("C:/Dissertation/ModelData/AVGTable","_shift_",shift[sh],".RTable", sep="");
                    save(avgTable, file = filenameavg);
                    filenamevar <-
paste("C:/Dissertation/ModelData/VARTable","_shift_",shift[sh],".RTable", sep="");
                    save(varTable, file = filenamevar);

                    };
               };
          };
     };
```