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Setpoint Tracking Predictive Control in Chemical Processes Based on System Identification

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ABSTRACT

SETPOINT TRACKING PREDICTIVE CONTROL IN CHEMICAL PROCESSES BASED ON SYSTEM IDENTIFICATION

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A Kraft recovery boiler in a pulp-paper mill provides a means for recovery of the heat energy in spent liquor and recovery of inorganic chemicals while controlling emissions. These processes are carried out in a combined chemical recovery unit and steam boiler that is fired with concentrated black liquor and which produces molten smelt. Since the recovery boiler is considered to be an essential part of the pulp-paper mill in terms of energy resources, the performance of the recovery boiler has to be controlled to achieve the highest efficiency under unexpected disturbances.

This dissertation presents a new approach for combining system identification technique with predictive control strategy. System identification is the process of building mathematical models of dynamical systems based on the available input and output data from the system. Predictive control is a strategy where the current control action is based upon a prediction of the system response at some number of time steps into the future. A new algorithm uses an i-step-ahead predictor integrated with the least-square technique to build the new control law. Based on the receding horizon predictive control approach, the tracking predictive control law is achieved and performs successfully on the recovery boiler of the pulp-paper mill. This predictive controller is designed from ARX coefficients that are computed directly from input and output data. The character of this controller is governed by two parameters. One parameter is the prediction horizon as in traditional predictive control and the other parameter is the order of the ARX model. A recursive version of the developed algorithm can be evolved for real-time implementation. It includes adaptive tuning of these two design parameters for optimal performance. The new predictive control is proven to be a significant improvement compared to a conventional PID controller, especially when the system is subjected to noise and disturbances.

To

Sin Chin Hiu

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This dissertation would not be possible without my parent's love, understanding, encouragement and support. Special thanks to my wife, Kallayanee Chinvorarat, and our daughter, Jittrerat (Bai-fern) Chinvorarat for their patience, love and sacrifice.

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ABBREVIATIONS

AR	Auto-Regressive
ARX	Auto-Regressive with eXogeneous input
MA	Moving-Average
ARMAX	Auto-Regressive Moving Average with eXogeneous input
ARIMAX	Auto-Regressive Integrated Moving-Average eXogenous input
ARIX	Auto-Regressive Integrated eXogenous input
FIR	Finite Impulse Response model
OKID	Observer/Kalman Filter Identification
CLID	Closed-Loop Identification
DMC	Dynamic Matrix Control
PCA	Predictive Control Algorithm
MAC	Model Algorithmic Control
GPC	Generalized Predictive Control
EPSAC	Extended Prediction Self-Adaptive Control
EHAC	Extended Horizon Adaptive Control
BTU	British Thermal Unit

LIST OF SYMBOLS

Unless otherwise stated the listed symbols are specified as follows.

A, B, C	open-loop system matrices
A_c, B_c, C_c	closed-loop system matrices
A_d, B_d, C_d, D_d	system matrices of the dynamic output feedback controller
E, F, G, H	Diophintine coefficient
H	Hankel matrix
H_H	Hessian
I	identity matrix
J	criterion function
K	Kalman filter gain
Y_S	open-loop system Markov parameter
Y_K	open-loop Kalman filter Markov parameter
Y_D	controller Markov parameter
Y_{SC}	close-loop system Markov parameter
Y_{KC}	close-loop Kalman filter Markov parameter
a_i , b_i	coefficient matrices of ARX model
d	time delay
e	a random, white noise process
g	gradient
h_p	prediction horizon
h_c	control horizon
h_m	minimum-cost horizon
n	number of states

n_i	number of inputs
n_o	number of outputs
p	ARX model order
r	reference input
u	control input
w	desired process output
x	system state
y	system output at time k

Greek Letters

ε	residual output
η	augment closed-loop system dynamics
θ_{okid} or θ_{clid}	coefficient matrix of ARX model
κ	process noise
υ	measurement noise
ξ	disturbances
Φ_{okid} or Φ_{clid}	input-output data matrix
χ	error covarience

Subscripts

k	k-th time step
i, j	discrete number

Superscripts

T	matrix transpose
-1	matrix inverse

Notation above a symbol

$\hat{}$	estimate
$\cdot$	time derivative

CHAPTER I

INTRODUCTION

1.1 Background and Problem Statement

It has been projected that the world consumption of paper will triple by the year 2000[1]. About 90% of the raw material for paper manufacture primarily comes from tree boles, the bark-free trunk. Bark that constitutes about 15% of a mature tree is considered one of the most serious contaminants in wood chips used for the manufacture of most pulp[2]. Black liquor is produced in the manufacture of the pulp by the Kraft pulping process using coniferous wood which contains lignins, carbohydrates, and other organic matter from the processed wood along with the pulping chemical used in processing. The amount of organic material dissolved in the spent pulping liquors has been estimated to be 45 million tons per year[3] and thus represents an enormous source of fuel.

Since the energy crisis, the American Paper Institute has started conducting annual surveys of the sources of energy used by the U.S. pulp and paper industry. The survey indicated that spent pulping liquors provided 32%, fuel oils 22%, natural gas 21%, coal 12%, electricity 5%, bark 5% , and hogged wood 2% of the industry total BTU requirements[4]. It is encouraging to note the growing use of wood wastes and processing wastes as a source of fuel in the pulp and paper industry.

Industrial processes in general are quite different; they are highly multivariable systems. Perturbations affect the plant structure more often than the measurement variables. Industrial processes have their own performance criteria and reliability requirements. The economic and psychological environments required for a successful implementation are often not met in practices. Many constraints prevent the implementation of on-line control schemes on production plant.

Kraft recovery technology is a mature, fairly well standardized, technology. The last major innovation was the introduction of the Tomlinson recovery boiler in the 1930's.

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Since then, there has been a gradual evolution toward larger, more efficient systems, but the basic technology has remained essentially the same. A Kraft recovery boiler in a pulp-paper mill provides a means of recovery of the heat energy in spent liquor and recovery of inorganic chemicals while controlling emissions. These processes are carried out in a combined chemical recovery unit and steam boiler that is fired with concentrated black liquor, and produces molten smelt. An acceptable operation of the Kraft recovery boiler requires simultaneous satisfaction of a number of objectives: high steam efficiency, stable operation, low emission and disturbance rejections due to the change of heating potential of black liquor.

Black liquor is a complex mixture of organic and inorganic material. The organic components of the black liquor are derived from the wood chips and will burn to produce a considerable quantity of heat. The inorganic materials from the cooking liquor are burned, undergo chemical changes, and are discharged from the furnace hearth as smelt.

Ideally there are three things that have control over black liquor as a fuel. They are flow rate, percentage solids, and heating value. In practice the solids and flow rate of black liquor are controlled. The third variable, the BTU value of the liquor changes depending on the type and properties of the wood being pulped and is beyond the control of the operator. Normally, the black liquor averages about 5,800 BTU's per pound of black liquor solids. This is an average figure and can change from about 5,000 to 6,500 BTU's per pound. Because of the changes in the heating potential of the black liquor, that comes from the property of wood from different seasons, the heat from the furnace can change even if the flow rate and percentage solids remain constant. However, changing liquor BTU values can be compensated by adjusting the quantity of air admitted to the boiler. The Proportional-Integral-Derivative (PID) control is the common commercial controller used today for Kraft recovery boilers in pulp-paper mills. However, the performance of the controller primarily depends on the controller gains. These gains are determined experientially by operators, and need to be improved due to the complexity of the processes. As new technology is developed, it is hoped that the resources of pulp-paper mills will be utilized more efficiently

System identification is the process of building mathematical models of dynamical systems based on the available input and output data from the systems[5-6]. This technique

is also important in many other disciplines, such as economics, communication, and system dynamics. Mathematical models allow researchers to understand more about the properties of the system, so that they can explain, predict or control the behavior of the system. For an unknown system, the system model must be identified before performing the control design. There are many system identification techniques regarding to different kinds of need such as the nature of the system and the purpose of identification. The system identification process normally begins by selecting a suitable model structure and then choosing the model parameters to minimize a defined cost objective that indicates the fitness of the model to the input and output data.

The concept of predictive control originated in the late seventies and has evolved to a mature level[7]. Predictive control belongs to the class of model-based controller design concepts which have remarkable features. It can be used to control a wide variety of processes, among which are non-minimum phase and unstable processes, without the designer having to take special precautions. Predictive controller design was developed specifically to address the non-minimum phase problem[8]. There are "three" principle design parameters: the control weight, the predictive horizon and the control horizon that guarantees stability of the predictive control law. Since predictive controllers belong to the class of model-based controller design methods, a model of the process must be available. In general, two phases in designing a control system can be distinguished as modeling and controller design. A model of a process normally can be constructed by the system identification technique. Therefore, combining the system identification technique and the controller design might be the best way to reduce the complexity of designing control system.

1.2 Objective

This dissertation combines the system identification and the predictive control law into one formulation and makes it suitable for the system engineer to implement the algorithm for a real time tracking control problem.

First, the concept of system identification is shown for both open-loop and closed-loop systems. By using the selecting model, one can realize mathematical model in a state space form that represents the dynamic behavior of the unknown system.

Second, the concept of predictive control is introduced. It starts with process mod-

els and prediction. Disturbances for both deterministic and stochastic are shown to incorporate in the model. Criterion function is formed in order to build the predictive control law with transient and steady state arguments.

Finally, by using a multi-step-ahead output prediction, the new predictive control law is derived from the identified matrices through a recursive Least-Square technique. The identified matrices minimize the output error between the estimated and real outputs. The algorithm computes the predictive control command directly from input/output time histories without explicitly identifying the system parameters. This approach has "two" design parameters: control horizon and the Auto Regressive with eXogeneous input (ARX) model order that related to the order of the system. By appropriate adjusting these two parameters, the predictive tracking controller can be achieved and implemented in real time.

1.3 Dissertation Outline

Chapter 2 shows the details of the pulp-paper mill process starting with a general description of the mill, and a detailed description of the function of the recovery boiler unit. Later, the recovery boiler is used as a model to investigate the performance of the developed controller.

Chapter 3 introduces the existing open-loop and closed-loop system identification techniques. The chapter starts by introducing several kinds of mathematical models used in the identification processes. By analyzing a sufficient quantity of input and output data through the system identification technique, a state space representation can be obtained by using an eigen-value realization technique. When the system is unstable, the closed-loop system identification that required to have a feedback control is the other technique to make the overall system stable before identifying. In addition, the Least-Squares technique for both batch and recursive solutions is also presented in this chapter.

Chapter 4 presents the general concept of predictive control design. By solving the Diophantine equation, an i-steps-ahead predictor with or without disturbances is obtained. This long range prediction form is suitable for building a predictive control law for unstable an non-minimal system. The predictive control law that minimizes a criteria function is also presented.

Chapter 5 proposes the tracking predictive control technique that implements system identification and predictive control into one formulation. First, an i-step-ahead prediction output is derived by using the Auto Regressive with eXogeneous input (ARX) model. By using the principle of a receding horizon, the new predictive control law can be obtained by integrating the predictive output equation through the system identification algorithm. The control law can be obtained by either off-line calculations or on-line operation, using the recursive Least-Squares technique.

Chapter 6 verifies the new control design algorithm by numerical simulation. Conventional Proportional-Integral-Derivative (PID) controller results are shown first, followed by the results of the new predictive tracking control. Comparison and evaluation of these two controllers performances are presented with and without external disturbances.

Finally, chapter 7 provides conclusions and prospects for the extension of this research.

CHAPTER II

PULP-PAPER MILL

2.1 Introduction

Paper is manufactured from wood[9], a naturally renewable resource, by a large industry with significant economic impact on the world economy. The manufacture of pulp and paper products is particularly challenging, as it is large in scale, highly non-linear, highly stochastic, and dominated by time delay. Wood consists mainly of fibers and lignin. The fibers give wood flexibility and strength, while the lignin acts as a cement or blinder. In order to make paper, the lignin must be dissolved from the fibers. This process employs white liquor (caustic soda and sodium sulfide) as the lignin dissolving agent under high temperature and pressure. During pulping about one half of the dry weight of wood is dissolved by the white liquor. The combination of spent white liquor and dissolved organic compound from the wood is called black liquor. Black liquor is removed from the pulp fibers at the washing and screening process.

At the time the black liquor is extracted from the pulp, it contains about 15% solids or fifteen pounds of solids and eighty-five pounds of water per one hundred pounds. This weak black liquor is concentrated in the multiple effect evaporators to about 50% solids. After concentration, the heavy black liquor is further concentrated in the cascade evaporators at a recovery boiler to about 65% solids. About 50% of wood are dissolved in the pulping process. The organic material in the wood and inorganic cooking chemicals used to cook the wood add up to about 3,300 pounds per ton of pulp produced.

With the black liquor concentrated to the desired level, the liquor is burned in the recovery boiler to produce heat and chemical smelt. The heat is used to generate steam and the smelt is added to water in the smelt dissolving tank to form green liquor. The green liquor contains nearly all of chemicals required for cooking process. These chemicals are not in the form to be used for cooking. The green liquor is converted to white liquor in the caustic room and can be used again in the cooking process.

2.2 Pulp-Paper Mill

The system blocks diagram is shown in Fig 2.1. Included in the pulp paper mill are Chemical Receiving, Chlorine Dioxide Plant, Wood Yard, Digesters, Washing and Screening, Bleach Plant, Evaporator Sets, Slakers and Causticizers, Lime Kilns, Recovery Boilers, Power Boilers, Boiler Feed-water, Turbine Generator, and Bleached Pulp Storage. The blocks and process arrows shown in heavy lines show the principle process flow in the pulp-paper mill. The following are brief descriptions of each block.

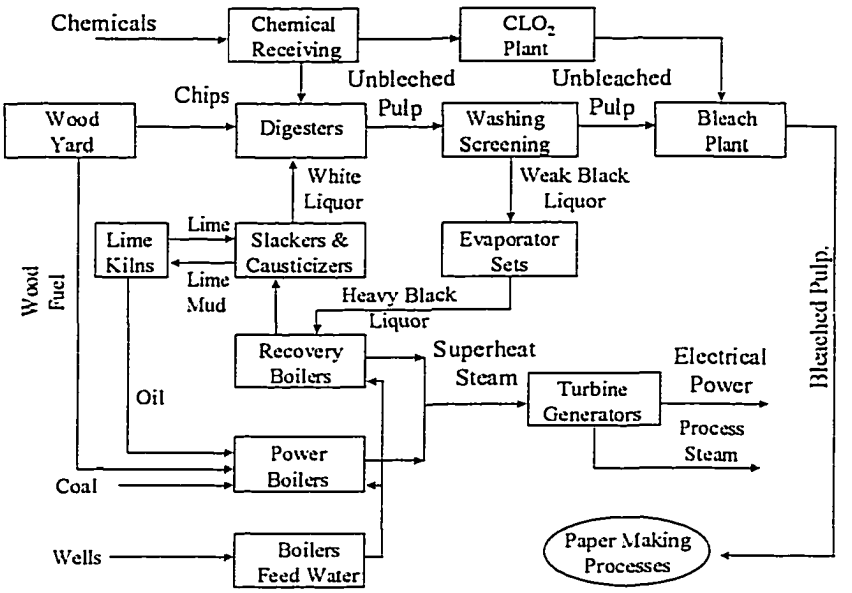

Fig 2.1 Simplified Block Diagram of a Pulp-Paper Mill

- Chemical Receiving

The inputs are various chemicals in bulk form. The bulk chemicals are prepared for use in suitable form and sent to the digesters, CLO2 plant, and to the bleach plant.

- Chlorine Dioxide Plant

The chemicals react to produce chlorine dioxide and some of the chlorine used in the bleach plant.

- Wood Yard

The wood yard receives pine and hardwood pulpwood logs, unscreened pine and hardwood chips, sawdust from sawmills, and whole tree hardwood chips. Pulpwood logs

are debarked and reduced to chips. All chips and sawdust are screened to remove fines and oversized pieces. Wood yard outputs to the pulp mill are screen pine and hardwood chips, screened whole tree chips, and screened hardwood and pine sawdust. Outputs to the power boiler wood fuel system are bark, and fines screened from the chips and sawdust.

- Digesters.

Screened chips and sawdust are combined with heated cooking chemicals in the digesters. These produce unscreened unbleached wood pulp that is sent to the washing and screening system.

- Washing and Screening

Unscreened, unbleached pulp is washed and screened to remove weak black liquor, undigested knots, and fiber bundles. Weak black liquor is made up of resinous wood residue and spent cooking liquors. Fiber bundles are fibers adhering to each other. The accepted washed pulp is sent to the bleach plant, the weak black liquor goes to the evaporator set, and the undigested knots and fiber bundles are recycled back to the digesters.

- Bleach Plant

The plant combines chemicals from the chlorine dioxide and chemical receiving plant, and unbleached pulp from the washing and screening system to produces bleached pulp.

- Lime Kiln

Lime mud is heated by burning fuel oil in the kiln. This converts it into lime for reuse in the slakers and causticizers.

- Slakers and Causticizers

The inputs consist of dissolved smelt (green liquor) from the recovery boiler, lime, and process water. A chemical reaction produces recovered cooking liquor (white liquor) for the digesters and lime mud that is sent to the lime kilns.

- Evaporator sets

Using 60 psi steam, the evaporators thicken the weak black liquor. The output is heavy black liquor that is the fuel input to the recovery boilers.

- Recovery Boilers

These recovery boilers receive heavy black liquor that is burned as fuel. Other inputs are boiler feedwater and air. The outputs are 1500 and 600 psi steam to the main steam headers and to the turbine generators, and dissolved smelt (molten chemical salts, dissolved in water) to the slakers and causticizers.

- Power Boilers

The inputs to the power boilers are feedwater, air, wood, coal or oil fuel. The output from the power boilers is 600 psi superheated steam to the main steam header and to the turbine generator.

- Turbine Generators

These turbine generators receive 600 psi steam from power boilers and 600 psi and 1500 psi steam from the recovery boilers. Steam is converted into electricity for use throughout the mill. Reduced pressure steam at 150 psi and 60 psi is extracted from the generators for use in all process operations.

- Boiler Feedwater

Input to the boiler feedwater is water from an artesian well and from a river. After screening, the water is used for selected process water. Water is demineralized in ion exchange units, and has oxygen removed by deaerators and chemical treatment. Afterwards it is suitable for use as boiler feedwater.

2.3 Recovery Boiler Operation

The recovery boiler has three primary functions. The first is to burn the organic material from the black liquor and form an inorganic chemical smelt. The second function is to utilize the heating potential of the organic matter in the black liquor to generate steam. Third, when operated correctly, the recovery boiler allows black liquor to be used in a way that is virtually harmless to the environment.

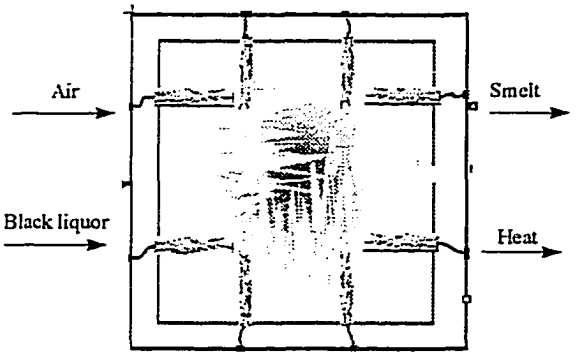

Figure 2.2 Furnace Unit in the Recovery Boiler

The recovery boilers are initially fired using fuel oil and are brought up to a furnace temperature of about 3,000 degree Fahrenheit. When the black liquor is injected and starts burning, the fuel oil is cut off. In the boiler, the high temperature from the steam evaporates the remaining water from the black liquor. By proper control of air, the carbon in the black liquor is burned to release heat. Oxygen in air provided by a fan and the high temperature causes the dried organic material in the black liquor to burn and sustain combustion. This burning mass forms a bed on the furnace hearth. The heat of combustion is used to convert feedwater into steam and to superheat steam.

The second major function of the recovery boiler is to generate steam. As mentioned previously, for each ton of pulp about 3,300 pounds of black liquor solids is produced. Of these 3,300 pounds, about 1,600-1,800 pounds are organic compounds derived from the wood and 1,500-1,700 pounds are chemicals that come from the white liquor. Due to the woody or organic content of the black liquor, the liquor has a heating potential of about 5,800 BTU per pound of black liquor solids. The chemical reaction of black liquor is shown as follow:

Chemical used in Recovery Boilers

Black Liquor + Oxygen ===> Smelt + Heat

Sodium Lignates

It takes about 1,000 BTU's to make one pound of 600 psig steam. Thus with perfect utilization of all the heat from each pound of black liquor solids, it would be possible to produce about six pounds of steam. However, in practice, heat is lost in a number of

places. It requires a considerable quantity of heat to evaporate the remaining water from the black liquor. Good recovery operation can achieve over 3 pounds of steam per pound of the black liquor solids.

2.4 Recovery Boilers Control

A recovery boiler can be controlled to obtain optimum performance of both steam generation and chemical recovery. In its simplest form a recovery boiler can be thought of as a device that takes black liquor and air and converts them into steam and green liquor.

2.4.1 Black Liquor

Black liquor is a complex mixture of organic and inorganic material. The organic components of the black liquor are derived from the wood chips and will be burned to produce a considerable quantity of heat. The inorganic materials from the cooking liquor are burned, undergo chemical changes, and are discharged from the furnace hearth as smelt.

Ideally there are three things that have control over black liquor as a fuel. They are flow rate, percentage solids, and heating value. In practice, the solids and flow rate of black liquor are controlled. The third variable, BTU value of the liquor, changes depending on the type of wood being pulped and is beyond the control of the operator. Normally the black liquor averages about 5,800 BTU's per pound of black liquor solids. This is an average figure and can change from about 5,000 to 6,500 BTU's per pound. Because the changes of heating potential of the black liquor, the heat from the furnace can change even if the flow rate and percentage solids remain constant. However, changing liquor BTU values can be compensated by adjusting the quantity of air admitted to the boiler.

2.4.2 Air

About five pounds of air are required for each pound of black liquor solids burned. The control of air is critical for optimum performance of the recovery boiler. At each firing rate a certain quantity of air is required to burn the black liquor. The air requirement has been determined by experience and is set within determined limits. During normal operations at a given firing rate, few adjustments are made to the total air flow. However, adjustments are necessary if the feed rate of black liquor solids changes or if the BTU value of the liquor changes. Too little air will cause incompletely burned furnace gases to escape and the bed to build up in size. By not completely burning all the fuel, the maxi-

mum heating potential of the liquor is not utilized and steam production per pound of black liquor solids is decreased. Too much air, on the other hand, causes an increased load of relatively cold air to enter the furnace and thus cools off the hot furnace gases which also can cause a decrease in the production of steam per pound of fuel.

2.5 Conclusion

Due to the complexity of paper making process, this chapter presents a general view of a pulp-paper mill and more specifically in the recovery boiler that is used as a model in Chapter 6. For more details of the recovery boiler operation, see reference 9.

CHAPTER III

SYSTEM IDENTIFICATION ALGORITHM

3.1 Introduction

There are normally two ways to obtain a model for a dynamics system. The first is an analytical approach. It normally starts from deriving governing equations based on physical laws, and then simplifies and/or solves the equations and transforms them into a desired form. One example is the finite element method. The second is an experimental approach. It processes the measured input and output data obtained from experiments or operations to find a model directly, such as some testing techniques. In general, the models obtained through analytical ways need further confirmation and refinement using experimental results before they are used for control design.

System identification is the technique that deals with building mathematical models for a dynamic system based on their input and output data[10]. Besides control, this technique is also important in many other disciplines, such as economics, communication, and system dynamics. Mathematical models allow researchers to understand more about the properties of the system, so that they can explain, predict or control the behavior of the system.

The system identification was first coined by Lotfi Zadeh in 1962. He defined system identification as:

Identification is the determination, on the basis on input and output, of a system within a specified class of systems, to which the system under test is equivalent.

This definition is highly systems oriented, and does not reflect a strong statistical flavor of system identification techniques. Nevertheless, the term caught on and soon become the standard terminology in a control community. In statistics, econometrics, geophysics, signal processing, etc., where models of dynamic systems are also built based on observed input output data. The choice of an identification method depends on the nature of the system and the purpose of identification. Most existing system identification meth-

ods apply for a stable system without requiring feedback terms for identification purposes. For identifying marginally stable or unstable systems, however, feedback control is required to ensure overall system stability. In many cases, a system, although stable, may be operated in closed loop and it is impossible to remove the existing feedback controller for security or production reasons. In other cases, such as economic and biological systems, the feedback effect may be inherent. Consequently, identification has to performed on a system operating in closed-loop.

There are basically two classes of system identification methods, the nonparametric system identification methods and the parametric system identification methods. The nonparametric methods were developed from the classical control theory. The frequency analysis technique plays a major role for this development. This methodology made it possible to determine the transfer function accurately, in a format that could be used for the control design. The parametric methods, however, are developed from modern control theory. Problems are introduced with these parametric models such as the identification processes have to solve parameter estimations and other related techniques. This development introduced a renewed interest in the field of parameter estimation techniques.

There are generally three ways to apply the identification methods[11-12]. One way that can always be applied is to treat the bounded plant input/output data exactly as if they were obtained from an open-loop experiment. This is called direct identification. Another is to treat the closed-loop system as a whole, and its dynamics can first be identified by some common methods. Then the open-loop plant dynamics may be determined from the identification closed-loop system dynamics using the knowledge of the feedback controller. This approach is called indirect identification. The third approach is called jointly input-output identification where the feedback controller may be considered as part of what is to be identified and then the input and output are considered as a joint process and the output of a system driven by noise only.

3.2 Type of Model Structure

A model of a system is a description of its properties, suitable for a certain purpose. The model need not have a true and accurate description of the system, nor need the users have to believe so, in order to serve its purpose. System identification deals with the

problem of building the mathematical models of dynamical systems relating to the input and output data. A crucial point in the process of identifying a system is the selection of a candidate model. The choice of the model structure will greatly influence the identification process and its results. The criterion of model equality is normally based on how well the model could perform when attempting to fit the measured data. A prior knowledge regarding the system would be very helpful in choosing model structure, although some information may be learned from analyzing measured data, if they are not a prior given.

The following, four different types of model structures will be briefly discussed, although, only one will be use later in this work, but it is worthwhile to introduce the differences and characteristics of the other models.

3.2.1 AR Model

In general, the AR model is a linear discrete time filter, which is excited by random impulse. AR means Auto-Regressive. The output is an autoregressive by itself. The model can be described as follows

$$y_k = \sum_{i=1}^{p} a_i y_{k-i} + e_k \tag{3.1}$$

where y_k are output at time k, y_{k-i} are the autoregressive part, a_i are the AR model parameter and e_k is a random, white noise process.

3.2.2 ARX Model

The ARX means the Auto-Regressive with eXogenous input. This model is an extension of the AR model by additional the exogenous input which is commonly used in developing recursive system identification technique. The model can be described as follows

$$y_k = \sum_{i=1}^{p} a_i y_{k-i} + \sum_{i=1}^{p} b_i u_{k-i} + e_k \tag{3.2}$$

where y_k are output at time k, y_{k-i} are the autoregressive part, u_{k-i} are the exogenous part, a_i and b_i are the AR model parameter and e_k is a random, whitenoise process.

3.2.3 MA Model

The MA model represents the Moving-Average term. The model can be described as follows

$$y_k = \sum_{i=1}^{p} a_i e_{k-i} + e_k \tag{3.3}$$

where a_i are the moving average parameters and e_{k-i} are white , gaussian noise term.

3.2.4 ARMAX Model

The ARMAX model includes the same extension as the AR model to an ARX model . The ARMAX means the AutoRegressive Moving Average with eXogeneous input model, the model can be described as

$$y_k = \sum_{i=1}^{p} a_i y_{k-i} + \sum_{i=1}^{p} b_i u_{k-i} + \sum_{i=1}^{p} c_i e_{k-i} + e_k \tag{3.4}$$

where a_i, b_i and c_i are ARMAX parameters. The model contains the moving average terms of noise dynamic which are difference from ARX model.

3.3 Least-Square Method

The Least-Squares method is a basic technique for parameter estimation. The method is particularly simple if the model has the property of being linear in the parameters. Karl Friedrich Gauss formulated the principle of Least-Squares at the end of the eighteenth century and used it to determine the orbits of planets and asteroids[13]. Guass stated that , according to this principle, the unknown parameters of a mathematical model should be chosen in such a way that

The sum of the squares of the differences between the actually observed and the computed values, multiplied by numbers that measure the degree of precision, is a minimum.

In great many different fields including the system identification applications, the Least-Squares method reached a significant achievement, and was modified according to different requirements.

3.3.1 Batch Least-Square Solution

The Batch Least-Square estimation is the technique to determine the parameters in such a way that the outputs computed from the model agree as closely as possible with the measured variables $y(i)$ in the sense of Least-Square.

For a mathematical model that can be written in the form

$$y(i) = \varphi_1(i)\theta_1 + \varphi_2(i)\theta_2 + \ldots + \varphi_n(i)\theta_n = \varphi^T(i)\theta \tag{3.5}$$

where y is the observed variable, $\theta_1, \theta_2, \ldots, \theta_n$ are parameters of the model to be determined, and $\varphi_1, \varphi_2, \ldots, \varphi_n$ are known functions that may depend on other known variable.

The Batch Least-Square solution can be obtained by minimizing the Least-Square lost function

$$\Theta(\theta, t) = \frac{1}{2}\sum_{i=1}^{t} (y(i) - \varphi^T(i)\theta)^2 \tag{3.6}$$

which results in

$$\hat{\theta} = (\Phi^T\Phi)^{-1}\Phi^T Y \tag{3.7}$$

3.3.2 Recursive Least-Square Estimation

In many cases it may be necessary to estimate a model on-line while the process is in operation. The model will be updated when the new observations are available. Hence for computing efficiency, it is desirable to arrange the algorithms in such a way that the results obtained previously can be used for on-line updating. This way of computing the estimates is called recursive.

Assume that the matrix $\Phi(t)$ has gull rank, that is $\Phi^T(t)\Phi(t)$ is nonsingular, for all $t \geq t_0$. Given $\hat{\theta}(t_0)$ and $W(t_0) = (\Phi^T(t_0)\Phi(t_0))^{-1}$, the Least-Squares estimate $\hat{\theta}(t)$ then satisfies the recursive equations

$$Z(t+1) = W(t+1)\varphi(t+1)^T = \frac{W(t)\varphi(t+1)^T}{[1 + \varphi(t+1)W(t)\varphi(t+1)^T]} \tag{3.8}$$

$$W(t+1) = W(t)[I - \varphi(t+1)Z(t+1)] \tag{3.9}$$

$$\hat{\theta}(t+1) = \hat{\theta}(t) + Z(t+1)[y(k+1) - \hat{y}(k+1)] \tag{3.10}$$

where $y(k+1)$ and $\hat{y}(k+1)$ are the observation and predicted values, respectively. Here no matrix inversion is needed and no approximation has been made in the derivation of the Recursive Least-Square. Therefore, the Recursive Least-Squares estimate and the off-line estimate are theoretically identical. This is the advantage of the Least-Squares method. The updating of $W(t+1)$ is not always numerically robust. Rounding errors may accumulate and make the computed $W(t+1)$ indefinite, even though it is theoretically always positive. When $W(t+1)$ becomes indefinite, the parameter estimates tend to diverge. A way to overcome this difficulty is to use a square root algorithm. Define $\aleph(t+1)$ through

$$W(t+1) = \aleph(t+1)\aleph(t+1)^T \tag{3.11}$$

and update $\aleph(t+1)$ instead of $W(t+1)$. The initial values $W(0)$ and $\hat{\theta}(0)$ can be obtained by the off-line Least-Squares method.

3.4 Open-Loop System Identification Algorithm

When the open-loop system needs to be identified, one normally excites a system directly by a random signal without a controller, and identifies the system from input and output data. Chen et al.[14-15] used the finite difference model named AutoRegressive with eXogeneous input (ARX) to identify a state space model from an open-loop system. This model, however, is derived through Kalman filter theories. The method requires a large number of the ARX model order which consumes a lot of computational time. Juang et al.[16] also developed a technique of implementing a state observer so that the ARX model order can be reduced.

3.4.1 Algorithm for Open-Loop System Identification

Given a linear, stochastic, finite dimensional, discrete-time , time invariance system . The system can be expressed by

$$x_{k+1} = Ax_k + Bu_k + \kappa_k \tag{3.12}$$

$$y_k = Cx_k + \upsilon_k \tag{3.13}$$

where $x_k \in R^{n\times 1}$ is the state vector, $u_k \in R^{s\times 1}$ is the input vector, $y_k \in R^{m\times 1}$ is the output vector, $[A, B, C]$ are the state-space system matrices. The sequence of the process noise κ_k and the measurement noise υ_k are assumed to be Gaussian , white, zero-mean, and stationary with covariance Q and R respectively.

By introducing a typical Kalman filter, equation (3.12) and (3.13) can be rewritten in the form

$$\hat{x}_{k+1} = A\hat{x}_k + Bu_k + AK\varepsilon_k \tag{3.14}$$

$$\hat{y}_k = C\hat{x}_k + \varepsilon_k \tag{3.15}$$

where ε_k is the residual output and $\varepsilon_k = y_k - \hat{y}_k$, $\hat{x}_k$ and $\hat{y}_k$ are the estimated state vector and output vector respectively, K is the steady state Kalman filter gain. At steady-state, the error covariance, χ, reaches a constant value which satisfies the steady state algebraic Riccati equation

$$\chi = A\chi A^T - A\chi C^T[R + C\chi C^T]^{-1}C\chi A^T + Q \tag{3.16}$$

The steady-state Kalman filter gain matrix is

$$K = A\chi C^T[R + C\chi C^T]^{-1}. \tag{3.17}$$

The existence of K is guaranteed if the system is detectable and $(A, Q^{1/2})$ is stabilizable.

The system in equation (3.14) can be expressed in the form

$$\hat{x}_{k+1} = (A - AKC)\hat{x}_k + Bu_k + AKy_k \tag{3.18}$$

$$\hat{y}_k = C\hat{x}_k + \varepsilon_k \tag{3.19}$$

The z-transform of the open- loop state space equation (3.18) and (3.19) are

$$x(z) = (z - \bar{A})^{-1}[AKy(z) + Bu(z)] \tag{3.20}$$

$$y(z) = Cx(z) + \varepsilon(z) \tag{3.21}$$

By inserting (3.21) in (3.20) , one gets

$$y(z) = C(z - \bar{A})^{-1}[AKy(z) + Bu(z)] + \varepsilon(z) \tag{3.22}$$

Taking the inverse z-transform of equation (3.22) with $(z-\bar{A})^{-1} = \sum_{i=1}^{\infty} A^{i-1} z^{-i}$.

The new relation between the input and output with zero initial condition can be described as

$$\hat{y}_k = \sum_{i=1}^{\infty} C\bar{A}^{i-1} AKy_{k-i} + \sum_{i=1}^{\infty} C\bar{A}^{i-1} Bu_{k-i} + \varepsilon_k \tag{3.23}$$

where $\bar{A} = A - AKC$, however, one can use the Kalman filter gain K to make $\bar{A}$ be asymptotically stable ($\bar{A}^{i-1} = 0$) for a sufficiently large number of p, $\hat{y}_k$ approaches y_k. Thus equation (3.23) becomes

$$y_k = \sum_{i=1}^{p} C\bar{A}^{i-1} AKy_{k-i} + \sum_{i=1}^{p} C\bar{A}^{i-1} Bu_{k-i} + \varepsilon_k \tag{3.24}$$

by comparing with the typical form of ARX model described by equation (3.2), one can get

$$a_i = C\bar{A}^{i-1} AK \text{ and } b_i = C\bar{A}^{i-1} B \qquad i = 1,2,.....,p \tag{3.25}$$

The model described by equation (3.24) is the ARX model, which directly represents the relationship between the input and output of the open-loop system. The coefficient a_i and b_i can be estimated through the Least-Squares methods from random excitation input u_k and the corresponding output y_k. For a number of data points N, the Batch Least-Square solution is

$$\theta_{okid} = Y\Phi_{okid}^{T}(\Phi_{okid}\Phi_{okid}^{T})^{-1} \tag{3.26}$$

where

$$Y = [y_0 y_1 \ldots y_p \ldots y_{N-1}] \ , \qquad \theta_{okid} = [b_1 a_1 \ldots b_p a_p]$$

and

$$\Phi_{okid} = \begin{bmatrix} 0 & u_0 & \cdots & u_{p-1} & \cdots & u_{N-2} \\ 0 & y_0 & \cdots & y_{p-1} & \cdots & y_{N-2} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & u_0 & \cdots & u_{N-p-1} \\ 0 & 0 & 0 & y_0 & \cdots & y_{N-p-1} \end{bmatrix}$$

The open-loop system Markov parameters $Y_S(k) = CA^{k-1}B$ and Kalman filter Markov parameters $Y_K(k) = CA^{k-1}AK$ can be obtained from the coefficients a_i and b_i as

$$Y_S(k) = b_k + \sum_{i=1}^{k} a_i Y_S(k-i) \tag{3.27}$$

and

$$Y_K(k) = a_k + \sum_{i=1}^{k} a_i Y_K(k-i) \tag{3.28}$$

However, since the system Markov parameters are uniquely determined for each system, one can realize system matrices (A, B, C) directly from the open-loop system Markov parameters and the Kalman filter gain K from the Kalman filter Markov parameters which will be shown next section.

3.4.2 Realization of Open-Loop Plant and Kalman Filter Gain

In mathematical terms, realization can be thought of as a factorization of a parameter sequence of the open-loop system Markov parameters $Y_S(k) = CA^{k-1}B$ to obtain a set of (C, A, B) that preserves the prescribed relationship between the parameters in the sequence. The eigen system realization algorithm (ERA) can be applied to the combined system Markov parameters sequence to compute a realization of the open-loop plant matrices and Kalman filter gain. This can be done by first forming the Hankel matrix of $Y_S(k)$ as

$$H_c(k-1) = \begin{bmatrix} Y_S(k) & Y_S(k+1) & \dots & Y_S(k+s) \\ Y_S(k+1) & Y_S(k+2) & \dots & Y_S(k+s+1) \\ \dots & \dots & \dots & \dots \\ Y_S(k+r) & Y_S(k+r+1) & \dots & Y_S(k+r+s) \end{bmatrix} \tag{3.29}$$

Using the singular value decomposition , the Hankel matrix $H(0)$ is factorized as

$$H(0) = U\Sigma V^T \tag{3.30}$$

A realization for $n-th$ order discrete state space model can be shown to be

$$A = \Sigma_n^{-\frac{1}{2}} U_n^T H(1) V_n \Sigma_n^{-\frac{1}{2}} \tag{3.31}$$

$$B = \Sigma_n^{\frac{1}{2}} V_n^T X_{ni} \tag{3.32}$$

$$C = X_{no}^T U_n \Sigma_n^{\frac{1}{2}} \tag{3.33}$$

where Σ_n is the upper left hand nxn partition of Σ containing the n largest singular values which are in the monotonically non-increasing order along the diagonal. U_n and V_n are the matrices formed by the first n columns of singular vectors associated with the n singular values from U and V respectively. $X_{ni}^T = [I_{ni} 0_{ni} \dots 0_{ni}]$, and $X_{no}^T = [I_{no} 0_{no} \dots 0_{no}]$ while ni are the number of inputs, and no are the number of outputs.

The Kalman filter gain can also retrieved from the open-loop Kalman filter Markov parameters $Y_K(k)$ and A, C matrices through the Least-Square as follows

$$K = (O^T O)^{-1} O^T \begin{bmatrix} Y_K(1) \\ Y_K(1) \\ \dots \\ Y_K(k) \end{bmatrix} \tag{3.34}$$

where $$O = \begin{bmatrix} CA \\ CA^2 \\ \cdots \\ CA^k \end{bmatrix}$$

3.4.3 Computational Steps for Open-Loop System Identification

1. Collect all input u and output y data from the experiment.

2. Form an information matrix and the output vector. Use the Least-Square technique to estimate the ARX model parameters a_i, b_i and the observer Markov parameters by choosing the appropriate ARX model order p from equation 3.26.

3. Obtain the open-loop system Markov parameters $Y_S(k)$ and the Kalman filter Markov parameters $Y_K(k)$ from equation 3.27 and 3.28 respectively.

4. Realize the system matrices from the open-loop system Markov parameters by using eigensystem realization method from equation 3.31, 3.32, and 3.33 , and the Kalman filter gain from the Kalman filter Markov parameters and matrices A, C from equation 3.34.

3.5 Closed-Loop System Identification Algorithm

The closed-loop system and Kalman filter Markov parameters are first calculated from the estimated coefficient matrices of the ARX model. The open-loop system and Kalman filter Markov parameters are then derived from the closed-loop system, Kalman filter Markov parameters, and known controller Markov parameters. The open-loop state-space model is realized by using singular-value decomposition of a Hankel matrix formed by the open-loop system Markov parameters. Finally, an open-loop Kalman filter gain is calculated from the realized state-space model and open-loop Kalman filter Markov parameters through the Least-Square method.

3.5.1 Algorithm for Closed-Loop System Identification

Given a linear, stochastic, finite dimensional, discrete-time ,and time invariance system, the system can be expressed by

$$x_{k+1} = Ax_k + Bu_k + \kappa_k \tag{3.35}$$

$$y_k = Cx_k + \upsilon_k \tag{3.36}$$

where $x_k \in R^{nx1}$ is the state vector, $u_k \in R^{sx1}$ is the input vector, $y_k \in R^{mx1}$ is the output vector,and $[A, B, C]$ are the state-space system matrices. The sequence of the process noise κ_k and the measurement noise υ_k are assumed to be Gaussian , white, zero-mean, and stationary with covariance Q and R respectively.

One can derive a steady-state filter innovation model

$$\hat{x}_{k+1} = A\hat{x}_k + Bu_k + AK\varepsilon_k \tag{3.37}$$

$$y_k = C\hat{x}_k + \varepsilon_k \tag{3.38}$$

where $\hat{x}_k$ is the priori estimated state, K is the steady-state Kalman filter gain, and ε_k is the residual after filtering ($\varepsilon_k = y_k - C\hat{x}_k$). The existence of K is guaranteed if the system is detectable and $(A, Q^{1/2})$ is stabilizable.

On the other hand, any kind of dynamic output feedback controller can be modeled as

$$p_{k+1} = A_d p_k + B_d y_k \tag{3.39}$$

$$u_k = C_d p_k + D_d y_k + r_k \tag{3.40}$$

where A_d, B_d, C_d and D_d are the system matrices of the dynamic output feedback controller, p_k is the controller state, and r_k is the reference input to the closed-loop system.

From equation (3.37) to (3.40), the augment closed-loop system dynamics becomes

$$\eta_{k+1} = A_c\eta_k + B_c r_k + A_c K_c \varepsilon_k \tag{3.41}$$

$$y_k = C_c\eta_k + \varepsilon_k \tag{3.42}$$

where

$$\eta_k = \begin{bmatrix} \hat{x}_k \\ p_k \end{bmatrix}, \qquad A_c = \begin{bmatrix} A + BD_dC & BC_d \\ B_dC & A_d \end{bmatrix}, \quad B_c = \begin{bmatrix} B \\ 0 \end{bmatrix}$$

$$A_c K_c = \begin{bmatrix} AK + BD_d \\ B_d \end{bmatrix}, \quad C_c = \begin{bmatrix} C & 0 \end{bmatrix} \tag{3.43}$$

K_c can be considered as the Kalman filter gain for the closed-loop system and the existence of the steady-state K_c is guaranteed when the closed-loop system matrix A_c is nonsingular. Substitute equation (3.42) into (3.41) yields

$$\eta_{k+1} = \bar{A}\eta_k + B_c r_k + A_c K_c y_k \tag{3.44}$$

where $\bar{A} = A_c - A_c K_c C_c$ is guaranteed to be asymptotically stable because the steady-state Kalman filter gain K_c exists. The z transform of equation (3.42) and (3.44) yields

$$\eta(z) = (z - \bar{A})^{-1}[A_c K_c y(z) + B_c r(z)] \tag{3.45}$$

$$y(z) = C_c \eta(z) + \varepsilon(z) \tag{3.46}$$

Substituting equation (3.45) into (3.46) , one has

$$y(z) = C_c (z - \bar{A})^{-1}[A_c K_c y(z) + B_c r(z)] + \varepsilon(z) \tag{3.47}$$

The inverse z transform of equation (3.47) with

$$(z - \bar{A})^{-1} = \sum_{i=1}^{\infty} \bar{A}^{i-1} z^{-i}$$

with zero initial condition yields

$$y_k = \sum_{i=1}^{\infty} C_c \bar{A}^{i-1} A_c K_c y_{k-i} + \sum_{i=1}^{\infty} C_c \bar{A}^{i-1} B_c r_{k-i} + \varepsilon_k \tag{3.48}$$

Since $\bar{A}$ is asymptotically stable, $\bar{A}^i \approx 0$ if $i > p$ for a sufficient large number p. Thus equation (3.48) becomes

$$y_k \approx \sum_{i=1}^{p} c_i y_{k-i} + \sum_{i=1}^{p} d_i r_{k-i} + \varepsilon_k \tag{3.49}$$

where

$$c_i = C_c \bar{A}^{i-1} A_c K_c \quad , \quad d_i = C_c \bar{A}^{i-1} B_c \tag{3.50}$$

The model described by equation (3.49) is the ARX model , which directly repre-

sents the relationship between the input and output of the closed-loop system. The coefficient matrices c_i and d_i which are the closed-loop ARX model parameters can be estimated through Least-Square methods from random excitation input r_k and the corresponding output y_k. For a number of data point N, the Batch Least-Square solution is

$$\theta_{clid} = Y\Phi_{clid}^{T}(\Phi_{clid}\Phi_{clid}^{T})^{-1} \tag{3.51}$$

where

$$Y = [y_0 y_1 \ldots y_p \ldots y_{N-1}] \quad , \quad \theta_{clid} = [d_1 c_1 \ldots d_p c_p]$$

$$\Phi_{clid} = \begin{bmatrix} 0 & r_0 & \cdots & r_{p-1} & \cdots & r_{N-2} \\ 0 & y_0 & \cdots & y_{p-1} & \cdots & y_{N-2} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & r_0 & \cdots & r_{N-p-1} \\ 0 & 0 & 0 & y_0 & \cdots & y_{N-p-1} \end{bmatrix}$$

The z transform of the open-loop state-space model (3.37) can be used to derived $u(z)$ and $y(z)$ as

$$\hat{x}(z) = (z-A)^{-1}[Bu(z) + AK\varepsilon(z)] \tag{3.52}$$

Substituting equation (3.52) to the z transform of equation (3.38) produces

$$\begin{aligned} y(z) &= C(z-A)^{-1}[Bu(z) + AK\varepsilon(z)] + \varepsilon(z) \\ &= \sum_{k=1}^{\infty} Y_S(k) z^{-k} u(z) + \sum_{k=0}^{\infty} Y_K(k) z^{-k} \varepsilon(z) \end{aligned} \tag{3.53}$$

again where $Y_S(k) = CA^{k-1}B$ is the open-loop system Markov parameters, $Y_K(k) = CA^{k-1}AK$ is Kalman filter Markov parameters, and $Y_K(0) = I$ is an identity matrix. Similarly, substituting the dynamic output feedback controller equation (3.39) into (3.40), and the closed-loop state-space model equation (3.41) to (3.42) produces

$$u(z) = \sum_{k=0}^{\infty} Y_D(k) z^{-k} y(z) + r(z) \tag{3.54}$$

$$y(z) = \sum_{k=1}^{\infty} Y_{SC}(k)z^{-k}u(z) + \sum_{k=0}^{\infty} Y_{KC}(k)z^{-k}\varepsilon(z) \tag{3.55}$$

where $Y_D(k) = C_d A_d^{k-1} B_d$ is the controller Markov parameter, $Y_{SC}(k) = C_c A_c^{k-1} B_c$ is the closed-loop system Markov parameters, and $Y_{KC}(k) = C_c A_c^{k-1} A_c K_c$ is the closed-loop Kalman filter Markov parameters. Note that $Y_D(0) = D_d$ and $Y_{KC}(0) = I$.

The z transform of the ARX model equation (3.49) yields

$$\left(I - \sum_{i=1}^{p} c_i z^{-1}\right) y(z) = \sum_{i=1}^{p} d_i z^{-i} r(z) + \varepsilon(z) \tag{3.56}$$

Applying long division to equation (3.55)

$$y(z) = \{d_1 z^{-1} + (d_2 + c_1 d_1)z^{-2} + [d_3 + c_1(d_2 + c_1 d_1) + c_2 d_1]z^{-3} + \ldots\} r(z)$$

$$+ \{I + c_1 z^{-1} + (c_1 c_1 + c_2)z^{-2} + [c_1(c_1 c_1 + c_2) + c_2 c_1 + c_3]z^{-3} + \ldots\} \varepsilon(z)$$

After comparing with equation (3.54), the closed-loop system and Kalman filter Markov parameters can be recursively calculated from the estimated coefficient matrices of the ARX model:

$$Y_{SC}(k) = d_k + \sum_{i=1}^{k} c_i Y_{SC}(k-i) \tag{3.57}$$

$$Y_{KC}(k) = \sum_{i=1}^{k} c_i Y_{KC}(k-i) \tag{3.58}$$

Note that $Y_{SC}(0) = 0$, $Y_{KC}(0) = I$, and $c_i = d_i = 0$, when $i > p$.

The open-loop system Markov parameters $Y_S(k)$, and the Kalman filter Markov parameters $Y_K(k)$ can be obtained by substituting equation (3.54) into (3.55), yields

$$y(z) = \left(\sum_{k=1}^{\infty} Y_S(k)z^{-k}\right)\left(\sum_{k=0}^{\infty} Y_D(k)z^{-k}y(z)\right) + \sum_{k=1}^{\infty} Y_S(k)z^{-k}r(z) + \sum_{k=0}^{\infty} Y_K(k)z^{-k}\varepsilon(z)$$

$$= \sum_{k=1}^{\infty} \delta_k(k) z^{-k} y(z) + \sum_{k=1}^{\infty} Y_S(k) z^{-k} r(z) + \sum_{k=0}^{\infty} Y_K(k) z^{-k} \varepsilon(z) \tag{3.59}$$

where $\delta_k = \sum_{i=1}^{k} Y_S(i) Y_D(k-i)$.

Rearranging equation (3.59) in the form

$$\left(I - \sum_{k=1}^{\infty} \delta_k z^{-k} \right) y(z) = \sum_{k=1}^{\infty} Y_S(k) z^{-k} r(z) + \sum_{k=0}^{\infty} Y_K(k) z^{-k} \varepsilon(z) \tag{3.60}$$

By applying long division to equation (3.60) , and comparing it with equation (3.55) . The closed-loop system Markov parameters can be described recursively in the open-loop system and the controller Markov parameter $Y_D(k) = C_d A_d^{k-1} B_d$

$$Y_{SC}(j) = Y_S(j) + \sum_{k=1}^{j} \delta_k Y_{SC}(j-k) = Y_S(j) + \sum_{k=1}^{j} \sum_{i=1}^{k} Y_S(i) Y_D(k-i) Y_{SC}(j-k) \tag{3.61}$$

The closed-loop Kalman filter Markov parameters can be recursively expressed in terms of the open-loop system and Kalman filter Markov parameters, and controller Markov parameters

$$\begin{aligned} Y_{KC}(j) &= Y_K(j) + \sum_{k=1}^{j} \delta_k Y_{KC}(j-k) \\ &= \left(Y_K(j) + \sum_{k=1}^{j} \sum_{i=1}^{k} Y_S(i) Y_D(k-i) Y_{KC}(j-k) \right) \end{aligned} \tag{3.62}$$

Finally, rearranging equation (3.61) and (3.62), yields

$$Y_S(j) = Y_{SC}(j) - \sum_{k=1}^{j} \sum_{i=1}^{k} Y_S(i) Y_D(k-i) Y_{SC}(j-k) \tag{3.63}$$

$$Y_K(j) = Y_{KC}(j) - \sum_{k=1}^{j} \sum_{i=1}^{k} Y_S(i) Y_D(k-i) Y_{KC}(j-k) \tag{3.64}$$

The open-loop state-space model can be realized by using singular-value decomposition for a Hankel matrix formed by the open-loop system Markov parameters. In addition, the open-loop Kalman filter gain can be formed from the open-loop Kalman filter

Markov parameters and state-space matrices A, C as described in section 3.4.2.

3.5.2 Computational Steps for Closed-Loop System Identification

1.Collect the reference input r and the corresponding output y data from the experiment.

2. Form an information matrix and the output vector using the Least-Square technique to estimate the ARX model parameters c_i, d_i by choosing the appropriate ARX model order p from equation 3.51.

3.Compute the closed-loop system $Y_{SC}(k)$ and Kalman filter Markov parameters $Y_{KC}(k)$ from the estimated coefficient matrices of the ARX model from equation 3.57 and 3.58.

4.Compute the open-loop system $Y_S(k)$ and Kalman filter Markov parameters $Y_K(k)$ from the closed-loop system $Y_{SC}(k)$, Kalman filter Markov parameters $Y_{KC}(k)$, and controller Markov parameters $Y_D(k)$ calculated from the known controller dynamics from equation 3.62, 3.63 and 3.64.

5.Realize the open-loop state-space system matrices from the open-loop system Markov parameters by using the Singular-Value Decomposition method from equation 3.31, 3.32 and 3.33.

6.Estimate the open-loop Kalman filter gain from the open-loop Kalman filter Markov parameters and realized system matrices A, C from equation 3.34.

3.6 Coordinate Transformation

For any dynamic system, the realized state-space model is not unique even though its system Markov parameter is unique. The state-space model needs to be compared with the analytical model in the same coordinate. A unique transformation matrix is derived to transform any realized state-space model in a form usually used for a structural dynamic system; therefore, any identified system parameter can be compared with the corresponding analytical parameter. This kind of transformation will exist only when one-half of the states are measured directly. If this condition is not satisfied, other transformation matrices may exist, but they usually are not unique.

Consider a structural dynamic system

$$\tilde{M}\ddot{v} + \tilde{D}\dot{v} + \tilde{S}v = \Omega u \tag{3.65}$$

where v is displacement, u control force, Ω control influence matrix and $\tilde{M}$, $\tilde{D}$ and $\tilde{S}$ are mass , damping and stiffness matrices, respectively. The state-space model can be described as

$$\dot{x} = A_{mo}x + B_{mo}u$$

$$y = C_{mo}x \tag{3.66}$$

where $x = \begin{bmatrix} v \\ \dot{v} \end{bmatrix}$, $A_{mo} = \begin{bmatrix} 0 & I \\ -\tilde{M}^{-1}\tilde{S} & -\tilde{M}^{-1}\tilde{D} \end{bmatrix}$, $B_{mo} = \begin{bmatrix} 0 \\ \tilde{M}^{-1}\Omega \end{bmatrix}$, and C_{mo} is the output matrix. If half of the states can be measured directly, then $C_{mo} = \begin{bmatrix} I & 0 \end{bmatrix}$. Convert the realized discrete-time system $\begin{bmatrix} A & B & C \end{bmatrix}$ to a continuous-time system $\begin{bmatrix} A_c & B_c & C \end{bmatrix}$. If A is diagonalized by matrix Ψ, then

$$\Psi^{-1}A\Psi = \Lambda$$

$$A_c = \Psi\frac{\ln(\Lambda)}{T_s}\Psi^{-1}$$

$$B_c = (A - I)^{-1}A_cB$$

where T_s is a sampling time. It is also assumed that the matrix $\begin{bmatrix} C \\ CA_c \end{bmatrix}$ is full rank.

Let the transformation matrix Υ be

$$\Upsilon = \begin{bmatrix} \Upsilon_1 & \Upsilon_2 \end{bmatrix} = \begin{bmatrix} C \\ CA_c \end{bmatrix}^{-1} \tag{3.67}$$

then the following transformation are

$$\Upsilon^{-1}\Upsilon = \begin{bmatrix} C \\ CA_c \end{bmatrix}\begin{bmatrix} \Upsilon_1 & \Upsilon_2 \end{bmatrix} = \begin{bmatrix} C\Upsilon_1 & C\Upsilon_2 \\ CA_c\Upsilon_1 & CA_c\Upsilon_2 \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

$$\Upsilon^{-1}A_c\Upsilon = \begin{bmatrix} C \\ CA_c \end{bmatrix} A_c \begin{bmatrix} \Upsilon_1 & \Upsilon_2 \end{bmatrix} = \begin{bmatrix} CA_c\Upsilon_1 & CA_c\Upsilon_2 \\ CA_c^2\Upsilon_1 & CA_c^2\Upsilon_2 \end{bmatrix} = \begin{bmatrix} 0 & I \\ \Upsilon & \Upsilon \end{bmatrix}$$

$$C\Upsilon = \begin{bmatrix} C\Upsilon_1 & C\Upsilon_2 \end{bmatrix} = \begin{bmatrix} I & 0 \end{bmatrix}$$

Note that $C\Upsilon = C_{mo}$. Finally, the identified continuous-time model $\begin{bmatrix} A_c & B_c & C \end{bmatrix}$ can be transformed in the triple form $\begin{bmatrix} \Upsilon^{-1}A_c\Upsilon & \Upsilon^{-1}B_c & C\Upsilon \end{bmatrix}$. Rewrite the transformation result in the state-space form, so both the identified and analytical models are in the same coordinate.

CHAPTER IV

PREDICTIVE CONTROL

4.1 Introduction

The concept of predictive control was introduced simultaneously by Richalet and Cutler and Ramaker in the late seventies[17]. Predictive control belongs to the class of model-based controller design concepts. That process model is explicitly used to design the controller. Predictive control is not the only model-based controller design method. Others are pole-placement control and linear quadratic (LQ) control. In Figure 4.1, u denotes the controller output, y denotes the process output and w denotes the desired output.

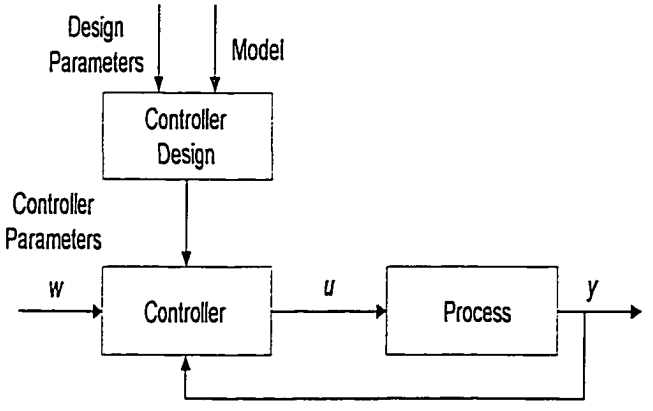

Figure 4.1 Model-Based Control

If the process is linear and there are no constraints and the desired process output is simple, then all of the above -mentioned model-based controllers can yield approximately the same results. This can be explained by the fact that these controller design methods yield linear controllers which after some manipulation are of the same structure and have a sufficient number of degrees of freedom. The controller parameters are, however, determined using a different design methodology, or philosophy. From a designer's point of

view the methods differ in the design parameters that are used to obtain the desired behavior of the control system.

Section 4.2 starts with the process models and prediction that the Diophentine equation is used to derived an i-step-ahead predictor. In section 4.3, disturbance models are presented. Two types of disturbances are "deterministic" and "stochastic" disturbances. Section 4.4 explains an approach to process modeling and prediction. The i-step-ahead predictor can be written in matrix form which has prediction horizon as a parameter. In section 4.5, criterion functions in terms of controller output weighting and structuring the controller output are explained. Section 4.6 explains the predictive control law derivation and the computational steps are also explained in section 4.7.

4.2 Process Models and Prediction

In order to predict the process output over the prediction horizon, an i-step-ahead predictor is required[18]. An i-step-ahead prediction of the process output must be a function of all data up to $t = k$ (defined as the vector ψ), the future controller output sequence u and a model of the process $\tilde{P}$. Such an i-step-ahead predictor can thus be described by

$$y(k+i) = f(u, \psi, \tilde{P}) \tag{4.1}$$

where f is the function. Clearly, i-step-ahead predictors depend on the model of the process. A general process model is the transfer-function model

$$y(k) = \frac{q^{-d}B_p(q^{-1})}{A_p(q^{-1})}u(k-1) \tag{4.2}$$

where d is the time delay of the process in samples ($d \geq 0$) and the polynomials A_p and B_p are given by

$$A_p(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_{n_A} q^{-n_A}$$

$$B_p(q^{-1}) = b_0 + b_1 q^{-1} + \ldots + b_{n_B} q^{-n_B}$$

where n_A and n_B are the degrees of the polynomials A_p and B_p respectively.

The transfer-function models have the following advantages. First, a minimal

number of parameters are required to describe a linear process. Second, stable and unstable processes can be described by using transfer-function models. The main disadvantages are an assumption about the order of the process must be made, and the prediction of the output of a process described by a transfer-function model is very complicated.

The process output at $t = k+i$ based on model (4.2) can be obtained by substituting $k+i$ for k in equation (4.2)

$$y(k+i) = \frac{q^{-d}B_p(q^{-1})}{A_p(q^{-1})}u(k+i-1) \tag{4.3}$$

Using the Certainly Equivalence principle , and replacing the true d, A_p and B_p by their estimates $\hat{d}, \hat{A}_p$ and $\hat{B}_p$ yields

$$\hat{y}(k+i) = \frac{q^{-\hat{d}}\hat{B}_p(q^{-1})}{\hat{A}_p(q^{-1})}u(k+i-1) \tag{4.4}$$

where the symbol ^ denotes estimation. Equation (4.4) can be rewritten as

$$\hat{y}(k+i) = q^{-\hat{d}}\hat{B}_p(q^{-1})u(k+i-1) - q(\hat{A}_p - 1)\hat{y}(k+i-1) \tag{4.5}$$

Note that $q(\hat{A}_p - 1) = a_1 + a_2q^{-1} + \ldots + a_{n_A}q^{-n_A+1}$ since $\hat{A}_p$ is assumed to be monic. Now , $\hat{y}(k+i)$ for $i \geq 1$ can be computed recursively using equation (4.5). It also notes that the i-step-ahead predictor (4.5) runs independently of the process. One way to improve the prediction is to compute the predictions using equation (4.4) and (4.5) with $\hat{y}(k)$ replaced by the measured process output $y(k)$. Rewrite equation (4.4) as

$$\hat{A}_p(q^{-1})\hat{y}(k+i) = q^{-\hat{d}}\hat{B}_p(q^{-1})u(k+i-1) \tag{4.6}$$

Introducing the following identity

$$\frac{1}{\hat{A}_p} = E_i + q^{-i}\frac{F_i}{\hat{A}_p} \Rightarrow E_i\hat{A}_p = 1 - q^{-1}F_i \tag{4.7}$$

where E_i has a degree less than or equal to $i-1$ and F_i is of degree $n_A - 1$. Equation (4.7) is so called a Diophantine equation whose solution can be computed manually using

long division or a recursive algorithm by a computer. Note that if $n_A > 0$, E_i contains the first i elements of the impulse response of $1/\hat{A}_p$.

Multiplying equation (4.6) by E_i and using (4.7) yields

$$\hat{y}(k+i) = q^{-d}E_i\hat{B}_p(q^{-1})u(k+i-1) + F_i\hat{y}(k) \tag{4.8}$$

Using equation (4.5) with $\hat{y}(k)$ replaced by $y(k)$ to correct the model for differences between the model and the process yields

$$\hat{y}(k+i) = q^{-d}E_i\hat{B}_p(q^{-1})u(k+i-1) + F_iy(k) \tag{4.9}$$

Multiply equation (4.7) by $\hat{B}_p$

$$E_i\hat{B}_p = \frac{\hat{B}_p}{\hat{A}_p} - q^{-i}\frac{F_i\hat{B}_p}{\hat{A}_p} \tag{4.10}$$

Substituting the factor $E_i\hat{B}_p$ in equation (4.9) by the right-hand side of (4.10) yields the following predictor-corrector model

$$\hat{y}(k+i) = \frac{q^{-d}\hat{B}_p}{\hat{A}_p}u(k+i-1) + F_i[y(k) - \hat{y}(k)] \tag{4.11}$$

4.3 Disturbance Models

In order to take disturbances into account when predicting the output of the process, the disturbances must also be modeled. For this purpose, the model equation (4.2) is extended with a disturbance term $\xi(k)$ that represents the totality of all disturbances and is assumed to be located at the output of the process

$$y(k) = \frac{q^{-d}B_p(q^{-1})}{A_p(q^{-1})}u(k-1) + \xi(k) \tag{4.12}$$

The disturbance $\xi(k)$ may in general be a sum of deterministic and stochastic disturbances. Prediction of the process output at $t = k+i$ is realized by

$$y(k+i) = \frac{q^{-d}B_p(q^{-1})}{A_p(q^{-1})}u(k+i-1) + \xi(k+i) \tag{4.13}$$

4.3.1 Deterministic disturbances

Deterministic disturbances mostly can be characterized by

$$\phi_{\xi}(q^{-1})\xi(k) = 0 \tag{4.14}$$

where $\phi_{\xi}(q^{-1})$ is a polynomial. Note that only the class of the disturbances characterized need to be known and not its exact waveform. The i-step-ahead prediction characterized by equation (4.14) is given by

$$\phi_{\xi}(q^{-1})\xi(k+i) = 0 \tag{4.15}$$

Because the i-step-ahead prediction for the process output must be a function of u and of all data up to time $t = k$, the following Diophantine equation is used in order to write $\xi(k+i)$ as a function of data up to time $t = k$

$$\frac{1}{\phi_{\xi}} = E_i + q^{-i}\frac{F_i}{\phi_{\xi}} \Rightarrow E_i\hat{\phi}_{\xi} = 1 - q^{-1}F_i \tag{4.16}$$

Multiply equation (4.15) by E_i yields using equation (4.16)

$$\xi(k+i) = F_i\xi(k) \tag{4.17}$$

Note that $\xi(k)$ in equation (4.17) can be computed from

$$\xi(k) = y(k) - \frac{q^{-d}B_p}{A_p}u(k-1) \tag{4.18}$$

The i-step-ahead predictor for the process (4.12) and the disturbances described by equation (4.14), become (using equation (4.16) and (4.18) and after applying the certainty equivalence principle):

$$\hat{y}(k+i) = \frac{q^{-d}\hat{B}_p(q^{-1})}{\hat{A}_p(q^{-1})}u(k+i-1) + F_i[y(k) - \hat{y}(k)] \tag{4.19}$$

4.3.2 Stochastic disturbances

A stochastic disturbance appearing on the output of the process is assumed to be

$$\xi(k) = \frac{C_p}{D_p}e(k) \tag{4.20}$$

where $e(k)$ is a discrete white noise sequence with zero mean and variance σ^2. C_p and D_p are monic polynomials with degree n_C and n_D respectively. The prediction of the disturbance at $t = k+i$ is given by

$$\xi(k+i) = \frac{C_p}{D_p}e(k+i) \tag{4.21}$$

Separation of future and past terms is realized by using Diophantine equation

$$\frac{C_p}{D_p} = E_i + q^{-i}\frac{F_i}{D_p} \tag{4.22}$$

where E_i and F_i are polynomials with degrees

$$n_{E_i} = i-1 \qquad \text{if } n_D > 0,$$

$$n_{E_i} = min(i-1, n_C) \quad \text{if } n_D = 0 \text{ and } n_{F_i} = max(n_C - i, n_D - 1)$$

Equation (4.22) becomes $$\xi(k+i) = E_i e(k+i) + \frac{F_i}{D_p}e(k) \tag{4.23}$$

Recall equation (4.12) with $\xi(k)$ given by (4.21)

$$y(k) = \frac{q^{-d}B_p(q^{-1})}{A_p(q^{-1})}u(k-1) + \frac{C_p}{D_p}e(k) \tag{4.24}$$

Multiplying (4.23) by F_i/C_p and rearranging the result yields

$$\frac{F_i}{D_p}e(k) = \frac{F_i}{C_p}\left[y(k) - \frac{q^{-d}B_p}{A_p}u(k-1)\right] \tag{4.25}$$

Equation (4.25) shows that the second term in (4.23) can be computed from data available at $t = k$ and is thus known. The i-step-ahead predictor for the process output (4.12) now becomes, using (4.17), (4.23) and (4.25)

$$y(k+i) = \frac{q^{-d}B_p}{A_p}u(k+i-1) + \frac{F_i}{C_p}\left[y(k) - \frac{q^{-d}B_p}{A_p}u(k-1)\right] + E_i e(k+i) \tag{4.26}$$

This predictor, however, contains a term that is unknown ($E_i e(k+i)$). The best i-step-ahead predictor can be obtained by taking the conditional expectation $\mathcal{E}$ of $y(k+i)$ given $\aleph$ (all data up to $t = k$) and the future controller output sequence u

$$\hat{y}(k+i) = \mathcal{E}[y(k+i)|u, \aleph]$$

$$= \frac{q^{-d}B_p}{A_p}u(k+i-1) + \frac{F_i}{C_p}\left[y(k) - \frac{q^{-d}B_p}{A_p}u(k-1)\right] \qquad (4.27)$$

The prediction error $\varepsilon(k+i)$ for the i th predictor is given by

$$\varepsilon(k+i) = y(k+i) - \hat{y}(k+i) = E_i e(k+i) \qquad (4.28)$$

Since this prediction error consists of future noise only, and because $e(k)$ is assumed to be white noise with zero mean, the variance of the prediction error is minimal. For this reason equation (4.27) will be called the minimum-variance (MV) i-step-ahead predictor. Applying the certainty equivalence principle finally yields

$$\hat{y}(k+i) = \frac{q^{-\hat{d}}\hat{B}_p}{\hat{A}_p}u(k+i-1) + \frac{F_i}{\hat{C}_p}[y(k) - \hat{y}(k)] \qquad (4.29)$$

where $\hat{y}(k)$ is given by

$$\hat{y}(k) = \frac{q^{-\hat{d}}\hat{B}_p}{\hat{A}_p}u(k-1) \qquad (4.30)$$

Remarks

1. The polynomials $\hat{A}_p$, $\hat{B}_p$ and the estimated time delay $\hat{d}$ can either be estimated by a suitable identification method or can be based on the true transfer function of the continuous process.

2. The polynomials $\hat{C}_p$ and $\hat{D}_p$ in the disturbance model are often used as design parameters which provides an easy and efficient way to tune the regulator behavior and the robustness of the control system.

3. If $T = \hat{C}_p = \hat{D}_p$, equation (4.22) shows that, for all i, $E_i = 1$ and $F_i = 0$. As a result (4.29) is no longer corrected for modeling errors and disturbances. Note that the choice $T = \hat{C}_p = \hat{D}_p$ corresponds to an assumption that the process output is disturbed by white noise only.

In order to split up the i-step-ahead predictor in parts that are known at time $t = k$ and future signals, another Diophantine equation is introduced

$$\frac{\hat{B}_p}{\hat{A}_p} = G_i + q^{-i+\hat{d}}\frac{H_i}{\hat{A}_p} \tag{4.31}$$

where, G_i and H_i are polynomials. The degree of G_i is less than or equal to $i-\hat{d}-1$ and the degree of H_i equals $\max(n_B - i + \hat{d}, n_A - 1)$. Also note that the coefficients of G_i are equal to the first $i-\hat{d}$ coefficients of the impulse response of the model. This implies that $g_{i+1,j} = g_{i,j}$ making it unnecessary to distinguish separate coefficients for G_i. Therefore, G_i can be simplified into

$$G_i(q^{-1}) = g_0 + g_1 q^{-1} + \ldots + g_{i-1} q^{-i+1} \tag{4.32}$$

Using equation (4.31) the i-step-ahead predictor (4.29) becomes (with $i \geq \hat{d} + 1$):

$$\hat{y}(k+i) = G_i u(k+i-\hat{d}-1) + \frac{H_i}{\hat{A}_p} u(k-1) + \frac{F_i}{T}[y(k) - \hat{y}(k)] \tag{4.33}$$

Because the degree of G_i is less than or equal to $i-\hat{d}-1$, the term $G_i u(k+i-\hat{d}-1)$ involves future controller output only. The other terms in (4.33) do not depend on future controller outputs and hence are fully determined at $t = k$.

4.4 Approach to Process Modeling and Prediction

It was shown in the previous section that the transfer-function model can be used to model a wide variety of processes. This is due to the fact that the degree of A_p and B_p in equation (4.2) can be chosen arbitrarily. Also, the stochastic disturbance model (4.20) can be used to describe all stationary random processes with rational spectral density. It can also be used to derived i-step-ahead predictors for deterministic disturbances characterized by $\phi_\xi(q^{-1})\xi(k) = 0$. The stochastic disturbance model (4.20) is quite general and hence suited for predictive controller design. Another argument for using (4.20) to

describe the disturbance is that if C_p and D_p can be chosen by the designer of the control system, one will obtain a powerful tool for tuning the robustness and regulator behavior of the close-loop system. The above-mentioned considerations have resulted in the subsequent use of the following process model for deriving the i-step-ahead predictors

$$y(k) = \frac{q^{-d}B_p}{A_p}u(k-1) + \frac{C_p}{D_p}e(k) \tag{4.34}$$

where the degrees of the polynomials are arbitrary and the disturbance model polynomials C_p and D_p can be chosen by the designer. The MV i-step-ahead predictor based on this model after applying the certainty equivalence principle is given by

$$\hat{y}(k+i) = G_i u(k+i-\hat{d}-1) + \frac{H_i}{\hat{A}_p}u(k-1) + \frac{F_i}{T}[y(k)-\hat{y}(k)] \tag{4.35}$$

where $\hat{y}(k)$ is given by (4.30) and T by definition equal to $\hat{C}_p$. G_i, H_i are obtained by solving (4.31) and F_i is obtained by solving (4.22) with C_p and D_p replaced by T and $\hat{D}_p$ respectively. Table 4.1 shows how the parameters of the general process model (4.34) must be selected to obtain the model that is used in some well-known predictive controllers.

Controller	Model	A_p	B_p	C_p	D_p
DMC	FSR	1	Arbitrarily	1	Δ
PCA	FIR	1	Arbitrarily	1	Δ
MAC	FIR	1	Arbitrarily	1	Δ
GPC	ARIMAX	Arbitrarily	Arbitrarily	Arbitrarily	$A_p\Delta$
EPSAC	ARIMAX	Arbitrarily	Arbitrarily	Arbitrarily	$A_p\Delta$
EHAC	ARIX	Arbitrarily	Arbitrarily	1	$A_p\Delta$

Table 4.1 Process models used by some well-known predictive controllers

Note that DMC is Dynamic Matrix Control, PCA is Predictive Control Algorithm, MAC is Model Algorithmic Control, GPC is Generalized Predictive Control[19-21], EPSAC is Extended Prediction Self-Adaptive Control, EHAC is Extended Horizon Adaptive Control, FIR is Finite Impulse Response model, ARIMAX is Auto-Regressive Integrated Moving-Average eXogenous model, and ARIX is Auto-Regressive Integrated eXogenous model.

Matrix Notation

For convenience, the i-step-ahead predictors (4.35) for $i = \hat{d} + 1, \ldots, h_p$ can be written in matrix notation

$$[\hat{y}] = [G][u] + [H][\tilde{u}] + [F][c] \tag{4.36}$$

where

$$[\hat{y}] = [\hat{y}(k+\hat{d}+1), \ldots, \hat{y}(k+h_p)]^T$$

$$[u] = [u(k), \ldots, u(k+h_p-\hat{d}-1)]^T$$

$$[\tilde{u}] = [\tilde{u}(k-1), \tilde{u}(k-2), \ldots]^T$$

$$[c] = [c(k), c(k-1), \ldots]^T$$

with $$\tilde{u}(k) = \frac{u(k)}{\hat{A}_p}$$

$$c(k) = \frac{y(k)-\hat{y}(k)}{T}$$

and the dimensions of $[\hat{y}]$, $[u]$, $[\tilde{u}]$ and $[c]$ are given by

$$[\hat{y}] = h_p - \hat{d} \times 1$$

$$[u] = h_p - \hat{d} \times 1$$

$$[\tilde{u}] = max(n_{H_i}) + 1 \times 1$$

$$[c] = max(n_{F_i}) + 1 \times 1$$

The matrices $[G]$, $[H]$ and $[F]$ are built up of the elements of the polynomials

G_i, H_i, F_i respectively

$$[G] = \begin{bmatrix} g_0 & 0 & \cdots & 0 \\ g_1 & g_0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ g_{H_p - \hat{d} - 1} & \cdots & \cdots & g_0 \end{bmatrix}$$

$$[H] = \begin{bmatrix} H_{\hat{d}+1} \\ \cdots \\ H_i \\ \cdots \\ H_{h_P} \end{bmatrix}$$

$$[F] = \begin{bmatrix} F_{\hat{d}+1} \\ \cdots \\ F_i \\ \cdots \\ F_{h_P} \end{bmatrix}$$

Note : h_P is called prediction horizon.

4.5 Criterion Functions

Criterion functions for use in predictive controllers are discussed in this section. Predictive controllers are obtained by the minimization of a criterion function that yields the predictive control law. Design objectives such as overshoot, rise time, settling time and damping ratio can be easily understood and specified. However, it is difficult to minimize criterion functions based on such objectives, because the relationship between the controller parameters and these criteria is in general highly nonlinear. Analytical solutions are seldom available. This is why mathematically convenient criterion functions are often used.

4.5.1 Controller Output Weighting

The foremost method of resolving problems with minimum-variance control results is to use the following criterion function

$$J = \sum_{i=1}^{h_P} [\hat{y}(k+i) - w(k+i)]^2 + \rho u(k+i-1)^2 \qquad (4.37)$$

where ρ is a weighting factor ($\rho > 0$). Here two conflicting objectives arise, the minimization of the tracking error and the minimization of the controller output. The weighting factor ρ is introduced and employed as a trade-off between these objectives. Increasing the weighting factor makes the controller output variance more important in the criterion function. Minimization of the criterion function results in a less active controller output. However, tracking of the trajectory by the process output becomes less important resulting in a slower process output.

The use of the weighting factor ρ as proposed in equation (4.37) has two major disadvantages.

1. Although the effect of ρ on the close-loop system is clear, it is hard to choose ρ such that the system behaves as desired because ρ depends on the process and must usually be determined by simulation in combination with the often-used trial-and-error method.

2. The use of ρ yields a steady-state error which is a function of ρ for type 0 processes. This is caused by the fact that for type 0 process, $u(k)$ is constant and nonzero in the steady state if the set point and disturbances are constant. Consequently, ρ affects the criterion function in the steady state and hence the steady-state controller output that is obtained by minimization of the criterion function.

4.5.2 Structuring the Controller Output

The second way to overcome the problem with minimum-variance control is by using prior information about the structure of the controller output required to drive the process output to the reference trajectory . An appropriate structure for the future controller outputs can be built from the relationship between the controller output and the output of the process.

4.5.2.1 Structuring the Controller Output using Steady State Arguments

Corollary 4.1 *If a stable and linear closed-loop system is driven by a disturbance* $\xi(k)$ *and a reference trajectory* $w(k)$ *which in the steady state satisfy* $\phi_\xi(q^{-1})\xi(k) = 0$ *and* $\phi_w(q^{-1})w(k) = 0$ *respectively, then* $y(k)$ *and* $u(k)$ *in the steady state satisfy*[22]

$$\phi(q^{-1})y(k) = 0$$

$$\phi(q^{-1})u(k) = 0 \tag{4.38}$$

where $\phi(q^{-1})$ *is the minimal polynomial of* $\phi_\xi(q^{-1})$ *and* $\phi_w(q^{-1})$: $\phi = min\{\phi_\xi, \phi_w\}$. *Further, if* B_p *and* ϕ *are coprime and* ϕ_{A_p} *is a factor of* A_p *and* ϕ, *then the controller outputs satisfies in the steady state* $\tilde{\phi}u(k) = 0$ *where* $\phi = \phi_{A_p}\tilde{\phi}$

Using this *a priori* information with respect to the controller output in the steady state, one can minimize the criterion function

$$J = \sum_{i=1}^{h_P} [\hat{y}(k+i) - w(k+i)]^2 \tag{4.39}$$

Under the constraint that the future controller outputs satisfy (4.38), and taking into account the following equality constraint:

$$\phi(q^{-1})u(k+i-1) = 0 \quad 1 < i \le h_P \tag{4.40}$$

The dimension of the optimization problem is reduced from h_P to 1. Once $u(k)$ is known, $u(k+i-1)$ for $i > 1$ can be computed using equation (4.40). Minimization of equation (4.39), taking into account equation (4.40), ensure that equation (4.38) is satisfied.

Thus, the controller outputs are structured using *a priori* information about the controller output in the steady state. This yields the desired steady state behavior. The transient behavior; however, is also a major importance. One can use the relationship between the controller output and the process output to determine the transient behavior.

4.5.2.2 Structuring the Controller Output using Transient Arguments

The control of the output of the process (4.34) in the absence of disturbances can be driven to a constant reference trajectory in $n_{B_p} + 1$ samples and $n_{A_p} + 1$ different con-

troller outputs are required. In addition, for $i > n_{A_p} + 1$, $u(k+i-1)$ is constant. Hence, the output of a controller satisfies

$$\Delta u(k+i-1) = 0 \qquad i > n_{A_p} + 1$$

and the process output satisfies:

$$y(k+i) = w(k+i) \quad i \geq n_{B_p} + 1$$

A predictive controller with the same behavior can be realized by minimization of

$$J = \sum_{i = n_B + 1}^{h_P} [\hat{y}(k+i) - w(k+i)]^2 \tag{4.41}$$

under the constraint

$$\Delta u(k+i-1) = 0 \qquad n_{A_p} + 1 < i \leq h_P \text{ and } h_P \to \infty \tag{4.42}$$

Comparing equation (4.41) and (4.42) with equation (4.39) and (4.40), the tracking error for $i = 1, \ldots, n_{B_p}$ is not included in equation (4.41) and $\Delta u(k+i-1)$ is not assumed to be zero for $i = 2, \ldots, n_{A_p} + 1$. In order to have both possibilities, two extra parameters must be introduced into equation (4.41) and (4.42), yields

$$J = \sum_{i = h_m}^{h_P} [\hat{y}(k+i) - w(k+i)]^2 \tag{4.43}$$

and

$$\Delta u(k+i-1) = 0 \qquad h_c < i \leq h_P \tag{4.44}$$

where h_m is the minimum-cost horizon and

h_c is the control horizon

Hence , one can state that if $h_P \to \infty$, $h_m = n_{B_p} + 1$ and $h_c = n_{A_p} + 1$, the predictive controller minimizing equation (4.43) and taking equation (4.44) into account, will cause the process output to settle to a constant reference trajectory in $n_{B_p} + 1$ samples using $n_{A_p} + 1$ distinct controller outputs.

In order to generalize the predictive controller results, the following criterion func-

tion can be considered

$$J = \sum_{i=h_m}^{h_P} [\hat{y}(k+i) - w(k+i)]^2 \tag{4.45}$$

which is minimized under the following constraint

$$\phi u(k+i-1) = 0 \quad h_c < i \le h_P \tag{4.46}$$

The following theorem can be stated.

Theorem 4.1: *if* $h_P \ge n_{A_p} + n_{B_p} + d + n_\phi$, $h_m = n_{B_p} + d + 1$, $h_c = n_{A_p} + n_\phi$, *disturbances are absent and the input/output behavior of the process is correctly estimated, then minimization of (4.45), taking (4.46) into account, yields a controller that drives the process output* $y(k)$ *in* $n_{B_p} + d + 1$ *samples to a reference trajectory specified by* $\phi_w w(k+i) = 0$ *for* $i \ge 1$ [22].

Remarks

The condition mentioned in the theorem 4.1 ensures that the sampled process output tracks the reference trajectory in $n_{B_p} + d + 1$ samples. However, the process output between the samples cannot in general track a non-constant reference trajectory because the controller output between the samples is in general constant.

Hence, h_P, h_m, and h_c can be selected such that the process output settles in $n_{B_p} + d + 1$ samples at a reference trajectory described by $\phi w(k+i) = 0$. A disadvantage is that the response time can only be influenced by changing the sample time. However, the servo behavior can be tuned by introducing two auxiliary signals, $y'(k)$ and $u'(k)$ defined by

$$y(k) = \frac{P_p(1)}{P_p(q^{-1})} y'(k) \tag{4.47}$$

$$u(k) = \frac{P(1)}{P(q^{-1})} u'(k) \tag{4.48}$$

where $P_p(q^{-1})$ is a monic polynomial in q^{-1}. If criterion function (4.45) is minimized, and taking equation (4.46) into account with $y(k)$ and $u(k)$ replaced by $y'(k)$ and $u'(k)$

respectively. Because the use of P_p is important for tuning the servo behavior of the control system, P_p will be incorporated directly in criterion function (4.45) and (4.46) yielding

$$J = \sum_{i=h_m}^{h_P} [P_p\hat{y}(k+i) - P_p(1)w(k+i)]^2 \tag{4.49}$$

and $$\phi P_p u(k+i-1) = 0 \qquad h_c < i \le h_P \tag{4.50}$$

Criterion function (4.49) shows that $P_p\hat{y}(k+i)$ must be predicted instead of $\hat{y}(k+i)$. It can be shown that the i-step-ahead predictor predicting $P_p\hat{y}(k+i)$ is given by

$$P_p\hat{y}(k+i) = \frac{q^{-\hat{d}}\hat{B}_p P_p(q^{-1})}{\hat{A}_p(q^{-1})}u(k+i-1) + \frac{F_i}{\hat{T}}[y(k) - \hat{y}(k)] \tag{4.51}$$

where $\hat{y}(k)$ is given by (4.30) and F_i is solved from

$$\frac{P_p T}{\hat{D}_p} = E_i + q^{-1}\frac{F_i}{\hat{D}_p} \tag{4.52}$$

where $n_{F_i} = max(n_{P_p} + n_T - i, n_{D_p} - 1)$. Separation of future and past can be realized by using

$$\frac{\hat{B}_p P_p}{\hat{A}_p} = G_i + q^{-i+\hat{d}}\frac{H_i}{\hat{A}_p} \qquad i \ge \hat{d} + 1 \tag{4.53}$$

where $n_{G_i} \le i - \hat{d} - 1$ and $n_{H_i} = max(n_{B_p} + n_{P_p} - i + \hat{d}, n_{A_p} - 1)$. Equation (4.51) can be rewritten as

$$P_p\hat{y}(k+i) = G_i u(k+i-\hat{d}-1) + \frac{H_i}{\hat{A}_p}u(k-1) + \frac{F_i}{\hat{T}}[y(k) - \hat{y}(k)] \tag{4.54}$$

Finally, collecting the i-step-ahead predictors in a matrix notation for $i = h_m, \ldots, h_P$ yields

$$[\hat{y}^*] = [G][u] + [H][\dot{\hat{u}}] + [F][c] + \xi_{det} \tag{4.55}$$

where

$$[\hat{y}^*] = [P_p\hat{y}((k+h_m), \ldots, P_p\hat{y}(k+h_P))]^T$$

$$[u] = [u(k), \ldots, u(k+h_P-\hat{d}-1)]^T$$

$$[\dot{\hat{u}}] = [\dot{\hat{u}}(k-1), \dot{\hat{u}}(k-2), \ldots]^T$$

$$[c] = [c(k), c(k-1), \ldots]^T$$

and

$$\dot{\hat{u}}(k) = \frac{u(k)}{\hat{A}}$$

$$c(k) = \frac{y(k)-\hat{y}(k)}{T}$$

The dimensions of $[\hat{y}^*]$, $[u]$, $[\dot{\hat{u}}]$ and $[c]$ are given by

$$[\hat{y}^*] = h_P - h_m + 1 \times 1$$

$$[u] = h_P - \hat{d} \times 1$$

$$[\dot{\hat{u}}] = max(n_{H_i}) + 1 \times 1$$

$$[c] = max(n_{F_i}) + 1 \times 1$$

Note that $[G]$ is not square if $h_m > \hat{d} + 1$ and also ξ_{det} describes the effect of deterministic disturbances on $P_p\hat{y}(k+i)$.

In order to select and examine different criterion functions, the following unified criterion function is used

$$J = \sum_{i=h_m}^{h_P} [P_p\hat{y}(k+i) - P_p(1)w(k+i)]^2 + \rho \sum_{i=1}^{h_P-\hat{d}} \left[\frac{Q_n}{Q_d}u(k+i-1)\right]^2 \tag{4.56}$$

where Q_n and Q_d are monic polynomial with no common factors. This criterion function is minimized under the constraint

$$\phi P_p u(k+i-1) = 0 \quad 1 \le h_c \le i \le h_P - \hat{d} \tag{4.57}$$

The controller output weighting Q_n and Q_d are introduced into the criterion function because it is quite useful to choose whereas ρ of (4.37) is difficult to choose.

4.6 Predictive Control Law

The optimal control law can be derived by the minimization of criterion function (4.56) subject to equation (4.57) with respect to the controller output sequence over the control horizon h_c : $u(k), \ldots, u(k+h_c-1)$. In this section, the criterion function can be minimize analytically by assuming that there are no constraints.

4.6.1 Derivation of the Predictive Control Law

If the criterion function J is optimized with respect to the vector u, then any local optimum u satisfies

$$g = \frac{\partial J}{\partial u} = 0$$

where g denotes the gradient . If the Hessian H_H is given by

$$H_H = \frac{\partial^2 J}{\partial u^2}$$

is positive definite $\forall u$, then any local optimum is the global minimum.

In order to calculate the gradient of equation (4.56) with respect to u, the criterion function (4.56) is rewritten in matrix notation

$$J = ([\hat{y}^*] - [w^*])^T([\hat{y}^*] - [w^*]) + \rho[u^*]^T[u^*] \tag{4.58}$$

where

$$[w^*] = [P_p(1)w(k+h_m), \ldots, P_p(1)w(k+h_P)]^T$$

$$[y^*] = [P_p\hat{y}(k+h_m), \ldots, P_p\hat{y}(k+h_P)]^T$$

$$[u^*] = [u^*(k), \ldots, u^*(k+h_P-\hat{d}-1)]^T \quad , \; u^*(k) = \frac{Q_n}{Q_d}u(k)$$

Introduce the vector $[\bar{u}]$

$$[\bar{u}] = [u(k), \ldots, u(k+h_c-1)]^T \tag{4.59}$$

Note that $[\bar{u}]$ contains only those elements of the controller output sequence that must be calculated. The other elements over the prediction horizon must satisfy (4.57).

The gradient of (4.58) with respect to $[\bar{u}]$ becomes

$$\frac{\partial J}{\partial[\bar{u}]} = 2\frac{\partial[\hat{y}^*]}{\partial[\bar{u}]}([\hat{y}^*]-[w^*]) + 2\rho\frac{\partial[u^*]}{\partial[\bar{u}]}[u^*] \tag{4.60}$$

The relationship between $[u]$ and $[\bar{u}]$ can be derived by solving

$u(k+h_c)$,....., $u(k+h_P-\hat{d}-1)$ from (4.57)

$$\phi P_p u(k+i-1) = 0 \quad 1 \le h_c < i \le h_P - \hat{d} \tag{4.61}$$

The required relationship can be obtained by using the following Diophantine equation

$$\frac{1}{\phi P_p} = E_{i-h_c} + q^{-i+h_c}\frac{F_{i-h_c}}{\phi P_p} \Rightarrow \phi P_p E_{i-h_c} = 1 - q^{-i+h_c}F_{i-h_c} \tag{4.62}$$

where the degree of F_{i-H_c} is given by $n_\phi + n_P - 1$. Using equation (4.61); equation (4.62) becomes

$$u(k+i-1) = F_{i-h_c}u(k+h_c-1) \qquad 1 \le h_c < i \le h_P - \hat{d} \tag{4.63}$$

Separation of future and past terms is realized by using

$$F_{i-h_c} = G_{i-h_c} + q^{-h_c}H_{i-h_c} \tag{4.64}$$

in which the degree of G_{i-h_c} and H_{i-h_c} are given by $n_G = min(h_c, n_\phi + n_P) - 1$ and $n_H = n_\phi + n_P - h_c - 1$ respectively. Using (4.64), equation (4.63) becomes

$$u(k+i-1) = G_{i-h_c}u(k+h_c-1) + H_{i-h_c}u(k-1) \tag{4.65}$$

with $1 \le h_c < i \le h_P - \hat{d}$. Note that if $\phi = \Delta$ and $P_p = 1$, then $G_{i-h_c} = 1$ and $H_{i-h_c} = 0$.

Now the relationship between $[u]$ and $[\bar{u}]$ becomes in a matrix notation

$$[u] = [M][\bar{u}] + [N][\widehat{u}] \tag{4.66}$$

in which $[M]$ is a matrix of dimension $h_p - \hat{d} \times h_c$

$$[M] = \begin{bmatrix} 1 & 0 & \dots & \dots\dots & \dots & 0 \\ 0 & 1 & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots & 0 & 1 \\ 0 & \dots & 0 & g_{1,n_G} & \dots & g_{1,0} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & g_{j,n_G} & \dots & g_{j,0} \end{bmatrix}$$

and $[N]$ is a matrix of dimension $(h_P - \hat{d}) \times (n_\phi + n_P - h_c)$

$$[N] = \begin{bmatrix} 0 & \dots & 0 \\ \dots & \dots & \dots \\ 0 & \dots & 0 \\ h_{1,0} & \dots & h_{1,n_H} \\ \dots & \dots & \dots \\ h_{j,0} & \dots & h_{j,n_H} \end{bmatrix}$$ where $j = h_P - h_c - \hat{d}$ and $[\widehat{u}]$ is given by

$$[\widehat{u}] = [u(k-1), \dots, u(k+h_c-n_\phi-n_P)]^T \tag{4.67}$$

Note that if $h_c = h_P - \hat{d}$, then $[M] = I$ and $[N] = 0$

The relationship between $[u]$ and $[u^*]$ is required

$$u^*(k+i-1) = \frac{Q_n}{Q_d} u(k+i-1) \quad 1 \le i \le h_P - \hat{d} \tag{4.68}$$

Separation of future and past elements is realized by using

$$\frac{Q_n}{Q_d} = \Phi_i + q^{-i}\frac{L_i}{Q_d} \tag{4.69}$$

where Φ_i and L_i are polynomials of degree $i-1$ and $max(n_{Q_n} - i, n_{Q_d} - 1)$ if $n_{Q_d} > 0$. If $n_{Q_d} = 0$, the degrees of $_i$ and L_i are given by $min(i-1, n_{Q_n})$ and $n_{Q_n} - i$ respectively. Note that, because Q_n and Q_d are monic, Φ_i is also monic.

Using (4.69) , (4.68) becomes

$$u^*(k+i-1) = \Phi_i u(k+i-1) + L_i \frac{u(k-1)}{Q_d}$$

Collecting $u^*(k+i-1)$ for $i = 1, \ldots, h_P - \hat{d}$ in matrix notation yields

$$[u^*] = [\Phi][u] + [L][\tilde{u}] \tag{4.70}$$

where $[\Phi]$ is a lower triangular matrix of dimension $(h_P - \hat{d}) \times (h_P - \hat{d})$ and $[L]$ is a matrix of dimension $(h_P - \hat{d}) \times n_L$ with $n_L = max(n_{Q_n}, n_{Q_d})$. The vector $[\tilde{u}]$ is given by

$$[\tilde{u}] = \left[\frac{u(k-1)}{Q_d}, \ldots, \frac{u(k-n_L)}{Q_d}\right]^T \tag{4.71}$$

Using (4.66) and (4.70, the relationship between $[\bar{u}]$ and $[u^*]$ is given by

$$[u^*] = [\Phi][M][\bar{u}] + [L][\tilde{u}] + [\Phi][N][\widehat{u}] \tag{4.72}$$

The partial derivative $\frac{\partial[u^*]}{\partial[\bar{u}]}$ now becomes $\frac{\partial[u^*]}{\partial[\bar{u}]} = [M]^T[\Phi]^T$

The partial derivative $\frac{\partial[\hat{y}^*]}{\partial[\bar{u}]}$ can be calculated by using the unified prediction model (4.55) . Utilizing (4.55) and (4.66), the relationship between $[\hat{y}^*]$ and $[\bar{u}]$ is given by

$$[\hat{y}^*] = [G][M][\bar{u}] + [H][\hat{u}] + [F][c] + \xi_{det} + [G][N][\widehat{u}] \tag{4.73}$$

The gradient (4.60) becomes

$$\frac{\partial J}{\partial[\bar{u}]} = 2[M]^T([G]^T[G] + \rho[K]^T[K])[M][\bar{u}] + 2[M]^T\{[G]^T([H][\hat{u}] + [F][c]$$

$$+ \xi_{det} + [G][N][\widehat{u}] - [w^*]) + \rho[\Phi]^T([L][\tilde{u}] + [K][N][\widehat{u}])\} \tag{4.74}$$

and the Hessian becomes

$$H_H = 2[M]^T([G]^T[G] + \rho[\Phi]^T[\Phi])[M] \tag{4.75}$$

Note that the Hessian is independent of $[\bar{u}]$ and, when singular, is positive defi-

nite. If $\rho \geq 0$, the Hessian is always positive definite. If, however, $\rho = 0$, the Hessian can in some special cases, be nonsingular. Assuming that the Hessian is singular, a global minimum of J with respect to $[\bar{u}]$ can be obtained by setting the gradient equal zero and solving for $[\bar{u}]$

$$[\bar{u}] = \{[M]^T([G]^T[G] + \rho[\Phi]^T[\Phi])[M]\}^{-1}[M]^T\{[G]([w^*] - [H][\tilde{u}] - [F][c] - \xi_{del} - [G][N][\widehat{u}]) - \rho[\Phi]([L][\tilde{u}] + [\Phi][N][\widehat{u}])\} \quad (4.76)$$

Note that the matrix to be inverted is of dimension $h_c \times h_c$. The first element of $\bar{u}(= u(k))$ is used to control the process. All other elements are not used and need not to be calculated.

4.7 Computational Steps for Predictive Control Law

1. From the model parameters, determine the prediction horizon h_P, the minimum-cost horizon h_m, the control horizons h_c, polynomial in criteria function P_p, Q_d, Q_n.

2. Compute the matrices $[G]$, $[F]$, $[H]$ in the prediction model equation (4.55).

3. Compute the matrices $[M]$, $[N]$ from equation (4.66) and matrices $[\Phi]$, $[L]$ from equation (4.70).

4. From the past and future input output data Solve for the first element of control $\bar{u}$ from equation (4.76).

5. Compute the feed-back control to get the output response.

CHAPTER V

PREDICTIVE CONTROL IN SYSTEM IDENTIFICATION APPROACH

5.1 Introduction

The state-space model has long been a fundamental element of modern control theory. In a state-space model, the relationship between the input and output variable is described in terms of an intermediate quantity called the state vector[23]. The state-space models can be derived analytically from the equations of motion or identified from experimental input-output data using system identification. Concurrent with the development of state-space based control methods are adaptive and predictive control (which are based on input-output models). A typical input-output model describes the current outputs as a linear combination of past input and output measurements. One such model is the Auto Regressive with eXogenous input model (ARX). An attractive feature of an ARX model is that its coefficients can be identified from input and output measurement. The identification process can be carried out recursively in real time if necessary.

Most of the researches that have been carried out in adaptive control are concerned with two classes of systems called Model Reference Adaptive Systems (MARS) and Self Tuning Regulators (STR)[24-26]. While the former evolved from deterministic servo problems, the latter arose in the context of stochastic regulation problems. The conventional approach for control system design of dynamic system normally can be distinguished into two phases: system modeling and controller design. For the system with unknown disturbances and considerable uncertainties, the conventional approach is not quick enough to catch up with the system changes. Therefore, on-line system identification and adaptive controller design become the significant solution for the controlled system[27-28].

Juang[29-30] proposed a deadbeat control design using an ARX model which showed the successful results to suppress the noise of dynamical system. This chapter will

propose the extension of the deadbeat control design to the tracking control design with and without system disturbances input to the system.

Section 5.2 starts with multi-step output predictor that can be derived from ARX model. The observer Markov parameters (OMP) or ARX parameters can be calculated by the recursive formulation. In section 5.3, a receding horizon predictive control concept is shown which uses the first element of controller output sequence to control the dynamic process . Section 5.4 shows the predictive control design which uses the algorithm derived from section 5.2 to form the control law. This approach still needs to calculate the OMP and ARX parameters first before forming the control step. In section 5.5, the standard Recursive Least-Square formulation is presented and able to combine system identification and predictive control in one formulation. The control parameters are updated in every sampling period. Since there is no matrix inversion needed to compute the control parameters, the method is applicable in real-time. Section 5.6 shows the extension of predictive control algorithm when feedback and feedforward for disturbance inputs are taken into consideration.

5.2 Multi-Step Output Prediction

A linear finite difference model for the $m \times 1$ output $y(k)$ and the $s \times 1$ input $u(k)$ at time k is described by:

$$y(k) = a_1 y(k-1) + a_2 y(k-2) + \ldots + a_p y(k-p) + b_0 u(k) + b_1 u(k-1) + b_2 u(k-2) + \ldots + b_p u(k-p) \quad (5.1)$$

This represents the relationship between the input and output and also means that the current output can be computed by the time series of the past input and past output. The finite difference model is also referred to as the ARX model as in (2.1). The coefficient matrices, $a_i (i = 1, 2, \ldots, p)$ of $m \times m$ and $b_i (i = 0, 1, 2, \ldots, p)$ of $m \times s$, are referred to as the observer Markov parameters (OMP) or ARX parameters. The matrix b_0 is the direct transmission term.

By shifting one time step ahead, one obtains

$$y(k+1) = a_1 y(k) + a_2 y(k-1) + \ldots + a_p y(k-p+1) + b_0 u(k+1) + b_1 u(k) + b_2 u(k-1) + \ldots + b_p u(k-p+1) \quad (5.2)$$

Lets define the following quantities

$$a_1^{(1)} = a_1 a_1 + a_2 \qquad b_1^{(1)} = a_1 b_1 + b_2$$

$$a_2^{(1)} = a_1 a_2 + a_3 \qquad b_2^{(1)} = a_1 b_2 + b_3$$

$$a_{p-1}^{(1)} = a_1 a_{p-1} + a_p \qquad b_{p-1}^{(1)} = a_1 b_{p-1} + b_p$$

$$a_p^{(1)} = a_1 a_p \qquad b_p^{(1)} = a_1 b_p \tag{5.3}$$

and

$$b_0^{(1)} = a_1 b_0 + b_1 \tag{5.4}$$

Substituting $y(k)$ from (5.1) to (5.2) yields

$$y(k+1) = a_1^{(1)} y(k-1) + a_2^{(1)} y(k-2) + \ldots + a_p^{(1)} y(k-p)$$

$$+ b_0 u(k+1) + b_0^{(1)} u(k) + b_1^{(1)} u(k-1) + b_2^{(1)} u(k-2) + \ldots + b_p^{(1)} u(k-p) \tag{5.5}$$

The output measurement at time step $k+1$ can be expressed as the sum of past input and output data with the absence of the output measurement at time step k. For the i-step ahead, one can express the output measurement at the time step $k+i$ by:

$$\begin{aligned} y(k+i) = {} & a_1^{(i)} y(k-1) + a_2^{(i)} y(k-2) + \ldots + a_p^{(i)} y(k-p) \\ & + b_0 u(k+i) + b_0^{(i)} u(k+i-1) + \ldots + b_0^{(i)} u(k) \\ & + b_1^{(i)} u(k-1) + b_2^{(i)} u(k-2) + \ldots + b_p^{(i)} u(k-p) \end{aligned} \tag{5.6}$$

where

$$a_1^{(i)} = a_1^{(i-1)} a_1 + a_2^{(i-1)} \qquad b_1^{(i)} = a_1^{(i-1)} b_1 + b_2^{(i-1)}$$

$$a_2^{(i)} = a_1^{(i-1)} a_2 + a_3^{(i-1)} \qquad b_2^{(i)} = a_1^{(i-1)} b_2 + b_3^{(i-1)}$$

$$a_{p-1}^{(i)} = a_1^{(i-1)} a_{p-1} + a_p^{(i-1)} \qquad b_{p-1}^{(i)} = a_1^{(i-1)} b_{p-1} + b_p^{(i-1)}$$

$$a_p^{(i)} = a_1^{(i-1)} a_p \qquad b_p^{(i)} = a_1^{(i-1)} b_p \tag{5.7}$$

and

$$b_0^{(i)} = a_1^{(i-1)} b_0 + b_1^{(i-1)} \tag{5.8}$$

Note that $a_j^{(0)} = a_j$ and $b_j^{(0)} = b_j$ for any possible integer 1,2,..... including 0 if applicable.

With some algebraic operation, Equation (5.8) becomes

$$b_0^{(0)} = b_0$$

$$b_0^{(k)} = b_k + \sum_{j=1}^{k} a_j b_0^{(k-i)} \qquad \text{for } k = 1, \ldots, p$$

$$b_0^{(k)} = \sum_{j=1}^{p} a_j b_0^{(k-j)} \qquad \text{for } k = p+1, \ldots, \infty \qquad (5.9)$$

Similar to (5.9) $a_1^{(i)} = a_1^{(i-1)} a_1 + a_2^{(i-1)}$ can also be written as

$$a_1^{(0)} = a_1$$

$$a_1^{(k)} = a_{k+1} + \sum_{j=1}^{k} a_j a_1^{(k-i)} \qquad \text{for } k = 1, \ldots, p-1$$

$$a_1^{(k)} = \sum_{j=1}^{p} a_j a_1^{(k-i)} \quad \text{for} \quad k = p, \ldots, \infty \qquad (5.10)$$

and also $a_{p-1}^{(i)} = a_1^{(i-1)} a_{p-1} + a_p^{(i-1)}$ can be written as

$$a_{p-1}^{(0)} = a_{p-1}$$

$$a_{p-1}^{(k)} = a_{k+p-1} + \sum_{j=1}^{k} a_j a_{p-1}^{(k-i)} \qquad \text{for } k = 1, \ldots, p-1$$

$$a_{p-1}^{(k)} = \sum_{j=1}^{p} a_j a_{p-1}^{(k-i)} \qquad \text{for } k = p, \ldots, \infty \qquad (5.11)$$

The quantities $b_0^{(j)}$ $(j = 0, 1, \ldots)$ are the pulse response sequence which and $a_1^{(j)}$ $(j = 0, 1, \ldots)$ are the observer gain Markov parameters that can be used to compute an observer for state estimation.

Let the index i be $i = 1, 2, \ldots, h_p, h_p + 1, \ldots, f-1$. and equation (5.7) can be written in matrix form.

$$y_f(k) = \gamma u_f(k) + \alpha y_p(k-p) + \beta u_p(k-p) \qquad (5.12)$$

where

$$y_f(k) = \begin{bmatrix} y(k) \\ y(k+1) \\ \cdots \\ y(k+h_p) \\ y(k+h_p+1) \\ \cdots \\ y(k+f-1) \end{bmatrix} \quad , \quad u_f(k) = \begin{bmatrix} u(k) \\ u(k+1) \\ \cdots \\ u(k+h_p) \\ u(k+h_p+1) \\ \cdots \\ u(k+f-1) \end{bmatrix}$$

$$y_p(k-p) = \begin{bmatrix} y(k-p) \\ y(k-p+1) \\ \cdots \\ y(k-1) \end{bmatrix} \quad , \quad u_p(k-p) = \begin{bmatrix} u(k-p) \\ u(k-p+1) \\ \cdots \\ u(k-1) \end{bmatrix} \tag{5.13}$$

and

$$\gamma = \begin{bmatrix} b_0 & & & & & & \\ b_0^{(1)} & b_0 & & & & & \\ \cdots & \cdots & & & & & \\ b_0^{(h_p)} & b_0^{(h_p-1)} & \cdots & b_0 & & & \\ b_0^{(h_p+1)} & b_0^{(h_p)} & \cdots & b_0^{(1)} & b_0 & & \\ \cdots & \cdots & \cdots & \cdots & \cdots & & \\ b_0^{(f-1)} & b_0^{(f-2)} & \cdots & b_0^{(f-h_p-1)} & b_0^{(f-h_p-2)} & \cdots & b_0 \end{bmatrix}$$

$$\beta = \begin{bmatrix} b_p & b_{p-1} & \cdots & b_1 \\ b_p^{(1)} & b_{p-1}^{(1)} & \cdots & b_1^{(1)} \\ \cdots & \cdots & \cdots & \cdots \\ b_p^{(h_p)} & b_{p-1}^{(h_p)} & \cdots & b_1^{(h_p)} \\ b_p^{(h_p+1)} & b_{p-1}^{(h_p+1)} & \cdots & b_1^{(h_p+1)} \\ \cdots & \cdots & \cdots & \cdots \\ b_p^{(f-1)} & b_{p-1}^{(f-1)} & \cdots & b_1^{(f-1)} \end{bmatrix}$$

$$\alpha = \begin{bmatrix} a_p & a_{p-1} & \cdots & a_1 \\ a_p^{(1)} & a_{p-1}^{(1)} & \cdots & a_1^{(1)} \\ \cdots & \cdots & \cdots & \cdots \\ a_p^{(h_p)} & a_{p-1}^{(h_p)} & \cdots & a_1^{(h_p)} \\ a_p^{(h_p+1)} & a_{p-1}^{(h_p+1)} & \cdots & a_1^{(h_p+1)} \\ \cdots & \cdots & \cdots & \cdots \\ a_p^{(f-1)} & a_{p-1}^{(f-1)} & \cdots & a_1^{(f-1)} \end{bmatrix} \tag{5.14}$$

The quantity $y_f(k)$ represents the future output with a total of f data points from the time step k to $k+f-1$, where $y_p(k-p)$ represent the past output.with the p data point from the time step $k-p$ to $k-1$.

Similarly, $u_f(k)$ has f future input data points starting from the time step k to $k+f-1$, where $u_p(k-p)$ has past output with the p data point from the time step $k-p$ to $k-1$. The matrix γ is commonly called the Toeplitz matrix which is formed from the parameters, $b_0, b_0^{(1)}, \ldots, b_0^{(f-1)}$ (the pulse response sequence).

The vector $y_f(k)$ consists of three terms. The first term is the future input vector $u_f(k)$ including inputs from time step k to $k+f-1$. Relative to the same time k, the second and third terms , $u_p(k-p)$ and $y_p(k-p)$, are past input and output vectors respectively, which past known data from $k-p$ to $k-1$.

5.3 Receding Horizon Predictive Control

Consider figure 5.1-b and 5.1-d and suppose that the current time is denoted by sample k which corresponds to the absolute time t. $u(k), y(k)$ and $w(k)$ denote the controller output, the process output and the desired process output at sample k, respectively.

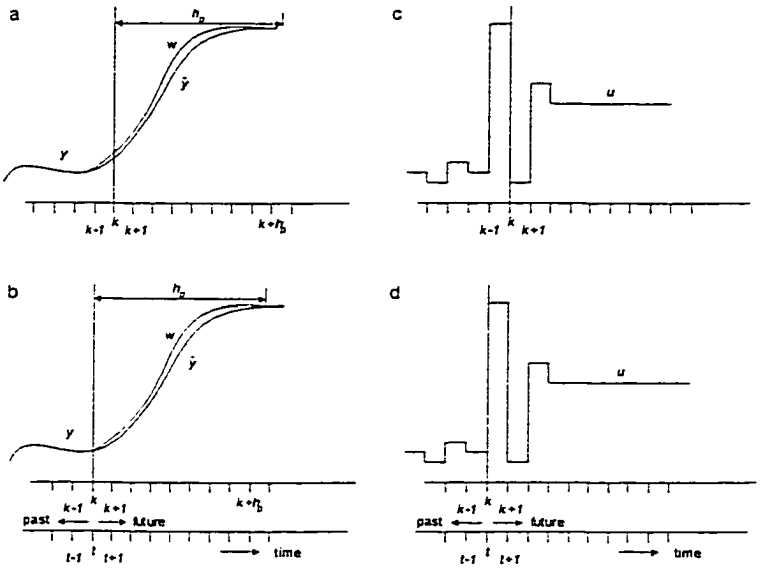

Figure 5.1 Receding Horizon Predictive Control. Parts *b* and *d* Denote the Situation at time t while Parts *a* and *c* Denote the Situation at time $t+1$

Define:

$$u = [u(k), \ldots, u(k+h_p-1)]^T$$

$$\hat{y} = [\hat{y}(k), \ldots, \hat{y}(k+h_p)]^T$$

$$w = [w(k), \ldots, w(k+h_p)]^T$$

where h_p is the predictive horizon and symbol $\hat{}$ denotes estimation. A predictive controller calculates such a future controller output sequence u, that predicted output of the process $\hat{y}$ is "close" to the desired process output w. The desired process output is often called the reference trajectory and it can be an arbitrary sequence of points[31].

Rather than using the controller output sequence determined to control the process in the next h_p samples, only the first element of this controller output sequence is used to control the process. At the next sample (hence, at $t+1$), the whole procedure is repeated using the latest measured information. Assuming that there are no disturbances and no modeling error, the predicted process output $\hat{y}(k+1)$ predicted at time t is exactly equal to the process output $y(k)$ measured at $t+1$. Again, a future controller output sequence is calculated such that the predicted process output is "close" to the reference trajectory.

5.4 Predictive Control Design

Assume that the close loop control action starts at time step k. The system before time k is open loop. When the control action is turn on at time k and end at time $k+h_p$, the control steps beyond the step $k+h_p$ are all zero. Equation (5.12) becomes:

$$y_f(k+h_p) = \gamma' u_f(k) + \alpha' y_p(k-p) + \beta' u_p(k-p) \tag{5.15}$$

where

$$y_f(k+h_p) = \begin{bmatrix} y(k+h_p) \\ y(k+h_p+1) \\ \cdots \\ y(k+h_p+p-1) \end{bmatrix}$$

and

$$\gamma' = \begin{bmatrix} b_0^{(h_p)} & b_0^{(h_p-1)} & \cdots & b_0 & & & \\ b_0^{(h_p+1)} & b_0^{(h_p)} & \cdots & b_0^{(1)} & b_0 & & \\ \cdots & \cdots & \cdots & \cdots & \cdots & & \\ b_0^{(h_p+p-1)} & b_0^{(h_p+p-2)} & \cdots & b_0^{(p-1)} & b_0^{(p-2)} & \cdots & b_0 \end{bmatrix}$$

$$\beta' = \begin{bmatrix} b_p^{(h_p)} & b_{p-1}^{(h_p)} & \cdots & b_1^{(h_p)} \\ b_p^{(h_p+1)} & b_{p-1}^{(h_p+1)} & \cdots & b_1^{(h_p+1)} \\ \cdots & \cdots & \cdots & \cdots \\ b_p^{(h_p+p-1)} & b_{p-1}^{(h_p+p-1)} & \cdots & b_1^{(h_p+p-1)} \end{bmatrix}$$

$$\alpha' = \begin{bmatrix} a_p^{(h_p)} & a_{p-1}^{(h_p)} & \cdots & a_1^{(h_p)} \\ a_p^{(h_p+1)} & a_{p-1}^{(h_p+1)} & \cdots & a_1^{(h_p+1)} \\ \cdots & \cdots & \cdots & \cdots \\ a_p^{(h_p+p-1)} & a_{p-1}^{(h_p+p-1)} & \cdots & a_1^{(h_p+p-1)} \end{bmatrix} \tag{5.16}$$

Let γ' be partitioned into two parts then equation (5.15) becomes

$$y_f(k+h_p) = \gamma_o u_f(k+h_p) + \gamma_c u_f(k) + \alpha' y_p(k-p) + \beta' u_p(k-p) \tag{5.17}$$

where

$$u_f(k+h_p) = \begin{bmatrix} u(k+h_p) \\ u(k+h_p+1) \\ \cdots \\ u(k+h_p+p-1) \end{bmatrix}, \quad u_f(k) = \begin{bmatrix} u(k) \\ u(k+1) \\ \cdots \\ u(k+h_p-1) \end{bmatrix}$$

and

$$\gamma_o = \begin{bmatrix} b_0 & 0 & \cdots & 0 \\ b_0^{(1)} & b_0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ b_0^{(p-1)} & b_0^{(p-2)} & \cdots & b_0 \end{bmatrix}$$

$$\gamma_c = \begin{bmatrix} b_0^{(h_p)} & b_0^{(h_p-1)} & \cdots & b_0^{(1)} \\ b_0^{(h_p+1)} & b_0^{(h_p)} & \cdots & b_0^{(2)} \\ \cdots & \cdots & \cdots & \cdots \\ b_0^{(h_p+p-1)} & b_0^{(h_p+p-2)} & \cdots & b_0^{(p)} \end{bmatrix} \tag{5.18}$$

Both γ_o and γ_c are formed by system pulse response (system markov parameters). By the given input and output sequence, equation (5.17) must satisfied and be able to identify coefficient matrices γ_o, γ_c, α', and β'.

By the idea of receding horizon control, equation (5.17) provides multiple solution for $u_f(k)$ with the minimum-norm solution expressed by

$$u_f(k) = \gamma_c^{\dagger} y_f(k+h_p) - \gamma_c^{\dagger}\gamma_o u_f(k+h_p) - \gamma_c^{\dagger}\alpha' y_p(k-p) - \gamma_c^{\dagger}\beta' u_p(k-p) \tag{5.19}$$

or in the matrix form

$$u_f(k) = \left[-\gamma_c^{\dagger}A' \; -\gamma_c^{\dagger}\beta' \; \gamma_c^{\dagger} \; -\gamma_c^{\dagger}\gamma_o\right] \begin{bmatrix} y_p(k-p) \\ u_p(k-p) \\ y_f(k+h_p) \\ u_f(k+h_p) \end{bmatrix} \tag{5.20}$$

To simplify equation (5.20), the following notation is defined

$$P_c = \left[-\gamma_c^{\dagger}\alpha' \; -\gamma_c^{\dagger}\beta'\right] \text{ and } P_o = \left[\gamma_c^{\dagger} \; -\gamma_c^{\dagger}\gamma_o\right] \tag{5.21}$$

$$\varphi_p(k-p) = \begin{bmatrix} y_p(k-p) \\ u_p(k-p) \end{bmatrix} \text{ and } \varphi_p(k+h_p) = \begin{bmatrix} y_f(k+h_p) \\ u_f(k+h_p) \end{bmatrix} \tag{5.22}$$

Equation (5.20) becomes

$$u_f(k) = \left[P_c \; P_o\right] \begin{bmatrix} \varphi_p(k-p) \\ \varphi_p(k+h_p) \end{bmatrix} \tag{5.23}$$

Equation (5.23) is another form of finite difference model for system identification. For any given input and output data, there exists a set of P_c and P_o satisfying equation (5.23).

In order to develop the predictive control law, equation (5.20) will give the control vector $u_f(k)$ by setting $y_f(k+h_p)$ (which is the future output equal to the reference trajectory $w(k)$) and setting the future control vector $u_f(k+h_p)$ to zero. Hence the $\varphi_f(k+h_p)$ will be in the form

$$\varphi_f(k+h_p) = \begin{bmatrix} w(k) \\ 0 \end{bmatrix} \tag{5.24}$$

and equation (5.23) can be rewritten as

$$u_f(k) = \left[-\gamma_c^{\dagger}\alpha' \; -\gamma_c^{\dagger}\beta' \; \gamma_c^{\dagger} \; -\gamma_c^{\dagger}\gamma_o\right] \begin{bmatrix} y_p(k-p) \\ u_p(k-p) \\ w(k) \\ 0 \end{bmatrix} \tag{5.25}$$

The first element of the control action at $u(k)$ will be in the form

$$u(k) = (\text{the first } m \text{ rows of } [P_c \; P_o]) \begin{bmatrix} \varphi_p(k-p) \\ \varphi_f(k+h_p) \end{bmatrix}$$

$$= \begin{bmatrix} P_{c1} & P_{o1} \end{bmatrix} \begin{bmatrix} \varphi_p(k-p) \\ \varphi_f(k+h_p) \end{bmatrix} \tag{5.26}$$

where the first m rows of $u_f(k)$ is the input vector $u(k)$ at time k. P_{c1} and $_{o1}$ are the first m rows of P_c and P_o, respectively.

Let us form the following matrices

$$U(k) = \begin{bmatrix} u(k) & u(k+1) & \dots & u(N-p-h_p+1) \end{bmatrix},$$

$$\Phi_p(k-p) = \begin{bmatrix} y(k-p) & y(k-p+1) & \dots & y(N-2p-h_p+1) \\ u(k-p) & u(k-p+1) & \dots & u(N-2p-h_p+1) \\ \dots & \dots & \dots & \dots \\ y(k-1) & y(k) & \dots & y(N-p-h_p) \\ u(k-1) & u(k) & \dots & u(N-p-h_p) \end{bmatrix},$$

$$\Phi_f(k+h_p) = \begin{bmatrix} y(k+h_p) & y(k+h_p+1) & \dots & y(N-p+1) \\ u(k+h_p) & u(k+h_p+1) & \dots & u(N-p+1) \\ \dots & \dots & \dots & \dots \\ y(k+h_p+p-1) & y(k+h_p+p) & \dots & y(N) \\ u(k+h_p+p-1) & u(k+h_p+p) & \dots & u(N) \end{bmatrix} \tag{5.27}$$

where N is the data length used for estimation of coefficient matrices P_{c1} and P_{o1}. Equation (5.26) can be solved by corroborating equation (5.27) in the form

$$U(k) = \begin{bmatrix} P_{c1} & P_{o1} \end{bmatrix} \begin{bmatrix} \Phi_p(k-p) \\ \Phi_f(k+h_p) \end{bmatrix} \tag{5.28}$$

In order to be able to solve Least-Squares solution, the data length N must be chosen large enough such that the matrix $U(k)$ of $m \times (N-p-h_p-k+2)$ has rank m,both

$\Phi_p(k-p)$ of $p(m+s)\times(N-p-h_p-k+2)$ and $\Phi_f(k+h_p)$ of $p(m+s)\times(N-p-h_p-k+2)$ have rank $pm+n$. Equation (5.28) produces the following Least-Squares solution.

$$\begin{bmatrix} P_{c1} & P_{o1} \end{bmatrix} = U(k)\begin{bmatrix} \Phi_p(k-p) \\ \Phi_f(k+h_p) \end{bmatrix}^{\dagger} \tag{5.29}$$

5.5 Recursive Least-Square Algorithm

Recursive Least-Square technique can be used to solve the Least-Square problem in equation (5.29). For computation efficiency, it is desirable to arrange the algorithms in such a way that the results obtained previously can be used for on-line updating. There are many recursive algorithms available to solve a Least-Square problem. The classical Least-Square method is the most straightforward approach[32]. The classical recursive method is briefly described as follows:

Rewriting equation (5.28) in a compact matrix form

$$\begin{aligned} u(k) &= \begin{bmatrix} P_{c1} & P_{o1} \end{bmatrix}\begin{bmatrix} \varphi_p(k-p) \\ \varphi_f(k+h_p) \end{bmatrix} \\ &= P\bar{\varphi}(k-1) \end{aligned} \tag{5.30}$$

where

$$P = \begin{bmatrix} P_{c1} & P_{o1} \end{bmatrix},\ \varphi(k) = \begin{bmatrix} y(k) \\ u(k) \end{bmatrix},$$

$$\bar{\varphi}(k-1) = \begin{bmatrix} \varphi(k-p) \\ \cdots \\ \varphi(k-1) \\ \varphi(k+h_p) \\ \cdots \\ \varphi(k+h_p+p-1) \end{bmatrix} \tag{5.31}$$

First, define the following quantities

$$(k) = \frac{\bar{\varphi}^T(k)W(k-1)}{1+\bar{\varphi}^T(k)W(k-1)\bar{\varphi}(k)} \tag{5.32}$$

$$\hat{u}(k+1) = \hat{P}(k)\bar{\varphi}(k) \tag{5.33}$$

Then compute the following quantities

$$(k) = W(k-1)[I-\bar{\varphi}(k)Z(k)] \tag{5.34}$$

$$\hat{P}(k+1) = \hat{P}(k) + [u(k+1)-\hat{u}(k+1)]Z(k) \tag{5.35}$$

Here, no matrix inversion is needed and no approximation has been made in the derivation of the Recursive Least-Square. Therefore, the Recursive Least-Squares estimate and the off-line estimate are theoretically identical. The initial values of $W(0)$ and $\hat{P}(1)$ can be either obtained from performing a Batch Least-Squares after collecting a sufficient number of data or assigned as $\vartheta I_{2p(m+s)}$ of $W(0)$ and $0_{m\times 2p(m+s)}$ of $\hat{P}(1)$ (where ϑ is a large positive number).

5.6 Feedback and Feedforward for Disturbance Input

In addition to the control input, there might be other disturbance inputs applied to the system. Some of the types of disturbances come from known sources that can be measured. Another type of disturbance is not known, but its correlation is known. This section presents the predictive feedback control design including feedforward from the measurable disturbance inputs.

With the disturbance input involved, the finite difference model in equation (5.1) becomes

$$\begin{aligned} y(k) &= a_1y(k-1)+a_2y(k-2)+\ldots+a_py(k-p) \\ &+ b_{c0}u_c(k)+b_{c1}u_c(k-1)+b_{c2}u_c(k-2)+\ldots+b_{cp}u_c(k-p) \\ &+ b_{d0}u_d(k)+b_{d1}u_d(k-1)+b_{d2}u_d(k-2)+\ldots+b_{dp}u_d(k-p) \end{aligned} \tag{5.36}$$

where the subscripts c and d are used to signify the corresponding quantities associated with the control input and the disturbance input, respectively. From section 5.2 , Equation (5.36) can be rewritten in the form

$$y_f(k+h_p) = \gamma_o u_{cf}(k+h_p)+\gamma_c u_{cf}(k)+\gamma'_d u_{d(h_p+p)}(k)+\alpha' y_p(k-p)$$

$$+\beta'_c u_{cp}(k-p) + \beta'_d u_{dp}(k-p) \tag{5.37}$$

where

$$y_p(k+h_p) = \begin{bmatrix} y(k+h_p) \\ y(k+h_p+1) \\ \cdots \\ y(k+h_p+p-1) \end{bmatrix}, \quad u_{d(h_p+p)}(k) = \begin{bmatrix} u_d(k) \\ u_d(k+1) \\ \cdots \\ u_d(k+h_p+p-1) \end{bmatrix}$$

$$u_{cf}(k+h_p) = \begin{bmatrix} u_c(k+h_p) \\ u_c(k+h_p+1) \\ \cdots \\ u_c(k+h_p+p-1) \end{bmatrix}, \quad u_{cf}(k) = \begin{bmatrix} u_c(k) \\ u_c(k+1) \\ \cdots \\ u_c(k+h_p-1) \end{bmatrix} \tag{5.38}$$

The form of matrix γ'_d associated with the disturbances u_d is similar to γ' defined in equation (5.16). The matrix γ'_c is a $ps \times h_p m_c$ matrix where m_c is the number of control inputs and γ'_d is a $ps \times h_p m_d$ matrix where m_d is the number of disturbance inputs. The form of β'_c and β'_d are also similar but corresponding to different type of forces. γ_c, γ_o, and β'_c are quantities associated with the control force $u_c(k)$.

Similar to equation (5.19), one can derive control force $u_{cf}(k)$ as

$$\begin{aligned} u_{cf}(k) = {} & \gamma_c^{\dagger} y_f(k+h_p) - \gamma_c^{\dagger}\gamma_o u_{cf}(k+h_p) - \gamma_c^{\dagger}\gamma'_d u_{d(h_p+p)}(k) \\ & - \gamma_c^{\dagger}\alpha' y_p(k-p) - \gamma_c^{\dagger}\beta'_c u_{cp}(k-p) - \gamma_c^{\dagger}\beta'_d u_{dp}(k-p) \end{aligned} \tag{5.39}$$

or in matrix form

$$u_{cf}(k) = \begin{bmatrix} -\gamma_c^{\dagger}\alpha' & -\gamma_c^{\dagger}\beta'_c & -\gamma_c^{\dagger}\beta'_d & -\gamma_c^{\dagger} & -\gamma_c^{\dagger}\gamma_o & -\gamma_c^{\dagger}\gamma'_d \end{bmatrix} \begin{bmatrix} y_p(k-p) \\ u_{cp}(k-p) \\ u_{dp}(k-p) \\ y_f(k+h_p) \\ u_{cf}(k+h_p) \\ u_{d(h_p+p)}(k) \end{bmatrix} \tag{5.40}$$

To simplify equation (5.40) define the following notation

$$P'_c = \left[-\gamma_c^{\dagger}\alpha' \; -\gamma_c^{\dagger}\beta'_c \; -\gamma_c^{\dagger}\beta'_d\right] \quad ,$$

$$P'_{co} = \left[-\gamma_c^{\dagger} \; -\gamma_c^{\dagger}\gamma_o\right] \quad ,$$

and $$P'_d = \left[-\gamma_c^{\dagger}\gamma'_d\right] \tag{5.41}$$

and

$$\varphi_p(k-p) = \begin{bmatrix} y_p(k-p) \\ u_{cp}(k-p) \\ u_{dp}(k-p) \end{bmatrix} \text{ and } \varphi_{cp}(k+h_p) = \begin{bmatrix} y_f(k+h_p) \\ u_{cf}(k+h_p) \end{bmatrix} \tag{5.42}$$

where P'_c is a $h_p m_c \times p(s+m_c+m_d)$ matrix, P'_{co} is a $h_p m_c \times p(s+m_c)$ matrix. The quantity $\varphi_p(k-p)$ is a $p(f+m_c+m_d)\times 1$ column vector whereas $\varphi_{cp}(k+h_p)$ is a $p(f+m_c)\times 1$ column vector. Equation (5.40) becomes

$$u_{cf}(k) = \left[P'_c \; P'_{co} \; P'_d\right] \begin{bmatrix} \varphi_p(k-p) \\ \varphi_{cp}(k+h_p) \\ u_{d(h_p+p)}(k) \end{bmatrix} \tag{5.43}$$

For any given input and output data, there exists a set of P'_c , P'_{co} and P'_d satisfying equation (5.43). By using the same approach as mentioned in section 5.4, the predictive control law, equation (5.40) will give the control vector $u_{cf}(k)$ by setting $y_f(k+h_p)$ which is the future output equal to the reference trajectory $w(k)$ and the future control vector $u_f(k+h_p)$ is equal to zero. However the term $u_{d(h_p+p)}(k)$ which is the future disturbance can be known only if the disturbance is measurable and known beforehand. Otherwise, the equation (5.40) will not be true.

Equation (5.40) can be rewritten as

$$u_{cf}(k) = \left[-\gamma_c^{\dagger}\alpha' \; -\gamma_c^{\dagger}\beta'_c \; -\gamma_c^{\dagger}\beta'_d \; -\gamma_c^{\dagger} \; -\gamma_c^{\dagger}\gamma_o \; -\gamma_c^{\dagger}\gamma'_d\right]\begin{bmatrix} y_p(k-p) \\ u_{cp}(k-p) \\ u_{dp}(k-p) \\ w(k) \\ 0 \\ u_{d(h_p+p)}(k) \end{bmatrix} \tag{5.44}$$

The first element of the control action at $u(k)$ will be in the form

$$u(k) = (\text{the first } m \text{ rows of } [\,P'_c \;\; P'_o \;\; P'_d\,])\begin{bmatrix} \varphi_p(k-p) \\ \varphi_f(k+h_p) \\ u_{d(h_p+p)}(k) \end{bmatrix}$$

$$= \left[P'_{c1} \; P'_{o1} \; P'_{d1}\right]\begin{bmatrix} \varphi_p(k-p) \\ \varphi_f(k+h_p) \\ u_{d(h_p+p)}(k) \end{bmatrix}$$

$$= P\bar{\varphi}(k-1) \tag{5.45}$$

where

$$\bar{\varphi}(k-1) = \begin{bmatrix} \varphi_p(k-p) \\ \varphi_f(k+h_p) \\ u_{d(h_p+p)}(k) \end{bmatrix} \tag{5.46}$$

where the first m rows of $u_{cf}(k)$ is the input vector $u(k)$ at time k. P'_{c1}, P'_{o1}, and P'_{d1} are the first m rows of P'_c, P'_o and P'_d, respectively.

Forming the following matrices

$$U(k) = \left[u_c(k) \; u_c(k+1) \; \ldots \; u_c(N-p-h_p+1)\right],$$

$$\Phi_p(k-p) = \begin{bmatrix} y(k-p) & y(k-p+1) & \dots & y(N-2p-h_p+1) \\ u_c(k-p) & u_c(k-p+1) & \dots & u_c(N-2p-h_p+1) \\ u_d(k-p) & u_d(k-p+1) & \dots & u_d(N-2p-h_p+1) \\ \dots & \dots & \dots & \dots \\ y(k-1) & y(k) & \dots & y(N-p-h_p) \\ u_c(k-1) & u_c(k) & \dots & u_c(N-p-h_p) \\ u_d(k-1) & u_d(k) & \dots & u_d(N-p-h_p) \end{bmatrix},$$

$$\Phi_f(k+h_p) = \begin{bmatrix} y(k+h_p) & y(k+h_p+1) & \dots & y(N-p+1) \\ u_c(k+h_p) & u_c(k+h_p+1) & \dots & u_c(N-p+1) \\ \dots & \dots & \dots & \dots \\ y(k+h_p+p-1) & y(k+h_p+p) & \dots & y(N) \\ u_c(k+h_p+p-1) & u_c(k+h_p+p) & \dots & u_c(N) \end{bmatrix},$$

$$U_{d(h_p+p)}(k) = \begin{bmatrix} u_d(k) & u_d(k+1) & \dots & u_d(N-p-h_p+1) \\ \dots & \dots & \dots & \dots \\ u_d(k+h_p+p-1) & u_d(k+h_p+p) & \dots & u_d(N) \end{bmatrix} \quad (5.47)$$

The following equation can be obtained similar to what was done ifor (5.29)

$$\left[\left[P'_{c1}\ P'_{o1}\ P'_{d1}\right]\right] = U(k)\begin{bmatrix} \Phi_p(k-p) \\ \Phi_f(k+h_p) \\ U_{d(h_p+p)}(k) \end{bmatrix}^{\dagger} \quad (5.48)$$

5.7 Computational Steps for New Predictive Control

The following is the computational steps for predictive control design.

1. From some sufficient the input output data, form the vector $\bar{\varphi}(k-1)$ as shown in equation (5.31) (in the case of no disturbances input) or equation (5.46) when system has disturbances input .

2. Using the Recursive Least-Square algorithm equation (5.32)-(5.35) to compute the matrix P.

3. From the reference trajectory $w(k)$, form the vector $\varphi_f(k+h_p)$ from equation (5.24).

4. Compute the predictive control vector from the equation (5.26) (in case of no disturbances input) or equation (5.45) when system has disturbances input .

CHAPTER VI

NUMERICAL VALIDATIONS

6.1 Introduction

Industrial processes generally are highly multivariable systems where perturbations affect the plant structure more often than the measured variables. Industrial processes have their performance criteria and reliability requirements. The economic and psychological environment required for a successful implementation is often not met in practice. Many constraints will prevent the implementation of on-line control schemes in production plant. An acceptable operation of a Kraft recovery boiler requires simultaneous satisfaction of a number of objectives : high steam efficiency, stable operation, low emission and disturbance rejection due to the changes in heating potential in black liquor.

This chapter will present numerical results that show the performance of the Kraft recovery boiler, particularly the output response from the boiler furnace. The sets of input output data are available from a simulator that simulates the actual operation of the recovery boiler. There are seven parts of simulator: the feedwater system, black liquor system, smelt spout cooling water, auxiliary oil burners, rapid drain system, combustion air and fuel gas system, and the green liquor system. The recovery boiler furnace is used as a model to demonstrate the feasibility of system identification and controller design.

Since the objective is to use a set of input output data to perform the new close-loop controller for the furnace at a steady state condition and to obtain the desired output as setting points, random inputs will be used to excite the furnace model to obtain the output data. There are mainly two inputs for the furnace, the air flow and black liquor flow and two corresponding outputs: the smelt flow and the heat. The conventional PID controller is initially used under the condition that the furnace model is derived from the system identification technique. The predictive control however, is applied to control the furnace by exciting the system at the first stage and perform the close-loop control by using a Least-Square technique.

6.2 Numerical Results

The numerical results for both PID controller and predictive controller are shown in this section. System disturbance is another input of the system to investigate the system disturbance rejection capability of both controllers.

6.2.1 Mathematical Model of Recovery Boiler Furnace

The model of the recovery boiler furnace is derived from the simulator that simulates the actual operation of the recovery boiler. Random signals of 3000 data points are used to be the input signal to the boiler furnace. By using the system identification technique (as described in Chapter 3), the state space model of the recovery boiler furnace is derived in the form:

$$A = \begin{bmatrix} 0.2785 & -0.0512 & 0.0342 & 0.0670 & 0.0034 & 0.0015 \\ 0.1131 & 0.2085 & 0.0880 & 0.0528 & -0.0154 & -0.0007 \\ 0.0373 & 0.0876 & 0.3086 & 0.1423 & 0.0306 & -0.0038 \\ 0.0865 & -0.0163 & 0.0677 & 0.2527 & 0.0225 & -0.0192 \\ -0.0022 & 0.0140 & 0.0075 & 0.0829 & 0.2189 & -0.0539 \\ 0.0001 & 0.0009 & 0.0079 & -0.0174 & -0.0026 & 0.2468 \end{bmatrix}$$

$$B = \begin{bmatrix} 1.4042 & -0.2065 & 0.2717 \\ -0.2551 & -0.0590 & 1.0597 \\ 0.0327 & 0.3035 & -0.0093 \\ -0.0383 & -0.0254 & -0.0284 \\ 0.0012 & -0.0064 & -0.0032 \\ -0.0001 & -0.0011 & -0.0004 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.4732 & 1.0291 & -0.0713 & -0.0097 & 0.0033 & 0.0001 \\ 1.3595 & -0.3494 & 0.0106 & -0.0364 & -0.0011 & -0.0006 \\ -0.0551 & 0.1826 & 0.2979 & -0.0569 & 0.0020 & 0.0005 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Figure 6.1 shows the original random input and output data from the model of recovery furnace respectively. Figure 6.2 illustrates the identified signal that corresponds to the output of the model at the first 1,500 data points. Figure 6.3 shows the predicted out-

put from the verified model that is tested by the rest 1,500 data points output.

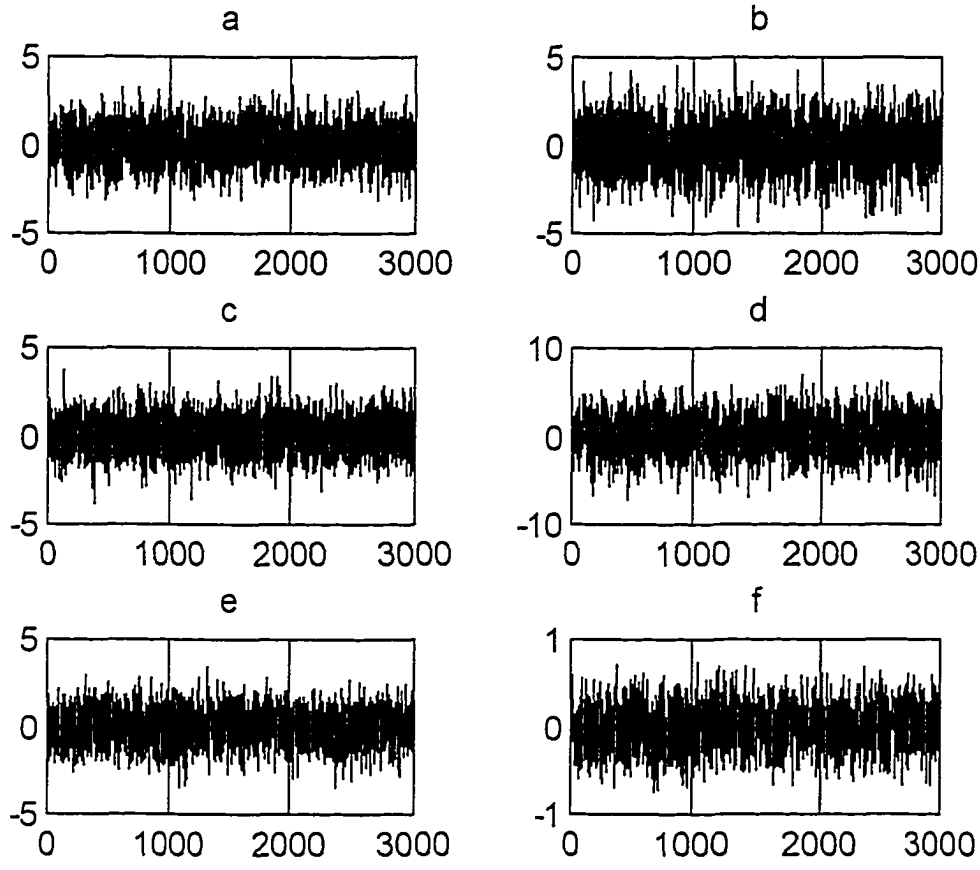

Figure 6.1 a,c, and e Shown the Random Signal Used to Excite the Simulator Model. b,d, and f are the Outputs from the Simulator (Smelt Flow, Heat Flow, and Steam Flow Respectively)

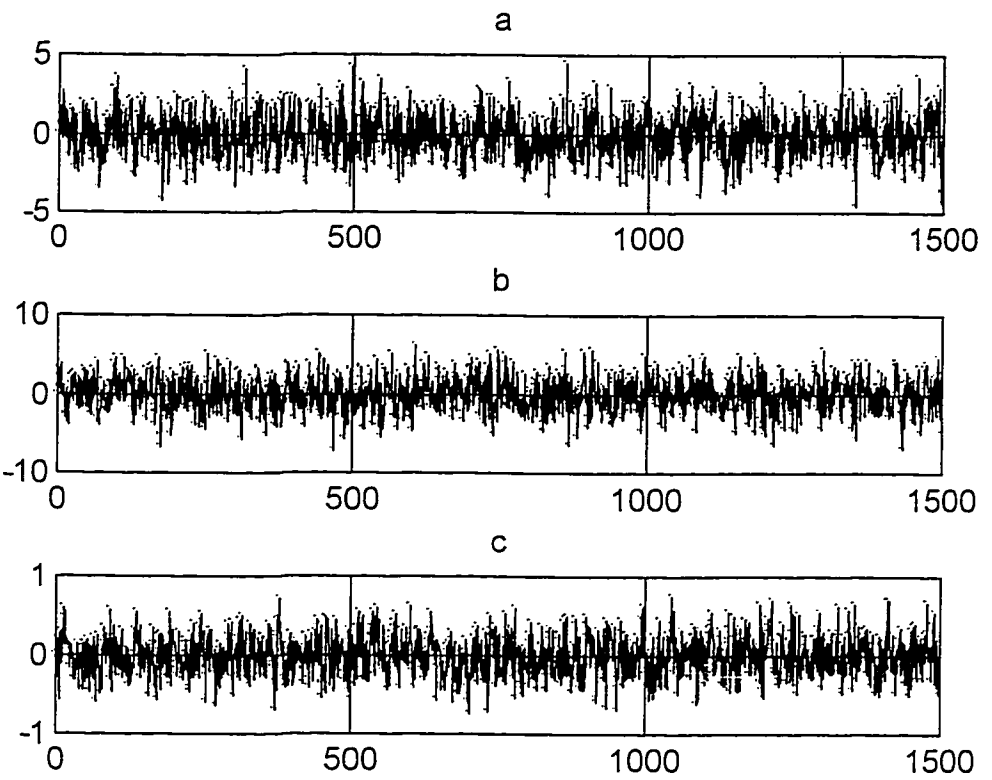

Figure 6.2 a,b, and c Illustrates the Identified Signals (dash line) which Fit to the Actual Outputs of the Simulator (Smelt Flow, Heat Flow, and Steam Flow Respectively) at the First 1,500 Data Points

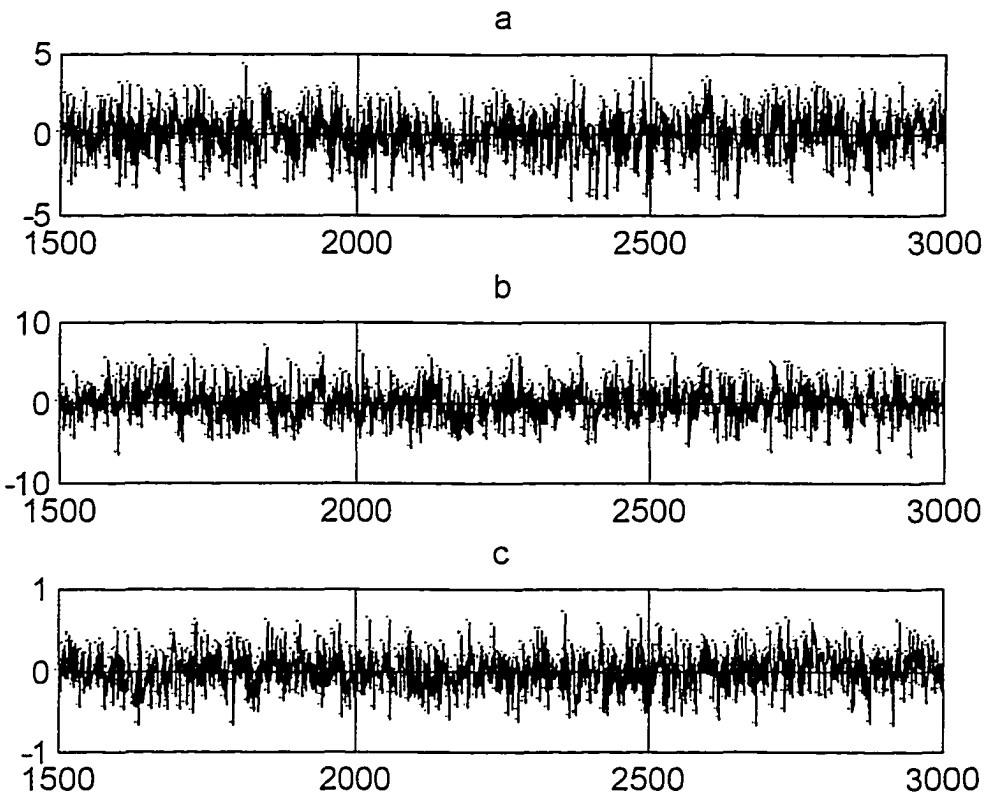

Figure 6.3 a,b, and c Illustrates the Signals (dash line) Predicted from the Identified Model which Correspond to the Actual Outputs of the Simulator (Smelt Flow, Heat Flow, and Steam Flow Respectively) by Using the Later 1,500 Data Points to Verify the Result

6.2.2 Result for Conventional PID Control

The proportional-integral-derivative (PID) controller is the most popular and commercially available controller used in the process industry. The integral controller increases the system type, which reduces steady state error. The derivative controller increases the damping and, hence, the stability of the system. Most of the recovery boilers used in pulp-paper mill still use a PID controller to control the boiler operation. As mentioned in Chapter II, the PID controller gains normally come from the experience of the operators and can be set to automatic or manual operation mode depending on the situation.

There are three inputs-outputs from the recovery boiler furnace according to the simulator. The inputs are the black liquor flow, the total air flow, and the steam flow (in units of pound per second, respectively). The outputs are the smelt flow (in units of pound per second), the heat (in unit of BTU per second), and the steam flow. The internal process inside the recovery boiler furnace is described in Chapter 2. The only inputs and outputs used to evaluate the performances of the controller in this chapter are the black liquor, the total air flow, the smelt flow, and the heat.

This section presents the simulation results of the PID controller that is used successfully to control the boiler parameters. The PID gains used for this simulation are based on the simulator that are not unique. The proportional (K_P), integral (K_I), and derivative (K_D) gains are 2, 1.5, and 0.8 respectively. At the steady state operation, the black liquor and the total air flow act as the inputs of the recovery boiler furnace at the rate about 20-22 and 100-102 pound per second respectively. These are converted to the heat at the rate about 60,000-61,000 BTU per second. The smelt flow acts as another output and is approximately 38-42 percent of the black liquor flow.

By using the identified state space model of the recovery boiler furnace, the PID controller performance shows the outputs of the recovery boiler furnace in terms of smelt flows and heat flows respectively (see figure 6.4 and 6.5). Figure 6.6 and 6.7 shows the control inputs (black liquor flow and total air flow) of the boiler furnace.

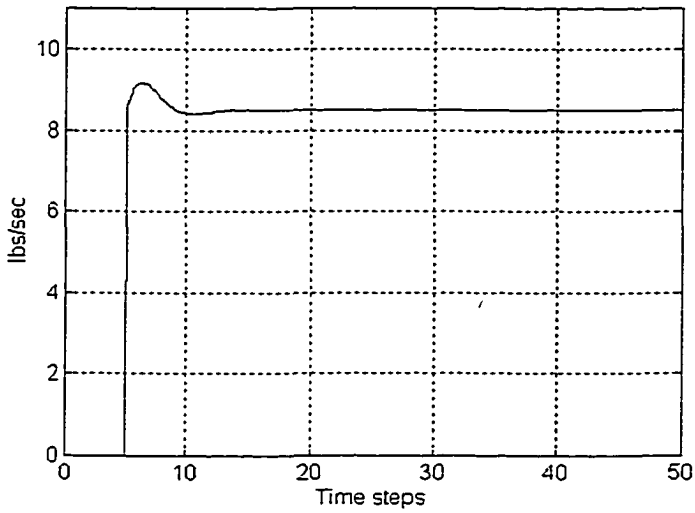

Figure 6.4 Output of the Recovery Boiler Furnace as Smelt Flow by Using the PID Controller ($K_P = 2$, $K_I = 1.5$, $K_D = 0.8$)

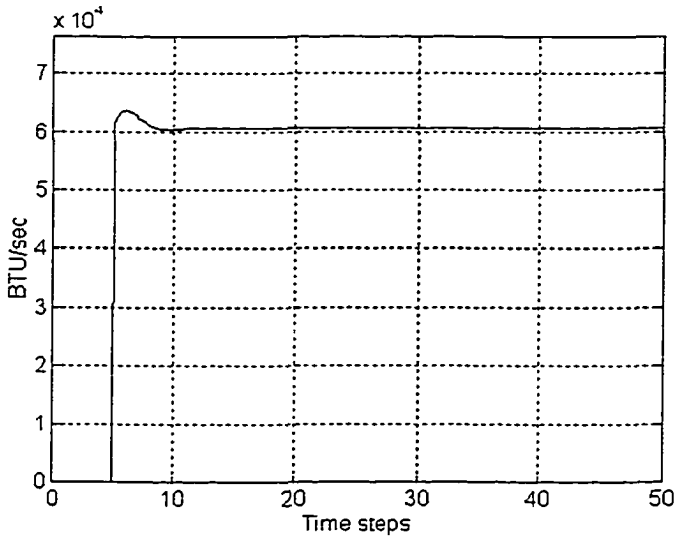

Figure 6.5 Output of the Recovery Boiler Furnace as Heat Flow by Using the PID Controller ($K_P = 2$, $K_I = 1.5$, $K_D = 0.8$)

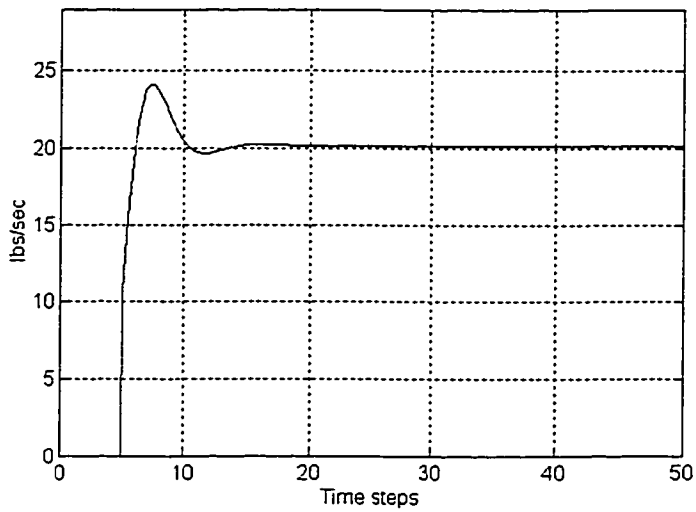

Figure 6.6 Input of the Recovery Boiler Furnace as Black Liquor Flow by Using the PID Controller ($K_P = 2$, $K_I = 1.5$, $K_D = 0.8$)

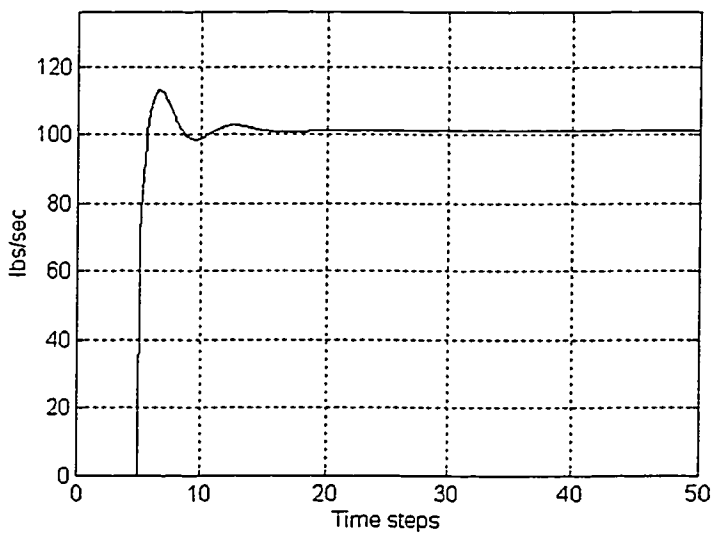

Figure 6.7 Input of the Recovery Boiler Furnace as Total Air Flow by Using the PID Controller ($K_P = 2$, $K_I = 1.5$, $K_D = 0.8$)

6.2.3 Result for Predictive Control

The new tracking predictive control is applied to the recovery boiler furnace in order to verify the performance of this controller and compare to the conventional PID controller in the previous section. By assuming that one does not know the mathematical model of the boiler furnace, the system parameters need to be first identified by choosing the appropriate value of the ARX model order, as well as the prediction horizon. The controller is performed after the identifying process of some proper data points is completed.

For a single-output system, it is known that the order of an ARX model is the same as its equivalent state-space, $p = n$. However for multiple-outputs, a state-space model of order n for the recovery boiler furnace, the relation of $p_{min} h_P \geq n$ can be used for the controller to bring the output to the desired values. In addition, the identification accuracy will be improved as p increases. Figure 6.8 and 6.9 show the tracking output of the new controller in terms of heat and smelt flows respectively while using the minimum prediction horizon (h_P) as 1 and the ARX model (p) as 6. Figure 6.10 and 6.11 depict the control input in term of Black liquor flow and Total Air flow respectively. Figure 6.12 and 6.13 show the output of the recovery boiler furnace when prediction horizon (h_P) is increased to 6 while the ARX model (p) remaines at 6. Figure 6.14 and 6.15 are the input of the recovery boiler furnace corresponding to the output in Figure 6.12 and 6.13. Finally, the prediction horizon (h_P) is increased to 20 while the ARX model (p) remaines at 6 and the output and input of the recovery boiler furnace are shown in Figure 6.16, 6.17, 6.18 and 6.19 respectively.

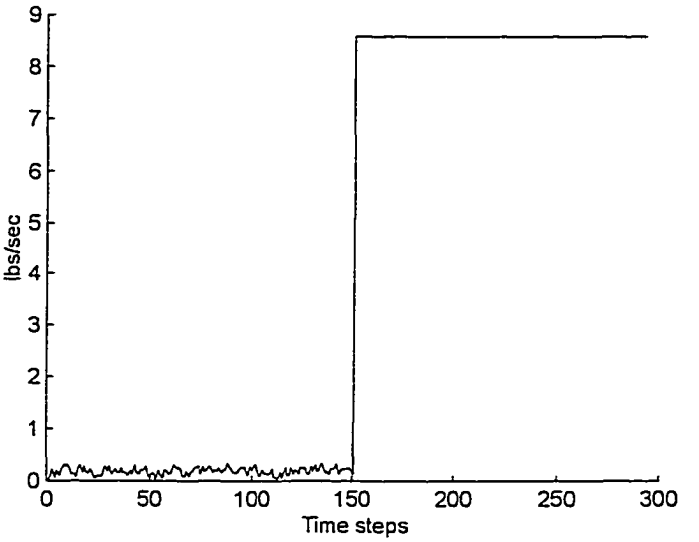

Figure 6.8 Output of the Recovery Boiler Furnace as Smelt Flow by the Predictive Controller when $h_P = 1$ and $p = 6$

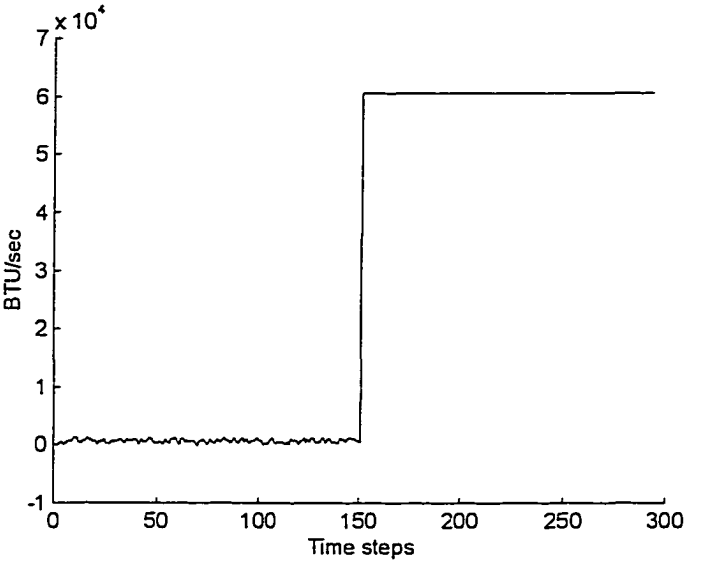

Figure 6.9 Output of the Recovery Boiler Furnace as Heat Flow by the Predictive Controller when $h_P = 1$ and $p = 6$

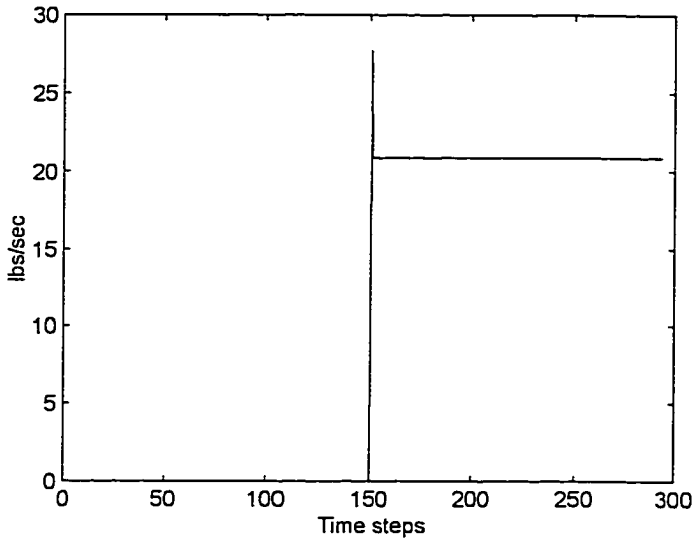

Figure 6.10 Input of the Recovery Boiler Furnace as Black Liquor Flow by the Predictive Controller when $h_P = 1$ and $p = 6$

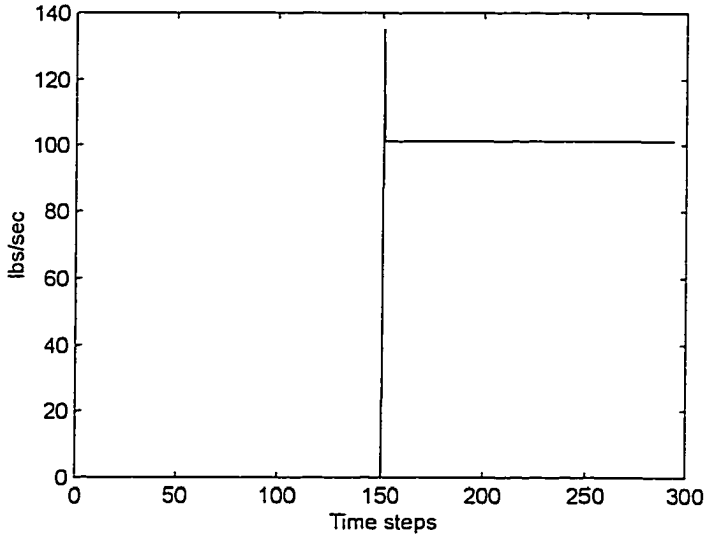

Figure 6.11 Input of the Recovery Boiler Furnace as Total Air Flow by the Predictive Controller when $h_P = 1$ and $p = 6$

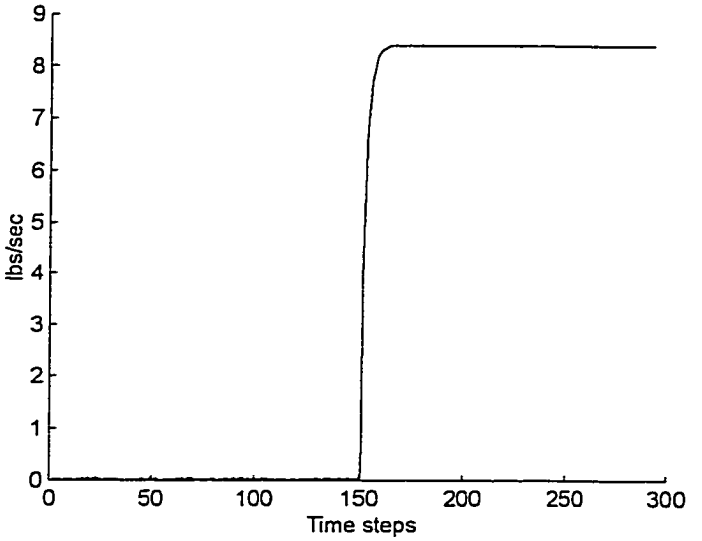

Figure 6.12 Output of the Recovery Boiler Furnace as Smelt Flow by the Predictive Controller when $h_P = 6$ and $p = 6$

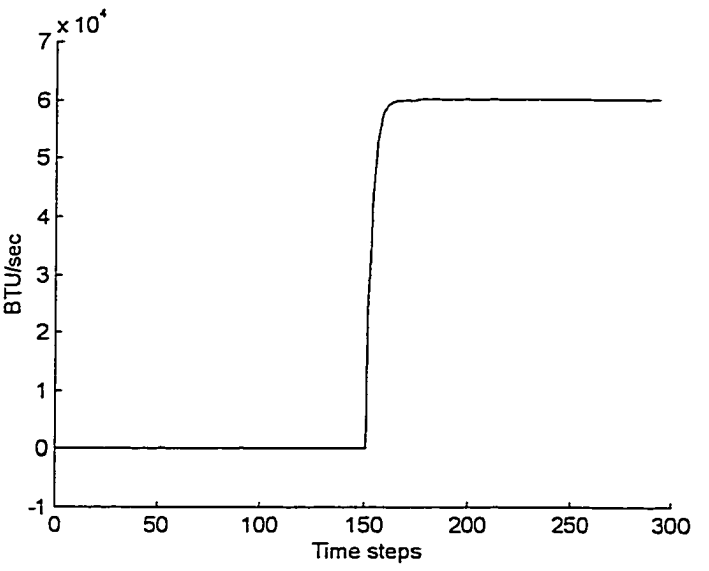

Figure 6.13 Output of the Recovery Boiler Furnace as Heat Flow by the Predictive Controller when $h_P = 6$ and $p = 6$.

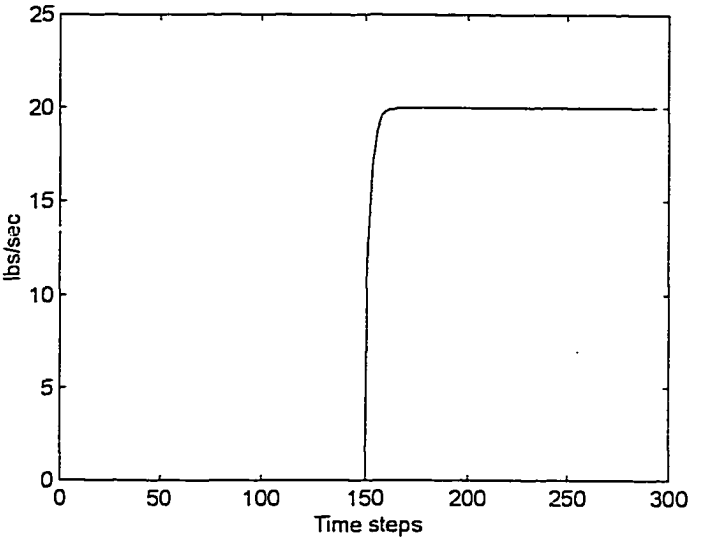

Figure 6.14 Input of the Recovery Boiler Furnace as Black Liquor Flow by the Predictive Controller when $h_P = 6$ and $p = 6$

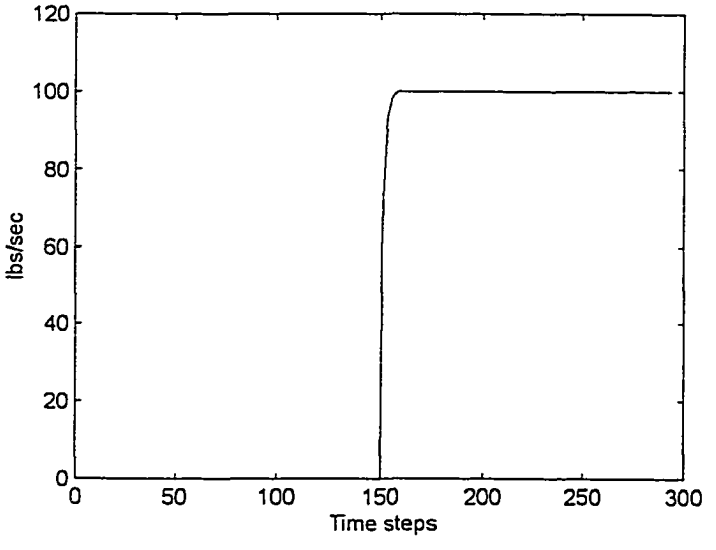

Figure 6.15 Input of the Recovery Boiler Furnace as Total Air Flow by the Predictive Controller when $h_P = 6$ and $p = 6$

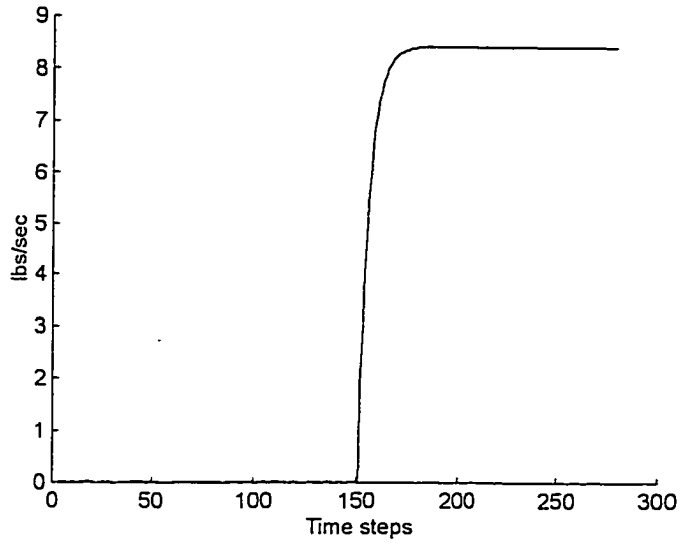

Figure 6.16 Output of the Recovery Boiler Furnace as Smelt Flow by the Predictive Controller when $h_P = 20$ and $p = 6$

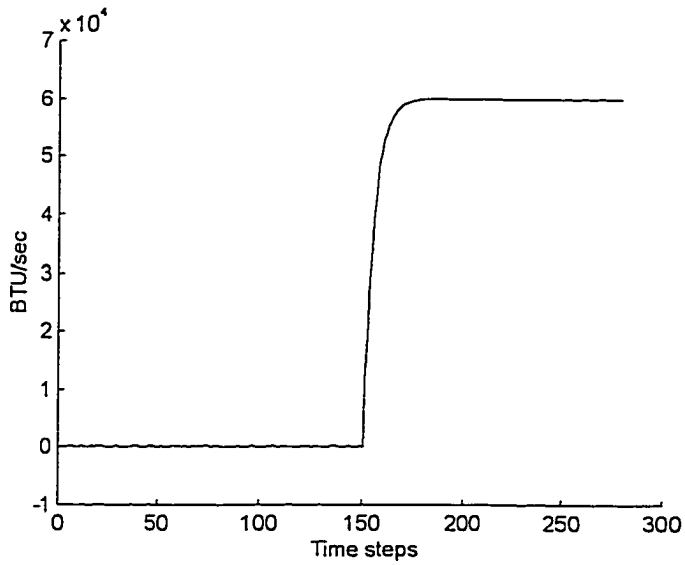

Figure 6.17 Output of the Recovery Boiler Furnace as Heat Flow by the Predictive Controller when $h_P = 20$ and $p = 6$

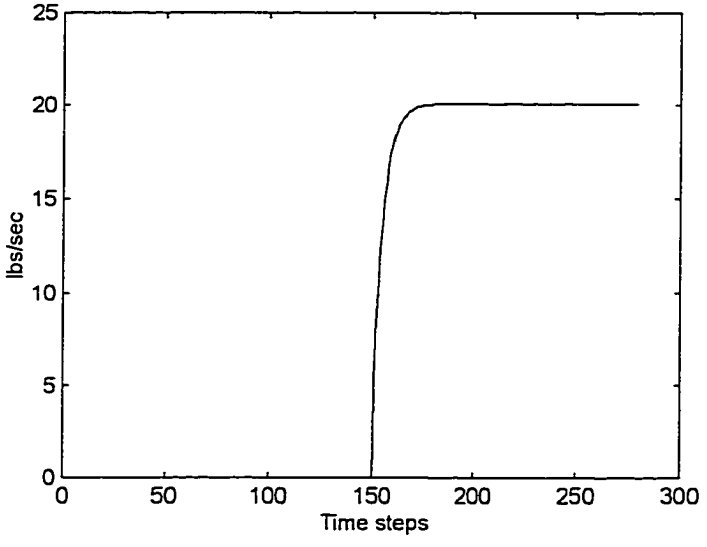

Figure 6.18 Input of the Recovery Boiler Furnace as Black Liquor Flow by the Predictive Controller when $h_P = 20$ and $p = 6$

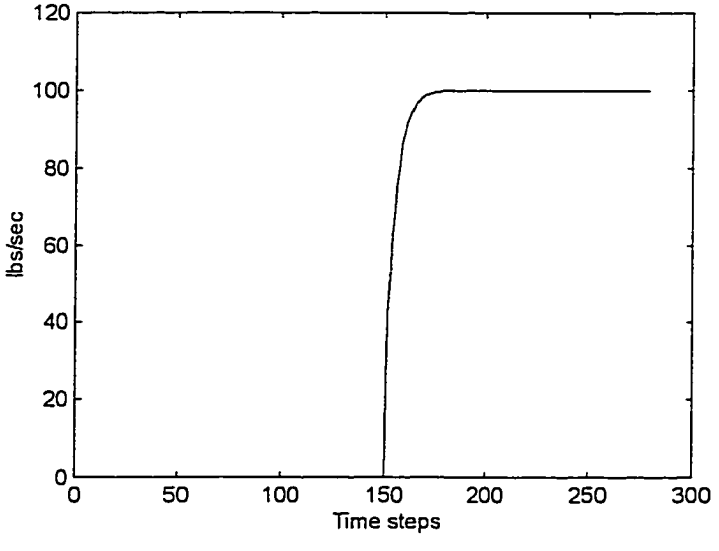

Figure 6.19 Input of the Recovery Boiler Furnace as Total Air Flow by the Predictive Controller when $h_P = 20$ and $p = 6$

As mentioned in Chapter 2, the heat potential of black liquor varies from the quality of the wood. A disturbance in Figure 6.20 is generated in terms of the percentage deviate from the standard heat potential value. This is accomplished in order to test the disturbance rejection capability of the predictive controller compared with the PID controller when the system is disturbed by some unknown input. Figure 6.21 and 6.22 show the outputs of the boiler furnace as the smelt flow and the heat flow respectively. By using the same PID gains as in previous section, the PID controller can reject the disturbances well. The control inputs of the furnace that are changed to maintain the desired outputs also are shown in figure 6.23 and 6.24.

The same disturbance shown in Figure 6.20 is injected into the system to investigate the disturbance rejection capability of this predictive controller. Note that the predictive horizon in this case is 1 while the ARX model is 6. One cannot use the predictive horizon greater than 1 because the disturbances are always random and cannot be predicted. Figure 6.25 and 6.26 show the output of the boiler furnace as the smelt flow and the heat flow respectively. Figure 6.27 and 6.28 depict the change in control input which attempt to maintain the desired value.

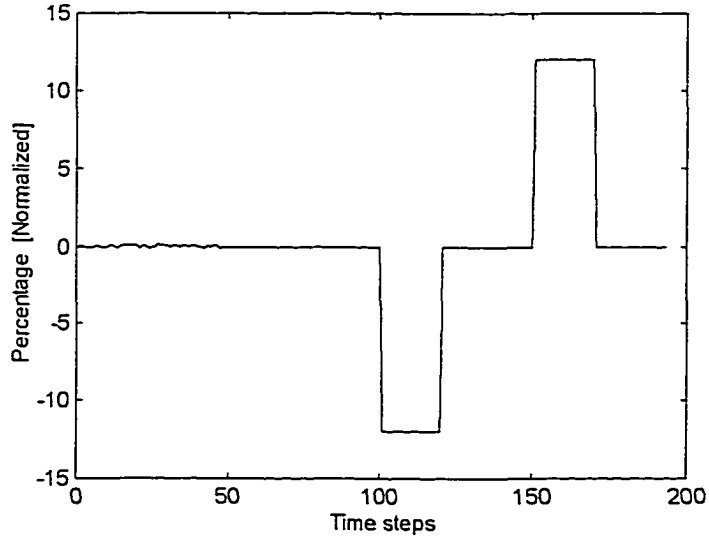

Figure 6.20 Disturbances in Terms of Percentage Changes of Heat Potential Value

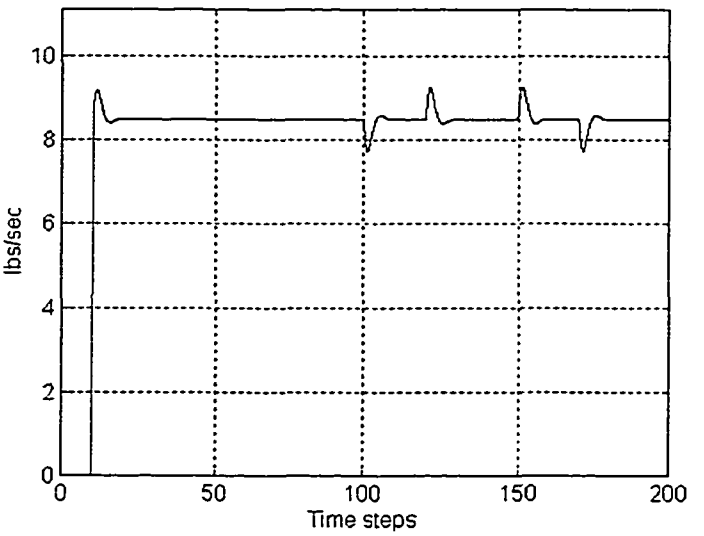

Figure 6.21 Output of the Recovery Boiler Furnace as Smelt Flow, Subjected to a Disturbance by Using the PID Controller ($K_P = 2$, $K_I = 1.5$, $K_D = 0.8$)

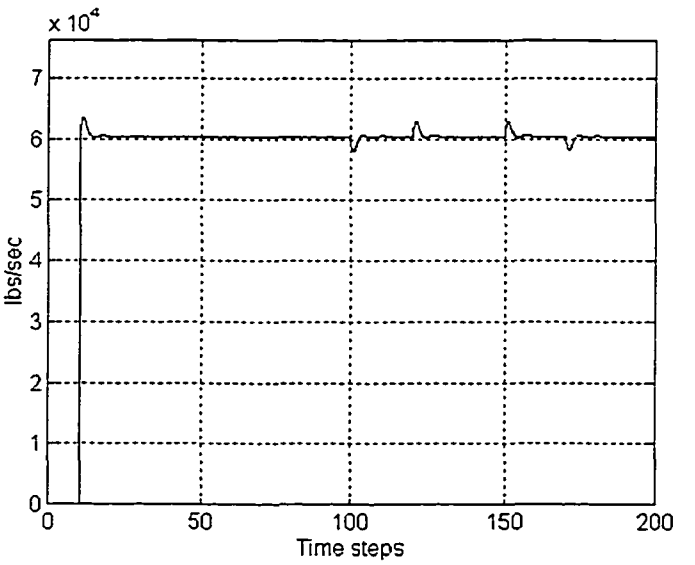

Figure 6.22 Output of the Recovery Boiler Furnace as Heat Flow, Subjected to a Disturbance by Using the PID Controller ($K_P = 2$, $K_I = 1.5$, $K_D = 0.8$)

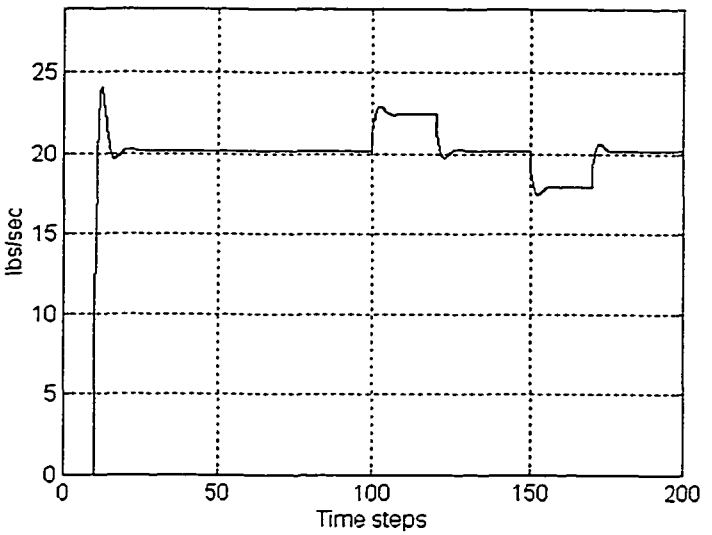

Figure 6.23 Input of the Recovery Boiler Furnace as Black Liquor Flow, Subjected to a Disturbance by Using the PID Controller ($K_P = 2$, $K_I = 1.5$, $K_D = 0.8$)

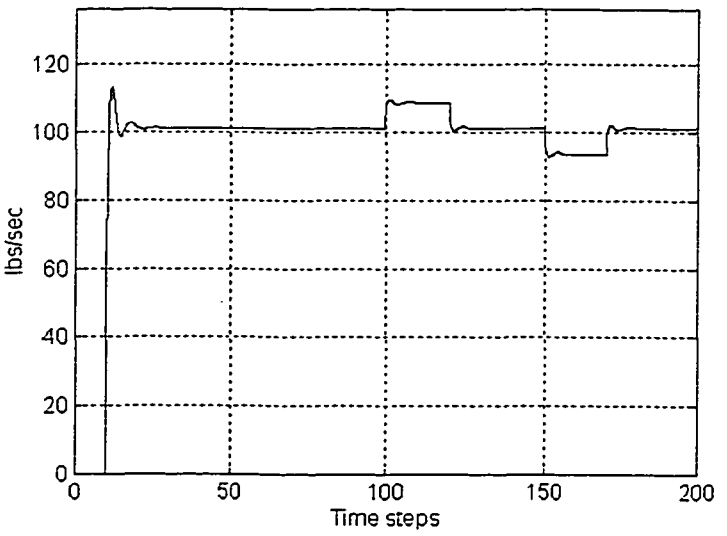

Figure 6.24 Input of the Recovery Boiler Furnace as Total Air Flow, Subjected to a Disturbance by Using the PID Controller ($K_P = 2$, $K_I = 1.5$, $K_D = 0.8$)

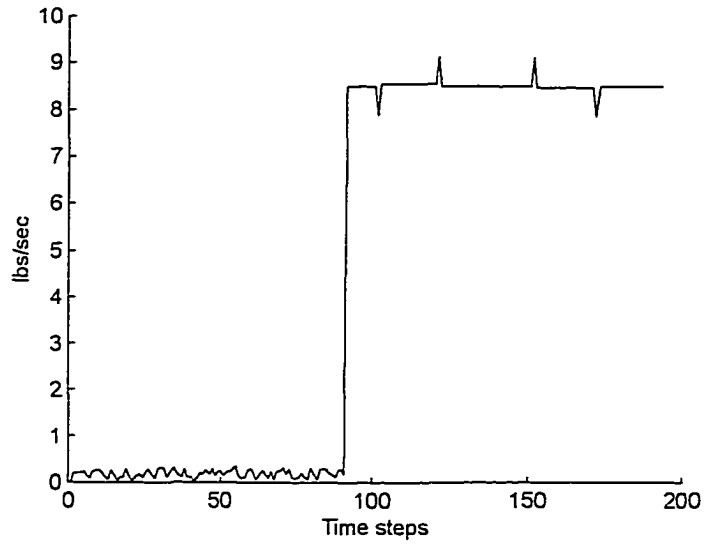

Figure 6.25 Output of the Recovery Boiler Furnace as Smelt Flow, Subjected to a Disturbance by the Predictive Controller when $h_P = 1$ and $p = 6$

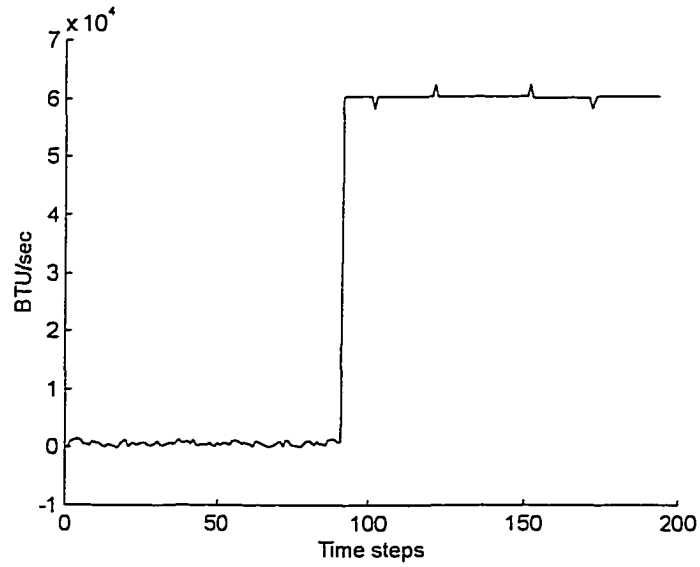

Figure 6.26 Output of the Recovery Boiler Furnace as Heat Flow, Subjected to a Disturbance by the Predictive Controller when $h_P = 1$ and $p = 6$

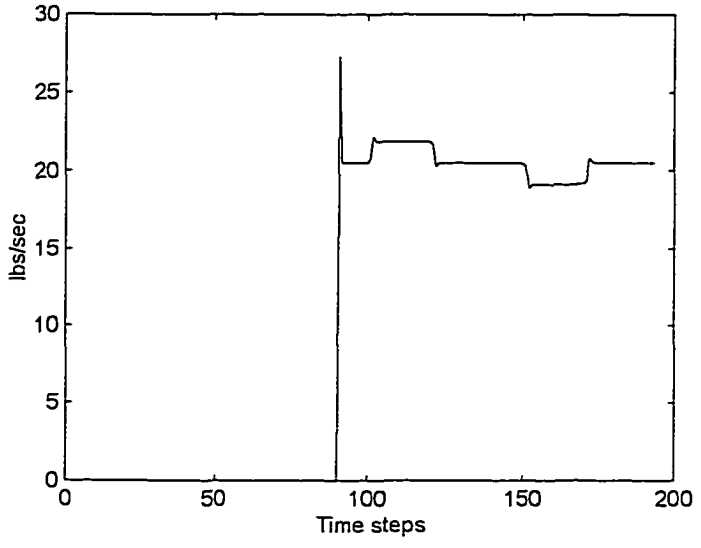

Figure 6.27 Input of the Recovery Boiler Furnace as Black Liquor Flow, Subjected to a Disturbance by the Predictive Controller when $h_P = 1$ and $p = 6$

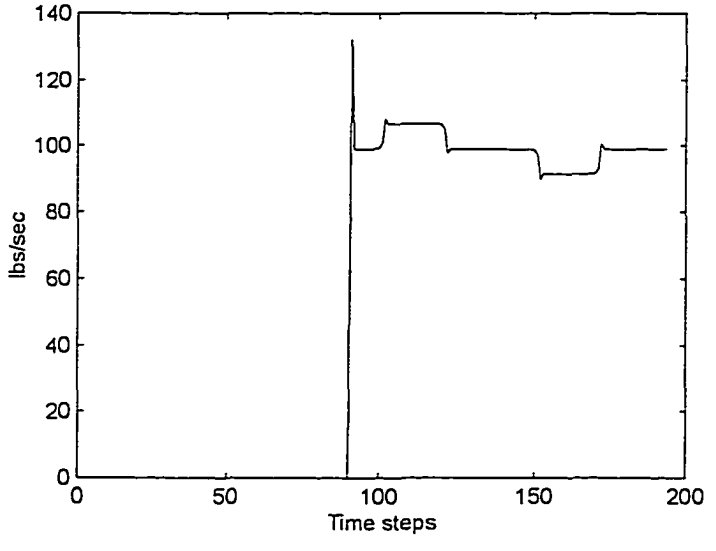

Figure 6.28 Input of the Recovery Boiler Furnace as Total Air Flow, Subjected to a Disturbance by the Predictive Controller when $h_P = 1$ and $p = 6$

6.3 Conclusion Remarks

This chapter shows the result of the PID controller and the new predictive controller to control the recovery boiler furnace. By performing the new predictive controller, a better result is shown in terms of tracking performance as well as the disturbance rejection. However, the control inputs of the predictive controller are larger than the conventional PID controller especially when performing the minimum prediction horizon. A smaller value of h_p implies a shorter prediction horizon, and more control energy is needed to bring the state of the system to the desired value in shorter time. A large value of h_p implies a longer prediction horizon, and less control energy is required. The predictive control also shows the dominant results of the disturbances rejection over the PID controller. The system responses is both robust and stable when these two controllers are subjected to the same disturbances.

CHAPTER VII

CONCLUSIONS

7.1 Summary

Industrial processes in general are highly multivariable systems, where perturbations affect the plant structure more often than measured variables. In the control engineering field, a conventional PID controller is commercially used most in industries. However, each have their performance criteria and reliability requirements. The multivariable system like that of the recovery boiler of pulp-paper mill, can be controlled by the conventional PID controller. However, performing PID controller requires a skill from the operators on an individual basis and the controller performance can not be adapted to compensate for all disturbances significantly, especially the long time delay system.

The setpoint tracking predictive control by system identification approach is proposed in this dissertation. The new algorithm uses the i-step-ahead predictor integrated with the least-square technique to build the new control law. Based on the receding horizon predictive control principle, the tracking predictive control law is achieved and performs successfully on the recovery boiler plant of a pulp-paper mill. This predictive controller is basically designed from ARX coefficients, which are computed directly from input and output data. The character of this controller is governed by two parameters. One parameter is the prediction horizon as in traditional predictive control. The other parameter is the order of the ARX model.

When the order of the ARX model is at a minimum, the identification is sensitive to the noise. The identification accuracy will be improved as the order of the ARX model increases. However the order of the ARX model for a multiple output system can be less than the order of the system. In addition, as the prediction horizon increases, the system response will move from the minimum time solution toward the minimum energy solution, making it less susceptible to the noise. In the predictive control, the control input at any time step is part of a sequence of minimum-norm control actions; therefore, the result-

ant control sequence is only an approximation of the truly energy-optimal solution.

The predictive controller formulated in this dissertation is an interesting combination of both a feedforward and a feedback control. The controller is feedforward in that at each time step it determines its action to bring the future output to the desired value in a finite number of time steps. The predictive component is a feedforward action, which takes advantage of the knowledge of the system to guide its control action by looking ahead. On the other hand, the form of the controller is clearly feedback because the current control input is a linear combination of actual input and output measurements. This feedback feature gives the controller the ability to handle unexpected disturbance as well as a certain degree of robustness with respect to both noises and modeling errors. In addition, this feedback action compensates for the inherent sensitivity of the feedforward action.

The design of the predictive controller depends on both parameters: the prediction horizon and the order of the ARX model. This controller is essentially designed from the input output model, that can be identified directly from input output data. The calculation can be carried out in real time by recursive least square technique if necessary. In addition, the prediction horizon and the order of the ARX model parameters can also be tuned in real time. However, this predictive controller is not optimum in the sense of traditional predictive control which satisfies a variety of design objectives and constraints. This type of design is useful in engineering applications where trade-off between design simplicity versus optimality is an issue.

7.2 Further Extension of The Research

Unlike the traditional state-space dynamic compensator designs satisfying a variety of design objective and constraints, the tracking predictive control that is developed in this dissertation does not integrate the design objective into the algorithm. The controller behaves from the minimum time control to minimum energy control solution depending on the control horizon value. Even though the control inputs, in general, are larger than the conventional PID controller, the control inputs of this predictive control are smaller than PID controller when system disturbances are taken into account. Therefore, incorporating the design objective into the algorithm will be an interesting extension of this work.

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VITA

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IMAGE EVALUATION TEST TARGET (QA-3)

150mm

6"

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