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The Effects of Metacognitive Training on Algebra Students' Calibration Accuracy, Achievement, and Mathematical Literacy

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THE EFFECTS OF METACOGNITIVE TRAINING ON ALGEBRA STUDENTS'

CALIBRATION ACCURACY, ACHIEVEMENT, AND MATHEMATICAL LITERACY

by

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ABSTRACT

THE EFFECTS OF METACOGNITIVE TRAINING ON ALGEBRA STUDENTS' CALIBRATION ACCURACY, ACHIEVEMENT, AND MATHEMATICAL LITERACY

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This dissertation describes an empirical study that investigated how metacognitive training influenced lower achieving Algebra students' calibration accuracy, achievement, and development of mathematics literacy. Multiple methods were used to collect and analyze the data. Close analysis of students' work and classroom observations revealed that students that were exposed to the metacognitive training had significantly higher prediction accuracy and made gains in their understanding of the mathematics word problems than did students who did not receive the metacognitive training. Overall, however, both the intervention and comparison groups improved in their academic performance and became more mathematically literate and accurate in their metacognitive judgments. These findings suggested that explicit instruction of self-regulation strategies was beneficial for improving metacognitive judgments among lower achieving Algebra students in this study. Results further suggest that the problem-solving strategy enhanced mathematics learning for both groups. Further research is warranted to better understand students' metacognitions as they engage in the problem-solving process.

This dissertation is dedicated to my family, for reminding me what is truly important in life.

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CHAPTER 1

INTRODUCTION

Research has consistently shown that students are more likely to succeed academically if they self-regulate their learning processes. Panadero and Alonso-Tapia (2014) define self-regulation as "the control that students have over their cognition, behavior, emotions, and motivation through the use of personal strategies to achieve the goals they have established" (p. 450). As students encounter a variety of problems, different strategies are necessary. Students plan, monitor, and reflect on their problem-solving processes to adapt a variety of appropriate strategies to successfully solve tasks.

Self-regulation has been extensively investigated as a predictor of students' mathematical problem-solving (Garcia, Rodrigues, Gonzalez-Castro, Gonzalez-Pienda, & Torrance, 2016; Pennequin, Sorel, Nanty, & Fontaine, 2010), mathematics achievement (Bol, Campbell, Perez, & Yen, 2016; Cleary & Chen, 2009; Perels, Dignath, & Schmitz, 2009, Zimmerman, Moylan, Hudesman, White, & Flugman, 2011) and metacognition (Bol et al., 2016; Chen & Chiu, 2016). Math educators should explicitly teach, model, and practice self-regulation strategies with their students, specifically lower achieving students. Self-regulation skills, such as self-monitoring performance, are not only important to academic success, but are also key components in becoming a life-long learner.

Calibration

Specific self-regulated learning (SRL) processes that underlie academic success are metacognition and calibration. Metacognition is the monitoring of one's learning processes. One type of metacognitive monitoring is calibration. Calibration is a metacognitive process that requires students to think about and make judgments of their own performance (Bol & Hacker,

2012). Fairly consistent results have been discovered between calibration accuracy and mathematics achievement. First, students struggle to accurately judge their learning in mathematics (Hacker & Bol, 2018). Second, students' mathematics achievement is related to their judgments of learning in math. Third, students that are more accurate in their judgments tend to be higher achievers and students that are less accurate in their judgments tend to be lower achievers (Hacker & Bol, 2018; Ozsoy, 2012). Lastly, lower achieving mathematics students often overestimate their judgments while higher achieving mathematics students lean towards underestimating their judgments (Bol, Riggs, Hacker, Dickerson, & Nunnery, 2010; Garcia et al., 2016; Ozsoy, 2012).

Lower achieving students have a *history* of scoring in the lowest quartile on mathematics assessments (Rinne & Mazzocco, 2014), and there appears to be a significant number of lower-level learners that are not proficient in mathematics (Kastberg, Chan, & Murray, 2016). Research has shown that lower achieving students seem to benefit the most from learning self-regulation strategies and training (Montague, Krawec, Enders, & Dietz, 2014; Zimmerman et al., 2011). Since self-regulation and calibration accuracy have been found to be linked to improved mathematics achievement (Garcia et al., 2016; Ozsoy, 2012), helping lower achieving students to monitor their mathematics learning is a valuable skill that is integrated into self-regulated learning (SRL) frameworks (Bol et al., 2010; Bol et al., 2016;).

Problem Solving

"Math problem solving is an increasingly critical skill in today's mathematics curriculum" (Krawec, Huang, Montague, Kressler, & Melia de Alba, 2013, p. 81). Developing students' problem-solving abilities is related to their academic success and is a valuable life skill. By being aware of their thoughts and performance while engaging in the problem-solving

process, students can monitor and assess their knowledge and learning (Schoenfeld, 1985). George Polya defined problem solving as finding "a way where no way is known, off-hand..." (1945, p. 1). Polya (1945) also offered four essential steps to problem-solving: understanding the problem, devising a plan, carrying out the plan, and looking back. His problem-solving process prompts self-regulated learning and involves metacognitive awareness, often through questioning strategies. For example, some students that struggle with problem solving may not necessarily understand the question. To better understand the question, Polya proposed students use metacognitive and explicit questioning strategies such as *do you understand all the words* or *can you restate the problem.* The second step, devising a plan, may entail looking for a pattern or drawing a picture to better understand the question. Research has shown that visual representations of math problems facilitate student comprehension (Krawec, 2014; (Schoenfeld, 1985) and illustrate how math concepts are applied (Dexter & Hughes, 2011; Edens & Potter, 2007; Montague et al., 2014). The third step, carrying out the plan, involves knowing if you have the necessary skills to complete the task and follow through with the task at hand. The last step, looking back, encourages students to reflect on their problem-solving process by determining what strategies worked and did not work.

Problem-solving has been extensively studied in mathematics. Polya's four problem-solving steps continue to underlie most, if not all, adopted approaches for problem solving in school mathematics. In fact, research has shown that explicit problem-solving and strategy instruction can improve students' mathematics performance, especially among lower achieving students (Krawec et al., 2013; Montague et al., 2014; Xin, Jitendra, & Deatline-Buchman, 2005; Schoenfeld, 1985). For this research study, Polya's problem-solving process was explicitly

taught to lower achieving mathematics students to determine how it influenced their achievement, calibration accuracy, and development of mathematics literacy.

Mathematical Literacy

Mathematical literacy (ML) is the ability "to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena" (Organization for Economic Cooperation and Development (OECD), 2013, p. 25). These abilities are portrayed in the Principles and Standards for School Mathematics with the assumption that "students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge" (National Council Teachers of Mathematics (NCTM), 2000, p. 11). Proficiency in mathematics literacy necessitates students to analyze and communicate ideas by posing and interpreting solutions to mathematical problems (OECD, 2013). Explicitly teaching lower achieving students to problem solve in mathematics may be one way to promote and develop students' mathematical literacy.

Lower Achieving Students

In the present study, participants were lower achieving students. A variety of terms have been used in research to describe students that consistently score at or below the 25th percentile on mathematics achievement tests. Some examples are underserved (Grant, Crompton, & Ford, 2015), low achievers (Ozsoy, 2012), at-risk (Geary, 2011; Xin et al., 2005), underprepared (Bol et al., 2016), or at-risk students (Zimmerman et al., 2011). It is recognized that the terms used to refer to these students are controversial and sometimes used to infer *reasons* (e.g., poverty, education system, transience, family issues, learning disability, health issues, motivation, etc.) for students' low achievement scores. The specific factors that lead to students struggles in

mathematics, however, are beyond the scope of this study. The term 'lower achieving students' will be used throughout to refer to students who consistently score at or below the 25th percentile on standardized mathematics achievement tests and is not intended to suggest any student deficits.

Purpose Statement and Research Questions

Lower achieving students struggle with self-regulating their mathematics learning, resulting in low achievement scores, poor judgments of their learning, and stagnate development of mathematics literacy. Metacognitive training is a promising avenue of study to improve lower achieving students' regulation of their own knowledge, achievement scores, and development of mathematics literacy. Thus, metacognitive training to guide students during problem-solving activities is an important strategy for promoting mathematics literacy.

The intervention implemented in the present study placed particular emphasis on explicitly teaching students how to comprehend, represent, and plan to solve mathematical word problems using Polya's problem-solving process with metacognitive questioning. Implementing effective problem solving and self-regulated learning could advance students' mathematics achievement, calibration accuracy, and development of mathematical literacy. Therefore, the purpose of this study was to investigate the impact of metacognitive training on lower achieving Algebra students' calibration accuracy, achievement, and development of mathematics literacy when solving word problems. The following research questions were investigated:

1. How does metacognitive training influence the achievement scores of lower achieving Algebra students?

2. How does metacognitive training influence the calibration accuracy of lower achieving Algebra students?

3. How does the metacognitive training influence lower-achieving Algebra students development of mathematics literacy?

It was predicted that the students that were exposed to metacognitive training would have better calibration accuracy and higher mathematics achievement scores than students exposed to the problem-solving strategy without metacognitive training. It was also predicted that students that were exposed to metacognitive training would show greater development in their mathematical literacy.

Significance of Study

Results of this study may be of value to researchers and practitioners by demonstrating the importance of metacognitive training on lower achieving mathematics students' calibration accuracy, achievement, and mathematics literacy development. Greater insight may be established to determine the impact of metacognitive training on secondary students' problem-solving. Such results may provide educators with a metacognitive strategy to implement into mathematics instruction to improve lower achieving students' mathematics literacy. This information may be valuable in combating persistent low math literacy rates (OECD, 2013). This study may provide insight to lower achieving mathematics students' calibration accuracy. Research suggests that lower achieving students are challenged to effectively self-regulate (Garcia et al., 2016; Ozsoy, 2012). The recommendation is to teach explicitly how to monitor and control their cognitions and metacognitions as they engage in problem solving activities (Krawec et al., 2013). Making students aware of their learning and the quality of their performance through calibration can be valuable knowledge towards improving mathematics performance.

Organization of the Dissertation

   The dissertation is organized into five chapters. The beginning of Chapter 2 delves into the role of self-regulation and calibration on students' performance and learning of mathematics. Later in the chapter, the primary focus is on the theoretical framework and empirical studies related to problem-solving in mathematics and mathematics literacy. Chapter 3 delineates the research design and methodology of the study, including the instrument, used to gather the data, the procedures followed, and determinations of the sample selected for study. An analysis of the data and a description of the findings are presented in Chapter 4. Chapter 5 contains the conclusions, implications, and recommendations for future research.

CHAPTER 2

Literature Review

This chapter reviews the theoretical and empirical literature addressing the role of self-regulation and calibration on students' performance and learning of mathematics. It also focuses on the theoretical framework and empirical studies related to problem-solving in mathematics and mathematics literacy. It begins with an overview of the self-regulation model, followed by its role in calibration accuracy and mathematics achievement. Problem-solving in mathematics is then discussed as being a fluid process between procedural steps, followed by the influence visual representations can have in problem solving. Mathematics literacy is presented in the latter part of the chapter, with a review of how literacy should be positioned among the disciplines, specifically in mathematics and problem solving.

Self-Regulation Framework

"Self-regulation (or self-regulated learning) refers to processes that learners use to systematically focus their thoughts, feelings, and actions, on the attainment of their goals" (Schunk, 2012, p. 400). Proficient self-regulated learners control their learning processes by being aware of their strengths and weaknesses, making personal adjustments to their learning, and achieving desired outcomes (Isaacson & Fujita, 2006). These students are proactive learners that, despite obstacles, find ways to succeed (Zimmerman, 1990).

Zimmerman and Campillo (2003) present a cyclical model of self-regulation, which provides an appropriate framework for promoting students' learning in an academic context. The three-phase cyclical learning process encompasses forethought, performance, and self-reflection (see Figure 1). The forethought phase is the process that "sets the stage for action," (Schunk, 2012, p. 411). It precedes learning or performance and incorporates goal setting,

strategic planning, task analysis, and self-motivation. The second phase of Zimmerman and

Campillo's (2003) self-regulation process is the performance phase. The performance phase is

the process of performing the task at hand. It involves self-control and self-observation. Self-

control refers to implementing methods or strategies that were selected during the forethought

phase, such as imagery, task strategies, and time management. Self-observation refers to

consciously tracking one's progress through a task and involves metacognitive monitoring. The

third phase of the self-regulation process is the self-reflection phase. The self-reflection phase is

one's response to their efforts on the task and encompasses self-judgment and self-reaction.

Self-judgment refers to comparing one's performance against a standard, and includes beliefs

about the causes of successes or failures. Self-reaction is the feeling of self-satisfaction or failure

after completing a task. These feelings drive either an adaptive behavior, such as modifying

strategies, or a defensive behavior, such as avoiding the task all together.
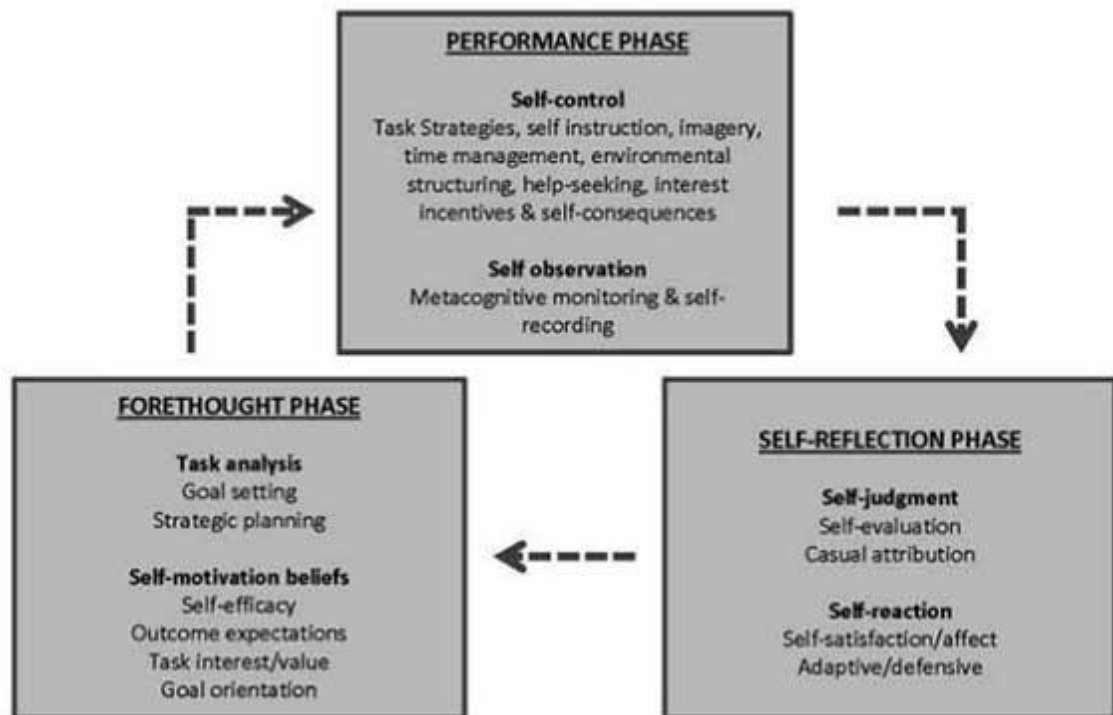
Figure 1. Zimmerman and Campillo's (2003) three cyclical phases of self-regulation.

Self-Regulation and Mathematics Achievement

Several studies have indicated that self-regulated training can improve students' mathematics achievement (Cleary & Kitsantas, 2017; Martin & Elliot, 2016; Perels et al., 2009; Schmitz & Perels, 2011). For example, Martin and Elliot (2016) examined how one component of self-regulation, goal-setting, could influence students' mathematics achievement. Elementary and secondary students were randomly assigned to either a treatment or control group. The treatment group was involved in a goal setting procedure in which the students determined their personal best, the score they wanted to accomplish or surpass on the mathematics test. The control group was not involved in the goal setting procedure. The results revealed that the students in the goal setting group scored significantly higher on the mathematics test than the students in the control group.

Another study also focused on the forethought phase, but examined a different component: the relationship between motivation and mathematics performance. Cleary and Kitsantas (2017) assessed three self-report measures of motivation: self-efficacy, task interest, and school connectedness. Statistical analysis revealed four variables emerging as unique predictors of students' mathematical performance: socioeconomic status, prior academic achievement, self-efficacy beliefs, and SRL behaviors, with prior achievement contributing the most. Self-efficacy, however, accounted for unique variation in SRL and mathematics grades suggesting "that self-efficacy acts as a critical factor in understanding academic outcomes" (Cleary & Kitsantas, 2017, p. 101). Although task interest and school connectedness were not strong predictors of academic achievement, they were strong predictors of students' efficacy beliefs to regulate their behaviors.

Some research studies have implemented all three phases of self-regulation to determine its influence on mathematics achievement (Perels et al., 2009; Schmitz & Perels, 2011). For example, Perels et al. (2009) investigated the effects self-regulation training had on middle school students' mathematical achievement. Students in the experimental group were taught eight self-regulation strategies and mathematical content over a period of three weeks. Students in the control group were taught three problem solving strategies and mathematical content over three weeks. The results revealed that after the intervention, both groups improved regarding their mathematical competencies (achievement). Only the group exposed to the self-regulation strategies, however, showed a statistically significant increase in their mathematical achievement (Perels et al., 2009). It is possible that these differences are related to the number of strategies each group was exposed to. The participants in the intervention group were exposed to eight SRL strategies while students in the control group were exposed to only three problem-solving strategies. In addition, the SRL strategies were implemented in conjunction with mathematical content instruction, while the mathematical content was not implemented in conjunction with the problem-solving strategies.

Also implementing all three phases of self-regulation, Schmitz and Perels (2011) investigated how using standardized diaries to monitor self-regulation during math homework influenced middle school mathematics student's self-regulation, self-efficacy, and performance. Using a pretest/posttest experimental design, participants were assigned to either an experimental or control group. The experimental group consisted of working with a learning diary for seven weeks. The learning diary was completed every day and invited students to observe and reflect on their mood and learning behaviors outside of school. The control group did not complete

learning diaries. Linear trend analysis revealed a significant positive linear trend for self-regulation, self-efficacy, and mathematics performance.

The results from the previous studies show that self-regulated training, in a natural learning environment, can be used to enhance students' mathematics achievement and self-regulatory processes. Self-regulated training has also shown to be effective for improving performance among lower achieving students (Bol et al., 2016; Cleary, Velardi, & Schnaidman, 2017; Zimmerman et al., 2011). For example, Zimmerman and colleagues (2011) sought to enhance students' self-reflection in response to their academic feedback (quiz outcomes). At-risk college students in remedial and introductory math courses were randomly assigned to either the intervention or control group. Students in the control group received conventional instruction, but students in the intervention group received three different reflective components. Students in the intervention group were taught to 1) detect errors and adapt strategies such as using feedback to make changes and adjustments in their learning, and 2) correct quiz errors by completing a self-reflection form. The form required students to reflect on their judgement accuracy, explain their unsuccessful strategies, establish new strategies, make new confidence judgments, and solve a similar math problem. If students solved the similar math problem accurately they would receive a point that had been lost during the quiz. The quiz points were an incentive system that rewarded students for making subsequent attempts.

All students were periodically (three times) administered a five-question quiz and were asked to make self-efficacy and self-evaluative judgments before and after the quiz, respectively. Results revealed that the SRL instructional group significantly outperformed the control group. Although there were no significant differences on the first quiz, a significant difference was found on the second and third quizzes in favor of the intervention group. Similar results were

found for the final exams in the developmental classes, as well as analysis of the tests and final exams in the introductory math courses; the SRL experimental groups significantly surpassed the control groups. Zimmerman and colleagues (2011) proposed "Instead of viewing the reception of an academic grade as an end-point of learning, these students learned to view it as an opportunity for further learning" (p. 157).

Also focusing on lower achieving students, Bol et al. (2016) investigated how SRL training would affect math achievement and metacognitions of community college students enrolled in a developmental mathematics course. Students were placed in the developmental course based on mathematical deficiencies that were identified through placement testing. Placement in the developmental mathematics courses categorized these students as underprepared, because they were lower achieving mathematics students with deficiencies. An experimental design was used, in which all the mathematics students were randomly assigned to either a SRL treatment condition or a control condition. Over a three-week period, students in the treatment condition were required to set a weekly academic goal and plan their math study time for the week (forethought), assess their math study habits and time management skills (performance), and compare their observed behaviors to their goals using reflective journaling (self-reflection). The control group received traditional mathematics instruction.

The results revealed significant differences between the control and treatment groups on achievement and metacognitions. Specifically, the treatment group scored significantly higher in mathematics achievement than the control group. The treatment group scored above average on their final exams, while the control group scored below average on their final exams. In addition, the treatment group reported higher metacognitive self-regulation scores and higher time/study management scores when compared to the control group. In fact, students that

received the SRL training were more likely to complete the course unit than students that did not receive the SRL training (Bol et al., 2016).

Lower achieving middle school students also showed academic improvement from a Self-Regulation Empowerment Program. The program was an applied self-regulated learning intervention for improving motivation, strategic skills, and mathematics achievement. Cleary et al. (2017) examined the effectiveness of the Self-Regulation Empowerment Program among at-risk middle school students. Students in remedial mathematics classes that were exposed to the Self-Regulation Empowerment Program were provided instructional modules and guidelines over the course of 3-4 months. The modules and guidelines included foundational concepts of SRL, strategy learning and practice, and self-reflections. The results revealed significant group differences on measure of strategic and regulatory thinking. There were also significant differences between the intervention group and comparison group in achievement scores. Over a period of two years, the middle school students exposed to the Self-Regulation Empowerment Program exhibited a statistically significant more positive trend in achievement, supporting the importance of targeting students who are at-risk for underperforming in mathematics.

The studies described above provide support for self-regulatory training as an effective strategy for improving performance scores and learning outcomes for mathematics students, particularly lower achieving mathematics students. These studies, however, focus on the self-regulation learning process and students' success in collegiate (Bol et al., 2016; Zimmerman et al., 2011) or middle school (Cleary et al., 2017) math courses. More research is needed to examine the effectiveness of applied field-based self-regulation interventions in secondary school contexts and among lower achieving students.

Calibration Accuracy

Self-regulation is dependent on cognitive and metacognitive strategies, and during each phase of SRL, metacognitive monitoring plays an important role (Hacker & Bol, 2018). Metacognition is crucial for self-regulation because a student could regulate their own knowledge by being aware of and having control over their metacognitive processes (Paris & Paris, 2001). Accurate monitoring, control, and evaluation of one's metacognitive processes is critical for successful learning (Hacker & Bol, 2018). The study of calibration is used to help understand the accuracy of metacognitive monitoring.

Calibration is a metacognitive process that requires students to think about and make judgments of their own performance (Bol & Hacker, 2012). More specifically, calibration is a quantitative comparison of the degree to which a person's judgment of performance on a task corresponds with their actual performance on the task (Keren, 1991). Calibration is commonly measured by absolute accuracy, which is calculating the absolute value of the difference between the predicted score and the actual performance score (Hacker, Bol, & Bahbahani, 2008; Schraw, 2009). The closer the difference is to zero, the better calibrated the individual (Garavalia & Gredler, 2002). Consider a scenario in which a student predicts, prior to taking a test, that they will receive a 90 on the test and then actually scores a 92 on the test. This student would be considered well calibrated because their actual score was close to their predicted score. In another scenario, a student may predict their score as a 90 and subsequently scores a 70. The large difference in their predictive and actual test score indicates that they are poorly calibrated (Garavalia & Gredler, 2002).

The direction of the difference between the predicted score and actual score is also revealing and is known as bias (Schraw, 2009). If the difference between the two scores is a

positive or negative number then the individual is considered either overconfident or underconfident, respectively, in their predictions. The first student in the example above would be considered slightly underconfident because their prediction score (90) was lower than their actual score (92) resulting in a negative calibration value of -2. The student in the second example, however, would be considered very overconfident because their prediction (90) was greater than their actual performance (70), yielding a positive calibration score of 20.

Glenberg and Epstein (1985) first used the term calibration, rather calibration comprehension, when they discovered that students' predictions and performance were generally unrelated. In recent years, calibration studies in educational settings have increased in interest, specifically as they relate to student achievement mostly in elementary schools (Garcia et al., 2016; Labuhn, Zimmerman, & Hasselhorn, 2010; Ozsoy, 2012; Pennequin et al., 2010) and collegiate settings (Hacker, et al., 2008; Zimmerman et al., 2011). Few studies, however, have investigated students' calibration accuracy in middle (Bol et al., 2010; Rinne & Mazzocco, 2014), and secondary schools (Chiu & Klassen, 2010; Dupeyrat, Escribe, Huet, & Regner, 2011).

Calibration Accuracy and Mathematics Achievement

There is a strong relationship between students' calibration accuracy and mathematics performance (Chiu & Klassen, 2010; Digiacomo & Chen, 2016; Dupeyrat et al., 2011; Garcia, et al., 2016; Labuhn et al., 2010; Ozsoy, 2012; Pennequin et al., 2010). Research has shown that students' mathematics achievement is related to their judgments of learning in math, and most students struggle to accurately judge their mathematics learning (Garcia et al., 2016; Ozsoy, 2012). For instance, Garcia et al. (2016) examined fifth and sixth grade mathematics students' calibration accuracy patterns with respect to mathematics achievement and grade level. Using students' final academic grades in their mathematics class as the level of mathematics

achievement, the researchers discovered that high achieving students made more accurate judgments than the lower achieving students. Overall, the participants had low calibration accuracy, and there were no significant differences between grade levels.

In another study involving the relationship between calibration accuracy and achievement, Ozsoy (2012) investigated fifth grade students' mathematical calibration skills. In his study, fifth grade students completed a 30-item mathematics achievement test and a 28-item mathematical calibration instrument which was developed by the researcher. His results showed that overall, the participants had medium-high levels of calibration skills. Although there were no differences between boys and girls, he did discover that high mathematical achievers were significantly better calibrators than middle and low math achievers. Middle achievers were significantly better calibrators than low achievers.

Students tend to be overconfident when monitoring their abilities or knowledge, (Hacker & Bol, 2018) and this is known as bias. Investigating bias in self-assessments of competence among high-school mathematics students, Dupeyrat et al. (2011) gave a mathematics achievement goals survey and a perceived competence in mathematics question to 8th and 9th grade students. Comparing the results of the survey and questionnaire to students' mathematics achievement scores and the students' progress in mathematics, the researchers found that overall, students did not accurately assess their mathematics competence. More specifically, gender differences were revealed; girls underestimated their competence, while boys overestimated their competence when compared to their actual mathematics achievement. In addition, students who overrated their math competence generally had the lowest average achievement progress.

In a broader study, Chiu and Klassen (2010) examined mathematics self-concept, calibration, and achievement among a large sample of fifteen-year-olds. The study indicated that

students that were better calibrated had higher mathematics achievement. Also, students that overestimated their predictions had lower mathematics achievement and students that underestimated their predictions had higher mathematics achievement. Students that had higher mathematics achievement typically had mathematics scores that exceeded their country's mean.

Studies have shown that calibration accuracy can be improved through self-regulation strategies, such as metacognitive training (Pennequin et al., 2010). For instance, to better understand students' calibration accuracy while solving mathematics problems, Pennequin et al. (2010) asked elementary students to predict the number of problems they would solve correctly on a 12-item mathematics test. Prior to the test, half of the students were provided with metacognitive training related to solving math problems (treatment group) and half of the students were not (control group). The researchers discovered a significant difference when comparing pretest/posttest accuracy in the lower achievers that received metacognitive training. Lower achieving students that were exposed to metacognitive training were more accurate in their postdictions than students in the control group. Even though the difference was not significant, the treatment group had better calibration accuracy than the control group. In addition, all the students were overconfident in their predictions, supporting the claim that students struggle to accurately judge their mathematics learning.

Calibration accuracy can also be improved through other self-regulation strategies, such as feedback. For example, Labuhn et al. (2010) sought to investigate how feedback and self-evaluative standards would influence students' calibration accuracy and mathematics performance. Fifth grade students were randomly assigned to two groups; one of either individual feedback, social comparative feedback, or feedback control group; and one of either mastery learning standards, social comparison standards, or standards control group. Students

were asked to predict and postdict their performance on an eight-question mathematics test involving the order of operations. Their results revealed that overall, the students that received social comparison feedback or individual feedback showed significantly higher calibration accuracy than students in the control group. Students that were overconfident in their postdictions, however, were also overconfident in their predictions. Additional analysis revealed the overconfident students were lower mathematics achievers on both the pretest and posttest. The lower achievers that received social comparison feedback, however, scored significantly higher on the posttest than the other groups of lower achievers.

Other self-regulation strategies have also been found to improve students' calibration accuracy. For example, Digiacomo and Chen (2016) found that calibration accuracy and mathematics achievement can be improved through self-monitoring and self-reflection training. Middle school students were exposed to structured and guided questions to encourage them to reflect on their calibration and regulatory behaviors. "The intervention focused on learners' attention to metacognitive monitoring during the performance phase to facilitate more productive reflection" (DiGiacomo & Chen, 2016, p. 604). Despite having a small sample size of only 30 students, and a short intervention period of only three weeks, significant differences between the intervention group and comparison group were reported. The results showed that the intervention group had significantly higher math performance and better calibration accuracy than the comparison group.

Calibration Accuracy and Lower Achieving Students

As revealed in many of the previous studies, student's ability to accurately calibrate their learning is often difficult, this is particularly difficult for lower achieving students (Chiu & Klassen, 2010; Garavalia & Gredler, 2002; Hacker et al., 2008). Much of the previous research

revealed a relation between calibration accuracy and achievement group, suggesting that lower

achieving students may have limited insight to the capacity of their learning and knowledge

(Hacker & Bol, 2018). To this end, some researchers further investigated mathematics students'

calibration abilities and biases, specifically among lower achieving students or students in entry

level mathematics classes.

For example, in Rinne and Mazzocco's (2014) study, fifth through eighth grade students

were grouped by their mathematics achievement levels as either typical achievement, low

achievement, or "having a mathematics learning disability" (p. 3). A total of 56 arithmetic

equations were presented to the students. The students were asked to quickly judge how

confident they were that the presented arithmetic equation was accurate. Although students in

the higher grades were more accurate and better calibrated, the results showed that students with

mathematics learning disabilities had a greater number of incorrect responses and were poorly

calibrated when compared to their peers. Overall, the lowest achieving students, the students

with mathematics learning disabilities, were less accurate and overconfident in their responses.

Calibration accuracy and bias was also evaluated among at-risk mathematics students in

the Zimmerman and colleagues' (2011) study mentioned earlier. The researchers found that

students receiving self-reflection training were more accurate in their judgments before and after

task completion. An analysis of students' bias in their judgments revealed that the SRL group

was significantly more accurate than the control group. The results suggest that teacher explicit

instruction and student training involving self-regulation can improve students' judgments and

achievement, particularly among lower-level mathematics achievers.

Calibration researchers have shown a clear relationship between calibration accuracy and

mathematics achievement; overall students struggle to accurately judge their learning in

mathematics. Higher achieving students more accurately judge their learning and are generally a bit underconfident. Lower achieving students, however, have more difficulty accurately judging their learning and tend to be overconfident. It should be noted, however, that lower achieving students are not incapable of accurate monitoring (Hacker & Bol, 2018), rather, the degree to which they self-regulate and monitor their learning may differ (Bol & Garner, 2011) from other students. Lower achieving mathematics students may simply need to develop their skills that will assist them in accurately monitoring and assessing their knowledge and learning (Garofalo & Lester, 1985); calibration is one such skill. The previous studies provide support for a variety of interventions that are explicitly taught to students to improve their metacognitive awareness, because accurate monitoring of one's learning as an important factor for successful academic performance.

Problem-Solving

One way for students to monitor and assess their knowledge and learning is for them to be aware of their thoughts and performance while engaging in the problem-solving process. Alan Schoenfeld called this 'control' in his book entitled Mathematical Problem Solving (1985). He proposed four characteristics of mathematical problem-solving behavior and performance; resources, heuristics, control, and belief systems. Resources are facts, algorithmic procedures, routines, and understandings that can be used to solve a problem. Heuristics are strategies used to solve the problem such as drawing a figure, exploiting related problems, and verifying procedures. Control is one's planning, metacognitive acts, and monitoring while problem solving. Lastly, belief system is one's world view about the topic, mathematics, self, and the environment. Schoenfeld contends that problem-solving performance is not only what "students know, it is also a function of their perceptions of that knowledge" (p. 14). He argues that

"students' failures to solve problems were caused by malfunctions at the control level; poor decision-making" will result in failed solutions, regardless of how much mathematical content students might know.

Schoenfeld (1985) also proposed that problem-solving can take two forms. Problem-solving may involve routine access to subskills and relevant knowledge retrieved from scripts, schemata, or frames, which are consistent and reliable. This kind of problem-solving "comprises the foundation upon which competent problem-solving performance is built" (p. 68). On the other hand, problem-solving in which the individual does not have ready access to a solution, which includes *more than* routinized performance, involves a variety of factors. These factors may include informal and intuitive knowledge, deeper meaning of facts and definitions, the ability to execute algorithmic procedures, and possession and application of relevant competences and heuristics.

"Once nearly forgotten, heuristics have now become nearly synonymous with mathematical problem-solving" (Schoenfeld, 1985, p. 23). Heuristic strategies are rules of thumb for successful problem-solving performance and include drawing a figure, exploiting related problems, and verifying procedures. Schoenfeld credits Polya's book, *How to Solve It* (1945), as the revival of heuristics and a guide to useful problem-solving techniques.

Polya defined problem solving in a similar way, as finding "a way where no way is known, off-hand..." (1945, p. 1). He proposed that students need to work through details of a problem to reach a solution, and their critical thinking skills can often be gauged by how they engage in the problem-solving process. In 1945, George Polya offered a general (heuristic) four step problem-solving process that is effective for solving word problems: understand the problem, devise a plan, carry out the plan, and review and extend. Polya's seminal problem-

solving process is a cyclical model in which problem-solvers can progress through in many different ways, back tracking and skipping (see Figure 2).
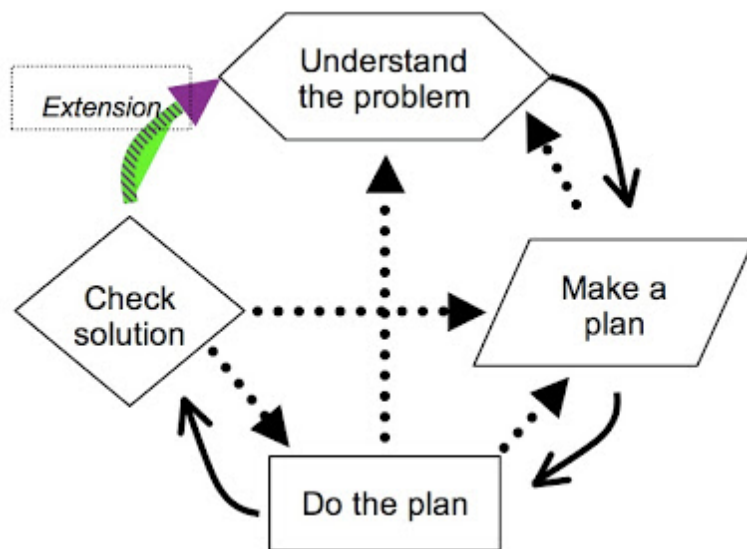
Figure 2. Visualization of Polya's problem-solving process (Golden, 2009).

Polya's problem-solving process stimulates self-regulated learning and involves metacognitive awareness, often through questioning strategies. The first principle of problem-solving requires students to put forth effort and energy to read and *understand the problem*. They must make sense of the information provided in the problem, whether it is in table, graph, figure, or text form. During this phase, students are in the forethought phase of self-regulation, because they are setting goals and planning strategically. They should be organizing the information, establishing goals, and constructing diagrams or other visual representations, to assist them in solving the problem. Students should also engage in metacognitive questions, such as: Do you understand all the words used in stating the problem? What are you asked to find or show? Can you restate the problem in your own words? Can you think of a picture or diagram that might help you understand the problem? Is there enough information to enable you

to find a solution? These metacognitive questions can assist students in understanding the problem.

While remaining in the forethought phase of self-regulation, the second principle of Poyla's problem-solving process, the planning phase, requires students to consider various solution approaches. Devising a plan or strategy is often the hardest step. During this phase of problem solving, mathematical concepts, knowledge, and facts are accessed and considered while conjectures are formulated.   Solution approaches are imagined and a strategy is determined. Four strategies commonly used to solve mathematics problems are guessing and checking, making a table, drawing a picture, and solving a simpler problem. Metacognitive questions students could use during this phase of problem-solving include: Do you know a related problem? Have you seen the same problem in a slightly different form? Is your diagram a good representation of the problem? Did you use all the data?

The third principle of problem-solving, carrying out the plan, coincides with the performance phase of self-regulation. During this phase the students engage in task strategies by implementing the plan that was selected and putting forth effort to stay mentally engaged. Students execute various procedures, construct and connect mathematical representations, carry out computations, and make sense of the new information. Students use metacognitive monitoring to determine if their plan of action is working to solve the problem. If not, they should discard the plan and choose another. Metacognitive questions students should ask themselves include: Can you connect the data and the unknown visually? Is your diagram a good representation of the problem? Do you need a formula or special notation? Do you know how to calculate the solution?

The final principle of Polya's problem-solving process is review and extend. During this phase students are engaging in the self-reflection phase of the self-regulation process. Students check their answer for accuracy and reasonableness, and a decision is made about the validity of their answer. During this final phase the student determines if they should discard their plan and choose another, requiring them to cycle back into the problem-solving process, or cycle forward based on their results. Students should ask themselves: Are your computations accurate? Is your answer reasonable? Where will you see this problem again? Which solution did I decide to use? Reflecting on their results, and the metacognitive process, can enable students to predict what strategy to use to solve future problems by considering the efficiency and effectiveness of various methods.

Problem-Solving in Mathematics

Problem-solving has been extensively studied in mathematics. Research has shown that explicit problem-solving and strategy instruction can improve students' mathematics performance among lower achieving students (Krawec et al., 2013; Montague et al., 2014; Xin et al., 2005). For example, Montague et al. (2014) implemented a problem-solving intervention among seventh grade mathematics students with varying abilities (students with learning disabilities, low-achieving students, or average-achieving students). The intervention required the students to read the word problems for understanding, visualize the problem by drawing a picture or a diagram, develop a plan to solve the problem, predict the answer, make computations, and check their work. Students in the intervention group "showed a significantly greater rate of growth on the curriculum-based measures" (Montague et al., 2014, p. 469) when compared to the comparison group. Synthesizing the results of this study involving seventh grade students, to an identical study involving eighth grade students, the researchers found the

intervention effect was stronger for the low-achieving and learning-disabled students than the average-achieving students. The researchers posit that these findings suggest specialized instruction in math problem solving may improve lower achieving students' mathematics achievement.

Elaborating on the data from the previous study, Krawec and colleagues (2013) sought to examine the treatment effects of the intervention on students' ability levels and their knowledge of problem-solving. Students completed a math problem-solving assessment that measured their knowledge, use, and control of the problem-solving process. Results indicated that students in the treatment group improved significantly from pretest to posttest on their reported strategy use, but the comparison group did not. In addition, after the intervention, students in the treatment group reported using significantly more strategies than their counterparts. Furthermore, the intervention was equally effective for students regardless of their ability level (average achieving students or students with learning disabilities).

Polya's problem-solving process was situated in a study by Xin et al. (2005). The researchers investigated the effect schema-based instruction and general strategy instruction had on middle school students' word problem-solving performance. Middle school students who had learning disabilities or were at-risk for mathematics failure participated in the study. Students in the schema-based instruction group were taught to read the problem for understanding, identify the problem type (multiplicative compare problem or proportion problem), use the schema diagram to represent the problem, transform the diagram to a math sentence, solve the problem, and look back to check their work. Students in the general strategy instruction group were taught to read the problem for understanding, draw a picture to represent the problem, solve the problem, and look back to check their work. Therefore, unlike the schema-based instruction

group, students in the general strategy instruction group did not receive instruction in recognizing the two different word problem types.

The results showed that students in the schema-based instruction group performed significantly better than students in the general strategy instruction group on all measures of acquisition, maintenance, and generalization. This study "used a schema-based instruction to systematically teach the structure of different problem types and directly show the linkage of the schematic diagram to problem solution" (Xin et al., 2005, p. 189). In addition, students in the schema-based instruction group that identified problem structure or type and applied schema knowledge to represent and solve the problems showed higher-order thinking skills.

Zollman (2009) examined how explicitly teaching students Polya's (1945) problem-solving hierarchy could influence mathematics students word problem solving process. In an action research project, ten elementary school teachers implemented a graphic organizer tool during a measurement unit with elementary grade students. The graphic organizer guided students through Polya's (1945) problem solving hierarchy: understand the problem, devise a plan, carry out the plan, and review and extend. The results showed that on average, word problem performance improved across all grades from pretest to posttest. In addition, the graphic organizers were efficient and effective for students at all achievement levels. Low-ability students now had a way to begin the problem-solving process, average-ability students had a way to organize the information and strategies, and high-ability students could improve their problem-solving communication skills. Lastly, graphic organizers which involve visualizations of the word problem allowed students to break the problem down into manageable parts, design and analyze multiple representations, and make connections about mathematics (Zollman, 2009).

Problem-solving is a focus of school mathematics. Students that have difficulty solving word problems often "lack knowledge of (or fail to use) problem-solving processes, particularly those necessary for representing the problem" (Montague et al., 2014, p. 470). Problem-solving researchers have shown, however, that teaching lower achieving students the problem-solving process and related strategies to solve word problems, can improve their mathematics performance. Thus, students, especially lower achieving students, should be explicitly taught strategies for representing word problems, such as visual representations, and how to apply them during the problem-solving process.

Visualizations

Visualizations are diagrams that transform "problem information to a representation that shows the relationships among problem parts" (Montague et al., 2014, p. 470). Visual representations can take many forms (graphic organizer, graph, table, picture, figure, etc.) during the problem-solving process. In fact, Stylianou (2002) found that expert problem-solvers, professional mathematicians, use visualizations to help them complete problem-solving activities. Other research has shown that the use of visual representations improved students' comprehension of the content (Dexter & Hughes, 2011) and problem-solving abilities (Edens & Potter, 2007), especially for lower achieving math students (Krawec, 2014).

To emphasize the importance of visualizations in problem-solving, Edens and Potter (2007) examined how fourth and fifth-grade students' drawing tasks related to their mathematical problem-solving. More specifically, the researchers were investigating students proportional thinking of math problem solving through students' drawings. The students were required to represent numerical information graphically and derive answers from their representations. Analysis revealed that there was a significant relationship between students that

used schematic visual representations and their problem-solving scores. Further analysis showed that the more proportional the visual representation was, the more accurate their problem solving.

Visual representations are important mathematical tools that help lower achieving mathematics students with the problem-solving process (Gersten & Clarke, 2007). Visual representations offer students a way to organize their mathematical thinking, identify relevant information, and assist in developing a solution to the problem (Draper & Wimmer, 2015), tasks that lower achieving students struggle with (Geary, 2011; Gersten & Clarke, 2007). Dexter and Hughes (2011) conducted a meta-analysis of graphic organizers and students with disabilities across multiple content areas. They extensively reviewed sixteen articles involving participants from grades four to twelve. They found that a variety of visualizations that required students to visually represent information, improved students' factual comprehension of the content. Additionally, visual representation was found to improve students' academic vocabulary, basic skills, and higher-order thinking skills (Dexter & Hughes, 2011).

Also investigating students' visual representations of mathematics word problems, Krawec (2014) examined the problem-solving solutions and visual representations of eighth grade students with varying mathematical abilities. The researcher analyzed students' work for accuracy and retrieval of relevant information. The results showed that average-achieving students demonstrated stronger problem-solving abilities and visually represented more relevant information than both low-achieving students and students with disabilities. Even more interesting, visual representation accuracy explained more of the variance in problem-solving accuracy than ability group. These results suggest that visual representation of mathematical

word problems is critical for accurate problems-solving, specifically for lower achieving students.

Many lower achieving mathematics students struggle with forming mental representations of mathematical concepts and have week abilities to access numerical meaning from symbols (Geary, 2011). Furthermore, because each mathematics problem is slightly different, there is no single way to solve a mathematics problem, making problem-solving even more difficult. Thus, teaching students not only the knowledge base, but strategies to use during problem-solving (Stylianou, 2002), such as visualizations, can be an effective method for assisting students in understanding concepts and transferring their skills and knowledge in various contexts. Students' ability to transfer their skills and knowledge to a variety of mathematics problems is a skill that is integrated in mathematics literacy practices.

Mathematics Literacy

Mathematics literacy (ML) is student's ability to "formulate, implement, and interpret mathematics in various contexts, including the capacity to perform reasoning mathematically and using the concepts, procedures, and facts to describe, explain or predict phenomena" (Wardono, Mariani, & Hendikawati, 2017, p. 1). Similarly, others have defined mathematics literacy as the ability to formulate, employ, connect, implement, and interpret mathematics in a variety of contexts (OECD, 2013), or more broadly as any individual that "has mathematical skills and abilities beyond pure mathematical content" (Lengnink, 2005, p. 247).

Mathematics literacy is imbedded in the disciplinary literacy framework. Disciplinary literacy is built on the premise that each content area has its own ways of understanding and knowing the material and that it is abstract and complex in nature (Moje, 2008; 2015). Shanahan and Shanahan (2008) illustrated a hierarchical model of how the development of literacy

progresses, placing disciplinary literacy at the most advanced level (see Figure 3). They propose

the base of the pyramid entails basic literacy skills, such as decoding, recognition, and

knowledge of high-frequency words. Literacy at the basic level is usually mastered during

primary grades, for slower learners during middle grades. The middle of the pyramid,

intermediate literacy, entails comprehension strategies, vocabulary understanding,

interpretations, and discourse within the discipline. At this level of literacy development, skills

are still more generalizable to other domains. Finally, disciplinary literacy is placed at the top of

the pyramid. Disciplinary literacy skills are more sophisticated and specialized to the discipline

but less generalizable.
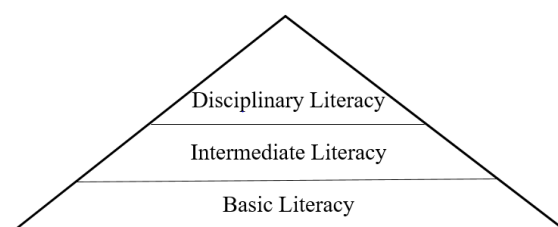
Figure 3. Shanahan and Shanahan (2008) Development of Literacy Progression

The disciplinary literacy framework has been suggested as an instructional perspective to

improving students understanding of mathematical concepts (Shanahan & Shanahan, 2008;

2012). Disciplinary literacy does not focus on traditional literacy perspectives, instead focusing

on engaging students in the practices, routines, and skills of the discipline (Moje, 2008; 2015;

Shanahan & Shanahan, 2008). These disciplinary practices are specific to the discipline and

cannot be generalized to other content areas. Therefore, disciplinary literacy instruction in

mathematics engages students in a deeper understanding of the content and employs practices of

mathematicians (Draper, 2008).

Disciplinary literacy researchers often focus on experts of a discipline to determine disciplinary practices. For example, Shanahan and Shanahan (2008) studied experts to better understand how they approached reading texts in their discipline. The experts were asked to read and think aloud about their process as they engaged with their disciplinary texts. The experts among each discipline "emphasized a different array of reading processes," (Shanahan & Shanahan, 2008, p. 49), which demonstrated the importance of disciplinary strategies for reading a variety of texts. More specifically, mathematicians stressed the importance of rereading and close reading as the most important strategies in order to understand the text. The experts emphasized that every word has precise meaning in math, and rereading allows for a deeper understanding of the text.

Not only is understanding the text of word problems important for problem solving in mathematics but creating visual representations of the problems are equally important. Researchers have found that expert mathematicians use visualizations to help them complete problem-solving activities. Stylianou (2002) investigated how visualizations are utilized by mathematicians during the problem-solving process. Through think alouds, interviews, and observations, Stylianau discovered that not only do mathematicians use visual representations to problem-solve, but they create their representations in systematic steps, pausing between sketching. During these pauses, the mathematicians were engaging in self-regulation and metacognitive monitoring. They were "thoroughly and systematically exploring the images they construct at each visual step, while closely monitoring the effectiveness of each visual step they take, altering and retracting their images when they find this necessary" (Stylianou, 2002, p. 315). This process provided the mathematicians with a complete understanding of the problem situation. Understanding the problem is an important factor in developing mathematics literacy.

It would be difficult to formulate, implement, or interpret mathematics if one does not understand the problem. Creating a visualization of the problem is a useful strategy for deeper understanding, and a common problem-solving activity used by expert mathematicians.

Disciplinary literacy researchers have studied mathematics experts to determine what constitutes the best practices and strategies that educators can implement in the classroom to support their students' development of mathematics literacy (Shanahan & Shanahan, 2008; Stylianou, 2002). These strategies include systematic processes, self-regulation, metacognitive monitoring, reading and rereading for understanding, and creating visual representations of the problem. "Making good use of metacognitive strategies allows for the transfer of mathematical literacy into new contexts" (Chen & Chiu, 2016, p265).

Mathematics researchers agree that developing students' mathematical literacy is a process (Friedman, Kazerouni, Lax, & Weisdorf, 2011; Lengnink, 2005; Lo'pezLeiva, Torres, & Khirsty, 2013; Wardono et al., 2017). The process that increases students' ability to become confident in handling, judging, and explaining mathematical applications. "Putting mathematical ideas and reasoning into words is a key element of mathematical literacy" (Friedman et al., 2011, p. 31). For instance, in a qualitative study involving elementary students' development of mathematical reasoning, Lo'pezLeiva et al. (2013) analyzed the social and communication processes of two groups of students working on probability tasks. The researchers sought to better understand what linguistic resources bilingual students use to make sense of probability problems. Fifteen hours of video data, students' work, and facilitators field notes involving seven students were analyzed for this study. Data analysis revealed that the students used a myriad of resources to develop their conceptual understanding and sense making. One student used meaningful linguistic resources, such as Spanglish words (Lo'pezLeiva et al., 2013), to

explain their understanding of the problem. Other students required a more concrete representation of the problem, such as a game-like activity or simulation, to develop their conceptual understanding. The researchers concluded that using multidimensional resources that capitalize and expand on struggling students' current ideas assisted in the development of students' mathematical literacy.

In another study, Friedman et al. (2011) investigated students' development of geometric concepts, vocabulary, and communication using a personal math concept chart to promote mathematical literacy. The chart required students to categorize, describe, draw a visual, and provide a real-life example and non-example of multiple math concepts or vocabulary terms. This action research study involved four classes of students in grades, first, second, third, and sixth. A pre-assessment and post-assessment was used to determine how the four-week intervention involving the personal math concept chart impacted their discipline specific language development. Data was collected from students' pre-assessment and post-assessments, pictorial descriptions, personal math concept charts, and teacher notes from informal math dialogue. Analysis of the data showed that the students provided longer, clearer, and more descriptive answers and explanations post-assessment (after using the personal math concept chart) than they did on the pre-assessment (prior to using the personal math concept chart). Additionally, all four teachers "found that the quality and depth of math discussion in class was increased throughout" (Friedman et al., 2011, p. 33) the intervention period. Furthermore, math surveys were conducted to determine whether students found the personal math concept chart beneficial to their learning. Overall, the students found the concept chart to be easy and helpful, specifically for daily work, and would like to have it available to them more often as a reference.

In a mixed methods study that involved both qualitative and quantitative data, Wardono

et al. (2017) sought to better understand how two instructional methods that engage students in the mathematizing process would influence their mathematics literacy. Mathematizing "is a process for mathematics phenomenon" (Wardono et al., 2017, p. 1). One could look at "mathematics relevant to a phenomenon" or build "a mathematical concept of a phenomenon" Wardono et al., 2017, p. 1). The researchers investigated how students cognitive style (reflective or impulsive) influenced their mathematizing process. Two classes of 8th grade junior high students were grouped to receive either problem-based instruction (experimental group) or scientific learning (control group) while solving real-life mathematics problems. Data was collected through documentation of observations, tests, and interviews. Analysis of the data revealed that students that engaged in the mathematizing process through problem-based instruction had improved mathematics literacy. Further analysis showed that students that had a reflective cognitive style towards the mathematizing process scored higher and developed their mathematics literacy more than the students that had an impulsive cognitive style towards the mathematizing process.

Kramarski and Mizrachi (2006) sought to quantitatively understand how instructional methods influenced middle school students' development of mathematical literacy and SRL strategies. Over a period of four weeks, a total of 86 students received either online or face to face discussion and metacognitive guidance or no metacognitive guidance during mathematical problem-solving. Metacognitive guidance provided students with a series of self-addressed metacognitive questions regarding comprehension, connection, strategy, and reflection when engaging in mathematical tasks. Students in the metacognitive groups received training in answering the metacognitive questions and were provided with an index card to guide them through the metacognitive process. Multiple-choice and open-ended pretests and posttests were

used to determine students level of mathematical literacy. A questionnaire was used to evaluate

students SRL strategies. The results of the study revealed that students that were exposed to

metacognitive guidance attained a higher level of mathematical literacy and SRL strategies than

the students that were not exposed to metacognitive guidance. This study supports the

importance of metacognitive training in developing students' mathematical literacy and SRL

strategies.

Students' development of mathematics literacy is a process that can be improved by

teaching them effective problem-solving strategies. However, teaching mathematics from a

disciplinary literacy perspective should demonstrate characteristics of expert mathematicians:

self-regulation, systematic problem-solving processes, metacognitive monitoring, reading and

rereading for understanding, and creating visual representations of the problem. Therefore,

students' development of mathematics literacy should focus on these characteristics.

Purpose Statement and Research Questions

This literature review provided empirical evidence for the connections between self-

regulation, problem-solving, visualizations, and mathematics literacy in the classroom.   No

studies could be found that directly show that teaching from a disciplinary literacy perspective

have improved students' way of thinking and learning mathematics. This study aims to fill the

gap between disciplinary literacy instruction and student outcomes in terms of their achievement,

calibration accuracy, and development of mathematics literacy. Thus, the purpose of this study

was to investigate the impact metacognitive training had on lower achieving Algebra students'

achievement, calibration accuracy, and development of mathematics literacy when solving word

problems.   The following research questions were investigated:

1. How does metacognitive training influence the achievement scores of lower achieving Algebra students?

2. How does metacognitive training influence the calibration accuracy of lower achieving Algebra students?

3. How does the metacognitive training influence lower-achieving Algebra students development of mathematics literacy?

It was predicted that the students that were exposed to metacognitive training would have better calibration accuracy, higher mathematics achievement scores, and greater development in their mathematics literacy than students exposed to the problem-solving strategy without metacognitive training.

Summary

Chapter 2 presented a review of the related literature regarding self-regulated learning, calibration, problem-solving, visualizations, and disciplinary literacy in mathematics. The literature review revealed that SRL is an effective strategy to improve student learning in mathematics classrooms. Students generally struggle to accurately judge their learning in mathematics and lower achieving students are typically overconfident. Polya' problem-solving process has shown to be an effective strategy for student learning and is integrated in self-regulated frameworks. Visualizations are one strategy, used by experts, that can assist students in better understanding the problem. As students advance their learning and understanding of mathematical concepts, they develop their mathematics literacy. Teaching students characteristics of mathematicians, such as systematic problem-solving, self-regulation, and visual representations, may improve students' mathematical literacy, in turn, their mathematics achievement and calibration accuracy.

CHAPTER 3

Methodology

Chapter 3 describes the design and the specific procedures that were used in the study. It begins with a restatement of this study's research questions followed by the research design, its rationale, and research variables. The population and sampling procedures for this study are then described in detail followed by the procedure, measures, and materials that were used in the study. Subsequently, data collection procedures and data analysis are described.

Research Questions

The following research questions were investigated:

1. How does metacognitive training influence the achievement scores of lower achieving Algebra students?

2. How does metacognitive training influence the calibration accuracy of lower achieving Algebra students?

3. How does metacognitive training influence lower-achieving Algebra students' development of mathematics literacy?

Design

A pretest/posttest quasi-experimental (Gall, Gall, & Borg, 2003) design was employed to compare the effectiveness of the metacognitive training group to the problem-solving group among lower achieving secondary mathematics students. The treatment was implemented over three weeks while students were solving algebra word problems as a warm up activity. The independent variable was the treatment condition, either the metacognitive training (MT) or the traditional problem-solving strategy (PSS). The dependent variables were students' calibration accuracy, achievement scores (score on the posttest), and mathematics literacy.

Participants

This study had a total sample size of 37 participants, using two classes with 15 and 18 participants in each class. Participants were male and female adolescents enrolled in an Algebra 1B course attending a local Southeast Virginia high school. All participants followed the Algebra 1B mathematics curriculum. One class was randomly assigned to the MT condition and the other class was the PSS condition.

The participants were lower achieving students and were assigned to the Algebra 1B course based on previous years grades and performance. This high school used a block schedule which means that students have longer class periods that typically meet fewer times each week. Students in the Algebra 1B courses, however, attended class everyday allowing for double the instructional time and covering half of the content of a traditional Algebra I course. Noteworthy, students who have higher mathematical ability usually take Algebra 1 in middle school; the participants in this study were 9th and 10th grade high school students. Moreover, eight students among the two classes had either an Individual Education Plan, a 504 plan, or a Behavior Improvement Plan.

Protection of subjects and participants. Since this study involved minors, participants' parents or guardians were informed of the study and could have chosen to "opt out" and not have their child's data included in the analysis (see Appendix A). The use of an opt-out, rather than an opt-in, form for parents is common practice and a typical procedure in this school district. All students participated in the instruction because warm-up activities were a regular part of the instruction; however, if the student or their parents decided not to participate in the research, their scores and responses were not used in the analysis. Students were provided with an introduction letter (see Appendix B) and were asked about their willingness to participate

(assent). To protect students' confidentiality, all identifying information was stripped from the data once it was collected.

Procedure

Word problem-solving warm up activities were implemented among the two classes. Over a period of three weeks, approximately the first thirty minutes of each lesson was dedicated to word problem-solving using either metacognitive training or problem-solving strategy. The time was allotted as follows: 10-15 minutes for students to work one warm up problem, 15-20 minutes for a class discussion about the problem solutions and for the cooperating teacher to model a solution to the problem using either MT or PSS. The cooperating teacher participated in a professional development session, provided by the researcher, to support the teacher in learning and implementing the metacognitive training and the problem-solving strategy.

Metacognitive training (MT). Metacognitive training was the same as the problem-solving strategy but included metacognitive questions. The five key features of the metacognitive training included 1) read the problem, 2) pull out important information, 3) draw a visualization, 4) solve the problem, and 5) check your work. Each one of the five steps were combined with two metacognitive questions to encourage students to think about each step of their problem-solving process. During the warm up activity, the students were provided with a worksheet that contained the warm-up problem and the MT procedure (see Appendix C) that reinforced and guided them through the MT process. After students work independently on the warm up activity, the cooperating teacher discussed and reviewed the five-step metacognitive process for solving mathematical word problems by way of modeling (Geary, 2011; Gersten & Clarke, 2007) and think aloud (Jacobse & Harskamp, 2012; Throndsen, 2011).

Problem-solving strategy (PSS). The problem-solving strategy was the same as the metacognitive training but did *not* include the metacognitive questions. The five key features of the problem-solving strategies included 1) read the problem, 2) pull out important information, 3) draw a visualization, 4) solve the problem, and (5) check your work. During the warm up activity, the students were provided with a worksheet that contained the warm-up problem and the PSS procedure (see Appendix D) that reinforced and guided them through the PSS process. After students worked independently on the warm up activity, the cooperating teacher discussed and reviewed the five-step problem solving strategy for mathematical word problems by way of modeling and thinking aloud. Table 1 provides a comparison of the PSS and MT problem-solving processes.

Table 1.

Comparison of Problem-Solving Strategies and Metacognitive Training

STEP	PSS	MT	Metacognitive Questions for MT
1	Read the problem	Read the problem	• Do you understand the problem? • Do you know what all the words mean?
2	Identify important information	Identify important information	• What is the unknown, what is being asked? • What are the data?
3	Draw a visualization	Draw a visualization	• Can I connect the data and the unknown visually? • Is my diagram a good representation of the problem?
5	Solve the problem	Solve the problem	• Do I need a formula or special notation? • Do I know how to calculate the solution?
6	Check your work	Check your work	• Are your computations accurate? • Is your answer reasonable?

Materials

Problem-solving questions. The cooperating teacher and researcher selected word

problems that directly related to multi-step problems-solving involving visualizations. All word

problems that were selected for the VisA instrument and warm up activities, were reviewed by a

content expert to determine appropriateness and level of difficulty. Word problems were

selected from a variety of resources including the current curriculum, previous curriculums, and

resources available online. See Table 2 for an example problem.

Table 2.
Example question from the VisA instrument

Page 5	Page 6
Name: **Problem** There are three rectangular tables. Each table seats six people, 2 people on each side and 1 person on each end. How many people can be seated at the tables if the tables are lined up end to end? **Question** On a scale of 1-3, how well do you think you can solve this problem? Circle one. 1 = I am sure I will solve this problem correctly 2 = I am not sure whether I will solve this problem correctly or incorrectly 3 = I am sure I cannot solve this problem correctly Please explain why…	**Problem** There are three rectangular tables. Each table seats six people, 2 people on each side and 1 person on each end. How many people can be seated at the tables if the tables are lined up end to end? **Question** Draw a sketch you can use to solve the problem.
Page 7	Page 8
Problem There are three rectangular tables. Each table seats six people, 2 people on each side and 1 person on each end. How many people can be seated at the tables if the tables are lined up end to end? **Question** Solve the problem.	**Problem** There are three rectangular tables. Each table seats six people, 2 people on each side and 1 person on each end. How many people can be seated at the tables if the tables are lined up end to end? **Question** On a scale of 1-3, how well do you think you solved this problem? Circle one. 1 = I am sure I solved this problem correctly 2 = I am not sure whether I solved this problem correctly or incorrectly 3 = I am sure I did not solve this problem correctly Please explain why…

Warm-up worksheets. During the warm-up activity, students were provided with a worksheet that contained the word problem and guided them in either the MT or PSS process (see Appendix C and D). The worksheets were intended to remind students to engage in the five-step process. In addition, the MT worksheet provided students with the metacognitive questions they should ask themselves for each step in the process.

Measures

Visualization & Accuracy (VisA) instrument. Typically, the valid and popular think aloud procedure has been used to assess students' metacognitions. Because the think aloud procedure is time-consuming, Jacobse & Harskamp (2012) sought to develop a less time-consuming method to assess students' metacognitions during problem solving; the Visualization and Accuracy (VisA) instrument. Comparing the VisA instrument to the think aloud procedure, they found that the think aloud measure and the VisA measure correlated highly with problem solving performance ($r(37) = 0.66$), and 43 percent of the variance was explained by both measures. More specifically, the VisA instrument explained 23 percent of the variance in students' word problem solving performance; making the VisA instrument a valid method for predicting students problem-solving performance (Jacobse & Harskamp, 2012).

The VisA instrument used in this study was revised to include 6 multi-step word problems, a prediction question, and a postdiction question. The researcher and cooperating teacher created the assessment instrument by selecting and creating word problems, appropriate for using visualizations, from the mathematics textbook and other available resources ensuring that the students have already learned the content. For each word problem, however, students were asked to divide their problem-solving over four phases:

1. Read the problem and rate your confidence for finding the correct answer (without calculating the answer);

2. Make a sketch or plan which can help you solve the problem;

3. Solve the problem and fill in the answer;

4. Rate your confidence for having found the correct answer;

Each of the four phases (listed above) were provided on a different page in the form of a booklet, therefore, the problem question was repeated on each page for student convenience (see Table 2). Students received approximately 60 minutes to solve all the word problems. The VisA instrument was administered to all participants using standardized procedure for both the pretest and posttest. The cooperating teacher was trained prior to administration. The pretest and posttest contained six different word problems but of equal difficulty. The VisA instrument was scored to determine students' achievement score and calibration accuracy. Components of the VisA instrument were used to determine students' development of mathematics literacy. The first and last page of the pretest and posttest contained an open-ended calibration item requesting participants to predict and postdict the number of questions correct (0-6).

Achievement. To determine students' achievement scores, all six word problems were scored as a 1 for a correct answer and 0 for incorrect answer. A sum score was computed (0-6) for each student for the total number of correct answers (actual score) to determine students' achievement score.

Calibration Accuracy. Participants were asked to make predictions and postdictions of how many problems they think they got correct on both the pretest and posttest. The directions for the pretest predictions were, "The following test involves six real-life problem-solving questions. Please estimate how many questions, out of 6, that you think you might get correct.

Number of questions I think I will get correct (0-6) _____." The directions for the pretest

postdictions were, "This test involved six real-life problem-solving questions. Now that you

have completed the questions, please estimate how many questions, out of 6, you think you got

correct. Number of questions I think I got correct (0-6) _____." The directions for the posttest

predictions and postdictions were the same as the pretest.

The participants' prediction calibration accuracy was computed by calculating the

absolute value of the difference between their total prediction scores and their total actual scores

(number correct) on the tests. Calibration bias was calculated by calculating the difference

between participants' predictions and their actual scores. The same computations were

calculated to determine participants' posttest calibration accuracy and bias.

Mathematics Literacy. Mathematics literacy was assessed using several sources of

quantitative and qualitative measures to triangulate findings. Quantitative measures were

interpreted from the VisA instrument, Analytic Scale for Problem Solving (Wilson, 1991;

Szetela & Nicol, 1992), and a Visualization rubric. Qualitative data was collected from

reviewing students' visualizations, classroom observations, and casual conversations with the

students and teacher, which allowed for a more complete understanding of students'

development of mathematics literacy. In an effort to get an overall view of the data (Creswell,

2007), the researcher reviewed students' representations three times while writing memos about

their work. The researcher's journal, memos, and notes from the students' work and classroom

interactions were reviewed to identify broad themes.

To maintain the integrity of the study, the researcher was present in the classroom every

other day during the intervention period and maintained a journal of field notes, memos, and

reflections (Hays & Singh, 2012). To minimize researcher bias, themes that emerged from

researcher observations and journal were cross-checked with the cooperating teacher.

Researcher observations and interactions with participants were documented and analyzed

simultaneously to ensure accuracy of interpreting results (Hays & Singh, 2012). This analysis

helped shape researcher questions, observations, and interactions for the next classroom visit.

Analyzing such a wide range of measures allowed the researcher to provide thick descriptions

and look deeply at students' progress toward becoming mathematically literate.

Analytic scale for problem-solving. Charles, Lester, & O'Daffer (1987; 1994)

propose that analytic scoring is a process-oriented view of evaluation, where the emphasis is on

the problem-solving process, and "involves the use of a scale to assign points to certain phases of

the process" (p.29). Analytic scales consider several phases of the problem-solving solving

process, allow for differential weighting of categories that make up the scale, and assign

numerical values to students' work for further analysis.

In the current study, students' word problems were scored using a modified version of

Wilson's (1991) Scale for Problem Solving (see Table 3). The scale assigns separate scores to

three different stages in problem solving: understanding the problem, solving the problem, and

answering the question. Notice there is an increase in emphasis on understanding and solving

the problem compared to answering the problem.

Table 3.
Wilson's (1991) Analytic Scale for Problem Solving

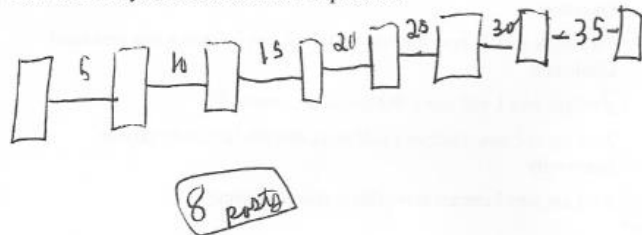| Analytic Scale for Problem Solving		
Understanding the problem	Solving the problem	Answering the problem
0. No attempt 1. Completely misinterprets the problem 2. Misinterprets major part of the problem 3. Misinterprets minor part of the problem 4. Complete understanding of the problem	0. No attempt 1. Totally inappropriate plan 2. Partially correct procedure but with major fault 3. Substantially correct procedure with minor omission or procedural error 4. A plan that could lead to a correct solution with no arithmetic errors	0. No answer or wrong answer based upon inappropriate plan 1. Copying error, computational error, partial answer for problem with multiple answers, no answer statement, answer labeled incorrectly 2. Correct Solution

Below are three examples from students' work to demonstrate how the analytic scale was applied. It is evident from the student work in Figure 4 that she has identified all the important data: namely, that there is a 35ft. fence, 5 ft. wide sections, and 1 post at each end. So, this student received *4 points for understanding*. This student used the visualization to solve the problem. She then checked her visual representation using select operations and calculations. She received *4 points for her solution*. Finally, this student answered the question correctly, provided a label for their answer, and circled their final answer; receiving *2 points for answer* and acquiring an overall, and maximum, score of 10.

Figure 4. Example of student's work

The student's work in Figure 5 had merit, even though the wrong answer was obtained. This student appeared to have identified all the important data, however, there was no clear indication of 1 fence post on each end; he received 3 points for understanding. This student used the visualization to solve the problem and he checked his visual representation using select operations and calculations. Unfortunately, the visualization and the calculations did not clearly represent the problem; he received 2 *points for his solution.* This student did not answer the question correctly nor did he provide a label for his answer. He received *0 points for his answer* and acquired an overall score of 5 out of 10 points.

Figure 5. Example of student's work

In the last example (see Figure 6), the student's work indicated they did not quite understand the problem. The student's sketch of the problem did not show the length of the fence as 35 ft. Even though the student identified that each section of fence was 5 ft. long, there were only four sections, suggesting 20 ft. of fence configured as a square with no 'ends,' not 35 ft. as the question indicated; she received *1 point for understanding.* Her plan was to create a visualization and make some calculations. The plan itself could have lead somewhere (it did in the first example), but it was not correct; receiving *1 point for solution.* Finally, this student

received *0 points for answer* because they provided a wrong answer based upon and inappropriate plan. She acquired an overall score of 2 points.

Problem

How many fence posts are needed to build a 35-ft. fence if each section of the fence is 5 ft. wide and there is 1 post at each end?

Question

Draw a sketch you can use to solve the problem.

5ft

5ft   35 ft.   5ft

5ft

How many fence posts are needed to build a 35-ft. fence if each section of the fence is 5 ft. wide and there is 1 post at each end?

Question

Solve the problem and circle the answer.

$$\frac{35}{5} = 7 \text{ posts}$$

Figure 6. Example of students' work

Students' scores on the analytic scale were further reviewed to verify the level of improvement at which understanding of the context of the problem, the solution procedure, and the answer requirements were evident between the pretest and posttest and across similar problems. Descriptive statistics for each component of the analytic scale were calculated to provide additional diagnostic information, more details about students' strengths and weaknesses, and specific determination about the effectiveness of the intervention for improving students' mathematics literacy. To check scorer reliability, a content area expert and the researcher scored a sample of the pretests and posttests. An intraclass correlation coefficient (ICC) was calculated to confirm interrater reliability.

Visualization rubric. A five-level rubric was used to evaluate students' ability to link important information from the word problem to a visual representation (Draper & Wimmer,

2015) (see Appendix E). Level 5, as in Figure 4, indicated that the diagram was a valid and appropriate linked representation of the word problem, while a level 2, as in Figure 6, suggested that the diagram was not a valid or linked representation of the problem and depicted multiple major identifiable errors. Level 1 was scored if no diagram was provided. To check scorer reliability, a content area expert and the researcher scored a sample of the pretests and posttests. An intraclass correlation coefficient was calculated to confirm interrater reliability.

Classroom observations. Classroom observations were conducted to observe students' natural occurring behaviors in the classroom setting, such as individual practice, collaboration, willingness to try problems, and perseverance. During warm up activities the researcher observed the class to better understand how the students engaged with the activity. To gain deeper insight to students' behaviors, observations were limited by looking for specific aspects of performance or attitude, and sometimes the researcher selected only a few students to observe. Although the researcher often had a plan during observations, she was also flexible enough to note other significant behaviors that may have been displayed. During instructional time, however, the researcher was an active participant in the classroom and engaged in casual and informal conversations with the students and teacher. This information was used to gain more insight into students' abilities to deal with the data and choose appropriate strategies and to better understand the teachers' beliefs about the students' abilities, attitudes, and behaviors. The researcher recorded the overall responses and findings briefly and objectively in a reflective journal.

Fidelity of Implementation. This study was conducted in a more ecologically valid context of the real-world classroom with the same teacher instructing both groups of students. To document fidelity of implementation, warm up lessons from both the intervention and

comparison groups were observed by the researcher. This ensured that the cooperating teacher

incorporated the intervention as he was trained to do. While observing the lessons, the

researcher used a checklist (Jitendra, Harwell, Dupuis, Karl, Lein, Simonson, & Slater, 2015)

developed to document the presence of the core features of metacognitive training (see Appendix

E). The same checklist was used in the control conditions to evaluate program differentiation

and determine whether the problem-solving strategies group was provided any key elements of

the metacognitive training. For each observation, the researcher evaluated whether the teacher

completed all five components and questions corresponding to MT and all five components for

the PPS.

CHAPTER 4

Results

In the present study, the researcher examined how metacognitive training during Algebra warm-up activities influenced lower achieving students' calibration accuracy, achievement, and development of mathematics literacy. In Chapter 4, I first report the verification of fidelity to evaluate group differentiation and diffusion of treatment. Second, I describe the data cleaning and assumption checking methods and decisions. Third, I report how the metacognitive training influenced students' mathematics achievement. Fourth, I examine how the metacognitive training influenced students' calibration accuracy and bias. Lastly, I report the influence metacognitive training had on lower achieving students' development of mathematics literacy.

Fidelity of Implementation

A checklist was used by the researcher during classroom observations to ensure that the cooperating teacher was implementing the intervention as he was trained to do so, and to evaluate program differentiation to determine whether the problem-solving strategies group received any key elements of the metacognitive training. The cooperating teacher accurately implemented the five key features that were shared among the metacognitive training and the problem-solving strategy which included 1) read the problem 2) pull out important information 3) draw a visualization 4) solve the problem and (5) check your work.

There were two metacognitive questions for each of the five key features in the MT group. On five different occasions, the researcher observed that the teacher did not address all ten metacognitive questions with the intervention group. In fact, on the first day of observation the teacher only addressed six of the ten questions. The researcher retrained the teacher, emphasizing the importance of addressing all ten metacognitive questions during every warm-up

activity. After the retraining, the teacher did much better, missing only one question on four different occasions throughout the duration of the study.

The same checklist was used to determine whether the problem-solving strategies group was provided any key elements of the metacognitive training. On seven different occurrences, the PSS group was inadvertently exposed to metacognitive questions. Table 4 provides a list of the questions and how many times the question was inadvertently asked to the comparison group.

Table 4.
Metacognitive Questions and Frequency Used in the Comparison Group

Metacognitive Question	Frequency
Do you know what all the words mean?	2
Do I need a formula or special notation?	3
What is the unknown?	1
What are the data?	5

Data Cleaning

Data cleaning was used to identify and correct errors in the data to minimize their impact on the study's results. For this study, data was collected over a period of 17 consecutive days. Exploratory descriptive statistics were used to check for missing data and outliers. The data from two participants was removed from the analysis, because they enrolled in the course two days after the intervention began. Two additional data points were removed from data analysis, because they were identified as an extreme value and outlier since they were both more than three standard deviations from the mean (Field, 2013). Three participants were missing some data, but remained in the analysis, leaving a total of 33 data points for analysis.

Descriptive Statistics

One independent variable, Group, was used in the analysis. Group was categorized into the intervention group ($N = 18$) and comparison group ($N = 15$). There were five dependent variables: test scores, prediction, postdiction, bias, and mathematics literacy. To determine students' test scores, all six word problems were scored as a 1 for a correct answer and 0 for incorrect answer. A sum score was computed (0-6) for each student for the total number of correct answers (actual score) to determine students' achievement score. Prediction calibration accuracy was computed by calculating the absolute value of the difference between participants' total prediction scores and their total actual scores on the tests. Calibration bias was calculated by calculating the difference between participants' predictions and their actual scores. The same computations were calculated to determine participants' posttest calibration accuracy and bias. Table 5 presents the descriptive statistics that were calculated for the dependent variables by group for both the pretest and posttest.

Table 5.

Descriptive Statistics for Calibration Accuracy and Achievement

	Group			
	Problem Solving		Metacognitive Training	
	M	*SD*	*M*	*SD*
Pretest				
Prediction	2.43	0.37	2.00	0.33
Postdiction	1.93	0.35	1.50	0.32
Test Scores	2.07	0.44	1.50	0.37
Posttest				
Prediction	2.57	0.40	1.25	0.25
Postdiction	1.93	0.50	1.00	0.26
Test Scores	3.21	0.54	2.44	0.37
PreBias	-1.00	0.77	0.88	0.34
PostBias	-0.50	0.72	0.75	0.31

Mathematics literacy was more qualitative and is described in depth later. However, some additional dependent variables were used to analyze mathematics literacy: analytic scale for problem solving scores, component scores of the analytic scale for problem solving, and visualization rubric scores. The analytic scale was used to determine a wholistic view of three components: student understanding, solution, and answer for each problem. The analytic scale scores ranged from 0-10. Individual component scores of the analytic scale range from 0-4 for understanding, 0-4 for solution, and 0-2 for answer. A rubric was used to score students' visual representations as an aspect of their mathematics. Visualization scores range from 1-5. Table 6 shows the descriptive statistics for the dependent variables involving mathematics literacy.

Table 6.
Descriptive Statistics for Mathematics Literacy

	Group			
	Problem Solving		Metacognitive Training	
	M	*SD*	*M*	*SD*
Pretest				
Analytic Scale	4.81	2.42	3.97	2.88
Understanding	2.21	1.06	1.74	1.18
Solution	2.07	0.95	1.68	1.14
Answer	0.69	0.54	0.56	0.61
Visualizations	2.68	0.96	2.28	0.82
Posttest				
Analytic Scale	6.87	3.02	5.66	3.08
Understanding	2.67	1.34	2.49	1.19
Solution	2.63	1.35	2.26	1.12
Answer	1.12	0.76	0.84	0.59
Visualizations	2.95	1.08	2.42	0.90

Assumption Checking

Univariate and exploratory descriptive statistics were used to check for underlying assumptions of an analysis of covariance (ANCOVA). Groups were independent samples, meaning that no participant was in both groups. Shirpiro-Wilk tests were used to determine normality of the dependent variables by group. The dependent variables prediction accuracy and achievement scores were normally distributed. The dependent variable, postdiction accuracy, was not normally distributed, however, ANCOVA is robust to violations of normality. Through visual inspection of scatterplots, a linear relationship between the covariates and the corresponding dependent variables, by group, appeared to be linear. Homogeneity of regression

slopes was significant for achievement scores by group, suggesting that the relationship between the pretest achievement scores and posttest achievement scores were not consistent between both groups. Homogeneity of regression slopes was not significant for prediction or postdiction accuracy, suggesting consistency between groups. Levene's test for homogeneity of variance between groups was significant for all three dependent variables. Three ANCOVAs were performed to determined differences between groups across the three dependent variables: achievement, prediction accuracy, and postdiction accuracy.

Achievement

There were no significant pretest differences between the groups before the intervention for achievement, $F(1, 30) = 1.00$, $p > 0.05$. The data was analyzed to determine how the metacognitive training influenced the achievement scores of lower achieving Algebra students. To determine students' achievement scores, all six word problems from the VisA instrument were scored as a 1 for a correct answer and 0 for incorrect answer. A sum score was computed (0-6) for each student for the total number of correct answers (actual score) to determine students' achievement score.

Using the data from Table 5, an ANCOVA was performed to determine if there was a statistically significant difference between the intervention and comparison groups in terms of achievement, while controlling for their pretest scores. Not surprisingly, the results revealed that the covariate, pretest scores, were significantly related to students posttest achievement scores, $F(1, 30) = 14.33$, $p < 0.00$, partial $\eta^2 = 0.32$. If a student had a high score on the pretest, they were more likely to score high on the posttest, in turn, if a student scored low on the pretest, they were more likely to score low on the posttest. There was no significant effect of achievement scores between groups, after controlling for the effect of pretest scores, $F(1, 30) = 0.86$, $p > 0.05$,

partial $\eta^2 = 0.03$. This means there was no significant differences between the groups on the posttest when considering their pretest scores. Interestingly, both the intervention and comparison groups improved in achievement scores from pretest to posttest (see Figure 7).
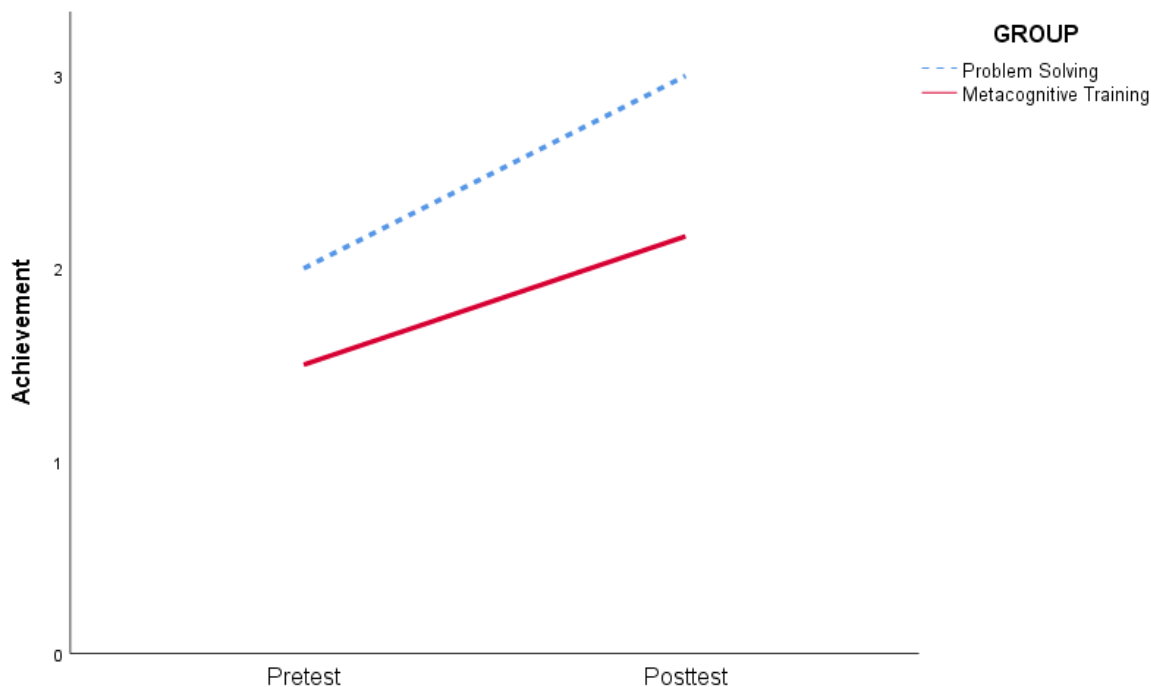

Figure 7. Average pretest and posttest achievement scores by group.

Calibration Accuracy

The data was analyzed to determine how metacognitive training influenced the calibration accuracy of lower achieving Algebra students. I organized the results by prediction accuracy, postdiction accuracy, and bias. The participants' prediction calibration accuracy was computed by calculating the absolute value of the difference between their total prediction scores and their total actual scores (number correct) on the tests. The participants' postdiction calibration accuracy was computed by calculating the absolute value of the difference between their total postdiction scores and their total actual scores (number correct) on the tests.

Calibration bias was calculated as the signed difference between participants' predictions or postdictions and their actual scores (Schraw, 2009).

  Prediction Accuracy. There were no significant pretest differences between the groups before the intervention prediction accuracy, $F(1, 30) = 0.73$, $p > 0.05$. Using the data from Table 5, another ANCOVA was performed to determine if there was a statistically significant difference between the intervention and comparison groups in terms of prediction accuracy, while controlling for their pretest predictions. There was a significant difference in students' prediction accuracy between groups, after controlling for the effect of pretest differences, $F(1, 29) = 6.49$, $p < 0.05$, partial $\eta^2 = 0.18$. The intervention group was significantly more accurate in their predictions on the posttest (see Figure 8). It must be noted that lower scores mean better calibration accuracy.
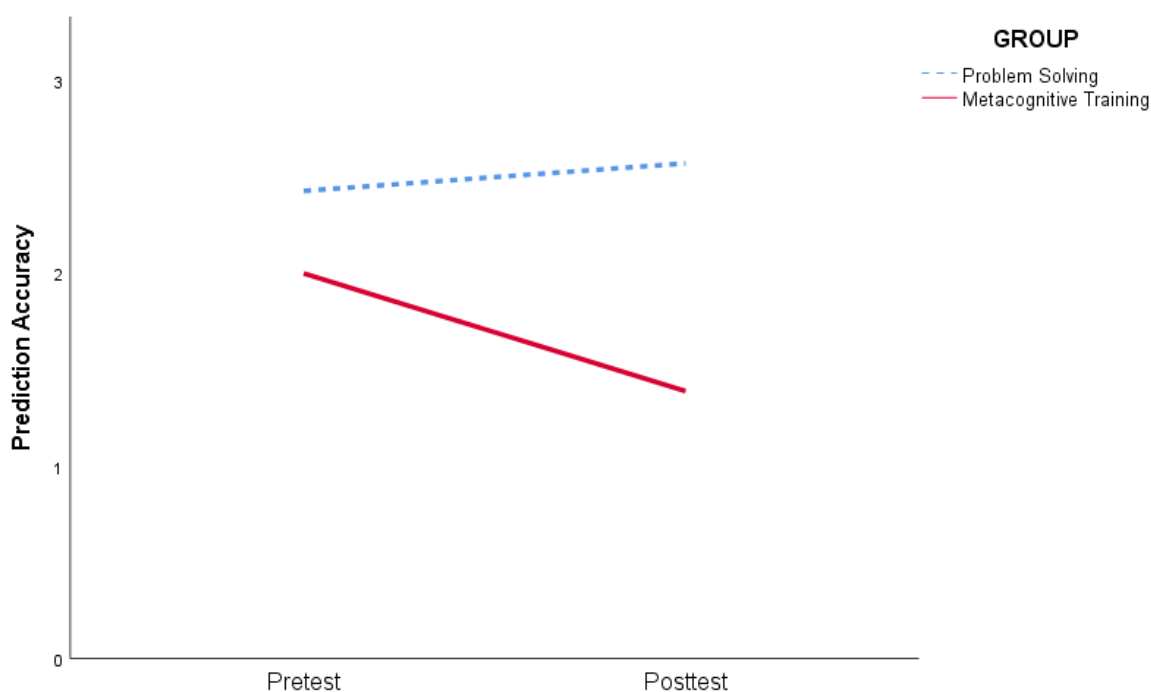
Figure 8. The effect of group on participants' prediction accuracy. Note: Lower scores mean better calibration accuracy.

Postdiction Accuracy. There were no significant pretest differences between the groups before the intervention for postdiction accuracy, $F(1, 30) = 0.81$, $p > 0.05$. Using the data from Table 5, a third ANCOVA was performed to determine if there was a statistically significant difference between the intervention and comparison groups in terms of postdiction accuracy, while controlling for their pretest postdictions. There was no significant difference in students' postdiction accuracy between groups, after controlling for the effect of pretest differences, $F(1, 27) = 6.49$, $p > 0.05$, partial $\eta^2 = 0.09$. The intervention group was more accurate in their postdictions than the comparison group, and became even more accurate in their postdictions after the intervention. The comparison groups' postdiction accuracy remained exactly the same from pretest to posttest (see Figure 9).
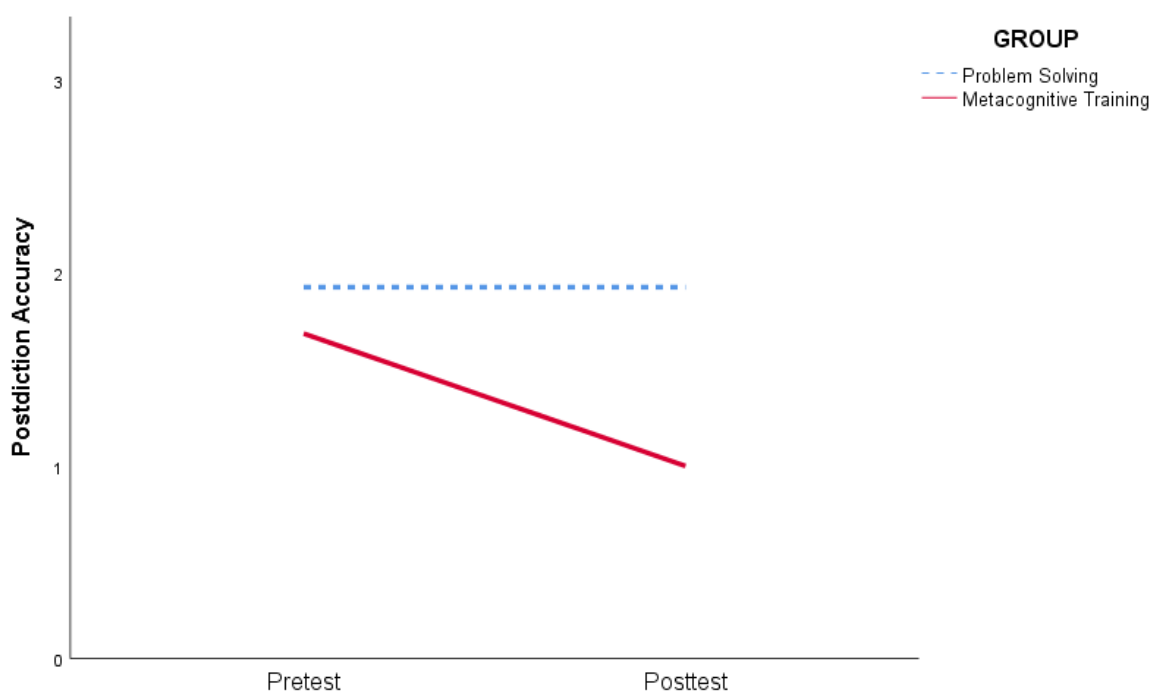
Figure 9. The effect of group on participants' postdiction accuracy. Note: Lower scores mean better calibration accuracy.

Calibration Bias. Another way to look at calibration accuracy is through bias scores. As was described in the method section, bias scores were derived from participants' prediction and postdiction accuracy. Participants that make judgments that are higher than their actual scores are overconfident and participants that make judgements that are lower than their actual scores are underconfident. On average, students in the comparison group were underconfident in their predictions ($M = -1.00$, $SD = 0.77$) and students in the intervention group were overconfident in their predictions ($M = 0.88$, $SD = 0.34$). Students in the comparison group were more accurate in their postdictions ($M = -0.50$, $SD = 0.72$) than the students in the intervention group ($M = 0.75$, $SD = 0.31$). Figure 10 provides a visual representation of students' bias scores on the posttest.
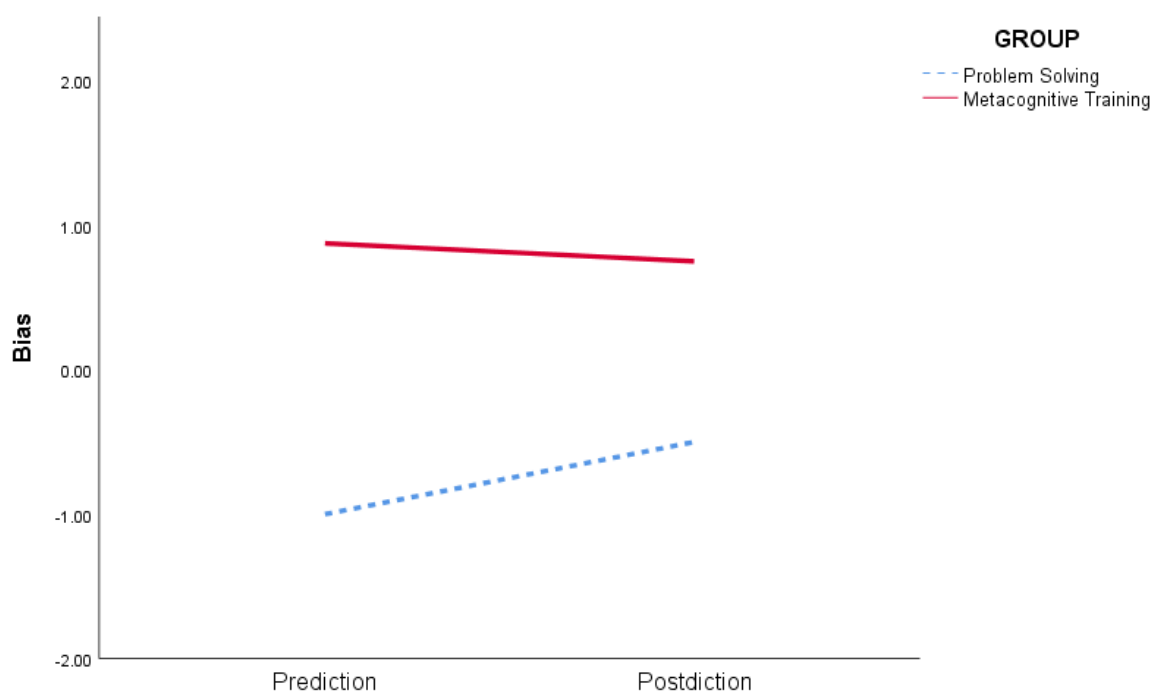

Figure 10. Students' prediction and postdiction bias scores on the posttest. Note: Scores closer to zero mean better calibration accuracy.

Mathematics Literacy

Both quantitative and qualitative analyses were used to determine how metacognitive training influenced lower-achieving Algebra students development of mathematics literacy. Mathematics literacy is typically assessed as a spectrum of students' ability. Multiple aspects of data analyses were conducted to determine students' development of mathematics literacy. Data from the analytic scale for problem-solving, components of the analytic scale, visualizations, informal conversations, and classroom observations were analyzed to look deeply at students' development of mathematics literacy.

Analytic Scale for Problem Solving. Students' problems were scored using Wilson's (1991) version of the Analytic Scale for Problem Solving. The analytic scale was used to determine a wholistic view of three components: student understanding, solution, and answer for each problem. Descriptive statistics for the analytic scale are available in Table 6. An intraclass correlation coefficient (ICC) was calculated to confirm interrater reliability. The ICC for the pretest and posttest were .94 and .88, respectively, which is considered to be excellent reliability (Cicchetti, 1994). An ANCOVA was performed to determine if there was a statistically significant difference between the intervention and comparison groups in terms of students' analytic problem-solving scores, while controlling for their pretest analytic scores. There was no significant difference in students' analytic scores between groups, after controlling for the effect of pretest differences, $F(1, 30) = 0.14$, $p > 0.05$, partial $\eta^2 = 0.34$. The intervention group scored lower than the comparison group on both the pretest and the posttest. Both groups, however, improved their scores on the analytic scale after the intervention, overall; 81% of the students improved on the analytic scale (see Figure 11). This finding suggests a wholistic view of

students understanding, solution, and answer to the problems improved in both groups from pretest to posttest; these are key features of mathematics literacy.
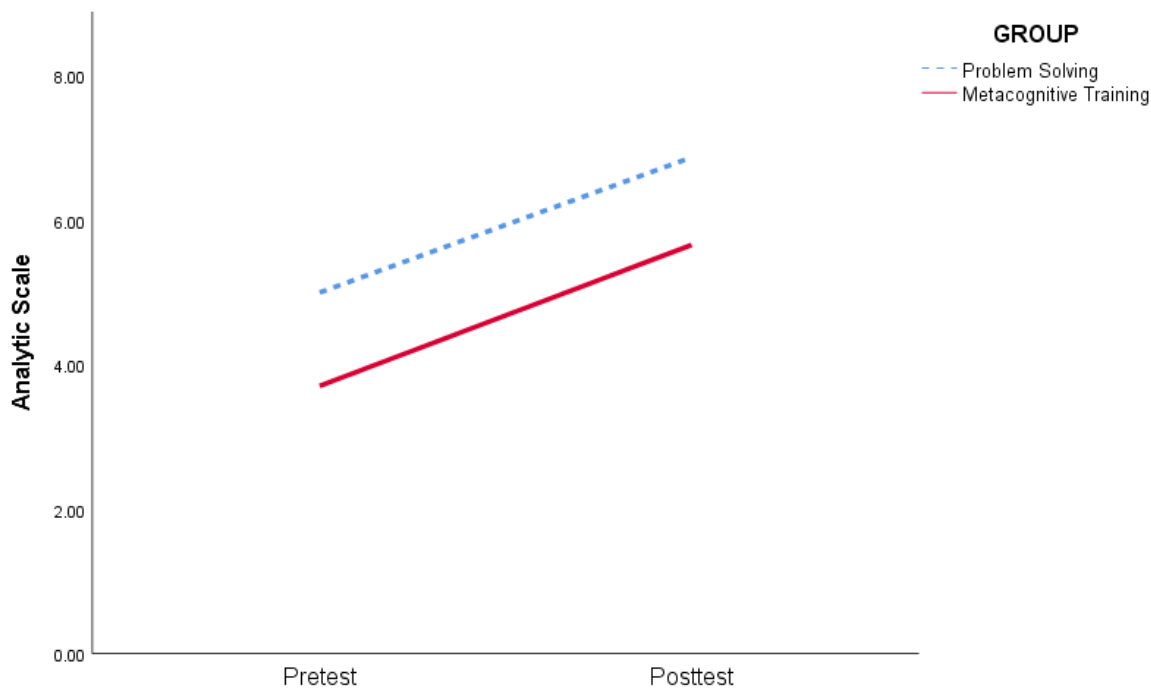
Figure 11. Students' average scores on the Analytic Scale for Problem Solving from pretest to posttest by group.

The following example demonstrate Ken's (pseudonym) understanding, solution, and answer to corresponding problems before and after the intervention. In Figure 12, it is noticeable on the pretest that Ken chose not to, or was not capable of, creating a visual representation of the problem, which would demonstrate some understanding. Ken did not offer a solution procedure or rationale for his answer. In addition, Ken's answer was incorrect, was not labeled, nor was it circled, as prescribed in the question. By simply presenting only a number answer suggests Ken guessed the answer, assuming, that is, that the 16 noted on the paper was a solution and not the Ken's work.

Problem

Kim has 2 sweatshirts, 4 sweatpants, and 3 headbands. How many outfits could she make to wear jogging?

Question

Draw a sketch you can use to solve the problem.

Problem

Kim has 2 sweatshirts, 4 sweatpants, and 3 headbands. How many outfits could she make to wear jogging?

Question

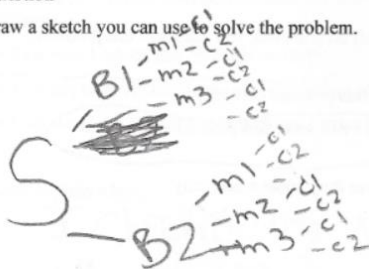Solve the problem and circle the answer.

16

Figure 12. An example of Ken's response to question number six on the pretest.

Problem

You can have a sandwich with your choice of 2 different kinds of cheeses, two different kinds of breads, and 3 different kinds of meats. How many choices do you have?

Question

Draw a sketch you can use to solve the problem.

Problem

You can have a sandwich with your choice of 2 different kinds of cheeses, two different kinds of breads, and 3 different kinds of meats. How many choices do you have?

Question

Solve the problem and circle the answer.

12

Figure 13. An example of Ken's response to a similar question on the posttest.

On Ken's posttest, however, he demonstrated understanding of the problem through the visualization he created (see Figure 13). His visualization made the words to the problem more concrete, provided him a visual representation of the problem, showed his problem-solving process, and anchored a solution. The visualization also provided evidence of how he solved the problem and how his appropriate plan led to an accurate solution to the problem. Although

unlabeled, Ken showed additional understanding of the question because he circled the number

12, indicating it was his final answer.

Figures 14 and 15 demonstrate another example of a students' improved understanding,

solution, and answer to similar problems before and after the intervention. On the pretest, Cay's

(pseudonym) sketch is not an accurate representation of the problem, suggesting she did not fully

understand the problem. Cay did not represent that the path was 27 meters long, nor that the

bushes were three meters apart. Cay calculated that 3 x 9 = 27, perhaps signifying that there

should be nine bushes, but the sketch does not represent this; there are only eight bushes, four on

each side of the path. Cay does acknowledge that there needs to be an even number of bushes,

so instead of nine bushes she concluded eight bushes would be sufficient. Cay misinterpreted a

major part of the problem, and her solution was inappropriate. Although her answer was

incorrect based upon an inappropriate plan, she did circle and label her answer (see Figure 14).

Problem

Marja plants rosebushes alongside the path to her house. The path is 27 meters long. She plants a rosebush on both sides of the path at the beginning and then plants a rosebush on both sides of the path every 3 meters apart. How many rosebushes does Marja need?

Question

Draw a sketch you can use to solve the problem.
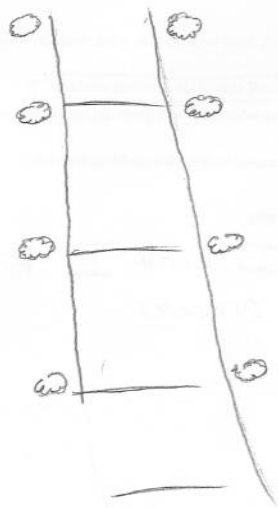
Problem

Marja plants rosebushes alongside the path to her house. The path is 27 meters long. She plants a rosebush on both sides of the path at the beginning and then plants a rosebush on both sides of the path every 3 meters apart. How many rosebushes does Marja need?

Question

Solve the problem and circle the answer.

$27 = 3 \times 9$ ...but then it has to be an even #, so 8 rose bushes.

Figure 14. An example of Cay's response to a question on the pretest.

On the posttest, however, Cay demonstrated complete understanding of the problem through their visual representation. Notice how she sketched the fence as 35 ft. long with each section of the fence 5 ft. wide (see Figure 15). Not only did she specify that there were fence posts on each end of the fence, but she even sketched fence posts every 5 feet apart. Cay showed her problem-solving procedure by numbering and totaling the fence posts. Her answer was correct, labeled, and circled, suggesting a complete and accurate understanding, solution, and answer for the problem.
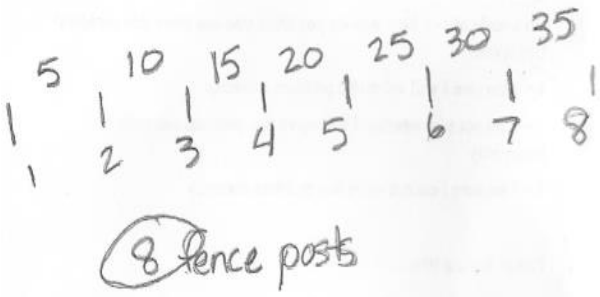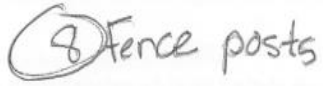
Figure 15. An example of Cay's response to a similar question on the posttest.

Components of Analytic Scale. The analytic scale provided a wholistic view of students understanding, solution, and answer while solving word problems. Students' scores on the analytic scale were further reviewed to verify the level of improvement at which understanding of the context of the problem, the solution procedure, and the answer requirements were evident between the pretest and posttest with similar problems. Descriptive statistics for each component of the analytic scale were calculated to provide additional diagnostic information, more details about students' strengths and weaknesses, and specific determination about the effectiveness of

the intervention for improving students' mathematics literacy. Table 6 shows the descriptive

statistics for each component of the analytic scale.

   Understanding. The intervention group scored lower than the comparison group on both

the pretest and the posttest for the component of understanding the problem. Both groups,

however, improved their understanding after the intervention (see Figure 16). It is important to

note that the intervention group made greater gains in understanding after the intervention than

the comparison group, an increase of 0.75 and 0.46, respectively. This is a valuable finding

because understanding the problem is important for developing a solution and correctly

answering the problem (Stylianou, 2002) and is a key aspect for developing one's mathematics

literacy.

Figure 16. Students' understanding scores on the Analytic Scale for Problem Solving from
pretest to posttest by group.

Solution. The intervention group scored lower than the comparison group on both the pretest and the posttest for the component of solving the problem (see Figure 17). Both groups, however, improved their solutions after the intervention by almost the same amount (0.56 and 0.58).
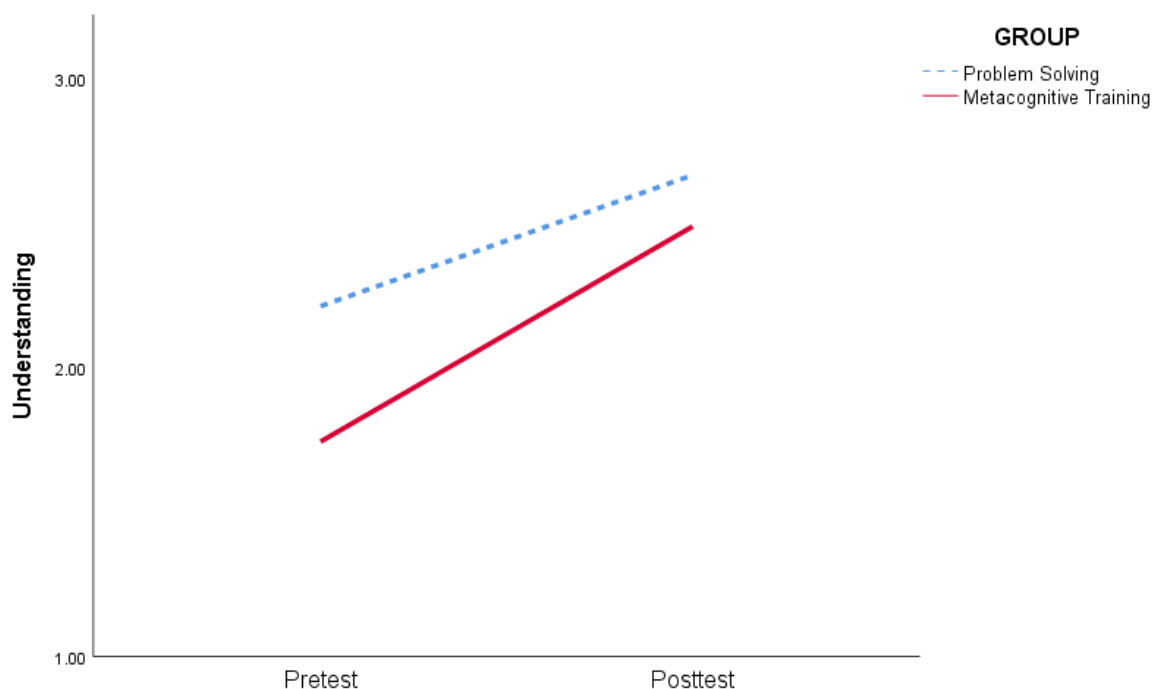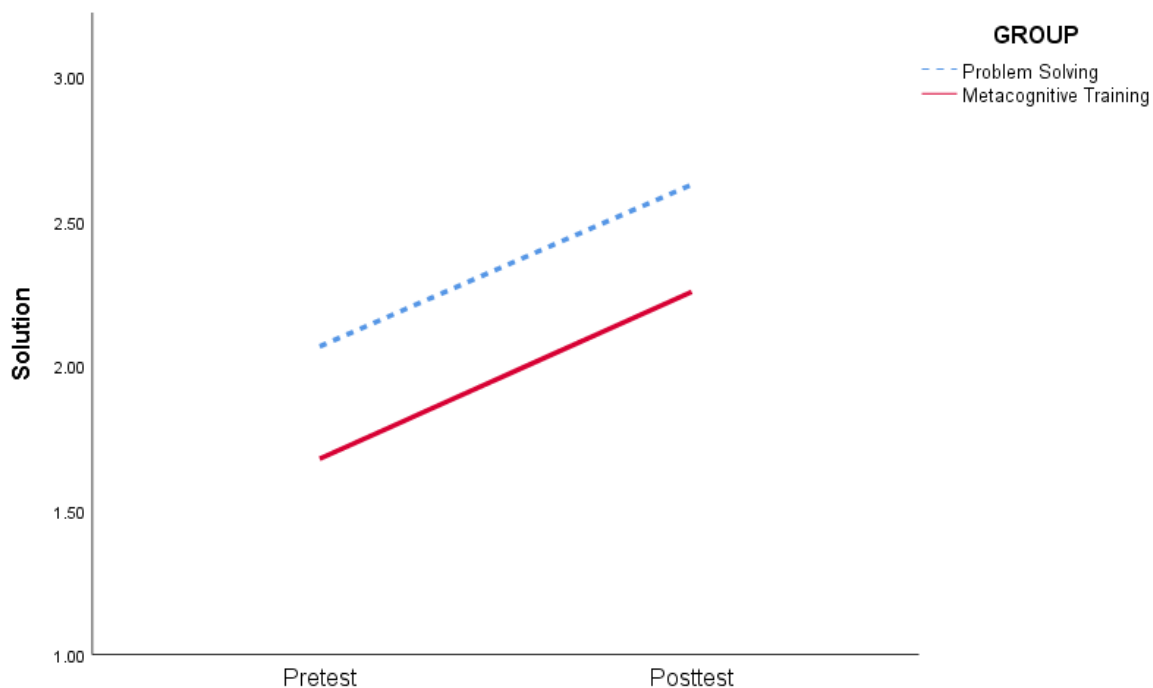

Figure 17. Students' solution scores on the Analytic Scale for Problem Solving from pretest to posttest by group.

Answer. The intervention group scored lower than the comparison group on both the

pretest and the posttest for the component of answering the problem (see Figure 18). Both

groups improved their answers after the intervention with the comparison group making greater

gains. Students in the intervention group showed little improvement in the component of

answering the problem, an increase of only 0.28.
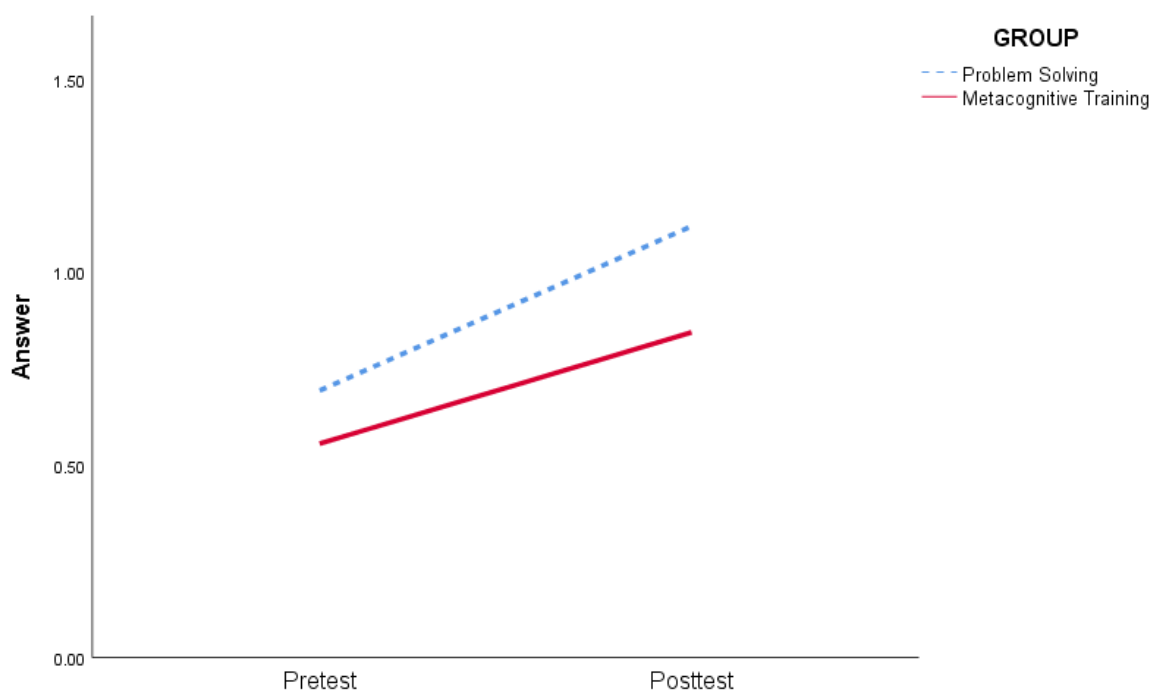
Figure 18. Students' answer scores on the Analytic Scale for Problem Solving from pretest to
posttest by group.

A comparison of students' understanding, solutions, and answers showed that students in

the comparison group scored higher than the intervention group on all components of the

analytic scale (understanding, solution, answer) on the pretest (see Figure 19), suggesting

students in the comparison group may have been more developed in their mathematics literacy

from the onset of the study. It is also noticeable, as shown in Table 6, that students in both the

intervention and comparison groups improved on all components of the analytic scale from

pretest to posttest. It is important to note, however, that the metacognitive training group made

greater gains in their understanding of the problem, improved about the same in their solutions,

and showed less gain in their answers than the problem-solving strategy group.
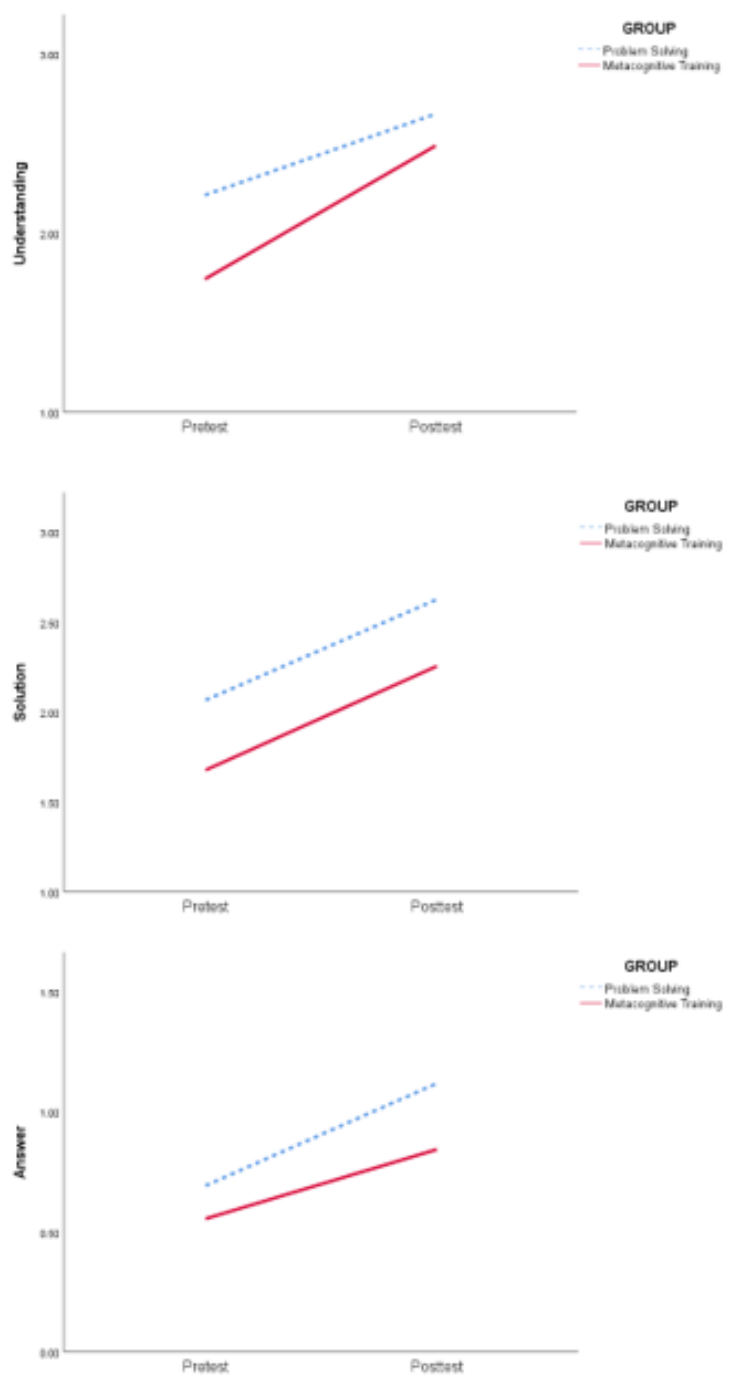
Figure 19. Comparison of the three components of the Analytic Scale for Problem Solving from pretest to posttest by group.

Visualizations. A five-level rubric was used to score students' visual representations as an aspect of their mathematics literacy. An intraclass correlation coefficient (ICC) was calculated to confirm interrater reliability. Descriptive statistics for students' visualization scores are available in Table 6. The ICC for the pretest and posttest were .82 and .80, respectively, which is considered to be good reliability (Cicchetti, 1994). One last ANCOVA was performed to determine if there was a statistically significant difference between the intervention and comparison groups in terms of students' visualizations, while controlling for their pretest answer scores. There was no significant difference in students' visualizations between groups, after controlling for the effect of pretest differences, $F(1, 30) = 0.32$, $p > 0.05$, partial $\eta^2 = 0.40$. The intervention group scored lower than the comparison group on both the pretest and the posttest. Both groups, however, improved their visualizations after the intervention (see Figure 20).
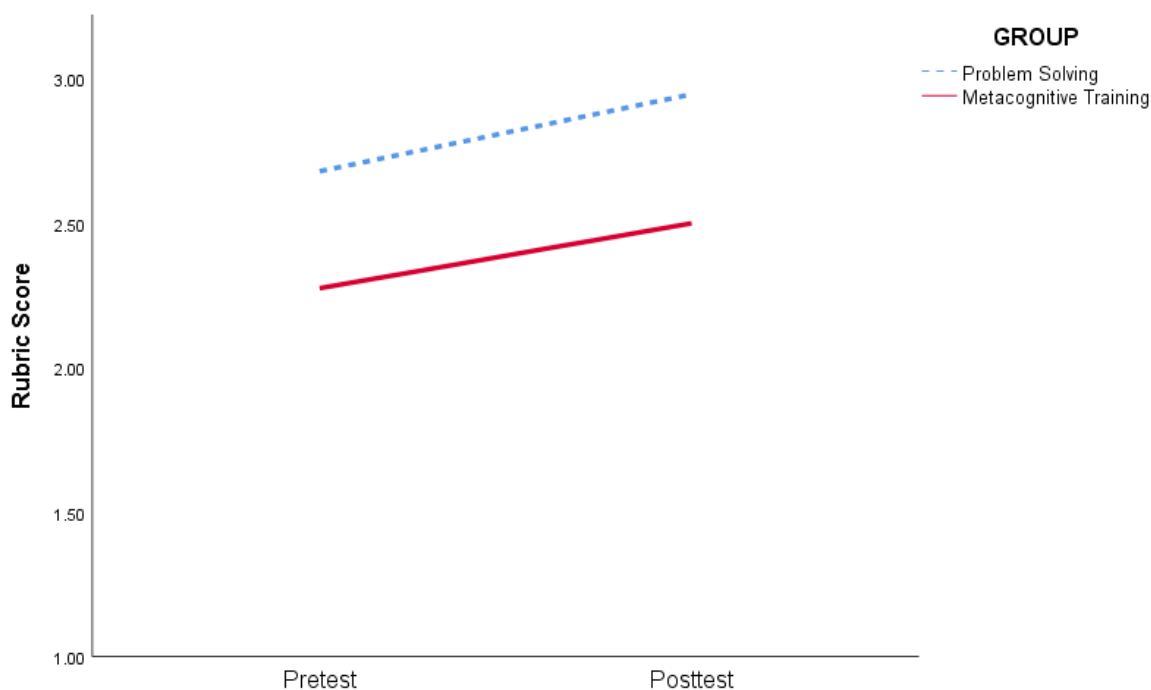
Figure 20. Students' average visual representations score by group using a five-level rubric.

From pretest to posttest, 69% of the students in the intervention group improved on the visualization scale, and 60% of the students in the comparison group improved on this scale. More specifically, the intervention group improved from an average score of 2.25 on the pretest, to an average score of 2.42 on the posttest. The comparison group also improved from pretest to posttest, 2.68 to 2.95, respectively. Student improvement in creating accurate visual representations of math problems is important for understanding the problem and demonstrates development of mathematics literacy.

Classroom Observations. Classroom observations were conducted by the researcher for the full class period, three to four days a week, for the entire duration of the study. The researcher was an active participant in the classes and engaged in informal conversations with the teacher. Through casual conversation, towards the beginning of the intervention, the teacher emphasized multiple times that his students often misbehaved and may not be cooperative with the study. He was also highly concerned that his students would not be able to successfully understand and complete the word problems, because his students had a history of struggling with mathematics content, specifically word problems. Approximately a week later, the cooperating teacher did not express any concerns regarding students' behaviors and expressed little concern about a few students struggling with the content. After the intervention ended, the teacher appeared 'pleasantly surprised' that the students' behaviors were appropriate and compliant during the warm up activities. The teacher did not express any concerns regarding students' behavior, participation, or understanding of the content.

The researcher also engaged in informal, casual conversations and interactions with the students. The researcher-student conversations and interactions allowed the researcher to build rapport with the students and establish a safe learning environment. During classwork time, the

researcher noticed that one individual student would constantly come to them for assistance with classwork problems, often showing no attempt at solving the problem. The researcher encouraged him to try to solve the problem on his own, and then when he was unsure what to do next to return to the researcher for some guidance. Over the next week, the number of times the student sought the researcher's help did not reduce, however, when the student did seek assistance, he had already attempted or completed the problem successfully. The researcher spoke with the student privately, praising him for successfully attempting and solving the problems on his own, further suggesting that he needs to have more confidence in himself and his math skills. The student responded, "I only ask for help when I don't know what to do." The researcher corrected this misconception demonstrating to him that most of his answers were correct, and that he does not need affirmation from others, suggesting he maintain confidence in his answers. The researcher suggested that he save his help-seeking behavior for when he really does not know what to do on a problem. The student responded by saying that he "just likes to know if I'm doing it right."

Another noteworthy conversation between a student and the researcher involved the posttest. When the researcher arrived to pick the posttest up from the cooperating teacher, a participating student stated "The questions were easier this time. I just knew what to do." This student, exposed to the metacognitive training, did not get any problems correct on the pretest, but correctly answered four out of the six of the questions on the posttest, demonstrating the usefulness of metacognitive questioning, and his development in mathematical literacy.

Classroom observations, casual conversations, researcher-student interactions, students' work, and the researchers' journal, containing field notes, memos, and reflections, revealed two

broad themes in regard to mathematics literacy: students' vocabulary knowledge and problem-solving strategy use.

Vocabulary knowledge. Since the students were engaged in word problems, there were a significant number of words in each problem for students to understand and interpret. Observations of the warm-up activities revealed that many students expressed difficulties with content specific vocabulary words and formulas. For example, the researcher observed multiple students asking the teacher:

"What are kilometers?"

"What do they mean square meters?"

"What is a vertices?"

"What do they mean by three times as many?"

"What is a triangular lot?"

In addition, another warm up activity question involved three people in a race and required students to determine "How many different ways they could finish." Students did not understand the question as being a combination problem. During other warm-up activities, the students often asked the teacher to provide them with mathematical formulas, such as the area of a square, area of a circle, the Pythagorean Theorem, and the area and perimeter of a rectangle. One student even said to the researcher, "Not knowing the formulas made the problem difficult." It was apparent that students struggled with content specific vocabulary terms and formulas.

Problem-solving strategies. Students individual work on the warm-up activities was followed by a class discussion and example from the teacher. Observation of the class discussions revealed that students used a variety of strategies to complete their warm-up activities. Many students shared or demonstrated their problem-solving strategy to the class.

Most students created a visualization, as requested by the researcher, to demonstrate their

understanding and to solve the problem (see Figure 21). The question was:

There are some bicycles and tricycles at the playground. There are 7 seats and 19 wheels.
How many bicycles and tricycles are there?
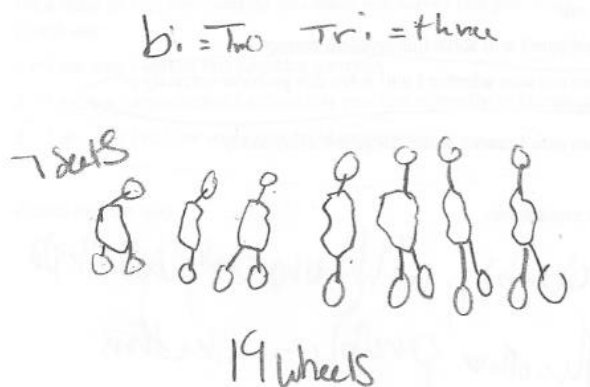
Figure 21. Example of Jon's work when using a visual representation to solve the problem.

The student whose work is in Figure 20, Jon, shared his problem-solving strategy with

the class. He said he knew that a bicycle had two wheels and a tricycle has three wheels. He

also knew that there was a total of seven seats. To begin with, he decided to draw the seven seats

and then make the seats into bicycles and count the wheels, which totaled 14. Jon then decided

to add one wheel to each bicycle, making it a tricycle, until he had a total of 19 wheels. Lastly,

he counted how many bicycles and tricycles he created, and accurately answered the question on

a different page stating, "two bi five tri."

A couple of students, however, were particularly fond of solving the problems

algebraically rather than creating a visualization (see Figure 22). It is obvious that Libby had a

complete understanding of the problem, as demonstrated through her accuracy in setting up the

problem algebraically. In the first equation, x represented roses which were $2.00 each, and y

represented carnations which were $0.75 each, and the bouquet totaled $20.50. A florist was

putting together a bouquet with a total of 14 flowers, second equation. Students were asked to

determine how many of each flower was needed for the bouquet. Libby admitted she entered the

two formulas into her calculator to determine the solution. The question was:

The florist advertises roses for $2.00 each and carnations for $0.75 each. If John pays $20.50
excluding tax, for a bouquet of 14 flower for Teresa, how many of each flower is in the bouquet?

Figure 22. Example of Libby's work when solving the problem algebraically.

Other students preferred to "make a chart and look for a pattern" (see Figure 23). Kevin

informed the class about his table for the bicycle and tricycle problem mentioned earlier. He

said he selected two numbers that totaled seven the total number of seats between the two types

of bikes (he inaccurately labeled this "wheels" on the chart). Kevin then multiplied the number

of seats by how many wheels and summed the wheels together to get a total. He noticed as the

number of tricycles increased the number of total wheels also increased; resulting in an accurate

solution.

Figure 23. Example of Kevin's work when solving the problem by making a chart and looking for a pattern.

Many students reported using "random guess and check" to solve the word problems (see Figure 24). In this example, Asia was solving the flower bouquet problem mentioned earlier. She knew there was a total of 14 flowers in the bouquet, so she started with seven of each flower and calculated the total ($19.25), which was not enough. She randomly picked 10 roses and 4 carnations (total 14) and calculated the total for each flower. Noticing that they totaled $20.00 with the roses, leaving only $0.50 left to buy carnations, which totaled $3.00, she decided to select another combination of 14. Asia then selected eight roses and six carnations, calculated the costs, and concluded it was the correct combination of flowers.

Figure 24. Example of Asia's solution using guess and check method.

Students used other interesting strategies to demonstrate their understanding of the problem. Some students used an organized list to keep their work manageable during a difficult problem. The student's work in Figure 25 shows their calculations on the right and a visual representation of the process (building a wall) on the left. It is interesting that this student was the only student that built the wall from bottom to top.

Figure 25. Example of student work when solving the problem by making an organized list.

Another student created a very concrete visualization of the problem (see Figure 26). The student created 60 markers, put a circle around 30 of them for elimination, and circled an additional five for elimination. The remaining markers, not circled, was the answer to the word problem. The question was:

A hitchhiker set out on a journey of 60 miles. He used a map to calculate the distance. He walked the first 5 miles and then got a lift from a taxi driver. When the taxi driver dropped him off he still had half of his journey to travel. How far had he traveled in the taxi?
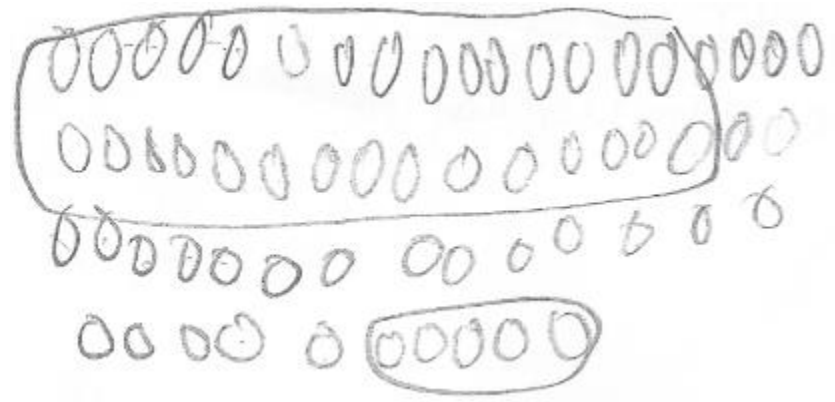
Figure 26. Example of student work when solving the problem by using a concrete representation.

In the last example (see Figure 27), a student used a simple number sentence to solve the word problem. Using a calculator, they individually added the cost of one apple or one orange until they reached the total cost they were looking for. It should be noted, however, that this student used apple and oranges and the problem was asking for oranges and bananas. The question was:

A grocer mixes oranges and bananas to make a 10-pound fruit basket. The oranges cost $0.75 per pound and the bananas cost $0.60 cents per pound. How many pounds of each should he use if the basket is to cost $6.90?
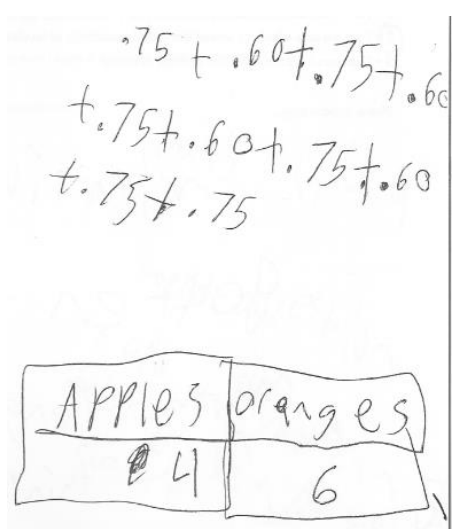
Figure 27. Example of student work when solving the problem by using a simple number sentence.

Classroom discussions of the warm up activities provided the researcher deeper insight to the students' vocabulary knowledge and problem-solving strategy usage. Observations of these discussions revealed limitations in vocabulary knowledge, specifically for content specific terms and formulas. The broad spectrum of visualizations were representative of the numerous available strategies that can be utilized by students for problem-solving, providing a glimpse into their development of mathematics literacy.

Summary

In Chapter 4, I reported the findings of how metacognitive training during Algebra warm-up activities influenced lower achieving students' calibration accuracy, achievement, and development of mathematics literacy. The results revealed there was no significant difference in achievement scores between groups, and in fact both the intervention and comparison groups improved in achievement scores from pretest to posttest. Regarding calibration accuracy, the intervention group was more accurate than the comparison group in their predictions and postdictions. Overall, students in the intervention group were overconfident in their calibrations, while students in the comparison group underconfident in their calibrations.

Students' data from the intervention group provided deeper insight into their development of mathematics literacy. An in-depth analysis of students' understanding, solutions, and answers of the mathematical word problems from pretest to posttest revealed that students that were exposed to the metacognitive training showed improvement in all analytic components of problem solving. The results were triangulated using analysis of students' visualizations of the math problems and classroom observations. The results suggest that explicitly teaching students Polya's problem-solving process with metacognitive questioning was an effective strategy for developing their mathematical literacy. In Chapter 5 I elaborate on these

results by providing a discussion by research question, important conclusions drawn from findings, limitations of the study, and recommendations for further research.

CHAPTER 5

Discussion

Chapter 5 contains a discussion of the results, research, and major findings for each question. In addition, discussions of implications for practice followed by limitations of the study are included. The conclusion of the chapter contains recommendations for further research.

Metacognitive Training and Achievement

It was hypothesized that metacognitive training with the problem-solving strategy could enhance students' mathematics achievement. Quantitative analyses showed that students who were exposed to the problem-solving strategy with metacognitive training did not score higher on the achievement test than the comparison group. It is possible that the experimental manipulation, metacognitive questions, was not powerful enough to augment the problem-solving strategy. Students in the intervention group may not have applied the metacognitive questions when they were solving the problem because they were too engaged in the problem-solving process itself. Perhaps, teaching all of the students the problem-solving strategy first, then implementing metacognitive questions with the intervention group, would have revealed different results.

This study is not the only one to show no differences in students' achievement scores. Labuhn and colleagues (2010) discovered similar results when exploring the effects of metacognitive feedback and standards on students' problem-solving scores. They found that extensive metacognitive training with middle school students showed no significant differences in mathematics achievement between groups for metacognitive standards or feedback. Likewise, Huff and Nietfeld (2009) found that their intervention may have focused only on monitoring but

not control of learning, resulting in increased calibration accuracy but not performance. In this study it is possible that students who received metacognitive training may have improved their monitoring abilities, but they did not improve their content knowledge or problem-solving abilities.

The findings of the current study contrast with the research that is available, which suggests a relationship between metacognitive training and mathematics achievement (Cleary et al., 2017; DiGiacomo and Chen, 2016; Kramarski & Mizrachi, 2006; Montague et al., 2014; Pennequin et al., 2010). For example, Pennequin et al. (2010) investigated the effects of metacognitive knowledge and skills training on students problem-solving performance. Students were taught to create representations of the problem, develop problem solving strategies, identify key words for interpretation, identify mathematical expressions, and apply the metacognitive knowledge and skills individually. The researchers found that metacognitive training improved students' mathematical word problem-solving performance. Likewise, DiGiacomo and Chen (2016) also investigated the effect of metacognitive training on mathematics performance. Students in the intervention group were taught SRL strategies, were provided feedback about their performance, and completed a worksheet designed to elicit self-reflection. Students in the comparison group used a computer program that was part of their math curriculum. The researchers found that the treatment group, exposed to metacognitive training, had significantly higher math performance than the control group. Although there are mixed research results regarding metacognitive training and achievement, this study contributes to the literature and may offer deeper insight about effective metacognitive strategies that improve student mathematics performance.

Although there were no significant differences between groups on achievement, overall, the problem-solving strategy instruction itself was effective for improving students' achievement, with or without the metacognitive questions, which is supported by previous research (Krawec et al., 2013; Montague et al., 2014; Polya, 1945; Xin et al., 2005). Both groups improved about the same amount in academic performance from pretest to posttest. One explanation for the improvement in both groups is that students were monitoring their learning by engaging in metacognitive processes simply by participating in the five-step problem-solving process. When students 'check their work,' they are using metacognitive processes to review their solution by determining if their computations were accurate, if their visualization is a good representation of the problem, or if their answer was reasonable. Therefore, *all* the students may have been engaging in the metacognitive questioning, either implicitly or explicitly, through the problem-solving process. Research has found students' achievement to improve after engaging in metacognitive processes during problem-solving activities (Cleary et al., 2017; Krawec et al., 2013; Montague et al., 2014; Xin et al., 2005). For example, Cleary et al. (2017) found that mathematics students' achievement scores improved over time for a group of students that were exposed to metacognitive questions and reflections. Likewise, Montague and colleagues (2014) found that specialized instruction involving math word problem solving was effective for improving students' performance, particularly among lower achieving students. If students naturally engage in metacognitive questions when problem-solving, then diffusion of treatment is a concern. Future research could investigate which metacognitive questions and strategies students naturally engage in when problem-solving in mathematics classrooms.

Students mathematics achievement may have improved by engaging in the problem-solving solving process because students were taught a plan (PSS) and a strategy for solving

mathematics word problems. Schoenfeld (1985) referred to a plan as control and strategies as heuristics. He proposed that both control and heuristics were necessary for successful problem solving. Additionally, planning for the task using the problem-solving steps may have provided students "a more complete mental representation of the task" (Hacker & Bol, 2018, p.15). It is possible when lower achieving students encounter a word problem, they begin to solve the problem without developing a plan of action. The five problem-solving steps all the students used required them to think about their problem-solving process, engaging them in thinking about their learning. Asking students to identify important information, draw a representation of the problem, and check their work, required them to make connections between words, symbols, and visuals. For example, when students identify important information in a word problem, they are engaging in a metacognitive process to access and connect their prior knowledge, as well as understand and distinguish between relevant and irrelevant information (Cleary & Kitsantas, 2017; Schoenfeld, 1985). They may have even read the problem multiple times, a metacognitive strategy, to ensure understanding of the problem, information, and question. Creating a visual representation of a word problem requires students to think deeply about their understanding of the problem to appropriately link the important information and the unknown visually. Previous research has emphasized the importance of visual representations of math problems, because they provide examples of how math concepts are applied (Dexter & Hughes, 2011; Edens & Potter, 2007; Montague et al., 2014; Polya, 1945; Schoenfeld, 1985) and facilitate student comprehension (Krawec, 2014). In fact, Krawec (2014) concluded that visual representations of math word problems was critical for accurate problems-solving, especially for lower achieving students, a claim that is supported by the data found in the present study.

Metacognitive Training and Calibration Accuracy

It was hypothesized that students that were exposed to metacognitive training with the problem-solving strategy would be better calibrated than students exposed to the problem-solving strategy without metacognitive training. Calibration accuracy was measured by quantitative analyses of students' predictions, postdictions, and bias. Predictions are based, in part, on students' judgments (confidence) of their abilities, knowledge, and skills. Faulty assessments of confidence lead to inaccurate predictions of performance (Glenberg & Epstein, 1985). In turn, sound assessments of confidence lead to accurate predictions of performance.

Postdictions, on the other hand, are based on students' beliefs about how they performed on a test. Postdictions are usually more accurate than predictions because predictions typically have an uncertainty to them (Foster, Was, Dunlosky, & Isaacson, 2017), whereas, postdictions are grounded on students' judgments of their abilities of the task at hand (Glenberg & Epstein, 1985; Hacker & Bol, 2018). For example, completing a test provides students with additional information about the skills and content knowledge needed for mastery of the test which, in turn, should allow them to better assess what they knew against what was tested. In this study, quantitative analysis revealed significant differences between the groups in prediction accuracy, but not for postdiction accuracy. In other words, students' that were exposed to metacognitive questions made significantly more accurate predictions about their mathematics performance than the comparison group.

Lower achieving students may not take full advantage of the additional information that is provided to them from completing the test, hence, inaccurate postdictions. This is a reasonable explanation, because researchers have shown that lower achieving students lack the ability to self-regulate their own knowledge (Chiu & Klassen, 2010; Garavalia & Gredler, 2002; Hacker et al., 2008; Hawthorne, 2014; Rinne & Mazzocco, 2014). In other words, the lower achieving

students in this study were not properly assessing their knowledge of the content material or their ability to apply the necessary skills needed to master the test. Similarly, Glenberg and Epstein (1985) proposed that the knowledge subjects use in arriving at a confidence rating is imperfectly matched to the knowledge necessary to perform successfully on a test. The researchers found that their intervention was effective for assisting students in analysis of their knowledge prior to taking the test, but after taking the test, the students felt it was more difficult than they thought it would be. Similarly, Hawthorne (2014) argued that domain knowledge is needed for calibration accuracy, especially for lower achieving students. They determined, the same skills needed to achieve competence are the same skills needed to evaluate ones' competence (Hawthorne, 2014). Improving the accuracy of lower achieving mathematics students' judgments of their performance is an important focus of research because they may not be aware of their mathematical knowledge, abilities, or deficits.

Although students that were exposed to the metacognitive training were not properly assessing their knowledge of the content material or their ability to apply the necessary skills needed to master the test, they were able to accurately judge the *potential* of their knowledge. In this study, it is possible that the metacognitive training raised students' general awareness of the importance of metacognition, a key component in becoming a life-long learner. This is especially important for lower achieving students who typically struggle in assessing their abilities. The intervention groups' predictive accuracy showed that the metacognitive training may have allowed students to adequately self-assess and judge the *potential* of their knowledge concerning mathematics problem-solving. Few research studies support this claim (Hawthorne, 2014; Pennequin et al., 2010; Schneider, Castleberry, Vuk, & Stowe, 2014). For example, Pennequin and colleagues (2010) found significant differences for prediction accuracy by group,

discovering that metacognitive training closed the achievement gap between 'normal achievers' and 'low achievers' with regard to predicting their own problem-solving performance when solving mathematical word-problems. The researchers explained that the students' improvement in prediction accuracy was related to them learning the importance of problem-solving strategies, which strategies to use, and how to apply them. Likewise, Schneider et al. (2014) found that "students in the lowest performance quartile were the best predictors of examination performance" (p3). It is important to note, that once students learn to accurately judge the potential of their knowledge (predictions) they may reach a ceiling effect (Hacker et al., 2000; Schneider et al., 2014) by showing no additional improvement in their predictive judgments. In addition, as students become more aware of how they apply skills, cognizant of their weaknesses, and knowledgeable of the content, their postdictions should improve.

Research has shown that lower achieving students are typically overconfident in their judgments (Bol et al., 2010; Garcia et al., 2016; Hacker & Bol, 2018; Pennequin et al., 2010). In a study of over 500 mathematics students, Garcia et al. (2016) found that low achievers were more overconfident than other students. Bol et al. (2010) reported similar results, that lower achieving middle school mathematics students were overconfident. The participants in the current study, however, were all lower achieving secondary students, therefore, one would posit that most of the participants would be overconfident. Interestingly, this was not the case, as the students in the metacognitive training group were consistently overconfident in their judgments and the students in the comparison group were steadily underconfident. Reflecting on the literature, it can be argued that because the students in the metacognitive training group scored lower than the comparison group on both the pretest and the posttest, they may struggle more

with mathematics problem-solving, and therefore were *more* lower achieving students than the students in the comparison group.

Another explanation for students' overconfidence could be wishful thinking (Finn & Metcalfe, 2014; Foster et al., 2017) or desired grade (Serra & DeMarree, 2016). Foster and colleagues (2017) found that students did not account for past exam performance when making predictions about future exam performance. Since past performance is a good predictor of future performance, this finding suggests that students were not reporting how they think they will perform, but rather how they want to perform. Finn and Metcalfe (2014) proposed that students may overvalue their effort in attempting a task and undervalue information needed to complete the task. This claim is consistent with the data found in this study; it is possible that the students in the intervention group increased their metacognitive awareness, which resulted in overvaluing their abilities and effort which, in turn, led to overconfidence. A regression analyses conducted by Serra and DeMarree (2016) revealed that students' desired grade was a stronger predictor of exam predictions and course predictions than actual grade. Perhaps, the students exposed to metacognitive training in this study focused on the grade they wanted to receive rather than the grade they thought they would earn when they were challenged to make judgements of their performance.

Metacognitive Training and Development of Mathematics Literacy

It was hypothesized that students that were exposed to metacognitive training with the problem-solving strategy would show greater development in their mathematics literacy than the students that were exposed to the problem-solving strategy. Mathematics literacy was assessed using several sources of both quantitative and qualitative measures to better understand and explain students' progress toward becoming mathematically literate. The data from this study

showed (1) that students that were exposed to the metacognitive training made greater gains than the comparison group in understanding of the problem, (2) that there were little differences between the groups in students' solutions or visualizations, (3) the comparison group made greater gains in answering the problems, and (4) students in both groups portrayed a variety of ability levels in their mathematics literacy.

Students that were exposed to the metacognitive training made greater gains than the comparison group in their understanding of the problem. It is possible that metacognitive training prompted the students to become more aware of their learning process, resulting in deeper understanding of the problems. More simply, it is possible that the students that were in the metacognitive training were more aware of the details of the word-problems. In turn, these details may have assisted the students in identifying relevant from irrelevant information in the word-problems. This finding is supported by other research. Xin and colleagues (2005) found that schema instruction in conjunction with diagrams helped students that were at risk for mathematics failure to differentiate between relevant and irrelevant information during problem-solving. Being aware of one's learning process is necessary to analyze and interpret mathematics is found to be a characteristic of expert mathematicians. For example, Stylianou (2002) found that expert mathematicians continuously engaged in metacognitive processes while problem solving. Experts closely monitored the details and effectiveness of each step and strategy while adjusting their strategies as needed during problem-solving activities (Stylianou, 2002). Chen and Chiu (2016) proposed that metacognitive training may force students to clarify concepts or generate new alternatives. In their study regarding calibration and mathematic literacy, they found that students that carry out the design and restructure their own knowledge develop at a higher level of mathematics literacy (Chen & Chiu, 2016). In relation to the present

study, the students that were exposed to the metacognitive training group were more aware of their learning, which allowed them to gain a deeper understanding of the problems.

It is also plausible that the metacognitive training group developed in their understanding, because of multiple exposures to metacognitive questioning, dual training (Kramarski & Mizrachi, 2006). As posited earlier, it is possible that the 5-step problem-solving process exposed all students to metacognitive questioning, implicitly. In this case, however, the metacognitive training group was also explicitly exposed to metacognitive questions. If students were engaging in metacognitive questions during the problem-solving process, it is unknown if the questions were helpful or relevant. In addition to the unknown questions the students may have been asking themselves, the metacognitive training group received guided, relevant questions that related to the task at hand. Other studies have also shown effects of dual training; metacognitive self-questioning and online discussion (Kramarski & Mizrachi, 2006) and guidelines in group settings (Bol, Hacker, Walck, & Nunnery, 2012).

Although students in the metacognitive training group made greater gains in their understanding of the word problems, there were nearly no differences between the groups in students' solutions or visualizations of the problems. These results are consistent with the previous studies regarding students' problem-solving achievement scores (Labuhn et al., 2010; Huff & Nietfeld, 2009; Schoenfeld, 1985). As mentioned earlier, it is possible that the experimental manipulation, metacognitive questions, was not powerful enough to overcome the problem-solving strategy. Students in the intervention group may not have applied the metacognitive questions when they were solving the problem or drawing their visualizations, because they were too engaged in the problem-solving process itself.

It is possible that the visualizations did not directly link the problem representations and its solution. The visualizations may have been effective for assisting students in understanding the problem, but not for solving and answering the problem correctly. Xin et al. (2005) found similar results and proposed that the linage of the visualization to problem solution may not be apparent to lower achieving students, because the students may have perceived the visualization as an external visual aid and did not use it as the solution. Schoenfeld (1985) called this routinized performance and argued that problem-solving that relies on routine procedures does not enhance deeper understanding of the content. He elaborated that specific heuristics were better for training tasks but were worse on transfer tasks than general heuristics.

Moreover, the metacognitive questions focused on self-regulation rather than content knowledge, a finding proposed by Chen and Chiu (2016). They found that encouraging students to plan, monitor, and regulate their problem-solving process was different than focusing on the knowledge aspect. Guthrie and colleagues (1996) found similar results in a study involving elementary students' literacy development in science classrooms. They found that students that were more self-regulated made greater gains in their literacy engagement.

Although the students that were exposed to metacognitive training improved the same amount as the comparison group in their solutions and visualizations, they made less gains on the component of answers. It is possible that students in the intervention group did not spend as much of their time performing calculations and reviewing their progress as they did planning, organizing, and representing the information. This finding is consistent with the research that suggests students spend little time reviewing and checking their work (Garcia et al., 2016). For example, Garcia and colleagues (2016) found that students reported spending a large amount of time thinking about solutions and little time reviewing and correcting mistakes. Klauda and

Guthrie (2015) discovered that student engagement and motivation does not necessarily increase student achievement, especially for struggling learners. They proposed that struggling learners may get caught up in strategies and the thinking process which may not "facilitate gains in achievement" (Guthrie & Klauda, 2015, p. 266). It is possible that the students in this study were overly focused on the strategies they were using to solve the problem and not as concerned about the accuracy of the answer they provided.

Lastly, as suggested earlier, it is possible that the students' ability levels were different between groups, signifying that students in the intervention group were less developed in their mathematics literacy than the comparison group. Hiebert (1984) proposed three levels of problem solving. The first level, or "site" (Hiebert, 1984, p. 499), students link symbolic representations with referents to create meaning. At the second site, students link the problem to a procedure or algorithm with the problem. During the last site, students connect the solution to the problem to a real-world or concrete context. The metacognitive training may have assisted students in improving their symbolic representations, however, it did not assist them in making the connection to a real-world context. Perhaps because students were not required to justify their solutions. The students may have had surface understanding, in which they memorized mathematics facts but no relational understanding of the concepts (Schoenfeld, 1985). Additionally, the metacognitive training group may have planned more, but they did not evaluate their progress and results, possibly because they had difficulties transferring their skills beyond planning. This suggests a potential relationship between strategies and mathematics literacy and that students need to take an active role in their learning process.

A book by the National Research Council, entitled *Adding it up: Helping children learn mathematics* (2001) proposed that learning mathematics entails five strands which include

conceptual understanding (comprehension of concepts), procedural fluency (skill in carrying out procedures), strategic competence (ability to solve math problems), adaptive reasoning (reflection, explanation, and justification), and productive disposition (efficacy). This framework could help to explain the results of this study. Students that were exposed to the metacognitive training made greater gains in understanding, made the same gains in solution, and made less gains in answers than the comparison group on the components of the analytic scale for problem-solving. Perhaps the metacognitive training assisted students in conceptual understanding and procedural fluency but not with strategic competence, adaptive reasoning, or productive disposition. In addition, *Adding it up* (2001) proposes that these strands must be "intertwined" (p. 5), it is possible that all of the strands were either not addressed or not intertwined in this study resulting in insignificant development of students' mathematics literacy.

Classroom observations and review of students' visualizations exhibited a variety of abilities levels across both groups which may explain the lack of differences between groups. A broad range of problem solving visualizations, from abstract to concrete, were identified and provided details about students' strengths and weaknesses. Overall, students in both groups could represent the problem in many forms as a graph, sketch, or table, suggesting they used different strategies flexibly, however, most of students work showed only one attempt at the problems and a lack of mistakes being corrected. Garcia et al. (2016) found similar results. In their study, students used different strategies to organize the information, which showed a clear relationship between data and facts, however, student answers were incorrect, and they showed limited signs of correction and editing. Likewise, Jacobse and Harskamp (2012) pointed out that building a representation of the problem is important in mathematics problem-solving and establishing a relationship between variables help students to solve the problem. They concluded

that visual representations provide insight into students' exploration and analysis of the problem (Jacobse & Harskamp, 2012).

Classroom observations showed that both groups portrayed difficulty in understanding vocabulary that was presented to them in the word problems. This is a concern because an important element of mathematical literacy is the ability to put "mathematical ideas and reasoning into words" (Friedman et al., 2011, p. 31). In fact, Schuth, Kohn, and Weinert (2017) found academic vocabulary to be a significant contributor to academic achievement in mathematics. Developing students' academic vocabulary is important for assisting them in development of mathematics literacy and should be a focus of mathematics classroom instruction.

The most interesting finding, however, is that students in both the intervention group and comparison group made gains in their development of mathematics literacy. Both groups improved on all measures and sources used to assess mathematics literacy. From pretest to posttest, students in both groups developed a better understanding of the problems, demonstrated their solutions, and provided more complete and accurate answers to the problems. As explained earlier, the problem-solving strategy itself was sufficient to assist students in understanding, applying, analyzing, and evaluating the mathematics, in addition to improving students' mathematics literacy. The problem-solving strategy was equally effective for students, regardless of ability level. Overall, knowledge of strategies for problem solving transferred to improved mathematics literacy, suggesting a relationship between strategies and mathematics literacy. High school classrooms have a broad range of ability levels, so explicitly teaching a problem-solving procedure that emphasizes higher-order concepts and skills can benefit all students by making them more mathematically literate.

Implications for Practice

Overall, findings from this study have several implications for practice. All students, including lower achieving mathematics, students require instruction designed to meet their needs (Grant et al., 2015; Labuhn et al., 2010). The findings here provide support for the effect self-regulation strategies (metacognitive questioning) and problem-solving strategies (five step process) have on lower achieving students' achievement scores, calibration accuracy, and mathematical literacy. These findings extend from school-based practices and are seen in other research that indicates self-regulation strategies, (Bol et al., 2016; Chen & Chiu, 2016; Cleary et al., 2017; Kramarski & Mizrachi, 2006) and problem-solving strategies, are teachable (Montague et al., 2014; Schoenfeld, 1985; Stylianou, 2002). In addition, explicit instruction of strategies is beneficial for lower achieving students, particularly in mathematics (Gersten & Clarke, 2007; Hacker & Bol, 2018; Montague et al., 2014), and should be embedded within the curriculum (Hattie & Donoghue, 2016). This finding is also valuable because low achievement in mathematics is an ongoing problem in the United States, and state and national mathematics assessments typically include problem-solving (NAEP, 2018).

Teacher training should raise awareness to the importance of self-regulated learning and metacognition, so that these techniques may be integrated consciously and effectively into the classroom. Teachers are in a unique position to support students to be more aware of their learning processes. They can assist students in monitoring their own learning by explicitly teaching learners self-regulation theory and strategies, while allowing them to practice during class time. The responsibility is on the preservice teacher programs to educate pre-service teachers about the importance of self-regulation, metacognition, and mathematics literacy on students' learning and academic outcomes. Current administrators and mathematics teachers

should seek out professional development opportunities to acquire training regarding teaching self-regulation strategies to students. Self-regulation is a significant component in problem-solving and mathematics achievement and, based on the present results, is effective for enhancing metacognitive judgments and mathematics literacy when explicitly taught (Bol et al., 2016) to lower achieving mathematics students using guided practice and visual aids.

Mathematics literacy is a broad concept and is directly influenced by many different variables; prior knowledge, vocabulary, personal experiences, specific skills, emotion, and the ability to communicate (read, write, listen, speak) and think critically. In order to improve students' performance and development of mathematics literacy, it has been suggested that teachers teach from a disciplinary literacy perspective (Colwell & Enderson, 2016; Draper & Wimmer, 2015). Mathematics teachers should establish a learning environment that integrates disciplinary literacy and mathematics content, in a variety of contexts and in a unified manner, to support students' development of mathematical literacy (Colwell & Enderson, 2016).

Lastly, it is imperative that teachers, specifically in mathematics, keep a positive caring attitude to avoid students' sense of learned helplessness, discontent, and negative beliefs. "If student-teacher relationships are not synergistic, they may not promote effective mathematics learning" (Grant et al., 2015, p. 113). Teachers should build students' confidence, provide a welcoming classroom environment, and motivate them to engage in mathematics. Motivation has been found to significantly influence mathematics achievement (Cleary & Chen, 2009; Cleary & Kitsantas, 2017). Take for example the student that continued to see the researcher for assistance and verification with their classwork. A few choice words of encouragement and confidence from the researcher, reduced the students need for seeking help, and increased his attempts to solve the problems independently and accurately. Negative attitudes from teachers

can decrease student motivation, specifically in mathematics, which can also influence the learning cycle of self-regulation (Cleary & Kitsantas, 2017). These concepts, motivation and learned helplessness, however, have been researched extensively and are beyond the scope of this study.

Limitations

Though some limitations have already been noted in the discussion of results, they are described here more generally, and others should be acknowledged. One potential limitation to this study is selection bias. Always an uncontrollable variable in quasi-experimental designs, the researcher could not randomly assign participants to conditions because the classes were already predetermined. A potential confounding variable is temporal validity, that is, the time of day the participants were in the mathematics classes, a variable that in this study could not be controlled. For example, there may be differences between having class after lunch or as the last class of the day. Differences were evident between the classes in all measures, but there were no statistically significant differences identified, suggesting that the classes were similar. Caution should be taken when generalizing the results of this study, because the sample size was small, (37 participants) and classes were unequal (18 & 15). Small samples reduced the power of the study and may explain why there were no significant results, suggesting perhaps a ceiling effect. The small sample did, however, afford a detailed evaluation of students work from four perspectives; statistically, holistically, specifically, and visually.

The length of the intervention is another limitation. The results of this study should not be generalized to the effects of implementing SRL strategies over a longer period of time or the course of an entire academic year, because the students were exposed to the intervention for a total of only 17 days. Although other studies have revealed successful results from short SRL

interventions (Bol et al., 2016; Chen & Chiu, 2016; Pennequin et al., 2010; Perels et al., 2009), the short time period utilized in this study is a possible area of concern.

The cooperating teacher's expressed concerns about student behaviors, ability, and effort prior to the administration of the study. Although his concerns diminished throughout the intervention period, these beliefs could have been a contributing factor to the general culture of the classroom. In addition, the students in this study were classified as lower achieving students by class enrollment, but they were limited to only the cooperating teacher's students. Other classes, subjects, or topics within mathematics could have proven valuable if utilized within the study parameters.

Lastly, it is possible that fidelity of implementation of the metacognitive training questions may have influenced the results of this study. Diffusion of treatment may have occurred, as the cooperating teacher erroneously asked some metacognitive questions to the intervention group early in the study. When discovered, the teacher received additional training, and the study was accurately administered for the remainder of the intervention. If some of the students in the comparison group inherently utilized metacognitive questioning, there is also the possibility of treatment diffusion.

It should be noted, however, that many of the limitations, such as sample size, unequal classrooms, and length of intervention, provide a glimpse into what is actually happening in the classroom. Because this study portrays a realistic representation of research in real-world contexts, it offers ecological validity.

Recommendations for Further Research

Additional research is needed to fully understand the influence metacognitive training has on students' performance, calibration accuracy and mathematics literacy. Although this study

focused on lower achieving secondary algebra students, the small sample did not allow for analysis by gender, race, age, or special education services. Researchers have found differences across these factors. Jitendra et al. (2015) found significant differences among students problem-solving performance by race. Montague et al. (2014) found significant differences by ability level with regard to the problem solving of students with and without learning disabilities. Ozoy (2012) found differences among boys and girls. Additional research regarding these factors could reveal differences based by group concerning students' monitoring and regulating their learning processes.

This study was conducted over a short period of three weeks with a small sample. Some studies have had significant results during short intervention periods (Bol et al., 2016; Chen & Chiu, 2016; Pennequin et al., 2010; Perels et al., 2009) and with small sample sizes (DiGiacomo & Chen, 2016; Xin et al., 2005). To generalize these conclusions, future research could longitudinally investigate the effects of self-regulated learning strategies over time or replicate this study with a larger sample size.

To clearly determine the differences the explicit metacognitive questions and implicit metacognitive questions may have on outcome variables, it is recommended that a baseline is established (Dugard, File, & Todman, 2012). All students could be taught the problem-solving strategy for a few weeks and then implement the metacognitive questions with the intervention group for a few weeks. Allowing students to learn the problem-solving strategy first would eliminate implicit metacognitive questions, and perhaps ease the difficulty of the problem-solving process, which may show the influence that explicit metacognitive questions have on students' performance, calibration accuracy, and mathematics literacy.

Further research could compare additional groups to see the effects of metacognitive training on students' performance, judgments, and mathematics literacy. For example, researchers could compare four groups; one that receives metacognitive training, one that receives the problem-solving strategy, one that receives both metacognitive training and the problem-solving strategy, and one that receives neither metacognitive training nor the problem-solving strategy. These group comparisons could identify cause and effect relationships between variables and provide additional empirical evidence to the effects of metacognitive training.

Providing the teacher with a variety of mathematics activities may offer deeper insight to how self-regulation could vary for different types of tasks. Research involving different types of activities implemented into the curriculum could account for the novelty effect and shed some light onto which activities are most effective for developing students' mathematical literacy, improving their academic achievement, and making them more aware of the learning processes. In fact, the five-step problem-solving process used in this study could be compared to other types of problem-solving strategies.

National Council and Teachers of Mathematics (2000) developed Principles and Standards for School Mathematics (PSSM) that are intended to guide and improve mathematics education nationally. Within the PSSM there are five process standards: problem solving, representations, reasoning and proof, communication, and connections. This study showed that students' representations improved more than their solutions. Therefore, future research could focus on how students' representations might improve students' solutions during problem-solving.

Additional qualitative research involving participants' perceptions and reflections about their problem-solving process, metacognitive questions, and visual representations is warranted.

To add depth to a study, students could illustrate the perspectives of their problem-solving process through verbal and written reflections and explanations about their decisions and visual representations. It has been suggested that having students explain in writing how a pictorial representation should be "read" by others assists in developing mathematical literacy (Old Dominion University, 2018).

Additionally, different measures to assess the *way* students solved the word problems should be examined further, to gain understanding of students' experiences and the impact it had on their mathematics learning (Ferguson, 2017). A collection of assessment and qualitative data from students (e.g., observations, student work, grades, reflections, interviews) would provide a holistic view of students' self-regulatory processes and development of their mathematics literacy.

To further gauge the relationships between strategy use, achievement, self-regulation, metacognition, and mathematics literacy, specific case studies of students' data could be evaluated (Ferguson, 2017). Lastly, a variety of measures across different groups and contexts is an important direction for future research to better understand students' development of mathematics literacy, performance, and metacognitive awareness.

Conclusion

The primary goal of this study was to test whether explicitly teaching a problem-solving strategy with metacognitive questioning effectively improved participants' math achievement scores, ability to monitor their learning, and develop their mathematics literacy. The study's participants were lower achieving high school mathematics students enrolled in a public school. The effective of metacognitive questions was examined to determine its influence on students' academic performance, calibration accuracy, and mathematics literacy.

The first research question addressed the impact of metacognitive questions on mathematics performance. Although there were no significant findings between groups for achievement, the data in this study revealed that lower achieving students, in both the comparison and intervention groups, made academic gains from the word problem-solving training itself. The problem-solving process supported students in developing a plan, identifying relevant information, and making connections, tasks that many lower achieving mathematics students have difficulties with (Geary, 2011; Gersten & Clarke, 2007). Overall, these results suggest that the problem-solving strategy, with or without metacognitive questions, was sufficient for improving students' mathematics achievement. Perhaps, with additional time and training, enhanced self-regulation differences would materialize.

In addressing the question of whether metacognitive questions influenced students' calibration accuracy, significant results were found for prediction accuracy. The research here demonstrated that students' expectations for performance were synchronized with their actual performance. In other words, students exposed to metacognitive questioning were able to accurately judge the potential of their knowledge. These students also improved in their judgements of performance and became less overconfident. Enhancing students' metacognitions is not an easy task to achieve, since students do not inherently self-regulate (Finn & Metcalfe, 2014), and self-regulation is even more difficult in mathematics (Winne & Muis, 2011). Metacognitive knowledge, however, can be improved through instruction (Gutierrez & Schraw, 2015) and practice (Pennequin et al. 2010; Serra & DeMarree, 2016) when it is embedded into the daily activities of mathematics classrooms and explicitly taught to students.

Lastly, the influence metacognitive questioning would have on students' development of mathematics literacy was assessed. Two important aspects of students' development in

mathematics literacy were observed: better understanding of the problem and the ability to use a broad variety of visual representations. The metacognitive questions played a valuable role in engaging students in understanding of the problem. Understanding of the problem, needed to create a solution and answer, is the first step to promoting students' literacy in mathematics. Students also portrayed a wide variety of visualizations of the problems, which demonstrated students' ability to use flexible strategies across multiple contexts. Embedding self-regulatory strategies, such as metacognitive questioning, helped to improve students' understanding and flexibility, signifying development in mathematics literacy.

Self-regulation strategies, such as the problem-solving strategy presented in this study, are needed to promote academic achievement, metacognitive awareness, and mathematics literacy, and are equally effective regardless of ability level. The intervention, however, played a critical role in engaging students in being more aware of their learning processes. Future research should continue investigating the relationship between metacognitive awareness on students' academic performance, calibration accuracy, and mathematics literacy.

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APPENDIX A

PARENT LETTER

Dear Parents,

I am presently enrolled in a doctoral program at Old Dominion University, and research is one program requirement. My research topic relates to improving students' mathematics achievement through problem solving training. I truly believe that success in mathematics begins teaching students effective problem-solving techniques that they can use in the real world. Therefore, in your student's Algebra class their teacher will be using a problem-solving strategy to assist them in better understanding mathematics.

In addition, students will be asked to make judgments about how confident they are about solving a problem correctly. They will be asked to predict how likely they will solve the problem correctly and, afterwards, how confident they are that they solved the problem correctly. Developing these judgment skills may help our students improve study habits, test performance, grades, and SOL scores. Overall, I am interested in how problem-solving strategies influence students' judgments and mathematics achievement.

I will be collecting data from two Algebra classes. Your child will be in one of two classes. This study will have minimal impact on classroom instruction and instructional time. It will occur for 30-40 minutes during the warm-up period for three weeks. All data collected will be used strictly for the purpose of the research and will not be released for any other purpose. Your child will not be exposed to any risk. All data will be handled with confidentiality so that student names will not be released. This research has been approved by Chesapeake Public Schools and Old Dominion University.

If you have any questions regarding the use of this data, or how the research is being conducted, please do not hesitate to contact me, Deana Ford, at 904-536-5028 or Dr. Linda Bol at 757-683-4584. If you have any other questions or concerns please contact Dr. Jill Stefaniak, current chair of the Darden College of Education Human Subject Committee at jstefani@odu.edu or 757-6836696.

Your child's participation is strictly voluntary and your child will in no way be penalized if you choose not to let him or her participate in the study. I believe this research will be extremely beneficial and I hope that you will permit your child to participate.

Please return this form only if you **do not** want your child to participate in this research project.

I _____(guardian name) **<u>do not</u>** give permission for

my student _____(student name) to participate in this research.

APPENDIX B

STUDENT LETTER

Dear Student,

I am a doctoral student at Old Dominion University conducting a research project. My project focuses on improving students' mathematics achievement by better understanding the problem-solving process. I truly believe that success in mathematics begins with learning effective problem-solving techniques that you can use in the real world.

I need your help in getting information to improve mathematics problem-solving instruction. I will be collecting some information during your Algebra class. Before and after you learn the problem-solving process, I am going to ask you to predict how confident you are at solving real world word problems. I will also ask you to tell me how well you think you solved the problem after you finished.

The potential benefit of your participation in this research is improvement in your mathematics performance and learning. There are no foreseen risks to your participation. I will maintain strict confidentiality and remove any information that might identify you. The results of this study may be used in reports, presentations, or publications, but you will not be identified. Your teacher has approved this project and your participation is voluntary; therefore, your participation or responses will not have any consequences for you.

If you have any questions regarding the use of this data, or how the research is being conducted, please do not hesitate to contact me, Deana Ford, at 904-536-5028 or Dr. Linda Bol at 757-683-4584. If you have any other questions or concerns please contact Dr. Jill Stefaniak, current chair of the Darden College of Education Human Subject Committee at jstefani@odu.edu or 757-6836696. By proceeding, you agree to participate. Thank you very much for your participation.

APPENDIX C

EXAMPLE WORKSHEET FOR METACOGNTIVE TRAINING

Solving Word Problems   Name_____

A farmer has both pigs and chickens on his farm. There are 78 feet and 27 heads. How many pigs and how many chickens are there?

1. Read the problem for understanding. Ask yourself (1) Do you understand the problem? (2) Do you know what all the words mean?

2. Identify the important information or data. Ask yourself (1) What is the unknown, what is being asked? (2) What are the data?

3. Draw a picture or diagram to visualize the problem. Ask yourself (1) Can I connect the data and the unknown visually? (2) Is my diagram a good representation of the problem?

4. Solve the problem. Ask yourself (1) Do I need a formula or special notation? (2) Do I know how to calculate the solution?

5. Check your work. Ask yourself (1) Are your computations accurate? (2) Is your answer reasonable?

APPENDIX D

EXAMPLE WORKSHEET FOR THE PROBLEM-SOLVING STRATEGY

Solving Word Problems   Name_____

A farmer has both pigs and chickens on his farm. There are 78 feet and 27 heads. How many pigs and how many chickens are there?

1. Read the problem.

2. Identify the important information or data.

3. Draw a picture or diagram to visualize the problem.

4. Solve the problem.

5. Check your work.

APPENDIX E

VISUAL REPRESENTATION RUBRIC

Ability Level	Visualization (model of a problem that links important information to a visual representation)
5	Diagram is a valid and appropriately linked representation of the problem.
4	Diagram generally represents the problem with few minor identifiable errors.
3	Diagram may represent the problem with few major identifiable errors.
2	Diagram is not a valid or linked representation of the problem depicting multiple major identifiable errors.
1	No diagram was provided.

APPENDIX F

PROCEDURAL FIDELITY AND ADHEARENCE TO THE METACOGNITIVE TRAINING
Group:　　　　MT _____　　PSS _____
Date: _____

Step	Check if Observed	Metacognitive Questions
Read the problem		
		Do you understand the problem?
		Do you know what all the words mean?
Identify important information		
		What is the unknown, what is being asked?
		What are the data?
Draw a visualization		
		Can I connect the data and the unknown visually?
		Is my diagram a good representation of the problem?
Solve the problem		
		Do I need a formula or special notation?
		Do I know how to calculate the solution?
Check your work		
		Are your computations accurate?
		Is your answer reasonable?

VITA

Deana J. Ford
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Curriculum & Instruction: Teaching & Learning
Education Building, 3rd Floor
Norfolk, VA
dford005@odu.edu
904-536-5028

Education
Old Dominion University, Norfolk, VA

Ph.D., Curriculum & Instruction: Teaching & Learning, November 2018
Cognate: Research Methodology

Dissertation: The Effects of Metacognitive Training on Lower Achieving Algebra Students' Calibration Accuracy, Achievement, and Mathematical Literacy.

Dr. Linda Bol, Chair

Research Interests: self-regulation, metacognition, calibration, STEM, achievement, struggling learners, secondary education, mathematics vocabulary

MS, Secondary Education, August 2014

Problems Paper: Art Students' Perceptions of their Skill, Academic, and Personal Development.

Dr. Robert Lucking, Chair

BS, Psychology, May 2001

License & Professional Training

State of Florida Professional Educator's Certificate, 2014-2019
        Endorsement in Mathematics (6-12) 1095868

Preparing Future Faculty Certificate, 2015

Teaching

Academic Appointments

Old Dominion University, Norfolk, VA

Adjunct Faculty, Darden College of Education, Fall 2017-Present
Graduate Teaching Assistant, Darden College of Education, Fall 2014 – Spring 2017
University Supervisor, Department of Teaching and Learning, Fall 2016-Spring 2017

Center for Allied Health and Nursing Education, Jacksonville, FL

Instructor, College Algebra, Fall 2011 – Spring 2013

Courses Taught

Old Dominion University, 2014 - Present

TLED 360: Classroom Management and Discipline – This course examines theories, research and practices involved in classroom management, motivation and discipline. It explores techniques for organizing and arranging classroom environments that are most conducive to learning. Instructor, Fall 2015 – Spring 2018.

FOUN722: Introduction to Applied Statistics and Data Analysis – This course provides an introduction to basic topics in statistical analysis, including descriptive statistics and simple inferential statistics such as correlation, regression, t-tests, one-way analysis of variance, and chi-square. Teaching Assistant, Spring 2016.

FOUN813: Program Evaluation in Education – This course examines procedures and problem in the design and utilization of program evaluation in education. It identifies evaluation purposes and the methods of evaluation especially as affected by organizational behavior, ethical consideration, and political influences. Evaluation methodology includes, but is not limited to, design considerations, data utilization, and teacher evaluation. Both quantitative and qualitative strategies were covered. Teaching Assistant, Summer 2016.

STEM101: Inquiry Approaches to Teaching STEM – This course provides mathematics and science students with the opportunity to explore teaching in a real classroom setting. Master teachers introduce students to examples of high-quality inquiry-based lessons and model the pedagogical concepts to which they are being introduced. In this course, with the guidance of the master teacher, students engage in two classroom observations and prepare and teach three inquiry-based lessons in an upper elementary school classroom. Teaching Assistant, Fall & Spring 2014.

STEM102: Inquiry Based STEM Lesson Design – This course continues the exploration of inquiry-based lesson design in STEM education. In this course, students build upon and practice lesson design skills developed in STEM101 while also becoming familiar with exemplary mathematics or science curricula at the middle school level. With the guidance of the master teacher, students engage in one observation and prepare and teach three inquiry-based lessons in a middle school classroom. Students incorporate and demonstrate their content knowledge in developing the inquiry-based lessons. At the end of this course, students are generally ready to make a decision about whether they want to pursue a pathway to teacher licensure through the MonarchTeach program. Instructor, Spring 2014.

Center for Allied Health and Nursing Education, 2011-2013

GE202: Math for Problem Solving & Research – This general education course focuses on arithmetical accuracy and problem solving. Mathematical concepts including ratios, proportions, and basic algebraic equations will be presented. Basic dosage calculations and IV rates will also be determined. In addition, an overview of statistics will be provided in this course to assist with review of research relative to evidence-based patient care.

Statistical Consulting

Data Analysis, *WHRO Ready to Learn Transmedia Initiative* (2015). WHRO received a Ready to Learn Grant from the Corporation for Public Broadcasting to expand children's learning through the use of transmedia.

Data Analysis, Dissertation, (2015). *The Effects the Virginia Department of Education Individual Students Alternative Education Plan had on students passing the General Educational Development (GED) Test.*

Workshop Presentations

River City Science Academy, Jacksonville, FL

Differentiated Learning. Middle and Secondary Teacher Workshop, January 2011
State Assessment Strategies. Middle and Secondary Teacher Workshop, February 2010
Implementing Literacy in Mathematics. Middle and Secondary Teacher Workshop, September 2009
Mathematics Teaching Techniques. Middle and Secondary Teacher Workshop, January 2009
Classroom Management. Middle and Secondary Teacher Workshop, October 2008

Research

Publications

Refereed Article

Grant, M., Crompton, H., & **Ford, D.J.** (2015). Black male students and the algebra project: Mathematics identity as participation. *Journal of Urban Mathematics Education*, 8(2), 87-118.

Invited

Ford, D.J. & Bol, L. (2018). *The effects of self-regulation strategies on middle school students' calibration accuracy and achievement.* Invited article for Studying and Self-Regulated Learning, Special Interest Group, American Educational Research Association.

Technical Report

Ford, D.J. & Overbaugh, R. (2015). *Online video games and children's understanding of mathematical concepts and game perceptions, Summer 2015 Evaluation.* Report to WHRO Ready to Learn Transmedia Initiative.

Conferences

National
American Educational Research Association, New York, NY, April 2018
Poster: The Effects of Self-Regulation Strategies on Middle School Students' Calibration Accuracy and Achievement; Received Graduate Student Research Award

American Educational Research Association, New York, NY, April 2018
Studying and Self-Regulated Learning Special Interest Group Mentoring Program.
Mentor: Jeffery Greene, The University of North Carolina at Chapel Hill

Regional
Eastern Educational Research Association, Hilton Head Island, SC, February 2016
      Poster: Online Video Games and Children's Understanding of Mathematical Concepts and Game Perceptions

Local
Graduate Research and Achievement Day, Norfolk, VA, March 2018
      Poster: Mathematics Vocabulary: A Summative Content Analysis of Algebra I High-Stakes Test

Service

University Service

Old Dominion University, Norfolk, VA

      Proctor, Comprehensive Exams, Darden College of Education, February 2018
      Tutor, Preservice Teachers Praxis, Mathematics: Content Knowledge, Fall 2015-Spring 2016
      Parent workshop for Space and Naval Warfare Systems Command (SPAWAR) and Norfolk State University, Girls Day Out, July 2015
      Workshop: *Glogerm-There are germs all around us*. Student Virginia Education Association, STEM Day. March 2015

Community Service

Creekside Elementary School, Suffolk, VA
      Mathematics Games: Keeping students' interest and making math fun, 2017 (volunteer)
      Various activities, 2017 (volunteer)

River City Science Academy, Jacksonville, FL
      Weekend Camp for Improving State Assessment Scores, 2009-2011

Awards & Grants

Graduate Student Research Award, Studying and Self-Regulated Learning SIG of AERA, New York, NY, 2018
Travel Grant, Teaching and Learning, Old Dominion University, Norfolk, VA, 2018
Travel Grant, Darden College of Education, Old Dominion University, Norfolk, VA, 2018
Dissertation Fellowship Award, 2017, finalist, Old Dominion University, Norfolk, VA, 2017
Returning Adult Award, Argosy University, Honolulu, HI, 2006 - 2007
Teacher of the Month, River City Science Academy, Jacksonville, FL March 2009
Computer Science Engineering Mathematics Scholarship, Old Dominion University, Norfolk, VA, 2000-2001

Certificates & Appreciations

Appreciation, Student Virginia Education Association
Certificate of Achievement – Office of Graduate Studies
Certificate, Preparing Future Faculty
Certificate, Graduate Teacher Assistant Instructor Institute
Certificate, Golden Key International Honour Society
Biography, The National Dean's List Book

Professional Affiliations

American Educational Research Association, SIG- Studying and Self-Regulated Learning
National Council of Teachers of Mathematics
Eastern Educational Research Association
Golden Key International Honour Society

Other Professional Experience

River City Science Academy, Jacksonville, FL
      Mathematics Department Chair, 2009-2011
      Secondary Mathematics Teacher, 2008-2011

Seacoast Christian Academy, Jacksonville, FL
      Secondary Mathematics Teacher, 2004-2006
      Psychology Teacher, 2005-2006

Virginia Beach Psychiatric Center, Virginia Beach, VA
      Mental Health Counselor, 2001-2003
      Group Meeting Counselor for Adults and Adolescents, 2002-2003