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# Transmitter Optimization in Multiuser Wireless Systems with Quality of Service Constraints

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# TRANSMITTER OPTIMIZATION IN MULTIUSER WIRELESS SYSTEMS WITH QUALITY OF SERVICE CONSTRAINTS

by

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
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# ABSTRACT

## TRANSMITTER OPTIMIZATION IN MULTIUSER WIRELESS SYSTEMS WITH QUALITY OF SERVICE CONSTRAINTS

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In this dissertation, transmitter adaptation for optimal resource allocation in wireless communication systems are investigated. First, a multiple access channel model is considered where many transmitters communicate with a single receiver. This scenario is a basic component of a wireless network in which multiple users simultaneously access the resources of a wireless service provider. Adaptive algorithms for transmitter optimization to meet Quality-of-Service (QoS) requirements in a distributed manner are studied. Second, an interference channel model is considered where multiple interfering transmitter-receiver pairs co-exist such that a given transmitter communicates with its intended receiver in the presence of interference from other transmitters. This scenario models a wireless network in which several wireless service providers share the spectrum to offer their services by using dynamic spectrum access and cognitive radio (CR) technologies. The primary objective of dynamic spectrum access in the CR approach is to enable use of the frequency band dynamically and opportunistically without creating harmful interference to licensed incumbent users. Specifically, CR users are envisioned to be able to provide high bandwidth and efficient utilization of the spectrum via dynamic spectrum access in heterogeneous networks. In this scenario, a distributed method is investigated for combined precoder and power adaptation of CR transmitters for dynamic spectrum

sharing in cognitive radio systems. Finally, the effect of limited feedback for transmitter optimization is analyzed where precoder adaptation uses the quantized version of interference information or the predictive vector quantization for incremental updates. The performance of the transmitter adaptation algorithms is also studied in the context of fading channels.

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To my family.

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## CHAPTER I

### INTRODUCTION

Current wireless networks consist of various wireless service providers which offer a wide range of services from traditional voice service to delivery of multimedia contents. Wireless networks have become less expensive and more ubiquitous with increasing demand for wireless services and applications. With the worldwide success of cellular telephone systems in wide area networks (WANs) and increasing deployment of wireless local area networks (WLANs) for home and office use, wireless communications is the fastest growing segment of the communication industry. Many new wireless applications such as wiring replacement (HomeRF, Bluetooth) networks, paging networks, wide area email access, sensor networks, and remote telemedicine are emerging from research ideas to concrete systems. However, the increasing demand for various wireless systems and services are tempered by scarce radio frequency (RF) spectrum and its usage fee or cost. The wireless applications which cannot be deployed with a regular revenue base, such as cellular telephone systems, cannot usually afford to pay for the RF spectrum and must therefore share use of various unlicensed bands such as the Unlicensed National Information Infrastructure (the UNII) bands at 5GHz and industrial, scientific and medical (ISM) bands at 900MHz and 2.4GHz. It is worth noting that the practical wireless systems are often multi-user in nature because system designs are often constrained by physical resources such as scarce RF spectrum and power, and thus a system that allows multiple users to share resources is often the most economical for service providers in

licensed bands such as in cellular systems, and is the most efficient way of utilizing precious RF spectrum in unlicensed bands.

Unfortunately, shared spectrum use by multiple users implies mutual interference among systems whose owners and/or offered services and/or traffic types may be completely different, and spending development, maintenance and development costs for equipment and services for interference mitigation from another system might not be an attractive perspective for the wireless industry. Thus, interference mitigation through transmitter optimization along with power control has been and continues to be an important research topic for wireless systems and networks.

As noted, the wireless systems are more public with the transmitter radiating a signal that can be received by any antenna in close proximity, interference and noise are much more prevalent in wireless communication systems [2–4]. The recent advances in the field of cognitive radios – software defined radios with some artificial intelligence – facilitate the wireless users by allowing them to adapt their operating parameters such as frequency, transmit power, output waveforms as well as their demodulation methods based on their operating environment [5–7]. The transmitter and receiver adapt their waveforms (or signatures) in response to interference conditions to enhance the system performance through interference avoidance methods [4, 8, 9], where the transmitter radio is instructed to adapt its waveform via feedback from the receiver [10–12]. Furthermore, transmitter optimization for interference avoidance through signature (or precoder or waveform or codeword) adaptation can be either *centralized* where the optimal transmitter parameters are obtained at the common receiver and assigned to individual users [13–17] or *decentralized*

where users independently update their transmitter parameters in response to feedback from the receiver by broadcast [8, 9, 18] and can be obtained global optimal solution in a distributed manner.

It is noted that the signature update is a transmitter optimization (or signal design) method to avoid the interference experienced from other users, in which the target SINRs may not be met. Therefore, combined signal design for interference avoidance and power control to meet QoS requirements has attracted research from both academia and industry.

It is worth noting that, in centralized systems, the network overhead as well as computational complexity is increased significantly for large systems, and thus the distributed implementation of algorithms is more desirable. Furthermore, communication networks are becoming increasingly decentralized in decision making and dynamic spectrum sharing. Individual nodes in such distributed networks are required to adapt their transmitted signals in a way that reduces interference and thus facilitates multi-user communications. Moreover, the rapid growth of wireless technologies and its pervasive use in daily life is creating the scarcity in radio frequency (RF) spectrum. In addition, the transmit power in wireless networks is a key element in the management of interference, energy, and connectivity. Therefore, the RF spectrum and power are regarded as the scarce resources and their efficient utilization is highly recommended in order to increase spectral efficiency and thus the system capacity, and active management of wireless resources using transmitter optimization with power control are fundamental steps in multi-user wireless systems, which is the subject matter of this dissertation.

Therefore, this dissertation provides transmitter optimization algorithms (for interference avoidance) and power control (for QoS requirement or to limit the interference to other users) for resource allocation in wireless systems by investigating their applications to general vector channels. The gradient descent approach for signature update for interference avoidance and power adaptation to meet QoS requirements in terms of target SINRs for multiple access channel in uplink of wireless systems is investigated. Then, for the same system, a game theoretic approach is applied for joint signature adaptation for transmitter optimization with power adaptation to meet QoS requirements for which the game has an optimal point called a Nash Equilibrium. Then, a generalized method for transmitter optimization using precoder adaptation with power control in interference systems is investigated, and the method is applicable for cognitive radio systems. Furthermore, high computational complexity at the receiver for large systems in centralized implementation motivates this research to use limited feedback from receiver to the individual transmitter so that individual transmitters adjust their operating parameters (such as transmit waveform and/or powers) in distributed manner and reach at optimal point. The effect of limited feedback in transmitter optimization is also investigated.

## I.1 SYSTEM MODEL

With ever increasing diversity of wireless services and applications, the communication model can be considered as an *interference channel* [19–21] where neither transmitters nor receivers cooperate and each transmitter-receiver pair attempts to communicate in the presence of interference from all other users, or *multiple access*

*channel* [2, 3, 22] where transmitters do not cooperate but compete with each other for the same resources, and each transmitter represents a different user sending independent information and the joint receiver (such as a common base station or access point) must decode information from all users, or *broadcast channel* where a single transmitter communicates with many receivers.

In this dissertation, interference channel and multiple access channel scenarios are considered, and the system models for them are presented in the following section.

### I.1.1 Interference Channel and Transmitter Adaptation

In the interference channel scenario, multiple transmitters communicate with their corresponding receivers and interfere with each other as depicted in Figure 1. The  $K$ -links interfering system is considered which comprised of  $K$  transmitters and  $K$  receivers operating in a signal space of dimension  $N$ , and each node is equipped with only one antenna.

The  $N$ -dimensional received signal at the  $k$ th receiver over one signaling interval is described as follows:

$$\mathbf{r}_k = \mathbf{H}_{k,k}\mathbf{x}_k + \sum_{j \neq k, j=1}^K \mathbf{H}_{k,j}\mathbf{x}_j + \mathbf{w}_k \quad (\text{I.1.1})$$

where  $\mathbf{H}_{k,j}$  is the  $N \times N$  dimensional channel matrix between transmitter  $j$  and receiver  $k$ ,  $\mathbf{x}_k$  is the  $N$ -dimensional transmitted signal vector, and  $\mathbf{w}_k$  is the  $N$ -dimensional additive white Gaussian noise (AWGN) at  $k$ th receiver with  $E[\mathbf{w}_k\mathbf{w}_k^\top] = \sigma_k^2\mathbf{I}$ , where  $E[\cdot]$  and  $[\cdot]^\top$ , respectively, denote the expectation and conjugate transpose operations. For a given link  $k$ , the first term on the right hand side of the equation



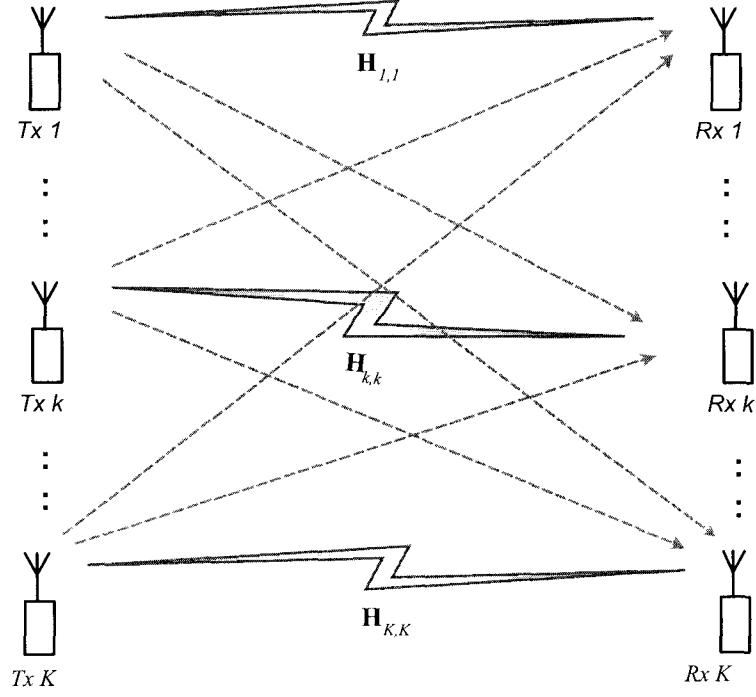


Fig. 1: Interference system model with  $K$  links.

(I.1.1) is the desired signal and the second term is interference from remaining  $K - 1$  links.

This type of system model has been used in analyzing radio resource management in [23, 24] and optimal transmission strategies in [25, 26] using a game theoretic approach for transmitter optimization. It is noted that the system model (I.1.1) incorporates FDM, TDM, as well as CDM scenarios with the proper choice of the set of transmitter and receiver functions [27], and the FDM scheme actually corresponds to OFDM (orthogonal frequency division multiplexing), whereas the TDM scheme actually corresponds to SCCP (single carrier with a cyclic prefix) and CDM scheme

covers code division multiple access (CDMA). It is also noted that this type of system model incorporates the scenario where incumbent primary users and secondary users co-exist and transmit simultaneously provided that the secondary users are restricted not to create harmful interference to incumbent primary users.

The interference system model (I.1.1) is a general one and can be modified to represent the multiple access channel scenario where multiple transmitters communicate with a single receiver. In other words, the multiple access channel model is a special case of the interference channel model, and is the subject matter of the following section.

### **I.1.2 Multiple Access Channel and Transmitter Adaptation**

In multiple access channel scenarios, many transmitters communicate with a common receiver and potentially interfere with each others' data stream at the receiver. For instance, the uplink of a cellular system after ignoring the out-of-the-cell interference, access-point, or infrastructure based wireless LAN and reporting of sensed information by sensor nodes to their common central unit/node are examples of the multi-user wireless systems as well as prototypical examples of multiple access channel.

For multiuser communication systems, uplink of a wireless system is considered with  $K$  active users communicating with a centralized base station in a signal space of dimension  $N$  where non-ideal channels between users and base station are explicitly considered for which the  $N$ -dimensional received signal at the base station receiver

for one signaling interval is given by the expression [4, 27]

$$\mathbf{r} = \sum_{k=1}^K \mathbf{H}_k \mathbf{x}_k + \mathbf{w} \quad (\text{I.1.2})$$

where  $\mathbf{H}_k$  is  $N \times N$  dimensional channel matrix between a user  $k$  and the base station,  $\mathbf{x}_k$  is the  $N$ -dimensional transmit signal vector of user  $k$ , i.e.

$$\mathbf{x}_k = \mathbf{S}_k \mathbf{P}_k^{\frac{1}{2}} \mathbf{b}_k \quad (\text{I.1.3})$$

with  $N \times M$  dimensional unit norm signature matrix  $\mathbf{S}_k = [\mathbf{s}_1^{(k)} \dots \mathbf{s}_k^{(k)} \dots \mathbf{s}_M^{(k)}]$ , the transmit frame of symbol vector  $\mathbf{b}_k = [b_1^{(k)} \dots b_k^{(k)} \dots b_M^{(k)}]^\top$  and the transmitted power matrix  $\mathbf{P}_k = \text{diag}\{p_1^{(k)} \dots p_k^{(k)} \dots p_M^{(k)}\}$  for user  $k$  which transmits  $M$  symbols, and  $\mathbf{w}$  is the AWGN that corrupts the received signal with zero-mean and covariance matrix  $\mathbf{W} = E[\mathbf{w}\mathbf{w}^\top]$ . As noted in the case of interference channel scenario, the system model (I.1.2), as in (I.1.1), still incorporates FDM, TDM, as well as CDM scenarios with the proper choice of the set of transmitter and receiver functions [27]. It is worth noting that the CDMA enables multiuser communications along with efficient utilization of available spectrum and transmitter power in wireless systems [4, 17, 28–32], and has been proposed for use in traditional as well as future generation wireless systems.

Now a simple scenario is considered where a user  $k$  has single signature  $\mathbf{s}_k$  (i.e., the transmit signal vector  $\mathbf{x}_k = b_k \sqrt{p_k} \mathbf{s}_k$ ), and the system model (I.1.2) becomes

$$\mathbf{r} = \sum_{k=1}^K b_k \sqrt{p_k} \mathbf{H}_k \mathbf{s}_k + \mathbf{w} \quad (\text{I.1.4})$$

and for ideal channel consideration (i.e., the channel matrix  $\mathbf{H}_k = \mathbf{I}, \forall k$ ), the received signal (I.1.4) can be written as

$$\mathbf{r} = \sum_{k=1}^K b_k \sqrt{p_k} \mathbf{s}_k + \mathbf{w} = \mathbf{S} \mathbf{P}^{1/2} \mathbf{b} + \mathbf{w} \quad (\text{I.1.5})$$

where  $\mathbf{S} = [\mathbf{s}_1, \dots, \mathbf{s}_k, \dots, \mathbf{s}_K]$  is the  $N \times K$  signature matrix having as columns the unit-norm signatures  $\{\mathbf{s}_k\}_{k=1}^K$  of active users in the system,  $\mathbf{P} = \text{diag}\{p_1, \dots, p_k, \dots, p_K\}$  is the  $K \times K$  diagonal matrix containing transmit powers of active users, and  $\mathbf{b} = [b_1 \dots b_k \dots b_K]^\top$  is the vector containing the information symbols transmitted by users.

Consider that a receiver uses a linear receiver vector  $\mathbf{c}_k$ , which is assumed to be unit norm, to estimate the symbol transmitted by a given user  $k$ , and the estimate is computed as

$$\tilde{b}_k = \mathbf{c}_k^\top \mathbf{r} = \underbrace{b_k \sqrt{p_k} \mathbf{c}_k^\top \mathbf{s}_k}_{\text{desired signal}} + \underbrace{\mathbf{c}_k^\top \left( \sum_{\ell=1, \ell \neq k}^K b_\ell \sqrt{p_\ell} \mathbf{s}_\ell + \mathbf{n} \right)}_{\text{interference + noise}} \quad (\text{I.1.6})$$

Then, from the perspective of user  $k$ , the signal-to-interference-plus-noise-ratio (SINR) for the received signal in (I.1.5) can be expressed as

$$\gamma_k = \frac{p_k (\mathbf{c}_k^\top \mathbf{s}_k)^2}{\sum_{\ell=1, \ell \neq k}^K p_\ell (\mathbf{c}_k^\top \mathbf{s}_\ell)^2 + E[(\mathbf{c}_k^\top \mathbf{n})^2]} \quad (\text{I.1.7})$$

Formally, the denominator of the SINR in (I.1.7) is defined as the user interference

function

$$i_k = \sum_{\ell=1, \ell \neq k}^K p_\ell (\mathbf{c}_k^\top \mathbf{s}_\ell)^2 + E[(\mathbf{c}_k^\top \mathbf{n})^2] = \mathbf{c}_k^\top \underbrace{\left( \sum_{\ell=1, \ell \neq k}^K p_\ell \mathbf{s}_\ell \mathbf{s}_\ell^\top + \mathbf{W} \right)}_{\mathbf{R}_k} \mathbf{c}_k = \mathbf{c}_k^\top \mathbf{R}_k \mathbf{c}_k \quad (\text{I.1.8})$$

where

$$\mathbf{R}_k = \sum_{\ell=1, \ell \neq k}^K p_\ell \mathbf{s}_\ell \mathbf{s}_\ell^\top + \mathbf{W} = \underbrace{\mathbf{S} \mathbf{P} \mathbf{S}^\top + \mathbf{W}}_{\mathbf{R}} - p_k \mathbf{s}_k \mathbf{s}_k^\top = \mathbf{R} - p_k \mathbf{s}_k \mathbf{s}_k^\top \quad (\text{I.1.9})$$

is the correlation matrix of the interference-plus-noise experienced by user  $k$ , and

$$\mathbf{R} = E[\mathbf{r} \mathbf{r}^\top] = \mathbf{S} \mathbf{P} \mathbf{S}^\top + \mathbf{W} \quad (\text{I.1.10})$$

is the correlation matrix of the received signal in equation (I.1.5). It is important to note that the interference function  $i_k$  in (I.1.8) for a given user  $k$  depends explicitly on user  $k$ 's receiver filter  $\mathbf{c}_k$  as well as on all the other user signatures  $\mathbf{s}_\ell$  and powers  $p_\ell$ , for  $\ell \neq k$ . Furthermore, the interference function  $i_k$  depends implicitly on user  $k$ 's signature, since the receiver filter  $\mathbf{c}_k$  depends on  $\mathbf{s}_k$ , however, it does not depend on user  $k$ 's power. It is also noted that interference function has been defined in previous work on power control [33] as well as both power and signature adaptation [34, 35]

In the case of matched filter (MF) receivers  $\mathbf{c}_k = \mathbf{s}_k$ ,  $\forall k$ , the interference function (I.1.8) can be expressed as

$$i_k^{\text{MF}} = \mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k \quad (\text{I.1.11})$$

and the user  $k$  SINR expression (I.1.7) can be expressed as

$$\gamma_k^{\text{MF}} = \frac{p_k}{\mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k} \quad (\text{I.1.12})$$

By looking at the equation (I.1.12), it can be seen that the maximization of SINR of user  $k$  is equivalent to minimization of denominator term of SINR for a given power  $p_k$ . Therefore, in this case, user  $k$  signature is updated by replacing it with the eigenvector corresponding to the minimum eigenvalue of the matrix  $\mathbf{R}_k$  which implies maximization of the user SINR  $\gamma_k$  [9], and the method is known as the Eigen algorithm.

The MMSE (minimum mean square error) algorithm for interference avoidance is an alternative to the matched filter based approach. The idea behind the MMSE filter [36] is to minimize the mean squared error (MSE) between the filter output and the transmitted information symbol, and is the optimal linear multiuser detector that maximizes also the SINR [4, 36, 37]. The unit norm MMSE receiver filter  $\mathbf{c}_k$  is [4, 36]

$$\mathbf{c}_k = \frac{\mathbf{R}_k^{-1} \mathbf{s}_k}{(\mathbf{s}_k^\top \mathbf{R}_k^{-2} \mathbf{s}_k)^{1/2}}, \quad \forall k \quad (\text{I.1.13})$$

In this case, the interference expression (I.1.8) for user  $k$  becomes

$$i_k^{\text{MMSE}} = \frac{\mathbf{s}_k^\top \mathbf{R}_k^{-1} \mathbf{s}_k}{\mathbf{s}_k^\top \mathbf{R}_k^{-2} \mathbf{s}_k} \quad (\text{I.1.14})$$

and the corresponding SINR expression (I.1.7) for user  $k$  with MMSE receivers is [36]

$$\gamma_k^{\text{MMSE}} = p_k \mathbf{s}_k^\top \mathbf{R}_k^{-1} \mathbf{s}_k \quad (\text{I.1.15})$$

In this case, user signatures are replaced by their corresponding MMSE receiver filters  $\mathbf{c}_k$  which implies the maximization of the user SINR  $\gamma_k^{\text{MMSE}}$  [18].

Formally, interference avoidance algorithms (the Eigen algorithm and the MMSE algorithm) for transmitter optimization are stated as follows:

1. Start with randomly initialized user signature ensemble specified by  $\{\mathbf{s}_k\}_{k=1}^K$ , powers  $\{p_k\}_{k=1}^K$ , the noise covariance matrix  $\mathbf{W}$  and the tolerance  $\epsilon$ .
2. For each user  $k = 1, \dots, K$ 
  - a) Compute the autocorrelation matrix  $\mathbf{R}_k$  of the interference-plus-noise experienced by user  $k$
  - b) Replace current signature of user  $k$  with the minimum eigenvector of  $\mathbf{R}_k$   
for *Eigen algorithm*

OR

Replace current signature of user  $k$  with the MMSE receiver filter for *MMSE interference avoidance*
3. Repeat Step 2. until a fixed point is reached

Numerically, a fixed point of the algorithm is defined with respect to a stopping criteria, and a fixed point is reached when the difference between two consecutive values of the stopping criteria is within a specified tolerance. For distributed implementation of the algorithm, the stopping criteria can use the local information available to individual users. Therefore, for distributed implementation, numerically the fixed point is reached when the Euclidean distance between a given signature and its corresponding replacement is within some specified tolerance  $\epsilon$ . For centralized

implementation, the algorithm stops if the change in sum capacity is within specified tolerance  $\epsilon$  [4]. The convergence of the algorithms has been investigated analytically in [8,38]. At the fixed point, the signatures correspond to a Generalized Welch Bound Equality (GWBE) signature ensemble [4, 9, 17, 18] in the case of overloaded system (i.e.,  $K > N$ ), and they satisfy the equality

$$\mathbf{SPS}^\top = \frac{K}{N}\mathbf{I} \quad \text{if } K > N \quad (\text{I.1.16})$$

Otherwise, in the case of underloaded systems (the number of users is less than the number of dimensions, i.e.,  $K < N$ ) and equally loaded (the number of users is equal to the number of dimensions, i.e.,  $K = N$ ), the choice of orthogonal signatures results in the optimal fixed point for interference avoidance algorithms.

Application of interference avoidance algorithms to multiple access uplink scenarios in which users are assigned multiple signatures for transmission similar to [39,40], and for non-ideal channels have been investigated in [41,42]. In this case the algorithms converge to sum capacity with the maximization of signature ensembles.

It is worth noting that signals, with non-orthogonal signatures, transmitted at the same time by different transmitters interfere with each other at the receiver and thus create multiple access interference (MAI) [37]. Therefore, in wireless systems, minimization of the MAI effects to increase spectral efficiency and meet specified QoS requirements for reliable communication are central problems. In order to solve those problems, the transmitters in an uplink of the system may adjust their signatures and/or powers, and numerous algorithms have been proposed for signature and/or power adaptation in the systems [4, 17, 30–32]. Furthermore, uplink signals in the



system are also affected by fading channels and propagation through multipath which further degrades their quality at the receiver. Algorithms for signature optimization for uplink systems with multipath are discussed in [42–44]. However, these algorithms focus exclusively on the optimization of uplink signatures, and do not adapt power and specify QoS requirements for users in the system.

In order to have successful communication, the instantaneous SINR of a link should be greater than or equal to minimum SINR. If the instantaneous SINR is lower than minimum SINR requirement, the user should be either dropped or handed over to other base station. The alternative approach of this scheme is to adjust the power level through an adaptive algorithm in order to meet target SINR along with precoder adaptation. Therefore, in order to perform optimal resource allocation in wireless systems, wireless systems need active management of wireless resources through adaptive algorithms by using precoder/signature update for interference mitigation and power control to match the target SINR in an uplink of a wireless system. Where as in interference systems, power control along with precoder adaptation helps to control the interference created to other links/users.

## **I.2 PROBLEM STATEMENT**

The main objectives of this dissertation are to:

- i. Investigate adaptive algorithms for decentralized based precoder adaptation with power control in multiple access channels by using:
  - a. Gradient descent based approach, and
  - b. Game theory with incremental strategies.

- ii. Investigate adaptive algorithms for decentralized precoder adaptation with power control in interference channels with their applications to cognitive radio systems for dynamic spectrum sharing.
- iii. Analyze the effect of limited feedback for transmitter adaptation in decentralized implementation of the algorithms.
- iv. Analyze the effect of fading channels in the implementation of transmitter adaptation algorithms.

### **I.3 DISSERTATION OUTLINE**

The dissertation is organized as follows: Chapter II describes the gradient descent based signature adaptation for interference avoidance and power control for target SINR matching for active users in the uplink of a wireless system. There are two scenarios; one is with ideal channel and the other is with non-ideal channel between the user and the base station. the channel between each user and the base station is assumed to be known. A matched filter is assumed to be used at the receiver which simplifies the receiver complexity. In an uplink system, transmitters' symbols are assumed to be independent of each other. In order to implement the proposed algorithms in a distributed manner, the common receiver broadcasts the correlation matrix of the received signal or the received signal itself to the users. The work presented in Chapter II and III maintains these assumptions.

Chapter III presents the incremental strategies for transmitter optimization by adapting signature and power control using a game theoretic approach for interference avoidance and power control in non-ideal channel scenario. It is noted that the

gradient descent based algorithm does not always converge to optimal point, since it might be trapped at sub-optimal points. Therefore the game theoretic approach for signature and power adaptation is applied since it gives a socially optimal point called a Nash equilibrium. In this approach, the optimal point for the algorithm is presented. We also present the tracking ability of algorithm in terms of number of active users and/or different QoS requirements in terms of different target SINR requirements.

Chapter IV describes the transmitter optimization and power control in interference systems, and the proposed approach is applicable to both conventional and cognitive radio systems. In dynamic spectrum access, unlicensed secondary users access the spectrum (which is not licensed to them) without creating harmful interference to licensed primary users. Both analytical and simulation results are presented for an optimal point of the proposed algorithm.

Chapter V presents the effect of the transmitter optimization with limited feedback. The simulation results are presented for two scenarios: First, by quantizing the interference information and feeding it back to transmitter for the signature adaptation. In this case, the interference information is quantized using both uniform and non-uniform quantization and compare them in terms of distortion introduced by them. Second, the effect of limited feedback is studied by using predictive vector quantization (PVQ) for incremental updates of precoders. The effect of PVQ is also compared with that of random vector quantization (RVQ) of precoders.

Chapter VI deals with the average channel and each channel realization of fast fading channels for distributed transmitter optimization algorithms as well as with the comparison between them in terms of outage probabilities.

Chapter VII concludes the dissertation with discussions and future directions of research work.

Most of the results in this dissertation have been presented in part at various IEEE conferences and published in part in IEEE transactions. The work presented in Chapter II was published in proceedings of The IEEE 41st Annual Asilomar Conference on Signals, Systems, and Computers [45] and the 5th IEEE Consumer Communications and Networking Conference [46]. The results presented in Chapter III were presented in part at the 2008 IEEE Global Telecommunications Conference [47] and published in the IEEE Transactions on Systems, Man, and Cybernetics: Part B [48]. Results of Chapter IV were presented in part at The 2nd IEEE International Workshop on Dynamic Spectrum Access and Cognitive Radio Networks [49]. Chapter V was presented in part at the IEEE 41st Annual Asilomar Conference on Signals, Systems and Computers [50].

This dissertation uses IEEE Transaction style for the bibliography and citation.

## CHAPTER II

### GRADIENT DESCENT BASED TRANSMITTER ADAPTATION WITH POWER CONTROL

This chapter presents distributed Gradient Descent (GD) based transmitter adaptation for interference avoidance, and power update to meet a QoS requirement. Specifically, this chapter deals with optimization of uplink precoders and powers for a multiple access wireless system in distributed manner in which the target SINRs are met to satisfy the QoS requirements of active users.

Section II.1 deals with the problem statement for precoder adaptation with power control for an uplink of a wireless system in multiple access scenario. We investigate the gradient descent based transmitter optimization with power control for ideal channel case in Section II.2, and the method is extended to non-ideal channel scenario in Section II.3 where the channel between users and the base station are explicitly considered. In both cases, we perform the GD-based precoder update and power control to meet minimum SINR requirement. Simulation results are also presented which show the convergence of the proposed algorithm, and tracking ability of the algorithm for variable number of active users and/or variable QoS requirements.

#### II.1 SYSTEM MODEL AND PROBLEM STATEMENT

It is worth noting that the precoder (or waveform) design method in transmitter optimization performs interference avoidance which may not satisfy the user QoS requirement. In order to have reliable communication, the instantaneous SINR should

be greater than or equal to target SINR for each user which is known as QoS requirement. If the calculated SINR is less than the required minimum SINR, there will be errors in transmission and the transmitter may need to re-transmit the information. In such cases, the transmitter will create burden/overhead in the network resulting in high interference to other active users. In this case, users who do not satisfy minimum SINR requirements either should be dropped [51,52] from the system or handed over to other base station [53,54]. However, in this chapter, an alternative approach is considered in which the user can satisfy minimum SINR requirement by adapting its transmit power along with precoder adaptation to meet QoS requirements.

A method is investigated for the precoder adaptation for transmitter optimization and power control for QoS requirement with specified user target SINR  $\gamma_k^*$  requirement,  $\forall k$ , where the target SINRs are admissible in the multiple access channel of uplink wireless system with signal dimension (or processing gain in the case of CDMA system)  $N$  if and only if the sum of their effective bandwidths

$$e(\gamma_k^*) = \frac{\gamma_k^*}{1 + \gamma_k^*}, \quad \forall k \quad (\text{II.1.1})$$

is less than the signal dimension  $N$  [17,55,56], i.e.

$$\sum_{\forall k} \frac{\gamma_k^*}{1 + \gamma_k^*} \leq N \quad (\text{II.1.2})$$

It is important to note that the Eigen-algorithm and the MMSE algorithm presented in Chapter I may result in abrupt change in user precoders and/or powers [4,32,57], which motivates us to find alternative methods where the precoder are

replaced in the direction of optimal ones using gradient descent approach. In order to derive a distributed algorithm in which individual users adjust their precoders for transmitter optimization (or interference avoidance) and their powers to achieve specified target SINRs  $\{\gamma_k\}$ ,  $\forall k$ , a method is proposed with two separate updates for all users which consist of

- a gradient based precoder update to decrease the effective interference and
- power update designed to meet the specified target SINRs.

## II.2 IDEAL CHANNEL SCENARIO

In this section, a simple scenario is considered where the channel between a user and the base station is considered as ideal, and the received signal at the base station receiver is given in (I.1.5). The SINR expression for matched filter receivers is given in (I.1.12) where the denominator is the effective interference, i.e.

$$i_k = \mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k = \frac{\mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k}{\mathbf{s}_k^\top \mathbf{s}_k} \quad (\text{II.2.1})$$

It is well known that the Rayleigh quotient of a matrix is minimized by the minimum eigenvector corresponding to the minimum eigenvalue of the matrix [58], and for a user  $k$  decrease in interference  $i_k$  results in increase in SINR. It is noted that  $i_k$  is a quadratic form with a positive definite matrix  $\mathbf{R}_k$ . As a consequence,  $i_k$  is strictly convex over the  $N$ -dimensional unit sphere  $\{\mathbf{s}_k | \mathbf{s}_k \in \mathbb{R}^N, \|\mathbf{s}_k\| = 1\}$ , and has the global minimum point equal to the minimum eigenvalue of  $\mathbf{R}_k$  achieved for precoder  $\mathbf{s}_k$  equal to the corresponding minimum eigenvector of  $\mathbf{R}_k$  [58]. Therefore, the global

minimum point can be approached using the gradient descent update iteration

$$\mathbf{s}_k(n+1) = \mathbf{s}_k(n) - \mu \frac{\partial i_k}{\partial \mathbf{s}_k} \big|_{\text{instant } n} \quad (\text{II.2.2})$$

where  $n$  denotes the  $n$ th time instant and  $\mu$  is suitably chosen algorithmic gradient constant.

It is worth noting that, in practical systems, the precoder and power should be changed in small increments/decrements allowing corresponding changes in the receiver matched filter. This will avoid steep changes that may not be tracked by the receiver and could result in increased probability of error at the receiver or even connection loss between transmitter and receiver. Such incremental updates is derived which uses GD-based adaptation for the user precoder and user power. For incremental changes in user precoders, the gradient of interference function  $i_k$  with respect to user precoder  $\mathbf{s}_k$  is

$$\frac{\partial i_k}{\partial \mathbf{s}_k} \big|_{\text{instant } n} = 2\mathbf{R}_k(n)\mathbf{s}_k(n) \quad (\text{II.2.3})$$

Then, from equations (II.2.2) and (II.2.3), the actual precoder that is unit norm update for the next instance  $n+1$  is as

$$\mathbf{s}_k(n+1) = \frac{[\mathbf{I} - 2\mu\mathbf{R}_k(n)]\mathbf{s}_k(n)}{\|[\mathbf{I} - 2\mu\mathbf{R}_k(n)]\mathbf{s}_k(n)\|} \quad (\text{II.2.4})$$

It is worth noting that the precoder update procedure in (II.2.4) does not need complex calculations like determining maximum or minimum eigenvectors and/or taking the inverse of  $\mathbf{R}_k$  for the MMSE updates for interference avoidance [42–44].



Therefore, this method has computational advantages over other interference avoidance updates. Furthermore, the convergence of the GD-base approach is guaranteed since the process of increment or decrement in precoder guarantees the global convergence [4, 57, 59] because of convexity.

After precoder adaptation, the new SINR at instant  $n + 1$  is

$$\gamma_k(n + 1) = \frac{p_k(n)}{\mathbf{s}_k^\top(n + 1)\mathbf{R}_k(n)\mathbf{s}_k(n + 1)} \quad (\text{II.2.5})$$

and the power update can be performed to match the specified target SINR  $\gamma_k^*$  for user  $k$  as

$$p'_k(n + 1) = \gamma_k^* \mathbf{s}_k^\top(n + 1)\mathbf{R}_k(n)\mathbf{s}_k(n + 1) \quad (\text{II.2.6})$$

If the instantaneous SINR  $\gamma_k(n + 1)$  in (II.2.5) is below the specified target SINR, the power update equation (II.2.6) results in increase in user power. Similarly, if the instantaneous SINR  $\gamma_k(n + 1)$  in (II.2.5) after precoder update is above the specified target SINR, the power update equation (II.2.6) results in decrease in user power. Furthermore, it is important to note that when the resulting power value  $p'_k(n + 1)$  in (II.2.6) is above the maximum allowed power  $p_k^{max}$  by which user  $k$  is allowed to transmit, the new power value  $p_k(n + 1)$  will be updated with the maximum allowed power value  $p_k^{max}$ , i.e.

$$p_k(n + 1) = \min\{p_k^{max}, p'_k(n + 1)\} \quad (\text{II.2.7})$$

The power value is updated to meet target SINR where the newly adapted power should not be greater than the maximum allowed power level  $p_k^{max}$  for given user  $k$ .

### II.2.1 Algorithm

Based on the above mentioned precoder and power updates, respectively, in equations (II.2.4) and (II.2.6), the gradient-descent interference avoidance with SINR matching algorithm is formally stated as follows:

**Algorithm: The GD-Based Algorithm for Ideal Channels**

1. Input data: initial user precoder matrix  $\mathbf{S}$ , power matrix  $\mathbf{P}$ , noise covariance matrix  $\mathbf{W}$ , desired target SINRs  $\{\gamma_1^*, \gamma_2^*, \dots, \gamma_K^*\}$ , maximum user power  $p_k^{max}$ , and the constant  $\mu$ .
2. If admissibility condition in equation (II.1.2) is satisfied continue with Step 3, else STOP.
3. For each user  $k = 1, \dots, K$  do
  - Compute  $\mathbf{R}_k$  using equation (I.1.9).
  - Update user  $k$ 's precoder to reduce effective interference using equation (II.2.4).
  - Compute power required to match specified target SINR using equation (II.2.6).
  - If  $p_k(n+1)$  implied by equation (II.2.6) is less than  $p_k^{max}$  update user  $k$ 's power with this value. Otherwise assign power to  $p_k^{max}$ .
4. Perform the iteration until a fixed point is reached.

Step 3 of the algorithm consists of  $K$  individual user updates in which all user precoders and powers are updated one time, and is called an ensemble iteration. In

this algorithm, the  $\mathbf{R}$  is assumed to be fed back through some error-free feedback channel so that individual users can adapt their precoders to avoid interference and powers to meet their target SINRs, and thus the proposed algorithm is implemented in distributed manner.

The algorithm stops at fixed point when the stopping criteria is met. Generally, a fixed point of the algorithm is reached when the precoder and power updates at this point imply no change in the value of the stopping criterion. Specifically as stopping criterion, the difference between the actual SINR at the receiver and the target SINR, as well as the Euclidean distance between a given precoder and its corresponding replacement are used.

### II.2.2 Simulations and Numerical Results

This section presents the results obtained from extensive simulations for various scenarios to show the convergence of the algorithm as well as the variation of user powers and SINRs. The fixed-point properties are also illustrated with optimal GWBE ensembles [16].

### II.2.3 Algorithm Convergence

First of all, the effect of algorithmic parameter  $\mu$  in the performance are investigated. The algorithm has run for the gradient descent based transmitter adaptation with matching target SINRs with initial random user precoders, random powers, and admissible target SINRs. With the extensive simulations, it has observed that the algorithm reaches a GWBE ensemble of user precoders and powers [16] within some tolerance limits that can be adjusted through parameters  $\mu$  and  $\epsilon$  as it is the case

in general with gradient-based algorithms. As expected, the result is consistent with that of related interference avoidance (Eigen and MMSE) algorithms which always converge to GWBE ensembles of user precoders and powers from random initialization [4]. In the simulation, for the stopping criteria and convergence of the algorithm, both criteria were used which are mentioned in Section II.2.1, i.e.

- the Euclidean distance between a given precoder and its corresponding replacement in two consecutive iterations, and
- the difference between the actual SINR and the desired target SINR.

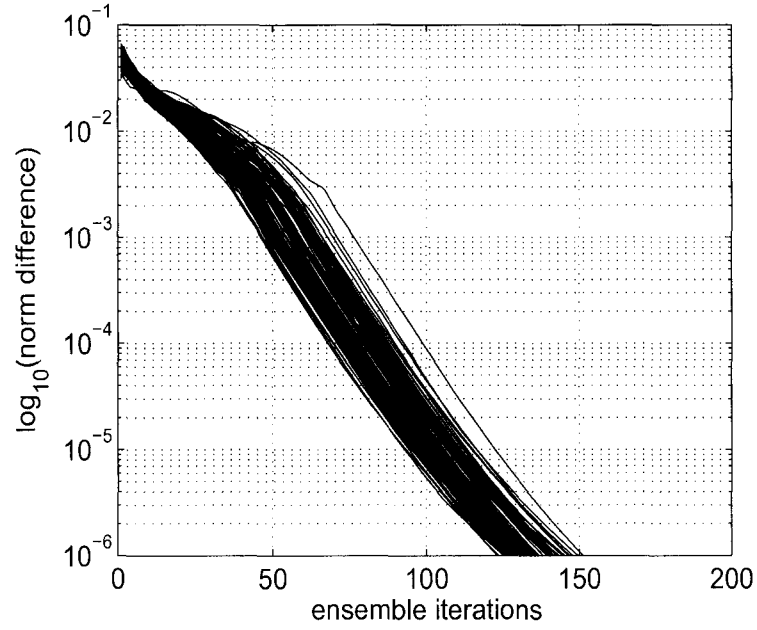
When the first criterion is used to stop the algorithm at fixed point, the speed of convergence to tight norm difference tolerances ( $\|\mathbf{s}_k(n+1) - \mathbf{s}_k(n)\| \leq 10^{-6}, \forall k$ ) depends, as it was expected, on the value of the gradient constant  $\mu$ . It is observed that for smaller  $\mu$  value, the algorithm takes more steps to reach to a fixed point. When the second criterion is used, it is noted that the convergence speed of the algorithm does not really depends on algorithm/gradient constants. Instead, the precision with which the target SINRs are met after the algorithm settles down and no more changes in user precoder and powers occur depends on the specified target SINR values. It has observed the following with the help of extensive simulations: *First*, when target SINR values, which are admissible in the system, are selected in such a way that the sum of user effective bandwidths in equation (II.1.2) is not close to  $N$  (that is sum of user effective bandwidths is much smaller than  $N$ ), then the given target SINRs can be achieved with arbitrary precision as shown in Figure 2 which is typical for all the simulations. *Second*, when the sum of effective bandwidths corresponding to the specified and admissible target SINRs is very close to the upper

bound  $N$ , then the instantaneous SINRs are reached only with limited precision of the order of  $10^{-2}$ , as shown in Figure 3, which is also typical for all simulations we have performed.

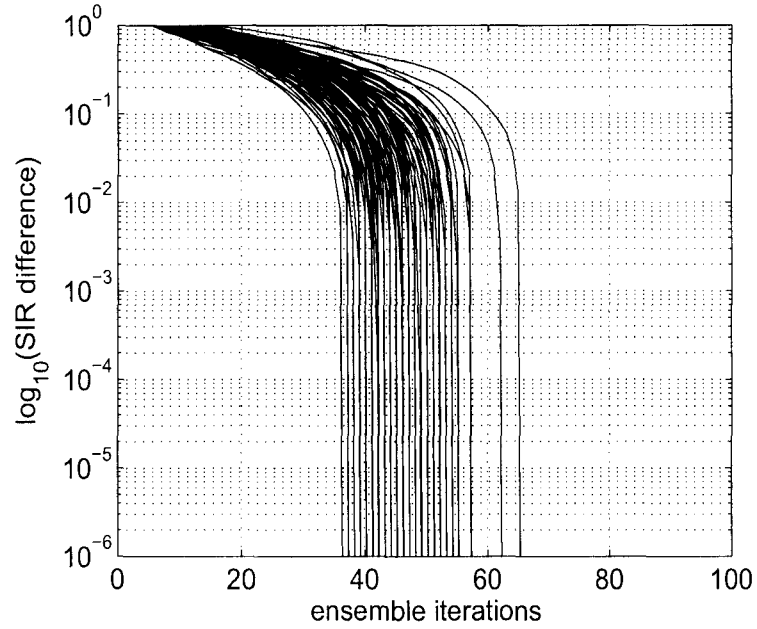
In Figure 2 and 3, it is noted that the convergence occurs slower in both precoders and SINRs when the sum of user effective bandwidths is very close to the upper bound  $N$  when the same value of the gradient constant  $\mu$  is used. This is another important characteristic observed in all simulation results we have performed. It has also observed that the number of precoder updates per user does not increase with proportionately increasing number of users and signal dimensions, and the number of ensemble iterations needed for convergence stays approximately constant for given ratio of number of users and signal dimensions. We have considered a simulation scenario with signal dimension  $N$  ranging from 5 to 50 with the increment of 5 and the ratio  $K/N \simeq 1.5$ , and observed that the algorithm converges in 300 and 400 ensemble iterations when the sum of effective bandwidths was close to its upper bound  $N$  in equation (II.1.2), and it converged in 60 to 80 ensemble iterations when the sum of effective bandwidths was not close to its upper bound.

#### II.2.4 Variation of User Powers and SINRs, and Fixed-Point Properties

This section shows the variation in user power and SINR. An uplink wireless system operating in a signal space of dimension  $N = 5$  with  $K = 7$  users is considered, and the AWGN with variance  $\sigma^2 = 0.1$ . User precoder matrix is initialized randomly and initial user powers are also selected randomly between 0 and the maximum allowed power  $p_k^{max} = 10, \forall k$ . The tolerance for precoder convergence is assumed to be  $\epsilon = 10^{-6}$ .



(a)



(b)

Fig. 2: Precoder and SINR convergence for 100 trials of the proposed algorithm for a system with  $K = 15$  users in  $N = 10$  dimensions, target SINRs for all users equal to 1.95, and gradient constant  $\mu = 10^{-3}$ . Sum of effective bandwidths is 9.9153 – roughly 10% below the upper bound – and target SINRs can be achieved with arbitrary precision.

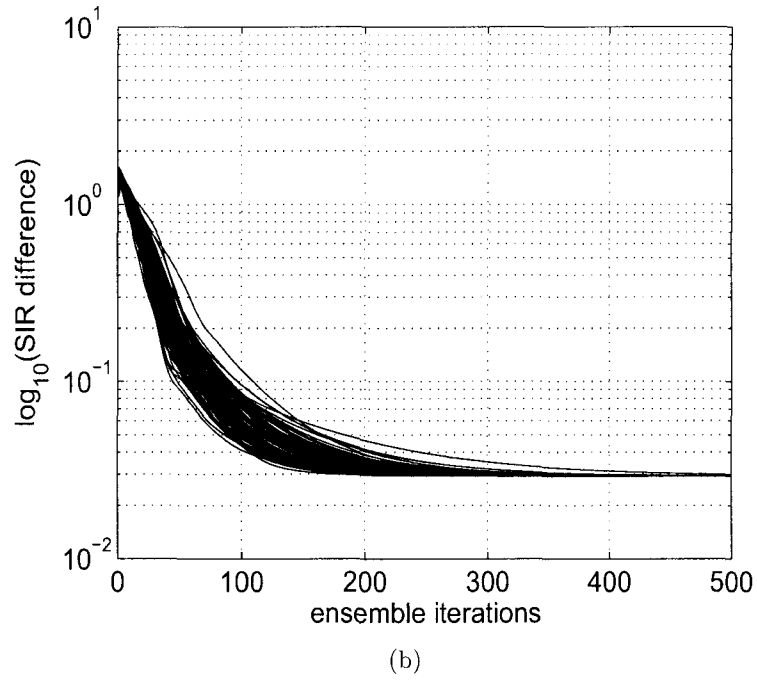
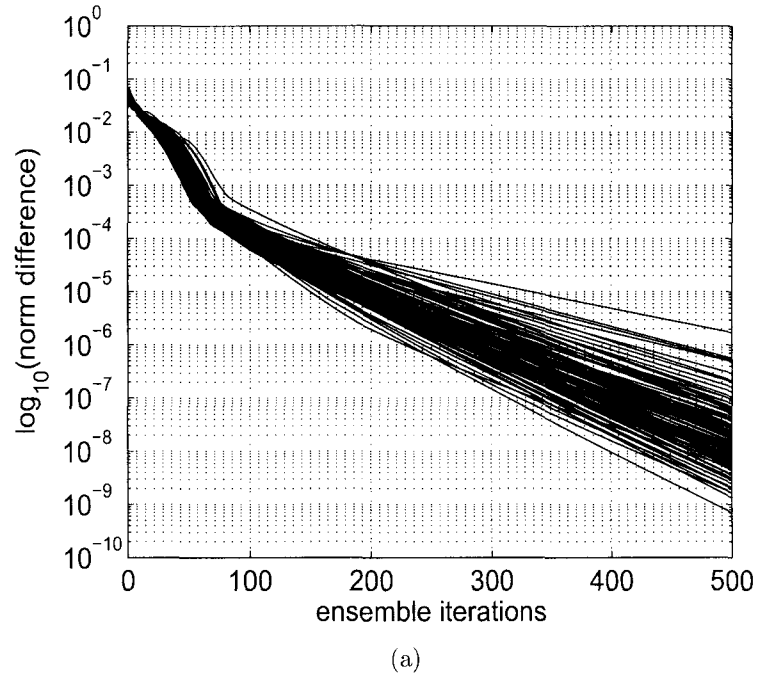


Fig. 3: Precoder and SINR convergence for 100 trials of the proposed algorithm for a system with  $K = 15$  users in  $N = 10$  dimensions, target SINRs for all users equal to 1.99, and gradient constant  $\mu = 10^{-3}$ . Sum of effective bandwidths is 9.9833 – only about 1% below the upper bound – and target SINRs are achieved with limited precision.

Two scenarios are considered: In the first scenario, users target SINRs, which satisfy the admissibility condition in equation (II.1.2), are as following

$$\{\gamma_1^*, \gamma_2^*, \gamma_3^*, \gamma_4^*, \gamma_5^*, \gamma_6^*, \gamma_7^*\} = \{3.25, 3, 2.75, 2.5, 2.25, 2, 1.75\} \quad (\text{II.2.8})$$

There are no over-sized users and the sum of effective bandwidths is 4.9577 which is less than the upper bound  $N = 5$ , and is not close to  $N$ . In this scenario, a gradient constant  $\mu = 0.01$  is considered and then the algorithm is applied. The algorithm converged in approximately 30 ensemble iterations (corresponding to 210 individual precoder updates). The user SINR variation during the update is shown in Figure 4(a). The gradient constant is then changed to  $\mu = 0.001$  and the algorithm is applied, and the resulting user SINR variation is shown in Figure 4(b). As seen from Figure 4(b), smaller  $\mu$  value slows down the convergence, however, it also reduces the variance of SINR variations which might be a desirable feature in practical implementations.

It is noted that, regardless of chosen gradient constant  $\mu$ , the algorithm yields same final precoder matrix  $\mathbf{S}_1$  in equation (II.2.11) and power matrix

$$\mathbf{P}_1 = \text{diag}\{9.0345, 8.8605, 8.6634, 8.4384, 8.1788, 7.8758, 7.5179\}$$

and the weighted correlation matrix  $\mathbf{S}_1 \mathbf{P}_1 \mathbf{S}_1^\top + \mathbf{W} = 11.814 \mathbf{I}_5$ , which is within the tolerance of  $\mathcal{O}(10^{-3})$  with the corresponding GWBE values implied by the algorithms in [16].



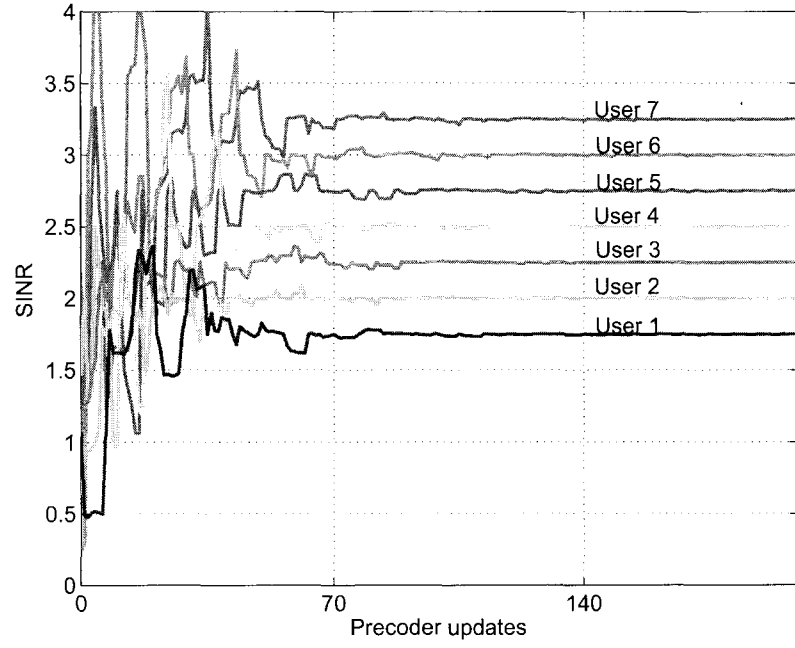
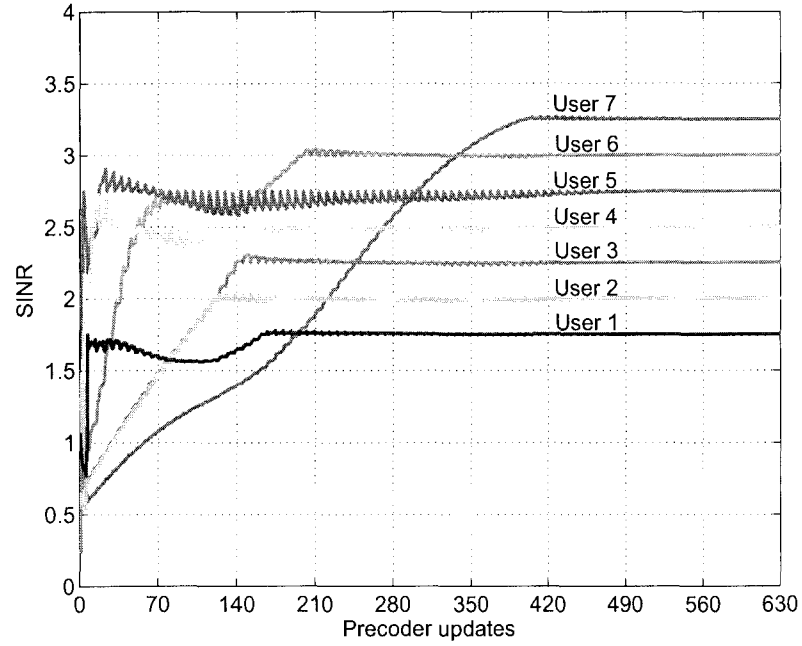
(a)  $\mu = 10^{-2}$ (b)  $\mu = 10^{-3}$ 

Fig. 4: SINR variation for the system with  $K = 7$  users in  $N = 5$  dimensions, target SINRs  $\{3.25, 3, 2.75, 2.5, 2.25, 2, 1.75\}$ , for different gradient constants  $\mu$ . One ensemble iteration is equal to 7 precoder updates in this case.

In the second scenario, the target SINRs for users are as

$$\{\gamma_1^*, \gamma_2^*, \gamma_3^*, \gamma_4^*, \gamma_5^*, \gamma_6^*, \gamma_7^*\} = \{15, 1.5, 1.5, 1.5, 1.5, 1.5, 1.5\} \quad (\text{II.2.9})$$

which imply that user 1 is over-sized according to [16]. The algorithm was applied with above setup, and the algorithm yielded optimal precoder matrix  $\mathbf{S}_2$  in equation (II.2.12) and power matrix

$$\mathbf{P}_2 = \text{diag}\{1.5, 0.6, 0.6, 0.6, 0.6, 0.6, 0.6\}$$

such that over-sized user 1 is orthogonal to all the other users as can be seen from the precoder correlation matrix  $\mathbf{S}_2^\top \mathbf{S}_2$  in equation (II.2.13). It is noted that this result agrees with the properties of optimal precoder ensembles in [16] which indicates that over-sized users must have private channels over which they transmit at the minimum power required to achieve their corresponding target SINRs. It is also noted that weighted correlation matrix  $\mathbf{S}_2 \mathbf{P}_2 \mathbf{S}_2^\top$  is not exactly in the form given in [16], however, can be converted to the same form using the linear transformation implied by the left singular vectors [58] of the matrix  $\mathbf{S}_2$  in (II.2.12) to align the signal space axes to the over-sized user. The transformation is applied which results in the transformed precoder matrix  $\tilde{\mathbf{S}}_2$  as shown in equation (II.2.14) which is in the form of [17] in which non-over-sized users 2–7 share dimensions 1–4 and over-sized user 1 uses signal dimension 5. For non-over-sized users, the weighted correlation matrix [16] is as

$$\sum_{j=2}^7 p_j \tilde{\mathbf{s}}_j \tilde{\mathbf{s}}_j^\top = 0.9 \mathbf{I}_4 \quad (\text{II.2.10})$$

Precoder matrix yielded by the algorithm for the first scenario in Section II.2.4, with no oversized users:

$$\mathbf{S}_1 = \begin{bmatrix} -0.2771 & 0.8654 & -0.2527 & -0.2680 & 0.4095 & 0.2838 & -0.4029 \\ -0.7015 & 0.2836 & 0.3208 & 0.6844 & -0.4232 & 0.0921 & 0.1549 \\ -0.0844 & 0.2588 & -0.5990 & 0.1284 & 0.0325 & -0.9008 & 0.4330 \\ 0.0864 & 0.1489 & 0.6783 & 0.1127 & 0.8062 & -0.1877 & 0.4843 \\ -0.6454 & -0.2856 & -0.1202 & -0.6562 & 0.0466 & 0.2537 & 0.6258 \end{bmatrix} \quad (\text{II.2.11})$$

Precoder matrix yielded by the algorithm for the second scenario in Section II.2.4, with oversized user 1:

$$\mathbf{S}_2 = \begin{bmatrix} -0.5468 & 0.6511 & 0.0681 & 0.1032 & 0.3167 & 0.3663 & -0.6147 \\ 0.1139 & 0.6327 & 0.2169 & 0.6543 & -0.5777 & 0.2072 & 0.4780 \\ -0.5551 & -0.1594 & -0.2750 & 0.5482 & 0.1084 & -0.7526 & 0.2409 \\ 0.0896 & 0.2679 & 0.8638 & -0.1405 & 0.6293 & -0.3753 & 0.3369 \\ -0.6098 & -0.2804 & 0.3558 & -0.4908 & -0.3978 & 0.3401 & 0.4714 \end{bmatrix} \quad (\text{II.2.12})$$

Correlation matrix of precoders yielded by the algorithm for the second scenario in Section II.2.4, showing that over-sized user 1 is orthogonal to the other, non-over-sized users:

$$\mathbf{S}_2^\top \mathbf{S}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0.3571 & 0.4938 & 0.1036 & 0.2936 & -0.1782 \\ 0 & 0.3571 & 1 & -0.2977 & 0.2685 & 0.0737 & 0.4543 \\ 0 & 0.4938 & -0.2977 & 1 & -0.1790 & -0.3534 & 0.1027 \\ 0 & 0.1036 & 0.2685 & -0.1790 & 1 & -0.4568 & -0.4201 \\ 0 & 0.2936 & 0.0737 & -0.3534 & -0.4568 & 1 & -0.2736 \\ 0 & -0.1782 & 0.4543 & 0.1027 & -0.4201 & -0.2736 & 1 \end{bmatrix} \quad (\text{II.2.13})$$

The results of second scenario is concluded by noting that user 1 had no *a priori* knowledge of its over-sized status in the system, and performed the same adaptations as the other non-over-sized users which resulted in the right ensemble of precoders and powers. However, in [16] it is required to have *a priori* knowledge of the over-sized status to obtain the right precoders and powers for over-sized users.

Transformed precoder matrix for second scenario in Section II.2.4, with signal space axes aligned to the over-sized user:

$$\tilde{\mathbf{S}}_2 = \begin{bmatrix} 0 & -0.4839 & -0.9595 & 0.0362 & -0.1317 & -0.0183 & -0.5712 \\ 0 & 0.8151 & -0.0334 & 0.3565 & 0.3689 & 0.2707 & -0.7057 \\ 0 & 0.3184 & -0.2540 & 0.7068 & -0.8097 & 0.1802 & 0.3827 \\ 0 & 0.0071 & 0.1173 & -0.6099 & -0.4370 & 0.9455 & -0.1713 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (\text{II.2.14})$$

### II.3 NON-IDEAL CHANNEL SCENARIO

This section extends the application of gradient descent approach for interference avoidance methods from ideal channel to non-ideal dispersive multiple access channels. Specifically, the adaptation of uplink precoders and transmit powers for an uplink wireless system in which the channels between users and the base station are considered explicitly and are assumed to be known. Moreover, the channels are assumed to be slow fading where channel fading matrix  $\mathbf{H}$  remains fixed throughout the encoding frame. Similar to [44], in order to decode the information transmitted by a given user  $k$ , the receiver uses an “inverse-channel” observation obtained by equalizing the received signal in (I.1.4) with the given user  $k$  channel matrix

$$\begin{aligned} \mathbf{r}_k &= \mathbf{H}_k^{-1} \mathbf{r} \\ &= \underbrace{b_k \sqrt{p_k} \mathbf{s}_k}_{\text{desired signal}} + \overbrace{\mathbf{H}_k^{-1} \left( \sum_{k \neq \ell, \ell=1}^K b_\ell \sqrt{p_\ell} \mathbf{H}_\ell \mathbf{s}_\ell + \mathbf{n} \right)}^{\text{interference + noise}} \end{aligned} \quad (\text{II.3.1})$$

This is processed by a matched filter receiver corresponding to user  $k$ 's precoder to obtain the decision variable for user  $k$ , i.e.

$$\begin{aligned} d_k &= \mathbf{s}_k^\top \mathbf{r}_k \\ &= b_k \sqrt{p_k} + \mathbf{s}_k^\top \mathbf{H}_k^{-1} \left( \sum_{\ell=1, \ell \neq k}^K b_\ell \sqrt{p_\ell} \mathbf{H}_\ell \mathbf{s}_\ell + \mathbf{n} \right) \end{aligned} \quad (\text{II.3.2})$$

For user  $k$ , the SINR expression for the received signal vector in (II.3.2) can be written as

$$\gamma_k = \frac{p_k}{\mathbf{s}_k^\top \mathbf{H}_k^{-1} \left( \sum_{\ell=1, \ell \neq k}^K p_\ell \mathbf{s}_\ell \mathbf{H}_\ell \mathbf{H}_\ell^\top \mathbf{s}_\ell^\top + \mathbf{W} \right) \mathbf{H}_k^{-\top} \mathbf{s}_k} = \frac{p_k}{\mathbf{s}_k^\top \tilde{\mathbf{R}}_k \mathbf{s}_k} \quad (\text{II.3.3})$$

where the matrix

$$\tilde{\mathbf{R}}_k = \mathbf{H}_k^{-1} \left( \sum_{\ell=1, \ell \neq k}^K p_\ell \mathbf{s}_\ell \mathbf{H}_\ell \mathbf{H}_\ell^\top \mathbf{s}_\ell^\top + \mathbf{W} \right) \mathbf{H}_k^{-\top} \quad (\text{II.3.4})$$

in the denominator of the SINR expression (II.3.3) is the correlation matrix of the interference-plus-noise that affects user  $k$ 's symbol in the “inverse-channel” observation, and is related to the correlation matrix of the received signal in equation (I.1.4) as

$$\tilde{\mathbf{R}} = \sum_{\ell=1}^K p_\ell \mathbf{H}_\ell \mathbf{s}_\ell \mathbf{s}_\ell^\top \mathbf{H}_\ell^\top + \mathbf{W} \quad (\text{II.3.5})$$

It can also be written

$$\begin{aligned}
\tilde{\mathbf{R}}_k &= \mathbf{H}_k^{-1} \left( \tilde{\mathbf{R}} - p_k \mathbf{H}_k \mathbf{s}_k \mathbf{s}_k^\top \mathbf{H}_k^\top \right) \mathbf{H}_k^{-\top} \\
&= \mathbf{H}_k^{-1} \tilde{\mathbf{R}} \mathbf{H}_k^{-\top} - p_k \mathbf{s}_k \mathbf{s}_k^\top
\end{aligned} \tag{II.3.6}$$

It is worth noting that both  $\tilde{\mathbf{R}}_k$  and  $\tilde{\mathbf{R}}$  are also positive definite matrices because of the presence of the positive definite noise covariance matrix  $\mathbf{W}$ . The denominator in (II.3.3) represents the effective interference+noise power that is present in user  $k$ 's decision variable, i.e.

$$i_k = \mathbf{s}_k^\top \tilde{\mathbf{R}}_k \mathbf{s}_k \tag{II.3.7}$$

The main goal in this setup is to derive a distributed algorithm in which individual users adjust their corresponding precoders using gradient-based updates for interference avoidance and powers to meet a specified set of target SINRs  $\{\gamma_1^*, \dots, \gamma_k^*, \dots, \gamma_K^*\}$ . It is noted that  $K$  active users with specified target SINRs are admissible in the uplink system with processing gain  $N$ , if and only if the admissibility condition in (II.1.2) is satisfied. The admissibility condition (II.1.2) is derived for ideal user channels in [17] and it has been extended to multipath non-ideal channel scenario in [56].

### II.3.1 Transmitter Adaptation and Power Control

As mentioned previously, the transmitted power for a given user  $k$  is a valuable resource, and user  $k$  would be interested in transmitting with the minimum power that satisfies its specified target SINR. In order to achieve this goal, each user  $k$

will first apply transmitter adaptation using GD-based precoder update to reduce the effective interference that corrupts its transmitted symbol at the receiver which eventually increases its SINR. The precoder adaptation for a given user increases its SINR, however, it is not guarantee that it meets target values. Therefore, the power update is necessary which adjusts the transmitted power such that the specified target SINR  $\gamma_k^*$  is satisfied. If the calculated SINR after precoder adaptation is equal to the target SINR, we do not need power update. Otherwise, there are two cases:

- If the SINR after precoder update is above its target value  $\gamma_k^*$  then the power update must decrease user  $k$  power.
- If the SINR after precoder update is below its target value  $\gamma_k^*$  then the power update must increase user  $k$  power.

An incremental update is performed in user precoder by using GD-based approach. It is noted that the effective interference function  $i_k$  in equation (II.3.7) is a quadratic form since the matrix  $\tilde{\mathbf{R}}_k$  is positive semidefinite. Therefore,  $i_k$  in (II.3.7) is a convex function over the  $N$ -dimensional unit sphere  $\{\mathbf{s}_k | \mathbf{s}_k \in \mathbb{R}^N, \|\mathbf{s}_k\| = 1\}$ , and it is decreased by the GD update iteration with actual precoder (i.e., unit norm precoder) update as in the case of ideal channel scenario

$$\mathbf{s}_k(n+1) = \frac{[\mathbf{I} - 2\mu_s \tilde{\mathbf{R}}_k(n)]\mathbf{s}_k(n)}{\|[\mathbf{I} - 2\mu_s \tilde{\mathbf{R}}_k(n)]\mathbf{s}_k(n)\|} \quad (\text{II.3.8})$$

As noted previously, the GD-based precoder update in equation (II.3.8) has a numerical advantage over the methods (such as minimum eigenvector or the MMSE updates for interference avoidance [42–44]) since it does not require computationally

intensive calculations.

After performing the precoder update, the value of the effective interference function is

$$i'_k(n) = \mathbf{s}_k(n+1)^\top \tilde{\mathbf{R}}_k(n) \mathbf{s}_k(n+1) \quad (\text{II.3.9})$$

which requires the user  $k$  power update in order for its SINR to satisfy the specified target value  $\gamma_k^*$  as

$$p'_k(n) = \gamma_k^* i'_k(n) \quad (\text{II.3.10})$$

However, the value  $p'_k(n)$  may not be close to  $p_k(n)$  which leads to abrupt change in power and might result in connection breakage or error in transmission. Thus, the “lagged” update is used to avoid abrupt variations as

$$p_k(n+1) = \min\{p_k^{max}, [(1 - \mu_p)p_k(n) + \mu_p p'_k(n)]\} \quad (\text{II.3.11})$$

where  $p_k^{max}$  is the maximum allowed power level for a given user  $k$ , and  $0 < \mu_p < 1$  a suitably chosen algorithmic constant, which adapts user  $k$  power to a new value that is a combination of the current power  $p_k(n)$  and the calculated power  $p'_k(n)$  required to meet the given target SINR after the effective interference function has been reduced by the incremental precoder update. It is worth noting that for smaller  $\mu_p$  value the lag value in power update will be more distinct and the power change will be with smaller incremental.



### II.3.2 Algorithm

Based on the precoder and power updates defined in the previous section, the proposed algorithm for GD based transmitter adaptation for multipath channels consists of two distinct stages performed sequentially by active users in the system. That is individual users perform incremental adaptation of their precoder followed by power control. A formal statement of the GD based transmitter adaptation for uplink channels for non-ideal channels is given below:

1. Initial Data:
  - Precoders  $\mathbf{s}_k$ , powers  $p_k$ , channel matrices  $\mathbf{H}_k$ , and target SINRs  $\gamma_k^*$  for active users  $k = 1, \dots, K$ .
  - Noise covariance matrix  $\mathbf{W}$
  - Constants  $\mu_s$ ,  $\mu_p$ , and tolerance  $\epsilon$ .
2. IF admissibility condition in equation (II.1.2) is satisfied GO TO Step 3, ELSE STOP. The desired target SINRs are not admissible.
3. FOR each user  $k = 1, \dots, K$  DO
  - (a) Compute corresponding  $\tilde{\mathbf{R}}_k$  using equation (II.3.6).
  - (b) Update user  $k$ 's precoder using equation (II.3.8).
  - (c) Update user  $k$ 's power using equation (II.3.11).
4. REPEAT Step 3 until all user SINRs are within specified tolerance  $\epsilon$  of their corresponding target value.

The optimal point of the algorithm is met when the difference between target SINR and the calculated SINR is within the specified tolerance  $\epsilon$ . Extensive simulations of the GD interference avoidance algorithm have shown that, when initialized with random user precoders and powers, and admissible target SINRs, the algorithm results in the SINRs to their corresponding specified target values, and the algorithm converges to a fixed point.

### II.3.3 Simulations and Numerical Results

In this section, in order to support theoretical claims, the extensive simulations for various scenarios are presented. Both diagonal channel [44] and circulant channel [43] matrices are considered for the simulations. In particular, simulation results are presented to show the convergence of the algorithm, and also to see the tracking ability of the algorithm for variable number of active users and their variable QoS requirement on the fly.

### II.3.4 Variation of user SINRs and Powers

In this section, the simulations are performed to look at the variation of user SINRs and the user powers and convergence of the algorithm.

For the simulation, the uplink of a wireless system is considered with  $K = 5$  users in a signal space dimension  $N = 3$  and AWGN with  $\sigma^2=0.1$  that is  $\mathbf{W} = 0.1\mathbf{I}_N$ . The constants for the algorithm were chosen as  $\mu_s=0.1$  and  $\mu_p=0.01$ , and the tolerance  $\epsilon=0.001$ . The maximum allowed power level  $p_k^{max}$ ,  $\forall k$ , was set to 10.

The user precoders and channel matrices of users were initialized randomly, and user powers were initialized to 0.1. The different target SINRs were set up for different

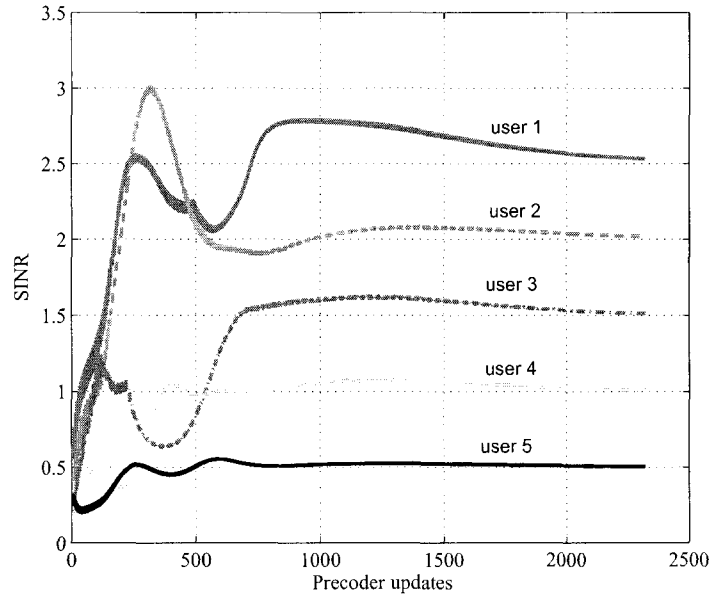


Fig. 5: SINR Variation for the system with  $K = 5$  users in  $N = 3$  signal space dimensions for target SINRs  $\{2.5, 2.0, 1.5, 1.0, 0.5\}$ .

users

$$\{\gamma_1^*, \gamma_2^*, \gamma_3^*, \gamma_4^*, \gamma_5^*\} = \{2.5, 2.0, 1.5, 1.0, 0.5\} \quad (\text{II.3.12})$$

such that they satisfy the admissibility condition in (II.1.2).

The proposed algorithm was applied for this simulation setup, and the results for SINR and power variation are plotted in Figure 5 and 6. The user SINRs and powers increases sharply at first and then decreases to settle down to the target SINRs. The power is also settled down that is minimized for all users. From these results, it is concluded that the proposed algorithm converges to a fixed point.

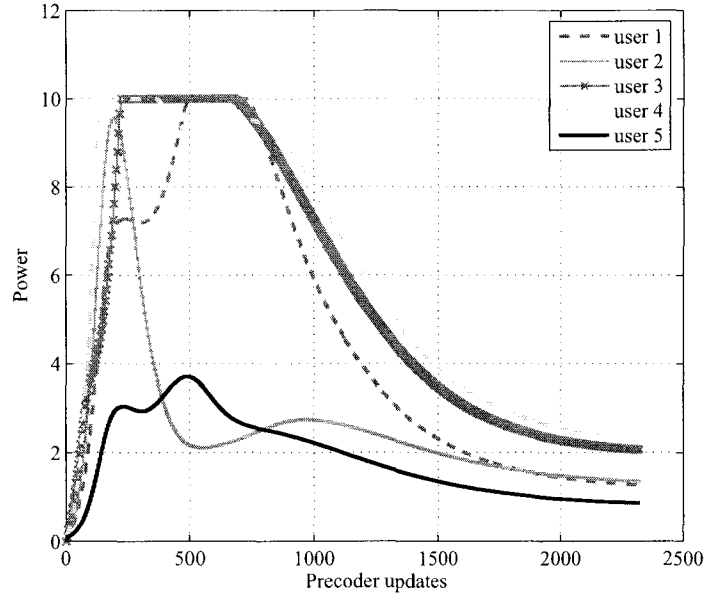


Fig. 6: Power Variation for the system with  $K = 5$  users in  $N = 3$  signal space dimensions for target SINRs  $\{2.5, 2.0, 1.5, 1.0, 0.5\}$ .

### II.3.5 Tracking Variable Number of Active Users

This section illustrates the tracking ability of the proposed algorithm for variable number of active users in the system. For this case, the simulation was started with  $K = 5$  users in  $N = 3$  signal space dimension with different user target SINRs in (II.3.12) which satisfies the admissibility condition in (II.1.2)

The proposed algorithm was applied for random initialization of precoder, channel and powers and the constants mentioned in section II.3.4, and at the fixed point of the algorithm, user 5 was dropped from the system so that the total number of active user in the system became  $K = 4$  with target SINRs (as we had before), that is

$$\{\gamma_1^*, \gamma_2^*, \gamma_3^*, \gamma_4^*\} = \{2.5, 2.0, 1.5, 1.0\}$$

which also satisfy the admissibility condition (II.1.2). The algorithm was applied for this new setup. Once the system reached the optimal fixed point, a new user was added in the system such that the total number of users in the system became  $K = 5$  without changing signal space dimension (i.e.  $N = 3$ ) with new target SINR for newly added user  $\gamma_5^* = 0.8$  keeping others' target SINRs as they were. After the admission of the new user with its target SINR, the admissibility condition in (II.1.2) should still be satisfied. Then the algorithm is applied in this new setup until a fixed point is reached.

The variation of user SINRs and powers during the tracking of variable number of active users in the system are plotted as shown in Figure 7 and 8. From these plots, it is observed that there are spikes on SINRs and the powers while adding and dropping the users to and from the system. After few ensemble iterations, the algorithm settled down to optimal fixed point as shown in Figure 7 and 8.

### II.3.6 Tracking Variable Target SINRs of Active Users

This section illustrates the tracking ability of the proposed algorithm for variable target SINRs (i.e., changing QoS requirements on the fly) for active users in the system. For this case, the simulation was started with  $K = 5$  users in signal space dimension  $N = 3$  with different user target SINRs as in (II.3.12) which are admissible in the system. The proposed algorithm was applied for the simulation setup presented in section II.3.4, and when the algorithm reached the fixed point, user 5 changed its target SINR to  $\gamma_5^* = 0.3$  in which target SINRs still satisfy the admissibility condition (II.1.2). The algorithm was ran for this new setup, and once the system reached optimal fixed point, again user 5 changed its target SINR to new value to  $\gamma_5^* = 0.7$ .

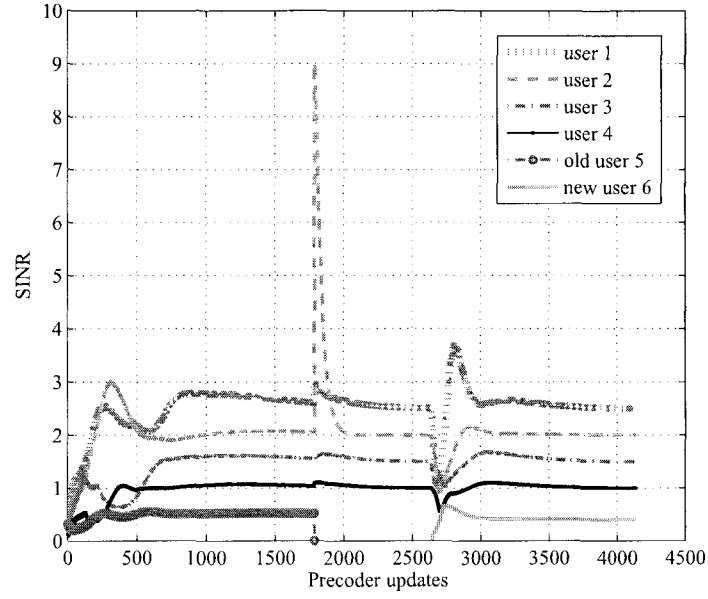


Fig. 7: Variation of user SINRs for the tracking example where one user is dropped from the system followed by subsequent addition of another user.

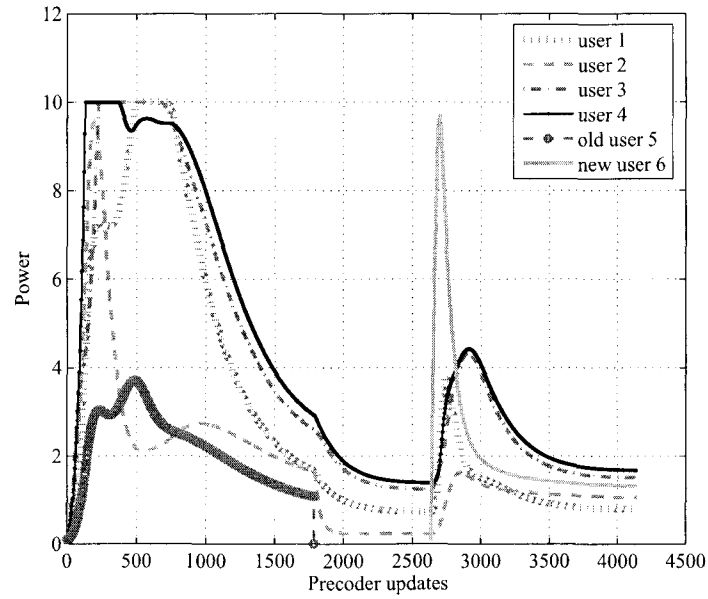


Fig. 8: Variation of user powers for the tracking example where one user is dropped from the system followed by subsequent addition of another user.

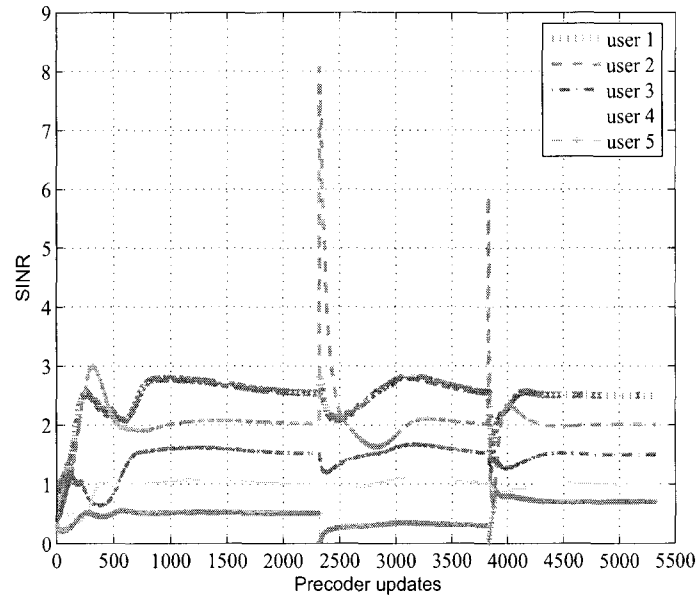


Fig. 9: Variation of user SINRs for  $K = 5$  user in  $N = 3$  signal space dimensions tracking variable number of active users in the system.

Then, the algorithm was applied to this new setup.

The variation of user SINRs and powers are plotted as shown in Figure 9 and 10 for tracking of variable number of active users in the system. As in the previous simulation results, it has observed that there are spikes on SINRs and powers as the target SINR changed. However, after few ensemble iteration, the algorithm settled down to optimal fixed point satisfying the target SINRs of active users in the system.

Extensive simulations of the gradient-based algorithm have shown few instances where users distribute their power inefficiently and the algorithm gets trapped in sub-optimal points

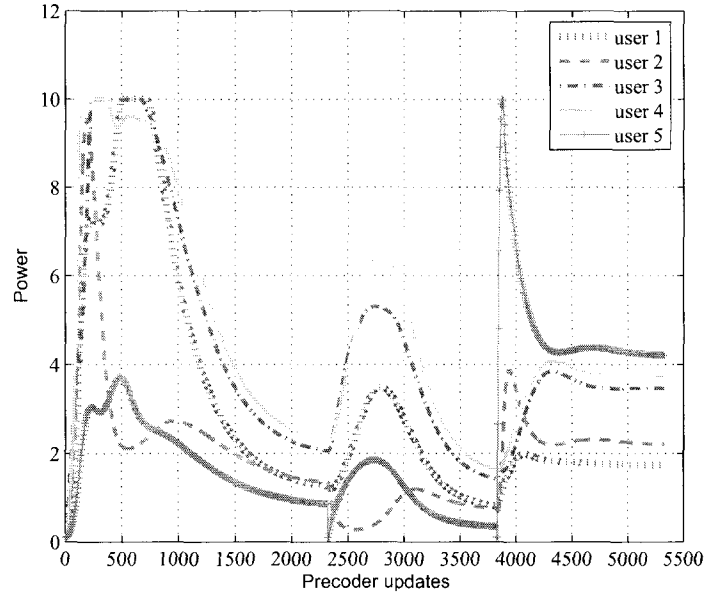


Fig. 10: Variation of user powers for  $K = 5$  user in  $N = 3$  signal space dimensions for tracking variable number of active users in the system.

## II.4 CHAPTER SUMMARY

In this chapter, the GD-based transmitter optimization is investigated for active users in the uplink of wireless systems with ideal channel scenario where the users adapt precoders and powers to meet minimum SINR requirements. Then, the approach is extended to non-ideal channel scenario where the channels between users and the base station are taken explicitly into account. The simulation results have shown that the proposed algorithm for both ideal and non-ideal channels converges to a fixed point. Furthermore, the algorithm can keep track of the number of active users and/or variable target SINRs in the system. Thus the proposed algorithm can be implemented in dynamic wireless system where the number of active users and/or QoS requirements changes on the fly.



# CHAPTER III

## INCREMENTAL STRATEGIES FOR TRANSMITTER ADAPTATION WITH POWER CONTROL

This chapter presents incremental strategies for transmitter optimization and power update using a game theoretic approach. Specifically, separable non-cooperative game is proposed to perform interference avoidance using incremental precoder adaptation sub-game and power control to meet QoS requirements using incremental power control sub-game for uplink of a wireless system with a non-ideal channel scenario. The work in this chapter is motivated by the fact that the extensive simulations of the gradient-based algorithm presented in Section II.3 (and/or [45]) have shown few instances where this algorithm gets trapped in sub-optimal points where users distribute their power inefficiently and are difficult to escape when the channel gains are really low compared to others for a given user in some bands. The given user puts most of its power into the lowest gain band and can not meet its required SINR. In Section III.1, a brief literature review relevant to this work and game theory is presented. The system model and formalized the game theoretic approach is presented in Section III.2. The Nash equilibrium of two sub-games is investigated and then the joint game followed by presentation of an algorithm in Section III.4. Simulation results are presented in Section III.5 to show the convergence of the algorithm as well as tracking ability of the proposed algorithm with respect to variable number of active users and/or variable QoS requirements of active users in the system.

### III.1 GAME THEORY AND RELATED WORK

Recently, game theory has been an emergent tool for wireless resource allocation [60]. There are several reasons for using a game theoretic approach for resource allocation in wireless systems. *First*, game theory helps scale up the network. That is, one can model the resource allocation using game for small-scale network, often 2- or 3-player games that are easy to describe in order to understand regarding basic issues such as rationality. Finding the efficient techniques for describing and analyzing the game such as Nash equilibrium becomes easy for such small games. Once everything is set up for a small-scale network, one can apply the model in a large-scale network, which will have a Nash equilibrium as in a small-scale network. Therefore, the game theory has been regarded as the tool to the scalability of wireless network in every aspect. *Second*, game theory has recently been regarded as a tool to evaluate the performance of the wireless networks. Performance can be measured in terms of the number of steps needed to reach the game at Nash equilibrium, which is seen as a *socially* optimal point of the game. The last but not least is learning in a game. There has been a great deal of work in game theoretic approach on learning to play well in different settings. Learning to play optimally in a reinforcement learning setting, where an agent interacts with an unknown (but fixed) environment, is one of the greatest advantages of game theoretic approach. After few steps, the player faces a trade-off between exploration and exploitation.

Since optimal resource allocation in distributed systems where users compete for resources is not straightforward, users operate using a game theory where rational users learn to respond optimally according to their operating environment. With

proper choice of utility functions the game leads to an equilibrium point called the Nash equilibrium [61] which leads to efficient global use of resources. A new algorithm is presented that is extended adaptive interference avoidance technique [32, 59] for the practical case where channels between users and the base station are explicitly considered. As noted previously, the distributed implementation of the algorithm is possible provided that the correlation matrix of the received signal is made available to individual users.

Game theory is a formal model of an interactive situation where several players are involved. It is also important to point out that a game with only one player is usually called a decision problem. The game is formally consist of following three basic elements [62]:

- **Set of players** which are the active agents in the system. That is a player is an agent who makes decisions in a game
- **Set of strategies** for players or the strategic actions available to them. In a game of strategic form, a strategy is one of the given possible actions of a player.
- **Player's payoff (or utility or cost) function** that maps the strategies or their preferences. That is a payoff is a number, also called utility that reflects the desirability of an outcome to a player.

In a non-cooperative game, the players are assumed to be rational, that is, they play a game always to choose an action, which gives the outcome she/he most prefers without taking care of opponents who are also rational. In other words, players in

non-cooperative games make choices out of their own interest without interaction with others. However, it is noted that the cooperation can arise in non-cooperative games, when players find their own best interests. A separable game is a particular type of non-cooperative game [62] in which the player payoff (or utility or cost) functions are separable with respect to variables that define user strategies [34, 63].

In this chapter, following the [32, 34], game theoretic approach is considered for precoder and power control of active users in the uplink wireless system which have QoS requirements to be satisfied in non-ideal channels, and show that the global optimal fixed point of the game analytically.

### III.2 SYSTEM MODEL AND PROBLEM STATEMENT

An uplink of a wireless system with  $K$  active users is considered in a signal space of dimension  $N$  where multipath channels between users and base station are explicitly considered. The  $N$ -dimensional received signal vector at the base station corresponding to one signaling interval is given by the expression in (I.1.4). As mentioned in Section II.3, the receiver uses an “inverse-channel” observation obtained by equalizing the received signal with the given user  $k$  channel matrix as in (II.3.1). Again it is noted that the denominator term in SINR expression in (II.3.3) represents the effective interference+noise power that is present in user  $k$ ’s decision variable

$$i_k = \mathbf{s}_k^\top \tilde{\mathbf{R}}_k \mathbf{s}_k \quad (\text{III.2.1})$$

The interference function  $i_k$  for a given user  $k$  depends implicitly on  $\mathbf{s}_k$  (for matched filter receiver case) as well as on all the other users precoders and powers

$\mathbf{s}_\ell, p_\ell, \forall \ell \neq k$ , but does not depend on user  $k$ 's power.

Main goal in this setup is to derive a distributed algorithm in which individual users adjust their corresponding precoders and powers to meet specified target SINRs  $\{\gamma_1^*, \dots, \gamma_k^*, \dots, \gamma_K^*\}$  that must be admissible as defined in [17] and/or satisfy equation (II.1.2) using separable non-cooperative game theoretic approach.

### III.3 JOINT PRECODER ADAPTATION AND POWER CONTROL GAME (JPAPCG)

The uplink of a wireless system with multipath is formulated and described by equations (II.3.1) or (II.3.2) as a separable non-cooperative game in which active users in the system are the players of the game, and adaptation of their precoders and powers are their corresponding strategies with strategy spaces formally defined, respectively, by the  $N$ -dimensional sphere with unity radius for the precoder strategies

$$\mathcal{S}_k = \{\mathbf{s}_k | \mathbf{s}_k \in \mathbb{R}^N, \|\mathbf{s}_k\| = 1\} \quad \forall k = 1, \dots, K \quad (\text{III.3.1})$$

and by the set corresponding to the real interval  $(0, P_k^{max}]$  for the power strategies

$$\mathcal{P}_k = \{p_k | p_k \in (0, P_k^{max}]\} \quad \forall k = 1, \dots, K \quad (\text{III.3.2})$$

where  $P_k^{max}$  is the maximum allowed power level for users for the power strategies.

Following [32], the payoff function is considered, which is cost function in this context, for a given user  $k$  to be the product between its power and its corresponding

interference function, that is

$$u_k = p_k i_k = \underbrace{p_k}_{f_k(p_k)} \underbrace{\mathbf{s}_k^\top \tilde{\mathbf{R}}_k \mathbf{s}_k}_{g_k(\mathbf{s}_k)} \quad \forall k = 1, \dots, K. \quad (\text{III.3.3})$$

Formally the joint precoder adaptation and power control game (JPAPCG) is defined as

$$\text{JPAPCG} = \langle \mathcal{K}, \{\mathcal{S}_k \times \mathcal{P}_k\}_{k \in \mathcal{K}}, \{u_k(\cdot)\}_{k \in \mathcal{K}} \rangle \quad (\text{III.3.4})$$

where the components of the game are as follows:

1.  $\mathcal{K} = \{1, \dots, K\}$  is the set of players which are the active users in the system.
2.  $\mathcal{S}_k$  is the set of precoder strategies for player  $k$  in (III.3.1), and  $\mathcal{P}_k$  is the set of power strategies for player  $k$  in (III.3.2).
3.  $u_k : \mathcal{S} \times \mathcal{P} \rightarrow (0, \infty)$  is the user cost function that maps the joint strategy spaces  $\mathcal{S} = \mathcal{S}_1 \times \dots \times \mathcal{S}_K$  and  $\mathcal{P} = \mathcal{P}_1 \times \dots \times \mathcal{P}_K$  to the set of positive real numbers.

As noted in [32, 34], the cost function in (III.3.3) is separable with respect to the two parameters that define the user strategy – the corresponding precoder and power. Again, it is noted that the interference function (i.e.,  $i_k$  or  $g_k(\mathbf{s}_k)$ ) does not depend on the user power however it depends on the user precoder [32, 34]. Thus, the cost function can be expressed as a product of two independent/separable functions: power strategies  $f_k(p_k)$  and/or precoder strategy  $g_k(\mathbf{s}_k)$ . This property leads to JPAPCG as a separable game with two separate sub-games:

1. Precoder Adaptation Sub-Game (PASG) and

## 2. Power Control Sub-Game (PCSG)

which minimize the user cost functions with optimal strategies. These two sub-games are further investigated in the following sections with best response strategies in terms of precoder and power updates that will minimize the user cost functions. It is also investigated the existence of Nash equilibria for the two sub-games, and the result in [32, 34] are used to show that the joint precoder adaptation and power control game has a Nash equilibrium provided that Nash equilibria for the two sub-games exist.

### III.3.1 Precoder Adaptation Sub-Game (PASG)

In this sub-game, for given fixed user powers, individual users adjust only their precoders in their corresponding strategy spaces (III.3.1) to minimize their corresponding cost functions. Formally, the PASG for given set of powers is defined as

$$\text{PASG} = \langle \mathcal{K}, \{\mathcal{S}_k\}_{k \in \mathcal{K}}, \{u_k(\cdot)\}_{k \in \mathcal{K}} \rangle \quad (\text{III.3.5})$$

Specifically, in this game, for a given set of powers, each user adjust its strategy to minimize its corresponding cost function subject to unit norm constraint on precoder, that is

$$\min_{\mathbf{s}_k} u_k |_{\{p_1, p_2, \dots, p_K\}=\text{fixed}} \quad \text{subject to} \quad \mathbf{s}_k^\top \mathbf{s}_k = 1, \quad \forall k = 1, \dots, K \quad (\text{III.3.6})$$

Next the following formal definitions from game theory are stated in the context of the problem in order to investigate the existence of a Nash equilibrium for the

PASG and identify the best response strategies for players.

*Definition 1 (Nash equilibrium for the PASG):* The precoder ensemble  $\{\mathbf{s}_1, \dots, \mathbf{s}_{k_1}, \mathbf{s}_k, \mathbf{s}_{k+1}, \dots, \mathbf{s}_K\}$  is a Nash equilibrium of the PASG if, for every user  $k \in \mathcal{K}$ , we have that

$$u_k(\mathbf{s}_1, \dots, \mathbf{s}_{k_1}, \mathbf{s}_k, \mathbf{s}_{k+1}, \dots, \mathbf{s}_K) \geq u_k(\mathbf{s}_1, \dots, \mathbf{s}_{k_1}, \mathbf{s}_k', \mathbf{s}_{k+1}, \dots, \mathbf{s}_K), \quad \forall \mathbf{s}_k' \in \mathcal{S}_k \quad (\text{III.3.7})$$

*Definition 2 (Best Response for the PASG):* The best response function of user  $k$  to the other users' strategies is the set

$$B_k^{\mathbf{s}} = \{\mathbf{s}_k \in \mathcal{S}_k | u_k(\mathbf{s}_1, \dots, \mathbf{s}_{k_1}, \mathbf{s}_k, \mathbf{s}_{k+1}, \dots, \mathbf{s}_K) \geq u_k(\mathbf{s}_1, \dots, \mathbf{s}_{k_1}, \mathbf{s}_k', \mathbf{s}_{k+1}, \dots, \mathbf{s}_K) \\ \forall \mathbf{s}_k' \in \mathcal{S}_k\} \quad (\text{III.3.8})$$

*Definition 3 (Convex Game):* A game is convex for a closed, convex, and bounded joint strategy space  $\mathcal{S}$  if the cost function of each user  $k$  is convex in  $\mathbf{s}_k$  for every fixed  $\mathbf{s}_j$ , such that  $k \neq j$ .

The cost function in (III.3.3) is a quadratic form in the user precoder  $\mathbf{s}_k$  for given fixed user power, which implies that is differentiable two times with respect to  $\mathbf{s}_k$ , and after differentiating two times, one can get

$$\frac{\partial^2 u_k}{\partial \mathbf{s}_k^2} = 2p_k \tilde{\mathbf{R}}_k \quad (\text{III.3.9})$$

It is noted that the correlation matrix  $\tilde{\mathbf{R}}_k$  of the interference-plus-noise corrupting user  $k$ 's inverse-channel observation (II.3.1) is symmetric and positive definite. This implies that, for a given set of powers (i.e.,  $p_k > 0$ ), the user cost function is convex



and that the PASG is a convex game, which ensures that it has a Nash equilibrium [32].

As discussed in [32], the best response of PASG in terms of precoder updates is found by solving the constrained optimization problem (III.3.6) in which user  $k$ 's precoder is replaced by the eigenvector corresponding to the minimum eigenvalue of its corresponding interference-plus-noise matrix  $\tilde{\mathbf{R}}_k$  [4]. This procedure minimizes the effective interference that affects user  $k$ 's "inverse-channel" observation. Thus, at fixed point, which is a Nash equilibrium, all user precoders will be the minimum eigenvectors of their corresponding interference-plus-noise matrices. If the following determinant relationship implied by the Kuhn-Tucker conditions for constrained optimum is satisfied [32], the Nash equilibrium implied by this procedure is optimal with respect to minimization of user  $k$ 's cost function:

$$D_k^s = (-1) \begin{vmatrix} 2p_k(\tilde{\mathbf{R}}_k - \gamma_k^* \mathbf{I}_N) & 2\mathbf{s}_k \\ 2\mathbf{s}_k^\top & 0 \end{vmatrix} > 0, \quad \forall k = 1, \dots, K \quad (\text{III.3.10})$$

This is also referred as determinant condition for optimality.

### III.3.2 Power Control Sub-Game (PCSG)

In this sub-game, the user precoders are fixed, and the individual users adjust only their powers in their corresponding strategy spaces (III.3.2) to minimize their corresponding cost functions.

Formally, the PCSG for a given set of precoders is defined as

$$\text{PCSG} = \langle \mathcal{K}, \{\mathcal{P}_k\}_{k \in \mathcal{K}}, \{u_k(\cdot)\}_{k \in \mathcal{K}} \rangle \quad (\text{III.3.11})$$

Specifically, in this game, for a given set of precoders, each user adjust its power strategy to minimize its corresponding cost function subject to QoS requirement in terms of target SINR, that is

$$\min_{p_k} u_k |_{\{\mathbf{s}_1, \mathbf{s}_2, \dots, \mathbf{s}_K\}=\text{fixed}} \quad \text{subject to} \quad p_k = \gamma_k^* \mathbf{s}_k^\top \tilde{\mathbf{R}}_k \mathbf{s}_k, \quad \forall k = 1, \dots, K \quad (\text{III.3.12})$$

Similar to PASG, some formal definitions are made to investigate the existence of a Nash equilibrium for the PCSG and identify the best response strategies for players.

*Definition 4 (Nash equilibrium for the PCSG):* The set of powers  $\{p_1, \dots, p_{k_1}, p_k, p_{k+1}, \dots, p_K\}$  is a Nash equilibrium of the PCSG if, for every user  $k \in \mathcal{K}$ , we have that

$$u_k(p_1, \dots, p_{k_1}, p_k, p_{k+1}, \dots, p_K) \geq u_k(p_1, \dots, p_{k_1}, p_k', p_{k+1}, \dots, p_K), \quad \forall p_k' \in \mathcal{P}_k \quad (\text{III.3.13})$$

*Definition 5 (Best Response for the PCSG):* The best response function of user  $k$  to the other users' strategies is the set

$$B_k^p = \{p_k \in \mathcal{S}_k | u_k(p_1, \dots, p_{k_1}, p_k, p_{k+1}, \dots, p_K) \geq u_k(p_1, \dots, p_{k_1}, p_k', p_{k+1}, \dots, p_K) \\ \forall p_k' \in \mathcal{P}_k\} \quad (\text{III.3.14})$$

In this sub-game, the user cost function is linear in  $p_k$ , which may also be regarded as a convex function. Following [32], a Nash equilibrium exists and the best response strategy is to update power to match target SINR, that is,  $p_k = i_k \gamma_k^*$ , for all  $k = 1, \dots, K$ .

### III.3.3 Nash Equilibrium for the JPAPCG

As mentioned in previous sections, the precoder ensemble  $\{\mathbf{s}_1, \dots, \mathbf{s}_K\}$  is a Nash equilibrium for the PASG and power set  $\{p_1, \dots, p_K\}$  is a Nash equilibrium for the PCSG. Since Nash equilibria exist for both the PASG and the PCSG, this implies that a Nash equilibrium for the JPAPCG also exists, and is implied by the best response strategies of the PASG and the PCSG. It is noted that, following the approach in [32] based on the result of [34, Theorem 1], a Nash equilibrium solution for the JPAPCG exists. It is also important to point out that precoder ensemble is not unique at Nash equilibrium. However, with the random initialization of initial precoders, equilibrium precoder ensemble preserves norms and cross correlations resulting in a new precoder ensemble, which is also a Nash equilibrium for the system. It is also noted that a precoder ensemble corresponding to a Nash equilibrium is optimal with respect to constrained minimization of the user cost function if the sufficient conditions in (III.3.10) are satisfied.

Furthermore, at a Nash equilibrium user precoder is an eigenvector corresponding to a minimum eigenvalue of its corresponding interference-plus-noise correlation matrices  $\tilde{\mathbf{R}}_k$ , that is

$$\tilde{\mathbf{R}}_k \mathbf{s}_k = \lambda_k \mathbf{s}_k, \quad \forall k = 1, \dots, K \quad (\text{III.3.15})$$

where  $\lambda_k$  is a minimum eigenvalue of a  $\tilde{\mathbf{R}}_k$ . At optimal Nash equilibrium, the equation (III.3.15) can be expressed in term of target SINR  $\gamma_k^*$  and thus the power at equilibrium is

$$p_k = \mathbf{s}_k^\top \tilde{\mathbf{R}}_k \mathbf{s}_k \gamma_k^* = \lambda_k \gamma_k^*$$

Then, using the relationship between  $\tilde{\mathbf{R}}$  and  $\tilde{\mathbf{R}}_k$  from equation (II.3.6), one can express (III.3.15) in terms of  $\tilde{\mathbf{R}}$  as

$$\mathbf{H}_k^{-1} \tilde{\mathbf{R}} \mathbf{H}_k^{-\top} \mathbf{s}_k = p_k \frac{1 + \gamma_k^*}{\gamma_k^*} \mathbf{s}_k \quad (\text{III.3.16})$$

This implies that, at Nash equilibrium, the user precoder  $\mathbf{s}_k$  is eigenvector of the matrix  $\mathbf{H}_k^{-1} \tilde{\mathbf{R}} \mathbf{H}_k^{-\top}$  with corresponding eigenvalue  $p_k \frac{1 + \gamma_k^*}{\gamma_k^*}$ , for all  $k = 1, \dots, K$ .

One can also express (III.3.16) as [58]

$$\mathbf{H}_k^{\top} \tilde{\mathbf{R}}^{-1} \mathbf{H}_k \mathbf{s}_k = \frac{1}{p_k} \frac{\gamma_k^*}{1 + \gamma_k^*} \mathbf{s}_k \quad (\text{III.3.17})$$

Motivated by the approach in [23] (where an optimal precoder ensemble and powers that maximize the sum capacity of the multiaccess vector channel), at the optimal Nash equilibrium of the JPAPCG, all user cost functions are minimized subject to the specified unit norm and target SINR constraints for which target SINRs are met with minimum transmitted power for all active users. This also extends the similar result obtained for uplink systems with ideal channels in [32] to the non-ideal channel case where the channel between users and the basestation are explicitly considered.

### III.4 ALGORITHM FOR INCREMENTAL STRATEGIES

The strategies that define the optimal Nash equilibrium solution of the joint precoder adaptation and power control game discussed in the previous section may

lead to abrupt changes of the user precoder and/or power which are not desirable in practical implementations and similar to [32, 59] the proposed algorithm uses the incremental updates:

- Precoder update of user  $k$  at step  $n$  of the algorithm is:

$$\mathbf{s}_k(n+1) = \frac{\mathbf{s}_k(n) + m\beta\mathbf{x}_k(n)}{\|\mathbf{s}_k(n) + m\beta\mathbf{x}_k(n)\|} \quad (\text{III.4.1})$$

where  $\mathbf{x}_k$  is the minimum eigenvector of corresponding interference+noise correlation matrix  $\tilde{\mathbf{R}}_k$ ,  $\beta$  is a parameter that limits how far in terms of Euclidian distance the updated precoder can be from the old precoder, and  $m = \text{sgn}(\mathbf{s}_k^\top \mathbf{x}_k)$  gives sign.

- Power update of user  $k$  at step  $n$  of the algorithm is:

$$p_k(n+1) = \min\{p_k^{max}, [p_k(n) - \mu[p_k(n) - \gamma_k^* i_k(n)]]\} \quad (\text{III.4.2})$$

where  $p_k^{max}$  is maximum allowed power level for a given user  $k$ , and  $0 < \mu < 1$  is suitably chosen constant.

The proposed algorithm consists of two distinct stages performed sequentially by active users in the system: individual users perform incremental adaptation of their precoders followed by their incremental power control. The algorithm is formally stated below:

1. Input Data:

- Precoders  $\mathbf{s}_k$ , powers  $p_k$ , channel matrices  $\mathbf{H}_k$ , and target SINRs  $\gamma_k^*$  for all

- active users  $k = 1, \dots, K$ .
- Noise covariance matrix  $\mathbf{W}$
  - Constants  $\beta$ ,  $\mu$ , and tolerance  $\epsilon$ .
2. IF admissibility condition in equation (II.1.2) is satisfied GO TO Step 3. OTHERWISE STOP, the desired system configuration is not admissible.
  3. FOR each user  $k = 1, \dots, K$  DO
    - (a) Compute corresponding  $\tilde{\mathbf{R}}_k(n)$  using equation (II.3.6) and determine its minimum eigenvector  $\mathbf{x}_k(n)$ .
    - (b) Update user  $k$ 's precoder using equation (III.4.1).
    - (c) Update user  $k$ 's power using equation (III.4.2).
  4. IF change in cost function is larger than  $\epsilon$  for any user then GO TO Step 3 OTHERWISE a Nash equilibrium is reached.
  5. IF optimality condition (III.3.10) is true then STOP: an optimal Nash equilibrium has been reached. OTHERWISE GO TO Step 3.

It is worth noting that the checking the optimality condition (III.3.10) in Step 5 ensures that the optimal Nash equilibrium is reached and that the algorithm does not stop at a sub-optimal point. Numerically, a fixed point of the algorithm may be reached when the precoder and power updates result in decreases of the user cost functions that are smaller than the specified tolerance, but if the optimality condition (III.3.10) is not satisfied the return to Step 3 and the incremental updates

that will follow will move the system away from the sub-optimal Nash equilibrium toward the optimal one.

It is also noted that the algorithm is implementable in a distributed manner where individual users update precoder and power using common feedback information broadcast by the receiver [10–12]. In order for user  $k$  to perform the updates the interference+noise correlation matrix  $\tilde{\mathbf{R}}_k$  is needed, which can be obtained from the correlation matrix  $\tilde{\mathbf{R}}$  of received signal by subtracting its contribution  $(p_k \mathbf{H}_k \mathbf{s}_k \mathbf{s}_k^\top \mathbf{H}_k^\top)$  as shown by equation (II.3.6).

### III.5 SIMULATIONS AND NUMERICAL RESULTS

This section presents numerical results obtained from simulations that illustrate convergence and tracking properties of the proposed algorithm.

#### III.5.1 Algorithm Convergence

Extensive simulations are performed with the proposed algorithm to study convergence of the JPAPCG game to the Nash equilibrium. It is noted that the convergence speed of the algorithm depends on the values of the corresponding increments specified by the algorithm constants  $\mu$  and  $\beta$ .

In the first experiment, simulation are performed to study the convergence speed for different values the algorithm constants  $\mu$  and  $\beta$  for fixed values of active users  $K$  and signal dimensions  $N$ . A wireless system with  $K = 6$ ,  $N = 5$ , and noise matrix  $\mathbf{W} = 0.1\mathbf{I}_N$  has been considered. The algorithm was applied starting from a random ensemble of user precoders and channel matrices. Once the algorithm stopped at Nash equilibrium one user was added/removed to/from the system with

one more/less corresponding user precoder ensemble. The algorithm was run for 1,000 independent trials and recorded the number of ensemble iterations for all cases needed for convergence within tolerance  $\epsilon = 0.001$ , and calculated their corresponding average values. Then, in Figure 11, the average number of ensemble iterations for different  $\mu$  and  $\beta$  values was plotted in which it is noted that convergence of the game to the Nash equilibrium is mostly determined by  $\beta$  value and is not very sensitive to changing  $\mu$  values. It is also noted that for  $\mu = 0.1$  and  $\beta = 0.1$  the game reaches to the Nash equilibrium in less than 70 ensemble iterations as shown in Figure 11.

In the second experiment, simulations are performed to look at the performance of the algorithm in terms of convergence speed for increasing  $K$  and  $N$  by keeping their ratio  $K/N$  (also referred to as system load factor in CDMA systems) fixed. As mentioned in previous experiment, the algorithm constants were chosen to be  $\mu = 0.1$  and  $\beta = 0.1$  and the algorithm was run for 1,000 independent trials. In each trail, the number of ensemble iterations required for convergence was recorded within the given tolerance  $\epsilon = 0.001$  for given load factor. Similar scenarios have been considered as in the first experiment: the algorithm was applied for random initialization of user precoder ensemble until it reached to Nash equilibrium. Next, starting from an ensemble of precoders corresponding to an optimal Nash equilibrium, one precoder added/removed corresponding to one more/less user in the ensemble so that there were one more/less active users in the system. The results are plotted as shown in Figure 12 where it is noted that average number of ensemble iterations to reach a Nash equilibrium does not change significantly with increasing values of  $K$  and  $N$  for given ratio  $K/N$ . The convergence speed of the algorithm to reach to Nash equilibrium is almost the same for light and average load factor ( $K/N = 1.2$



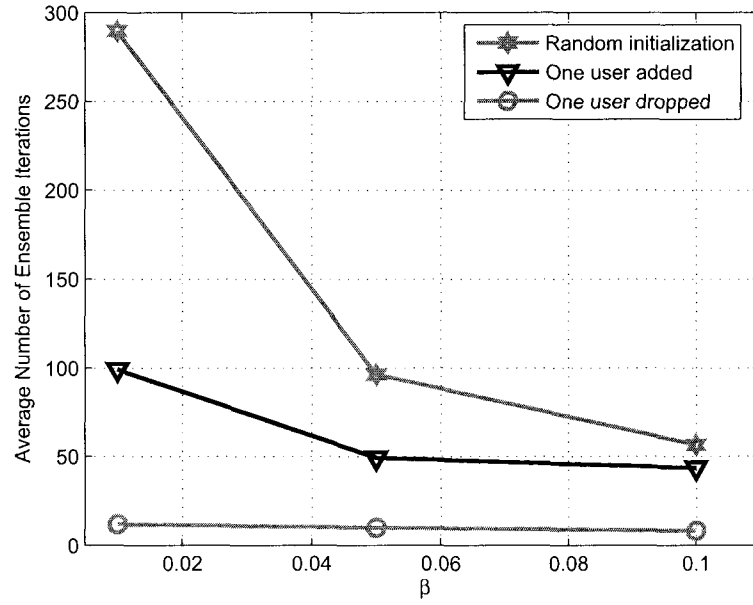
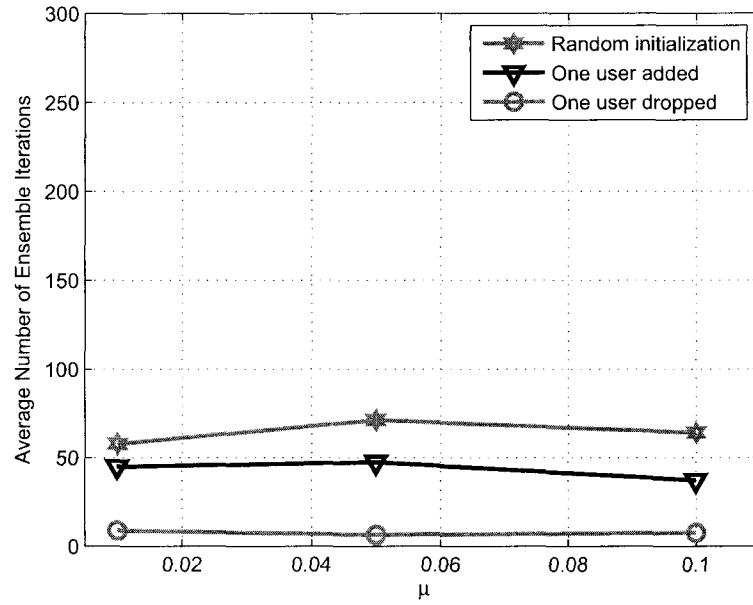
(a) Varying  $\beta$  and fixed  $\mu = 0.1$ (b) Varying  $\mu$  and fixed  $\beta = 0.1$ 

Fig. 11: Average number of ensemble iterations for convergence to optimal Nash equilibrium of JPAPCG for  $K = 6$  and  $N = 5$  in 1,000 trials.

and 1.8), and is slightly higher for higher load factor ( $K/N = 2.4$ ) as it is expected since more users would be competing for the same resources for their transmissions.

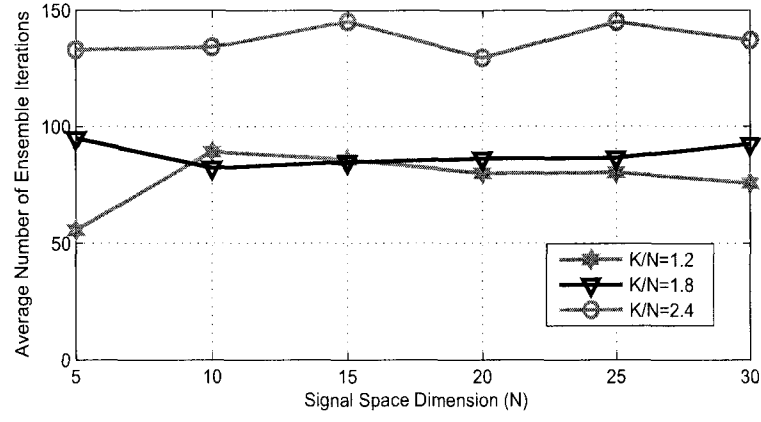
### III.5.2 User SINRs, Powers and Costs Variation, and Tracking Ability of the Algorithm for Variable Number of Active Users

In the first experiment, convergence of the algorithm is illustrated from random initialization of precoders, channels and powers matrices. An uplink of wireless system with  $K = 5$  users in a signal space of dimension  $N = 3$  and AWGN with covariance matrix  $\mathbf{W} = 0.1\mathbf{I}_3$  was considered. The algorithm constants were chosen to be  $\beta = 0.01, \mu = 0.01$ , and tolerance  $\epsilon = 0.001$ . User precoders and channel matrices were initialized randomly, and the target SINRs were selected as

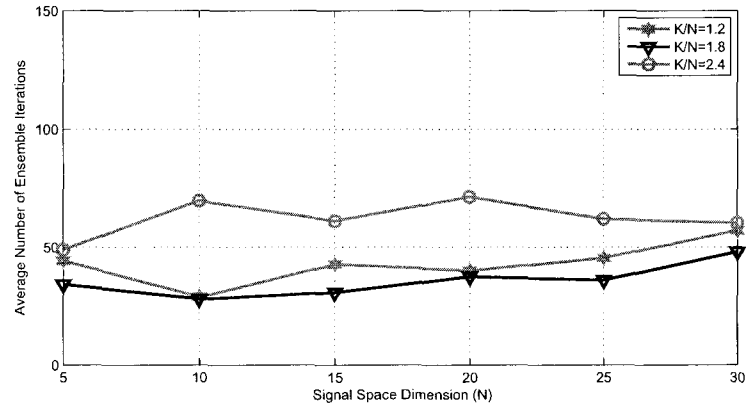
$$\{\gamma_1^*, \gamma_2^*, \gamma_3^*, \gamma_4^*, \gamma_5^*\} = \{1.5, 1.4, 1.3, 1.2, 1.1\}$$

which satisfy the admissibility condition in equation (II.1.2). The algorithm was applied for this simulation setup, and the variation of the user SINRs, powers and cost functions are plotted in Figures 13, 14, and 15, respectively. The plots show fluctuation at first followed by gradual adaptation and convergence to the specified target SINR values. This example shows that the proposed algorithm converges to a Nash equilibrium point.

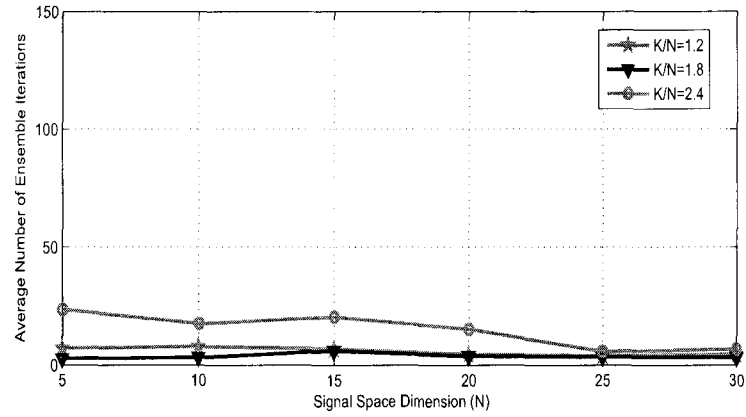
In the second experiment, the tracking ability of the algorithm is illustrated when the number of active user in the system changes in the fly. In this case, once the algorithm reached at optimal fixed point for first experiment, user 5 was dropped from the system leaving the total number of active users  $K = 4$ , and the target SINRs



(a) Random initialization



(b) One user added to the system



(c) One user dropped from the system

Fig. 12: Average number of ensemble iterations for convergence to optimal Nash equilibrium of JPAPCG for fixed  $\mu = 0.1$  and  $\beta = 0.1$  and increasing  $K$  and  $N$  in 1,000 trials

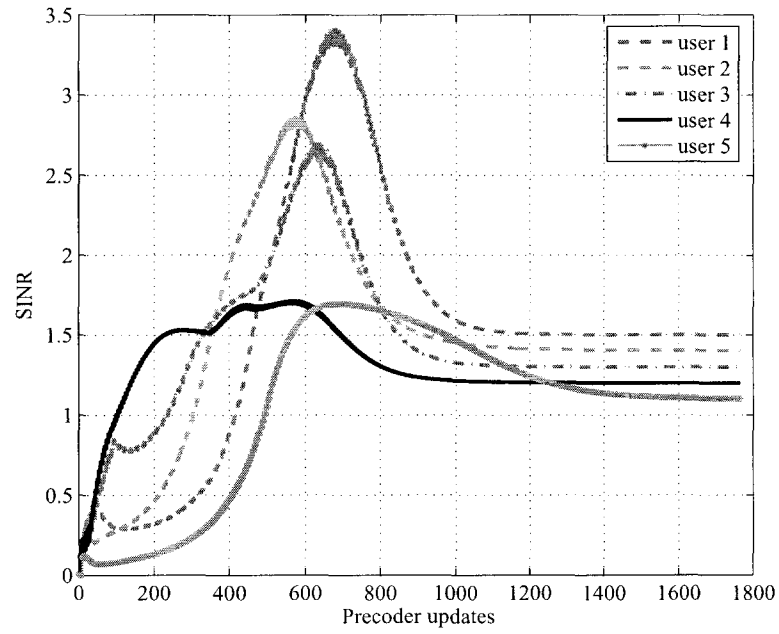


Fig. 13: SINR Variation for the system with  $K = 5$  users in  $N = 3$  signal space dimensions from random initialization.

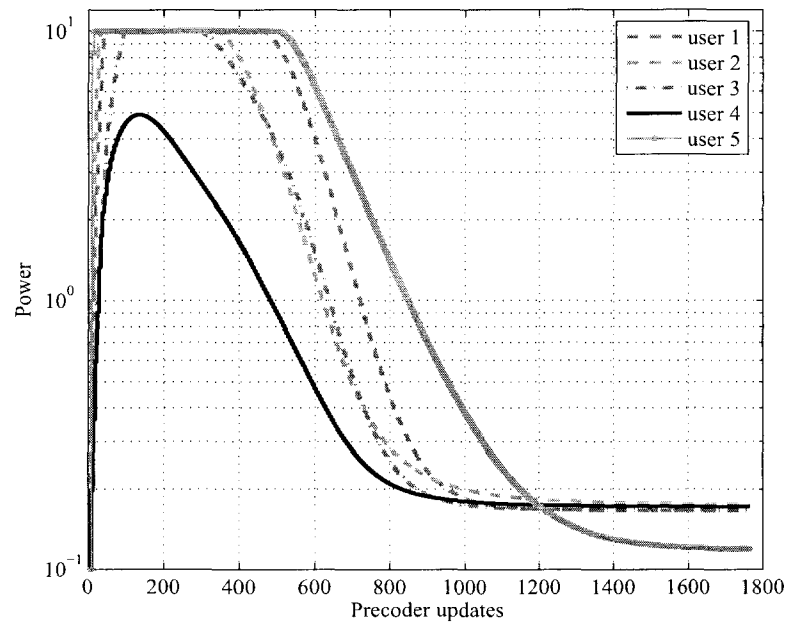


Fig. 14: Power Variation for the system with  $K = 5$  users in  $N = 3$  signal space dimensions from random initialization.

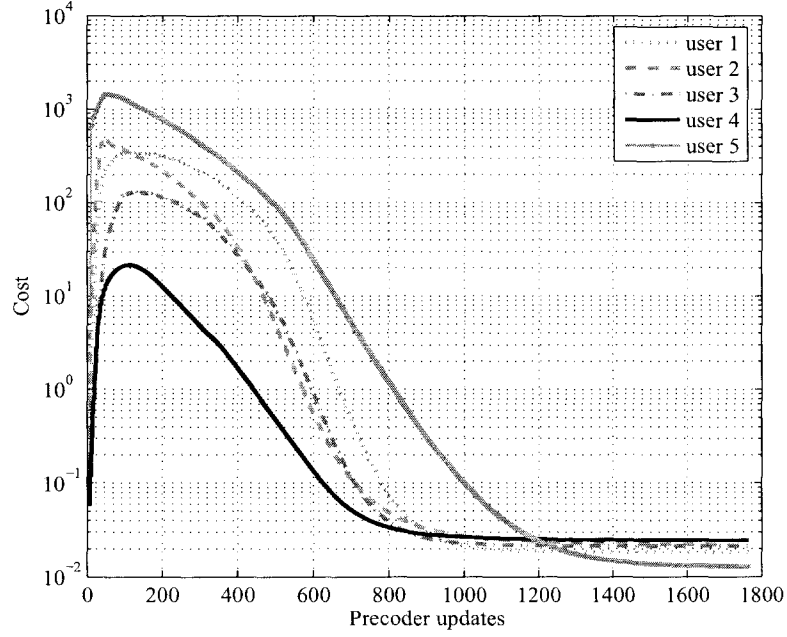


Fig. 15: Cost Variation for the system with  $K = 5$  users in  $N = 3$  signal space dimensions from random initialization.

for remaining active users as  $\{1.5, 1.4, 1.3, 1.2\}$ . Then the algorithm was applied to this new configuration. After the system reached at new optimal fixed point a new user was added to the system with randomly generated precoder, power and channel matrix, and with a target SINR  $\gamma_5^* = 1.0$ . For this new configuration, the algorithm was applied until it reached to the optimal point.

Then, the variation of the user SINRs, powers and costs are plotted as shown in Figures 16, 17, and 18 where a sudden sharp change appeared in user SINRs, powers and costs when one user is dropped from the system as well as when the new user is added to the system, that are gradually compensated by the algorithm which brings the users SINRs to the specified target SINR values. The plots in Figures 16, 17, and 18 show the tracking ability of the algorithm for variable number of active users in

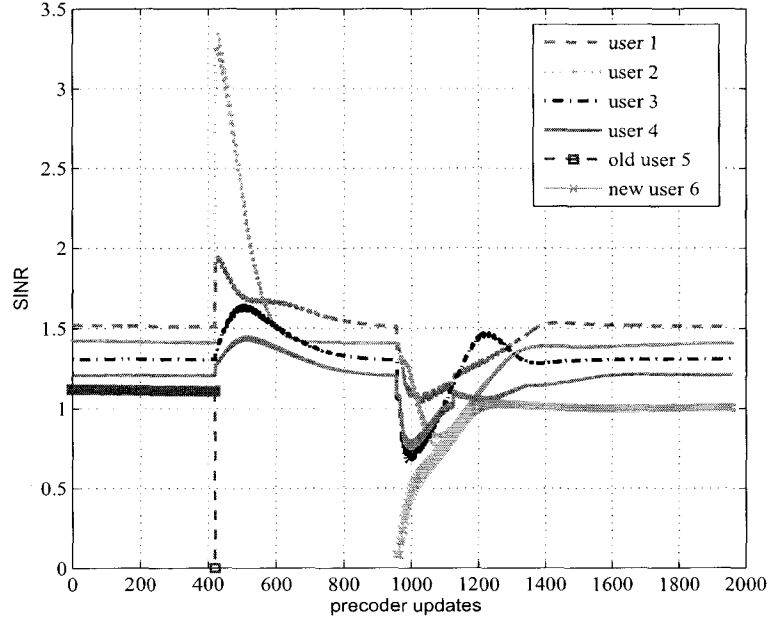


Fig. 16: Variation of user SINRs for the tracking example where one user is dropped from the system followed by subsequent addition of another user.

multipath uplink of the wireless system. This shows that the proposed algorithm is applicable to real time dynamic wireless systems where the users can join and leave the system.

### III.5.3 Tracking Ability of the Algorithm for Variable Target SINRs

In order to show that tracking ability of the algorithm for variable target SINR, another scenario with  $K = 5$  users in a signal space of dimension  $N = 4$  and AWGN with covariance matrix  $\mathbf{W} = 0.1\mathbf{I}_N$  was considered. The algorithm constants are  $\beta = 0.01$ ,  $\mu = 0.01$ , and tolerance  $\epsilon = 0.01$  and channels matrices were chosen randomly for all users. Initial user powers were  $p_k = 0.1$ ,  $\forall k$  and initial precoders were selected randomly. The maximum allowed power level  $p_k^{max}$ ,  $\forall k$ , was set to 10.

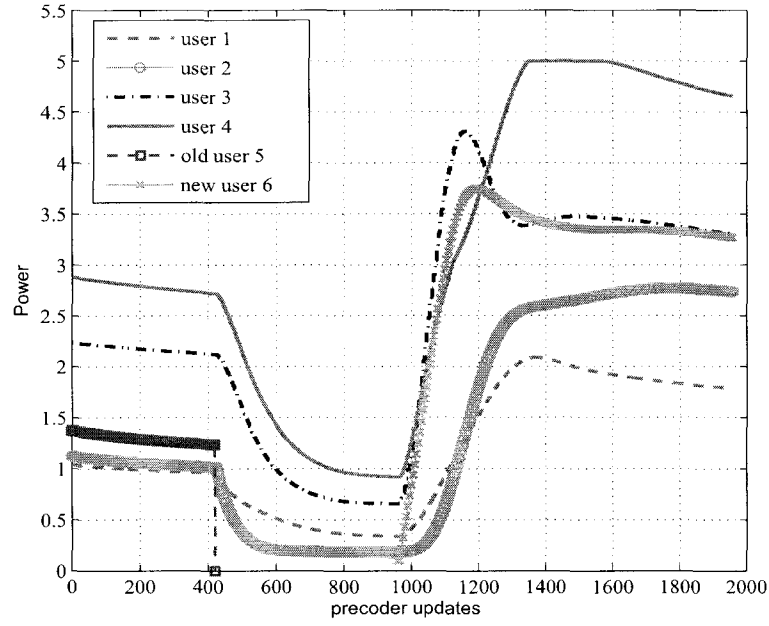


Fig. 17: Variation of user powers for the tracking example where one user is dropped from the system followed by subsequent addition of another user.

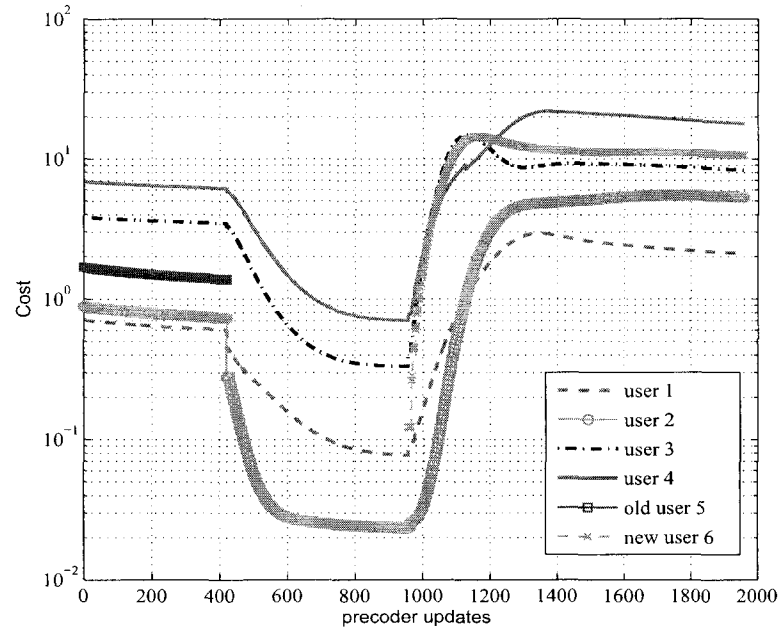


Fig. 18: Variation of user costs for the tracking example where one user is dropped from the system followed by subsequent addition of another user.

In this case, user target SINRs were chosen as

$$\{\gamma_1^*, \gamma_2^*, \gamma_3^*, \gamma_4^*, \gamma_5^*\} = \{5.0, 4.0, 3.0, 2.0, 1.0\}$$

which are admissible in the system. The objective is to illustrate the tracking ability of the proposed incremental algorithm for the users who keep changing their corresponding target SINRs based on their needs. The algorithm was applied for this simulation setup, and once the algorithm is reached at the optimal Nash equilibrium, the target SINR of user 5 was changed to  $\gamma_5^* = 2.5$  and the algorithm was applied until it reached again to Nash equilibrium. Then after, the target SINR of user 5 was changed to a new value  $\gamma_5^* = 1.75$ , and the algorithm was applied for this setup until it reached to Nash equilibrium. The variation of the user SINRs, powers and costs for this scenario are plotted, respectively, in Figures 19, 20, and 21 which show the tracking ability of the algorithm in terms of variable target SINRs. Thus, the proposed algorithm can be used in dynamic systems where users may change their QoS requirements on the fly.

### III.6 PERFORMANCE COMPARISON

In this section, the performance of the proposed non-cooperative game is compared with the existing method proposed by Buzzi et al. in [1]. It is worth noting that the authors in [1] have compared their proposed method with many other existing ones and have shown that their method outperforms all of them.

In order to compare, similar simulation scenario presented in [1] is considered which consists of an uplink of a wireless system consists of  $K = 1, 4, 7, 10, 13, 16, 19, 22$



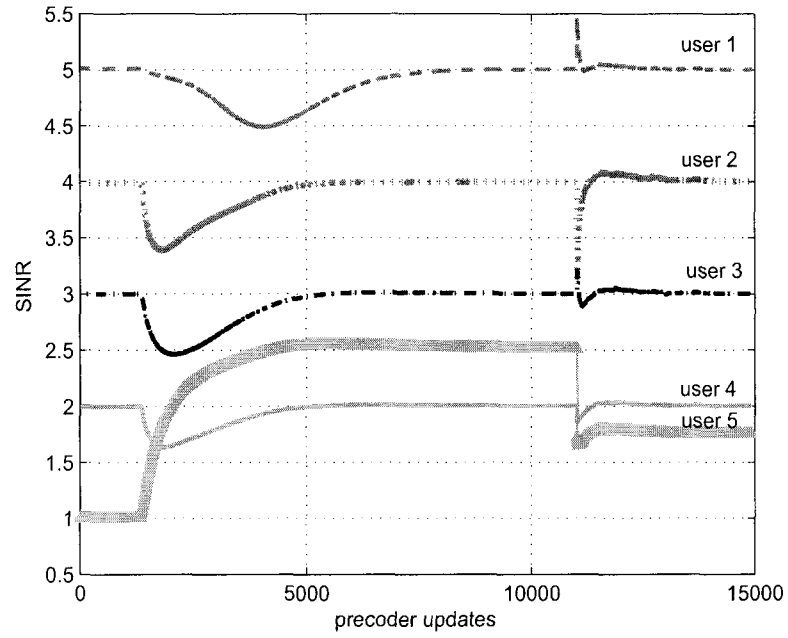


Fig. 19: Variation of user SINRs for the tracking SINR example with  $K = 5$  users in  $N = 3$  signal space dimensions.

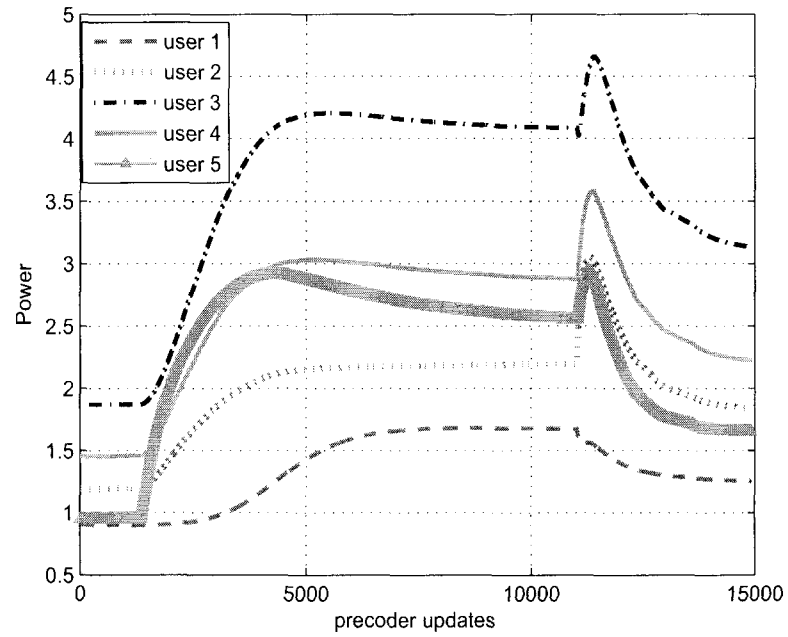


Fig. 20: Variation of user powers for the tracking SINR example with  $K = 5$  users in  $N = 3$  signal space dimensions.

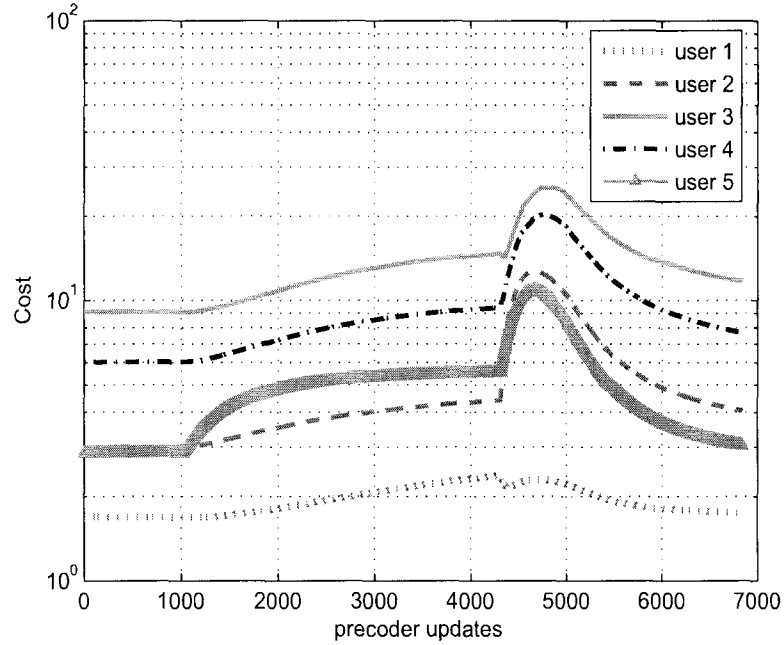


Fig. 21: Variation of user costs for the tracking SINR example with  $K = 5$  users in  $N = 3$  signal space dimensions.

and signal space dimension  $N = 15$ . In each case, the same target SINRs  $\gamma_k^* = 6.689 = 8.25\text{dB}$  are taken for all active users wherein users may have random positions which is incorporated by using different channel gains generated randomly. The AWGN level is chosen to be  $N_0 = 10^{-9}\text{W/Hz}$  and the maximum allowed power  $P_k^{max}$  is  $-25\text{dBW}$  for all users  $k = 1, \dots, K$ .

Figures 22 and 23 show the average user transmit power and the average achieved SINR at the receiver output versus the number of users, for the game in [1] and for the non-cooperative game considered in this chapter. The results presented are of averaging over 1,000 independent realizations for the simulation scenarios. More precisely, for each trial, channel coefficients, initial power and precoder for each user were generated randomly.

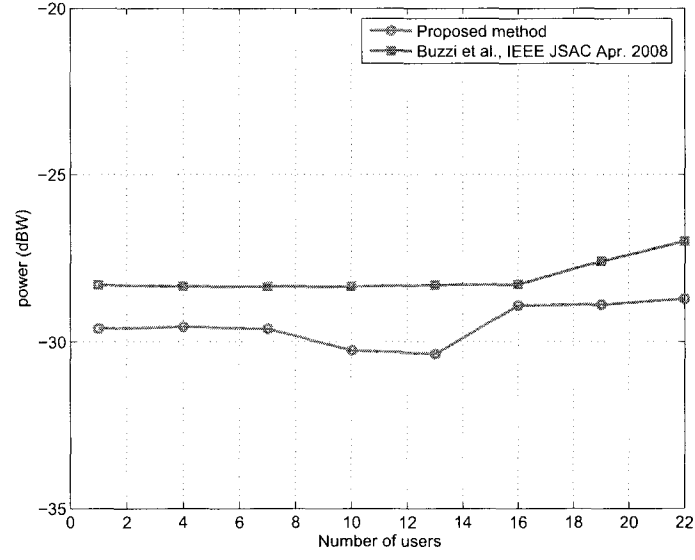


Fig. 22: Average transmit power versus number of active users for the proposed non-cooperative game and for the game in references [1] for the signal space dimension (system processing gain)  $N = 15$ .

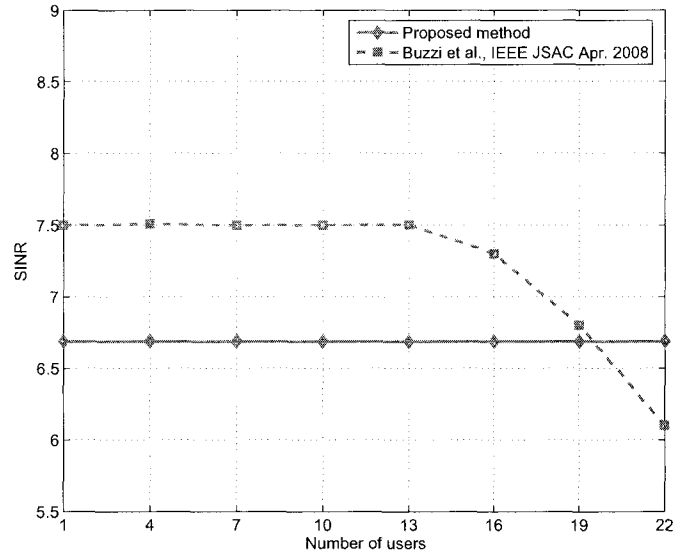


Fig. 23: Achieved average output SINR versus number of active users for the proposed non-cooperative game and for the game in references [1] for the signal space dimension (system processing gain)  $N = 15$ .

By observing the plots in Figures 22 and 23, it is seen that the proposed approach converges to a social optimal point (Nash equilibrium) where transmit power is lower in the case presented in [1]. This became possible because the proposed algorithm uses interference avoidance algorithm with power control to meet target SINRs. Furthermore, the SINR is always constant in our scheme since the target SINR is always met within the given tolerance regardless of the system load (i.e., underloaded or overloaded). However, in the case of overloaded scenarios for the game in [1], the achieved SINR is below the target SINR. It has shown that the game proposed in this chapter largely outperforms the game proposed in the literature [1].

### III.7 CHAPTER SUMMARY

In this chapter, game theory based algorithm is presented for joint precoder adaptation and power control in uplink wireless systems with non-ideal channels between users and base station. The proposed algorithm uses incremental precoder and power updates in the direction of the optimal strategies that minimize cost functions and has a smooth transition to a steady state configuration where specified SINR targets are achieved with minimum power. The algorithm can also keep track of variable target SINRs for active users and variable number of active users in the system, and therefore is useful for dynamic wireless system with changing number of active users and/or QoS requirements. It has also shown that the analytical optimal point which is Nash equilibrium in the game theoretic approach. Furthermore, proposed approach outperforms the methods existing in the literature in terms of transmit power and QoS.

## CHAPTER IV

### TRANSMITTER ADAPTATION WITH POWER CONTROL IN INTERFERENCE SYSTEMS

In this chapter, transmitter adaptation with power control in interference system where each user-transmitter pair communicates in the presence of other interfering links and creates interference to other user-receivers is presented. The proposed method is applicable to both conventional wireless system and cognitive radio systems. As centralized methods for resource allocation tend to be computationally expensive in large-scale networks, the interest is to optimize the link parameters with local information and reasonable computational burden leading to a decentralized approach in interference systems. A distributed method, where each link attempts to selfishly maximize its own mutual information based on the knowledge of its own channel matrix and the covariance of the total interference and the noise at its own receiver.

Section IV.1 presents the background and a literature review relevant to this research problem. Section IV.2 deals with a system model, problem statement and operating constraints. Achievable rate maximization in spectrum underlay is presented in Section IV.3 followed by the rate maximization in spectrum overlay in Section IV.4 along with simulation results.

## IV.1 BACKGROUND AND RELATED WORK

Conventional radio frequency (RF) spectrum allocation is based on the specific band assignments designated for a particular service (or a service provider) for long time and vast geographic area. The usage of statistically allocated RF spectrum is ranging from 15% to 85% in the bands below 3 GHz that are favored in non-line-of-sight propagation and even lesser in the bands above 3 GHz leading to spectrum scarcity [7, 64–66]. In other words, the existing “command-and-control” spectrum allocations defined by government regulatory agencies prohibit the unlicensed access to licensed spectrum which results in underutilization of significant amount of spectrum in almost all currently deployed frequency bands. Wireless technologies and devices are increasing day-by-day and becoming ubiquitous in our daily life leading to strain the effectiveness of the traditional spectrum policies. As a consequence, the need of more efficient usage of limited wireless resources is the central step in next generation (XG) wireless networks. The inefficient usage of the RF spectrum can be improved through dynamic and opportunistic access of licensed bands by unlicensed secondary users [67, 68]. In particular, cognitive radio (CR) [5, 7, 69] has been proposed to alleviate the spectrum scarcity through dynamic spectrum access. It is noted that the dynamic spectrum access for spectrum sharing has two basic approaches [68]. One is a *spectrum underlay* approach where CR users are allowed to coexist and transmit simultaneously with primary users sharing the same licensed bands but by imposing transmit power constraint on CR users so as not to cause any harmful interference to the active primary user-receivers. In this approach, CR users are not required to sense for spectrum opportunities however they are not allowed to transmit with

higher than the preset power mask even if the primary system is completely idle. The other is *spectrum overlay* technique whereby a secondary CR users are required to sense and identify the spectrum opportunities in licensed bands before using them for given time and geographic location, and exploit those opportunities dynamically. If primary users are active in a given frequency band for given time and location, the band will not be used by secondary CR users. In this approach CR users can transmit with high data rate as primary users once they find the idle spectrum bands. It is noted that the main goal in both approaches is to access the licensed spectrum dynamically and/or opportunistically by respecting the primary user transmissions.

In this chapter, the combined precoder adaptation and power control is presented for both spectrum underlay and overlay scenarios for dynamic spectrum sharing in cognitive radio networks. For spectrum underlay, it is considered that the secondary users coexist and transmit simultaneously with primary users without violating the interference power limit at primary user-receivers. For spectrum overlay, the information related to spectrum opportunities is assumed to be available at CR transmitters using a spectrum sensing method as mentioned in [70] so that they use available idle spectrum bands opportunistically. Interference system is considered where a given transmitter creates interference to all receivers. In general, finding the capacity of interference channel is an open problem [20, 71]. One can model the interference as colored noise and use a whitening transform [72] to find the capacity in the presence of external interference.

The related previous works include [25, 26, 73–75], where users allocate their powers over frequency-selective channels in a competitive way. The optimal point in

terms of the Nash equilibrium in the Gaussian interference channel has been characterized in [25, 26, 73] for selfish users with their power spectra to maximize their own rates. The simulation results in [74] has shown the inefficiency of Nash equilibrium for two-user power control game, and the results in [75] has shown that the reduction in Nash equilibrium inefficiency in a spectrum sharing game using the repeated game equilibria. It is emphasized that the model used in these studies is different from this research, since the user decision parameters in [25, 26, 73–75], are either the precoder or the transmitted power adaptations over the spectrum rather than combined adaptation of precoder and power of the transmitted signals. Moreover, all the previous related works have not considered the interference constraint at primary user-receivers and the methods are not applicable to spectrum underlay scenarios.

## IV.2 SYSTEM MODEL AND PROBLEM STATEMENT

This chapter considers an interference system for wireless communications with  $K$  CR links and  $L$  primary links where each link consist of transmitter-receiver pair as shown in Figure 24.

### IV.2.1 System Model

Consider a primary wireless system operating in a signal space of dimension  $N$  where secondary CR users share those signal space dimensions dynamically and opportunistically. That is, the licensed bandwidth will be accessed by secondary CR users dynamically through the use of signal spaces. In a spectrum underlay, secondary CR users coexist with primary users and use the same signal spaces but satisfy the interference constraint. Whereas in a spectrum overlay, secondary CR



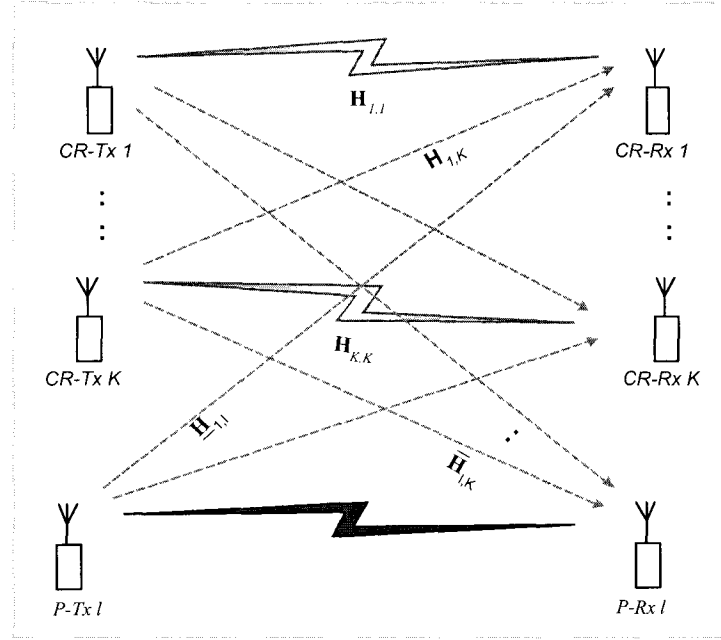


Fig. 24: System model with one primary (licensed) link and  $K$  secondary CR links.

users avoid the signal spaces occupied by primary users in order not to harm primary user transmissions.

As the block transmission, which includes schemes such as orthogonal frequency division multiplexing (OFDM) and code division multiple access (CDMA), is capacity-lossless strategy [72, 76], it is considered that the user transmitter adopts the block transmission making the proposed approach general and independent of the CR system implementation. The  $N$ -dimensional transmitted signal by a CR user-transmitter  $i$  can be expressed as

$$\mathbf{x}_i = \mathbf{C}_i \mathbf{d}_i = \bar{\mathbf{C}}_i \mathbf{P}_i^{\frac{1}{2}} \mathbf{d}_i \quad (\text{IV.2.1})$$

where  $\mathbf{d}_i$  is  $N$ -dimensional information symbol block transmitted by CR user  $i$  with  $E[\mathbf{d}_i \mathbf{d}_i^\top] = \mathbf{I}$ , and  $\mathbf{C}_i$  is  $N \times N$  dimensional precoding matrix and is expressed as  $\mathbf{C}_i = \bar{\mathbf{C}}_i \mathbf{P}_i^{\frac{1}{2}}$  in terms of its normalized  $N \times N$  dimensional form  $\bar{\mathbf{C}}_i^1$  and  $N \times N$  dimensional diagonal transmit power matrix  $\mathbf{P}_i = \text{diag}\{p_1^{(i)}, p_2^{(i)}, \dots, p_n^{(i)}, \dots, p_N^{(i)}\}$ .

Similar to equation (I.1.1), the  $N$ -dimensional received signal at the desired CR receiver of link  $i$  is described in terms of the vector channel model as [77]

$$\mathbf{y}_i = \mathbf{H}_{i,i} \mathbf{x}_i + \underbrace{\sum_{j \neq i, j=1}^K \mathbf{H}_{i,j} \mathbf{x}_j}_{\mathbf{z}_i} + \mathbf{q}_i + \mathbf{n}_i \quad (\text{IV.2.2})$$

where  $\mathbf{H}_{i,j}$  is the  $N \times N$  dimensional channel matrix between transmitter  $j$  and receiver  $i$  (i.e.,  $\mathbf{H}_{i,i}$  is the channel matrix for the CR link  $i$ ),  $\mathbf{x}_i$  is the  $N$ -dimensional transmitted signal vector as in (IV.2.1),  $\mathbf{q}_i = \sum_{\ell=1}^L \mathbf{H}_{i,\ell} \mathbf{x}_\ell$  represents the interference experienced at  $i$ th CR receiver from  $L$  licensed user-transmitters with channel matrices  $\mathbf{H}_{i,\ell}$ ,  $\forall \ell$ , and  $\mathbf{n}_i$  is the  $N$ -dimensional additive white Gaussian noise at  $i$ th receiver with  $E[\mathbf{n}_i \mathbf{n}_i^\dagger] = \sigma_i^2 \mathbf{I}$ . The first term of right side in (IV.2.2) is the desired signal for link  $i$ , and the remaining terms consist of interference from other links and the noise. In this chapter, it is assumed that:

- individual transmitter-receiver pairs transmit and receive symbols independently without any collaboration at either transmitter or receiver,
- the co-channel interference from other interfering links is unknown and thus treated as noise (i.e., no interference cancelation techniques are employed at receivers), and

---

<sup>1</sup>The  $N \times N$  dimensional matrix  $\bar{\mathbf{C}}_i$  is said to be normalized when its each column has unit norm.

- the channels are assumed to be known and vary sufficiently slow and can be considered as time invariant during the period of each symbol transmission, and varies block by block independently.

Therefore, by treating the multiuser interference (MUI) as noise, the received signal in (IV.2.2) can be expressed from the perspective of a single CR link  $i$  as

$$\mathbf{y}_i = \mathbf{H}_{i,i}\mathbf{x}_i + \mathbf{z}_i = \mathbf{H}_{i,i}\mathbf{C}_i\mathbf{d}_i + \mathbf{z}_i \quad (\text{IV.2.3})$$

where  $\mathbf{z}_i$  is the interference-plus-noise vector that corrupts CR  $i$ 's transmission and is assumed to be identified by the CR  $i$  receiver in a preliminary sensing operation, and hence its autocorrelation matrix  $E[\mathbf{z}_i\mathbf{z}_i^\dagger] = \mathbf{R}_{z_i}$  is known.

In this chapter with an aim of reducing complexities at receivers, we are not interested in joint decoding of the interfering signals rather are interested to maximize achievable rates over the transmit-covariance matrices.

$$\mathbf{Q}_i = E[\mathbf{x}_i\mathbf{x}_i^\dagger] = \mathbf{C}_i\mathbf{C}_i^\dagger = \bar{\mathbf{C}}_i\mathbf{P}_i\bar{\mathbf{C}}_i^\dagger, \quad \forall i = 1, \dots, K \quad (\text{IV.2.4})$$

It is noted that the covariance matrix consists of both precoder and power of the corresponding link.

## IV.2.2 Operating Constraints

The transmit power in wireless networks is a key element in the management of interference, energy, and connectivity. The transmit power along with the RF

spectrum is regarded as the scarce resources and its efficient utilization is highly recommended in order to increase spectral efficiency and the system capacity. Therefore, the CR transmitter  $i$  is subject to an average transmit power constraint this implies that

$$\text{Trace}[\mathbf{Q}_i] = \text{Trace}[\mathbf{C}_i \mathbf{C}_i^\dagger] = \text{Trace}[\mathbf{P}_i] \leq P_i^{\max} \quad (\text{IV.2.5})$$

where  $P_i^{\max}$  is an upper limit set on the CR link  $i$ 's average transmit power.

In the spectrum underlay, CR users may operate in the same frequency bands with licensed primary users provided that the interference generated by CR systems is below the maximum level tolerated by the licensed users, i.e.

$$I_\ell \leq P_\ell^{IT}, \quad \forall \ell = 1, \dots, L \quad (\text{IV.2.6})$$

where  $I_\ell$  is the interference power experienced from CR users at primary receiver  $\ell$ , and the interference (and noise) level experienced from primary system at primary receiver  $\ell$  are assumed to be already taken into account. It is worth noting that the maximum permissible interference level  $P_\ell^{IT}$  should be supplied either by the governmental regulatory agencies or the licensed primary operator and may be obtained based on *interference temperature* constraints [78, 79].

In order to estimate the interference power  $I_\ell$ , the received signal from  $K$  CR transmitters at the primary receiver  $\ell$  is given as

$$\mathbf{r}_\ell = \sum_{i=1}^K \bar{\mathbf{H}}_{\ell,i} \mathbf{x}_i = \sum_{i=1}^K \bar{\mathbf{H}}_{\ell,i} \mathbf{C}_i \mathbf{d}_i \quad (\text{IV.2.7})$$

where  $\bar{\mathbf{H}}_{\ell,i}$  is the  $N \times N$  channel matrix between CR transmitter  $i$  and primary

receiver  $\ell$ . The interference power can be obtained as

$$I_\ell = \text{Trace} \left[ \sum_{i=1}^K \bar{\mathbf{H}}_{\ell,i} \mathbf{Q}_i \bar{\mathbf{H}}_{\ell,i}^\dagger \right] \quad (\text{IV.2.8})$$

Then, one can write the interference constraint in equation (IV.2.6) as

$$\text{Trace} \left[ \sum_{i=1}^K \bar{\mathbf{H}}_{\ell,i} \mathbf{Q}_i \bar{\mathbf{H}}_{\ell,i}^\dagger \right] \leq P_\ell^{IT} \quad (\text{IV.2.9})$$

It is considered that the CR users are aware of the total interference caused to the primary users and the interference threshold  $P_\ell^{IT}$ ,  $\forall \ell$ . This is possible either by broadcasting the related interference information by primary users or by using extra sensors (which have ranging capabilities) at CR users to measure it or by using an external detection agent [80].

In the spectrum overlay, the spectral shapes of transmitted waveforms need to comply some design and governmental spectral regulation requirements. In order to comply with a specific power mask in each signal space, the spectral mask constraint for user  $i$  can be expressed as [25, 26]

$$p_n^{(i)} = [\mathbf{Q}_i]_{n,n} \leq p_i^{mask}(n), \quad \forall n = 1, \dots, N, \quad \forall i = 1, \dots, K \quad (\text{IV.2.10})$$

It is important to note that the power mask in each signal space dimension should not be violated in order to comply with regulation requirements which leads to the condition

$$\sum_{n=1}^N p_i^{mask}(n) \geq P_i^{max} \quad (\text{IV.2.11})$$

that help to protect the primary user transmissions from the interference created by CR links.

Furthermore, in the spectrum overlay, the signal space dimensions  $n_p \in \{1, \dots, N\}$  which are occupied by primary users should not be used in secondary CR transmissions. This can be ensured by setting  $p_i^{mask}(n_p) = 0$  for  $n_p \in \{1, \dots, N\}$  to avoid possible interference to primary users. This avoidance process will be carried out at CR transmitter  $i$  based on the spectrum opportunities obtained from spectrum sensing and analysis process [70], and the spectrum opportunities are assumed to be available at CR transmitter. It is noted that the spectrum sensing process in CR systems is a fundamental step however it is out of the scope of this chapter. In spectrum overlay case, it must be pointed out that the interference from primary users in (IV.2.2) will be zero since secondary CR users avoid the bands used by primary users.

### IV.2.3 Problem Formulation

In this framework, the problem of resource allocation for CR link  $i$  consists of finding the precoder matrix  $\mathbf{C}_i$  along with the transmit power that optimizes some meaningful objective function subject to specified operating constraints. From an individual CR link perspective, the achievable rate corresponding to each CR link is considered as a performance measure of wireless applications, and for the CR link described by equation (IV.2.3), it is expressed as [20]

$$\mathcal{R}_i = \frac{1}{2} \log_2 |\mathbf{R}_{y_i}| - \frac{1}{2} \log_2 |\mathbf{R}_{z_i}| \quad [\text{bits/transmission}] \quad (\text{IV.2.12})$$

where

$$\mathbf{R}_{y_i} = \mathbf{H}_{i,i} \mathbf{Q}_i \mathbf{H}_{i,i}^\dagger + \mathbf{R}_{z_i} \quad (\text{IV.2.13})$$

is the correlation matrix of the received signal  $\mathbf{y}_i$  in (IV.2.3) at the CR  $i$  receiver.

The rate expression (IV.2.12) can also be expressed in compact form using the fact that  $|(\mathbf{I} + \mathbf{AB})| = |(\mathbf{I} + \mathbf{BA})|$  as

$$\begin{aligned} \mathcal{R}_i &= \frac{1}{2} \log_2 |\mathbf{R}_{z_i} + \mathbf{H}_{i,i} \mathbf{Q}_i \mathbf{H}_{i,i}^\dagger| - \frac{1}{2} \log_2 |\mathbf{R}_{z_i}| \\ &= \frac{1}{2} \log_2 |\mathbf{I} + \mathbf{R}_{z_i}^{-1} \mathbf{H}_{i,i} \mathbf{Q}_i \mathbf{H}_{i,i}^\dagger| \\ &= \frac{1}{2} \log_2 |\mathbf{I} + \mathbf{C}_i^\dagger \mathbf{H}_{i,i}^\dagger \mathbf{R}_{z_i}^{-1} \mathbf{H}_{i,i} \mathbf{C}_i| \end{aligned} \quad (\text{IV.2.14})$$

The decentralized optimal resource allocation which maximizes the achievable rate  $\mathcal{R}_i$  for CR link by choosing an appropriate precoder matrix and power in the presence of the other links those also want to maximize their own achievable rates can be formally written as the following constrained optimization problem

$$\begin{aligned} &\max_{\mathbf{Q}_i \geq 0} \quad \mathcal{R}_i \\ &\text{subject to} \quad \left\{ \begin{array}{l} \text{Trace}[\mathbf{Q}_i] \leq P_i^{\max} \\ [\mathbf{Q}_i]_{n,n} \leq p_i^{\text{mask}}(n) \\ \text{Trace} \left[ \sum_{i=1}^K \bar{\mathbf{H}}_{\ell,i} \mathbf{Q}_i \bar{\mathbf{H}}_{\ell,i}^\dagger \right] \leq P_\ell^{IT} \end{array} \right. \end{aligned} \quad (\text{IV.2.15})$$

It is noted that the relation in equation (IV.2.11) should be satisfied so that the primary user transmissions are not hampered from the interference created by the CR users. Otherwise average transmit power constraint can be omitted. Then the

optimization problem can be written as

$$\begin{aligned} & \max_{\mathbf{Q}_i \geq 0} \quad \mathcal{R}_i \\ & \text{subject to } \begin{cases} \text{Trace}[\mathbf{Q}_i] \leq P_i^{max}, & \forall i \\ \text{Trace} \left[ \sum_{i=1}^K \bar{\mathbf{H}}_{\ell,i} \mathbf{Q}_i \bar{\mathbf{H}}_{\ell,i}^\dagger \right] \leq P_\ell^{IT}, \forall \ell \end{cases} \end{aligned} \quad (\text{IV.2.16})$$

It is noted that the optimization problem (IV.2.16) is different from the traditional rate maximization problem under average power constraint only which has a water filling solution [81] and that a related problem with power and spectral constraints has been studied in [25,26]. However, the approach proposed in [25,26] has not taken account of interference constraint (i.e., the interference power experienced at primary receivers from CR users), which might not be realistic assumption for CR systems, and thus the method may not be applicable to spectrum underlay scenario. But, the resource allocation approach proposed in this chapter is suitable for CR system which can switch from spectrum underlay to overlay and vice versa.

### IV.3 RATE MAXIMIZATION IN SPECTRUM UNDERLAY

In this scenario, the achievable rate maximization problem (IV.2.16) for individual CR links is considered in which each transmitter chooses an appropriate precoder and power in the presence of the other interfering links.

As mentioned previously, the dissemination of the interference power contribution  $P_{\ell,i}^{IT}$  of CR link  $i$  can be done by broadcasting or feeding back the related information from coordinator. However, this research adopts the way of setting the upper limit



of interference power contribution for each link as  $P_{\ell,i}^{IT} = P_{\ell}^{IT}/K$  to obtain a fully distributed solution.

#### IV.3.1 Distributed Solution in Spectrum Underlay Using Primal Decomposition Approach

With primal-decomposition of overall interference power constraint contribution to per-link constraint, the constrained optimization problem (IV.2.16) can be written in terms of precoder as

$$f(P_{\ell,i}^{IT}) \equiv \mathcal{R}_i \quad \text{subject to} \quad \begin{cases} \text{Trace} [\mathbf{C}_i \mathbf{C}_i^{\dagger}] \leq P_i^{max}, & \forall i \\ \text{Trace} [\mathbf{C}_i^{\dagger} \bar{\mathbf{H}}_{\ell,i}^{\dagger} \bar{\mathbf{H}}_{\ell,i} \mathbf{C}_i] \leq P_{\ell,i}^{IT}, & \forall \ell \end{cases} \quad (\text{IV.3.1})$$

It is noted that the desired optimization problem for per-link in terms of semi-distributed solution can be expressed as

$$\max \quad f(P_{\ell,i}^{IT}) \quad \text{subject to} \quad \begin{cases} \sum_{i=1}^K P_{\ell,i}^{IT} \leq P_{\ell}^{IT}, & \forall \ell \\ P_{\ell,i}^{IT} \geq 0, & \forall i, \ell \end{cases} \quad (\text{IV.3.2})$$

It is noted that the maximization of (IV.3.1) results in maximization of (IV.3.2). The Lagrangian solution of (IV.3.1) is presented, which is the rate maximization problem in a distributed approach, and it can be written in terms of  $\mathbf{Q}_i$  as

$$\begin{aligned} \tilde{\mathcal{L}}_i(\mathbf{Q}_i, \mu_i \lambda_{\ell,i}) &= \frac{1}{2} \log_2 |\mathbf{R}_{z_i} + \mathbf{H}_{i,i} \mathbf{Q}_i \mathbf{H}_{i,i}^{\dagger}| - \frac{1}{2} \log_2 |\mathbf{R}_{z_i}| - \\ &\quad - \mu_i \text{Trace} [\mathbf{Q}_i] - \lambda_{\ell,i} \text{Trace} [\bar{\mathbf{H}}_{\ell,i} \mathbf{Q}_i \bar{\mathbf{H}}_{\ell,i}^{\dagger}] + \mu_i P_i^{max} + \lambda_{\ell,i} P_{\ell,i}^{IT} \end{aligned} \quad (\text{IV.3.3})$$

and equivalently it can be expressed in terms of  $\mathbf{C}_i$  as

$$\begin{aligned} \tilde{\mathcal{L}}_i(\mathbf{C}_i, \mu_i \lambda_{\ell,i}) &= \frac{1}{2} \log_2 |\mathbf{I} + \mathbf{C}_i^\dagger \mathbf{H}_{i,i}^\dagger \mathbf{R}_{z_i}^{-1} \mathbf{H}_{i,i} \mathbf{C}_i| - \text{Trace} \left[ \mathbf{C}_i^\dagger (\mu_i \mathbf{I} + \lambda_{\ell,i} \bar{\mathbf{H}}_{\ell,i}^\dagger \bar{\mathbf{H}}_{\ell,i}) \mathbf{C}_i \right] + \\ &\quad + \mu_i P_i^{max} + \lambda_{\ell,i} P_{\ell,i}^{IT} \end{aligned} \quad (\text{IV.3.4})$$

where  $\mu_i$  and  $\lambda_{\ell,i}$  are, respectively, the Lagrange multipliers associated with average power constraint and interference constraint. These two Lagrange multiplier can be found by using the ellipsoid algorithm [82, 83] which will satisfy both power and interference constraints.

**Proposition 1** [84]: *The optimal  $\mathbf{C}_i$  solving the maximization problem (IV.3.4) is a generalized eigen-matrix of  $\mathbf{H}_{i,i}^\dagger \mathbf{R}_{z_i}^{-1} \mathbf{H}_{i,i}$  and  $(\mu_i \mathbf{I} + \lambda_{\ell,i} \bar{\mathbf{H}}_{\ell,i}^\dagger \bar{\mathbf{H}}_{\ell,i})$ . That is,*

$$\mathbf{H}_{i,i}^\dagger \mathbf{R}_{z_i}^{-1} \mathbf{H}_{i,i} \mathbf{C}_i = (\mu_i \mathbf{I} + \lambda_{\ell,i} \bar{\mathbf{H}}_{\ell,i}^\dagger \bar{\mathbf{H}}_{\ell,i}) \mathbf{C}_i \Delta_i \quad (\text{IV.3.5})$$

*Proof:* A similar result is proved in [84] in the context of multiple-input- multiple-output (MIMO) system. Thus the results are used for the single antenna-based system and the proof is omitted here. ■

It is noted that the Proposition 1 specifies the optimal precoders  $\mathbf{C}_i$  but not the optimal power allocation. Without loss of generality, the columns of the precoder are converted into unit norms such that  $\mathbf{C}_i = \bar{\mathbf{C}}_i \mathbf{P}_i^{\frac{1}{2}}$ , where  $\mathbf{P}_i$  is diagonal and non-negative power matrix.

One can write from [58] that

$$\Delta_i^{(1)} = \bar{\mathbf{C}}_i^\dagger \mathbf{H}_{i,i}^\dagger \mathbf{R}_{z_i}^{-1} \mathbf{H}_{i,i} \bar{\mathbf{C}}_i \quad (\text{IV.3.6})$$

and

$$\bar{\mathbf{C}}_i^\dagger (\mu_i \mathbf{I} + \lambda_{\ell,i} \bar{\mathbf{H}}_{\ell,i}^\dagger \bar{\mathbf{H}}_{\ell,i}) \bar{\mathbf{C}}_i = \Delta_i^{(2)} \quad (\text{IV.3.7})$$

where both  $\Delta_i^{(1)}$  and  $\Delta_i^{(2)}$  are diagonal. Moreover, from (IV.3.5), (IV.3.6) and (IV.3.7), one can write

$$\Delta_i^{(1)} = \Delta_i \Delta_i^{(2)} \quad (\text{IV.3.8})$$

Then, (IV.3.3) can be expressed as

$$\max_{\mathbf{P}_i \geq 0} \quad \frac{1}{2} \log_2 |\mathbf{I} + \mathbf{P}_i \Delta_i^{(1)}| - \text{Trace} [\mathbf{P}_i \Delta_i^{(2)}] \quad (\text{IV.3.9})$$

Therefore, the optimal power allocation with spectral mask constraint in each signal space dimension for the problem (IV.3.9) is given as [25, 26]

$$p_n^{(i)} = [\mathbf{P}_i]_{n,n} = \left[ \frac{[\Delta_i]_{(n,n)} - 1}{[\Delta_i^{(1)}]_{(n,n)}} \right]_{p_i^{mask(n)}}^{p_i^{mask(n)}}, \quad \forall n = 1, \dots, N \quad (\text{IV.3.10})$$

where the Euclidean projection  $[s]_a^b$  with  $a \leq b$  is defined as [82]:  $[s]_a^b = a$  if  $s \leq a$ ,  $[s]_a^b = b$  if  $s \geq b$ , and  $[s]_a^b = s$  otherwise. It is noted that the other non-diagonal elements of  $\mathbf{P}_i$  are zeros.

The spectral mask determines the upper limit of transmit power in each signal space dimension which protect the primary user transmissions.

### IV.3.2 Algorithm

Based on the analysis above, formally an algorithm is stated for resource allocation by using combined precoder adaptation and power control for CR links as

follows:

### Algorithm: CR Resource Allocation

#### 1. Input:

- Precoder matrices  $\mathbf{C}_i$  and find  $\mathbf{Q}_i = \mathbf{C}_i \mathbf{C}_i^\dagger = \bar{\mathbf{C}}_i \mathbf{P}_i \bar{\mathbf{C}}_i^\dagger$ .
- Channel matrices  $\mathbf{H}_{i,i}$ ,  $\mathbf{H}_{i,j}$  and  $\bar{\mathbf{H}}_{\ell,i}$ ,  $P_i^{max}$ ,  $P_\ell^{IT}$ ,  $\mathbf{q}_i$ , the noise variances  $\sigma_i$  and
- the desired tolerance  $\epsilon$ ,  $\mu_i$  and  $\lambda_{\ell,i}$ .

#### 2. Output:

- Optimal precoder and transmit power of CR links

#### 3. Precoder Adaptation and Power Control

- DO FOR  $i = 1, \dots, K$  CR links
  - (a) Compute the generalized eigen matrix  $\mathbf{C}_i$  and eigenvalues  $\Delta_i$  of the matrix pair  $\mathbf{H}_{i,i}^\dagger \mathbf{R}_{z_i}^{-1} \mathbf{H}_{i,i}$  and  $(\mu_i \mathbf{I} + \lambda_{\ell,i} \bar{\mathbf{H}}_{\ell,i}^\dagger \bar{\mathbf{H}}_{\ell,i})$ .
  - (b) Normalize  $\mathbf{C}_i$  such that  $\mathbf{C}_i = \bar{\mathbf{C}}_i \mathbf{P}_i^{\frac{1}{2}}$ .
  - (c) Compute  $\Delta_i^{(1)}$  using (IV.3.6).
  - (d) Allocate the power  $\mathbf{P}_i$  using (IV.3.10).
  - (e) Set  $\mathbf{Q}_i = \bar{\mathbf{C}}_i \mathbf{P}_i \bar{\mathbf{C}}_i^\dagger$

#### 4. Repeat Step 3 until the algorithm converges to a fixed point.

It is noted that a fixed point of the algorithm is implied by a stopping criterion and it is reached when the difference between two consecutive values of the criterion is within the specified tolerance  $\epsilon$ . One obvious criterion for such distributed approach is obtained using the achievable rate  $\mathcal{R}_i$  of the CR link in (IV.2.12). The overall algorithm essentially optimizes the rate of each link  $i$  in distributed manner until the system converges to KKT point of the problem (IV.3.3).

**Proposition 2.** *The algorithm 1 converges to a fixed point which is a KKT point of problem (IV.3.3).*

*Proof:* It is worth mentioning that both objective function  $\mathcal{R}_i$  and constraints are convex in  $\mathbf{Q}_i$ . In order to find a KKT point, we differentiate  $\mathcal{L}_i(\mathbf{Q}_i, \mu_i \lambda_{\ell,i})$  in (IV.3.3) with respect to  $\mathbf{Q}_i$  as [85]

$$\frac{\partial \mathcal{L}_i(\mathbf{Q}_i, \mu_i)}{\partial \mathbf{Q}_i} = [\mathbf{H}_{i,i}^\dagger (\mathbf{R}_{z_i} + \mathbf{H}_{i,i} \mathbf{Q}_i \mathbf{H}_{i,i}^\dagger)^{-1} \mathbf{H}_{i,i}]^\dagger - \mu_i \mathbf{I} - \lambda_{\ell,i} \bar{\mathbf{H}}_{\ell,i}^\dagger \bar{\mathbf{H}}_{\ell,i} \quad (\text{IV.3.11})$$

where it is noted that  $(\mathbf{R}_{z_i} + \mathbf{H}_{i,i} \mathbf{Q}_i \mathbf{H}_{i,i}^\dagger)^{-1} \forall i = 1, \dots, K$  are Hermitian matrices.

The optimal KKT point of the problem (IV.3.3) can be obtained using

$$\frac{\partial \mathcal{L}_i(\mathbf{Q}_i, \mu_i)}{\partial \mathbf{Q}_i} = 0 \quad (\text{IV.3.12})$$

That is, at the fixed KKT point, the covariance matrix is

$$\mathbf{Q}_i = \mathbf{H}_{i,i}^{-1} [(\mathbf{H}_{i,i} \mathbf{S}^{-\dagger} \mathbf{H}_{i,i}^\dagger) - \mathbf{R}_{z_i}] \mathbf{H}_{i,i}^{-\dagger} \quad (\text{IV.3.13})$$

where the matrix  $\mathbf{S} = \mu_i \mathbf{I} + \lambda_{\ell,i} \bar{\mathbf{H}}_{\ell,i}^\dagger \bar{\mathbf{H}}_{\ell,i}$ . It is worth mentioning that the channel matrices are assumed to be full ranked and thus invertible<sup>2</sup>. At the optimal KKT point, the covariance matrix is as (IV.3.13) and the optimal power allocation for each CR user-transmitter is as presented in equation (IV.3.10). It is also noted that the covariance matrix and power allocation for a given CR link will be different for different initial values of the algorithm (e.g., initial precoders, channels, noise and so on).

#### IV.4 RATE MAXIMIZATION IN SPECTRUM OVERLAY

This section presents the process of precoder adaptation and power control for rate maximization in spectrum overlay approach where secondary CR users avoid the signal spaces used in primary user transmissions. In spectrum overlay framework, the interference power constraint is not considered since the secondary CR users will avoid the signal space dimensions occupied by primary users (i.e., they do not coexist and transmit simultaneously in the same signal spaces with primary users), and the achievable rate maximization problem (IV.2.16) can be written as

$$\max_{\mathbf{Q}_i \geq 0} \quad \mathcal{R}_i \quad \text{subject to} \quad \text{Trace}[\mathbf{Q}_i] \leq P_i^{max}, \forall i \quad (\text{IV.4.1})$$

It is noted that the similar optimization problem (IV.4.1) has been solved in [25, 26] using game theory which is different from this approach. In an interference network scenario, individual CR links maximize their corresponding achievable rates

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<sup>2</sup>Noninvertible channel gain matrix can be made invertible as in [44, Theorem 1] which does not change the capacity. Therefore, it is assumed that all channels are invertible with no loss of generality.

by choosing suitable precoder and power.

The rate optimization problem (IV.4.1) in terms of  $\mathbf{C}_i$  can be written as

$$\begin{aligned} \max_{\mathbf{C}_i} \quad & \frac{1}{2} \log_2 |\mathbf{I} + \mathbf{C}_i^\dagger \mathbf{H}_{i,i}^\dagger \mathbf{R}_{z_i}^{-1} \mathbf{H}_{i,i} \mathbf{C}_i| \\ \text{subject to} \quad & \text{Trace} [\mathbf{C}_i^\dagger \mathbf{C}_i] \leq P_i^{max}, \quad \forall i \end{aligned} \quad (\text{IV.4.2})$$

Now, the Lagrangian of (IV.4.2) can be expressed as

$$\begin{aligned} \mathcal{L}_i(\mathbf{C}_i, \mu_i) = & \frac{1}{2} \log_2 |\mathbf{I} + \mathbf{C}_i^\dagger \mathbf{H}_{i,i}^\dagger \mathbf{R}_{z_i}^{-1} \mathbf{H}_{i,i} \mathbf{C}_i| - \\ & \underbrace{- \mu_i \text{Trace} [\mathbf{C}_i^\dagger \mathbf{C}_i]}_{\text{Trace}[\mathbf{C}_i^\dagger (\mu_i \mathbf{I}) \mathbf{C}_i]} + \mu_i P_i^{max} \end{aligned} \quad (\text{IV.4.3})$$

where  $\mu_i$  is the Lagrange multiplier corresponding to the power constraint. In problem (IV.4.3), the optimal  $\mu_i$  needs to be find by solving the following dual function using binary search algorithm, i.e.

$$\min_{\mu_i \geq 0} \max_{\mathbf{C}_i} \mathcal{L}_i(\mathbf{C}_i, \mu_i) \quad (\text{IV.4.4})$$

Equivalently, one can write (IV.4.4) in terms of covariance matrix  $\mathbf{Q}_i$  as

$$\min_{\mu_i \geq 0} \max_{\mathbf{Q}_i \geq 0} \mathcal{L}_i(\mathbf{Q}_i, \mu_i) \quad (\text{IV.4.5})$$

The problem (IV.4.3) and (IV.3.4) are similar except that a  $(\mu_i \mathbf{I} + \lambda_{\ell,i} \mathbf{H}_{\ell,i}^\dagger \mathbf{H}_{\ell,i})$  in (IV.3.4) is replaced by a  $\mu_i \mathbf{I}$  in (IV.4.3). Then the generalized eigen value problem becomes a regular eigen value problem, and the optimal power allocation with

spectral mask constraint in each signal space dimension for the problem (IV.3.9) is given as in (IV.3.10).

As mentioned previously, signal spaces which are used for primary transmissions will be avoided by CR user-transmitters by setting  $p_i^{max}(n_p) = 0, \forall n_p$ , which is required to ensure not to disturb primary users. It is also noted that the solution in the case of spectrum overlay in (IV.3.10) can be used to power control in conventional wireless system when no spectral mask constraints are imposed (i.e.,  $p_i^{mask}(n) = +\infty, \forall n, \forall i$ ) with the assumption that  $[\Delta_i^{(1)}]_{(n,n)} > 0$ . In this case, the power allocation can be done by the classical simultaneous waterfilling solutions as in [81, 86]. Therefore, the proposed approach is applicable to both conventional wireless systems and cognitive radio systems in spectrum underlay as well as spectrum overlay scenarios.

It is noted that the Algorithm 1 presented for spectrum underlay can be used for spectrum overlay by replacing generalized eigen value problem by a regular eigen value problem. Following the similar steps as in the case of spectrum underlay, at the KKT point, the covariance matrix in spectrum underlay is as

$$\mathbf{Q}_i = \mathbf{H}_{i,i}^{-1}[\mu_i(\mathbf{H}_{i,i}\mathbf{H}_{i,i}^\dagger) - \mathbf{R}_{z_i}]\mathbf{H}_{i,i}^{-\dagger} \quad (\text{IV.4.6})$$

Similar to spectrum underlay, the channel matrices are assumed to be full ranked and thus invertible. It is also pointed out that the covariance matrix and power allocation will be different with different initial values of the inputs.



## IV.5 SIMULATIONS AND NUMERICAL EXAMPLES

In order to illustrate the proposed algorithm, a communication system with  $K = 20$  CR links and  $L = 1$  primary link operating in a signal space of dimension  $N = 30$  is considered. The diagonal channel matrices were generated which represents the fading coefficients of the path using uniform random distribution in the interval  $(0, 1]$ , and were assumed to be fixed for one signaling interval.

The background noise is assumed to be white with covariance matrix equal to  $0.1\mathbf{I}_N$ , the tolerable interference power level is  $P_\ell^{IT} = 0.7$ , the power allowed for transmission is  $P_i^{max} = 3$ ,  $\forall i$ , and the tolerance is set to be  $\epsilon = 0.001$ . The precoder matrices  $\mathbf{C}_i$ ,  $\forall i$ , were initialized randomly so that the initial covariance matrices were  $\mathbf{Q}_i = \mathbf{C}_i \mathbf{C}_i^\dagger$ ,  $\forall i$ , and satisfied  $\text{Trace}[\mathbf{Q}_i] \leq P_i^{max}$ .

The first scenario considers the spectrum underlay where 20 CR links co-exist with 1 primary link and can use all signal spaces for their transmissions by satisfying imposed average transmit power and interference constraints to CR transmitters. In this case, the interference from primary system to CR users is considered to be  $E[\mathbf{q}_i \mathbf{q}_i^\top] = 0.05\mathbf{I}$ .

Then, the Algorithm 1 was applied (taking account of both average power and interference constraints) for the above simulation setup to optimize the precoders and power of CR links. The variations in achievable rates are plotted for CR links in Figure 25. By satisfying both power and interference constraints, all CR user-transmitters maximize their achievable rates, and the proposed algorithm reached at KKT fixed point.

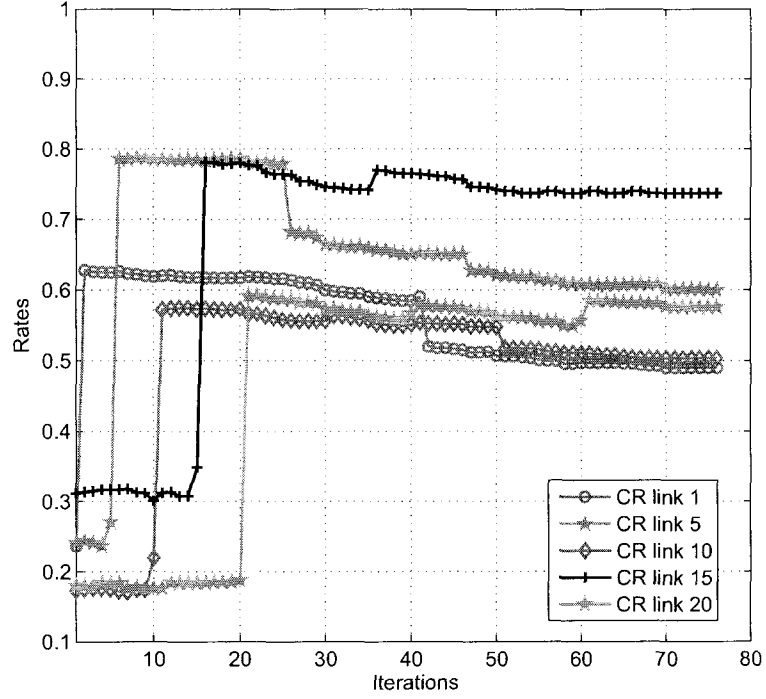


Fig. 25: Achievable rates of the secondary CR links versus iterations in spectrum underlay with spectral mask, power and interference constraints.

The second scenario considers the spectrum overlay where the primary link has occupied 6 signal spaces (1, 2, 3, 12, 13 and 14) out of 30 which are avoided by secondary CR user-transmitters by setting  $p_i^{mask}(n_p) = 0$  for  $\forall n_p \in \{1, 2, 3, 12, 13, 14\}$ . Thus the interference from primary link to CR users is taken as  $\mathbf{q}_i = 0$  (i.e.,  $E[\mathbf{q}_i \mathbf{q}_i^T] = 0$ ) in the case of spectrum overlay. Then, Algorithm 1 was applied without considering interference constraint for the above simulation setup for precoder and power adaptation by using the procedure mentioned in Section IV.4. As in the previous scenario, the resulting achievable rate variations for CR links are plotted as shown in Figure 26. It is noted that the proposed algorithm reached at KKT fixed point

and the fixed point might not be unique for different random initialization of input parameters in the algorithm. It is also noted that results similar to those shown in Figure 26 are presented in [25,26] and are obtained by application of the simultaneous Iterative Water Filling Algorithm (IWFA), which was derived using game theoretic approach. Once again it is worth noting that the approach presented in this chapter is applicable to both spectrum underlay and overlay whereas the approach presented in [25,26] might not be applicable to spectrum underlay.

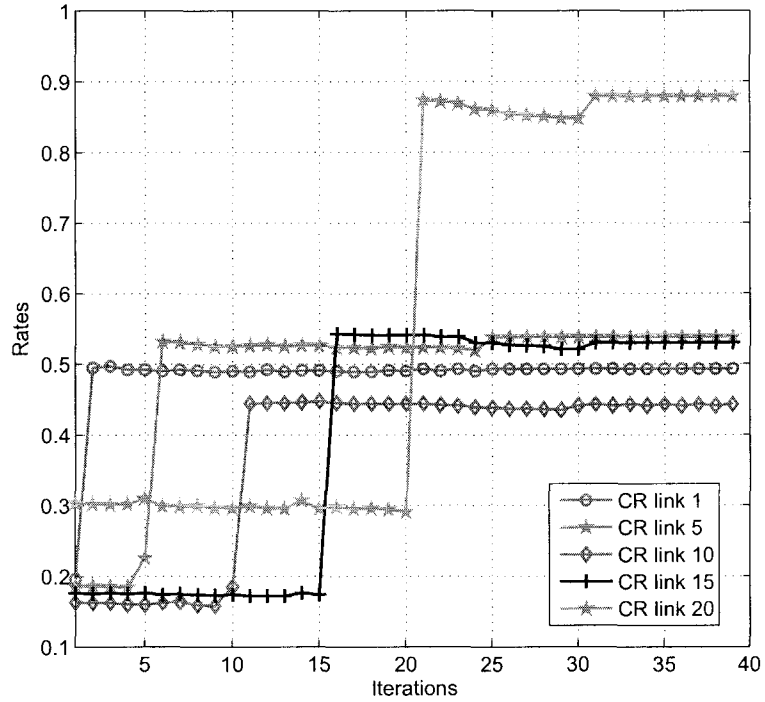


Fig. 26: Achievable rates of the secondary CR links versus iterations in spectrum overlay with spectral mask and average transmit power constraints.

It is noted that if the cross channel gains are higher than the intended link channel gains, the algorithm takes longer time to converge or might not be able to converge

to a fixed point. The higher cross channel gains implies that the cross transmitter-receivers are closer than the intended transmitter-receiver pairs. If this case happens in the case of spectrum underlay, the secondary CR users are not allowed to coexist and transmit simultaneously with primary users or will be allowed to coexist with primary users only if they satisfy both power and interference constraints.

## IV.6 CHAPTER SUMMARY

In this chapter, a new approach has been presented for combined precoder and power adaptation for multi-user spectrum sharing over interference CR systems. The rate maximization problem has been solved for spectrum underlay scenario of CR systems by taking average transmit power and interference constraints since CR users can transmit simultaneously and coexist with primary users in the same signal space dimensions. Then, the approach has been particularized to spectrum overlay scenario where CR transmitters are imposed only transmit power constraint as the CR users avoid the signal space dimension occupied by primary users. Specifically, the optimal resource allocation for achievable rate maximization of CR links as a constrained optimization problem has been formulated, where the bandwidth use in terms of signal space dimensions has been optimized by the CR transmitters under imposed constraints that restrict the operation of CR user-transmitters to protect the primary user-transmissions. The proposed algorithm converged to KKT point and has been illustrated with numerical results obtained from simulations.

# CHAPTER V

## TRANSMITTER ADAPTATION WITH LIMITED FEEDBACK

In the implementation of transmitter adaptation algorithms in distributed manner, users in the system update their corresponding precoders individually provided that the correlation matrix of received signal (or interference information) is made available to them. The conventional approach using fixed transmitters and heavy signal processing at the receiver is changing to a new one in which adaptive transmitters use feedback from receiver(s) to adjust to varying operating environment such as in emerging wireless cognitive radios and adaptive wireless networks [87, 88]. Specifically, feedback in an adaptive wireless systems is used to cooperate between transmitter and receiver to improve the quality of the desired signal at the receiver [89, 90]. However, the feedback channel has limited capacity in practice and thus the information to be fed back needs to be quantized.

In this chapter, two ways of providing the limited feedback from receiver to the transmitter are presented: feedback using *interference information quantization* and using *predictive vector quantization*. In both cases, the objective is to feed back the least amount of information used to update the precoder in decentralized manner and use the least number of bits to quantize. Comparison of different quantization techniques are presented which are obtained from a Monte Carlo Simulation.

## V.1 INTERFERENCE INFORMATION QUANTIZATION

For a precoder update, individual users do not require complete knowledge about all other active users in the system in terms of precoders and powers but they only need the correlation matrix  $\mathbf{R}_k$  of interference-plus-noise experienced at the receiver. For a given user, this matrix can be obtained by subtracting its contribution from the correlation matrix  $\mathbf{R}$  of the received signal provided that this matrix is available at individual users. Thus, distributed implementation of interference avoidance algorithms require that only the correlation matrix of the received signal,  $\mathbf{R}$ , be made available through the feedback channel. Furthermore, as mentioned in previous chapters, the users have access to  $\mathbf{R}$  instantly, and it can be obtained by either periodic broadcast of  $\mathbf{R}$  by the basestation or periodic broadcast of the received signal  $\mathbf{r}$  and then compute its correlation matrix at the individual users. However, in this chapter it is assumed that the base station broadcasts the quantized  $\mathbf{R}$ .

It is worth noting that  $\mathbf{R}$  is an  $N \times N$  symmetric matrix, and that the values of only  $N(N+1)/2$  elements need to be actually transmitted over the feedback channel. Whereas in case of  $\mathbf{r}$  broadcast, one needs only  $N$  elements to be transmitted but need to compute correlation matrix at mobile users, which might be costly in terms of computation and energy for individual users. Therefore, it is assumed that the quantized  $\mathbf{R}$  is fed back.

In the study for interference avoidance, an Eigen algorithm with scalar quantization [22] of the  $\mathbf{R}$  matrix is considered and assumed that the quantized matrix  $\bar{\mathbf{R}}$  (whose elements are the quantized version of the corresponding elements in  $\mathbf{R}$ ) is used during the interference avoidance algorithm to compute the interference-plus-noise

correlation matrices of active users in the system, i.e.

$$\bar{\mathbf{R}}_k = \bar{\mathbf{R}} - p_k \mathbf{s}_k^\top \mathbf{s}_k \quad (\text{V.1.1})$$

is used instead of  $\mathbf{R}_k$ .

In order to see the effect of limited feedback, the system performance in terms of sum capacity of the system is analysed, where the sum capacity for non-quantized  $\mathbf{R}$  is defined as [4]

$$C_s = \frac{1}{2} \log_2 |\mathbf{R}| - \frac{1}{2} \log_2 |\mathbf{W}| \quad (\text{V.1.2})$$

and with quantized version of  $\mathbf{R}$  is as

$$\bar{C}_s = \frac{1}{2} \log_2 |\bar{\mathbf{R}}| - \frac{1}{2} \log_2 |\mathbf{W}| \quad (\text{V.1.3})$$

It is noted that the main objective is to get sum capacity closer to the sum capacity obtained without quantization. The distortion introduced by quantization can be expressed as

$$D = E[(C_s - \bar{C}_s)^2] = E \left[ \log_2 \frac{|\mathbf{R}|}{|\bar{\mathbf{R}}|} \right] \quad (\text{V.1.4})$$

In this setup, main aim is to minimize the distortion introduced by quantization by choosing suitable number of bits.

In scalar quantization, the set of real numbers is partitioned into  $L$  disjoint subsets  $\{\mathcal{R}\}_{i=1}^L$ , and generally  $L$  is chosen to be power of 2. It is noted that the quantization is nonlinear and non-invertible. For  $L$  quantization levels, one needs  $B = \log_2 L$  bits to encode. Depending on the choice of quantization regions, one can have uniform quantization where quantization widths are equal and non-uniform quantization

where the widths are not necessarily equal. The optimal quantizer minimizes the distortion by optimal selection of output levels and corresponding input levels, and the optimal quantizer is known as *Lloyd-Max* [22].

### V.1.1 Algorithm

Formally the algorithm is as follows:

#### **Algorithm: The Eigen Algorithm with Quantized Information**

1. **Input:** Initial precoder matrix  $\mathbf{S}$ , user power matrix  $\mathbf{P}$ , noise covariance matrix  $\mathbf{W}$ , desired tolerance  $\epsilon$ , and  $B$ -bit increment codebook.
2. For each user  $k = 1, \dots, K$ 
  - (a) Determine the true  $\mathbf{R}$  matrix at the receiver and quantize it to obtain matrix  $\bar{\mathbf{R}}$ .
  - (b) Compute matrix  $\bar{\mathbf{R}}_k$  using equation (V.1.1).
  - (c) Replace current precoder of user  $k$  with the minimum eigenvector of  $\bar{\mathbf{R}}_k$ .
3. Repeat Step 2 until a fixed point is reached
4. **Output:** Updated precoder matrix  $\bar{\mathbf{S}}$ .

According to [4,8], the convergence of Eigen algorithm with quantized interference information is not guaranteed. Thus, the sum capacity is used for a stopping criterion of the algorithm. The algorithm reached to fixed point when the difference between two consecutive values of the sum capacity are within the specified tolerance.



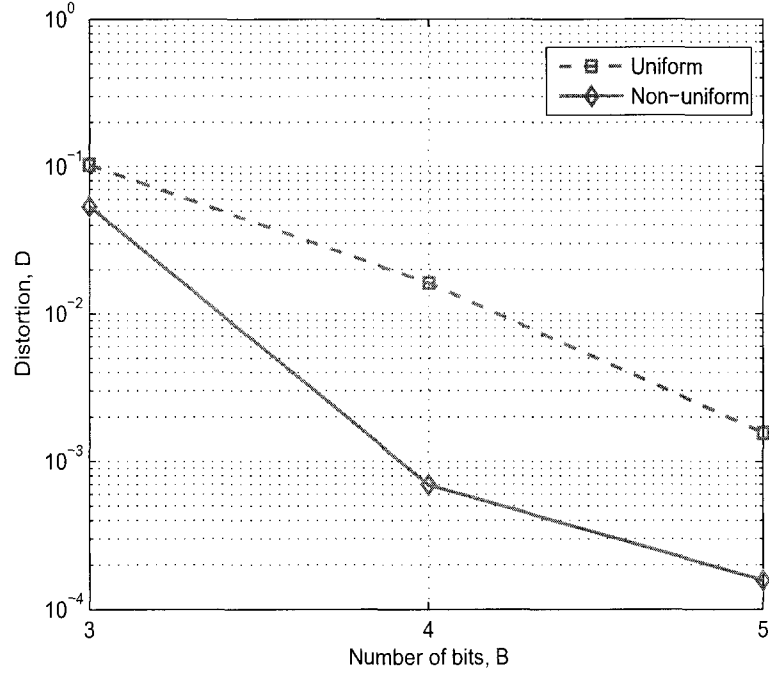


Fig. 27: Distortion versus the number of bits for uniform and non-uniform quantization of the interference information for  $K = 15$  and  $N = 10$ .

### V.1.2 Simulations and Numerical Results

Extensive simulations are performed for both uniform and non-uniform quantization of  $\mathbf{R}$  using different number of bits for 100 independent trials and plotted the average distortion versus the quantization bits as shown in Figure 27. It is noted that the distortion introduced by non-uniform quantization in sum capacity is lower than that introduced by uniform quantization as shown in Figure 27.

Then, having looked at the sum capacity variations for non-uniform quantization using three different bits, as shown in Figure 28, shows that 3-bit non-uniform quantization gives sufficient approximation in the performance measure.

It is noted that one can choose a suitable number of bit as per the specified

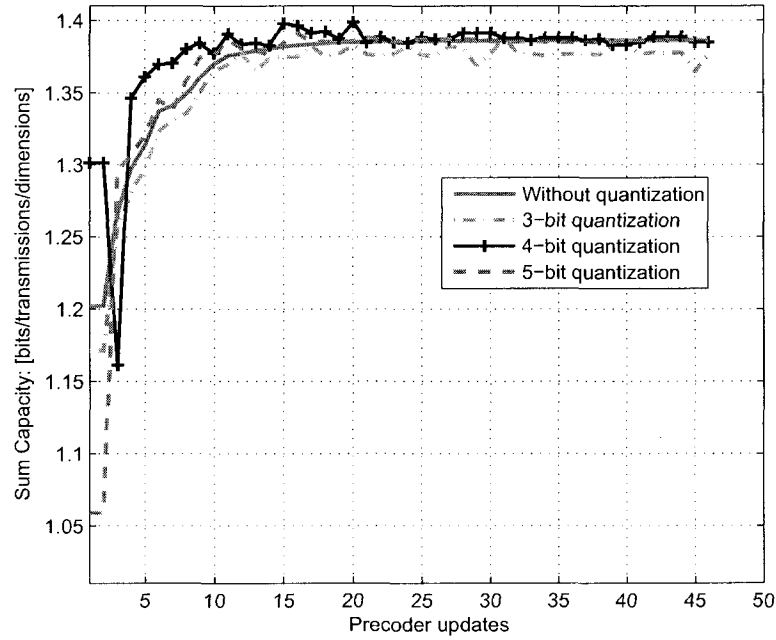


Fig. 28: Variation of sum capacity for the interference avoidance algorithm with *non-uniform* quantization of the interference information for  $K = 15$  and  $N = 10$ .

upper limit in distortion, and can apply non-uniform quantization to the  $\mathbf{R}$  and relay the quantized information through feedback channel so that individual transmitters adapt their corresponding precoders.

## V.2 PREDICTIVE VECTOR QUANTIZATION FOR PRECODER ADAPTATION

As mentioned previously, the optimal precoder can be obtained when a receiver can estimate or compute the covariance matrix of the interference-plus-noise. Then the receiver relays the covariance matrix or the optimal precoder to the transmitter via a error-free feedback channel. As the feedback channel has a limited capacity

in practice, in this section, it is of interest to present vector quantization for user precoders for transmitter adaptation.

In this section, predictive vector quantization (PVQ) [91] is studied for quantizing user precoders in the context of distributed incremental algorithms for precoder adaptation [92] and compare its performance with the random vector quantization (RVQ) method discussed in [10]. The codebook of precoders is known *a priori* at both the transmitter and receiver in the system. For given  $B$  feedback bits, the receiver for a given user selects a incremental precoder vector, which gives the small distortion in sum capacity, from  $2^B$ -precoder codebook and transmits back the corresponding value/index to the transmitter via an error-free feedback channel.

In this setup, the precoder adaptation is considered which uses incremental updates given in equation (III.4.1), i.e.

$$\mathbf{s}_k(n+1) = \frac{\mathbf{s}_k(n) + m\beta\mathbf{x}_k(n)}{\|\mathbf{s}_k(n) + m\beta\mathbf{x}_k(n)\|} \quad (\text{V.2.1})$$

where  $\mathbf{x}_k$  is the minimum eigenvector of corresponding interference+noise correlation matrix corresponding to user  $k$

$$\tilde{\mathbf{R}}_k = \mathbf{S}\mathbf{P}\mathbf{S}^\top + \mathbf{W} - p_k\mathbf{s}_k\mathbf{s}_k^\top \quad (\text{V.2.2})$$

and determines the direction of the increment. The parameter  $\beta$  determines the increment size (that is, how far in terms of Euclidian distance the updated precoder is from the old one) while  $m = \text{sgn}(\mathbf{s}_k^\top \mathbf{x}_k)$  gives sign of the increment. It is noted that, for distributed implementation of the precoder update (V.2.1), users expect to

receive the information of their corresponding increment over a feedback channel.

### V.2.1 Predictive Vector Quantization (PVQ)

PVQ is discussed in [91] in the context of beam forming for MIMO systems for feeding back information about channel prediction/estimation errors. In this section, PVQ is applied for the incremental precoder update (V.2.1). Specifically, a given user  $k$  at instant  $n$  uses a quantized version  $\bar{\mathbf{q}}_k$  of its corresponding increment vector  $m\beta\mathbf{x}_k(n)$  and updates its precoder as follows:

$$\mathbf{s}_k(n+1) = \frac{\mathbf{s}_k(n) + \bar{\mathbf{q}}_k(n)}{\|\mathbf{s}_k(n) + \bar{\mathbf{q}}_k(n)\|}, \quad \forall k = 1, \dots, K \quad (\text{V.2.3})$$

The quantized increment is obtained by applying  $B$ -bit vector quantization where an increment codebook that is known to both transmitter and receiver is used. This codebook is obtained prior to running the PVQ updates and its use implies that only  $B$  bits need to be feed back from receiver to transmitter for this update.

The effect of quantization and feedback limited to  $B$  bits is studied in terms of global system performance measured through the information-theoretic sum capacity given by

$$C_{\text{sum}}(\mathbf{S}) = \frac{1}{2} \log_2 |\mathbf{S}\mathbf{P}\mathbf{S}^\top + \mathbf{W}| - \frac{1}{2} \log_2 |\mathbf{W}| \quad (\text{V.2.4})$$

The distortion is measured in sum capacity  $\bar{C}_{\text{sum}}$  corresponding to the precoders  $\bar{\mathbf{S}}$  yielded by the algorithm with  $C_{\text{sum}}$  corresponding to optimal Welch Bound Equality (WBE) precoders [16]. The distortion measure is expressed as

$$D(\bar{\mathbf{S}}) = E[(C_{\text{sum}} - \bar{C}_{\text{sum}})^2] \quad (\text{V.2.5})$$

The main objective here is to maximize the sum capacity where the distortion caused by PVQ does not exceed the given upper limit. That is, the PVQ-based sum capacity maximization problem can be written as

$$\max_{\bar{\mathbf{S}}} \quad \frac{1}{2} \log_2 |\bar{\mathbf{S}}\mathbf{P}\bar{\mathbf{S}}^\top + \mathbf{W}| - \frac{1}{2} \log_2 |\mathbf{W}| \quad (\text{V.2.6})$$

$$\text{subject to} \quad D(\bar{\mathbf{S}}) \leq D^{\max}$$

where the  $D^{\max}$  is the allowed upper limit in the distortion.

Following [93,94], it is noted that the predictive vector quantization for precoders can be done using 1 bit PVQ corresponding to a simple up/down feedback scheme where the distortion does not exceed its given limit.

In addition, the distortion is compared which is introduced when RVQ [10] is used for quantizing the user signature updates  $\mathbf{s}_k(n+1)$  in (V.2.1) with that of when PVQ is used. In RVQ, precoders are quantized instead of just their increments in PVQ. Similar to [10], it has considered that the precoders are independent and Gaussian distributed in RVQ, and the unquantized optimal precoders are the eigenvectors corresponding to the minimum eigenvalues of their corresponding interference-plus-noise matrices. The simulation results plotted in Figure 29 shows that PVQ results in smaller distortion than the RVQ ones. Therefore, PVQ method is further investigated in the following section.

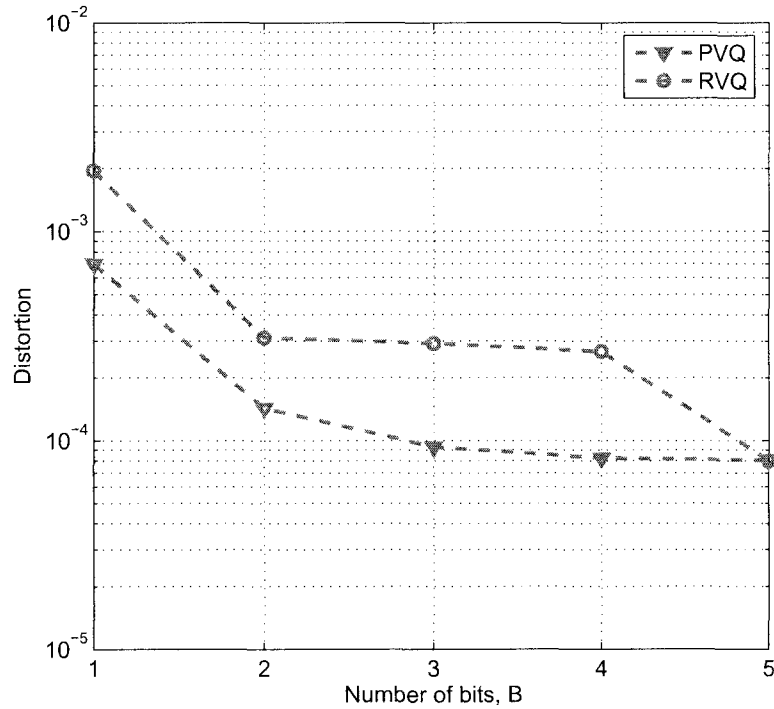


Fig. 29: Average sum capacity distortion for different number of quantization bits for PVQ and RVQ of precoders in which we take the number of users  $K = 6$ , signal space dimension  $N = 5$ , and  $\mathbf{W} = 0.1\mathbf{I}_N$ .

### V.2.2 The Algorithm

Based on the above analysis, formally an algorithm is presented for PVQ based precoder adaptation as below:

#### Algorithm: PVQ Precoder Adaptation

1. **Input:** Initial precoder matrix  $\mathbf{S}$ , user power matrix  $\mathbf{P}$ , noise covariance matrix  $\mathbf{W}$ , increment constant  $\beta$ , desired tolerance  $\epsilon$ ,  $B$ -bit increment codebook.
2. For each user  $k = 1, \dots, K$

- (a) Determine the minimum eigenvector  $\mathbf{x}_k(n)$  of  $\mathbf{R}_k(n)$ .
  - (b) Determine quantized increment  $\bar{\mathbf{q}}_k(n)$  in codebook.
  - (c) Update user  $k$ 's precoder using equation (V.2.3).
3. Repeat Step 2 until a fixed point is reached
  4. **Output:** Updated precoder matrix  $\bar{\mathbf{S}}$ .

It is noted that, numerically, a fixed point of the algorithm is observed when the difference between two consecutive values of the stopping criterion is within the specified tolerance  $\epsilon$ . For distributed implementation of the algorithm, the stopping criterion uses local information that is available to individual users such as the Euclidean distance between a given precoder and its corresponding replacement to be within the specified tolerance  $\epsilon$ . It is also noted that the value of  $\epsilon$  must be much smaller than that of the increment  $\beta$ .

### V.2.3 Simulation and Numerical Results

This section presents numerical results obtained from simulations to look performance of the proposed algorithm for different cases.

The codebook was generated independently for incremental updates for different number of bits and assumed to be known at both receiver and transmitter. It is considered that the transmitter uses the CDMA technology to transmit their information.

The initial precoders were initialized randomly and shared between transmitter and receiver, and the channels are assumed to be ideal ones. The generated user

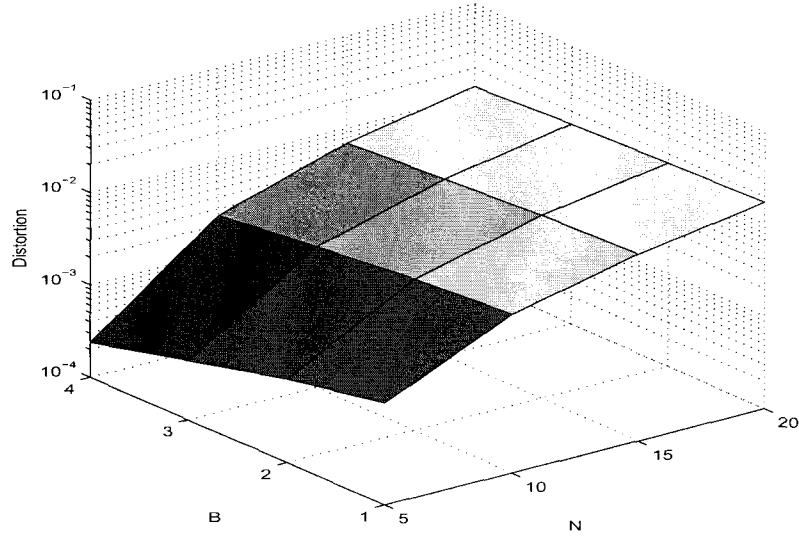


Fig. 30: Average sum capacity distortion by PVQ versus the number of bits and signal space dimensions.

powers are assumed to be fixed during the entire simulation. White noise is considered to be a covariance matrix as  $\mathbf{W} = 0.1\mathbf{I}_N$ . Algorithm constants were chosen to be  $\beta = 0.01$ ,  $\mu = 0.01$ , the distortion limit  $D^{\max} = 0.1$ , and tolerance  $\epsilon = 0.001$ . For an ideal channel scenario, channels were initialized as identity matrices of dimension  $N$ .

The first experiment illustrates the variation of distortion for different number of bits and different number of dimensions by keeping the ratio (known as load factor in CDMA systems)  $\frac{K}{N} = 2.4$  which is highly overloaded scenario. The algorithm was run for 1,000 independent realizations and their average values were calculated. Figure 30 shows the average distortion variations in sum capacity introduced by PVQ versus the number of bits and number of signal space dimensions.



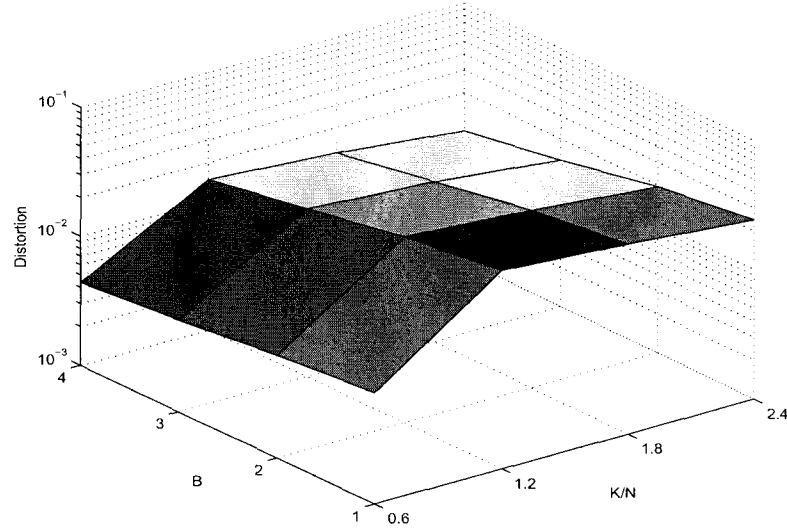


Fig. 31: Average sum capacity distortion versus the different number of bits and load factor  $\frac{K}{N}$ .

From Figure 30, it is noted that, as expected, the distortion decreases with increasing number of bits however there is no high difference between the distortion introduced by 1 bit and 4 bits. It is also noted that the distortion increases with the increasing signal space dimensions (or number of users) in the system.

In the second scenario, the simulation was performed for different load factors and quantization bits for PVQ of precoder. Underloaded system were considered which consists of  $\frac{K}{N} = 0.6$  i.e.  $K < N$  and the overloaded system (i.e.,  $K > N$ ) which includes light, moderate, and high numbers of transmitters in the system, respectively, with the load factor  $\frac{K}{N} = 1.2, 1.8$ , and  $2.4$ . The variation of distortion is plotted in Figure 31.

From Figures 31, it is observed that the distortion increases with increasing load

factor, and the distortion does not change significantly as the number of bits increases from 1 to 4 for the given value of load factor.

From Figure 30 and 31, it is observed that the distortion increases with increasing load factor, and the distortion does not change significantly as the number of bits changes from 1 to 4 for the given load factor. It is concluded that the sum capacity maximization problem for limited feedback can be solved by using 1 bit PVQ in which the distortion does not exceed its upper limit 0.1.

### V.3 CHAPTER SUMMARY

In the first section of this chapter, transmitter adaptation were studied for sum capacity maximization problem with limited feedback when only quantized interference information is available. The results have shown that non-uniform quantization outperforms uniform quantization when scalar quantization of interference information is used. It is noted that 3-bit non-uniform quantization gives close approximation to the non-quantized system performance.

In the second part of this chapter, PVQ was studied for precoder adaptation with limited feedback and its performance was compared with that of RVQ in which the PVQ outperforms the RVQ in terms of introduced distortion. Then, the PVQ for different load factor has been studied where, for given  $B$  feedback bits, the receiver for a given user selects a incremental precoder vector, which gives the small distortion in sum capacity, from  $2^B$ -precoder codebook (known *a priori* at both the transmitter and receiver) and transmits back the corresponding  $B$  bits to the transmitter via an error-free feedback channel. The performance of a PVQ scheme has been studied in

terms of sum capacity for different number of bits and load factors. For  $B = 1, \dots, 4$ , the distortion value introduced by quantization does not exceed 0.1. As seen in the simulation results, a 1 bit PVQ corresponding to a simple up/down feedback scheme can be considered as an optimal where the distortion does not exceed its given limit.

## CHAPTER VI

### TRANSMITTER ADAPTATION AND POWER CONTROL WITH FADING CHANNELS

In previous chapters, the transmitter adaptation is presented for interference mitigation and power control under specified target SINR requirement where the channels between transmitters and a receiver are assumed to be known and fixed for the entire duration of transmission. However, in wireless communications, the channels experience fading. The fading may vary with time, geographical position and/or radio frequency which may either be multipath induced fading (due to multipath propagation) or shadow fading (due to shadowing from obstacles affecting the wave propagation). Since the time variations appear to be unpredictable, channels in wireless systems are often modeled as a stochastic process. Usually, fading is modeled as a time-varying random change in the amplitude and phase of the transmitted signal. Fading channel models are generally used to model the random effects of electromagnetic transmission of information in cellular networks, broadcast communication and underwater acoustic communications.

The performance of the wireless system heavily depends on channels. As mentioned in previous chapters, the SINRs are used as performance measure characteristic of the system. It is also noted that the SINR is measured at the output of the receiver and thus directly related to the data detection process, and the bit-error-rate has one-to-one relationship with target SINRs matching criteria.

In this chapter, the outage probability in multiaccess fading channels is analyzed in the context of uplink of a wireless system.

## VI.1 OUTAGE PROBABILITY

The performance of wireless systems operating over fading channels can be evaluated using the outage probability denoted by  $P_{out}$  and defined as the probability that the output SNR for a given user  $k$ ,  $\gamma_k$ , falls below a certain specified threshold (i.e, target SINR),  $\gamma_k^*$ . Formally, the outage probability is

$$P_{out} = \int_0^{\gamma_k^*} P_{\gamma_k}(\gamma_k) d\gamma_k \quad (\text{VI.1.1})$$

which is the cumulative distribution function evaluated at  $\gamma_k^*$  where  $P_{\gamma_k}(\gamma_k)$  denotes the probability density function (PDF) of  $\gamma_k$ .

For the case with quasi-static fading channels which has a large coherence time, channels can be estimated and assumed to be known. That is, channels can be assumed to vary sufficiently slowly and can be considered as time invariant during the period of each symbol transmission and vary block by block independently. Then one can apply joint transmitter adaptation and power control to meet target SINR requirements in straightforward way.

For the dynamic channel, where the coherence time of the channel is small, it may not be possible to estimate channel characteristics and apply the algorithm for joint transmitter adaptation and power control. That is, by the time channel has been estimated and the algorithm for joint transmitter adaptation and power control applied, the channel under consideration may have already changed to new values.

This is possible when a communicating device is traveling on a vehicle moving with high speed. In this case, a practical approach would be to use average characteristics of the channel and apply our algorithm. That is, precoder ensemble for transmitter adaptation and power control will be applied over the average channels regardless of the actual channel realizations.

Instantaneous SINR is considered as random variable and find Complementary Cumulative Distributed Functions (CCDFs) to compare the performance of the algorithm with the help of Monte Carlo simulations. Based on SINRs CCDFs, the performance of the algorithm is compared for average channel and for each channel realizations.

It is noted that SINR CCDFs show that the probability of exceeding user SINR by its value in abscissa for a given precoder ensemble. An outage occurs whenever SINR is below a given target value, and the probability of outage  $P_{out}$  can be identified from the corresponding CCDF plot.

## VI.2 FADING CHANNEL MODEL AND SIMULATION RESULTS

This section shows the application of transmitter adaptation to fading channels with average and each channel realizations.

It is worth noting that the information dissemination over wireless channels is a complicated phenomenon and characterized by various effects, such as multipath and shadowing. As the wireless fading channels depend on the particular propagation environment and the underlying communication scenario, a precise mathematical model

of the phenomenon is either unknown or too complex. However, one can use statistical modeling and characterization to incorporate different effects. In this chapter, indoor wireless channels are the focus [95]. Similar to [4, 95], the frequency selective fading channel model with flat fading are considered that represents Rayleigh fading. In the Rayleigh fading environment, the amplitude scaling  $\alpha_\ell$  for the channel between a user  $\ell$  and basestation is a Rayleigh random variable with PDF

$$f_\ell(\alpha_\ell) = \frac{\alpha_\ell}{\sigma_\ell^2} \exp^{-\frac{\alpha_\ell^2}{2\sigma_\ell^2}} \quad (\text{VI.2.1})$$

where  $E[\alpha_\ell^2] = 2\sigma_\ell^2$  which relates the  $\sigma_\ell^2$  to the second moment of the Rayleigh random variable. Specifically, the second moment of the random variable represents the eigenvalue corresponding to user  $\ell$  average channel, that is,  $E[\alpha_\ell^2] = 2\sigma_\ell^2 = \lambda_\ell$ .

To apply this algorithm, a transmitter needs to estimate the autocorrelation matrix of the received signal in addition to channel between the transmitter and basestation. The transmitter adjusts its waveforms based on the received interference-plus-noise (obtained by subtracting the contribution of a given user from the autocorrelation matrix). This problem has been studied in Chapter V.

It is noted that, for slowly fading indoor channels, it may be possible to apply our algorithms in a straightforward way. But, fast fading channels are of interest where their average values as well as actual channel values are used. In this case when the channel between a given user and basestation is dynamic and can not be estimated rapidly to make it like a quasi-static to apply this algorithm, one can measure the average channel characteristics and use the optimal precoders ensemble. For the system with a frequency selective fading channel model, the average value

of the channel gains given by the equation (VI.2.1) is taken and apply ensemble of precoders and power control to meet QoS requirement. Next, precoder ensemble is applied for the real channel realizations with power control to meet individual users' QoS requirements. It is noted that given precoder ensemble, which is applied in average and actual channel realizations, might not be optimal one. To look at system performance, extensive simulations were performed in terms of SINR CCDFs.

In order to perform simulations, an uplink of a wireless system with  $K = 5$  number of users operating in signal space of dimension  $N = 4$ , AWGN at the basestation receiver with correlation matrix  $\mathbf{W} = 0.1\mathbf{I}_N$  and user target SINRs  $\gamma_k^* = \{1.5, 2.5, 3.0, 4.0, 5.0\}$  was considered. The algorithm was applied and the SINR CCDFs were plotted as shown in Figures 32 and 33. 500 set of precoder ensembles were generated and the simulation was performed for 1,000 independent realizations of Rayleigh fading channels for the given system.

Two cases for average channel realizations are considered: First, *ideal average channels*, where all channel eigenvalues are equal to 1 which implies that second moment of Rayleigh random variables in equation (VI.2.1) are also equal to 1. Second, *non-ideal average channels*, in which channel eigenvalues generated from uniform distribution in the interval  $[0.6, 1.4]$  are used which includes both attenuation and boosting of the signal.

From Figures 32 and 33, the best performance is obtained with precoder ensembles optimal for each channel realization, however, it should be noted that this might not be always possible for rapidly varying channels. It is noted that, because of the computational burden in each channel realization and transmitter adaptation, selecting sets of precoder optimal for the average channel might be a reasonable



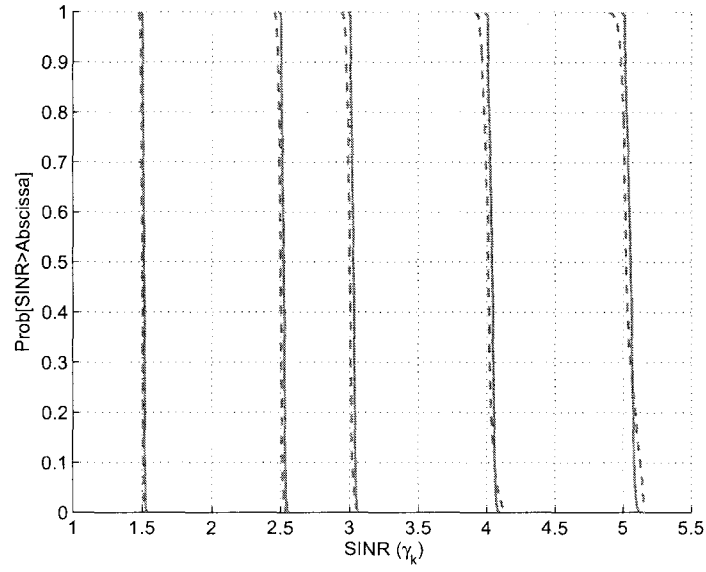


Fig. 32: SINR CCDFs for multiaccess fading channels comparing precoder ensembles optimal for the average channel (dashed line) and precoder ensembles optimal for each channel realization (solid line). Average channels are assumed to be ideal.

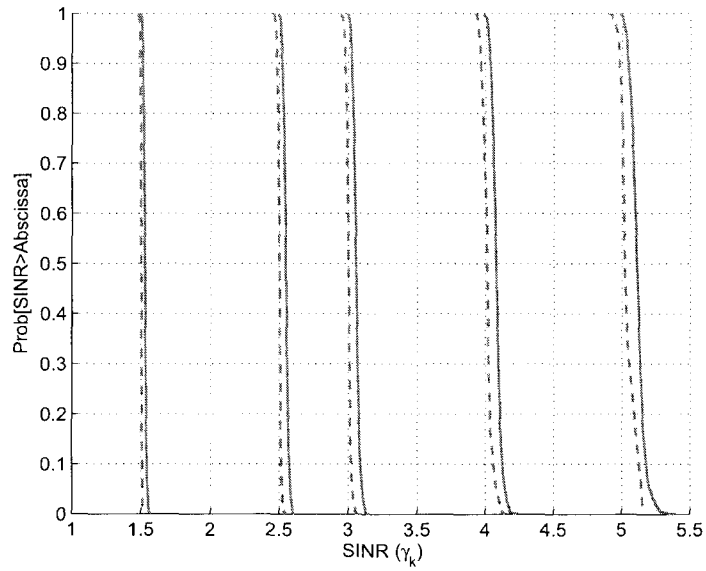


Fig. 33: SINR CCDFs for multiaccess fading channels comparing precoder ensembles optimal for the average channel (dashed line) and precoder ensembles optimal for each channel realization (solid line). Average channels are assumed to be non-ideal.

choice as it results in well enough system performance.

### **VI.3 CHAPTER SUMMARY**

In this chapter, the performance of the algorithm was analyzed in the context of fading channels. The application of the algorithm is straightforward in the case of slowly fading channels which are assumed to be known and stable for the entire duration of transmission. In the case of fast varying channels, during the process of finding optimal precoder with required QoS service, the channel would be changed to different values and thus, the algorithm can be used to compute precoder ensembles with average channel parameters and the performance of the algorithm has been presented with the help of Monte Carlo simulations.

## CHAPTER VII

### CONCLUSIONS AND FUTURE WORK

The number of wireless devices and applications that access unlicensed parts of the radio spectrum are increasing at a prolific rate. Further deployment of wireless devices and networks that use the same parts of the unlicensed spectrum thus needs to take account for possible external interference. In unlicensed bands, cooperation among devices in terms of shared medium access cannot be guaranteed. Furthermore, there is lack of any enforcing body pushing for optimal resource allocation between the unlicensed band technologies and devices, thus the development of adaptive algorithms for resource management in unlicensed RF bands is of importance. Interference avoidance is one promising solution in this direction for wireless resource management. Emerging trends in the wireless industry, such as cognitive radios and adaptive wireless networks are shifting the design paradigm for wireless communication systems. The traditional approach using fixed transmitters and heavy signal processing at the receiver is changing to a new one in which adaptive transmitters use feedback from receiver(s) to adjust to varying channels and interference patterns in order to better suit the dynamic environment in which they operate under specified quality of service. Specifically, in an adaptive wireless communication system with feedback, the transmitter and receiver cooperate in order to improve the quality of the desired signal at the receiver [89].

This thesis is a result of implementation of transmitter optimization algorithms in distributed manner. Since the centralized methods for resource allocation tend

to be computationally expensive in large-scale networks. Furthermore, the need for optimizing the link parameters with local information and reasonable computational burden motivates a decentralized approach. A distributed method, where each link attempts to update its precoder and optimize its power based on the knowledge of its own channel matrix and the covariance of the total interference and the noise at its own receiver.

Chapter II considered the system model where multiple transmitters communicate with a single receiver, and the transmitters employ CDMA type signatures/waveforms for transmitting their information. A Gradient-Descent based precoder adaptation by each user in distributed manner for interference mitigation and power control where each user has QoS requirement in terms of minimum target SINR has been analyzed and simulated, and the adaptation is based on the received covariance matrix of interference information via error-free feedback channel. In this scenario, the distributed method and the centralized methods are equivalent in terms of precoder ensemble. The proposed algorithm has been analyzed for ideal channel scenario and then has been extended to non-ideal channel scenario where channels between users and base station were considered explicitly.

Chapter III investigated the incremental strategies for precoder adaptation and power control in distributed manner for a system model that is similar to the model of Chapter II. Specifically, non-cooperative separable games for precoder adaptation and power control were used where individual users have their QoS requirements to satisfy. The incremental algorithm has presented which can keep track of variable QoS requirement and variable number of users in the system on the fly. Therefore the algorithm presented is applicable to dynamic wireless systems. The performance

of proposed algorithm was also compared with methods existing in the literature [1] and our algorithm outperforms the existing one in terms of power and QoS (i.e., target SINR). Specifically, the proposed algorithm needed lower power to satisfy the specified QoS than the method available in the literature.

Chapter IV considered the interference system where individual transmitters have their intended receivers, and communication occurs in the presence of other interfering links. A distributed algorithm was proposed which requires absolutely no coordination between the links and allows them to maximize their desired rates. The algorithm presented in this chapter is applicable to traditional wireless systems as well as future generation cognitive radio systems. Furthermore, in cognitive radio system, the method can be implemented in both spectrum overlay and underlay approaches.

In Chapter V, the effect of quantization of information used in distributed precoder adaptation was explored. In the first section, the distortion introduced by quantization (i.e., non-uniform and uniform scaler) of interference information was presented. The non-uniform quantization of interference information with 3-bits resulted the sum capacity closer to that obtained when unquantized interference information was used. In the next section, two different vector quantization methods were compared: PVQ and RVQ. As the PVQ outperforms the RVQ in terms of distortion introduced in sum capacity, the PVQ approach for precoder adaptation was analyzed and simulated for different load factors and quantization bits. It was shown that PVQ with 1 bit for precoder adaptation is sufficient to get distortion in sum capacity lower than the value of 0.1 for sparse to highly overloaded systems.

In Chapter VI, the performance of the algorithm in the context of fading channels

was analyzed. Average channel realization with precoder ensemble gives reasonable system performance, and the performance has been presented with the help of Monte Carlo simulations.

Finding out a method based on PVQ for joint precoder adaptation and power control to meet QoS requirement in MAC is part of future work. The other interesting areas of future work would be developing the similar mechanisms for downlink (broadcast channels) using uplink-downlink duality. In interference systems, the areas of future work would be to develop a game theoretic approach for social optimal resource allocation.

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### REFEREED JOURNAL PUBLICATIONS

- **D. B. Rawat**, D. C. Popescu, G. Yan, and S. Olariu. Enhancing VANET Performance by Joint Adaptation of Transmission Power and Contention Window Size, IEEE Transactions on Parallel and Distributed systems, 2010, to appear.
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