Extension of Kaprekar's Algorithm to Arbitrary Bases

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Abstract

Kaprekar's Algorithm was developed in 1949 by D.R. Kaprekar. One takes a 4 digit number in base 10 and creates two new numbers, one with the digits of the orginal number in descending order and the other with these digits in ascending order. Then one subtracts the number with digits in ascending order from the number with digits in descending order. The algorithm shows that repeating this process will always converge to 6174 no matter what the originating number.

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For example, start with number 3524:
5432-2345=3087
8730-378=8352
8532-2358=6174
7641-1467=6174
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We began to investigate this algorithm for 3 digit numbers instead of 4 to see if we could find this same type of fixed point in other bases.

In our early work, we focused on 3 and 4 digit numbers in base 10 in order to better understand how the Kaprekar algorithm works. We proved that all 3 digit numbers converge to 495, and all 4 digit numbers converge to 6174. Once we had a good understanding of how the Kaprekar algorithm worked in base 10, we worked on generalizing these results for an arbitrary base n.

For 3 digit numbers in an arbitrary base n, we found that all Kaprekar numbers were of the form $[\alpha-1][n-1][n-\alpha]$, where n is the base and α is the difference between the largest and smallest digit. We were also able to prove that when n is even, the algorithm converges to one fixed point, and we can compute this fixed point. We proved that for odd bases, the algorithm always converges to a cycle of two numbers, and we can compute these two numbers.

Other work includes generalizing the Kaprekar algorithm for 4 digit numbers in an arbitrary base. We plan on looking at cycles for 4 digit numbers in arbitrary bases. We hope to find similar patterns like we did with the 3 digit cases. So far we know that we are not going to be able to generalize the 4 digit cases as even and odd like we did with the 3 digit case.