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PERFORMANCE OPTIMIZATION OVER WIRELESS LINKS WITH OPERATING CONSTRAINTS

by

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ABSTRACT

PERFORMANCE OPTIMIZATION OVER WIRELESS LINKS WITH OPERATING CONSTRAINTS

Shiny Abraham

Old Dominion University, 2012

Director: Dr. Dimitrie C. Popescu

Wireless communication is one of the most active areas of technological innovations and groundbreaking research ranging from simple cellular phones to highly complex military monitoring devices. The emergence of radios with cognitive capabilities like software defined radios has revolutionized modern communication systems by providing transceivers which can vary their output waveforms as well as their demodulation methods. This adaptability plays a pivotal role in efficient utilization of radio spectrum in an intelligent way while simultaneously not interfering with other radio devices operating on the same frequency band. Thus, it is safe to say that current and future wireless systems and networks depend on their adaptation capability which in turn presents many new technical challenges in hardware and protocol design, power management, interference metrics, distributed algorithms, Quality of Service (QoS) requirements and security issues. Transmitter adaptation methods have gained importance, and numerous transmitter optimization algorithms have been proposed in recent years. The main idea behind these algorithms is to optimize the transmitted signals according to the patterns of interference in the operating environment such that some specific criterion is optimized. In this context,

the objective of this dissertation is to propose transmitter adaptation algorithms in conjunction with power control for wireless systems focusing on performance optimization based on operating constraints. Specifically, this dissertation achieves joint transmitter adaptation and power control in the uplink and downlink of wireless systems with applications to Multiple-Input-Multiple-Output(MIMO) wireless systems and cognitive radio networks. In addition, performance of the proposed algorithms are evaluated in the context of fading channels, taking into consideration the time-varying nature of wireless channels.

To my family, friends and teachers.

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Last but certainly not least, I thank God for his countless blessings.

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Chapter I

INTRODUCTION

The evolution of cellular technology from wired to wireless in the late twentieth century is undoubtedly a revolutionary milestone in the history of communication networks and has undergone exponential growth in the last decade. Cellular phones and wireless local area networks (WLANs) have become an integral part of our daily lives and coupled with applications like smart home appliances, video teleconferencing and remote tele-medicine, have ensured that the essence of wireless technology permeates our very existence. The broader impact of wireless technology in a global context can also be observed in military applications where identification and tracking of enemy targets, detection of chemical and biological attacks, support of unmanned robotic vehicles and counter-terrorism are explicit consequences of the wireless boom [1]. The transition from wired to wireless technology brought with it significant challenges that must be addressed in order for this vision to become reality. Unlike wired networks, the wireless environment is unpredictable with noise and interference levels being the primary causes of degraded performance and signal impairments.

Current and future generation wireless systems and networks depend greatly on transceiver adaptation capability; this is enabled by the emergence of cognitive and software defined radios [2,3]. This adaptive capability results in versatile transmitters that vary their waveforms and versatile receivers that can vary their filters for a more efficient use of available radio resources. Numerous algorithms for transmitter adaptation have been proposed in recent years both for uplink wireless systems where multiple transmitters communicate with a common receiver [4–9] and for downlink systems where a single transmitter sends information to multiple receivers [10–13]. Various algorithms for specific wireless systems such as CDMA, OFDM or MIMO have also been proposed [4, 7–9].

I.1 RESEARCH MOTIVATION

An important attribute of future generations of wireless systems is their adaptation capability that results in systems which are more efficient in their use of the limited frequency spectrum along with interference mitigation among systems that use the same frequency bands. The need for adaptive capability in wireless systems was brought about by the emergence of cognitive and software defined radio platforms that have enabled adaptive radios with versatile transmitters to vary their waveforms and versatile receivers to vary their filters over time [3].

Efficient use of radio resources in future generations of wireless systems is vital in order to provide a wide range of services for mobile users, from multimedia transmissions performed in real time to transmission of data that can tolerate delay and which is not performed in real time. Radio spectrum is a scarce resource that is tightly regulated by the Federal Communications Commission (FCC) and the National Telecommunications and Information Administration (NTIA), and it forms a major source of investment in many countries where spectral licenses are auctioned to the highest bidder [2]. In order to get a reasonable return on its investment, the spectrum has to be utilized efficiently and also be reused over and over in the same geographical area. This presents technical challenges like interference, reduced capacity and degraded performance in its practical implementation. Effective methods of radio resource management include optimum utilization of the allocated spectrum and transmitter power control. These methods result in interference mitigation and increased system capacity with the transmitter power control in particular contributing to extending the battery life in mobile stations by ensuring that they transmit at the minimum power level necessary to achieve a specified Quality of Service (QoS). In addition to spectral auctions, unlicensed frequency bands are available to radio services that meet certain predefined regulatory requirements. A major drawback of unlicensed bands is the interference generated when many unlicensed devices occupy the band and operate at close proximities.

Traditional approaches to combating interference usually involve measurement and/or prediction of the channel followed by appropriate selection of modulation methods and signal processing algorithms for reliable reception [14]. This resulted in a relatively rigid hardware infrastructure with an associated set of standards that lacked flexibility, scalabity and adaptability. However, the emergence of software defined radios enabled greater degrees of freedom in combating interference without being restricted by physical limitations. The concept of interference avoidance in adaptive systems is based on the ability of the transmitting radio to vary its waveform (or signature) in response to interference conditions when instructed by the receiver via feedback mechanisms.

I.2 MODELING WIRELESS LINKS

Depending on the number of active transmitters and receivers, wireless communication links can be classified as :

- *Downlink (Broadcast Channels)* where a single transmitter communicates with many receivers as shown in Figure 1.
- *Uplink (Multiple Access Channels)* where transmitters do not cooperate but compete with each other for the same resources, and multiple transmitters (mobile terminals) send independent information to a single receiver (base station receiver or access point) as shown in Figure 2.
- *Mutually interfering links (Interference Channels)* where neither transmitters nor receivers cooperate and each transmitter-receiver pair attempts to communicate in the presence of interference from all other users as shown in Figure 3.

Multiple-input multiple-output (MIMO) wireless systems employ multiple antennas at both the transmitter and receiver which exploit spatial dimensions in wireless channels to provide increased capacity and diversity, as well as to mitigate interference [15].

The interference channel shown in Figure 3 can be represented in a general signal-space formulation which makes it applicable to a wide variety of wireless transmission schemes and scenarios (such as OFDM, CDMA, or multiple antenna/MIMO systems) [5, 6].

We consider a wireless system with K active transmitters that use block transmissions for sending information [5, 6] such that the N-dimensional vector $\mathbf{x}_k =$

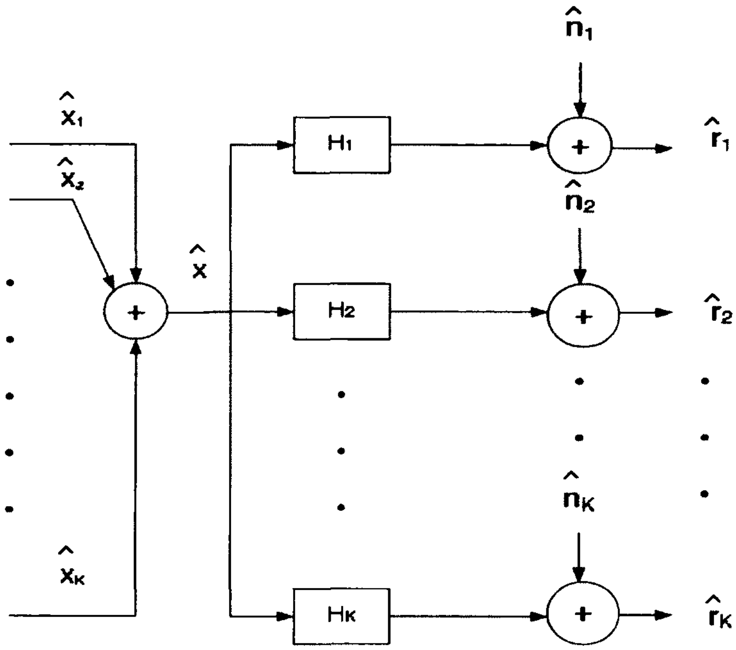


FIG. 1: Block Diagram for Downlink Scenario.

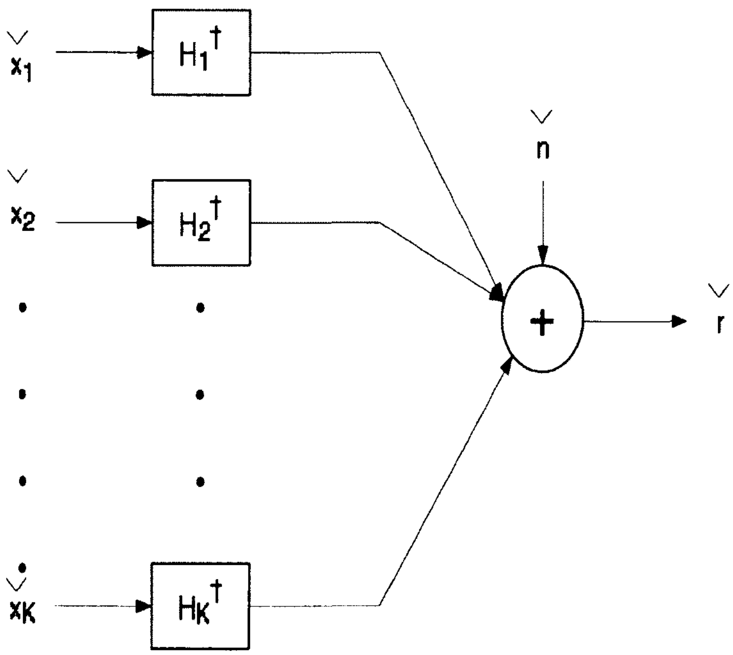


FIG. 2: Block Diagram for Uplink Scenario.

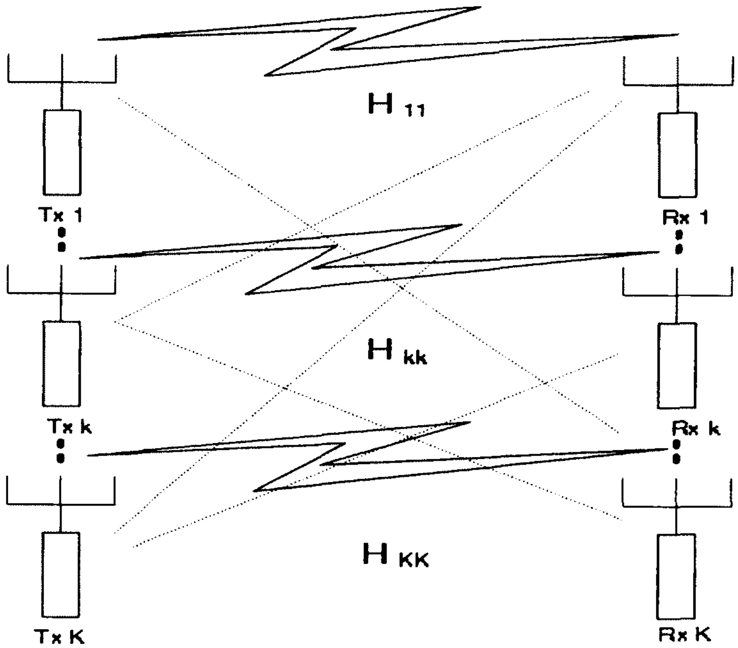


FIG. 3: Interference system model with K links.

$[x_{1k} \dots x_{Nk}]^\top$ transmitted by a given user k is written in terms of the information symbols vector $\mathbf{b}_k = [b_1^{(k)} \dots b_{M_k}^{(k)}]^\top$ as

$$\mathbf{x}_k = \sum_{m=1}^{M_k} b_m^{(k)} \mathbf{s}_m^{(k)} \sqrt{p_m^{(k)}} = \mathbf{S}_k \mathbf{P}_k^{1/2} \mathbf{b}_k, \quad k = 1, \dots, K \qquad \text{(I.2.1)}$$

where $\mathbf{s}_m^{(k)}$ is an N-dimensional vector that precodes symbol m of user k that is transmitted with power p_k. In compact form, $\mathbf{S}_k = [\mathbf{s}_1^{(k)} \dots \mathbf{s}_m^{(k)} \dots \mathbf{s}_{M_k}^{(k)}]$ is the $N \times M_k$ precoding matrix corresponding to user k and $\mathbf{P}_k = \text{diag}\{p_1^{(k)}, \dots, p_{M_k}^{(k)}\}$ is the $M_k \times M_k$ transmitted power matrix of user k. Symbols transmitted by distinct users are assumed to be uncorrelated, that is $E[\mathbf{b}_k \mathbf{b}_\ell^\top] = \mathbf{0}_{M_k \times M_\ell}$ and $E[\mathbf{b}_k \mathbf{b}_k^\top] = \mathbf{I}_{M_k \times M_k}$. We also assume that the columns of the precoding matrix are normalized to unit norm, $\mathbf{s}_m^{(k)\top} \mathbf{s}_m^{(k)} = 1$, $\forall m = 1, \dots, M_k$, such that the average transmitted power corresponding to user k is given by

$$E\{\|\mathbf{x}_i\|^2\} = \text{Trace}\,[\mathbf{P}_k] \leq P_k^{\max} \tag{I.2.2}$$

where $P_k^{\max}$ is the upper limit on user k transmit power. We note that this approach is general and covers many wireless scenarios as the N elements of the transmitted vector $\mathbf{x}_k$ may be sent over N distinct dimensions defined by non-overlapping pulses (or "chips") in typical CDMA systems [16], tones of different frequencies in multi-carrier and OFDM systems [17], spatial dimensions in multiple antenna and MIMO systems [8], or even wavelets [18].

We assume that the K active users in the system transmit to K receivers through vector channels described by matrices $\mathbf{H}_{kk}$ of dimension $N \times N$ which embed all characteristics of the physical channels between transmitters and receivers such as attenuation, multipath, or multiple antennas [9,14,19–21]. The $\mathbf{H}_{kk}$ matrices are assumed known at the receivers as well as fixed for the entire duration of the transmission, and the N-dimensional received signal vector at the kth receiver corresponding to one signalling interval is given by the expression

$$\mathbf{r}_k = \mathbf{H}_{kk}\mathbf{x}_k + \underbrace{\sum_{j=1, j\neq k}^{K} \mathbf{H}_{kj}\mathbf{x}_j + \mathbf{n}_k}_{\mathbf{i}_k \,=\, \text{interference + noise on wireless link } k} \tag{I.2.3}$$

where $\mathbf{H}_{kj}$ is the $N \times N$ dimensional channel matrix between transmitter j and receiver k and $\mathbf{n}_k$ is the N-dimensional additive white Gaussian noise at receiver k with zero mean and positive definite covariance matrix $E[\mathbf{n}_k\mathbf{n}_k^\top] = \mathbf{W}_k$. The correlation matrix of the interference+noise $\mathbf{i}_k$ corrupting link k is given by

$$\mathbf{R}_{ik} = E[\mathbf{i}_k\mathbf{i}_k^\top] = \sum_{j=1, j\neq k}^{K} \mathbf{H}_{kj}\mathbf{S}_k\mathbf{P}_k\mathbf{S}_k^\top\mathbf{H}_{kj}^\top + \mathbf{W}_k \tag{I.2.4}$$

and

$$\mathbf{R}_{rk} = E[\mathbf{r}_k \mathbf{r}_k^\top] = \mathbf{H}_{kk} \mathbf{S}_k \mathbf{P}_k \mathbf{S}_k^\top \mathbf{H}_{kk}^\top + \mathbf{R}_{ik} \tag{I.2.5}$$

is the correlation matrix of the received signal $\mathbf{r}_k$. This adaptive wireless system forms the basic system model referred to in the following chapters.

I.3 TRANSMITTER ADAPTATION AND POWER CONTROL IN WIRELESS SYSTEMS

Recent advances in the field of software defined radios have revolutionized wireless systems through incorporating artificial intelligence by allowing users to adapt their operating parameters such as frequency, transmit power and output waveforms as well as their demodulation methods based on their operating environment. Transmitter adaptation in wireless systems has been investigated in the context of specific wireless systems like CDMA, OFDM and MIMO [4, 7–9], and the main idea behind this is to optimize the transmitted signals according to the patterns of interference in the operating environment such that some specific criterion is optimized. Transmitter optimization for interference avoidance through signature (precoder, waveform or codeword) adaptation can be either centralized or decentralized. In the former adaptation scenario, a common receiver acquires optimal transmitter parameters from all the wireless links and in turn assigns them to individual users whereas in the latter adaptation scenario, users independently update their transmission parameters in response to feedback from the receiver. In general, centralized systems result in large network overhead, and their computational complexity increases with size. Hence, a decentralized or distributed adaptation scheme is preferred especially in applications where decision making and dynamic spectrum sharing are vital requirements.

Traditionally, transmitter power control is implemented by regulating the transmitted power to provide each user with an acceptable connection by limiting the interference caused by other users, and the power control problem requires that a vector of users transmitter powers be computed such that a specified set of constraints is met.

Until recently, transmitter adaptation and power control were treated as distinct problems. Efficient utilization of radio resources require transmitter adaptation in addition to transmitter power control, and this has resulted in the need for joint transmitter adaptation and power control in wireless systems. More recently, game-theoretic approaches have been used in the design of joint codeword and power control in wireless systems. We note that, while the uplink wireless scenario has been studied extensively over the past years and algorithms for uplink transmitter adaptation are presented in [9, 22–24], the downlink scenario has received less attention and few works discuss downlink transmitter adaptation [11, 20, 25]. Transmit beamforming and receiver combining techniques have been employed to make use of the significant diversity that is available in MIMO systems [26, 27]. When users are expected to meet specified target SINRs at the receiver, beamforming is complemented with transmitter power control, and joint beamforming and power control problems have been discussed in the context of single-input single-output (SISO) [28], single-input multiple output (SIMO) [29] and multiple-input single-output (MISO).

In addition to efficient spectrum utilization, interference mitigation and power control, it is of great importance for wireless communication systems to satisfy certain Quality of Service (QoS) requirements that ensure acceptable performance and quality of the system. In the following chapters we will come across a very widely used QoS parameter, the target Signal-to-Interference+Noise-Ratio (SINR) and perform joint transmitter adaptation and power control based on this QoS parameter. Another QoS parameter we come across in this dissertation is the achievable rate of link k given by [30]

$$\mathcal{R}_k = \frac{1}{2} \log \det \mathbf{R}_{rk} - \frac{1}{2} \log \det \mathbf{R}_{ik} \tag{I.3.1}$$

which implies convergence of an algorithm to socially optimal ensemble of precoders that maximizes $\mathcal{R}_k$.

I.4 PROBLEM STATEMENT

In this dissertation, performance optimization of wireless systems is achieved with the help of adaptive transmitters and operating constraints at the receiver. The main objectives of this dissertation are to:

1. Propose algorithms for downlink transmitter adaptation based on greedy SINR maximization and study their fixed point properties.

2. Achieve joint transmitter adaptation and power control using constrained optimization theory and propose specific algorithms for downlink and uplink scenarios.

3. Investigate convergence of the proposed algorithms.

4. Analyze the effect of fading channels in the implementation of the proposed joint transmitter adaptation and power control algorithms.

I.5 DISSERTATION ROAD MAP

Chapter II proposes algorithms for downlink transmitter adaptation based on greedy SINR maximization and interference avoidance. The proposed algorithms are implemented using precoder adaptation based on collaborative receivers, inverse channel assumptions and matched filter receiver techniques followed by their fixed point properties for various scenarios like ideal and non-ideal channels, white and colored noise at all user receivers and point-to-point communication.

Chapter III studies joint transmitter adaptation and power control in downlink wireless systems using constrained optimization theory. The adaptation scenario is formulated as a constrained optimization problem that aims to minimize the total interference subject to target SINR and unit norm codeword constraints. Using necessary and sufficient conditions of the optimization problem, a joint transmitter adaptation and power control algorithm is proposed which performs incremental updates of the codeword and power values that culminate in an optimal codeword and power ensemble. Simulation results illustrate how the proposed algorithm can be used for tracking varying target SINRs and/or a variable number of active users. As an extension of this approach, a general framework based on block transmissions and linear precoders is considered, and the joint precoder adaptation and power control objective is formulated as a constrained optimization problem that minimizes the total sum of Minimum Mean Square Errors (MMSE's) subject to target SINR constraints. Necessary and sufficient conditions that must be satisfied by the optimal precoder and power matrices are determined, and the resulting incremental algorithms are implemented until a fixed point is reached where the specified target

SINRs are achieved with minimum transmitted power. Simulation results illustrate convergence of the algorithm and dependence of convergence speed on the algorithm constants.

Chapter IV presents a similar study of joint precoder adaptation and power control in uplink scenarios with target SINR requirements. The uplink adaptation scenario is formulated as multiple constrained optimization problems corresponding to the multiple active transmitters, and this differs from the downlink scenario that has a single constrained optimization problem corresponding to the base station transmitter. A detailed analysis of necessary and sufficient conditions is presented followed by the proposed joint precoder adaptation and power control algorithm which is implemented in a distributed fashion until a fixed point is reached where the specified target SINRs are achieved with minimum transmitted power. Simulation results illustrate convergence of the algorithm followed by its application in cognitive radio networks.

Chapter V analyzes performance of the incremental algorithm for joint transmitter adaptation and power control in wireless systems in the context of fading channels. Multipath Rayleigh fast fading channels are considered where the channel between any given user and the base station is dynamic with a small coherence time such that it is not practical to estimate channel characteristics. Average characteristics of the multipath fading channels are used to compute precoder and power matrices for which target SINR values are satisfied; then, Monte Carlo simulations are performed to study the performance for actual and average channel realizations. The performance measure used to evaluate performance of the algorithm in this work is the probability of outage.

Chapter VI concludes the dissertation with final discussions and future directions of research work.

Most of the results in this dissertation have been presented previously. The work on algorithms for transmitter adaptation in downlink wireless systems in Chapter II was presented at the 2009 IEEE Canadian Conference on Electrical and Computer Engineering (CCECE 2009) [31]. The results from Chapter III on joint transmitter adaptation and power control in downlink wireless systems was presented in part at the 2010 IEEE Radio and Wireless Symposium (RWS 2010) [32], the 2011 IEEE Conference on Information Sciences and Systems (CISS 2011) [33] and submitted for review to the Journal of Wireless Communications and Mobile Computing [34]. Work on uplink wireless systems in Chapter IV was presented in part at the 2010 IEEE Asilomar Conference on Signals, Systems [35] and the extension to Cognitive Radio Networks was accepted for publication in the Elsevier Journal on Physical Communications [36]. Work on fading channels in Chapter VII was submitted for review to the 2013 IEEE Radio and Wireless Symposium (RWS 2013) [37].

This dissertation uses IEEE Transaction style for the bibliography and citation.

Chapter II

ALGORITHMS FOR TRANSMITTER ADAPTATION IN DOWNLINK WIRELESS SYSTEMS

In this chapter we discuss algorithms for downlink transmitter adaptation that are based on greedy SINR maximization through interference avoidance [14]. The first proposed algorithm is based on the collaborative approach introduced in [30] where the signals received by all active users are combined and used for joint decoding, and the proposed algorithm yields socially optimal precoder ensembles which maximize individual SINRs at the collaborative receiver as well as sum capacity. Since collaboration can be difficult to achieve in the downlink scenario we discuss also two alternative algorithms for precoder adaptation where no collaboration among receivers is needed. In the first one "inverse channel" observations similar to [24] are used to obtain the decision variables and decode received signals by users. In this case the received signal by a given user is first equalized by multiplication with its inverse channel matrix followed by matched filter detection. The second alternative algorithm is based on the interference avoidance procedure in [25] and uses matched filters and channel information to obtain the decision variable directly from the received signal.

II.1 DOWNLINK SYSTEM MODEL

The downlink system model is a particular case of the system model presented in section I.2 with a single transmitter and multiple receivers with single codewords per user. Assuming that there are K active users in the system the signal transmitted by the base station is expressed similar to equation I.2.1 as

$$\mathbf{x} = \sum_{\ell=1}^{K} b_\ell \sqrt{p_\ell} \mathbf{s}_\ell = \mathbf{S}\mathbf{P}^{1/2}\mathbf{b}, \tag{II.1.1}$$

and the received signal by a given user k is given by

$$\mathbf{r}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{n}_k = \mathbf{H}_k \mathbf{S}\mathbf{P}^{1/2}\mathbf{b} + \mathbf{n}_k. \tag{II.1.2}$$

The $N \times N$ vector channel matrix $\mathbf{H}_k$ is the mathematical representation of the physical channel between the base station transmitter and user k receiver and is a particular case of the channel matrix $\mathbf{H}_{kk}$ defined in section I.2 with a single transmitter and multiple receivers. Channel matrices $\mathbf{H}_1, \ldots, \mathbf{H}_K$ are assumed invertible and known at the receiver as well as fixed for the entire duration of the transmission.

In this setup we discuss algorithms that adapt the precoders $\{\mathbf{s}_1, \ldots, \mathbf{s}_K\}$ assigned to users through various approaches based on greedy SINR maximization and interference avoidance.

II.2 COLLABORATIVE PRECODER ADAPTATION

Reference [30] considers a wireless system with multiple transmitters and receivers that collaborate and presents an algorithm for precoder adaptation based on greedy SINR maximization through interference avoidance. This algorithm converges to a socially optimal ensemble of precoders which maximizes sum capacity. We note that the downlink wireless system considered in our paper and described in the previous section is a particular case of the general wireless scenario considered in [30] where the system has multiple receivers but only a single transmitter. Following [30] and assuming collaboration among receivers, we may form the KN-dimensional vector $\mathbf{r}$ by grouping the received signals by all users as shown below

$$\underbrace{\begin{bmatrix} \mathbf{r}_1 \\ \vdots \\ \mathbf{r}_K \end{bmatrix}}_{\mathbf{r}} = \underbrace{\begin{bmatrix} \mathbf{H}_1 \\ \vdots \\ \mathbf{H}_K \end{bmatrix}}_{\mathbf{H}} \mathbf{S}\mathbf{P}^{1/2}\mathbf{b} + \underbrace{\begin{bmatrix} \mathbf{n}_1 \\ \vdots \\ \mathbf{n}_K \end{bmatrix}}_{\mathbf{n}} \tag{II.2.1}$$

or in compact form

$$\mathbf{r} = \mathbf{H}\mathbf{S}\mathbf{P}^{1/2}\mathbf{b} + \mathbf{n}. \tag{II.2.2}$$

The signal in equation (II.2.2) has correlation matrix

$$\mathbf{R} = E[\mathbf{r}\mathbf{r}^\top] = \mathbf{H}\mathbf{S}\mathbf{P}\mathbf{S}^\top\mathbf{H}^\top + \mathbf{W} \tag{II.2.3}$$

where matrix $\mathbf{W}$ is the $NK \times NK$ correlation matrix of the aggregated noise vector $\mathbf{n}$ containing the noise vectors at each receiver and is expressed in terms of the covariance matrices $\mathbf{W}_\ell$ of individual noise vectors $\mathbf{n}_\ell$, $\ell = 1, \ldots, K$.

Using the approach proposed for the multi-receiver system in [30] we rewrite (II.2.2) from a given user k perspective as

$$\mathbf{r} = \underbrace{\sqrt{p_k}\mathbf{H}\mathbf{s}_k b_k}_{\text{desired signal}} + \underbrace{\sum_{\ell=1,\ell\neq k}^{K} \sqrt{p_\ell}\mathbf{H}\mathbf{s}_\ell b_\ell + \mathbf{n}}_{\text{interference+noise}} \tag{II.2.4}$$

and assume that an MMSE receiver is used to decode user k from the received signal $\mathbf{r}$. Denoting $\mathbf{y}_k = \sqrt{p_k}\mathbf{H}\mathbf{s}_k$, the expression of the MMSE receiver for user k is [30]

$$\mathbf{c}^* = \frac{\mathbf{R}_k^{-1}\mathbf{y}_k}{\sqrt{\mathbf{y}_k^\top\mathbf{R}_k^{-2}\mathbf{y}_k}} = \frac{\mathbf{R}_k^{-1}\mathbf{H}_k\mathbf{s}_k}{\sqrt{\mathbf{s}_k^\top\mathbf{H}_k^\top\mathbf{R}_k^{-2}\mathbf{H}_k\mathbf{s}_k}} \tag{II.2.5}$$

Algorithm 1 –Collaborative Precoder Adaptation Algorithm

1: Initial Data

- All user precoder and power matrices $\mathbf{S}$ and $\mathbf{P}$, as well as all channel and noise covariance matrices $\mathbf{H}_k$ and $\mathbf{W}_k$, $\forall k$.

2: **for** each user $k = 1, \ldots, K$ **do**
3: Calculate matrix $\mathbf{H}^\top \mathbf{R}_k^{-1} \mathbf{H}$ and determine its maximum eigenvector $\mathbf{x}_k$.
4: Replace user k's current precoder $\mathbf{s}_k$ by $\mathbf{x}_k$.
5: **end for**
6: REPEAT Step 2 UNTIL a fixed point is reached.

where $\mathbf{R}_k = \mathbf{R} - \mathbf{y}_k \mathbf{y}_k^\top$ is the correlation of the interference+noise corrupting user k's signal. The corresponding SINR expression for user k is

$$\gamma_k^{(col)} = \mathbf{y}_k^\top \mathbf{R}_k^{-1} \mathbf{y}_k = p_k \mathbf{s}_k^\top \mathbf{H}^\top \mathbf{R}_k^{-1} \mathbf{H} \mathbf{s}_k. \tag{II.2.6}$$

The greedy SINR maximization procedure in [30] replaces user k precoder $\mathbf{s}_k$ with a new one that maximizes the SINR expression (II.2.6). We note that the right-hand side of (II.2.6) contains the Rayleigh quotient of matrix $\mathbf{H}^\top \mathbf{R}_k^{-1} \mathbf{H}$ multiplied by the desired user k power p_k and for fixed user power is maximized when $\mathbf{s}_k$ is replaced by the eigenvector corresponding to the maximum eigenvalue (also referred to as the maximum eigenvector) of $\mathbf{H}^\top \mathbf{R}_k^{-1} \mathbf{H}$. Based on this procedure the algorithm for downlink precoder adaptation is formally stated as **Algorithm 1**.

We note that Step 2 of **Algorithm 1** defines an ensemble iteration, in which all precoders are updated one time. We also note that this algorithm monotonically increases the sum capacity of the multiple access channel (II.2.2) given by [30]

$$\mathcal{C}_{sum} = \frac{1}{2} \log \det \mathbf{R} - \frac{1}{2} \log \det \mathbf{W} \tag{II.2.7}$$

and converges precoder ensembles that maximize $\mathcal{C}_{sum}$.

Algorithm 1 may be implemented at the base station transmitter which needs to know (in addition to the user precoders and powers which are already known) also the downlink channel matrices, $\mathbf{H}_k$, for all active users in the system along with their corresponding noise covariance matrices $\mathbf{W}_k$. We note that channel state information for the downlink can be made available at the transmitter by using either direct channel feedback in the case of frequency-division duplex (FDD) systems or the reciprocal channel information from the uplink for time-division duplex (TDD) systems [38], while knowledge of noise covariance matrices at each receiver may be obtained over a dedicated feedback channel. However, in order to take advantage of the optimal precoders and to obtain the maximum SINR at the receiver collaborative decoding using all received signals is required which may not be practical. Alternative algorithms in which users are not required to perform collaborative decoding to achieve maximum SINR are presented in the following sections.

II.3 PRECODER ADAPTATION BASED ON INVERSE CHANNEL OBSERVATIONS

Assuming that the user channel matrices are invertible, in this approach the receiver at a given user k uses an "inverse channel" observation to decode the transmitted information symbol by the base station. The "inverse-channel" observation vector is obtained similar to [24] by equalizing the received signal through multiplication with the inverse of the given user k channel matrix, $\tilde{\mathbf{r}}_k = \mathbf{H}_k^{-1}\mathbf{r}_k$, and has the expression

$$\tilde{\mathbf{r}}_k = b_k\sqrt{p_k}\mathbf{s}_k + \sum_{\ell=1,\ell\neq k}^{K} b_\ell\sqrt{p_\ell}\mathbf{s}_\ell + \mathbf{H}_k^{-1}\mathbf{n}_k. \tag{II.3.1}$$

The "inverse-channel" observation is processed by a matched filter corresponding to user k's precoder to obtain the decision variable for user k, $d_k = \mathbf{s}_k^\top \tilde{\mathbf{r}}_k$, and is expressed as

$$d_k = b_k\sqrt{p_k} + \sum_{\ell=1,\ell\neq k}^{K} b_\ell\sqrt{p_\ell}\mathbf{s}_k^\top \mathbf{s}_\ell + \mathbf{s}_k^\top \mathbf{H}_k^{-1}\mathbf{n}_k \tag{II.3.2}$$

which implies that the expression of the SINR for user k is

$$\gamma_k^{(ic)} = \frac{p_k}{\mathbf{s}_k^\top \underbrace{\mathbf{H}_k^{-1}\left(\mathbf{Q}_k - p_k\mathbf{H}_k\mathbf{s}_k\mathbf{s}_k^\top\mathbf{H}_k^\top\right)\mathbf{H}_k^{-\top}}_{\mathbf{Z}_k}\mathbf{s}_k} \tag{II.3.3}$$

where $\mathbf{Q}_k$ represents the correlation matrix of the received signal at user k receiver in equation (II.1.2). In order to perform greedy maximization of the SINR in this case we note that the denominator in equation (II.3.3) contains the Rayleigh quotient of matrix $\mathbf{H}_k^{-1}\left(\mathbf{Q}_k - p_k\mathbf{H}_k\mathbf{s}_k\mathbf{s}_k^\top\mathbf{H}_k^\top\right)\mathbf{H}_k^{-\top}$ which should be minimized in this case to achieve maximum SINR. We note that $\mathbf{Z}_k$ represents the correlation matrix of the interference+noise that corrupts the desired signal from user k in the "inverse channel" observation, and replacing the user precoder $\mathbf{s}_k$ by the minimum eigenvector of $\mathbf{Z}_k$ will ensure maximization of user k SINR. Based on this procedure we can define a second algorithm for downlink precoder adaptation which is formally stated as **Algorithm 2**

Similar to **Algorithm 1**, Step 2 defines an ensemble iteration in which all precoders are updated one time, but unlike **Algorithm 1** for which convergence to a fixed point has been established analytically in [30] based on the monotonic increase

Algorithm 2 –Inverse Channel Precoder Adaptation Algorithm

1: Initial Data

- User precoder and power matrices $\mathbf{S}$, $\mathbf{P}$, channel and noise covariance matrices $\mathbf{H}_k$, $\mathbf{W}_k$, $\forall k$.

2: **for** each user $k = 1, \ldots, K$ **do**
3: Calculate matrix $\mathbf{Z}_k$ and determine its minimum eigenvector $\mathbf{u}_k$.
4: Replace user k's current precoder $\mathbf{s}_k$ by $\mathbf{u}_k$.
5: **end for**
6: REPEAT Step 2 UNTIL a fixed point is reached.

of the sum capacity C_{sum} in (II.2.7), for **Algorithm 2** we have only empirical evidence of convergence. A fixed point of the algorithm is reached when the difference between two consecutive values of a stopping criterion is within a specified tolerance value ϵ, and we ran extensive simulations to assess the convergence of **Algorithm 2** using the norm difference between a given precoder and its replacement as stopping criterion. Numerical results have shown that precoders converge to within tight norm difference tolerances when starting with randomly initialized user precoders, and for $\epsilon = 10^{-3}$ precoder convergence varied from tens of ensemble iterations for low values of the signal space dimension to several hundred ensemble iterations for large values of the signal space dimensions. We have also looked at the convergence of user SINRs which occurs much faster than precoder convergence, and simulations have shown that typically this occurs in $10 \div 15$ ensemble iterations and does not depend on the dimension of the signal space. **Algorithm 2** may also be implemented at the base station and requires similar information as **Algorithm 1**. However, collaborative decoding using all received signals is not required, and users decode their received signals independently in this case.

II.4 PRECODER ADAPTATION WITH MATCHED FILTER RECEIVERS

This approach for downlink precoder adaptation has been proposed in [24], and in this case users employ linear filters matched to the expression of their corresponding received precoder to obtain the decision variable. For a given user k the receiver filter has the expression

$$\mathbf{f}_k = \frac{1}{\sqrt{\mathbf{s}_k^\top \mathbf{H}_k^\top \mathbf{H}_k \mathbf{s}_k}} \mathbf{H}_k \mathbf{s}_k, \qquad k = 1, \ldots, K, \tag{II.4.1}$$

and its decision variable in this case is $\tilde{d}_k = \mathbf{f}_k^\top \mathbf{r}_k$ which implies that the SINR at user k receiver is

$$\gamma_k^{(mf)} = p_k \frac{\mathbf{f}_k^\top (\mathbf{H}_k \mathbf{s}_k \mathbf{s}_k^\top \mathbf{H}_k^\top) \mathbf{f}_k}{\mathbf{f}_k^\top \tilde{\mathbf{Z}}_k \mathbf{f}_k} \tag{II.4.2}$$

where $\tilde{\mathbf{Z}}_k = \mathbf{Q}_k - p_k \mathbf{H}_k \mathbf{s}_k \mathbf{s}_k^\top \mathbf{H}_k^\top$ represents the correlation matrix of the interference+noise that corrupts the desired signal from user k. We note that for a given user precoder sk, the user SINR $\gamma_k^{(mf)}$ is maximized by the choice of receiver filter $\mathbf{f}_k^*$ which implies maximization of the ratio of the two quadratic forms defined by matrices $\mathbf{H}_k \mathbf{s}_k \mathbf{s}_k^\top \mathbf{H}_k^\top$ and $\tilde{\mathbf{Z}}_k$. As discussed in [24], this ratio is maximized by the largest eigenvalue of the matrix pencil defined by the pair of matrices

$$(\mathbf{H}_k \mathbf{s}_k \mathbf{s}_k^\top \mathbf{H}_k^\top, \tilde{\mathbf{Z}}_k) \tag{II.4.3}$$

and implies that the optimum receiver filter $\mathbf{f}_k^*$ is the eigenvector corresponding to the largest generalized eigenvalue of the matrix pair (II.4.3), that is

$$(\mathbf{H}_k \mathbf{s}_k \mathbf{s}_k^\top \mathbf{H}_k^\top) \mathbf{f}_k^* = \lambda_k^* \tilde{\mathbf{Z}}_k \mathbf{f}_k^*. \tag{II.4.4}$$

Algorithm 3 –Matched Filter Receiver based Precoder Adaptation Algorithm

1: Initial Data

- User precoder and power matrices $\mathbf{S}$, $\mathbf{P}$, channel and noise covariance matrices $\mathbf{H}_k$, $\mathbf{W}_k$, $\forall k$.

2: **for** each user $k = 1, \ldots, K$ **do**
3: Determine the SINR maximizing filter $\mathbf{f}_k^*$.
4: Replace user k's current precoder $\mathbf{s}_k$ by $\mathbf{s}_k^*$.
5: **end for**
6: REPEAT Step 2 UNTIL a fixed point is reached.

Using the same assumption as in the previous section, namely that the user channel matrices are invertible, the precoder update for user k is obtained using the expression of the SINR maximizing filter $\mathbf{f}_k^*$ as $\mathbf{s}_k^* = \mathbf{H}_k^{-1}\mathbf{f}_k^*$. We note that this precoder update is similar to the uplink MMSE interference avoidance update [14], and based on it we define a third algorithm for downlink precoder adaptation using greedy SINR maximization which is formally stated as **Algorithm 3**.

As in the case of the other two algorithms defined in previous sections, Step 2) defines an ensemble iteration in which all precoders are updated one time. Convergence of **Algorithm 3** to a fixed point was investigated in [24] where numerical results obtained from simulations are used to establish empirical convergence similar to **Algorithm 2**. We note that no analytical convergence proof is available for **Algorithm 2** and **Algorithm 3** yet, and this will be the object of future research. We also note that **Algorithm 3** may be implemented at the base station and requires similar information as **Algorithm 1** and **Algorithm 2** with independent decoding of received signals by users and no need for collaborative decoding.

II.5 FIXED POINT PROPERTIES

In order to study the fixed-points properties of the proposed algorithms we performed simulations for various scenarios and looked at the correlation properties of the resulting precoder ensembles given by the matrix $\mathbf{SPS}^\top$. In the case of ideal channels for all users, that is $\mathbf{H}_k = \mathbf{I}_N, \forall k$ and white noise at all receivers, that is noise covariance matrices are expressed as $\mathbf{W}_k = \sigma_k \mathbf{I}_N, \forall k$, all algorithms resulted in precoder ensembles having the same correlation matrix $\mathbf{SPS}^\top = (K/N)\mathbf{I}_N$. This solution corresponds to Welch Bound Equality (WBE) precoder ensembles [39] for which the total squared correlation (TSC) of the ensemble, defined as the sum of squared correlations among all precoders in the system weighted by their corresponding powers, is minimized. This result can be confirmed analytically as it can easily be shown that in the case of ideal channel matrices the precoder updates implied by the three algorithms are essentially the same: the precoder of a given user k is replaced by the minimum eigenvector of matrix $\mathbf{SPS}^\top - p_k \mathbf{s}_k \mathbf{s}_k^\top$ and iterating for all users leads to WBE precoder ensembles [14]. We note that WBE ensembles maximize sum capacity of the multiple access channel corresponding to the dual uplink CDMA scenario with ideal channels and white noise at the receiver.

With ideal channel matrices and colored noise with the same covariance matrix at all user receivers, that is $\mathbf{W}_k = \overline{\mathbf{W}}, \forall k$ where $\overline{\mathbf{W}}$ is no longer a scaled identity matrix, the three algorithms yield precoder ensembles with different correlation properties. Among the three resulting precoder ensembles the one corresponding to **Algorithm 2** has the same correlation properties $\mathbf{SPS}^\top$ as those of the dual uplink scenario with ideal user channels and same noise covariance matrix $\overline{\mathbf{W}}$ at the receiver.

This result can also be confirmed analytically as one can easily note that with ideal channels and same noise covariance matrix at all receivers the precoder updates of **Algorithm 2** are similar to those for the dual uplink scenario with ideal channels and colored noise with covariance $\overline{\mathbf{W}}$ at the receiver [14]. We note that when noise is colored but has different covariance matrices for different users, a meaningful comparison with a dual uplink scenario is difficult as the corresponding noise covariance matrix in the uplink scenario cannot be clearly established.

We have also looked at point-to-point communication scenarios where the channel and noise covariance matrices are the same for all users and observed that the algorithms yield in general precoder ensembles with different correlations and that only the ensembles implied by **Algorithm 2** have the same correlation as those of the dual uplink CDMA scenario. This was also expected as in the point-to-point scenario the precoder updates implied by **Algorithm 2**, and those corresponding to the uplink scenario [24] are similar.

Simulations have also shown that in the case of interference limited systems, where the power of the noise corrupting the received signals corresponding to all users is small compared to the power of the transmitted signal, that is Trace$[\mathbf{W}_k]$ $\ll$ Trace$[\mathbf{P}]$, $\forall k$, **Algorithm 1** and **Algorithm 2** yield very similar results with precoder correlations getting closer to those corresponding to WBE ensembles, that is $\mathbf{S}\mathbf{P}\mathbf{S}^\top = (K/N)\mathbf{I}_N$, as the power of the noise gets closer to zero regardless of the user channel matrices. We note that, while it is not obvious why WBE ensembles correspond to fixed points of **Algorithm 1** for interference-limited scenarios, for **Algorithm 2** this can be easily confirmed analytically as we note that in the case of very small noise power at the receiver the "inverse channel" observation in equation

(III.1.9) is essentially identical for all users, $k = 1, \ldots, K$, and is the same as the uplink channel equation considered in [14] where it is shown that precoder adaptation based on greedy SINR maximization and minimum eigenvector replacement results in WBE precoder ensembles.

In the most general case of non-ideal channels where user channel matrices are only assumed to be invertible but can take any form (diagonal, circulant, etc.) and the noise covariance matrices $\mathbf{W}_k$ are different for distinct users, simulations have shown that the three proposed algorithms result in general in precoder ensembles that have different correlation properties. In such scenarios, additional criteria are needed to decide which of the resulting precoder ensembles is more desirable, and this will be the object of future investigations.

II.6 CHAPTER SUMMARY

In this chapter we considered the downlink of a wireless system and presented three algorithms for precoder adaptation based on various greedy SINR maximization procedures. **Algorithm 1** uses a collaborative approach and yields precoder ensembles that maximize the sum capacity which is a global measure for the wireless system when collaboration among user receivers is assumed. Since receiver collaboration may be difficult to achieve in the downlink scenario we also discuss two alternative algorithms where collaboration is not needed: **Algorithm 2** is based on "inverse channel" observations and minimum eigenvector precoder replacement similar to the one proposed for uplink scenarios in [24], while **Algorithm 3** uses matched filters and channel knowledge along with a maximum generalized eigenvector precoder replacement as outlined in [25]. The proposed algorithms yield precoder ensembles that usually have different correlation properties, and additional criteria should be specified in order to determine which of the ensembles is desirable in downlink scenarios.

Chapter III

JOINT TRANSMITTER ADAPTATION AND POWER CONTROL IN DOWNLINK WIRELESS SYSTEMS

In this chapter we present a new approach that combines transmitter adaptation with power control in downlink wireless systems with SINR constraints at the receiver terminals. We formulate this as a constrained optimization problem, and we derive necessary and sufficient conditions for minimizing total interference subject to codeword norm constraints and target SINR constraints. We then present an algorithm that incrementally adapts the codewords and powers to meet the specified SINR targets with minimum powers, and we study the tracking ability of the proposed algorithm through simulations.

The proposed approach can be extended to more general scenarios by using a framework based on block transmissions and linear precoders that is applicable to many wireless transmission schemes and scenarios (such as OFDM, CDMA, or multiple antenna/MIMO systems [5,6]). The problem of joint precoder adaptation and power control subject to specific target SINR requirements is also formulated as a constrained optimization problem. We study the necessary conditions that must be satisfied by the optimal precoder and power matrices and present an incremental algorithm that jointly adapts transmitter precoders and power values until a fixed point is reached where the specified target SINRs are achieved with minimum transmitted power. We analyze the dependence of convergence speed on the algorithm constants through simulations.

III.1 SINR AND INTERFERENCE EXPRESSIONS FOR THE DOWNLINK SYSTEM MODEL

We consider the downlink of a wireless system with a system model similar to that in section I.2 but with a single transmitter and K receivers for which the N-dimensional signal vector from the base station transmitter is given by equation (II.1.1), and the N-dimensional received signal by a given user k is given by equation (II.1.2).

Our goal for this system is to establish a procedure for joint downlink transmitter adaptation and power control which reduces the total interference experienced by the active receivers and ensures that they operate with specific signal-to-interference+noise-ratios (SINRs). We note that the downlink transmitter has knowledge of all codewords and transmit powers for all users and that channel state information for the downlink can be acquired at the base station transmitter by using either direct channel feedback in the case of frequency-division duplex (FDD) systems or the reciprocal channel information from the uplink for time-division duplex (TDD) systems [38].

In order to establish the expression of the interference that corrupts the desired information signal at a given receiver k we rewrite the received signal in equation (II.1.2) to distinguish between the desired signal and the corresponding interference+noise:

$$\mathbf{r}_k = \underbrace{\mathbf{H}_k b_k \sqrt{p_k} \mathbf{s}_k}_{\text{desired signal}} + \underbrace{\mathbf{H}_k \sum_{\ell=1,\ell\neq k}^{K} b_\ell \sqrt{p_\ell} \mathbf{s}_\ell + \mathbf{n}_k}_{\text{interference + noise } (\mathbf{z}_k)} \tag{III.1.1}$$

where the interference+noise $\mathbf{z}_k$ experienced by user k has correlation matrix

$$\mathbf{Z}_k = E[\mathbf{z}_k\mathbf{z}_k^\top] = \mathbf{H}_k \left(\sum_{\ell=1,\ell\neq k}^{K} \mathbf{s}_\ell p_\ell \mathbf{s}_\ell^\top \right) \mathbf{H}_k^\top + \mathbf{W}_k. \tag{III.1.2}$$

With knowledge of user codewords, transmitted power values, channel matrices, and noise correlation matrices, the correlation matrix $\mathbf{Z}_k$ is available at the downlink transmitter, and parallel decomposition of user k downlink channel can be obtained. This is accomplished by applying a whitening transformation followed by a singular value decomposition (SVD) of the transformed channel matrix. The whitening transformation can be written as

$$\mathbf{T}_k = \triangle_k^{-1/2}\mathbf{E}_k^\top \tag{III.1.3}$$

where the matrices $\mathbf{E}_k$ and $\triangle_k$ are obtained from the diagonal decomposition of the correlation matrix $\mathbf{Z}_k = \mathbf{E}_k\triangle_k\mathbf{E}_k^\top$. In the transformed coordinates, equation (III.1.1) is equivalent to

$$\begin{aligned} \tilde{\mathbf{r}}_k &= \mathbf{T}_k\mathbf{r}_k = \mathbf{T}_k\mathbf{H}_k b_k\sqrt{p_k}\mathbf{s}_k + \mathbf{T}_k\mathbf{z}_k \\ &= \tilde{\mathbf{H}}_k b_k\sqrt{p_k}\mathbf{s}_k + \mathbf{w}_k \end{aligned} \tag{III.1.4}$$

where $\tilde{\mathbf{H}}_k = \mathbf{T}_k\mathbf{H}_k$ is the equivalent channel matrix corresponding to user k and $\mathbf{w}_k = \mathbf{T}_k\mathbf{z}_k$ is the equivalent white noise term with identity covariance matrix $E[\mathbf{w}_k\mathbf{w}_k^\top] = \mathbf{T}_k\mathbf{Z}_k\mathbf{T}_k^\top = \mathbf{I}_{N_k}$.

Parallel decomposition of user k's channel is now achieved by using SVD of the transformed channel matrix

$$\tilde{\mathbf{H}}_k = \mathbf{U}_k\mathbf{D}_k\mathbf{V}_k^\top \tag{III.1.5}$$

where the matrix of left singular vectors $\mathbf{U}_k$ is of dimension $N_k \times N_k$, the matrix of right singular vectors $\mathbf{V}_k$ is of dimension $N \times N$ and the matrix of singular values

$\mathbf{D}_k$ is of dimension $N_k \times N$. Assuming a rich scattering environment, the channel matrix $\tilde{\mathbf{H}}_k$ has full rank [40, Sec. 10.2] $\text{rank}(\tilde{\mathbf{H}}_k) = N_k$, and the singular value matrix $\mathbf{D}_k$ may be partitioned as

$$\mathbf{D}_k = \begin{bmatrix} \bar{\mathbf{D}}_k & \mathbf{0}_{N_k \times (N - N_k)} \end{bmatrix} \tag{III.1.6}$$

where $\bar{\mathbf{D}}_k$ is a $N_k \times N_k$ diagonal matrix containing the singular values and the zero matrix has the appropriate dimension. Premultiplying by $\mathbf{U}_k^\top$ in equation (III.1.4) we obtain the equivalent expression for (III.1.1)

$$\bar{\mathbf{r}}_k = \mathbf{D}_k \bar{\mathbf{s}}_k b_k \sqrt{p_k} + \bar{\mathbf{w}}_k \tag{III.1.7}$$

where $\bar{\mathbf{s}}_k = \mathbf{V}_k^\top \mathbf{s}_k$ and $\bar{\mathbf{w}}_k = \mathbf{U}_k^\top \mathbf{w}_k$.

By defining the left inverse of matrix $\mathbf{D}_k$ as

$$\mathbf{D}_k^\dagger = \begin{bmatrix} \bar{\mathbf{D}}_k^{-1} \\ \mathbf{0}_{(N-N_k) \times N_k} \end{bmatrix} \tag{III.1.8}$$

we can use it to "invert" the channel in equation (III.1.7) to obtain the equivalent expression

$$\hat{\mathbf{r}}_{k,inv} = \mathbf{D}_k^\dagger \bar{\mathbf{r}}_k = \underbrace{\bar{\mathbf{s}}_k b_k \sqrt{p_k}}_{\text{desired signal}} + \underbrace{\mathbf{D}_k^\dagger \bar{\mathbf{w}}_k}_{\text{interference + noise}} \tag{III.1.9}$$

which can be used to obtain the decision variable for user k's symbol by correlation with the corresponding transmit codeword

$$\hat{b}_k = \bar{\mathbf{s}}_k^\top \hat{\mathbf{r}}_{k,inv} = \bar{\mathbf{s}}_k^\top \bar{\mathbf{s}}_k b_k \sqrt{p_k} + \bar{\mathbf{s}}_k^\top \mathbf{D}_k^\dagger \bar{\mathbf{w}}_k. \tag{III.1.10}$$

The average power for user k's interference is

$$i_k = \bar{\mathbf{s}}_k^\top \mathbf{D}_k^\dagger (\mathbf{D}_k^\dagger)^\top \bar{\mathbf{s}}_k \tag{III.1.11}$$

with the corresponding user k SINR being

$$\gamma_k = \frac{p_k}{i_k}. \tag{III.1.12}$$

We note that $\mathbf{D}_k^{\dagger}(\mathbf{D}_k^{\dagger})^{\top}$ can be expressed as

$$\mathbf{D}_k^{\dagger}(\mathbf{D}_k^{\dagger})^{\top} = \begin{bmatrix} \bar{\mathbf{D}}_k^{-2} & \mathbf{0}_{N_k \times (N-N_k)} \\ \mathbf{0}_{(N-N_k)\times N_k} & \mathbf{0}_{(N-N_k)\times(N-N_k)} \end{bmatrix} \tag{III.1.13}$$

which induces the following partition on the transformed codeword vector $\bar{\mathbf{s}}_k$ corresponding to user k as

$$\bar{\mathbf{s}}_k = \begin{bmatrix} \bar{\mathbf{s}}_k^{(1)} \\ \bar{\mathbf{s}}_k^{(2)} \end{bmatrix} \tag{III.1.14}$$

where $\bar{\mathbf{s}}_k^{(1)}$ and $\bar{\mathbf{s}}_k^{(2)}$ are of dimension $N_k \times 1$ and $(N - N_k) \times 1$, respectively. Hence, we can rewrite the interference expression (III.1.11) as

$$i_k = \bar{\mathbf{s}}_k^{(1)^{\top}} \bar{\mathbf{D}}_k^{-2} \bar{\mathbf{s}}_k^{(1)} \tag{III.1.15}$$

The total interference power affecting all users' symbols is

$$\mathcal{I} = \sum_{k=1}^{K} i_k = \sum_{k=1}^{K} \bar{\mathbf{s}}_k^{(1)^{\top}} \bar{\mathbf{D}}_k^{-2} \bar{\mathbf{s}}_k^{(1)} \tag{III.1.16}$$

III.2 JOINT TRANSMITTER ADAPTATION AND POWER CONTROL AS A CONSTRAINED OPTIMIZATION PROBLEM

With the base station transmitter having knowledge of all the user codewords, transmit powers, and channel matrices, it may obtain an optimal ensemble of codewords and transmit powers that satisfy target SINR constraints by solving the constrained minimization of the sum of interference functions, that is

$$\min_{\left\{\begin{array}{l}\mathbf{s}_1,\ldots,\mathbf{s}_K\\ p_1,\ldots,p_K\end{array}\right\}} \mathcal{I} \text{ subject to} \begin{cases} \gamma_k = \gamma_k^* \\ \mathbf{s}_k^\top \mathbf{s}_k = 1 \end{cases} k = 1,\ldots,K \tag{III.2.1}$$

In order to solve this constrained optimization problem we use expression (III.1.16) for the sum of interference functions and note that the unit norm constraint on the original codeword vectors $\mathbf{s}_k^\top \mathbf{s}_k = 1$ can be translated in a unit norm constraint on $\bar{\mathbf{s}}_k^{(1)}$, that is $\bar{\mathbf{s}}_k^{(1)^\top} \bar{\mathbf{s}}_k^{(1)} = 1$, if the elements of $\bar{\mathbf{s}}_k^{(2)}$ are set to zero. This can be safely done as the $N - N_k$ dimensions of user k channel correspond to zero singular values and any transmitted signal energy will be wasted on those dimensions. Under this assumption we write the Lagrangian function for the constrained optimization problem (III.2.1) as

$$L = \mathcal{I} + \sum_{k=1}^{K} \lambda_k \left(\frac{p_k}{i_k} - \gamma_k^* \right) + \sum_{k=1}^{K} \xi_k (\bar{\mathbf{s}}_k^{(1)^\top} \bar{\mathbf{s}}_k^{(1)} - 1) \tag{III.2.2}$$

where λ_k and ξ_k are Lagrange multipliers associated with the constraints in equation (III.2.1). The necessary conditions for minimizing the Lagrangian (III.2.2) are obtained by differentiating with respect to the corresponding variables, $\bar{\mathbf{s}}_k^{(1)}$, p_k, and multipliers λ_k, ξ_k, $k = 1,\ldots,K$, and by equating the corresponding partial derivatives to zero. Differentiating with respect to $\bar{\mathbf{s}}_k^{(1)}$ leads to the eigenvalue/eigenvector equation

$$\frac{\partial L}{\partial \bar{\mathbf{s}}_k^{(1)}} = 0 \quad \Longrightarrow \quad \bar{\mathbf{D}}_k^{-2} \bar{\mathbf{s}}_k^{(1)} = \nu_k \bar{\mathbf{s}}_k^{(1)} \tag{III.2.3}$$

We note that, since $\bar{\mathbf{D}}_k$ is the diagonal matrix containing the non-zero singular matrix of $\tilde{\mathbf{H}}_k$ as implied by its SVD in (III.1.5) and (III.1.6), its eigenvectors are canonical vectors with one element equal to 1 in the position of the corresponding eigenvalue and all the other elements equal to 0, and that their corresponding eigenvalues ν_k are equal to the inverse squared of the singular values of of $\tilde{\mathbf{H}}_k$.

Differentiating now the Lagrangian (III.2.2) with respect to the multiplier λ_k we obtain a second necessary condition for the constrained optimization problem (III.2.1)

$$\frac{\partial L_k}{\partial \lambda_k} = \frac{p_k}{\bar{\mathbf{s}}_k^{(1)^T} \bar{\mathbf{D}}_k^{-2} \bar{\mathbf{s}}_k^{(1)}} - \gamma_k^* = 0 \tag{III.2.4}$$

The two equations (III.2.3) and (III.2.4) implied by the necessary conditions of the optimization problem (III.2.1) indicate that the following (necessary) conditions must be satisfied at the optimal point:

- Since $\bar{\mathbf{s}}_k^{(1)}$ is a canonical vector, $\bar{\mathbf{s}}_k$ will also be a similar canonical vector with $(N - N_k) \times 1$ zeros appended to it, which implies that user k codeword, calculated as $\mathbf{s}_k = \mathbf{V}_k \bar{\mathbf{s}}_k$, will correspond to a right singular vector of the channel matrix $\tilde{\mathbf{H}}_k$.

- For a given $\bar{\mathbf{s}}_k^{(1)}$, the power value p_k matches the specified target SINR γ_k^*, that is $p_k = \gamma_k^* \bar{\mathbf{s}}_k^{(1)^T} \bar{\mathbf{D}}_k^{-2} \bar{\mathbf{s}}_k^{(1)}$.

Among all the right singular vectors of the channel matrix $\tilde{\mathbf{H}}_k$, the meaningful choice for the codeword $\mathbf{s}_k$ is the right singular vector $\mathbf{x}_k$ corresponding to the maximum singular value σ_k^* which implies that the corresponding $\nu_k^* = \sigma_k^{*-2}$ minimizes the term i_k corresponding to user k in the sum interference function $\mathcal{I}$, since this will require minimum transmit power for matching user k target SINR, that is

$$p_k^*|_{\mathbf{s}_k = \mathbf{x}_k} = \gamma_k^* \sigma_k^{*-2} \quad k = 1, \ldots, K \tag{III.2.5}$$

Thus, a codeword matrix $\mathbf{S}$ whose columns $\mathbf{s}_k$ are right singular vectors of their corresponding channel matrices $\tilde{\mathbf{H}}_k$ along with the power matrix $\mathbf{P} = \mathrm{diag}[\gamma_1^* \sigma_1^{*-2}, \ldots, \gamma_K^* \sigma_K^{*-2}]$, satisfies the necessary conditions (III.2.3)-(III.2.4) and corresponds to a stationary point of the constrained optimization problem (III.2.1).

In order to investigate whether the $\tilde{\mathbf{S}}_1$ and $\mathbf{P}$ satisfying (III.2.3)-(III.2.4) are also optimal with respect to the constrained optimization of the cost function (III.2.1), we apply the approach in [41, Ch. 5] that checks the second order optimality conditions based on evaluation of the bordered Hessian matrix at the critical point. For the Lagrangian expression (III.2.2), the bordered Hessian matrix is expressed as [41, Ch. 5].

$$\mathbf{B}_h = \left[\begin{array}{c:c} \mathbf{0}_{(2K\times 2K)} & \mathbf{A}_{2K\times(NK+K)} \\ \hdashline \mathbf{A}^{\top}_{(NK+K)\times 2K} & \mathbf{C}_{(NK+K)\times(NK+K)} \end{array}\right] \tag{III.2.6}$$

where the matrices $\mathbf{A}$ and $\mathbf{C}$ are given below

$$\mathbf{A} = \left[\begin{array}{ccc:ccc} -2\mathbf{p}_1^* \bar{\mathbf{s}}_1^{(1)^{\top}} & \cdots & 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & -2\mathbf{p}_K^* \bar{\mathbf{s}}_K^{(1)^{\top}} & 0 & \cdots & 1 \\ \hline 2\bar{\mathbf{s}}_1^{(1)^{\top}} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 2\bar{\mathbf{s}}_K^{(1)^{\top}} & 0 & \cdots & 0 \end{array}\right], \tag{III.2.7}$$

$$\mathbf{C} = \left[\begin{array}{ccc:ccc} \mathbf{Q}_1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{Q}_K & 0 & \cdots & 0 \\ \hline 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 \end{array}\right], \tag{III.2.8}$$

and $\mathbf{Q}_k = 2\bar{\mathbf{D}}_k^{-2} - 2\nu_k^* \mathbf{I}_{N_k} \quad \forall k = 1, \cdots, K.$

At the optimal point of (III.2.1), the following conditions must be satisfied by the bordered Hessian matrix:

$$|\mathbf{B}_{h,\ell}| > 0; \quad \forall \ell = 4K+1, 4K+2, \cdots, NK+3K, \tag{III.2.9}$$

where $|\mathbf{B}_{h,\ell}|$ is the ℓ-th principal determinant of the bordered Hessian matrix $\mathbf{B}_h$. Thus, provided that (III.2.9) is verified, the matrices $\bar{\mathbf{S}}_1$ and $\mathbf{P}$, satisfying (III.2.3)-(III.2.4) are a solution of the constrained optimization problem (III.2.1).

III.3 JOINT TRANSMITTER ADAPTATION AND POWER CONTROL ALGORITHM

Based on the necessary conditions established in the previous section, we propose an iterative procedure that uses incremental updates for joint adaptation of the codeword and power matrices until an optimal solution is reached. At each step, one codeword vector is adapted in the direction of the right singular vector that corresponds to its maximum singular value, followed by an incremental update of its corresponding power toward the value that matches its corresponding target SINR γ_k^*. This procedure is formally stated in **Algorithm 4** and uses the following updates:

- At step j of the algorithm, column k of the codeword matrix $\mathbf{S}$ is updated to:

$$\mathbf{s}_k(j+1) = \frac{\mathbf{s}_k(j) + \alpha(j)\beta\mathbf{x}_k(j)}{\|\mathbf{s}_k(j) + \alpha(n)\beta\mathbf{x}_k(j)\|} \tag{III.3.1}$$

  where $\mathbf{x}_k(j)$ is the first right singular vector of $\tilde{\mathbf{H}}_k$ corresponding to its maximum singular value, β is a parameter that limits how far in terms of Euclidian distance the updated codeword can be from the old one and $\alpha(j) = \text{sgn}[\mathbf{s}_k^\top(j)\mathbf{x}_k(j)]$. This update results in a decrease of the interference function i_k and an increase in user k SINR.

- Following the codeword update, the power value for user k is updated to:

$$p_k(n+1) = (1-\mu)p_k(n) + \mu p'_k(n) \tag{III.3.2}$$

where

$$p'_k(n) = \gamma_k^* \mathbf{s}_k(n+1)^\top \mathbf{D}_k^\dagger (\mathbf{D}_k^\dagger)^\top \mathbf{s}_k(n+1), \tag{III.3.3}$$

and $0 < \mu < 1$ is a suitable constant that defines the size of the power increment. This is a "lagged" power update in which the new power value is obtained as a combination of the current power $p_k(n)$ and the power $p'_k(n)$ required to meet the specified target SINR after its corresponding effective interference function has been reduced by the incremental codeword update (III.3.1). We note that, the smaller the μ constant is, the more pronounced the lag in the power update and the smaller the incremental power change will be.

A similar iterative procedure but with different updates was used for joint transmitter adaptation and power control in CDMA systems in [42] and its convergence to a fixed point is established using a game-theoretic framework, where the fixed point corresponds to a Nash equilibrium. In the context of the constrained optimization problem (III.2.1), the fixed point of the proposed procedure corresponds to a stationary point where the necessary conditions (III.2.3) and (III.2.4) are satisfied, while the optimal Nash equilibrium is implied by the solution of the optimization problem (III.2.1), which satisfies also the additional bordered Hessian condition (III.2.9).

Numerically, a fixed point of **Algorithm 4** is reached when the codeword and power updates result in changes of the sum interference function $\mathcal{I}$ that are smaller than the specified tolerance ϵ. We note that, as it is the case with incremental/adaptive algorithms in general, the convergence speed of **Algorithm 4** depends

Algorithm 4 – Joint Transmitter Adaptation and Power Control

1: Input Data

- Codeword and power matrices $\mathbf{S}$, $\mathbf{P}$, downlink channel matrices $\mathbf{H}_k$, target SINR values γ_k^*, and noise covariance matrices $\mathbf{W}_k$, $k = 1, \ldots, K$.
- Constants β, μ, and tolerance ϵ.

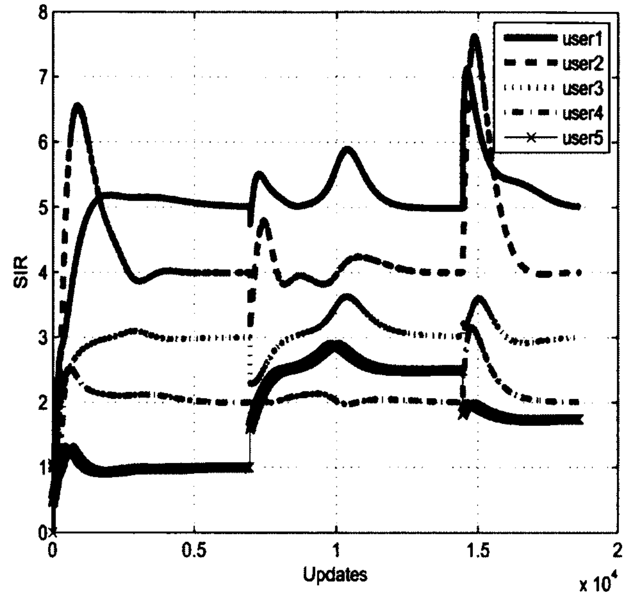
2: Initialize iteration counter $j = 0$.
3: **for** each user $k = 1, ..., K$ **do**
4:    Increment iteration counter $j = j + 1$
5:    Apply the whitening transformation as shown in equation (III.1.3) followed by the SVD in equation (III.1.5) to obtain the equivalent problem in equation (III.1.9).
6:    Update user k's codeword using equation (III.3.1).
7:    Update user k's power using equation (III.3.2).
8: **end for**
9: **if** change in sum interference function $\mathcal{I}$ is larger than specified tolerance ϵ **then**
10:    GO TO Step 8
11: **else**
12:    STOP: a fixed point has been reached.
13: **end if**
14: **if** optimality condition (III.2.9) is true **then**
15:    STOP: an optimal solution has been reached.
16:    OUTPUT $\mathbf{S}(j)$ and $\mathbf{P}(j)$.
17: **else**
18:    GO TO Step 8.
19: **end if**

on the values of the corresponding increments specified by the algorithm constants β and μ, as well as by the value of the tolerance ϵ.

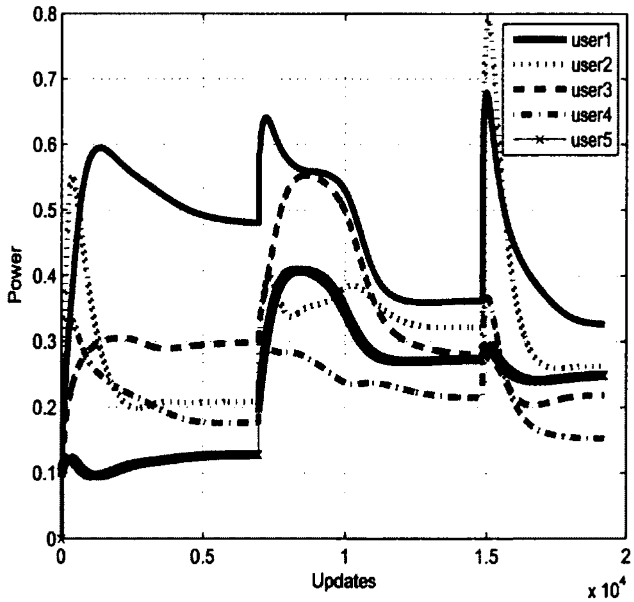
In order to illustrate the proposed algorithm we consider a downlink wireless system with $N = 10$ transmit antennas at the base station, $K = 5$ active users with $N_k = 4$ antennas each and white noise with covariance matrix $\mathbf{W}_k = 0.1\mathbf{I}_4$ at all receivers. The power matrix is initialized to $\mathbf{P} = 0.1\mathbf{I}_5$ while the codeword matrix $\mathbf{S}$ and the user channel matrices are initialized randomly. The algorithm parameters are set to $\beta = 0.02$, $\mu = 0.01$, tolerance $\epsilon = 0.02$ and the target SINRs are initialized to $\gamma^* = \{5, 4, 3, 2, 1\}$.

In the first experiment we simulate the algorithm for fixed number of active users with variable target SINRs. Once the codewords and powers that meet the specified targets are obtained using the proposed algorithm, user 5 increases its target SINR from 1 to 2.5. As a result of this change the algorithm starts updating user codewords and powers until a new fixed point that meets the new specified set of target SINRs is reached, when user 5 decreases target SINR to 1.75 and initiates new updates for codewords and power. The algorithm adjusts their values until a new fixed point is reached where the target SINRs are once again met for all users. The variation of user SINRs and powers for this experiment are plotted in Figure 4 from which we note that each time a change in the target SINR of user 5 occurs there is a sharp change in all the user's SINR and power values which is then compensated by the algorithm.

In the second experiment we start from the same initializations as before (including the same initial target SINRs), but after convergence to the fixed point where specified target SINRs are met, user 5 becomes inactive. Its corresponding codeword

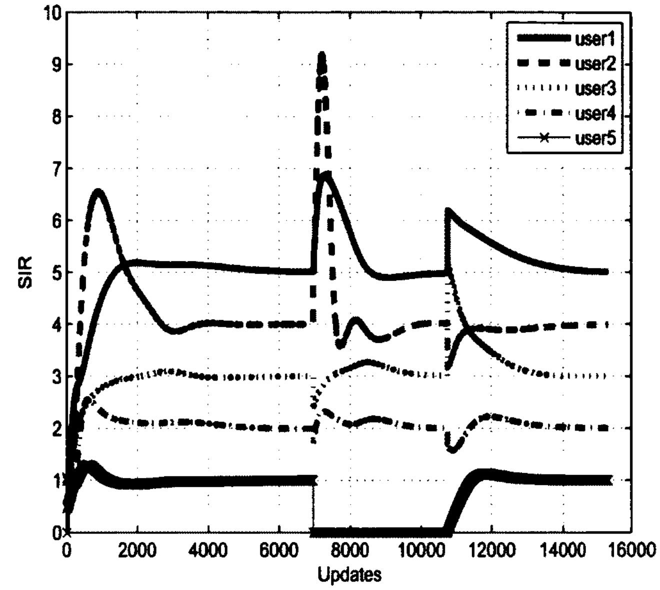


(a) SINR variation

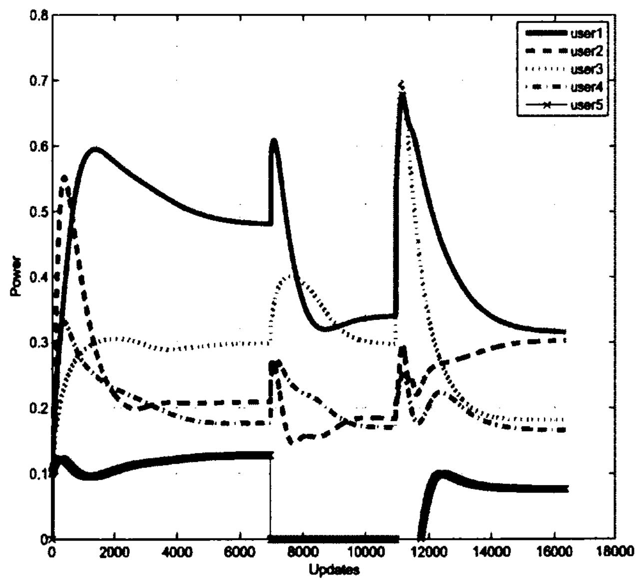


(b) Power variation

FIG. 4: Variation of user SINRs and powers for the variable SINR tracking example.

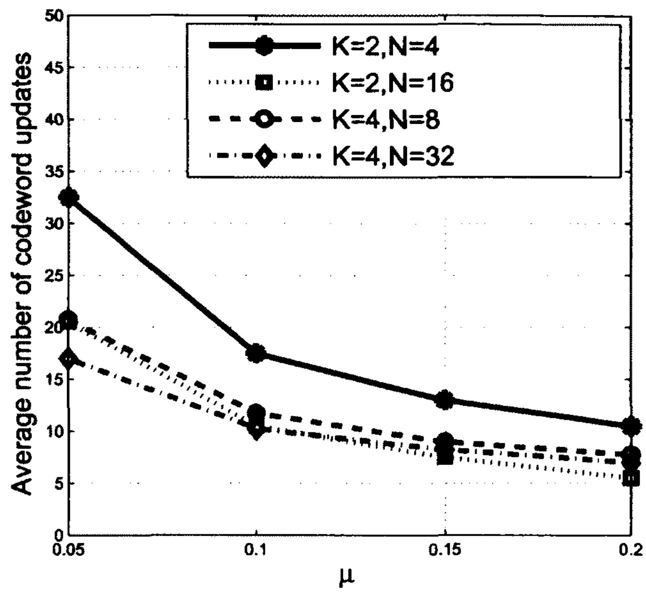


(a) SINR variation

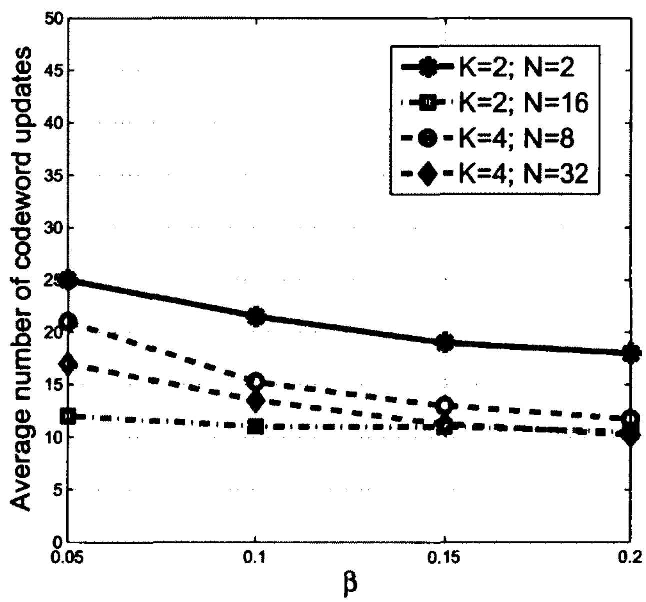


(b) Power variation

FIG. 5: Variation of user SINRs and powers for variable number of active users in the system.



(a) Varying μ for β=0.2



(b) Varying β for μ=0.1

FIG. 6: Dependence of convergence speed on the increment constants μ and β in 100 trials.

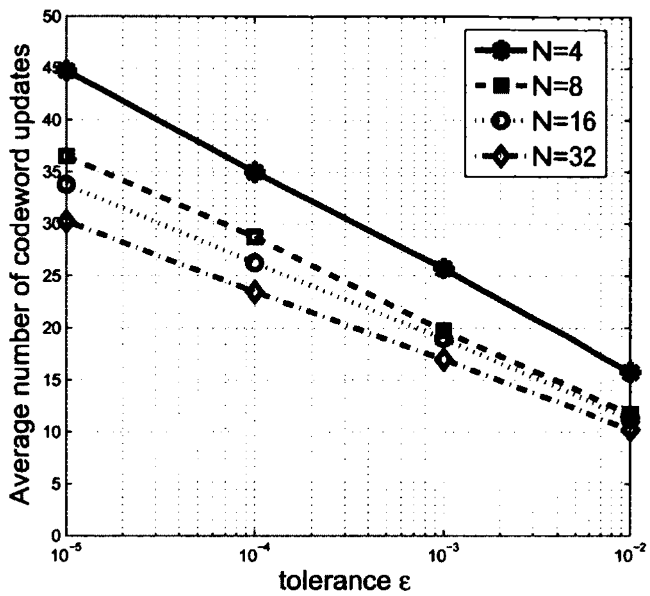


FIG. 7: Dependence of convergence speed on the tolerance ϵ in 100 trials.

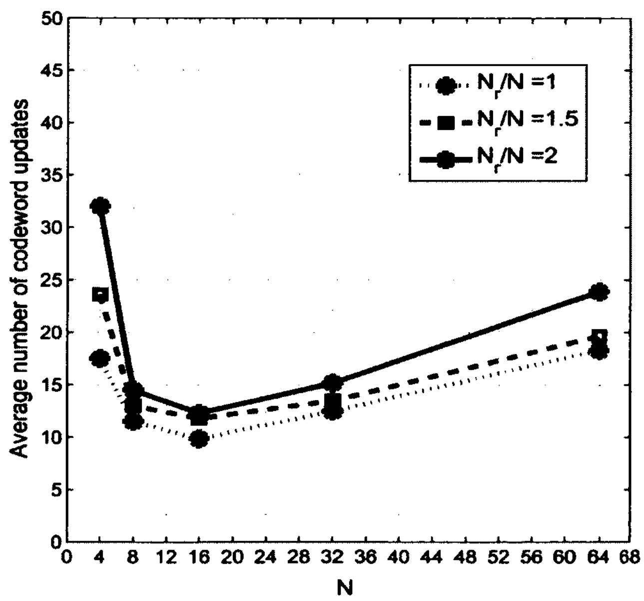


FIG. 8: Convergence speed for increasing number of antennas in 100 trials.

and power are dropped from **S** and **P** matrices which determines the algorithm to update the codewords and powers for remaining users until a new fixed point is reached where the target SINRs of active users $\gamma^* = \{5, 4, 3, 2\}$ are satisfied. Then, a new user becomes active in the system and its (randomly initialized) codeword and power are added to the **S** and **P** matrices under new user 5 with new target SINR equal to 0.5. This determines the algorithm to update again all codewords and powers until a new fixed point is reached where the target SINRs for all users are satisfied. The variation of user SINRs and powers for this experiment are plotted in Figure 5 from where we note that, similar to the previous experiment, each time a change in the number of active users in the system occurs there is a sharp change in all active user's SINR and power values which is compensated by the proposed algorithm.

In order to evaluate the convergence speed of the proposed algorithm for joint transmitter adaptation and power control we performed numerous simulations over various scenarios. In the simulation experiments performed, we selected specific values for K, N and N_k; for the algorithm increment constants β and μ, and for the tolerance ϵ, we generate a set of channel matrices, and for each selection we ran 100 trials of the algorithm recording the number of iterations j needed for convergence within the specified tolerance ϵ when starting with randomly initialized codeword and power matrices. We note that the actual number of iterations depends on K as one update of the codeword and power matrices **S** and **P** consist of K iterations of the **FOR** loop: 3 in **Algorithm 4**. Thus, for better visualization of the convergence speed results we will look at the average number of updates of the codeword and power matrices until convergence rather than the average number of actual iterations j; that is, we will show the number of codeword and power matrices updates j/K in our plots.

In the third experiment we studied the convergence speed for different values of the increment constants β and μ and varying K and N. The results of this experiment are plotted in Fig. 6, where we note that convergence speed is affected mostly by changes in the value of μ and that it does not depend significantly on the value of β. We also note that for algorithm constants $\beta = 0.2$ and $\mu = 0.1$, the optimal equilibrium is reached in an average number of fewer than 25 updates in all considered scenarios.

In the fourth experiment we studied the dependence of convergence speed on the algorithm tolerance ϵ for fixed values for K, N, N_k, μ and β as shown in Fig. 7. It can be observed that the lower the value of ϵ, the longer it takes for **Algorithm 4** to reach the optimal fixed point. This result was expected considering the fact that a smaller ϵ implies a higher precision for the fixed-point, which, with fixed increments will be reached through an increased number of updates.

In the fifth experiment we studied the convergence speed of the algorithm for increasing number of antennas in the system $N_r = \sum_{k=1}^{K} N_k$ and N such that the ratio N_r/N stays constant which is shown in Fig. 8, from which we note that the average number of updates needed for convergence does not significantly change for the ratio $N_r/N = 1$; however for higher values of the ratio, we notice an increase in the average number of codeword updates with increasing N.

III.4 SINR AND MMSE EXPRESSIONS WITH LINEAR PRECODERS IN THE DOWNLINK SYSTEM MODEL

We consider the downlink of a wireless system with a system model similar to that in section I.2 but with a single transmitter and K receivers for which the N-dimensional signal vector from the base station transmitter is expressed as the sum of individual transmitted signals in equation (I.2.1) and is given by

$$\mathbf{x} = \sum_{k=1}^{K} \sum_{m=1}^{M_k} b_m^{(k)} \mathbf{s}_m^{(k)} \sqrt{p_m^{(k)}} = \sum_{k=1}^{K} \mathbf{S}_k \mathbf{P}_k^{1/2} \mathbf{b}_k, \tag{III.4.1}$$

and the N-dimensional received signal by a given user k is

$$\mathbf{r}_k = \mathbf{H}_k \mathbf{x} + \mathbf{n}_k = \mathbf{H}_k \left(\sum_{\ell=1}^{K} \mathbf{S}_\ell \mathbf{P}_\ell^{1/2} \mathbf{b}_\ell \right) + \mathbf{n}_k. \tag{III.4.2}$$

We note that each user k has a precoder matrix $\mathbf{S}_k$ when compared to the system model in Section III.1 where each user has a codeword vector $\mathbf{s}_k$. In order to decode the information symbols intended for it a given user k employs a bank of MMSE receiver filters such that its corresponding vector of decision variables $\mathbf{d}_k = [d_1^{(k)} \dots d_{M_k}^{(k)}]^\top$ is

$$\begin{aligned} \mathbf{d}_k &= \mathbf{C}_k^\top \mathbf{r}_k \\ &= \underbrace{(\mathbf{R}_k^{-1} \mathbf{H}_k \mathbf{S}_k \mathbf{P}_k^{1/2})^\top}_{\text{MMSE receiver [43]}} \times \mathbf{r}_k , \quad k = 1, \dots, K \end{aligned} \tag{III.4.3}$$

where

$$\mathbf{R}_k = E[\mathbf{r}_k \mathbf{r}_k^\top] = \mathbf{H}_k \left(\sum_{\ell=1}^{K} \mathbf{S}_\ell \mathbf{P}_\ell \mathbf{S}_\ell^\top \right) \mathbf{H}_k^\top + \mathbf{W}_k \tag{III.4.4}$$

is the correlation matrix of the received signal in (III.4.2). We note that the MMSE receiver corresponding to an individual symbol m of user k, $b_m^{(k)}$, is $\mathbf{c}_m^{(k)} =$

$\mathbf{R}_k^{-1}\mathbf{H}_k\mathbf{s}_m^{(k)}\sqrt{p_m^{(k)}}$ and can also be written in the following form

$$\mathbf{c}_m^{(k)} = [\mathbf{R}_m^{(k)}]^{-1}\mathbf{H}_k\mathbf{s}_m^{(k)} \frac{\sqrt{p_m^{(k)}}}{1 + p_m^{(k)}\mathbf{s}_m^{(k)\top}\mathbf{H}_k^\top[\mathbf{R}_m^{(k)}]^{-1}\mathbf{H}_k\mathbf{s}_m^{(k)}} \tag{III.4.5}$$

where $\mathbf{R}_m^{(k)}$ is the correlation matrix of the interference+noise which affects m-th decision variable of user k

$$\mathbf{R}_m^{(k)} = \mathbf{R}_k - p_m^{(k)}\mathbf{H}_k\mathbf{s}_m^{(k)}\mathbf{s}_m^{(k)\top}\mathbf{H}_k^\top. \tag{III.4.6}$$

The SINR for symbol m of user k is

$$\begin{aligned} \gamma_m^{(k)} &= \frac{p_m^{(k)}\mathbf{s}_m^{(k)\top}\mathbf{H}_k^\top\mathbf{R}_k^{-1}\mathbf{H}_k\mathbf{s}_m^{(k)}}{1 - p_m^{(k)}\mathbf{s}_m^{(k)\top}\mathbf{H}_k^\top\mathbf{R}_k^{-1}\mathbf{H}_k\mathbf{s}_m^{(k)}} \\ &= p_m^{(k)}\mathbf{s}_m^{(k)\top}\mathbf{H}_k^\top[\mathbf{R}_m^{(k)}]^{-1}\mathbf{H}_k\mathbf{s}_m^{(k)}, \end{aligned} \tag{III.4.7}$$

and the corresponding MMSE expression is

$$\begin{aligned} \varepsilon_m^{(k)} = \frac{1}{1+\gamma_m^{(k)}} &= 1 - p_m^{(k)}\mathbf{s}_m^{(k)\top}\mathbf{H}_k^\top\mathbf{R}_k^{-1}\mathbf{H}_k\mathbf{s}_m^{(k)} \\ &= \frac{1}{1 + p_m^{(k)}\mathbf{s}_m^{(k)\top}\mathbf{H}_k^\top[\mathbf{R}_m^{(k)}]^{-1}\mathbf{H}_k\mathbf{s}_m^{(k)}}. \end{aligned} \tag{III.4.8}$$

Our goal in this setup is to formulate an algorithm for joint adaptation of the precoder and power matrices of all users such that the SINR value for all symbols of a given user k is equal to a specified target SINR γ_k^*, for all $k = 1, \ldots, K$.

III.5 JOINT PRECODER ADAPTATION AND POWER CONTROL AS A CONSTRAINED OPTIMIZATION PROBLEM

In order to formulate joint precoder adaptation and power control as a constrained optimization problem we must choose a suitable criterion (or cost function) to be optimized when the downlink transmitter updates the precoder and power matrices.

In the context of the MMSE receivers we choose this cost function to be the sum of MMSE errors corresponding to all symbols of all users; that is,

$$\mathcal{U} = \sum_{k=1}^{K} \sum_{m=1}^{M_k} \varepsilon_m^{(k)}. \tag{III.5.1}$$

Using the MMSE expression (III.4.8) this can be rewritten as

$$\mathcal{U} = \sum_{k=1}^{K} \sum_{m=1}^{M_k} \frac{1}{1 + p_m^{(k)} \mathbf{s}_m^{(k)\top} \mathbf{H}_k^\top [\mathbf{R}_m^{(k)}]^{-1} \mathbf{H}_k \mathbf{s}_m^{(k)}}, \tag{III.5.2}$$

and we note that, while the transmitter does not have access to the actual MMSE errors which are computed at the user receivers, it does have knowledge of all the user precoder and transmit power matrices $\mathbf{S}_k$, $\mathbf{P}_k$, $\forall k$. In addition, we assume that the transmitter has knowledge of the downlink channels $\mathbf{H}_k$ and of the received signal correlation matrices $\mathbf{R}_k$ through a feedback channel [44,45] so that it is able to easily determine the cost function using expression (III.5.2). Thus, we formally define the joint precoder adaptation and power control in terms of the following constrained optimization problem:

$$\min_{\mathbf{S}_k, \mathbf{P}_k} \mathcal{U} \text{ subject to } \begin{cases} p_m^{(k)} \mathbf{s}_m^{(k)\top} \mathbf{H}_k^\top \mathbf{R}_m^{(k)-1} \mathbf{H}_k \mathbf{s}_m^{(k)} = \gamma_k^*, \\ \mathbf{s}_m^{(k)\top} \mathbf{s}_m^{(k)} = 1, \\ m = 1, \ldots, M_k, \ k = 1, \ldots, K \\ \sum_{k=1}^{K} \text{Trace}\left[\mathbf{P}_k\right] \leq P_{\max}. \end{cases} \tag{III.5.3}$$

We note that formal constraints in (III.5.3) are implied by the target SINR values γ_k^* specified for all users in the system, as well as by the unit norm constraints imposed on columns of the precoder matrices and the average power constraint of the downlink transmitter.

In order to solve the constrained optimization problem (III.5.3) we use the Lagrange multipliers method [41, Ch. 5] and define the corresponding Lagrangian function

$$
\begin{aligned}
L = \mathcal{U} &+ \sum_{k=1}^{K}\sum_{m=1}^{M_k} \zeta_m^{(k)} \left(p_m^{(k)} \mathbf{s}_m^{(k)\top} \mathbf{H}_k^\top [\mathbf{R}_m^{(k)}]^{-1} \mathbf{H}_k \mathbf{s}_m^{(k)} - \gamma_k^*\right) \\
&+ \sum_{k=1}^{K}\sum_{m=1}^{M_k} \xi_m^{(k)} (\mathbf{s}_m^{(k)\top}\mathbf{s}_m^{(k)} - 1) + \rho\left(P_{\max} - \sum_{k=1}^{K}\sum_{m=1}^{M_k} p_m^{(k)}\right).
\end{aligned}
\tag{III.5.4}
$$

We note that L is a function of the M_k columns of user k precoder matrix $\mathbf{s}_1^{(k)}, \ldots, \mathbf{s}_{M_k}^{(k)}$, powers $p_1^{(k)}, \ldots, p_{M_k}^{(k)}$, and multipliers $\zeta_1^{(k)}, \ldots, \zeta_{M_k}^{(k)}$, $\xi_1^{(k)}, \ldots, \xi_{M_k}^{(k)}$, and ρ associated with the constraints in equation (III.5.3). The necessary conditions for minimizing the Lagrangian are as follows:

$$\frac{\partial L}{\partial \mathbf{s}_m^{(k)}} = 0,\ k = 1, \ldots, K; m = 1, \ldots, M_k \tag{III.5.5}$$

$$\frac{\partial L}{\partial p_m^{(k)}} = 0,\ k = 1, \ldots, K; m = 1, \ldots, M_k \tag{III.5.6}$$

$$\frac{\partial L}{\partial \zeta_m^{(k)}} = 0,\ k = 1, \ldots, K; m = 1, \ldots, M_k \tag{III.5.7}$$

$$\frac{\partial L}{\partial \xi_m^{(k)}} = 0,\ k = 1, \ldots, K; m = 1, \ldots, M_k \tag{III.5.8}$$

$$\frac{\partial L}{\partial \rho} \geq 0. \tag{III.5.9}$$

To these one must add the complementary slackness condition

$$\rho\left(P_{\max} - \sum_{k=1}^{K}\sum_{m=1}^{M_k} p_m^{(k)}\right) = 0 \tag{III.5.10}$$

which is needed in the case of inequality constraints to ensure that, at the optimum point, the Lagrangian and the original cost function have the same value.

Condition (III.5.5) leads to the eigenvalue/eigenvector equation

$$\frac{\partial L}{\partial \mathbf{s}_m^{(k)}} = 0 \implies \mathbf{H}_k^\top [\mathbf{R}_m^{(k)}]^{-1} \mathbf{H}_k \mathbf{s}_m^{(k)} = \lambda_m^{(k)} \mathbf{s}_m^{(k)} \qquad k = 1, \ldots, K; m = 1, \ldots, M_k \tag{III.5.11}$$

where the corresponding eigenvalue $\lambda_m^{(k)}$ can be written in terms of the Lagrange multipliers $\zeta_m^{(k)}$, $\xi_m^{(k)}$, the user power $p_m^{(k)}$, and the corresponding MMSE $\varepsilon_m^{(k)}$. Furthermore, condition (III.5.7) implies

$$\frac{\partial L}{\partial \zeta_m^{(k)}} = 0 \implies p_m^{(k)} = \frac{\gamma_k^*}{\mathbf{s}_m^{(k)\top} \mathbf{H}_k^\top [\mathbf{R}_m^{(k)}]^{-1} \mathbf{H}_k \mathbf{s}_m^{(k)}} \qquad k = 1, \ldots, K; m = 1, \ldots, M_k. \tag{III.5.12}$$

The two equations (III.5.11) and (III.5.12) implied by the necessary conditions of the constrained optimization problem (III.5.3) indicate that, at the optimal point, user k precoder and power matrices satisfy the following (necessary) conditions:

- Any column m of user k precoder matrix, $\mathbf{s}_m^{(k)}$, is an eigenvector of corresponding matrix $\mathbf{H}_k^\top [\mathbf{R}_m^{(k)}]^{-1} \mathbf{H}_k$.
- For the given $\mathbf{s}_m^{(k)}$ the power value corresponding to symbol m of user k, $p_m^{(k)}$, matches the specified target SINR γ_k^*.

We note that, among all possible eigenvectors of matrix $\mathbf{H}_k^\top [\mathbf{R}_m^{(k)}]^{-1} \mathbf{H}_k$ the meaningful choice for $\mathbf{s}_m^{(k)}$ is the eigenvector $\mathbf{x}_m^{(k)}$ corresponding to its maximum eigenvalue[1] $\bar{\lambda}_m^{(k)}$ since, according to the SIR expression (III.4.7), this will require minimum transmit power for matching the specified target SINR

$$\bar{p}_m^{(k)}|_{\mathbf{s}_m^{(k)} = \mathbf{x}_m^{(k)}} = \frac{\gamma_k^*}{\bar{\lambda}_m^{(k)}}. \tag{III.5.13}$$

[1]This is also referred to it as the maximum eigenvector.

Furthermore, using equation (III.4.6) along with the SIR expression in terms of $\mathbf{R}_k^{-1}$ in equation (III.4.7), we note that, with this choice of power, the maximum eigenvector $\mathbf{x}_m^{(k)}$ of $\mathbf{H}_k^\top[\mathbf{R}_m^{(k)}]^{-1}\mathbf{H}_k$ matrix is also an eigenvector of the $\mathbf{H}_k^\top\mathbf{R}_k^{-1}\mathbf{H}_k$ matrix, satisfying

$$\mathbf{H}_k^\top\mathbf{R}_k^{-1}\mathbf{H}_k\mathbf{x}_m^{(k)} = \frac{1}{\bar{p}_m^{(k)}}\frac{\gamma_k^*}{1+\gamma_k^*}\mathbf{x}_m^{(k)}, \tag{III.5.14}$$

$$m = 1, \ldots, M_k$$

Since $\bar{p}_m^{(k)}$ is the minimum transmit power required to match the specified target $\forall m$, we have that $\mathbf{x}_m^{(k)}$ is the eigenvector corresponding to the maximum eigenvalue $\bar{\nu}_k$ of matrix $\mathbf{H}_k^\top\mathbf{R}_k^{-1}\mathbf{H}_k$, $m = 1, \ldots, M_k$, which in turn implies that the power values for all symbols of user k will be identical at the optimal point

$$\bar{p}_m^{(k)} = \frac{1}{\bar{\nu}_k}\frac{\gamma^*}{1+\gamma^*}, \quad m = 1, \ldots, M_k. \tag{III.5.15}$$

Thus, an ensemble of precoder and power matrices where all columns of a given user precoder $\mathbf{s}_m^{(k)}$ are maximum eigenvectors of their corresponding matrices $\mathbf{H}_k^\top\mathbf{R}_k^{-1}\mathbf{H}_k$, $m = 1, \ldots, M_k$ and user k power matrix is a scaled identity matrix

$$\mathbf{P}_k = \frac{1}{\bar{\nu}_k}\frac{\gamma_k^*}{1+\gamma_k^*}\mathbf{I}_{M_k} \tag{III.5.16}$$

with $\bar{\nu}_k$ being the maximum eigenvalue of matrix $\mathbf{H}_k^\top\mathbf{R}_k^{-1}\mathbf{H}_k$, satisfies the necessary conditions (III.5.11) and (III.5.12) and is a stationary point of the constrained optimization problem (III.5.3). Since at this point the SINRs for all M_k symbols of a user k are equal, this implies that all their corresponding MMSEs are also equal and given by

$$\bar{\varepsilon}_m^{(k)} = \frac{1}{1+\gamma_k^*}, \quad m = 1, \ldots, M_k \tag{III.5.17}$$

which in turn implies that the value of the cost function at this point is

$$\bar{\mathcal{U}} = \sum_{k=1}^{K} \frac{M_k}{1 + \gamma_k^*}. \tag{III.5.18}$$

Due to the one-to-one relationship (III.4.8) between SINR and MMSE values when MMSE receivers are used [43] we note that the value $\bar{\mathcal{U}}$ in (III.5.18) is also the minimum value of the cost function $\mathcal{U}$ given the target SINR values γ_k^*. This indicates that this stationary point is in fact the optimal point of the constrained optimization problem (III.5.3) where the cost function is minimized subject to the specified constraints and can also be verified using the second order optimality conditions based on the bordered Hessian matrix similar to the uplink wireless system in Section IV.2.

III.6 JOINT PRECODER ADAPTATION AND POWER CONTROL ALGORITHM

Necessary conditions (III.5.11) and (III.5.12) suggest potential updates for user k precoder and power matrices: at some given instant replace column m of user k precoder, $\mathbf{s}_m^{(k)}$, by the eigenvector $\mathbf{x}_m^{(k)}$ corresponding to maximum eigenvalue $\bar{\lambda}_m^{(k)}$ of matrix $\mathbf{H}_k^\top \mathbf{R}_m^{(k)-1} \mathbf{H}_k$ followed by updating the corresponding power to the new value impled by (III.5.12). These potential updates may result in steep changes in the transmitter that may not be tracked by the receiver and could result in increased probability of error or even connection loss between the base station transmitter and user k receiver (outage); in order to avoid that we use incremental updates where $\mathbf{s}_m^{(k)}$ is updated in the direction of the desired maximum eigenvector $\mathbf{x}_m^{(k)}$ by the equation

$$\mathbf{s}_m^{(k)}(n+1) = \frac{\mathbf{s}_m^{(k)}(n) + \alpha(n)\beta \mathbf{x}_m^{(k)}(n)}{||\mathbf{s}_m^{(k)}(n) + \alpha\beta \mathbf{x}_m^{(k)}(n)||} \tag{III.6.1}$$

where $\alpha(n) = \mathrm{sgn}[\mathbf{s}_m^{(k)\top}(n) \cdot \mathbf{x}_m^{(k)}(n)]$, and β is a parameter that defines the increment size by limiting how far, in terms of Euclidian distance, the updated precoder column is from the old one. Upon adaptation of $\mathbf{s}_m^{(k)}$ the corresponding SINR (III.4.7) is increased, and the required power value according to (III.5.12) becomes

$$p_m^{(k)'}(n) = \frac{\gamma_k^*}{\mathbf{s}_m^{(k)\top}(n+1)\mathbf{H}_k^\top[\mathbf{R}_m^{(k)}(n)]^{-1}\mathbf{H}_k\mathbf{s}_m^{(k)}(n+1)}. \tag{III.6.2}$$

Since the value $p_m^{(k)'}(n)$ may not be close to the current power value $p_m^{(k)}(n)$ and in order to avoid abrupt power variations we apply a "lagged" power update given by

$$p_m^{(k)}(n+1) = p_m^{(k)}(n) + \mu[p_m^{(k)'}(n) - p_m^{(k)}(n)] \tag{III.6.3}$$

with $0 < \mu < 1$ a suitably chosen constant defining the size of the power increment. This updates the power value corresponding to symbol m of user k to a new value which is a combination of the current power $p_m^{(k)}(n)$ and the power $p_m^{(k)'}(n)$ required to meet the specified constraint after the incremental codeword update (III.6.1). We note that the smaller the μ constant, the smaller the incremental power change, and the power will always be updated toward the value needed to match the target SINR (that is, if the power value $p_m^{(k)'}(n)$ needed to match the target SINR is lower than the current power then the power will be decreased and vice versa).

We note that, empirically, we have observed that the incremental updates implied by equations (III.6.1)–(III.6.3) result in a monotonic decrease of the cost function $\mathcal{U}$, that is

$$\mathcal{U}(n) \geq \mathcal{U}(n+1). \tag{III.6.4}$$

This observation in conjunction with the fact that

$$\mathcal{U}(n) \geq \bar{\mathcal{U}} \quad \forall n, \tag{III.6.5}$$

Algorithm 5 – Joint Precoder Adaptation and Power Control Algorithm for Multi-User Downlink Scenario

1: Input Data

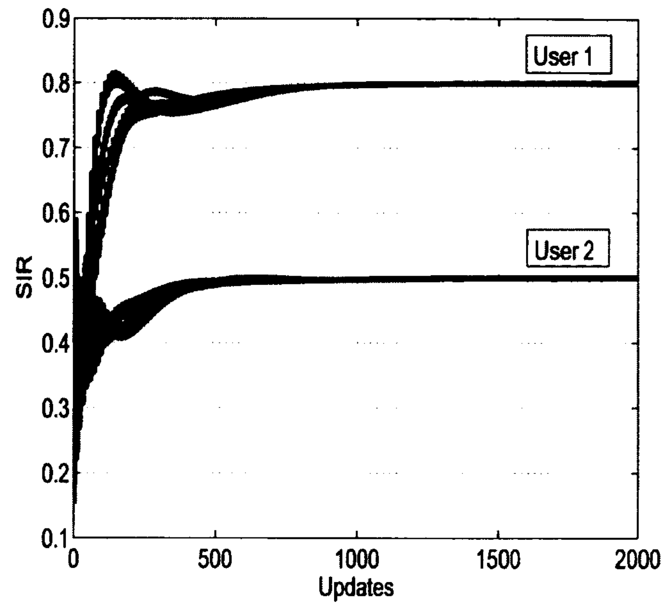
- Precoder and power matrices $\mathbf{S}_k$, $\mathbf{P}_k$, downlink channel matrices $\mathbf{H}_k$, target SINR values γ_k^*, and noise covariance matrices $\mathbf{W}_k$, $k = 1, \ldots, K$.
- Constants β, μ, and tolerance ϵ.

2: **for** each user precoder matrix $k = 1, ..., K$ **do**
3:     **for** each column $m = 1, \ldots, M_k$ of user k precoder matrix **do**
4:         Compute corresponding $\mathbf{H}_k^\top[\mathbf{R}_m^{(k)}(n)]^{-1}\mathbf{H}_k$ using equation (III.4.6) and determine its maximum eigenvector $\mathbf{x}_m^{(k)}$.
5:         Update column m of user k precoder using equation (III.6.1).
6:         Update transmit power for symbol m of user k using equation (III.6.3).
7:     **end for**
8: **end for**
9: **if** change in the cost function $\mathcal{U}$ is larger than specified tolerance ϵ **then**
10:     GO TO Step 2
11: **else**
12:     STOP: a fixed point has been reached.
13: **end if**
14: **if** the cost function $\mathcal{U}$ is larger than the lower bound in (III.6.5) **then**
15:     GO TO Step (2) for further iterations
16: **else**
17:     STOP:an optimal point has been reached.
18: **end if**

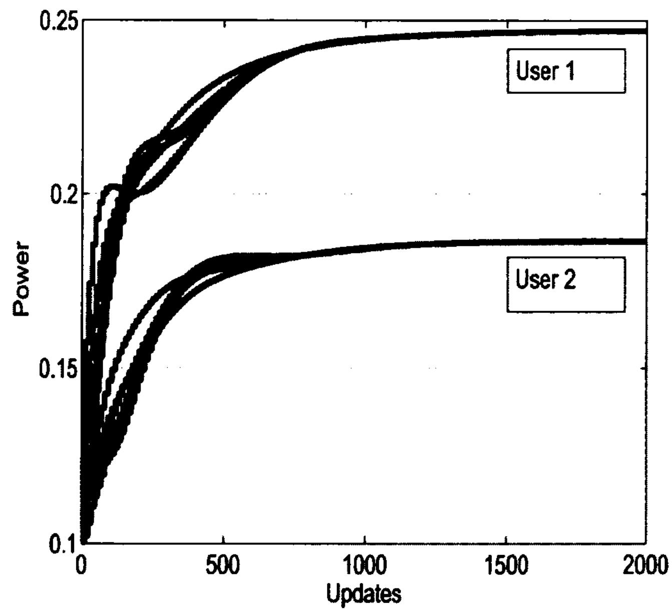
implies that iterative application of the updates (III.6.1)–(III.6.3) is guaranteed to converge to a fixed point as formally stated in **Algorithm 5**

Extensive simulations have confirmed that it always converges to the optimal point where the cost function is equal to the lower bound in equation (III.5.18).

We present results obtained from simulations of **Algorithm 5** for a system with $K = 2$ active users in a signal space of dimension $N = 6$ and blocks lengths $M_1 = M_2 = 6$. The precoder and power matrices, $\mathbf{S}_k$ and $\mathbf{P}_k$ ($\forall k = 1, 2$), are initialized randomly, and white noise with the same covariance matrix $\mathbf{W}_1 = \mathbf{W}_2 = 0.1\mathbf{I}_6$ is



(a) SINR variation



(b) Power variation

FIG. 9: Variation of symbol SINRs and powers for one run of the proposed algorithm from random initialization.

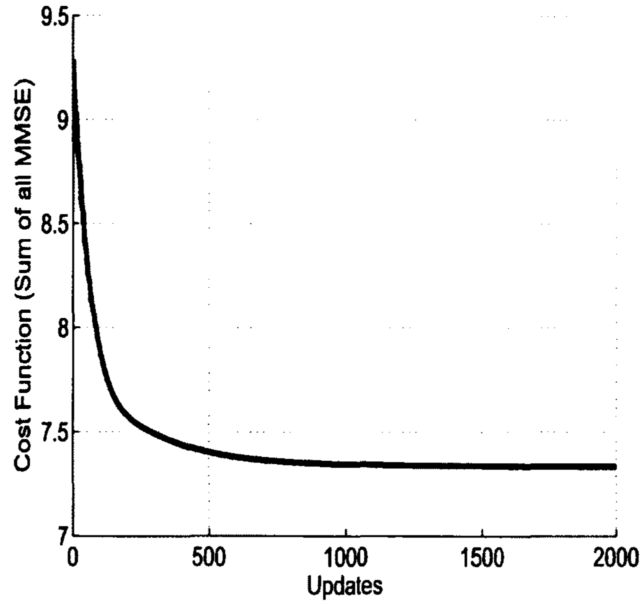
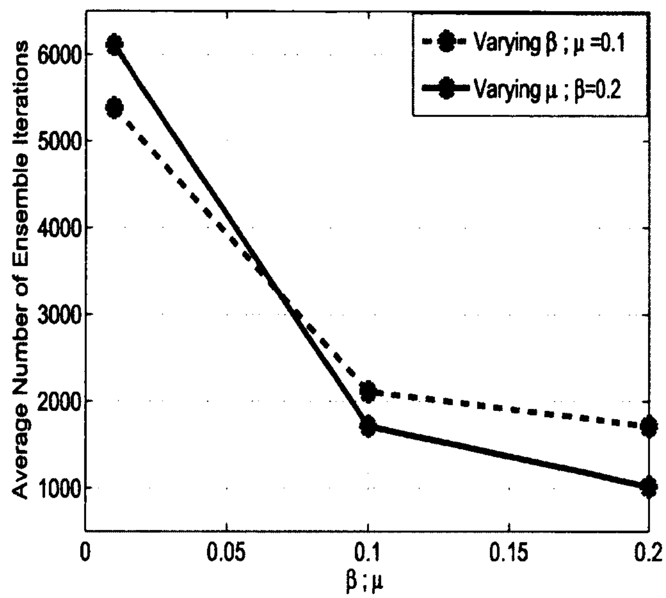


FIG. 10: Variation of transmitter cost function for one run of the proposed algorithm from random initialization.

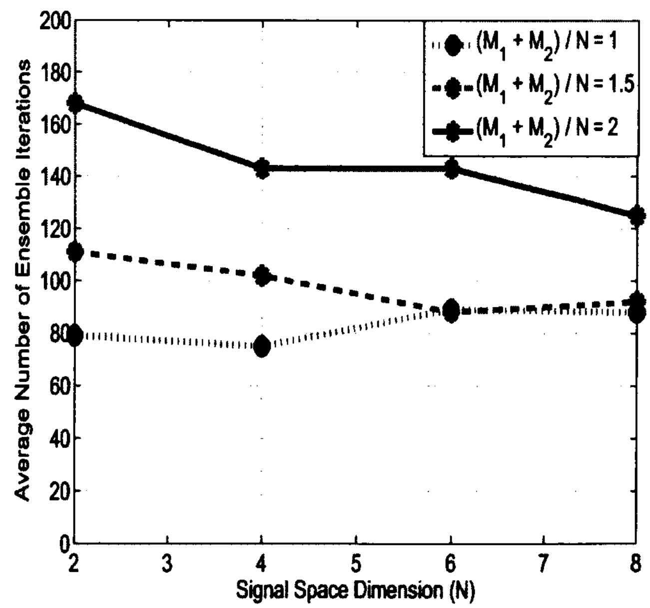
considered. The algorithm parameters are set to $\mu = 0.1$, $\beta = 0.2$, and $\epsilon = 0.001$, and the user target SINRs are initialized to $\gamma_1^* = 0.8$ and $\gamma_2^* = 0.5$. The channel matrices are chosen to be circulant and are also initialized randomly.

The variation of the user SINRs and powers for this example are plotted in Figures 9(a) and (b), and the variation of the cost function for this example is plotted in Figure 10. We note the monotonic decrease of the cost function to its lower bound $\bar{\mathcal{U}}$ which in this case is found to be equal to 7.33.

Convergence of the proposed algorithm for several values of the algorithm constants β and μ are plotted in Figure 11(a), and it can be observed that the β and μ variations closely follow each other. Convergence for varying signal dimensions N and block lengths M_1 and M_2 are plotted in Figure 11(b), and it is seen that



(a) Varying β for $\mu = 0.1$ and μ for $\beta = 0.2$



(b) Varying $M_1 + M_2/N$ for $\beta = 0.2$ and $\mu = 0.1$

FIG. 11: Average number of iterations for convergence of the proposed algorithm in 100 trials.

the average number of ensemble iterations needed for convergence within tolerance $\epsilon = 0.001$ does not vary significantly for increasing N and $M_1 + M_2$.

III.7 CHAPTER SUMMARY

In this chapter we studied the problem of joint transmitter adaptation and power control for downlink wireless systems with multiple users that operate with specified target SINR values. This problem is cast as a constrained optimization problem for which necessary and sufficient conditions for the optimal solution are identified. A new algorithm for joint transmitter adaptation and power control in downlink wireless systems is also presented in the paper. The proposed algorithm uses incremental updates for the codeword and power matrices which result in optimal values for these matrices for which the specified target SINR values are achieved with minimum transmitted power, and it can be used for tracking changing target SINRs and/or variable number of active users. Convergence of the proposed algorithm is studied numerically through simulations over various scenarios.

We also investigated joint precoder adaptation and power control in downlink wireless systems with target values imposed on the SINRs at the receiver terminals. Using a general framework based on block transmissions and linear precoders, we formulated this as a constrained optimization problem and discussed the conditions that must be satisfied by the optimal solution. We also proposed an incremental algorithm for joint updates of the transmit precoder and power matrices which we illustrated with numerical results obtained from simulations.

Chapter IV

JOINT TRANSMITTER ADAPTATION AND POWER CONTROL IN UPLINK WIRELESS SYSTEMS

In this chapter we study transmitter adaptation in multi-user wireless systems in a general framework which assumes that block transmission is used for sending information by active transmitters and is applicable to most current transmission schemes including CDMA, OFDM, and MIMO systems. Specifically, we formulate the problem of joint transmitter adaptation and power control subject to target SINR requirements as a constrained optimization problem in which the transmit precoder and power matrices are jointly optimized subject to specific mathematical constraints implied by constraints on the precoder matrices and by the desired target SINRs. We also present an incremental algorithm that jointly adapts transmitter precoders and power values in a distributed fashion until a fixed point is reached where the specified target SINRs are achieved with minimum transmitted power. Convergence of the algorithm is illustrated using simulation results followed by an application of the proposed algorithm in Cognitive Radio Networks.

IV.1 UPLINK SYSTEM MODEL

We consider the uplink of a wireless system with a system model as defined in Section I.2 with K active transmitters and a single base station receiver such that the N-dimensional vector transmitted by a given user k is given by equation (I.2.1) and the N-dimensional received signal vector at the receiver corresponding to one signalling interval is given by the expression

$$\mathbf{r} = \sum_{k=1}^{K} \mathbf{H}_k \mathbf{S}_k \mathbf{P}_k^{1/2} \mathbf{b}_k + \mathbf{n}. \tag{IV.1.1}$$

We rewrite the received signal in (IV.1.1) from the perspective of the m-th symbol of user k as

$$\mathbf{r} = b_m^{(k)} \mathbf{H}_k \mathbf{s}_m^{(k)} \sqrt{p_m^{(k)}} + \underbrace{\sum_{n=1, n\neq m}^{M_k} b_n^{(k)} \mathbf{H}_k \mathbf{s}_n^{(k)} \sqrt{p_n^{(k)}} + \sum_{\ell=1, \ell\neq k}^{K} \mathbf{H}_\ell \mathbf{S}_\ell \mathbf{P}_\ell^{1/2} \mathbf{b}_\ell + \mathbf{n}}_{\text{interference } + \text{ noise } = \ \mathbf{i}_m^{(k)}}. \tag{IV.1.2}$$

The first term is the desired signal corresponding to symbol m of user k, and the remaining terms represent the interference and noise corrupting it at the receiver with correlation matrix

$$\mathbf{R}_m^{(k)} = E[\mathbf{i}_m^{(k)} \mathbf{i}_m^{(k)\top}] = \mathbf{R} - p_m^{(k)} \mathbf{H}_k \mathbf{s}_m^{(k)} \mathbf{s}_m^{(k)\top} \mathbf{H}_k^\top \tag{IV.1.3}$$

where

$$\mathbf{R} = E[\mathbf{r}\mathbf{r}^\top] = \sum_{k=1}^{K} \mathbf{H}_k \mathbf{S}_k \mathbf{P}_k \mathbf{S}_k^\top \mathbf{H}_k^\top + \mathbf{W} \tag{IV.1.4}$$

is the correlation matrix of the received signal in (IV.1.2).

In order to decode the information transmitted by a given user k, the receiver uses a bank of MMSE receiver filters such that the vector of decision variables $\mathbf{d}_k = [d_1^{(k)} \dots d_{M_k}^{(k)}]^\top$ for a given user k is obtained as [43]

$$\mathbf{d}_k = \mathbf{C}^\top \times \mathbf{r} \tag{IV.1.5}$$

where $\mathbf{C} = \mathbf{R}^{-1} \mathbf{H}_k \mathbf{S}_k \mathbf{P}_k^{1/2}$ is the matrix corresponding to the bank of MMSE receiver filters. The expression of the MMSE receiver for the m-th symbol of the k-th user is given by

$$\begin{aligned}\mathbf{c}_m^{(k)} &= \mathbf{R}^{-1}\mathbf{H}_k\mathbf{s}_m^{(k)}\sqrt{p_m^{(k)}} \\ &= \mathbf{R}_m^{(k)-1}\mathbf{H}_k\mathbf{s}_m^{(k)}\frac{\sqrt{p_m^{(k)}}}{1+p_m^{(k)}\mathbf{s}_m^{(k)\top}\mathbf{H}_k^\top\mathbf{R}_m^{(k)-1}\mathbf{H}_k\mathbf{s}_m^{(k)}}.\end{aligned} \tag{IV.1.6}$$

Using expression (IV.1.6) of the individual receiver filter we get expressions of its corresponding SINR [43]

$$\begin{aligned}\gamma_m^{(k)} &= \frac{p_m^{(k)}\mathbf{s}_m^{(k)\top}\mathbf{H}_k^\top\mathbf{R}^{-1}\mathbf{H}_k\mathbf{s}_m^{(k)}}{1-p_m^{(k)}\mathbf{s}_m^{(k)\top}\mathbf{H}_k^\top\mathbf{R}^{-1}\mathbf{H}_k\mathbf{s}_m^{(k)}} \\ &= p_m^{(k)}\mathbf{s}_m^{(k)\top}\mathbf{H}_k^\top\mathbf{R}_m^{(k)-1}\mathbf{H}_k\mathbf{s}_m^{(k)}\end{aligned} \tag{IV.1.7}$$

and MMSE

$$\begin{aligned}\varepsilon_m^{(k)} &= \frac{1}{1+\gamma_m^{(k)}} = 1-p_m^{(k)}\mathbf{s}_m^{(k)\top}\mathbf{H}_k^\top\mathbf{R}^{-1}\mathbf{H}_k\mathbf{s}_m^{(k)} \\ &= \frac{1}{1+p_m^{(k)}\mathbf{s}_m^{(k)\top}\mathbf{H}_k^\top\mathbf{R}_m^{(k)-1}\mathbf{H}_k\mathbf{s}_m^{(k)}}.\end{aligned} \tag{IV.1.8}$$

Our goal in this setup is to derive an algorithm by which individual users adjust their corresponding precoding matrices and powers such that the SINR value for all symbols of a given user k is equal to a specified target SINR γ_k^*, for all $k = 1, \ldots, K$, that satisfies the admissibility condition [43]

$$\sum_{k=1}^{K} M_k \frac{1}{1+\gamma_k^*} < N. \tag{IV.1.9}$$

IV.2 JOINT PRECODER ADAPTATION AND POWER CONTROL AS A CONSTRAINED OPTIMIZATION PROBLEM

To formulate the transmitter adaptation and power control problem as a distributed constrained optimization problem we use the total MMSE corresponding to a given user k as its cost function

$$\mathcal{U}_k = \sum_{m=1}^{M_k} \varepsilon_m^{(k)} = \sum_{m=1}^{M_k} \frac{1}{1 + p_m^{(k)} \mathbf{s}_m^{(k)\top} \mathbf{H}_k^\top \mathbf{R}_m^{(k)-1} \mathbf{H}_k \mathbf{s}_m^{(k)}}. \tag{IV.2.1}$$

and we formally define the joint precoder and power adaptation problem for the given user k subject to the specified target SIR constraints as

$$\min_{\mathbf{S}_k, \mathbf{P}_k} \mathcal{U}_k \text{ subject to } \begin{cases} p_m^{(k)} \mathbf{s}_m^{(k)\top} \mathbf{H}_k^\top \mathbf{R}_m^{(k)-1} \mathbf{H}_k \mathbf{s}_m^{(k)} = \gamma_k^*, \\ \mathbf{s}_m^{(k)\top} \mathbf{s}_m^{(k)} = 1, \ m = 1, \ldots, M_k \\ \text{Trace}\left[\mathbf{P}_k\right] \leq P_k^{\max}. \end{cases} \tag{IV.2.2}$$

In order to solve this constrained optimization problem we use the Lagrange multipliers method [41, Ch. 5] and define the corresponding Lagrangian function.

$$\begin{aligned} L_k = {} & \mathcal{U}_k + \sum_{m=1}^{M_k} \zeta_m^{(k)} \left(p_m^{(k)} \mathbf{s}_m^{(k)\top} \mathbf{H}_k^\top \mathbf{R}_m^{(k)-1} \mathbf{H}_k \mathbf{s}_m^{(k)} - \gamma_k^* \right) \\ & + \sum_{m=1}^{M_k} \xi_m^{(k)} (\mathbf{s}_m^{(k)\top} \mathbf{s}_m^{(k)} - 1) + \rho \left(P_k^{\max} - \sum_{m=1}^{M_k} p_m^{(k)} \right). \end{aligned} \tag{IV.2.3}$$

We note that L_k is a function of the M_k columns of user k precoder matrix $\mathbf{s}_1^{(k)}, \ldots, \mathbf{s}_{M_k}^{(k)}$, powers $p_1^{(k)}, \ldots, p_{M_k}^{(k)}$, and multipliers $\zeta_1^{(k)}, \ldots, \zeta_{M_k}^{(k)}$, $\xi_1^{(k)}, \ldots, \xi_{M_k}^{(k)}$, and ρ associated with the constraints in equation (IV.2.2). The necessary conditions for minimizing the Lagrangian are as follows:

$$\frac{\partial L_k}{\partial \mathbf{s}_m^{(k)}} = 0, \ m = 1, \ldots, M_k \tag{IV.2.4}$$

$$\frac{\partial L_k}{\partial p_m^{(k)}} = 0, \ m = 1, \ldots, M_k \tag{IV.2.5}$$

$$\frac{\partial L_k}{\partial \zeta_m^{(k)}} = 0, \ m = 1, \ldots, M_k \tag{IV.2.6}$$

$$\frac{\partial L_k}{\partial \xi_m^{(k)}} = 0, \ m = 1, \ldots, M_k \tag{IV.2.7}$$

$$\frac{\partial L_k}{\partial \rho} \geq 0. \tag{IV.2.8}$$

To these one must add the complementary slackness condition

$$\rho \left(P_k^{\max} - \sum_{m=1}^{M_k} p_m^{(k)} \right) = 0 \tag{IV.2.9}$$

which is needed in the case of inequality constraints to ensure that at the optimum point the Lagrangian and the original cost function have the same value.

Condition (IV.2.4) leads to the eigenvalue/eigenvector equation

$$\begin{gathered} \frac{\partial L_k}{\partial \mathbf{s}_m^{(k)}} = 0 \implies \mathbf{H}_k^\top \mathbf{R}_m^{(k)-1} \mathbf{H}_k \mathbf{s}_m^{(k)} = \lambda_m^{(k)} \mathbf{s}_m^{(k)} \\ m = 1, \ldots, M_k \end{gathered} \tag{IV.2.10}$$

where the corresponding eigenvalue $\lambda_m^{(k)}$ can be written in terms of the Lagrange multipliers $\zeta_m^{(k)}$, $\xi_m^{(k)}$, the user power $p_m^{(k)}$, and the corresponding MMSE $\varepsilon_m^{(k)}$.

Furthermore, condition (IV.2.6) implies

$$\begin{gathered} \frac{\partial L_k}{\partial \zeta_m^{(k)}} = 0 \implies p_m^{(k)} = \frac{\gamma_k^*}{\mathbf{s}_m^{(k)\top} \mathbf{H}_k^\top \mathbf{R}_m^{(k)-1} \mathbf{H}_k \mathbf{s}_m^{(k)}} \\ m = 1, \ldots, M_k. \end{gathered} \tag{IV.2.11}$$

The two equations (IV.2.10) and (IV.2.11) implied by the necessary conditions of the constrained optimization problem (IV.2.2) indicate that, at the optimal point, user k precoder and power matrices satisfy the following (necessary) conditions:

- Any column m of user k precoder matrix, $\mathbf{s}_m^{(k)}$, is an eigenvector of corresponding matrix $\mathbf{H}_k^\top \mathbf{R}_m^{(k)-1} \mathbf{H}_k$.

- For the given $\mathbf{s}_m^{(k)}$ the power value corresponding to symbol m of user k, $p_m^{(k)}$, matches the specified target SINR γ_k^*.

We note that, among all possible eigenvectors of matrix $\mathbf{H}_k^\top \mathbf{R}_m^{(k)-1} \mathbf{H}_k$ the meaningful choice for $\mathbf{s}_m^{(k)}$ is the eigenvector $\mathbf{x}_m^{(k)}$ corresponding to its maximum eigenvalue[1] $\bar{\lambda}_m^{(k)}$, since, according to the SIR expression (IV.1.7), this will require minimum transmit power for matching the specified target SINR

$$\bar{p}_m^{(k)}|_{\mathbf{s}_m^{(k)}=\mathbf{x}_m^{(k)}} = \frac{\gamma_k^*}{\bar{\lambda}_m^{(k)}}. \tag{IV.2.12}$$

Furthermore, using equation (IV.1.3) along with the SINR expression in terms of $\mathbf{R}^{-1}$ in equation (IV.1.7), we also have that, with this choice of power, the maximum eigenvector of $\mathbf{H}_k^\top \mathbf{R}_m^{(k)-1} \mathbf{H}_k$ matrix is also an eigenvector the $\mathbf{H}_k^\top \mathbf{R}^{-1} \mathbf{H}_k$ matrix, satisfying

$$\mathbf{H}_k^\top \mathbf{R}^{-1} \mathbf{H}_k \mathbf{x}_m^{(k)} = \frac{1}{\bar{p}_m^{(k)}} \frac{\gamma^*}{1+\gamma^*} \mathbf{x}_m^{(k)}, \qquad m = 1, \ldots, M_k. \tag{IV.2.13}$$

Noting that the power value $\bar{p}_m^{(k)}$ is the minimum power needed to match the specified target, this implies that $\mathbf{x}_m^{(k)}$ corresponds actually to the maximum eigenvalue $\bar{\nu}_k$ of matrix $\mathbf{H}_k^\top \mathbf{R}^{-1} \mathbf{H}_k$. As a consequence, an ensemble where all columns of a given user precoder $\mathbf{s}_m^{(k)}$ are maximum eigenvectors of their corresponding matrices $\mathbf{H}_k^\top \mathbf{R}_m^{(k)-1} \mathbf{H}_k$, $m = 1, \ldots, M_k$ and user k power matrix is a scaled identity matrix

$$\mathbf{P}_k = \frac{1}{\bar{\nu}_k} \frac{\gamma^*}{1+\gamma^*} \mathbf{I}_{M_k} \tag{IV.2.14}$$

with $\bar{\nu}_k$ being the maximum eigenvalue of matrix $\mathbf{H}_k^\top \mathbf{R}^{-1} \mathbf{H}_k$, satisfies the necessary conditions (IV.2.10) and (IV.2.11) and is a stationary point of the constrained optimization problem (IV.2.2). We note that, in addition to all SINRs of user k being

[1] We also refer to it as the maximum eigenvector.

achieved with minimum power, at this stationary point all MMSE values corresponding to all user k symbols are equal to

$$\bar{\varepsilon}_m^{(k)} = \frac{1}{1+\gamma_k^*}, \quad m = 1, \ldots, M_k \tag{IV.2.15}$$

which in turn implies that the value of the user k cost function at this point is

$$\overline{\mathcal{U}}_k = \frac{M_k}{1+\gamma_k^*} \tag{IV.2.16}$$

and corresponds to the minimum value of the user k cost function for the given target SINR constraint.

In order to decide whether the $\mathbf{S}_k$ and $\mathbf{P}_k$ satisfying (IV.2.10)-(IV.2.11) are also optimal with respect to the constrained optimization of the cost function (IV.2.2), we apply the approach in [41, Ch. 5] that checks the second order optimality conditions based on evaluation of the bordered Hessian matrix at the critical point. For the Lagrangian expression (IV.2.3), the bordered Hessian matrix is expressed as [41, Ch. 5]

$$\mathbf{B}_h^{(k)} = \left[\begin{array}{c|c} \mathbf{0}_{(2M_k\times 2M_k)} & \mathbf{A}^{(k)}_{(2M_k+1)\times(NM_k+M_k)} \\ \hline \mathbf{A}^{(k)\top}_{(NM_k+M_k)\times(2M_k+1)} & \mathbf{C}^{(k)}_{(NM_k+M_k)\times(NM_k+M_k)} \end{array}\right] \tag{IV.2.17}$$

where the matrices $\mathbf{A}$ and $\mathbf{C}$ are given below

$$\mathbf{A}^{(k)} = \left[\begin{array}{ccc|ccc} 2\gamma_k^*\mathbf{s}_1^{(k)\top} & \cdots & 0 & \bar{\lambda}_1^{(k)} & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 2\gamma_k^*\mathbf{s}_{M_k}^{(k)\top} & 0 & \cdots & \bar{\lambda}_{M_k}^{(k)} \\ \hline 2\mathbf{s}_1^{(k)\top} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 2\mathbf{s}_{M_k}^{(k)\top} & 0 & \cdots & 0 \\ \hline 0 & \cdots & 0 & -1 & \cdots & -1 \end{array}\right], \tag{IV.2.18}$$

$$
\mathbf{C}^{(k)} = \left[\begin{array}{ccc|ccc}
\mathbf{C}_1^{'} & \cdots & 0 & \mathbf{c}_1^{''} & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & \mathbf{C}_{M_k}^{'} & 0 & \cdots & \mathbf{c}_{M_k}^{''} \\
\hline
(\mathbf{c}_1^{''})^\top & \cdots & 0 & c_1^{'''} & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & (\mathbf{c}_{M_k}^{''})^\top & 0 & \cdots & c_{M_k}^{'''}
\end{array}\right], \tag{IV.2.19}
$$

where the block elements in (IV.2.19) are given by

$$
\begin{aligned}
\mathbf{C}_m^{'} &= 8\gamma_k^{*2}\bar{\varepsilon}_m^{(k)3}\mathbf{s}_m^{(k)}\mathbf{s}_m^{(k)\top} \\
\mathbf{c}_m^{''} &= 4\gamma_k^{*}\bar{\lambda}_m^{(k)}\bar{\varepsilon}_m^{(k)3}\mathbf{s}_m^{(k)} \\
c_m^{'''} &= 2\bar{\lambda}_m^{(k)2}\bar{\varepsilon}_m^{(k)3}
\end{aligned} \tag{IV.2.20}
$$

for all $m = 1, \ldots, M_k$.

Details on the derivation of the bordered Hessian matrix (IV.2.17) and of its block matrices (IV.2.18)-(IV.2.19) are presented in the appendix.

We note that, at the optimal point of the constrained optimization problem (IV.2.2), the following conditions must be satisfied by the bordered Hessian matrix [41, Ch. 5]:

$$
\begin{aligned}
&(-1)^{(2M_k+1)}|\mathbf{B}_{h,\ell}| > 0, \\
&\text{for} \quad \ell = (4M_k+3), (4M_k+4), \cdots, M_k(N+3)+1,
\end{aligned} \tag{IV.2.21}
$$

where $|\mathbf{B}_{h,\ell}|$ is the ℓ-th principal determinant of the bordered Hessian matrix $\mathbf{B}_h$. Thus, provided that (IV.2.21) is verified, the matrices $\mathbf{S}_k$ and $\mathbf{P}_k$, satisfying (IV.2.10)-(IV.2.11) are a solution of the constrained optimization problem (IV.2.2).

IV.3 JOINT PRECODER ADAPTATION AND POWER CONTROL ALGORITHM

Using the necessary conditions established in the previous section we propose an iterative procedure that uses incremental updates for joint adaptation of the precoder and power matrices of all active users in the system until the optimal ensemble of precoder and power matrices is reached. At each step, the procedure incrementally updates one column m of a precoder matrix k in the direction of the maximum eigenvector of its corresponding $\mathbf{H}_k^\top \mathbf{R}_m^{(k)-1}\mathbf{H}_k$ matrix, followed by an incremental update of its associated transmit power toward the value that matches its target SINR γ_k^*.

At a given update step n, precoder column $\mathbf{s}_m^{(k)}$ is updated as follows:

$$\mathbf{s}_m^{(k)}(n+1) = \frac{\mathbf{s}_m^{(k)}(n) + \alpha(n)\beta\mathbf{x}_m^{(k)}(n)}{\|\mathbf{s}_m^{(k)}(n) + \alpha(n)\beta\mathbf{x}_m^{(k)}(n)\|} \tag{IV.3.1}$$

where $\mathbf{x}_m^{(k)}$ is the maximum eigenvector of matrix $\mathbf{H}_k^\top[\mathbf{R}_m^{(k)}(n)]^{-1}\mathbf{H}_k$, $\alpha(n) = \mathrm{sgn}[\mathbf{s}_m^{(k)\top}(n)\mathbf{x}_m^{(k)}(n)]$, and β is a parameter that defines the increment size by limiting how far, in terms of Euclidian distance, the updated precoder column is from the old one.

After the precoder update (IV.3.1) we use equation (IV.2.11) to obtain the power value needed to match the target SINR

$$p_m^{(k)\prime}(n) = \frac{\gamma_k^*}{\mathbf{s}_m^{(k)\top}(n+1)\mathbf{H}_k^\top[\mathbf{R}_m^{(k)}(n)]^{-1}\mathbf{H}_k\mathbf{s}_m^{(k)}(n+1)}, \tag{IV.3.2}$$

and we update the transmit power $p_m^{(k)}$ as follows:

$$p_m^{(k)}(n+1) = p_m^{(k)}(n) + \mu[p_m^{(k)\prime}(n) - p_m^{(k)}(n)] \tag{IV.3.3}$$

with $0 < \mu < 1$ a suitably chosen constant defining the size of the power increment. We note that, the smaller the μ constant, the smaller the incremental power change, and that the power will always be updated toward the value needed to match the target SINR (that is, if the power value $p_m^{(k)'}(n)$ needed to match the target SINR is lower than the current power then the power will be decreased and vice versa).

We have empirically observed that the incremental updates implied by equations (IV.3.1)–(IV.3.3) result in a monotonic decrease of the sum of all user cost functions

$$\mathcal{U} = \sum_{k=1}^{K} \mathcal{U}_k = \sum_{k=1}^{K} \sum_{m=1}^{M_k} \varepsilon_m^{(k)}. \tag{IV.3.4}$$

That is, the difference between $\mathcal{U}(n)$ before step n updates (IV.3.1)–(IV.3.3) are applied and $\mathcal{U}(n+1)$ after the updates is always non-negative

$$\Delta\mathcal{U} = \mathcal{U}(n) - \mathcal{U}(n+1) \geq 0. \tag{IV.3.5}$$

Thus, given the constrained minima of $\overline{\mathcal{U}}_k$ in (IV.2.16), and that according to [46] $\mathcal{U}$ is lower bounded, we have that

$$\mathcal{U}(n) \geq \overline{\mathcal{U}} = \sum_{k=1}^{K} \overline{\mathcal{U}}_k, \tag{IV.3.6}$$

which implies that we can use the global cost function $\mathcal{U}$ as a formal stopping criterion in Algorithm 6. Thus, a fixed point of the algorithm is reached when the updates result in changes of the cost function that are smaller than the specified tolerance ϵ, and the check of the bordered Hessian sufficient condition (IV.2.21) in Line 26 of **Algorithm 6** guarantees that iterations will not stop in a suboptimal point and will reach the optimal point discussed in the previous section.

Extensive simulations of **Algorithm 6** have confirmed that it always converges to the optimal point where the global cost function is equal to the tight lower bound in

Algorithm 6 – Joint Precoder Adaptation and Power Control in Uplink Wireless Systems

1: Input Data

- Initial precoder and power matrices $\mathbf{S}_k$, $\mathbf{P}_k$, for all active users $k = 1, \ldots, K$.
- Corresponding channel matrices $\mathbf{H}_k$ and desired target SINR values γ_k^*, for $k = 1, \ldots, K$.
- Noise covariance matrix at the receiver $\mathbf{W}$.
- Algorithm constants β, μ, and tolerance ϵ.

2: **if** admissibility condition in equation (IV.1.9) is satisfied **then**
3: GO TO Step 8.
4: **else**
5: STOP: desired system configuration is not admissible.
6: **end if**
7: Initialize iteration counter $n = 0$.
8: **for** each user $k = 1, ..., K$ **do**
9: Increment iteration counter $n = n + 1$
10: **for** each column $m = 1, \ldots, M_k$ of user k precoder matrix **do**
11: Compute corresponding $\mathbf{H}_k^T[\mathbf{R}_m^{(k)}(n)]^{-1}\mathbf{H}_k$ using equation (IV.1.3) and determine its maximum eigenvector $\mathbf{x}_m^{(k)}$.
12: Update column m of user k precoder using equation (IV.3.1).
13: Update transmit power for symbol m of CR transmitter k using equation (IV.3.3).
14: **end for**
15: **end for**
16: **if** change in global cost function $\mathcal{U}$ is larger than specified tolerance ϵ **then**
17: GO TO Step (8)
18: **else**
19: STOP: a fixed point has been reached.
20: **end if**
21: **if** global cost function $\mathcal{U}$ is larger than the tight lower bound in (IV.3.6) **then**
22: GO TO Step (8)
23: **else**
24: STOP: a sub-optimal fixed point has been reached.
25: **end if**
26: **if** optimality condition (IV.2.21) is true **then**
27: STOP, an optimal point has been reached.
28: OUTPUT $\mathbf{S}(n)$ and $\mathbf{P}(n)$.
29: **else**
30: GO TO Step (8)
31: **end if**

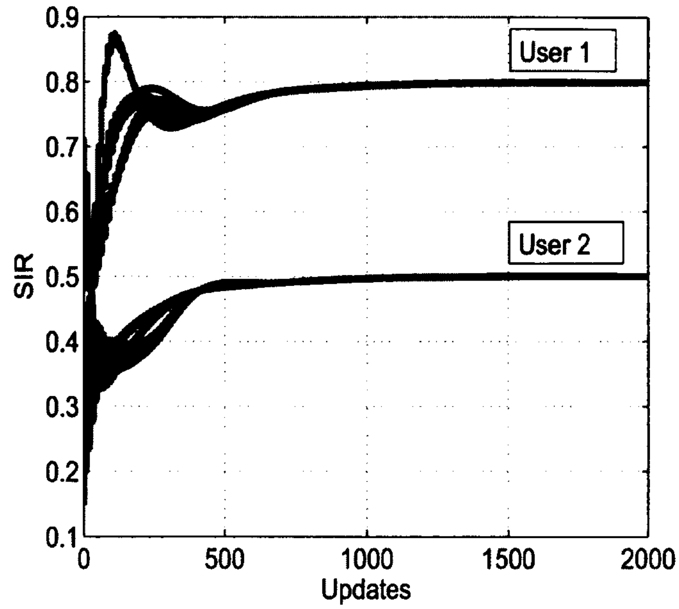
(IV.3.6), and all individual user cost functions are minimized subject to the specified constraints as discussed in Section IV.2. An analytical proof of convergence is currently under investigation. We note that, as it is the case with incremental/adaptive algorithms in general, the convergence speed of the algorithm depends on the values of the corresponding increments specified by the algorithm constants β and μ, and that the smaller these values the slower the algorithm convergence.

We present numerical results obtained using **Algorithm 6** which show the variation of user powers, SINRs, and cost functions, and support its convergence.

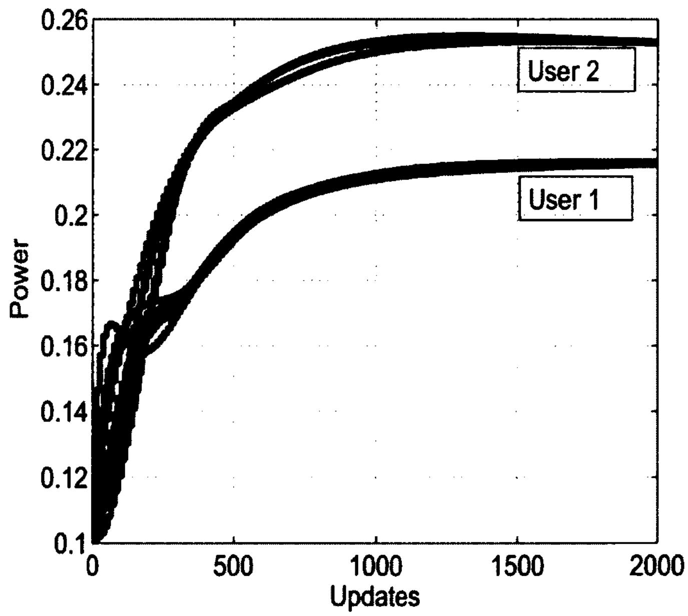
We consider a wireless system with $K = 2$ active users in a signal space of dimension $N = 6$ transmitting blocks of size $M_1 = M_2 = 6$ symbols and target SINRS $\gamma_k^* = \{0.8, 0.5\}$. The user channel matrices are taken to be circulant, and the background noise covariance matrix is $\mathbf{W} = 0.1\mathbf{I}_6$. The algorithm parameters are set to $\mu = 0.1$, $\beta = 0.2$ $\epsilon = 0.001$ Codeword and Power matrices: $\mathbf{S}_k$ and $\mathbf{P}_k$ ($\forall k = 1, \dots, K$) are initialized randomly, and white noise is considered with covariance matrix $\mathbf{W} = 0.1\mathbf{I}_6$.

In the first experiment, variations of user SINRs and powers are plotted in Figure 12. This confirms that all symbols of a given user have the same transmitted power and meet the specified target SINRs.

In the second experiment we plotted the variation of individual and global cost functions for the same example as seen in Figure 13. We note that this shows the monotonic decrease of the global cost function U to the lower bound (IV.3.6) whose value is 7.33 for this example. We also note that Figures 12 and 13 are typical for all simulations we have run, with various values for N, K, Mk, and admissible γ_k^* that satisfy (IV.1.9).

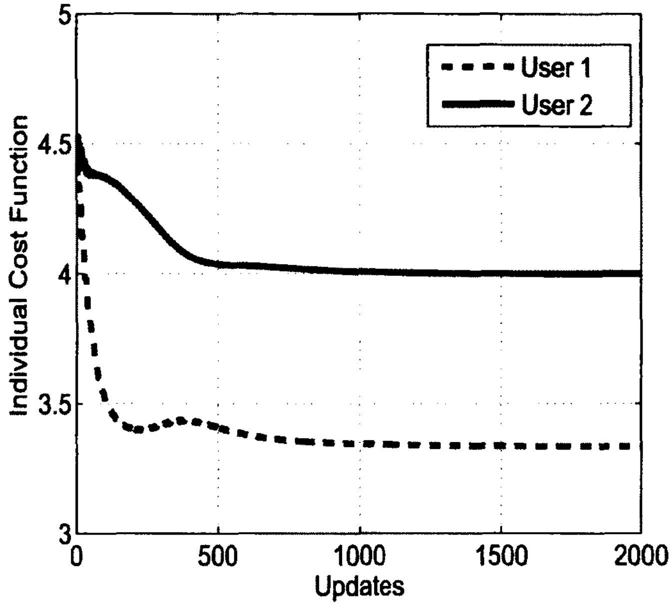


(a) SINR variation

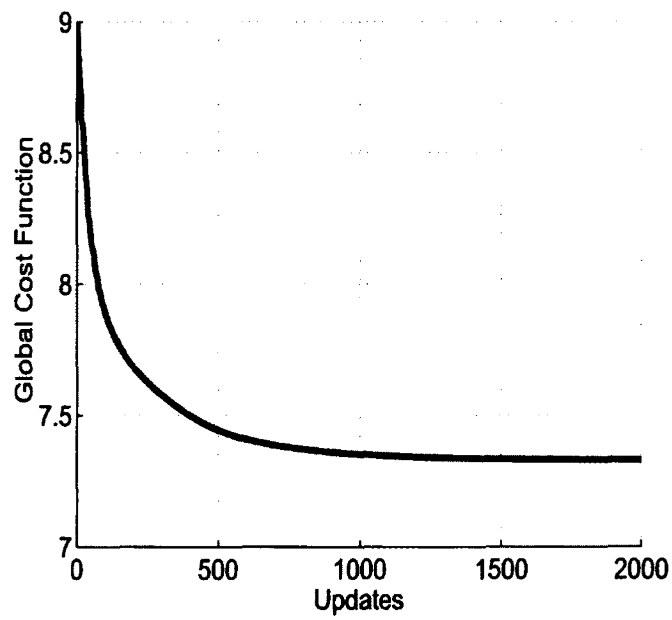


(b) Power variation

FIG. 12: Variation of symbol SINRs and powers values of the proposed algorithm from random initialization.



(a) User Cost Function



(b) Global Cost Function

FIG. 13: Variation of individual and global cost functions of the proposed algorithm from random initialization.

IV.4 JOINT TRANSMITTER ADAPTATION AND POWER CONTROL IN COGNITIVE RADIO NETWORKS

An application of **Algorithm 6** to the uplink of a Cognitive Radio (CR) Network with target values imposed on the signal-to-interference+noise-ratios (SINR) at the CR receiver is discussed in this section.

CRs enable access to the licensed spectrum by secondary transmitters (SU) with specific restrictions on the interference that these can cause to the incumbent licensed users of the spectrum [47]. They are made possible through implementations in software defined radio platforms [48] which provide adaptable radios with versatile transmitters and receivers that can vary their transmitted waveforms and receiver filters for efficient use of the available radio spectrum. Unlike traditional communication systems which rely heavily on receiver signal processing, cognitive radios use receiver feedback [49] to optimize spectrum utilization through transmitter adaptation.

A common approach taken for transmitter adaptation in CRs is based on precoder optimization [50] where the problem of transmitter adaptation is cast in a general signal space framework that is applicable to a wide range of scenarios which include OFDM-based transmissions and MIMO systems as particular cases [51–53]. For multiuser systems that share the same bandwidth and signaling interval resources at the physical layer, precoder optimization has also been investigated using game-theoretic approaches in [5, 6], where the problem of maximizing mutual information on each link is solved for specific operating constraints such as transmit power and spectral mask design, while accommodating practical implementation aspects like maximization of transmission rate on each link with an average error probability constraint.

When the SUs are expected to meet specified operating constraints such as target SINR or interference constraints, transmitter adaptation is augmented with a power control mechanism. In this direction, we note the recent work for downlink CR systems in [54] which discusses the design of block diagonalizing precoders based on a minimum mean-squared error (MMSE) criterion with power constraints. A resource allocation problem for spectrum underlay in cognitive wireless networks is discussed in [55] where a joint rate and power allocation scheme with Quality of Service (QoS) and interference constraints is proposed. A similar scheme for spectrum overlay is presented in [56] where the secondary user's effective throughput is maximized while exploiting the sensing result and accuracy for efficient resource allocation.

We consider the uplink of a CR network as illustrated in Figure 14 where multiple secondary users are transmitting to a secondary base station receiver in the presence of a licensed primary user system. We note that the transmission between the primary (licensed) user has highest priority and that the secondary CR network is expected to limit the interference caused to the primary network while attempting to provide service to the secondary users with a specified quality of service determined by specific target SINR values at the secondary CR receiver. This can be accomplished through joint adaptation of the precoder and transmit power.

In order to evaluate the convergence speed of the proposed algorithm for joint transmitter adaptation and power control, we performed simulations over various scenarios, noting that, as it is the case in general with incremental algorithms, the convergence speed depends on the values of the increment steps specified by the

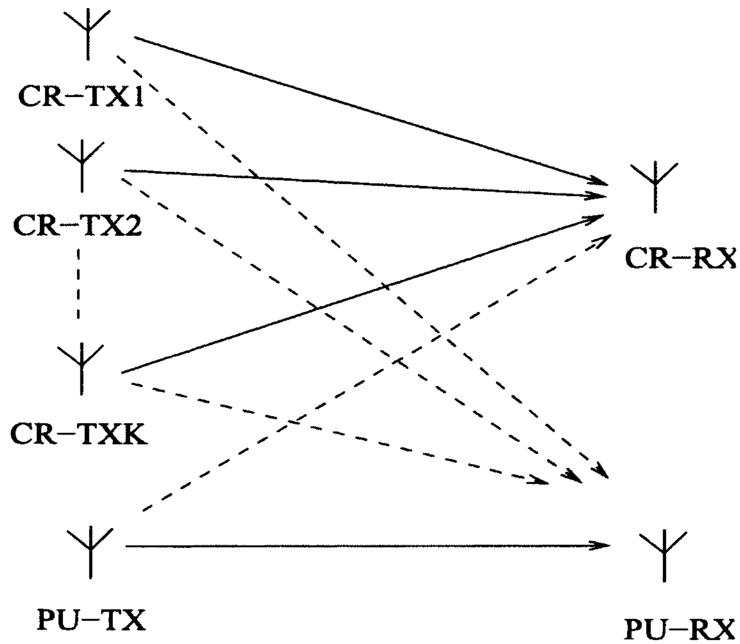


FIG. 14: Cognitive Radio Network Model.

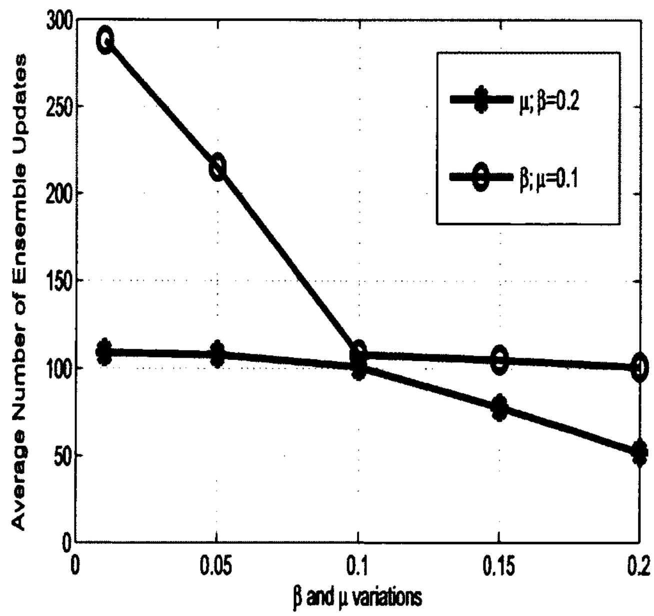


FIG. 15: Dependence of convergence speed on the increment constants μ and β in 100 trials.

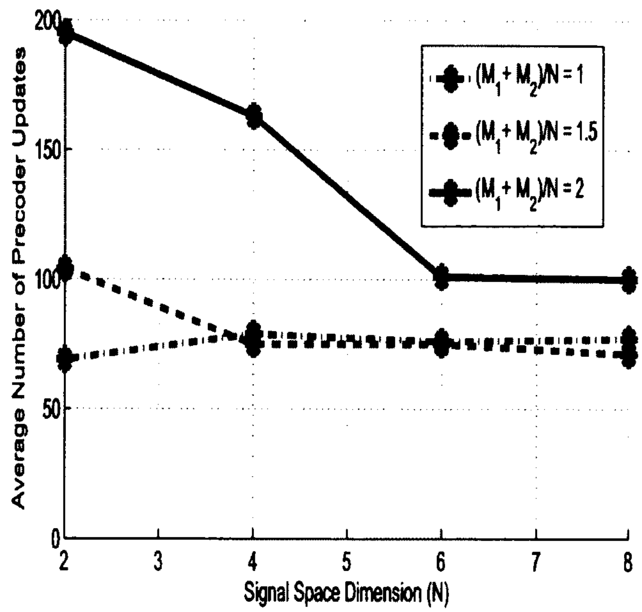


FIG. 16: Convergence speed for varying $(M_1 + M_2)/N$ with $\beta = 0.2$ and $\mu = 0.1$.

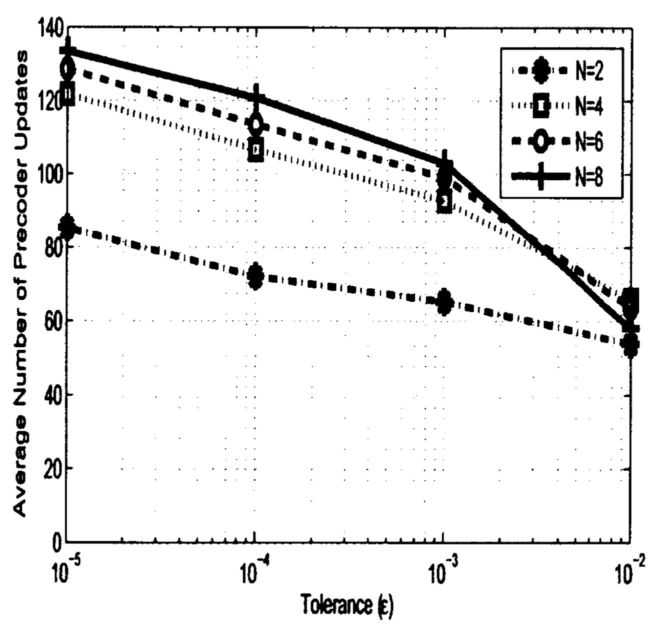


FIG. 17: Dependence of convergence speed on the tolerance ϵ in 100 trials

algorithm constants β and μ, as well as by the value of the tolerance ϵ.

In the following simulation experiments we considered a CR system with $K = 2$ active transmitters and selected specific values for the number of signal dimensions N, the precoder sizes M_1 and M_2, for the algorithm constants β and μ, and for the tolerance ϵ, and we ran 100 trials of the algorithm recording the number of iterations n needed for convergence. We note that, the actual number of iterations depends on the number of active users and their corresponding precoder sizes: one update for precoder k consists of M_k updates of all of its columns, and one update of the ensemble of precoders consists of $\sum_{k=1}^{K} M_k$ iterations corresponding to the **FOR** loop: 8 in **Algorithm 6**. Thus, for better visualization of the convergence speed results we will look at the average number of ensemble updates until convergence rather than the average number of actual iterations n in our plots. This is obtained by dividing the total number of iterations recorded, n, to $\sum_{k=1}^{K} M_k$ which represents the number of iterations/ensemble.

In the third experiment we studied convergence of the proposed algorithm for different values of the algorithm constants μ and β, precoder sizes $M_1 = M_2 = 6$ and target SINRs $\gamma_1^* = 0.8$ and $\gamma_1^* = 0.5$. The results of this experiment are plotted in Fig. 15 from where we note that convergence speed is affected mostly by changes in the value of β and that it does not depend significantly on the value of μ.

In the fourth experiment we studied the convergence speed of the algorithm for fixed algorithm constants $\beta = 0.2$ and $\mu = 0.1$ and tolerance $\epsilon = 0.001$ for varying precoder sizes and signal dimensions available such that the ratio of the total number of precoder columns to the number of signal dimensions $\sum_{k=1}^{K} M_k/N$ is constant. The results of this experiment are shown in Fig. 16 from which we note that the average number of ensemble updates needed for convergence within tolerance $\epsilon = 0.001$ does not vary significantly for increasing N and $M_1 + M_2$.

In the final experiment performed we studied the dependence of convergence speed on the algorithm tolerance ϵ for a different number of signal dimensions N and fixed algorithm constants $\beta = 0.2$ and $\mu = 0.1$. The results of this experiment are shown in Fig. 17 which confirms our expectations that the lower the value of ϵ, the slower the convergence of the proposed algorithm to the optimal fixed point. This result was expected considering the fact that a smaller ϵ implies a higher precision for the fixed-point, which, with fixed increments will be reached through an increased number of updates.

IV.5 CHAPTER SUMMARY

In this chapter we study joint transmitter adaptation and power control in multiuser uplink wireless systems with target SINR values as a distributed constrained optimization problem, and we discuss necessary conditions that must be satisfied by the optimal solution. We also present an incremental algorithm that adapts user transmitters and power values in a distributed fashion until a fixed point is reached where the specified target SINRs are achieved with minimum transmitted power. Numerical results obtained from simulations that illustrate the convergence of the proposed algorithm are also presented along with an extension of the algorithm to Cognitive radio networks.

IV.6 APPENDIX

Necessary conditions for optimality have been discussed in Section III.5 for downlink wireless systems and Section IV.2 for uplink wireless systems . Turning to the sufficient conditions, we use the Bordered Hessian Matrix approach [41, Ch. 5] to determine second order optimality conditions, and a detailed analysis of the above in the uplink scenario is presented in the appendix. We consider the Lagrangian expression (III.2.2) for a given user k and note that there are three sets of constraints

$$\begin{aligned} g_m^{(1)} &= p_m^{(k)} \mathbf{s}_m^{(k)\top} \mathbf{H}_k^\top [\mathbf{R}_m^{(k)}]^{-1} \mathbf{H}_k \mathbf{s}_m^{(k)} - \gamma_k^*,\ m = 1, \ldots, M_k \\ g_m^{(2)} &= \mathbf{s}_m^{(k)\top} \mathbf{s}_m^{(k)} - 1,\ m = 1, \ldots, M_k \\ g^{(3)} &= P_k^{\max} - \textstyle\sum_{m=1}^{M_k} p_m^{(k)}, \end{aligned} \tag{IV.6.1}$$

along with five sets of variables: ρ, $\mathbf{s}_m^{(k)}, p_m^{(k)}, \zeta_m^{(k)}, \xi_m^{(k)}$, $m = 1, \ldots, M_k$. The Bordered Hessian Matrix for the Lagrangian expression (III.2.2) can be written in a compact block matrix form as [41, Ch. 5]

$$\mathbf{B}_h^{(k)} = \left[\begin{array}{c|c} \mathbf{0}_{(2M_k \times 2M_k)} & \mathbf{A}^{(k)}_{(2M_k+1)\times(NM_k+M_k)} \\ \hline \mathbf{A}^{(k)\top}_{(NM_k+M_k)\times(2M_k+1)} & \mathbf{C}^{(k)}_{(NM_k+M_k)\times(NM_k+M_k)} \end{array} \right]. \tag{IV.6.2}$$

The expression of the block matrix $\mathbf{A}_k$ is

$$\mathbf{A}^{(k)} = \begin{bmatrix} \mathbf{a}_1', & \cdots & ,0 & a_1'', & \cdots & ,0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0, & \cdots & ,\mathbf{a}_{M_k}' & 0, & \cdots & ,a_{M_k}'' \\ \mathbf{a}_1''', & \cdots & ,0 & 0, & \cdots & ,0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0, & \cdots & ,\mathbf{a}_{M_k}''' & 0, & \cdots & ,0 \\ 0, & \cdots & ,0 & -1, & \cdots & ,-1 \end{bmatrix} \tag{IV.6.3}$$

where

$$\begin{aligned}
\mathbf{a}'_m &= 2\bar{p_m}^{(k)}\mathbf{s}_m^{(k)\top}\mathbf{H}_k^\top[\mathbf{R}_m^{(k)}]^{-1}\mathbf{H}_k\mathbf{s}_m^{(k)} = 2\gamma_k^* \\
a''_m &= \mathbf{s}_m^{(k)\top}\mathbf{H}_k^\top[\mathbf{R}_m^{(k)}]^{-1}\mathbf{H}_k\mathbf{s}_m^{(k)} = \bar{\lambda}_m^{(k)} \\
\mathbf{a}'''_m &= 2\mathbf{s}_m^{(k)\top}
\end{aligned} \tag{IV.6.4}$$

for $m = 1, \ldots, M_k$.

The expression of the block matrix $\mathbf{C}_k$ is

$$\mathbf{C}^{(k)} = \begin{bmatrix}
\mathbf{C}'_1, & \cdots & ,0 & \mathbf{c}''_1, & \cdots & ,0 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0, & \cdots & ,\mathbf{C}'_{M_k} & 0, & \cdots & ,\mathbf{c}''_{M_k} \\
(\mathbf{c}''_1)^\top, & \cdots & ,0 & c'''_1, & \cdots & ,0 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0, & \cdots & ,(\mathbf{c}''_{M_k})^\top & 0, & \cdots & ,c'''_{M_k}
\end{bmatrix} \tag{IV.6.5}$$

where

$$\begin{aligned}
\mathbf{C}'_m &= 8\bar{p_m}^{(k)2}\bar{\varepsilon}_m^{(k)3}(\mathbf{H}_k^\top[\mathbf{R}_m^{(k)}]^{-1}\mathbf{H}_k\mathbf{s}_m^{(k)}\mathbf{s}_m^{(k)\top}\mathbf{H}_k^\top[\mathbf{R}_m^{(k)}]^{-1}\mathbf{H}_k) \\
&= 8\gamma_k^{*2}\bar{\varepsilon}_m^{(k)3}\mathbf{s}_m^{(k)}\mathbf{s}_m^{(k)\top} \\
\mathbf{c}''_m &= 4\gamma_k^*\bar{\varepsilon}_m^{(k)3}\mathbf{H}_k^\top[\mathbf{R}_m^{(k)}]^{-1}\mathbf{H}_k\mathbf{s}_m^{(k)} = 4\gamma_k^*\bar{\lambda}_m^{(k)}\bar{\varepsilon}_m^{(k)3}\mathbf{s}_m^{(k)} \\
c'''_m &= 2\bar{\varepsilon}_m^{(k)3}(\mathbf{s}_m^{(k)\top}\mathbf{H}_k^\top[\mathbf{R}_m^{(k)}]^{-1}\mathbf{H}_k\mathbf{s}_m^{(k)})^2 = 2\bar{\lambda}_m^{(k)2}\bar{\varepsilon}_m^{(k)3}
\end{aligned} \tag{IV.6.6}$$

$m = 1, \ldots, M_k$.

Note that the optimum values of $\mathbf{s}_m^{(k)}$ and $p_m^{(k)}$ are considered in the bordered Hessian matrix, and the sufficient conditions for the Lagrangian (III.2.2) are given by

$$\begin{gathered}
(-1)^{(2M_k+1)}|\mathbf{B}_{h,\ell}| > 0, \\
\text{for} \quad \ell = (4M_k+3), (4M_k+4), \cdots, M_k(N+3)+1,
\end{gathered} \tag{IV.6.7}$$

where $|\mathbf{B}_{h,\ell}|$ is the ℓ-th principal determinant of the bordered Hessian matrix $\mathbf{B}_h$. Thus, provided that (IV.2.21) is verified, the matrices $\mathbf{S}_k$ and $\mathbf{P}_k$, satisfying (III.2.3)-(III.2.4) are a solution of the constrained optimization problem (III.2.1).

Second order optimality conditions for the downlink constrained optimization problem can also be verified as shown above.

Chapter V

JOINT TRANSMITTER ADAPTATION AND POWER CONTROL IN FADING CHANNELS

In the previous chapters, transmitter adaptation and power control was achieved based on certain target SINR constraints under the assumption that the wireless channel parameters are known and do not change while an active transmission is ongoing. However, this assumption may not be satisfied in practical scenarios where multipath propagation and mobility result in time-varying fading wireless channels. We note that the effects of fading may be mitigated by using appropriately designed signal processing algorithms that adapt both transmitters and receivers based on the actual channel characteristics.

However, in many instances, it may not be possible to estimate channel characteristics due to its time varying nature. In other words, by the time the channel parameters are estimated and its performance analyzed, the channel under consideration may change to new values. A realistic approach in this case would be to use the average characteristics of the channel in the transmitter adaptation and power control algorithm. In this chapter, performance of the incremental algorithm for joint transmitter adaptation and power control in both downlink and uplink scenarios are studied in the context of fading channels. Specifically, average characteristics of the multipath fading channels are used to compute precoder and power matrices for which target SINR values are satisfied; then, Monte Carlo simulations are performed to study the performance for actual channel realizations. The performance measure used to evaluate performance of the algorithm in this work is the probability of outage.

V.1 SYSTEM MODEL

For the downlink and uplink scenarios considered in Chapters III and IV, propagation between a given transmitter and receiver is modeled by a vector channel described by matrix $\mathbf{H}_k$ of dimension $N \times N$ which is assumed to be a circulant matrix corresponding to the multipath channel response $\mathbf{h}_k = [h_{k1}, \ldots, h_{kN}]^\top$. We note that a stable and known multipath channel is assumed in Chapters III and IV which implies a deterministic and known $\mathbf{H}_k$ matrix.

However, this assumption is usually not satisfied in practice, where fading makes the parameters of channel matrices random variables rather than deterministic. In this chapter we assume that the multipath channel response corresponds to a fading channel with known statistics (mean and variance) and that only the average channel matrix $\overline{\mathbf{H}}_k$, corresponding to the average channel parameters $\bar{\mathbf{h}}_k = [\bar{h}_{k1}, \ldots, \bar{h}_{kN}]^\top$, is known.

For the downlink scenario in Chapter III, the N-dimensional received signal by a given user k is given by the expression

$$\mathbf{r}_k = \overline{\mathbf{H}}_k \mathbf{x}_k + \mathbf{n}_k = \overline{\mathbf{H}}_k \mathbf{S}\mathbf{P}^{1/2}\mathbf{b} + \mathbf{n}_k, \tag{V.1.1}$$

and in order to distinguish between the desired signal and the corresponding interference+noise, it can be rewritten as

$$\mathbf{r}_k = \underbrace{\overline{\mathbf{H}}_k b_k \sqrt{p_k}\mathbf{s}_k}_{\text{desired signal}} + \underbrace{\overline{\mathbf{H}}_k \sum_{\ell=1,\ell\neq k}^{K} b_\ell \sqrt{p_\ell}\mathbf{s}_\ell + \mathbf{n}_k}_{\text{interference + noise } (\mathbf{z}_k)} \tag{V.1.2}$$

where the interference+noise $\mathbf{z}_k$ experienced by user k has correlation matrix

$$\mathbf{Z}_k = E[\mathbf{z}_k\mathbf{z}_k^\top] = \overline{\mathbf{H}}_k\left(\sum_{\ell=1,\ell\neq k}^{K} \mathbf{s}_\ell p_\ell \mathbf{s}_\ell^\top\right)\overline{\mathbf{H}}_k^\top + \mathbf{W}_k. \tag{V.1.3}$$

Following the sequence of steps as shown in Section III.1 to perform parallel decomposition of user k downlink channel matrix $\overline{\mathbf{H}}_k$ by applying the whitening transformation followed by an SVD of the transformed channel matrix, partitioning the singular value matrix and the transformed codeword vector, we obtain the total interference power affecting all the user's symbols as

$$\mathcal{I} = \sum_{k=1}^{K} i_k = \sum_{k=1}^{K} \bar{\mathbf{s}}_k^{(1)^\top}\tilde{\mathbf{D}}_k^{-2}\bar{\mathbf{s}}_k^{(1)} \tag{V.1.4}$$

where $\tilde{\mathbf{D}}_k$ is the partitioned singular value matrix corresponding to the SVD of $\overline{\mathbf{H}}_k$.

For the uplink scenario in Chapter IV, the N-dimensional received signal vector at the common receiver corresponding to one signaling interval is given by the expression

$$\mathbf{r} = \sum_{k=1}^{K} \overline{\mathbf{H}}_k\mathbf{S}_k\mathbf{P}_k^{1/2}\mathbf{b}_k + \mathbf{n}, \tag{V.1.5}$$

and it can also be expressed from the perspective of the m-th symbol of user k as

$$\begin{aligned}\mathbf{r} = {} & b_m^{(k)}\overline{\mathbf{H}}_k\mathbf{s}_m^{(k)}\sqrt{p_m^{(k)}} + \\ & \underbrace{+\sum_{n=1,n\neq m}^{M_k} b_n^{(k)}\overline{\mathbf{H}}_k\mathbf{s}_n^{(k)}\sqrt{p_n^{(k)}} + \sum_{\ell=1,\ell\neq k}^{K}\overline{\mathbf{H}}_\ell\mathbf{S}_\ell\mathbf{P}_\ell^{1/2}\mathbf{b}_\ell + \mathbf{n}}_{\text{interference + noise } = \ \mathbf{i}_m^{(k)}}.\end{aligned} \tag{V.1.6}$$

The first term is the desired signal corresponding to symbol m of user k, and the remaining terms represent the interference and noise corrupting it at the receiver with correlation matrix

$$\mathbf{R}_m^{(k)} = E[\mathbf{i}_m^{(k)}\mathbf{i}_m^{(k)\top}] = \mathbf{R} - p_m^{(k)}\overline{\mathbf{H}}_k\mathbf{s}_m^{(k)}\mathbf{s}_m^{(k)\top}\overline{\mathbf{H}}_k^\top \tag{V.1.7}$$

where

$$\mathbf{R} = E[\mathbf{r}\mathbf{r}^\top] = \sum_{k=1}^{K} \overline{\mathbf{H}}_k \mathbf{S}_k \mathbf{P}_k \mathbf{S}_k^\top \overline{\mathbf{H}}_k^\top + \mathbf{W} \tag{V.1.8}$$

is the correlation matrix of the received signal in (V.1.6).

Following the sequence of steps as shown in section IV.1, the SINR for symbol m of user k is given by

$$\begin{aligned} \gamma_m^{(k)} &= \frac{p_m^{(k)} \mathbf{s}_m^{(k)\top} \overline{\mathbf{H}}_k^\top \mathbf{R}_k^{-1} \overline{\mathbf{H}}_k \mathbf{s}_m^{(k)}}{1 - p_m^{(k)} \mathbf{s}_m^{(k)\top} \overline{\mathbf{H}}_k^\top \mathbf{R}_k^{-1} \overline{\mathbf{H}}_k \mathbf{s}_m^{(k)}} \\ &= p_m^{(k)} \mathbf{s}_m^{(k)\top} \overline{\mathbf{H}}_k^\top [\mathbf{R}_m^{(k)}]^{-1} \overline{\mathbf{H}}_k \mathbf{s}_m^{(k)}, \end{aligned} \tag{V.1.9}$$

and the corresponding MMSE expression is

$$\begin{aligned} \varepsilon_m^{(k)} = \frac{1}{1+\gamma_m^{(k)}} &= 1 - p_m^{(k)} \mathbf{s}_m^{(k)\top} \overline{\mathbf{H}}_k^\top \mathbf{R}_k^{-1} \overline{\mathbf{H}}_k \mathbf{s}_m^{(k)} \\ &= \frac{1}{1 + p_m^{(k)} \mathbf{s}_m^{(k)\top} \overline{\mathbf{H}}_k^\top [\mathbf{R}_m^{(k)}]^{-1} \overline{\mathbf{H}}_k \mathbf{s}_m^{(k)}}. \end{aligned} \tag{V.1.10}$$

Our goal in this setup is to study the joint transmitter adaptation and power control algorithms in Sections III.3 and IV.3 in the context of multipath fading channels where average characteristics of the channel are used to compute optimal precoding and power matrices with specified target SINR values γ_k^*, for all $k = 1, \ldots, K$.

V.2 THE JOINT TRANSMITTER ADAPTATION AND POWER CONTROL ALGORITHM IN FADING CHANNELS

The proposed algorithms for joint transmitter adaptation and power control in the downlink and uplink scenarios are derived as shown in Sections III.3 and IV.3 considering only the average channel matrix $\overline{\mathbf{H}}_k$ for all $k = 1, \ldots, K$.

Performance of **Algorithm 4** and **Algorithm 6** for joint transmitter adaptation and power control in the context of fading channels is studied using the probability of outage which is defined as the probability that the output SINR for a given user k, γ_k, falls below the specified target SINR γ_k^*. When the statistics of the fading channel parameters is known, the probability of outage can be evaluated analytically and is implied by the cumulative distribution function (CDF) evaluated at γ_k^*, that is

$$P_{\text{out},k} = \int_0^{\gamma_k^*} P_{\gamma_k}(\gamma_k) d\gamma_k \tag{V.2.1}$$

where $P_{\gamma_k}(\gamma_k)$ denotes the probability density function (PDF) of the SINR γ_k which is implied by the statistics of the fading channel parameters.

Numerically, the probability of outage is evaluated through Monte Carlo simulations, and we study the performance of **Algorithm 4** and **Algorithm 6** by evaluating the probability of outage in the case of a specific fading channel model. We note that a wireless communication channel can experience fading due to several factors among which the most common are short term fading (also referred to as fast fading) and long term fading (also referred to as slow fading). The former is due to multipath propagation and user mobility while the latter is due to shadowing and exponential path-loss.

V.3 PERFORMANCE ANALYSIS

Monte Carlo simulations are performed to evaluate the probability of outage for **Algorithm 4** and **Algorithm 6**. In our simulations we consider a fast fading scenario where the channel between a given user k and the base station is dynamic with

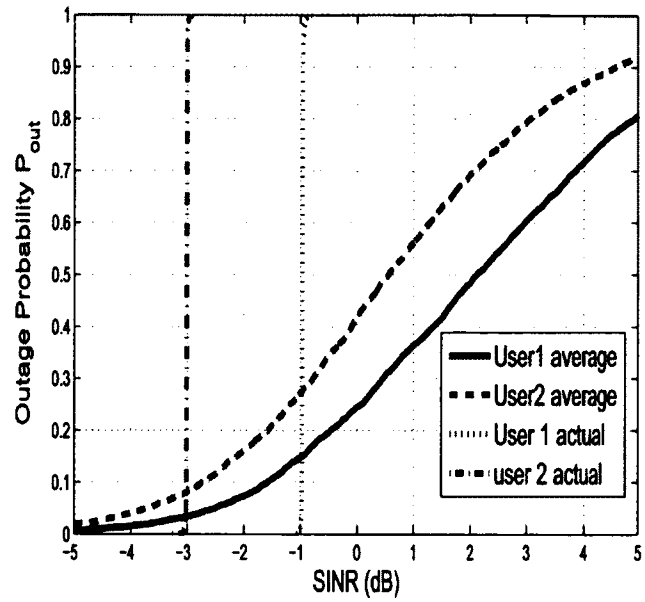


FIG. 18: Outage probability for average and actual downlink channel realizations.

a small coherence time such that it is not practical to estimate channel characteristics for use in conjunction with joint transmitter adaptation and power control, so the average channel parameters are used as discussed in the previous section. Specifically, we consider multipath Rayleigh fading channels where the amplitude scaling h_{ik} for path i of user k channel is a Rayleigh random variable with PDF [59]

$$f(h_{ik}) = \frac{h_{ik}}{\sigma_k^2} e^{-\frac{h_{ik}^2}{2\sigma_k^2}} \tag{V.3.1}$$

where $E[h_{ik}^2] = 2\sigma_k^2$.

We performed simulations to analyze the outage probability for a system wireless with $K = 2$ active users in a signal space of dimension $N = 6$ and with precoders of dimension $M_1 = M_2 = 4$. The precoder and power matrices are initialized randomly, and white noise is considered with covariance matrix $\mathbf{W} = 0.1\mathbf{I}_6$. The algorithm

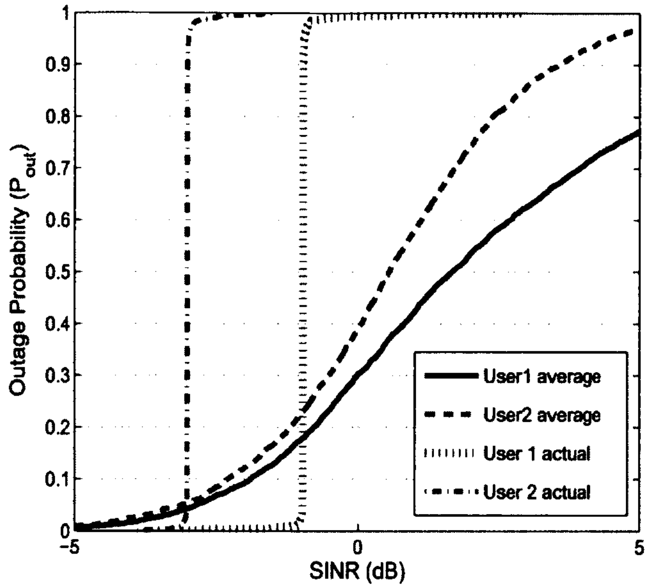


FIG. 19: Outage probability for average and actual uplink channel realizations.

parameters for this experiment are set to $\mu = 0.1$, $\beta = 0.2$, $\epsilon = 0.001$, and the user target SINRs are initialized to $\gamma_k^* = \{0.8, 0.5\}$. The channels matrices are circulant with elements having a Rayleigh distribution.

We start by applying **Algorithm 4** for downlink average channels and **Algorithm 6** for uplink average channels, computing the optimal precoder and power matrices that meet the target SINR requirements. Next, the optimal precoder and power matrices obtained for average channels are used to determine the SINR values for 1,000 independent realizations of the user channels. Results of this experiment are plotted in Figures 18 and 19 which show that the target SINR value directly influences the outage probability as the higher this is the higher the probability of outage will be. For the downlink system model illustrated by Figure 18, we observe that the probability of outage for user 1 with target SINR equal to 0.8(or -0.9691 dB) is close to 15% while for user 2 with target SINR equal to 0.5 (or -3.01 dB) is only 8%. Similarly, for the uplink system model, we observe from Figure 19 that the probability of outage for user 1 with target SINR equal to 0.8 is close to 19% while for user 2 with target SINR equal to 0.5 it is only 5.5%.

V.4 CHAPTER SUMMARY

In this chapter we investigated the performance of joint transmitter adaptation and power control algorithms for downlink and uplink wireless systems with target SINR requirements in the context of fading channels. The algorithms are implemented using the average values of channel parameters for which optimal precoder and power matrices that meet the target SINR values are obtained. These matrices are then used to evaluate the probability of outage using Monte Carlo simulations.

Chapter VI

CONCLUSIONS AND FUTURE RESEARCH

In this dissertation we have provided a comprehensive theoretical analysis of performance optimization over wireless links with operating constraints. The prominent goal achieved by the work presented in this dissertation is efficient utilization of radio resources by joint transmitter adaptation and power control. The proposed algorithms result in wireless systems with highly adaptive capabilities while combating interference and satisfying target Quality of Service requirements. The algorithms are developed in a general signal space framework which makes them applicable to a wide variety of communication scenarios (such as OFDM, CDMA, or multiple antenna/MIMO systems [5,6]).

Downlink transmitter adaptation based on greedy SINR maximization and interference avoidance is presented in Chapter II, and three algorithms are proposed based on a collaborative, "inverse channel"and a matched filter based approach. Fixed point properties of the proposed algorithms yield interesting results for various scenarios, like Welch Bound Equality (WBE) precoder ensembles for ideal channels with white noise at all receivers and point-to-point communication scenarios which yield the same correlations for the "inverse channel" algorithm precoder emsembles as those of the dual uplink scenario.

This is followed by joint transmitter adaptation and power control in downlink wireless systems where a constrained optimization problem is formulated based on codeword norm and target SINR constraints. The proposed **Algorithm 4** minimizes total interference based on the optimal codeword and power ensembles that satisfy necessary and sufficient conditions derived in Chapter III. Simulation results illustrate the tracking ability of **Algorithm 4** for a varying number of active users and for variable SINRs. Convergence speed of **Algorithm 4** is evaluated based on varying algorithmic constants, and all simulations consider the average number of updates of the codeword and power matrices. It is observed that the convergence speed is affected by changes in the increment constant μ, the algorithm tolerance ϵ and increasing number of antennas in the system. The joint precoder adaptation and power control algorithm is proposed for a general framework with linear precoders and block transmissions in downlink wireless systems using a constrained optimization problem that minimizes total sum MMSEs based on target SINR requirements. Extensive simulations of the proposed **Algorithm 5** have confirmed that the incremental updates result in a monotonic decrease of the total sum of MMSEs to its lower bound as illustrated in the cost function convergence plot.

Chapter IV presents a similar study on joint precoder adaptation and power control in uplink wireless systems where a constrained optimization problem is formulated at each active transmitter and transmit precoder and power matrices are jointly optimized subject to specific mathematical constraints implied by constraints on the precoder matrices and by the desired target SIRs. The joint precoder adaptation and power control algorithm is implemented in a distributed fashion until a fixed point is reached where the specified target SINRs are achieved with minimum transmitted power. A detailed discussion of second order optimality conditions in the context of uplink wireless systems has been presented this chapter. The constrained optimization problem is solved using the Lagrange multipliers method [41, Ch. 5], and the bordered Hessian matrix is evaluated at the critical point in order to determine if the precoder and power ensembles satisfying the necessary conditions are also optimal with respect to the constrained optimization problem. Extensive simulations of the proposed **Algorithm 6** illustrate that it always converges to the optimal point where the global cost function is equal to the tight lower bound where the global cost function is defined as the sum of all individual user cost functions. Simulation results illustrate convergence of **Algorithm 6** for SINRs, user powers and cost functions. This was followed by an application of **Algorithm 6** to the uplink of a Cognitive Radio (CR) Network with target values imposed on the signal-to-interference+noise-ratios (SINR) at the CR receiver. Convergence speed of the proposed algorithm for joint transmitter adaptation and power control was evaluated in the context of an uplink cognitive radio network based on the average number of ensemble updates until convergence.

In the previous chapters, transmitter adaptation and power control was achieved based on certain target SINR constraints under the assumption that the wireless channel parameters are known and do not change while an active transmission is ongoing. However, in many instances, it may not be possible to estimate characteristics of the channel due to its time varying nature, and this challenge motivated the work in this chapter, where the average characteristics of the channel in the transmitter adaptation and power control algorithm are used to compute optimal precoding and power matrices with specified target SINR values. Performance of the proposed algorithms for both downlink and uplink scenarios are evaluated using the probability of outage measure which is defined as the probability that the output SINR for a given user k, γ_k, falls below the specified target SINR γ_k^*. Multipath Rayleigh fading channels are considered, and the probability of outage is evaluated through Monte Carlo simulations which show that the target SINR value directly influences the outage probability and the higher this is the higher the probability of outage.

VI.1 FUTURE WORK

This dissertation presents lot of promising results which can be implemented successfully in the design of future wireless communication systems provided some additional theoretical issues are addressed.

Analysis of feedback mechanisms for the wireless system models considered in this work is an interesting area of future work, and it provides scope for various quantization schemes.

Future work on implementation of the proposed algorithms in Cognitive Radio Networks includes analysis of how potential interference between primary and secondary users operating in adjacent channels can be formally incorporated in the constrained optimization problem describing joint transmitter adaptation and power control and how fading channels affect the performance of the proposed algorithm.

Performance of the proposed algorithms in the context of fading channels can be evaluated using additional performance measures such as target satisfaction probability.

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