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UNIVERSITY OF NORTHERN COLORADO

Greeley, Colorado

The Graduate School

SENSITIVITY COMPARISONS AND AVERAGE RUN LENGTHS OF THE MEWMS AND MEWMV CONTROL CHARTS USING INDIVIDUAL OBSERVATIONS WITH SINGULAR MEAN SHIFTS AND VARIANCE CHANGES

A Dissertation Submitted in Partial Fulfillment
Of the Requirements for the Degree of
Doctor of Philosophy

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ABSTRACT

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In statistical process control (SPC), continued development of techniques look for new monitoring charts for processes with multiple correlated variables. Two such charts are the multivariate exponentially weighted moving standard deviation (MEWMS) and multivariate exponentially weighted moving variance (MEWMV). Originally developed by Huwang, Yeh, and Wu (2007), and furthered by Hawkins and Maboudou-Tchao (2008), these control charts monitor the trace elements of the respective covariance matrices for a change in values of the multivariate process using individual observations. Originally, control chart parameters were developed during the simulation process for p = 2 and p = 3 process variables. Using computer simulations of 10,000 replications, further development of the MEWMS and MEWMV used p = 5 and p = 10 correlated variables with individual observations to develop control chart parameters and determine the sensitivity of the MEWMS and MEWMV compared to the multivariate CUSUM and MEWMA charts in their detection of a singular shift of mean, a singular change in variance or a combination of the two.

Initial findings from this dissertation suggest that both the MEWMS and MEWMV control charts are highly sensitive to small changes of a single element in the covariance matrix and sensitive to changes in a single element of the observation vector. When comparing the MEWMS and MEWMV control charts to the MCUSUM and

MEWMA control charts popularly used today, it was found that the MEWMS and MEWMV control charts are less sensitive to mean shifts than the MCUSUM or MEWMA. However, it was also shown that the MEWMS and MEWMV are much more sensitive to changes in elements of the covariance matrix than the MCUSUM, while the MEWMA control chart is insensitive to any changes of variance elements.

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This is dedicated to my friends and family. I wish I could name you all, but there are far too many that have gone through this process with me to list. I must thank my parents, Joe and Joan Eshelman, who have supported me and encouraged me to perform at my very best while helping me realize that I am only human. Words can't describe how much I appreciate everything you've done for me over the years. Special thanks to Lisa Hull, who has put up with four plus years of stress, toil and tears. I couldn't do this without you, and I'm glad you're here to share this with me.

I would like to dedicate this dissertation to two people in my life that couldn't be here to share in this. First, my grandfather, Harry Edgar Eshelman, without whom I would not have my name and whose last words to me were, “Dr. Eshelman…I like the sound of that”. And last, but certainly not least, Dr. Cheryl E. Fagerberg, the adopted sister I never knew I needed. I have only one word to share with her: Finally.

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CHAPTER I

INTRODUCTION

Background Information

Statistical process control, or SPC, was developed as a method of controlling process characteristics in the manufacturing process, but has evolved as applications expand. Before the formal introduction of SPC by Walter Shewhart in 1925, quality control was individually overseen by those tradesmen producing goods. This formal introduction of SPC worked to control explainable causes of variation of goods. Shewhart's baseline control chart gave rise to multiple charts that expanded and improved upon the detection capabilities of the original control chart. Major developments in SPC followed Shewhart's initial chart, and industry over the last two decades has demanded further control charts' development.

Shewhart's (1925) original control chart used the mean of product characteristics to determine if the process characteristic fell within specifications. While the values of these product specifications are specified by design engineers, production characteristic values are generally unknown. Sampling items drawn from the production line provide data for estimation of product characteristics. While these measurements are not exact, they do provide an adequate determination of a process characteristics' compliance with functionality.

Sampling from the production line helped to create a baseline measurement for testing process characteristics. Further tests were developed to monitor ranges, moving ranges, and multiple variables, or attributes, of manufacturing processes. The purpose of these control charts was similar to that of Shewhart's (1925); to monitor a manufacturing process and signal when a process had gone out of control (OOC). Development of a multivariate control chart allowed for the monitoring of multiple related process characteristics.

One of these multivariate control charts was the multivariate exponentially weighted moving average (MEWMA). Lowery, Woodall, Champ and Rigdon (1992) developed and introduced the MEWMA. The MEWMA is the multivariate expansion of the exponentially weighted moving average (EWMA) introduced by Roberts in 1959. Comparisons by Lowery et al. (1992) and Montgomery (2005) of the MEWMA to the multivariate cumulative sum (MCUSUM) control chart suggest little difference in the power of the MEWMA and the MCUSUM control charts' ability to detect a process shift in the mean vector.

Current research of MEWMA control charts has explored the effects of individual observations in process control. One such study was that of Huwang, Yeh, and Wu (2007) which studied the ability of two modified MEWMA control charts to detect uniform changes in the covariance matrix, rather than the mean vector alone. The first modified MEWMA chart studied by Huwang et al. was the multivariate exponentially weighted mean squared (MEWMS) chart. This control chart was designed to detect a uniform change in all the variance components while the mean vector remained in control and plotted the trace of the exponentially weighted covariance matrix monitoring variance changes with correlations between the process characteristics. The second modified MEWMA chart used by Huwang et al. monitored a uniform change in all variance components as well as a shift in the mean vector. The second chart, called the multivariate exponentially weighted moving variance (MEWMV) chart, used the trace of the covariance matrix, while also calculating an exponentially weighted moving average for the mean vector. This simulation study measured the overall performance of the MEWMS and MEWMV when a single variance element changed with appropriate covariance values and uniform correlation between process characteristics and compared performance to previous MEWMA studies using the average run length (ARL).

ARL is defined by Montgomery as, "Essentially…the average number of points that must be plotted [on a control chart] before a point indicates an out-of-control condition" (Montgomery, 2005). With every control chart, there exists a distribution that describes the behavior, or shape, of the run length distribution. Similar to other distributions, run length has a mean and standard deviation. The mean of the distribution is the ARL. When a process is in control, the ARL is known as the in-control ARL (ARL_0).

The purpose of this research paper is to identify the sensitivity of the MEWMS to detect a change of a single element of the covariance matrix and MEWMV to detect a change of a single element of the covariance matrix and/or an individual mean shift in the mean vector and compare these findings to the MEWMA and MCUSUM control charts. Original research on the MEWMS and MEWMV charts by Huwang et al.(2007) used p=2 and p=3 process characteristics. Further research on the MEWMV was conducted by Hawkins and Maboudou-Tchao (2008) which algebraically simplified the control chart statistics. This study examined the sensitivity and ARL properties of the MEWMS and MEWMV charts for singular characteristic changes when p=2, p=3, p=5, and p=10 process characteristics, which expanded on the research of Huwang et al. and Hawkins and Maboudou-Tchao. Previous studies have shown that the original MEWMA control chart and MCUSUM control chart can detect small shifts in the mean vector quickly; however, discussion concerning detection of a change in variance is relatively new, and little information is currently published.

Notations and Assumptions

For this dissertation, underlined lower case letters denote a vector ($\underline{x}$). A matrix is denoted as underlined upper case letters $(\underline{X})$. The vector of observations is assumed to be of dimension of $p\times1$ unless otherwise stated. The simulated sample covariance matrix is designated $\underline{S}$. The vector of observations is assumed p-variable normally distributed with mean vector $\underline{\mu}$ and covariance matrix $\underline{\Sigma}$, which is a $p \times p$ positive definite matrix.

Purpose of the Study

This dissertation examines and discusses the measurement of average run length (ARL) of the MEWMS when a single element in $\underline{\Sigma}$ has changed while maintaining an in-control mean vector ($\underline{\mu}$). Additionally, this dissertation will examine the ARL of the MEWMV when both $\underline{\mu}$ and $\underline{\Sigma}$ have experienced single element changes where p=2, p=3, p=5, and p=10 process characteristics for individual observations. The purpose of this simulation is to develop the sensitivity and ARL measurements of the MEWMS that can be applicable in a situation when the covariance of a multivariate process changes while the mean vector has not. This simulation study will also develop the ARL measurement and sensitivity of a singular change for the MEWMV when the covariance matrix changes and mean vector shifts in a single position. Developing these run length tables will assist in identification of a change and allow for a timely correction back to in-control specifications. Simulation studies were performed to develop the run length distributions for both in control (IC) and OOC run lengths using the modified MEWMA control charts introduced by Huwang et al.(2007). The ARL of the distributions were calculated to determine the general form of the distribution and to develop control chart limits.

Significance of the Study

Should these control charts prove capable of detecting a singular change in the covariance matrix and detecting a singular change in both the covariance matrix and the mean vector (where applicable), these advances will contribute to improved control chart development. If the MEWMS detects a change in the covariance matrix and the MEWMV detects a change in the covariance matrix as well as a shift in the mean vector, both charts will be a significant contribution to the field of SPC. It has been suggested that any changes in the covariance matrix may lead to a shift in the mean vector (Yeh, Lin, Zhou, & Venkataramani, 2003). If a change in covariance does correlate with a mean vector shift, use of the MEWMS and MEWMV will be more effective in detecting changes in process characteristics faster than monitoring the mean vector alone. With the identification of a change in the covariance matrix, the phase I process could become shorter allowing the phase II process to begin earlier. In addition, detecting an OOC signal using the MEWMS or MEWMV could lead to earlier corrections to the process than if using the MEWMA or MCUSUM charts alone.

Statement of Research Questions

The following questions are addressed in this study:

Q1 Does the MEWMS control chart monitor for a singular change in the covariance matrix and mean shift more effectively than the MEWMA or the MCUSUM control charts?

Q2 Does the MEWMV control chart monitor for a singular change in the covariance matrix and mean shift more effectively than the MEWMA or the MCUSUM control charts?

Q3 What are the appropriate values for the control chart parameters for the MEWMS and MEWMV to create an ARL_0 approximately equal to 370 (per Huwang et al. (2007) and Hawkins and Maboudou-Tchao (2008))?

Limitations of the Study

The distribution run lengths will be compared with studies from Hawkins and Maboudou-Tchao (2008), Huwang, Yeh and Wu (2007), Hawkins, Choi, and Lee (2007), Montgomery (2005) and Jones (2002). These scenarios were used as the basis for simulation development and the average run length values of these scenarios were used for comparison. Other control charts used for comparison are the MCUSUM and MEWMA. Comparison of run length to all current studies published would be unnecessary due to similar results as the aforementioned studies.

Restrictions to the MEWMS and MEWMV charts development include the weighting parameters, ω, of the equation for test statistic calculation. The development of the MEWMS and MEWMV is similar to the MEWMA in that a weighting value is given to the most recent observation, and the value of one minus this weight given to the prior observation. In this simulation study, weights for the current observations will measure as $\omega = \{0.1, 0.3, 0.5, 0.7, 0.9\}$, similar to Huwang et al. (2007). While simulations could examine every value of weights, the weighting values are infinite due to a continuous range of values, and may give similar values within deciles of weighting. Research using the MEWMA suggests that the value of the weighting parameter is directly related to the size of shift attempted to detect; that is, small values of weighting for small shift sizes and large weighting values for large shift sizes (Stumbos & Sullivan, 2002). Hawkins et al. (2007) suggests a range of weights (0.46 to .5) that will return ARL_0's in the range of 370.

A similar restriction exists for the MEWMV chart. In the development of the MEWMV control chart, a weighted moving average was calculated for the covariance matrix in Hawkins and Maboudou-Tchao (2008). The weighting parameter used for this equation was also ω. Values for ω were restricted to ω = {0.1, 0.3, 0.5, 0.7, 0.9}. Similar selections were made for the weighting parameter ω for the MEWMS charts.

Another limitation to the development of the MEWMS and MEWMV charts is the correlation levels, ρ, among the variables. This simulation study will use uniform correlations between variables such that ρ = {0.1, 0.3, 0.5, 0.7, 0.9}, similar to the study of Huwang et al.(2007). The purpose of using a uniform correlation matrix is that it ensures a positive definite matrix for the generation of the observations. Other research in multivariate control charts have used levels of correlation between variables of 0.5 (medium correlation) and 0.9 (high correlation). While using the full range of correlation values is possible in computer simulation, there are infinite values of ρ. Restricting the number and levels of correlation removed unnecessary simulations.

Additional limitations to this dissertation include limiting the mean shift and variance changes to singular changes across a single component value. The values used in this simulation were initially used by Huwang et al. (2007) using p=2 and p=3 process characteristics. The expansion of this dissertation to the research of Huwang, Yeh and Wu worked with p=2, p=3, p=5 and p=10 process characteristics for the MEWMS control chart. Research from Hawkins and Maboudou-Tchao (2008) dealt with a broader range of process characteristics, ranging from p=2 to p=50 process characteristics. Expansion beyond Huwang, Yeh, and Wu's uniform shift of means or uniform change in variances resulted in infinite unnecessary combinations for simulation.

Definition of Terms

Average Run Length (ARL): The average number of successive points produced in a control chart before a signal of an out-of-control situation.

In-control ARL (ARL_0): The ARL produced from a process that has no deviation from original specification, i.e. no shift or change has occurred.

Out-of-control ARL (ARL_1): The ARL produced when a change, or shift, in the process has occurred.

Variance Change: Similar to a change in mean values of a process, the covariance of the multivariate process has changed due to the production of more extreme values while maintaining a constant mean vector.

CHAPTER II

REVIEW OF LITERATURE

Chapter two discusses the development of the control charts from the univariate standard deviation and variance to control charts monitoring the covariance matrix. In addition, the MEWMA and MCUSUM are discussed, leading to the development of Huwang, Yeh, and Wu's (2007) MEWMS and Hawkins and Maboudou-Tchao's (2008) MEWMV control charts. For any of these control charts, the choice of which chart to use will depend upon the sample size, number of variables measured, and change or shift of interest. In many cases, SPC is not interested in a decrease in the process variance or standard deviation; as such changes imply improvements (precision) in production and measurement techniques.

Control charts are used as graphical representations of trends in parameters in SPC. A center line is used as the mean value for development in phase I of SPC to determine a known baseline value of comparison. From this center line, control limits are developed using probability distributions associated with the type of parameter being monitored; typically working within ± 3σ. In the case of standard deviations, a Gamma function is used to determine a correcting value to create an unbiased calculation of the approximation of the standard deviation. The control chart associated with s^2 used the χ^2 distribution probabilities to create control limits (Montgomery, 2005). While many different control charts exist, few monitor process variance, standard deviation, or in the multivariate case, covariance.

S charts are used to monitor the standard deviation in a univariate control setting.

$$s = \sqrt{\frac{\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2}{n-1}} \tag{II.1}$$

Montgomery (2005) points out that equation (II.1) is not an unbiased estimator for the population standard deviation, σ. For the univariate case, using the standard deviation from m-samples of size n, the average is calculated:

$$\overline{s} = \frac{\sum_{i=1}^{m} s_i}{m} \tag{II.2}$$

While this approximation of σ is still biased, equation (II.2) defines the center line and is used to determine control limits for the *s*-control chart. Typically, the centerline and control limits are determined by:

$$UCL = B_6\sigma \tag{II.3.a.i}$$
$$CL = c_4\sigma \tag{II.3.a.ii}$$
$$LCL = B_5\sigma \tag{II.3.a.iii}$$

when σ is known and c_4, B_5 and B_6 are available from most SPC texts, such as Montgomery (2005). However, when σ is unknown, the control limits become:

$$UCL = B_4\overline{s} \tag{II.3.b.i}$$
$$CL = \overline{s} \tag{II.3.b.ii}$$
$$LCL = B_3\overline{s} \tag{II.3.b.iii}$$

where B_4 and B_3 are constants provided in most SPC texts.

Once Phase I of SPC is complete and the process is stable and assumed in control, Phase II of SPC begins. From each new Phase II sample taken, the standard deviation is calculated and plotted on a chart using the control limits from equation (II.3). The process continues until an OOC signal occurs, at which time corrections can be made.

The development of the s^2 control chart is similar to the s-control chart. Sample data collected in Phase I are used to calculate s^2 for *m*-samples of size *n* using the equation:

$$s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1} \tag{II.4}$$

The average of the *m*-variances are computed and used as the center line:

$$\overline{s^2} = \frac{\sum_{i=1}^{m} s_i^2}{m} \tag{II.5}$$

As discussed earlier, the variance of the samples is distributed as χ^2. Using the χ^2 distribution and a level of significance of $\alpha/2$, the s^2 control chart parameters are:

$$UCL = \frac{\overline{s^2}}{n-1} \chi^2_{\alpha/2, n-1} \tag{II.6.a}$$

$$CL = \overline{s^2} \tag{II.6.b}$$

$$LCL = \frac{\overline{s^2}}{n-1} \chi^2_{1-(\alpha/2), n-1} \tag{II.6.c}$$

Upon completion of phase I measurements, phase II processes are measured and plotted on the Shewhart style control chart using the calculated control limits from (II.6).

Development of the Univariate Cumulative Summation (CUSUM)

Another chart designed for SPC is the cumulative summation chart, or CUSUM. The CUSUM control chart was designed to monitor an individual observation or the mean of a logical subgroup. Originally developed in 1954 by E.S. Page, the CUSUM was touted as “a fundamental change in the classical procedure [of SPC]” (Barnard, 1959). Page (1961) made additional contributions to the development, with the expanded explanation of the CUSUM which suggested that the downward direction in measurements may be considered good, so the use of a lower control limit of zero is useful. In the case of monitoring standard deviation and variance, this concept holds true, as any decline in measurements implies an improvement in measurement techniques.

A modified CUSUM technique is that of the tabular CUSUM control chart (Montgomery, 2005). If μ_0 were considered a target value, the tabular CUSUM creates statistics by accumulating the deviation from the target value. This control chart does not use a center line for reference, but rather begins at a starting state of zero. Tabular CUSUM statistics are given by:

$$C_i^+ = max[0, x_i - (\mu_0 - K) + C_{i-1}^+] \quad \text{(II.7.a)}$$

$$C_i^- = max[0, (\mu_0 - K) - x_i + C_{i-1}^-] \quad \text{(II.7.b)}$$

where *x* is the observed characteristic, $C_0^+ = C_0^- = 0$, and K is a reference value expressed as one-half the shift size, δ, times the standard deviation, σ:

$$K = \frac{\delta}{2}\sigma \quad \text{(II.8)}$$

(Montgomery, 2005).

The final component used in the tabular CUSUM is the decision interval of the control chart, referred to as *H* (Montgomery, 2005). The value of *H* is considered the upper control limit for C_i^+ and C_i^- and, “…a reasonable value for *H* is five times the process standard deviation σ” (Montgomery 2005, p. 391). Koning and Does (2000) showed that the CUSUM performs very well when individual observations are used and small mean shifts have occurred.

Another technique in CUSUM charts uses modifications that transform the observations into standardized data for monitoring. The standardized charts allow for use of values of *K* and *H* that do not change due to scale dependency, or dependency on σ (Montgomery, 2005). The design allows for development of a CUSUM that monitors process variability. Using a standardized value:

$$v_i = \frac{\sqrt{|y_i|} - 0.822}{0.349} \tag{II.9}$$

where $y_i = {(x_i - \mu_0)}/{\sigma}$, Hawkins (1981) suggests that the standardized CUSUM is sensitive to changes in variance rather than mean changes. The test statistics for the two-sided standardized CUSUM are:

$$S_i^+ = max[0, v_i - k + S_{i-1}^+] \tag{II.10.a}$$

$$S_i^- = max[0, -k - v_i + S_{i-1}^-] \tag{II.10.b}$$

where $S_0^+ = S_0^- = 0$. The statistics are plotted on a Shewhart style control chart, using the parameter *H* as an upper control limit that signals OOC.

First developed by Lucas and Crosier (1982), the FIR is designed to identify a shift earlier in the process by starting C_0^+ and C_0^- or S_0^+ and S_0^- at one half of the value of *H* $\left(\frac{H}{2}\right)$, or a 50% head start. Head start values are determined prior to measurement and give increased sensitivity to the control chart (Montgomery, 2005).

Development of the Univariate Exponentially Weighted Moving Average (EWMA)

Originally called the geometric moving average, the exponentially weighted moving average, or EWMA was developed by S.W. Roberts (1959). EWMA control charts were developed to monitor consecutive observations and place greater weight on the most recent observation. Similar to other control charts, the EWMA chart is plotted about a center line. Roberts used control limits of $\pm 3\sigma_{\bar{x}}$ when using sample means and

$\pm\frac{3}{2}\sigma_{\bar{x}}$when using standardized observations, where $\sigma_{\bar{x}}$ is the standard error of measurement $\left(\frac{\sigma}{\sqrt{n}}\right)$.

EWMA control charts were designed to detect a shift in the mean of a process. The shift size is designated as δ, such that the process is centered around $\mu_0 + \delta$. In an IC process situation, δ=0. When $\delta \neq 0$, the EWMA detects small shifts quickly.(Roberts, 1959; Prabhu & Runger, 1997). The general form of the EWMA statistic is:

$$z_i = \lambda x_i + (1-\lambda)z_{i-1} \quad \text{(II.11)}$$

where $0 \leq \lambda \leq 1$, is a constant weighting parameter and z_i is the observation value. Typically $z_0=\mu_0$ or, if the population mean is unknown, $z_0=\bar{\bar{x}}$ (Montgomery, 2005).

Control limit development for the EWMA is based on the weighting parameter, λ, and a width value, L. Research by Crowder (1989) and Lucas and Saccucci (1990) developed values of L for different ARLs. The control limits for the EWMA are:

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]} \quad \text{(II.12.a.i)}$$

$$CL = \mu_0 \quad \text{(II.12.a.ii)}$$

$$LCL = \mu_0 - L\sigma\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]} \quad \text{(II.12.a.iii)}$$

After the process has run for several periods, the control limits converge to:

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\lambda}{2-\lambda}} \quad \text{(II.12.b.i)}$$

$$CL = \mu_0 \quad \text{(II.12.b.ii)}$$

$$LCL = \mu_0 - L\sigma\sqrt{\frac{\lambda}{2-\lambda}} \quad \text{(II.12.b.iii)}$$

Montgomery (2005) suggests that using the control limits from (II.12.a) improve the sensitivity of the EWMA early in the process. Head start techniques work for the EWMA (Lucas & Saccucci; MacGregor, J.F., 1990) (Sweet, 1986)allowing for a faster detection of a shift.

Development of the Exponentially Weighted Mean Squared (EWMS)

The EWMS was designed to monitor the standard deviation of a process. Originally suggested by Wortham and Ringer (1971), Crowder and Hamilton (1992) discuss the specific use of a EWMA to monitor process variability. Sweet (1986) and Ng and Case (1989) discussed different methods of monitoring process variability. However, the use of the EWMA to monitor the mean squared deviation, called the EWMS, differs from these designs. Unlike the EWMA discussed earlier, which could monitor mean shifts using individual observations, the EWMS discussed by Crowder and Hamilton requires a sample of n > 1 to calculate the process standard deviation. Additionally, the statistic involves a linear transformation of the variance ($ln\ s^2$). MacGregor and Harris (1993) introduced another model of the EWMS which used the variance and the calculation of the statistic:

$$S_i^2 = \lambda(x_i - \mu)^2 + (1-\lambda)S_{i-1}^2 \qquad \text{(II.13)}$$

where *x* is the observed characteristic, λ is the weighting parameter, and μ is the IC target value of the process. It can be shown when *i* is large, $E(S_i^2) = \sigma^2$. As a result, s_i^2 has an approximate χ^2 distribution (Montgomery, 2005). Using this information, the root of the statistic (II.13) can be plotted on a control chart using:

$$UCL = \sigma_0\sqrt{\frac{\chi^2_{v,\alpha/2}}{v}} \qquad \text{(II.14.a)}$$

$$LCL = \sigma_0\sqrt{\frac{\chi^2_{v,1-\alpha/2}}{v}} \qquad \text{(II.14.b)}$$

where $\sigma_0$is the IC target value of the process and $v = \frac{(2-\lambda)}{\lambda}$ are the degrees of freedom for the χ^2 distribution (Montgomery, 2005). The statistic $\sqrt{S_i^2}$ is plotted on a Shewhart style control chart and monitored until OOC is signaled.

Development of the Exponentially Weighted Moving Variance (EWMV)

Several approaches have been taken to develop a EWMA that monitors variance components. Wortham and Ringer (1971) and Harris and Ross (1991) developed a control chart commonly known as the exponentially weighted moving variance (EWMV). Further development from MacGregor and Harris (1993) and Sparks (2003) discuss the calculation of the statistic:

$$S_i^2 = \omega(x_i - y_i)^2 + (1 - \omega)S_{i-1}^2 \quad \text{(II.15)}$$

where *x* is the observed characteristic, ω is the weighting paremeter, and y_i is the approximate process mean given by the EWMA (z_i) from equation (II.11). Using the approximate process mean is especially convenient when the mean varies as a result of process continuation (MacGregor & Harris).

MacGregor and Harris (1993) derived control limits for the EWMV using the χ^2 distribution and the expectation of the sample variance. The square root of the EWMV statistic $\left(\sqrt{S_i^2}\right)$ was plotted against the control limits in a Shewhart style control chart. MacGregor and Harris demonstrated the EWMV control chart was a useful chart when monitoring changes in variation as well as shifts in the mean vector. The EWMV control chart is especially useful when single observations are used, or when observations are auto correlated (MacGregor & Harris).

Development of the Multivariate CUSUM (MCUSUM)

Following the univariate design first introduced by Page (1954), multiple CUSUM charts being used to monitor multiple variables was common practice (Woodall & Ncube, 1985). Rather than working with multiple CUSUM charts, Woodall and Ncube (1985) suggested creating a single control chart to monitor multiple variables, called the

Multivariate Cumulative Summation (MCUSUM) control chart. Statistics for the MCUSUM were developed as two separate statistics:

$$\underline{s}_i = \max\left(0, \underline{s}_{i-1} + \left(\underline{x}_i - \underline{a}\right) - \underline{k}\right) \text{ [upper side]} \qquad \text{II.16.a}$$

$$\underline{t}_i = min\left(0, \underline{s}_{i-1} + \left(\underline{x}_i - \underline{a}\right) + \underline{k}\right) \text{ [lower side]} \qquad \text{II.16.b}$$

where $\underline{a}$ is the IC target value of the process and $\underline{k}$ is the head start value of the MCUSUM. The value of $\underline{k}$ must meet specific criterion; the length of $\underline{k}$ is determined by the covariance matrix $\underline{\Sigma}$ such that $\underline{k}'\underline{\Sigma}\,\underline{k} = k^2$ where k is the scalar value of the length of $\underline{k}$ that is determined by $\underline{\Sigma}$ which is used in equation (II.16) to bring the values of the equation closer to zero (Crosier, 1988). Simplified versions of equation (II.16) are:

$$C_i = \left\{\left(\underline{S}_{i-1} + \underline{X}_i\right)\underline{\Sigma}^{-1}\left(\underline{S}_{i-1} + \underline{X}_i\right)\right\}^{1/2} \text{ if } C_i \le k_1 \qquad \text{II.17.a}$$

or

$$C_i = \left\{\left(\underline{S}_{i-1} + \underline{X}_i\right)\left(1 - \frac{k_1}{C_i}\right)\right\}^{1/2} \text{ if } C_i > k_1 \qquad \text{II.17.b}$$

where $\underline{x}$ is the vector of observed characteristics, k_1>0, and $\underline{S_0} = \underline{0}$. The statistics were plotted on a Shewhart style control chart using the control parameter ±h. Development of an alternative MCUSUM by Crosier worked with the positive square root of the MCUSUM function described in equation (II.17) to develop ARL curves for the MCUSUM control chart. Most original designs involving the MCUSUM control chart suggest using multiple MCUSUM charts to monitor the *p*-process characteristics as discussed by Woodall and Ncube (1985), Pignatiello and Runger (1990), and Huwang, Yeh and Wu (2007).

Development of the Multivariate EWMA (MEWMA)

With the expansion of the exponentially weighted moving average to a multivariate application, Lowery et al. (1992) developed a control chart that works well for detecting small shifts in the mean or observation vector. Development of the multivariate exponentially weighted moving average (MEWMA) control chart is based on the observation vectors such that the first stage of statistics is calculated by:

$$\underline{Z}_i = r\underline{x}_i + (1 - r)\underline{Z}_{i-1} \qquad \text{(II.18)}$$

where $\underline{x}$ is the observed characteristic vector, $\underline{Z}_0=\underline{0}$ and r is the weighting value pre-determined for the MEWMA design. $\underline{Z}_i$ is then used in the calculation of the MEWMA control chart statistic:

$$T_i^2 = \underline{Z}_i{}' \underline{\Sigma}_{\underline{Z}_i}^{-1} \underline{Z}_i \qquad \text{(II.19)}$$

where $\underline{\Sigma}_{\underline{Z}_i} = \{r(1 - (1 - r)^{2i})/(2 - r)\}\underline{\Sigma}$ and is asymptotic to $\underline{\Sigma}_{\underline{Z}_i} = \{r/(2 - r)\}\underline{\Sigma}$. The new test statistic is plotted against a set control limit, h, where $h > 0$. Choosing h is determined by the choice of ARL_0. Hawkins, Choi and Lee (2007) and Prabhu and Runger (1997) published various table values for h for corresponding ARL's.

An advantage to using the MEWMA control chart to monitor process components is that a single control chart can monitor several process characteristics simultaneously. Kim and Reynolds (2005) discuss a situation when the MEWMA was used for unequal sample sizes. Hawkins, Choi, and Lee (2007) found that the MEWMA works well for monitoring process characteristics when the covariance matrix is full, rather than just the main diagonal of $\underline{\Sigma}$.

Development of the Multivariate Exponentially Weighted Mean Squared (MEWMS)

The first control chart used in this simulation study is the multivariate exponentially weighted mean squared (MEWMS) control chart. Using previous developments in the EWMS control chart, the MEWMS uses observation or mean vectors as values in the control chart development. Design of the MEWMS originally discussed by Huwang, Yeh and Wu (2007) focused on the trace of the covariance matrix produced from vectors using individual observations. Huwang et al.worked with scenarios where the variance elements changed uniformly for all observed variables.

The covariance matrix used by the MEWMS control chart employs the weighted covariance matrices from consecutive observations. Equations from Huwang, Yeh and Wu (2007) give the developmental equation:

$$\underline{S}_t = \omega \underline{x}_i \underline{x}_i' + (1-\omega)\underline{S}_{t-1} \qquad \text{(II.20.a)}$$

where $\underline{x}$ is the vector of observed characteristics, ω is the weighting parameter such that $0 < \omega < 1$ and $\underline{S}_0 = \underline{x}_1 \underline{x}_1'$. The simplified formula for this equation is:

$$\underline{S}_t = \sum_{i=1}^{t} c_i \underline{x}_i \underline{x}_i' \qquad \text{(II.20.b)}$$

where $c_1 = (1-\omega)^{t-1}, c_i = \omega(1-\omega)^{t-i}$ such that $\sum_{i=1}^{t} c_i = 1$ (Huwang, Yeh, & Wu). Using the value from equation (II.20), the test statistic is calculated by taking the trace of $\underline{S}_t$:

$$\text{Test Statistic 1: } T_{st} = trace[\underline{S}_t]$$

where $T_{S0} = 0$. The test statistic is plotted on a Shewhart style control chart using the control limits:

$$p \pm L\sqrt{2p \sum_{i=1}^{t} c_i^2} \qquad \text{(II.21)}$$

where $\sum_{i=1}^{t} c_i^2 = \frac{\omega}{2-\varpi} + \frac{2-2\omega}{2-\varpi}(1-\omega)^{2(t-1)}$ which converges to $\frac{\omega}{2-\omega}$ as $t \to \infty$ and L is provided by Huwang et al.for p=2 and p=3 process characteristics. One of the goals of this dissertation is to determine L through simulation that provide $ARL_0 \approx 370$ when p=5 and p=10 process characteristics.

Development of the Multivariate Exponentially Weighted Moving Variance (MEWMV)

The second control chart used in this simulation study is the multivariate application of the EWMV. First discussed by Sweet (1986) and MacGregor and Harris (1993), the multivariate exponentially weighted moving variance (MEWMV) uses the vector form of the EWMV statistic from (II.15) to develop the equation used by Huwang, Yeh and Wu (2007):

$$\underline{V}_t = \omega\left(\underline{x}_t - \underline{y}_t\right)\left(\underline{x}_t - \underline{y}_t\right)' + (1-\omega)\underline{V}_{t-1} \qquad \text{(II.22)}$$

where $\underline{x}$ is the observed characteristic vector, ω is the weighting value such that $0 < \omega < 1$ and $\underline{V}_0 = \left(\underline{x}_1 - \underline{y}_1\right)\left(\underline{x}_1 - \underline{y}_1\right)'$. The approximation for $\underline{y}_t$ is developed using the MEWMA of the process at time t from equation (II.18) as described by Lowery et al.. (1992). The test statistic for the MEWMV is found by taking the trace of the matrix resulting from equation (II.22) and is:

$$T_{Vt} = trace\left[\underline{V}_t\right] \qquad \text{(II.23)}$$

where $T_{V0} = 0$. Using equation (II.23) allows for the detection of a shift of the mean as well as detection of a change in the covariance matrix.

The control limits of for the statistic from equation (II.23) was based on the χ^2 distribution as explained by Huwang et al. (2007) to give the control limits of:

$$E\left[tr\left(\underline{V}_t\right)\right] \pm L\sqrt{Var\left[tr\left(\underline{V}_t\right)\right]} \tag{II.24}$$

where L is a constant and is dependent upon the values of p, ω, and λ from (II.22). The value of L was determined through simulation to correspond to a desired ARL_0 (Huwang, Yeh, and Wu). From equation (II.22) the linear expansion of the MEWMV equation is:

$$\left(\underline{x}_i - \underline{y}_i\right) = \underline{x}_i - \sum_{j=1}^{i} \lambda(1-\lambda)^{i-j}\underline{x}_j \tag{II.25.a}$$

$$= (1-\lambda)\underline{x}_i - \lambda(1-\lambda)\underline{x}_{i-1} - \cdots - \lambda(1-\lambda)^{i-1}\underline{x}_1 \tag{II.25.b}$$

where i=1, 2… t. From equation (II.25), the expansion to matrix form is:

$$\left(\underline{X} - \underline{Y}\right) = \begin{bmatrix} \left(\underline{x}_1 - \underline{y}_1\right)' \\ \vdots \\ \left(\underline{x}_t - \underline{y}_t\right)' \end{bmatrix} = \begin{bmatrix} 1-\lambda & 0 & 0 \\ \vdots & 1-\lambda & 0 \\ -\lambda(1-\lambda)^{t-1} & \cdots & 1-\lambda \end{bmatrix} * \begin{bmatrix} \underline{x}_1' \\ \vdots \\ \underline{x}_t' \end{bmatrix} \tag{II.26.a}$$

and let $\underline{M} = \begin{bmatrix} \lambda & 0 & 0 \\ \vdots & \lambda & 0 \\ \lambda(1-\lambda)^{t-1} & \cdots & \lambda \end{bmatrix}$

and $\underline{I}_p$ is a $p \times p$ identity matrix, then from (II.25.a)

$$\left(\underline{X} - \underline{Y}\right) = \begin{bmatrix} \left(\underline{x}_1 - \underline{y}_1\right)' \\ \vdots \\ \left(\underline{x}_t - \underline{y}_t\right)' \end{bmatrix} = \left(\underline{I}_t - \underline{M}\right)\underline{X} \tag{II.26.b}$$

Now, let $\underline{C} = \begin{bmatrix} (1-\omega)^{t-1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \omega \end{bmatrix}$, so equation (II.22) is now modified to:

$$\underline{V}_t = \left(\underline{X} - \underline{Y}\right)'\underline{C}\left(\underline{X} - \underline{Y}\right) = \underline{X}'\left(\underline{I}_t - \underline{M}\right)'\underline{C}\left(\underline{I}_t - \underline{M}\right)\underline{X} \tag{II.27.a}$$

and let $\underline{Q} = \left(\underline{I}_t - \underline{M}\right)'\underline{C}\left(\underline{I}_t - \underline{M}\right) = \underline{q}_{ij}$ where $1 \le$ i and j $\le$ t, such that:

$$\underline{V}_t = \underline{X}'\underline{Q}\underline{X} \tag{II.27.b}$$

Using the linear expansion of the MEWMV statistics allows for the development of the control chart limits. The control limits for the MEWMV test statistic (Test Statistic 2) as described by Huwang et al. are:

$$\text{UCL: } p\left[tr\left(\underline{Q}\right)\right]+L\sqrt{2p\sum_{j=1}^{t}\sum_{i=1}^{t}q_{ij}^{2}} \tag{II.28.a}$$

$$\text{LCL: } p\left[tr\left(Q\right)\right]-L\sqrt{2p\sum_{j=1}^{t}\sum_{i=1}^{t}q_{ij}^{2}} \tag{II.28.b}$$

where values for $\underline{Q}$ are calculated using techniques described in equation (II.27) and L is determined in simulation study to provide an ARL_0 of approximately 370 for $p=2$ and $p=3$ process characteristics per Huwang et al. The calculated test statistic (equation II.23) from the trace of equation (II.22) is then plotted on a Shewhart style control chart using the control limits from either (II.24) or (II.28). The OOC is signaled if the test statistic is plotted outside of the control limits defined by equations (II.24) or (II.28).

Yeh et al. (2003) first introduced the multivariate EWMA-V chart as an alternative to the |*S*| control chart and was the predecessor to the MEWMV chart which he co-authored (Huwang et al., 2007). In the study, the EWMA-V chart was developed to monitor the change in the process variability and the counterpart EWMA-M chart was developed to monitor the process mean (Yeh et al., 2003). Yeh et al. concluded that using these two control charts together were a better alternative than the MEWMA. Reynolds and Cho (2006) explored the use of the MEWMA using regression adjustments to monitor the change in the covariance matrix and the shift in the mean vector. Reynolds and Cho, as well as others including Huwang et al., have examined a single directional change, since the combination of directional shifts can vary in multiple combinations.

New developments by Hawkins and Maboudou-Tchao (2008) calculated a standardized test statistic that used the traditional height parameter, h. Hawkins and Maboudou-Tchao generated the control chart equation:

$$\text{Test Statistic 2:} c_n = tr(\underline{S}_n) - log|\underline{S}_n| - p$$

where $\underline{S}_n = (1-\omega)\underline{S}_{n-1} + \omega \underline{x}_n \underline{x}_n^{`}$, taken from equation (II.20.a) and $\underline{x}$ is the standardized observation vector. The statistic, c_n, is sequentially plotted on the control chart and compared to the height parameter, h. The process signals OOC when $c_n > h$. Hawkins and Maboudou-Tchao published values of h for ARL_0's ranging from 100 to 2000. Hawkins and Maboudou-Tchao's new control chart is an algebraically simplified statistic compared to the statistic and control limits developed by Huwang et al.(2007). Simulated control charts for the MEWMV were tested against Hawkins and Maboudou-Tchao's new design for sensitivity of a singular characteristic shift/change with individual observations.

<u>Development of the Control Limit Parameters, L/h</u>

In the cases of MEWMS and MEWMV the control limits h or control limit component, L, are pre-determined for values of ARL_0 approximately equal to 370. Values for L were published by Huwang et al. (2007) for p=2 and p=3 process characteristics as a mathematical component of the control limit. As stated in Huwang, Yeh, and Wu, the values of L were unknown and determined in simulation. Using the published values of L from Huwang, Yeh, and Wu, replication of p=2 and p=3 process characteristics will determine the relative accuracy of the values of L in the newly simulated MEWMS to give an accurate starting procedure for expanding to greater values of p. The unknown values of L for the MEWMS using p=5 and p =10 process characteristics were found in simulation, similar to Huwang, Yeh, and Wu. Values of *h* were published by Hawkins and Maboudou-Tchao (2008) for the MEWMV control charts, but values for ARL_0 of 370 were not. The unknown values for *h* were developed for *p*=2, *p*=3, *p*=5 and *p*=10 process characteristics in the MEWMV using the published values as starting values and adjusting values of *h* to achieve a target ARL_0 approximately equal to 370. Values of *L* and *h* were derived to create an ARL_0 approximately equal to 370 in the MEWMV and MEWMS to maintain comparable ARL curves. Knowing that the value for *L* and *h* will increase due to the increased number of process characteristics, *p*, and value of weighting parameter, ω (Huwang et al., 2007). The values of *L* and *h* were adjusted throughout the simulation to achieve ARL_0 accuracy at approximately 370.

Conclusions

While the control charts discussed here are only a small portion of the charts used in SPC, they are substantial contributions to control charts that use standard deviation to monitor process characteristics. The use of standard deviation or the covariance matrix as the test statistic is still new in the development of SPC. However, these measurements are used in many of the control charts. Since it is commonly believed that the shift of the mean is also related to the change in the variance components, it suggests that a change in variance components may lead to a change in the mean. With the development of the MEWMS and MEWMV control charts, new approaches to monitoring process characteristics were introduced.

Chapter three introduces the theoretical and methodological development of the MEWMS and MEWMV control charts first described by Huwang et al.(2007) and continued in the discussion of Hawkins and Maboudou-Tchao (2008). The defining characteristic of both the MEWMS and MEWMV control charts is the use of the trace of the covariance matrix as the test statistic used in the control charts. In many of the multivariate control charts discussed in this chapter, the covariance matrix is used as a part of the equation of the test statistic rather than the basis of the test statistic. The remainder of this dissertation discusses the expansion of Huwang, Yeh, and Wu's MEWMS research that used $p=2$ and $p=3$ correlated process characteristics and Hawkins and Maboudou-Tchao's MEWMV research to determine the sensitivity of the MEWMS and MEWMV to detect variance changes and mean shifts that occur when only a single element of each, or both, change.

CHAPTER III

METHODOLOGY

The purpose of this simulation study was to establish the sensitivity to a single variance change and/or single mean shift as well as ARL measurements with the MEWMS and MEWMV tests using multiple process characteristics. This dissertation expands on the previous research conducted by Huwang et al.(2007) and Hawkins and Maboudou-Tchao (2008). Original research explored the ARL of the MEWMS and MEWMV control charts and the development of the control chart parameters to determine measures of ARL_0. MEWMS control charts developed by Huwang et al. monitored for a uniform change in the process variance using the trace elements of the covariance matrices for individual observations with *p*=2 and *p*=3 correlated process characteristics. Similarly, the MEWMV control charts described by Huwang et al. monitored variance elements and mean vectors using the trace elements of the covariance matrix with a different developmental equation using *p*=2 and *p*=3 process characteristics and individual observations. Later research developed another MEWMV control chart that expanded upon the work of Huwang et al. and worked with sample sizes related to the number of process characteristics (Hawkins & Maboudou-Tchao). The author wished to determine the sensitivity and the run length distributions and control chart parameters for the MEWMS from processes with *p*=2, *p*=3, *p*=5, and *p*=10 correlated process characteristics when a single element changed in the variance structure or the observation vector, using individual observations. The author also wished to establish the run length distribution and control chart parameters of the MEWMV with p=2, p=3, p=5, and p=10 correlated process characteristics when a single change in variance and/or a single shift in the mean vector occurred using individual observations. Simulations also developed control chart parameters for the ARL distribution for p=5 and p=10 process characteristics. This chapter discusses the general development of the previous study and method development for the construction of the expanded MEWMS and MEWMV control charts.

Statement of Research Questions

The following questions are addressed in this study:

Q1 Does the MEWMS control chart monitor for a singular change in the covariance matrix and mean shift more effectively than the MEWMA or the MCUSUM control charts?

Q2 Does the MEWMV control chart monitor for a singular change in the covariance matrix and mean shift more effectively than the MEWMA or the MCUSUM control charts?

Q3 What are the appropriate values for the control chart parameters for the MEWMS and MEWMV to create an ARL_0 approximately equal to 370 (per Huwang et al.(2007) and Hawkins and Maboudou-Tchao (2008))?

Method Development

Run length distributions were derived through simulations using PROC IML in SAS version 9.1. This simulated data represented populations with known characteristics and known correlation coefficients and data were simulated to have multivariate normal distributions $\underline{X} \sim N_p\left(\underline{0}, \underline{I}\right)$. Initial runs were generated from a population with no shift and no variance change ($\underline{\mu}^{`} = \left[\mu_1 + 0 \quad \mu_2 + 0\right], \underline{\Sigma} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$) to develop the "Steady State" IC ARL as defined by Hawkins et al. (2007). Uniform shifts were introduced into the observation generating matrices before the control chart was tested, using the “Initial State” ARL design. Huwang et al.(2007) produced control charts using uniform changes in variances of 25%, 50%, 75%, 100% and 200% for the MEWMS control chart when *p*=2 and *p*=3 process characteristics. The same changes in variance values were used in this simulation, however, variance changes occurred in only one element of the *p*=2 and *p*=3 process characteristics. This simulation expanded the process to singular variance changes in *p*=5 and *p*=10 process characteristics. Changes in variance also coincided with uniform shifts of the mean to test the MEWMV control chart. Shift sizes for the mean vector varied by values of $\delta = \{.25, .50, 1.0, 2.0, 3.0\}$ per Huwang, Yeh, and Wu. These mean shifts were placed in the last element of the observation vectors. The variance changes were placed in the last element of the covariance matrix with the appropriate changes of the off diagonal elements. Values used in the change in the variance matrix changed by values of $\delta = \{1.0, 1.25,\ 1.50, 1.75, 2.0, 3.0\}$. The MEWMS and MEWMV control charts are discussed separately as tests to explain the step by step development of the simulation as well as the control chart processes for each.

Methodological Development of the MEWMS Control Chart

This simulation study expands upon the previous work of Huwang et al.(2007) which developed control chart parameters and studied the ARL of the MEWMS control chart using *p*=2 and *p*=3 process characteristics. This dissertation used *p*=2, *p*=3, *p*=5, and *p*=10 correlated process characteristics to develop the new control chart parameters and explore the sensitivity of the MEWMS chart to a single change in variance. Furthermore, this dissertation will explore the ARL distribution of these multivariate cases of the MEWMS when a single component of the distribution changes.

The number of process characteristics monitored in previous studies range from *p*=1 (EWMA) to *p*=20, where the most common value from previous MEWMA and MCUSUM studies is *p*=5.

Each sequence in the MEWMS testing was run using 10,000 replications. The mean run length, median run length, skewness and kurtosis of the 10,000 replications were calculated to produce an approximation of ARL for each situation. The ARL for each scenario of mean shift and variance change was then plotted on a graph to show the three-dimensional average run length curves for each test when detecting the various combinations of mean shift and variance change. ARL's produced in simulation were also plotted in comparative two dimensional graphs showing the comparative sensitivity to either mean shift or variance change for the four control charts.

The number of sampling subgroups of individual observations used in the simulation were *m*=1 to *m*=10,000. The numbers of subgroups were predetermined by Huwang et al.(2007) to be 20,000 to maintain proper MEWMA technique and to provide an ending observation point if the *p* process characteristics remain in control during the simulation. However, up to 10,000 observation vectors were generated using the Proc IML step in SAS 9.1 for use in the MEWMS control chart simulation. Previous simulation research have used *m*=5000 to *m*=100,000, with common subgroup sizes ranging from 10,000 to 20,000. Prior research in MEWMA development of ARL have used subgroups ranging in size from *m*=1 to *m*=50,000. Subgroups of value *m*=30 is widely used; however, Quesenberry (1993) stated that at least *m*=100 subgroups are needed for appropriate estimation in non-MEWMA studies. Subgroups are used in the Phase I process to establish approximation values of control limits. For the MEWMS control chart, however, the subgroups are used to limit the number of subsets used in simulation. Hawkins and Maboudou-Tchao (2008) worked with subgroups with sample sizes that were the desired ARL_0 divided by the number of sampling subgroups used. The number of sampling subgroups was twice the number of process characteristics monitored ($2 \times p$). However, it is not unusual in practice to have unequal sample sizes.

Sample sizes for the MEWMS simulation are n=1, or individual observations, per Huwang, Yeh and Wu (2007). From previous research, the smallest sample size used was n=1 (individual observation) to the largest simulated sample sizes of n=2000. Other discussions conclude that the product of sample size (n) and number of subsets (m) is a measurable guideline to work with. Yeh, Lin, Zhou, and Venkataramani (2003) suggest $n \times m$ ranging in values from 500 to 600 when $p \leq 10$. Another study using the MEWMA suggested a product value of $n \times m$ that ranges from 200 to 250 where the number of correlated process characteristics p is large, such as p=10 or p=20 (Lowery & Montgomery, 1995).

This simulation study used correlation values that were uniformly distributed across the off-diagonal elements of the covariance matrix using values of 0.1 to 0.9, $\rho = \{0.1, 0.3, 0.5, 0.7, 0.9\}$. Huwang et al.(2007) used a range of correlation such that $\rho = \{0.1, 0.2, \ldots, 0.9\}$. The selection of these correlation values reflects weak, moderate, and strong correlation because multicollinearity is a common occurrence in multivariate designs. Huwang, Yeh and Wu provided the general form of the population covariance matrix for the development of observation vectors:

$\underline{\Sigma} = \begin{bmatrix} \sigma_1^2 & \cdots & \rho\sigma_1\sigma_2 \\ \vdots & \ddots & \vdots \\ \rho\sigma_p\sigma_1 & \cdots & \sigma_p^2 \end{bmatrix}$. Using a uniform correlation matrix ensures a positive definite matrix for the generation of observations.

The choice of using the value of ρ uniformly across the covariance matrix was to ensure that a positive-definite matrix was used to create the observation vectors. Previous studies have tested the MEWMA with multicollinearity issues using correlation values of ρ=.50 (moderate) and ρ=.90 (high). While the correlation of the variables will not influence the trace of the statistics as designed by Huwang et al.(2007), the correlated values will influence findings from the MEWMA and MCUSUM used for comparisons. This situation will be discussed further in chapter four.

Values for the weighting parameter in the MEWMS control chart varied in values of $\omega = \{0.1, 0.3, 0.5, 0.7, 0.9\}$ similar to Huwang et al. (2007). Several discussions of weighting values have occurred for the EWMA and MEWMA control charts, with several studies stating that smaller values of ω are more effective at detecting small shifts in the mean vector with larger values detecting larger shifts (Lowery et al., 1992; Lowery & Montgomery, 1995). Earlier discussion stated that the individual values of the control limit parameter *L* were dependent upon values of ω and ρ. Table 1 shows the layout of the simulation design giving the levels of ω and ρ used in the calculations of the MEWMS control chart statistic from equation (II.20). Previous studies in the MEWMA and MCUSUM control chart designs have stated that while larger values of ω can detect larger shifts in the mean and smaller values can detect smaller shifts, the most consistent values of ω to detect any shift lies in the range of ω=.42 to ω=.56. With regards to the

MEWMA control chart, the value within this range that is most appropriate is still discussed.

Table 1

Levels of weighting, ω, and levels of correlation, ρ, for the generation of observation vectors for the MEWMS and MEWMV control charts.

ω	ρ	ω	ρ	ω	ρ	ω	ρ	ω	ρ
0.1	0.1	0.3	0.1	0.5	0.1	0.7	0.1	0.9	0.1
	0.3		0.3		0.3		0.3		0.3
	0.5		0.5		0.5		0.5		0.5
	0.7		0.7		0.7		0.7		0.7
	0.9		0.9		0.9		0.9		0.9

Methodological Development of the MEWMV Control Chart

Method development for the MEWMV control chart is similar to the development of the MEWMS. Techniques discussed in this section will expand upon the design described by Huwang et al.(2007) which worked with p=2 and p=3 correlated process characteristics and the further research of Hawkins and Maboudou-Tchao (2008) which dealt with sample sizes and large values of p. This dissertation expands on the design of the Hawkins and Maboudou-Tchao MEWMV control chart to determine the appropriate control limit parameters and examine the sensitivity of the control charts for single process characteristic shift/changes with individual measurements. Previous studies involving the MCUSUM (Crosier, 1988; Koning & Does, 2000) and MEWMA (Lowery et al., 1992; Lowery & Montgomery, 1995) worked with a variety of values for p, with common values ranging from p=2 to p=20; however, values ranging from p=5 to p =10 are most commonly used.

Sample size for the MEWMV simulation study is n=1, or individual observations, measuring one by one until OOC occurs. Huwang et al. (2007) designed the original study with individual observations of *p* correlated process characteristics as an observation vector, $\underline{x}_i$. Each of these observation vectors was treated as a separate independent subgroup, as discussed earlier. Sample sizes from previous studies involving MCUSUM, EWMA, MCUSUM and MEWMA all used various sample sizes that involved mean values/vectors of *n*=2 to *n*=2000. In designs of the MEWMV using sample sizes, the use of sample sizes at least twice the number of processes characteristics monitored was recommended (Hawkins & Maboudou-Tchao, 2008). The discussion of sample size and number of subgroups has long been discussed as to what combination is appropriate. Studies from MEWMA and MCUSUM have suggested the product of sample size and subgroups range from $n \times m$=200 to $n \times m$=3000.

Where the MEWMV initially differed from the MEWMS is in the development of the statistic used as the basis of the test statistic. The MEWMV developed by Huwang, Yeh and Wu (2007) used an exponentially weighted moving average as a mean vector to calculate the covariance matrix $\underline{S}_i$ from equation (II.22). Using a MEWMA statistic to calculate this value requires a secondary weighting parameter, λ. Similar to the weighting value ω, λ was used to create a weighted mean vector in the MEWMV to monitor for a shift in the mean vector while the MEWMV test statistic monitored for a change in the variance. With the publication of Hawkins and Maboudou-Tchao's (2008) new statistic calculation for the MEWMV, the use of the exponentially weighted moving average for the mean vector is no longer necessary. Instead, the new MEWMV calculation used the same covariance matrix calculation as Huwang, Yeh, and Wu's (2007) MEWMS (equation II.20.a), but the equation for the control chart statistic has differed (Test Statistic 2).

Construction of Simulations

Development of the simulation of the MEWMS control chart using PROC IML in SAS version 9.1 initially began with the development of observation vectors of size p=2, p=3, p=5 and p=10. Ten thousand observation vectors were generated for each level of parameter with various mean shift and correlation changes and used in the MEWMS, MEWMV, MEWMA and MCUSUM control charts to determine ARL's. Each observation matrix was run through each individual test where ARL's were calculated and ARL curves were generated for the various combinations of correlation (ρ) and control chart weighting (ω). Calculations for control chart limits were calculated within the PROC IML statements as well. Adjustments to the value of L were made in simulation to generate ARL_0's approximate to those defined ($ARL_{0=}$370) by Huwang et al.(2007).

Construction of ARL Measurements

The in-control average run length (ARL_0) was predetermined at a value of 370. Montgomery (2005), Jones (2002), and Huwang et al.(2007) used this predetermined value for ARL_0 in their studies. This value was initially obtained in univariate designs, where theory is discussed in Montgomery (2005). In an earlier study, Yeh, Lin, Zhou, and Venkataramani (2003) used an ARL_0 of 400. ARL_0 were based on control limits created using the control limit equation (II.21) and depending on the value of L used in each equation. Values of L were found during simulations by Huwang et al. to achieve the ARL_0 of approximately 370. Control limit parameters were developed in simulation by Hawkins and Maboudou-Tchao (2008) and new control limit parameters were developed in simulation to create the ARL_0 to be approximately 370. Previous studies in both univariate and multivariate control charts used ARL_0 values ranging from 185 to 2000 with 370 being the most commonly used.

Test Statistic 1, T_{st} from the trace of equation (II.20) plots sequentially on a Shewhart style control chart. This chart, referred to as the MEWMS chart, worked with control limits predetermined by Huwang, Yeh and Wu (2007) which were found through adjustments in simulation to determine the control limit parameter, L. The control limits parameter, L, for p=2 and p=3 process characteristics were published by Huwang, Yeh, and Wu. These existing limits are used to determine if the multivariate process is IC or OOC. Computation of the control limit parameter L was found for p=5 and p=10 process characteristics using the same adjustment techniques in simulation as described by Huwang, Yeh, and Wu. The purpose of these values of limits is to compute a simulated ARL_0 approximately equal to 370. The run length will be graphed and the ARL calculated based upon these distributions. Each simulation was run until an OOC signal was raised, or until 10,000 subsets were sampled. These simulations were run to 10,000 replications and the mean of these replications were used to calculate the ARL for each scenario.

Test Statistic 2, from the equation (II.28), also plots on a Shewhart style control chart, referred to as the MEWMV chart using values of h that gave ARL_0 approximately equal to 370. Appropriate values for h were determined in the simulation for p=2, p=3, p=5 and p=10 process characteristics using regression predicted values of h and making necessary adjustments in simulation to produce the ARL_0 of approximately 370. The comparison of ARL tables produced from the MEWMS and MEWMV with those of the multivariate MCUSUM and the MEWMA will follow with ARL_0 values matching.

Evaluation of ARL Measurements

To determine the ability of the MEWMS and MEWMV charts to monitor process variability for multiple correlated variables, simulations were run with p=2, p=3, p=5 and p=10 correlated process characteristics and the distribution of the ARL was compared to other multivariate control chart distributions. Assume the process produced observation vectors $\underline{x} \sim N_p\left(\underline{\mu}_p, \underline{\Sigma}_p\right)$, t=1, 2, 3… m. Shift in the process variance was measured using the trace of the covariance matrix and plotted on the respective control charts. ARLs of the MEWMS and MEWMV charts were studied to determine effectiveness for identifying shifts in variance. These simulation ARL charts were compared to ARL charts from previous studies by Lowery (1992), Montgomery (2005) and Huwang et al.(2007). Comparison of ARL values for the various control charts are discussed in chapter four.

CHAPTER IV

RESULTS

The purpose of this simulation study was to determine the sensitivity of the MEWMS and MEWMV control charts when a single element of the mean vector or covariance matrix changed. The ARLs of the MEWMS and MEWMV were graphed and compared to the ARLs of the MEWMA and MCUSUM control charts. Original development of the MEWMA and MCUSUM control charts was to detect small shifts in the mean vector. The development of the MEWMS and MEWMV control charts was designed to detect changes in the covariance matrix. Mean shifts and the variance changes were introduced in simulation and the sensitivity to detect these changes was monitored using the discussed control charts.

Method Development

Tables 3 through 15 display the resulting ARL_0 for the MCUSUM, MEWMA, MEWMS and MEWMV with the correlation between variables of 0.0. Tables 17 through 32 address the ARL values with all simulated correlation values. ARL values indicate that the MCUSUM showed little to no sensitivity to the change in variance components unless those changes were large. Variance changes in the MEWMA result in a sensitivity decrease of this control chart. A decrease in sensitivity was defined as an increase in ARL values resulting from the change in variance or mean shift compared to previous control chart runs. These tables are organized such that the mean shift is the first column and the variance change is the first row. For each of these charts, the ARL_0

was set for approximately 370. Table 4 shows the ARLs for the MCUSUM control charts.

Table 2

ARL values of the MCUSUM control charts when correlation=0.0

p	Mean	Variance					
		1	**1.25**	**1.5**	**1.75**	**2**	**3**
2	0	371.5742	334.8417	295.1528	263.4451	242.4911	172.9827
	0.25	110.7083	106.7214	102.3035	98.2189	93.2351	82.1249
	0.5	63.0545	61.0558	59.4789	57.9063	56.4184	52.5038
	1	34.0018	33.4491	32.8602	32.3493	32.0855	30.7247
	2	17.9540	17.7986	17.6359	17.7309	17.2147	16.8119
	3	12.1239	12.0525	11.9814	12.1036	11.9758	11.7506
3	0	375.7772	339.2914	309.3775	286.7104	281.8765	189.6674
	0.25	153.5514	145.0215	136.6566	131.6526	130.6541	101.5821
	0.5	83.6020	81.5229	79.6743	81.1798	77.1638	67.5579
	1	44.6231	43.5370	42.9011	44.6615	42.2722	39.0883
	2	22.7574	22.7188	22.5672	24.1955	22.1313	21.2824
	3	15.2500	15.3076	15.2640	13.5757	15.0665	14.4811
5	0	371.2534	353.7594	326.8422	294.6840	275.1339	200.7574
	0.25	239.6817	213.4887	193.8697	180.1815	160.5017	126.2775
	0.5	155.4436	141.7690	126.8379	119.5488	110.2096	87.4268
	1	73.3475	69.1376	65.9051	62.9714	60.0903	52.9791
	2	32.2086	30.9999	30.4664	30.1211	29.2457	27.9791
	3	19.9246	19.6755	19.2385	19.1548	18.8779	18.2778
10	0	380.2773	358.1375	337.7441	314.9554	291.7876	215.6140
	0.25	322.2781	276.7998	253.2137	229.4624	206.9162	155.5769
	0.5	253.0775	214.8527	192.7138	167.0369	148.3471	113.6289
	1	138.2853	119.3620	109.8242	97.2196	87.8516	72.2721
	2	48.0261	45.7510	44.8901	42.4210	40.7139	36.0343
	3	27.2047	25.9055	24.8293	24.8803	23.9260	22.9570

Tables 3 through 6 are the ARL tables for the MEWMA control chart with no correlation between the observed variables and increasing numbers of variables observed. With each increase in weighting values, the sensitivity of the MEWMA decreases. There is also a decrease in sensitivity with the increase in variance components. The decrease in sensitivity associated with the variance change is apparent in the table values when comparing equivalent variance value ARLs for increasing weighting values and mean shift combinations.

Table 3

ARL values of the MEWMA control charts when p=2 and correlation=0.0

Weight (value)	**Mean Shift**	**Variance**					
		1	**1.25**	**1.5**	**1.75**	**2**	**3**
0.1	0	371.6496	370.8668	374.3083	375.9602	372.8661	368.1014
	0.25	110.8831	130.1310	144.4593	163.7582	176.3766	217.8409
	0.5	32.4798	40.5109	48.0702	55.1239	62.4493	89.5308
	1	9.1762	11.2541	13.1290	15.0099	17.0668	25.1408
	2	2.9241	3.4907	4.0592	4.5261	5.1343	7.2038
	3	1.6373	1.8965	2.1637	2.4216	2.6573	3.6901
0.3	0	367.6546	368.7479	373.4068	369.3769	371.1513	371.9239
	0.25	189.8748	207.9583	228.4455	247.7523	251.1090	287.8718
	0.5	43.7327	79.8510	93.1831	107.6894	118.5208	160.3426
	1	13.4100	17.2997	21.3209	25.8704	30.0004	46.6551
	2	3.3565	4.0610	4.7936	5.5496	6.3996	9.5652
	3	1.7902	2.0900	2.4022	2.7211	3.0088	4.3064
0.5	0	370.3705	368.1954	372.6909	373.1647	375.9958	366.3845
	0.25	240.4617	257.4602	272.7399	282.1475	291.0797	316.1613
	0.5	100.9875	122.0901	140.8612	159.1507	170.7113	215.3747
	1	21.3927	28.8100	35.1956	42.9026	50.2943	77.2277
	2	3.8469	4.8897	6.0293	7.1516	8.5793	14.4460
	3	1.8434	2.2250	2.6036	2.9886	3.3739	5.2511
0.7	0	365.7777	371.5766	368.5220	370.0020	364.0761	363.4680
	0.25	277.9842	287.5723	298.2361	309.0823	316.3405	325.6169
	0.5	138.4604	162.0115	184.0619	198.5253	214.7689	252.6684
	1	33.6906	44.8513	56.2118	66.2183	77.4454	112.6582
	2	5.0211	6.5850	8.5256	10.7188	12.8674	23.2660
	3	1.9858	2.4463	3.0000	3.6397	4.2605	7.3334
0.9	0	370.9324	369.3624	366.7255	375.3362	366.7302	374.4812
	0.25	300.0910	309.3282	320.2460	331.8963	327.1194	348.3158
	0.5	178.7719	207.0211	219.9623	238.1180	249.8221	283.3871
	1	54.9644	69.2466	84.2809	100.6299	109.2904	152.1156
	2	7.2559	10.3441	13.6466	17.6173	21.2551	37.6965
	3	2.2714	3.0226	3.9803	4.9768	6.1089	11.3900

Table 4

ARL values of the MEWMA control charts when p= 3 and correlation=0.0

Weight (value)	Mean Shift	Variance					
		1	**1.25**	**1.5**	**1.75**	**2**	**3**
0.1	0	369.0617	367.3880	365.6250	376.4178	369.8173	362.4943
	0.25	129.4805	152.3469	166.9135	183.5927	197.6831	236.0523
	0.5	38.4309	47.2468	55.8626	65.2062	71.3292	102.2937
	1	10.2221	12.7039	14.8542	16.8749	19.4354	28.4258
	2	3.1954	3.8281	4.4530	5.0675	5.6784	7.9699
	3	1.7792	2.0720	2.3369	2.6570	2.9421	4.0612
0.3	0	372.6284	373.0501	368.3954	373.2871	369.0304	374.3740
	0.25	215.3815	237.9017	251.5433	263.9189	278.4508	304.5622
	0.5	79.6967	95.8145	114.7449	125.5584	142.0945	183.9392
	1	15.7787	20.9093	25.8828	31.2861	36.5946	57.9626
	2	3.6690	4.5024	5.3728	6.2709	7.2279	11.1648
	3	1.9300	2.2828	2.6086	2.9808	3.3232	4.7860
0.5	0	369.2408	374.1736	367.6642	372.2041	370.2731	372.9387
	0.25	258.3383	277.4758	298.2343	298.5209	310.0356	325.8661
	0.5	124.5923	146.5314	164.8319	183.8670	199.3293	231.4324
	1	26.0712	35.0094	45.1610	53.8657	61.7744	94.2752
	2	4.4018	5.6888	6.9897	8.5822	10.2193	17.8739
	3	2.0186	2.4553	2.8747	3.3567	3.8923	6.0919
0.7	0	373.3422	379.5295	377.9229	375.2852	375.8008	378.0775
	0.25	296.7437	310.3807	318.8310	325.4146	332.9409	350.1787
	0.5	170.9036	192.8274	211.7308	229.6629	240.1536	278.3285
	1	44.3409	58.4641	71.7746	85.4127	95.7593	137.0808
	2	5.9478	8.1307	10.8478	13.5311	16.5152	29.8088
	3	2.2154	2.8066	3.5138	4.1791	5.1169	8.8952
0.9	0	372.8597	382.1269	370.0412	372.9045	375.3414	376.5865
	0.25	318.1080	325.5133	335.0759	344.6064	341.7795	355.0202
	0.5	209.6866	233.6223	250.1950	265.4109	274.5724	298.8875
	1	69.4625	90.3744	107.0398	124.1297	138.6917	179.9458
	2	9.3173	13.4513	18.1589	23.3757	28.1266	49.7714
	3	2.6641	3.6900	4.9405	6.2069	7.7076	15.0767

Table 5 shows the MEWMA control chart ARLs when p=5 variables and correlation equals zero. With the increased number of variables, the sensitivity noticeably decreases. The increase in variance as well as the increase in weighting values also decreases the sensitivity of the MEWMA. These decreases in sensitivity with changes in variation suggest that the MEWMA is ill-equipped to detect potential variance changes. Upon further investigation of the control chart ARLs, the data agreed with previous studies stating weighting values are best at or below 0.5 (Hawkins, Choi, & Lee, 2007) as the sensitivity of each control chart tended to decrease as the weighting value exceeds 0.5.

Table 5

ARL values of the MEWMA control charts when p=5 and correlation=0.0

Weight (value)	**Mean Shift**	**Variance**					
		1	**1.25**	**1.5**	**1.75**	**2**	**3**
0.1	0	376.7386	370.7972	360.6733	370.6359	363.1053	367.1899
	0.25	153.0740	175.3105	192.9943	213.7551	233.8461	258.7401
	0.5	45.8958	58.8607	69.8900	78.2162	89.5633	123.8580
	1	12.1080	14.6743	17.3709	20.5224	22.8010	34.6630
	2	3.6598	4.4357	5.1339	5.9083	6.5041	9.2798
	3	1.9782	2.3303	2.6613	2.9974	3.2971	4.6710
0.3	0	372.5497	369.3572	368.4952	368.4648	372.3185	368.5296
	0.25	238.0758	268.0716	274.2624	280.9222	223.3120	318.2317
	0.5	99.6632	123.6593	141.1725	154.8659	168.2792	212.1276
	1	19.9441	26.8502	33.6341	40.0595	48.0611	75.2514
	2	4.2511	5.2627	6.3375	7.4931	8.6468	13.8997
	3	2.1513	2.5723	2.9823	3.4033	3.8238	5.5612
0.5	0	372.7452	374.9320	371.4578	375.9833	373.6538	373.0239
	0.25	283.7408	300.9425	317.6082	322.1854	324.3753	344.6030
	0.5	155.2012	178.7623	196.0513	216.6122	231.2815	265.2970
	1	35.6406	48.8357	60.7518	72.1889	83.9021	125.6059
	2	5.3065	6.9446	8.8621	11.2288	13.5480	23.6023
	3	2.3094	2.8260	3.3748	3.9459	4.6360	7.5049
0.7	0	370.5546	372.7157	374.0765	370.7994	368.5779	370.5988
	0.25	315.3130	323.1094	328.5912	339.0715	334.2596	352.5238
	0.5	201.3078	224.0005	239.2281	253.9738	268.8489	297.2178
	1	60.2898	78.6319	95.5637	111.0833	126.3234	172.3779
	2	7.7649	10.9239	14.5822	19.0306	23.0170	41.4551
	3	2.5873	3.4128	4.3240	5.3849	6.5587	12.1077
0.9	0	367.1131	369.2684	370.2500	370.2472	376.0947	372.2262
	0.25	329.4914	333.3696	338.1294	340.0997	354.5418	352.1373
	0.5	245.1107	260.6778	277.0030	284.0985	292.1234	320.9702
	1	91.9363	117.8771	137.8598	154.4452	165.9747	210.9600
	2	13.2685	19.2496	26.0283	32.9991	39.7862	69.0086
	3	3.3582	4.8308	6.4921	8.6306	10.9287	21.3349

Table 6

ARL values of the MEWMA control charts when p=10 and correlation=0.0

Weight (value)	Mean Shift	Variance					
		1	**1.25**	**1.5**	**1.75**	**2**	**3**
0.1	0	364.1486	366.5491	363.9491	364.8344	364.4578	368.4532
	0.25	188.5058	218.0343	234.9125	246.7333	256.1822	287.2313
	0.5	62.4149	77.4829	92.0867	103.7651	119.3280	158.9803
	1	15.2076	18.6989	22.6430	26.0148	30.2894	47.1012
	2	4.4789	5.3509	6.2615	7.1147	8.0390	11.5689
	3	2.3252	2.7588	3.1984	3.6509	4.0353	5.6670
0.3	0	370.7655	371.7732	369.2522	366.4591	372.3230	376.5283
	0.25	279.6550	295.4709	306.6438	311.6372	316.3217	335.9952
	0.5	138.7953	164.1999	181.6355	199.8561	215.5938	257.1628
	1	29.5246	39.6964	49.6487	60.9770	70.8918	106.7982
	2	5.3315	6.7620	8.2523	10.0018	11.6503	19.8461
	3	2.5697	3.0946	3.6291	4.1950	4.7767	7.3070
0.5	0	374.6690	372.8570	369.8187	368.7668	358.8111	371.1531
	0.25	313.7681	321.3419	331.0157	312.7036	341.9380	349.5404
	0.5	200.3871	221.4450	247.6468	254.8274	270.7206	297.4791
	1	55.9614	75.2410	90.1202	106.8804	122.9827	164.0532
	2	7.3105	10.2051	13.3195	16.8879	20.5919	37.7843
	3	2.8257	3.6124	4.3912	5.2807	6.2557	11.2148
0.7	0	370.4053	374.4450	372.4764	366.4285	365.9597	371.5959
	0.25	330.9481	337.7715	347.1672	342.0134	351.6262	356.0538
	0.5	242.0547	262.8538	279.2153	284.2329	294.1598	320.1170
	1	93.6109	117.5057	139.2012	157.9275	168.0584	213.6084
	2	12.0157	17.8217	24.1515	30.8013	37.9999	65.4243
	3	3.4699	4.7421	6.2956	7.8889	10.1940	20.0012
0.9	0	367.9667	364.2590	364.8485	361.1030	369.1765	368.5511
	0.25	344.8818	346.4252	352.9492	358.1487	356.8365	362.8801
	0.5	274.4297	288.3902	304.8318	310.4849	315.4124	331.8614
	1	140.5646	161.4818	182.3081	199.5052	211.9647	253.9660
	2	22.5538	32.4163	44.4861	54.1548	65.3057	102.4371
	3	5.1622	7.8449	10.8353	14.3815	18.2778	36.7158

Tables 7 through 10 show the MEWMS control chart ARLs when there is no correlation between the observed variables. As explained in earlier chapters, the MEWMS control chart was specifically designed to detect small changes in variance components. When comparing values of these tables to the values from earlier tables from the MCUSUM (Table 2) and MEWMA control charts (Tables 3 through 6), the sensitivity to singular changes in variance components has improved. The MEWMS is comparable to the MCUSUM in its sensitivity to detect a singular mean shift and less sensitive than the MEWMA control chart.

Table 7

ARL values of the MEWMS control charts when p=2 and correlation=0.0

Weight (value)	**Mean Shift**	**Variance**					
		1	**1.25**	**1.5**	**1.75**	**2**	**3**
0.1	0	368.3827	140.5713	74.9200	49.5542	38.5478	21.8099
	0.25	277.4577	116.7016	65.7657	46.7237	36.3629	21.2228
	0.5	144.7199	74.5444	50.1838	38.1785	31.4703	19.9858
	1	39.3032	31.5011	27.1286	23.9100	21.6624	16.3095
	2	13.0199	12.6209	12.1431	11.7548	11.3932	10.2391
	3	8.5934	8.4402	8.2824	8.1943	8.0813	7.6281
0.3	0	358.5521	157.4182	80.2247	50.1575	34.6424	15.6211
	0.25	294.8288	129.1671	71.0021	45.4013	32.1132	14.9036
	0.5	164.2253	84.8512	51.3526	35.0995	26.5089	13.7391
	1	42.3527	29.3655	22.2400	18.2330	15.6755	10.4285
	2	7.7118	7.3208	6.9618	6.7176	6.5028	5.7937
	3	4.8119	4.7472	4.6966	4.6832	4.6224	4.3934
0.5	0	369.5183	169.0379	90.1176	54.7769	38.0964	15.8215
	0.25	309.1548	142.5277	79.2825	48.9594	35.2830	14.8731
	0.5	182.2021	95.1728	57.0516	38.6122	28.6925	13.7662
	1	52.1277	33.2141	24.5332	19.1904	15.8310	10.0629
	2	7.8127	7.0203	6.5425	6.2156	5.9035	5.1337
	3	4.4164	4.3204	4.2364	4.1982	4.0936	3.9130
0.7	0	378.9924	178.5763	95.5451	59.2033	41.4626	16.3117
	0.25	311.7954	148.1873	86.4123	54.5318	37.1613	15.7140
	0.5	192.0490	100.7606	61.2124	41.7611	30.6559	14.0457
	1	59.9244	37.5107	26.4040	20.5851	17.1332	10.2352
	2	8.6210	7.6628	6.9130	6.4388	6.0582	5.1097
	3	4.7652	4.5696	4.4115	4.2686	4.1894	3.9385
0.9	0	371.5148	180.3400	100.7678	62.0094	42.4945	16.9624
	0.25	305.8320	155.7156	89.4862	55.9666	39.3328	16.2831
	0.5	202.4536	105.9986	64.3775	43.6224	31.8958	14.5905
	1	66.5039	40.6153	28.7736	22.0693	18.0197	10.4962
	2	9.8644	8.2631	7.4577	6.8357	6.2960	5.1646
	3	5.4190	5.0035	4.7619	4.6313	4.4288	4.0637

Table 8

ARL values of the MEWMS control charts when p=3 and correlation=0.0

Weight (value)	Mean Shift	Variance					
		1	**1.25**	**1.5**	**1.75**	**2**	**3**
0.1	0	364.7046	163.9110	91.9350	60.6368	45.7831	25.2989
	0.25	297.7395	137.6684	81.2659	56.8168	43.4039	24.6657
	0.5	164.2162	91.7241	61.2798	46.3335	37.2547	23.2650
	1	47.7400	48.3325	31.9266	28.2416	25.2918	18.8430
	2	15.1061	14.4963	13.9988	13.5932	13.1475	11.8126
	3	8.7399	8.6024	8.4552	8.3542	8.2537	7.8058
0.3	0	374.6656	182.0540	99.5520	62.0500	44.8222	18.4701
	0.25	314.1976	156.7581	88.5265	57.7997	40.7636	17.9919
	0.5	198.9730	104.0860	63.2342	43.6673	33.0441	16.2493
	1	53.3725	36.2944	27.1671	22.2561	18.6761	11.9797
	2	9.1037	8.5617	8.0939	7.6803	7.3697	6.5313
	3	4.4867	4.4369	4.3781	4.3598	4.3445	4.1246
0.5	0	376.8332	192.2310	109.8445	69.3940	46.9619	18.6894
	0.25	321.3747	167.2220	98.4888	63.1351	43.6824	17.7735
	0.5	211.5318	114.3692	72.0542	48.9173	35.1427	16.2402
	1	65.2887	43.1624	30.3203	23.6803	19.3676	11.7319
	2	9.4892	8.4595	7.7046	7.1629	6.8784	5.7346
	3	3.8153	3.7546	3.7312	3.6886	3.6477	3.4848
0.7	0	373.5688	206.0988	116.2881	73.9585	51.2971	19.9456
	0.25	318.4778	179.1428	104.4294	69.2443	47.1217	18.7417
	0.5	220.7141	125.4088	75.7287	51.4958	38.2220	16.8325
	1	76.2371	48.6549	33.6253	25.7361	20.5876	11.8904
	2	10.8757	9.2602	8.1806	7.4851	6.9163	5.7835
	3	3.7636	3.7029	3.6244	3.5875	3.5725	3.4146
0.9	0	369.3932	210.2705	124.9240	79.9332	52.5722	20.4273
	0.25	325.3132	181.8452	108.4759	71.5124	49.7855	321.3143
	0.5	225.2312	128.5499	79.9067	54.9038	40.4471	17.6592
	1	85.3276	51.7554	36.0369	27.3025	21.5941	12.2629
	2	12.5586	10.2508	8.9637	7.8983	7.3238	5.8634
	3	3.9542	3.8371	3.7431	3.7249	3.6569	3.4833

Table 9 is the ARL tables of the MEWMS control chart with *p*=5 variables. With the increase in number of observed variables, the sensitivity of the control chart notably decreases; similar to that of the MEWMA control chart. The behavior for all of the discussed control charts showed a decrease in sensitivity associated with the number of observed variables exceeding *p*=5. Table 10 also shows this behavior as the number of variables (*p*) has increased to ten. This behavior is discussed later in chapter five.

Table 9

ARL values of the MEWMS control charts when p=5 and correlation=0.0

Weight (value)	**Mean Shift**	**Variance**					
		1	**1.25**	**1.5**	**1.75**	**2**	**3**
0.1	0	366.7903	194.7938	119.5938	80.7726	59.8003	31.6943
	0.25	321.4128	175.4952	107.2300	75.0383	56.6582	30.8857
	0.5	205.5801	120.0719	80.7994	59.9196	47.7823	28.6516
	1	63.5910	49.1223	40.9720	35.6564	31.4792	22.8918
	2	18.5768	17.8001	17.1134	16.5313	16.0178	14.2805
	3	10.7486	10.5501	10.4381	10.2440	10.0591	9.5511
0.3	0	370.5375	215.7772	130.0818	82.8200	58.4756	23.5790
	0.25	320.3431	186.9199	116.7750	76.5529	53.5727	22.6003
	0.5	225.6829	132.7730	83.9714	57.9713	44.0414	20.2486
	1	72.8258	50.0555	37.3642	29.2079	24.1501	14.8767
	2	11.5943	10.5969	10.0279	9.4218	8.9642	7.6794
	3	5.3179	5.2383	5.1312	5.0793	5.0377	4.7834
0.5	0	371.1002	226.7370	140.5337	90.6757	63.7366	24.3496
	0.25	329.8448	199.7232	126.4343	83.5608	59.1370	23.4533
	0.5	238.3668	144.7926	93.2142	63.9769	46.7825	20.5910
	1	87.6877	58.4909	41.3385	31.2929	25.5514	14.6242
	2	12.3761	10.8223	9.8146	8.9834	8.3983	6.7655
	3	4.5158	4.4139	4.3432	4.2695	4.2318	4.0109
0.7	0	376.1902	236.0925	151.6356	100.4960	69.0116	25.5363
	0.25	341.9932	211.1002	135.3198	90.6923	63.5842	24.5762
	0.5	251.8061	159.8070	102.4930	69.4698	51.3797	21.5564
	1	102.5635	65.8613	47.9287	35.6786	27.5092	14.9054
	2	14.8175	12.1998	10.4583	9.5015	8.6693	6.8327
	3	4.5716	4.3676	4.2608	4.1733	4.0971	3.8660
0.9	0	379.8064	241.0659	156.6049	105.6131	73.4122	27.2722
	0.25	330.6585	220.6160	141.5888	95.1480	68.7005	26.0302
	0.5	260.0832	163.2501	107.2530	73.6361	54.1695	23.1039
	1	113.9535	72.7423	49.9720	37.4224	29.7208	15.4210
	2	17.8563	13.8222	11.6605	10.2680	9.1954	6.8867
	3	4.9704	4.5972	4.4830	4.3365	4.2571	3.9239

Table 10

ARL values of the MEWMS control charts when p=10 and correlation=0.0

Weight (value)	Mean Shift	Variance					
		1	**1.25**	**1.5**	**1.75**	**2**	**3**
0.1	0	368.8985	256.0400	171.7888	123.2713	91.6162	45.1758
	0.25	341.8178	234.5571	155.8205	113.1742	86.4505	43.5412
	0.5	260.4634	175.0243	123.6261	91.9782	72.3126	40.0765
	1	99.7267	76.7226	61.3997	52.0396	45.4321	31.7578
	2	25.2694	24.0034	23.1007	22.2208	21.4686	18.9685
	3	14.3442	14.1405	13.9497	13.7637	13.5024	12.7847
0.3	0	375.2021	247.6065	173.1477	120.4539	85.4187	34.8917
	0.25	341.8421	227.2758	156.9965	110.8712	80.1491	33.7021
	0.5	256.6446	175.9224	121.2221	88.7575	65.4425	30.3384
	1	105.6987	75.5658	57.5560	44.2358	36.4953	20.9229
	2	17.0551	15.5543	14.0941	13.1952	12.2079	10.1752
	3	6.9246	6.7815	6.6586	6.5556	6.4173	6.0494
0.5	0	373.3395	267.9068	189.0536	132.7437	95.1805	37.1473
	0.25	348.5448	239.0426	172.0731	122.2303	89.3182	35.2941
	0.5	268.7968	191.4741	136.3422	99.0537	72.7381	30.9782
	1	124.7393	90.0521	64.6082	50.2742	39.6037	20.9747
	2	19.9838	16.5650	14.4266	13.0143	11.8762	9.0534
	3	6.1031	5.8688	5.7281	5.5342	5.3909	5.0313
0.7	0	377.0577	276.0776	199.1482	139.8425	103.5882	39.4618
	0.25	345.6566	254.8410	183.7998	129.6533	96.8293	38.1886
	0.5	288.2449	204.3807	147.4138	105.7314	78.7617	33.4373
	1	143.2237	99.8988	73.1754	54.5842	43.2655	21.8008
	2	24.7902	19.4325	16.4170	14.0840	12.6500	9.0992
	3	6.5229	6.0779	5.7303	5.4727	5.3041	4.7591
0.9	0	372.0806	284.8961	204.4080	147.6741	108.0112	40.9920
	0.25	346.6738	258.3685	192.6026	138.4867	101.9592	39.1463
	0.5	288.7056	211.4792	154.2020	113.7607	84.8970	35.2396
	1	154.1752	109.9800	78.7796	59.3183	47.2818	23.3171
	2	30.0048	22.9783	18.7400	15.7675	13.6014	9.4810
	3	7.3417	6.6403	6.1791	5.8058	5.5545	4.8043

Tables 11 through 14 show the MEWMV control chart ARLs when there is no correlation between the observed variables. As explained in earlier chapters, the MEWMV control chart was designed as a mathematically simpler way to detect small changes in variance components compared to the MEWMS control chart. When comparing values of these tables to the values from earlier tables from the MCUSUM (Table 2) and MEWMA control charts (Tables 3 through 6), the sensitivity to singular changes in variance components has improved. The MEWMV is comparable to the MCUSUM and MEWMS in its sensitivity to detect a singular mean shift and less sensitive than the MEWMA control chart.

Table 11

ARL values of the MEWMV control chart when p=2 and correlation=0.0

Weight (value)	Mean Shift	Variance					
		1	**1.25**	**1.5**	**1.75**	**2**	**3**
0.1	0	376.0233	118.4028	79.1616	48.5828	32.4536	13.4006
	0.25	291.5077	127.4208	68.2770	42.3837	30.4788	12.9792
	0.5	167.1225	81.8355	49.3581	32.8402	24.3848	11.9606
	1	40.5231	27.3422	20.2016	16.3041	13.5119	8.6486
	2	6.2138	5.8057	5.4795	5.1612	4.9094	4.2946
	3	2.6534	2.6157	2.6112	2.6136	2.5770	2.5309
0.3	0	370.6940	155.0649	91.0643	54.4618	36.0417	14.2067
	0.25	298.5762	143.2002	76.9119	48.0673	33.2483	13.6650
	0.5	179.0741	92.2678	55.2680	37.1443	26.6070	12.5805
	1	50.4534	31.3985	23.2877	17.9486	14.8176	8.6398
	2	6.6228	5.8992	5.3892	5.1293	4.7722	4.0855
	3	2.4019	2.3810	2.3667	2.3410	2.3250	2.2991
0.5	0	370.4231	174.4174	94.3616	59.6903	39.2130	15.2740
	0.25	306.4156	150.2079	82.7974	52.4466	36.8731	14.4861
	0.5	193.7813	101.2005	59.3369	40.6626	29.5954	13.1588
	1	58.6460	36.3759	25.2134	19.5469	15.6036	8.9045
	2	7.5734	6.5756	5.8569	5.3383	5.0072	4.1076
	3	2.3598	2.3388	2.3461	2.3166	2.2812	2.2563
0.7	0	358.9030	182.4974	99.4969	61.5347	41.8273	16.0564
	0.25	312.4470	153.5485	86.6553	55.4304	39.1887	15.4875
	0.5	200.5399	106.3722	63.7570	43.8070	31.0567	13.7028
	1	65.8687	39.7443	28.2069	20.7382	16.8296	9.5361
	2	8.8365	7.3237	6.2919	5.7080	5.2185	4.2733
	3	2.4719	2.4075	2.3752	2.3713	2.3428	2.2868
0.9	0	373.8134	181.5188	98.5504	62.566	41.8841	15.8878
	0.25	315.3463	157.4319	86.9075	55.1299	38.8332	15.3722
	0.5	204.2787	104.6676	63.4214	42.6634	31.1321	13.5976
	1	65.8786	39.5374	27.5175	20.7408	16.8793	9.5018
	2	8.762	7.1874	6.2678	5.7627	5.2592	4.2285
	3	2.501	2.4797	2.4374	2.3819	2.3458	2.2722

Table 12

ARL values of the MEWMV control chart when p=3 and correlation=0.0

Weight (value)	Mean Shift	Variance					
		1	**1.25**	**1.5**	**1.75**	**2**	**3**
0.1	0	365.5406	156.0085	84.3530	55.1886	40.6241	20.3458
	0.25	286.5630	130.6575	75.0153	50.5992	38.5497	19.9587
	0.5	156.6836	84.1947	55.9224	41.3719	32.1869	18.4205
	1	43.0132	33.1875	27.3492	22.9366	20.3953	14.2216
	2	10.8954	10.2522	9.8025	9.3421	9.0381	7.7766
	3	5.3097	5.2312	5.1139	5.0363	5.0053	4.7068
0.3	0	367.2858	181.3339	99.4432	61.0825	41.5667	16.6802
	0.25	304.4154	152.5192	84.1924	54.4574	38.4249	15.8918
	0.5	191.3297	101.7521	61.2039	41.1050	30.9420	14.7200
	1	51.8702	35.0452	26.0266	20.7164	16.9844	10.3550
	2	7.6782	7.0598	6.5508	6.2288	5.9104	5.0382
	3	3.1369	3.1080	3.0714	3.0272	2.9898	2.8872
0.5	0	371.8887	192.0747	109.1977	68.3405	46.1300	17.6054
	0.25	313.1364	166.0847	94.5839	60.9473	42.5187	16.8888
	0.5	209.4093	113.6002	69.2799	46.7282	34.0380	15.0403
	1	64.9001	40.6251	29.0730	22.5165	18.0996	10.1693
	2	8.2270	7.2290	6.5864	6.1047	5.6974	4.7184
	3	2.8144	2.7677	2.7690	2.7055	2.6716	2.5603
0.7	0	366.0919	202.7449	115.2441	72.3803	49.8249	18.2482
	0.25	320.3871	174.8611	102.0684	64.8737	45.3255	17.6700
	0.5	215.9338	124.4806	76.3971	50.3756	36.2754	15.8309
	1	74.1678	46.1241	31.7644	24.0268	19.8801	10.7574
	2	9.8104	8.0598	7.1902	6.3098	5.8110	4.7612
	3	2.7746	2.7345	2.6714	2.6493	2.6024	2.5927
0.9	0	375.6673	209.4956	123.2860	76.9410	52.3737	19.6335
	0.25	324.2957	184.5680	108.7483	69.5827	49.0737	19.0289
	0.5	227.1673	129.0326	80.2110	53.8910	39.0316	16.8751
	1	83.7333	51.3620	34.9312	26.1768	20.8503	11.1038
	2	11.4566	9.2171	7.6204	6.8691	6.2591	4.8492
	3	3.0124	2.8639	2.7493	2.7329	2.6498	2.4797

Table 13

ARL values of the MEWMV control chart when p=5 and correlation=0.0

Weight (value)	**Mean Shift**	**Variance**					
		1	**1.25**	**1.5**	**1.75**	**2**	**3**
0.1	0	373.7841	186.7615	112.3777	74.8685	56.0670	27.7851
	0.25	310.0979	162.6429	99.1466	69.8992	52.1587	26.9911
	0.5	190.1646	113.4853	76.0995	56.7710	43.8846	24.8930
	1	58.9983	45.7265	37.2047	31.6962	27.7420	19.1384
	2	14.7508	14.0115	13.3959	12.8176	12.2833	10.6059
	3	7.2601	7.0970	6.9927	6.8985	6.7738	6.3808
0.3	0	373.6690	213.7529	125.5902	80.9385	57.0562	22.2250
	0.25	321.1424	186.9262	112.8417	74.4459	52.0088	21.5239
	0.5	216.7292	130.0902	81.6198	57.5799	43.1711	19.1506
	1	71.2923	48.5340	35.5963	27.9671	23.1163	13.4207
	2	10.4344	9.3720	8.7152	8.1785	7.6307	6.3943
	3	4.0568	3.9899	3.8844	3.8422	3.8073	3.5745
0.5	0	373.8105	226.6698	139.7917	90.2403	62.8826	23.5921
	0.25	326.2151	201.0761	126.1896	83.4223	56.9206	21.9314
	0.5	235.2258	145.7152	94.3653	63.6617	46.5348	19.7196
	1	87.6487	56.7736	39.9168	30.5982	24.4857	13.3343
	2	11.4471	9.9095	8.7582	7.8901	7.2395	5.8420
	3	3.5746	3.4817	3.4138	3.3550	3.2699	3.0799
0.7	0	367.0265	233.1168	148.7875	97.0935	68.7647	24.7553
	0.25	333.3312	207.8470	133.9688	87.5800	61.7890	23.7843
	0.5	243.8387	155.2402	99.9903	68.5096	50.4574	20.8522
	1	100.7520	65.6894	44.9436	33.7906	26.8164	13.8867
	2	13.7500	11.1211	9.5807	8.4717	7.7451	5.7678
	3	3.6351	3.4571	3.3732	3.2554	3.1586	2.9137
0.9	0	376.1865	241.8192	155.2425	103.5272	71.2654	26.0370
	0.25	337.9930	217.5698	138.6015	96.1774	66.0120	24.4940
	0.5	261.9441	163.5136	106.0955	73.0057	52.5807	21.8063
	1	112.2131	70.1442	49.5638	36.5139	28.6601	14.5651
	2	16.4225	12.9602	10.7240	9.1088	8.1685	6.0633
	3	3.9420	3.6692	3.4879	3.3532	3.2196	2.9555

Table 14

ARL values of the MEWMV control chart when p=10 and correlation=0.0

Weight (value)	Mean Shift	Variance					
		1	1.25	1.5	1.75	2	3
0.1	0	372.3180	224.9191	149.8656	106.8590	82.5067	40.4322
	0.25	324.6600	206.2601	136.6440	99.7664	77.4531	39.5621
	0.5	234.4543	151.5350	108.8000	81.5749	64.5575	36.4730
	1	85.7522	67.4176	54.6645	46.5753	41.2211	27.9184
	2	22.2684	20.8993	19.8817	18.9032	18.1450	15.7039
	3	11.1679	10.9337	10.8065	10.5720	10.3739	9.6545
0.3	0	370.2489	245.1118	169.9419	118.2671	86.9727	34.2204
	0.25	341.1880	226.3406	155.8670	109.5559	78.9247	32.3519
	0.5	252.2400	175.6435	122.2397	85.7643	65.5453	29.5323
	1	103.7257	73.7510	55.9921	44.3484	35.5382	19.9489
	2	16.2348	14.4607	13.1618	12.2069	11.2852	9.0430
	3	5.9269	5.7745	5.6017	5.5317	5.3469	4.9614
0.5	0	365.0418	266.6814	187.2428	129.4161	95.3646	35.6968
	0.25	341.6832	237.6193	167.5169	120.9802	88.6597	34.3376
	0.5	271.2823	189.3152	135.4328	97.5938	71.2594	29.9406
	1	124.8807	88.2400	64.1788	49.5698	38.7028	19.8675
	2	18.5326	16.1235	13.5388	12.0815	11.0676	8.2632
	3	5.3090	5.0787	4.8946	4.6767	4.5333	4.1148
0.7	0	368.5400	274.1937	196.7169	137.8552	103.5930	38.2206
	0.25	348.8134	257.4099	180.4856	130.6729	95.4546	36.3829
	0.5	289.3347	200.7862	144.5396	103.6783	78.9143	31.5076
	1	141.4855	98.6875	71.7971	54.4618	42.6949	20.8603
	2	23.8319	18.6773	15.2831	13.0455	11.7003	8.1520
	3	5.6595	5.2946	4.8479	4.5990	4.4907	3.9171
0.9	0	377.4626	280.1434	202.5275	149.2602	105.8911	40.2413
	0.25	347.7651	257.0848	187.4555	136.1891	103.1255	38.8120
	0.5	287.4860	209.3867	152.3494	113.0242	82.5817	33.3139
	1	156.0867	107.8264	78.9530	58.9709	46.2032	22.2801
	2	28.5107	21.3898	17.4808	14.3913	12.6980	8.5470
	3	6.4155	5.7284	5.1437	4.8434	4.5455	3.9915

Increase of Correlation and Effects on Control Charts

With the development of baseline ARL measurements, the effects of uniform increases in correlation were monitored. The increase in correlation across all related variables in the sample space led to different behaviors involving control chart sensitivity. The MCUSUM control chart showed a decrease in sensitivity with the increase of correlation between variables. The MEWMA control chart showed improved sensitivity with the increased correlation. The MEWMS control chart showed a similar decrease in sensitivity as the MCUSUM with the increased correlations. The MEWMV also showed a decrease in sensitivity with the increase in correlations. Tables 15 through 30 display the ARLs of the control charts with weighting values of 0.5 for the MEWMA, MEWMS and MEWMV control charts. The control charts produced from the use of ω=0.5 showed the greatest sensitivity to mean shifts and variance changes. Additionally, the behavior of the control charts was similar regardless of weighting value used, which is discussed in chapter five.

Each table was designed with an ARL_0 of approximately 370. Sensitivity changes are noticeable with the mean shifts and variance changes for each increase in correlation. This three-way combination of effect has greater effect on the sensitivity of these control charts versus any single effect. This behavior is shown in tables 15 through 30. Tables 15 through 18 are the ARLs of the MCUSUM control chart with increasing values of correlation for each table. For every number of observed variables, the sensitivity decreases with each increase of correlation.

Table 15

ARLs for the MCUSUM control chart with increasing correlation and p=2 variables

		Variance					
Correlation	**Mean Shift**	**1**	**1.25**	**1.5**	**1.75**	**2**	**3**
0	0	366.2611	320.5516	289.9584	257.3709	233.7057	165.7725
	0.25	98.5394	97.5543	93.5057	92.2373	88.4572	78.5781
	0.5	50.9171	50.4433	50.6717	50.4417	49.7234	47.5385
	1	24.9172	25.198	24.9253	25.0939	24.974	24.7801
	2	12.1511	12.3016	12.2607	12.2362	12.351	12.2689
	3	8.0433	8.1557	8.0286	8.0608	8.1082	8.0206
0.1	0	370.6408	331.6639	287.524	261.5893	239.0108	168.1747
	0.25	102.2598	100.2696	98.9147	94.309	91.6656	81.0186
	0.5	53.2159	53.8841	52.4526	51.9316	51.7784	49.5218
	1	26.9124	26.7285	26.9529	26.6514	26.4823	26.0947
	2	13.1902	13.2995	13.3799	13.2911	13.1149	13.1795
	3	8.8455	8.8538	8.8258	8.8744	8.8756	8.7575
0.3	0	361.6423	322.8531	292.6173	259.8879	238.8637	168.4856
	0.25	105.3131	103.0124	99.1086	95.994	92.4187	81.6027
	0.5	58.0932	56.4771	54.8993	55.516	53.4729	50.6177
	1	29.6097	29.4444	29.4866	29.031	28.8421	27.8291
	2	15.0504	14.9312	14.7763	14.8126	14.7975	14.8062
	3	10.0574	10.0949	9.997	9.9371	9.9638	9.9382
0.5	0	362.5619	330.9451	293.6715	263.4451	242.4911	172.9827
	0.25	110.4599	106.7214	101.2805	98.2189	93.2351	82.1249
	0.5	60.5786	60.1554	58.4266	57.0598	55.7513	51.7843
	1	32.2482	32.0032	31.61	31.0124	30.8838	29.7816
	2	16.6223	16.5172	16.3279	16.2609	16.4438	16.0083
	3	11.1097	11.1511	11.2288	11.1502	11.0408	10.8381
0.7	0	371.5742	334.8417	295.1528	258.7696	237.7272	168.2917
	0.25	110.7083	106.6331	102.3035	96.9654	92.1638	81.8605
	0.5	63.0545	61.0558	59.4789	57.9063	56.4184	52.5038
	1	33.5618	33.4491	32.8602	32.3493	31.6641	30.7247
	2	17.5643	17.4996	17.4169	17.2464	17.1984	16.7446
	3	11.9117	11.895	11.7809	11.6382	11.7108	11.5525
0.9	0	369.4245	325.5631	288.2353	251.0138	225.3173	157.8768
	0.25	105.8188	101.2994	96.7791	92.121	87.6596	74.6935
	0.5	61.8528	60.187	58.0068	56.5798	54.8822	50.3161
	1	34.0018	33.1275	32.7686	32.2947	31.5945	29.9355
	2	17.954	17.7986	17.6359	17.7309	17.2147	16.8119
	3	12.1239	12.0525	11.9814	12.1036	11.9758	11.7506

Table 16

ARLs for the MCUSUM control chart with increasing correlation and p=3 variables

		Variance					
Correlation	**Mean Shift**	**1**	**1.25**	**1.5**	**1.75**	**2**	**3**
0	0	369.6917	324.8908	290.712	256.2677	228.7261	148.5249
	0.25	151.4915	142.8724	133.1948	124.0251	117.562	90.7771
	0.5	80.0138	76.4152	75.073	72.5164	67.2851	58.4369
	1	35.2256	34.7989	34.5412	33.8869	32.6565	30.6504
	2	14.4672	14.4545	14.0983	14.21	14.083	13.4473
	3	8.5062	8.4046	8.4508	8.3799	8.3212	8.0018
0.1	0	372.2067	329.7674	297.8565	268.1205	266.0022	165.5906
	0.25	150.8033	144.5865	136.6566	128.5713	130.6541	95.805
	0.5	81.7829	79.8106	77.109	77.9909	75.6948	62.14
	1	38.2867	37.1127	37.3535	38.1993	36.4606	33.725
	2	16.7032	16.516	16.5285	14.3961	16.4354	15.5388
	3	10.1579	10.168	10.051	10.1201	9.9567	9.7152
0.3	0	370.9755	330.443	303.4673	278.2629	280.0901	181.1507
	0.25	150.1411	141.78	132.844	130.8031	127.9757	100.2223
	0.5	83.5448	80.8913	78.7059	81.1798	76.9023	65.064
	1	42.1003	41.5783	40.992	42.5043	40.6172	37.1175
	2	20.2942	19.9499	19.5759	21.4888	19.9656	18.7937
	3	12.9135	12.7906	12.7047	12.6926	12.4817	12.2631
0.5	0	371.7254	333.8596	309.3775	281.2038	279.0021	189.6674
	0.25	147.3291	139.4402	133.1913	131.6526	128.8044	101.5821
	0.5	83.602	81.5229	79.6743	79.6554	77.1638	67.5579
	1	44.2059	43.537	42.9011	44.6615	42.2722	39.0883
	2	22.371	22.0604	21.9782	22.9351	21.6512	21.0544
	3	14.7606	14.5472	14.6559	13.5757	14.6549	14.0603
0.7	0	371.2767	339.2914	307.236	286.7104	281.8765	185.3936
	0.25	138.4789	130.2293	124.5757	124.8061	119.0331	96.24
	0.5	80.9181	79.2598	76.513	76.139	72.4619	63.6743
	1	44.6231	43.4263	42.6457	43.8125	42.0896	38.561
	2	22.7574	22.7188	22.5672	24.1955	22.1313	21.2824
	3	15.25	15.3076	15.264	13.1516	15.0665	14.4811
0.9	0	370.0606	337.9045	301.2031	277.6626	271.1859	170.3369
	0.25	125.0334	115.2248	110.3347	110.9512	105.0544	83.4457
	0.5	72.8005	69.8557	67.7428	70.271	64.9611	56.9759
	1	40.3402	39.0414	37.9806	40.7009	38.2099	34.2366
	2	21.0127	20.9271	20.5492	22.7107	20.7202	19.6574
	3	12.0739	12.2311	12.0088	12.9964	12.9899	12.7816

Table 17

ARLs for the MCUSUM control chart with increasing correlation and p=5 variables

		Variance					
Correlation	**Mean Shift**	**1**	**1.25**	**1.5**	**1.75**	**2**	**3**
0	0	361.326	310.285	249.292	194.6463	149.2442	61.0516
	0.25	235.4156	197.2833	160.0904	124.0325	97.0013	42.714
	0.5	151.4989	125.1787	101.6401	79.5322	64.3735	31.1343
	1	67.9201	54.8344	44.4479	36.2749	30.2415	17.0777
	2	15.9983	13.4294	11.524	10.0524	8.9031	6.5033
	3	4.7164	4.2423	3.9388	3.7014	3.5554	3.1598
0.1	0	365.2628	336.0935	297.4537	259.8186	221.1861	121.0843
	0.25	235.339	213.4887	189.0454	165.0683	145.109	85.8263
	0.5	155.4436	141.769	124.6651	113.7403	97.5378	60.7448
	1	73.3475	66.6424	60.8299	56.0226	50.4232	33.3287
	2	24.2551	22.5424	20.9902	19.4927	17.6963	12.8367
	3	11.2512	10.0358	9.0987	8.3178	7.5138	5.8875
0.3	0	369.9611	346.6095	315.1087	284.3634	260.9624	183.2972
	0.25	226.5861	207.1214	193.8697	180.1815	160.5017	121.6521
	0.5	145.0173	136.2265	126.8379	119.5488	110.2096	87.3653
	1	73.2002	69.1376	65.9051	62.9714	59.5356	51.5567
	2	30.0905	29.2171	28.2	27.9642	27.6551	24.6951
	3	16.9457	16.3313	16.1936	15.9649	15.5225	14.6312
0.5	0	371.2534	337.9254	322.5595	294.684	275.1339	200.7574
	0.25	210.3901	196.3101	178.4367	169.2453	159.2733	126.2775
	0.5	127.0732	122.3719	114.0701	110.4866	104.7184	87.4268
	1	66.4011	64.8973	63.0712	60.59	58.501	52.9791
	2	32.2086	30.9999	30.4664	30.1211	29.2457	27.9791
	3	19.6025	19.5948	19.1755	18.9298	18.5113	18.1506
0.7	0	370.8555	344.3456	326.8422	294.0694	272.9461	200.613
	0.25	182.7727	170.1338	159.0352	151.2265	138.6321	115.1464
	0.5	107.5102	103.0045	99.6227	94.6135	91.1236	78.3533
	1	58.9361	57.2881	55.2125	55.3812	53.4056	49.2246
	2	30.2575	29.9243	29.1687	28.9067	28.7211	27.0418
	3	19.9246	19.6755	19.2385	19.1548	18.8779	18.2778
0.9	0	369.7126	353.7594	318.5612	290.7122	261.2403	180.8849
	0.25	139.876	133.7531	124.0729	121.8569	113.6485	93.9723
	0.5	84.5287	81.0673	77.7461	75.5475	72.7576	65.6819
	1	47.0144	45.5002	44.6984	43.752	42.9148	39.9148
	2	24.9894	24.0836	24.0436	23.8911	23.6027	23.0752
	3	16.5533	16.6118	16.4364	16.428	16.352	15.7213

Table 18

ARLs for the MCUSUM control chart with increasing correlation and p=10 variables

		Variance					
Correlation	**Mean Shift**	**1**	**1.25**	**1.5**	**1.75**	**2**	**3**
0	0	376.9464	295.8588	210.7881	149.0509	105.6349	40.0116
	0.25	314.3378	221.8322	148.5177	100.4653	70.9211	29.4795
	0.5	250.379	155.2152	97.6987	66.8245	49.3857	22.0815
	1	107.0693	62.1005	40.1165	29.7457	23.4853	12.6869
	2	13.7275	10.2749	8.4931	7.4005	6.5915	4.9539
	3	3.1088	2.9417	2.8515	2.7615	2.6776	2.5186
0.1	0	371.2464	315.2653	238.2984	172.5622	125.4237	48.0829
	0.25	314.6549	239.635	168.7383	118.8926	88.9828	35.7823
	0.5	246.5102	168.327	116.4451	80.3937	59.5879	26.2617
	1	115.8659	72.9521	49.8239	35.9166	28.1264	14.694
	2	17.4808	12.7965	10.4087	8.8681	7.7967	5.6975
	3	3.6937	3.5	3.2882	3.193	3.0356	2.7682
0.3	0	373.1786	346.2363	320.2697	294.978	263.7432	162.7574
	0.25	302.7846	276.7998	253.2137	224.7495	197.6663	120.6963
	0.5	244.28	214.8527	192.7138	167.0369	148.0158	92.0329
	1	138.2853	119.362	109.8242	97.2196	85.8601	54.4778
	2	46.4429	43.0122	39.2286	36.1239	31.5564	20.8203
	3	22.0371	19.3022	16.4605	14.4064	12.5838	8.8847
0.5	0	377.9741	358.1375	337.7441	314.9554	291.7876	214.2479
	0.25	290.463	269.4841	246.7271	229.4624	206.9162	155.5769
	0.5	212.5603	193.9191	176.7535	161.6062	148.3471	113.6289
	1	113.9964	105.6134	96.434	91.0443	87.7328	72.2721
	2	48.0261	45.751	44.8901	42.421	40.7139	36.0343
	3	27.2047	25.9055	24.8293	24.8803	23.8173	21.6943
0.7	0	367.4963	356.7046	335.6037	310.9566	290.893	215.614
	0.25	247.8557	223.6985	204.695	192.0828	179.6743	139.6234
	0.5	152.7583	139.2566	136.1285	126.8567	122.8593	101.1391
	1	82.8204	78.0286	76.1446	73.1483	70.2681	63.2428
	2	40.1401	39.192	38.8056	37.8019	37.2012	34.9828
	3	25.5964	24.9265	24.7342	24.315	23.926	22.957
0.9	0	369.9768	351.5196	330.3314	306.2211	276.3561	198.4335
	0.25	167.7415	155.6169	148.4614	137.9955	129.2741	107.4423
	0.5	101.2394	96.1189	91.4218	88.3395	84.7138	73.8242
	1	55.5709	54.2509	52.5236	51.5002	49.7001	46.5667
	2	28.9297	28.807	28.4578	27.7936	27.7643	26.4087
	3	19.5219	19.3616	19.0095	18.9348	18.7543	18.0705

Tables 19 through 30 display the ARLs for the MEWMA, MEWMS and MEWMV control charts with increasing correlation values. The tables displayed used a weighting value of 0.5 as this value appeared to give the most sensitive results. Full tables for all weighting values (ω={0.1, 0.3, 0.5, 0.7, 0.9}) are provided in Appendix B. Tables 19 through 22 show the ARL results for the MEWMA control chart for p=2 through p=10 variables. As stated earlier, the MEWMA control chart displayed a behavior different from that of the other control charts in that the sensitivity seemed to improve with the increase in correlation.

Table 19

ARLs for the MEWMA control chart with increasing correlation, ω=0.5 and p=2 variables

		Variance					
Correlation	**Mean Shift**	**1**	**1.25**	**1.5**	**1.75**	**2**	**3**
0	0	370.3705	368.1954	372.6909	373.1647	375.9958	366.3845
	0.25	240.4617	257.4602	272.7399	282.1475	291.0797	316.1613
	0.5	100.9875	122.0901	140.8612	159.1507	170.7113	215.3747
	1	21.3927	28.81	35.1956	42.9026	50.2943	77.2277
	2	3.8469	4.8897	6.0293	7.1516	8.5793	14.446
	3	1.8434	2.225	2.6036	2.9886	3.3739	5.2511
0.1	0	366.0201	372.3836	367.143	368.3464	372.556	369.8885
	0.25	236.8546	257.6873	275.4198	279.9168	296.5198	319.9509
	0.5	101.5824	120.9311	138.2788	156.5157	166.2894	211.524
	1	20.6535	27.7894	35.3913	42.6683	48.8967	75.9564
	2	3.7965	4.8722	5.9465	7.1	8.5081	14.3483
	3	1.8334	2.1814	2.5672	2.9513	3.4042	5.2175
0.3	0	367.6524	367.1812	371.8443	363.8221	367.1717	367.3403
	0.25	230.5226	254.4031	269.8788	275.9629	288.0467	309.0569
	0.5	93.9865	112.464	131.0977	145.972	161.1711	205.4468
	1	18.6875	25.0225	31.5189	38.4963	45.1691	71.8561
	2	3.4685	4.4047	5.3629	6.4671	7.5104	12.7392
	3	1.7148	2.0537	2.3859	2.75	3.117	4.732
0.5	0	377.8955	368.2749	370.9835	375.1042	373.534	372.28
	0.25	214.6002	236.1464	250.7139	262.793	274.5344	302.7118
	0.5	76.9975	95.4088	112.6861	127.7196	140.1174	187.8056
	1	14.6921	19.1801	24.5792	29.2272	35.1599	57.0595
	2	2.9162	3.6069	4.3181	5.1948	5.9913	10.0106
	3	1.5075	1.7589	2.0382	2.3031	2.5943	3.8427
0.7	0	367.2053	367.3395	372.6484	372.203	367.1948	372.074
	0.25	171.0971	193.2082	214.9245	226.4495	241.0809	275.0865
	0.5	50.5789	66.3023	77.9126	90.6472	103.0309	142.6108
	1	8.8391	11.6238	14.723	17.9046	21.8924	36.3271
	2	2.0671	2.4868	2.9292	3.4343	3.9171	6.1716
	3	1.1834	1.3579	1.5128	1.6985	1.8719	2.6495
0.9	0	373.9244	368.7999	368.3512	365.613	366.596	371.8752
	0.25	78.3645	97.199	113.9905	128.0798	143.5676	185.7698
	0.5	14.4863	19.6504	24.8159	30.5891	36.2644	57.7705
	1	2.9274	3.6292	4.3687	5.2651	6.1282	10.2955
	2	1.0966	1.2147	1.341	1.4896	1.6516	2.2647
	3	1.0004	1.0022	1.0105	1.0297	1.059	1.2568

Table 20

ARLs for the MEWMA control chart with increasing correlation, ω=0.5 n and p=3 variables

		Variance					
Correlation	**Mean Shift**	**1**	**1.25**	**1.5**	**1.75**	**2**	**3**
0	0	369.2408	374.1736	367.6642	372.2041	370.2731	372.9387
	0.25	258.3383	277.4758	298.2343	298.5209	310.0356	325.8661
	0.5	124.5923	146.5314	164.8319	183.867	199.3293	231.4324
	1	26.0712	35.0094	45.161	53.8657	61.7744	94.2752
	2	4.4018	5.6888	6.9897	8.5822	10.2193	17.8739
	3	2.0186	2.4553	2.8747	3.3567	3.8923	6.0919
0.1	0	369.6629	369.1169	370.8372	368.0195	365.5759	372.4248
	0.25	259.7028	280.1823	285.1929	300.1037	308.4467	326.1224
	0.5	119.4712	143.3317	164.4587	182.4862	195.6867	231.864
	1	25.0486	35.3579	43.1422	52.1877	61.6271	94.1413
	2	4.3446	5.5221	6.9162	8.4321	10.168	17.2586
	3	1.98	2.4169	2.8382	3.3159	3.7961	5.9939
0.3	0	368.3021	362.6002	369.9614	372.2596	368.3341	361.1685
	0.25	249.6221	264.7677	281.4484	291.337	301.3688	320.571
	0.5	108.5373	128.7948	147.4433	164.372	179.3022	224.0569
	1	21.1841	29.1254	36.2302	45.3128	53.0439	81.2905
	2	3.7344	4.7927	5.9087	7.136	8.4986	14.5049
	3	1.7903	2.1417	2.5071	2.871	3.3079	5.0958
0.5	0	365.6929	368.6659	371.7813	367.8308	373.8584	371.2315
	0.25	225.7406	248.0624	260.2715	273.4646	280.687	300.1277
	0.5	84.9419	103.9637	123.4149	136.2605	150.5341	194.8505
	1	15.2878	20.6207	26.2234	32.3572	38.8443	63.0576
	2	2.8773	3.6088	4.4127	5.2442	5.9998	1.9948
	3	1.4844	1.7452	2.0198	2.3008	2.6042	3.8222
0.7	0	369.4614	367.3676	362.6699	373.1111	362.737	364.6205
	0.25	177.2214	203.607	217.1052	235.0715	250.9929	274.3538
	0.5	51.2048	66.6444	80.7941	94.4032	106.1527	146.3028
	1	8.3949	10.961	14.2581	17.5357	21.1965	35.8234
	2	1.9435	2.313	2.7683	3.1836	3.6576	5.7482
	3	1.1416	1.2795	1.4357	1.6032	1.7652	2.4808
0.9	0	359.2019	369.0535	362.7214	377.5268	367.6526	370.5884
	0.25	73.6006	93.3264	109.1702	126.9154	138.8489	183.5669
	0.5	12.7234	17.2675	22.3849	27.6523	33.4155	54.4393
	1	2.5895	3.1711	3.8714	4.5596	5.2855	8.672
	2	1.043	1.128	1.229	1.3566	1.4824	2.0067
	3	1	1	1.0027	1.0076	1.0211	1.1515

Table 21

ARLs for the MEWMA control chart with increasing correlation, ω=0.5 and p=5 variables

		Variance					
Correlation	**Mean Shift**	**1**	**1.25**	**1.5**	**1.75**	**2**	**3**
0	0	372.7452	374.932	371.4578	375.9833	373.6538	373.0239
	0.25	283.7408	300.9425	317.6082	322.1854	324.3753	344.603
	0.5	155.2012	178.7623	196.0513	216.6122	231.2815	265.297
	1	35.6406	48.8357	60.7518	72.1889	83.9021	125.6059
	2	5.3065	6.9446	8.8621	11.2288	13.548	23.6023
	3	2.3094	2.826	3.3748	3.9459	4.636	7.5049
0.1	0	373.4918	369.9866	375.2187	373.8644	376.1303	376.4284
	0.25	286.3046	301.0284	306.0954	325.6194	326.8017	338.2255
	0.5	150.0598	175.6409	198.7678	212.9742	224.3799	264.1613
	1	34.1453	45.9567	58.4588	71.2538	81.8673	119.8178
	2	5.0814	6.7163	8.6938	10.7617	12.7737	22.9658
	3	2.2207	2.7218	3.2641	3.8409	4.4263	7.3713
0.3	0	370.947	369.3322	378.0759	377.6926	378.2022	373.6193
	0.25	273.1516	286.0911	300.5725	315.8136	320.4879	329.8599
	0.5	135.8968	155.3277	175.2971	190.8775	204.8444	246.4025
	1	26.9501	36.105	47.2453	56.7203	67.6967	103.4774
	2	4.1934	5.4122	6.7476	8.4007	9.992	17.7195
	3	1.9183	2.3222	2.7428	3.1858	3.6421	5.8009
0.5	0	369.4439	368.6081	373.1211	368.7574	373.1793	369.5423
	0.25	245.3166	264.9879	283.784	285.7841	304.9509	319.6945
	0.5	100.654	121.9166	142.6547	163.1751	176.8408	218.3726
	1	17.4083	24.1273	31.1256	39.0672	45.7386	75.2321
	2	3.0205	3.8397	4.6605	5.602	6.7168	11.6546
	3	1.5298	1.8109	2.098	2.4185	2.7185	4.0632
0.7	0	371.2309	374.1017	373.9573	371.8823	370.9117	372.344
	0.25	193.524	216.8243	236.409	255.5637	265.2085	287.1369
	0.5	59.134	76.2683	92.6084	110.3863	121.2932	165.1201
	1	8.9642	11.9382	15.4002	19.3599	23.5514	41.73
	2	1.9466	2.373	2.8128	3.2717	3.7833	5.9497
	3	1.1323	1.2777	1.4441	1.6101	1.7613	2.5081
0.9	0	381.5163	371.2156	373.6441	372.5822	379.1563	371.799
	0.25	82.2963	106.4128	122.0148	139.361	154.9683	201.6351
	0.5	13.2305	18.7497	23.9881	30.594	35.9657	59.7639
	1	2.5339	3.1503	3.8118	4.4992	5.2731	8.9241
	2	1.0341	1.1082	1.2152	1.3285	1.4536	1.9829
	3	1	1.0001	1.0018	1.0048	1.0135	1.1346

Table 22

ARLs for the MEWMA control chart with increasing correlation, ω=0.5 and p=10 variables

		Variance					
Correlation	**Mean Shift**	**1**	**1.25**	**1.5**	**1.75**	**2**	**3**
0	0	374.669	372.857	369.8187	368.7668	358.8111	371.1531
	0.25	313.7681	321.3419	331.0157	312.7036	341.938	349.5404
	0.5	200.3871	221.445	247.6468	254.8274	270.7206	297.4791
	1	55.9614	75.241	90.1202	106.8804	122.9827	164.0532
	2	7.3105	10.2051	13.3195	16.8879	20.5919	37.7843
	3	2.8257	3.6124	4.3912	5.2807	6.2557	11.2148
0.1	0	367.4767	375.4327	371.5565	370.8611	369.7443	363.7982
	0.25	307.1755	322.5432	330.6231	311.6893	333.4984	352.0829
	0.5	196.2034	216.0002	234.4162	250.5324	259.6175	287.6359
	1	52.7092	69.4323	86.4084	100.3793	117.1269	160.2979
	2	6.8003	9.359	12.2364	15.8169	18.7219	35.9932
	3	2.6748	3.3582	4.1111	4.9086	5.863	10.2108
0.3	0	368.3827	368.2622	371.8989	379.1253	365.0764	372.57
	0.25	299.9163	316.9391	327.2896	293.4591	329.8033	344.9743
	0.5	169.6831	192.448	215.6814	229.3702	244.1226	279.8167
	1	38.8789	53.6226	67.6599	79.4605	92.9438	136.5997
	2	5.1527	6.8585	8.7476	11.1145	13.7978	25.3137
	3	2.1888	2.6709	3.2589	3.8149	4.4849	7.4996
0.5	0	370.3542	370.5187	371.4888	375.0134	371.5573	371.8142
	0.25	270.2669	286.3665	298.2041	277.1015	315.7736	336.5209
	0.5	131.5032	157.033	174.2993	196.2066	209.9583	250.9278
	1	24.0025	34.1246	43.7567	53.594	64.0698	101.4844
	2	3.5549	4.5093	5.6865	6.9642	8.5367	15.5713
	3	1.6883	2.0235	2.3495	2.7468	3.1162	4.9049
0.7	0	371.1128	373.1457	374.1952	369.4183	367.1678	373.4935
	0.25	235.643	251.0421	263.9611	280.6967	292.6872	313.1594
	0.5	79.8523	45.9655	57.3914	69.4907	81.541	124.0144
	1	11.1228	15.5301	20.7209	25.8965	32.271	56.0646
	2	2.1567	2.6371	3.1772	3.7521	4.3835	7.3669
	3	1.1937	1.3685	1.5558	1.769	1.9512	2.8153
0.9	0	367.5997	366.996	369.2054	369.9712	379.3377	373.2089
	0.25	109.8859	133.3407	154.9891	170.9377	187.163	228.1023
	0.5	17.3512	24.6035	32.4631	40.0897	48.8584	79.5563
	1	2.8393	3.5657	4.3369	5.2542	6.2475	11.2803
	2	1.051	1.1547	1.277	1.4164	1.5709	2.1774
	3	1	1.0002	1.0012	1.0088	1.0226	1.1899

Tables 23 through 26 show the ARL results for the MEWMS control chart for *p*=2 through *p*=10 variables using, ω=0.5 as the weighting value. While the MEWMS control chart was developed to monitor the covariance matrix using individual observations, the number of variables was limited in the study by Huwang et al.(2007). Table comparisons to the research of Huwang et al.will follow in chapter five.

Table 23

ARLs for the MEWMS control chart with increasing correlation, ω=0.5 and p=2 variables

		Variance					
Correlation	**Mean Shift**	**1**	**1.25**	**1.5**	**1.75**	**2**	**3**
0	0	369.5183	169.0379	90.1176	54.7769	38.0964	15.8215
	0.25	309.1548	142.5277	79.2825	48.9594	35.283	14.8731
	0.5	182.2021	95.1728	57.0516	38.6122	28.6925	13.7662
	1	52.1277	33.2141	24.5332	19.1904	15.831	10.0629
	2	7.8127	7.0203	6.5425	6.2156	5.9035	5.1337
	3	4.4164	4.3204	4.2364	4.1982	4.0936	3.913
0.1	0	372.7511	175.4963	90.5015	56.8074	39.5208	15.9636
	0.25	307.8184	142.2708	81.054	51.0138	35.8999	15.4007
	0.5	188.8115	96.0216	58.8298	39.6734	29.3107	13.8523
	1	53.5473	34.747	25.0117	19.6774	16.3174	10.1441
	2	7.8185	7.2274	6.6931	6.3475	5.9766	5.2007
	3	3.4341	3.377	3.3864	3.3594	3.3019	3.2042
0.3	0	367.6517	180.9171	100.6855	63.5524	43.1598	17.4824
	0.25	314.6131	155.7582	90.08	57.2893	40.0226	16.7541
	0.5	204.8817	109.1722	64.8959	44.4016	32.5739	15.3904
	1	63.3563	40.0743	28.9851	22.5617	18.5043	11.0646
	2	8.9736	8.072	7.3564	6.8729	6.5002	5.5239
	3	3.6493	3.5844	3.5717	3.5132	3.4741	3.3519
0.5	0	371.4704	191.5056	111.7437	71.969	50.9426	20.8615
	0.25	327.1077	171.6751	100.9043	67.229	47.8034	19.6346
	0.5	225.925	124.6845	78.0677	53.0832	38.715	17.9535
	1	81.0638	50.5848	35.7784	27.1815	22.1638	12.863
	2	11.0498	9.7405	8.6924	8.0576	7.5416	6.1219
	3	4.0167	3.9725	3.906	3.8683	3.8368	3.6459
0.7	0	366.7071	196.9598	120.4681	79.4884	56.9342	23.2912
	0.25	325.5585	180.7525	110.3535	73.4048	53.0743	22.5376
	0.5	242.761	135.8119	86.3736	60.3472	44.9439	20.1375
	1	95.3846	60.0771	42.6421	32.2124	25.8399	14.7856
	2	13.9013	11.8472	10.279	9.4123	8.6963	6.9581
	3	4.5604	4.4512	4.3346	4.2656	4.2201	4.0065
0.9	0	371.7128	202.5594	125.4032	85.0299	61.3042	25.677
	0.25	333.8517	185.7399	113.2421	78.7231	57.2282	24.7019
	0.5	256.4009	144.8204	92.4401	66.6515	49.671	22.6018
	1	105.3247	68.5429	49.4335	37.7427	29.9099	16.8085
	2	17.1685	14.2913	12.3545	10.919	10.0452	7.8392
	3	5.217	5.041	4.9005	4.7772	4.6913	4.3693

Table 24

ARLs for the MEWMS control chart with increasing correlation, ω=0.5 and p=3 variables

		Variance					
Correlation	**Mean Shift**	**1**	**1.25**	**1.5**	**1.75**	**2**	**3**
0	0	376.8332	192.231	109.8445	69.394	46.9619	18.6894
	0.25	321.3747	167.222	98.4888	63.1351	43.6824	17.7735
	0.5	211.5318	114.3692	72.0542	48.9173	35.1427	16.2402
	1	65.2887	43.1624	30.3203	23.6803	19.3676	11.7319
	2	9.4892	8.4595	7.7046	7.1629	6.8784	5.7346
	3	3.8153	3.7546	3.7312	3.6886	3.6477	3.4848
0.1	0	376.7368	198.3863	114.321	71.9633	49.0907	19.3431
	0.25	317.9929	173.1141	99.3278	66.1699	45.5712	18.5883
	0.5	219.7735	120.5797	74.2445	49.8001	37.0254	16.6628
	1	69.0488	44.2981	31.8122	24.7886	20.2762	11.8009
	2	9.6724	8.6471	7.9125	7.4182	6.9245	5.9194
	3	3.8655	3.8538	3.8036	3.711	3.6826	3.5689
0.3	0	368.5131	217.3694	132.1676	86.1417	61.577	23.621
	0.25	330.5006	192.9148	119.2155	80.4872	56.0784	22.8964
	0.5	245.7052	147.4915	92.4907	63.6404	46.4746	20.4024
	1	95.3086	60.0778	42.6221	33.1056	26.091	14.5467
	2	13.0786	11.1621	9.8059	8.9648	8.3622	6.8431
	3	4.4585	4.3491	4.2478	4.1771	4.1263	3.9261
0.5	0	377.319	231.0414	149.8601	105.7195	76.529	31.4095
	0.25	341.0302	214.8665	141.1888	98.0426	72.2684	29.6144
	0.5	273.6777	169.5693	114.1094	80.6443	59.6588	26.5957
	1	127.1596	83.4057	59.582	44.6203	35.2483	19.1458
	2	19.9371	15.9227	13.876	12.2883	11.232	8.3837
	3	5.5462	5.3977	5.2224	5.0032	4.9397	4.5751
0.7	0	365.3635	237.6876	157.3152	114.1176	86.1916	37.5646
	0.25	337.4969	223.8548	149.7014	111.0676	81.3503	35.1495
	0.5	287.339	186.016	128.9759	94.6464	71.4435	32.8445
	1	149.55	101.839	74.0225	57.9215	45.1498	23.5275
	2	29.08	22.7921	19.117	16.3695	14.6864	10.4595
	3	7.2285	6.8201	6.5605	6.2277	6.0452	5.4242
0.9	0	369.5935	247.074	173.5036	124.8069	94.4795	43.0326
	0.25	353.1303	233.1118	164.195	120.139	92.324	41.7571
	0.5	301.4451	198.424	141.8704	103.7437	80.4807	38.4231
	1	173.991	119.1165	88.3299	66.1109	54.2044	28.2394
	2	38.4362	30.2181	24.8877	21.0006	18.324	12.9918
	3	9.7622	9.0039	8.3165	7.8336	7.4716	6.4133

Table 25

ARLs for the MEWMS control chart with increasing correlation, ω=0.5 and p=5 variables

		Variance					
Correlation	**Mean Shift**	**1**	**1.25**	**1.5**	**1.75**	**2**	**3**
0	0	371.1002	226.737	140.5337	90.6757	63.7366	24.3496
	0.25	329.8448	199.7232	126.4343	83.5608	59.137	23.4533
	0.5	238.3668	144.7926	93.2142	63.9769	46.7825	20.591
	1	87.6877	58.4909	41.3385	31.2929	25.5514	14.6242
	2	12.3761	10.8223	9.8146	8.9834	8.3983	6.7655
	3	4.5158	4.4139	4.3432	4.2695	4.2318	4.0109
0.01	0	368.341	235.4527	147.0522	97.1628	68.642	26.0689
	0.25	332.5651	209.5149	133.2406	87.7624	64.4377	25.227
	0.5	251.8943	154.6374	101.2675	70.7276	50.8744	22.2797
	1	95.988	64.7595	46.4312	34.5123	27.9878	15.4155
	2	13.6645	11.7844	10.3563	9.5314	8.8792	7.1516
	3	4.6489	4.545	4.5207	4.3952	4.3767	4.1181
0.3	0	365.7436	260.0636	183.2974	135.0901	100.9504	39.694
	0.25	343.5016	248.7772	175.2484	125.2315	95.0497	38.0949
	0.5	293.0441	202.1931	145.4314	107.7815	78.7987	33.6189
	1	153.9947	107.1775	76.1654	59.1294	45.4696	23.1729
	2	25.6045	20.3885	17.4694	14.9846	13.3595	9.7878
	3	6.5547	6.1607	5.9197	5.8001	5.6247	5.1193
0.5	0	376.3799	277.0673	206.9143	155.9218	124.1317	56.3281
	0.25	352.6281	264.5478	201.5675	149.9534	121.1618	54.4051
	0.5	317.2664	232.7684	172.3681	132.88	105.4208	49.1528
	1	197.083	145.8587	110.1634	87.9746	70.4263	35.3081
	2	49.2185	37.6247	31.2286	26.0618	22.4987	14.8513
	3	11.2469	9.9369	9.3641	8.7481	8.1209	6.9661
0.7	0	364.7485	283.0042	216.837	174.111	138.6715	68.1184
	0.25	361.7663	268.6965	208.1641	166.4854	135.5001	65.799
	0.5	322.1529	244.6224	192.7435	149.2818	121.6228	60.9391
	1	229.4345	171.5312	134.5478	107.7895	89.4841	47.7485
	2	71.7425	57.0857	46.3637	38.7383	33.6328	21.0446
	3	18.6734	16.617	14.651	13.0041	12.1063	9.6328
0.9	0	375.5708	284.2587	219.5361	178.0618	148.541	75.8836
	0.25	351.9447	276.1691	218.9725	174.9887	142.3103	75.3312
	0.5	330.2397	253.8401	202.61	162.7108	132.613	70.5146
	1	249.6394	191.9688	153.0703	121.6057	102.8083	56.6293
	2	94.0997	73.6869	61.4124	50.9076	43.422	28.0958
	3	27.957	23.7282	20.8367	18.7784	16.9901	12.9652

Table 26

ARLs for the MEWMS control chart with increasing correlation, ω=0.5 and p=10 variables

		Variance					
Correlation	**Mean Shift**	**1**	**1.25**	**1.5**	**1.75**	**2**	**3**
0	0	373.3395	267.9068	189.0536	132.7437	95.1805	37.1473
	0.25	348.5448	239.0426	172.0731	122.2303	89.3182	35.2941
	0.5	268.7968	191.4741	136.3422	99.0537	72.7381	30.9782
	1	124.7393	90.0521	64.6082	50.2742	39.6037	20.9747
	2	19.9838	16.565	14.4266	13.0143	11.8762	9.0534
	3	6.1031	5.8688	5.7281	5.5342	5.3909	5.0313
0.1	0	367.4271	280.7765	203.7198	149.7987	110.8858	44.3121
	0.25	353.3708	259.7294	188.8917	139.0342	104.296	41.8387
	0.5	295.8196	211.8068	155.4202	114.2367	86.2183	36.7649
	1	152.3766	108.5581	80.4947	61.8508	48.5662	24.4523
	2	25.1299	20.4506	17.5113	15.6116	13.7774	10.1951
	3	6.8709	6.5468	6.3141	6.0611	5.8735	5.398
0.3	0	370.2578	308.8629	252.3407	211.9074	172.8154	88.2209
	0.25	355.6593	301.5712	244.4578	206.0196	170.312	85.9147
	0.5	329.1472	273.7704	225.353	186.1404	154.5647	78.9418
	1	237.0305	192.5018	161.7138	130.3021	108.4318	56.5574
	2	76.5366	62.0345	50.4243	42.2097	36.1072	22.3993
	3	18.5795	15.888	14.0743	13.0692	11.9456	9.4049
0.5	0	375.9985	321.7518	274.8969	203.9534	202.773	120.5263
	0.25	366.5659	310.0225	269.2821	197.6112	198.7301	118.3066
	0.5	345.3114	292.0373	253.6813	190.3004	188.7827	110.4625
	1	279.8696	239.6935	203.4081	149.7437	150.6864	89.2533
	2	130.3914	109.4061	91.1528	71.5647	70.2294	42.8937
	3	45.0402	38.4931	33.0569	27.8607	26.3674	18.3752
0.7	0	366.2151	326.1645	282.0074	249.0506	217.5948	138.2499
	0.25	359.3871	321.4199	274.5415	243.64	214.7653	136.4802
	0.5	348.6328	309.2534	267.2678	234.3195	203.8242	131.4415
	1	301.1515	259.1629	228.5156	201.2071	174.6791	111.6526
	2	173.5865	146.9378	127.8032	110.9725	97.5168	62.7425
	3	73.4665	63.8969	55.5368	49.1209	43.3537	30.605
0.9	0	369.9474	320.3662	283.5195	249.124	219.1355	147.8451
	0.25	366.2403	312.2588	279.0917	248.4808	218.386	143.2353
	0.5	347.5633	303.7155	270.6761	243.1378	215.1485	139.4828
	1	306.4893	279.7466	236.467	211.6683	187.3844	125.522
	2	197.7279	169.0563	148.0542	131.4783	119.1652	79.0025
	3	99.4234	85.1908	75.198	65.9519	59.6777	42.83

Tables 27 through 30 show the ARLs for the MEWMV control chart when the weighting value is 0.5 and the correlation of the related variables uniformly increases. With these increases, the sensitivity of the MEWMV control chart decreases. As the number of variables (*p)* increased, the sensitivity decreased. The behavior of the MEWMV is similar to the MEWMS control chart and can be compared with tables 24 through 26.

Table 27

ARLs for the MEWMV control chart with increasing correlation, ω=0.5 and p=2 variables

		Variance					
Correlation	**Mean Shift**	**1**	**1.25**	**1.5**	**1.75**	**2**	**3**
0	0	370.4231	168.3056	94.3616	59.6903	39.213	15.274
	0.25	306.4156	150.2079	82.7974	52.4466	36.8731	14.4861
	0.5	193.7813	101.2005	59.3369	40.6626	29.5954	13.1588
	1	58.646	36.3759	25.2134	19.5469	15.6036	8.9045
	2	7.5734	6.5756	5.8569	5.3383	5.0072	4.1076
	3	2.3598	2.3388	2.3461	2.3166	2.2812	2.2563
0.1	0	372.05	168.2515	92.2001	54.5218	37.066	14.608
	0.25	305.9812	143.4321	78.1298	49.7369	34.192	13.9048
	0.5	186.2651	95.5964	57.1728	38.4967	27.8644	12.7746
	1	52.6743	32.7566	23.7568	18.0501	14.7747	8.6683
	2	6.7133	6.0822	5.5485	5.1416	4.8679	4.1631
	3	2.4379	2.4009	2.3815	2.389	2.3487	2.3696
0.3	0	371.249	180.8555	98.9625	59.2089	42.0788	16.2229
	0.25	314.1935	153.934	87.5457	55.6364	39.1401	15.6975
	0.5	204.1971	106.5039	64.1625	43.5312	31.1234	13.9106
	1	62.0806	39.205	27.7583	20.8058	17.059	9.7318
	2	7.7873	6.8211	6.2071	5.7194	5.3728	4.4275
	3	2.5905	2.58	2.5534	2.519	2.5345	2.463
0.5	0	364.9711	188.825	109.7278	70.2184	49.4487	19.0149
	0.25	323.9093	166.9922	97.3308	64.1033	44.7973	18.0637
	0.5	227.0175	120.9063	75.7344	50.8956	37.4225	16.1789
	1	79.0661	49.1046	33.959	25.4802	20.4211	11.4762
	2	9.897	8.3899	7.4514	6.695	6.1447	5.0773
	3	2.9424	2.8779	2.827	2.84	2.7813	2.6909
0.7	0	368.5045	196.2104	117.9044	76.8631	56.0422	21.7797
	0.25	328.1747	176.1627	109.6234	73.0808	50.9289	21.1102
	0.5	237.66	135.2891	83.8052	57.9497	43.3185	18.9677
	1	92.3766	59.4885	40.8978	30.9538	24.5065	13.3357
	2	12.5682	10.363	9.1013	8.1335	7.3469	5.7207
	3	3.4283	3.3575	3.2742	3.1815	3.1401	2.9348
0.9	0	367.1552	198.2293	121.6069	81.8893	59.9007	24.3346
	0.25	325.9063	182.33	114.6612	78.2185	56.1006	23.3793
	0.5	250.0991	143.0079	92.5166	64.3619	47.6305	21.207
	1	104.5493	67.2628	46.8969	35.4035	28.3473	14.923
	2	16.1132	13.0308	11.1981	9.7687	8.8611	6.6875
	3	4.0759	3.9183	3.796	3.6595	3.589	3.303

Table 28

ARLs for the MEWMV control chart with increasing correlation, ω=0.5 and p=3 variables

		Variance					
Correlation	**Mean Shift**	**1**	**1.25**	**1.5**	**1.75**	**2**	**3**
0	0	371.8887	192.0747	109.1977	68.3405	46.13	17.6054
	0.25	313.1364	166.0847	94.5839	60.9473	42.5187	16.8888
	0.5	209.4093	113.6002	69.2799	46.7282	34.038	15.0403
	1	64.9001	40.6251	29.073	22.5165	18.0996	10.1693
	2	8.227	7.229	6.5864	6.1047	5.6974	4.7184
	3	2.8144	2.7677	2.769	2.7055	2.6716	2.5603
0.1	0	376.4452	199.4878	113.5183	71.1372	47.8313	17.9788
	0.25	319.5334	172.0839	98.8684	62.4821	43.1886	17.323
	0.5	216.5401	119.4023	71.7616	49.1162	35.7477	15.3045
	1	68.3496	43.3426	30.9612	23.532	18.8288	10.682
	2	8.7115	7.5124	6.7818	6.2996	5.888	4.8037
	3	2.8798	2.8622	2.8089	2.7609	2.7182	2.6176
0.3	0	373.642	220.1916	131.868	87.5895	61.4702	22.4104
	0.25	333.841	195.5298	121.9321	79.8189	56.359	21.9752
	0.5	244.9305	146.4242	92.9571	61.925	45.9337	19.3629
	1	95.1208	61.2626	41.6807	31.3624	24.7922	13.1129
	2	11.7276	10.006	8.809	7.8354	7.1134	5.6297
	3	3.427	3.308	3.247	3.185	3.1328	2.9638
0.5	0	372.6467	231.2948	150.397	102.4983	74.7136	29.3008
	0.25	338.4733	213.0628	139.2634	96.505	71.4495	28.0721
	0.5	267.0448	168.2999	113.9364	79.1457	58.8662	24.9746
	1	126.3012	82.229	57.8451	43.7583	33.7924	17.5266
	2	18.6632	15.044	12.528	11.0458	9.8517	7.204
	3	4.4763	4.3005	4.0866	3.9457	3.8082	3.4911
0.7	0	363.5554	235.0278	162.3712	115.2852	85.2361	35.9804
	0.25	343.1779	223.5756	149.6113	109.984	80.3344	34.3652
	0.5	284.1284	183.4117	126.0253	93.0177	69.8512	30.6978
	1	152.5428	100.8867	72.8431	54.6947	43.1399	22.2817
	2	27.4731	21.709	17.7086	15.0923	13.2269	9.2558
	3	6.2372	5.7147	5.3809	5.104	4.9241	4.2852
0.9	0	369.4852	244.4491	167.1854	123.1293	93.3247	41.2264
	0.25	345.5391	228.9969	159.956	116.6622	88.2259	38.6627
	0.5	299.4138	196.2103	138.0549	102.1653	77.8239	36.5865
	1	174.7867	114.8473	85.8192	65.0027	52.6099	26.3793
	2	36.6492	28.4668	23.5324	19.7444	17.2135	11.5921
	3	8.5349	7.6544	7.1219	6.5484	6.2798	5.3162

Table 29

ARLs for the MEWMV control chart with increasing correlation, ω=0.5 and p=5 variables

		Variance					
Correlation	**Mean Shift**	**1**	**1.25**	**1.5**	**1.75**	**2**	**3**
0	0	373.8105	226.6698	139.7917	90.2403	62.8826	23.5921
	0.25	326.2151	201.0761	126.1896	83.4223	56.9206	21.9314
	0.5	235.2258	145.7152	94.3653	63.6617	46.5348	19.7196
	1	87.6487	56.7736	39.9168	30.5982	24.4857	13.3343
	2	11.4471	9.9095	8.7582	7.8901	7.2395	5.842
	3	3.5746	3.4817	3.4138	3.355	3.2699	3.0799
0.1	0	375.8842	233.0281	146.8762	98.0591	68.8502	25.2216
	0.25	331.7788	209.9058	135.0472	89.7255	62.7123	23.5096
	0.5	252.8574	153.4463	101.9263	68.0604	50.3287	21.2266
	1	96.5434	63.7254	44.4146	34.1089	26.2645	14.0947
	2	12.8043	10.7432	9.3613	8.5338	7.7346	6.1368
	3	3.7808	3.6552	3.5996	3.4597	3.394	3.1849
0.3	0	375.4065	257.52	182.4541	133.5005	99.652	38.9134
	0.25	350.8066	244.3691	173.3104	122.3278	93.5201	36.97
	0.5	289.9207	206.048	144.8713	105.2254	78.8718	32.556
	1	152.0954	106.0814	75.9282	57.8955	45.08	22.2917
	2	24.9009	19.3398	16.3654	13.8916	12.3134	8.7203
	3	5.5531	5.1915	5.0129	4.7476	4.6183	4.1684
0.5	0	376.8813	279.8288	206.5585	158.2779	121.0793	55.6862
	0.25	353.7196	263.9341	198.7299	151.4063	116.4311	53.0996
	0.5	322.0953	234.606	174.3115	133.3166	107.4692	47.9849
	1	199.2293	147.0846	109.8028	87.0584	67.8079	34.027
	2	48.2804	37.0199	30.1595	24.4349	21.0939	13.775
	3	10.1214	9.0329	8.2404	7.6745	7.0869	5.8644
0.7	0	371.3792	278.3	217.5065	175.9035	137.761	66.2676
	0.25	354.3616	268.9956	207.7015	166.8073	134.0458	65.5641
	0.5	326.078	250.0087	192.9173	150.7359	122.3859	60.4657
	1	228.1652	172.4365	133.8405	105.2048	85.6834	45.563
	2	70.5389	56.3774	44.198	36.6308	31.4652	20.1587
	3	17.9515	15.1729	13.4893	11.9654	10.9851	8.431
0.9	0	378.3523	288.3565	221.2859	182.2447	148.3304	76.1904
	0.25	372.8929	279.0063	218.8781	174.3426	144.9382	74.4308
	0.5	339.5463	261.1774	205.6659	162.5949	135.3801	69.8601
	1	251.6022	192.3074	148.9691	122.832	100.9874	55.1712
	2	93.4105	73.2809	60.433	49.5451	42.8915	26.5078
	3	27.3144	22.8568	19.7948	17.3728	15.9373	11.5811

Table 30

ARLs for the MEWMV control chart with increasing correlation, ω=0.5 and p=10 variables

		Variance					
Correlation	**Mean Shift**	**1**	**1.25**	**1.5**	**1.75**	**2**	**3**
0	0	365.0418	266.6814	187.2428	129.4161	95.3646	35.6968
	0.25	341.6832	237.6193	167.5169	120.9802	88.6597	34.3376
	0.5	271.2823	189.3152	135.4328	97.5938	71.2594	29.9406
	1	124.8807	88.24	64.1788	49.5698	38.7028	19.8675
	2	18.5326	16.1235	13.5388	12.0815	11.0676	8.2632
	3	5.309	5.0787	4.8946	4.6767	4.5333	4.1148
0.1	0	372.8315	280.2132	201.9263	152.8614	112.4398	43.5127
	0.25	343.6836	257.5999	190.9888	138.943	103.5855	41.4056
	0.5	293.3913	215.941	156.4577	114.1287	86.5482	35.4395
	1	150.4359	110.8892	79.8674	60.6335	47.546	23.1225
	2	24.8374	19.6511	16.9712	14.5645	13.0581	9.4054
	3	6.1708	5.76	5.5079	5.3053	5.1209	4.494
0.3	0	373.6416	303.8848	254.8512	211.7648	174.4856	89.463
	0.25	360.078	300.3611	250.0582	203.1993	170.1542	86.2764
	0.5	329.7731	272.2847	227.1017	185.2624	151.9506	78.1425
	1	239.7097	193.6175	158.4583	130.7688	107.0555	55.2876
	2	75.6414	62.445	50.5728	41.9667	35.1535	21.2608
	3	17.5846	14.764	13.3945	12.0084	11.0687	8.526
0.5	0	367.309	309.8344	270.4415	237.4971	198.4373	118.267
	0.25	359.9981	308.7101	267.0641	231.5018	196.4481	115.6111
	0.5	343.5166	288.7435	251.6207	215.3497	183.5457	109.4941
	1	279.3086	232.5052	196.598	171.9751	147.7223	86.3451
	2	128.4731	108.0439	92.3284	79.5887	69.6851	42.1607
	3	43.6059	36.3119	31.6683	28.127	24.9893	17.482
0.7	0	370.6011	317.303	280.4651	244.8688	216.7472	134.5701
	0.25	368.4809	318.4434	276.1816	242.8376	210.5174	132.5863
	0.5	357.0258	306.8277	268.889	232.4519	203.4911	126.6893
	1	301.9282	258.3869	229.3845	197.4507	150.2084	107.2985
	2	172.3146	147.2356	124.2069	106.356	95.4311	62.5897
	3	72.0654	63.0123	54.3336	47.7508	42.1199	29.9954
0.9	0	366.7545	322.7002	287.9627	253.8993	220.4715	144.7043
	0.25	368.6802	319.1355	283.8412	248.3164	216.922	146.6583
	0.5	350.9629	307.9	274.8603	240.7716	209.3703	137.7761
	1	314.421	271.5681	240.8898	206.6933	186.8942	123.0649
	2	199.2592	172.026	147.5747	129.2987	115.1195	77.4385
	3	96.4258	83.7563	75.3518	65.8962	59.2482	40.7927

Method Comparison

Graphically, the ARLs of the MCUSUM, MEWMA, MEWMS and MEWMV control charts all behave differently, except for the MEWMS and MEWMV graphs. The MEWMV control chart is a derivation of the MEWMS and shows similar ARL curves. Figures 4.1 through 4.33 show the various ARL graphs of the MCUSUM, MEWMA, MEWMS and MEWMV control charts when using the weighting value of ω=0.5, where applicable. These graphs display the behavioral characteristics of each type of control chart; especially the decrease in sensitivity that corresponds with changes in correlation, number of observed variables, or weighting values. Figures 4.1 through 4.4 show the ARL curves when the number of variables increased for the MCUSUM control chart with no correlation between variables. For the complete selections of graphical outputs, refer to Appendix C.

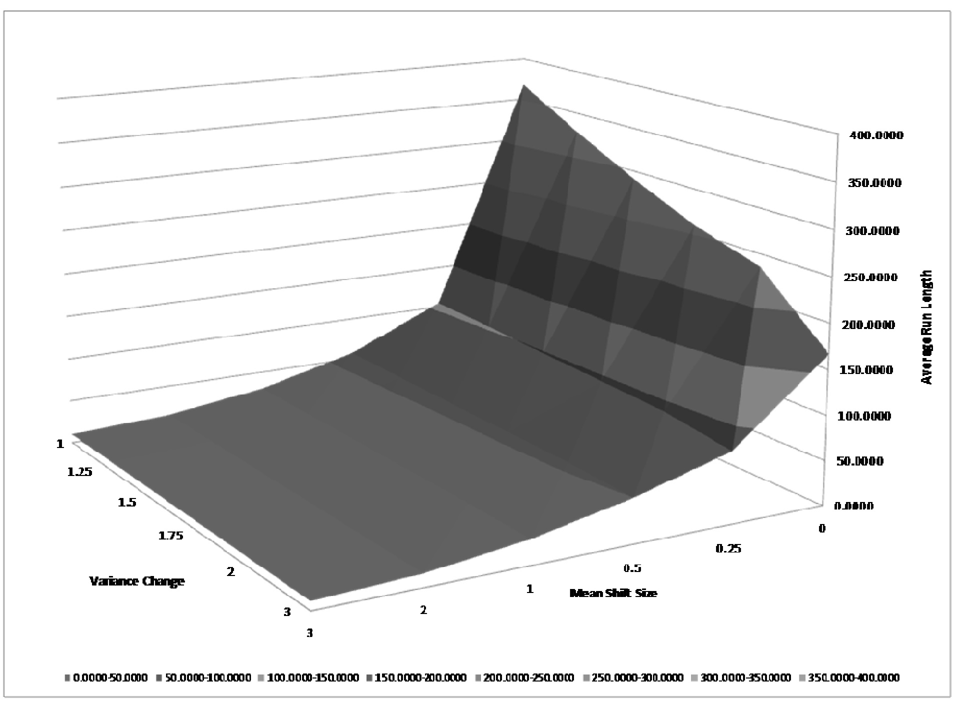

Figure 4.1

ARL curve for the MCUSUM control chart when p=2 and correlation=0.0

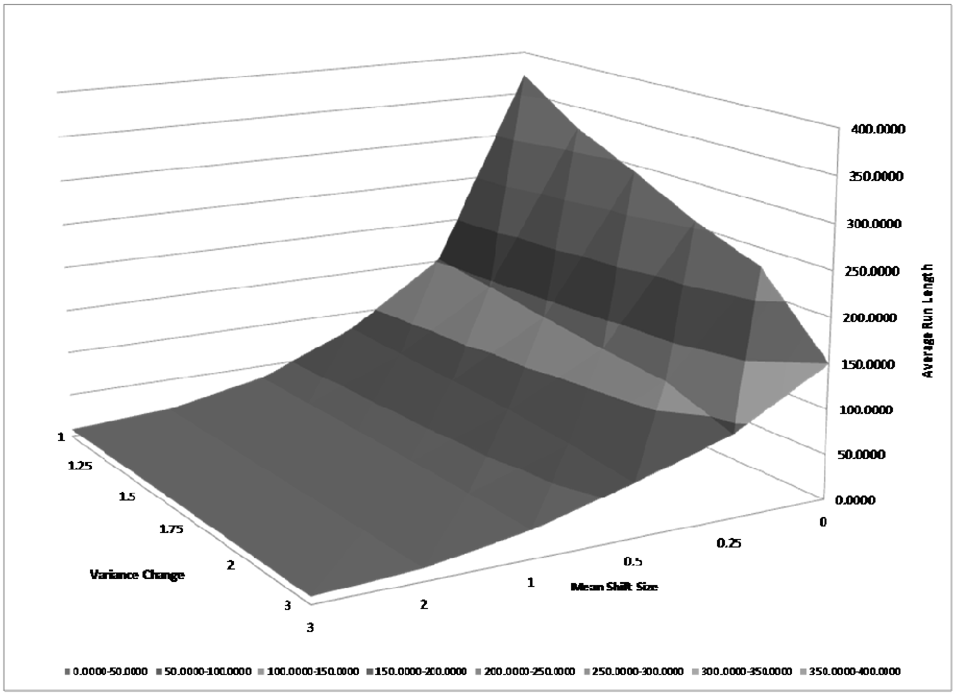

Figure 4.2

ARL curve for the MCUSUM control chart when p=3 and correlation=0.0

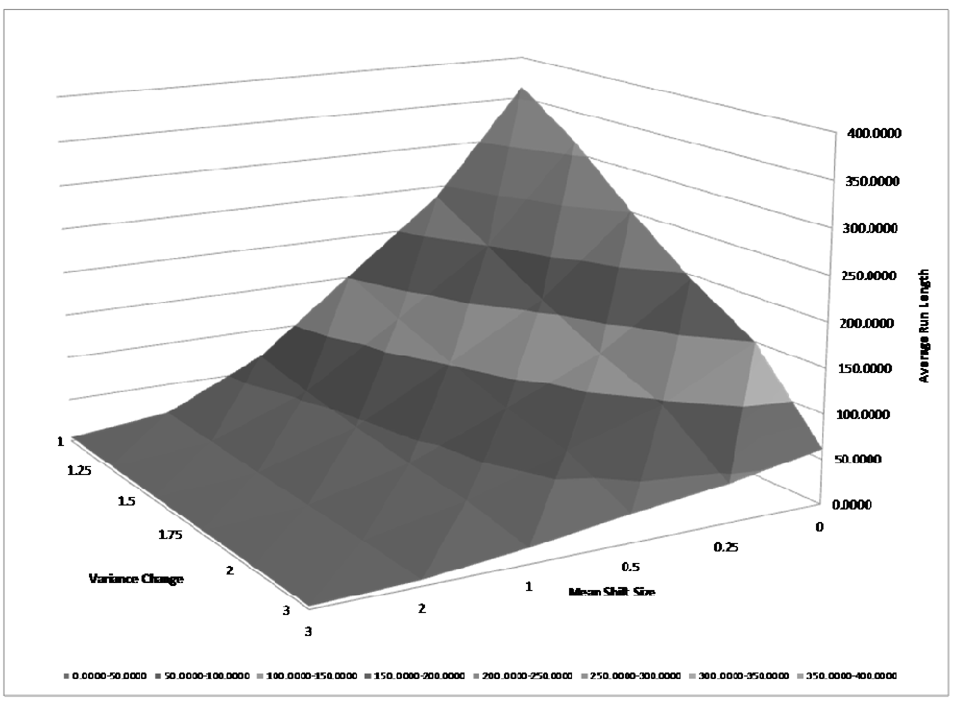

Figure 4.3

ARL curve for the MCUSUM control chart when p=5 and correlation=0.0

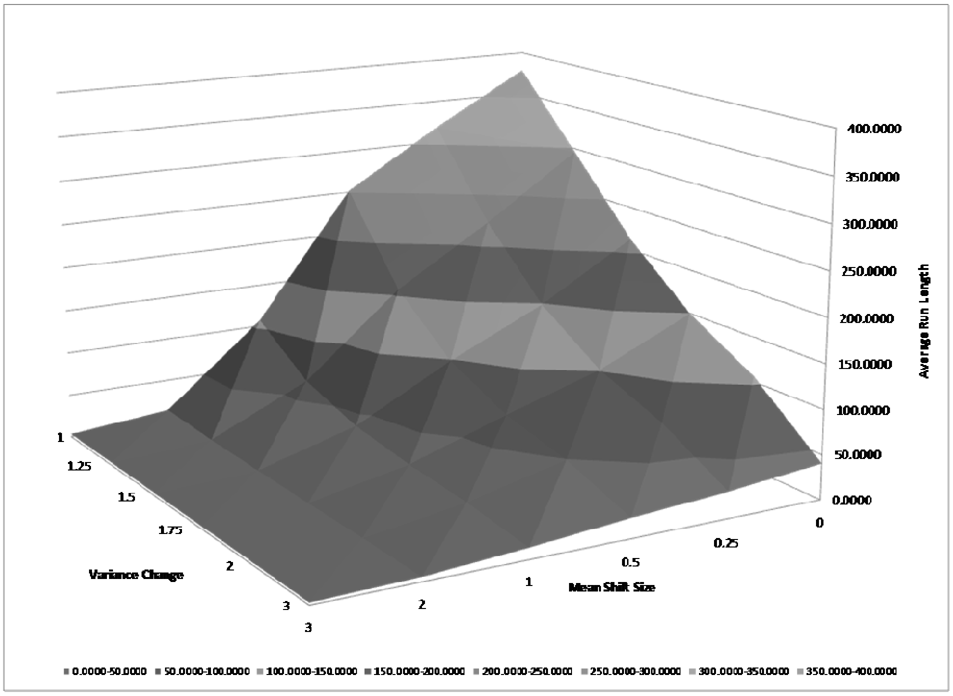

Figure 4.4

ARL curve for the MCUSUM control chart when p=10 and correlation=0.0

Figures 4.5 through 4.9 are the MCUSUM control chart ARLs when the correlation equals 0.5. These figures convey the decrease in sensitivity of the MCUSUM control chart as correlation increases. With the increase in correlation, the MCUSUM displays an effect that changes the surface of the ARL curve to resemble a plane, rather than the slopes shown in figures 4.1 through 4.4. Graphically, the sensitivity of the MCUSUM is apparent in the mean shift compared to the variance change. Throughout much of this simulation, the most significant detection of a variance change was for large changes, specifically when the variance change was equal to three.

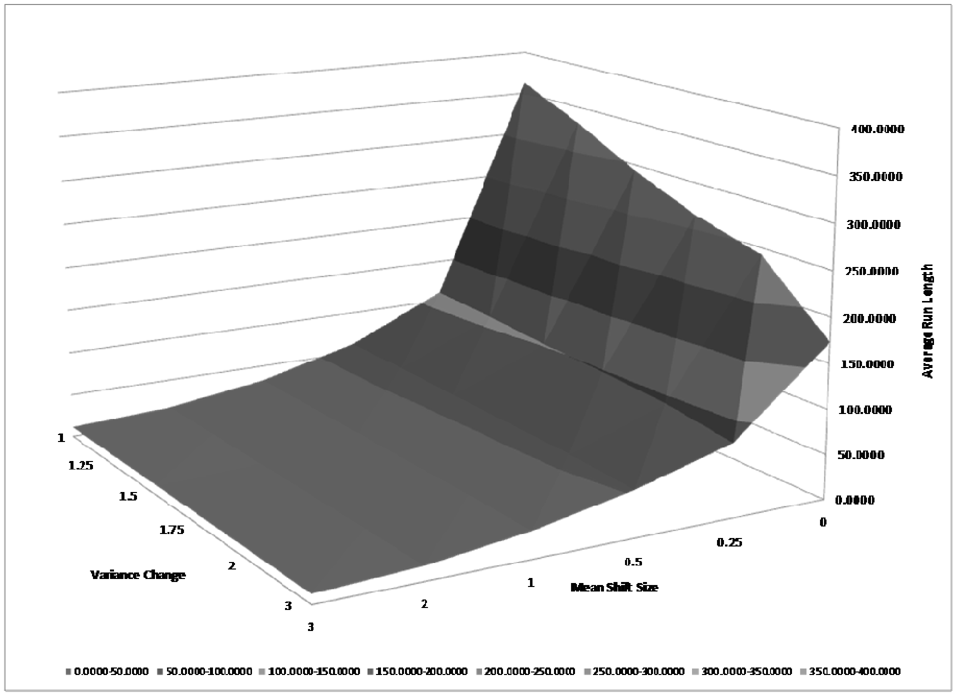

Figure 4.5

ARL curve for the MCUSUM control chart when p=2 and correlation=0.5

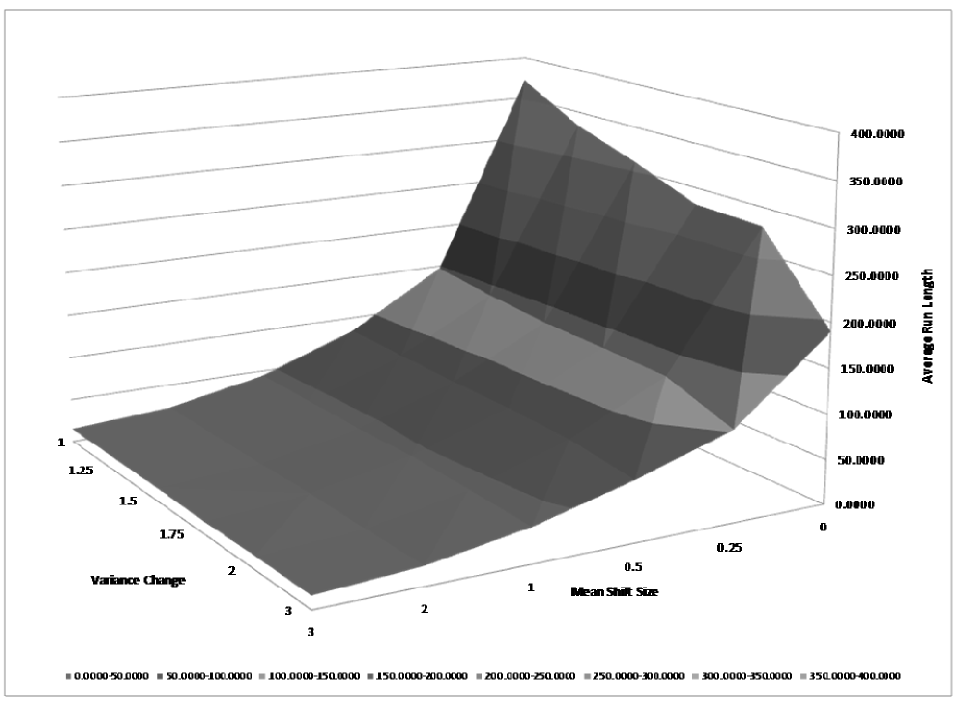

Figure 4.6

ARL curve for the MCUSUM control chart when p=3 and correlation=0.5

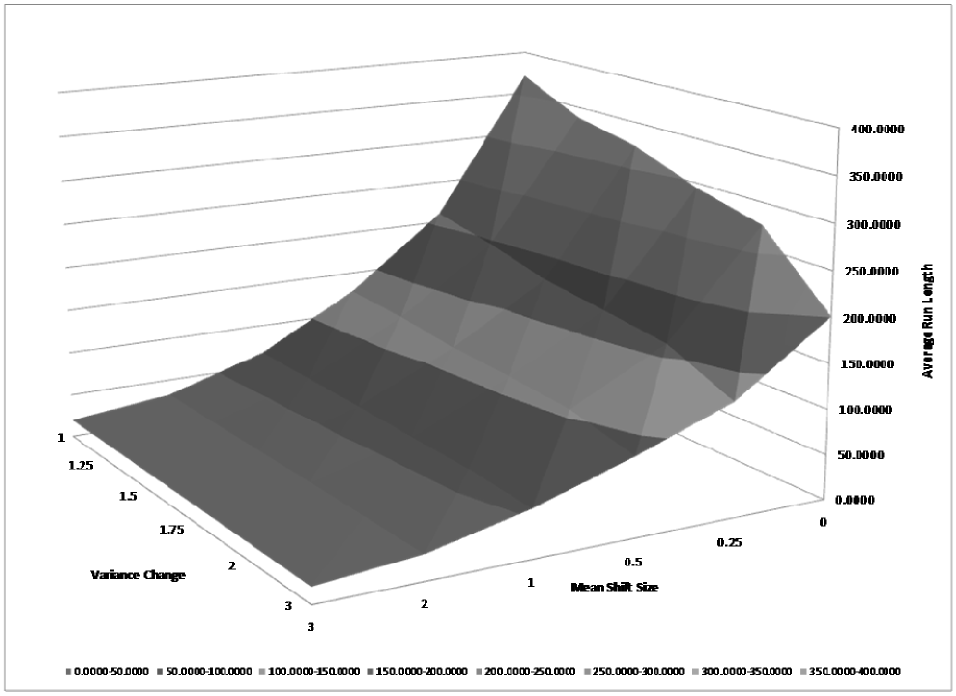

Figure 4.7

ARL curve for the MCUSUM control chart when p=5 and correlation=0.5

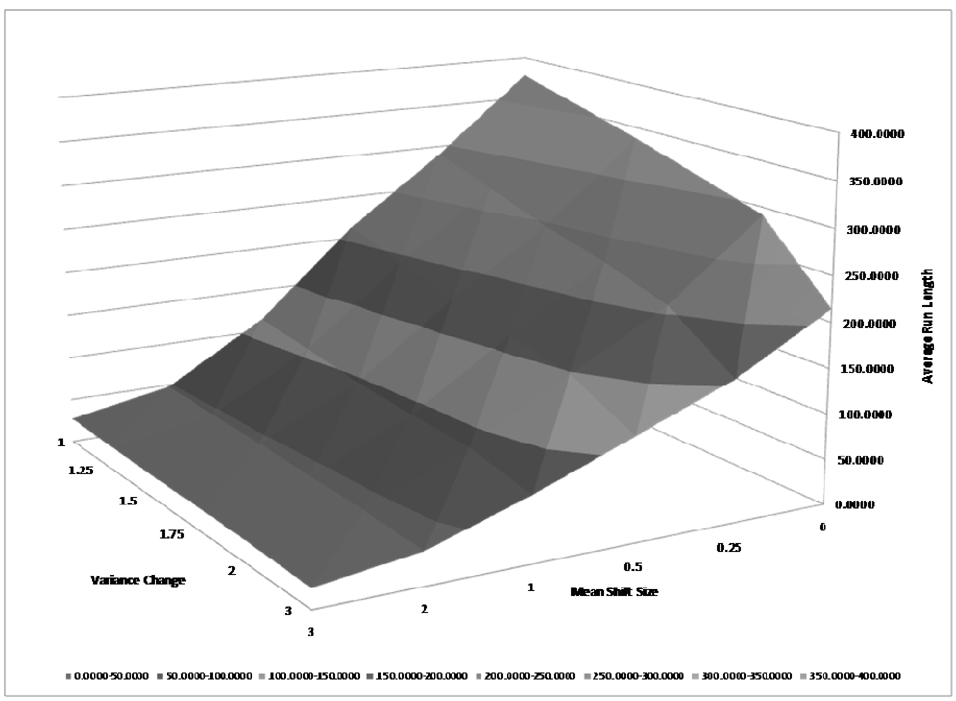

Figure 4.8

ARL curve for the MCUSUM control chart when p=10 and correlation=0.5

The MEWMA control chart displayed a behavior different from that of the other control charts. While the MCUSUM control chart showed very little sensitivity to a change in variance and the MEWMS and MEWMV control charts were specifically designed for detecting a change in variance, the MEWMA shows no sensitivity to this change in variance. Similar to the behavior of the MCUSUM control charts, as the number of variables increases, the sensitivity of the control chart decreases. The similarities of the MEWMA control chart to the MEWMS and MEWMV control charts is that the sensitivity of the MEWMA control chart decreases with the increase in weighting values.

Figures 4.9 through 4.12 displays the ARL curves of the MEWMA control chart where correlation values equal 0.0 and $\omega=0.5$, with the number of observed variables increasing. These figures show that with each increase in observed variables, the sensitivity does decrease, but only slightly, in comparison to the other control charts. In all figures involving the MEWMA, it is shown that there is no sensitivity to variance changes. A complete list of the MEWMA control chart ARL curves is available in Appendix C.

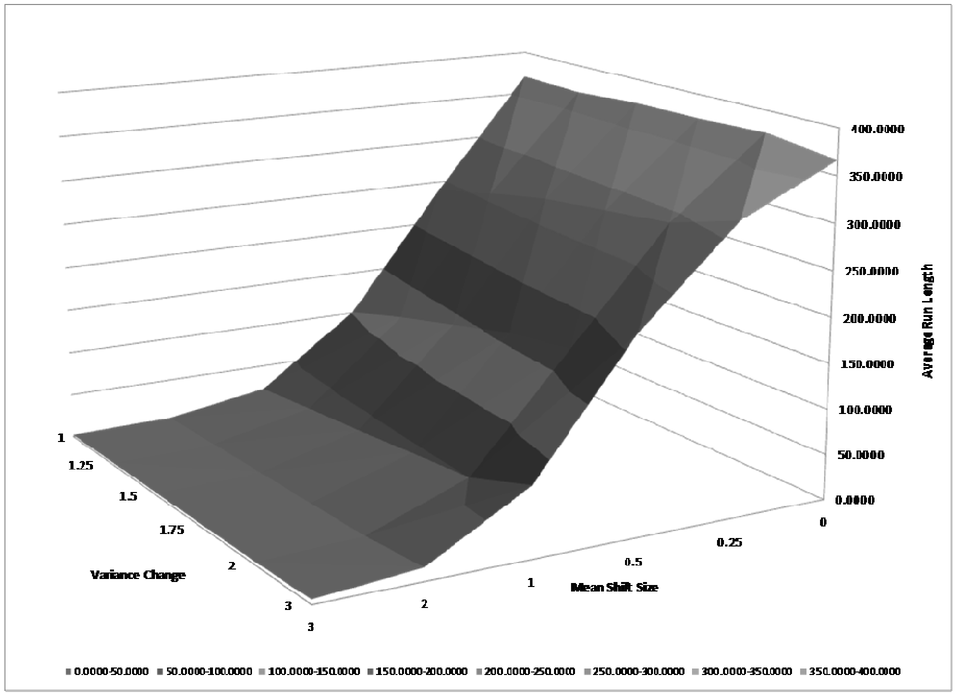

Figure 4.9

ARL curve for the MEWMA control chart when p=2, correlation=0.0 and weight=0.5

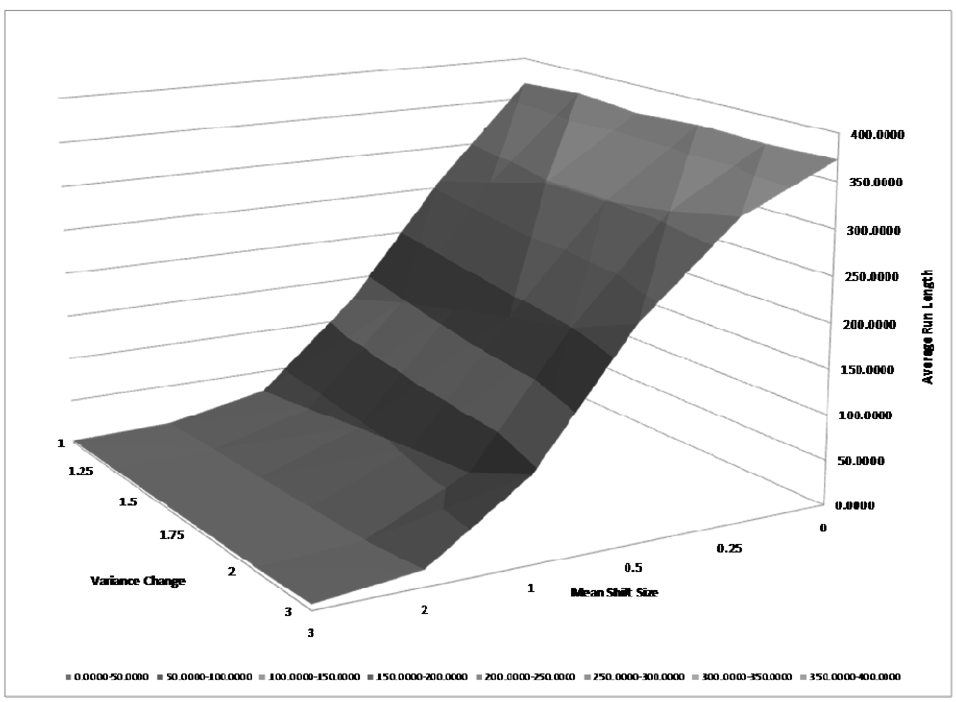

Figure 4.10

ARL curve for the MEWMA control chart when p=3, correlation=0.0, and weight=0.5

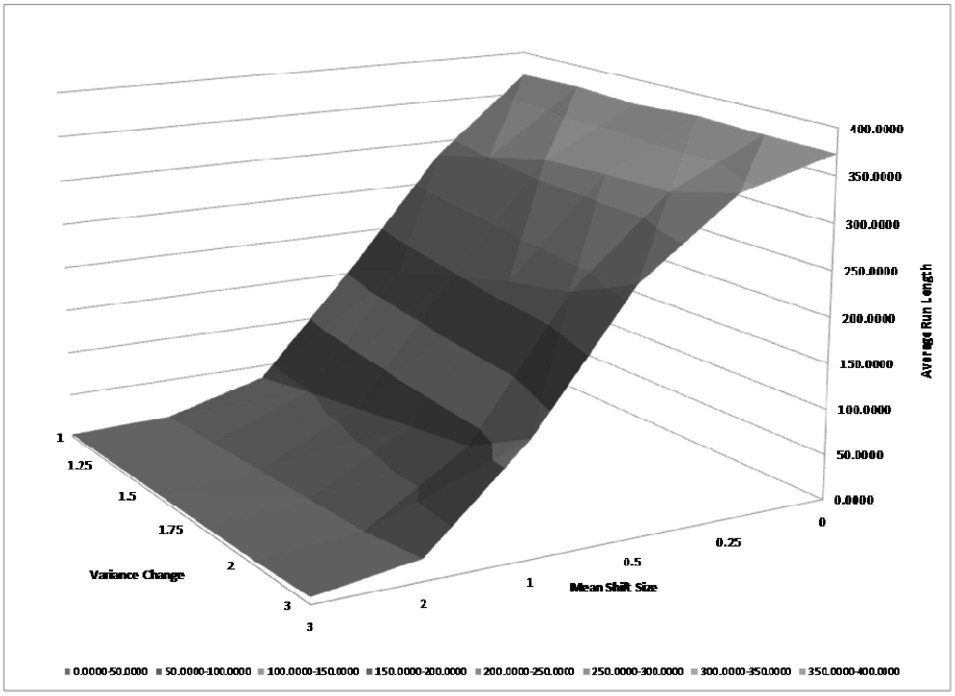

Figure 4.11

ARL curve for the MEWMA control chart when p=5, correlation=0.0 and weight=0.5

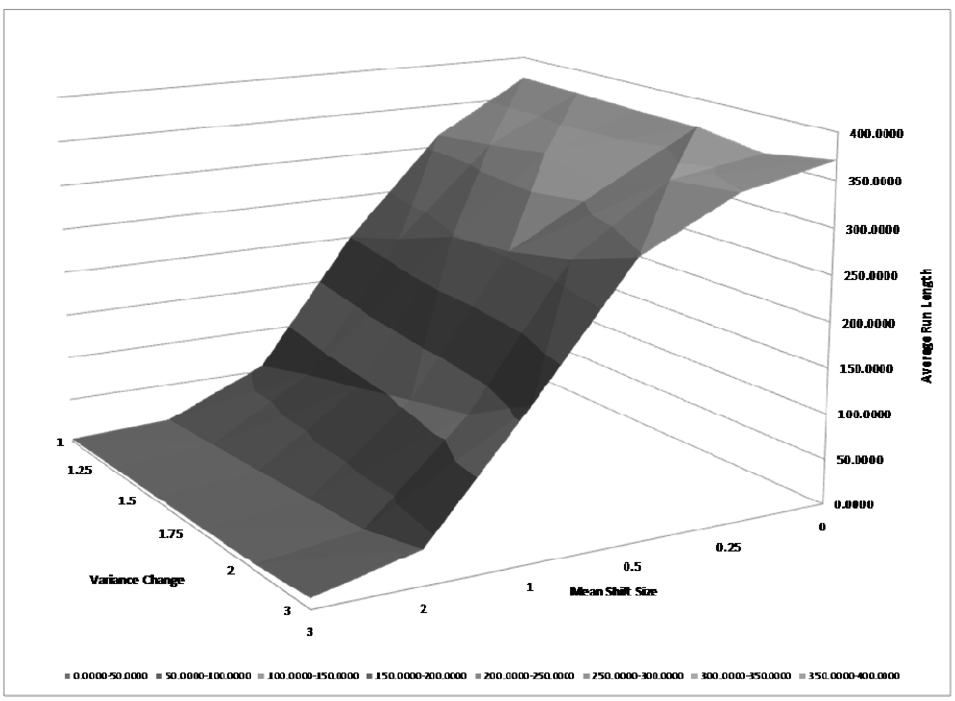

Figure 4.12

ARL curve for the MEWMA control chart when p=10, correlation=0.0 and weight=0.5

Figures 4.13 through 4.16 displays the MEWMA control chart where correlation was 0.5. In this simulation, it was shown that as correlation values increased uniformly across the covariance matrix, the sensitivity of the MEWMA control chart actually increased. The resulting curve show little change from figures 4.11 through 4.13. However, looking at tables 21 though 24, there is a decrease in detection sensitivity.

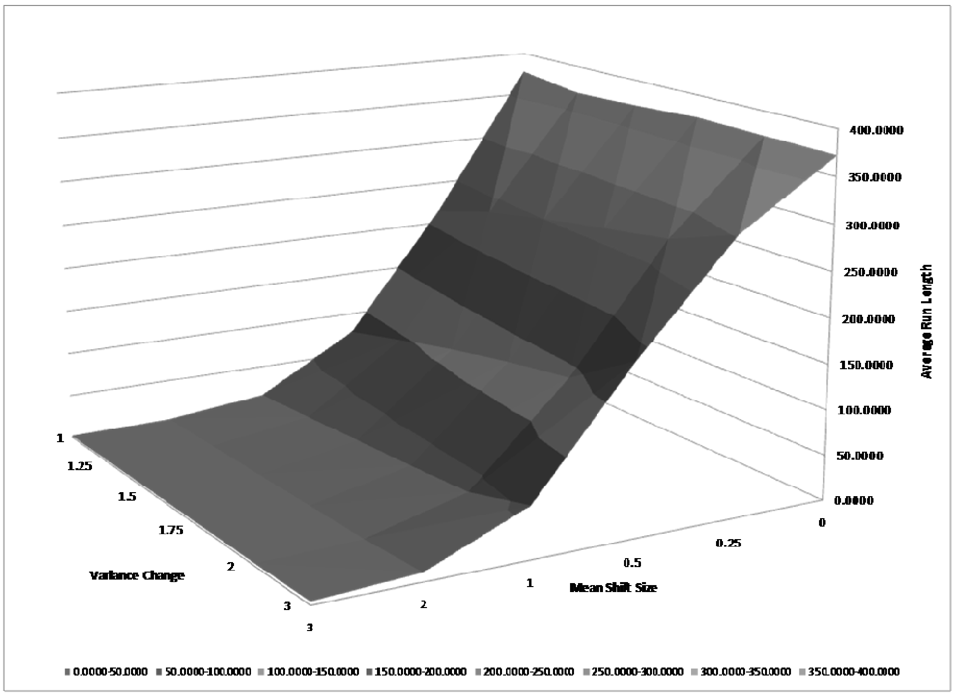

Figure 4.13

ARL curve for the MEWMA control chart when p=2, correlation=0.5 and weight=0.5

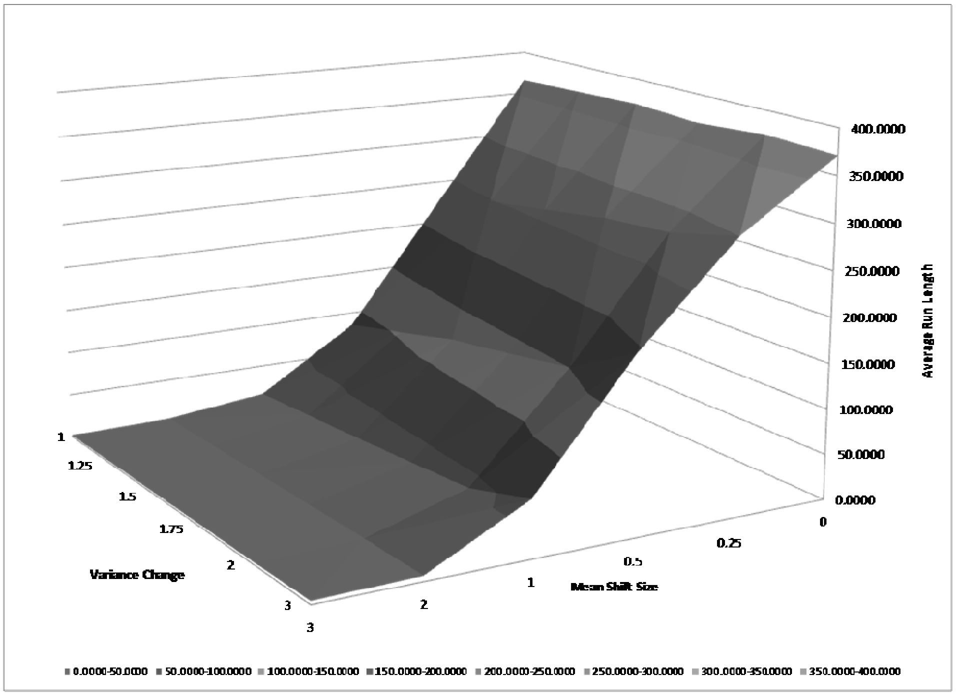

Figure 4.14

ARL curve for the MEWMA control chart when p=3, correlation=0.5 and weight=0.5

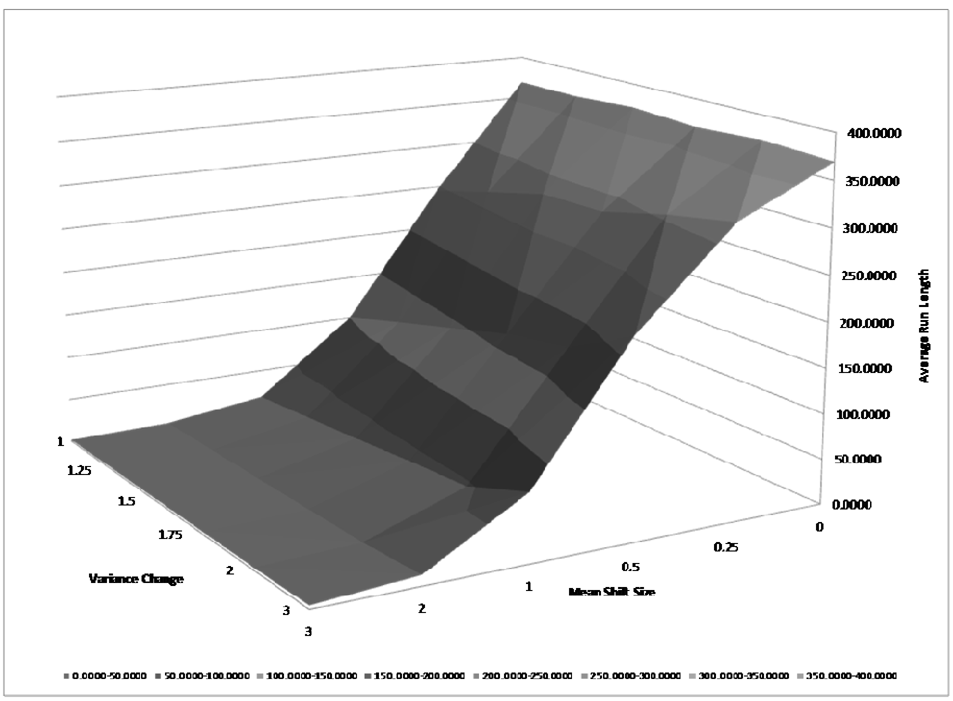

Figure 4.15

ARL curve for the MEWMA control chart when p=5, correlation=0.5 and weight=0.5

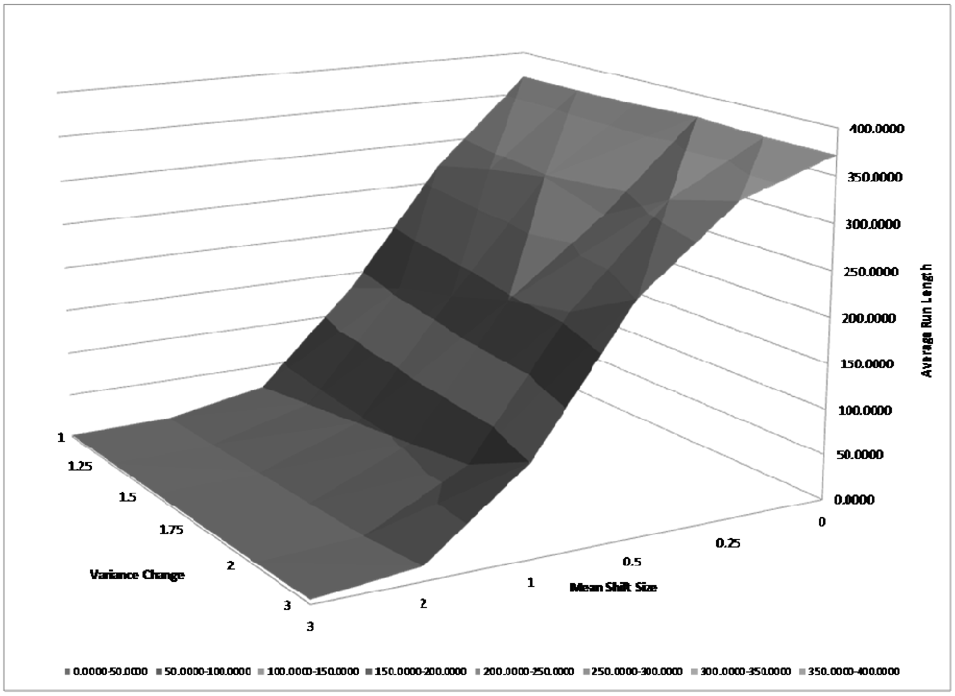

Figure 4.16

ARL curve for the MEWMA control chart when p=10, correlation=0.5 and weight=0.5

The MEWMA control chart shows little sensitivity to any changes beyond the mean shift. As the purpose of this control chart was the detection of mean shifts, this dissertation has reaffirmed the developmental finding of Lowery et al. (1992). This finding is discussed in depth in chapter five.

With the development of the MEWMS control chart, Huwang et al.(2007) built a control chart capable of detecting both a mean shift and a variance change. Figures 4.17 through 4.20 show the ARL curves of the MEWMS control chart with *p* variables and correlations equal to 0.0 using the weighting value of 0.5. Under these conditions, the MEWMV control chart is sensitive to both mean shifts and variance changes. With the increase of observed variables (p), the sensitivity to detect the mean shift decreases. To view the entire list of MEWMS control chart curves, please refer to Appendix C.

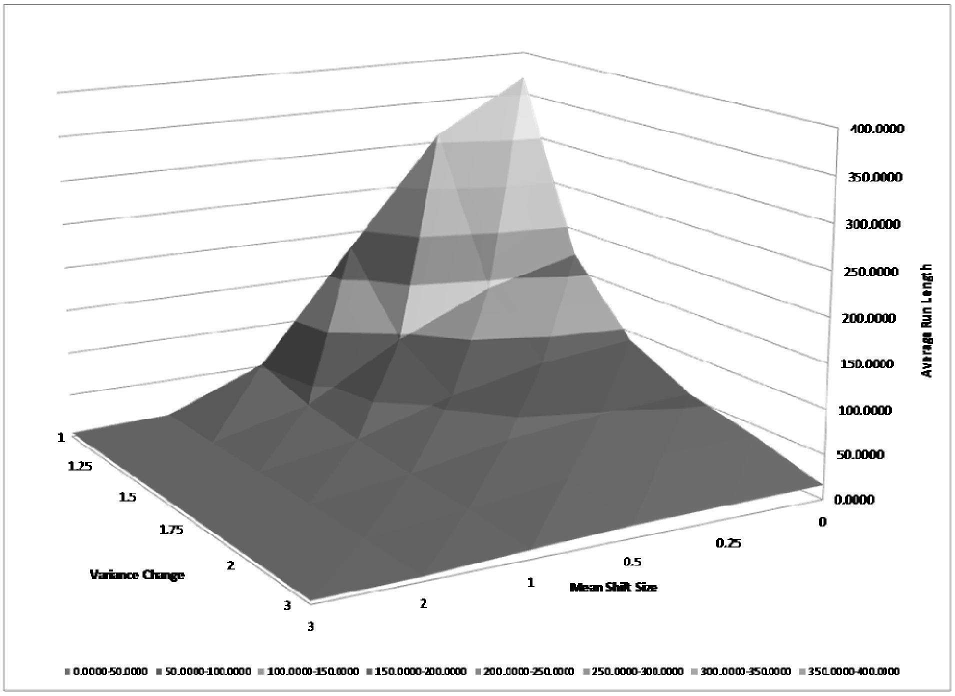

Figure 4.17

ARL curve for the MEWMS control chart when p=2, correlation=0.0 and weight=0.5

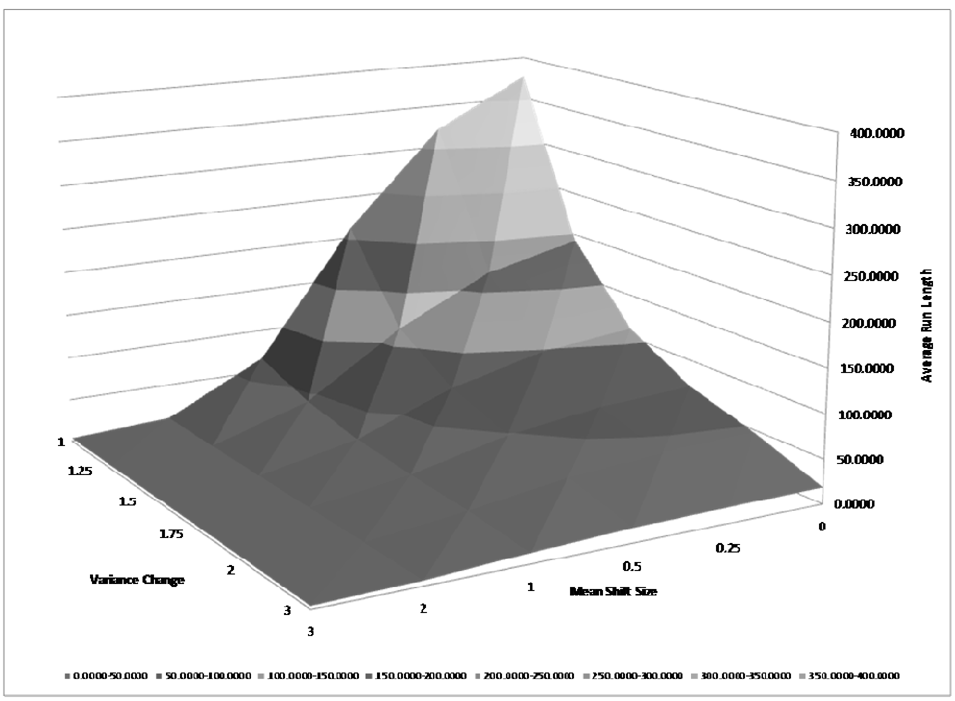

Figure 4.18

ARL curve for the MEWMS control chart when p=3, correlation=0.0 and weight=0.5

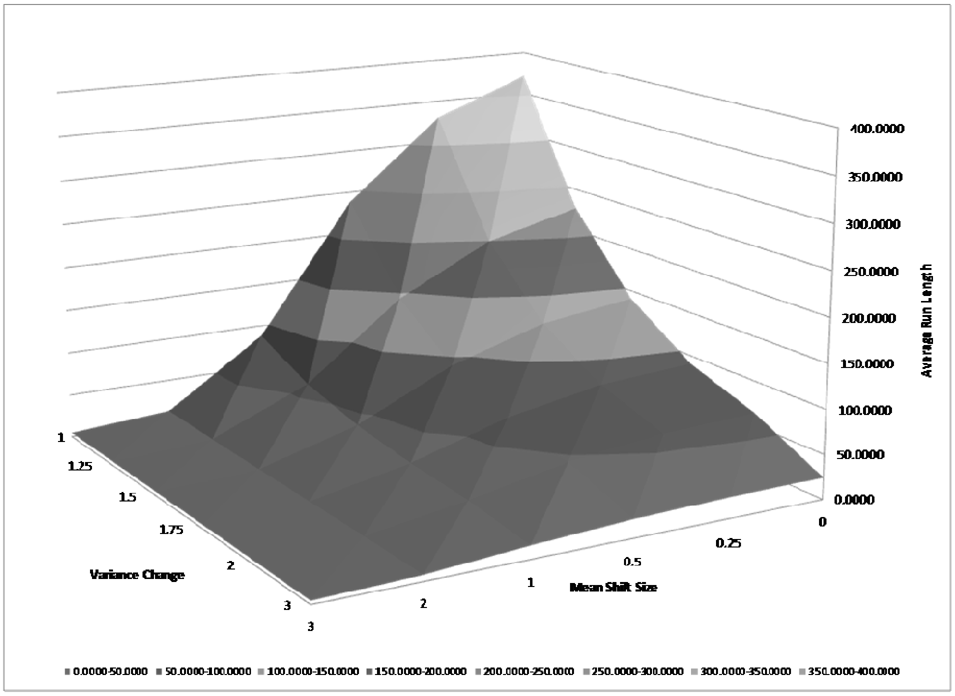

Figure 4.19

ARL curve for the MEWMS control chart when p=5, correlation=0.0 and weight=0.5

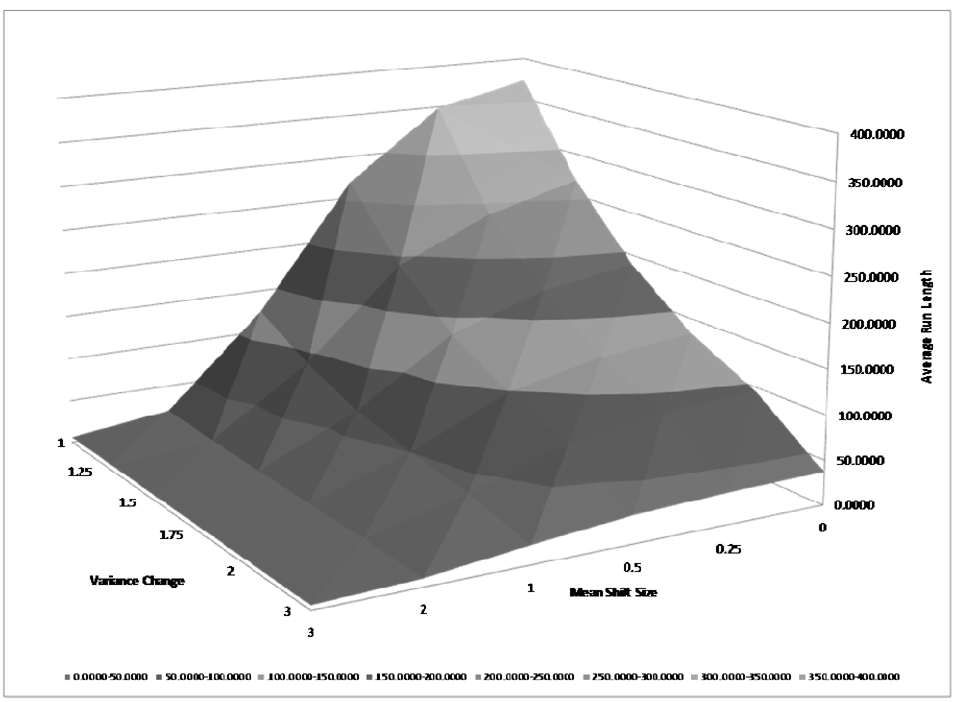

Figure 4.20

ARL curve for the MEWMS control chart when p=10, correlation=0.0 and weight=0.5

Figures 4.21 through 4.24 show the MEWMS control chart with correlation equal 0.5, the increasing number of observed variables, and weighting value of 0.5. These figures show the effect of uniformly increased correlation on the detection sensitivity of the MEWMS control chart. With the increased correlation, the sensitivity of the MEWMS to detect a mean shift is dramatically decreased. This decrease in sensitivity is seen as early as *p*=3 observed variables, and as *p* increases, the slope of the graphs resembles a plane, rather than the decreasing curves seen in figures 4.17 through 4.20.

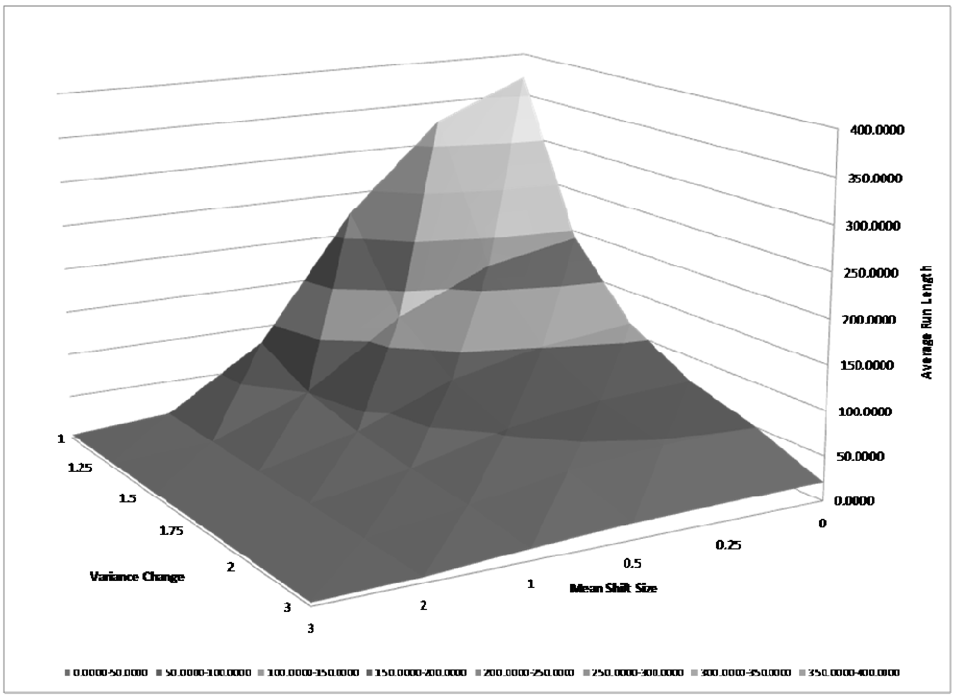

Figure 4.21

ARL curve for the MEWMS control chart when p=2, correlation=0.5 and weight=0.5

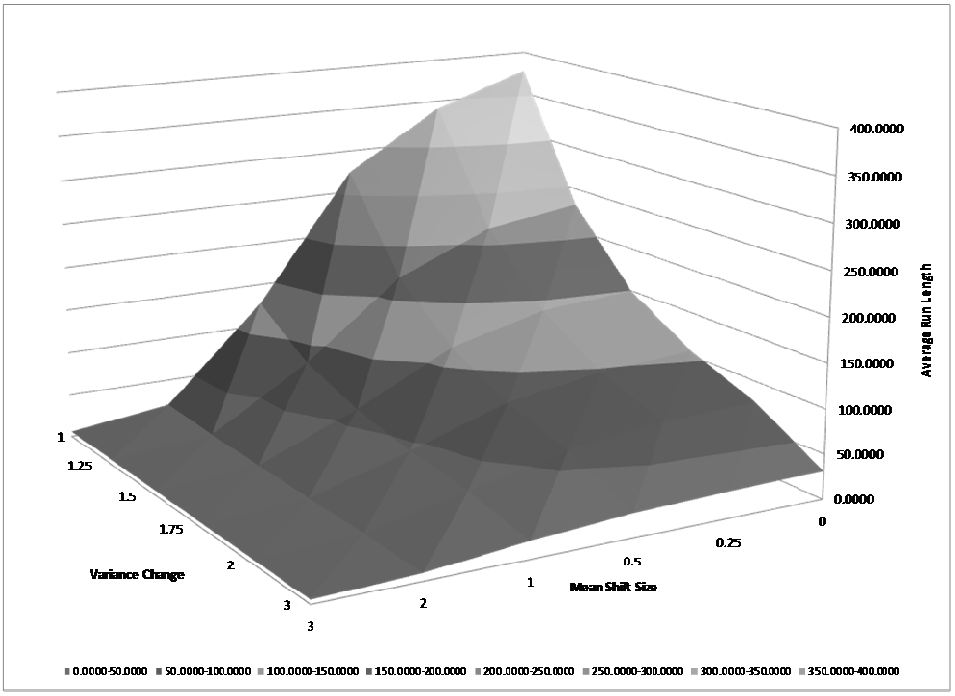

Figure 4.22

ARL curve for the MEWMS control chart when p=3, correlation=0.5 and weight=0.5

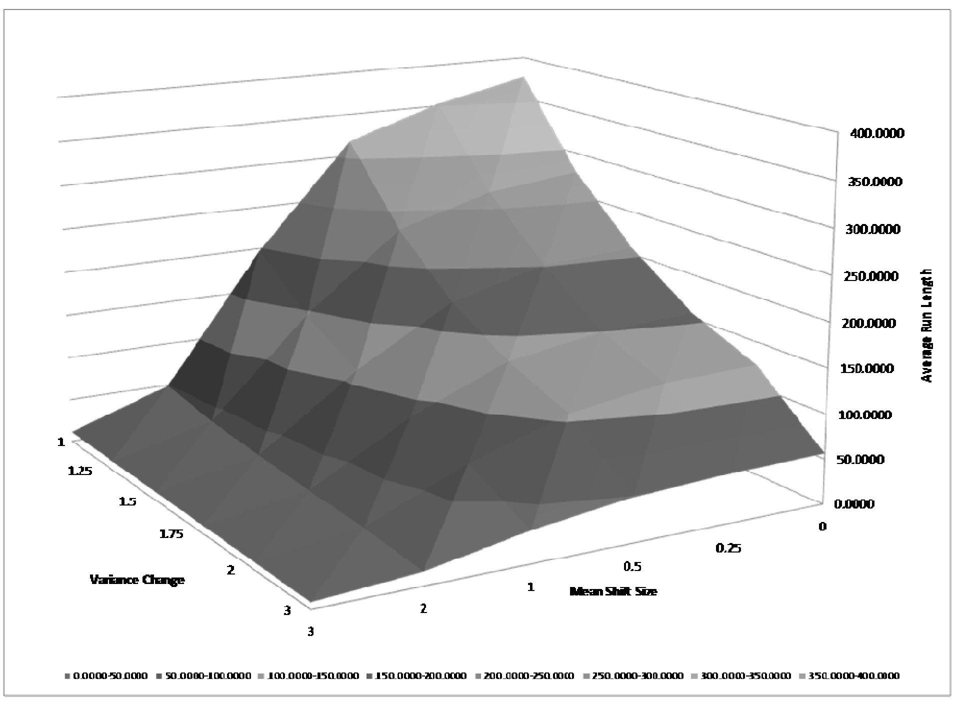

Figure 4.23

ARL curve for the MEWMS control chart when p=5, correlation=0.5 and weight=0.5

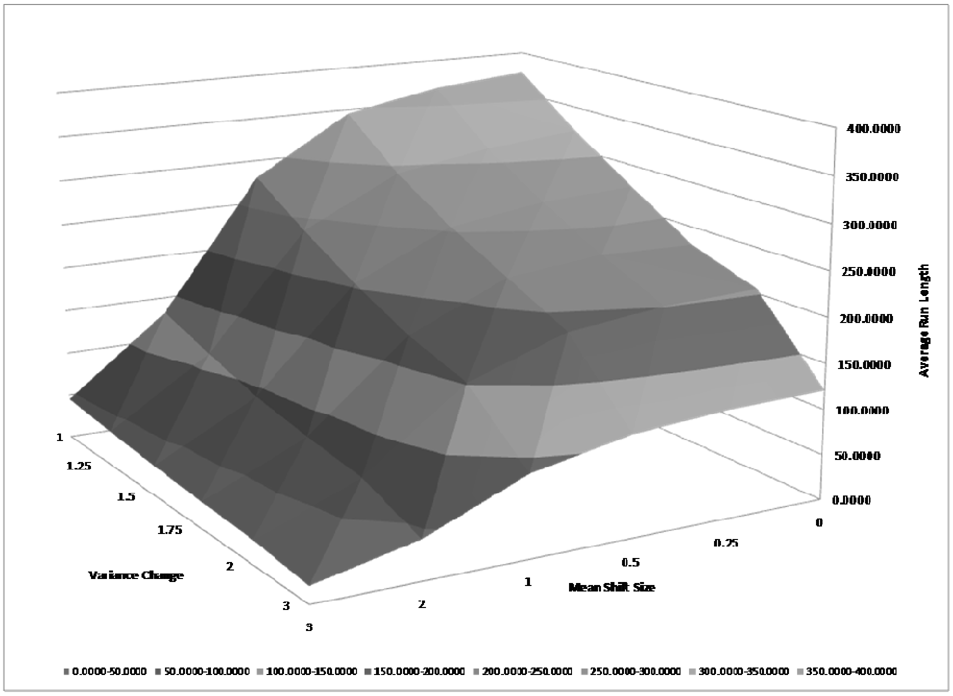

Figure 4.24

ARL curve for the MEWMS control chart when p=10, correlation=0.5 and weight=0.5

As the above figures show, with the increase in variables (*p*), the MEWMS control chart displays a decrease in the sensitivity to detect a mean shift and variance change. However, the sensitivity of the detection of a variance change is still greater than that of the MCUSUM. Comparisons to the MEWMA control chart for detection of a variance change will be ignored, as the MEWMA shows no sensitivity to a variance change.

Figures 4.1 to 4.24 and tables 1 through 12, with the tables in Appendix B and graphs in Appendix C, provide sufficient information to answer the first question posed in this dissertation: “Does the MEWMS control chart monitor for a singular change in the covariance matrix and mean shift more effectively than the MEWMA or the MCUSUM control charts?”

The MEWMS control chart shows an increased sensitivity to detect a singular change in the covariance matrix compared to the MCUSUM control chart. The

MCUSUM control chart shows sensitivity to a singular change in the covariance matrix, especially compared to the MEWMA which is not sensitive to a singular change in the covariance matrix. The sensitivity of the MEWMS control chart to detect a mean shift is less than the MCUSUM and the MEWMA control charts. The MCUSUM and MEWMA control charts were both developed to detect small mean shifts and were not specifically designed to detect small changes in the covariance matrix. Figures 4.25 and 4.26 display the sensitivity comparisons of the MCUSUM, MEWMA and MEWMS control charts for mean shifts and variance changes using p=2 observed variables. The figures for p=2 observed variables displays the most differentiation of behaviors of the compared control charts. In all cases where the dimension of the control charts increased, this behavior was observed, but differentiation decreased.

Figures 4.25 and 4.26 show the projection of the three dimensional figures onto a single dimension of the graph. These projections were layered atop one another for comparison. Figure 4.25 shows the sensitivity comparison of the three control charts to a mean shift when the variance is held at 1.0. Figure 4.26 shows the sensitivity comparisons of the three control charts for a variance change when the mean is held at 0.0. For both of these figures, the weighting values were held at 0.5 for the MEWMA and MEWMS control charts.

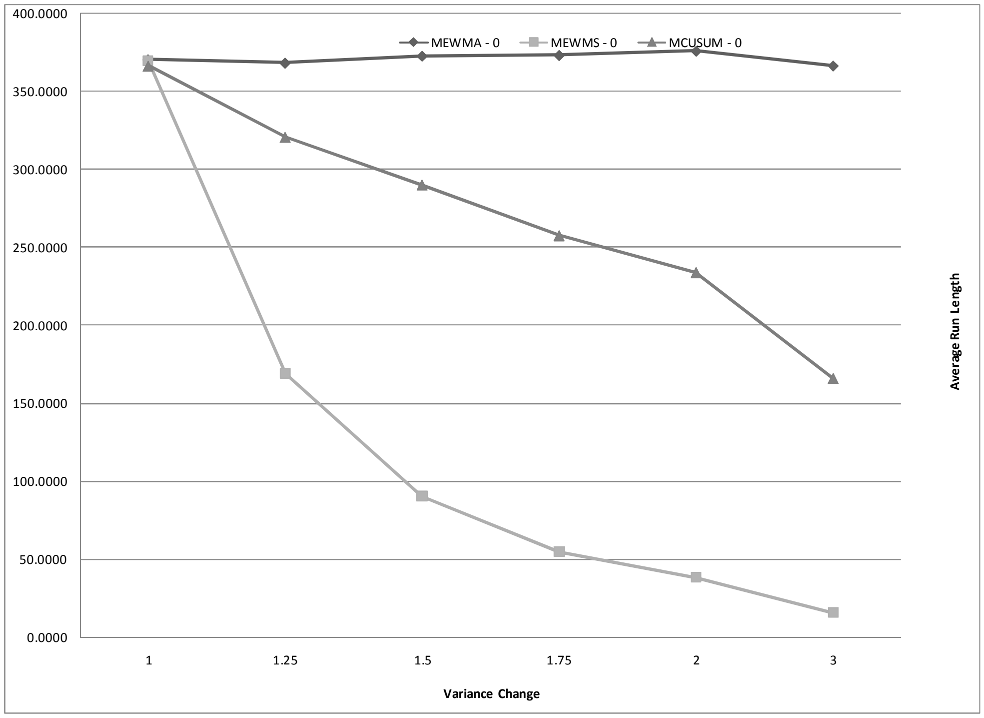

Figure 4.25

ARLs for the MCUSUM, MEWMA, and MEWMS control charts when the mean is held constant and covariance components change with weight=0.5

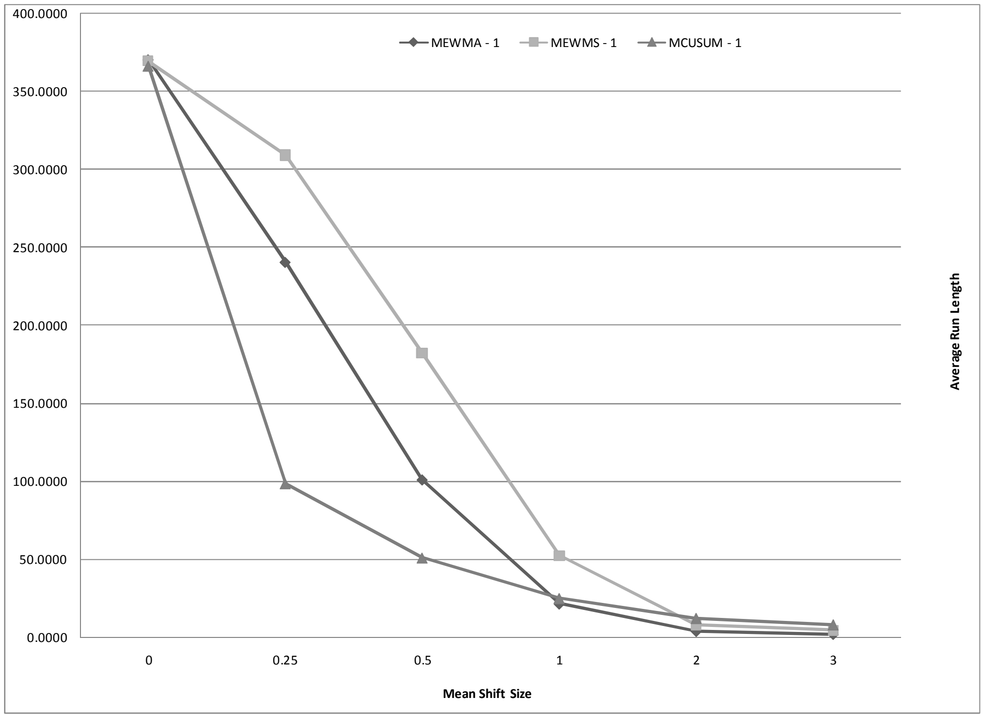

Figure 4.26

ARLs for the MCSUM, MEWMA and MEWMS control charts when variance is held constant and mean component shifts with weight=0.5

Hawkins and Maboudou-Tchao (2008) introduced the MEWMV as a less complicated equation for a similar control chart as Huwang, Yeh, and Wu's MEWMS (2007). Like the MEWMS, the MEWMV was developed to detect both mean shifts and variance changes. Figures 4.27 through 4.30 show the ARL curves for the MEWMV control chart with correlations equal to 0.0 and weighting value of 0.5. With the addition of variables (*p*), the sensitivity decreases, most noticeably in the detection of a mean shift. For the complete collection of MEWMV ARL curves, please refer to Appendix C.

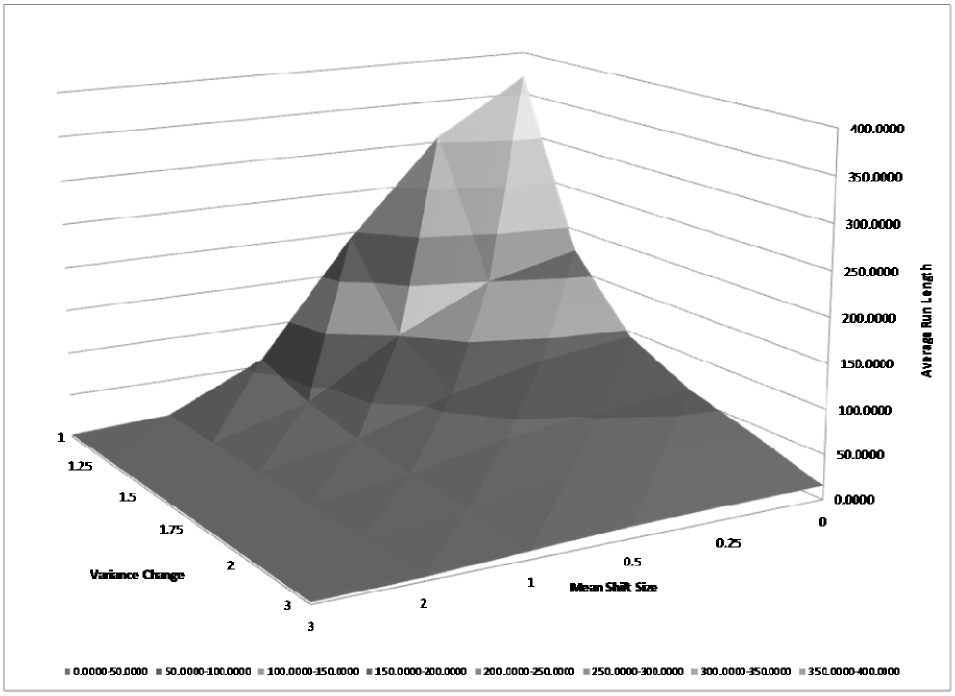

Figure 4.27

ARL curve for the MEWMV control chart when p=2, correlation=0.0 and weight=0.5

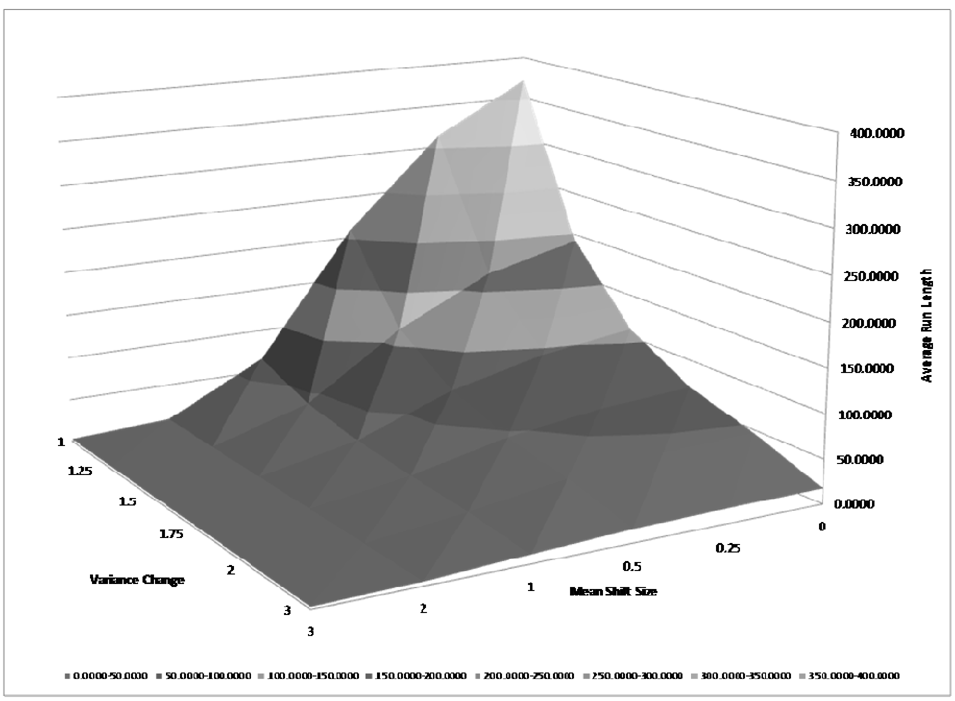

Table 4.28

ARL curve for the MEWMV control chart when p=3, correlation=0.0 and weight=0.5

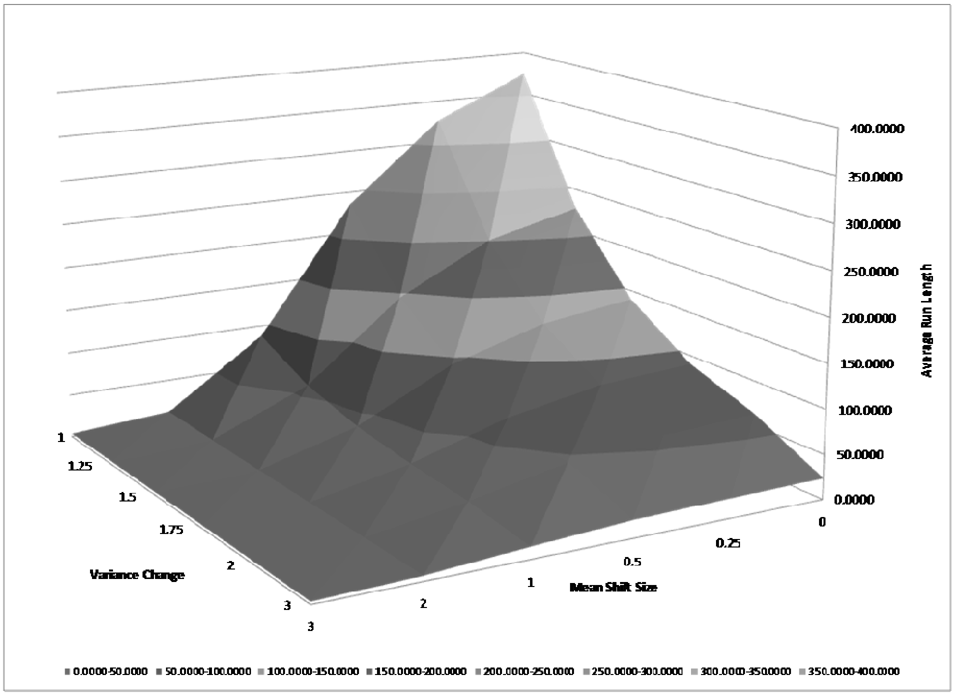

Figure 4.29

ARL curve for the MEWMV control chart when p=5, correlation=0.0 and weight=0.5

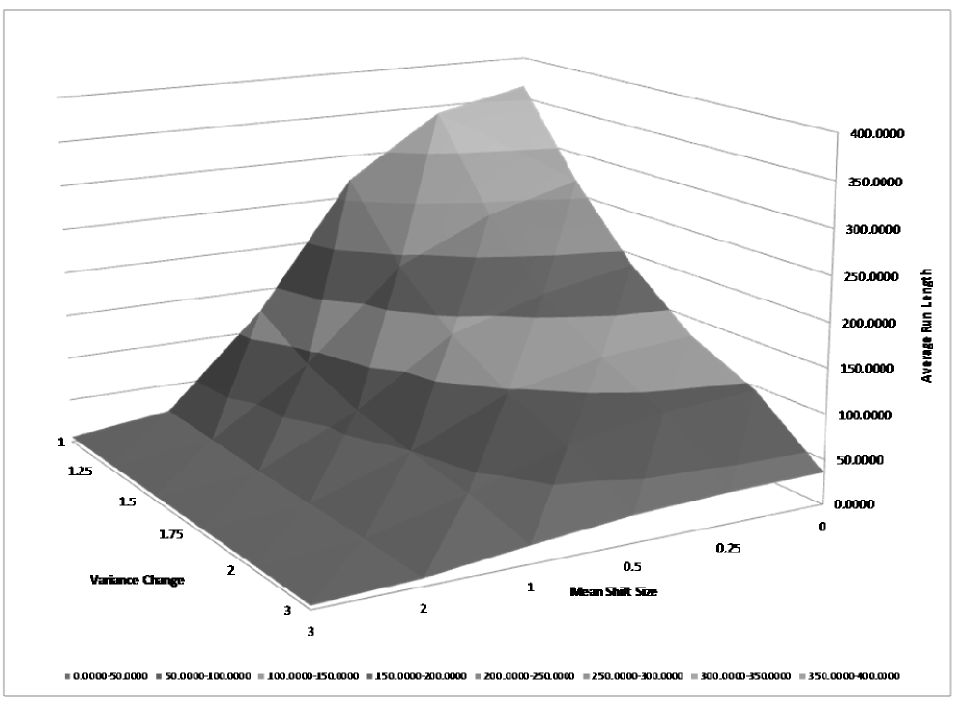

Figure 4.30

ARL curve for the MEWMV control chart when p=10, correlation=0.0 and weight=0.5

Figures 4.31 through 4.34 show the MEWMV control chart curves as the number of observed variables increased with correlation of 0.5 between the variables and weighting value of 0.5. Like the MEWMS control chart curves, the uniform increase in correlations decreases the sensitivity of the MEWMV control chart. This decrease is seen in the detection of a mean shift more so than in the detection of a variance change.

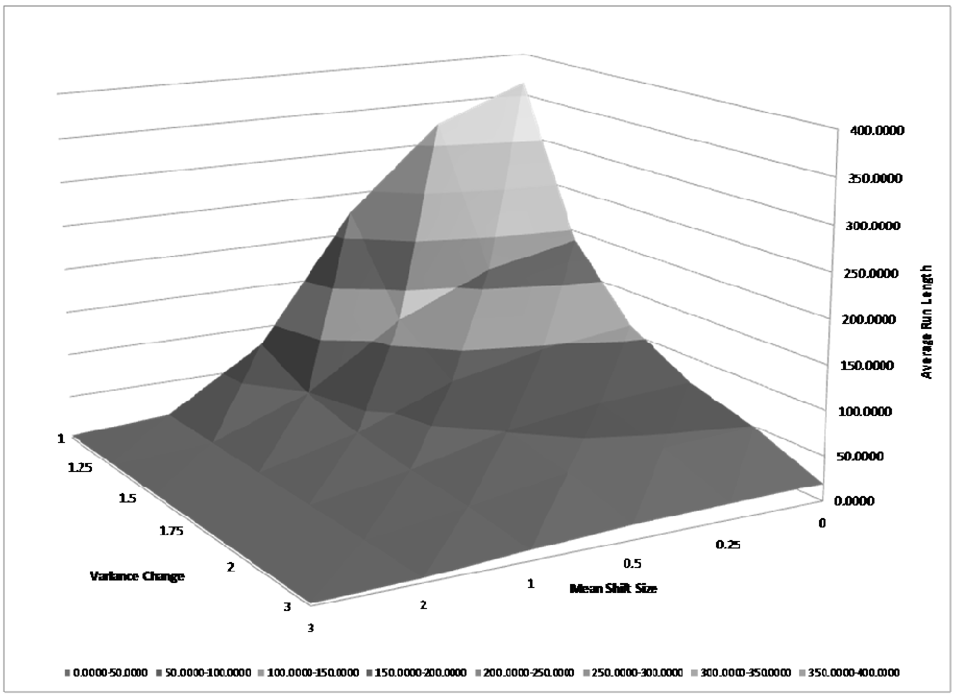

Figure 4.31

ARL curve for the MEWMV control chart when p=2, correlation=0.5 and weight=0.5

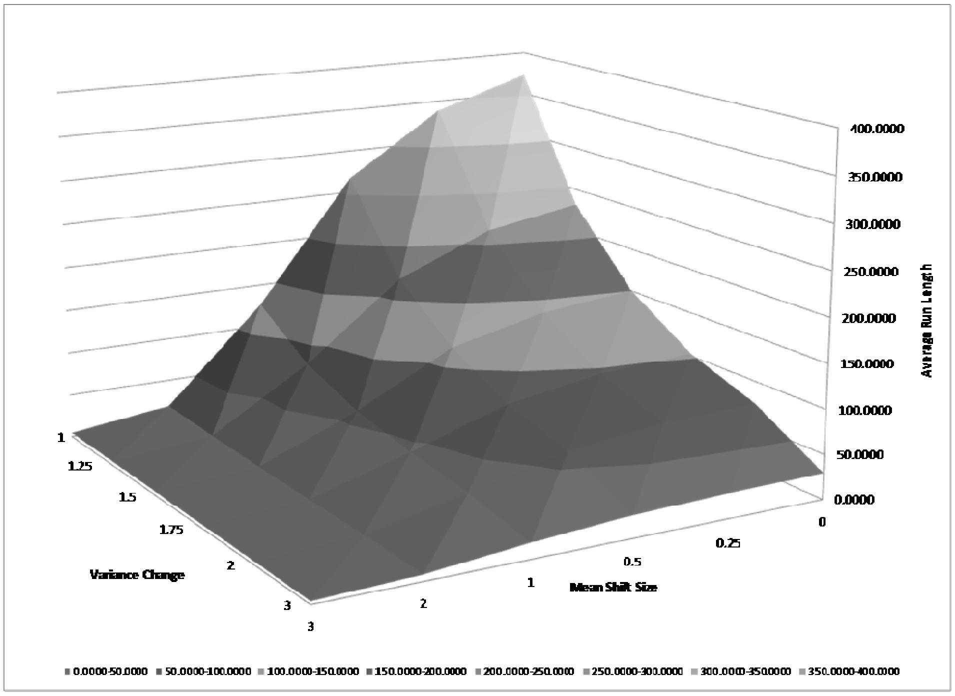

Figure 4.32

ARL curve for the MEWMV control chart when p=3, correlation=0.5 and weight=0.5

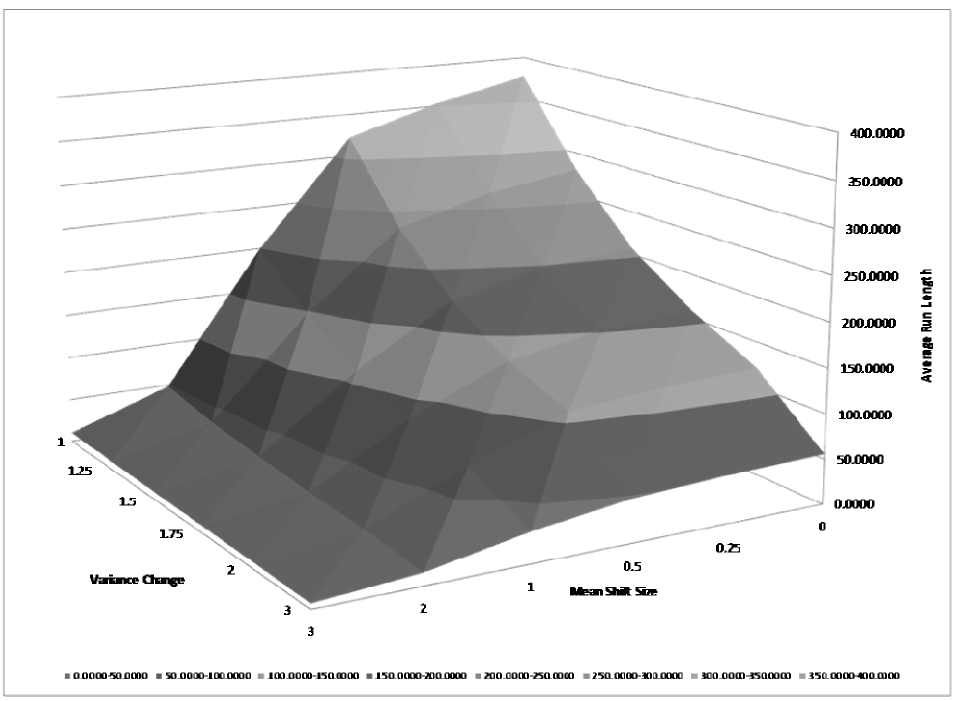

Figure 4.33

ARL curve for the MEWMV control chart when p=5, correlation=0.5 and weight=0.5

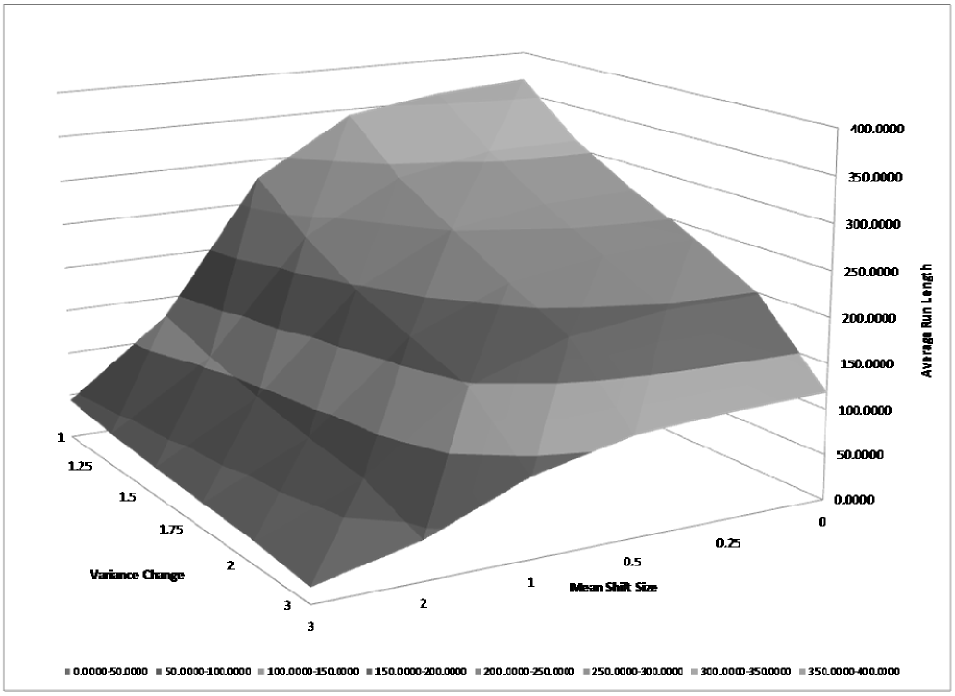

Figure 4.34

ARL curve for the MEWMV control chart when p=10, correlation=0.5 and weight=0.5

Additionally, figures 4.35 and 4.36 displayed the sensitivity comparisons of the MCUSUM, MEWMA and MEWMV control charts for mean shifts and variance changes. Figure 4.34 shows the sensitivity comparison of the three control charts to a mean shift when the variance is held at 1.0. Figures 4.35 and 4.36 display the projections of the three dimensional graphs into a single plane for comparison. Figure 4.35 shows the sensitivity comparisons of the three control charts for a variance change when the mean is held at 0.0. For both of these figures, the weighting values were held at 0.5 for the MEWMA and MEWMV control charts.

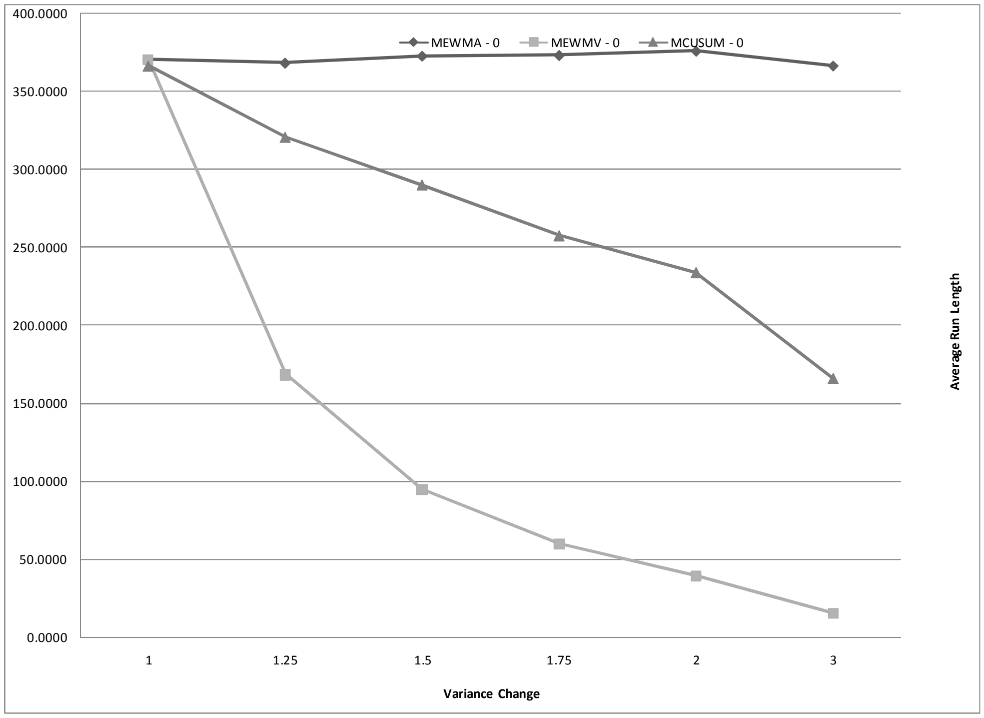

Figure 4.35

ARLs for the MCUSUM, MEWMA, and MEWMV control charts when the mean is held constant and covariance components change with weight=0.5

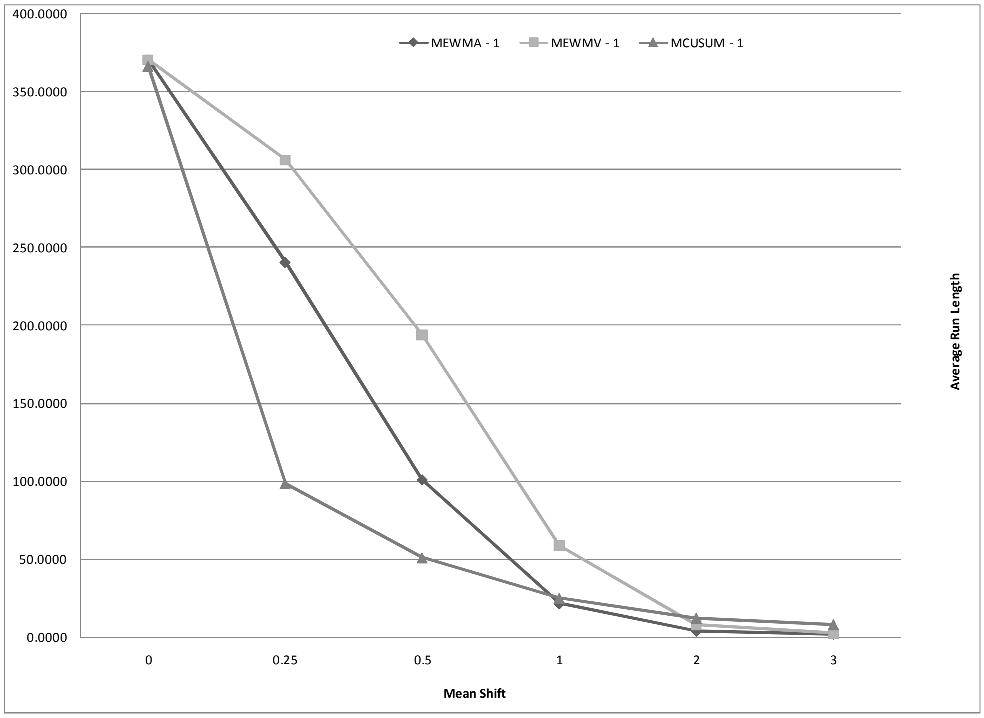

Figure 4.36

ARLs for the MCSUM, MEWMA and MEWMV control charts when variance is held constant and mean component shifts with weight=0.5

Using tables 1 through 8 and tables 27 through 30 as well as figures 4.1 through 4.16 and figures 4.27 through 4.36, the comparisons of the MCUSUM, MEWMA and MEWMV answer question two of this dissertation: “Does the MEWMV control chart monitor for a singular change in the covariance matrix and mean shift more effectively than the MEWMA or the MCUSUM control charts?”

Compared to the MEWMA and MCUSUM control charts, the MEWMV control chart showed a greater sensitivity to detection of a singular change in the covariance matrix than either the MEWMA or MCUSUM control charts. As earlier discussed, the MEWMA is insensitive to the changes of a covariance matrix element and the MCUSUM was insensitive to small changes in the elements of the covariance matrix.

The MEWMV was also capable of detecting a singular mean shift, but was not as sensitive to this detection as either the MEWMA or MCUSUM control charts. In this dissertation, it was shown that as the number of observed variables increased, the sensitivity to detection of a mean shift declined. This characteristic held true for all of the studied control charts. When monitoring multiple related variables, the use of individual observations may not have the precision desired in the detection of an OOC measurement. Future studies in this area will be discussed in chapter five.

The final question posed in this dissertation was, “What are the appropriate values for the control chart parameters for the MEWMS and MEWMV control charts to create an ARL_0 approximately equal to 370 (per Huwang et al.(2007) and Hawkins and Maboudou-Tchao (2008))?” These values were either not previously posted for the re-creation of the control charts or not studied for publication. Using simulations in SAS and multiple trials to determine the critical values for the MEWMS and MEWMV control charts, Table 31 shows the MEWMS control chart critical values for each initial run with a uniform correlation across the covariance matrix and an ARL_0 was set for approximately 370.

Table 31

Control limits for the MEWMS control chart for p-variables when correlation is equal across the covariance matrix.

		Correlation					
p	**Weight**	**0.00**	**0.10**	**0.30**	**0.50**	**0.70**	**0.90**
2.0	0.1	2.873	2.877	3.306	3.331	3.682	4.092
	0.3	3.861	3.898	4.174	4.641	5.194	5.778
	0.5	4.398	4.451	4.794	5.384	6.038	6.722
	0.7	4.731	4.777	5.181	5.805	6.551	7.275
	0.9	4.891	4.951	5.366	6.024	6.785	7.566
3.0	0.1	2.792	2.824	3.120	3.628	4.252	4.933
	0.3	3.662	3.722	4.240	5.068	5.997	6.961
	0.5	4.111	4.201	4.846	5.855	6.931	8.102
	0.7	4.391	4.491	5.225	6.342	7.513	8.751
	0.9	4.535	4.636	5.405	6.581	7.809	9.097
5.0	0.1	2.744	2.809	3.314	4.195	5.202	6.258
	0.3	3.439	3.563	4.510	5.868	7.339	8.859
	0.5	3.808	3.975	5.151	6.781	8.511	10.275
	0.7	4.047	4.225	5.548	7.324	9.231	11.121
	0.9	4.164	4.349	5.748	7.605	9.548	11.549
10.0	0.1	2.732	2.859	3.848	5.375	7.022	8.759
	0.3	3.210	3.472	5.258	7.571	9.957	12.405
	0.5	3.499	3.834	6.035	8.752	11.573	14.358
	0.7	3.678	4.055	6.522	9.492	12.526	15.495
	0.9	3.763	4.177	6.756	9.845	12.984	16.179

Table 32 shows the control chart limits for the MEWMV control chart when the correlations are uniform across the covariance matrix and the ARL_0 was set approximately 370. In the research of Hawkins and Maboudou-Tchao (2008), ARL_0 values ranged from 100 to 2,000. In their research, the common value of $ARL_0 = 370$ was left out. Since this is considered a commonly accepted value of ARL_0, this dissertation focused on establishing the control limit values to define ARL_0 of approximately 370.

Table 32

Control limits for the MEWMV control chart for p-variables when correlation is equal across the covariance matrix.

		Correlation					
p	**Weight**	**0.00**	**0.10**	**0.30**	**0.50**	**0.70**	**0.90**
2	0.1	0.254	0.262	0.322	0.427	0.560	0.733
	0.3	1.617	1.642	1.833	2.160	2.540	2.975
	0.5	3.130	3.180	3.530	4.110	4.780	5.477
	0.7	4.751	4.820	5.340	6.180	7.151	8.150
	0.9	6.475	6.540	7.248	8.850	9.600	10.935
3	0.1	0.068	0.090	0.221	0.459	0.743	1.073
	0.3	1.847	1.910	2.376	3.120	3.950	4.874
	0.5	3.639	3.757	4.594	5.858	7.271	8.775
	0.7	5.501	5.681	6.895	8.750	10.752	12.835
	0.9	7.487	7.710	9.290	11.685	14.308	17.065
5	0.1	0.012	0.060	0.411	0.968	1.612	2.310
	0.3	2.308	2.459	3.591	5.250	7.052	8.952
	0.5	4.474	4.759	6.747	9.550	12.550	15.680
	0.7	6.710	7.130	10.011	13.950	18.180	22.450
	0.9	9.080	9.610	13.429	18.495	24.015	29.469
10	0.1	0.122	0.266	1.303	2.790	4.420	6.064
	0.3	3.250	3.725	6.899	11.020	15.390	19.820
	0.5	6.079	6.922	12.365	19.100	26.190	33.310
	0.7	8.970	10.170	17.985	27.353	37.190	47.229
	0.9	12.000	13.590	23.769	35.910	48.520	61.150

The purpose of these tables was to create a starting point of analysis when values of the scenarios are known. When identifying particular control chart requirements for monitoring real data, having an established starting control limit value allows for faster fine-tuning times to establish working control limits. Additionally, the increasing values of the control limits identified another aspect of sensitivity attached to the MEWMS and MEWMV control charts. The MEWMS control chart based its primary statistic on the trace of the weighted sample covariance matrix. As a result, the critical value is forced to a greater value due to the increase in elements of the equations. Similar effects exist for the MEWMV control chart, as the primary statistic is calculated using the determinant of the covariance matrix.

Summary

As the number of variable numbers increased, the sensitivity of each control chart decreased. The decrease in sensitivity was most noted in the detection of mean shifts. Figure 4.34 displayed the best example of this effect. Similar behavior was displayed with the increase in correlation of covariance elements. As the weighting value increases, the behavior of the sensitivity of the control chart changed. The sensitivity of the MEWMS and MEWMV control charts improved as the weighting value increased from 0.1 to 0.5; however, the sensitivity decreases as the weighting values increase to values of 0.7 and 0.9.

In all four methods described in this dissertation, the MCUSUM and MEWMA control charts displayed the highest sensitivity to mean shifts. The MEWMS and MEWMV control charts displayed nearly identical sensitivity to mean shifts as one another. Only when the weighting value increased to 0.9 did the MEWMS and MEWMV show equivalent sensitivity as the MEWMA control chart in the detection of mean shifts. However, the overall sensitivity of all control charts was decreased with the weighting values exceeded 0.5 in the simulations.

CHAPTER V

DISCUSSION

This chapter provides a discussion of the method development and comparisons of the MEWMS and MEWMV control charts to the MEWMS and MEWMV results provided by Huwang et al.(2007) and Hawkins and Maboudou-Tchao (2008) respectively. Further discussion concerning study limitations, suggestions, and future research are addressed.

Summary of Major Findings

The major findings associated with this dissertation are discussed as part of each individual control chart. With each chart, certain behaviors were observed. These different behaviors allow for comparison of the various control charts. The first finding was that the MEWMS and MEWMV control charts were more sensitive to changes in a single element of the covariance matrix compared to the MCUSUM and MEWMA control charts. The formulas used for the MEWMS and MEWMV control charts were specifically derived to detect this change in variance components where the MCUSUM and MEWMA control charts were designed to detect mean shifts.

The second finding was that the MEWMS and MEWMV control chart were sensitive to mean shifts. While the sensitivity of the MEWMS and MEWMV control charts is not as great as the MCUSUM and MEWMA control charts, the ability of the MEWMS and MEWMV control charts to detect both mean shifts and variance changes allows for greater overall detection capability. This ability to detect mean shifts as well as changes in the covariance matrix gives the MEWMS and MEWMV greater potential for monitoring processes and signaling an OOC situation faster than the MCUSUM or MEWMA may. The development of the MEWMS and MEWMV control charts allows for the monitoring of two types of variables. Since the variance is dependent upon the mean, this two-way monitoring is more effective.

The third finding was as the number of observed variables increased, the sensitivity of all four discussed control charts decreased. This decrease in sensitivity is most noted in the detection of the mean shifts. The MEWMA control chart was the least affected by the increase in number of observed variables. As the number of observed variables increased, the sensitivity to detect smaller shifts significantly decreased. This may be in part due to using a single observation as a test point, rather than using a sample value. Discussions concerning this result are addressed in the further research portion of this chapter. Similar decreases in sensitivity are shown with the increase in correlations of the variables, but were not specifically within the scope of this study.

Method Development

The general trend developing from method development was that sensitivity to detect individual changes in the mean shift and variance components was dependent upon three main properties: 1) the number of variables observed; 2) the strength of correlation between the related variables; and 3) the weighting values used, where applicable. One purpose of this dissertation was to replicate simulations and establish the effects of an increased number of observed variables on the MEWMS and MEWMV control charts with individual observations. Huwang et al.(2007) used p = 2 and p = 3 variables with the MEWMS control chart and Hawkins and Maboudou-Tchao (2008) performed simulations using from two variables up to fifty variables with the MEWMV control chart.

The sensitivity trends related to the number of variables was observed in simulation with all four control charts discussed in this dissertation. Regardless of control chart, as the number of observed variables (p) increased, the sensitivity of the control chart decreased. Decreases in sensitivity were more observable in the detection of mean shifts, rather than variance changes. However, a minor decrease in sensitivity was also observed in the detection of variance change. In the case of the MEWMA control chart, detection of variance changes did not exist.

Behaviors of the ARL charts were also affected by the strength of correlation values used in the simulation. As the correlation values increased through the simulations, the sensitivity decreases. This decrease is most noticeable in the detection of mean shifts, similar to the increased number of observed variables. Decreases in sensitivity were also observable with the detection of the variance change. While this behavior was noted, the true effects of a correlation increase or decrease associated with a variance change will be discussed in the limitations and opportunities sections of this dissertation.

The final developmental factor that influenced the simulation outcomes was that of the weighting values used in the MEWMA, MEWMS and MEWMV control chart scenarios. For these control charts, it was displayed that the most sensitive weighting value for detection of either a mean shift or variance change was $\omega = 0.5$. As the weighting value moved from the 0.5 value, the sensitivity declined; however, Table 33 shows an example of how the sensitivity was affected more from different weighting values, depending upon the control chart used. When combining effects of the number of variables, the weighting value used in each control chart and the correlations between the numbers of variables, the sensitivity of all discussed control charts decreased greatly.

Table 33

ARL values for the MEWMA, MEWMS and MEWMV control charts with constant variance, various mean shifts and different weighting values and p = 2 variables.

		Variance = [1]				
		Weight				
Test	Mean	0.1	0.3	0.5	0.7	0.9
MEWMA	0	371.6496	367.6546	370.3705	365.7777	370.9324
	0.25	110.8831	189.8748	240.4617	277.9842	300.091
	0.5	32.4798	43.7327	100.9875	138.4604	178.7719
	1	9.1762	13.41	21.3927	33.6906	54.9644
	2	2.9241	3.3565	3.8469	5.0211	7.2559
	3	1.6373	1.7902	1.8434	1.9858	2.2714
MEWMS	0	368.3827	358.5521	369.5183	378.9924	371.5148
	0.25	277.4577	294.8288	309.1548	311.7954	305.832
	0.5	144.7199	164.2253	182.2021	192.049	202.4536
	1	39.3032	42.3527	52.1277	59.9244	66.5039
	2	13.0199	7.7118	7.8127	8.621	9.8644
	3	8.5934	4.8119	4.4164	4.7652	5.419
MEWMV	0	376.0233	370.694	370.4231	358.903	373.8134
	0.25	291.5077	298.5762	306.4156	312.447	315.3463
	0.5	167.1225	179.0741	193.7813	200.5399	204.2787
	1	40.5231	50.4534	58.646	65.8687	65.8786
	2	6.2138	6.6228	7.5734	8.8365	8.762
	3	2.6534	2.4019	2.3598	2.4719	2.501

As Table 33 shows, the MEWMA control chart decreases in overall sensitivity as the weighting value increases. The MEWMS and MEWMV control charts show sensitivity behaviors that as the weighting value moves away, both positively and negatively, from 0.5 the sensitivity of the control charts decreases. This behavior is displayed in all scenarios for all sizes of observed variables. The complete lists of tables are shown in Appendix B.

Method Comparison

The MCUSUM control chart, as designed, was highly sensitive to detect small shifts in mean values, including a small shift of a single element of a mean or observation vector. However, the MCUSUM was relatively insensitive to detect a change in a single element of the covariance matrix. It was capable of detecting a large change in a single covariance element, as is shown in simulation, to detect a large variance component change. A large change in the variance component in the covariance matrix was defined as a shift of size three, or a 200% increase in variance.

The MEWMA control chart showed behavior similar to the MCUSUM control chart in its ability to detect small shifts in the mean vector. Compared to the MEWMS and MEWMV control charts, the MEWMA control chart was more sensitive to a change in the mean/observation vector components. The exception to this greater sensitivity was when the weighting values for the MEWMA control chart equation (II.18, p. 18) was above 0.5. Using these values, the sensitivity of the MEWMA, MEWMS and MEWMV control charts were comparable. An unexpected behavior was the complete inability of the MEWMA control chart to detect a change in the covariance matrix, regardless of size. This behavior was observed with the MEWMA control chart only, making it the only control chart insensitive to any change in the variance components.

The MEWMS control chart displayed sensitivity to both mean shifts and variance changes. However, the sensitivity to a mean shift was showed to be less than that of the MCUSUM or MEWMA control charts. Unlike the MCUSUM or MEWMA control charts, the MEWMS control chart’s ability to detect small changes in a single element of the covariance matrix is increased. This increased ability makes the MEWMS control chart a significantly better control chart to simultaneously monitor both mean shift and variance change scenarios than the MCUSUM or MEWMA.

The MEWMV control chart displayed identical sensitivity to the mean shift and variance component changes as the MEWMS control chart. The behavior of the MEWMV control chart should theoretically be identical to the behavior of the MEWMS control chart, since the MEWMV control chart was derived from the MEWMS. Like the MEWMS control chart, the MEWMV control chart was highly sensitive to small changes in a single element of the covariance matrix. Like the MEWMS control chart the MEWMV control chart was also sensitive to mean shifts, but did not display the same level of sensitivity as the MCUSUM or MEWMA control charts.

Comparison to Previous Research

The research presented here both replicated and expanded on the research of the MEWMS control chart of Huwang et al.(2007) and the MEWMV control chart of Hawkins and Maboudou-Tchao (2008). In the study by Huwang, Yeh, and Wu, the number of variables used in simulation were p=2 and p=3. This dissertation expanded beyond the original study and used p=5 and p=10 variables to determine effects of greater dimensions on the sensitivity of the MEWMS control chart. With smaller dimensions, the MEWMS control chart is highly sensitive to singular mean shifts and singular variance changes using individual observations. As the number of observed variables increases, the sensitivity to detection of a mean shift decreases.

Huwang et al.(2007) looked at uniform variance changes that occurred uniformly across the entire covariance matrix. This dissertation evaluated the MEWMS control chart while monitoring for a single variance element change of the covariance matrix.

Findings discussed by Huwang et al. stated that the MEWMS was highly sensitive to changes in variance, especially variance changes that were very small. A uniform variance change across the entire dimension of the covariance matrix is an unlikely event in reality. For this reason, this dissertation explored if one element of the covariance matrix changed.

The MEWMV control chart described by Hawkins and Maboudou-Tchao (2008) used real data and simplified the MEWMS control chart. Hawkins and Maboudou-Tchao also expanded on the number of observed variables used, p = {2, 3, 4, 5, 10, 15, 20, 25, 50}, but limited the weighting value of the MEWMV control chart to values less than 0.5; ω = {.05, .10, .15, .20, .25, .30}. The weighting values for the MEWMV in this simulation were increased to expand beyond ω = 0.5 to monitor for effects on the MEWMV control chart ARLs. Results showed that values above the 0.5 limits defined by Hawkins and Maboudou-Tchao had decreased sensitivity. This previous research explored a various number of ARL_0 values, but never explored the common ARL_0 = 370. This dissertation expanded the MEWMV control chart to define the common ARL_0 of 370.

Hawkins and Maboudou-Tchao (2008) also used sample sizes equal to twice the size of the number of observed variables. In this dissertation, sample sizes were reduced to individual observations to determine the MEWMV control chart capability to detect small mean shifts and variance changes compared to earlier research. Using individual observations was associated with determining the sensitivity of the MEWMV control chart compared to its predecessor, the MEWMS which used individual observations.

Results from the single observation MEWMV control chart compared to the MEWMV control chart defined by Hawkins and Maboudou-Tchao (2008) were similar. The general shapes of the ARL curves were similar; however, the use of single observations changed the control limits of the tests. Monitoring with individual observations influenced the sensitivity of the MEWMV control chart, particularly when monitoring for a mean shift when the dimension was greater than p=5. For this reason, certain other characteristics of the MEWMV control chart changed; especially the control limit values used in simulation.

Hawkins and Maboudou-Tchao (2008) did not define control limits for an ARL_0 of 370. Comparisons of critical values were made against published values for ARL_0=250 and ARL_0=500. With these values as comparisons to the critical values found in simulation, it was shown that some differences arose. The use of single observations caused a reduction in the critical value, h, compared to published critical values of Hawkins and Maboudou-Tchao. The end result was that comparisons of critical values became difficult.

Study Limitations and Suggestions for Further Research

The first limitation of this study was the use of a singular element change in either the mean shift or variance change. Using this limitation tested the true ability of the MEWMS and MEWMV control charts to detect minor changes, but may not exist in reality. With the change of a single element of the covariance matrix, it is reasonable to assume that this change would affect other related variables; especially on occasions when variables were highly correlated. Future study may look at scenarios with various effects of change within the covariance matrix. Real data may be required for this study.

A second limitation was that correlations were uniformly increased throughout the covariance matrix. Simulation data that is complete and sufficient is difficult to create on such a varied scale, and for this reason the uniform correlation changes were used. Future research could use real-world data where correlations reflect realistic changes as a single element of the covariance matrix. Acknowledging that correlations will change differently as different variance elements change, the sensitivity of these tests may improve.

Another limitation was that mean shift and variance changes were positive. Hawkins and Maboudou-Tchao (2008) explored decreasing variance changes using samples. Further studies could work with negative directional changes to determine the sensitivity of the MEWMS and MEWMV control charts with individual observations. While a decrease in variance may indicate a more consistent measurement, it may also help to identify other measurement errors not considered.

This investigation restricted the sizes of mean shift, variance change and dimension changes. These increases were not of uniform size, giving a distorted view of the overall sensitivity of the MEWMS and MEWMV control charts. Further research could pick smaller shift sizes and variance changes to identify the general regions of change to the ARL tables/graphs that identify significant ARL_1 measures. Current research has looked at shift and change sizes of 0.2 incrementally with the MEWMV control chart with individual observations. Other studies associated with mean shift size and variance change size may explore dimensional effects on individual observations. That the MEWMS or MEWMV control charts lose the sensitivity of mean shift or variance change detection based on dimension is currently unknown.

The use of individual observations was another limitation to this study. Individual observations provided sufficient detection of small changes in variance and mean shifts when the dimension, p, was less than five. However, when the number of observed variables increases, this study suggests that individual observations may not adequately detect small changes quickly. Future studies could look at the development of the MEWMS using samples to determine the sensitivity to mean shifts with larger numbers of variables. Hawkins and Maboudou-Tchao (2008) performed initial studies with the MEWMV concerning sample sizes of two times the number of observed variables. Studies beyond this work could look at minimum sample sizes that are useful for large numbers of variables, as two times the number of observed variables may become too large to succinctly detect mean shifts or variance changes.

A final suggestion of further research is that of ideal weighting values. While results suggest that using the weighting value of 0.5 in the MEWMS and MEWMV was the most efficient in detection, this may not be the case. In using large increases in weighing values, the overall best value was potentially overlooked in this simulation study. Further research may restrict weighting values to a narrow range around ω=0.5 to discover a weighting value(s) that produce smaller ARL_1 values than those published in this dissertation.

Conclusions

This dissertation expanded upon two new developments in control charts. The MEWMS control chart was studied using greater dimension of the covariance matrix, and the MEWMV control chart was studied using individual observations. The goal of this investigation was to determine the sensitivity of each control chart when either a single mean element shifted or a single covariance element increased. Both the MEWMS and MEWMV control charts were found to be sensitive to covariance changes when using small numbers of observed variables, *p*. Both control charts displayed sensitivity to mean shifts as well, but lost sensitivity of detection as the number of observed variables increased beyond $p = 5$ when using individual observations.

If a researcher were to monitor the variance components of a manufacturing process, the decision to use individual observations should be considered carefully. In cases where there are few correlated variables to monitor, the use of the MEWMS or MEWMV control charts with individual observations may be appropriate. In the case where more variables are being monitored, it may best suit the researcher to explore the MEWMV control charts using sample mean vectors initially described by Hawkins and Maboudou-Tchao (2008).

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APPENDIX A
Sample SAS Code

```
/* Multiple CUSUM control chart code*/
dm log 'clear'; dm output 'clear';
          options nonumber nodate;

Title 'CUSUM2000';
proc iml;
          create control var {count};

                    do i= 1 to 10000;
                              Flag = 0;          /*Flag to stop iterations*/
                              count=0;           /*Count variable to measure ARL*/
                              D=0;               /*Value holder for calculated values*/
                              var=2;             /*number of variables in simulation*/

                              do while (Flag=0);
                                        m={0, 0};          /*Mean or observation vector*/
                                        l={1 1, 1 1};      /*Covariance matrix*/
                                        s={1 .0,.0 1};     /*Correlation values*/

/*Begin observation generation*/
                              seed = 0;
                              n = 1;
                              sigma =l#s;
                              p = nrow(sigma);
                              b = repeat(m`,n,1);
                                        q = root(sigma);
                              z =normal(repeat(seed,n,p));
                              y =z*q + b;
                                        out=y;
                              j=count+1;
                                        k=2*j;
                    fir={0, 0};        /*Fast initial response values (not used in simulation)*/

                    R=D+(y)-fir`;
                              if R > 0 then D=R;
                                        else D=0;

                    T = R;
                    h = 12.20;                   /*Critical value for test*/

          count = count + 1;

                    flag = (T > h);
                    if count > 9999 then flag = 1;
                    end;
```

```
                append var {count};
        end;
quit;

proc means data = control MEAN;             /*Calculation of ARL*/
        var count; output out = stats;
        run;



/****************************************************************
        var=2, 3, 5, 10          *Dimension of matrices
        m={0, 0}, {0, .25}, {0, .5}, {0, 1}, {0, 2}, {0, 3}
                * Mean or observation vector (dimension expands with *var)

        l={1 1, 1 1}, {1 1.118, 1.118 1.25}, {1 1.225, 1.225 1.5}, {1 1.33, 1.33 1.75}
                {1 1.44, 1.44 2}, {1 1.72, 1.72 3}
                 * Covariance matrices (dimension expanded with increase in *var)

        s={1 0, 0 1}, {1 .1, .1 1}, {1 .3, .3 1}, {1 .5, .5 1}, {1 .7, .7 1}, {1 .9, .9 1}
                *Correlations of related variables (dimension expanded with *var)
****************************************************************/
```

```
/* MEWMA control chart code*/
dm log 'clear'; dm output 'clear';
options nonumber nodate;

Title ‘omega1\MEWMA2000’;
proc iml;
create control var {count};
    do i= 1 to 10000;
        Flag = 0;       /*Flag to stop iterations*/
        count=0;        /*Count variable to measure ARL*/
        D=0;            /*Value holder for calculated values*/
        var=2;          /*number of variables in simulation*/

        do while (Flag=0);
            m={0, 0};
            l={1 1, 1 1};
            s={1 .0,.0 1};
            omega=.1;

/*Begin observation generation*/
            seed = 0;
            n = 1;
            sigma =l#s;

            p = nrow(sigma);
            b = repeat(m`,n,1);
                q = root(sigma);
            z =normal(repeat(seed,n,p));
            y =z*q + b;
            out=y;

            j=count+1;
                k=2*j;

/*Calculation of test statistic*/
        beta= 1-omega;
        gam= (1-omega)**k;
            del = ((omega * (1-gam))/(2-omega));
        sig = del * sigma;
            siginv = inv(sig);

            h = 10.17;      /*MEWMA critical value*/

            B=omega * y;
            Z=C + B;
            C = beta * Z;
```

```
                T = Z * siginv * Z`;
        count = count + 1;
                flag = (T > h);
        if count > 9999 then flag = 1;
        end;
                append var {count};
    end;
quit;

proc means data = control MEAN;          /*Calculation of ARL*/
    var count; output out = stats;
    run;



/*****************************************************************
    var=2, 3, 5, 10          *Dimension of matrices
    m={0, 0}, {0, .25}, {0, .5}, {0, 1}, {0, 2}, {0, 3}
        * Mean or observation vector (dimension expands with *var)

    l={1 1, 1 1}, {1 1.118, 1.118 1.25}, {1 1.225, 1.225 1.5}, {1 1.33, 1.33 1.75}
        {1 1.44, 1.44 2}, {1 1.72, 1.72 3}
         * Covariance matrices (dimension expanded with increase in *var)

    s={1 0, 0 1}, {1 .1, .1 1}, {1 .3, .3 1}, {1 .5, .5 1}, {1 .7, .7 1}, {1 .9, .9 1}
        *Correlations of related variables (dimension expanded with *var)

    omega=.1, .3, .5, .7, .9          * Weighting value used for every level
*****************************************************************/
```

```
/* MEWMS control chart code*/
dm log 'clear'; dm output 'clear';
options nonumber nodate;

Title ‘omega1\MEWMS2000’;

proc iml;
create control var {count};
        do i= 1 to 10000;
                Flag = 0;
                count=0;
                D=0;
                var=2;
                        do while (Flag=0);
                                m={0, 0};
                                l={1 1,         1 1};
                                s={1 .0,.0 1};
                                omega=.1; crit=2.8725;      /*MEWMS critical value*/

/*Begin observation generation*/
                        seed = 0;
                        n = 1;
                        sigma =l#s;
                        p = nrow(sigma);
                        b = repeat(m`,n,1);
                                q = root(sigma);
                        z =normal(repeat(seed,n,p));
                        y =z*q + b;
                                out=y;
                        c= y*y`;
                        j=count+1;
                                k=2*j;

                        V=omega*c + (1-omega)*D;
                                D=V;
                        T=trace(V);



/*Control chart- limit development*/
                w=(omega/(2-omega))+(2-2*omega)/(2-omega) * (1-omega)**(2*(count-
1));
                        hi=var+ (crit)*sqrt(2*var*w);
                        low = var - (crit)*sqrt(2*var*w);

                count = count + 1;
```

```
                flag = (T > hi | T<low);
            if count > 9999 then flag = 1;
            end;
                append var {count};
        end;
quit;

proc means data = control MEAN;              /*Calculation of ARL*/
        var count; output out = stats;
        run;




/****************************************************************
        var=2, 3, 5, 10           *Dimension of matrices
        m={0, 0}, {0, .25}, {0, .5}, {0, 1}, {0, 2}, {0, 3}
            * Mean or observation vector (dimension expands with *var)

        l={1 1, 1 1}, {1 1.118, 1.118 1.25}, {1 1.225, 1.225 1.5}, {1 1.33, 1.33 1.75}
            {1 1.44, 1.44 2}, {1 1.72, 1.72 3}
             * Covariance matrices (dimension expanded with increase in *var)

        s={1 0, 0 1}, {1 .1, .1 1}, {1 .3, .3 1}, {1 .5, .5 1}, {1 .7, .7 1}, {1 .9, .9 1}
            *Correlations of related variables (dimension expanded with *var)

        omega=.1, .3, .5, .7, .9          * Weighting value used for every level
****************************************************************/
```

```
/* MEWMS control chart code*/
dm log 'clear'; dm output 'clear';
options nonumber nodate;

Title 'omega9\MEWMV2000';

proc iml;
create control var {count};
do i= 1 to 10000;
        Flag = 0;
        count=0;
        D=0;
        var=2;
                do while (Flag=0);
                        m={0, 0};
                        l={1 1, 1 1};
                        s={1 .0,.0 1};

                        omega=.9; h =6.475;             /*MEWMV critical value*/

/*Begin observation generation*/
                        seed = 0;
                        n = 1;
                        sigma =l#s;
                        p = nrow(sigma);
                        b = repeat(m`,n,1);
                                q = root(sigma);
                        z =normal(repeat(seed,n,p));
                        y =z*q + b;
                                out=y;
                        c= y*y`;

                        j=count+1;
                                k=2*j;

/*Control chart- limit development*/
                        V=omega*c + (1-omega)*D;
                                D=V;
                        U=det(V);
                        q=log(U);
                        T=trace(V);
                        r=T-q-var;


                count = count + 1;
                        flag = (r > h);
```

```
            if count > 9999 then flag = 1;
                  end;
            append var {count};
      end;
quit;

proc means data = control MEAN;            /*Calculation of ARL*/
      var count; output out = stats;
      run;

/*****************************************************************
      var=2, 3, 5, 10          *Dimension of matrices
      m={0, 0}, {0, .25}, {0, .5}, {0, 1}, {0, 2}, {0, 3}
            * Mean or observation vector (dimension expands with *var)

      l={1 1, 1 1}, {1 1.118, 1.118 1.25}, {1 1.225, 1.225 1.5}, {1 1.33, 1.33 1.75}
            {1 1.44, 1.44 2}, {1 1.72, 1.72 3}
             * Covariance matrices (dimension expanded with increase in *var)

      s={1 0, 0 1}, {1 .1, .1 1}, {1 .3, .3 1}, {1 .5, .5 1}, {1 .7, .7 1}, {1 .9, .9 1}
            *Correlations of related variables (dimension expanded with *var)

      omega=.1, .3, .5, .7, .9         * Weighting value used for every level
*****************************************************************/
```