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An Investigation into the Analysis of Truncated Standard Normal Distributions Using Heuristic Techniques

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AN INVESTIGATION INTO THE ANALYSIS OF TRUNCATED STANDARD NORMAL DISTRIBUTIONS USING HEURISTIC TECHNIQUES

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ABSTRACT

AN INVESTIGATION INTO THE ANALYSIS OF TRUNCATED STANDARD NORMAL DISTRIBUTIONS USING HEURISTIC TECHNIQUES

John Walter Ralls
Old Dominion University, 2014
Director: Dr. C. Ariel Pinto

Standard normal distributions (SND) and truncated standard normal distributions (TSND) have been widely used and accepted methods to characterize the data sets in various engineering disciplines, financial industries, medical fields, management, and other mathematic and scientific applications. For engineering managers, risk managers and quality practitioners, the use of the standard normal distribution and truncated standard normal distribution have particular relevance when bounding data sets, evaluating manufacturing and assembly tolerances, and identifying measures of quality. In particular, truncated standard normal distributions are used in areas such as component assemblies to bound upper and lower process specification limits.

This dissertation presents a heuristic approach for the analysis of assembly-level truncated standard normal distributions. This dissertation utilizes unique properties of a characteristic function to analyze truncated assemblies. Billingsley (1995) suggests that an inversion equation aids in converting the characteristic functions for a given truncated standard normal distribution to its corresponding probability density function. The heuristic for the inversion characteristics for a single doubly truncated standard normal distribution uses a known truncated standard normal distribution as a probability density function baseline. Additionally, a heuristic for the analysis of TSND assemblies building from the initial inversion heuristic was developed. Three examples are used to further demonstrate the heuristics developed by this dissertation.

Mathematical formulation, along with correlation and regression analysis results, support the alternate hypotheses presented by this dissertation. The correlation and regression analysis provides additional insight into the relationship between the truncated standard normal distributions analyzed. Heuristic procedures and results from this dissertation will also serve as a benchmark for future research.

This research contributes to the body of knowledge and provides opportunities for continued research in the area of truncated distribution analysis. The results and proposed heuristics can be applied by engineering managers, quality practitioners, and other decision makers to the area of assembly analysis.

This dissertation is dedicated to my wife, family and friends whom have loved and supported me throughout the course of this research process and whose continual encouragement enabled my success.

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I am thankful for the support of my family and for their patience with me as I worked to complete this dissertation. To my wife Lindsey, thank you for your encouragement and support. To my parents Stephen and Laura Ralls, thank you for keeping me on task and for setting the example. To other family members and friends, thank you for enduring with me on this journey.

Finally, I thank God for providing me the strength, discipline, knowledge and perseverance to finish this work and for providing me with the needed support along the way.

NOMENCLATURE

cdf	Cumulative Distribution Function
pdf	Probability Density Function
Z	Standard Score (i.e., z-score) $= \dfrac{x-\mu}{\sigma}$
USL	Upper Specification Limit
LSL	Lower Specification Limit
n	Sample Size
μ	Mean
μ_t	Truncated Mean
σ	Standard Deviation
σ_t	Truncated Standard Deviation
σ^2	Variance
σ^2_t	Truncated Variance
x	A Random Variable
$\varphi(t)$	Characteristic Function
∞	Infinity
α	significance level

Note – This nomenclature list provides a representative sample of nomenclature used within this dissertation. The scope of this dissertation is not intended to include general, referenced, or other nomenclature common to this field. Please refer to applicable references for nomenclature details beyond the scope of this work.

TABLE OF CONTENTS

Page

Page

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

1.1 Problem Statement

Engineering, manufacturing, financial analysis, risk management, insurance and numerous other industries deal with assembly relationships when assessing their specific areas of interest. Whether that area of interest deals with the assembly of machined parts having an upper and lower specification, financial and portfolio analysis, or analysis variables affecting insurance (e.g., weather conditions, location, age, risk factors, etc.), these areas deal in assessment of truncated standard normal distributions. Numerous probability distributions have been utilized across these fields to accurately describe phenomena readily seen in typical, everyday occurrences.

Research in truncated standard normal distribution assemblies is lacking. As a result, heuristics and analysis methods are limited or non-existent, and the practical application of data or tools in this field is not readily identifiable. The use of the assembly-level truncated standard normal distributions have particular relevance when bounding data sets, evaluating tolerances, identifying quality measures, and for decision makers. Also lacking are assembly-level truncation tables for varying assembled truncated standard normal distributions for two pair combinations.

While an assembly may have numerous parts, the subassembly portions can generally be simplified and reduced to a manageable size. In their simplest form assemblies should be able to be reduced into at least two parts. Therefore, one of the initial problems addressed by this research is focused on providing decision makers a heuristic to analyze the assembled truncated standard normal distributions for two parts. This research question and others are presented in the next section.

1.2 Research Questions

This research is designed to address the following questions:

1. What are the research gaps relative to truncated standard normal distribution analysis and is there an opportunity to address a portion of these gaps?
2. Does the analysis of two truncated standard normal distributions (i.e., assemblies) provide a quality indicator and/or an enhanced understanding of characteristics of truncated distributions with respect to assemblies?
3. To what extent can heuristic techniques be employed to aid in truncated standard normal distribution analysis? What relationships can be inferred from the analysis of truncated standard normal distributions?
4. Can qualitative or quantitative data sets be developed to assist decision makers and/or quality practitioners with an enhanced understanding of truncated standard normal distributions (single and assemblies)?
5. Will correlations, goodness-of-fit, or other testing methods provide meaningful data from truncated standard normal distribution (single and/or assemblies) and other known distributions?

1.3 Research Contributions to the Body of Knowledge

This research addresses important gaps in the body of knowledge including:

- A lack of understanding related to the distribution characteristics resulting from the assembly of two truncated standard normal distribution (e.g., final assembly characteristics between two piece parts for identical TSND).
- A lack of heuristics or other methods/frameworks for engineering managers, quality practitioners and other decision makers.
- The characteristics/relationships between assembled parts utilizing truncated standard normal distributions (e.g., via correlation and regression analysis).
- Qualitative or quantitative data often found in quality tables or other properties for truncated standard normal distributions (using characteristic functions).

This research contributes to the body of knowledge by:

- Providing a practical heuristic based method for characteristic function inversion of a single doubly truncated standard normal distribution.
- Providing heuristic and mathematical formulations associated with assembly-level truncation between at least two distributions.
- Providing an approach to the assembly-level truncated standard normal distribution analysis through the inversion of the distributions assembled characteristic function. This approach provides an alternative method for engineering managers, quality and other practitioners to analyze and respond to process variation decision making.

- Expounding on the relationship between truncated standard normal distributions and their assembly using empirical analysis methods (e.g., mathematical formulation, characteristic function evaluations, heuristics, etc.).
- Providing decision makers and quality practitioners with qualitative and quantitative data for analysis of data sets using truncated distribution assemblies.
- Providing observations and evaluations relative to the additive relationship of truncated distributions (e.g., graphical, by inspection, etc.).
- Providing correlation and regression analysis results for a given truncated standard normally distributed sub-assembly and a truncated final assembly. These forms of analysis aid in identifying relationships between the analyzed distributions.

CHAPTER 2

BACKGROUND OF THE STUDY

2.1 Literature Review Overview

An extensive literature review was performed in the following primary areas of research: truncated distributions, selective assembly, heuristics, and assembly sequencing. The review is primarily centered on my interest in assembly and design. Specifically, this research interest included a review of methodologies that could be utilized by an engineering manager, quality practitioner, or other decision maker. While researching these topics it became evident that assembly methods and sequencing spanned multiple interdisciplinary fields with numerous secondary areas of consideration for this research topic. The primary areas of research that were examined dealt with applications that were associated with assemblies and decision making.

Secondary areas of literature review included tolerance design, optimal target setting, extreme value theory (EVT), storage management systems, inventory management systems, and a limited review of simulation methods. These secondary areas of investigation are addressed in limited capacity in this literature review and provide context and application insight to this research.

Hart (2005) states that research can generally be classified according to its design features and its intended outcomes. Hart (2005) also identifies that the literature review is important because without it you will not acquire an understanding of your topic. The literature review aided in the completion of a comparative review of scholarly works to assess research gaps and to gain insight into TSNDs and other areas of application.

Hart (2005) described the following research techniques which were utilized as part of this dissertation:

- Construction of parameters for the review topic (e.g., literature mapping)
- Identification of issues in research design (e.g., research gap analysis)
- Identification if an approach for the literature review process
- Presentation of methods, fallacies in arguments, and/or identification of other aspects for the literature review process.

The literature review for this dissertation began its focus in three main areas with the purpose of identifying knowledge gaps. The initial focus of my review was on assembly selection and sequencing techniques. Findings from that review were generally reduced to two major areas: assembly selection/sequencing/systems (i.e., physical methods) and applications (i.e., industrial and/or academic application). That review identified and assisted in bounding the context and scope of this research.

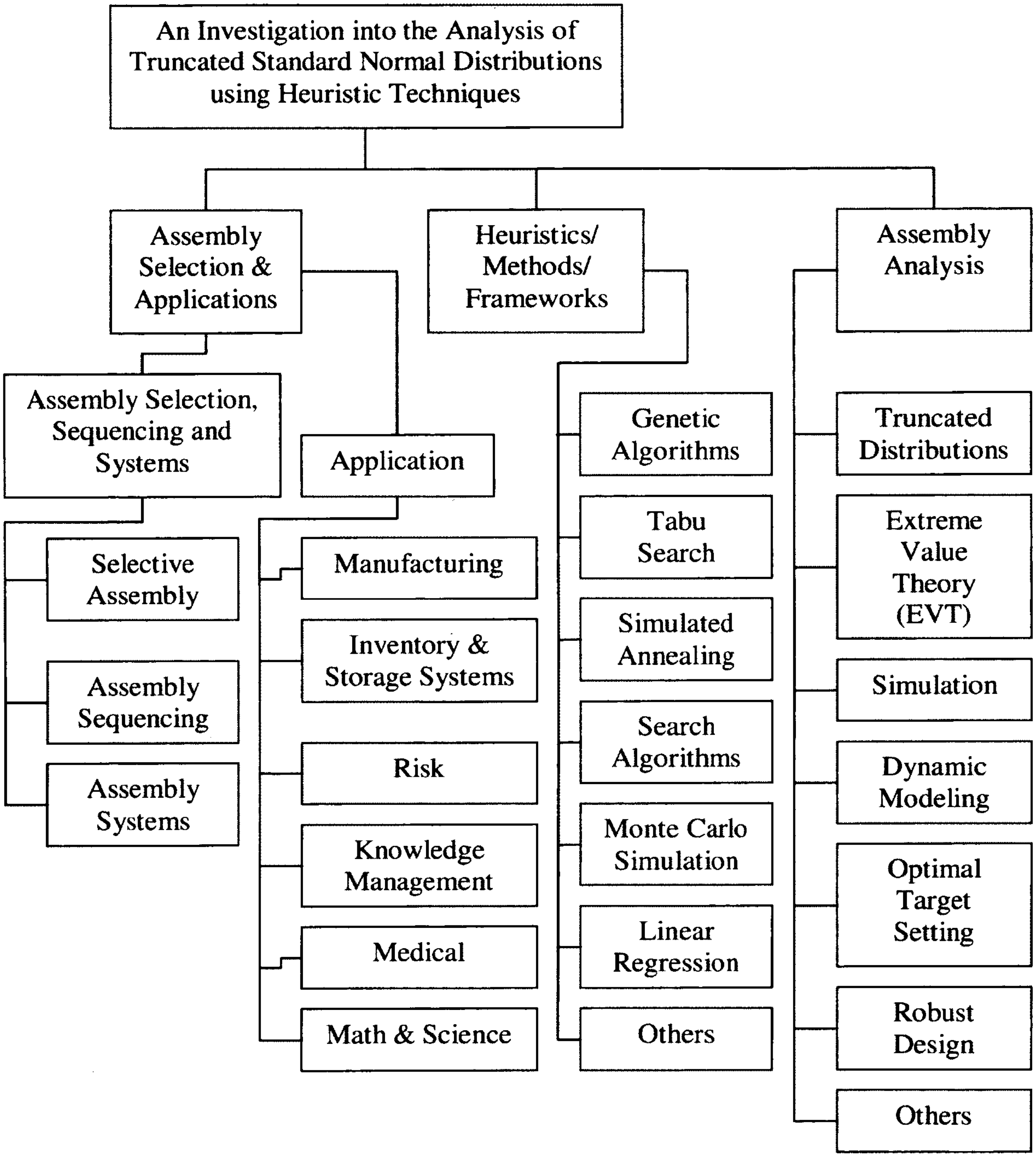
Given the interrelated nature of the literature review the second main area for my review involved the evaluation of heuristics/frameworks/methods used as part of assembly selection. The primary reason for this was to identify decision making, sequencing, or other methods that have been used in various applications and to identify predominant methods used in assembly assessments.

Finally, the most extensive portion of my review and a significant portion of this research centered on the analysis of methods associated with assemblies. The primary focus of this research being with truncated standard normal distribution and their analysis. Secondary insights revealed numerous other analysis methods such as dynamic modeling, EVT, simulation, and robust design techniques.

The literature review identified that a knowledge gap exists relative to the relationship associated with the assembly of truncated portions of standard normal distributions. It also identified the applied use of the characteristic function as a means to determine the probability density for a truncated standard normal distribution. Additional gaps exist relative to comparative analysis of truncations, approximation methods, heuristics and application methods were also evaluated. The literature review that follows identifies a breakdown and high-level review of an extensive sample of scholarly works from this field.

An overview of the literature mapping performed for this dissertation is shown in Figure 1. The research method for this work is addressed in Chapter 3.

Figure 1: Literature Map



2.2 Assembly Selection and Applications

2.2.1 Selective Assembly, Sequencing, Systems and Applications

Work in the area of subassembly design information appears to be very limited. Selective assembly appears to be the predominant literature available regarding assembly selection design. Several works initially appeared to be relevant or near relevant to this field of research, they are:

Whitney (2005) was identified as a scholar in the area of mechanical assemblies. This work is comprehensive and describes the methods of designing workstations and systems for assemblies. Whitney's work provided some insight in subassemblies but focused primarily on mechanical assemblies, part interrelationships, assembly sequencing, design for assembly techniques, and product architecture. The utility of this work in this dissertation came in the form of general assembly insight and enhanced understanding of mechanical assemblies.

Kannan and Jayabalan (2001) proposed a method for lot partitioning using selective assembly groups. They also examined an example of three mating parts with different standard deviations and provided steps for group tolerances of these assembled parts. This particular work did not address assembled parts or associated truncation analysis addressed by this work.

Selective Assembly is a means by which high-precision assemblies may be fabricated from relatively low precision components (Pugh, 1986). In Pugh's conference preceding on the partitioning of selective assembly he introduces the idea of partitioning a component population into groups prior to random assembly. Later Pugh discusses how these selective assemblies can be used to assemble components that could not meet specifications if they were not selected in such a fashion. Pugh (1986) indicates that selective assembly works by dividing component distributions into two or more groups, randomly choosing components and limiting their group creation by discarding groups beyond three standard deviations.

Cittolin (1997) used filter and assembly sequencing methods to group and sequence assembly combination. Review of this literature was limited to applications of methods dealing with the selection of relevant possibilities associated with assembly sequencing minimization. This study did not address truncations. The paper also compares its approach with other methods.

Pugh (1992) identifies the use of statistical selective assembly as a means to produce high-precision assemblies from relatively low-precision components. Pugh (1992) also elaborates on the random selection of components from with a group assembly as a means to meet specification when a group of components has a high variability. In this paper Pugh discusses the systematical truncation and normal distributions in addressing component distributions. Other author such as Desmond and Setty (1961) and Mansoor (1961) have also provided input with regard to selective assembly. Selective assembly partitioning (e.g., truncation) was identified as a primary area of consideration within this dissertation.

In 1994, Malakooti's study identified that one of the problem's in design of assembly line balancing (ALB) dealt with the allocation of work elements. This problem was termed assembly line balancing and specifically documents that the failure of workstations and other unforeseen circumstances can result in unnecessary idling of the production line. This particular study addresses aspects of ALB through the use of single and multiple decision making criteria which included quantities of stations, buffer size, cycle time, and total cost of operation. Assembly line balancing has potential applications of truncation analysis with assemblies. In this work Malakooti also provides several examples with computational experiment results. As a result, it can then be inferred that an applications of truncation analysis toward this knowledge gap would support improvements in the area of assembly line buffering.

As a contrast to Malakooti, Lee 1994 presents a method for the automatic generation of assembly sequencing. Lee's work states that by adjusting the assembly coefficients of subassembly selection indices according to a given assembly environment, an optimal assembly sequence can be generated. Truncation analysis application in the area of assembly planning was not identified by Lee.

So and Scott (1994) studied a production control model for a product comprised of matching components (i.e., a heart valve). The study addressed aspects of part assemblies assuming "N" possible categories. In their study So and Scott identify high level concepts of assembly but did not include aspects or discussion of truncation, EVT, or other specific work assembly methods. A greedy heuristic sequencing rule for other general cases was used by the authors.

Whitney (2006) identifies key characteristics associated with mechanical assemblies, data flow chains and tolerance analysis. His research focuses on utilizing key characteristics for conveying design intent. Whitney (2006) focused on complex assemblies at the design level.

In 2007, Lee and Shin presented a method for the automatic determination of assembly partial orders from a liaison graph representation. This work identified an approach for the extraction of subassemblies. The application of this literature to this dissertation was limited to knowledge gap identification and insight into industry assembly and subassembly methods. Additionally, Agard and Kusiak (2004) utilized data mining algorithms for the selection of subassemblies. Neither of these works appeared to address the knowledge gap addressed by this dissertation.

Kwon, Kim, and Chandra (1999) identified a selective assembly procedure for components composed of two mating parts. While this product focuses on product clearance, the focus of this research dealt with component characteristics of a normal distribution with equal variance. This study presented limited application to truncated portions or assemblies.

De Fazio, Rhee and Whitney (1999) presented an assembly sequence analysis (ASA) for applications involving design-for-assembly (DFA). The paper detailed subassembly partitioning based on criterion based searches. The paper also identified genetic algorithm search techniques for us in assembly sequencing.

Abe, Murayama, Oba, and Narutaki (1999) reviewed part removal verifications associated with disassembly sequences related to assembly planning systems. Their research focused on reducing verification times associated with subassemblies. As part of their research, they employed a genetic algorithm and heuristics to aid in the generation of assembly sequencing. While not specifically focused on truncation assembly analysis, Abe et al. (1999) provided application insight and documentation of industry use of heuristics as part of subassembly analysis.

Lee and Saitou (2007) presented a systematic approach to early product design in order to achieve a cost-effective design. Their work identified that critical dimensions were adjusted through subassembly partitioning as part of the assembly process. The paper also identified that a genetic algorithm was used in selection processes. The application of this literature to this dissertation was limited to knowledge gap identification and insight into assembly sequencing. Truncation analysis methods presented in this work could potentially be applied in this area.

2.3 Assembly Analysis

2.3.1 Truncated Distributions

Work in the area of truncated distributions continues to progress. Research and studies in this area aid quality practitioners, engineers, and decision makers in multiple fields. For example, Johnson and Thomopolous (undated) presented reference tables for use by works for left-truncated normal distributions. Similarly, Khasawneh, Bowling, Kaewkuekool, and Cho (2005) presented greater detail on Truncated standard distributions for singly truncated and doubly truncated cases in two separate scholarly works. In another work Johnson and Thomopoulos (undated) provided a slightly different approach toward addressing an approximation method for doubly-truncated cases using a computer model. None of these works utilized a distributions characteristic function or addressed assembly level distribution approaches.

Dhrymes (2005) developed the moments of truncated distributions in dummy endogenous variable models. An interesting aspect of this study to this research was the approach to normalization of a truncated distribution used within the study. Dummy endogenous variables were also used to address the mean and variance of the distribution.

Finally, the author formulated theoretical equations for determination of the moments of truncated distributions.

In a study in 2003 Ostermeier examined incremental truncations as a method for pairing DNA. As part of this literature review a myriad of industries were included for relevant aspects of truncation and assembly, in this case DNA. A Key point made in this scholarly work was that the experimental determination of the distributions used would require extensive, cost-prohibitive, sequencing. Additionally, the author examined the use of incremental truncation libraries and also a uniform distribution of truncation lengths. The author also provided a comparative review of different truncation methods along with comparison of different DNA truncations.

Horrace (2005) formalized analytical results on the n-dimensional multivariate truncated normal distributions. His paper focused on one-sided truncations at arbitrary points and provided results related to linear transformations along with supporting proofs and mathematical theory. The application of this document was directed toward the field of economics. The specific application of Horrace (2005) to this research was with respect to the comparative review and research gap identification support.

2.3.2 Characteristic Functions and Inversion Theorems

A literature review in the area of Characteristic functions and their inversion was the result of the EVT study. As part of this review the details related to a distributions characteristic function were identified. Relevant equations from this review are identified in Appendix A. Billingsley (1995) provided over-arching support for both characteristic functions and general inversion principles. S. Sheffield (2011) amplified the work provided by Billingsley (1995). Abadir and Magdalinos (2002) provided specific insight into the characteristic function of a singly doubly truncated normal distribution and applications.

Shephard (1992), Kawata (1969), Bernadic and Candel (2012), and Abate and Whitt (1991) provided examples for the application and inversion of a Characteristic function. Inversion principles in these references along with inversion formulas identified by Billingsley (1995) were utilized in Appendix A and adapted for the evaluation of assemblies. Billingsley (1995) and S. Sheffield (2011) identify that characteristic function for the sum of two characteristic functions is the product of their respective characteristic functions (i.e., similar to moment generating functions).

2.3.3 Extreme Value Theory (EVT) and Value At Risk (VaR)

Castillo, Hadi, Balakrishnan, and Sarabia (2005) provided overarching insight in the area of Extreme Value Theory. Use of this resource with a sampling of journal articles and other literature enabled knowledge gap identification and served as grounding for technical fundamentals in the area of truncated distribution analysis (i.e., through principles identified in scholarly reviews regarding EVT).

Raschke (2012) examined right truncation exponential distributions and an estimator for finite sample sizes of truncation points. Raschke also introduced the use of an inverse mean squared error to evaluate the estimator's behavior. Raschke comments on EVT as it relates to truncated distributions as it relates to sample size. Monte Carlo simulations and examples were used by the author to examine different truncation points and sample sizes.

Blanchet and Liu (2012) introduced change of measure techniques for rare-event-analysis of heavy tailed. Monte Carlo simulations were used by the authors to aid in the estimation of rare event probabilities and to present a "good" Markovian approximation of conditional distribution of the rate event being analyzed.

Kuwahara and Mura (2008) used a weighted stochastic simulation algorithm (SSA) and a Monte Carlo simulation method to analyze rare events of biochemical systems. Case studies are used to analyze the proposed method and effectiveness along with an explanation of the proposed algorithm using weight (SSA).

Drees et al. (2005) estimated the far tail portions of distributions functions using EVT as a framework. The authors developed weighted approximations to the tail of the distribution and other empirical data. An Anderson-Darling type test of the null hypothesis was used to demonstrate that the distribution belongs to an EVT domain of attraction.

Using Monte Carlo experiments Stoyanov and Rachev (2007) reviewed the impacts of tail behavior for varying sample sizes (in addition to value-at-risk). The effects on the tail distributions were further analyzed along with the convergence rate as part of their analysis. The authors concluded that a simple tail truncation improves the convergence rate and that asymptotic distribution reliability improves with large sample sizes (e.g., 5000+) for specific cases.

Peng and Qi (2009) studied maximum likelihood estimates of extreme value indices between -1 and -1/2. They also generalized irregular cases and cases of an unknown extreme value index. Peng et al. (2009) in addition to Chavez-Demoulin and Roerhl (2004) provided a general overview on the understanding and application of EVT.

Bermudez and Kotz (2009) examined varying methods for the use of the generalized Pareto distribution (GPD) and their application to estimation methods. This literature focused on applications to EVT and its approach was to review and identify options of GPD parameter estimation. The first paper focused on methods such as maximum likelihood (ML), method of moments (MOM), and probability weighted moments (PWM). The second paper (a continuation) focused on the application of methods to real world data.

Brazauskas and Kleefeld (2009) proposed a method for fitting generalized Pareto distribution (GPD) associated with trade-offs between robustness and efficiency. Using a "trimmed moments" method as a basis the authors used simulations and their method to fit GPD to historical data. Utility was provided following application to areas of risk measurement and ratemaking. The authors utilized a large sample size to provide a mean and relative efficiency between various methods.

Carpinteri, Cornetti, and Puzzi (2005) used extreme value theory in the form of a statistical model to evaluate materials. Prior comparisons using EVT and a Multi-Fractional Scaling Law (MFSL) are used in their evaluation. A model and correlation between for their area of interest is drawn (e.g., fracture energy and crack surface parameters). The authors further used experimental data available in literature to confirm their approach. The utility of this work toward this dissertation was relative to problem solving and decision making approaches.

Brooks, Clare, Dalle Molle, and Persand (2005) examined various EVT models for VaR. The authors used GPD, ML and a semi-nonparametric methodology in their reviews. Comparative analysis was performed by the authors including nonparametric tail index estimates of GPD threshold levels. The relevance of this literature to this dissertation was in application and understanding of sample and comparative approaches. Simulation was further used in data analysis as part of the authors approach.

Debolt, Guillou, and Rached (2005) used a generalized probability weighted moment method (GPWM) to study the asymptotic behavior of estimation tools presented. The authors provided proofs and generalized weighted moment estimators. This work was used in conjunction with Bermudez and Kotz (2009) to better understand research gaps that exist in assembly methods. An understanding of the extremes was intended to better support an understanding of TSNDs.

2.3.4 Simulation

Yanoff and Weirich (2010) discuss the philosophical and epistemological implications of simulation, simulation representation, and policy decisions. The paper argued that simulation is "an important new tool for the social sciences" and that simulation "shares features with both models and experiments." The key purpose of review of this literature was in expanding my breadth of knowledge in the philosophical approaches that could be applied to this research.

Bradley and Gupta (2004) analyzed data associated with the sum of "n" independent non-identically distributed uniform random variables. In this work the authors use Fourier theory to derive an explicit formula for this approach by inverting the characteristic function. This research is one example associated with approaches used in the summation of a uniform distribution. However, no research has been identified in the areas of TSND assembly analysis using characteristic function inversion heuristics.

In a 1999 study Kosfeld and Quinn evaluated storage and retrieval system strategies to improve production throughput capabilities. The study identified that the use of simulation models allowed prioritization and performance prediction for different strategies. From the proceedings of this winter simulation conference, the authors addressed a method of locating empty bins for storage in order to increase throughput. Although, this research did not address subassemblies it did address storage system modeling and throughput. The study base lined simulations using known parameters to benchmark their model. The study then performed throughput simulations to estimate performance improvement from their methods. These approaches were considered when developing the research approach for this dissertation.

Bates, Buck, Riccomagno, and Wynn (1996) identified experimental design and modeling as part of optimization and sensitivity analysis of large systems. The study provides an example for simulation (i.e., emulation) in large system analysis.

Breedis (2001) presented a simplified approach to subassembly design using Monte Carlo analysis. The primary focus of this review was on the author's methods and problem approach. The study identified key variables for evaluation of the simulations performed.

2.3.5 Dynamic Modeling

Dynamic Variation Reduction was developed by R. Musa (2007) as part of a dissertation relating to strategic and dynamic variation reduction for assembly lines. Musa (2007) proposed a method to reduce variation for assemblies by developing inspection plans based on:

- Historical data for existing products, or simulated data for newly developed products
- Monte Carlo Simulation and Optimization Search Techniques
- Sought to minimize the cost function for the total of inspection, rework, scrap, and failure costs

Musa's research developed methods to utilize data in near real time to dynamically reduce variation by assigning the inspected subassembly parts together and he also proposed mating inspected subassembly items through the use of dynamic rolled yield throughput maximization (DTM). Musa (2007) also proposed heuristics for inspection based DTM.

Musa, Sturges, and Chen (2006) identified an inspection methodology for inspection planning using CAD data and simulation. The author proposed a methodology for out-of-tolerance quality characteristics for subassembly. Monte Carlo simulation was used as part of their model development.

Musa and Chen (undated and 2006) presented work on a dynamic throughput maximization study performed after inspection of a batch of subassemblies. This work presented the authors' approach using meta-heuristic algorithms. The study also compared ant colony heuristics to simulated annealing (SA) algorithms. The primary focus of this review was toward a review of the heuristic application used in subassembly design.

Musa, Chen, and Ghoniem (undated) extended previous work from Musa et al. (2006) regarding dynamic variation reduction and throughput via development of a mathematical part matching model for variation reduction. In this study the authors propose a 3-rule heuristic and another model for throughput maximization.

Huang, Liu, and Musa (2004) proposed a method for process plan evaluation to provide rapid evaluation for process plan decision making. The authors approach uses Monte Carlo simulation to aid in the analysis through analysis of deviations assuming normal or uniform distributions. This research did not address truncated distributions or their assembly.

Das and Sarin (1988) used a dynamic programming approach along with a heuristic procedure to address part arrival dates in a multi-job stochastic assembly system. Application of this literature was limited to review of the heuristic approach by the authors.

Seidmann and Tenenbaum (1994) developed a dynamic part-allocation policies for a Flexible Manufacturing System (FMSs) having finite storage capacity. The paper evaluates modeling approaches to evaluate part-routing policies. Additionally, several closed-loop heuristic policies were proposed and provide near optimal FMS performance results. This journal article was examined for application to truncation analysis and part allocation.

Gutierrez, Hausman, and Lee (1995) studied a matching problem and dynamic control rules relative to optimal system performance. The authors proposed a heuristic and provided examples of performance improvements relative to their proposed heuristic. The authors identified a computationally infeasible dynamic programming formulation along with a myopic control procedure for general application to sorting and matching problems.

Selection techniques for a dynamic model framework along with alternative model framework for multistage stochastic programming models were reviewed (Puelz, 2002). The author used empirical test from historical data to benchmark the framework.

2.3.6 Robust Design

Carlson and Doyle (2002) studied aspects of robust design and complexity dealing with highly optimized tolerance (HOT). This particular study focuses on highly structured and robust designs. This work also performed a comparative review by leveraging examples and model systems.

In a 2001 study by Caleb Li and Chou the optimal process mean and associated variables were identified to aid in minimizing the expected quality loss for the works identified parameters. The variables considered by these authors were those quality characteristics typically associated with quality (e.g., smaller-the-better, nominal-the-best, etc). The approach examined direct and indirectly measurable quality characteristics.

2.3.7 Optimal Target Setting

This dissertation reviewed the area of Optimal Target setting for general oversight and applicability to TSNDs. In general the techniques used in optimal target setting could have applicability at the application level. The following articles and summaries expand current knowledge in the area of optimal target setting:

Bouchard, Elie, and Imbert (2010) studied Markovian optimal stochastic control problem under stochastic target constraints. The direct approach was merely reviewed for applicability and bounding of the research gap from this dissertation.

In Yang, Gui, Kong, and Wang (2009) the authors present a quality prediction model for optimal-setting control of a manufacturing process in a metallurgical industry. Yang et al. (2009) identifies the use of a kind of hierarchical strategy for determination of an optimal set point for raw material portioning. The authors compare the efficiency improvement to an example system in an alumina smelting.

In 2003, Bai and Kwong studied the use of target setting values and heuristics to develop "inexact" optimal target settings. In this particular approach the authors utilized a fuzzy optimization model for target value determination and an inexact genetic algorithm was used to solve the problem. Both heuristics and optimal target setting were used as part of this work.

Ohtsubo (2004) evaluated risk minimization for Markov decisions with a target set. Ohtsubo's study considered the risk associated threshold probability along with the passage time for a target set. The paper also identified the use of value iteration methods and presented a policy improvement method (e.g., a heuristic).

Kim, Michekena, and Papalambros (2003) used target cascading to model a multi-level optimization problem. The authors utilized design targets (cascaded to lower levels) by partitioning their problem into small sub-problems. The authors then formulated an optimization model to minimize deviations from their propagated targets. While the authors do not specifically cite the use of heuristics or a specific simulation technique the authors presented a coordination strategy (e.g., essentially a heuristic) to address their problem. The authors took steps toward simplification of their models (e.g., smaller model structures) and to reduce their model and analysis complexity.

Krzysztofowicz (1990) presented a critique of a target setting problem with exponential utility. The framework of this critique was reviewed in consideration as part of this dissertation. The utility and reason for review was primarily with respect to the application and decision steps/considerations as part of the problem formulation and analysis.

In a 2006 work Cooper, Georgiopoulos, Kim, and Papalambros utilized "analytical target setting" to perform target setting within the context of an enterprise. The paper addresses a partitioned decision making process. The paper was reviewed as part of an introspective approach for the comparative review and heuristic development for this dissertation.

Huang, Cheung, and Liang (2006) utilized a multi-agent system to solve for optimal design using analytical target cascading. This approach and methodology was cited as having gained more ground as a methodology for an optimal design approach. The primary use of this literature was to identify other gaps and approaches which may lend insight into the research gaps from this dissertation.

Li's (2004) research focused on optimal manufacturing settings to minimize quality loss for the identified production system. The author's work found that the use of smaller tolerances for both sides or adjustment of the process mean were unsuccessful at minimizing the quality loss. The author also used a pokayoke procedure and truncated quadratic loss function to solve the solution when setting the process mean at an optimal point to minimize the expected quality loss.

Bisgaard (1997) explored the experimental determination of tolerance limits of mating components of an assembled product. Bisgaard provided a functional approach for setting tolerances in assembly when applied to high-volume products. The research concluded that the application may be reasonable for setting tolerances when the data can be reasonably amortized or in higher risk applications. It is important to note that while this study did address tolerance design it did not address the use of truncated distributions in any aspect of its application.

Ramirez-beltran (1995) demonstrated a real-world application of an integer programming problem. The focus of this study was on finding an optimal solution for a labor cost problem. The paper utilized a matrix method for optimization and a branch and bound heuristic algorithm. The author utilized a numerical example to demonstrate the utility of the method (and its effectiveness).

Baykasoğlu (2001) used mathematical programming tools to model multiple objective optimization problems. Baykasoğlu's study cites a trend in industry to solve these types of mathematical problems using heuristic optimization techniques (e.g., Tabu search, genetic algorithm, and simulated annealing). A multi-objective Tabu search heuristic was proposed in this study and results presented demonstrate the proposed heuristics utility.

Nussbaum, Sepúlveda, Singer, and Laval (1998) studied approximation methodologies for sequencing and resource allocation problems. The authors presented a declarative problem solving framework for specification and sequencing problem solving. A focus of this study was on optimization heuristics and procedures and their parameterization.

2.3.8 Other Methods and Applications

Mease, Nair, and Sudjuanto (2004) described a statistical formulation for determination of optimal binning strategies for various loss functions and distributions and compare the results to heuristics. This research provides direct insight into the knowledge gap existing among truncated distribution along with binning and quality applications that could be applied in future expansion of the research presented by this dissertation.

Moorhead and Wu (1998) addressed parameter design methodology by developing a model and data-analysis strategy for a general loss function. The authors presented a methodology that utilized a location-scale model and their study cited approximation as a form of utility in substantiating their model performance. The authors also identified that the utility of their method extended the scope of parameter design of nominal-the-best to include a more general loss function (including subjective interpretation of improved generality).

Xiaoping and Jingjing (2009) presented a model and algorithm for evaluation storage bins for a transport problem. Binning applications have relevance to assembly techniques and the case study presented by this work utilized existing (known) outcomes to approach the idea of storage bin availability improvement. In Xiaoping and Jingjing (2009) also studied the control of optimization methodologies related to storage bin capacity in transport problems. Relevance of these papers to this dissertation was primarily in approaches used for identification and review of the heuristic.

Zhu and Oommen (1997) studied a problem in which a detection function was used to evaluate an object with "N" locations (bins) for the purpose of maximizing resource allocations. One of the main observations from this study is that the target distribution is assumed to be unobservable, where as prior research focus on known target distributions. The relevance of this is in understanding prior approaches considered when attempting to understand evaluations of unknown distributions. Here the authors seek to obtain "good" rather than optimal selection criteria for their process.

Jun, Jacobson, and Swisher (1999) used discrete event simulation to improve patient flow and for resource allocation. The paper used modeling techniques relative to discrete-event-simulation. Liu and Cheung (1997) also studied continuous review inventory models. The focus included review of exponentially distributed variables along with other key operating characteristics for the inventory model. The authors utilized numerical examples to validate their model and provided a level of demonstration of its effectiveness.

Mazzola and Schantz (1995) developed an optimal allocation model of a single facility production environment. Branch and bound heuristics along with Tabu Search heuristics were utilized in their approach. The primary utility of this study was the understanding and review of definitions employed with heuristics employed in the author's research.

Wilson and Roach (2000) identified a methodology for the automatic generation of computerized solutions to the container stowage problem. The methodology presented heuristic rules for "good" but not optimal solutions. The primary focus of this literature review was for application to assembly planning and heuristics. A re-occurring trend in the application of heuristics in these areas appears to be relative to practical application (i.e., good solutions versus optimal).

Pourbabai (1992) utilized a mixed non-Markovian queuing network with infinite capacity nodes to model an automated assembly system problem. The study focused on identifying the minimum required local storages by using a stochastic optimization model and a heuristic algorithm to solve for and approximate results for the simulation study. Pourbabai (1992) also discusses a strategy for the selection of a required amount of local storages for workstations of a flexible assembly line system. While this research is relevant to this research topic, it does not specifically address the research gap identified by this area of research.

Pourbabai (1989) described the design of a finite capacity assembly model and quality control station that used a Markovian queuing system performance model and an optimization model to select optimal storage sizes. This study also utilized a Poisson arrival process as part of the performance model. The paper identifies a simulation model and focuses on observations noted as part of the simulation results; findings presented suggest that explicitly considering random variables dependencies makes performance analyses of a complicated stochastic network difficult.

2.4 Heuristics, Frameworks, and Other Methodologies

Heuristics techniques can be broadly characterized as exploratory problem solving techniques. Merriam-Webster.com identifies the following heuristic definition:

"Involving or serving as an aid to learning, discovery, or problem-solving by experimental and especially trial-an-error methods."

Heuristics is a very broad knowledge base that aids in effective problem solving and serves as a way to "frame new problems" (Michalewicz and Fogel, 1998). A review of Michalewicz et al. (1998) uncovered various useful problem-solving heuristics and approach techniques. The development of a heuristic or improving heuristics could include a variety of techniques found similar to prior heuristic research, such as:

Michalewicz et al. (1998) provided insight in the area of heuristics. This literature provided insight into heuristics such as simulated annealing, tab search, model overviews; various search methods, and other algorithms that served as a foundational basis for numerous aspects of this research. For the purpose of this paper the basis of numerous heuristic definitions were cited from this source.

Chiang, Kouvelis, and Urban (2002) developed optimal and heuristic solutions methodologies for evaluation of workflow interference. The paper focused on application of these methodologies from a facility layout perspective by examining branch and bound heuristics along with Tabu search heuristics. While facility applications are relevant to assembly (i.e., storage of assemblies) the primary utility of this study dealt with application and heuristics approaches.

Lozano, Adenso-Diaz, Eguia, and Onieva (1999) used a Tabu search heuristic in a cellular manufacturing design. The heuristic proposed systematically explored feasible machine cell configurations in part family determinations. The heuristic was benchmarked against two simulated annealing approaches and other heuristics.

A 1997 study by Salhi developed a constructive heuristic for a location problem. The author tested the proposed heuristic against other location problem methods.

Cao and Ho (1987) model a production line with limited storage capacity as a cyclic network with finite buffers. In this analysis Cao and Ho identify a new technique called "perturbation analysis of discrete event dynamic systems." The paper identifies that its main purpose is to investigate perturbation analysis of a closed queuing network with blocking and its application to the optimization of the system throughput in a tandem production line with a finite storage capacity. The simulation results identified that the estimate of the derivative of the throughput and the estimate of the derivative of the time required to complete a finite number of services is unbiased. Finally, the paper also utilized Monte Carlo simulation as a viable method for this optimization approach.

Rochat and Semet (1994) evaluated a vehicle routing problem using two proposed heuristics to find a "good" solution. This study was considered to further evaluate heuristics in a similar application and for evaluation techniques used to compare the heuristic against a baseline configuration.

Naddor (1975) identified heuristic decisions for inventory policy. The heuristic involved knowledge of the mean, standard deviation of demand along with other variables for the model. This brief article provides an overview of the heuristic and limited application.

Park, Kang, and Park (1996) proposed an algorithm associated with integer programming formulation of a bandwidth packing problem. A heuristic was proposed and utilized a column generation technique as part of the algorithm. The authors further tested the algorithm using random problems. Of particular interest is that the authors compare their heuristic to a previously benchmarked method. The authors also provide a brief discussion of "good" vs. optimal solutions.

Patterson and Rolland (2002) explored network design and presented a heuristic with a methodology that utilized an adaptive reasoning technique. The authors also generalized their formulation and measured its effectiveness. The primary utility of this study toward this research dealt with the heuristic approach methodology.

Zhang, Wang, Cheok, and Nee (2003) proposed a knowledge-based selection procedure/rules (e.g., heuristic) to provide a unique name based search mechanism geared toward component reuse (i.e., reapplication).

Meller and Bozer (1996) presented a heuristic (i.e., simulated annealing) for facility layout. The significance of this particular study with respect to this dissertation was the approach method for performance comparison and application utility of the study toward heuristic and algorithm development. This study primarily focused on production facilities and achieving a good solutions for a series of 200 plus problem sets and provided a relatable and practical application and approach for the methods developed by the authors.

Thakur, Nair, Wen, and Tarasewich (2000) used Beam Search (BS) based heuristics to identify optimal or near optimal product lines. The authors test their heuristic on 300 simulated problems with applications. They also compare their search technique with a Genetic Algorithm (GA) based heuristic and conclude that their BS based heuristic is more effective than the GA used in identifying optimal or near optimal solutions quickly. The authors provide examples to illustrate their heuristic and their model.

Coverdale and Wharton (1976) identified an improve heuristic for a Nonlinear Cutting Stock Problem. This particular study focuses on the cutting operations by constraining material cutting patterns to improve residual scheduling via pattern enumeration. The results of the paper also compare heuristic performance using different methods for the analysis.

Rubin (1990) proposed a mixed-integer model and suggested heuristics to obtain a suboptimal but "good" solution to reduce computational cost using linear programming. A linear programming heuristic based method was the second method used as part of this review. The results were compared using a Monte Carlo simulation with Gaussian data.

Kulm (1977) identified that the absences of theoretical or empirical hypothesis raised questions on two different problem-solving heuristics. The critique also raised questions on understanding a clear or consistent meaning of the term heuristic.

Nair, Thakur, and Wen (1995) used beam search heuristics to improve upon prior heuristics for the product line design and selection. Nair et al. used computations from over 400 simulations to demonstrate improvement in five defined performance measures. Their solutions resulted in improve optimality for the design simulation resulting in "good" solutions.

Barish (1962) examined the present and future scopes of management science, operations research, and industrial engineering. The framework and approach used to identify conceptual relations between provided useful backgrounds to the author on similar comparative approaches used in these fields. Application of this literature review was primarily from an introspective approach for this dissertation.

2.4.1 Search Methods (Local and Exhaustive)

Exhaustive search methods are those methods that "check each and every solution in a search space until the best global solution has been found (Michalewicz and Fogel, 1998). Michalewicz et al. also suggest that exhaustive search methods are usually not practical for real world applications due to the large search areas, potential quantity of feasible search possibilities, and uncertainty in obtaining knowing whether the best solution has been found for a given search. They later note that local search methods/algorithms present a more reasonable alternative to exhaustive search techniques for providing satisfying results from defining the current solution, transformation and formulation of a new solution and its merit evaluation, solution exchange or retention, repetition of technique until no transformation improves the current solution.

2.4.2 Algorithm and Optimization Techniques

Michalewicz et al. (1998) identify a Greedy Algorithm as a type of algorithm that attacks "a problem by constructing complete solutions in a series of steps." The simplicity of this type of algorithm lends itself to greater application. They indicate that Greedy Algorithms perform the following:

- Assign Values for all of the decision variables
- Make the best available decision based on an assumed heuristic and available information
- Shortfalls – local optimum at each step may not result in a global optimum

Aggarwal, Orlin, and Tai (1997) explored applications of genetic algorithms to demonstrate the utility of knowledge based mechanisms. Application of this study was limited to understanding the utility of genetic algorithms in a given application and heuristic comparison methods.

A 1994 study by Park and Kim developed a heuristic algorithm to address aspects of production planning problems for an assembly system. The particular focus of this study was on assembly systems operating on a make-to-order basis. In particular, this study utilized packaging examples of automobile subassemblies toward the minimization of inventory holding costs. This review considered the process and application of heuristics which were considered to better understand the type and application of specific heuristics dealing with assembly line systems.

Kannan, Jayabalan and Jeevanantham (2003) utilized genetic algorithms to find the best combination of the selective assembly groups necessary to minimize assembly variation. This method focused on linear assembly. The paper itself focuses on minimizing component tolerances and variation.

Ponnambalam, Aravindan, and Rao (2003) presented a mixed model sequencing problem using genetic algorithms for assembly lines. Their focus was the investigation of genetic algorithms and also performed a comparison of exiting vs. proposed GA's by consideration of variation at multiple assembly levels (e.g., raw materials, product, subassembly, etc.). The dissertation application of this study was primarily focused on the method and heuristic approach by the authors.

Sanderson (1997) used a tolerance model to estimate part configurations based on maximum likelihood using a filter algorithm. Sanderson then stated that the resulting configurations could then be used to evaluate the ability to assemble as it relates to clearance likelihood from the problem constraints. This was also applied to the ability to assemble of subassemblies.

Kwok, Driessen and Phillips (2002) utilized a matching algorithm to address a problem associated with multiple-target-multiple-agent scenarios. The study was primarily focused towards robotics; however, focus was applied to optimal assignment algorithms. The paper also addresses heuristics on a limited basis. Klincewicz (1990) solved a freight transportation problem using facility location techniques. Of specific interest from this review was the method employed by the author in heuristic evaluation. Since facility location problems are potentially derivative of the large assembly sequencing or selection process this paper provided relative insight to support the approach for this dissertation. A flow chart of the basic heuristic model was developed and computational efforts were performed to identify the impacts when compared to a known optimal solution.

2.4.3 Branch and Bound

Branch and Bound is a heuristic that works on the idea of successive partitioning of the search space (Michalewicz et al., 1998). The authors also find that this type of heuristic eliminates areas of interest by evaluating successive partitions of a search space and eliminating a bounded region that does is beyond the constraints of the next branch being compared within the problem. Michalewicz et al. (1998) also note that the heuristic allows for the search to be minimized without performing a detailed analysis of a portion of the problem.

2.4.4 Simulated Annealing

Bohachevsky, Johnson and Stein (1986) was reviewed for initial applicability and potential to this research. Bohachevsky et al. (1986) described generalized simulated annealing for the "optimization of functions having many local extrema" and methods for improved optimums of other problems. This paper identifies simulated annealing as an optimization derived from "the annealing process of metals in which final crystalline configurations are possible depending on the rate of the cooling process."

Ohlemüller (1997) used simulated annealing for solving a minisum location problem. Tabu search was utilized in this study. Efficiency of the method was presented along with results relative to the expected deviation. Finally, the author's study is consistent with other approaches of finding "good" solutions (e.g., vs. optimal).

2.4.5 Tabu Search

Tabu search (TS) is a meta-heuristic that is "based on the premise that problem solving, in order to qualify as intelligent, must incorporate adaptive memory and responsive exploration (Glover and Laguna, n.d.)." In Glover and Laguna's short article in their 1997 book they indicate that the "adaptive memory feature of TS allows the implementation of procedures that are capable of searching the solution space economically and effectively." It is interesting to note that Tabu search heuristics are not memory less like some semi-random search processes. Glover and Laguna (n.d.) identifies that

Fred Glover is generally regarded as the originator of Tabu Search meta-heuristics. Glover's search name "Tabu" is aptly named because the memory attributes "forces the search to explore new areas of the search space (Michalewicz et al, 1998)."

Glover and Laguna (1997) presented one of the earliest comprehensive looks at Tabu search. Given the re-occurrence of Tabu-search in other literature this work was reviewed to gain insight into this meta-heuristic approach and its application to problem solving and decision making.

Glover (1990) examined the characteristics of heuristic procedures used as frameworks for analyzing difficult optimization problems. While the research included the review of several types of heuristics the author focused specific attention on Tabu search heuristics. Glover (1990) discusses four major heuristic methods (e.g., neural networks, simulated annealing, genetic algorithms, and Tabu search). The author also discussed target analysis as a method for determining good decision rules as a means to improve heuristic effectiveness. Markland (1990) summarized glovers work relative to these four major areas.

Punnen and Aneja (1995) studied a resource-constrained assignment problem and developed a Tabu search heuristic. The primary utility of this work to this dissertation was in expansion of heuristic test method understanding. The authors used computational results to demonstrate the effectiveness of their method from other algorithms.

Gendreau, Hertz, Laporte (1996) developed a Tabu search heuristic for a stochastic vehicle routing problem with random demands and probabilities. Tabu search heuristics proposed were compared against a known optimal solution. The authors provided a model confidence factor and average deviation to an optimal solution (e.g., "good" vs. optimal). Similarly, Gendreau, Hertz and Laporte (1994) described a Tabu search heuristic for vehicle routing problem with various restrictions. The heuristic utilized a generalized procedure and performed numerical test on a set of benchmark problems to demonstrate the viability of their heuristic.

Logendran and Sonthinen (1997) developed a Tabu search heuristics and statistical experimentation to present a "good" solution for solving a problem within a flexible manufacturing system. In their work they identify a six part Tabu-search heuristic. The application of this study to this literature review was primarily focused on heuristic development and application overview in the area of flexible manufacturing systems.

Dell'Amico (1996) analyzed the performance of lower and upper bounds for a flow-shop problem with two machines. This study used a Tabu search algorithm and proved the effectiveness of the proposed bounds through computational results. Although this study was focused on machine scheduling the applications relevance was targeted toward the understanding of knowledge gaps related to assemblies.

Moccellin and Nagano (1998) evaluated the relative performance of Tabu search procedures. Their focus was in the area of flow shop sequencing (which has application to assemblies). Moccellin et al. (1998) presented methods to improve heuristics by obtaining an initial solution using the traveling salesman problem and then Tabu search methods to improve the initial solution.

Consiglio and Zenios (1999) presented a multimodal Tabu search procedure with empirical results.

2.4.7 Additional Techniques

Finite-Element and Difference Methods were investigated for applicability and references such as Grieme (2011), Simpson (2008), and [96] Asvadurov, Druskin, Guddati and Knizhnerman (2003) were explored for further relevance to this dissertation.

Brown and Spillane (1989) described a knowledge-based design aid for fabrication of a low-cost boiler component. Of particular interest in this study was that the design approach they used was a pseudo-random search technique to improve the design cost (Brown et al, 1989). Application to this research was focused on the heuristics and their use of "applications" as part of testing their design aid.

Bracker and Pearson (1986) developed a planning process with comparison to a specified area of interest. The authors used multivariate analysis of variance in their determinations. The primary use of this study in this dissertation was relative to gaining insight into approaches and hypothesis testing examples.

Kozan (2000) developed an analytical framework for the examination of inventory strategies for an assembly plant. The model addressed minimization strategy along with material management efficiency. Kozan (2000) leveraged this work off of prior work in the area of vehicle routing problems and used a genetic algorithm in its implementation. Historical data was used to measure the heuristic efficiency.

Phoomboplab and Ceglarek (2007) proposed a design synthesis framework for dimensional management of a multi-stage assembly system. Applications from this work included tolerance optimization, fixture layout, and part-to-part joint design. Of note, this work presented a methodology to illustrate a subassembly design configuration and framework (e.g., heuristic for part assembly).

2.5 Research Hypotheses

The null and alternate hypotheses for this research are:

H_0: No relationship/correlation exists to assess the additive relationship of a truncated standard normal distribution with another identical distribution.

H_1: Analysis of the relationships between additive truncated standard normal distributions and a given truncated standard normal distribution will provide meaningful correlation data.

H_2: Regression analysis between an additive truncated standard normal distributions and a given truncated standard normal distribution will provide meaningful data regarding the relationship between these distributions.

H_3: A heuristic based approach for the analysis of a truncated standard normal distributions using its characteristic function and inversion factor can produce results equivalent to $f(z)dz = \int_{Z_L}^{Z_U} \frac{1}{\sqrt{2\pi}} e^{\left(-\frac{1}{2}*z^2\right)} dz$.

These hypotheses will be tested in a later chapter along with the establishment of a heuristic framework/approach for the assembly of truncated normal distributions, and compilation of a comparative analysis of the subject matter. In addition to these research hypotheses, a comparative review will be performed to identify the dominant methods identified in the literature review along with relevant research gaps.

CHAPTER 3

METHODOLOGY

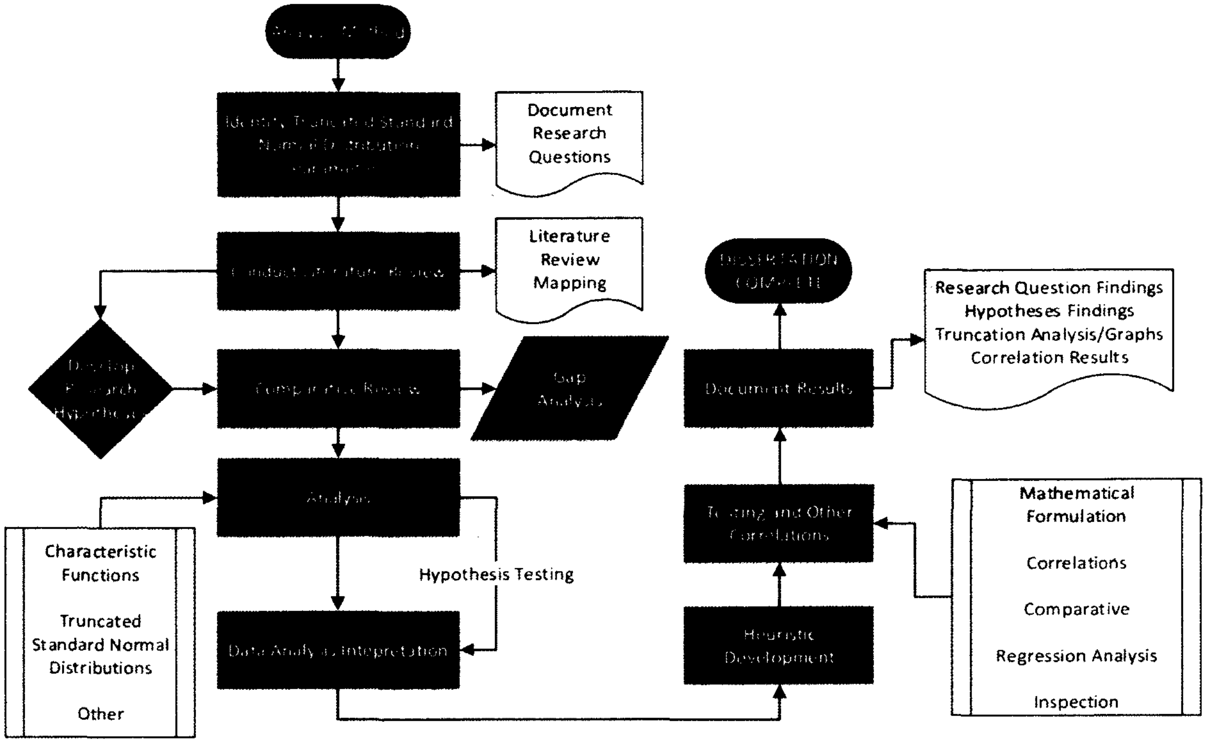
3.1 Research Method Overview

The research approach for this dissertation employs a literature review, comparative analysis, truncated standard normal distribution analysis, application demonstrations, heuristic development and hypotheses testing. An overview of the process employed for this research is depicted in Figure 2.

In this dissertation the literature review follows the initial development of the research questions. These reviews aid in the development of the research hypotheses identified in Chapter 2. Further analysis/comparative reviews aid in the identification of knowledge gaps to substantiate the research hypotheses. Mathematical formulations, correlation and regression analysis tests, along with observation and inspection provide insight into the research questions posed by this dissertation.

This research provides an alternative approach and techniques for solving single doubly truncated standard normal distributions through use of its characteristic function. Mathematical formulations of this phenomenon using an inversion factor are presented in Appendix A. This approach provides new evidence and performs empirical analysis not previously identified by prior work.

Figure 2: Research Process Overview



Eighty-one combinations of a single doubly truncated normal distribution will be evaluated and a baseline inversion factor will be developed to baseline the analysis results to methods identified by Khasawneh, et al. (2005). These combinations were evaluated in 0.1 increments ranging from an USL = 4 to a LSL = -4. Combinations for the assembly of identical doubly truncated standard normal distributions will use the same range with an overall assembly USL =8 and LSL = -8 (i.e. two assembled distributions each with an USL =4 and LSL =-4).

The analysis results are evaluated mathematically and compared against known TSND baselines using correlation and regression analysis. Mathematical inspection and analysis observations provide further quantitative and qualitative data. The results of the analysis will be documented in Appendix B through H, as applicable.

Testing of the research hypotheses is performed following the data analysis. The heuristics and data analysis results serve to "reject" or "fail to reject" the null hypothesis of this research. Alternate hypotheses evaluations were also conducted to determine if there was sufficient or insufficient data to "support" the final conclusions for each hypothesis.

Specifically, this research investigates analysis methods and heuristics for a truncated standard normal distributions' characteristic function and seeks to provide an approach to test the results. The research hypothesis test approach is addressed further in Section 3.4.

The research approach employed by this dissertation primarily utilized a quantitative research along with deductive and inductive modes of reasoning to investigate TSNDs. Creswell (2003) identifies that elements of a quantitative approach involve "reduction to specific variables and hypotheses and questions" in addition to "measurement and observation, and the test of theories." Creswell (2003) also identifies the characteristics associated with deductive and inductive modes of reasoning. The quantitative data analysis techniques utilized in this research include mathematical formulations and their associated statistical analysis. Example data was also generated and evaluated as part of this approach using various analysis techniques. Other evaluations included comparative reviews, data interpretation, and heuristic development. Refer to Sections 3.2, 3.3 and 3.4 for additional details related to research gap identification, TSND analysis and hypothesis testing.

3.2 Comparative Review and Research Gaps

A comparative review was performed of a sample of more than 100 relevant scholarly works in the areas pertaining to assembly, truncated standard normal distributions, heuristics, EVT and other applicable subsets of this dissertation. A comprehensive review of these results is shown in Appendices F and G.

The comparative review aided in classifying and categorizing a representative sample of scholarly works for the purpose of identify truncation methods, heuristics and analysis methods relevant to TSND within the body of knowledge. In general the categorization of comparative review was based on the judgment of the author. Attempts were made to group and categorize literature as objectively as practical. Appendix E categorization information aided in the review and comparative analysis of the literature reviewed. Scholarly works were categorized and grouped based on concepts presented explicitly and implicitly. For example, in some cases a scholarly work may have addressed heuristic steps without explicitly sighting a procedure or approach as a heuristic. As a result, those instances were categorized using good judgment with objective intentions. A primary focus of this review was to identify research and knowledge gaps in this engineering management discipline. The following general areas were analyzed:

- Comparative Review – Selective Assembly/Heuristics/Truncation with specific categorization based on data source.
- Comparative Review Heuristics/Benchmarking/Truncation with specific categorization based on a benchmarking emphasis.

- Comparative Review – Heuristic Type with specific emphasis toward testing methods.
- Comparative Review – Testing Methods & Truncated Assembly.
- Comparative Review – Heuristic Data Sources of Truncated Assembly

The results shown in Figures 3 through 7 and Appendices F and G identify observations of various data partitions for the review variables evaluated. The following observations are made from the data:

- Heuristic procedures represent a knowledge generation method (28% from Appendix G, Table G.1) and widely used problem solving/approach techniques used to expand the body of knowledge. Beyond heuristic procedures, examples/case studies also serve as a widely used and accepted methods for knowledge creation. Of the benchmarking methods identified in Appendix G, Table G.1, 43% were involved heuristics in a broader level of review.
- Statistical means to benchmark quantitative and/or qualitative results (e.g., correlation), and efficiency improvements all represent examples for testing problems in this knowledge area.
- Appendix G, Tables G.2 and G.4 identify that the majority of testing methods identified in the literature review was performed using some form of mathematical computations/model and/or via comparative analysis. Table G.6 identifies heuristics and models as primary analysis techniques for the research.
- Appendix G, Table G.3 reinforces the knowledge gap relative to truncation/selective assembly and heuristics and although data is limited data sources leveraged "example" data as a means of analysis.

- Simulation, historical data, or example data are also widely used data sources for analyses (i.e. Appendix G, Table G.5).

A review of Appendices F and G also shows that in the area of truncated standard normal distribution analysis that there is little data related to heuristics, analysis for truncation of assemblies, and alternative methods for truncated standard normal distribution using characteristic functions. Figures 3 through 7 provide results for comparative review compilations for select areas of focus in this dissertation.

Figure 3: Comparative Review – Selective Assembly/Heuristics/Truncation

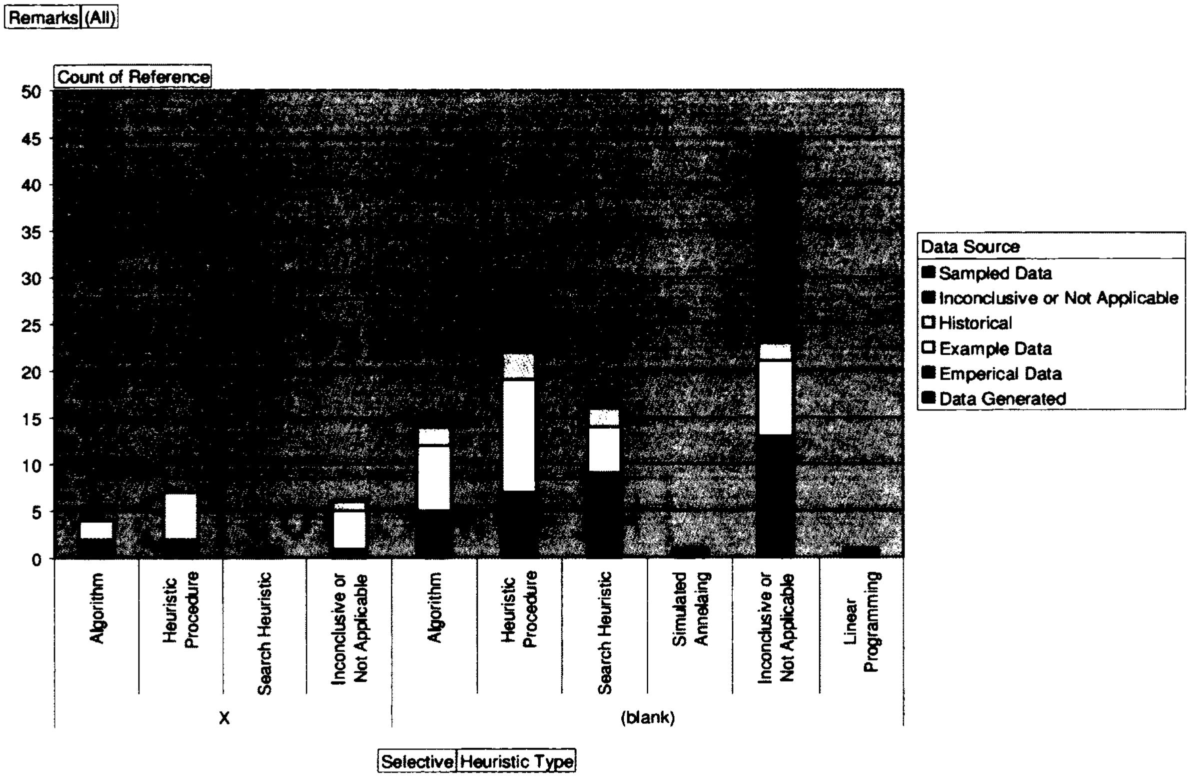


Figure 4: Comparative Review Heuristics/Benchmarking/Truncation

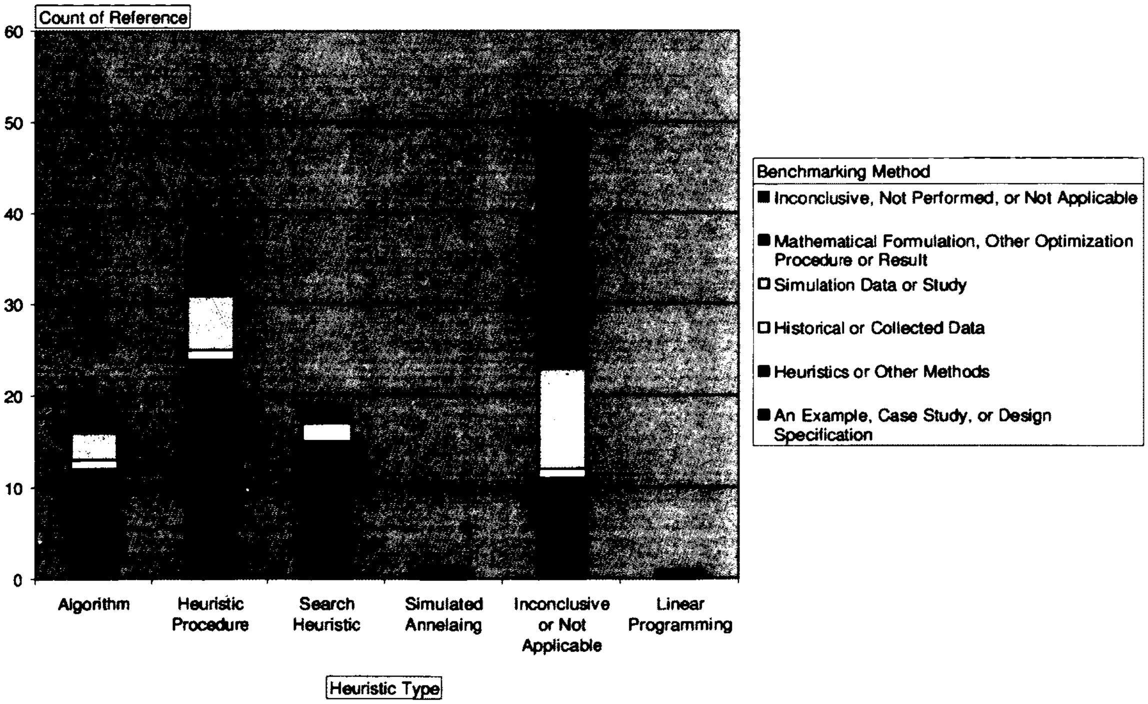


Figure 5: Comparative Review – Heuristic Type & Testing Methods

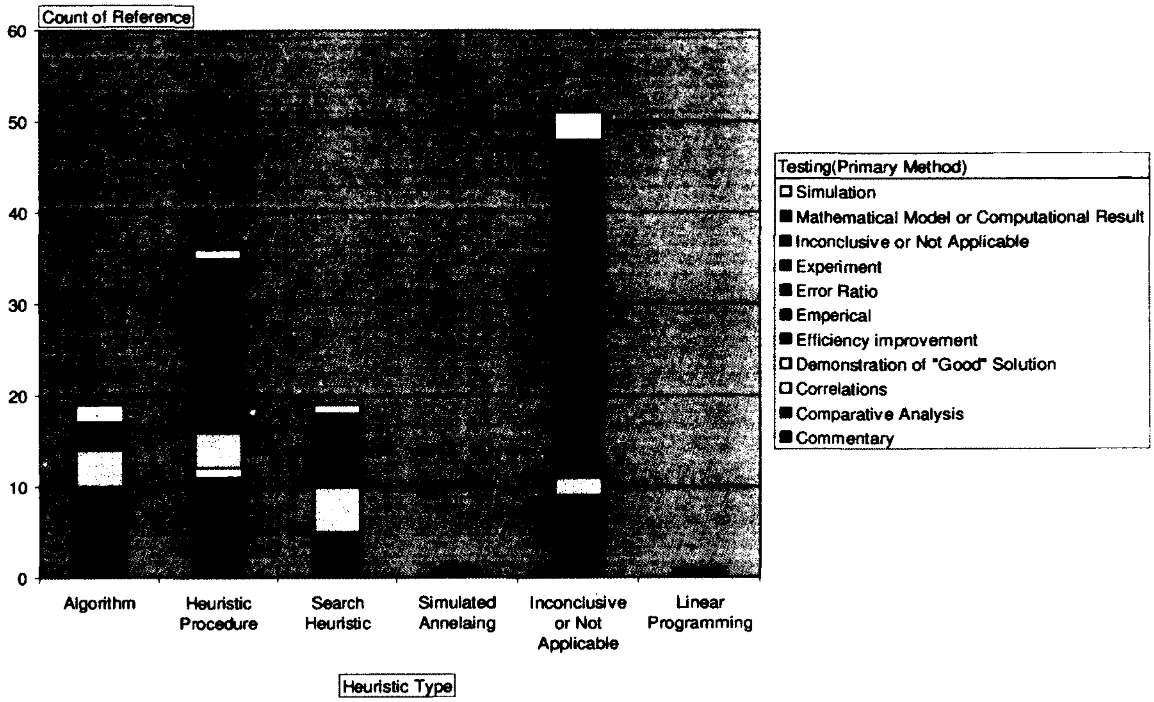


Figure 6: Comparative Review – Testing Methods & Truncated Assembly

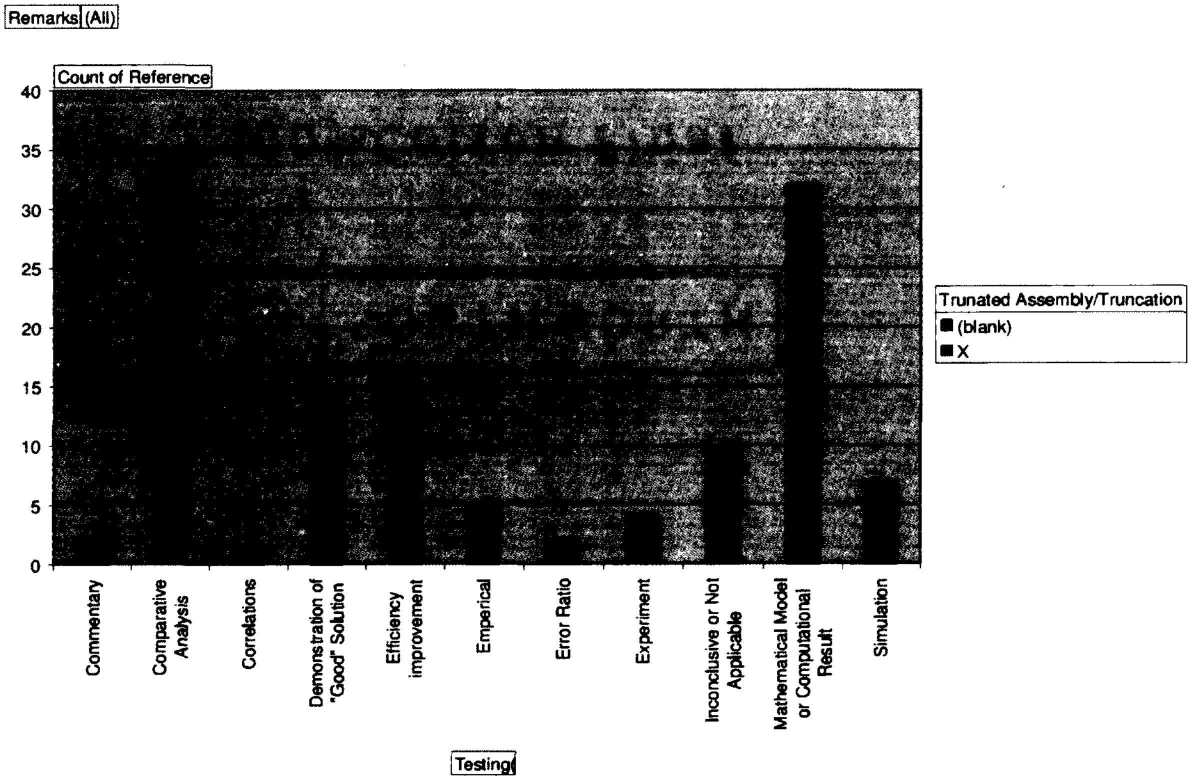
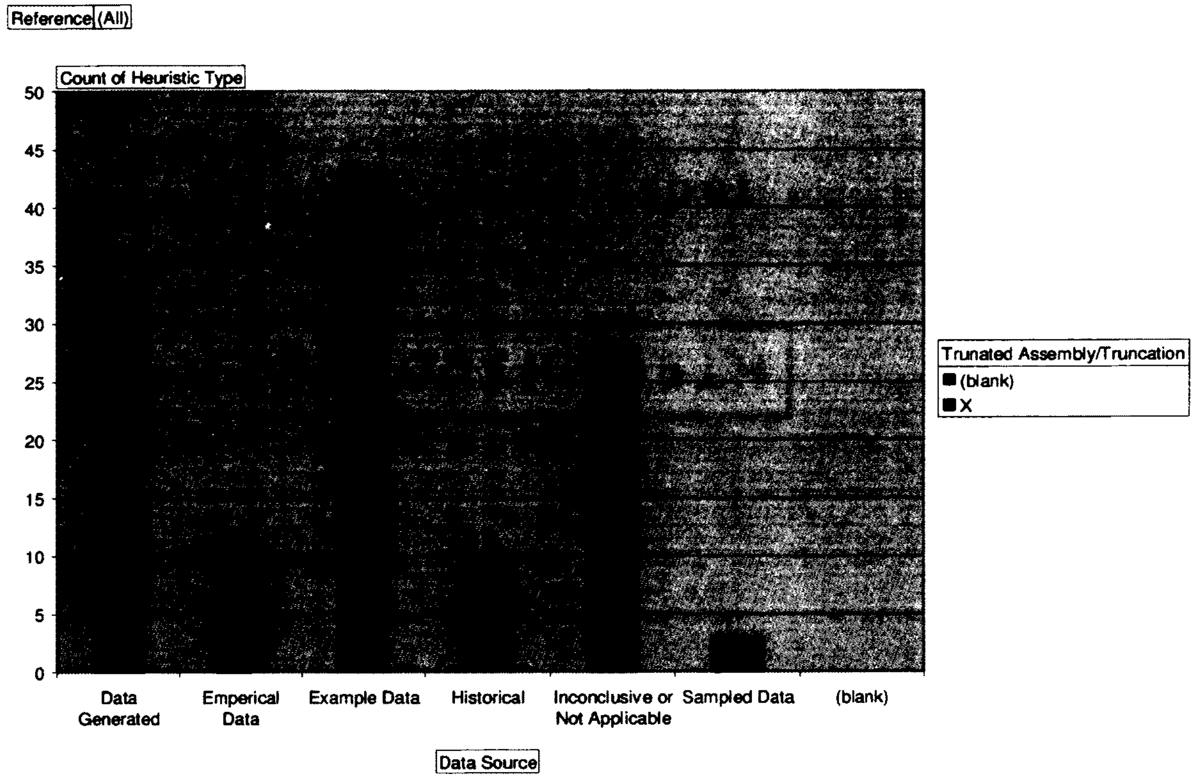


Figure 7: Comparative Review – Heuristic Data Sources of Truncated Assembly



3.3 Truncated Standard Normal Distribution Analysis

One facet of this dissertation research focuses on the analysis of truncated standard normal distributions. As part of this research the literature mapping identified a gap in analytical approaches for the computation of a truncated distribution using a distributions characteristic function. Original work presented by this dissertation provides an empirical basis for the proposed approximation of a truncated standard normal distribution assembly using an inversion factor. Heuristic procedures are developed by this research and documented in Appendices I and J. A summary-level of these heuristics are shown in Figures 8 and 10.

Unique aspects of a distributions characteristic function are leveraged by this research in the analysis of truncated standard normal distributions. P. Billingsley (1995) identifies that for a given "characteristic function φ uniquely determines the measure of μ it comes from." Therefore, it can be inferred that an inversion formula can be used to identify the result of two doubly truncated normal distributions. This research uniquely identifies a means to obtain the result of such an inversion of a truncated standard normal distribution and provides inversion factors for this inversion with a baseline against known truncated standard normal distributions. This research also proposes an evaluation method to compute the result of two assembled truncated standard normal distributions through the use of the inversion of the combination of their respective characteristic functions and proposes the use of inversion factors established for a given truncated standard normal distribution upper and lower specification limit.

Appendix I documents a baseline inversion heuristic for a truncated standard normal distribution from a characteristic function. Appendix J expands Appendix I heuristic at the assembly level (for identical distributions). Figure 9 provides a simplified visual representation of two assemblies with a given upper and lower specification.

Equations and relevant calculations are found in Appendix A. Examples of computational results are found in Appendix D. Analysis results for single and assembled TSND parameters are identified in Appendix B. Section 3.4 addresses the analysis testing (e.g. correlation and regression analysis) that was investigated beyond observations and inspections from the mathematical results.

Figure 8: Heuristic - Analysis for Truncated Standard Normal Distribution

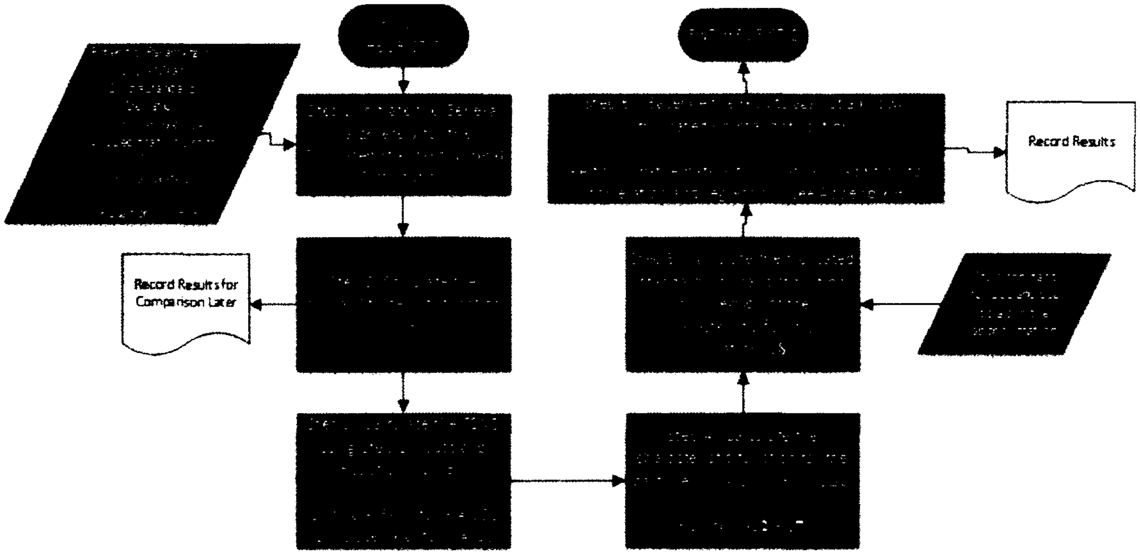


Figure 9: Truncation Assembly-level Example (Simplified)

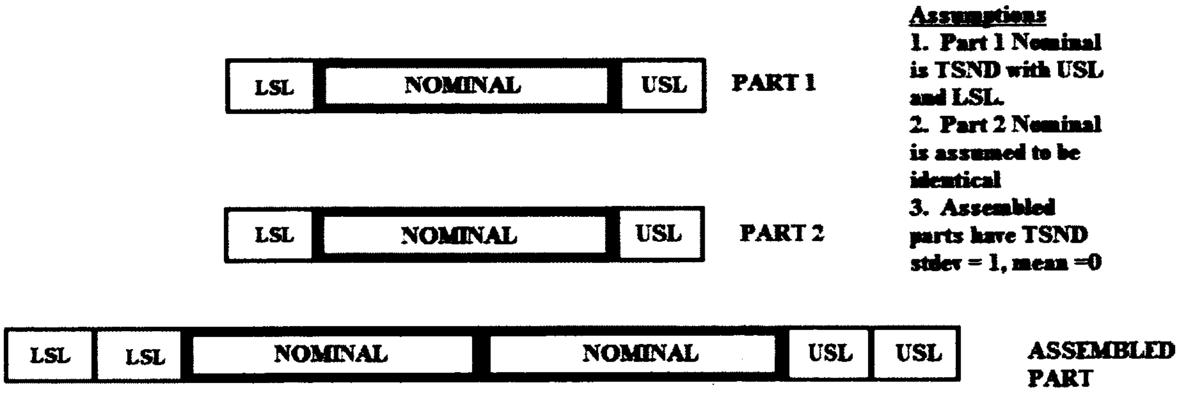
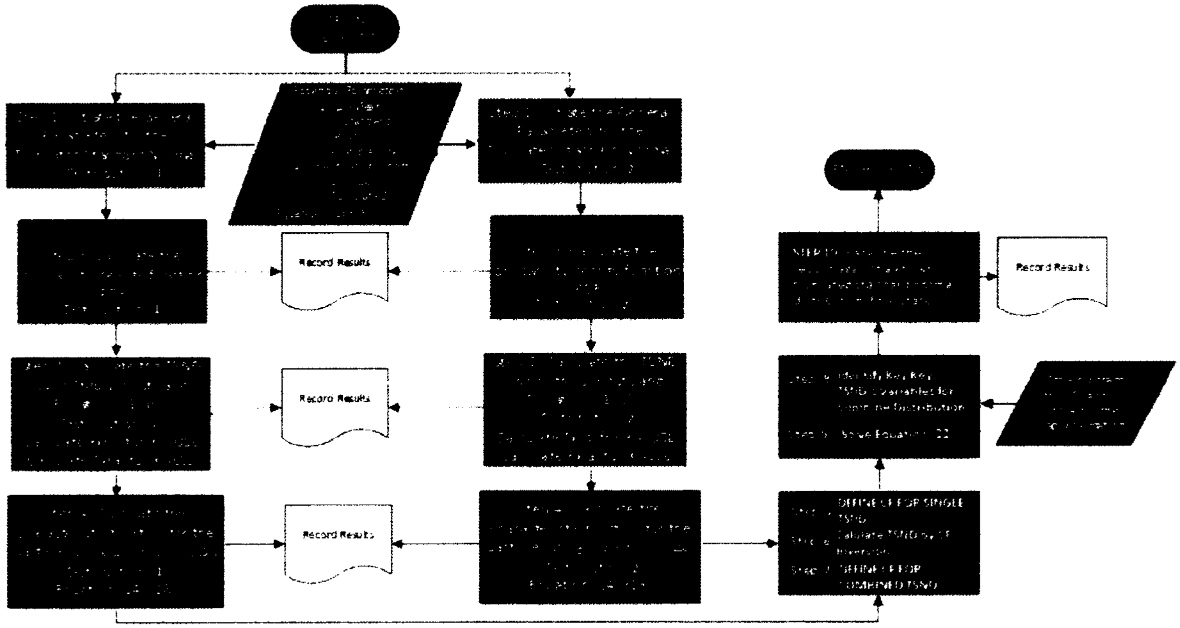


Figure 10: TSND Assembly CF Inversion Heuristic



3.4 Hypothesis Testing

This section outlines the hypothesis testing steps performed as part of this dissertation for each research hypothesis. Various elements of the analysis of the truncated standard normal distributions are performed mathematically and serve as a logical axiom and baseline for this research (i.e., Appendix I heuristic logic). For example, the analysis testing in Appendix H for truncated standard normal distributions (single distribution) using inversion techniques for its characteristic function was established using a known TSND baseline. Logically this method is applied to assemblies by expanding on the mathematical formulations in Appendix A.

This dissertation indentified null and alternative hypotheses to be tested. Mathematical formulation in addition to the structure hypothesis tests aid in the investigation into the analysis of truncated standard normal distributions. This dissertation uses National Institute of Standards and Technology (NIST) 2014 guidance that identifies "the p-value is the probability of the test statistic being at least as extreme as the one observed given that the null hypothesis is true."

Hypothesis tests will be performed for each hypothesis as follows:

1. H_0 will be tested by developing identical distributions for combinations of distributions with specification limits ranging from -4 to 4. Distributions combinations increments will be analyzed at increments of 0.1 per distributions (i.e., 81 combinations of two identical distributions). The assembled distributions will be analyzed in increments of 0.2 for specification limits ranging from -8 to 8. The Appendix A equations were used to identify the characteristic function and other equation inputs and results. Correlation analysis (i.e. Pearson's correlation

coefficient) and regression analysis will be performed to assess the relationship and linear relationship between variables. P-values were also analyzed as part of statistical testing. Statistical testing will be performed with commonly accepted statistical software (Minitab® et al.). The final assessment of this hypothesis will be made following evaluation of the alternative hypotheses. In addition the correlation analysis and regression analysis from those tests would serve to "reject" or "fail to reject" this null hypothesis.

   a. NIST (2014) identifies that "The choice of α is somewhat arbitrary, although in practice values of 0.1, 0.05, and 0.01 are common." As a result, a significance level of $\alpha = 0.05$ was assumed for the analysis performed in this dissertation.
   b. Data results will be reviewed against the evaluation criteria in addition to the results of the alternate hypotheses to evaluate the hypothesis test results.

2. H_1 will be tested by following the generation of identical distributions for combinations of distributions with specification limits ranging from -4 to 4. The distributions combination increments will be analyzed at increments of 0.1 per distributions (i.e., 81 combinations of two identical distributions). Assembled distributions will be analyzed in increments of 0.2 for specification limits ranging from -8 to 8. This hypothesis will be tested by generating the distributions for a TSND (with adjusted standard deviation) and TSND (using Khasawneh et al. 2005 methods) and evaluating these distributions using TSNDs assemblies (using

characteristic function inversion). These distributions will be compared as follows:

   a. Direct comparison by correlation between a TSND (e.g., fT(z) - adju stdev) and TSND assembly based on its characteristic function. Evaluations will be performed using Pearson's correlation coefficient, regression analysis, p-value, and via mathematical formulation and observation/inspection.
   b. Correlations will also be performed between Ratio1 and Ratio 2 as shown in Appendices B and C. Correlations will also be performed between Ratio 3 and Ratio 4 as shown in Appendices B and C. The ratios represent ratios between TSND assembly-level truncated distributions (with a standard deviation of 1 and an alternative which utilizes a standard deviation of square root of the sum of the squares of each distributions standard deviation).
   c. A significance level of $\alpha = 0.05$ was assumed based on NIST (2014) guidance.
   d. Data results will be reviewed against the evaluation criteria to evaluate the hypothesis test results.

3. H_2 will be tested by following the generation of identical distributions for combinations of distributions with specification limits ranging from -4 to 4. The distributions combination increments will be analyzed at increments of 0.1 per distributions (i.e., 81 combinations of two identical distributions). Assembled distributions will be analyzed in increments of 0.2 for specification limits ranging

from -8 to 8. This hypothesis will be tested by generating the distributions for a TSND (with adjusted standard deviation) and TSND (using Khasawneh et al. 2005 methods) and a evaluating these distributions using TSNDs assemblies (using characteristic function inversion). These distributions will be evaluated as follows:

   a. Regression analysis between a TSND (e.g., fT(z) - adju stdev) and TSND (e.g.. fT(z) – assy) assembly based on its characteristic function.
   b. Regression analysis between a TSND (e.g., fT(z) - standard) and TSND (e.g.. fT(z) – assy) assembly based on its characteristic function.
   c. A significance level of $\alpha = 0.05$ was assumed based on NIST (2014) guidance.
   d. Data results will be reviewed against the evaluation criteria to evaluate the hypothesis test results.

4. H_3: A heuristic based approach for the analysis of a truncated standard normal distributions using its characteristic function and inversion factor can produce results equivalent to $f(z)dz = \int_{z_L}^{z_U} \frac{1}{\sqrt{2\pi}} e^{\left(-\frac{1}{2}*z^2\right)} dz$.
   a. This research hypothesis will be verified by demonstrating the results are equivalent. Correlation and regression analysis will further confirm that the values have a strong correlation. Regression models will confirm that the data model equations are equivalent.
   b. A significance level of $\alpha = 0.05$ was assumed based on NIST (2014) guidance.

CHAPTER 4

RESULTS

This chapter will discuss the results of the mathematical formulations, correlations, regression analysis and heuristics developed as part of this research.

4.1 TSND Analysis Results and Heuristics

Appendix A provides a summary of the equations utilized as part of this research to investigate the analysis of truncated standard normal distributions. Appendices I and J provide the final heuristics developed as part of this research. Mathematical axioms are leveraged as part of the formulation applications to truncated standard normal distribution assemblies. Correlation analysis and regression analysis identify relationships between various distributions. These methods of evaluation only aid in identifying distribution relationships between the alternative analysis formulation presented and other methods for a single doubly truncated normal distributions.

4.2 Hypothesis Testing Results

Mathematical formulation and observation along with statistical analysis software (Minitab® et al.) were used to test the research hypotheses of this dissertation. Correlations were used as a means to compare different distribution results and to gain insight into any observations between distributions. Regression analysis was used to provide additional insight the relationship between distribution analysis methods. Hypothesis testing was performed using a significance value of $\alpha = 0.05$ along in conjunction with mathematical formulations, observation, and inspection. While an $\alpha = 0.05$ was specified for analysis testing the results of this dissertation generally indicate that the results were significant to the 0.01 level (e.g., $p < 0.001$).

The testing of each research hypothesis involved data analysis, observation, and interpretation. Hypothesis testing results are documented in an Appendix C, D, and H. The research hypotheses were tested as follows:

H_0: No relationship/correlation exists to assess the additive relationship of a truncated standard normal distribution with another identical distribution.

This null hypothesis is rejected. Mathematical formulation along with correlation and regression analysis performed as part of alternate hypothesis analysis generally indicate a statistically significant and strong positive relationship ($p < 0.001$) for all distributions analyzed (where a p-value could be calculated). Additionally, observations and inspections of mathematical formulations support this conclusion. Regression analysis provides further insight into the relationship between assembly-level truncation analysis (using two different methods). R-values ranging from 99.13% to 100% for cubic line model plots further support this conclusion. Appendices B and C document the results and other corresponding analysis.

H_1: Analysis of the relationships between additive truncated standard normal distributions and a given truncated standard normal distribution will provide meaningful correlation data.

Mathematical formulations presented in Appendix A along with correlation analysis testing results support alternate hypothesis H_1. Correlation data generally indicates a statistically significant and strong positive relationship ($p < 0.001$) for all analyzed distributions (where a p-value could be calculated). Meaningful results are defined as either a statistically significant relationship, positive correlation/relationship, or any other observed, calculated, or identified parameter which provides data or indications not previously understood by the body of knowledge. Additionally, observations and inspections of mathematical formulations support this conclusion. Appendices B and C document the results and other corresponding analysis.

H_2: Regression analysis between additive truncated standard normal distributions and a given truncated standard normal distribution will provide meaningful data regarding the relationship between these distributions.

Mathematical formulations presented in Appendix A along with regression analysis testing results support alternate hypothesis H_2. Regression analysis generally indicates a statistically significant and strong positive relationship ($p < 0.001$) for all analyzed distributions (where a p-value could be calculated). Meaningful results are defined as either a statistically significant relationship, positive correlation/relationship, or any other observed, calculated, or identified parameter which provides data or indications not previously understood by the body of knowledge. Additionally, observations and inspections of mathematical formulations support this conclusion. Appendices B and C document the results and other corresponding analysis.

H_3: A heuristic based approach for the analysis of a truncated standard normal distributions using its characteristic function and inversion factor can produce results equivalent to $f(z)dz = \int_{Z_L}^{Z_U} \frac{1}{\sqrt{2\pi}} e^{\left(-\frac{1}{2}*z^2\right)} dz$.

Mathematical formulations presented in Appendix A along with Appendix H analysis support alternate hypothesis H4. Appendix H results confirm that the baseline mathematical formulations via correlation and regression analysis. Appendix H regression results confirm that the results of the equations for a given X value are equivalent. This is confirmed graphically as well as through the examination of the fitted line plot equation for cubic model from the regression analysis. In addition to these results the correlation and regression analysis generally indicates a statistically significant and strong positive relationship ($p < 0.001$) for the distributions analyzed (i.e., -1 to 1, -2 to 2, -3 to 3, and -4 to 4). Additionally, observations and inspections of mathematical formulations support this conclusion. Appendix H documents the results and other corresponding analysis.

4.3 Simulation Examples

Examples of mathematical formulations used in this dissertation were developed using industry software (i.e., NtRand). This industry software was utilized for the purpose of generating three examples of random data sets with a population of 10,000 samples for a given USL, LSL, standard deviation, and mean. These data sets were then analyzed using mathematical formulations presented in Appendix A and using statistical software (Minitab® et al.).

The simulations performed were for sample distributions generated from -4 to 4, -3 to 3, and -2 to 2. Identical truncated standard normal distribution assemblies were used for each analysis of assemblies. Combination of these assemblies was performed using the distributions characteristic function. Results of this analysis are found in Appendix D. A summary of the results is also identified below in Tables D.1-D.9.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

For engineering managers, risk managers and quality practitioners, the use of the standard normal distribution and truncated standard normal distribution have particular relevance when bounding data sets, evaluating manufacturing and assembly tolerances, and identifying measures of quality. In particular, truncated standard normal distributions are used in areas such as component assemblies to bound upper and lower process specification limits. This research provided an alternative approach to the analysis of TSNDs using an inversion factor and applied that insight to address the relationship of truncated distributions.

Heuristic procedures were developed to characterize the approach of this dissertation along with mathematical formulation and data analysis. The heuristics, correlations, regression analysis and other investigations performed provided additional insight into these distributions. Appendix A also documents the equations that form a part of the heuristic procedures in Appendices I and J. Additionally, truncation assembly data was provided in Appendix B to address two pair TSND combinations.

This dissertation presents a heuristic approach for the analysis of assembly-level truncated standard normal distributions. Specifically, this dissertation utilized the unique properties of a distributions characteristic function as method for the analysis of truncated assemblies. A comparative review was performed to aid in the identification of traditionally accepted analysis and evaluation methods dealing with part truncation.

In addition to the mathematical formulations for TSND assemblies in this dissertation practical application of the theory was also presented. Three examples of varying specification limits for a sample size of n = 10,000 were developed to reinforce the research framework presented. The analysis results for these examples are presented in Appendix D.

In general, the mathematical formulations performed in conjunction with the correlation and regression analysis results support the alternate hypotheses of this research. The approach presented also provides a framework and baseline for future efficiency and heuristic improvements along with conceptual expansion toward the potential application to other distributions.

5.1 Research Question Conclusions

The research questions, literature review, comparative analysis, TSND analysis, hypothesis testing, and other evaluations assisted with interrogatory review. The following statements and conclusions are provided:

Research Question 1: What are the research gaps relative to truncated standard normal distribution analysis and is there an opportunity to address a portion of these gaps?

- This question poses a contextual question aimed at addressing TSND research gaps. The question was posed as a means to narrow the focus of this research (relative to assemblies) and to initiate a framework for future expansion of this work. The literature review and comparative analysis results confirmed the existence of research gaps as compared to the sample population of scholarly works reviewed.

Research Question 2: Does the analysis of two truncated standard normal distributions (i.e., assemblies) provide a quality indicator and/or an enhanced understanding of characteristics of truncated distributions with respect to assemblies?

- This research question focused on the analysis of two truncated standard normal distributions as a means to gain insight into assemblies. An assembly in its simplest form contains at least two pieces. This is important to engineering managers and other decision makers as it serves as the foundation for understanding more elaborate assemblies. Baseline and assembly level TSND mathematical formulations along with correlation and regression analysis provide insight into the relationships analyzed.

Research Question 3: To what extent can heuristic techniques be employed to aid in truncated standard normal distribution analysis? What relationships can be inferred from the analysis of truncated standard normal distributions?

- As identified earlier in this dissertation a heuristic serves as an aid for learning, discovery and problem-solving. The use of heuristics was considered as a method of knowledge generation. Development of a "heuristic" provides a method for which analysis of truncated standard normal distributions could be performed by the practitioner. Heuristics provide a method of solving problems.

Understanding the TSND analysis relationships also serves as a benchmark for future efficiency improvement or expanded evaluations and comparisons.

Research Question 4: Can qualitative or quantitative data sets be developed to assist decision makers and/or quality practitioners with an enhanced understanding of truncated standard normal distributions (single and assemblies)?

- This question was initially focused on capturing a framework of assemblies and single truncated analysis using CF. Qualitative data would come from a "comparative review" or possible graphs whereas quantitative data is apparent in the analytical portions of the Appendices in this dissertation. Both of these approaches provide practical methods of enhancing TSND knowledge by a practitioner.

Research Question 5: Will correlations, goodness-of-fit or other testing methods provide meaningful data from truncated standard normal distribution (single and/or assemblies) and other known distributions?

- Correlation and regression analysis testing was performed in addition to the mathematical formulations, observations, and data inspections of TSNDs. Statistically significant strong positive relationships were identified in analyses performed. Regression analysis and correlation analysis for various ratios of assembly distributions were also evaluated for normal distributions. The test methods presented (e.g. regression analysis) aid in identifying relationships between distributions analyzed. Further evaluations beyond TSND distributions were considered outside the scope of this work and provide an avenue for future research in this area.

5.2 Research Assumptions and Limitations

This research includes various assumptions and limitations that form an integral part of the research. The following assumptions and/or limitations apply to this research:

- This research focuses on truncated standard normal distributions. While this phenomenon generally exists in many engineering, financial and related industries it is important to also understand that that sample distributions may be normal even though the population as a whole may be better characterized by another distribution. This limitation could also be the focus of future research in this field.
- General statistical analysis tools were utilized in this research (e.g., Minitab®, NtRand, etc.); however, this software is assumed to be a reliable tool used within industry that provides consistent and repeatable results.
- This research scope was limited to the evaluation of identical doubly truncated standard normal distributions.
- Sample size evaluations were limited and represent a future research opportunity to provide additional research fidelity and improved accuracy through focused sample sizes in specific truncation areas of evaluation (e.g. sample sizes with increments smaller than 0.1 or 0.2).
- Statistical Significance values assumed an $\alpha = 0.05$ as a generally accepted significance level per NIST (2014).

- For the purposes of evaluating "$f_T(z)$ - adj. stdev", a $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2} = 1.414214$ was assumed based on similar guidance by Weisstein, E.W (1994-2014).
- The research analysis was limited to mathematical formulation, correlation analysis, and regression analysis. As a result statistical analysis and significance (e.g., $p < 0.001$) does not imply that x causes y.
- It is not the intent of this research to attempt to characterize the population or variations, permutations or other circumstances that may exist in nature.
- Mathematical formulations assumptions were based on mathematical axioms concerning the baseline inversion of a CF using a typical TSND and its application.

5.3 Future Research Opportunities

Elements of this dissertation research provide various opportunities for continued or further research in the area of truncated distribution analysis. While this research focused on the analysis of truncated standard normal distributions expansion of this work toward the evaluation of other distributions could be considered. This research could also be further expanded by:

- Enhancement and improvement of the heuristics developed by this work.
- Refinement of the data as a function of sample size.
- Evaluation of the application of normalization concepts to concepts presented.
- Investigation into the inversion factors for alternative distributions (e.g., Weibull).

- The research analysis was limited to mathematical formulation, correlation analysis, and regression analysis. Alternative analysis methods could be considered to further investigate the analysis of TSNDs.
- Expansion to part binning and storage assembly of truncated piece parts.
- Further expansion into mathematical inversion of CF beyond the concepts presented in this research.
- Expansion of heuristic approach to include search techniques such as Tabu, beam, and/or other heuristic techniques.
- Expansion of comparative reviews to identify interrelationships between various methods (e.g. benchmarking, testing, heuristic type, etc).

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APPENDICES

APPENDIX A: EQUATIONS

This appendix documents the equations utilized as part of the truncated standard normal distribution analysis used by the research. This section provides general and specific equations used for the evaluation of truncated standard normal distributions and corresponding assemblies. The following equations were generally or specifically applied in this dissertation and heuristic procedures documented in Appendices I and J.

General Equations:

$$f_T(z) = \int_{z_L}^{z} \frac{f(z)}{\left(\int_{z_L}^{z_U} f(z)dz \right)} dz \text{, } z_L \le z \le z_u \qquad \text{(EQUATION 1)}$$

Equation 1 Reference: Khasawneh et al. (2005)

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{\left(-\frac{1}{2} * z^2\right)} \qquad \text{(EQUATION 2)}$$

Equation 2 Reference: Khasawneh et al. (2005)

$$f(z)dz = \int_{Z_L}^{Z_U} \frac{1}{\sqrt{2\pi}} e^{\left(-\frac{1}{2} * z^2\right)} dz$$

$$f(z)dz = \int_{Z_L}^{Z_U} \frac{1}{\sqrt{2\pi}} e^{\left(-\frac{1}{2} * z^2\right)} dz \qquad \text{(EQUATION 3)}$$

Equation 3 Reference: Khasawneh et al. (2005)

$$z = \frac{x - \mu}{\sigma} \qquad \text{(EQUATION 4)}$$

Equation 4 Reference: Khasawneh et al. (2005)

$$\mu_{T_1}(z) = \int_{z_{L_1}}^{z_{U1}} z f_{T1}(z)dz \qquad \text{(EQUATION 5)}$$

Equation 5 Reference: Khasawneh et al. (2005)

$$f(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\left(\frac{(x-\mu)^2}{2\sigma^2}\right)} \qquad \text{(EQUATION 6)}$$

Equation 6 Reference: Johnson et al.(undated) and Billingsley (1995), adapted

$$\varphi(t) = \varphi_x(t) := E\left[e^{itX}\right] = \int_{-\infty}^{\infty} e^{itx}\mu(dx)$$

(EQUATION 7)

Equation 7 Reference: Billingsley (1995)

$$\left[e^{itX}\right] = \cos(t) + i\sin(t)$$

(EQUATION 8)

Equation 8 Reference: Sheffield (2011)

$$\varphi_y(\tau) := \frac{1}{F_x(b) - F_x(a)}\int_a^b f_x(u)e^{iu\tau}du$$

(EQUATION 9)

Equation 9 Reference: Abadir et al. (2002)

$$\varphi_{X+Y} = \varphi_x\varphi_y$$

(EQUATION 10)

Equation 10 Reference: Sheffield (2011) and Billingsley (1995), adapted

$$\varphi(t) = \int_a^b f_x(u)e^{iu\tau}du \ = e^{iu\tau - \frac{\sigma^2 t^2}{2}}$$

(EQUATION 11)

Equation 14 Reference: Srinivasa Varadhan (2000)

CF Inversion Equations:

$$\varphi(\tau) = \frac{1}{F_x(b) - F_x(a)}\int_a^b f_x(u)e^{iu\tau}du$$

(EQUATION 12)

Equation 15 Reference: Adapted from Equation 9 with slight nomenclature change

$$\varphi(\tau) = \frac{e^{iu\tau - \frac{\sigma^2 t^2}{2}}}{F_x(b) - F_x(a)}$$

(EQUATION 13)

Equation 16 Reference: By inspection a combination of Equations 11 and 12

$$f_t(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-itx}\varphi(\tau)dt$$

(EQUATION 14)

Equation 17 Reference: Billingsley (1995)

$$f_t(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-itx}\left(\frac{e^{iu\tau - \frac{\sigma^2 t^2}{2}}}{F_x(b) - F_x(a)}\right)dt$$

(EQUATION 15)

Equation 18 Reference: By inspection the incorporation of Equation 13 into Equation 14

$$f_t(x) \approx \frac{1}{2\pi}\left(C_{Tc}\right)\left(\frac{e^{\frac{\sigma^2 x^2}{2}}}{F_x(b)-F_x(a)}\right), \text{ where } C_{TC} = \frac{1}{\sqrt{2\pi}} \qquad \text{(EQUATION 16)}$$

Equation 16 Reference: Solved by the author by setting Equation 1 equal to Equation 15. C_{TC} = Equation 15 results/ Equation 1 results for a given z value. See Equation 17

$$f_t(z) \approx \frac{1}{2\pi}\left(C_{Tc}\right)\left(\frac{e^{-\frac{\sigma^2 z^2}{2}}}{F_x(b)-F_x(a)}\right) \approx \int_{z_L}^{z} \frac{f(z)}{\left(\int_{z_L}^{z_U} f(z)\,dz\right)}dz \qquad \text{(EQUATION 17)}$$

, where $C_{TC} = \dfrac{1}{\sqrt{2\pi}}$

Equation 17 Reference: Set Equation 16 equal to Equation 1. This baselines this equation by this author and identifies C_{TC}

For an Assembly:

Given Equation 14

$$\text{And } \varphi_z(\tau) := \left(\frac{1}{F_x(b)-F_x(a)}\int_a^b f_x(u)e^{iu\tau}du\right)\left(\frac{1}{F_y(b)-F_y(a)}\int_a^b f_y(u)e^{iu\tau}du\right) \qquad \text{(EQUATION 18)}$$

Equation 18 Reference: Solved by applying equation 10 and 12 and via inspection. $\varphi_z = \varphi_{x} + \varphi_y$

$$\text{Then } f_t(z)_{assy} = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-itx}\left[\left(\frac{e^{iu\tau-\frac{\sigma^2 z^2}{2}}}{F_x(b)-F_x(a)}\right)_{d1}\left(\frac{e^{iu\tau-\frac{\sigma^2 z^2}{2}}}{F_x(b)-F_x(a)}\right)_{d2}\right]dt \qquad \text{(EQUATION 19)}$$

Equation 19 Reference: Solved by applying Equations 14 and 21 and via inspection. Where d1 is distribution one and d2 is distribution two.

$$\therefore f_t(z)_{assy} = \frac{1}{2\pi}\left(\frac{1}{\left(F_x(b)-F_x(a)\right)_{d1} * \left(F_x(b)-F_x(a)\right)_{d2}}\right)\int_{-\infty}^{\infty} e^{-itx}\left[\left(e^{iu\tau-\frac{\sigma^2 t^2}{2}}\right)_{d1}\left(e^{iu\tau-\frac{\sigma^2 t^2}{2}}\right)_{d2}\right]dt \qquad \text{(EQUATION 20)}$$

Equation 20 Reference: Continuation of Equation 19

$$\text{Where } f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-itx}\left[e^{iu\tau-\frac{\sigma^2 t^2}{2}}\right]dt = e^{-\frac{\sigma^2 t^2}{2}} \qquad \text{(EQUATION 21)}$$

Equation 21 Reference: Applied from Billingsley (1995), Equation 11, and Equation 14. For comparison $\sigma=1$ would result in a inversion of CF similar to Billingsley (1995) Equation 26.21. Adjust for z.

$$\therefore f_t(z)_{assy} \approx \frac{1}{2\pi}(C_{Tc})\left(\frac{\left(e^{-\frac{\sigma^2 t^2}{2}}\right)_{d1}\left(e^{-\frac{\sigma^2 t^2}{2}}\right)_{d2}}{(F_x(b)-F_x(a))_{d1} * (F_x(b)-F_x(a))_{d2}}\right), \text{ where } C_{TC} = \frac{1}{\sqrt{2\pi}} \qquad \text{(EQUATION 22)}$$

Equation 22 Reference: Solved by this author by applying the Equation 17 approach with Equation 20 and 21. Assumes C_{TC} remains constant given Equations 10 and 11. Given the values for σ =1, μ=0 the values of x and z and therefore are interchangeable in the notation for this example.

Appendix B, Table B.1 identifies the following variables not cited in the Nomenclature Section of this work. They following calculated variables are identified:

- *"$f_T(z)$ Standard"* is identified as a truncated standard normal distribution where (σ = 1, μ = 0). Results in this column reflect calculations using Equations 1-5 in Appendix A.
- *"ft(z) – ASSY"* is identified as a truncated standard normal distribution assembly, where (σ = 1, μ = 0)
- "$f_T(z)$ - adj. stdev" is identified as a truncated standard normal distribution assembly where (σ = $\sqrt{\sigma_1^2 + \sigma_2^2}$ = 1.414214, μ = 0)
- "NORMPDFASSY" is identified as a normal distribution (i.e., Equation 2), where (σ = 1, μ = 0).
- "PDFASSY" is identified as a normal distribution (i.e., Equation 2),where (σ = $\sqrt{\sigma_1^2 + \sigma_2^2}$ = 1.414214, μ = 0).
- "Ratio 1" is identified as ft(z) - ASSY/NORMPDFASSY
- "Ratio 2" is identified as fT(z)/NORMPDFASSY
- "Ratio 3" is identified as fT(z)/ft(z) – ASSY
- "Ratio 4" is identified as ft(z) - ASSY/PDFASSY
- "Ratio 5" is identified as fT(z)/PDFASSY

APPENDIX B: TSND ANALYSIS RESULTS

This appendix documents the results summary for TSND analysis performed as part of this dissertation. The results include mathematical results and also evaluate various ratios generated using the equations and research steps from Appendix A. Tabular results include the combinatorial rang of assemblies for identical truncated standard normal distribution combinations ranging from specification limits of -4 to 4. Table B.1 summarizes the TSND results of this research.

Table B.1 – TSND Analysis Results

Range	DISTRIBUTION 1 $\mu = 0, \sigma=1$, Ctc = 0.39894228, USL/LSL (unless otherwise noted)				DISTRIBUTION 2 $\mu = 0, \sigma=1$, Ctc = 0.39894228, USL/LSL (unless otherwise noted)				ASSY $\mu = 0, \sigma=1$, Ctc = 0.39894228, USL/LSL (unless otherwise noted)							RATIOS				
Simulation Number	*X*	*pdf*	$\varphi_z(w\ \mu\&\sigma)$	*ft(z)*	*X*	*pdf*	$\varphi(w\ \mu\&\sigma)$	*ft(x)*	*X*	$\varphi_z(w\ \mu\&\sigma)$	*ft(z) - ASSY*	$f_T(z)$ - *adj. stdev* $\mu = 0$, $\sigma=1.4142$	$f_T(z)$ *Standard*	*NORMPDF ASSY*	*PDFASSY*	*Ratio 1* *ft(z) - ASSY NORM PDFASSY*	*Ratio 2* *fT(z) NORMPDFASSY*	*Ratio 3* *fT(z) ft(x) - ASSY*	*Ratio 4* *ft(z) - ASSY PDFASSY*	*Ratio 5* *fT(z) PDFASSY*
1	-4	0.00013383	0.00033548	0.00013384	-4	0.00013383	0.00033548	0.00013384	-8	1.1255E-07	4.49E-08	3.1746E-08	5.05E-15	5.05227E-15	3.1746E-08	1.12521E-07	1.59149E-07	1.12521E-07	1.59149E-07	0.999999985
2	-3.9	0.00019866	0.00049799	0.00019867	-3.9	0.00019866	0.00049799	0.00019867	-7.8	2.4799E-07	9.893E-08	6.9948E-08	2.45E-14	2.45286E-14	6.9948E-08	2.47928E-07	3.50668E-07	2.47928E-07	3.50668E-07	0.999999985
3	-3.8	0.00029195	0.00073185	0.00029197	-3.8	0.00029195	0.00073185	0.00029197	-7.6	5.356E-07	2.137E-07	1.5107E-07	1.14E-13	1.14416E-13	1.5107E-07	5.35467E-07	7.57361E-07	5.35467E-07	7.57361E-07	0.999999985
4	-3.7	0.00042478	0.00106483	0.00042481	-3.7	0.00042478	0.00106483	0.00042481	-7.4	1.1339E-06	4.523E-07	3.1982E-07	5.13E-13	5.12775E-13	3.1982E-07	1.13358E-06	1.60333E-06	1.13358E-06	1.60333E-06	0.999999985
5	-3.6	0.0006119	0.00153391	0.00061194	-3.6	0.0006119	0.00153391	0.00061194	-7.2	2.3529E-06	9.387E-07	6.6365E-07	2.21E-12	2.20799E-12	6.6365E-07	2.35228E-06	3.32704E-06	2.35228E-06	3.32704E-06	0.999999985
6	-3.5	0.00087268	0.00218763	0.00087274	-3.5	0.00087268	0.00218763	0.00087274	-7	4.7857E-06	1.909E-06	1.3499E-06	9.13E-12	9.13472E-12	1.3499E-06	4.78451E-06	6.76718E-06	4.78451E-06	6.76718E-06	0.999999985
7	-3.4	0.00123222	0.00308891	0.0012323	-3.4	0.00123222	0.00308891	0.0012323	-6.8	9.5414E-06	3.806E-06	2.6912E-06	3.63E-11	3.63096E-11	2.6912E-06	9.53895E-06	1.34918E-05	9.53895E-06	1.34918E-05	0.999999985
8	-3.3	0.00172257	0.00431811	0.00172268	-3.3	0.00172257	0.00431811	0.00172268	-6.6	1.8646E-05	7.439E-06	5.2593E-06	1.39E-10	1.38668E-10	5.2593E-06	1.86414E-05	2.63662E-05	1.86414E-05	2.63662E-05	0.999999985
9	-3.2	0.00238409	0.0059764	0.00238424	-3.2	0.00238409	0.0059764	0.00238424	-6.4	3.5717E-05	1.425E-05	1.0074E-05	5.09E-10	5.08814E-10	1.0074E-05	3.57083E-05	5.05056E-05	3.57083E-05	5.05056E-05	0.999999985
10	-3.1	0.00326682	0.00818922	0.00326703	-3.1	0.00326682	0.00818922	0.00326703	-6.2	6.7063E-05	2.675E-05	1.8916E-05	1.79E-09	1.79378E-09	1.8916E-05	6.70463E-05	9.48298E-05	6.70463E-05	9.48298E-05	0.999999985
11	-3	0.00443185	0.0111097	0.00443213	-3	0.00443185	0.0111097	0.00443213	-6	0.00012343	4.924E-05	3.4813E-05	6.08E-09	6.07588E-09	3.4813E-05	0.000123394	0.000174528	0.000123394	0.000174528	0.999999985
12	-2.9	0.00595253	0.01492173	0.00595291	-2.9	0.00595253	0.01492173	0.00595291	-5.8	0.00022266	8.883E-05	6.2803E-05	1.98E-08	1.97732E-08	6.2803E-05	0.000222602	0.000314846	0.000222602	0.000314846	0.999999985
13	-2.8	0.00791545	0.01984235	0.00791595	-2.8	0.00791545	0.01984235	0.00791595	-5.6	0.00039372	0.0001571	0.00011105	6.18E-08	6.18262E-08	0.00011105	0.000393619	0.000556732	0.000393619	0.000556732	0.999999985
14	-2.7	0.01042093	0.02612306	0.01042159	-2.7	0.01042093	0.02612306	0.01042159	-5.4	0.00068241	0.0002722	0.00019248	1.86E-07	1.85736E-07	0.00019248	0.000682242	0.000964958	0.000682242	0.000964958	0.999999985
15	-2.6	0.01358297	0.03404961	0.01358383	-2.6	0.01358297	0.03404961	0.01358383	-5.2	0.00115938	0.0004625	0.00032701	5.36E-07	5.36104E-07	0.00032701	0.001159082	0.001639398	0.001159082	0.001639398	0.999999985
16	-2.5	0.0175283	0.04393972	0.01752941	-2.5	0.0175283	0.04393972	0.01752941	-5	0.0019307	0.0007702	0.00054457	1.49E-06	1.48672E-06	0.00054457	0.00193021	0.002730074	0.00193021	0.002730074	0.999999985
17	-2.4	0.02239453	0.05613832	0.02239595	-2.4	0.02239453	0.05613832	0.02239595	-4.8	0.00315151	0.0012573	0.00088891	3.96E-06	3.9613E-06	0.00088891	0.003150712	0.004456345	0.003150712	0.004456345	0.999999985
18	-2.3	0.02832704	0.07100985	0.02832883	-2.3	0.02832704	0.07100985	0.02832883	-4.6	0.0050424	0.0020116	0.00142225	1.01E-05	1.01409E-05	0.00142225	0.005041122	0.007130126	0.005041122	0.007130126	0.999999985
19	-2.2	0.03547459	0.08892725	0.03547684	-2.2	0.03547459	0.08892725	0.03547684	-4.4	0.00790806	0.0031549	0.00223054	2.49E-05	2.49425E-05	0.00223054	0.007906052	0.011182263	0.007906052	0.011182263	0.999999985
20	-2.1	0.0439836	0.11025751	0.04398638	-2.1	0.0439836	0.11025751	0.04398638	-4.2	0.01215672	0.0048498	0.00342891	5.89E-05	5.89431E-05	0.00342891	0.012153639	0.017190018	0.012153639	0.017190018	0.999999985
21	-2	0.05399097	0.13534386	0.05399439	-2	0.05399097	0.13534386	0.05399439	-4	0.01831796	0.0073078	0.00516675	0.000134	0.00013383	0.00516675	0.018313319	0.025902225	0.018313319	0.025902225	0.999999985
22	-1.9	0.06561581	0.16448488	0.06561997	-1.9	0.06561581	0.16448488	0.06561997	-3.8	0.02705527	0.0107935	0.00763119	0.000292	0.000291947	0.00763119	0.02704842	0.038257088	0.02704842	0.038257088	0.999999985
23	-1.8	0.07895016	0.19791124	0.07895516	-1.8	0.07895016	0.19791124	0.07895516	-3.6	0.03916886	0.0156261	0.01104793	0.000612	0.000611902	0.01104793	0.039158934	0.055386111	0.039158934	0.055386111	0.999999985
24	-1.7	0.09404908	0.23576101	0.09405504	-1.7	0.09404908	0.23576101	0.09405504	-3.4	0.05558325	0.0221745	0.01567776	0.001232	0.001232219	0.01567776	0.055569172	0.078596632	0.055569172	0.078596632	0.999999985
25	-1.6	0.11092083	0.27805491	0.11092786	-1.6	0.11092083	0.27805491	0.11092786	-3.2	0.07731453	0.030844	0.02180726	0.002384	0.002384088	0.02180726	0.077294947	0.109325411	0.077294947	0.109325411	0.999999985
26	-1.5	0.1295176	0.32467303	0.1295258	-1.5	0.1295176	0.32467303	0.1295258	-3	0.10541258	0.0420535	0.02973257	0.004432	0.004431848	0.02973257	0.105385872	0.149057011	0.105385872	0.149057011	0.999999985
27	-1.4	0.14972747	0.37533487	0.14973695	-1.4	0.14972747	0.37533487	0.14973695	-2.8	0.14087627	0.0562015	0.03973543	0.007915	0.007915452	0.03973543	0.140840577	0.199203886	0.140840577	0.199203886	0.999999985
28	-1.3	0.17136859	0.42958457	0.17137945	-1.3	0.17136859	0.42958457	0.17137945	-2.6	0.1845429	0.073622	0.052052	0.013583	0.013582969	0.052052	0.184496149	0.260950009	0.184496149	0.260950009	0.999999985
29	-1.2	0.19418605	0.48678309	0.19419836	-1.2	0.19418605	0.48678309	0.19419836	-2.4	0.23695778	0.0945325	0.06683609	0.022395	0.02239453	0.06683609	0.236897744	0.335066444	0.236897744	0.335066444	0.999999985
30	-1.1	0.21785218	0.54610902	0.21786598	-1.1	0.21785218	0.54610902	0.21786598	-2.2	0.29823506	0.1189786	0.0841199	0.035475	0.035474593	0.0841199	0.298159504	0.42171463	0.298159504	0.42171463	0.999999985
31	-1	0.24197072	0.60656908	0.24198605	-1	0.24197072	0.60656908	0.24198605	-2	0.36792605	0.1467813	0.10377688	0.053991	0.053990967	0.10377687	0.367832838	0.520260087	0.367832838	0.520260087	0.999999985
32	-0.9	0.26608525	0.66701906	0.26610211	-0.9	0.26608525	0.66701906	0.26610211	-1.8	0.44491443	0.1774952	0.12549215	0.07895	0.078950158	0.12549214	0.444801711	0.629124301	0.444801711	0.629124301	0.999999985
33	-0.8	0.28969155	0.72619504	0.2897099	-0.8	0.28969155	0.72619504	0.2897099	-1.6	0.52735923	0.2103859	0.14874645	0.110921	0.110920835	0.14874645	0.527225626	0.745704086	0.527225626	0.745704086	0.999999985
34	-0.7	0.31225393	0.78275412	0.31227371	-0.7	0.31225393	0.78275412	0.31227371	-1.4	0.61270401	0.2444335	0.17281872	0.149727	0.149727466	0.17281872	0.612548786	0.866384542	0.612548786	0.866384542	0.999999985
35	-0.6	0.3332246	0.83532312	0.33324571	-0.6	0.3332246	0.83532312	0.33324571	-1.2	0.69776472	0.2783678	0.19681086	0.194186	0.194186055	0.19681086	0.697587944	0.986663307	0.697587944	0.986663307	0.999999985
36	-0.5	0.35206533	0.88255281	0.35208763	-0.5	0.35206533	0.88255281	0.35208763	-1	0.77889945	0.3107359	0.21969565	0.241971	0.241970725	0.21969564	0.778702124	1.101390613	0.778702124	1.101390613	0.999999985
37	-0.4	0.36827014	0.92317482	0.36829347	-0.4	0.36827014	0.92317482	0.36829347	-0.8	0.85225175	0.3399993	0.24038533	0.289692	0.289691553	0.24038532	0.852035839	1.205113285	0.852035839	1.205113285	0.999999985
38	-0.3	0.38138782	0.95605804	0.38141198	-0.3	0.38138782	0.95605804	0.38141198	-0.6	0.91404698	0.364652	0.25781523	0.333225	0.333224603	0.25781523	0.913815408	1.292493857	0.913815408	1.292493857	0.999999985
39	-0.2	0.39104269	0.98026077	0.39106747	-0.2	0.39104269	0.98026077	0.39106747	-0.4	0.96091117	0.3833481	0.2710337	0.36827	0.36827014	0.2710337	0.960667725	1.358761434	0.960667725	1.358761434	0.999999985

40	-0.1	0.39695255	0.99507551	0.39697769	-0.1	0.39695255	0.99507551	0.39697769	-0.2	0.99017527	0.3950228	0.27928791	0.391043	0.391042694	0.2792879	0.989924413	1.400141881	0.989924413	1.400141881	0.999999985
41	0	0.39894228	1.00006335	0.39896755	0	0.39894228	1.00006335	0.39896755	0	1.0001267	0.3989928	0.2820948	0.398942	0.39894228	0.28209479	0.999873319	1.414213541	0.999873319	1.414213541	0.999999985
42	0.1	0.39695255	0.99507551	0.39697769	0.1	0.39695255	0.99507551	0.39697769	0.2	0.99017527	0.3950228	0.27928791	0.391043	0.391042694	0.2792879	0.989924413	1.400141881	0.989924413	1.400141881	0.999999985
43	0.2	0.39104269	0.98026077	0.39106747	0.2	0.39104269	0.98026077	0.39106747	0.4	0.96091117	0.3833481	0.2710337	0.36827	0.36827014	0.2710337	0.960667725	1.358761434	0.960667725	1.358761434	0.999999985
44	0.3	0.38138782	0.95605804	0.38141198	0.3	0.38138782	0.95605804	0.38141198	0.6	0.91404698	0.364652	0.25781523	0.333225	0.333224603	0.25781523	0.913815408	1.292493857	0.913815408	1.292493857	0.999999985
45	0.4	0.36827014	0.92317482	0.36829347	0.4	0.36827014	0.92317482	0.36829347	0.8	0.85225175	0.3399993	0.24038533	0.289692	0.289691553	0.24038532	0.852035839	1.205113285	0.852035839	1.205113285	0.999999985
46	0.5	0.35206533	0.88255281	0.35208763	0.5	0.35206533	0.88255281	0.35208763	1	0.77889945	0.3107359	0.21969565	0.241971	0.241970725	0.21969564	0.778702124	1.101390613	0.778702124	1.101390613	0.999999985
47	0.6	0.3332246	0.83532312	0.33324571	0.6	0.3332246	0.83532312	0.33324571	1.2	0.69776472	0.2783678	0.19681086	0.194186	0.194186055	0.19681086	0.697587944	0.986663307	0.697587944	0.986663307	0.999999985
48	0.7	0.31225393	0.78275412	0.31227371	0.7	0.31225393	0.78275412	0.31227371	1.4	0.61270401	0.2444335	0.17281872	0.149727	0.149727466	0.17281872	0.612548786	0.866384542	0.612548786	0.866384542	0.999999985
49	0.8	0.28969155	0.72619504	0.2897099	0.8	0.28969155	0.72619504	0.2897099	1.6	0.52735923	0.2103859	0.14874645	0.110921	0.110920835	0.14874645	0.527225626	0.745704086	0.527225626	0.745704086	0.999999985
50	0.9	0.26608525	0.66701906	0.26610211	0.9	0.26608525	0.66701906	0.26610211	1.8	0.44491443	0.1774952	0.12549215	0.07895	0.078950158	0.12549214	0.444801711	0.629124301	0.444801711	0.629124301	0.999999985
51	1	0.24197072	0.60656908	0.24198605	1	0.24197072	0.60656908	0.24198605	2	0.36792605	0.1467813	0.10377688	0.053991	0.053990967	0.10377687	0.367832838	0.520260087	0.367832838	0.520260087	0.999999985
52	1.1	0.21785218	0.54610902	0.21786598	1.1	0.21785218	0.54610902	0.21786598	2.2	0.29823506	0.1189786	0.0841199	0.035475	0.035474593	0.0841199	0.298159504	0.42171463	0.298159504	0.42171463	0.999999985
53	1.2	0.19418605	0.48678309	0.19419836	1.2	0.19418605	0.48678309	0.19419836	2.4	0.23695778	0.0945325	0.06683609	0.022395	0.02239453	0.06683609	0.236897744	0.335066444	0.236897744	0.335066444	0.999999985
54	1.3	0.17136859	0.42958457	0.17137945	1.3	0.17136859	0.42958457	0.17137945	2.6	0.1845429	0.073622	0.052052	0.013583	0.013582969	0.052052	0.184496149	0.260950009	0.184496149	0.260950009	0.999999985
55	1.4	0.14972747	0.37533487	0.14973695	1.4	0.14972747	0.37533487	0.14973695	2.8	0.14087627	0.0562015	0.03973543	0.007915	0.007915452	0.03973543	0.140840577	0.199203886	0.140840577	0.199203886	0.999999985
56	1.5	0.1295176	0.32467303	0.1295258	1.5	0.1295176	0.32467303	0.1295258	3	0.10541258	0.0420535	0.02973257	0.004432	0.004431848	0.02973257	0.105385872	0.149057011	0.105385872	0.149057011	0.999999985
57	1.6	0.11092083	0.27805491	0.11092786	1.6	0.11092083	0.27805491	0.11092786	3.2	0.07731453	0.030844	0.02180726	0.002384	0.002384088	0.02180726	0.077294947	0.109325411	0.077294947	0.109325411	0.999999985
58	1.7	0.09404908	0.23576101	0.09405504	1.7	0.09404908	0.23576101	0.09405504	3.4	0.05558325	0.0221745	0.01567776	0.001232	0.001232219	0.01567776	0.055569172	0.078596632	0.055569172	0.078596632	0.999999985
59	1.8	0.07895016	0.19791124	0.07895516	1.8	0.07895016	0.19791124	0.07895516	3.6	0.03916886	0.0156261	0.01104793	0.000612	0.000611902	0.01104793	0.039158934	0.055386111	0.039158934	0.055386111	0.999999985
60	1.9	0.06561581	0.16448488	0.06561997	1.9	0.06561581	0.16448488	0.06561997	3.8	0.02705527	0.0107935	0.00763119	0.000292	0.000291947	0.00763119	0.02704842	0.038257088	0.02704842	0.038257088	0.999999985
61	2	0.05399097	0.13534386	0.05399439	2	0.05399097	0.13534386	0.05399439	4	0.01831796	0.0073078	0.00516675	0.000134	0.00013383	0.00516675	0.018313319	0.025902225	0.018313319	0.025902225	0.999999985
62	2.1	0.0439836	0.11025751	0.04398638	2.1	0.0439836	0.11025751	0.04398638	4.2	0.01215672	0.0048498	0.00342891	5.89E-05	5.89431E-05	0.00342891	0.012153639	0.017190018	0.012153639	0.017190018	0.999999985
63	2.2	0.03547459	0.08892725	0.03547684	2.2	0.03547459	0.08892725	0.03547684	4.4	0.00790806	0.0031549	0.00223054	2.49E-05	2.49425E-05	0.00223054	0.007906052	0.011182263	0.007906052	0.011182263	0.999999985
64	2.3	0.02832704	0.07100985	0.02832883	2.3	0.02832704	0.07100985	0.02832883	4.6	0.0050424	0.0020116	0.00142225	1.01E-05	1.01409E-05	0.00142225	0.005041122	0.007130126	0.005041122	0.007130126	0.999999985
65	2.4	0.02239453	0.05613832	0.02239595	2.4	0.02239453	0.05613832	0.02239595	4.8	0.00315151	0.0012573	0.00088891	3.96E-06	3.9613E-06	0.00088891	0.003150712	0.004456345	0.003150712	0.004456345	0.999999985
66	2.5	0.0175283	0.04393972	0.01752941	2.5	0.0175283	0.04393972	0.01752941	5	0.0019307	0.0007702	0.00054457	1.49E-06	1.48672E-06	0.00054457	0.00193021	0.002730074	0.00193021	0.002730074	0.999999985
67	2.6	0.01358297	0.03404961	0.01358383	2.6	0.01358297	0.03404961	0.01358383	5.2	0.00115938	0.0004625	0.00032701	5.36E-07	5.36104E-07	0.00032701	0.001159082	0.001639398	0.001159082	0.001639398	0.999999985
68	2.7	0.01042093	0.02612306	0.01042159	2.7	0.01042093	0.02612306	0.01042159	5.4	0.00068241	0.0002722	0.00019248	1.86E-07	1.85736E-07	0.00019248	0.000682242	0.000964958	0.000682242	0.000964958	0.999999985
69	2.8	0.00791545	0.01984235	0.00791595	2.8	0.00791545	0.01984235	0.00791595	5.6	0.00039372	0.0001571	0.00011105	6.18E-08	6.18262E-08	0.00011105	0.000393619	0.000556732	0.000393619	0.000556732	0.999999985
70	2.9	0.00595253	0.01492173	0.00595291	2.9	0.00595253	0.01492173	0.00595291	5.8	0.00022266	8.883E-05	6.2803E-05	1.98E-08	1.97732E-08	6.2803E-05	0.000222602	0.000314846	0.000222602	0.000314846	0.999999985
71	3	0.00443185	0.0111097	0.00443213	3	0.00443185	0.0111097	0.00443213	6	0.00012343	4.924E-05	3.4813E-05	6.08E-09	6.07588E-09	3.4813E-05	0.000123394	0.000174528	0.000123394	0.000174528	0.999999985
72	3.1	0.00326682	0.00818922	0.00326703	3.1	0.00326682	0.00818922	0.00326703	6.2	6.7063E-05	2.675E-05	1.8916E-05	1.79E-09	1.79378E-09	1.8916E-05	6.70463E-05	9.48298E-05	6.70463E-05	9.48298E-05	0.999999985
73	3.2	0.00238409	0.0059764	0.00238424	3.2	0.00238409	0.0059764	0.00238424	6.4	3.5717E-05	1.425E-05	1.0074E-05	5.09E-10	5.08814E-10	1.0074E-05	3.57083E-05	5.05056E-05	3.57083E-05	5.05056E-05	0.999999985
74	3.3	0.00172257	0.00431811	0.00172268	3.3	0.00172257	0.00431811	0.00172268	6.6	1.8646E-05	7.439E-06	5.2593E-06	1.39E-10	1.38668E-10	5.2593E-06	1.86414E-05	2.63662E-05	1.86414E-05	2.63662E-05	0.999999985
75	3.4	0.00123222	0.00308891	0.0012323	3.4	0.00123222	0.00308891	0.0012323	6.8	9.5414E-06	3.806E-06	2.6912E-06	3.63E-11	3.63096E-11	2.6912E-06	9.53895E-06	1.34918E-05	9.53895E-06	1.34918E-05	0.999999985
76	3.5	0.00087268	0.00218763	0.00087274	3.5	0.00087268	0.00218763	0.00087274	7	4.7857E-06	1.909E-06	1.3499E-06	9.13E-12	9.13472E-12	1.3499E-06	4.78451E-06	6.76718E-06	4.78451E-06	6.76718E-06	0.999999985
77	3.6	0.0006119	0.00153391	0.00061194	3.6	0.0006119	0.00153391	0.00061194	7.2	2.3529E-06	9.387E-07	6.6365E-07	2.21E-12	2.20799E-12	6.6365E-07	2.35228E-06	3.32704E-06	2.35228E-06	3.32704E-06	0.999999985
78	3.7	0.00042478	0.00106483	0.00042481	3.7	0.00042478	0.00106483	0.00042481	7.4	1.1339E-06	4.523E-07	3.1982E-07	5.13E-13	5.12775E-13	3.1982E-07	1.13358E-06	1.60333E-06	1.13358E-06	1.60333E-06	0.999999985
79	3.8	0.00029195	0.00073185	0.00029197	3.8	0.00029195	0.00073185	0.00029197	7.6	5.356E-07	2.137E-07	1.5107E-07	1.14E-13	1.14416E-13	1.5107E-07	5.35467E-07	7.57361E-07	5.35467E-07	7.57361E-07	0.999999985
80	3.9	0.00019866	0.00049799	0.00019867	3.9	0.00019866	0.00049799	0.00019867	7.8	2.4799E-07	9.893E-08	6.9948E-08	2.45E-14	2.45286E-14	6.9948E-08	2.47928E-07	3.50668E-07	2.47928E-07	3.50668E-07	0.999999985
81	4	0.00013383	0.00033548	0.00013384	4	0.00013383	0.00033548	0.00013384	8	1.1255E-07	4.49E-08	3.1746E-08	5.05E-15	5.05227E-15	3.1746E-08	1.12521E-07	1.59149E-07	1.12521E-07	1.59149E-07	0.999999985

APPENDIX C: CORRELATION AND REGRESSION ANALYSIS

This appendix documents the correlation testing and regression analysis performed as part of the scope of this dissertation. Tables C.1 through C.5 summarize the correlation and regression results identified in Figures C.1 through C.120. The results are as follows.

Table C.1 - Pearson Correlation for ft(z) - ASSY, fT(z) - standard

$\sigma = 1$	$\mu = 0$ $n = 81$	Correlations: ft(z) - ASSY, fT(z) - standard	
USL	LSL	Pearson correlation of ft(z) - ASSY and fT(z) - adju stdev	P-Value
8	-8	0.968	$p < 0.001$
7.8	-7.8	0.968	$p < 0.001$
7.6	-7.6	0.968	$p < 0.001$
7.4	-7.4	0.968	$p < 0.001$
7.2	-7.2	0.968	$p < 0.001$
7	-7	0.967	$p < 0.001$
6.8	-6.8	0.967	$p < 0.001$
6.6	-6.6	0.967	$p < 0.001$
6.4	-6.4	0.967	$p < 0.001$
6.2	-6.2	0.967	$p < 0.001$
6	-6	0.967	$p < 0.001$
5.8	-5.8	0.968	$p < 0.001$
5.6	-5.6	0.968	$p < 0.001$
5.4	-5.4	0.968	$p < 0.001$
5.2	-5.2	0.968	$p < 0.001$
5	-5	0.968	$p < 0.001$
4.8	-4.8	0.968	$p < 0.001$
4.6	-4.6	0.969	$p < 0.001$
4.4	-4.4	0.969	$p < 0.001$
4.2	-4.2	0.970	$p < 0.001$
4	-4	0.971	$p < 0.001$
3.8	-3.8	0.972	$p < 0.001$
3.6	-3.6	0.974	$p < 0.001$
3.4	-3.4	0.976	$p < 0.001$
3.2	-3.2	0.978	$p < 0.001$
3	-3	0.980	$p < 0.001$
2.8	-2.8	0.983	$p < 0.001$
2.6	-2.6	0.985	$p < 0.001$
2.4	-2.4	0.988	$p < 0.001$
2.2	-2.2	0.991	$p < 0.001$
2	-2	0.993	$p < 0.001$
1.8	-1.8	0.995	$p < 0.001$
1.6	-1.6	0.997	$p < 0.001$
1.4	-1.4	0.998	$p < 0.001$
1.2	-1.2	0.999	$p < 0.001$
1	-1	0.999	$p < 0.001$
0.8	-0.8	1.000	$p < 0.001$
0.6	-0.6	1.000	$p < 0.001$
0.4	-0.4	1.000	$p < 0.001$
0.2	-0.2	1.000	Note 1

Note 1: For values referencing this note the p-value could not be calculated

Note 2: Standard deviation is 1, unless otherwise noted in Appendix B

Table C.2 - Pearson Correlation for ft(z) - ASSY, fT(z) - adju stdev

σ = 1	μ = 0 n = 81	Correlations: ft(z) - ASSY, fT(z) - adju stdev	
USL	LSL	Pearson correlation of ft(z) - ASSY and fT(z) - standard	P-Value
8	-8	1	Note 1
7.8	-7.8	1	Note 1
7.6	-7.6	1	Note 1
7.4	-7.4	1	Note 1
7.2	-7.2	1	Note 1
7	-7	1	Note 1
6.8	-6.8	1	Note 1
6.6	-6.6	1	Note 1
6.4	-6.4	1	Note 1
6.2	-6.2	1	Note 1
6	-6	1	Note 1
5.8	-5.8	1	Note 1
5.6	-5.6	1	Note 1
5.4	-5.4	1	Note 1
5.2	-5.2	1	Note 1
5	-5	1	Note 1
4.8	-4.8	1	Note 1
4.6	-4.6	1	Note 1
4.4	-4.4	1	Note 1
4.2	-4.2	1	Note 1
4	-4	1	Note 1
3.8	-3.8	1	Note 1
3.6	-3.6	1	Note 1
3.4	-3.4	1	Note 1
3.2	-3.2	1	Note 1
3	-3	1	Note 1
2.8	-2.8	1	Note 1
2.6	-2.6	1	Note 1
2.4	-2.4	1	Note 1
2.2	-2.2	1	Note 1
2	-2	1	Note 1
1.8	-1.8	1	Note 1
1.6	-1.6	1	Note 1
1.4	-1.4	1	Note 1
1.2	-1.2	1	Note 1
1	-1	1	Note 1
0.8	-0.8	1	Note 1
0.6	-0.6	1	Note 1
0.4	-0.4	1	Note 1
0.2	-0.2	1	Note 1

Note 1: For values referencing this note the p-value could not be calculated
Note 2: Standard deviation is 1, unless otherwise noted in Appendix B

Table C.3 - Pearson Correlation of Analysis Ratios

σ = 1	M = 0 n = 81	Correlations: RATIO 1, RATIO 2		Correlations: RATIO 3, RATIO 4	
USL	LSL	Pearson correlation of RATIO 1 and RATIO 2	P-Value	Pearson correlation of RATIO 3 and RATIO 4	P-Value
8	-8	1	Note 1	1	Note 1
7.8	-7.8	1	Note 1	1	Note 1
7.6	-7.6	1	Note 1	1	Note 1
7.4	-7.4	1	Note 1	1	Note 1
7.2	-7.2	1	Note 1	1	Note 1
7	-7	1	Note 1	1	Note 1
6.8	-6.8	1	Note 1	1	Note 1
6.6	-6.6	1	Note 1	1	Note 1
6.4	-6.4	1	Note 1	1	Note 1
6.2	-6.2	1	Note 1	1	Note 1
6	-6	1	Note 1	1	Note 1
5.8	-5.8	1	Note 1	1	Note 1
5.6	-5.6	1	Note 1	1	Note 1
5.4	-5.4	1	Note 1	1	Note 1
5.2	-5.2	1	Note 1	1	Note 1
5	-5	1	Note 1	1	Note 1
4.8	-4.8	1	Note 1	1	Note 1
4.6	-4.6	1	Note 1	1	Note 1
4.4	-4.4	1	Note 1	1	Note 1
4.2	-4.2	1	Note 1	1	Note 1
4	-4	1	Note 1	1	Note 1
3.8	-3.8	1	Note 1	1	Note 1
3.6	-3.6	1	Note 1	1	Note 1
3.4	-3.4	1	Note 1	1	Note 1
3.2	-3.2	1	Note 1	1	Note 1
3	-3	1	Note 1	1	Note 1
2.8	-2.8	1	Note 1	1	Note 1
2.6	-2.6	1	Note 1	1	Note 1
2.4	-2.4	1	Note 1	1	Note 1
2.2	-2.2	1	Note 1	1	Note 1
2	-2	1	Note 1	1	Note 1
1.8	-1.8	1	Note 1	1	Note 1
1.6	-1.6	1	Note 1	1	Note 1
1.4	-1.4	1	Note 1	1	Note 1
1.2	-1.2	1	Note 1	1	Note 1
1	-1	1	Note 1	1	Note 1
0.8	-0.8	1	Note 1	1	Note 1
0.6	-0.6	1	Note 1	1	Note 1
0.4	-0.4	1	Note 1	1	Note 1
0.2	-0.2	1	Note 1	1	Note 1

Note 1: For values referencing this note the p-value could not be calculated
Note 2: Standard deviation is 1, unless otherwise noted in Appendix B

Table C.4 – Regression Analysis for ft(z) - ASSY, fT(z) - standard

σ = 1 Note 2	μ = 0 n = 81	Regression for: ft(z) - ASSY, fT(z) - standard		
USL	LSL	R-sq (adj)	P-Value	Fitted Line Plot Equation for Cubic Model
8	-8	99.14%	p < 0.001	Y = 0.007522 + 2.915 X - 10.14 X**2 + 13.46 X**3
7.8	-7.8	99.14%	p < 0.001	Y = 0.007806 + 2.908 X - 10.10 X**2 + 13.41 X**3
7.6	-7.6	99.14%	p < 0.001	Y = 0.008111 + 2.901 X - 10.06 X**2 + 13.35 X**3
7.4	-7.4	99.13%	p < 0.001	Y = 0.008441 + 2.893 X - 10.02 X**2 + 13.29 X**3
7.2	-7.2	99.13%	p < 0.001	Y = 0.008800 + 2.885 X - 9.973 X**2 + 13.22 X**3
7	-7	99.13%	p < 0.001	Y = 0.009190 + 2.876 X - 9.923 X**2 + 13.14 X**3
6.8	-6.8	99.13%	p < 0.001	Y = 0.009616 + 2.865 X - 9.869 X**2 + 13.06 X**3
6.6	-6.6	99.13%	p < 0.001	Y = 0.01008 + 2.854 X - 9.810 X**2 + 12.97 X**3
6.4	-6.4	99.13%	p < 0.001	Y = 0.01060 + 2.842 X - 9.744 X**2 + 12.88 X**3
6.2	-6.2	99.13%	p < 0.001	Y = 0.01117 + 2.829 X - 9.671 X**2 + 12.77 X**3
6	-6	99.14%	p < 0.001	Y = 0.01180 + 2.814 X - 9.590 X**2 + 12.65 X**3
5.8	-5.8	99.14%	p < 0.001	Y = 0.01251 + 2.797 X - 9.500 X**2 + 12.51 X**3
5.6	-5.6	99.15%	p < 0.001	Y = 0.01331 + 2.778 X - 9.398 X**2 + 12.36 X**3
5.4	-5.4	99.16%	p < 0.001	Y = 0.01422 + 2.757 X - 9.283 X**2 + 12.19 X**3
5.2	-5.2	99.17%	p < 0.001	Y = 0.01525 + 2.732 X - 9.152 X**2 + 11.99 X**3
5	-5	99.19%	p < 0.001	Y = 0.01642 + 2.705 X - 9.002 X**2 + 11.77 X**3
4.8	-4.8	99.21%	p < 0.001	Y = 0.01778 + 2.672 X - 8.830 X**2 + 11.51 X**3
4.6	-4.6	99.24%	p < 0.001	Y = 0.01934 + 2.635 X - 8.630 X**2 + 11.21 X**3
4.4	-4.4	99.28%	p < 0.001	Y = 0.02116 + 2.593 X - 8.399 X**2 + 10.87 X**3
4.2	-4.2	99.33%	p < 0.001	Y = 0.02327 + 2.543 X - 8.131 X**2 + 10.47 X**3
4	-4	99.39%	p < 0.001	Y = 0.02572 + 2.485 X - 7.820 X**2 + 10.00 X**3
3.8	-3.8	99.46%	p < 0.001	Y = 0.02857 + 2.418 X - 7.460 X**2 + 9.468 X**3
3.6	-3.6	99.54%	p < 0.001	Y = 0.03186 + 2.341 X - 7.048 X**2 + 8.855 X**3
3.4	-3.4	99.62%	p < 0.001	Y = 0.03564 + 2.254 X - 6.581 X**2 + 8.161 X**3
3.2	-3.2	99.71%	p < 0.001	Y = 0.03994 + 2.156 X - 6.061 X**2 + 7.390 X**3
3	-3	99.79%	p < 0.001	Y = 0.04478 + 2.048 X - 5.496 X**2 + 6.558 X**3
2.8	-2.8	99.86%	p < 0.001	Y = 0.05017 + 1.933 X - 4.899 X**2 + 5.685 X**3
2.6	-2.6	99.92%	p < 0.001	Y = 0.05607 + 1.813 X - 4.291 X**2 + 4.807 X**3
2.4	-2.4	99.95%	p < 0.001	Y = 0.06244 + 1.692 X - 3.696 X**2 + 3.962 X**3
2.2	-2.2	99.98%	p < 0.001	Y = 0.06919 + 1.574 X - 3.137 X**2 + 3.186 X**3
2	-2	99.99%	p < 0.001	Y = 0.07622 + 1.462 X - 2.633 X**2 + 2.508 X**3
1.8	-1.8	100.00%	p < 0.001	Y = 0.08339 + 1.359 X - 2.196 X**2 + 1.942 X**3
1.6	-1.6	100.00%	p < 0.001	Y = 0.09056 + 1.266 X - 1.829 X**2 + 1.490 X**3
1.4	-1.4	100.00%	p < 0.001	Y = 0.09755 + 1.185 X - 1.532 X**2 + 1.142 X**3
1.2	-1.2	100.00%	p < 0.001	Y = 0.1042 + 1.115 X - 1.295 X**2 + 0.8829 X**3
1	-1	100.00%	p < 0.001	Y = 0.1103 + 1.057 X - 1.112 X**2 + 0.6942 X**3
0.8	-0.8	100.00%	p < 0.001	Y = 0.1157 + 1.009 X - 0.9725 X**2 + 0.5600 X**3
0.6	-0.6	100.00%	p < 0.001	Y = 0.1203 + 0.9714 X - 0.8702 X**2 + 0.4670 X**3
0.4	-0.4	100.00%	p < 0.001	Y = 0.1471 + 0.7627 X - 0.3293 X**2
0.2	-0.2	100.00%	p < 0.001	Y = 0.1985 + 0.5026 X

Note 1: For values referencing this note the p-value could not be calculated
Note 2: Standard deviation is 1, unless otherwise noted in Appendix B

Table C.5 - Regression Analysis for ft(z) - ASSY, fT(z) – adju stdev

$\sigma = 1$ Note 2	$\mu = 0$ n = 81	Regression for: ft(z) - ASSY vs. fT(z) - adju stdev		
USL	LSL	R-sq (adj)	P-Value	Fitted Line Plot Equation for Cubic Model
8	-8	100.00%	Note 1	Y = - 0.000000 + 1.414 X + 0.000000 X**2
7.8	-7.8	100.00%	Note 1	Y = - 0.000000 + 1.414 X + 0.000000 X**2
7.6	-7.6	100.00%	Note 1	Y = 0.000000 + 1.414 X + 0.000000 X**2
7.4	-7.4	100.00%	Note 1	Y = 0.000000 + 1.414 X + 0.000000 X**2
7.2	-7.2	100.00%	Note 1	Y = - 0.000000 + 1.414 X
7	-7	100.00%	$p < 0.001$	Y = - 0.000000 + 1.414 X
6.8	-6.8	100.00%	Note 1	Y = 0.000000 + 1.414 X - 0.000000 X**2
6.6	-6.6	100.00%	Note 1	Y = 0.000000 + 1.414 X
6.4	-6.4	100.00%	Note 1	Y = - 0.000000 + 1.414 X - 0.000000 X**2
6.2	-6.2	100.00%	Note 1	Y = - 0.000000 + 1.414 X
6	-6	100.00%	Note 1	Y = 0.000000 + 1.414 X
5.8	-5.8	100.00%	$p < 0.001$	Y = 0.000000 + 1.414 X
5.6	-5.6	100.00%	$p < 0.001$	Y = 0.000000 + 1.414 X
5.4	-5.4	100.00%	$p < 0.001$	Y = 0.000000 + 1.414 X
5.2	-5.2	100.00%	$p < 0.001$	Y = 0.000000 + 1.414 X
5	-5	100.00%	$p < 0.001$	Y = - 0.000000 + 1.414 X
4.8	-4.8	100.00%	$p < 0.001$	Y = 0.000000 + 1.414 X
4.6	-4.6	100.00%	$p < 0.001$	Y = 0.000000 + 1.414 X
4.4	-4.4	100.00%	$p < 0.001$	Y = 0.000000 + 1.414 X
4.2	-4.2	100.00%	$p < 0.001$	Y = 0.000000 + 1.414 X
4	-4	100.00%	$p < 0.001$	Y = 0.000000 + 1.414 X
3.8	-3.8	100.00%	$p < 0.001$	Y = 0.000000 + 1.414 X
3.6	-3.6	100.00%	$p < 0.001$	Y = 0.000000 + 1.414 X
3.4	-3.4	100.00%	$p < 0.001$	Y = 0.000000 + 1.414 X
3.2	-3.2	100.00%	$p < 0.001$	Y = 0.000000 + 1.414 X
3	-3	100.00%	$p < 0.001$	Y = 0.000000 + 1.414 X
2.8	-2.8	100.00%	$p < 0.001$	Y = 0.000000 + 1.414 X
2.6	-2.6	100.00%	$p < 0.001$	Y = 0.000000 + 1.414 X
2.4	-2.4	100.00%	$p < 0.001$	Y = 0.000000 + 1.414 X
2.2	-2.2	100.00%	$p < 0.001$	Y = 0.000000 + 1.414 X
2	-2	100.00%	$p < 0.001$	Y = 0.000000 + 1.414 X
1.8	-1.8	100.00%	$p < 0.001$	Y = 0.000000 + 1.414 X
1.6	-1.6	100.00%	$p < 0.001$	Y = 0.000000 + 1.414 X
1.4	-1.4	100.00%	$p < 0.001$	Y = 0.000000 + 1.414 X
1.2	-1.2	100.00%	$p < 0.001$	Y = 0.000000 + 1.414 X
1	-1	100.00%	$p < 0.001$	Y = 0.000000 + 1.414 X
0.8	-0.8	100.00%	$p < 0.001$	Y = 0.000000 + 1.414 X
0.6	-0.6	100.00%	$p < 0.001$	Y = 0.000000 + 1.414 X
0.4	-0.4	100.00%	$p < 0.001$	Y = 0.000000 + 1.414 X
0.2	-0.2	100.00%	$p < 0.001$	Y = 0.000000 + 1.414 X

Note 1: For values referencing this note the p-value could not be calculated
Note 2: Standard deviation is 1, unless otherwise noted in Appendix B

Figure C.1 - TSND Assembly Comparison (USL = 8, LSL = -8)

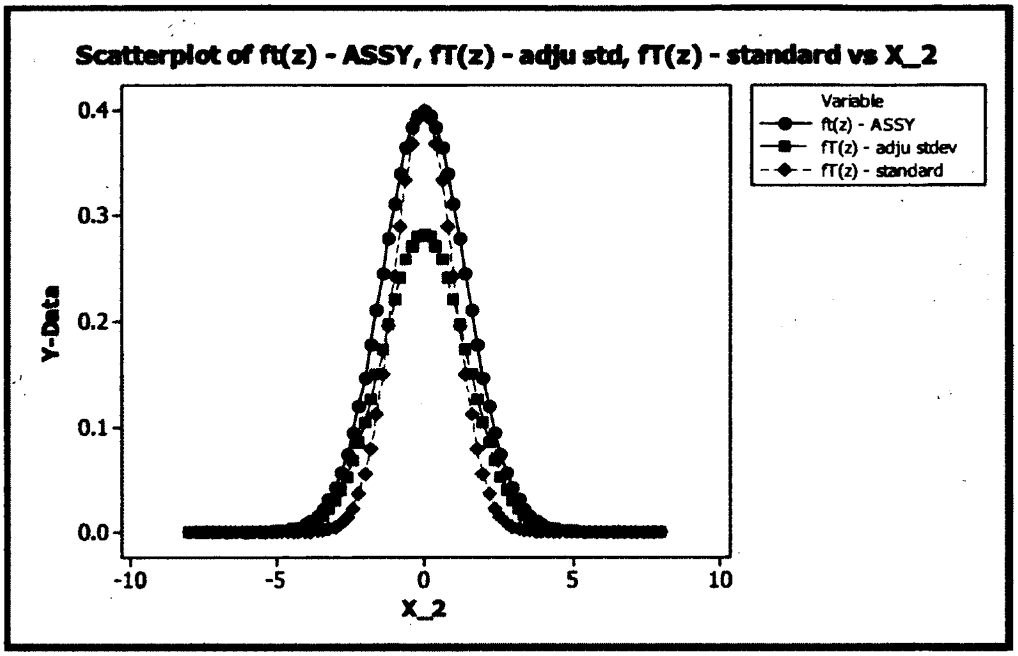
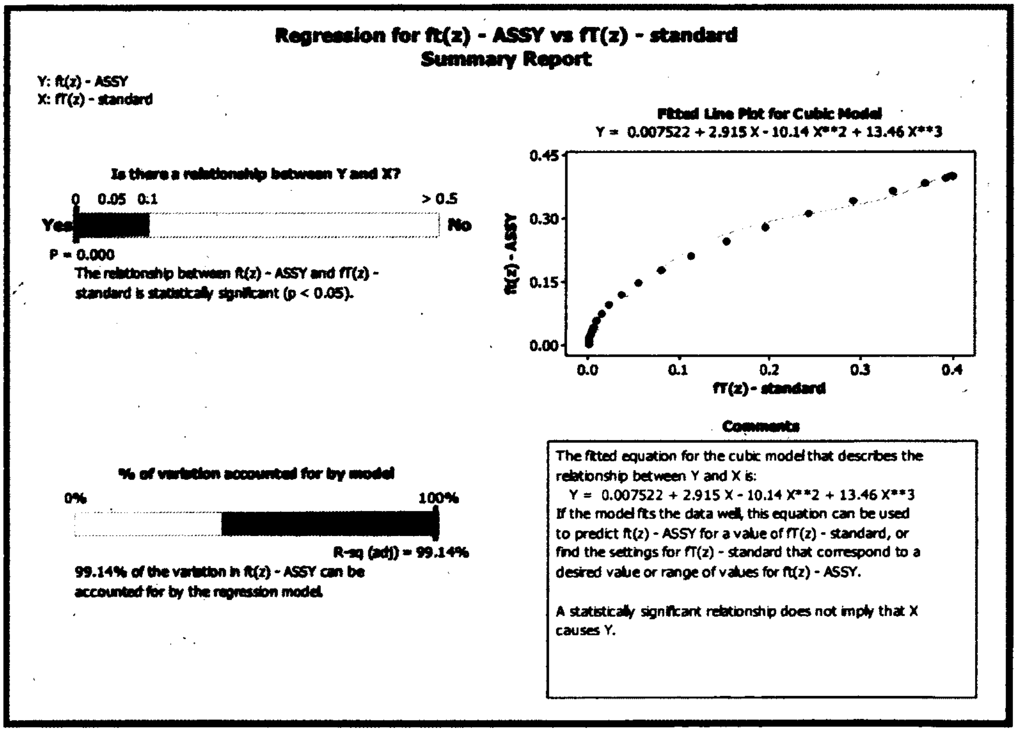


Figure C.2 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 8, LSL = -8)



Regression for ft(z) - ASSY vs fT(z) - standard
Summary Report

Y: ft(z) - ASSY
X: fT(z) - standard

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant ($p < 0.05$).

% of variation accounted for by model

R-sq (adj) = 99.14%

99.14% of the variation in ft(z) - ASSY can be accounted for by the regression model.

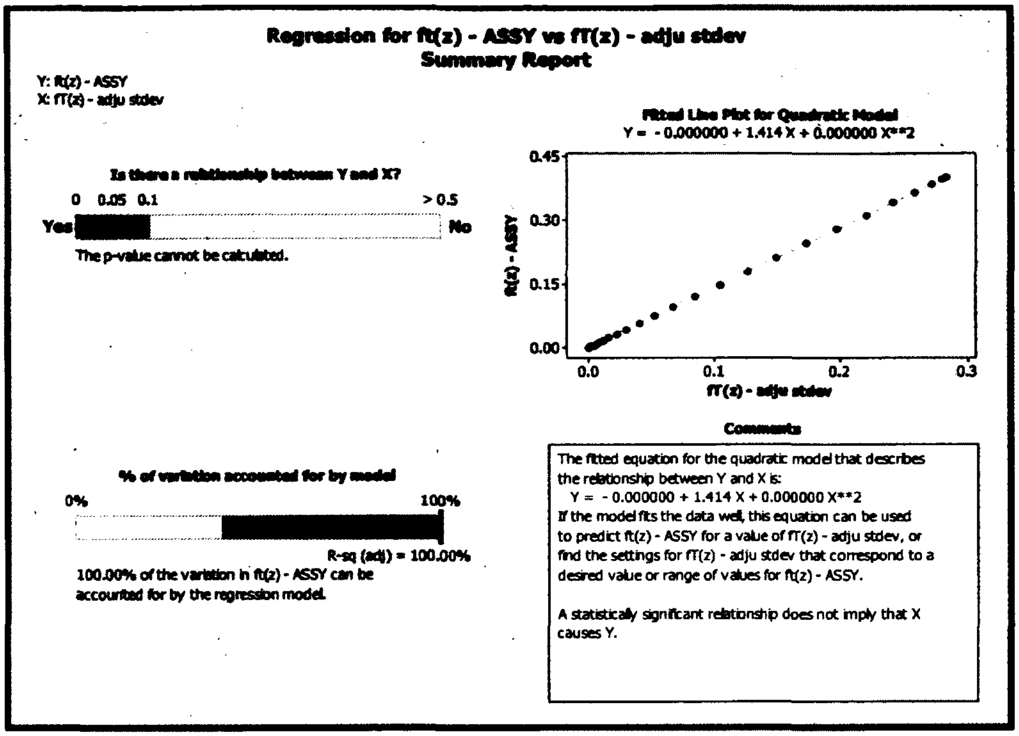
Fitted Line Plot for Cubic Model
Y = 0.007522 + 2.915 X - 10.14 X**2 + 13.46 X**3

Comments

The fitted equation for the cubic model that describes the relationship between Y and X is:
Y = 0.007522 + 2.915 X - 10.14 X**2 + 13.46 X**3
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.3 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 8, LSL = -8)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

Is there a relationship between Y and X?

The p-value cannot be calculated.

% of variation accounted for by model

R-sq (adj) = 100.00%

100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Comments

The fitted equation for the quadratic model that describes the relationship between Y and X is:
Y = - 0.000000 + 1.414 X + 0.000000 X**2
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.4 - TSND Assembly Comparison (USL = 7.8, LSL = -7.8)

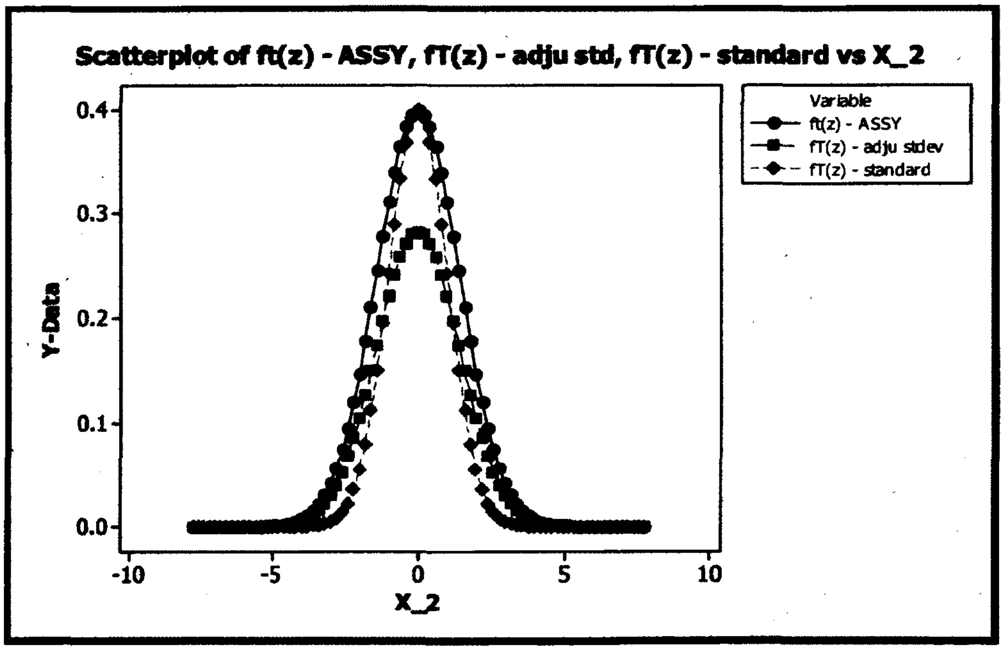
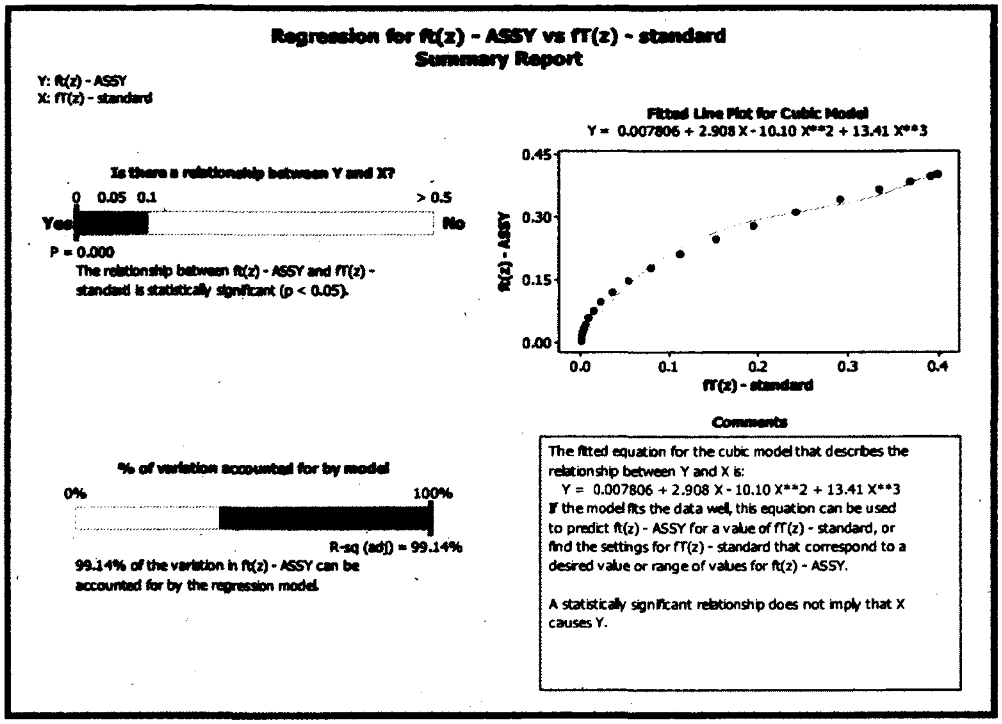


Figure C. 5 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 7.8, LSL = -7.8)



Regression for ft(z) - ASSY vs fT(z) - standard
Summary Report

Y: ft(z) - ASSY
X: fT(z) - standard

Is there a relationship between Y and X?

0 0.05 0.1 > 0.5

Yes No

P = 0.000

The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant (p < 0.05).

% of variation accounted for by model

0% 100%

R-sq (adj) = 99.14%

99.14% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Fitted Line Plot for Cubic Model

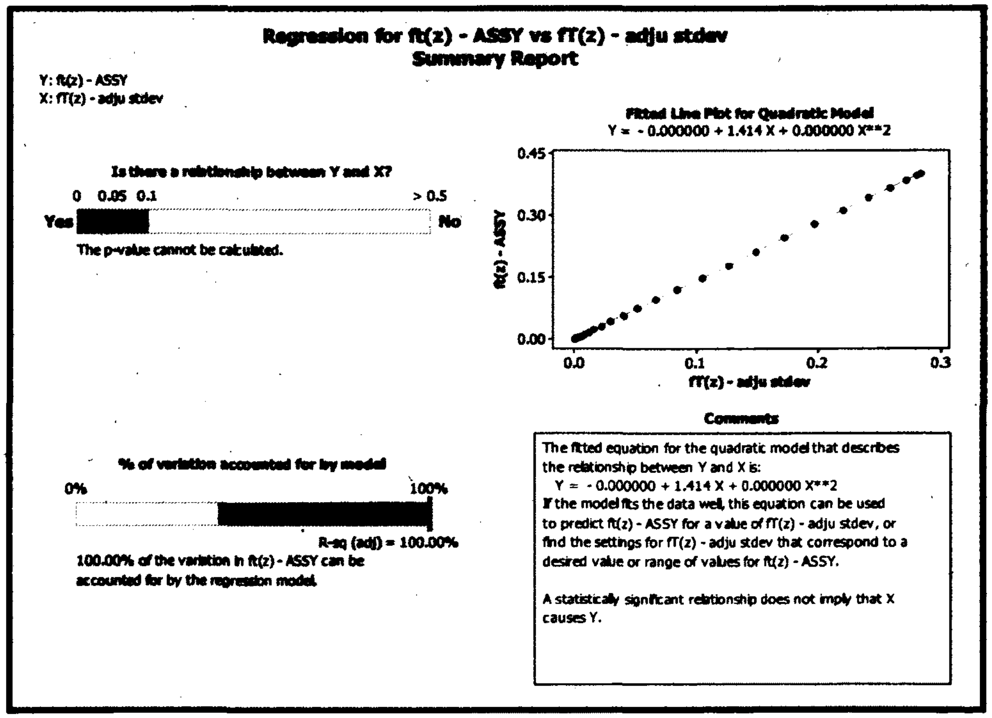
Y = 0.007806 + 2.908 X - 10.10 X**2 + 13.41 X**3

Comments

The fitted equation for the cubic model that describes the relationship between Y and X is:
Y = 0.007806 + 2.908 X - 10.10 X**2 + 13.41 X**3
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.6 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 7.8, LSL = -7.8)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

Is there a relationship between Y and X?

0 0.05 0.1 > 0.5

Yes No

The p-value cannot be calculated.

% of variation accounted for by model

0% 100%

R-sq (adj) = 100.00%

100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Fitted Line Plot for Quadratic Model

Y = - 0.000000 + 1.414 X + 0.000000 X**2

Comments

The fitted equation for the quadratic model that describes the relationship between Y and X is:
Y = - 0.000000 + 1.414 X + 0.000000 X**2
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.7 - TSND Assembly Comparison (USL = 7.6, LSL = -7.6)

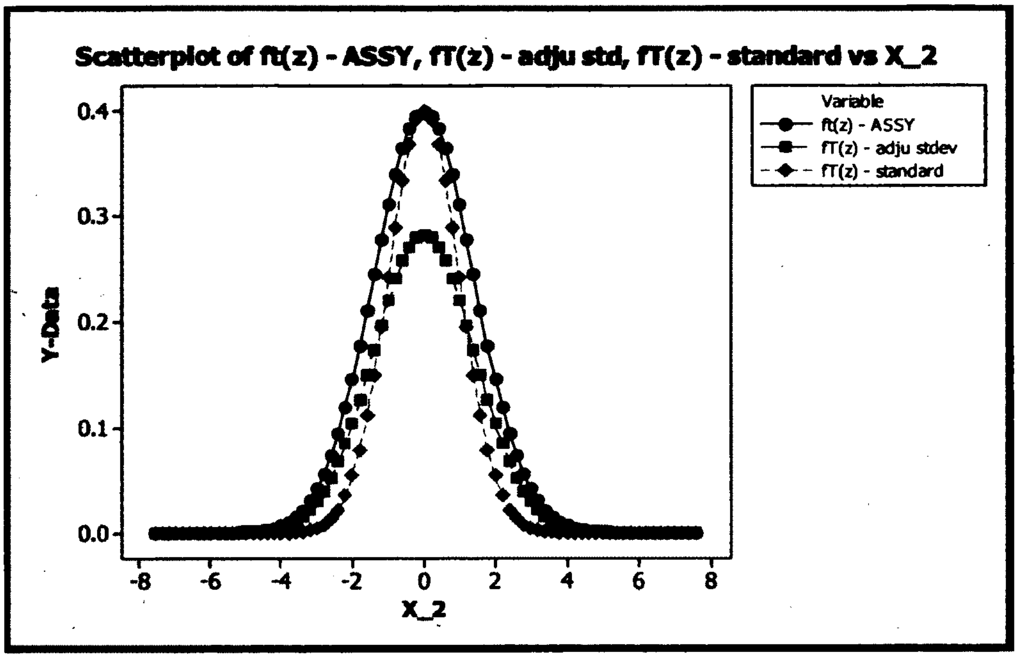
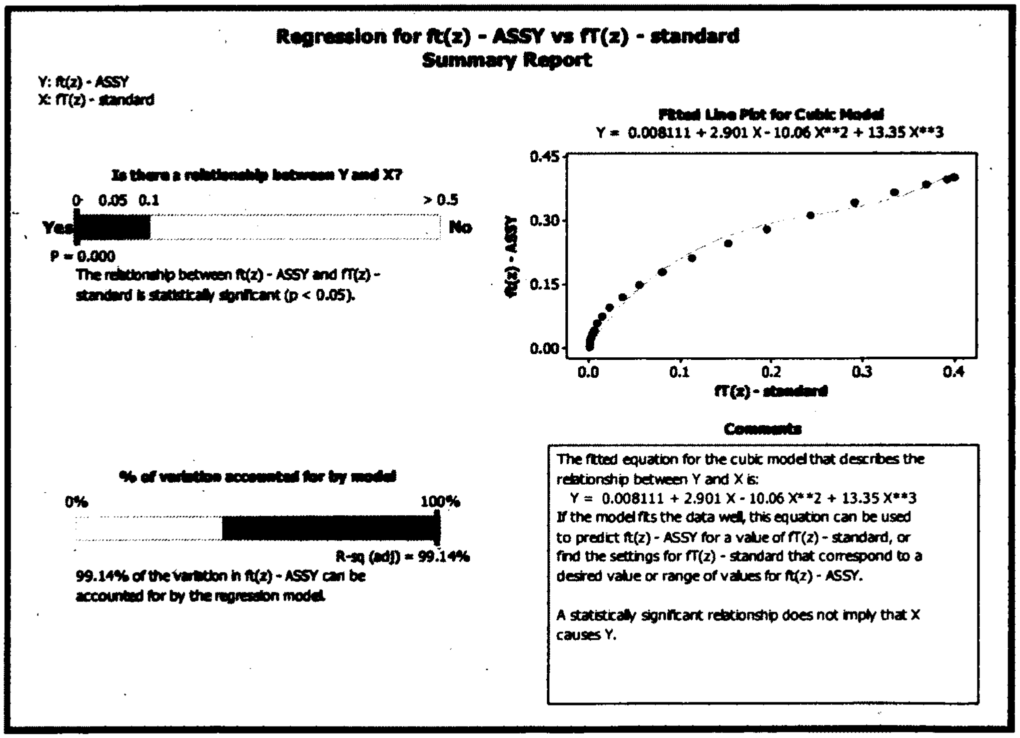


Figure C. 8 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 7.6, LSL = -7.6)



The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant (p < 0.05).

99.14% of the variation in ft(z) - ASSY can be accounted for by the regression model.

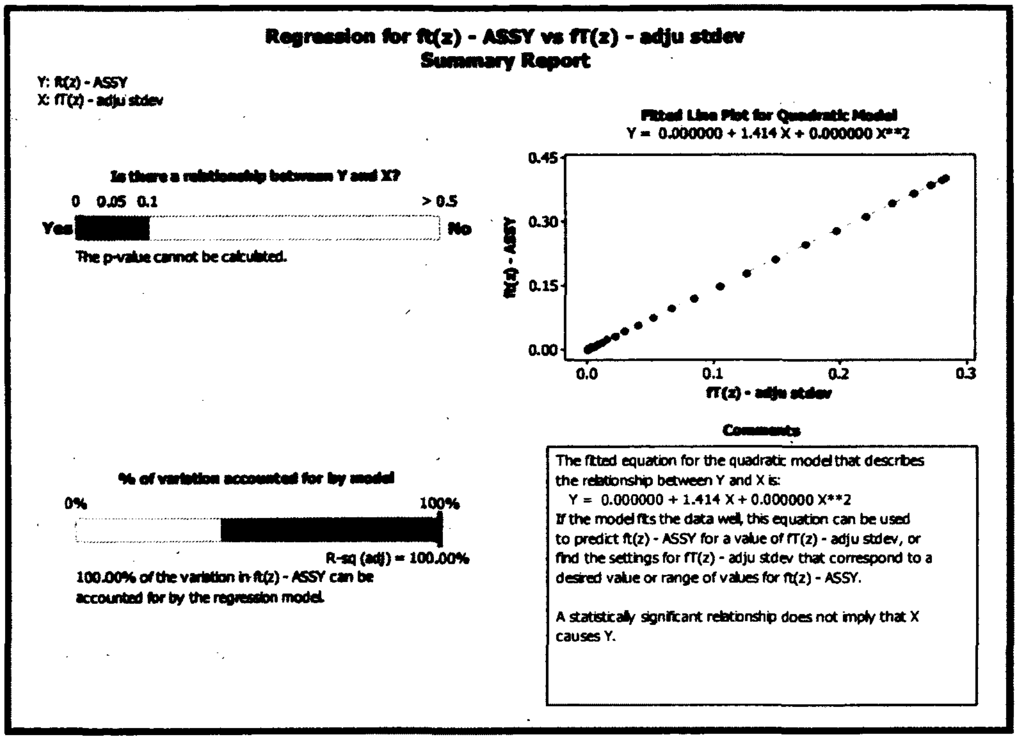
The fitted equation for the cubic model that describes the relationship between Y and X is:

Y = 0.008111 + 2.901 X - 10.06 X**2 + 13.35 X**3

If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.9 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 7.6, LSL = -7.6)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

The p-value cannot be calculated.

R-sq (adj) = 100.00%

100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Comments

The fitted equation for the quadratic model that describes the relationship between Y and X is:

Y = 0.000000 + 1.414 X + 0.000000 X**2

If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.10 - TSND Assembly Comparison (USL = -7.4, LSL = -7.4)

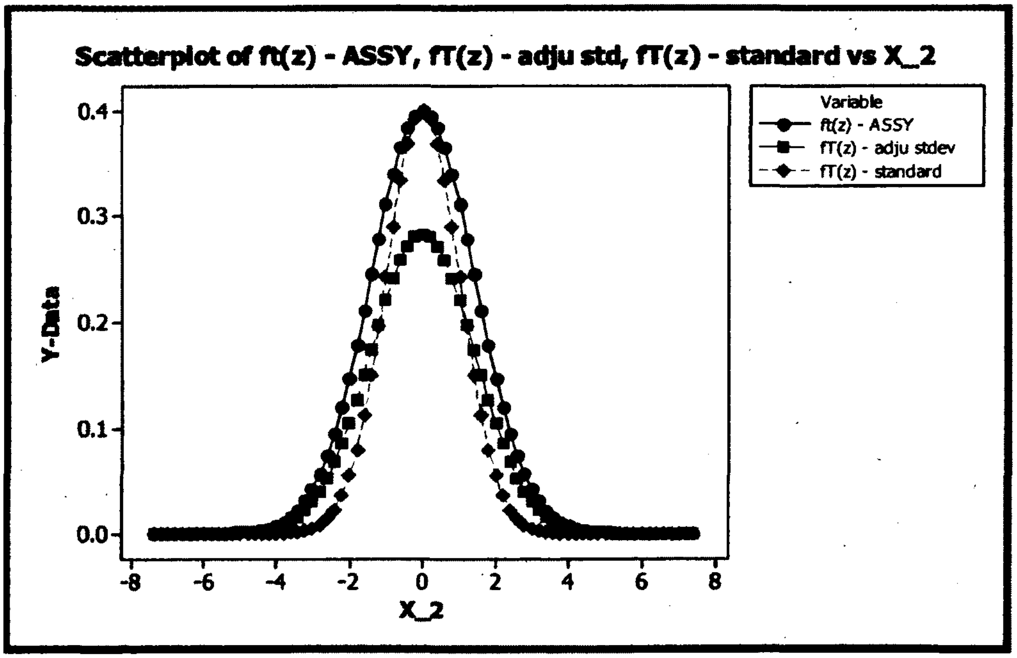
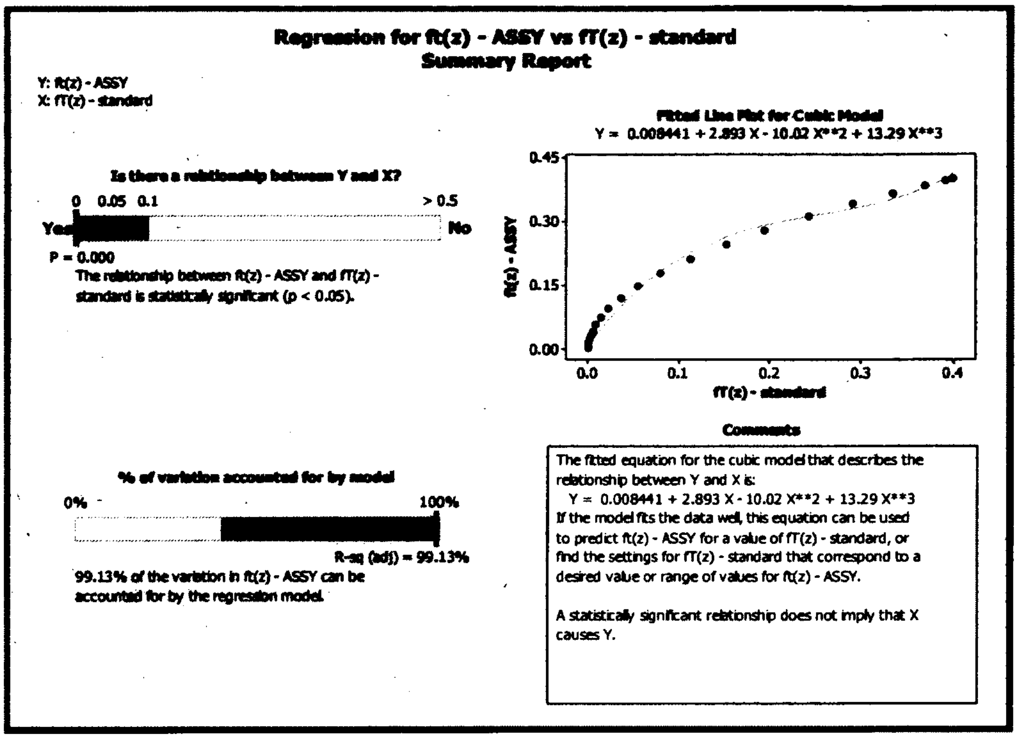


Figure C.11 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 7.4, LSL = -7.4)



Regression for ft(z) - ASSY vs fT(z) - standard
Summary Report

Y: ft(z) - ASSY
X: fT(z) - standard

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant ($p < 0.05$).

% of variation accounted for by model

R-sq (adj) = 99.13%
99.13% of the variation in ft(z) - ASSY can be accounted for by the regression model.

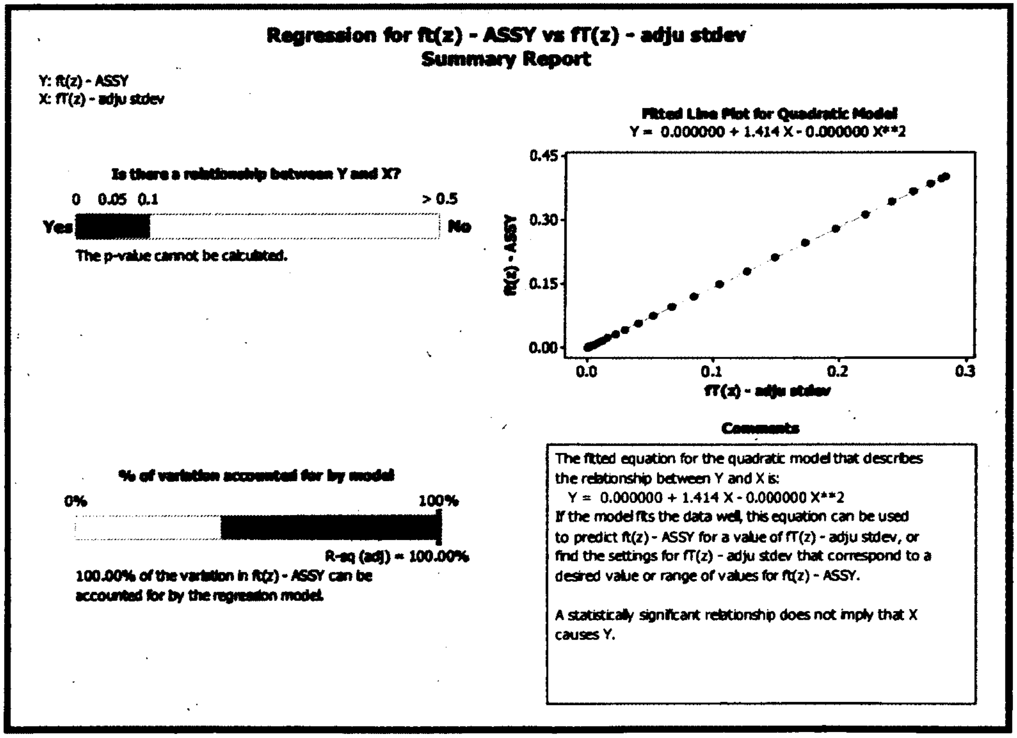
Fitted Line Plot for Cubic Model
Y = 0.008441 + 2.893 X - 10.02 X**2 + 13.29 X**3

Comments

The fitted equation for the cubic model that describes the relationship between Y and X is:
Y = 0.008441 + 2.893 X - 10.02 X**2 + 13.29 X**3
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.12 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 7.4, LSL = -7.4)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

Is there a relationship between Y and X?

The p-value cannot be calculated.

% of variation accounted for by model

R-sq (adj) = 100.00%
100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Fitted Line Plot for Quadratic Model
Y = 0.000000 + 1.414 X - 0.000000 X**2

Comments

The fitted equation for the quadratic model that describes the relationship between Y and X is:
Y = 0.000000 + 1.414 X - 0.000000 X**2
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.13 - TSND Assembly Comparison (USL = 7.2, LSL = -7.2)

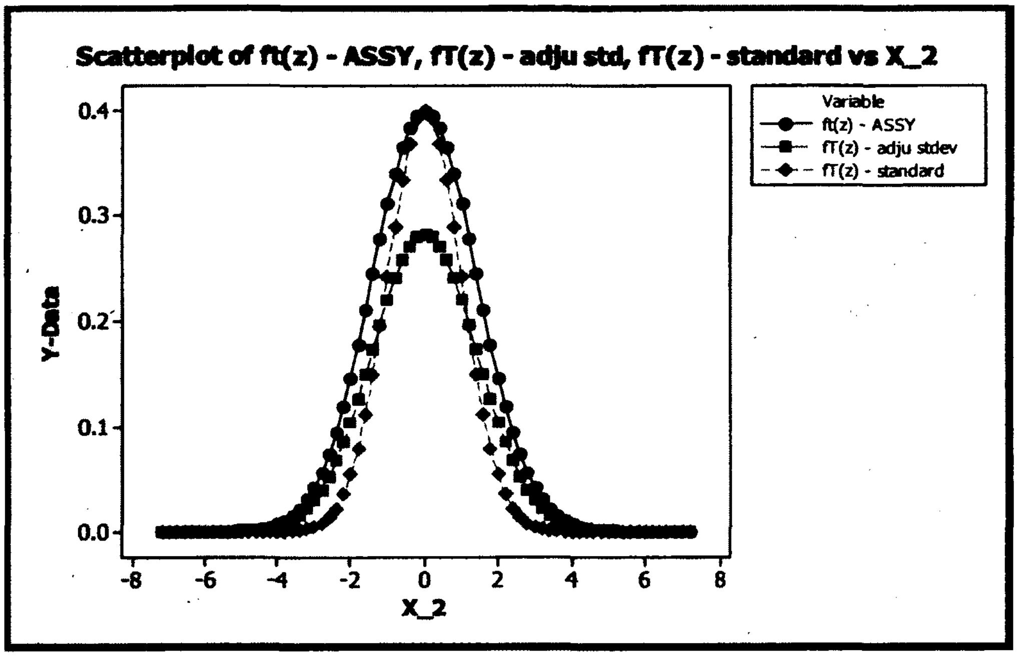
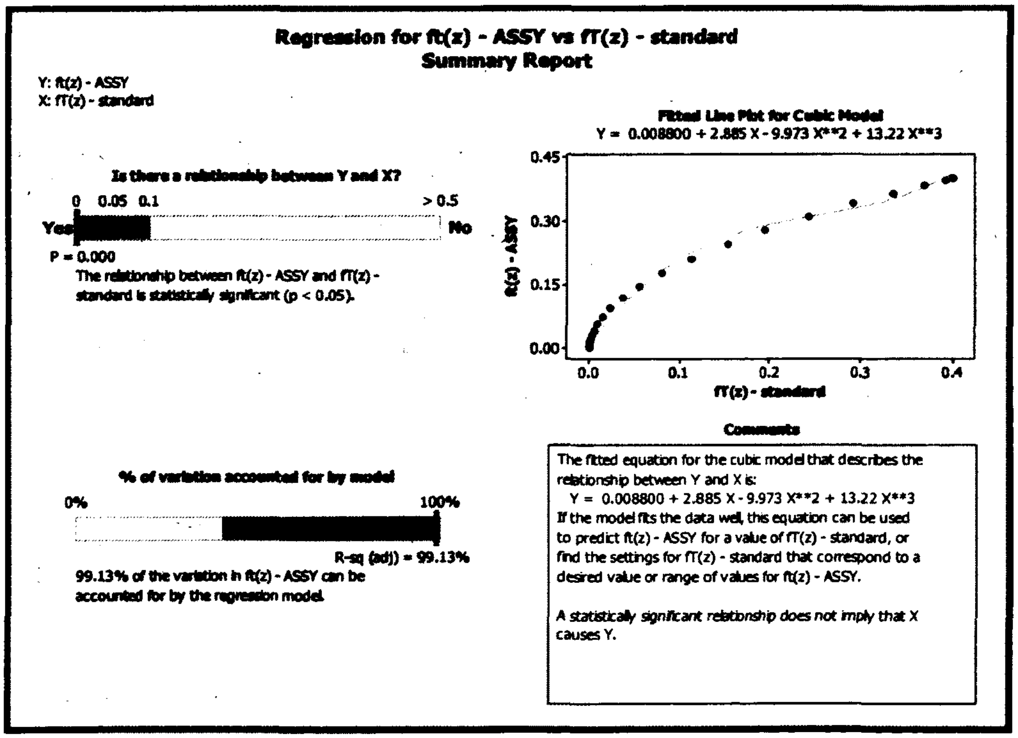


Figure C.14 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 7.2, LSL = -7.2)



P = 0.000

The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant ($p < 0.05$).

R-sq (adj) = 99.13%

99.13% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Y = 0.008800 + 2.885 X - 9.973 X**2 + 13.22 X**3

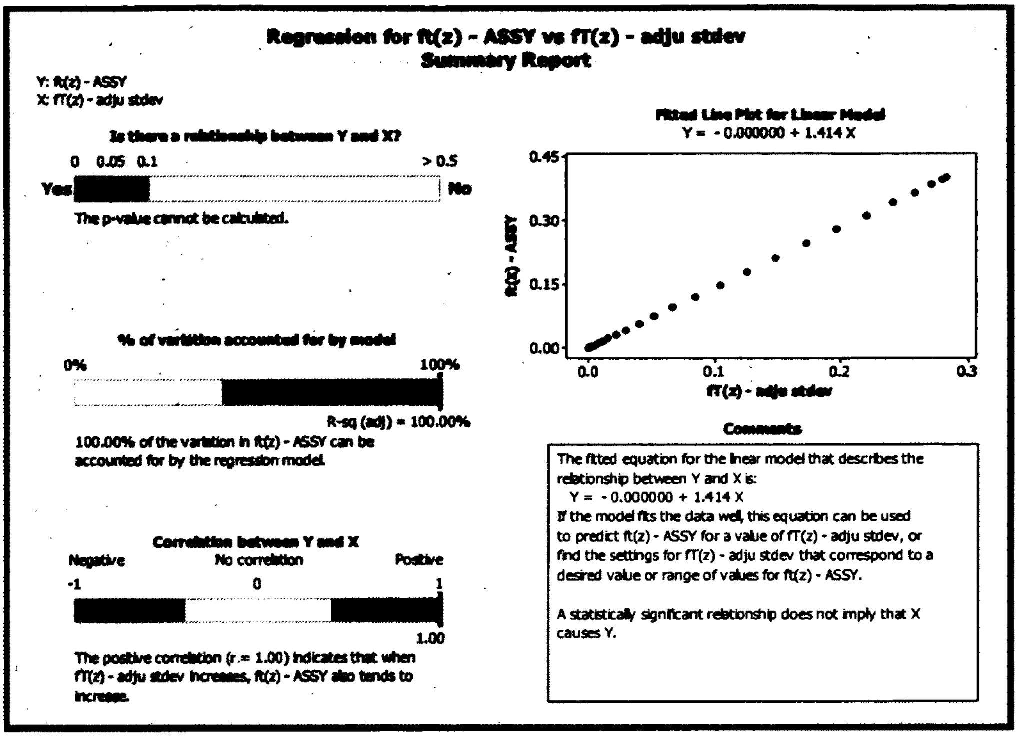
The fitted equation for the cubic model that describes the relationship between Y and X is:

Y = 0.008800 + 2.885 X - 9.973 X**2 + 13.22 X**3

If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.15 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 7.2, LSL = -7.2)



The p-value cannot be calculated.

100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

The positive correlation (r = 1.00) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

The fitted equation for the linear model that describes the relationship between Y and X is:

Y = - 0.000000 + 1.414 X

If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.16 TSND Assembly Comparison (USL = 7, LSL = -7)

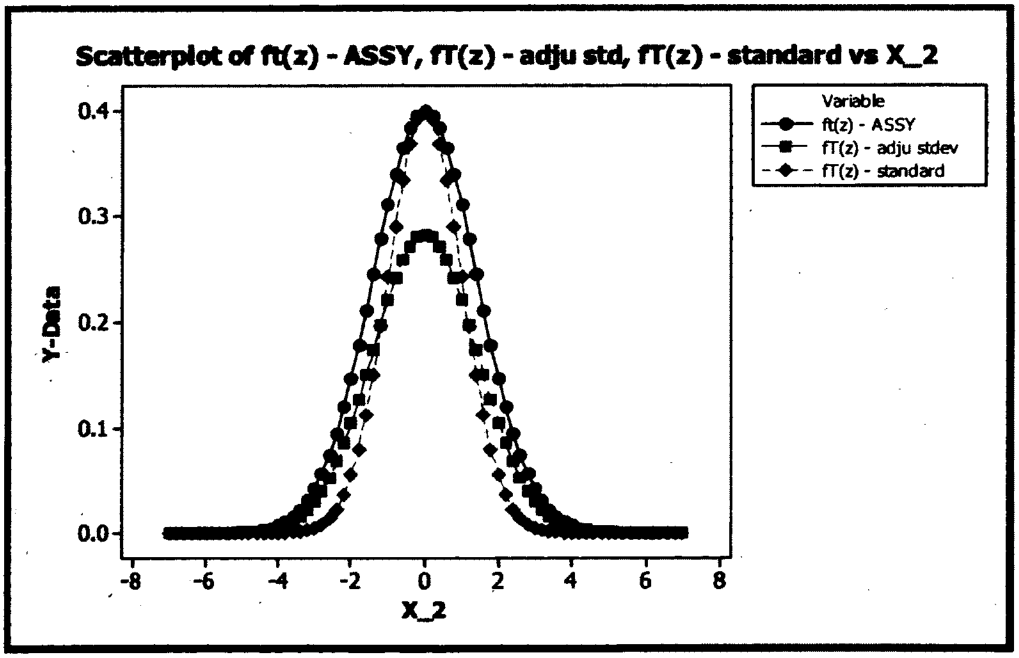
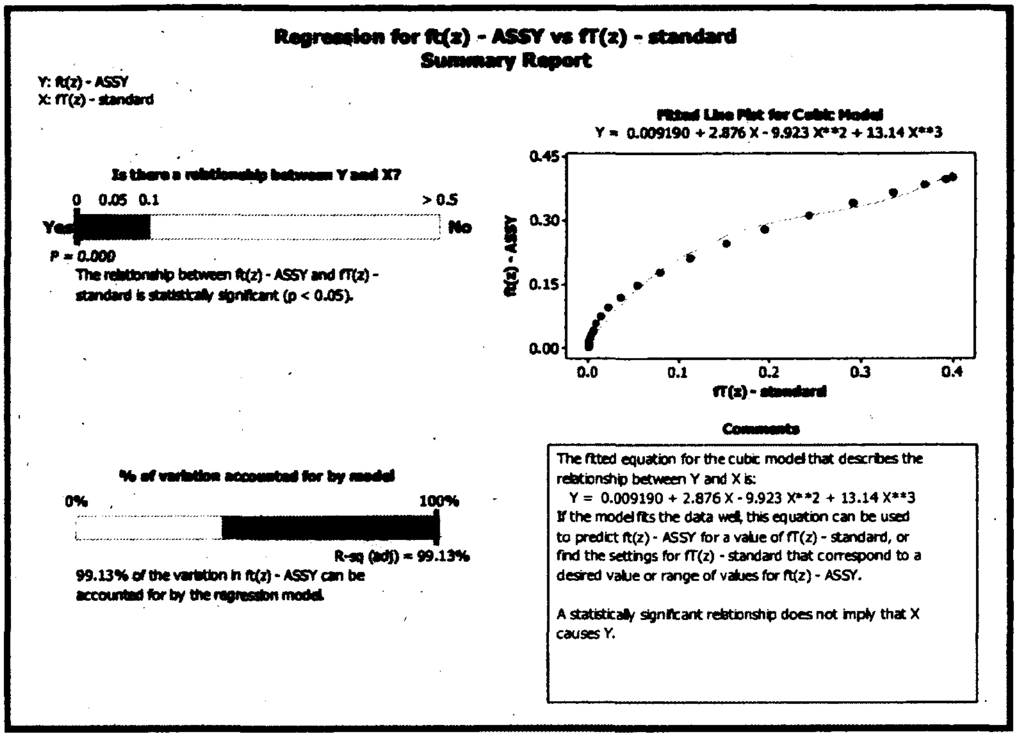


Figure C.17 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 7, LSL = -7)



Regression for ft(z) - ASSY vs fT(z) - standard
Summary Report

Y: ft(z) - ASSY
X: fT(z) - standard

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant (p < 0.05).

% of variation accounted for by model

R-sq (adj) = 99.13%
99.13% of the variation in ft(z) - ASSY can be accounted for by the regression model.

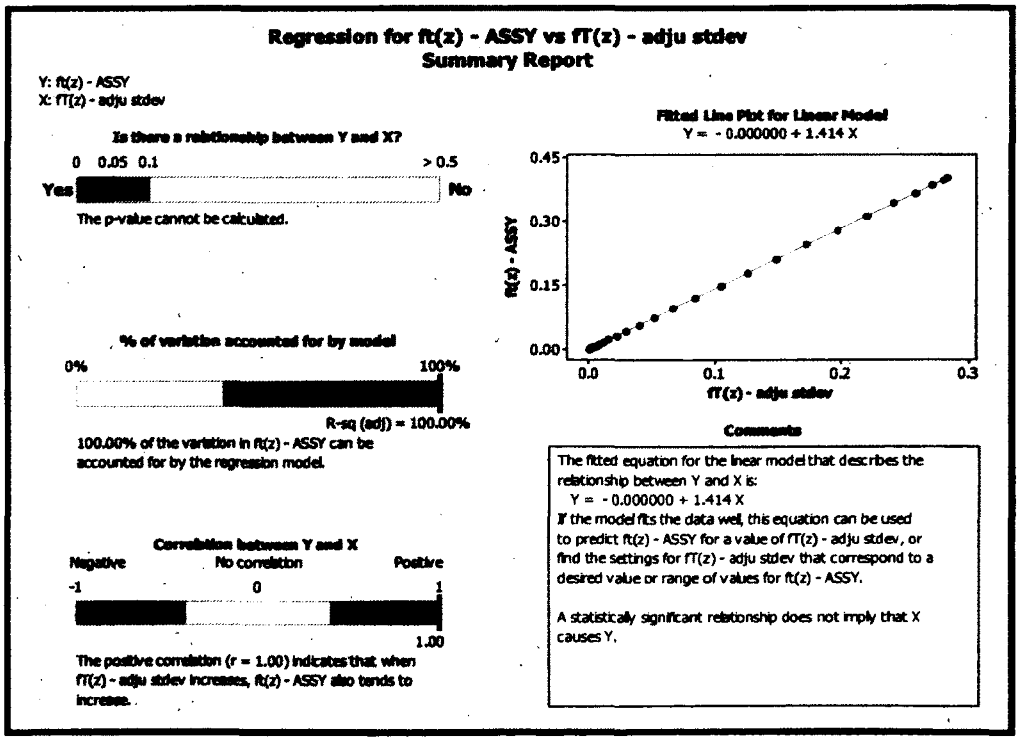
Fitted Line Plot for Cubic Model
Y = 0.009190 + 2.876 X - 9.923 X**2 + 13.14 X**3

Comments

The fitted equation for the cubic model that describes the relationship between Y and X is:
Y = 0.009190 + 2.876 X - 9.923 X**2 + 13.14 X**3
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.18 -ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 7, LSL = -7)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

Is there a relationship between Y and X?

The p-value cannot be calculated.

% of variation accounted for by model

R-sq (adj) = 100.00%
100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Correlation between Y and X

The positive correlation (r = 1.00) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

Fitted Line Plot for Linear Model
Y = - 0.000000 + 1.414 X

Comments

The fitted equation for the linear model that describes the relationship between Y and X is:
Y = - 0.000000 + 1.414 X
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.19 - TSND Assembly Comparison (USL = 6.8, LSL = -6.8)

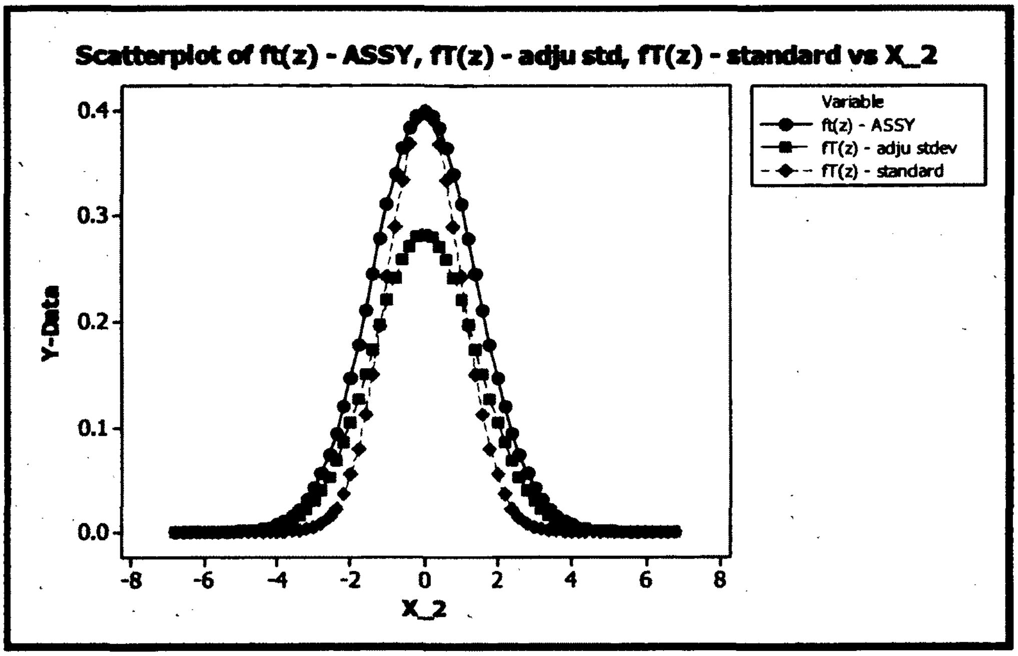
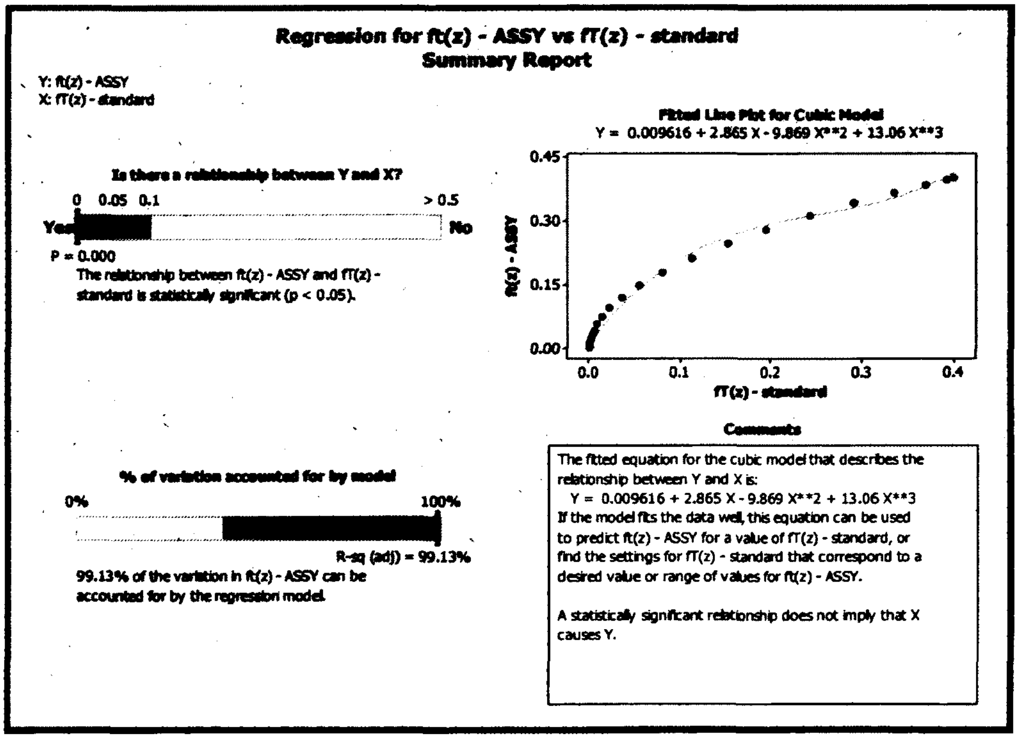


Figure C.20 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 6.8, LSL = -6.8)



The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant (p < 0.05).

99.13% of the variation in ft(z) - ASSY can be accounted for by the regression model.

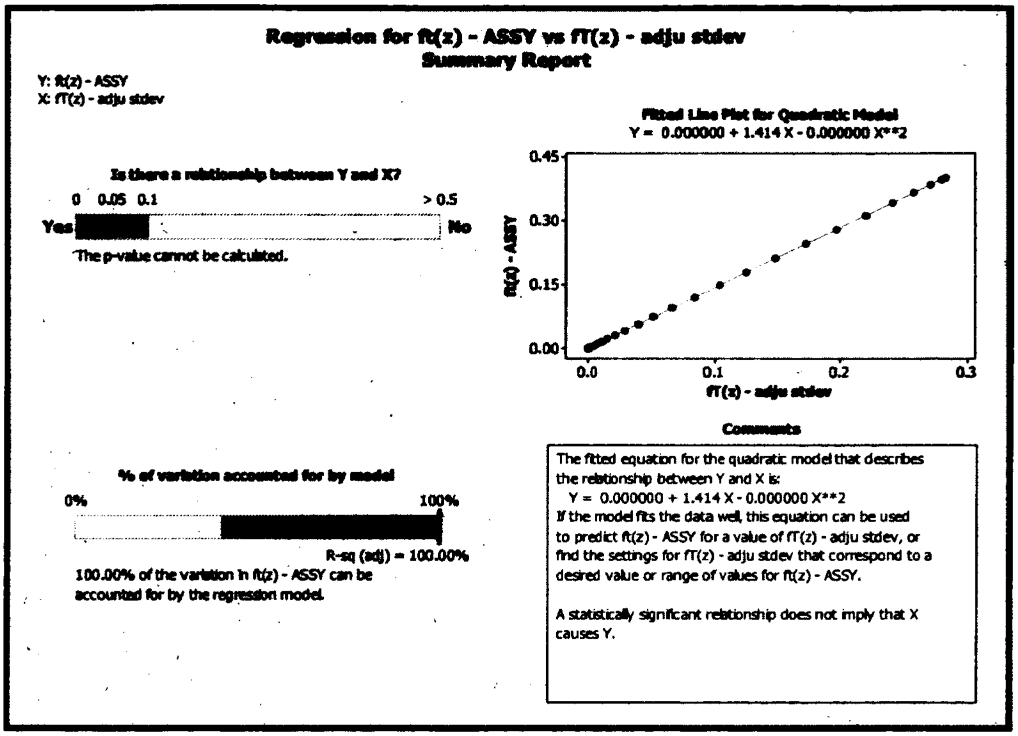
The fitted equation for the cubic model that describes the relationship between Y and X is:

Y = 0.009616 + 2.865 X - 9.869 X**2 + 13.06 X**3

If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.21 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 6.8, LSL = -6.8)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

Is there a relationship between Y and X?

The p-value cannot be calculated.

% of variation accounted for by model

R-sq (adj) = 100.00%

100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Fitted Line Plot for Quadratic Model
Y = 0.000000 + 1.414 X - 0.000000 X**2

Comments

The fitted equation for the quadratic model that describes the relationship between Y and X is:
Y = 0.000000 + 1.414 X - 0.000000 X**2
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.22 - TSND Assembly Comparison (USL = 6.6, LSL = -6.6)

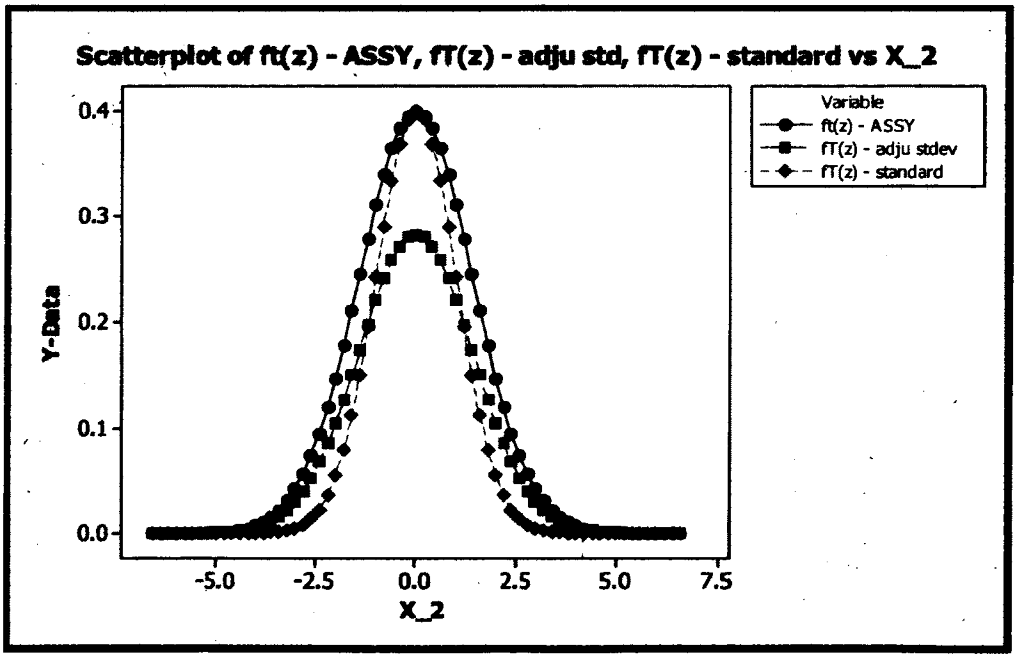
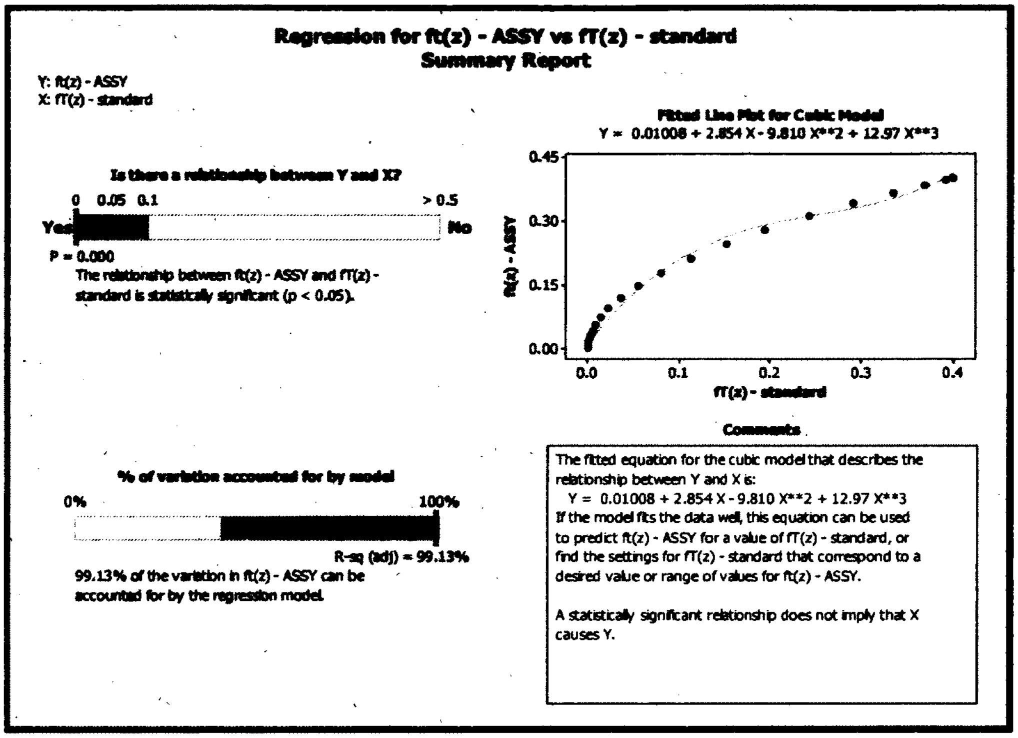


Figure C.23 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 6.6, LSL = -6.6)



Regression for ft(z) - ASSY vs fT(z) - standard
Summary Report

Y: ft(z) - ASSY
X: fT(z) - standard

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant (p < 0.05).

% of variation accounted for by model

R-sq (adj) = 99.13%
99.13% of the variation in ft(z) - ASSY can be accounted for by the regression model.

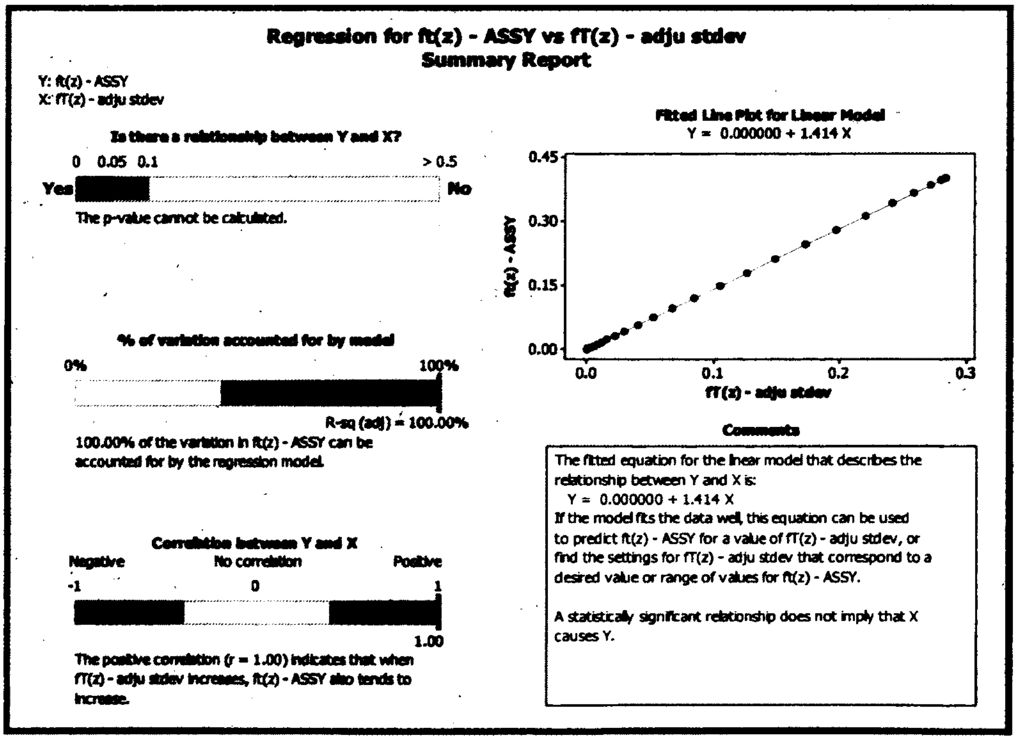
Fitted Line Plot for Cubic Model
Y = 0.01008 + 2.854 X - 9.810 X**2 + 12.97 X**3

Comments

The fitted equation for the cubic model that describes the relationship between Y and X is:
Y = 0.01008 + 2.854 X - 9.810 X**2 + 12.97 X**3
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.24 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 6.6, LSL = -6.6)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

Is there a relationship between Y and X?

The p-value cannot be calculated.

% of variation accounted for by model

R-sq (adj) = 100.00%
100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Correlation between Y and X

1.00
The positive correlation (r = 1.00) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

Fitted Line Plot for Linear Model
Y = 0.000000 + 1.414 X

Comments

The fitted equation for the linear model that describes the relationship between Y and X is:
Y = 0.000000 + 1.414 X
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.25 - TSND Assembly Comparison (USL = 6.4, LSL = -6.4)

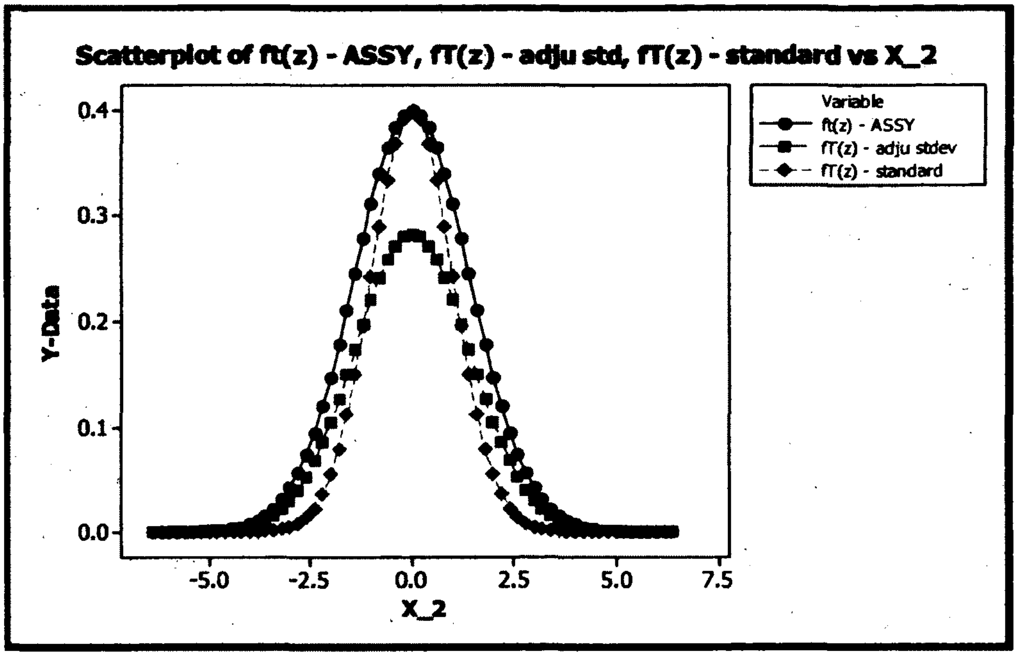
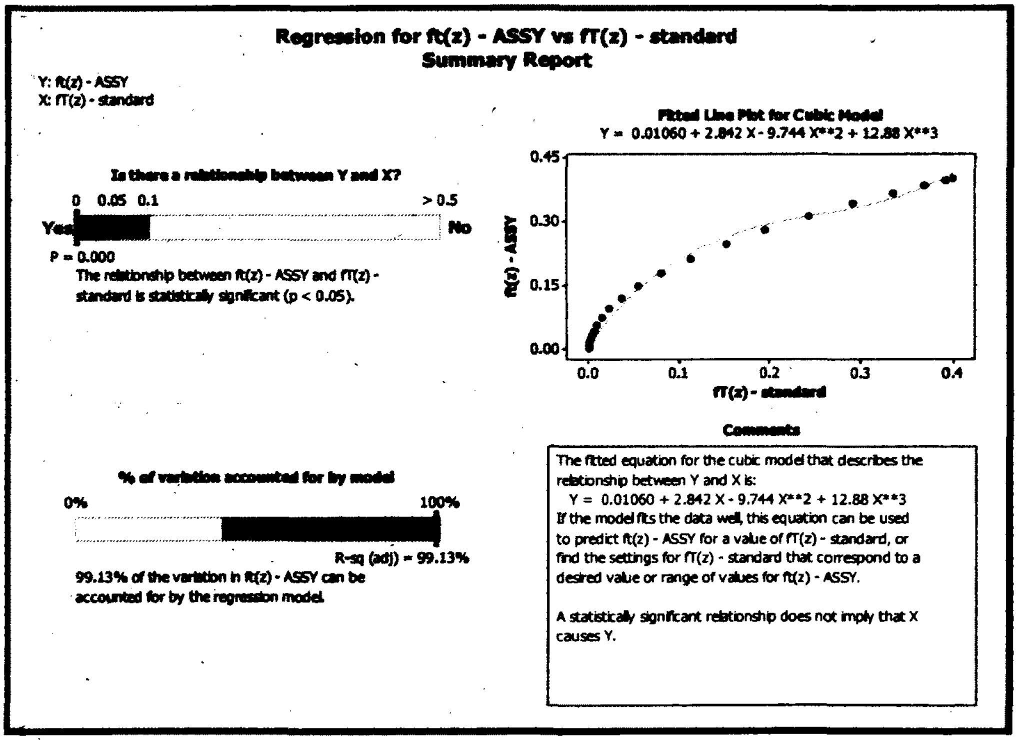


Figure C.26 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 6.4, LSL = -6.4)



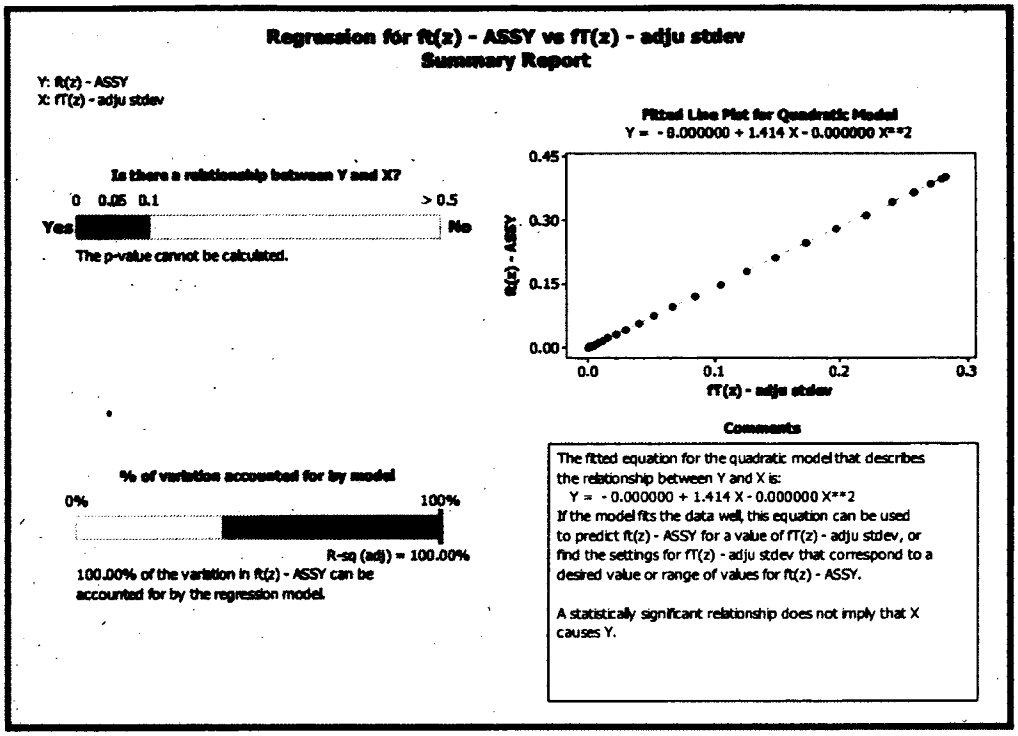
The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant (p < 0.05).

99.13% of the variation in ft(z) - ASSY can be accounted for by the regression model.

The fitted equation for the cubic model that describes the relationship between Y and X is:
Y = 0.01060 + 2.842 X - 9.744 X**2 + 12.88 X**3
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.27 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 6.4, LSL = -6.4)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

Is there a relationship between Y and X?

The p-value cannot be calculated.

% of variation accounted for by model

R-sq (adj) = 100.00%

100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Fitted Line Plot for Quadratic Model

Y = - 0.000000 + 1.414 X - 0.000000 X**2

Comments

The fitted equation for the quadratic model that describes the relationship between Y and X is:
Y = - 0.000000 + 1.414 X - 0.000000 X**2
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.28 - TSND Assembly Comparison (USL = 6.2, LSL = -6.2)

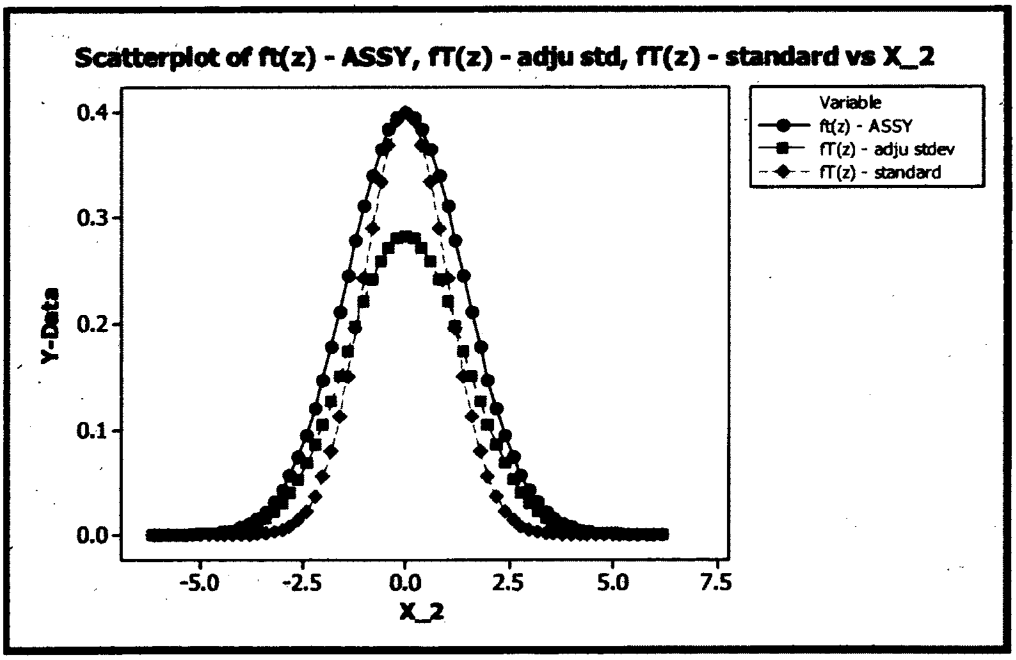
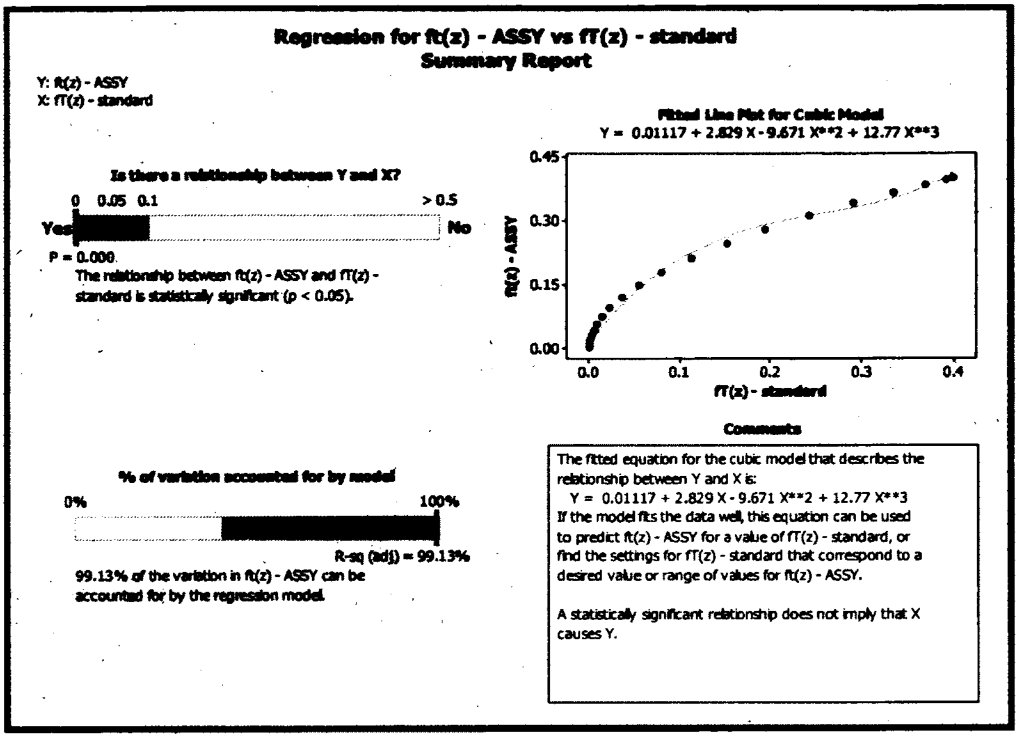


Figure C.29 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 6.2, LSL = -6.2)



Regression for ft(z) - ASSY vs fT(z) - standard
Summary Report

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant ($p < 0.05$).

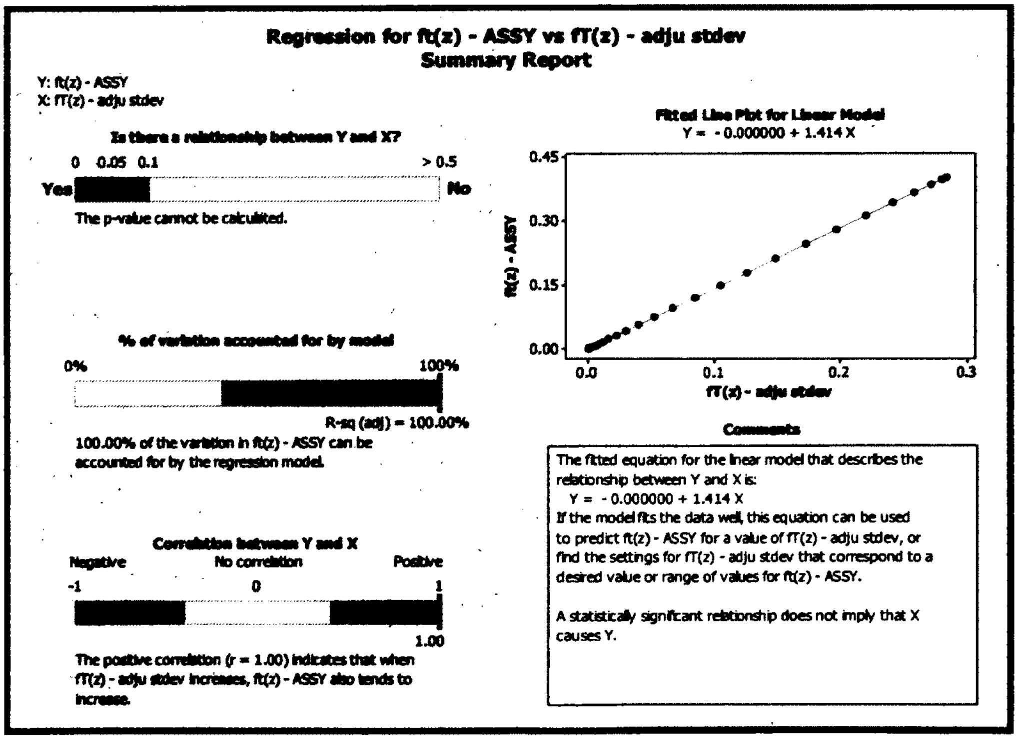
Y = 0.01117 + 2.829 X - 9.671 X**2 + 12.77 X**3

99.13% of the variation in ft(z) - ASSY can be accounted for by the regression model.

The fitted equation for the cubic model that describes the relationship between Y and X is:
Y = 0.01117 + 2.829 X - 9.671 X**2 + 12.77 X**3
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.30 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 6.2, LSL = -6.2)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

The p-value cannot be calculited.

Y = - 0.000000 + 1.414 X

100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

The positive correlation ($r = 1.00$) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

The fitted equation for the linear model that describes the relationship between Y and X is:
Y = - 0.000000 + 1.414 X
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.31 - TSND Assembly Comparison (USL = 6, LSL = -6)

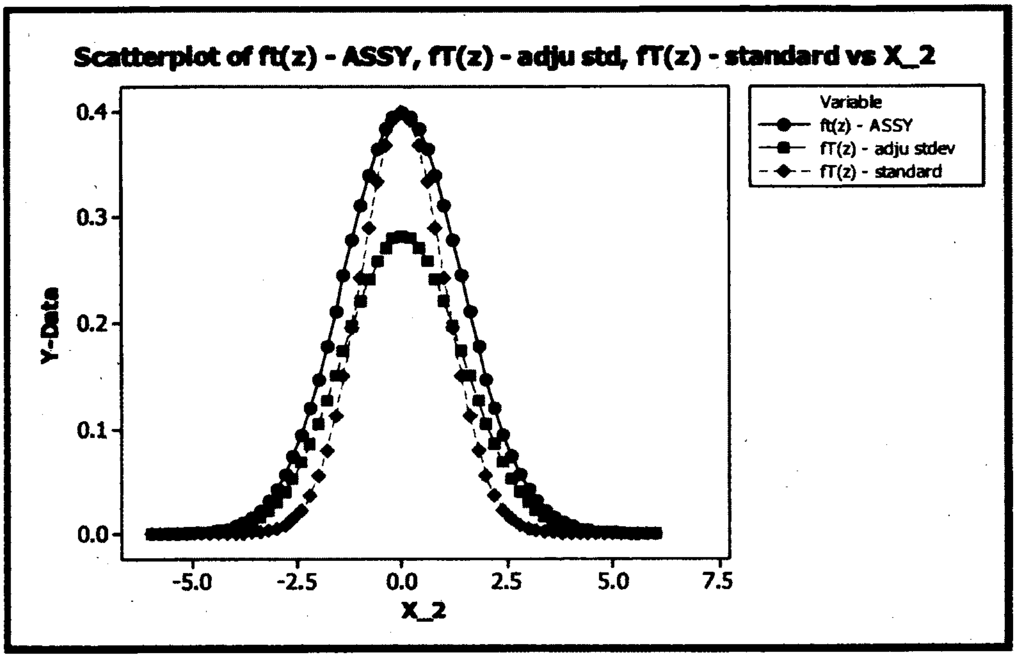
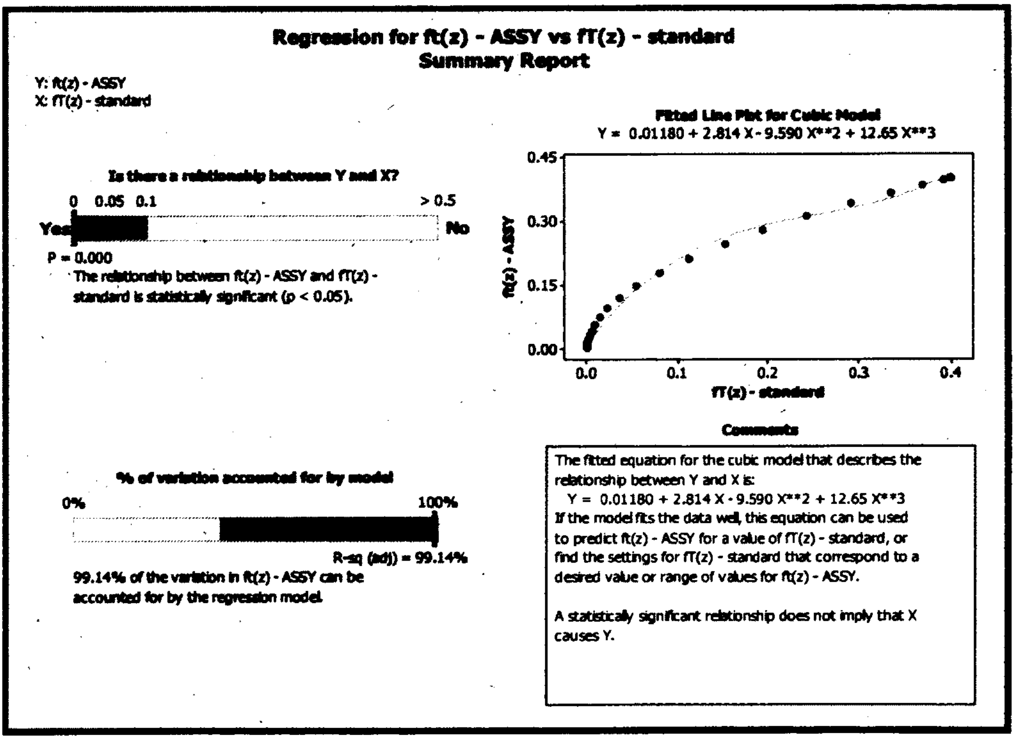


Figure C.32 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 6, LSL = -6)



The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant ($p < 0.05$).

99.14% of the variation in ft(z) - ASSY can be accounted for by the regression model.

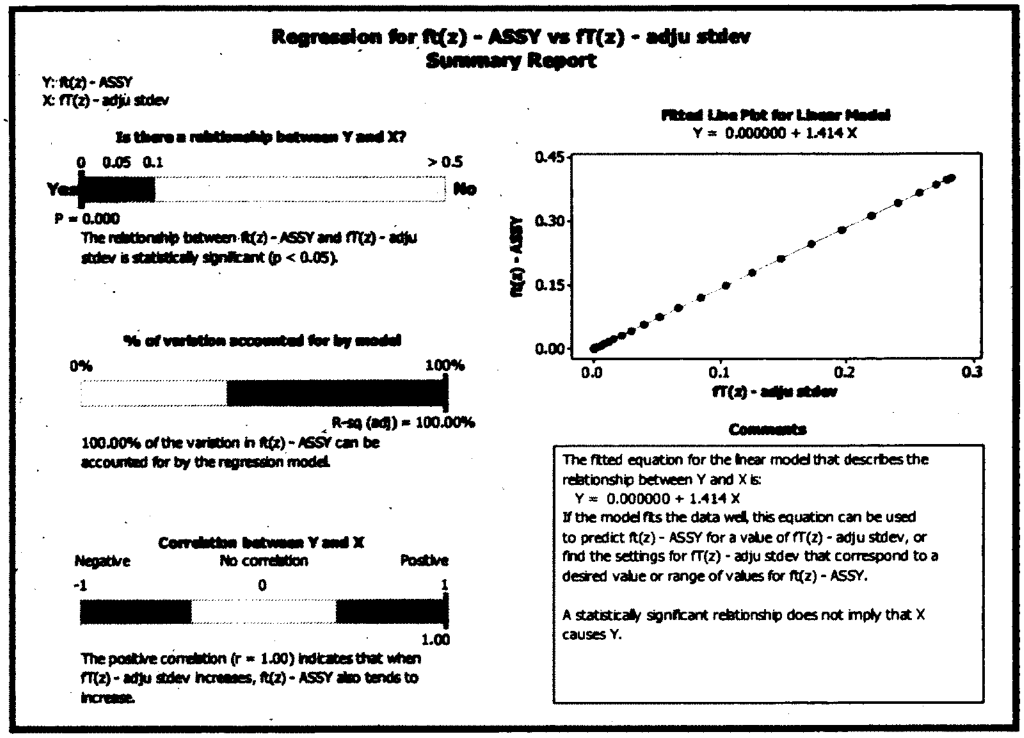
The fitted equation for the cubic model that describes the relationship between Y and X is:

Y = 0.01180 + 2.814 X - 9.590 X**2 + 12.65 X**3

If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.33 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 6, LSL = -6)



The relationship between ft(z) - ASSY and fT(z) - adju stdev is statistically significant ($p < 0.05$).

100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

The positive correlation ($r = 1.00$) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

The fitted equation for the linear model that describes the relationship between Y and X is:

$Y = 0.000000 + 1.414 X$

If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.34 - TSND Assembly Comparison (USL = 5.8, LSL = -5.8)

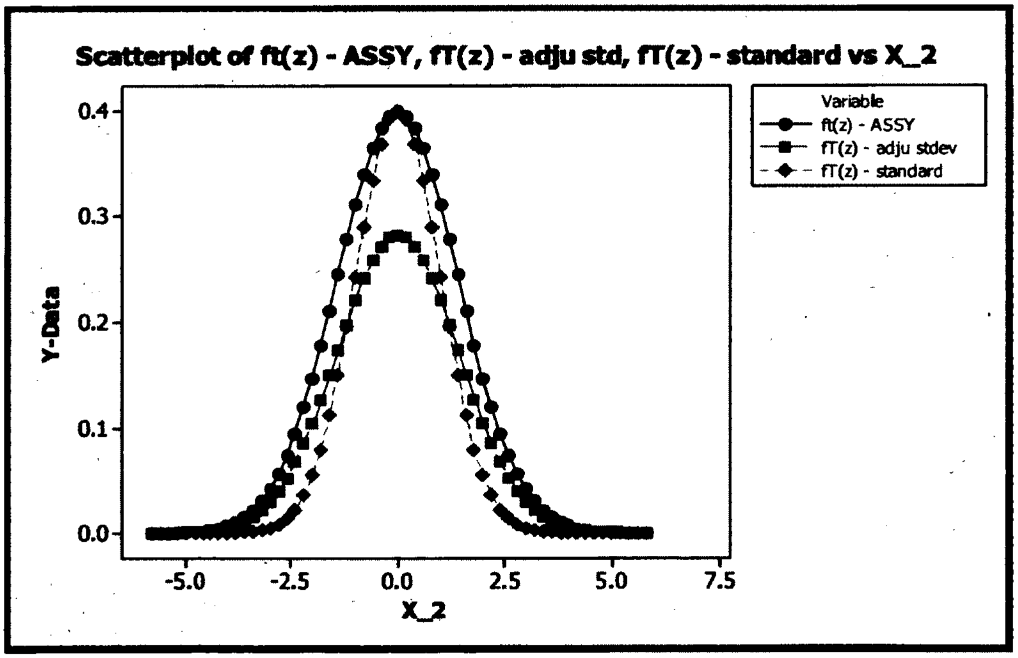
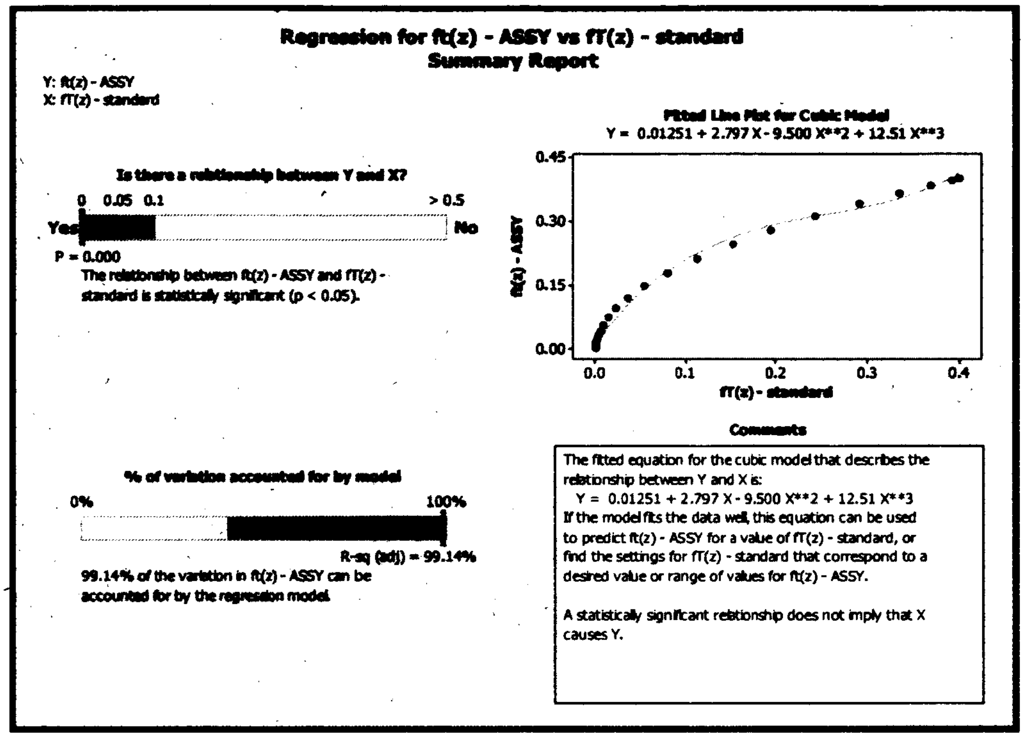


Figure C.35 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 5.8, LSL = -5.8)



Regression for ft(z) - ASSY vs fT(z) - standard
Summary Report

Y: ft(z) - ASSY
X: fT(z) - standard

Is there a relationship between Y and X?

P = 0.000

The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant (p < 0.05).

% of variation accounted for by model

R-sq (adj) = 99.14%

99.14% of the variation in ft(z) - ASSY can be accounted for by the regression model.

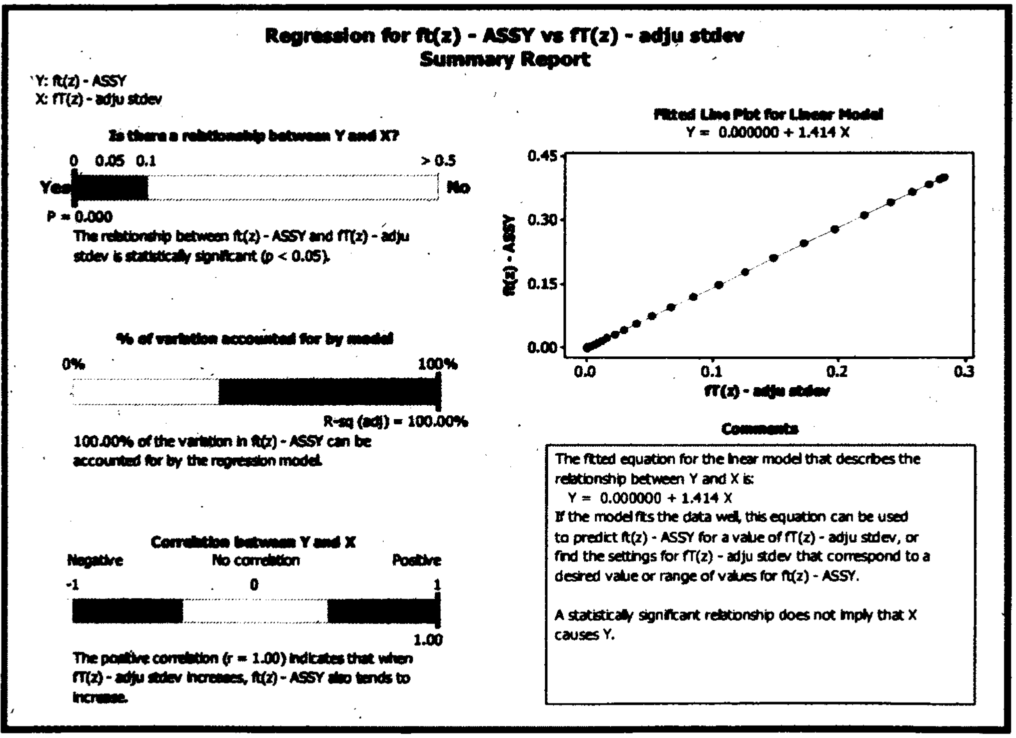
Fitted Line Plot for Cubic Model
Y = 0.01251 + 2.797 X - 9.500 X**2 + 12.51 X**3

Comments

The fitted equation for the cubic model that describes the relationship between Y and X is:
Y = 0.01251 + 2.797 X - 9.500 X**2 + 12.51 X**3
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.36 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 5.8, LSL = -5.8)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

Is there a relationship between Y and X?

P = 0.000

The relationship between ft(z) - ASSY and fT(z) - adju stdev is statistically significant (p < 0.05).

% of variation accounted for by model

R-sq (adj) = 100.00%

100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Correlation between Y and X

The positive correlation (r = 1.00) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

Fitted Line Plot for Linear Model
Y = 0.000000 + 1.414 X

Comments

The fitted equation for the linear model that describes the relationship between Y and X is:
Y = 0.000000 + 1.414 X
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.37 - TSND Assembly Comparison (USL = 5.6, LSL = -5.6)

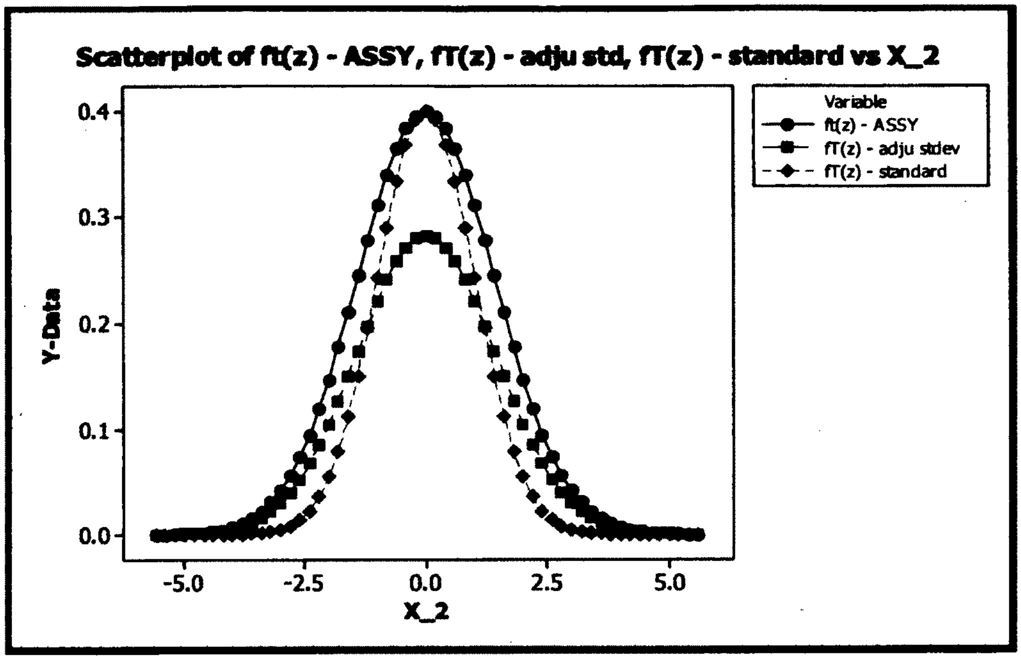
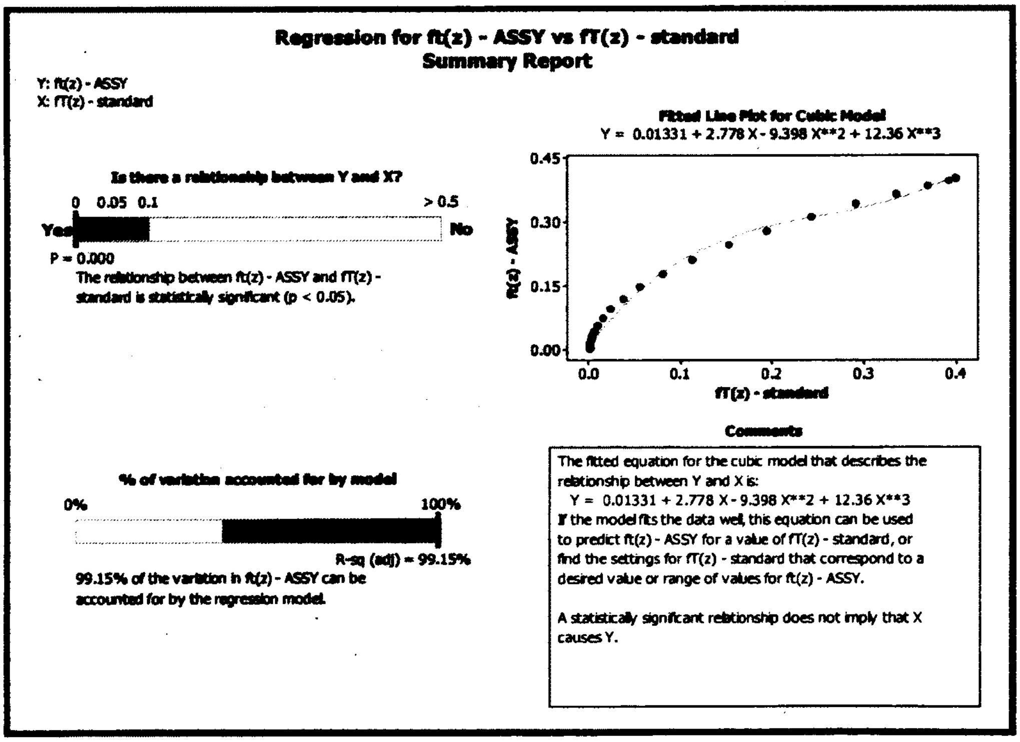


Figure C.38 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 5.6, LSL = -5.6)



P = 0.000

The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant (p < 0.05).

R-sq (adj) = 99.15%

99.15% of the variation in ft(z) - ASSY can be accounted for by the regression model.

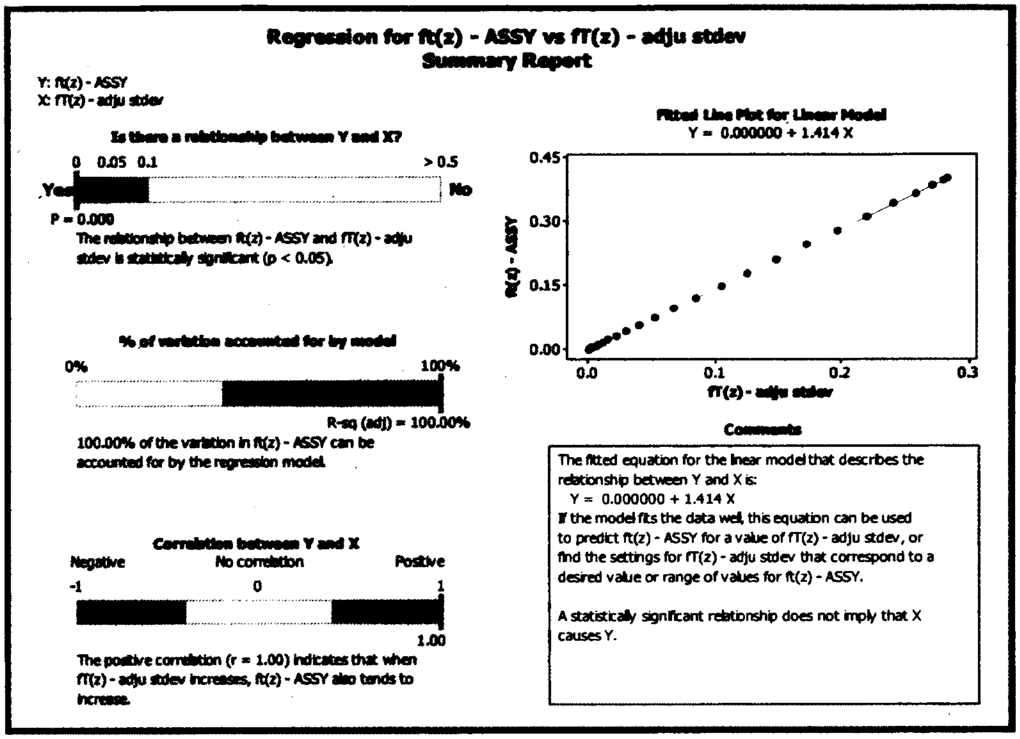
The fitted equation for the cubic model that describes the relationship between Y and X is:

Y = 0.01331 + 2.778 X - 9.398 X**2 + 12.36 X**3

If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.39 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 5.6, LSL = -5.6)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - adju stdev is statistically significant (p < 0.05).

% of variation accounted for by model

R-sq (adj) = 100.00%
100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Correlation between Y and X

The positive correlation (r = 1.00) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

Fitted Line Plot for Linear Model
Y = 0.000000 + 1.414 X

Comments

The fitted equation for the linear model that describes the relationship between Y and X is:
Y = 0.000000 + 1.414 X
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.40 - TSND Assembly Comparison (USL = 5.4, LSL = -5.4)

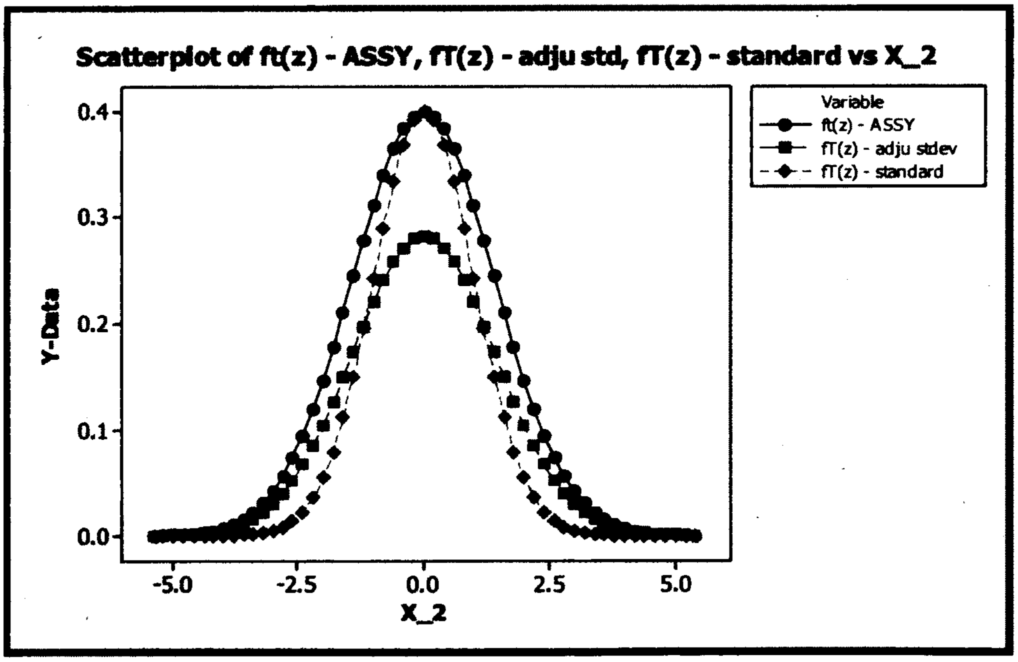
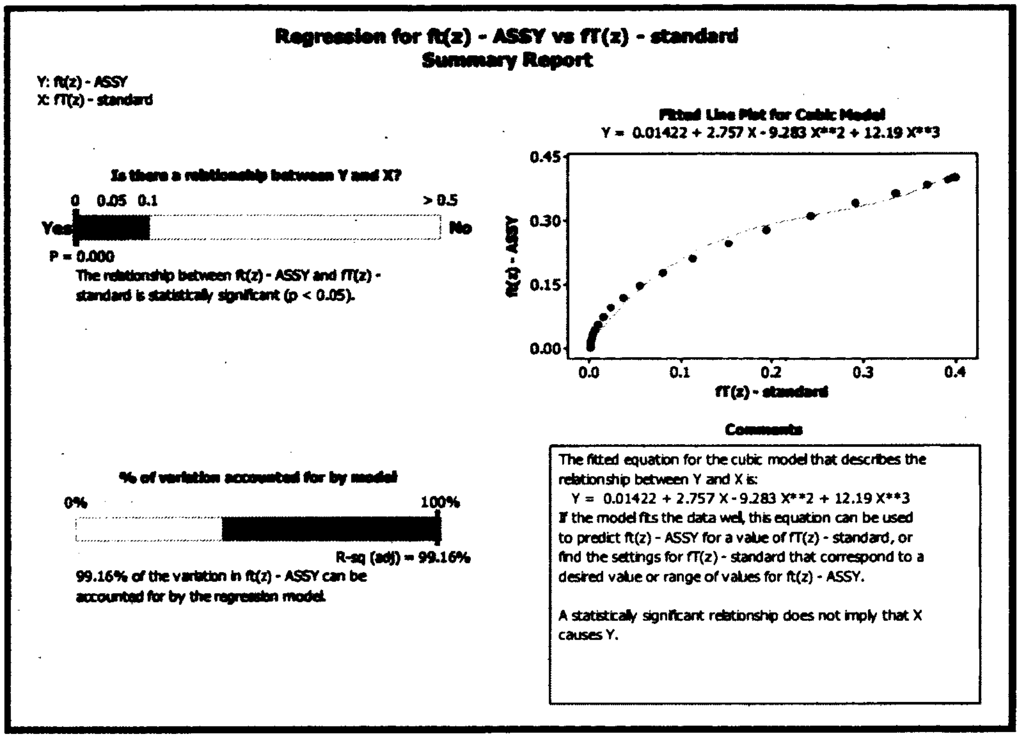


Figure C.41 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 5.4, LSL = -5.4)



Regression for ft(z) - ASSY vs fT(z) - standard
Summary Report

Y: ft(z) - ASSY
X: fT(z) - standard

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant ($p < 0.05$).

% of variation accounted for by model

R-sq (adj) = 99.16%
99.16% of the variation in ft(z) - ASSY can be accounted for by the regression model.

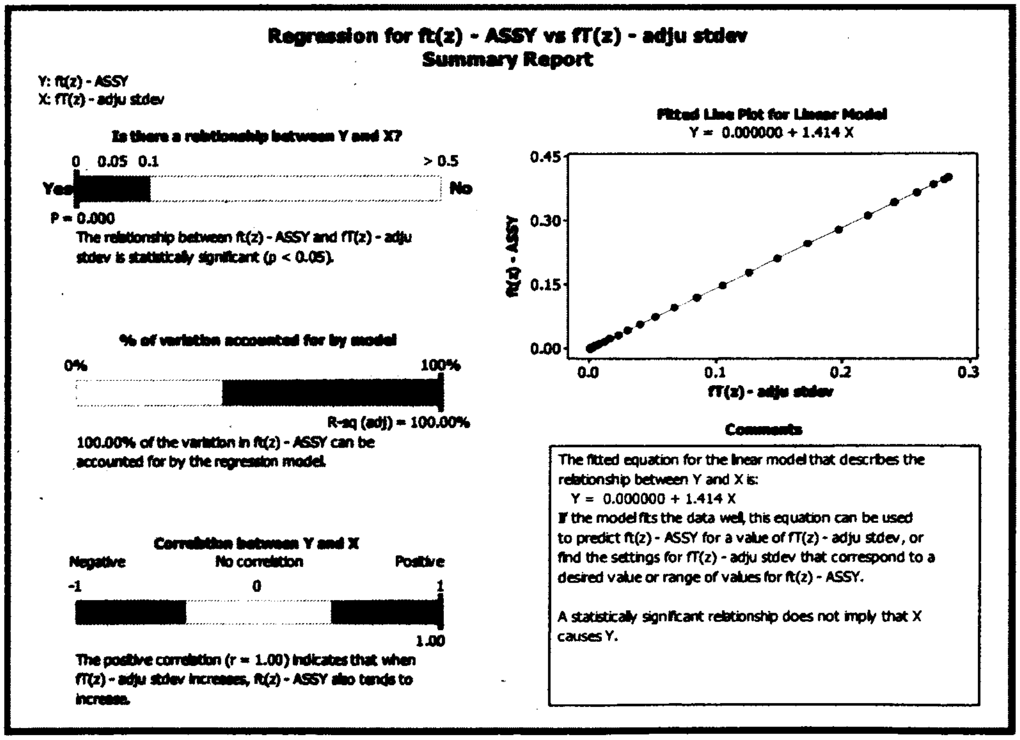
Fitted Line Plot for Cubic Model
Y = 0.01422 + 2.757 X - 9.283 X**2 + 12.19 X**3

Comments

The fitted equation for the cubic model that describes the relationship between Y and X is:
Y = 0.01422 + 2.757 X - 9.283 X**2 + 12.19 X**3
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.42 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 5.4, LSL = -5.4)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - adju stdev is statistically significant ($p < 0.05$).

% of variation accounted for by model

R-sq (adj) = 100.00%
100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Correlation between Y and X

1.00
The positive correlation (r = 1.00) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

Fitted Line Plot for Linear Model
Y = 0.000000 + 1.414 X

Comments

The fitted equation for the linear model that describes the relationship between Y and X is:
Y = 0.000000 + 1.414 X
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.43 - TSND Assembly Comparison (USL = 5.2, LSL = -5.2)

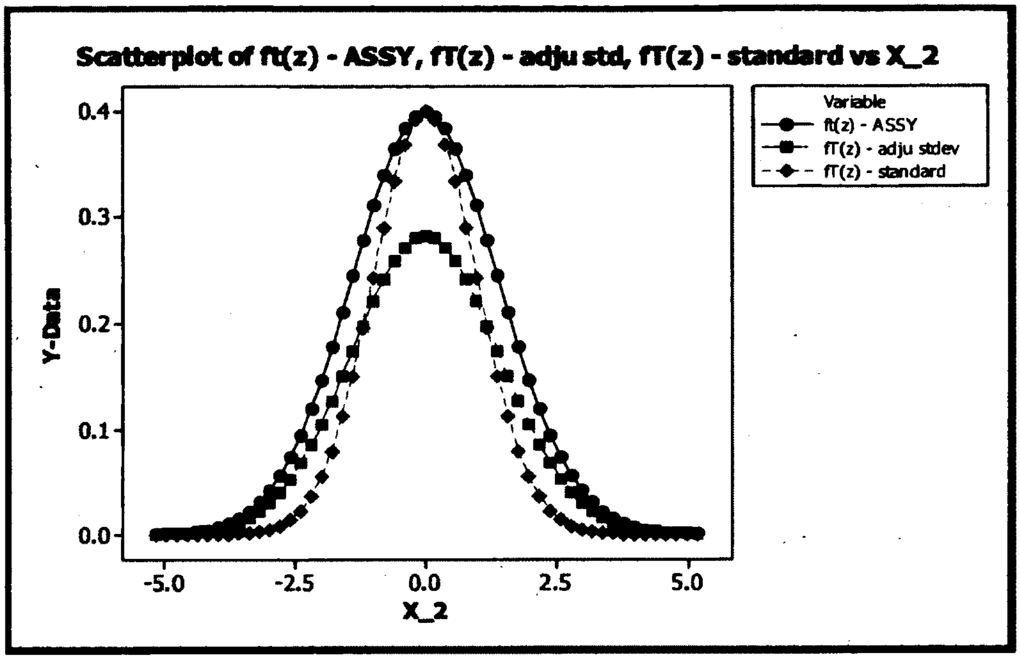
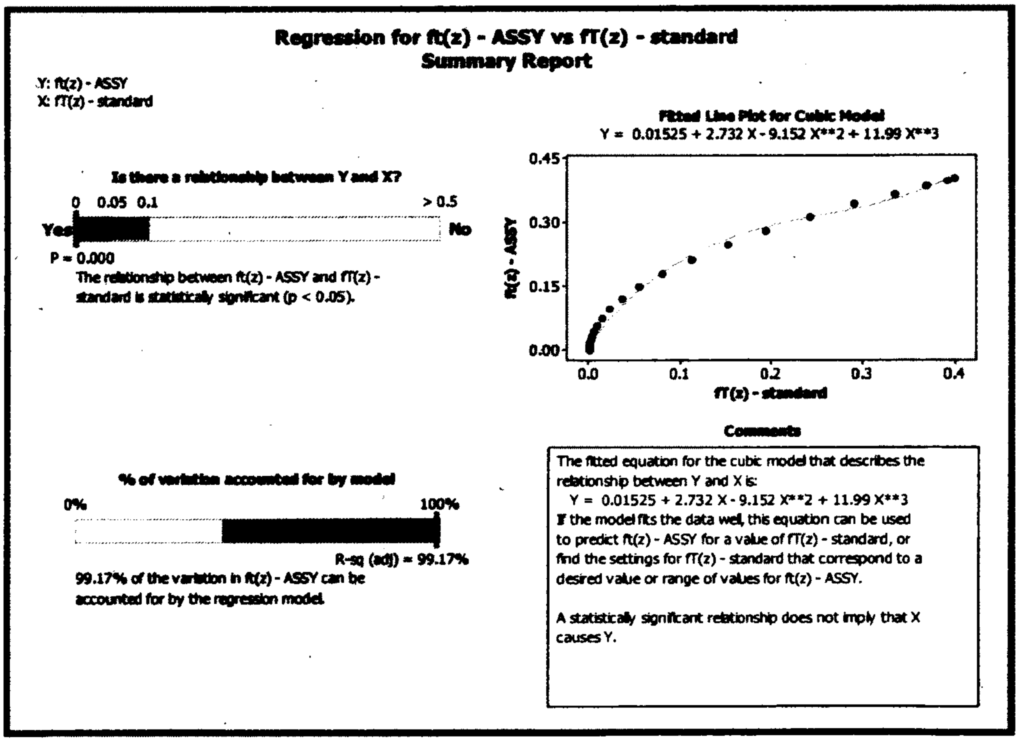


Figure C.44 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 5.2, LSL = -5.2)



P = 0.000

The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant (p < 0.05).

99.17% of the variation in ft(z) - ASSY can be accounted for by the regression model.

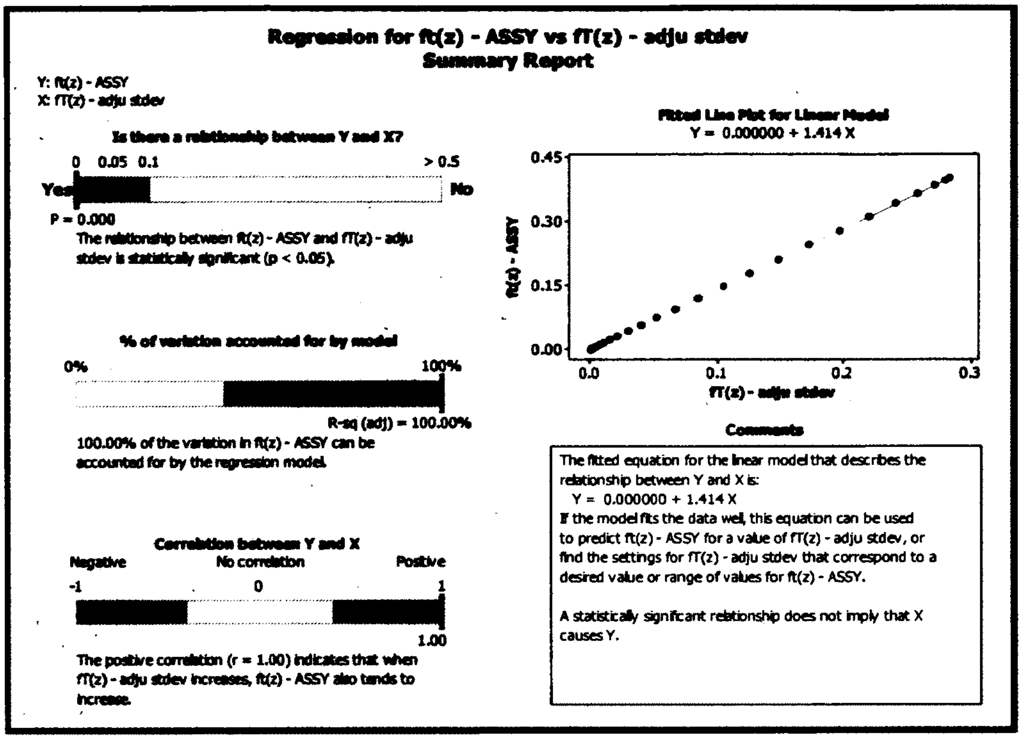
The fitted equation for the cubic model that describes the relationship between Y and X is:

Y = 0.01525 + 2.732 X - 9.152 X**2 + 11.99 X**3

If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.45 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 5.2, LSL = -5.2)


Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - adju stdev is statistically significant (p < 0.05).

% of variation accounted for by model

R-sq (adj) = 100.00%
100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Correlation between Y and X

The positive correlation (r = 1.00) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

Fitted Line Plot for Linear Model
Y = 0.000000 + 1.414 X

Comments

The fitted equation for the linear model that describes the relationship between Y and X is:
Y = 0.000000 + 1.414 X
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.46 - TSND Assembly Comparison (USL = 5, LSL = -5)

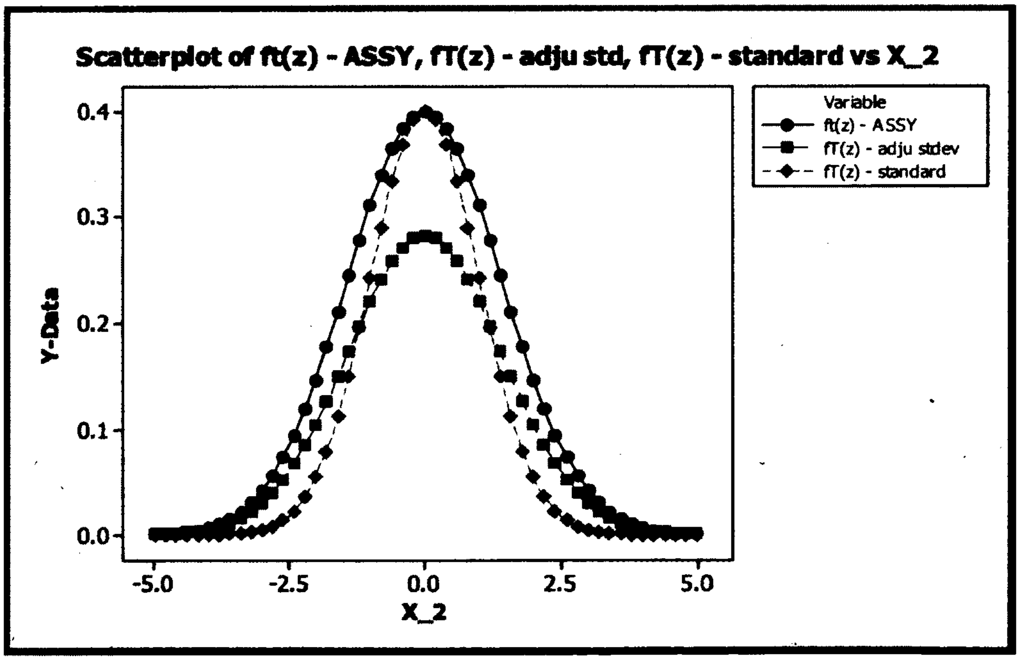
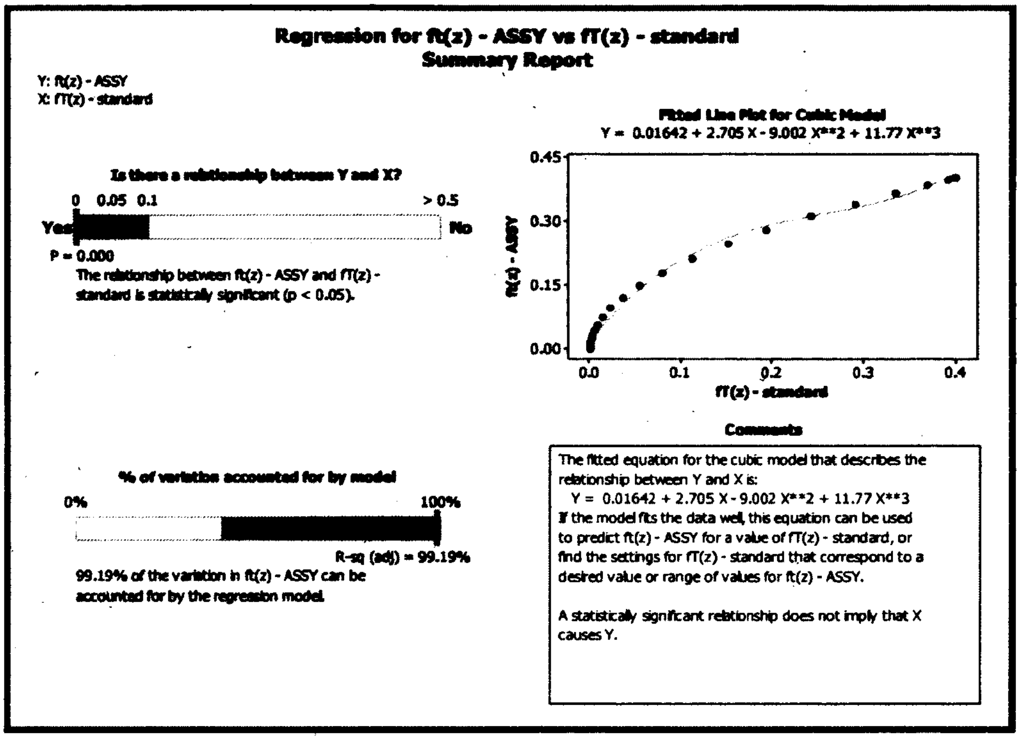


Figure C.47 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 5, LSL = -5)



Regression for ft(z) - ASSY vs fT(z) - standard
Summary Report

Y: ft(z) - ASSY
X: fT(z) - standard

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant (p < 0.05).

% of variation accounted for by model

R-sq (adj) = 99.19%
99.19% of the variation in ft(z) - ASSY can be accounted for by the regression model.

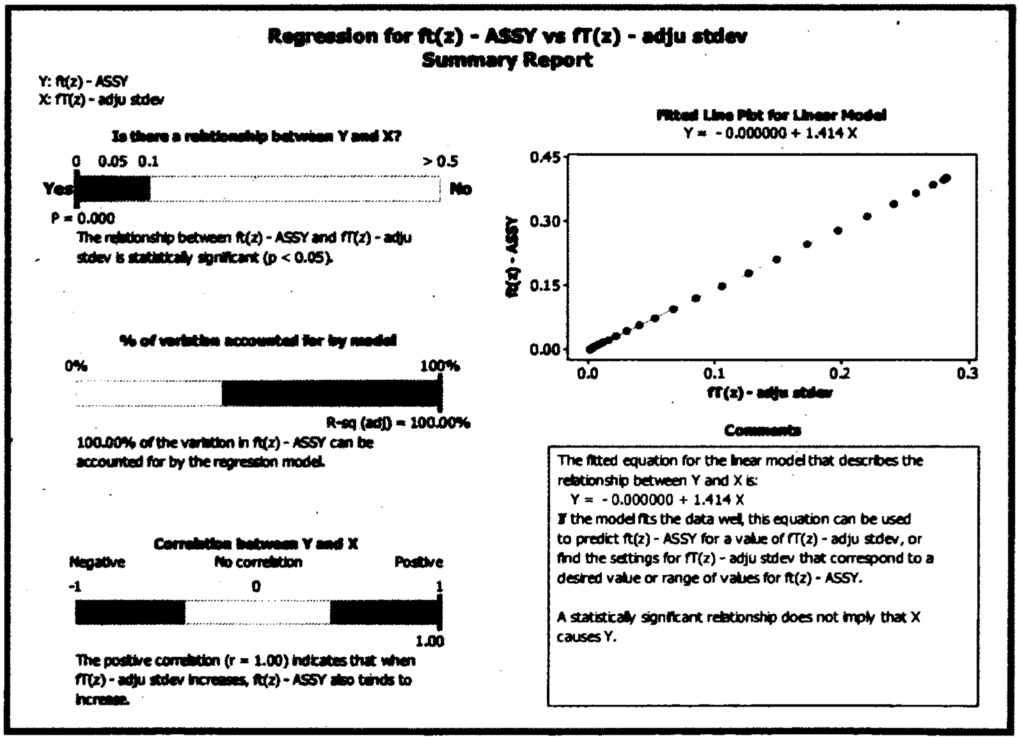
Fitted Line Plot for Cubic Model
Y = 0.01642 + 2.705 X - 9.002 X**2 + 11.77 X**3

Comments

The fitted equation for the cubic model that describes the relationship between Y and X is:
Y = 0.01642 + 2.705 X - 9.002 X**2 + 11.77 X**3
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.48 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 5, LSL = -5)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - adju stdev is statistically significant (p < 0.05).

% of variation accounted for by model

R-sq (adj) = 100.00%
100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Correlation between Y and X

The positive correlation (r = 1.00) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

Fitted Line Plot for Linear Model
Y = - 0.000000 + 1.414 X

Comments

The fitted equation for the linear model that describes the relationship between Y and X is:
Y = - 0.000000 + 1.414 X
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.49 - TSND Assembly Comparison (USL = 4.8, LSL = -4.8)

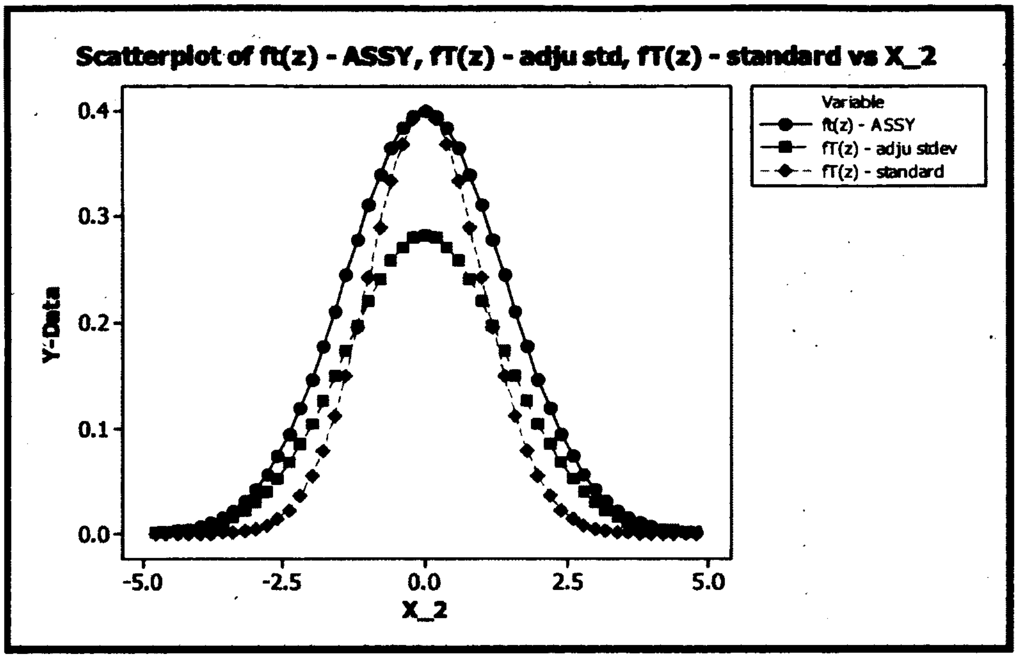
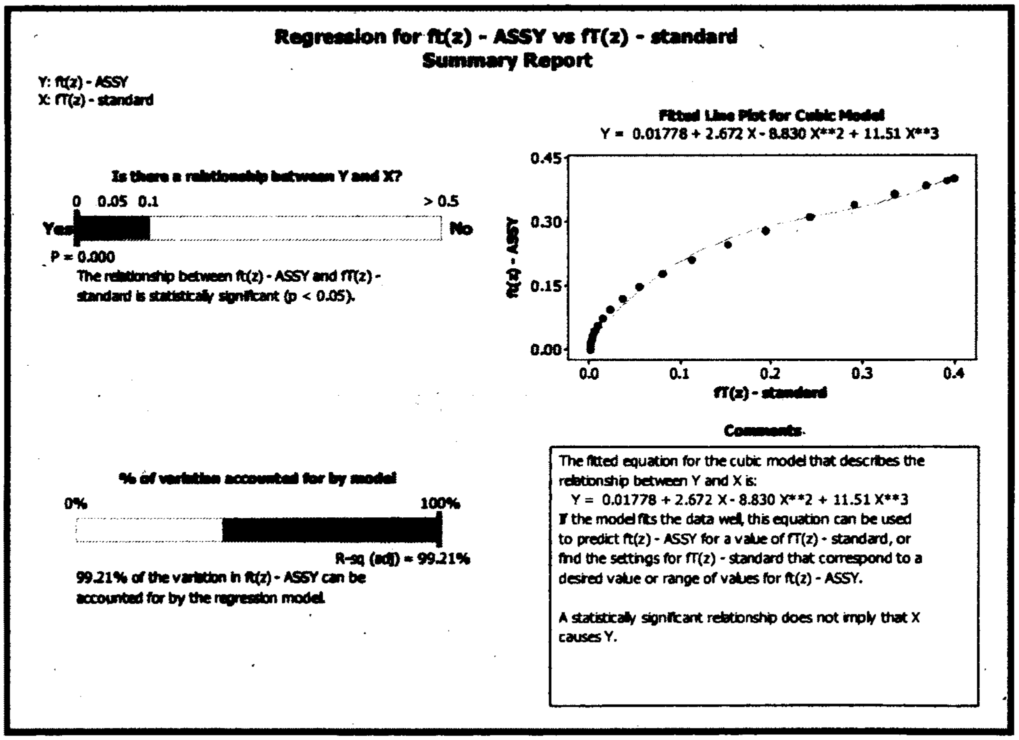


Figure C.50 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 4.8, LSL = -4.8)


Regression for ft(z) - ASSY vs fT(z) - standard
Summary Report

Y: ft(z) - ASSY
X: fT(z) - standard

Is there a relationship between Y and X?

P = 0.000

The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant (p < 0.05).

% of variation accounted for by model

R-sq (adj) = 99.21%

99.21% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Fitted Line Plot for Cubic Model

Y = 0.01778 + 2.672 X - 8.830 X**2 + 11.51 X**3

Comments

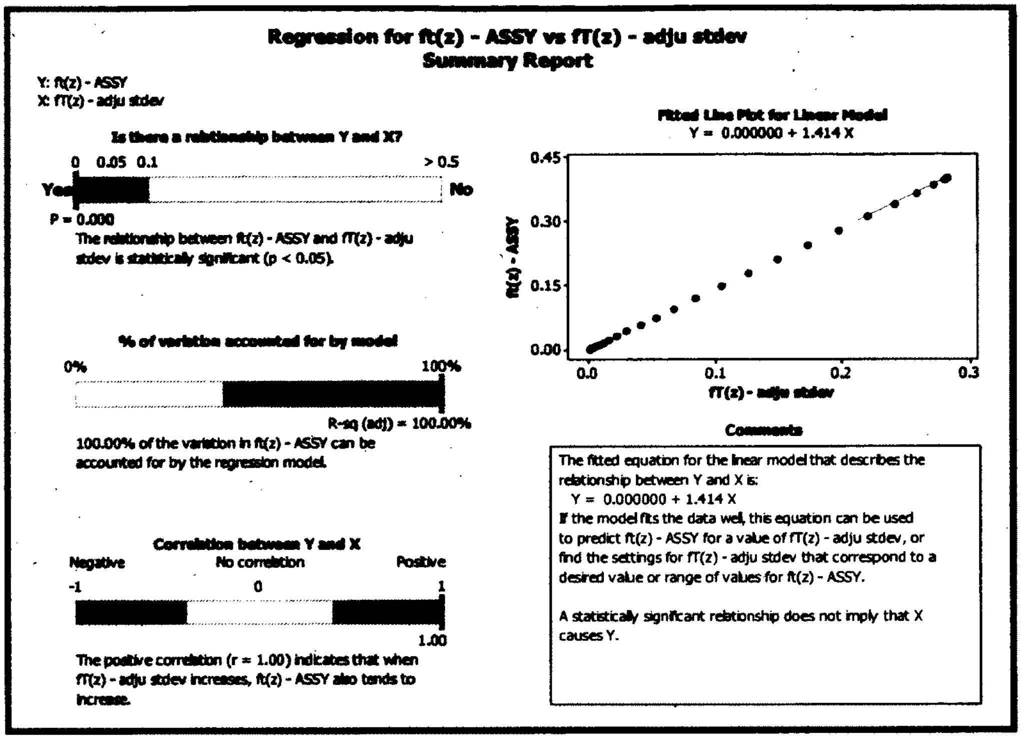
The fitted equation for the cubic model that describes the relationship between Y and X is:

Y = 0.01778 + 2.672 X - 8.830 X**2 + 11.51 X**3

If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.51 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 4.8, LSL = -4.8)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - adju stdev is statistically significant ($p < 0.05$).

% of variation accounted for by model

R-sq (adj) = 100.00%
100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Correlation between Y and X

The positive correlation (r = 1.00) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

Fitted Line Plot for Linear Model
Y = 0.000000 + 1.414 X

Comments

The fitted equation for the linear model that describes the relationship between Y and X is:
Y = 0.000000 + 1.414 X
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.52 - TSND Assembly Comparison (USL = 4.6, LSL = -4.6)

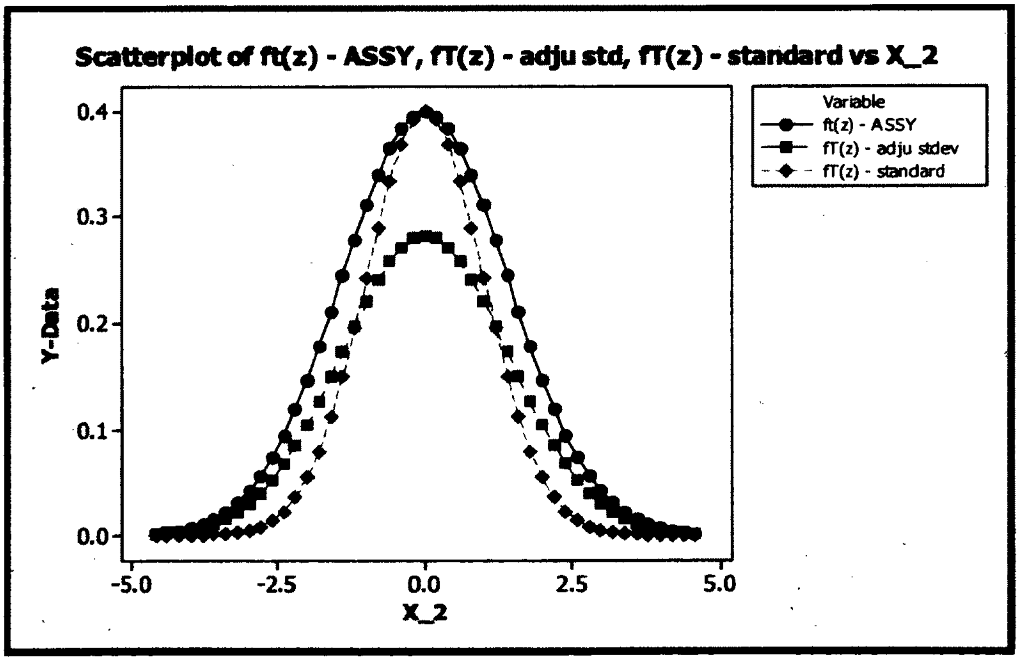
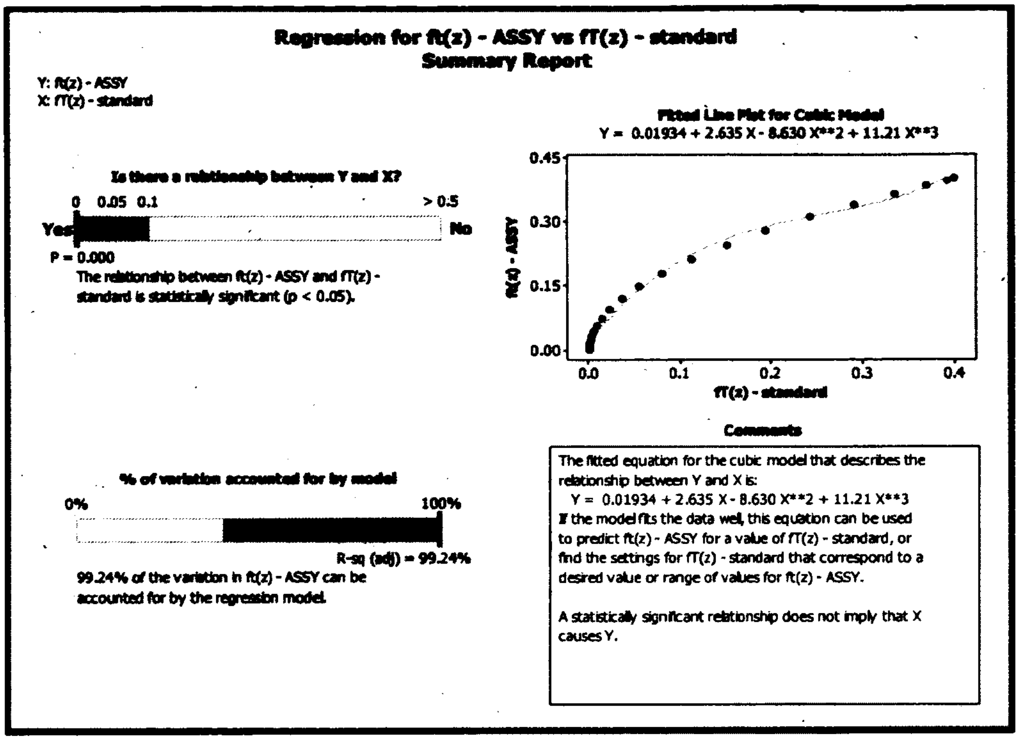


Figure C.53 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 4.6, LSL = -4.6)



Regression for ft(z) - ASSY vs fT(z) - standard
Summary Report

Y: ft(z) - ASSY
X: fT(z) - standard

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant ($p < 0.05$).

% of variation accounted for by model

R-sq (adj) = 99.24%
99.24% of the variation in ft(z) - ASSY can be accounted for by the regression model.

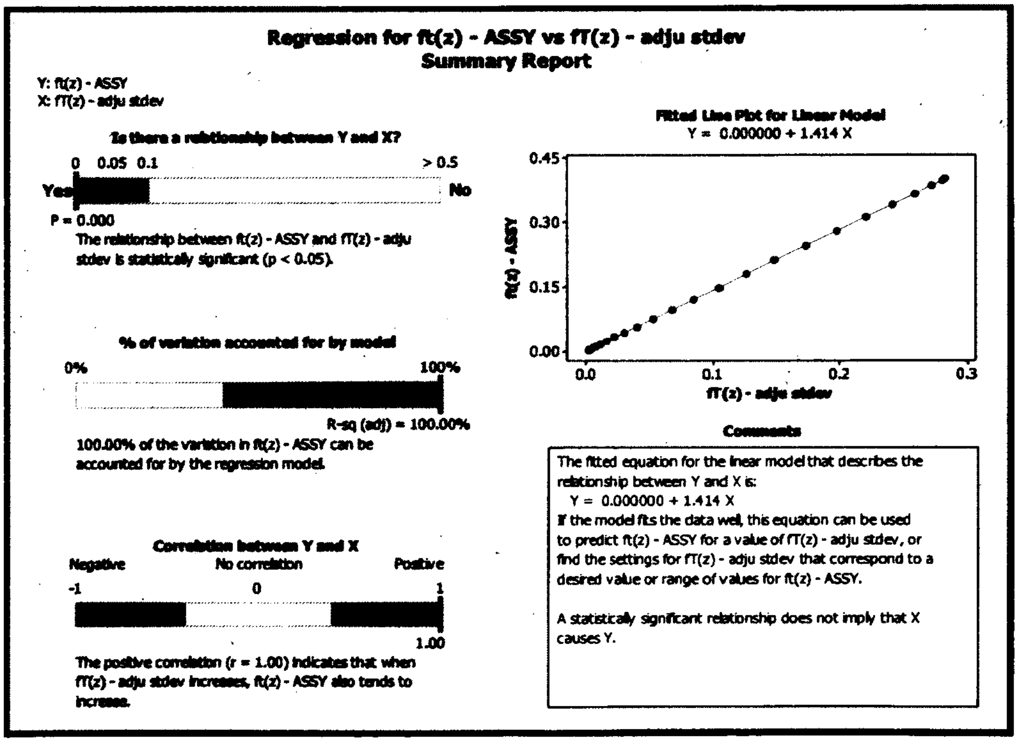
Fitted Line Plot for Cubic Model
Y = 0.01934 + 2.635 X - 8.630 X**2 + 11.21 X**3

Comments

The fitted equation for the cubic model that describes the relationship between Y and X is:
Y = 0.01934 + 2.635 X - 8.630 X**2 + 11.21 X**3
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.54 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 4.6, LSL = -4.6)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - adju stdev is statistically significant ($p < 0.05$).

% of variation accounted for by model

R-sq (adj) = 100.00%
100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Correlation between Y and X

1.00
The positive correlation (r = 1.00) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

Fitted Line Plot for Linear Model
Y = 0.000000 + 1.414 X

Comments

The fitted equation for the linear model that describes the relationship between Y and X is:
Y = 0.000000 + 1.414 X
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.55 - TSND Assembly Comparison (USL = 4.4, LSL = -4.4)

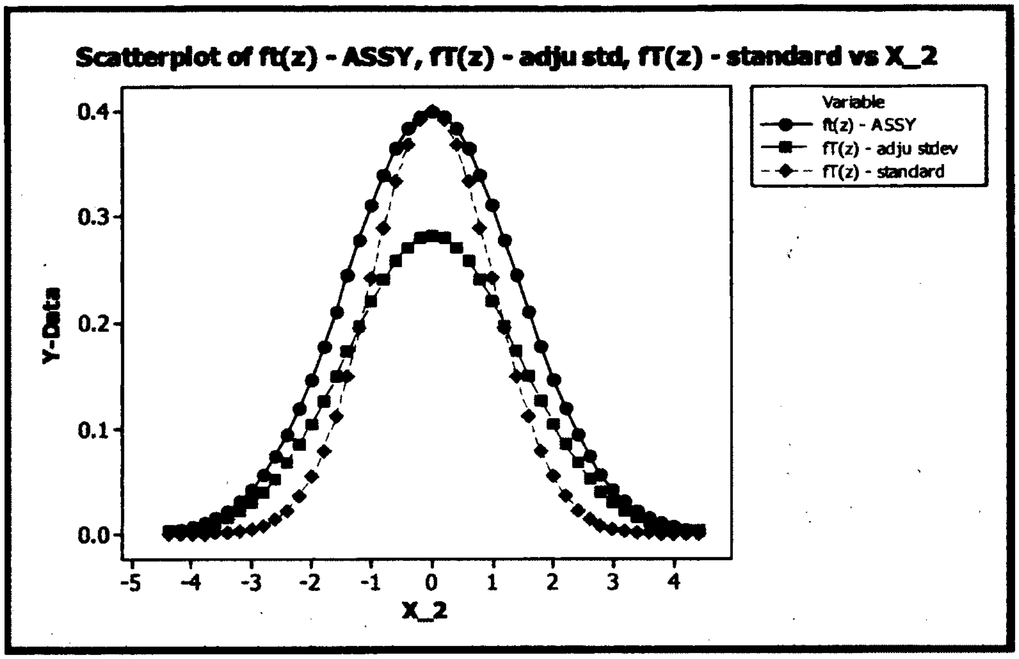
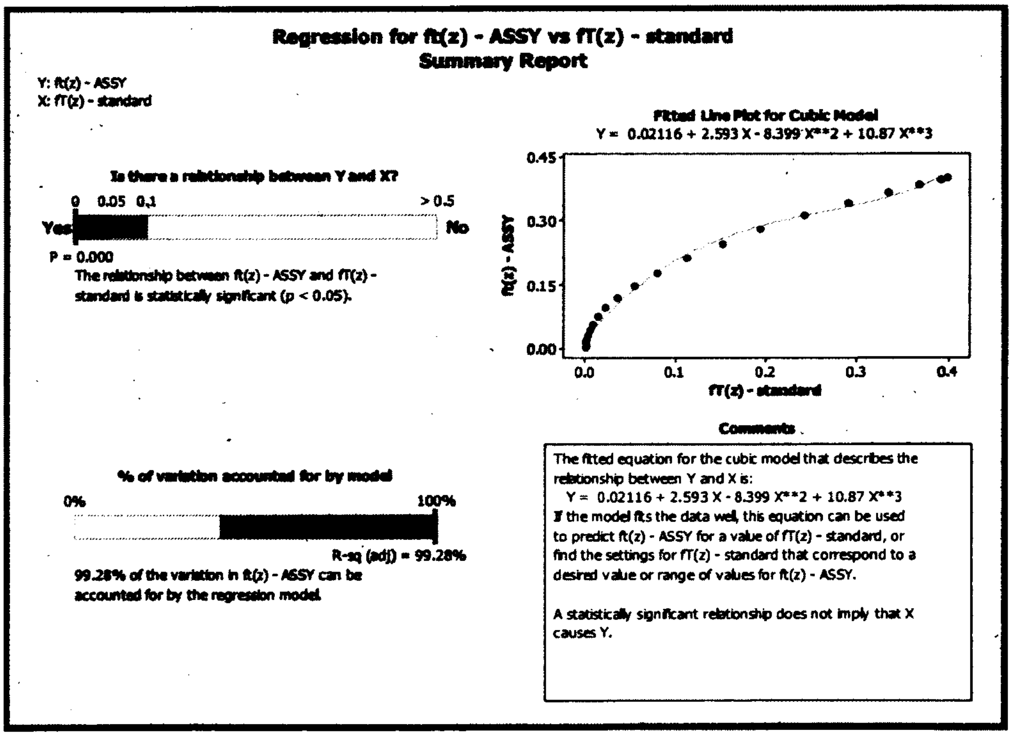


Figure C.56 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 4.4, LSL = -4.4)



P = 0.000

The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant ($p < 0.05$).

$Y = 0.02116 + 2.593 X - 8.399 X^{**}2 + 10.87 X^{**}3$

R-sq (adj) = 99.28%

99.28% of the variation in ft(z) - ASSY can be accounted for by the regression model.

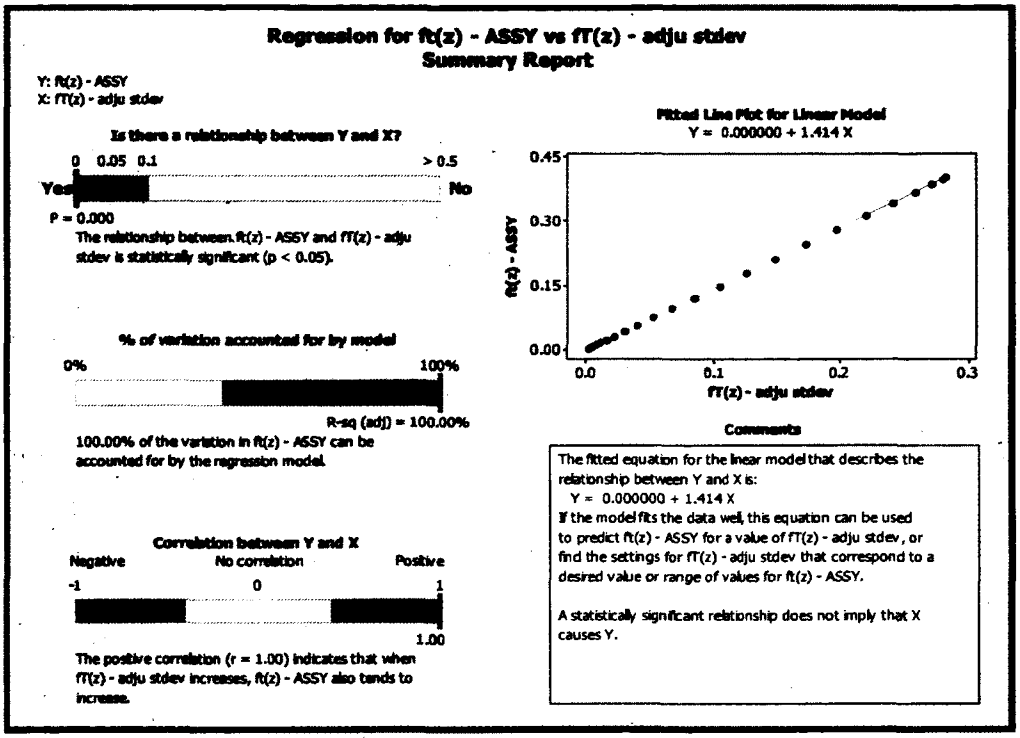
The fitted equation for the cubic model that describes the relationship between Y and X is:

$Y = 0.02116 + 2.593 X - 8.399 X^{**}2 + 10.87 X^{**}3$

If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.57 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 4.4, LSL = -4.4)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

Is there a relationship between Y and X?

P = 0.000

The relationship between ft(z) - ASSY and fT(z) - adju stdev is statistically significant (p < 0.05).

% of variation accounted for by model

R-sq (adj) = 100.00%

100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Correlation between Y and X

The positive correlation (r = 1.00) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

Fitted Line Plot for Linear Model
Y = 0.000000 + 1.414 X

Comments

The fitted equation for the linear model that describes the relationship between Y and X is:
Y = 0.000000 + 1.414 X
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.58 - TSND Assembly Comparison (USL = 4.2, LSL = -4.2)

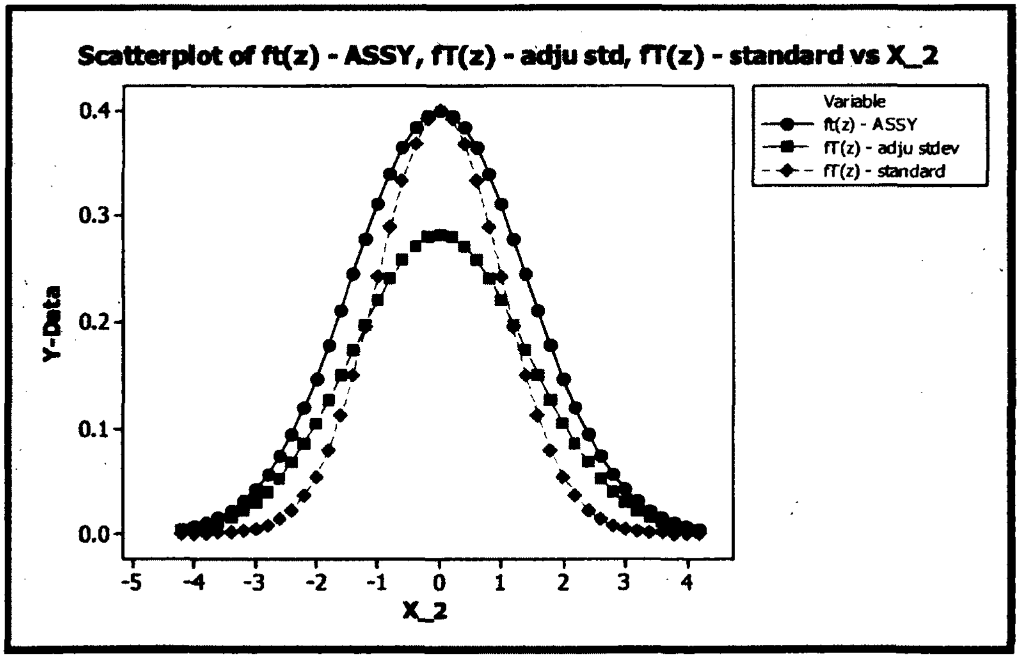
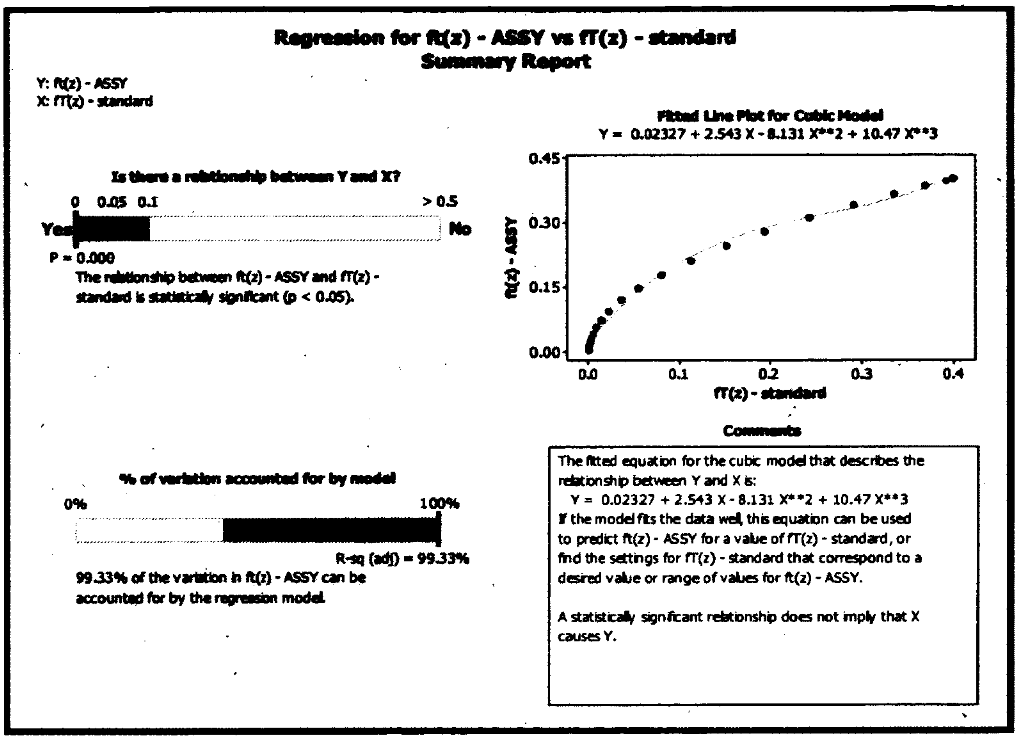


Figure C.59 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 4.2, LSL = -4.2)



Regression for ft(z) - ASSY vs fT(z) - standard
Summary Report

Y: ft(z) - ASSY
X: fT(z) - standard

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant ($p < 0.05$).

% of variation accounted for by model

R-sq (adj) = 99.33%
99.33% of the variation in ft(z) - ASSY can be accounted for by the regression model.

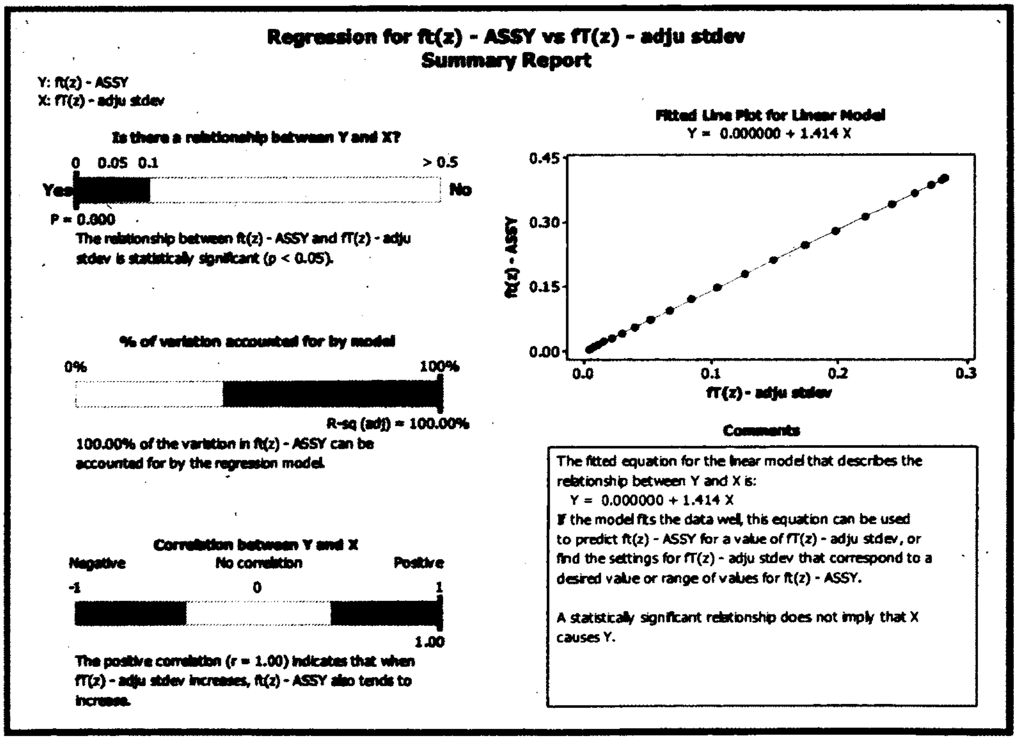
Fitted Line Plot for Cubic Model
Y = 0.02327 + 2.543 X - 8.131 X**2 + 10.47 X**3

Comments

The fitted equation for the cubic model that describes the relationship between Y and X is:
Y = 0.02327 + 2.543 X - 8.131 X**2 + 10.47 X**3
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.60 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 4.2, LSL = -4.2)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - adju stdev is statistically significant ($p < 0.05$).

% of variation accounted for by model

R-sq (adj) = 100.00%
100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Correlation between Y and X

The positive correlation (r = 1.00) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

Fitted Line Plot for Linear Model
Y = 0.000000 + 1.414 X

Comments

The fitted equation for the linear model that describes the relationship between Y and X is:
Y = 0.000000 + 1.414 X
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.61 - TSND Assembly Comparison (USL = 4, LSL = -4)

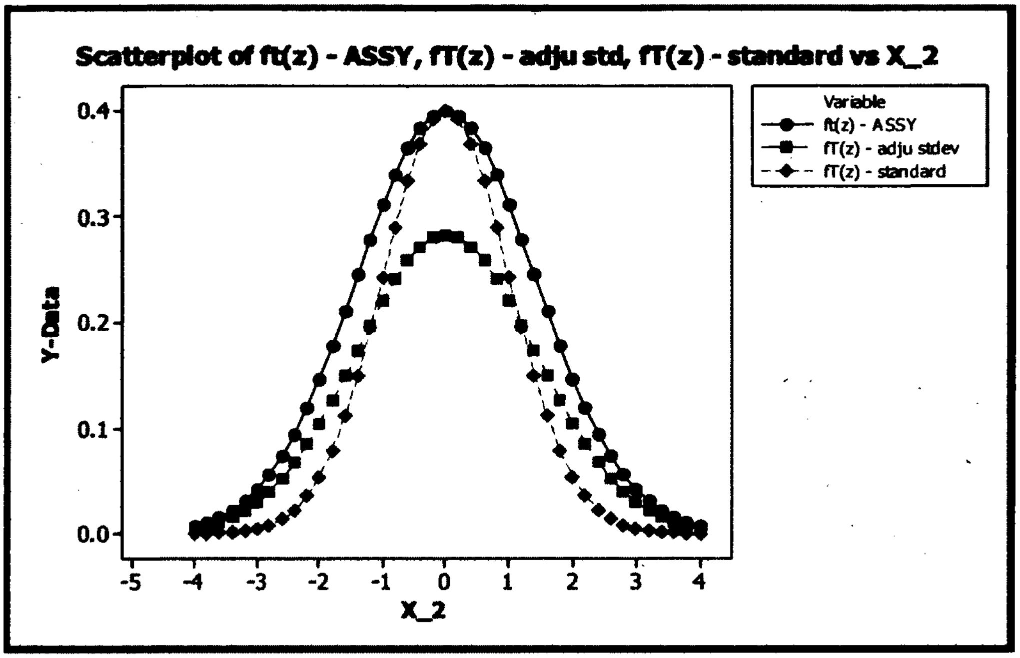
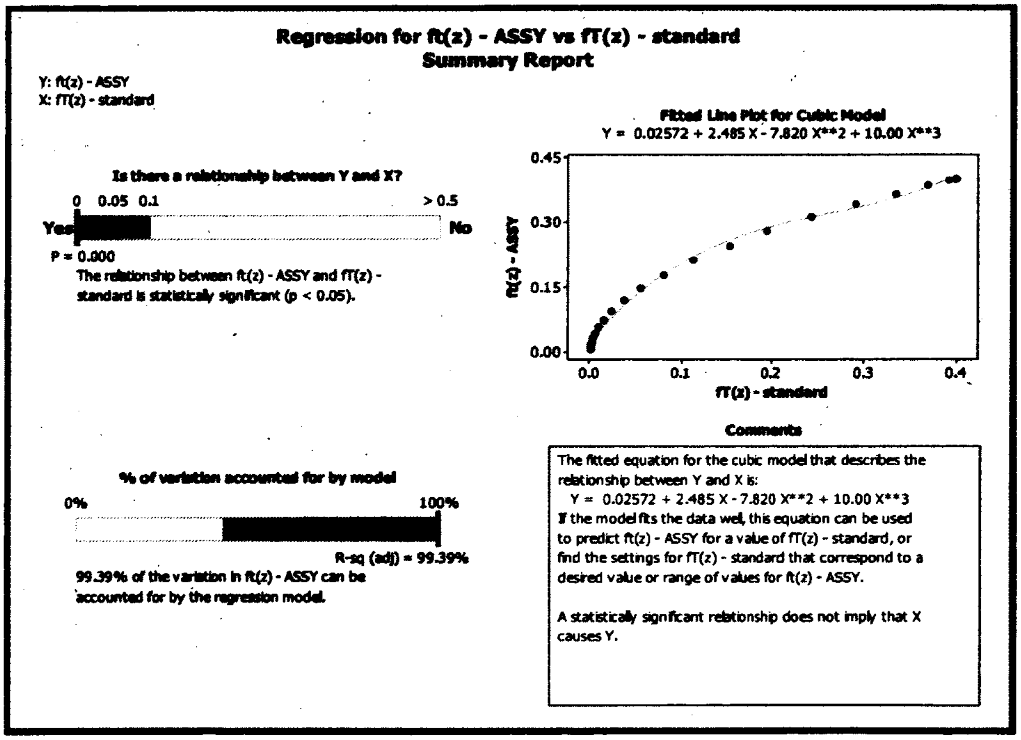


Figure C.62 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 4, LSL = -4)



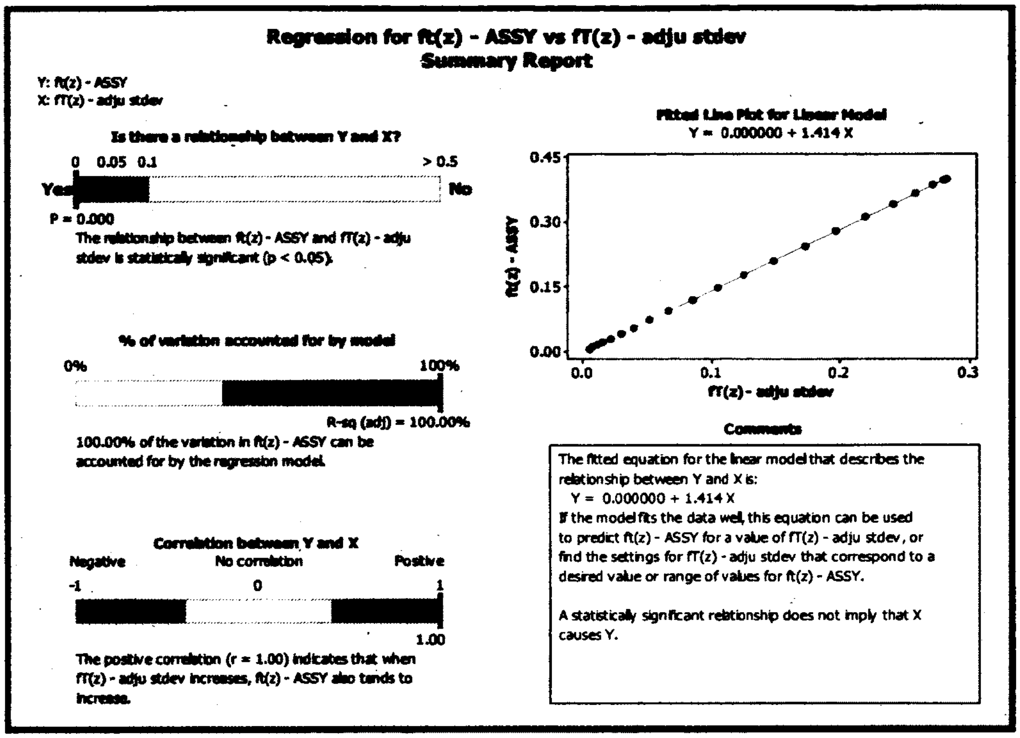
The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant ($p < 0.05$).

99.39% of the variation in ft(z) - ASSY can be accounted for by the regression model.

The fitted equation for the cubic model that describes the relationship between Y and X is:
Y = 0.02572 + 2.485 X - 7.820 X**2 + 10.00 X**3
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.63 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 4, LSL = -4)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - adju stdev is statistically significant (p < 0.05).

% of variation accounted for by model

R-sq (adj) = 100.00%
100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Correlation between Y and X

The positive correlation (r = 1.00) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

Fitted Line Plot for Linear Model
Y = 0.000000 + 1.414 X

Comments

The fitted equation for the linear model that describes the relationship between Y and X is:
Y = 0.000000 + 1.414 X
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.64 - TSND Assembly Comparison (USL = 3.8, LSL = -3.8)

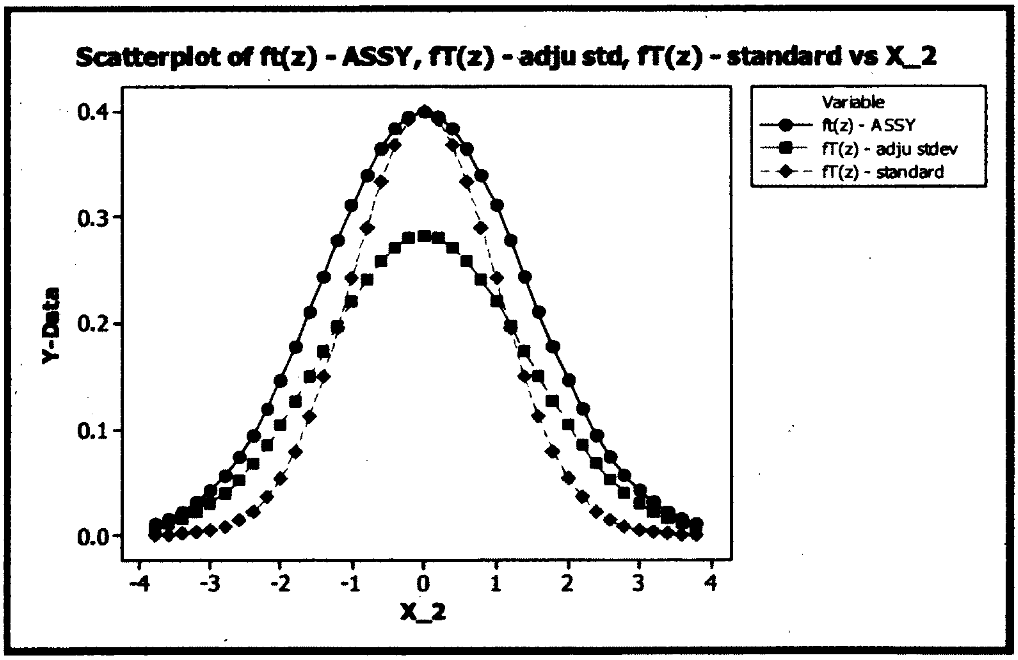
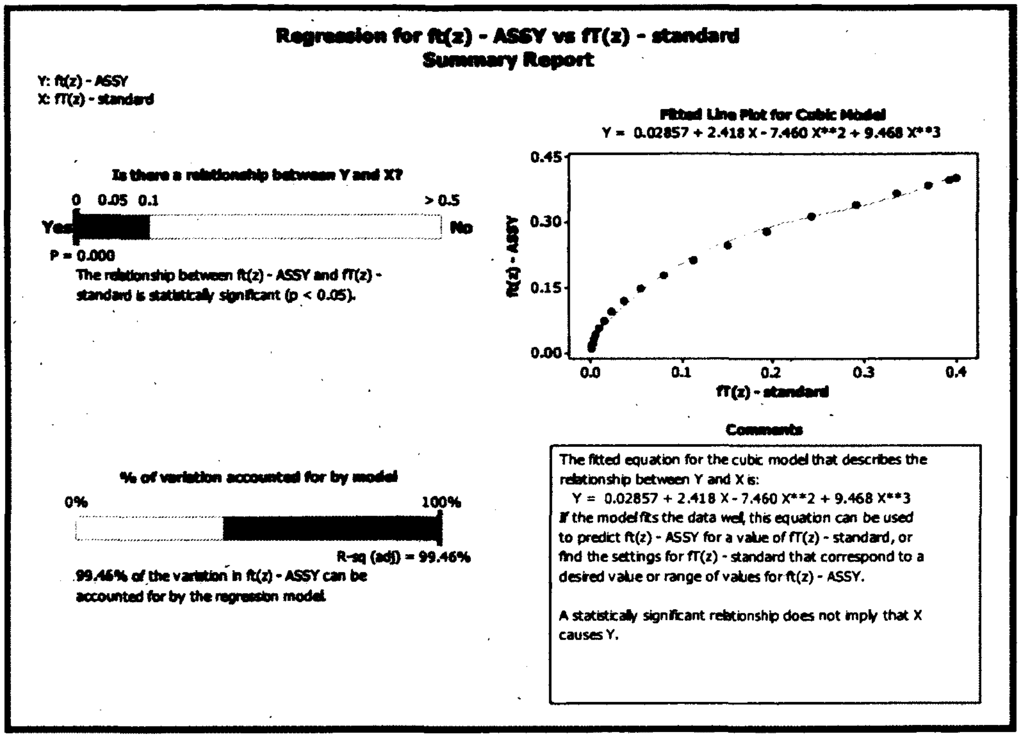


Figure C.65 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 3.8, LSL = -3.8)



Regression for ft(z) - ASSY vs fT(z) - standard
Summary Report

Y: ft(z) - ASSY
X: fT(z) - standard

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant ($p < 0.05$).

% of variation accounted for by model

R-sq (adj) = 99.46%
99.46% of the variation in ft(z) - ASSY can be accounted for by the regression model.

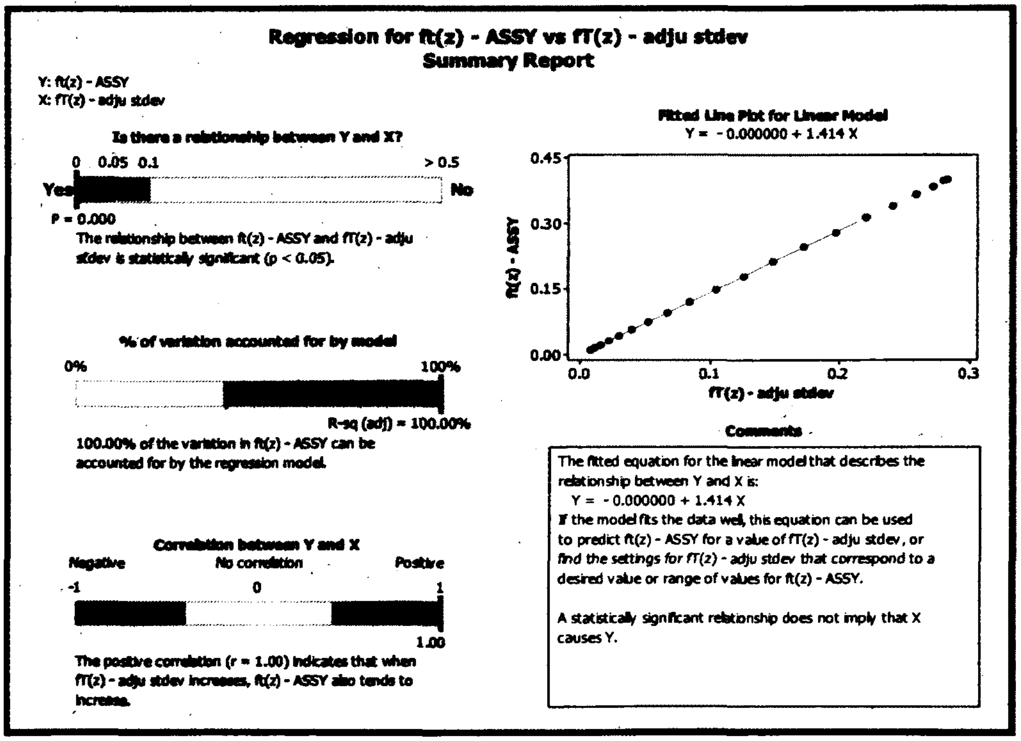
Fitted Line Plot for Cubic Model
Y = 0.02857 + 2.418 X - 7.460 X**2 + 9.468 X**3

Comments

The fitted equation for the cubic model that describes the relationship between Y and X is:
Y = 0.02857 + 2.418 X - 7.460 X**2 + 9.468 X**3
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.66 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 3.8, LSL = -3.8)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - adju stdev is statistically significant ($p < 0.05$).

% of variation accounted for by model

R-sq (adj) = 100.00%
100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Correlation between Y and X

1.00
The positive correlation (r = 1.00) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

Fitted Line Plot for Linear Model
Y = - 0.000000 + 1.414 X

Comments

The fitted equation for the linear model that describes the relationship between Y and X is:
Y = - 0.000000 + 1.414 X
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.67 - TSND Assembly Comparison (USL = 3.6, LSL = -3.6)

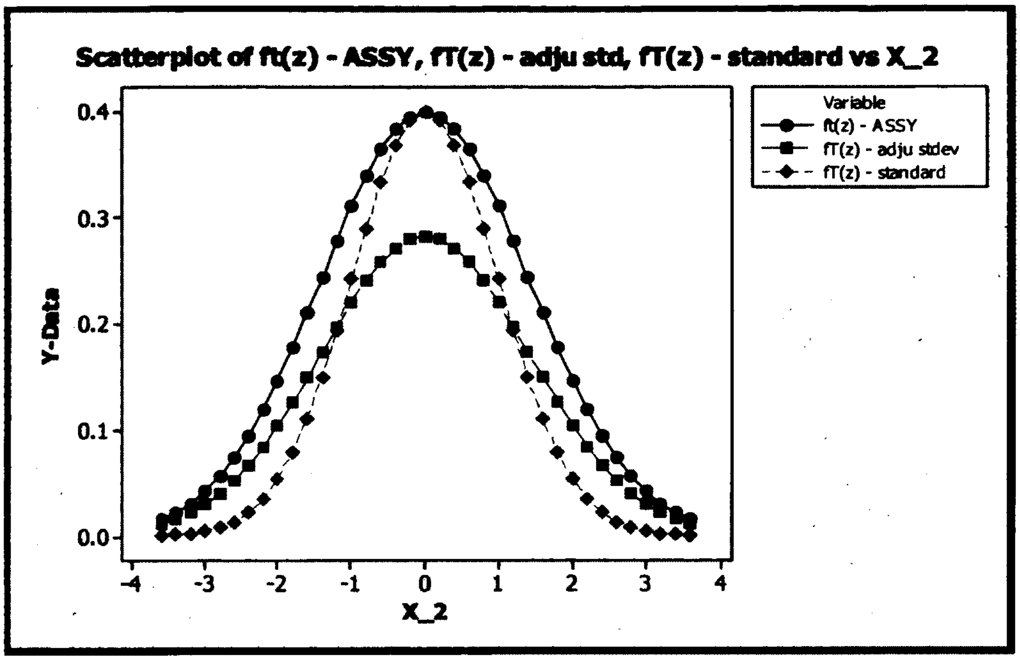
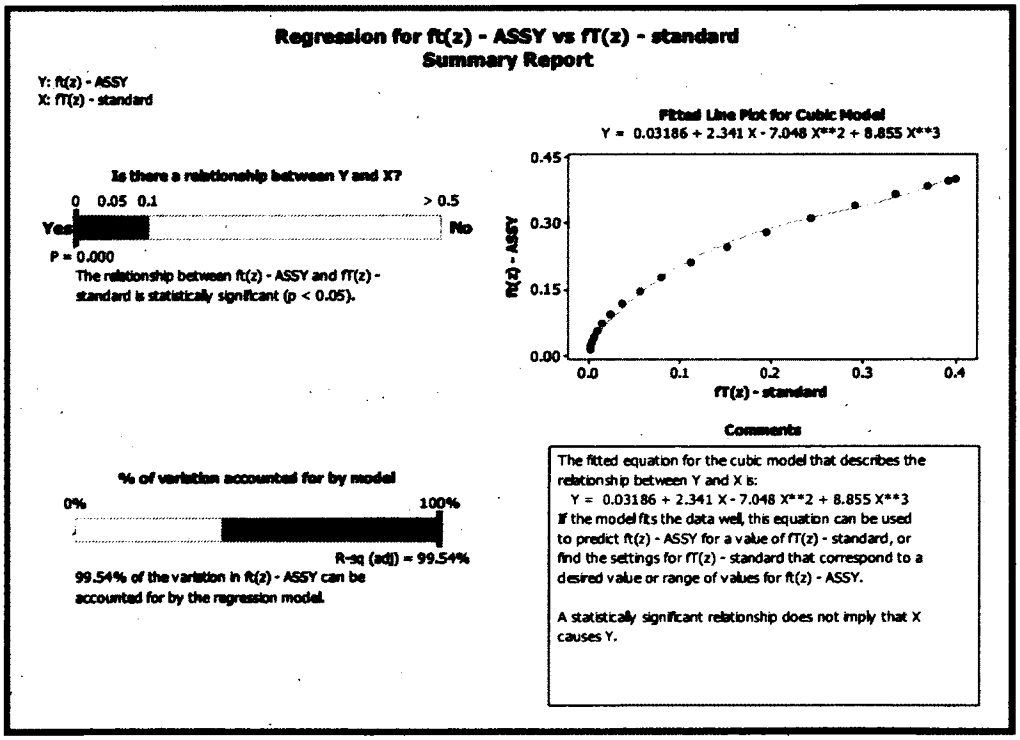


Figure C.68 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 3.6, LSL = -3.6)



P = 0.000

The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant (p < 0.05).

99.54% of the variation in ft(z) - ASSY can be accounted for by the regression model.

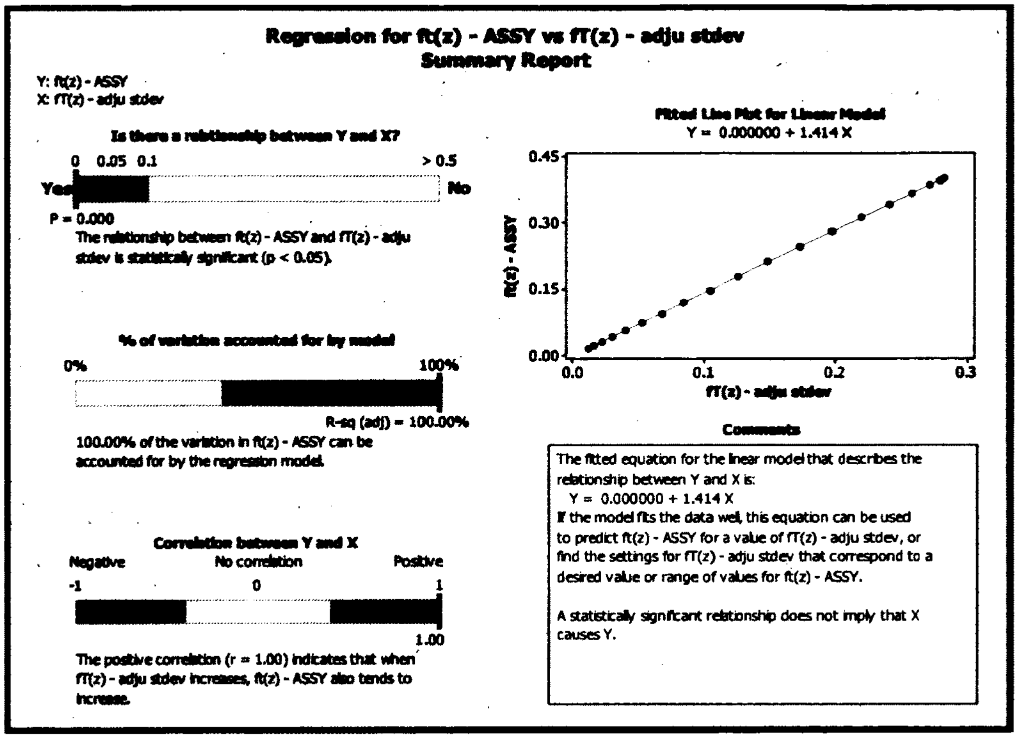
The fitted equation for the cubic model that describes the relationship between Y and X is:

Y = 0.03186 + 2.341 X - 7.048 X**2 + 8.855 X**3

If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.69 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 3.6, LSL = -3.6)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

Is there a relationship between Y and X?

P = 0.000

The relationship between ft(z) - ASSY and fT(z) - adju stdev is statistically significant ($p < 0.05$).

% of variation accounted for by model

R-sq (adj) = 100.00%

100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Correlation between Y and X

The positive correlation (r = 1.00) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

Fitted Line Plot for Linear Model

Y = 0.000000 + 1.414 X

Comments

The fitted equation for the linear model that describes the relationship between Y and X is:

Y = 0.000000 + 1.414 X

If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.70 - TSND Assembly Comparison (USL = 3.4, LSL = -3.4)

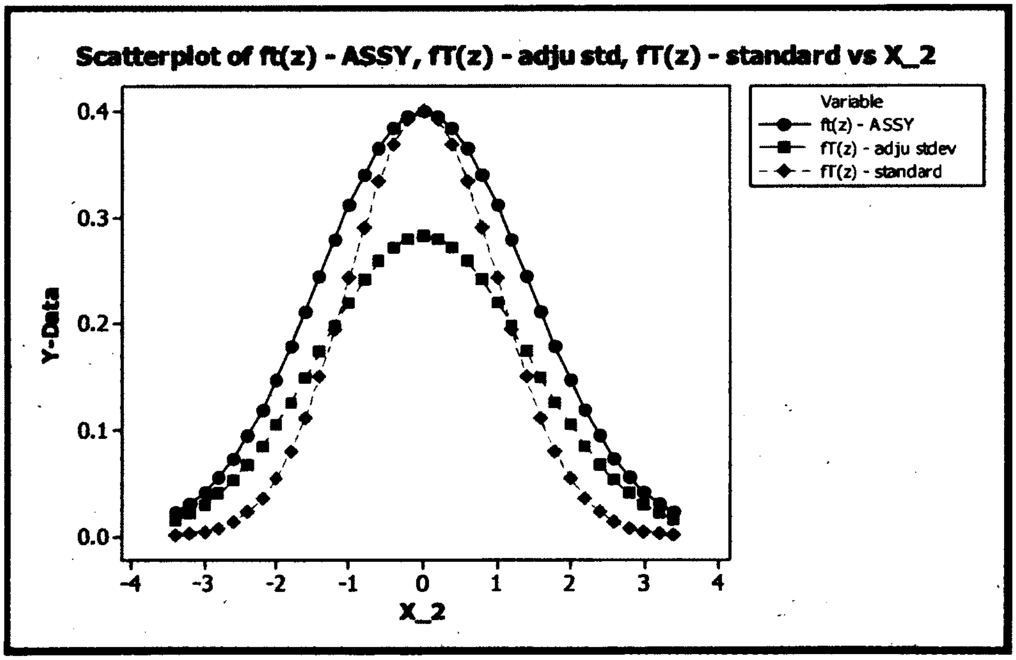
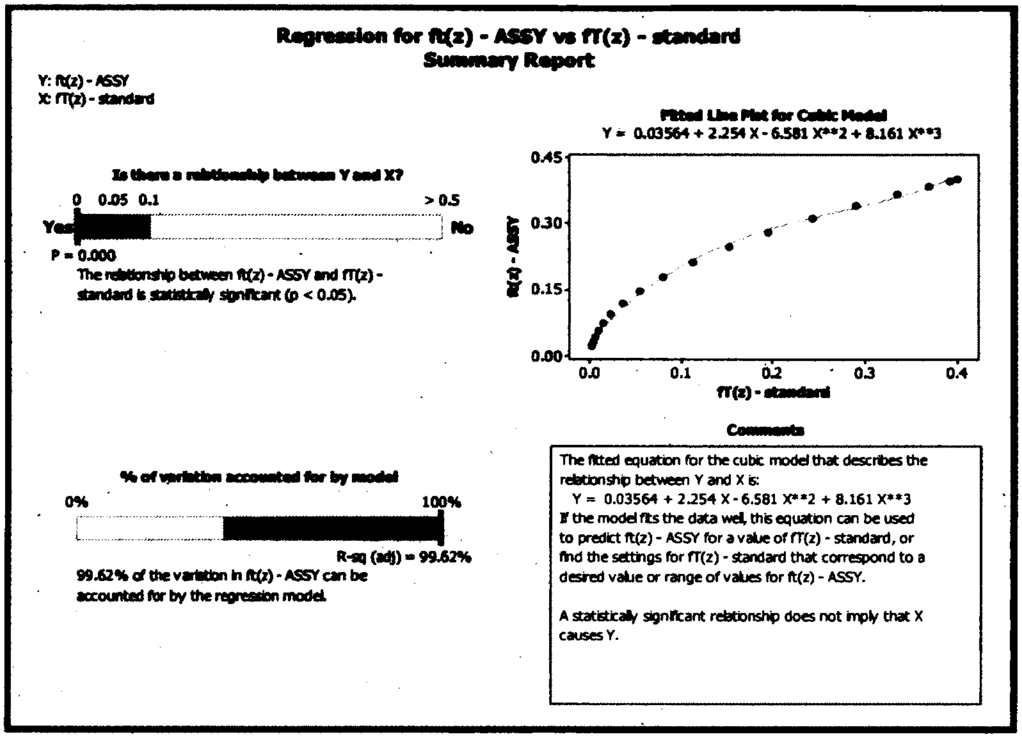


Figure C.71 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 3.4, LSL = -3.4)



Regression for ft(z) - ASSY vs fT(z) - standard
Summary Report

Y: ft(z) - ASSY
X: fT(z) - standard

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant (p < 0.05).

% of variation accounted for by model

R-sq (adj) = 99.62%
99.62% of the variation in ft(z) - ASSY can be accounted for by the regression model.

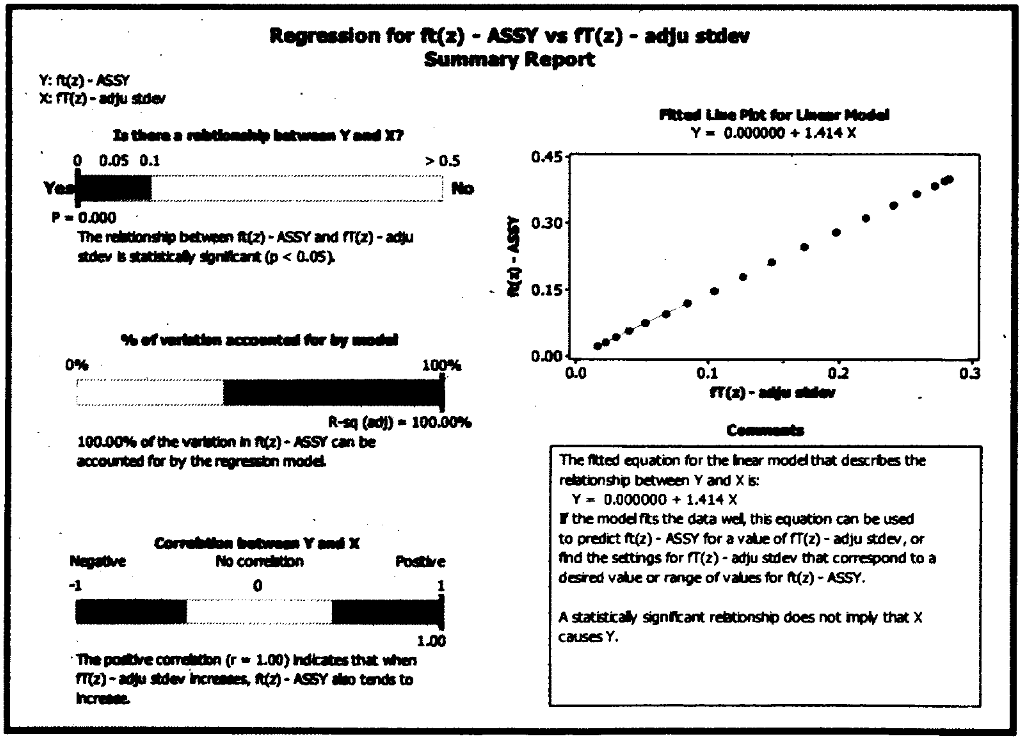
Fitted Line Plot for Cubic Model
Y = 0.03564 + 2.254 X - 6.581 X**2 + 8.161 X**3

Comments

The fitted equation for the cubic model that describes the relationship between Y and X is:
Y = 0.03564 + 2.254 X - 6.581 X**2 + 8.161 X**3
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.72 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 3.4, LSL = -3.4)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - adju stdev is statistically significant (p < 0.05).

% of variation accounted for by model

R-sq (adj) = 100.00%
100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Correlation between Y and X

The positive correlation (r = 1.00) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

Fitted Line Plot for Linear Model
Y = 0.000000 + 1.414 X

Comments

The fitted equation for the linear model that describes the relationship between Y and X is:
Y = 0.000000 + 1.414 X
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.73 TSND Assembly Comparison (USL = 3.2, LSL = -3.2)

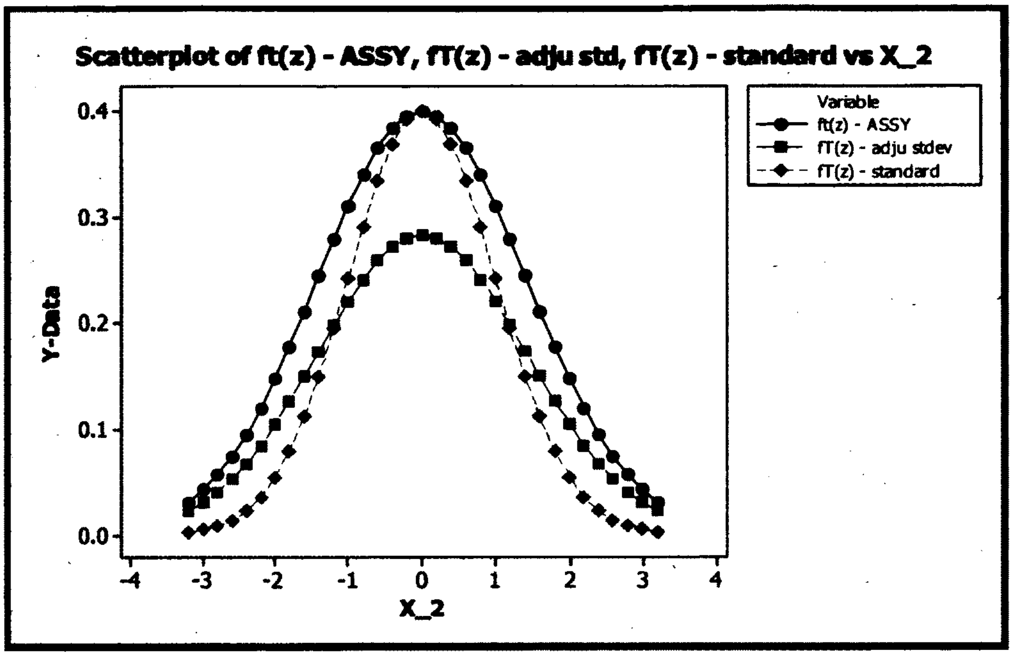
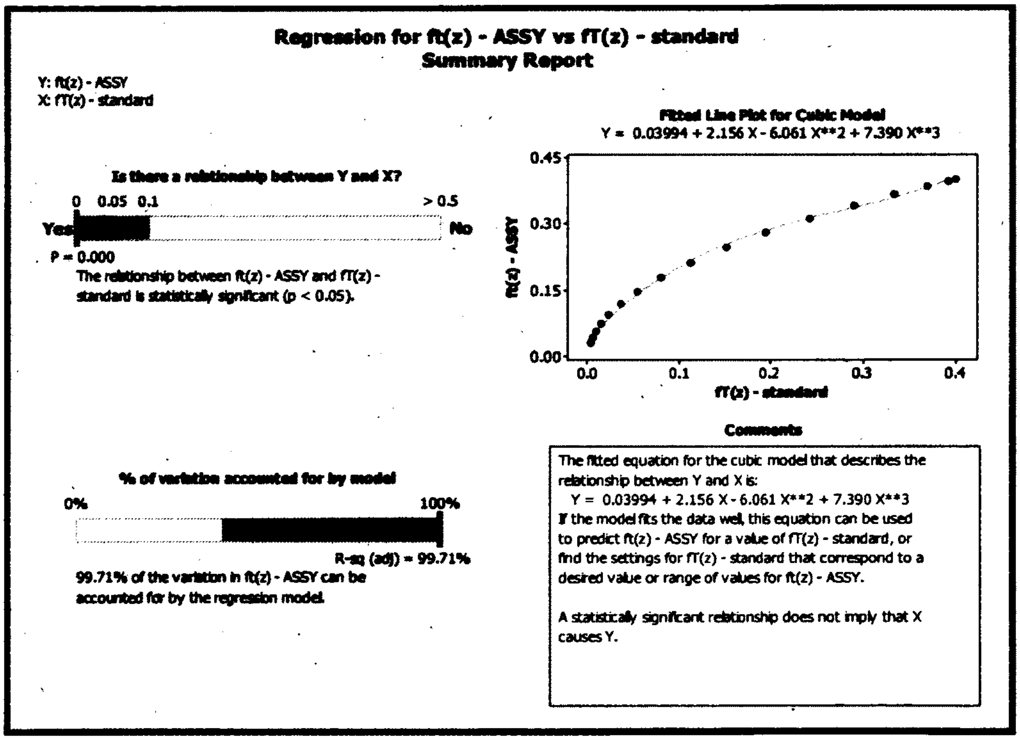


Figure C.74 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 3.2, LSL = -3.2)



The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant ($p < 0.05$).

99.71% of the variation in ft(z) - ASSY can be accounted for by the regression model.

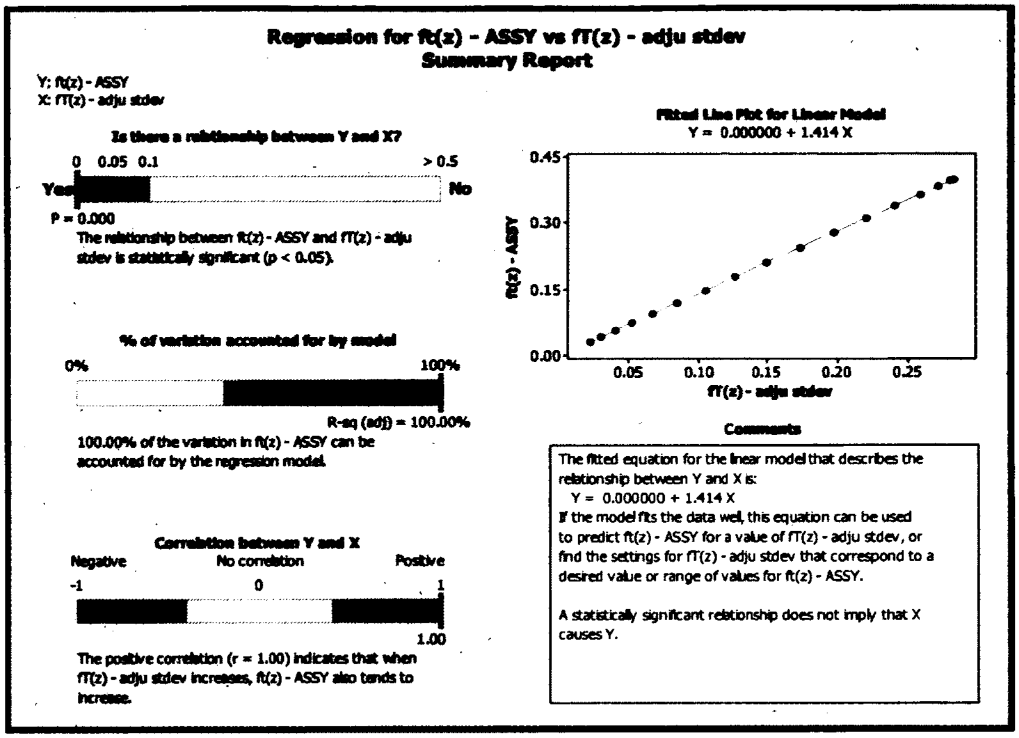
The fitted equation for the cubic model that describes the relationship between Y and X is:

Y = 0.03994 + 2.156 X - 6.061 X**2 + 7.390 X**3

If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.75 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 3.2, LSL = -3.2)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - adju stdev is statistically significant (p < 0.05).

% of variation accounted for by model

R-sq (adj) = 100.00%
100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Correlation between Y and X

The positive correlation (r = 1.00) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

Fitted Line Plot for Linear Model
Y = 0.000000 + 1.414 X

Comments

The fitted equation for the linear model that describes the relationship between Y and X is:
Y = 0.000000 + 1.414 X
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.76 - TSND Assembly Comparison (USL = 3, LSL = -3)

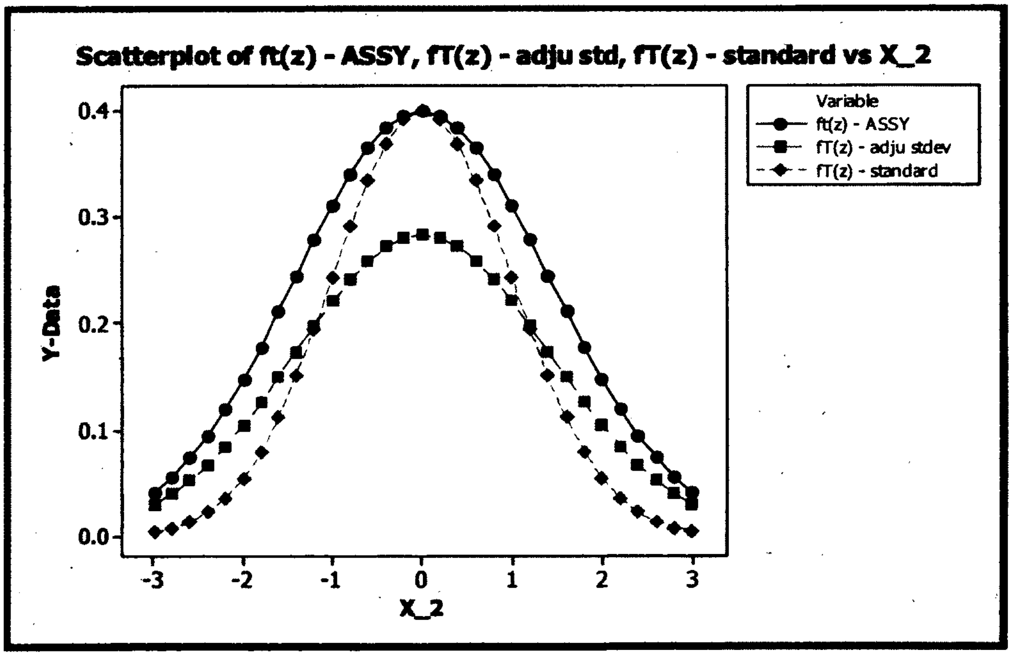
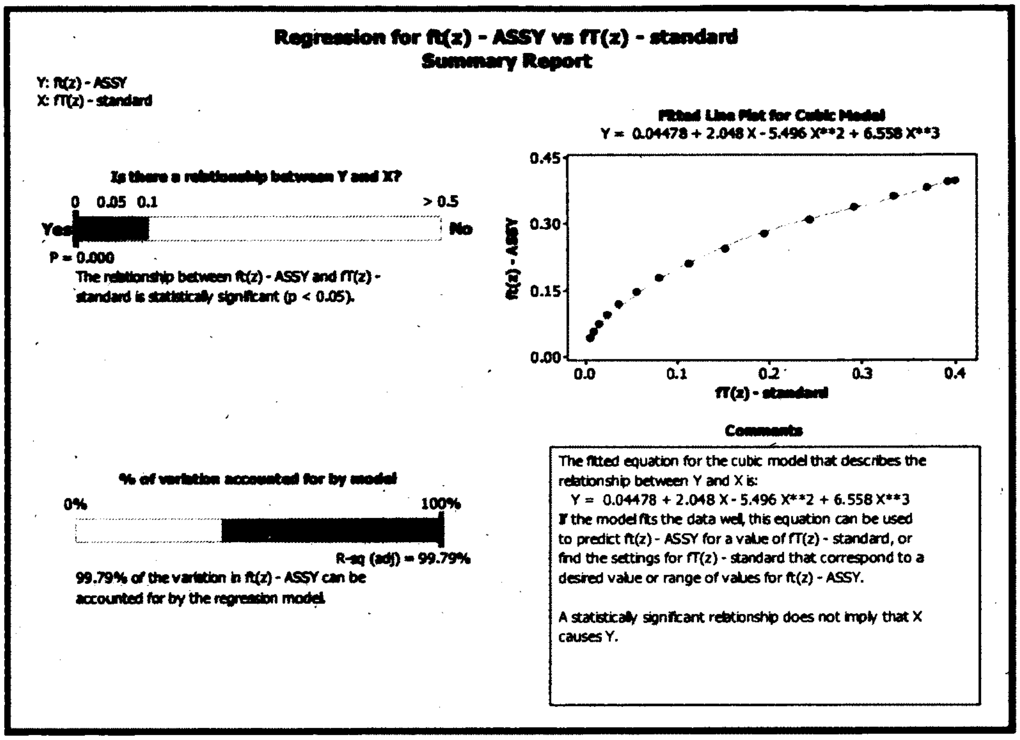


Figure C.77 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 3, LSL = -3)



Regression for ft(z) - ASSY vs fT(z) - standard
Summary Report

Y: ft(z) - ASSY
X: fT(z) - standard

Is there a relationship between Y and X?

0 0.05 0.1 > 0.5

Yes No

P = 0.000

The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant ($p < 0.05$).

% of variation accounted for by model

0% 100%

R-sq (adj) = 99.79%

99.79% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Fitted Line Plot for Cubic Model

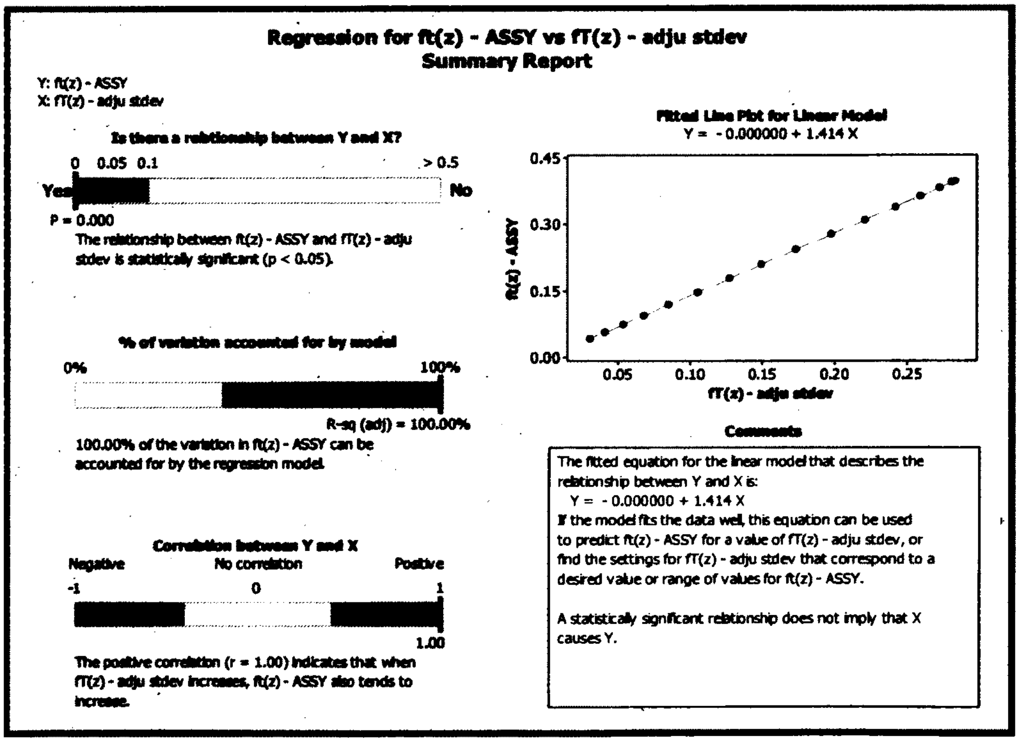
Y = 0.04478 + 2.048 X - 5.496 X**2 + 6.558 X**3

Comments

The fitted equation for the cubic model that describes the relationship between Y and X is:
Y = 0.04478 + 2.048 X - 5.496 X**2 + 6.558 X**3
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.78 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 3, LSL = -3)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

Is there a relationship between Y and X?

0 0.05 0.1 > 0.5

Yes No

P = 0.000

The relationship between ft(z) - ASSY and fT(z) - adju stdev is statistically significant ($p < 0.05$).

% of variation accounted for by model

0% 100%

R-sq (adj) = 100.00%

100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Correlation between Y and X

Negative No correlation Positive

-1 0 1

1.00

The positive correlation (r = 1.00) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

Fitted Line Plot for Linear Model

Y = - 0.000000 + 1.414 X

Comments

The fitted equation for the linear model that describes the relationship between Y and X is:
Y = - 0.000000 + 1.414 X
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.79 - TSND Assembly Comparison (USL = 2.8, LSL = -2.8)

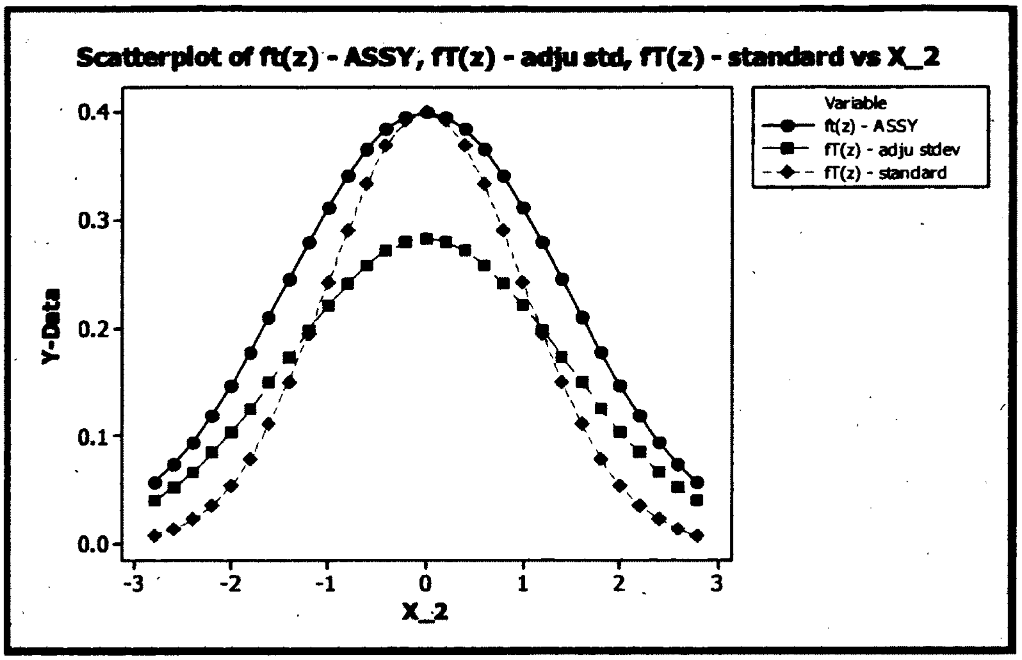
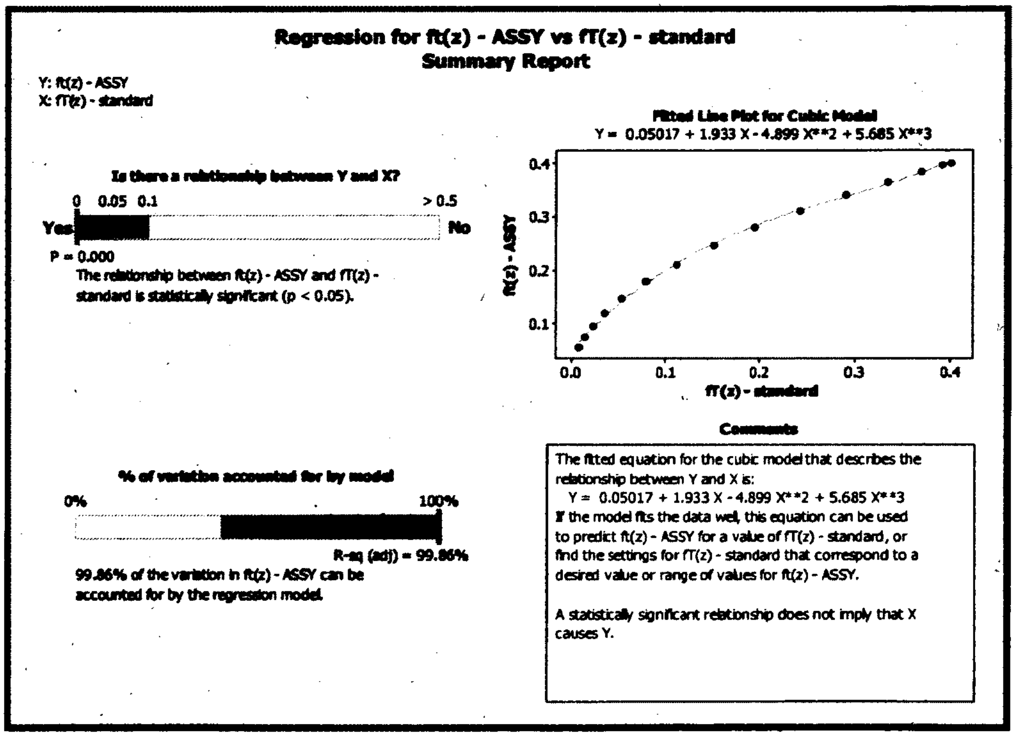


Figure C.80 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 2.8, LSL = -2.8)



P = 0.000

The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant (p < 0.05).

99.86% of the variation in ft(z) - ASSY can be accounted for by the regression model.

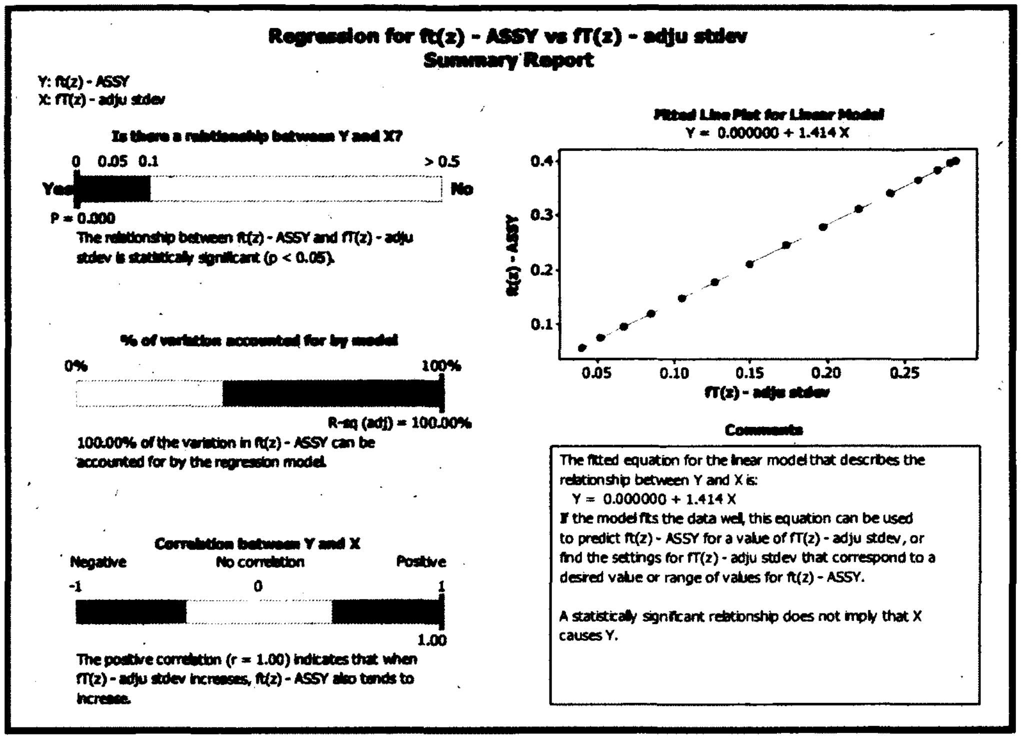
The fitted equation for the cubic model that describes the relationship between Y and X is:

Y = 0.05017 + 1.933 X - 4.899 X**2 + 5.685 X**3

If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.81 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 2.8, LSL = -2.8)



The relationship between ft(z) - ASSY and fT(z) - adju stdev is statistically significant (p < 0.05).

100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

The positive correlation (r = 1.00) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

The fitted equation for the linear model that describes the relationship between Y and X is:

Y = 0.000000 + 1.414 X

If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.82 - TSND Assembly Comparison (USL = 2.6, LSL = -2.6)

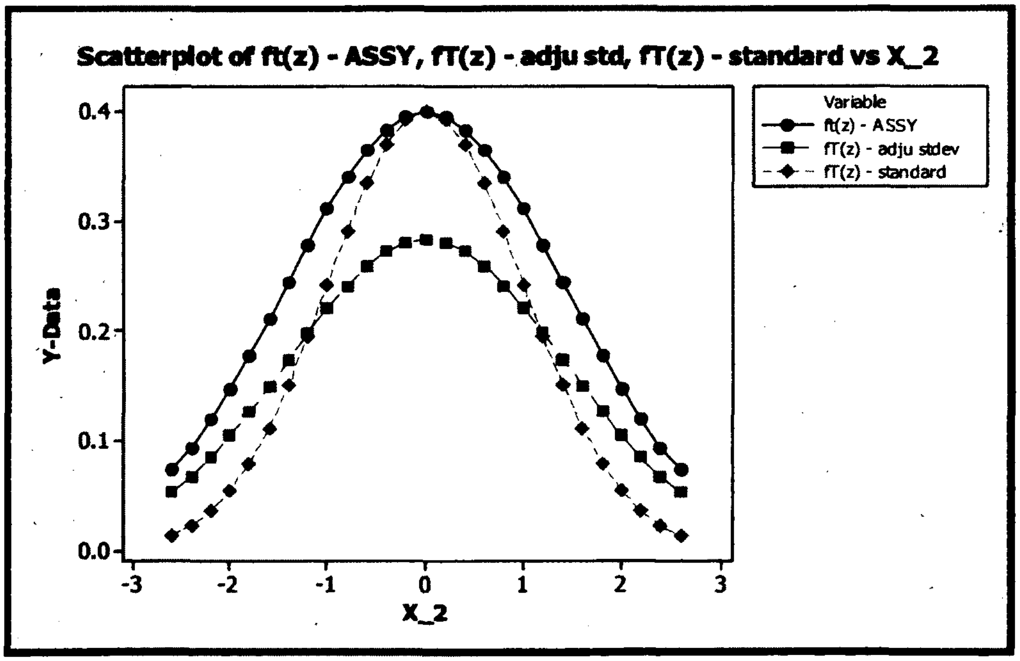
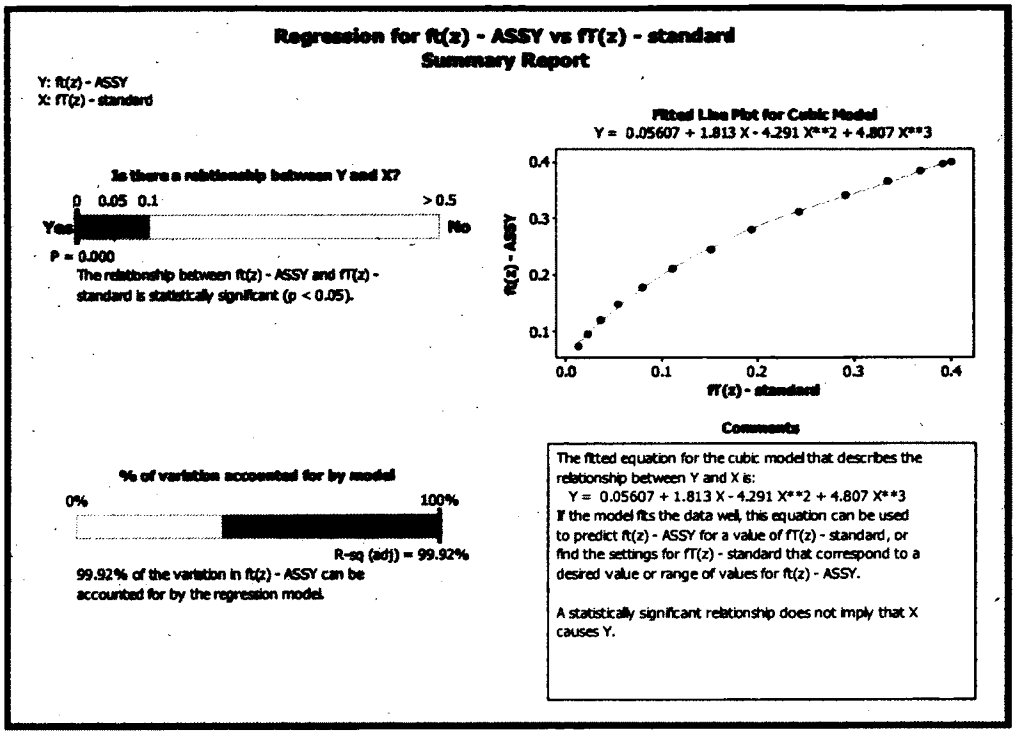


Figure C.83 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 2.6, LSL = -2.6)



Regression for ft(z) - ASSY vs fT(z) - standard
Summary Report

Y: ft(z) - ASSY
X: fT(z) - standard

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant ($p < 0.05$).

% of variation accounted for by model

R-sq (adj) = 99.92%
99.92% of the variation in ft(z) - ASSY can be accounted for by the regression model.

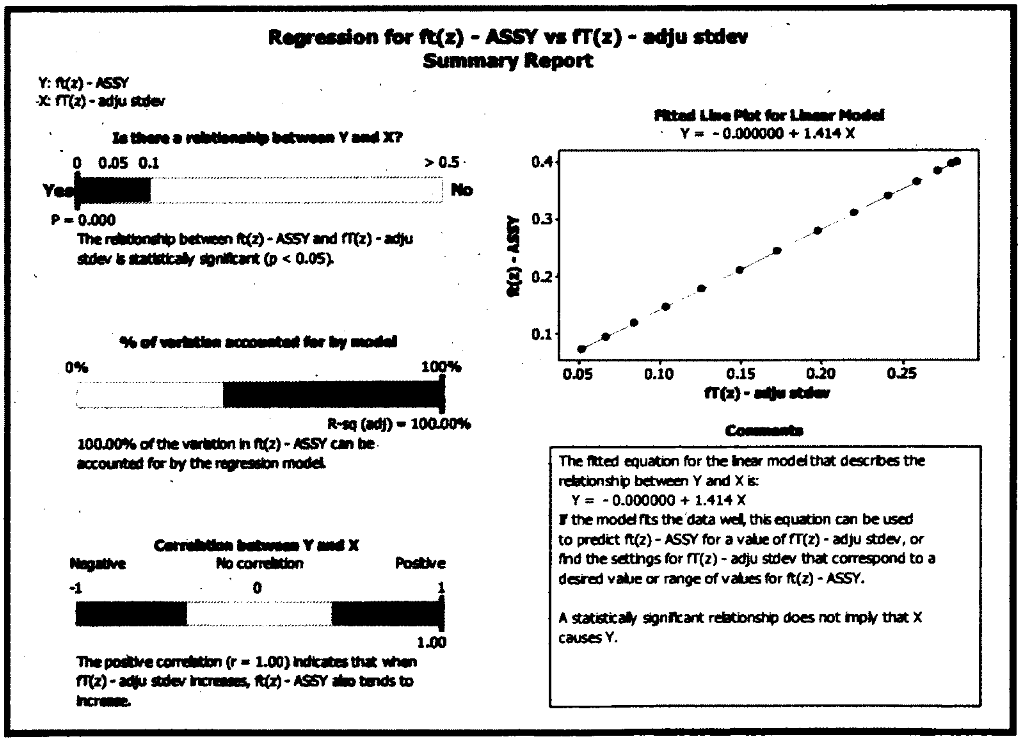
Fitted Line Plot for Cubic Model
Y = 0.05607 + 1.813 X - 4.291 X**2 + 4.807 X**3

Comments

The fitted equation for the cubic model that describes the relationship between Y and X is:
Y = 0.05607 + 1.813 X - 4.291 X**2 + 4.807 X**3
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.84 ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 2.6, LSL = -2.6)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - adju stdev is statistically significant ($p < 0.05$).

% of variation accounted for by model

R-sq (adj) = 100.00%
100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Correlation between Y and X

The positive correlation (r = 1.00) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

Fitted Line Plot for Linear Model
Y = - 0.000000 + 1.414 X

Comments

The fitted equation for the linear model that describes the relationship between Y and X is:
Y = - 0.000000 + 1.414 X
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.85 - TSND Assembly Comparison (USL = 2.4, LSL = -2.4)

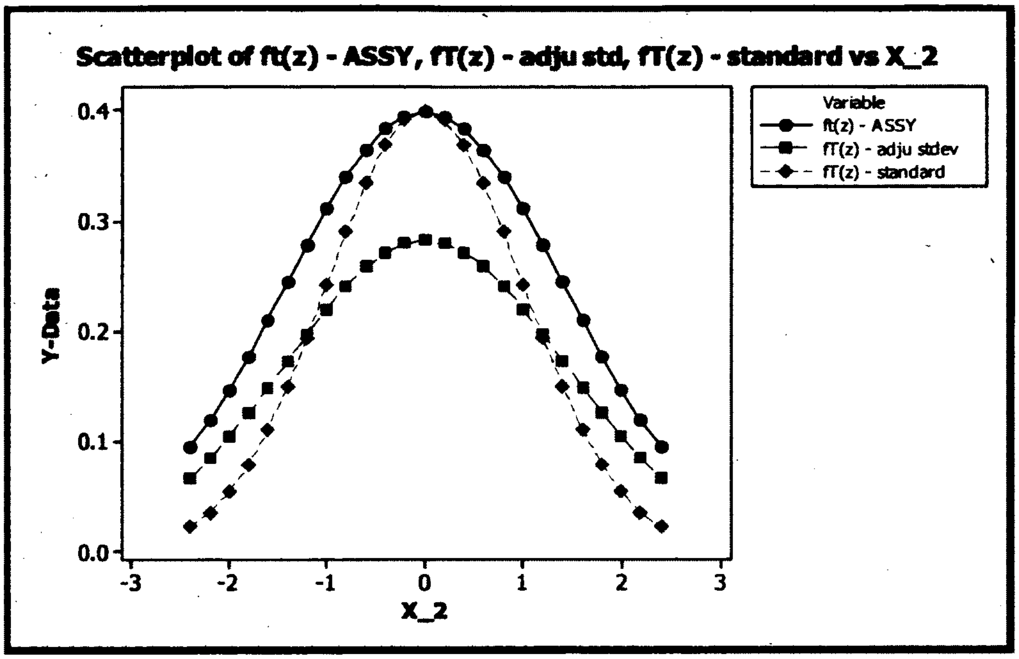
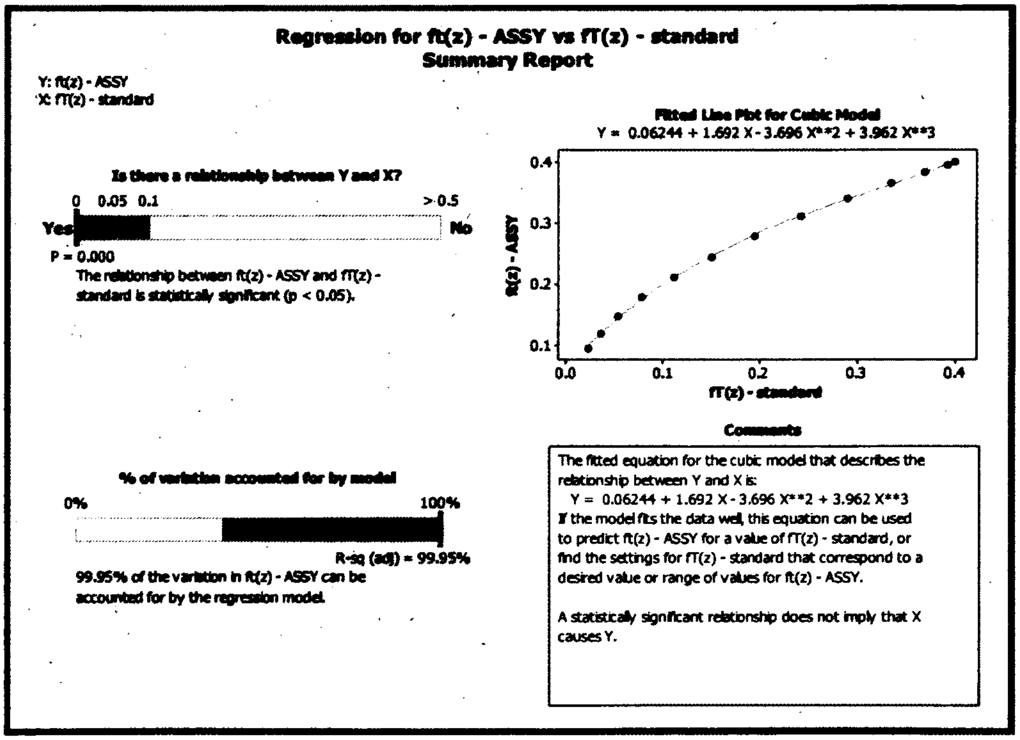


Figure C.86 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 2.4, LSL = -2.4)



Regression for ft(z) - ASSY vs fT(z) - standard
Summary Report

Y: ft(z) - ASSY
X: fT(z) - standard

Is there a relationship between Y and X?

P = 0.000

The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant ($p < 0.05$).

% of variation accounted for by model

R-sq (adj) = 99.95%

99.95% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Fitted Line Plot for Cubic Model

Y = 0.06244 + 1.692 X - 3.696 X**2 + 3.962 X**3

Comments

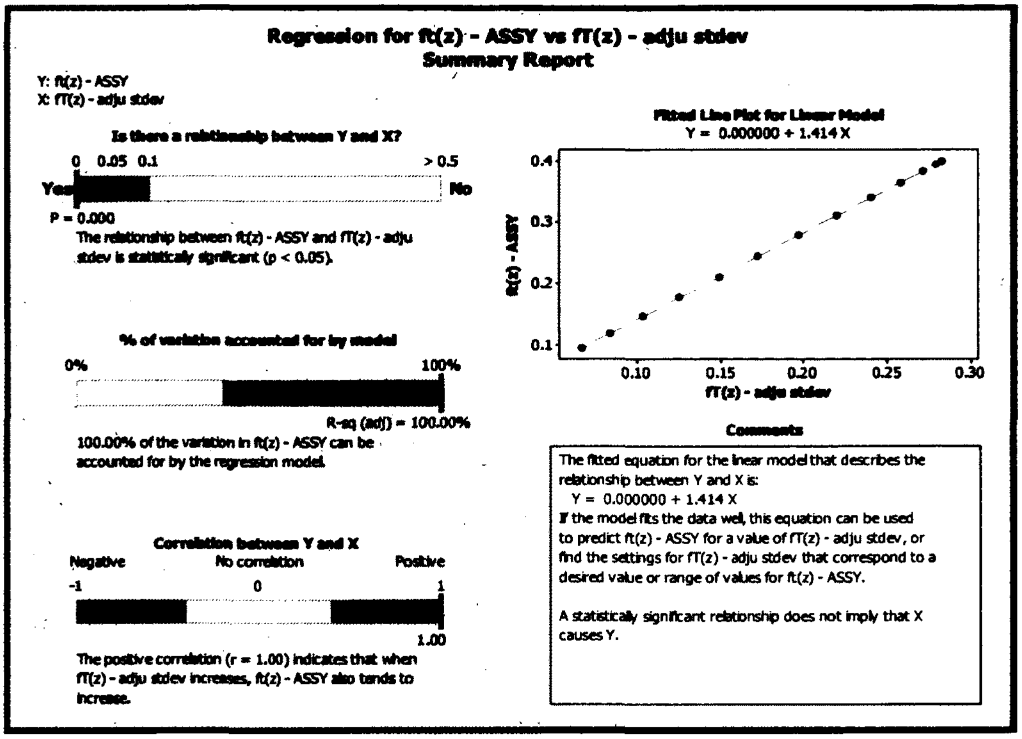
The fitted equation for the cubic model that describes the relationship between Y and X is:

Y = 0.06244 + 1.692 X - 3.696 X**2 + 3.962 X**3

If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.87 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 2.4, LSL = -2.4)



The relationship between ft(z) - ASSY and fT(z) - adju stdev is statistically significant ($p < 0.05$).

100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

The positive correlation ($r = 1.00$) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

The fitted equation for the linear model that describes the relationship between Y and X is:
$Y = 0.000000 + 1.414 X$
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.88 - TSND Assembly Comparison (USL = 2.2, LSL = -2.2)

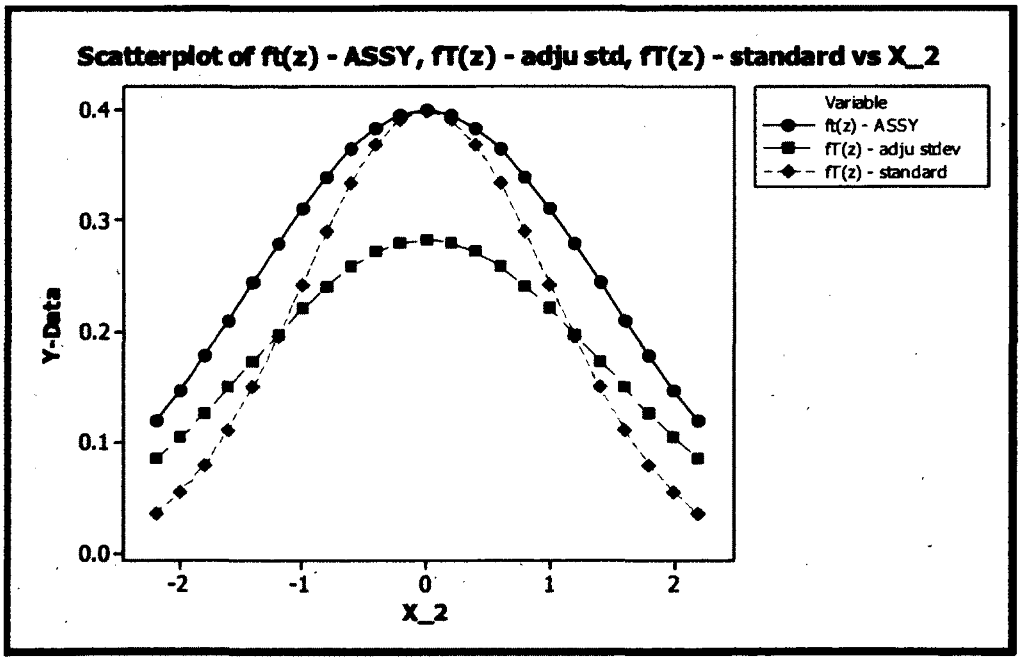
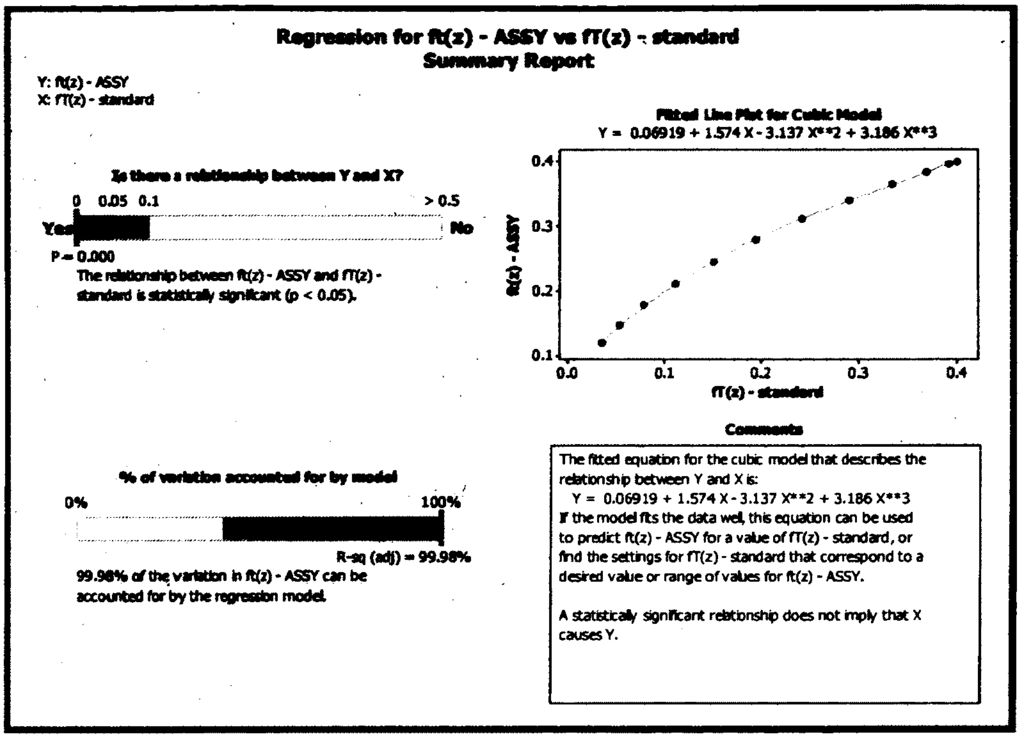


Figure C.89 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 2.2, LSL = -2.2)



Regression for ft(z) - ASSY vs fT(z) - standard
Summary Report

Y: ft(z) - ASSY
X: fT(z) - standard

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant (p < 0.05).

R-sq (adj) = 99.98%
99.98% of the variation in ft(z) - ASSY can be accounted for by the regression model.

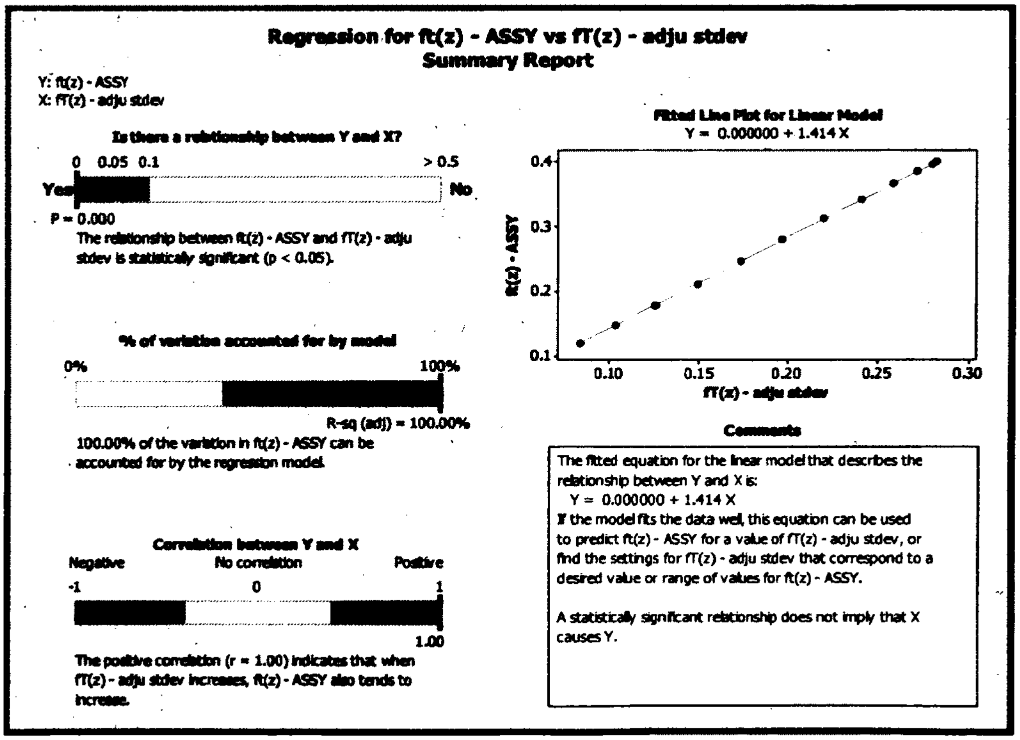
Fitted Line Plot for Cubic Model
Y = 0.06919 + 1.574 X - 3.137 X**2 + 3.186 X**3

Comments

The fitted equation for the cubic model that describes the relationship between Y and X is:
Y = 0.06919 + 1.574 X - 3.137 X**2 + 3.186 X**3
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.90 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 2.2, LSL = -2.2)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - adju stdev is statistically significant (p < 0.05).

R-sq (adj) = 100.00%
100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

The positive correlation (r = 1.00) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

Fitted Line Plot for Linear Model
Y = 0.000000 + 1.414 X

Comments

The fitted equation for the linear model that describes the relationship between Y and X is:
Y = 0.000000 + 1.414 X
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.91 - TSND Assembly Comparison (USL = 2, LSL = -2)

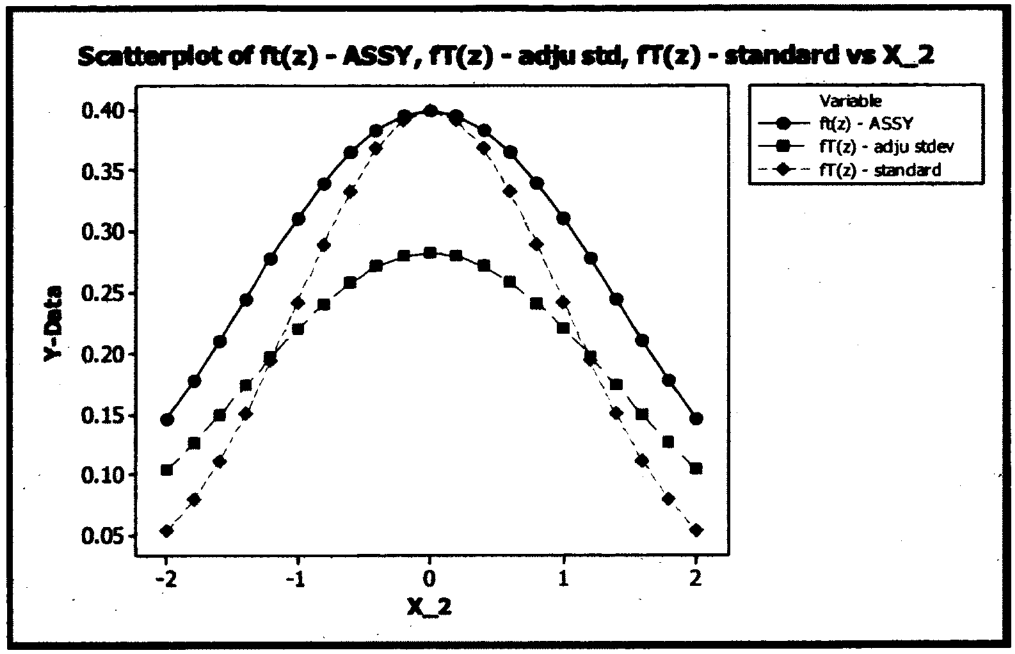
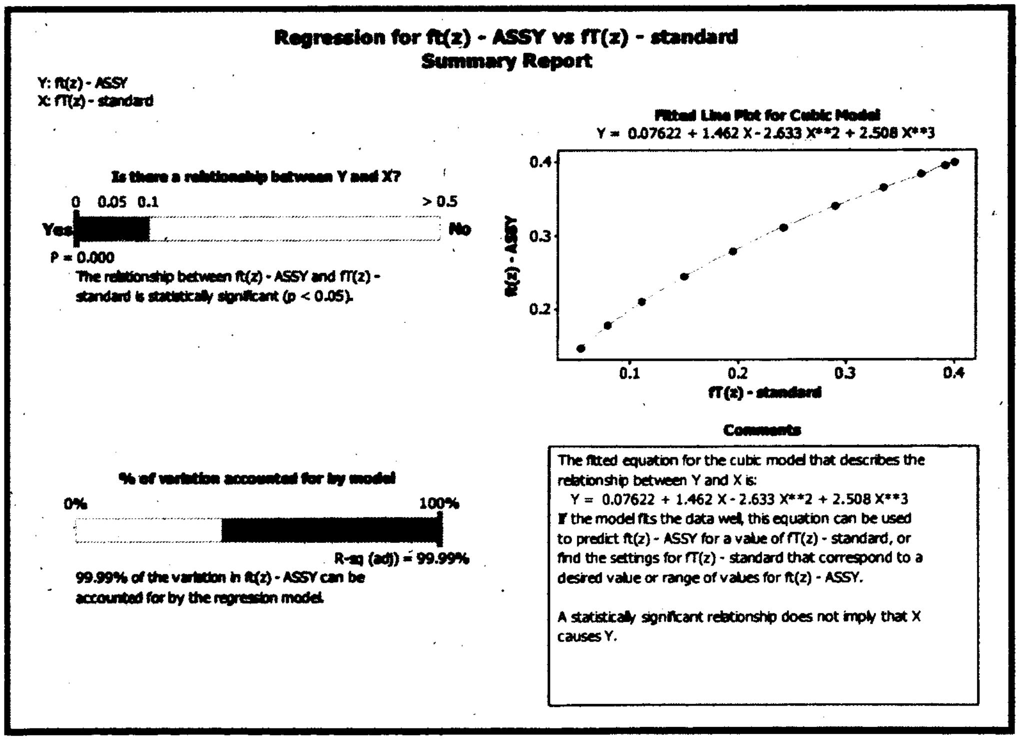


Figure C.92 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 2, LSL = -2)



P = 0.000

The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant (p < 0.05).

R-sq (adj) = 99.99%

99.99% of the variation in ft(z) - ASSY can be accounted for by the regression model.

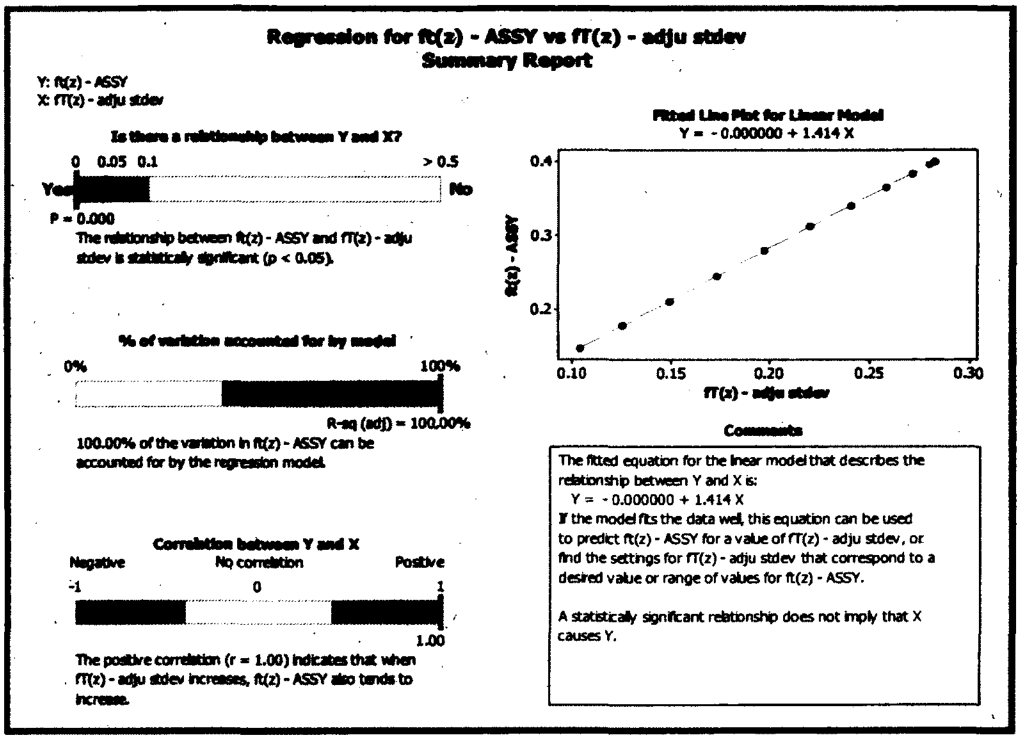
The fitted equation for the cubic model that describes the relationship between Y and X is:

Y = 0.07622 + 1.462 X - 2.633 X**2 + 2.508 X**3

If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.93 ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 2, LSL = -2)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

Is there a relationship between Y and X?

P = 0.000

The relationship between ft(z) - ASSY and fT(z) - adju stdev is statistically significant ($p < 0.05$).

% of variation accounted for by model

R-sq (adj) = 100.00%

100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Correlation between Y and X

1.00

The positive correlation ($r = 1.00$) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

Fitted Line Plot for Linear Model
Y = - 0.000000 + 1.414 X

Comments

The fitted equation for the linear model that describes the relationship between Y and X is:

Y = - 0.000000 + 1.414 X

If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.94 - TSND Assembly Comparison (USL = 1.8, LSL = -1.8)

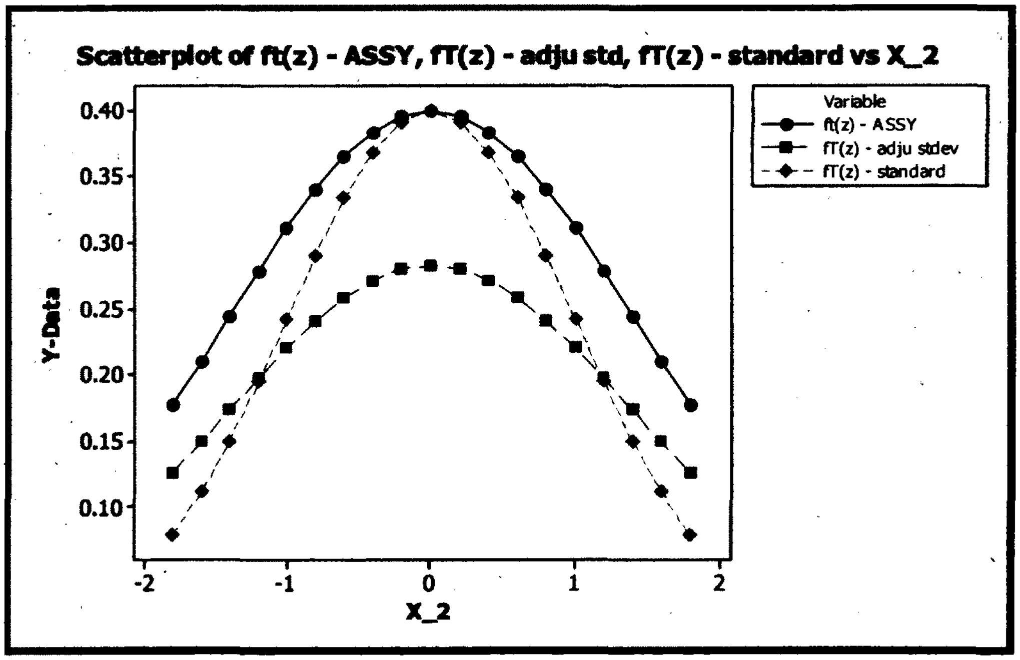
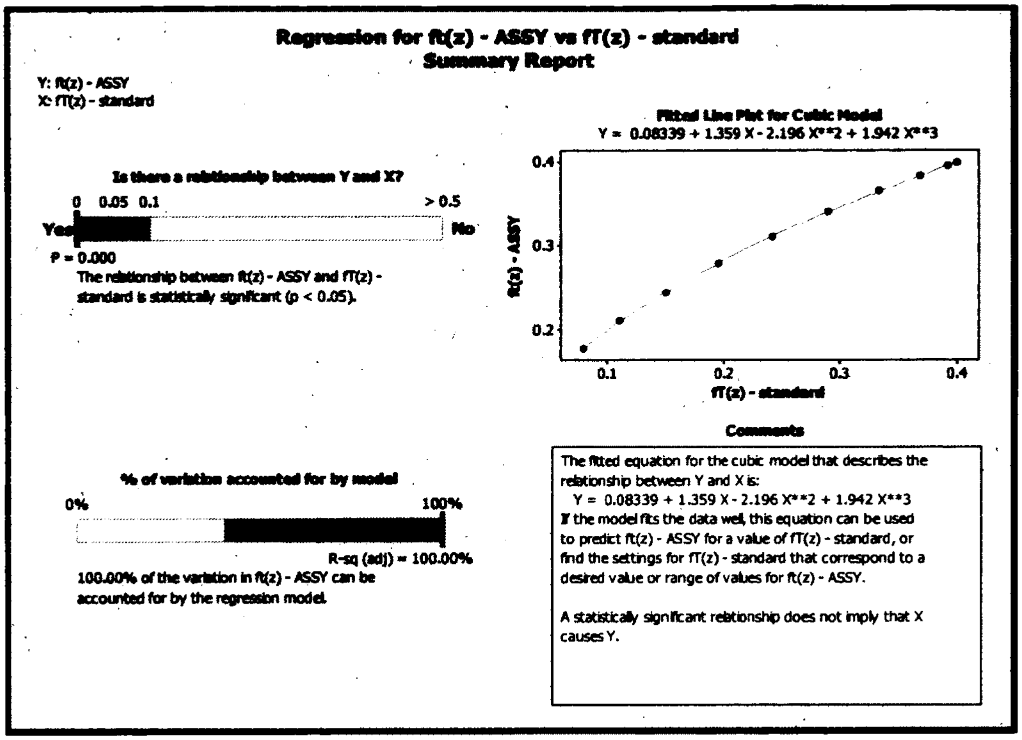


Figure C.95 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 1.8, LSL = -1.8)



Regression for ft(z) - ASSY vs fT(z) - standard
Summary Report

Y: ft(z) - ASSY
X: fT(z) - standard

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant ($p < 0.05$).

% of variation accounted for by model

R-sq (adj) = 100.00%
100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

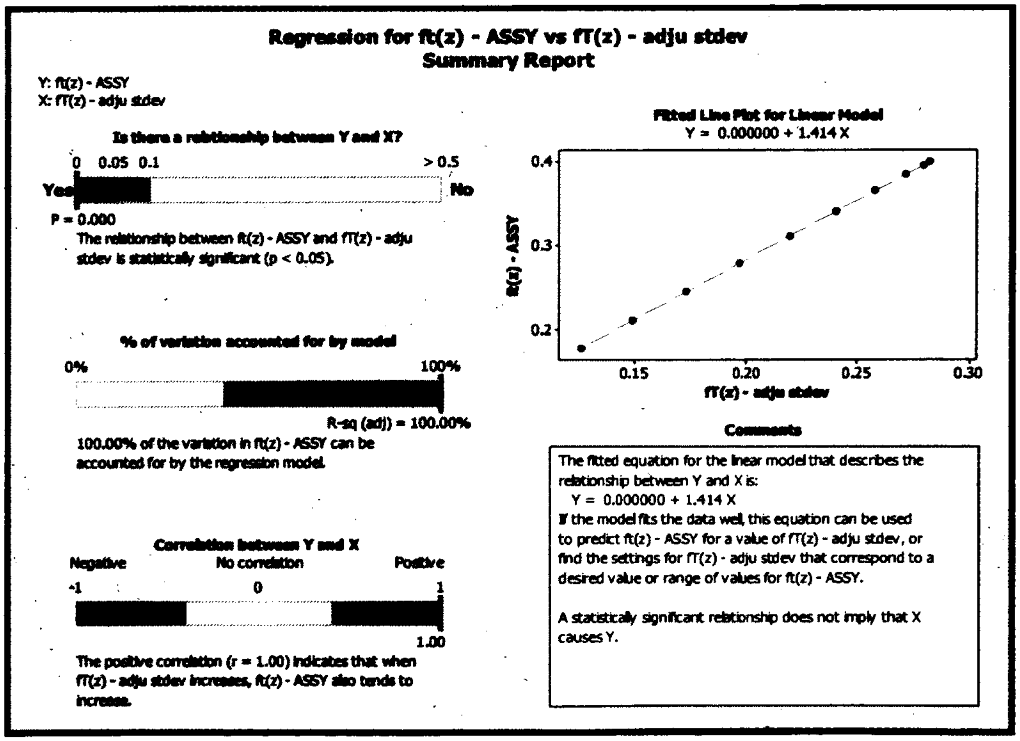
Fitted Line Plot for Cubic Model
Y = 0.08339 + 1.359 X - 2.196 X**2 + 1.942 X**3

Comments

The fitted equation for the cubic model that describes the relationship between Y and X is:
Y = 0.08339 + 1.359 X - 2.196 X**2 + 1.942 X**3
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.96 ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 1.8, LSL = -1.8)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - adju stdev is statistically significant ($p < 0.05$).

% of variation accounted for by model

R-sq (adj) = 100.00%
100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Correlation between Y and X

1.00
The positive correlation (r = 1.00) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

Fitted Line Plot for Linear Model
Y = 0.000000 + 1.414 X

Comments

The fitted equation for the linear model that describes the relationship between Y and X is:
Y = 0.000000 + 1.414 X
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.97 - TSND Assembly Comparison (USL = 1.6, LSL = -1.6)

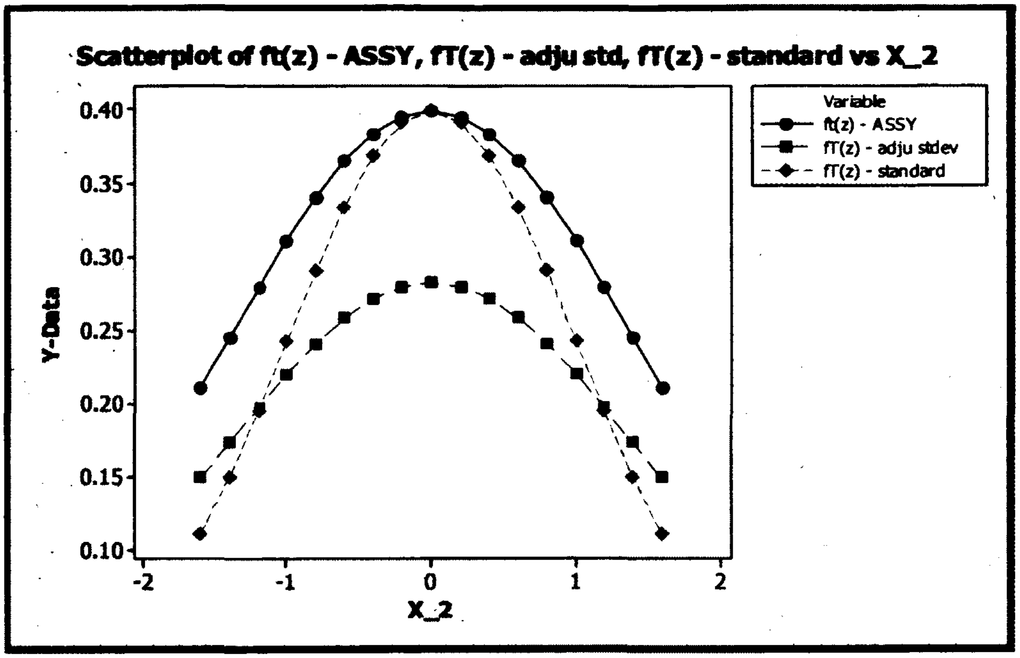
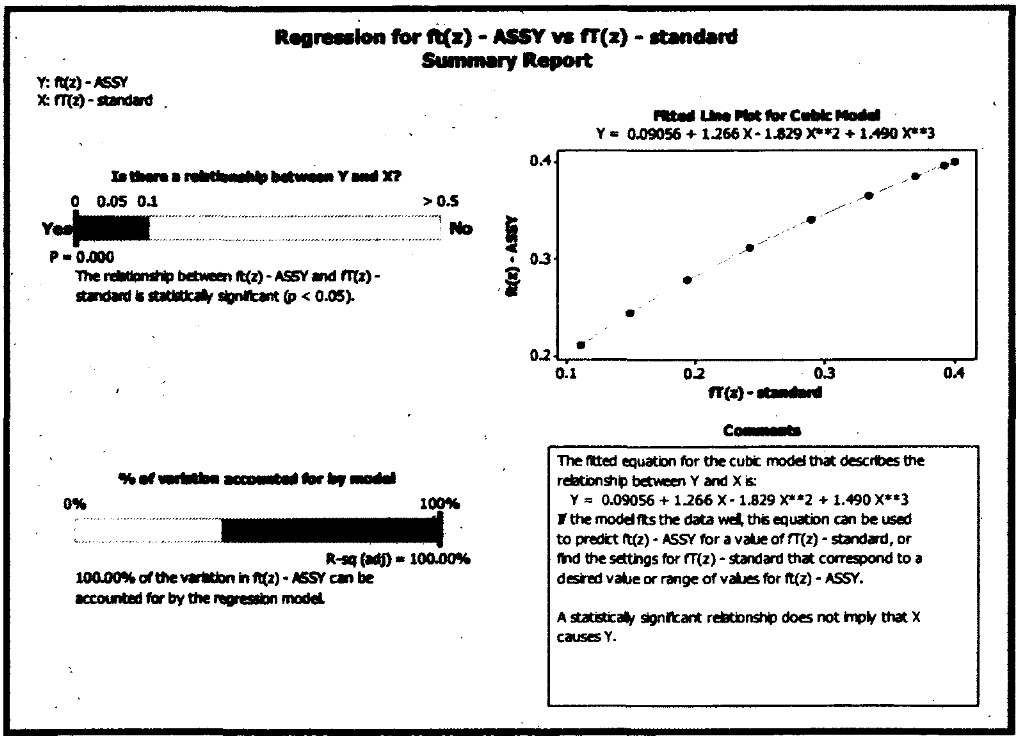


Figure C. 98 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 1.6, LSL = -1.6)



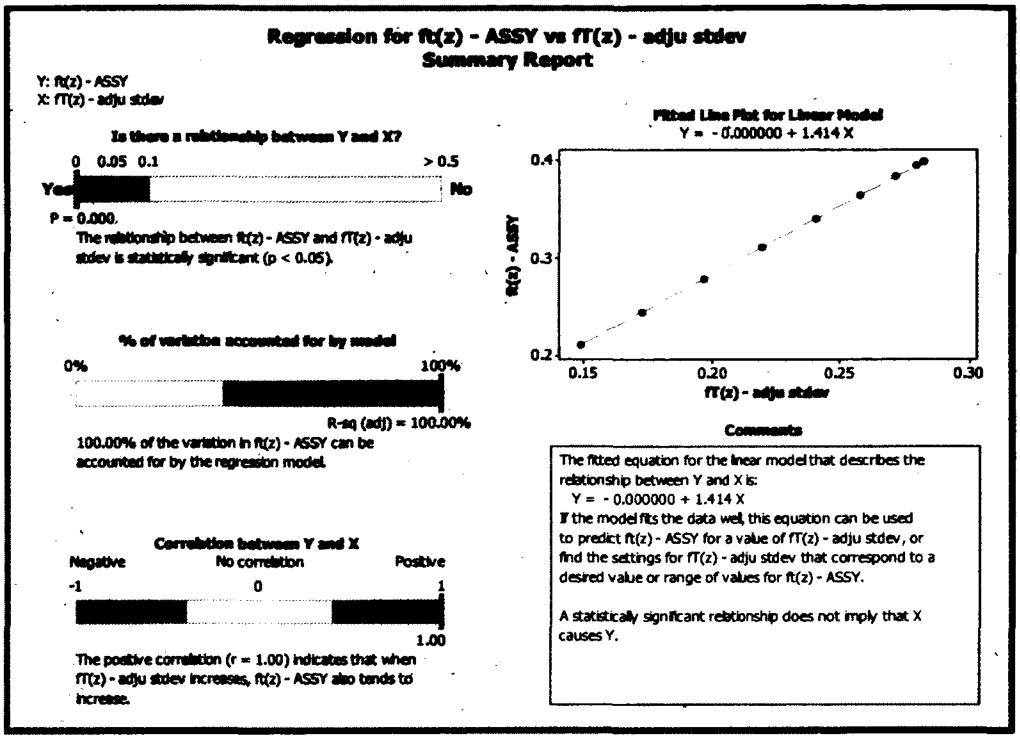
The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant (p < 0.05).

100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

The fitted equation for the cubic model that describes the relationship between Y and X is:
Y = 0.09056 + 1.266 X - 1.829 X**2 + 1.490 X**3
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.99 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 1.6, LSL = -1.6)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

Is there a relationship between Y and X?

P = 0.000.
The relationship between ft(z) - ASSY and fT(z) - adju stdev is statistically significant (p < 0.05).

% of variation accounted for by model

R-sq (adj) = 100.00%
100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Correlation between Y and X

The positive correlation (r = 1.00) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

Fitted Line Plot for Linear Model
Y = - 0.000000 + 1.414 X

Comments

The fitted equation for the linear model that describes the relationship between Y and X is:
Y = - 0.000000 + 1.414 X
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.100 - TSND Assembly Comparison (USL = 1.4, LSL = -1.4)

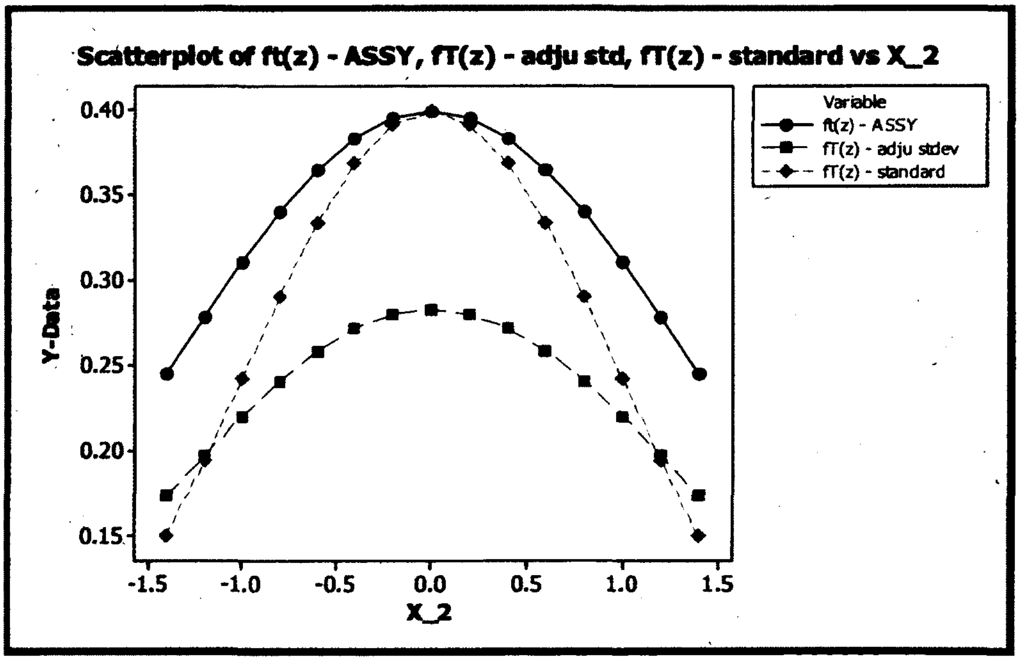
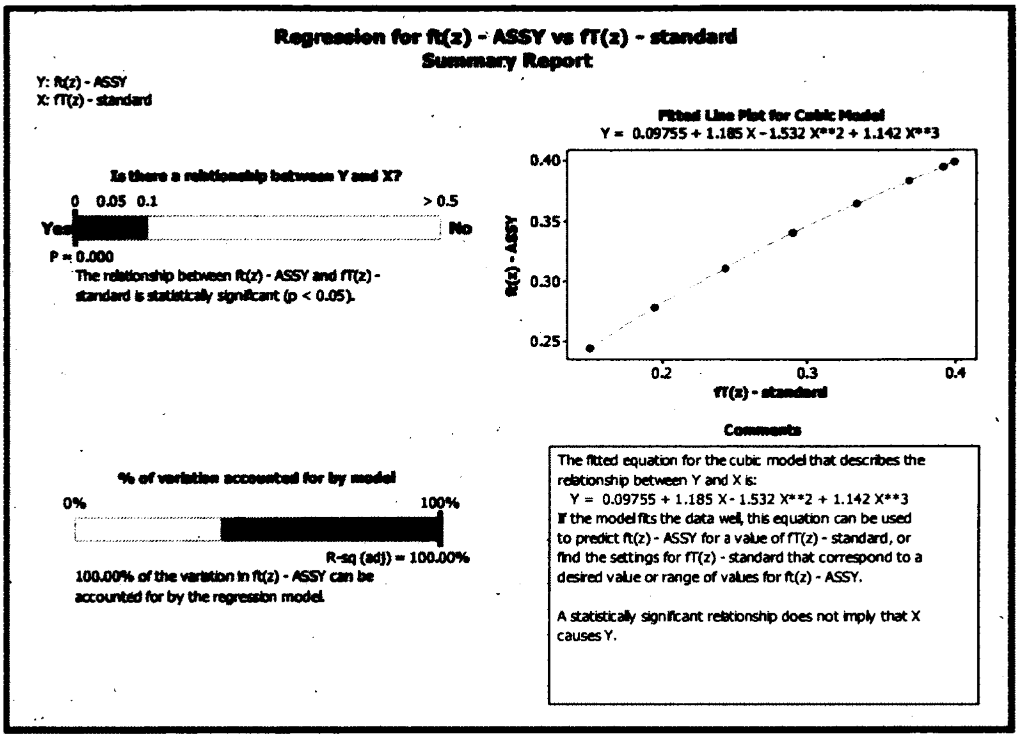


Figure C.101 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 1.4, LSL = -1.4)



Regression for ft(z) - ASSY vs fT(z) - standard
Summary Report

Y: ft(z) - ASSY
X: fT(z) - standard

Is there a relationship between Y and X?

P = 0.000

The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant ($p < 0.05$).

% of variation accounted for by model

R-sq (adj) = 100.00%

100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

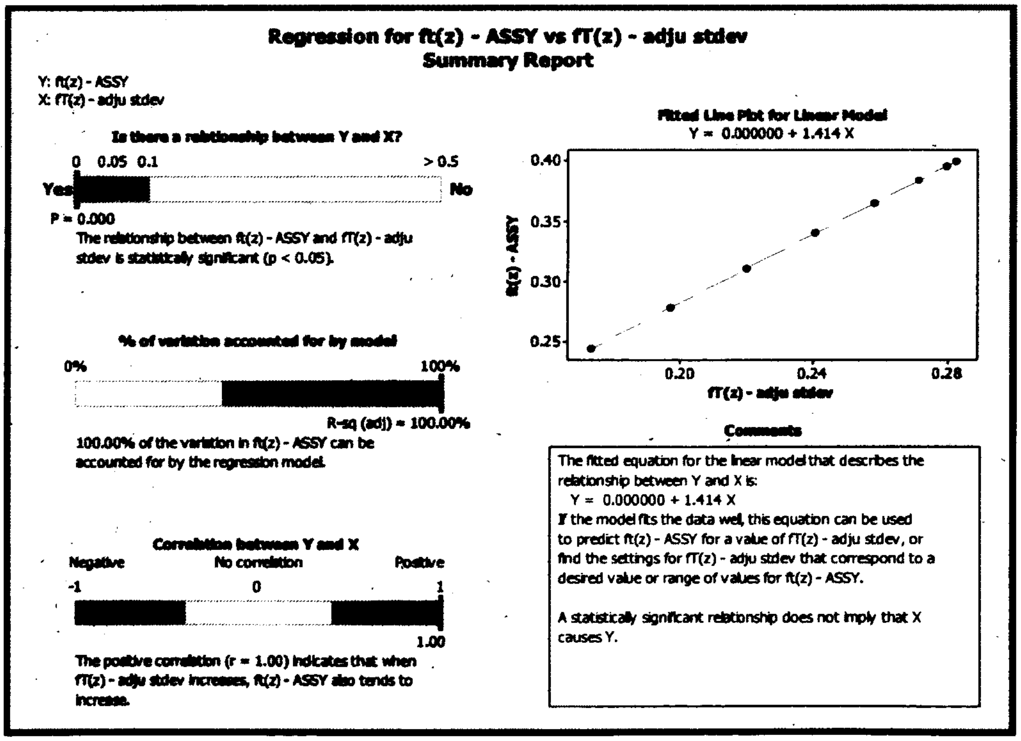
Fitted Line Plot for Cubic Model
Y = 0.09755 + 1.185 X - 1.532 X**2 + 1.142 X**3

Comments

The fitted equation for the cubic model that describes the relationship between Y and X is:
Y = 0.09755 + 1.185 X - 1.532 X**2 + 1.142 X**3
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.102 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 1.4, LSL = -1.4)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

Is there a relationship between Y and X?

P = 0.000

The relationship between ft(z) - ASSY and fT(z) - adju stdev is statistically significant ($p < 0.05$).

% of variation accounted for by model

R-sq (adj) = 100.00%

100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Correlation between Y and X

1.00

The positive correlation ($r = 1.00$) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

Fitted Line Plot for Linear Model
Y = 0.000000 + 1.414 X

Comments

The fitted equation for the linear model that describes the relationship between Y and X is:
Y = 0.000000 + 1.414 X
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.103 - TSND Assembly Comparison (USL = 1.2, LSL = -1.2)

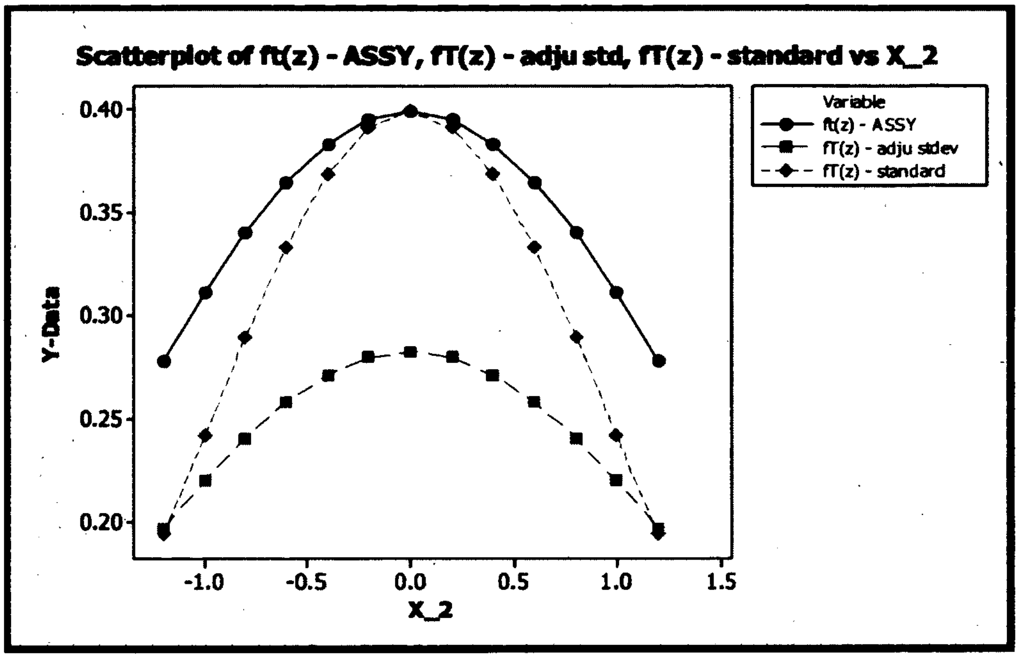
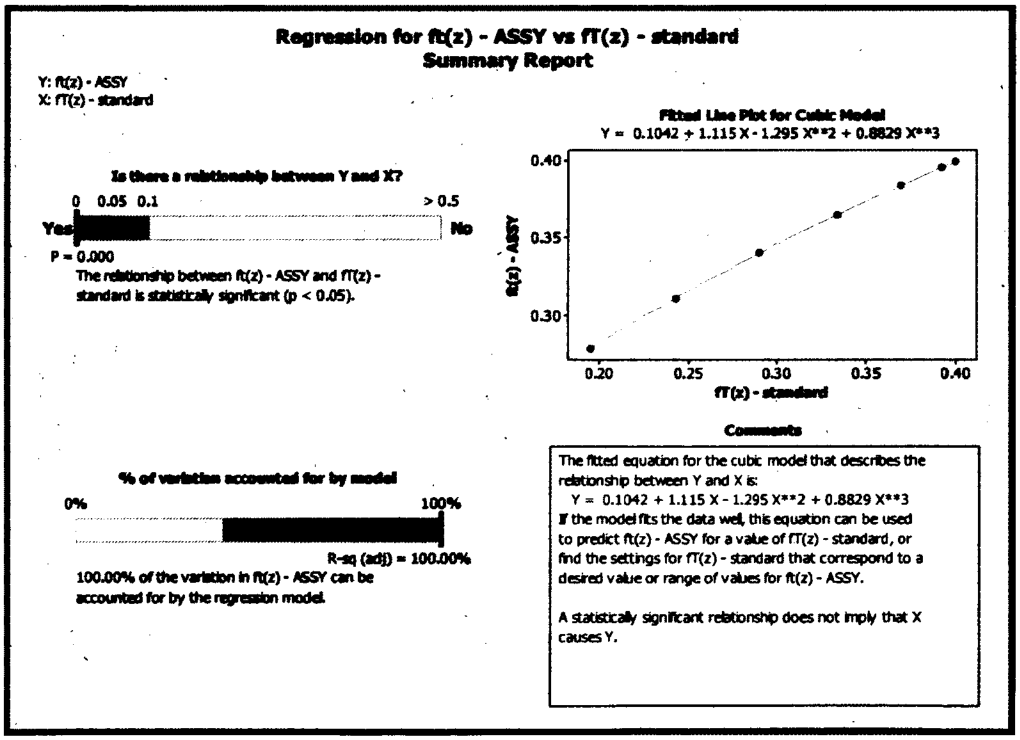


Figure C.104 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 1.2, LSL = -1.2)



P = 0.000

The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant (p < 0.05).

100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

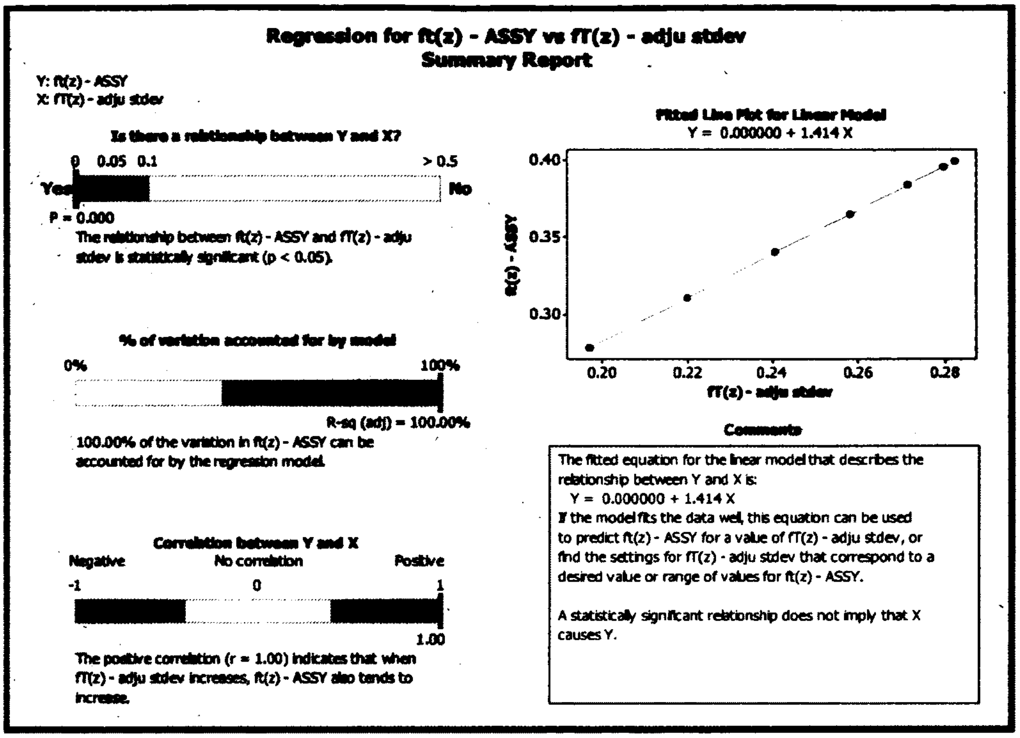
The fitted equation for the cubic model that describes the relationship between Y and X is:

Y = 0.1042 + 1.115 X - 1.295 X**2 + 0.8829 X**3

If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.105 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 1.2, LSL = -1.2)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

Is there a relationship between Y and X?

P = 0.000

The relationship between ft(z) - ASSY and fT(z) - adju stdev is statistically significant (p < 0.05).

% of variation accounted for by model

R-sq (adj) = 100.00%

100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Correlation between Y and X

The positive correlation (r = 1.00) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

Fitted Line Plot for Linear Model
Y = 0.000000 + 1.414 X

Comments

The fitted equation for the linear model that describes the relationship between Y and X is:
Y = 0.000000 + 1.414 X
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.106 - TSND Assembly Comparison (USL = 1.0, LSL = -1.0)

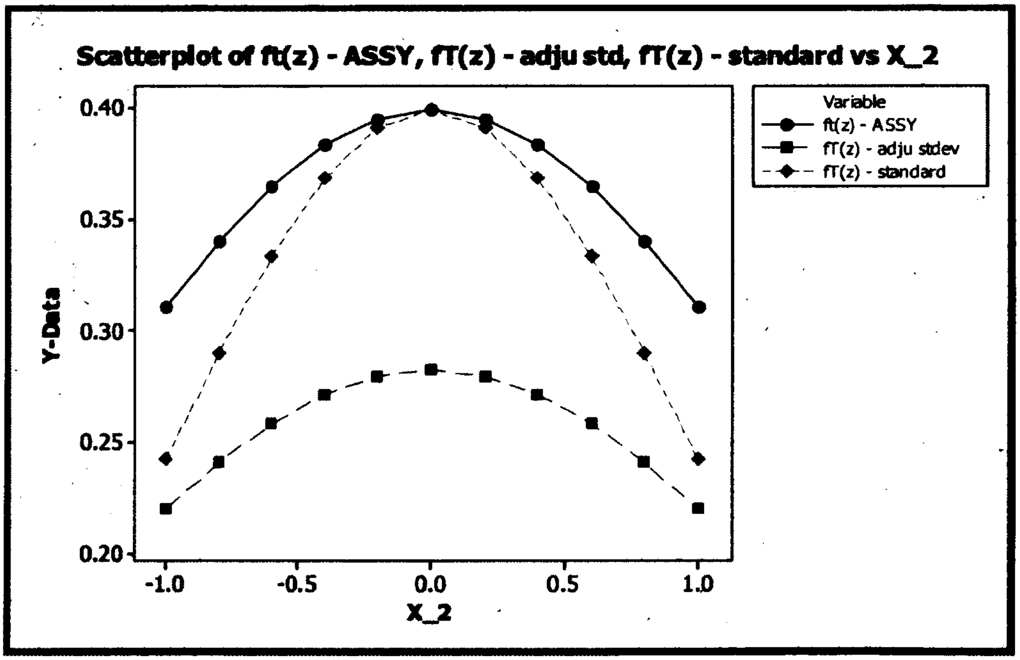
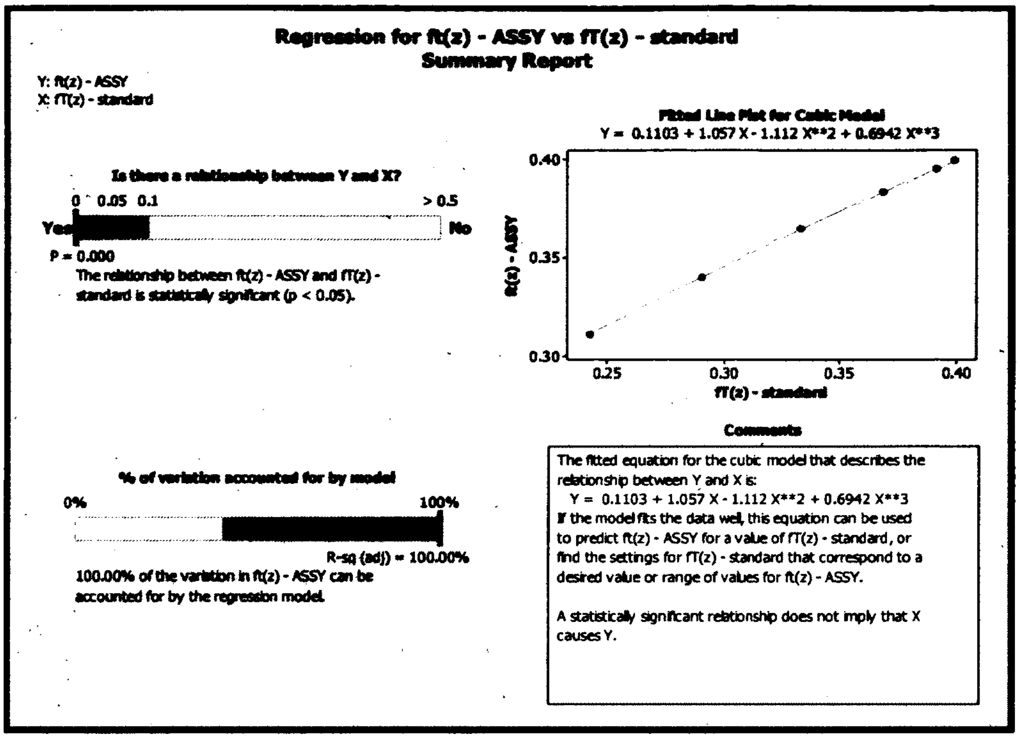


Figure C.107 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 1, LSL = -1)



Regression for ft(z) - ASSY vs fT(z) - standard
Summary Report

Y: ft(z) - ASSY
X: fT(z) - standard

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant (p < 0.05).

% of variation accounted for by model

R-sq (adj) = 100.00%
100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

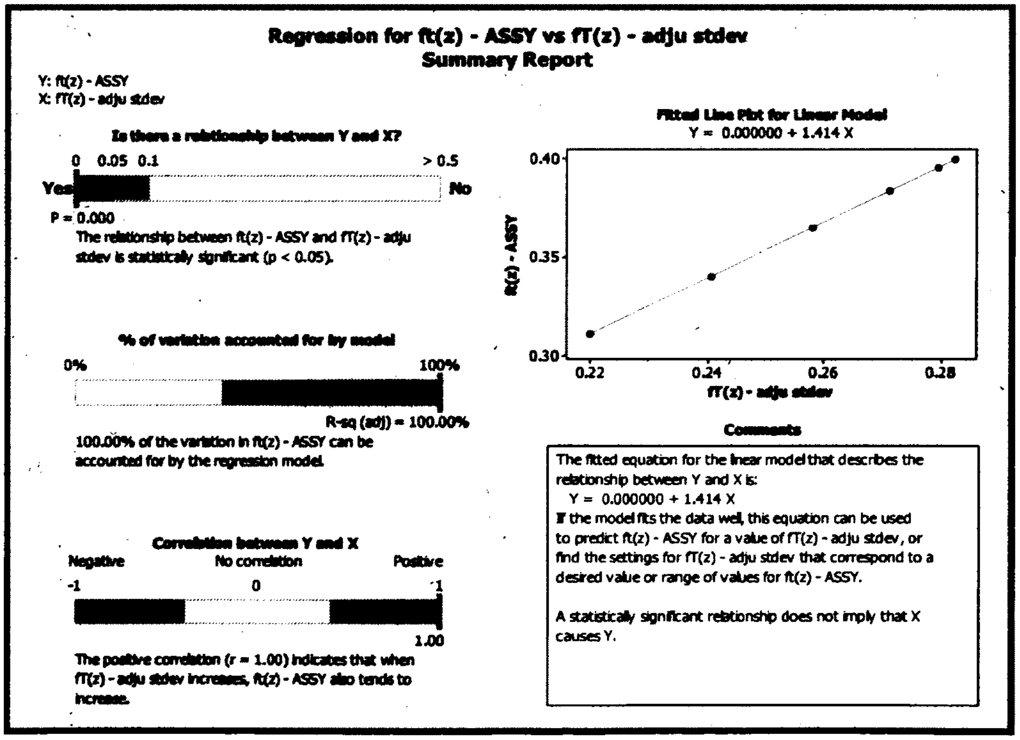
Fitted Line Plot for Cubic Model
Y = 0.1103 + 1.057 X - 1.112 X**2 + 0.6942 X**3

Comments

The fitted equation for the cubic model that describes the relationship between Y and X is:
Y = 0.1103 + 1.057 X - 1.112 X**2 + 0.6942 X**3
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.108 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 1, LSL = -1)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - adju stdev is statistically significant (p < 0.05).

% of variation accounted for by model

R-sq (adj) = 100.00%
100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Correlation between Y and X

1.00
The positive correlation (r = 1.00) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

Fitted Line Plot for Linear Model
Y = 0.000000 + 1.414 X

Comments

The fitted equation for the linear model that describes the relationship between Y and X is:
Y = 0.000000 + 1.414 X
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.109 - TSND Assembly Comparison (USL = 0.8, LSL = -0.8)

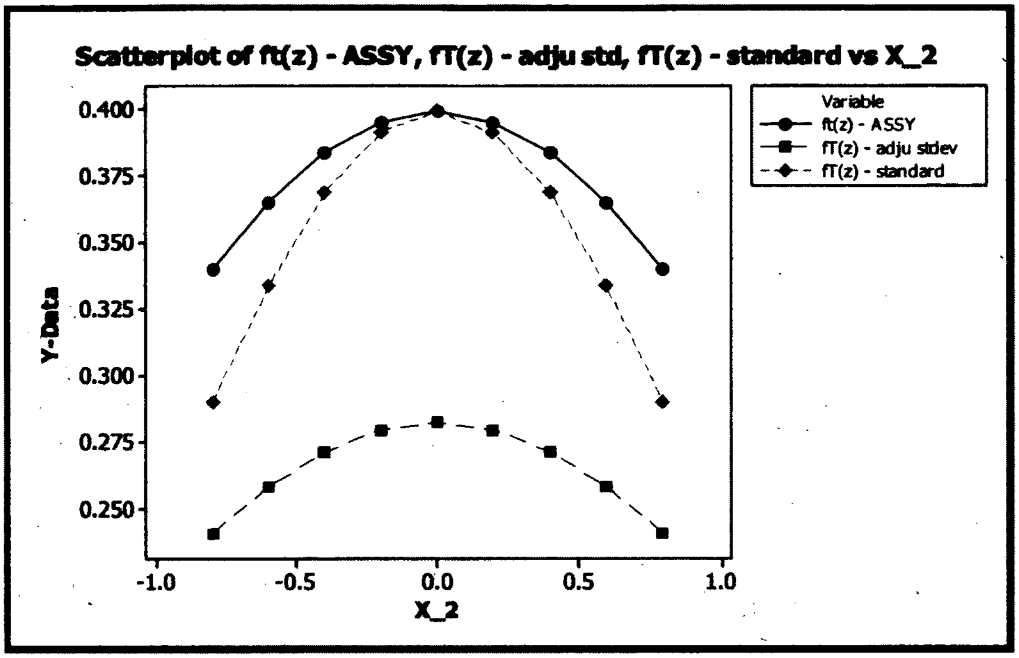
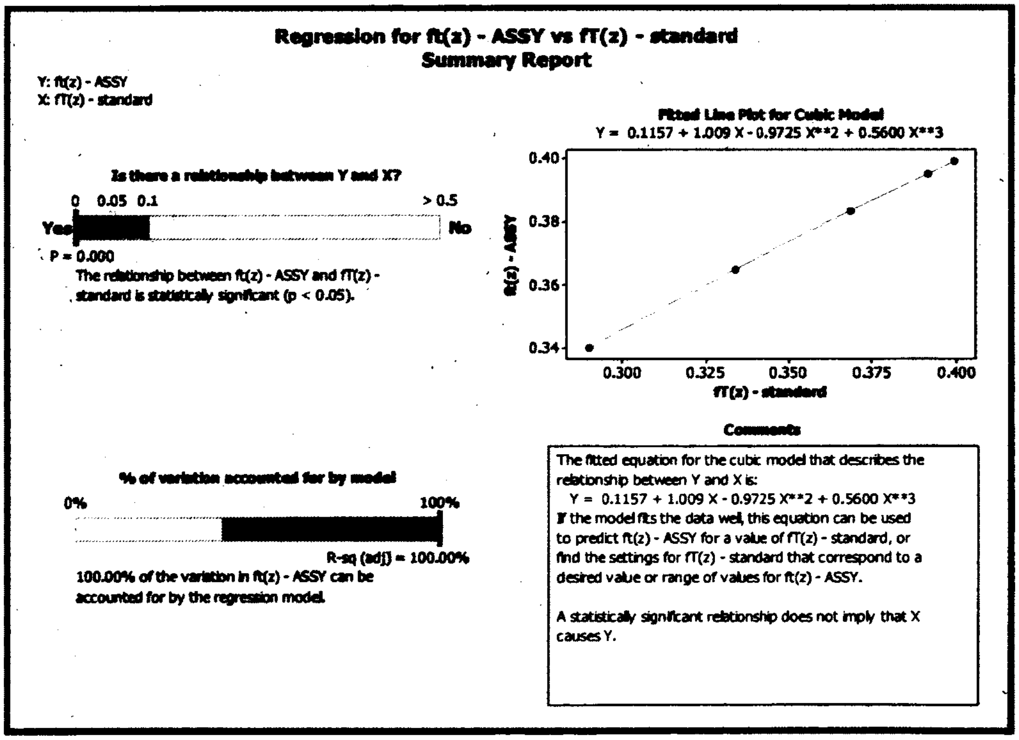


Figure C.110 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 0.8, LSL = -0.8)



Regression for ft(z) - ASSY vs fT(z) - standard
Summary Report

Y: ft(z) - ASSY
X: fT(z) - standard

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant (p < 0.05).

% of variation accounted for by model

R-sq (adj) = 100.00%
100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

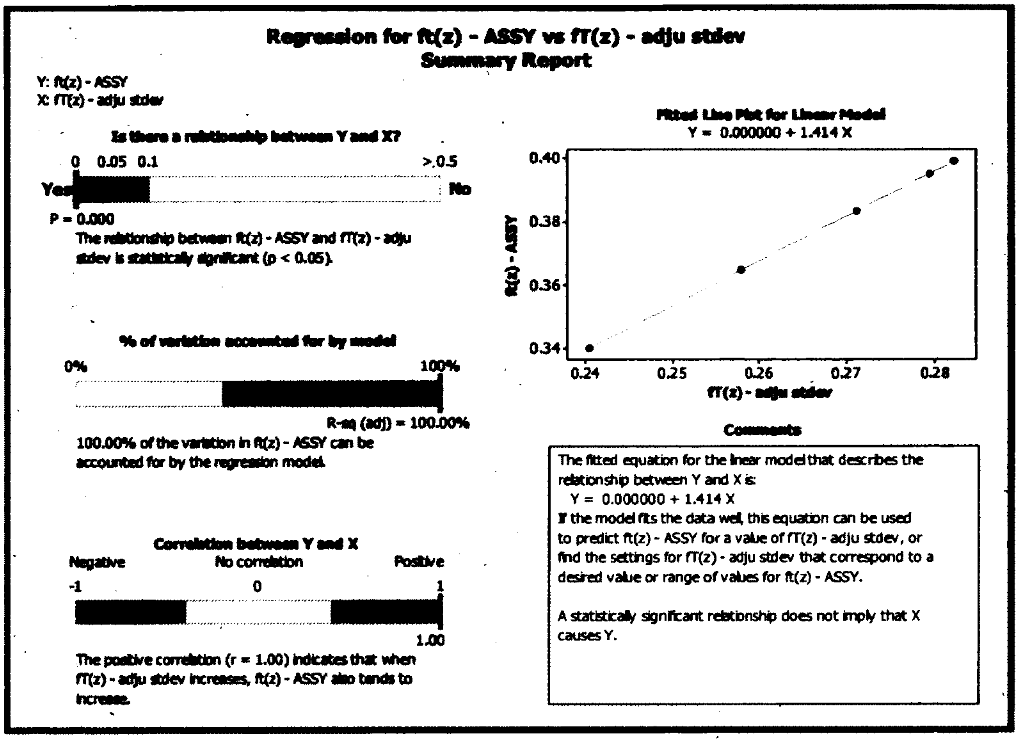
Fitted Line Plot for Cubic Model
Y = 0.1157 + 1.009 X - 0.9725 X**2 + 0.5600 X**3

Comments

The fitted equation for the cubic model that describes the relationship between Y and X is:
Y = 0.1157 + 1.009 X - 0.9725 X**2 + 0.5600 X**3
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.111 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 0.8, LSL = -0.8)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - adju stdev is statistically significant (p < 0.05).

% of variation accounted for by model

R-sq (adj) = 100.00%
100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Correlation between Y and X

The positive correlation (r = 1.00) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

Fitted Line Plot for Linear Model
Y = 0.000000 + 1.414 X

Comments

The fitted equation for the linear model that describes the relationship between Y and X is:
Y = 0.000000 + 1.414 X
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.112 - TSND Assembly Comparison (USL = 0.6, LSL = -0.6)

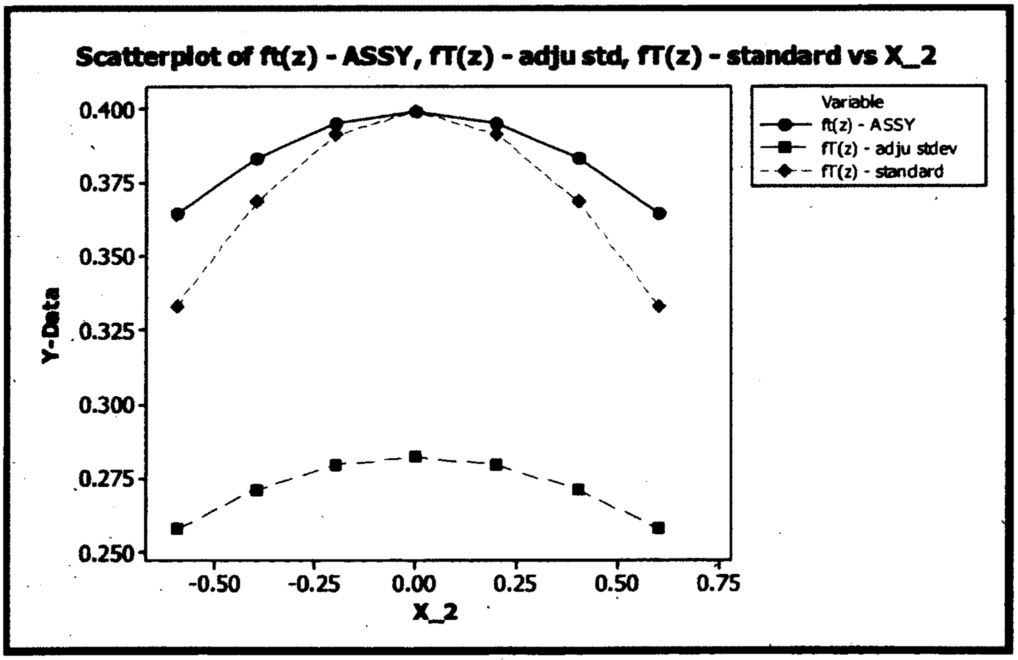
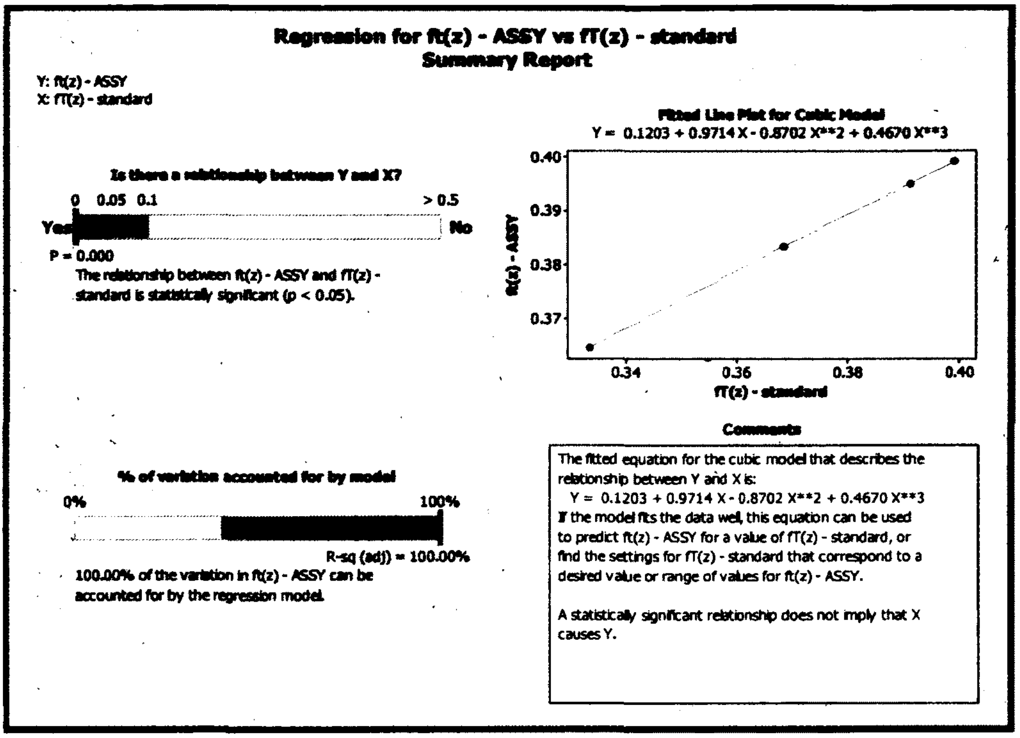


Figure C.113 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 0.6, LSL = -0.6)



Regression for ft(z) - ASSY vs fT(z) - standard
Summary Report

Y: ft(z) - ASSY
X: fT(z) - standard

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant ($p < 0.05$).

% of variation accounted for by model

R-sq (adj) = 100.00%
100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

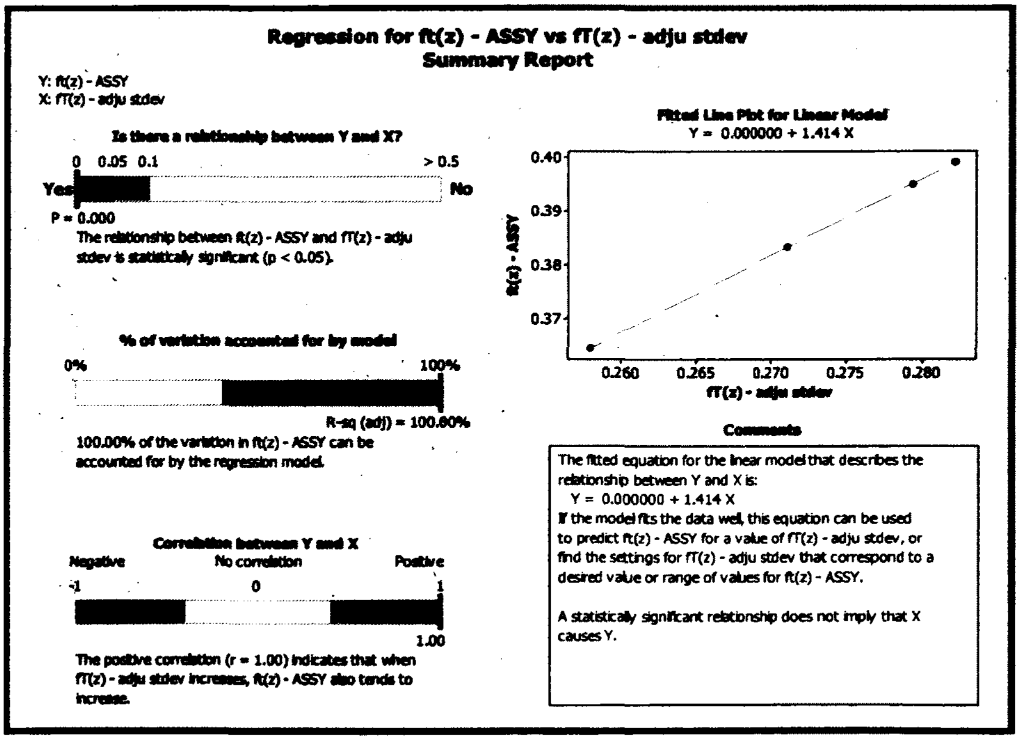
Fitted Line Plot for Cubic Model
Y = 0.1203 + 0.9714 X - 0.8702 X**2 + 0.4670 X**3

Comments

The fitted equation for the cubic model that describes the relationship between Y and X is:
Y = 0.1203 + 0.9714 X - 0.8702 X**2 + 0.4670 X**3
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.114 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 0.6, LSL = -0.6)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - adju stdev is statistically significant ($p < 0.05$).

% of variation accounted for by model

R-sq (adj) = 100.00%
100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Correlation between Y and X

The positive correlation (r = 1.00) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

Fitted Line Plot for Linear Model
Y = 0.000000 + 1.414 X

Comments

The fitted equation for the linear model that describes the relationship between Y and X is:
Y = 0.000000 + 1.414 X
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.115 - TSND Assembly Comparison (USL = 0.4, LSL = -0.4)

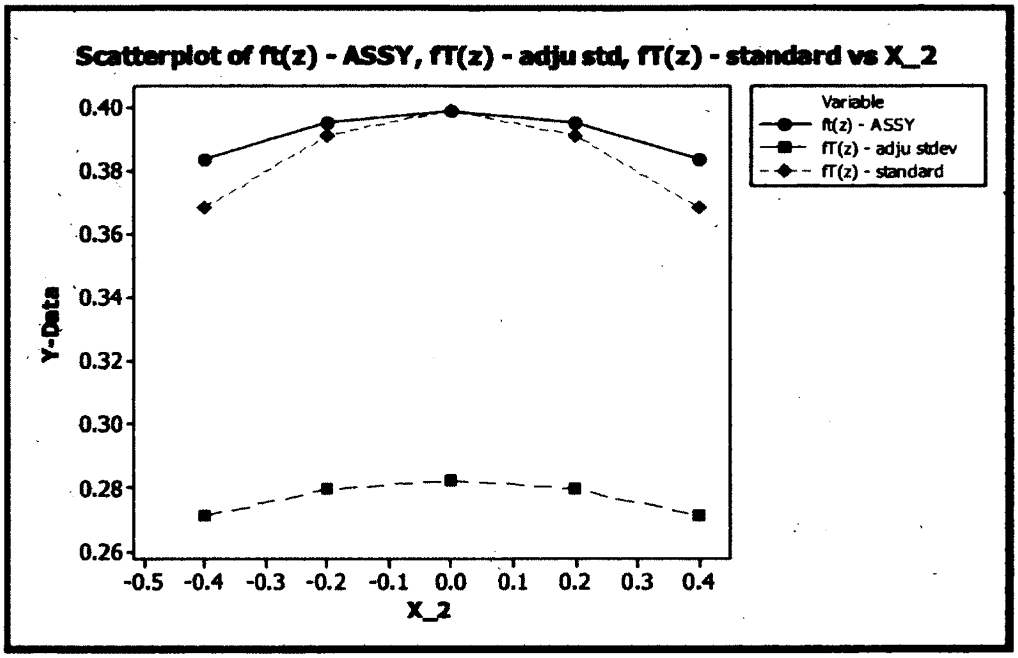
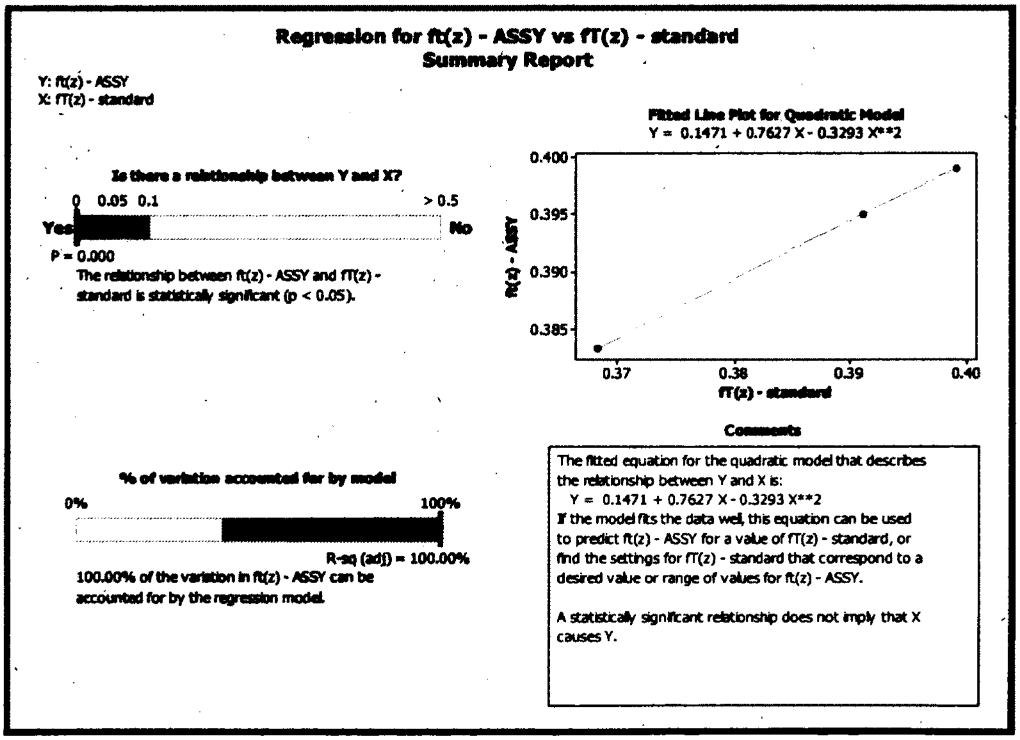


Figure C.116 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 0.4, LSL = -0.4)



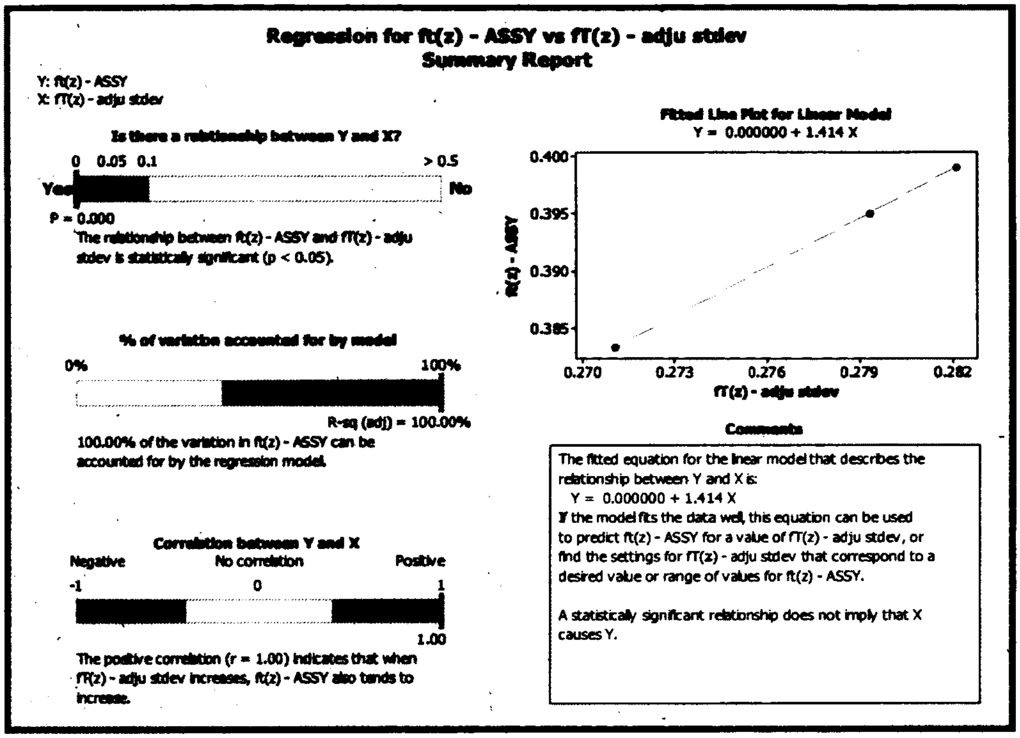
The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant (p < 0.05).

100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

The fitted equation for the quadratic model that describes the relationship between Y and X is:
Y = 0.1471 + 0.7627 X - 0.3293 X**2
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.117 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 0.4, LSL = -0.4)



The relationship between ft(z) - ASSY and fT(z) - adju stdev is statistically significant ($p < 0.05$).

100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

The positive correlation ($r = 1.00$) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

The fitted equation for the linear model that describes the relationship between Y and X is:

$Y = 0.000000 + 1.414 X$

If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.118 - TSND Assembly Comparison (USL = 0.2, LSL = -0.2)

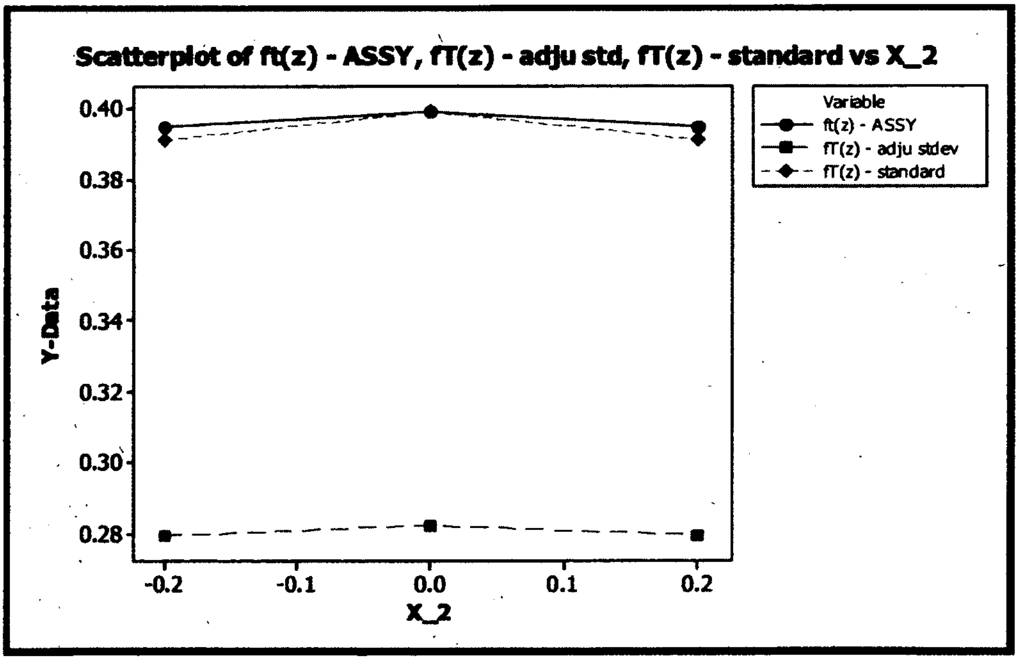
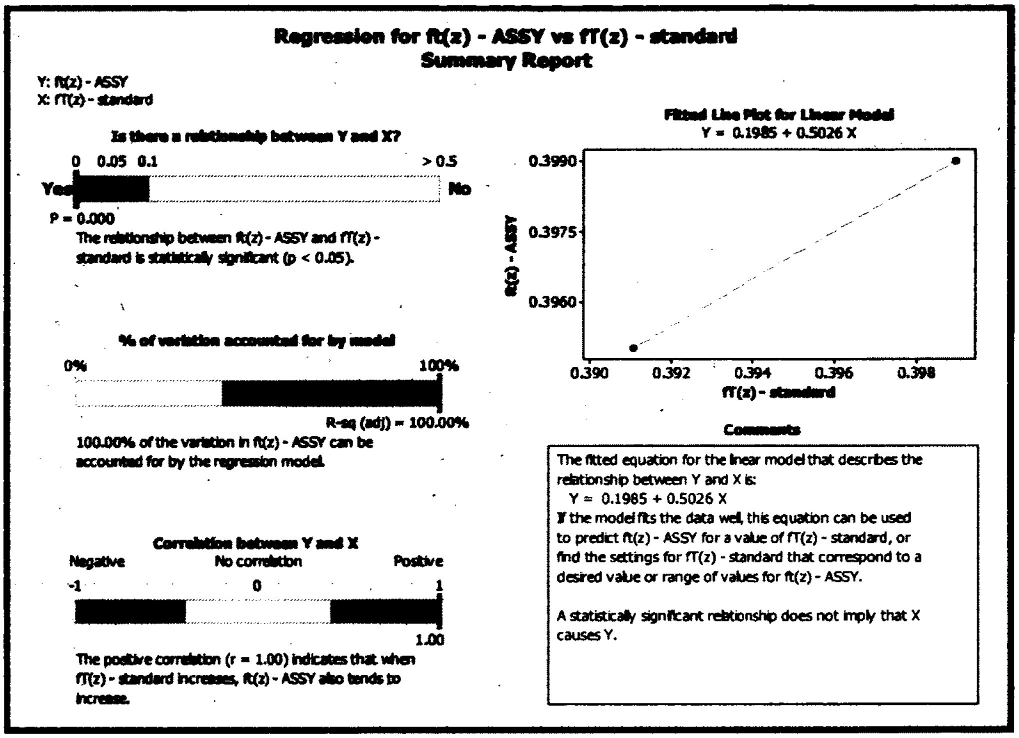


Figure C.119 - ft(z)-ASSY vs. ft(z) standard Regression (USL = 0.2, LSL = -0.2)



Regression for ft(z) - ASSY vs fT(z) - standard
Summary Report

Y: ft(z) - ASSY
X: fT(z) - standard

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - standard is statistically significant ($p < 0.05$).

% of variation accounted for by model

R-sq (adj) = 100.00%
100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Correlation between Y and X

The positive correlation (r = 1.00) indicates that when fT(z) - standard increases, ft(z) - ASSY also tends to increase.

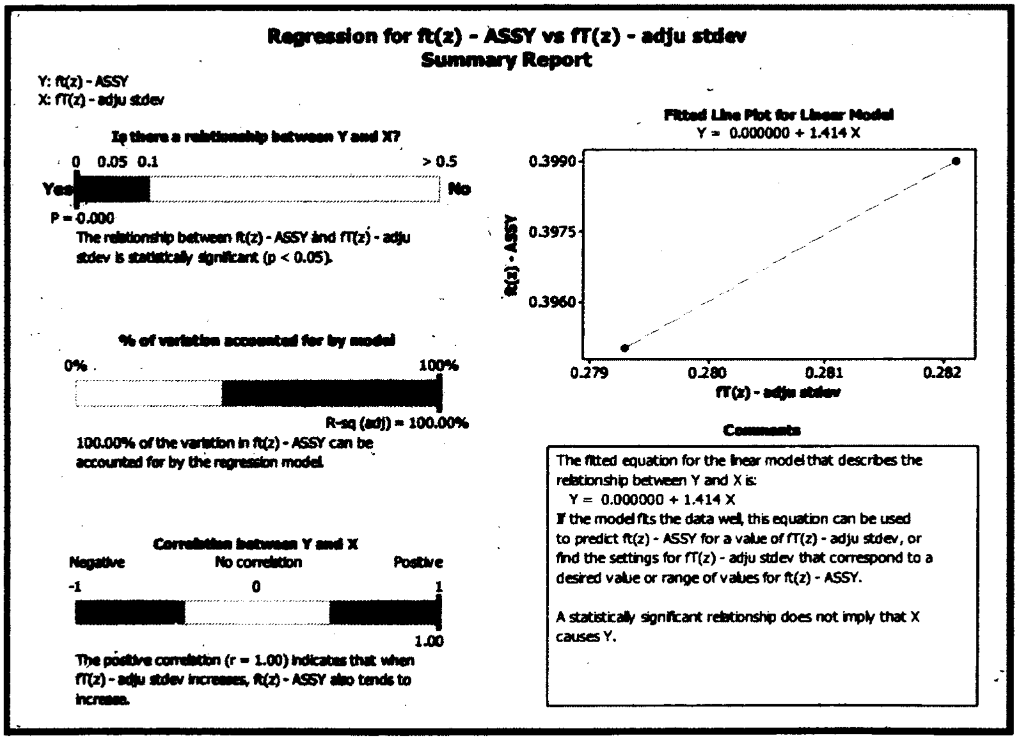
Fitted Line Plot for Linear Model
Y = 0.1985 + 0.5026 X

Comments

The fitted equation for the linear model that describes the relationship between Y and X is:
Y = 0.1985 + 0.5026 X
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - standard, or find the settings for fT(z) - standard that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

Figure C.120 - ft(z)-ASSY vs. ft(z) adju stdev Regression (USL = 0.2, LSL = -0.2)



Regression for ft(z) - ASSY vs fT(z) - adju stdev
Summary Report

Y: ft(z) - ASSY
X: fT(z) - adju stdev

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(z) - ASSY and fT(z) - adju stdev is statistically significant ($p < 0.05$).

% of variation accounted for by model

R-sq (adj) = 100.00%
100.00% of the variation in ft(z) - ASSY can be accounted for by the regression model.

Correlation between Y and X

The positive correlation (r = 1.00) indicates that when fT(z) - adju stdev increases, ft(z) - ASSY also tends to increase.

Fitted Line Plot for Linear Model
Y = 0.000000 + 1.414 X

Comments

The fitted equation for the linear model that describes the relationship between Y and X is:
Y = 0.000000 + 1.414 X
If the model fits the data well, this equation can be used to predict ft(z) - ASSY for a value of fT(z) - adju stdev, or find the settings for fT(z) - adju stdev that correspond to a desired value or range of values for ft(z) - ASSY.

A statistically significant relationship does not imply that X causes Y.

APPENDIX D: TSND ANALYSIS EXAMPLES

This appendix documents the analysis results for three simulated truncated standard normal distribution assemblies. Various examples and their results demonstrate the application of a truncated standard normal distribution characteristic function inversion using an inversion factor. This example has been baselined against calculation methods which employ methods found in References [104] and [105].

The three examples identified use the inversion factor verified from a single truncated standard normal distribution. Inversion factors for truncated standard normal distributions will be established for various combinations (i.e., USL = 8 to LSL = -8). For the purpose of this example, identical combinations will be used due to the multitude of combinations and to maintain simplicity in the calculations presented within the framework for this research.

Refer to Section 4 for additional information.

Example 1:

Simulation Input Parameters: $\mu = 0$, $\sigma = 1$, LSL = -2, USL = 2, n = 10,000 (sample size)

Table D.1 Truncated Distribution Simulation Range, 10,000 Samples (-2 to 2)

Bin	*Frequency*
-4	0
-3.9	0
-3.8	0
-3.7	0
-3.6	0
-3.5	0
-3.4	0
-3.3	0
-3.2	0
-3.1	0
-3	0
-2.9	0
-2.8	0
-2.7	0
-2.6	0
-2.5	0
-2.4	0
-2.3	0
-2.2	0
-2.1	0
-2	0
-1.9	120
-1.8	136
-1.7	118
-1.6	150
-1.5	197
-1.4	189
-1.3	202
-1.2	212
-1.1	238
-1	241
-0.9	256
-0.8	309
-0.7	304
-0.6	300
-0.5	277
-0.4	337
-0.3	306
-0.2	340
-0.1	307
-9.91E-15	365
0.1	313
0.2	316
0.3	352
0.4	340
0.5	352
0.6	331
0.7	328
0.8	282
0.9	298
1	290
1.1	261
1.2	226
1.3	230
1.4	223
1.5	169
1.6	192
1.7	164
1.8	160
1.9	140
2	129
2.1	0
2.2	0
2.3	0
2.4	0
2.5	0
2.6	0
2.7	0
2.8	0
2.9	0
3	0
3.1	0
3.2	0
3.3	0
3.4	0
3.5	0
3.6	0
3.7	0
3.8	0
3.9	0
4	0
More	0

Table D.2 - Pearson Correlation of Example 1

TSND RANGE	Pearson correlation of ft(z) a- ASSY and ft(z) - standard	Pearson correlation of ft(z) a- ASSY and ft(z) – adju stdev
USL = 2, LSL = -2	.973	1

Table D.3 - Regression Analysis of Example 1

TSND (USL = 2, LSL = -2)	R-sq (adj)	P-Value	Fitted Line Plot Equation for Cubic Model
ft(z) a- ASSY and ft(z) – standard	99.55%	$p < 0.001$	Y = 0.03426 + 2.546 X - 7.524 X**2 + 9.334 X**3
ft(z) a- ASSY and ft(z) – adju stdev	100%	$p < 0.001$	Y = 0.000000 + 1.545 X

Figure D.1 - Truncated Distribution Histogram, 10,000 Samples (-2 to 2)

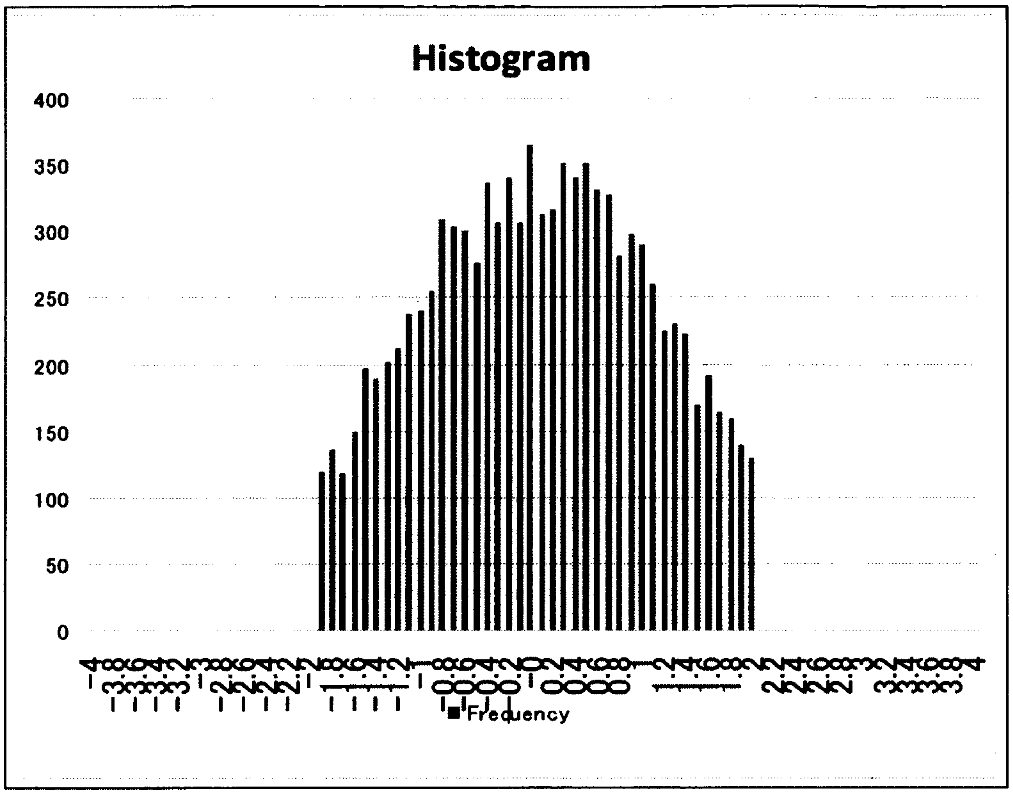
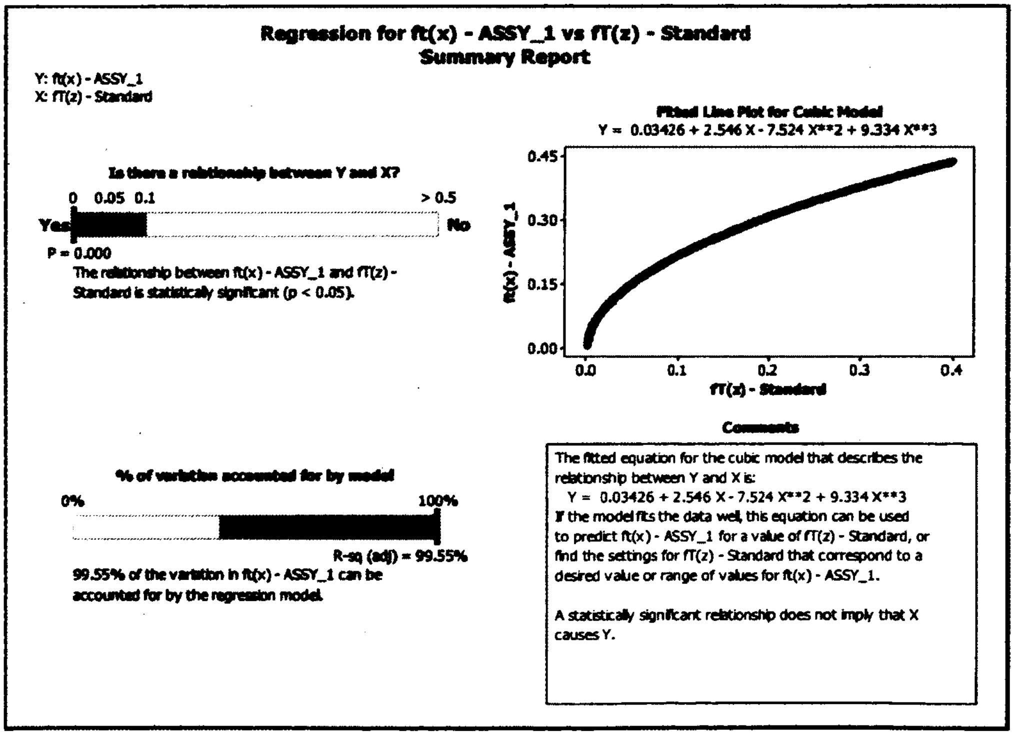


Figure D.2 - ft(z)-ASSY vs. ft(z) standard Regression, 10,000 Samples (-2 to 2)



Regression for ft(x) - ASSY_1 vs fT(z) - Standard
Summary Report

Y: ft(x) - ASSY_1
X: fT(z) - Standard

P = 0.000
The relationship between ft(x) - ASSY_1 and fT(z) - Standard is statistically significant (p < 0.05).

R-sq (adj) = 99.55%
99.55% of the variation in ft(x) - ASSY_1 can be accounted for by the regression model.

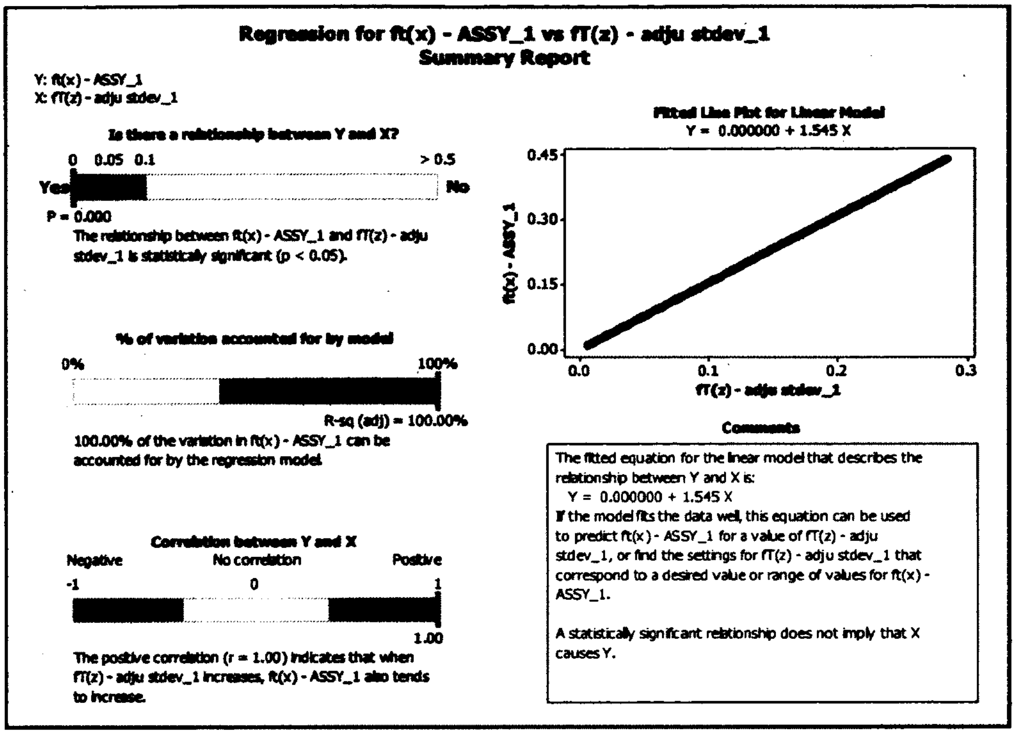
Y = 0.03426 + 2.546 X - 7.524 X**2 + 9.334 X**3

Comments

The fitted equation for the cubic model that describes the relationship between Y and X is:
Y = 0.03426 + 2.546 X - 7.524 X**2 + 9.334 X**3
If the model fits the data well, this equation can be used to predict ft(x) - ASSY_1 for a value of fT(z) - Standard, or find the settings for fT(z) - Standard that correspond to a desired value or range of values for ft(x) - ASSY_1.

A statistically significant relationship does not imply that X causes Y.

Figure D.3 - ft(z)-ASSY vs. ft(z) adju stdev Regression, 10,000 Samples (-2 to 2)



Regression for ft(x) - ASSY_1 vs fT(z) - adju stdev_1
Summary Report

Y: ft(x) - ASSY_1
X: fT(z) - adju stdev_1

Is there a relationship between Y and X?

P = 0.000

The relationship between ft(x) - ASSY_1 and fT(z) - adju stdev_1 is statistically significant (p < 0.05).

% of variation accounted for by model

R-sq (adj) = 100.00%

100.00% of the variation in ft(x) - ASSY_1 can be accounted for by the regression model.

Correlation between Y and X

The positive correlation (r = 1.00) indicates that when fT(z) - adju stdev_1 increases, ft(x) - ASSY_1 also tends to increase.

Fitted Line Plot for Linear Model
Y = 0.000000 + 1.545 X

Comments

The fitted equation for the linear model that describes the relationship between Y and X is:
Y = 0.000000 + 1.545 X
If the model fits the data well, this equation can be used to predict ft(x) - ASSY_1 for a value of fT(z) - adju stdev_1, or find the settings for fT(z) - adju stdev_1 that correspond to a desired value or range of values for ft(x) - ASSY_1.

A statistically significant relationship does not imply that X causes Y.

Example 2:

Simulation Input Parameters: $\mu = 0$, $\sigma = 1$, LSL = -3, USL = 3, n = 10,000 (sample size)

Table D.4 Truncated Distribution Simulation Range, 10,000 Samples (-3 to 3)

Bin	*Frequency*
-4	0
-3.9	0
-3.8	0
-3.7	0
-3.6	0
-3.5	0
-3.4	0
-3.3	0
-3.2	0
-3.1	0
-3	0
-2.9	9
-2.8	6
-2.7	10
-2.6	13
-2.5	17
-2.4	20
-2.3	23
-2.2	36
-2.1	31
-2	54
-1.9	57
-1.8	67
-1.7	60
-1.6	100
-1.5	129
-1.4	156
-1.3	153
-1.2	186
-1.1	192
-1	228
-0.9	242
-0.8	266
-0.7	326
-0.6	337
-0.5	338
-0.4	321
-0.3	374
-0.2	361
-0.1	392
-9.9E-15	400
0.1	365
0.2	386
0.3	401
0.4	392
0.5	371
0.6	365
0.7	315
0.8	321
0.9	303
1	259
1.1	221
1.2	215
1.3	188
1.4	138
1.5	156
1.6	123
1.7	106
1.8	88
1.9	89
2	58
2.1	57
2.2	39
2.3	37
2.4	29
2.5	18
2.6	14
2.7	18
2.8	11
2.9	7
3	6
3.1	0
3.2	0
3.3	0
3.4	0
3.5	0
3.6	0
3.7	0
3.8	0
3.9	0
4	0
More	0

Table D.5 - Pearson Correlation of Example 2

TSND RANGE	Pearson correlation of ft(z) a- ASSY and ft(z) - standard	Pearson correlation of ft(z) a- ASSY and ft(z) – adju stdev
USL = 3, LSL = -3	.972	1

Table D.6 - Regression Analysis of Example 2

TSND (USL = 3, LSL = -3)	R-sq (adj)	P-Value	Fitted Line Plot Equation for Cubic Model
ft(z) a- ASSY and ft(z) – standard	99.45%	$p < 0.001$	Y = 0.02832 + 2.369 X - 7.014 X**2 + 8.675 X**3
ft(z) a- ASSY and ft(z) – adju stdev	100%	Note 1	Y = - 0.000000+1.422 X+0.000000 X**2 - 0.000000 X**3

Note 1: For values referencing this note the p-value could not be calculated

Note 2: Standard deviation is 1, unless otherwise noted in Appendix B

Figure D.4 - Truncated Distribution Histogram, 10,000 Samples (-3 to 3)

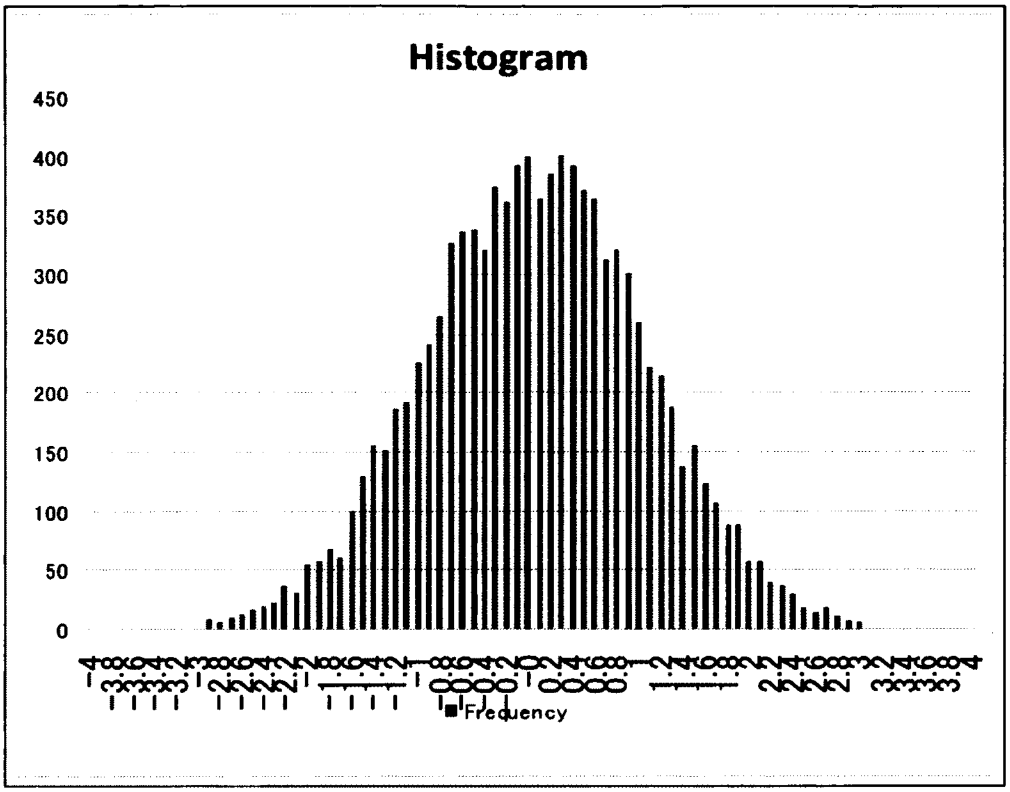
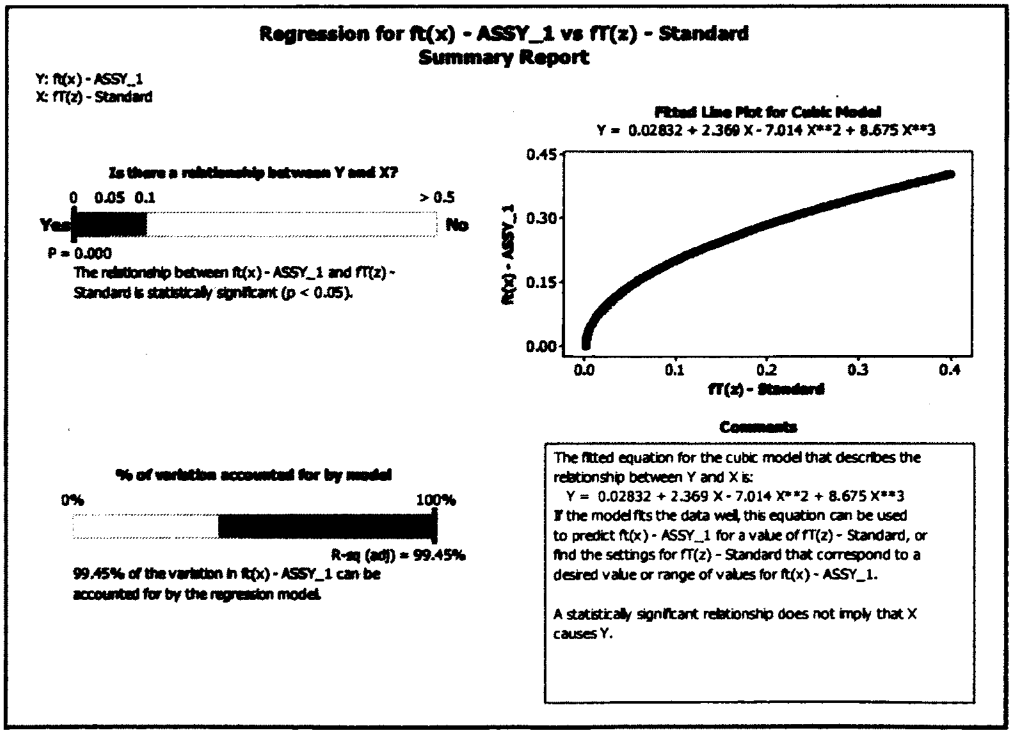


Figure D.5 - ft(z)-ASSY vs. ft(z) standard Regression, 10,000 Samples (-3 to 3)



Regression for ft(x) - ASSY_1 vs fT(z) - Standard
Summary Report

Y: ft(x) - ASSY_1
X: fT(z) - Standard

Is there a relationship between Y and X?

P = 0.000

The relationship between ft(x) - ASSY_1 and fT(z) - Standard is statistically significant (p < 0.05).

% of variation accounted for by model

R-sq (adj) = 99.45%

99.45% of the variation in ft(x) - ASSY_1 can be accounted for by the regression model.

Fitted Line Plot for Cubic Model

Y = 0.02832 + 2.369 X - 7.014 X**2 + 8.675 X**3

Comments

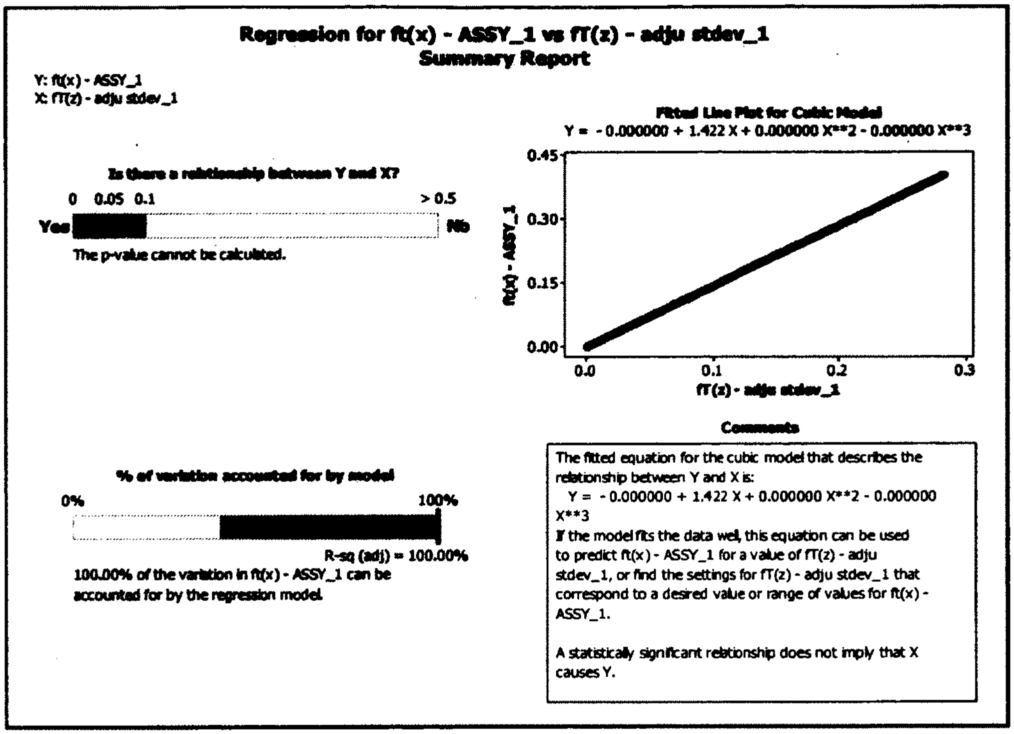
The fitted equation for the cubic model that describes the relationship between Y and X is:

Y = 0.02832 + 2.369 X - 7.014 X**2 + 8.675 X**3

If the model fits the data well, this equation can be used to predict ft(x) - ASSY_1 for a value of fT(z) - Standard, or find the settings for fT(z) - Standard that correspond to a desired value or range of values for ft(x) - ASSY_1.

A statistically significant relationship does not imply that X causes Y.

Figure D.6 - ft(z)-ASSY vs. ft(z) adju stdev Regression, 10,000 Samples (-3 to 3)



Regression for ft(x) - ASSY_1 vs fT(z) - adju stdev_1
Summary Report

Y: ft(x) - ASSY_1
X: fT(z) - adju stdev_1

The p-value cannot be calculated.

R-sq (adj) = 100.00%

100.00% of the variation in ft(x) - ASSY_1 can be accounted for by the regression model.

Comments

The fitted equation for the cubic model that describes the relationship between Y and X is:

Y = - 0.000000 + 1.422 X + 0.000000 X**2 - 0.000000 X**3

If the model fits the data well, this equation can be used to predict ft(x) - ASSY_1 for a value of fT(z) - adju stdev_1, or find the settings for fT(z) - adju stdev_1 that correspond to a desired value or range of values for ft(x) - ASSY_1.

A statistically significant relationship does not imply that X causes Y.

Example 3:

Simulation Input Parameters: $\mu = 0$, $\sigma = 1$, LSL = -4, USL = 4, n = 10,000 (sample size),

Table D.7 Truncated Distribution Simulation Range, 10,000 Samples (-4 to 4)

Bin	*Frequency*
-4	0
-3.9	0
-3.8	0
-3.7	0
-3.6	2
-3.5	2
-3.4	0
-3.3	3
-3.2	3
-3.1	1
-3	3
-2.9	6
-2.8	7
-2.7	8
-2.6	11
-2.5	14
-2.4	21
-2.3	21
-2.2	36
-2.1	28
-2	52
-1.9	57
-1.8	61
-1.7	59
-1.6	90
-1.5	123
-1.4	158
-1.3	156
-1.2	182
-1.1	191
-1	221
-0.9	241
-0.8	274
-0.7	326
-0.6	337
-0.5	344
-0.4	330
-0.3	371
-0.2	363
-0.1	399
-9.91E-15	403
0.1	369
0.2	388
0.3	411
0.4	398
0.5	372
0.6	370
0.7	315
0.8	317
0.9	301
1	253
1.1	223
1.2	221
1.3	188
1.4	143
1.5	142
1.6	122
1.7	101
1.8	85
1.9	87
2	56
2.1	52
2.2	37
2.3	38
2.4	25
2.5	14
2.6	19
2.7	11
2.8	12
2.9	6
3	7
3.1	3
3.2	4
3.3	3
3.4	2
3.5	0
3.6	0
3.7	0
3.8	1
3.9	0
4	0
More	0

Table D.8 - Pearson Correlation of Example 3

TSND RANGE	Pearson correlation of ft(z) a- ASSY and ft(z) - standard	Pearson correlation of ft(z) a- ASSY and ft(z) – adju stdev
USL = 4, LSL = -4	.972	1

Table D.9 - Regression Analysis of Example 3

TSND (USL = 4, LSL = -4)	R-sq (adj)	P-Value	Fitted Line Plot Equation for Cubic Model
ft(z) a- ASSY and ft(z) – standard	99.45%	p < 0.001	Y = 0.02827 + 2.352 X - 6.955 X**2 + 8.593 X**3
ft(z) a- ASSY and ft(z) – adju stdev	100%	Note 1	Y = 0.000000 + 1.414 X - 0.000000 X**2 + 0.000000 X**3

Note 1: For values referencing this note the p-value could not be calculated

Note 2: Standard deviation is 1, unless otherwise noted in Appendix B

Figure D.7 - Truncated Distribution Histogram, 10,000 Samples (-4 to 4)

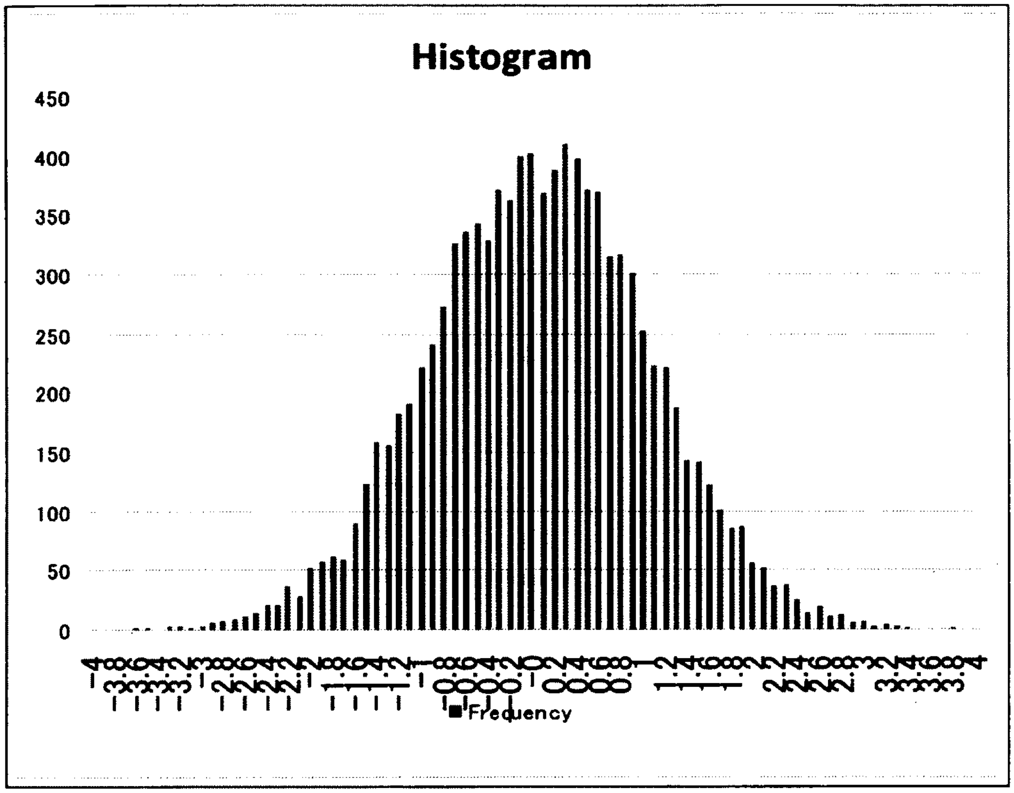
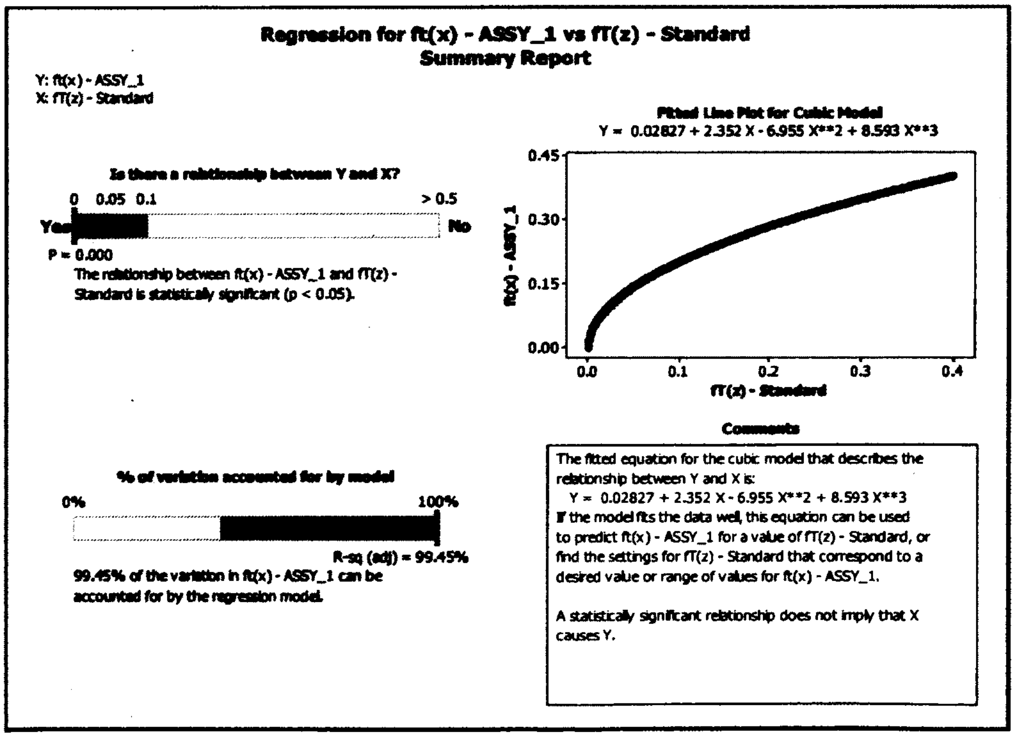


Figure D.8 - ft(z)-ASSY vs. ft(z) standard Regression, 10,000 Samples (-4 to 4)


Regression for ft(x) - ASSY_1 vs fT(z) - Standard
Summary Report

Y: ft(x) - ASSY_1
X: fT(z) - Standard

Is there a relationship between Y and X?

P = 0.000
The relationship between ft(x) - ASSY_1 and fT(z) - Standard is statistically significant (p < 0.05).

% of variation accounted for by model

R-sq (adj) = 99.45%
99.45% of the variation in ft(x) - ASSY_1 can be accounted for by the regression model.

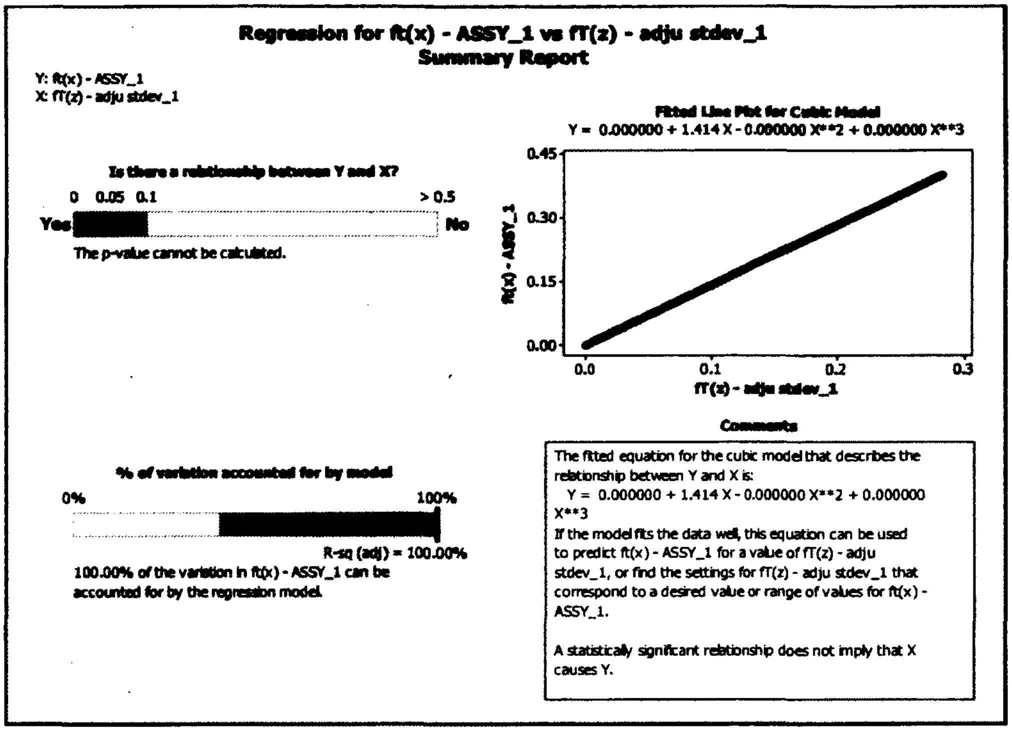
Fitted Line Plot for Cubic Model
Y = 0.02827 + 2.352 X - 6.955 X**2 + 8.593 X**3

Comments

The fitted equation for the cubic model that describes the relationship between Y and X is:
Y = 0.02827 + 2.352 X - 6.955 X**2 + 8.593 X**3
If the model fits the data well, this equation can be used to predict ft(x) - ASSY_1 for a value of fT(z) - Standard, or find the settings for fT(z) - Standard that correspond to a desired value or range of values for ft(x) - ASSY_1.

A statistically significant relationship does not imply that X causes Y.

Figure D.9 - ft(z)-ASSY vs. ft(z) adju stdev Regression, 10,000 Samples (-4 to 4)



Regression for ft(x) - ASSY_1 vs fT(z) - adju stdev_1
Summary Report

Y: ft(x) - ASSY_1
X: fT(z) - adju stdev_1

Is there a relationship between Y and X?

The p-value cannot be calculated.

% of variation accounted for by model

R-sq (adj) = 100.00%

100.00% of the variation in ft(x) - ASSY_1 can be accounted for by the regression model.

Fitted Line Plot for Cubic Model

Y = 0.000000 + 1.414 X - 0.000000 X**2 + 0.000000 X**3

Comments

The fitted equation for the cubic model that describes the relationship between Y and X is:
Y = 0.000000 + 1.414 X - 0.000000 X**2 + 0.000000 X**3
If the model fits the data well, this equation can be used to predict ft(x) - ASSY_1 for a value of fT(z) - adju stdev_1, or find the settings for fT(z) - adju stdev_1 that correspond to a desired value or range of values for ft(x) - ASSY_1.

A statistically significant relationship does not imply that X causes Y.

APPENDIX E: CATEGORIZATION INFORMATION

This appendix provides grouping information related to the comparative review and literature review conducted as part of this research. Categorizations generally focused on the primary method identified by the research in each field of categorization. The analysis results for the comparative reviews performed are identified in Appendices F and G. It is not the intent of this dissertation to define the general concepts presented in this appendix. Refer to relevant references for insight into that level of evaluation which is outside the scope of this dissertation. In order to reduce the degree of analysis subjectivity, the following serves to contextualize the groupings performed in this research:

Search Heuristics: Search Heuristic generally included beam search, pseudo random search, and tab search heuristics. Refer to Michalewicz and Fogel (1998) for additional heuristic summary information outside the scope of this work.

Heuristic Procedure: Heuristic procedures were generally grouped to include explicitly identified heuristic procedure, knowledge based procedure, Taguchi procedures, and other step by step instructions that are generally representative of a heuristic as defined above.

Algorithm: Algorithm groupings generally included the references to assignment algorithms, greedy algorithms, genetic algorithms, network based algorithms, and other general reference to mathematical steps and formulations. Refer to Michalewicz and Fogel (1998) for algorithm information which is outside the scope of this work.

Optimization: The grouping and identification of optimization techniques was identified if any of the following optimization methods were identified:

- Any Colony Optimization
- Perturbation Techniques
- Keifer-Wolfwitz Optimization Procedure
- Operations Research
- Highly Optimized Tolerance (HOT)
- Utility Maximization through Criteria Weighting
- Optimization Model
- Simplex Method (or variant)
- Pattern Enumeration
- Mixed Integer Programming
- Attribute Level Driven
- Linear
- Policy Space Procedure
- Mathematical formulation
- Analytical Target Cascading
- Simulated Annealing
- Heuristic Based
- Weighting
- Function

Benchmarking Methods:

- An Example, Case Study, Design Specification, and Case Study Compared to design specification: This benchmarking method was identified if the literature generally or specifically involved examples, case studies, or design specifications as part of the literature evaluations.
- Heuristics or Other Methods: This grouping was identified if heuristic performance or computational experiment comparisons were identified by the literature.
- Historical or Collected Data Comparisons
- Simulation Data or Study: Results of examples compared with simulation study or other simulation/study comparison
- Mathematical Formulation: Identified if the primary benchmarking method observed dealt with mathematical formulations and related comparisons.
- Inconclusive or Not Performed: No experimental comparisons were performed.

Data Source/Simulation:

- Historical: Historical data was generally grouped or identified as data that was used for analysis based on previously collected or possibly even analyzed data. Historical data was pre-existing data. In some cases historical data was used to compare an existing state with a proposed future or improved condition.
- Data Generated: Any reference to data that was simulated, generated, randomly created or proposed as part of a scholarly work. Example of data generation could include such data as Monte Carlo Simulation or random number generation.

- Empirical data: Empirical data was generally identified as data which may have involved real time results or other industry related data.
- Sampled Data: Sample data groups consisted of those groups pulled from identified sample data from a given process.
- Example Data: Example data was grouped as that data which was used for demonstration purposes. This field differs from data generation or historical data.
- Inconclusive – Identified when the data source was not easily or readily identifiable.

<u>Test Methods:</u> Test methods were generally grouped into one of the following categories:

- Efficiency Improvement: Methods in which tests were performed to show an improvement in efficiency over a given value, process, heuristic, or other measured result.
- Demonstration of "Good" Solution: This grouping included results which focused not on optimization but on obtaining reasonably accurate or balanced solutions.
- Comparative Analysis: Direct or interpreted tests by comparison
- Simulation: A test method in which data may have been generated or developed as a means to produce a data set or solve a solution.
- Correlations: Statistical analysis such a Pearson's Correlation Coefficient or other general method of comparing the relationships of one variable to another.
- Experiment: Test methods done by physical or theoretical method.
- Error Ratio: Regression or other analysis in which error ratios were evaluated

- Mathematical Model: A test method involving the evaluation or utilization of a mathematical or analytical model.
- Commentary: Qualitative testing focused on interpretation and judgment

“Meaningful results” are defined as: either a statistically significant relationship, positive correlation/relationship, or any other observed, calculated, or identified parameter which provides data or indications not previously understood by the body of knowledge.

APPENDIX F: LITERATURE REVIEW VARIABLES

This appendix provides variables utilized in the gap analysis of the subject dissertation. Refer to Chapter 2 for the literature review variables reviewed (e.g., truncation, Selective assembly, etc.). It should be noted that not all references were utilized in this review. An "X" denotes that the literature identified an explicit or implied identification of the literature review variable. Additionally, general calculation references, definitions, duplicative or other references were excluded from this review. All review variables analyzed are included in Table F.1. Refer to Chapter 2 for additional information.

Table F. 1 – Literature Review Table

Reference	Literature Review Variables										
	Truncated Assembly/ Truncation	Selective Assembly	Tolerance/ Parameter Design/ Quality Control	Heuristics/ Frameworks/ Heuristic Solutions	Optimal Target Setting/ Optimization/Max-Minimization	Inventory Management Systems/ Storage/ Delivery	Assembly Systems/ Flexible Manufacturing/ Production Sequencing	Knowledge Management	Complexity/ Robust Design/ Sensitivity Analysis	Dynamic Simulation/ Simulation	Extreme Value Theory (EVT)
Carlson, J.M., & Doyle, J. (2002)			X	X	X				X		
Malakooti, B.B. (1994)					X		X				
Park M.W., & Kim Y.D. (1999)				X	X		X				
Pourbabai, B. (1992)				X	X	X	X				
Pourbabai, B. (1989)			X		X	X	X				
Cao, X., & Ho, Y. (1987)			X	X	X	X	X		X	X	
Seidmann, A., & Tenenbaum, A (1994)			X	X	X		X		X	X	
Kosfeld, M.A., & Quinn, T.D. (1999)					X	X			X	X	
Brown, D.E., & Spillane, A.R. (1989)				X	X			X			
Thakur, L.S., Nair, S.K., Wen, K-W, & Tarasewich, P. (2000)				X	X				X	X	
Coverdale, I., & Wharton, F. (1976)			X	X	X	X					
Rubin, P.A. (1990)			X	X	X					X	

Literature Review Variables											
Reference	Truncated Assembly/ Truncation	Selective Assembly	Tolerance/ Parameter Design/ Quality Control	Heuristics/ Frameworks/ Heuristic Solutions	Optimal Target Setting/ Optimization/Max-Minimization	Inventory Management Systems/ Storage/ Delivery	Assembly Systems/ Flexible Manufacturing/ Production Sequencing	Knowledge Management	Complexity/ Robust Design/ Sensitivity Analysis	Dynamic Simulation/ Simulation	Extreme Value Theory (EVT)
Klum, G. (1977)				X							
Nair, S.K., Thakur, L.S., & Wen, K-W (1995)		X	X	X	X			X	X	X	
Barish, N.N. (1963)								X			
Naddor, E. (1975)				X	X	X					
So, K.C., & Scott, C.H (1994)			X	X	X	X	X			X	
Klincewicz, J.G. (1990)				X	X	X					
Bai, H., & Kwong, C.K (2003)			X	X	X						
Cooper, A.B., Georgiopoulos, P., Kim H.M., & Papalambros, P. Y. (2006)		X		X	X						
Kwok, K.S., Driessen, B.I., & Phillips, C.A (2002)				X	X				X		
Huang, G.Q., Qu, T., Cheung, D. W.L., & Liang, L. (2006)					X						X
Caleb Li, M.H (2004)	X		X		X		X		X		

Literature Review Variables											
Reference	Truncated Assembly/ Truncation	Selective Assembly	Tolerance/ Parameter Design/ Quality Control	Heuristics/ Frameworks/ Heuristic Solutions	Optimal Target Setting/ Optimization/Max-Minimization	Inventory Management Systems/ Storage/ Delivery	Assembly Systems/ Flexible Manufacturing/ Production Sequencing	Knowledge Management	Complexity/ Robust Design/ Sensitivity Analysis	Dynamic Simulation/ Simulation	Extreme Value Theory (EVT)
Ohtsubo, Y. (2004)				X	X						
Kim, H.M., Michelena, N.F., & Papalambros, P.Y (2003)			X		X					X	
Krzysztofowicz, R. (1990)					X				X		
Moorhead, P.R., & Wu, C.F.J. (1998)			X	X	X				X		
Musa, R., Sturges, R. H., & Chen, F. F. (2006)			X	X	X		X			X	
Musa, R., & Chen, F.F. (undated)		X		X	X					X	
Musa, R., Chen, F.F., & Ghoniem, A. S. (undated)				X	X		X			X	
Musa, R., & Chen, F.F. (2008)		X	X	X	X					X	
Huang, S.H., Liu, Q., & Musa, R. (2004)		X	X	X	X		X			X	
Zhang, W.Z., Wang, G.X., Cheok, B.T., & Nee, A.Y.C. (2003)				X			X	X			
Meller, R.D., & Bozer, Y.A (1996)				X	X						

Literature Review Variables

Reference	Truncated Assembly/ Truncation	Selective Assembly	Tolerance/ Parameter Design/ Quality Control	Heuristics/ Frameworks/ Heuristic Solutions	Optimal Target Setting/ Optimization/Max-Minimization	Inventory Management Systems/ Storage/ Delivery	Assembly Systems/ Flexible Manufacturing/ Production Sequencing	Knowledge Management	Complexity/ Robust Design/ Sensitivity Analysis	Dynamic Simulation/ Simulation	Extreme Value Theory (EVT)
Kwon, H-M, Kim, K-J, & Chandra, M. J. (1999)	X	X	X		X			X			
De Fazio, T.L., Rhee, S.J., & Whitney, D.E. (1999)				X			X				
Kannan, S.M., Jayabalan, V., & Jeevanantham, K. (2003)		X		X	X						
Ponnambalam, S.G., Aravindan, P., & Rao, M.S. (2003)				X	X		X				
Kannan, S.M., & Jayabalan, V. (2001)	X	X		X					X		
Pugh, G.A. (1986)	X	X					X				
Das, S.K., & Sarin, S.C. (1988)				X		X	X			X	
Cittolin, A. (1997)				X			X				
Pugh, G.A. (1992)	X	X								X	
Mansoor, E.M. (1961)		X	X				X				
Desmond, D. J., & Setty, C.A. (1961)		X	X	X							

Literature Review Variables											
Reference	Truncated Assembly/ Truncation	Selective Assembly	Tolerance/ Parameter Design/ Quality Control	Heuristics/ Frameworks/ Heuristic Solutions	Optimal Target Setting/ Optimization/Max-Minimization	Inventory Management Systems/ Storage/ Delivery	Assembly Systems/ Flexible Manufacturing/ Production Sequencing	Knowledge Management	Complexity/ Robust Design/ Sensitivity Analysis	Dynamic Simulation/ Simulation	Extreme Value Theory (EVT)
Whitney, D. E. (2006)			X				X		X		
Lee, S., & Shin, Y.G. (1990)				X	X		X				
Lee, B., & Saitou, K. (2007)				X	X		X		X	X	
Agard, B., & Kusiak, A. (2004)		X		X	X		X	X			
Abe, S., Murayama, T., Oba, F., & Narutaki, N. (1999)		X		X	X		X				
Lee, S. (1994)		X			X		X				
Bohachevsky, I.O., Johnson, M.E., & Stein, M.L. (1986)			X	X	X				X		
Sanderson, A.C. (1997)			X		X		X				
Phoomboplab, T., & Ceglarek, D. (2007)		X	X	X		X	X			X	
Gutierrez, G.J., Hausman, W.H., & Lee, H.L. (1995)			X	X	X		X			X	
Breedis, J.B. (2001)			X		X					X	

Literature Review Variables											
Reference	Truncated Assembly/ Truncation	Selective Assembly	Tolerance/ Parameter Design/ Quality Control	Heuristics/ Frameworks/ Heuristic Solutions	Optimal Target Setting/ Optimization/Max-Minimization	Inventory Management Systems/ Storage/ Delivery	Assembly Systems/ Flexible Manufacturing/ Production Sequencing	Knowledge Management	Complexity/ Robust Design/ Sensitivity Analysis	Dynamic Simulation/ Simulation	Extreme Value Theory (EVT)
Lozano, S., Adenso-Diaz, B., Eguia, I., & Onieva, L. (1999)				X							
Salhi, S. (1997)				X	X						
Puelz, A.V. (2002)					X					X	
Punnen, A.P., & Aneja, Y.P. (1995)				X							
Rochat, Y., & Semet, F. (1994)				X	X	X					
Gendreau, M., Laporte, G., & Séguin, R. (1996)				X	X	X					
Gendreau, M., Hertz, A., & Laporte, G. (1994)	X			X		X					
Logendran, R., & Sonthinen, A. (1997)			X	X	X	X	X				
Nussbaum, M., Sepúlveda, M., Singer, M., & Laval, E. (1998)				X	X					X	
Park, K., Kang, S., & Park, S. (1996)				X		X	X				

Literature Review Variables

Reference	Truncated Assembly/ Truncation	Selective Assembly	Tolerance/ Parameter Design/ Quality Control	Heuristics/ Frameworks/ Heuristic Solutions	Optimal Target Setting/ Optimization/Max-Minimization	Inventory Management Systems/ Storage/ Delivery	Assembly Systems/ Flexible Manufacturing/ Production Sequencing	Knowledge Management	Complexity/ Robust Design/ Sensitivity Analysis	Dynamic Simulation/ Simulation	Extreme Value Theory (EVT)
Kozan, E. (2000)				X	X	X					
Glover, F. (1990)				X	X						
Markland, R.E. (1990)				X							
Grüne-Yanoff, T., & Weirich, P. (2010)								X			X
Wilson, I.D., & Roach, P.A. (2000)				X	X	X	X				
Consiglio, A., & Zenios, S. (1999)			X	X	X					X	
Moccellin, J.V., & Nagano, M.S. (1998)				X	X		X			X	
Bastes, R.A., Buck, R.J., Riccomagno. E., & Wynn, H.P. (1996)				X					X		
Baykasoğlu. A. (2001)				X				X			
Dell'Amico, M. (1996)	X			X			X		X		
Patterson, R.A., & Rolland, E. (2002)				X				X		X	

Literature Review Variables											
Reference	**Truncated Assembly/ Truncation**	**Selective Assembly**	**Tolerance/ Parameter Design/ Quality Control**	**Heuristics/ Frameworks/ Heuristic Solutions**	**Optimal Target Setting/ Optimization/Max-Minimization**	**Inventory Management Systems/ Storage/ Delivery**	**Assembly Systems/ Flexible Manufacturing/ Production Sequencing**	**Knowledge Management**	**Complexity/ Robust Design/ Sensitivity Analysis**	**Dynamic Simulation/ Simulation**	**Extreme Value Theory (EVT)**
Chiang, W-C., Kouvelis, P., & Urban, T.L. (2002)				X	X	X	X				
Ramirez-Beltran, N.D. (1995)				X	X						
Agarwal, C.C., Orlin, J.B., & Tai, R.P. (1997)				X	X			X	X		
Bracker, J.S., & Pearson, J.N. (1986)				X					X		
Jun, J.B., Jacobson, S.H., & Swisher, J.R. (1999)				X	X		X	X		X	
Liu, L., & Chang, K.L. (1997)			X		X	X					
Mazzola, J.B., & Schantz, R.H. (1995)				X	X	X					
Ohlemüller, M. (1997)				X	X						
Bisgaard, S. (1997)		X	X				X				
Ostermeier, M. (2003)	X										
Bradley, D.M., & Gupta, R.C. (2002)	X										

Literature Review Variables											
Reference	Truncated Assembly/ Truncation	Selective Assembly	Tolerance/ Parameter Design/ Quality Control	Heuristics/ Frameworks/ Heuristic Solutions	Optimal Target Setting/ Optimization/Max-Minimization	Inventory Management Systems/ Storage/ Delivery	Assembly Systems/ Flexible Manufacturing/ Production Sequencing	Knowledge Management	Complexity/ Robust Design/ Sensitivity Analysis	Dynamic Simulation/ Simulation	Extreme Value Theory (EVT)
Dhrymes, P. J. (2005)	X		X		X					X	
Yang, C., Gui, W., Kong, L., & Wang, Y. (2009)				X	X			X		X	
Grieme, N. (2011)											
Zhu, Q., & Oommen, J. (1997)				X	X	X	X	X			
Bouchard, B., Elie, R., & Imbert, C. (2010)				X	X					X	
Simpson, D. P. (2008)	X		X	X	X					X	
Asvadurov, S., Druskin, V., Guddati, M., & Knizhnerman, L. (2003)	X			X	X						
Xiaoping, B. & Jingjing, M. (2009)				X		X	X		X	X	
Mease, D., Nair, V., & Sudjianto, A. (2004)		X		X	X	X				X	
Xiaoping, B. & Jingjing, M. (2009)					X	X					X
Glover, F., & Laguna, M (undated)				X	X						

	Literature Review Variables										
Reference	**Truncated Assembly/ Truncation**	**Selective Assembly**	**Tolerance/ Parameter Design/ Quality Control**	**Heuristics/ Frameworks/ Heuristic Solutions**	**Optimal Target Setting/ Optimization/Max-Minimization**	**Inventory Management Systems/ Storage/ Delivery**	**Assembly Systems/ Flexible Manufacturing/ Production Sequencing**	**Knowledge Management**	**Complexity/ Robust Design/ Sensitivity Analysis**	**Dynamic Simulation/ Simulation**	**Extreme Value Theory (EVT)**
Caleb Li, M.-H., & Chou, C.-Y.(2001)	X		X	X	X			X			
Johnson, A., & Thomopoulos, N.T. (undated)	X				X						
Khasawneh, M.T., Bowling, S.R., Kaewkuekool. S., & Cho, B.R. (2005)	X										
Johnson, A.C., & Thomopoulos, N.T. (undated)	X										
Khasawneh, M.T., Bowling, S.R., Kaewkuekool. S., & Cho, B.R. (2005)	X										
Raschke, M. (2012)	X									X	X
Blanchet, J., & Liu, J. (2012)										X	X
Kuwahara, H., & Mura, I. (2008)										X	X
Drees, H., de Haan. L., & Li, D. (2006)											X
Stoyanov, S., & Rachev, S. (2008)	X									X	X

Literature Review Variables											
Reference	Truncated Assembly/ Truncation	Selective Assembly	Tolerance/ Parameter Design/ Quality Control	Heuristics/ Frameworks/ Heuristic Solutions	Optimal Target Setting/ Optimization/Max-Minimization	Inventory Management Systems/ Storage/ Delivery	Assembly Systems/ Flexible Manufacturing/ Production Sequencing	Knowledge Management	Complexity/ Robust Design/ Sensitivity Analysis	Dynamic Simulation/ Simulation	Extreme Value Theory (EVT)
Chavez-Demoulin, V., & Roehrl. A. (2004)											X
Peng, L., & Qi, Y. (2009)	X										X
Bermudez, P.D.Z., & Kotz, S. (2010)	X										X
Bermudez, P.D.Z., & Kotz, S. (2010)	X		X								X
Horrace, W.C. (2005)	X										
Brazauskas, V., & Kleefeld, A. (2009)			X						X	X	X
Carpinteri, A., Cometti. P., & Puzzi. S. (2005)	X		X								X
Brooks, C., Clare, A.D., Dalle Molle. J.W., & Persand. G. (2005)			X							X	X
Diebolt. J., Guillou. A., & Rached. I. (2005)			X				X				X
Castillo, E., Hadi. A., Balakrishnan. N., Sarabia. J. (2005)	X		X							X	X

Literature Review Variables											
Reference	Truncated Assembly/ Truncation	Selective Assembly	Tolerance/ Parameter Design/ Quality Control	Heuristics/ Frameworks/ Heuristic Solutions	Optimal Target Setting/ Optimization/Max-Minimization	Inventory Management Systems/ Storage/ Delivery	Assembly Systems/ Flexible Manufacturing/ Production Sequencing	Knowledge Management	Complexity/ Robust Design/ Sensitivity Analysis	Dynamic Simulation/ Simulation	Extreme Value Theory (EVT)
Michalewicz, Z., & Fogel, D. (1998)				X	X						
Whitney, D. E. (2004)			X				X				
Billingsley, P. (1995)											
Sheffield, S. (2011)											
Abadir, K., & Magdalinos., T. (2002)	X										
Shephard, N.G. (1992)	X										
Kawata, T. (1969)											
Barnadic, M., & Candal, J. (2012)	X										

APPENDIX G: CATEGORIZATION AND COMPARATIVE ANALYSIS

This appendix provides a categorization and comparative analysis of the literature review variables for a sample set of data for the subject dissertation. It should be noted that not all references were utilized in this review. For example, general calculation references, definitions, duplicative or other references were excluded from this review. All review variables analyzed are included in Table G.7. Refer to Chapter 2 and Appendix E for additional information.

Table G.1 – Comparative Review Results Heuristic Type/Benchmark Method

Benchmark Method / Heuristic Type	An Example, Case Study, or Design Spec.	Heuristics or Other Methods	Historical or Collected Data	Simulation Data or Study	Mathematical Formulation, Other Optimization Procedure or Result	Inconclusive, Not Performed, or Not Applicable	Grand Total
Algorithm	4%	6%	1%	2%	2%	0%	15%
Heuristic Procedure	10%	9%	1%	5%	1%	3%	28%
Search Heuristic	2%	9%	2%	0%	0%	2%	15%
Simulated Annealing	0%	0%	0%	0%	1%	0%	1%
Inconclusive or Not Applicable	6%	2%	1%	9%	10%	12%	40%
Linear Programming	0%	1%	0%	0%	0%	0%	1%
Grand Total	23%	27%	4%	16%	14%	17%	127

Table G. 2 - Comparative Review Results Heuristic Type/Test Method

Testing (PM) Heuristic Type	Commentary	Comparative Analysis	Correlation	"Good" Solutions	Efficiency Evaluation	Empirical Analysis	Error Ratio	Experiment	N/A or Inc.	Math Model or Computation	Simulation	Grand Total
Algorithm	0%	8%	0%	3%	1%	0%	0%	1%	0%	1%	2%	15%
Heuristic Procedure	2%	7%	1%	3%	5%	0%	0%	2%	1%	7%	1%	28%
Search Heuristic	0%	4%	0%	4%	4%	0%	1%	0%	0%	2%	1%	15%
Simulated Annealing	0%	0%	0%	0%	1%	0%	0%	0%	0%	0%	0%	1%
Inconclusive or Not Applicable	0%	7%	0%	2%	2%	4%	1%	0%	7%	16%	2%	40%
Linear Programming	0%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%	1%
Grand Total	2%	27%	1%	12%	12%	4%	2%	3%	8%	25%	6%	127

Table G.3 - Comparative Review Results Sel. Assy, Heuristic Type, Data Source

Selective Assembly	Heuristic Type	Data Source						
		Data Generated	Empirical Data	Example Data	Historical	Inconclusive or Not Applicable	Sampled Data	Grand Total
Selective Assembly Identified	Algorithm	2%	0%	2%	0%	0%	0%	3%
	Heuristic Procedure	1%	1%	4%	0%	0%	0%	6%
	Search Heuristic	1%	0%	0%	0%	0%	0%	1%
	Inconclusive or Not Applicable	1%	0%	3%	1%	0%	0%	5%
	Total	4%	1%	9%	1%	0%	0%	14%
Selective Assembly Not Identified	Algorithm	3%	1%	6%	2%	1%	0%	12%
	Heuristic Procedure	3%	2%	9%	2%	4%	2%	23%
	Search Heuristic	4%	3%	4%	2%	1%	1%	14%
	Simulated Annealing	1%	0%	0%	0%	0%	0%	1%
	Inconclusive or Not Applicable	9%	2%	6%	2%	17%	0%	35%
	Linear Programming	1%	0%	0%	0%	0%	0%	1%
	Total	20%	8%	25%	7%	23%	2%	86%
Grand Total		24%	9%	34%	8%	23%	2%	127

Table G.4 - Comparative Review Results Test Method

Testing (Primary Method)	Truncation Assy / Truncation	No Truncation Assy /Truncation	Grand Total
Commentary	0%	2%	2%
Comparative Analysis	5%	22%	27%
Correlations	0%	1%	1%
Demonstration of "Good" Solution	1%	11%	12%
Efficiency improvement	1%	11%	12%
Empirical	3%	1%	4%
Error Ratio	1%	1%	2%
Experiment	0%	3%	3%
Inconclusive or Not Applicable	2%	6%	8%
Mathematical Model or Computational Result	9%	16%	25%
Simulation	1%	5%	6%
Grand Total	22%	78%	127

Table G.5 – Comparative Review Truncation and Data Source w/Heuristic

Truncation & Data Source	Heuristic Identified	Heuristic Type Not Identified	Grand Total
Data Generated	5%	20%	24%
Empirical Data	1%	8%	9%
Example Data	7%	27%	34%
Historical	0%	8%	8%
Inconclusive or Not Applicable	9%	13%	23%
Sampled Data	0%	2%	2%
Grand Total	22%	78%	127

Table G.6 – Comparative Review Optimization Techniques

Optimization Technique	Total
Analytical Target Cascading	1%
Analytical Target Setting	1%
Attribute Level Driven	1%
Function	1%
Heuristic Based	11%
Highly Optimized Tolerance (HOT)	1%
Inconclusive	2%
Keifer-Wolfowitz Optimization Procedure	1%
Linear	1%
Markovian queuing network model	1%
Mathematical Formulation	4%
Mixed Integer Programming	1%
Not Applicable	43%
Optimization Model	1%
Optimization Model	23%
Partitioned Decision Making Model	1%
Pattern Enumeration	2%
Semi-Markovian model generating such an optimal (deterministic) routing scheme	1%
Simplex Method (or variant)	1%
Simulated Annealing Optimization	1%
Stochastic Model	1%
Target Setting	1%
Utility Maximization through Criteria Weighting	1%
Value iteration and policy improvement methods	1%
Weighting	1%
(blank)	1%
Grand Total	127

Table G.7 – Categorization Table from Comparative Review

Categorization & Other Specifics								
Reference	Heuristic Type	Optimization Technique	Focus	Benchmarking Method	Data Source	Other	Testing (Primary Method)	Remarks
Carlson, J.M., & Doyle, J. (2002)	Algorithm	Highly Optimized Tolerance (HOT)	Hot Method	Historical or Collected Data	Historical	Not Applicable	Simulation	Review: Approach Driven Optimal Compression Algorithm (similar to)
Malakooti, B.B. (1994)	Inconclusive or Not Applicable	Optimization Model	Assembly Line Buffering	Heuristics or Other Methods	Historical	9 Tests Data was not generated	Efficiency improvement	Compared with other methods using Computational Experiments
Park M.W., & Kim Y.D. (1999)	Algorithm	Utility Maximization through Criteria Weighting	Assembly systems operating on make-to-order basis	Heuristics or Other Methods	Data Generated	Used an Additive Utility Function	Comparative Analysis	Network Based Algorithm Compared with other methods using Computational Experiments
Pourbabai, B. (1992)	Algorithm	Optimization Model	Identification of minimum required local storages.	Simulation Data or Study	Data Generated	"Approximated Solution",	Comparative Analysis	Review: Approach Driven for Solution Algorithm and Application Solution Algorithm
Pourbabai, B. (1989)	Inconclusive or Not Applicable	Markovian queuing network model	Design of an assembly system, linked by a transporter for just-in-time (JIT) and storage minimum	Simulation Data or Study	Data Generated	"Approximated Solution"	Simulation	Review: Approach Driven for Solution Algorithm and Application
Cao, X., & Ho, Y. (1987)	Algorithm	Keifer-Wolfowitz Optimization Procedure	Observations noted as part of the simulation results. Explicit random variable dependencies. Stochastic network	Mathematical Formulation, Other Optimization Procedure or Result	Data Generated	"Crude" Monte Carlo Estimate	Comparative Analysis	Review: Approach Driven for Solution Algorithm and Application Perturbation analysis of discrete event dynamic system Comparison to Kiefer-Wolfowitz Optimization Procedure
Seidmann, A., & Tenenbaum, A. (1994)	Heuristic Procedure	Semi-Markovian model generating such an optimal (deterministic) routing scheme	Evaluates modeling approaches to part-routing policies	Mathematical Formulation, Other Optimization Procedure or Result	Inconclusive or Not Applicable	"Near" optimal FMS performance results	Comparative Analysis	Review: Approach Driven for Solution Algorithm and Application Closed Loop Heuristic Policies Probabilistic Shortest Queue (PSQ) Scheduling Heuristic

Categorization & Other Specifics								
Reference	Heuristic Type	Optimization Technique	Focus	Benchmarking Method	Data Source	Other	Testing (Primary Method)	Remarks
Naddor, E. (1975)	Heuristic Procedure	Mathematical Formulation	Heuristic decision rules in probabilistic inventory systems requiring only the knowledge of the mean and standard deviation of demand, probability of no demand, carrying and replenishing costs, desired availability, and lead-time	An Example, Case Study, or Design Specification	Example Data	Not Applicable	Mathematical Model or Computational Result	Heuristic Decision Policy for probabilistic inventory systems Numerical results are given to illustrate the application of the rules
So, K.C., & Scott, C.H. (1994)	Algorithm	Optimization Model	Studied a production control model for a product comprised of matching components	Heuristics or Other Methods	Empirical Data	Not Applicable	Comparative Analysis	Greedy Heuristic Sequencing Rule
Klincewicz, J.G. (1990)	Heuristic Procedure	Linear	Heuristic Methods to solve freight transport problem and cost facility location problem.	Heuristics or Other Methods	Historical	Transport Problem	Demonstration of "Good" Solution	Empirical Testing, Compared with other methods using Computational Experiments
Bai, H., & Kwong, C.K. (2003)	Algorithm	Optimization Model	An "inexact" genetic algorithm approach and model were used for determination of target values for engineering requirements in Quality Function Deployment	Heuristics or Other Methods	Example Data	Not Applicable	Mathematical Model or Computational Result	Review: Approach Driven Genetic Algorithm Interactive approach to obtain inexact optimal target values
Cooper, A.B., Georgiopoulos, P., Kim H.M., & Papalambros, P. Y. (2006)	Heuristic Procedure	Partitioned Decision Making Model	A partitioned decision-making process is modeled to demonstrate how decisions at the top-level of the hierarchy analytical target setting are expected results from the lower level analytic target cascading.	An Example, Case Study, or Design Specification	Example Data	Measures of Effectiveness of Analytical Target Setting	Comparative Analysis	Example of deployment related to decision making and product development Heuristic Procedural Framework
Kwok, K.S., Driessen, B.J., & Phillips, C.A. (2002)	Algorithm	Optimization Model	Focus is on a maximum utilization problem of a set of mobile robots with limited sensor-range capabilities and limited travel distances (initially at random positions).	Mathematical Formulation, Other Optimization Procedure or Result	Example Data	Transport Problem Max Opt.	Comparative Analysis	Network Optimization Model Assignment Algorithm Compared with other optimization procedures

Categorization & Other Specifics								
Reference	**Heuristic Type**	**Optimization Technique**	**Focus**	**Benchmarking Method**	**Data Source**	**Other**	**Testing (Primary Method)**	**Remarks**
Huang, G.Q., Qu, T., Cheung, D. W.L., & Liang, L. (2006)	Inconclusive or Not Applicable	Analytical Target Cascading	Utilized a multi-agent system to solve for optimal design using analytical target cascading	An Example, Case Study, or Design Specification	Example Data	Set targets to extreme lower and upper bounds (normally 0 and ∞) for testing the boundaries of decision variables in minimization and maximization	Demonstration of "Good" Solution	Hierarchical system modeling Attribute Level Driven
Caleb Li, M.H. (2004)	Inconclusive or Not Applicable	Optimization Model	On optimal manufacturing settings to minimize quality loss for the identified production system	An Example, Case Study, or Design Specification	Example Data	Not Applicable	Error Ratio	Truncated asymmetrical quadratic loss function. Minimize expected quality loss
Ohtsubo, Y. (2004)	Heuristic Procedure	Value iteration and policy improvement methods	Evaluation of risk minimizing problems in undiscounted Markov decisions processes with a target set. Also utilizes infinite horizon cases to show that an optimal value function is a unique solution to an optimality equation	An Example, Case Study, or Design Specification	Example Data	Min Opt	Mathematical Model or Computational Result	Risk Minimization Policy Improvement Methods Policy Space Procedure Optimal Value Function is unique to optimality
Kim, H.M., Michelena, N.F., & Papalambros, P.Y (2003)	Inconclusive or Not Applicable	Optimization Model	Used target cascading to model a multi-level optimization problem using design targets (cascaded to lower levels) by partitioning their problem into small sub problems	An Example, Case Study, or Design Specification	Example Data	Analytical Example and Application	Comparative Analysis	Model Using Analytical Target Cascading
Krzysztofowicz, R. (1990)	Inconclusive or Not Applicable	Target Setting	Provides an critique and application extension of a decision model	Inconclusive, Not Performed, or Not Applicable	Inconclusive or Not Applicable	Exp. Utility Function	Mathematical Model or Computational Result	Decision Model Critique
Moorhead, P.R., & Wu, C.F.J. (1998)	Heuristic Procedure	Optimization Model	Developed a modeling and data-analysis strategy for a general loss function, in which the quality characteristic follows a location-scale model.	Heuristics or Other Methods	Empirical Data	Min Opt	Experiment	Extends a nominal-the-best parameter design to include loss functions of a much more general nature Taguchi Procedure Compared with other methods using Computational Experiments

Categorization & Other Specifics								
Reference	Heuristic Type	Optimization Technique	Focus	Benchmarking Method	Data Source	Other	Testing (Primary Method)	Remarks
Musa, R., Sturges, R. H., & Chen, F. F. (2006)	Algorithm	Optimization Model	Develops a methodology for inspection planning for a new product based on CAD data and simulation.	Heuristics or Other Methods	Data Generated	Inspection Plans	Simulation	Methodology for inspection planning for a new product based on CAD data Genetic Algorithm
Musa, R., & Chen, F.F. (undated)	Algorithm	Optimization Model	Revision of a previous model for inspection planning in agile production environment using Dynamic Throughput Maximization (DTM).	Simulation Data or Study	Data Generated	Solve Numerical Examples	Comparative Analysis	Meta-heuristic Algorithm Simulated Annealing Algorithm Compares results with and without implementation of the proposed integration
Musa, R., Chen, F.F., & Ghoniem, A. S. (undated)	Heuristic Procedure	Optimization Model	Development of a mathematical part matching model for variation reduction	Simulation Data or Study	Data Generated	Inspection Plans	Mathematical Model or Computational Result	Mathematical Model to match parts to achieve minimal variation using 3-rule heuristic procedure
Musa, R., & Chen, F.F. (2008)	Algorithm	Optimization Model	Exploits data of measured features of a batch of subassembly data to reduce variation in the final assembly to maximize the rolled yield throughput.	Simulation Data or Study	Data Generated	Maximum Optimization used commercial optimization solvers (e.g. opt. models)	Demonstration of "Good" Solution	Five Algorithms (simple greedy, simulated annealing, ant colony optimization) Greedy Algorithm Simulated Annealing
Huang, S.H., Lin, Q., & Musa, R. (2004)	Heuristic Procedure	Pattern Enumeration	Proposed a method for process plan evaluation to provide rapid evaluation for process plan decision making	Simulation Data or Study	Empirical Data	Assignment of Production Tolerances as part of heuristic procedural approach	Comparative Analysis	Monte Carlo Simulation by prediction of machining tolerances

Categorization & Other Specifics								
Reference	**Heuristic Type**	**Optimization Technique**	**Focus**	**Benchmarking Method**	**Data Source**	**Other**	**Testing (Primary Method)**	**Remarks**
Zhang, W.Z., Wang, G.X., Cheok, B.T., & Nee, A.Y.C. (2003)	Heuristic Procedure	Not Applicable	Proposed a knowledge-based selection procedure/rules (e.g. heuristic) to provide a unique name based search mechanism geared toward component reuse (i.e. reapplication)	Inconclusive, Not Performed, or Not Applicable	Inconclusive or Not Applicable	Not Applicable	Inconclusive or Not Applicable	Rules based selection procedures Knowledge Based Selection Procedure
Meller, R.D., & Bozer, Y.A (1996)	Algorithm	Simulated Annealing Optimization	Presented a heuristic (i.e simulated annealing) for facility layout	Heuristics or Other Methods	Historical	Examples of real life facility layout problems Min Opt	Demonstration of "Good" Solution	Improvement type layout algorithm Simulated Annealing Algorithm
Kwon, H-M, Kim, K-I, & Chandra, M. J. (1999)	Inconclusive or Not Applicable	Optimization Model	On product clearance the focus of this research dealt with component characteristics of a normal distribution with equal variance	Mathematical Formulation, Other Optimization Procedure or Result	Example Data	Cost Model	Mathematical Model or Computational Result	Component characteristics. Assumed to be normally distributed Formulas for rejection and mating
De Fazio, T.L., Rhee, S.I., & Whitney, D.E (1999)	Algorithm	Not Applicable	Detailed subassembly partitioning based on criterion based searches and identified genetic algorithm search techniques for us in assembly sequencing	An Example, Case Study, or Design Specification	Example Data	Search Heuristic	Efficiency improvement	Design For Assembly Method Assembly Sequence Analysis Criterion Based Searches Genetic Algorithm
Kannan, S.M., Jayabalan, V., & Jeevanantham, K (2003)	Algorithm	Heuristic Based	Utilized genetic algorithms to find the best combination of the selective assembly groups necessary to minimize assembly variation and focused on linear assembly	Mathematical Formulation, Other Optimization Procedure or Result	Example Data	Genetic Algorithms Min Opt	Demonstration of "Good" Solution	Method for selective grouping Genetic Algorithm

Categorization & Other Specifics								
Reference	Heuristic Type	Optimization Technique	Focus	Benchmarking Method	Data Source	Other	Testing (Primary Method)	Remarks
Ponnambalam, S.G., Aravindan, P., & Rao, M.S. (2003)	Algorithm	Heuristic Based	Investigation of genetic algorithms and also performed a comparison of exiting vs. proposed GA's by consideration of variation at multiple assembly levels (e.g. raw materials, product, subassembly, etc.)	Heuristics or Other Methods	Example Data	Genetic Algorithms	Experiment	Heuristic Comparison Sequencing Problem for Mixed Model Assembly Lines Genetic Algorithm
Kannan, S.M., & Jayabalan, V. (2001)	Heuristic Procedure	Not Applicable	Proposed a method for lot partitioning using selective assembly groups	Heuristics or Other Methods	Example Data	Lot Partitioning to aid in selective assembly	Comparative Analysis	Complex assy's with 3 mating parts Compared with other methods using Computational Experiments
Pugh, G.A (1986)	Inconclusive or Not Applicable	Not Applicable	Introduces the idea of partitioning a component population into groups prior to random assembly	Inconclusive, Not Performed, or Not Applicable	Example Data	Group Partitioning	Inconclusive or Not Applicable	Early initial framework of method
Das, S.K., & Sarin, S.C. (1988)	Heuristic Procedure	Not Applicable	Used a dynamic programming approach along with a heuristic procedure to address part arrival dates in a multi-job stochastic assembly system	Heuristics or Other Methods	Data Generated	Algorithm	Comparative Analysis	Heuristic Procedure Stochastic vs Deterministic
Cittolin, A (1997)	Heuristic Procedure	Not Applicable	Used filter and assembly sequencing methods to group and sequence assembly combination	Heuristics or Other Methods	Example Data	Assy Sequencing and used filtering methods	Experiment	Method Comparison
Pugh, G.A (1992)	Inconclusive or Not Applicable	Not Applicable	Discusses the systematical truncation and normal distributions in addressing component distributions	Simulation Data or Study	Data Generated	Assembly Procedure	Simulation	Truncation based on variances for selective assembly
Mansoor, E.M. (1961)	Inconclusive or Not Applicable	Not Applicable	Selective Assembly focus with application to a case history of a piston-cylinder fit and problem solving for a mis-matching issue	An Example, Case Study, or Design Specification	Example Data	Example of a piston	Mathematical Model or Computational Result	An application of selective assembly

Categorization & Other Specifics								
Reference	Heuristic Type	Optimization Technique	Focus	Benchmarking Method	Data Source	Other	Testing (Primary Method)	Remarks
Desmond, D. J., & Setty, C.A (1961)	Heuristic Procedure	Not Applicable	Proposed a method such that components should be balanced by quantities rather than by size.	An Example, Case Study, or Design Specification	Example Data	Numerical Example	Mathematical Model or Computational Result	Method proposed balancing quantities by quantities rather than by size
Whitney, D. E. (2006)	Inconclusive or Not Applicable	Not Applicable	Identifies key characteristics associated with mechanical assemblies, data flow chains and tolerance analysis. This research focuses on utilizing key characteristics for conveying design intent.	Inconclusive, Not Performed, or Not Applicable	Inconclusive or Not Applicable	Whitney covers assembly techniques and research supports the gap analysis and application of this dissertation focus.	Inconclusive or Not Applicable	Role of Key Characteristics in Mech Assy's
Lee, S., & Shin, Y.G. (1990)	Heuristic Procedure	Mathematical Formulation	Presented a method for the automatic determination of assembly partial orders from a liaison graph representation	An Example, Case Study, or Design Specification	Example Data	Method provides a new approach to assy via temp relationships Max opt	Experiment	Method for automatic determination of assy partial order from a liaison graph rep of an assy through ext. of preferred assy
Lee, B., & Saitou, K. (2007)	Algorithm	Optimization Model	Presents a systematic method that decomposes product geometry at an early stage of design, selects joint types, and generates subassembly partitioning to achieve the adjustment of the critical dimensions during assembly processes	An Example, Case Study, or Design Specification	Example Data	Key Characteristics used in Opt. Model	Comparative Analysis	Genetic Algorithm generates candidate joint assemblies based on joint library specific for an application domain Genetic Algorithm
Agard, B., & Kusiak, A (2004)	Heuristic Procedure	Optimization Model	Data mining was implemented as part of the subassembly selection process and utilized and integer programming model and applied a heuristic algorithm for the selection of the optimal subassembly structure.	An Example, Case Study, or Design Specification	Example Data	Example and automotive case study	Mathematical Model or Computational Result	Data mining algorithm
Abe, S., Murayama, T., Oba, F., & Narutaki, N. (1999)	Algorithm	Inconclusive	Focus on heuristics to aid in the generation of assembly sequencing	An Example, Case Study, or Design Specification	Example Data	GA and heuristics to select the expedient candidates	Demonstration of "Good" Solution	Method for reduction of verification time Genetic Algorithm

Categorization & Other Specifics								
Reference	**Heuristic Type**	**Optimization Technique**	**Focus**	**Benchmarking Method**	**Data Source**	**Other**	**Testing (Primary Method)**	**Remarks**
Lee, S. (1994)	Inconclusive or Not Applicable	Weighting	Presents a method for the automatic generation of assembly sequences from a liaison graph representation of an assembly through the recursive decomposition of assembly into subassemblies	An Example, Case Study, or Design Specification	Historical	Correspondence	Comparative Analysis	Data Source: CAD database
Bohachevsky, I.O., Johnson, M.E., & Stein, M.L. (1986)	Simulated Annealing	Function	Described generalized simulated annealing for the "optimization of functions having many local extrema" and methods for improved optimums of other problems	Mathematical Formulation, Other Optimization Procedure or Result	Data Generated	Sensitivity Analysis	Efficiency improvement	Generalized Simulated Annealing
Sanderson, AC. (1997)	Algorithm	Optimization Model	Used a tolerance model to estimate part configurations based on maximum likelihood using filter algorithm	An Example, Case Study, or Design Specification	Example Data	Tolerance Model: Max Opt	Comparative Analysis	App. Of tech to int. subassemblies used to evaluate assy spec. steps and discriminate at assy seq. plans Filter Algorithm
Phoomboplab, T., & Ceglarek, D. (2007)	Heuristic Procedure	Not Applicable	Proposed a design synthesis framework for dimensional management of a multi-stage assembly system	Simulation Data or Study	Data Generated	Monte Carlo Simulation	Simulation	Dimensional Mgmt Method for process design configuration
Gutierrez, G.J., Hausman, W.H., & Lee, H.L. (1995)	Heuristic Procedure	Optimization Model	Identified a computationally infeasible dynamic programming formulation along with a myopic control procedure for general application to sorting and matching problems	Heuristics or Other Methods	Empirical Data	Mathematical Model	Efficiency improvement	Heuristic for sorting comp into class sizes Compared with other methods using Computational Experiments
Breadia, IB. (2001)	Inconclusive or Not Applicable	Optimization Model	Presented a simplified approach to subassembly design using Monte Carlo Analysis	Simulation Data or Study	Data Generated	Monte Carlo Simulation	Simulation	Monte Carlo Analysis

Categorization & Other Specifics								
Reference	Heuristic Type	Optimization Technique	Focus	Benchmarking Method	Data Source	Other	Testing (Primary Method)	Remarks
Lozano, S., Adenso-Diaz, B., Eguia, I., & Onieva, L. (1999)	Search Heuristic	Not Applicable	Used a Tabu search heuristic in a cellular manufacturing design and proposed systematically explored feasible machine cell configurations in part family determinations	Heuristics or Other Methods	Empirical Data	Benchmarked against 2 simulated annealing approaches	Demonstration of "Good" Solution	Tabu Search for manufacturing cell design
Salhi, S. (1997)	Heuristic Procedure	Heuristic Based	Developed a constructive heuristic for a location problem. The author tested the proposed heuristic against other location problem methods	Heuristics or Other Methods	Data Generated	Perturbation heuristic	Efficiency improvement	Tested against reputable methods on a class location problems Perturbation heuristic and heuristic procedure
Puelz, A.V. (2002)	Inconclusive or Not Applicable	Stochastic Model	Presented selection techniques for a dynamic model framework for decision makers	Simulation Data or Study	Data Generated	Max Opt	Empirical	Primarily Financial but provided relative information on a stochastic convergence model for consideration
Punnen, A.P., & Aneja, Y.P. (1995)	Search Heuristic	Not Applicable	Studied a resource-constrained assignment problem and developed a Tabu search heuristic	Historical or Collected Data	Example Data	Resource Constrained Assignment Problem	Efficiency improvement	Tabu Search uses strategic oscillation, randomized short-term memory and multiple start as a means of search diversification
Rochat, Y., & Semet, F. (1994)	Search Heuristic	Optimization Model	Evaluated a vehicle routing problem using two proposed heuristics to find a "good" solution	Heuristics or Other Methods	Example Data	Vehicle Routing Problem	Demonstration of "Good" Solution	2 heuristics methods. Fast straightforward insertion procedure and a method based on Tabu search techniques. Tabu Search heuristic Compared with other methods using Computational Experiments
Gendreau, M., Laporte, G., & Séguin, R. (1996)	Search Heuristic	Heuristic Based	Developed a Tabu search heuristic for a stochastic vehicle routing problem with random demands and probabilities	Heuristics or Other Methods	Historical	Comparisons against known optimal solutions	Error Ratio	Tabu Search Compared with other methods using Computational Experiments
Gendreau, M., Hertz, A., & Laporte, G. (1994)	Search Heuristic	Heuristic Based	Described a Tabu search heuristic for vehicle routing problem with various restrictions	Heuristics or Other Methods	Example Data	Numerical tests on a set of benchmark problems	Comparative Analysis	Tabu Search - Tabu route. A search heuristic for the vehicle routing problem with capacity and route length restrictions

Categorization & Other Specifics								
Reference	Heuristic Type	Optimization Technique	Focus	Benchmarking Method	Data Source	Other	Testing (Primary Method)	Remarks
Logendran, R., & Sonthinen, A. (1997)	Search Heuristic	Optimization Model	Developed a Tabu search heuristics and statistical experimentation to present both a "good" solution for solving a problem within a flexible manufacturing system. The model also identifies assumptions and a six part Tabu-search heuristic.	Heuristics or Other Methods	Example Data	Comparison of 6 different versions of Tabu search based heuristics	Comparative Analysis	Tabu Search
Nussbaum, M., Sepúlveda, M., Singer, M., & Laval, E. (1998)	Search Heuristic	Optimization Model	Approximation methodlogies for sequencing and resource allocation problems	An Example, Case Study, or Design Specification	Empirical Data	Approximation methods	Mathematical Model or Computational Result	Architecture solving sequencing
Park, K., Kang, S., & Park, S. (1996)	Algorithm	Not Applicable	Proposed an algorithm associated with integer programming formulation of a bandwidth packing problem. A heuristic was proposed and utilized a column generation technique as part of the algorithm. The authors further tested the algorithm using rnd pro	An Example, Case Study, or Design Specification	Example Data	Tested the proposed algorithm on some random problems Application to Inventory & Assy Systems	Comparative Analysis	Algorithm to solve integer programming formulation technique to solve the linear programming relaxation is proposed algorithm includes the column generation technique
Kozan, E. (2000)	Heuristic Procedure	Mathematical Formulation	Developed an analytical framework for the examination of inventory strategies for an assembly plant. The model addressed minimization strategy along with material management efficiency.	Historical or Collected Data	Historical	Eff increase by about 30% Material handling Strategies	Efficiency improvement	Heuristic is an implementation of a genetic algorithm
Glover, F. (1990)	Heuristic Procedure	Heuristic Based	Examined the characteristics of heuristic procedures used as frameworks for analyzing difficult optimization problems	Heuristics or Other Methods	Example Data	Restricts the search to avoid unproductive retracting of paths. Also explores target analysis a method for determining good decision rules to enable heuristics to perform more effectively	Comparative Analysis	AI Heuristic relative to optimization problems, Tabu search

Categorization & Other Specifics								
Reference	Heuristic Type	Optimization Technique	Focus	Benchmarking Method	Data Source	Other	Testing (Primary Method)	Remarks
Markland, R.E. (1990)	Heuristic Procedure	Not Applicable	Commentary on Heuristic procedure presented by glover in [68]	Heuristics or Other Methods	Example Data	Neural networks, simulated annealing, GA, and Tabu Search	Commentary	Discussion of {68}
Grime-Yanoff, T., & Weirich, P. (2010)	Inconclusive or Not Applicable	Not Applicable	Discusses the philosophical and epistemological implications of simulation, simulation representation, and policy decisions. The paper argued that simulation is "an important new tool for the social sciences"	Inconclusive, Not Performed, or Not Applicable	Inconclusive or Not Applicable	Philosophical approach to strategic methodology. Historical Analysis	Inconclusive or Not Applicable	Application of strategy research to any strategy
Wilson, I.D., & Roach, P.A (2000)	Heuristic Procedure	Optimization Model	Presents a methodology for automatic generation of computerized solutions to the container stowage problem.	An Example, Case Study, or Design Specification	Example Data	Heuristic rules were developed	Demonstration of "Good" Solution	Optimization using Tabu Search
Consiglio, A., & Zenios, S. (1999)	Search Heuristic	Optimization Model	Formulates a hierarchical optimization model	An Example, Case Study, or Design Specification	Empirical Data	Empirical results	Efficiency improvement	Design specification meeting the portfolio results Tabu Search
Moccellin, J.V., & Nagano, M.S. (1998)	Search Heuristic	Mathematical Formulation	Focused on flow shop sequencing (which has application to assembly sequencing). Methods to improve heuristics by obtaining an initial solution using the traveling salesman problem and then Tabu search methods to improve the initial solution	An Example, Case Study, or Design Specification	Empirical Data	Evaluates the relative performance of the procedures	Demonstration of "Good" Solution	Tabu Search
Bates, R.A., Buck, R.J., Riccomagno, E., & Wynn, H.P. (1996)	Heuristic Procedure	Not Applicable	Identified experimental design and modeling as part of optimization and sensitivity analysis of large systems	An Example, Case Study, or Design Specification	Empirical Data	optimization and sensitivity analysis	Demonstration of "Good" Solution	System decomposition using sparse matrix method, experimental design, and modeling

Categorization & Other Specifics								
Reference	Heuristic Type	Optimization Technique	Focus	Benchmarking Method	Data Source	Other	Testing (Primary Method)	Remarks
Baykasoğlu, A (2001)	Search Heuristic	Not Applicable	Used mathematical programming tools to model multiple objective optimization problems	Heuristics or Other Methods	Example Data	Goal Programming	Comparative Analysis	Trend in literature is using modern heuristic optimization tech. GA, TS, and SA. Paper uses Multi. Obj Tabu Search.
Dell'Amico, M. (1996)	Search Heuristic	Not Applicable	Analyzed performance of lower and upper bounds for a flow-shop problem with two machines. This study used a Tabu search algorithm and proved the effectiveness of the proposed bounds through computational results	Heuristics or Other Methods	Data Generated	Flow Shop Problem w2 machines	Efficiency improvement	Truncation application using upper/lower bounds Tabu Search Compared with other methods using Computational Experiments
Patterson, R.A., & Rolland, E. (2002)	Heuristic Procedure	Not Applicable	Explored network design and presented a heuristic with a methodology that utilized an adaptive reasoning technique. The authors also generalized their formulation and measured its effectiveness	Heuristics or Other Methods	Example Data	Proposes a formulation and heuristic based on hierarchical decomposition	Mathematical Model or Computational Result	meta-heuristic over specialized heuristics for sub problems. Developed 80 test problems to model data
Chiang, W-C., Kouvelis, P., & Urban, T.L. (2002)	Search Heuristic	Heuristic Based	Developed optimal and heuristic solutions methodologies for evaluation of workflow interference. The paper focused on application of these methodologies from a facility layout perspective by examining branch and bound heuristics along with Tabu search he	Heuristics or Other Methods	Data Generated	Workflow interference from a facility layout perspective	Efficiency improvement	Optimal and heuristic solution methodologies are developed and evaluated Tabu Search Branch and Bound
Ramirez-Beltran, N.D. (1995)	Heuristic Procedure	Optimization Model	Demonstrated a real-world application of an integer programming problem. The focus of this study was on finding an optimal solution for a labor cost problem	An Example, Case Study, or Design Specification	Example Data	Real-world application of pure integer programming	Mathematical Model or Computational Result	Branch and bound algorithms are used and a state=space model was used to predict the stochastic behavior of monthly demands
Aggarwal, C.C., Orlin, J.B., & Tai, R.P. (1997)	Algorithm	Heuristic Based	Explored applications of genetic algorithms to demonstrate the utility of knowledge based mechanisms	Heuristics or Other Methods	Inconclusive or Not Applicable	Technique used an optimize	Comparative Analysis	Purpose if the paper was to demonstrate the power of knowledge based mechanisms in Genetic Algorithms

Categorization & Other Specifics								
Reference	Heuristic Type	Optimization Technique	Focus	Benchmarking Method	Data Source	Other	Testing (Primary Method)	Remarks
Bracker, J.S., & Pearson, J.N. (1986)	Heuristic Procedure	Not Applicable	Developed a planning process with comparison to a specified area of interest. The authors used multivariate analysis of variance.	An Example, Case Study, or Design Specification	Sampled Data	Heuristic Procedure	Comparative Analysis	Application is methodological as the study analyzes shortcomings of prior research and strategic planning.
Jun, J.B., Jacobson, S.H., & Swisher, J.R. (1999)	Heuristic Procedure	Optimization Model	Used discrete event simulation to improve patient flow and for resource allocation.	An Example, Case Study, or Design Specification	Sampled Data	Discussion of discrete event simulation	Efficiency improvement	Allocation of resources. Selected for application of flow sequencing Sample data in the form of a survey
Liu, L., & Cheng, K.L. (1997)	Inconclusive or Not Applicable	Optimization Model	Studied continuous review inventory models and included review of exponentially distributed variables along with other key operating characteristics for the inventory model	Mathematical Formulation, Other Optimization Procedure or Result	Example Data	Markov process for inventory models	Comparative Analysis	Application of optimization of system parameters
Mazzola, J.B., & Schantz, R.H. (1995)	Search Heuristic	Heuristic Based	Developed an optimal allocation model of a single facility production environment.	Heuristics or Other Methods	Data Generated	Generalizes an approach to resource application	Comparative Analysis	Research Application: Primarily focused on approach, knowledge area with respect to over-arching dissertation research and approach focus
Ohlemiller, M. (1997)	Search Heuristic	Heuristic Based	Used simulated annealing for solving a minimum location problem and efficiency of the method was presented along with results relative to the expected deviation	Heuristics or Other Methods	Data Generated	Tabu Search	Demonstration of "Good" Solution	Simulated Annealing. Is a good alt. for minimum location. The study compares those prior findings with Tabu search
Bisgaard, S. (1997)	Inconclusive or Not Applicable	Not Applicable	Explored the experimental determination of tolerance limits of mating components of an assembled product	Mathematical Formulation, Other Optimization Procedure or Result	Example Data	Design examples provided	Mathematical Model or Computational Result	Outlines theory and methods for exp. Of tolerance limits for mating components Ind. Indirect Application to selective assy
Ostermeier, M. (2003)	Inconclusive or Not Applicable	Not Applicable	Key points made in this research (not performed) were that "experimental determination of the distributions used would require extensive, cost-prohibitive," sequencing. This lends credence to the use of Monte Carlo simulation presented in this dissertation	Mathematical Formulation, Other Optimization Procedure or Result	Example Data	Non-specific. Paper presents incremental truncation of DNA	Mathematical Model or Computational Result	Research Application: Primarily focused on approach, knowledge area with respect to over-arching dissertation research and approach focus

Categorization & Other Specifics								
Reference	**Heuristic Type**	**Optimization Technique**	**Focus**	**Benchmarking Method**	**Data Source**	**Other**	**Testing (Primary Method)**	**Remarks**
Bradley, D. M., & Gupta, R.C. (2002)	Inconclusive or Not Applicable	Not Applicable	Analyzes data associated with the sum of "n" independent non-identically distributed uniform random variables	Inconclusive, Not Performed, or Not Applicable	Inconclusive or Not Applicable	Not truncated but sum of n non-ident uniform random var	Mathematical Model or Computational Result	Application considerations
Dhrymes, P. J. (2005)	Inconclusive or Not Applicable	Analytical Target Setting	Developed the moments of truncated distributions in dummy endogenous variable models. An interesting aspect of this study to this research was the approach to normalization of a truncated distribution used within the study	An Example, Case Study, or Design Specification	Example Data	Presents moments of truncated distributions in variable models	Comparative Analysis	
Yang, C., Gui, W., Kong, L., & Wang, Y. (2009)	Heuristic Procedure	Optimization Model	Present a quality prediction model for optimal-setting control of a manufacturing process in a metallurgical industry. Their approach used a "kind of hierarchical strategy for determination of an optimal set point for raw material postioning	Simulation Data or Study	Example Data	Presents a hierarchical inference strategy	Efficiency improvement	Strategy proposed based on biases to aid in determining an optimal set-point for real-time proportioning
Grieme, N. (2011)	Inconclusive or Not Applicable	Not Applicable	Finite difference methods	An Example, Case Study, or Design Specification	Example Data	Application of Finite element method	Inconclusive or Not Applicable	General solution for FEM
Zhu, Q., & Oommen, J. (1997)	Search Heuristic	Heuristic Based	Binning search philosophy based on detection function.	Historical or Collected Data	Historical	Not Applicable	Comparative Analysis	Application consideration as it relates to unknown truncation assys Authors assume that their method is the first use because of unknown target distribution.
Bouchard, B., Elie, R., & Imbert, C. (2010)	Heuristic Procedure	Optimization Model	Optimal stochastic control problem under stochastic target constraints	Simulation Data or Study	Example Data	Optimal control problem	Correlations	

Categorization & Other Specifics								
Reference	**Heuristic Type**	**Optimization Technique**	**Focus**	**Benchmarking Method**	**Data Source**	**Other**	**Testing (Primary Method)**	**Remarks**
Simpson, D.P. (2008)	Heuristic Procedure	Heuristic Based	Finite difference methods	Simulation Data or Study	Data Generated	Application review	Mathematical Model or Computational Result	Formulates simple sufficient conditions for optimality
Asvadurov, S., Druskin, V., Guddati, M., & Knizhnerman, L. (2003)	Heuristic Procedure	Heuristic Based	Finite difference methods	Inconclusive, Not Performed, or Not Applicable	Inconclusive or Not Applicable	Not Applicable	Comparative Analysis	Heuristic Framework
Xiaoping, B. & Jingjing, M. (2009)	Heuristic Procedure	Not Applicable	Presents a system analysis model and application algorithm for calculating availability of complex storage bins	Inconclusive, Not Performed, or Not Applicable	Inconclusive or Not Applicable	bins	Comparative Analysis	Application of availability and algorithm for heuristic
Mease, D., Nair, V., & Sudjianto, A. (2004)	Heuristic Procedure	Mathematical Formulation	Described a statistical formulation for determination of optimal binning strategies for various loss functions and distributions and compare the results to heuristics.	An Example, Case Study, or Design Specification	Example Data	binning	Mathematical Model or Computational Result	Statistical formulation and optimal binning under loss function and distributional assumptions
Xiaoping, B. & Jingjing, M. (2009)	Inconclusive or Not Applicable	Optimization Model	Reviews optimization methodologies as it relates to binning	Mathematical Formulation, Other Optimization Procedure or Result	Inconclusive or Not Applicable	Not Applicable	Efficiency improvement	
Glover, F., & Laguna, M. (undated)	Search Heuristic	Heuristic Based	Exploration of Tabu Search. Application for improved understanding of Tabu Search methods	Inconclusive, Not Performed, or Not Applicable	Inconclusive or Not Applicable	Not Applicable	Demonstration of "Good" Solution	Tabu Search
Caleb Li, M.-H., & Chou, C.-Y. (2001)	Heuristic Procedure	Optimization Model	Application of process optimization as it relates to quality characteristics. Application to target selection of dissertation and consideration of methods and quality indicators	An Example, Case Study, or Design Specification	Example Data	Taguchi loss function	Demonstration of "Good" Solution	
Johnson, A., & Thomopoulos, N.T. (undated)	Inconclusive or Not Applicable	Not Applicable	Left Truncated Normal Distributions	Inconclusive, Not Performed, or Not Applicable	Example Data	Tables	Mathematical Model or Computational Result	Truncated Dist

Categorization & Other Specifics								
Reference	**Heuristic Type**	**Optimization Technique**	**Focus**	**Benchmarking Method**	**Data Source**	**Other**	**Testing (Primary Method)**	**Remarks**
Khasawneh, M.T., Bowling, S.R., Kaewkuekool, S., & Cho, B.R. (2005)	Inconclusive or Not Applicable	Not Applicable	Tables of Standard Normal Doubly Truncated Distributions	Inconclusive, Not Performed, or Not Applicable	Inconclusive or Not Applicable	This method addresses doubly truncated distribution	Empirical	Truncated Dist
Johnson, A.C., & Thomopoulos, N.T. (undated)	Inconclusive or Not Applicable	Not Applicable	Methods and Equations for Doubly Truncated Normal Distributions	Inconclusive, Not Performed, or Not Applicable	Inconclusive or Not Applicable	This method addresses doubly truncated distribution	Empirical	Truncated Dist
Khasawneh, M.T., Bowling, S. R., Kaewkuekool, S., & Cho, B.R. (2005)	Inconclusive or Not Applicable	Not Applicable	Tables of Standard Normal Singly Truncated Distributions	Inconclusive, Not Performed, or Not Applicable	Inconclusive or Not Applicable	This method addresses a singly truncated distribution	Empirical	Truncated Dist
Raschke, M. (2012)	Inconclusive or Not Applicable	Not Applicable	Examined right truncation exponential distributions and an estimator for finite sample sizes of truncation points. Raschke also introduced the use of an inverse mean squared error to evaluate the estimators behavior	Simulation Data or Study	Data Generated	Not Applicable	Comparative Analysis	Monte Carlo Simulation
Blanchet, J., & Liu, J. (2012)	Inconclusive or Not Applicable	Not Applicable	Introduced change of measure techniques for rare-event-analysis of heavy tailed . Monte Carlo simulations were used by the authors to aid in the estimation of rare event probabilities and to present a "good" Markovian approximation of conditional distribution	Simulation Data or Study	Data Generated	Monte Carlo Simulation	Demonstration of "Good" Solution	
Kuwahara, H., & Mura, I. (2008)	Inconclusive or Not Applicable	Not Applicable	Used a weighted stochastic simulation algorithm (SSA) and a Monte Carlo simulation method to analyze rare events of biochemical systems	Simulation Data or Study	Data Generated	Weighted Stochastic Simulation method	Comparative Analysis	Heuristics not specifically discussed

Categorization & Other Specifics								
Reference	**Heuristic Type**	**Optimization Technique**	**Focus**	**Benchmarking Method**	**Data Source**	**Other**	**Testing (Primary Method)**	**Remarks**
Drees, H., de Haan, L., & Li, D. (2006)	Inconclusive or Not Applicable	Not Applicable	Anderson-Darling type test of the null hypothesis used to demonstrate distribution belongs to EVT domain of attraction	Simulation Data or Study	Data Generated	Anderson-Darling Type Test	Mathematical Model or Computational Result	Weighted approximations of the tail of the distributions and other empirical data
Stoyanov, S., & Rachev, S. (2008)	Inconclusive or Not Applicable	Not Applicable	Effects of tail distributions analyzed with convergence rate	Simulation Data or Study	Inconclusive or Not Applicable	Monte Carlo Simulation	Comparative Analysis	Approximation Model
Chavez-Demoulin, V., & Roehrl, A (2004)	Inconclusive or Not Applicable	Not Applicable	General overview of EVT	Inconclusive, Not Performed, or Not Applicable	Inconclusive or Not Applicable	Not Applicable	Inconclusive or Not Applicable	
Peng, L., & Qi, Y. (2009)	Inconclusive or Not Applicable	Not Applicable	Studied maximum likelihood estimates of extreme value indices between -1 and -1/2. They also generalized irregular cases and cases of an unknown extreme value index.	An Example, Case Study, or Design Specification	Inconclusive or Not Applicable	Maximum Likelihood	Mathematical Model or Computational Result	
Bermudez, P.D.Z., & Kotz, S. (2010)	Inconclusive or Not Applicable	Not Applicable	This paper focused on methods such as maximum likelihood (ML), method of moments (MOM), and probability weighted moments (PWM).	Heuristics or Other Methods	Data Generated	GPD, ML, MOM	Mathematical Model or Computational Result	Examined varying methods for the use of the generalized Pareto distribution (GPD) and their application to estimation methods. This literature focused on applications to EVT and its approach was to review and identify options of GPD parameter estimation.
Bermudez, P.D.Z., & Kotz, S. (2010)	Inconclusive or Not Applicable	Not Applicable	This focused on the application and methods of [114] to real world data. The others provide close by providing criteria for a decision maker to aid in determination of an appropriate method for their application.	Heuristics or Other Methods	Data Generated	Application of Real World Data	Mathematical Model or Computational Result	Examined varying methods for the use of the generalized Pareto distribution (GPD) and their application to estimation methods. This literature focused on applications to EVT and its approach was to review and identify options of GPD parameter estimation.

Categorization & Other Specifics								
Reference	**Heuristic Type**	**Optimization Technique**	**Focus**	**Benchmarking Method**	**Data Source**	**Other**	**Testing (Primary Method)**	**Remarks**
Horace, W.C. (2005)	Inconclusive or Not Applicable	Not Applicable	Mathematical Formulation	Mathematical Formulation, Other Optimization Procedure or Result	Inconclusive or Not Applicable	Mathematical Proof	Mathematical Model or Computational Result	
Brazauskas, V., & Kleefeld, A. (2009)	Inconclusive or Not Applicable	Not Applicable	Proposed a method for fitting GPD associated with trade-offs between robustness and efficiency. Using a "trimmed moments" method as a basis the authors used simulations and their method to fit GPD to historical data. Utility was provided following application	Simulation Data or Study	Data Generated	Large sample sizes used to provide a mean and relative efficiency b/w various methods	Mathematical Model or Computational Result	Monte Carlo, 10,000 samples
Carpinteri, A., Cornetti, P., & Puzzi, S. (2006)	Inconclusive or Not Applicable	Not Applicable	Used extreme value theory in the form of a statistical model to evaluate materials. Prior comparisons using EVT and a Multi-Fractional Scaling Law (MFSL) are used in their evaluation	Inconclusive, Not Performed, or Not Applicable	Empirical Data	MFSL and EVT, Model and Correlations	Empirical	A model and correlation between for their area of interest is drawn (e.g. fracture energy and crack surface parameters)
Brooks, C., Clare, A.D., Dalle Molle, J.W., & Persand, G. (2005)	Inconclusive or Not Applicable	Not Applicable	Examined various EVT models for VaR. The authors used GPD, ML and a semi-nonparametric methodology in their reviews.	Simulation Data or Study	Data Generated	Comparative Analysis	Comparative Analysis	Monte Carlo
Diebolt, J., Guillou, A., & Rached, I. (2005)	Inconclusive or Not Applicable	Not Applicable	Used a generalized probability weighted moment method (GPWM) to study the asymptotic behavior of estimation tools presented	Mathematical Formulation, Other Optimization Procedure or Result	Empirical Data	Mathematical Proof	Mathematical Model or Computational Result	
Castillo, E., Hadi, A., Balakrishnan, N., Sarabia, J. (2005)	Inconclusive or Not Applicable	Not Applicable	Not Applicable	Inconclusive, Not Performed, or Not Applicable	Inconclusive or Not Applicable	Not Applicable	Inconclusive or Not Applicable	Overarching review and application only

Categorization & Other Specifics								
Reference	**Heuristic Type**	**Optimization Technique**	**Focus**	**Benchmarking Method**	**Data Source**	**Other**	**Testing (Primary Method)**	**Remarks**
Michalewicz, Z., & Fogel, D. (1998)	Inconclusive or Not Applicable	Not Applicable	Not Applicable	Inconclusive, Not Performed, or Not Applicable	Inconclusive or Not Applicable	Not Applicable	Inconclusive or Not Applicable	Overarching review and application only
Whitney, D. E. (2004)	Inconclusive or Not Applicable	Not Applicable	Not Applicable	Inconclusive, Not Performed, or Not Applicable	Inconclusive or Not Applicable	Not Applicable	Inconclusive or Not Applicable	Overarching review and application only
Billingsley, P. (1995)	Inconclusive or Not Applicable	Not Applicable	General Probability	Mathematical Formulation, Other Optimization Procedure or Result	Inconclusive or Not Applicable	Not Applicable	Mathematical Model or Computational Result	Overarching review and application only
Sheffield, S. (2011)	Inconclusive or Not Applicable	Not Applicable	Characteristic Functions	Mathematical Formulation, Other Optimization Procedure or Result	Inconclusive or Not Applicable	Not Applicable	Mathematical Model or Computational Result	Overarching review and application only
Abadir, K., & Magdalinos., T. (2002)	Inconclusive or Not Applicable	Not Applicable	Characteristic Functions	Mathematical Formulation, Other Optimization Procedure or Result	Inconclusive or Not Applicable	Not Applicable	Mathematical Model or Computational Result	Characteristic Function of a Truncated Standard Normal Distribution

Categorization & Other Specifics								
Reference	**Heuristic Type**	**Optimization Technique**	**Focus**	**Benchmarking Method**	**Data Source**	**Other**	**Testing (Primary Method)**	**Remarks**
Shephard, N.G (1992)	Inconclusive or Not Applicable	Not Applicable	Characteristic Functions	Mathematical Formulation, Other Optimization Procedure or Result	Inconclusive or Not Applicable	Not Applicable	Mathematical Model or Computational Result	
Kawata, T. (1969)	Inconclusive or Not Applicable	Not Applicable	Inversion of Characteristic Functions	Mathematical Formulation, Other Optimization Procedure or Result	Inconclusive or Not Applicable	Not Applicable	Mathematical Model or Computational Result	
Bernadic, M., & Candel, J. (2012)	Inconclusive or Not Applicable	Not Applicable	Truncated Normal Distributions	Mathematical Formulation, Other Optimization Procedure or Result	Inconclusive or Not Applicable	Not Applicable	Mathematical Model or Computational Result	

APPENDIX H: RESEARCH HYPOTHESIS H_3 TESTING RESULTS

This appendix documents the results of research hypothesis H3 testing. Given that the CF inversion methods presented in this dissertation were developed using a single doubly truncated standard normal distribution as a baseline, by logical inference and inspections the results are identical. However, Tables H.1 and H.2 presented below further reinforce this logical inference through correlation and regression analysis under the varying USL and LSL's presented (i.e., Ct = 0.39894228, Fx(b) = 0.9997, Fx(a) = 3.16712 E-05, n = 81). A Pearson's correlation of 1 suggests a statistically significant strong positive correlation. Regression analysis between the two distributions across varying x-values identifies an adjusted R-square value of 100% at a p-value of <.001. The corresponding fitted line plot equation confirms that the values are identical. Table H.3 contains a summary of the hypothesis test table.

Table H.1 - H3 Hypothesis Pearson Correlation of fT(z) and ft(z) - CF

TSND RANGE	Pearson correlation of fT(z) and ft(z) - CF
USL = 4, LSL = -4	1
USL = 3, LSL = -3	1
USL = 2, LSL = -2	1
USL = 1, LSL = -1	1

Table H.2 - H3 Hypothesis Regression Results of fT(z) and ft(z) - CF

TSND RANGE	R-sq (adj)	P-Value	Fitted Line Plot Equation for Cubic Model
USL = 4, LSL = -4	100%	$p < 0.001$	Y = 0.000000 + 1.000 X - 0.000000 X**2
USL = 3, LSL = -3	100%	$p < 0.001$	Y = 0.000000 + 1.000 X
USL = 2, LSL = -2	100%	$p < 0.001$	Y = 0.000000 + 1.000 X
USL = 1, LSL = -1	100%	$p < 0.001$	Y = 0.000000 + 1.000 X

Table H.3 - H3 Hypothesis Test Summary Table

X	pdf	fT(z)	φ(w/μ&σ)	ft(z) - CF
-4	0.00013383	0.00013384	0.00033548	0.00013384
-3.9	0.000198655	0.00019867	0.00049799	0.00019867
-3.8	0.000291947	0.00029197	0.00073185	0.00029197
-3.7	0.00042478	0.00042481	0.00106483	0.00042481
-3.6	0.000611902	0.00061194	0.00153391	0.00061194
-3.5	0.000872683	0.00087274	0.00218763	0.00087274
-3.4	0.001232219	0.0012323	0.00308891	0.0012323
-3.3	0.001722569	0.00172268	0.00431811	0.00172268
-3.2	0.002384088	0.00238424	0.0059764	0.00238424
-3.1	0.003266819	0.00326703	0.00818922	0.00326703
-3	0.004431848	0.00443213	0.0111097	0.00443213
-2.9	0.005952532	0.00595291	0.01492173	0.00595291
-2.8	0.007915452	0.00791595	0.01984235	0.00791595
-2.7	0.010420935	0.01042159	0.02612306	0.01042159
-2.6	0.013582969	0.01358383	0.03404961	0.01358383
-2.5	0.0175283	0.01752941	0.04393972	0.01752941
-2.4	0.02239453	0.02239595	0.05613832	0.02239595
-2.3	0.028327038	0.02832883	0.07100985	0.02832883
-2.2	0.035474593	0.03547684	0.08892725	0.03547684
-2.1	0.043983596	0.04398638	0.11025751	0.04398638
-2	0.053990967	0.05399439	0.13534386	0.05399439
-1.9	0.065615815	0.06561997	0.16448488	0.06561997
-1.8	0.078950158	0.07895516	0.19791124	0.07895516
-1.7	0.094049077	0.09405504	0.23576101	0.09405504
-1.6	0.110920835	0.11092786	0.27805491	0.11092786
-1.5	0.129517596	0.1295258	0.32467303	0.1295258
-1.4	0.149727466	0.14973695	0.37533487	0.14973695
-1.3	0.171368592	0.17137945	0.42958457	0.17137945
-1.2	0.194186055	0.19419836	0.48678309	0.19419836
-1.1	0.217852177	0.21786598	0.54610902	0.21786598
-1	0.241970725	0.24198605	0.60656908	0.24198605
-0.9	0.26608525	0.26610211	0.66701906	0.26610211
-0.8	0.289691553	0.2897099	0.72619504	0.2897099
-0.7	0.312253933	0.31227371	0.78275412	0.31227371
-0.6	0.333224603	0.33324571	0.83532312	0.33324571
-0.5	0.352065327	0.35208763	0.88255281	0.35208763
-0.4	0.36827014	0.36829347	0.92317482	0.36829347
-0.3	0.381387815	0.38141198	0.95605804	0.38141198
-0.2	0.391042694	0.39106747	0.98026077	0.39106747
-0.1	0.396952547	0.39697769	0.99507551	0.39697769
0	0.39894228	0.39896755	1.00006335	0.39896755
0.1	0.396952547	0.39697769	0.99507551	0.39697769
0.2	0.391042694	0.39106747	0.98026077	0.39106747
0.3	0.381387815	0.38141198	0.95605804	0.38141198
0.4	0.36827014	0.36829347	0.92317482	0.36829347

X	pdf	fT(z)	φ(w/μ&σ)	ft(z) - CF
0.5	0.352065327	0.35208763	0.88255281	0.35208763
0.6	0.333224603	0.33324571	0.83532312	0.33324571
0.7	0.312253933	0.31227371	0.78275412	0.31227371
0.8	0.289691553	0.2897099	0.72619504	0.2897099
0.9	0.26608525	0.26610211	0.66701906	0.26610211
1	0.241970725	0.24198605	0.60656908	0.24198605
1.1	0.217852177	0.21786598	0.54610902	0.21786598
1.2	0.194186055	0.19419836	0.48678309	0.19419836
1.3	0.171368592	0.17137945	0.42958457	0.17137945
1.4	0.149727466	0.14973695	0.37533487	0.14973695
1.5	0.129517596	0.1295258	0.32467303	0.1295258
1.6	0.110920835	0.11092786	0.27805491	0.11092786
1.7	0.094049077	0.09405504	0.23576101	0.09405504
1.8	0.078950158	0.07895516	0.19791124	0.07895516
1.9	0.065615815	0.06561997	0.16448488	0.06561997
2	0.053990967	0.05399439	0.13534386	0.05399439
2.1	0.043983596	0.04398638	0.11025751	0.04398638
2.2	0.035474593	0.03547684	0.08892725	0.03547684
2.3	0.028327038	0.02832883	0.07100985	0.02832883
2.4	0.02239453	0.02239595	0.05613832	0.02239595
2.5	0.0175283	0.01752941	0.04393972	0.01752941
2.6	0.013582969	0.01358383	0.03404961	0.01358383
2.7	0.010420935	0.01042159	0.02612306	0.01042159
2.8	0.007915452	0.00791595	0.01984235	0.00791595
2.9	0.005952532	0.00595291	0.01492173	0.00595291
3	0.004431848	0.00443213	0.0111097	0.00443213
3.1	0.003266819	0.00326703	0.00818922	0.00326703
3.2	0.002384088	0.00238424	0.0059764	0.00238424
3.3	0.001722569	0.00172268	0.00431811	0.00172268
3.4	0.001232219	0.0012323	0.00308891	0.0012323
3.5	0.000872683	0.00087274	0.00218763	0.00087274
3.6	0.000611902	0.00061194	0.00153391	0.00061194
3.7	0.00042478	0.00042481	0.00106483	0.00042481
3.8	0.000291947	0.00029197	0.00073185	0.00029197
3.9	0.000198655	0.00019867	0.00049799	0.00019867
4	0.00013383	0.00013384	0.00033548	0.00013384

Figure H.1 - TSND Range (-4 to 4)

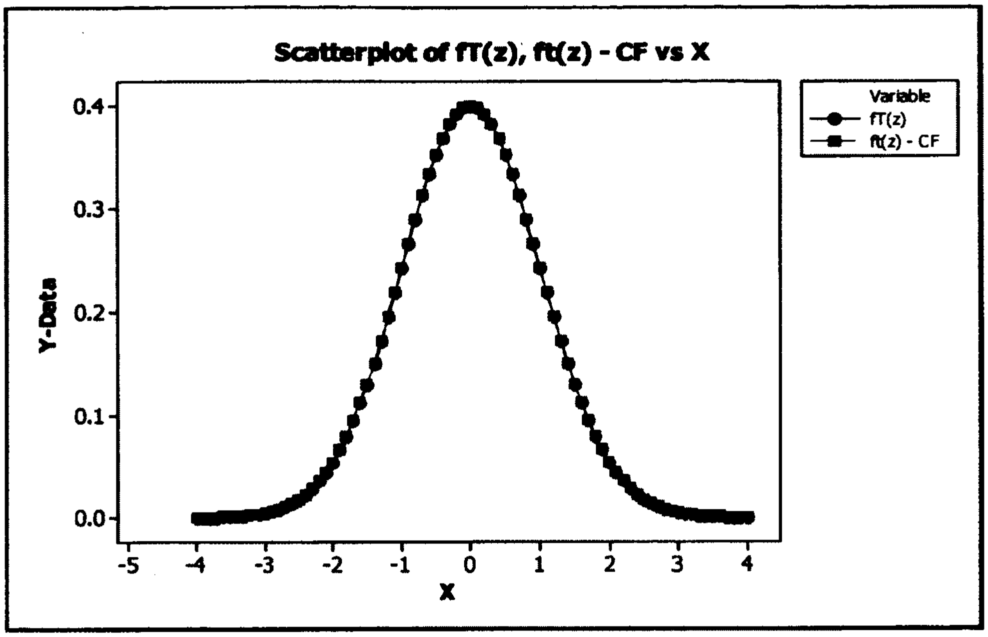
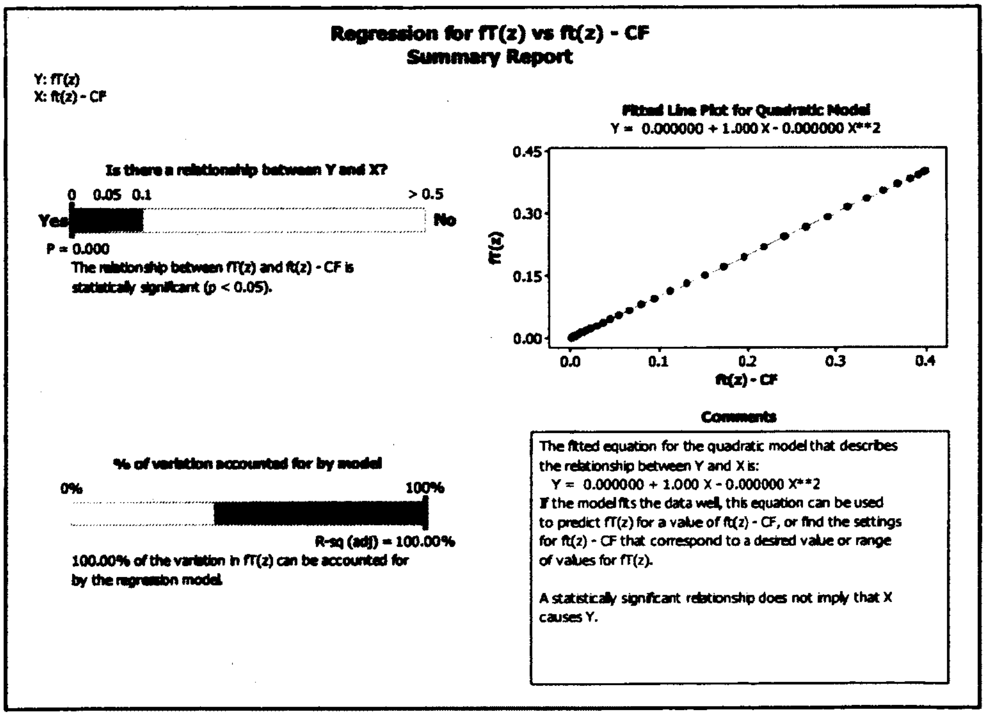


Figure H.2 - TSND Regression (-4 to 4)



P = 0.000
The relationship between fT(z) and ft(z) - CF is statistically significant ($p < 0.05$).

R-sq (adj) = 100.00%
100.00% of the variation in fT(z) can be accounted for by the regression model.

Comments

The fitted equation for the quadratic model that describes the relationship between Y and X is:
Y = 0.000000 + 1.000 X - 0.000000 X**2
If the model fits the data well, this equation can be used to predict fT(z) for a value of ft(z) - CF, or find the settings for ft(z) - CF that correspond to a desired value or range of values for fT(z).

A statistically significant relationship does not imply that X causes Y.

Figure H.3 - TSND Range (-3 to 3)

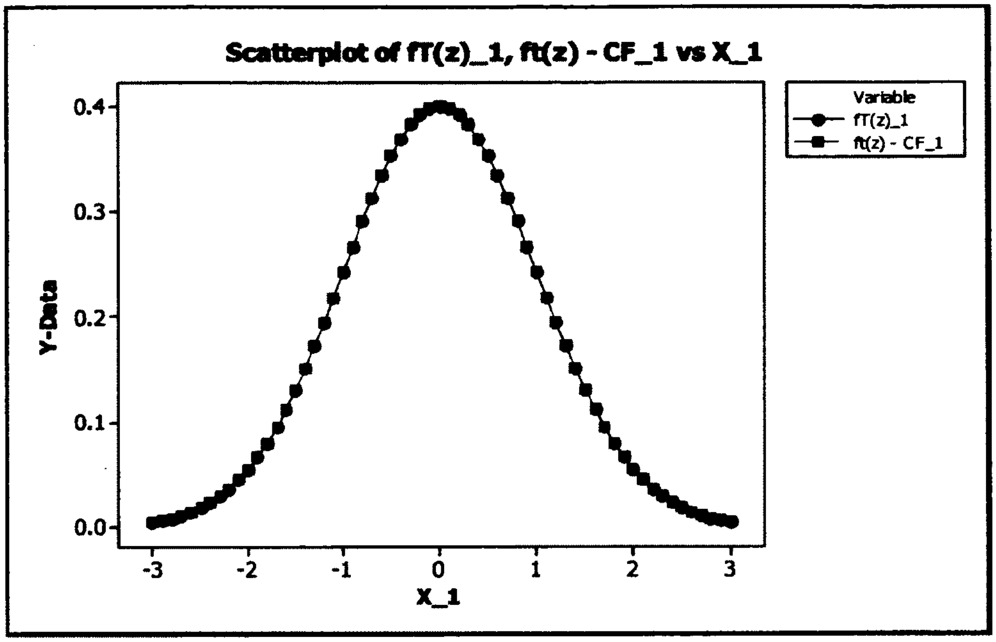
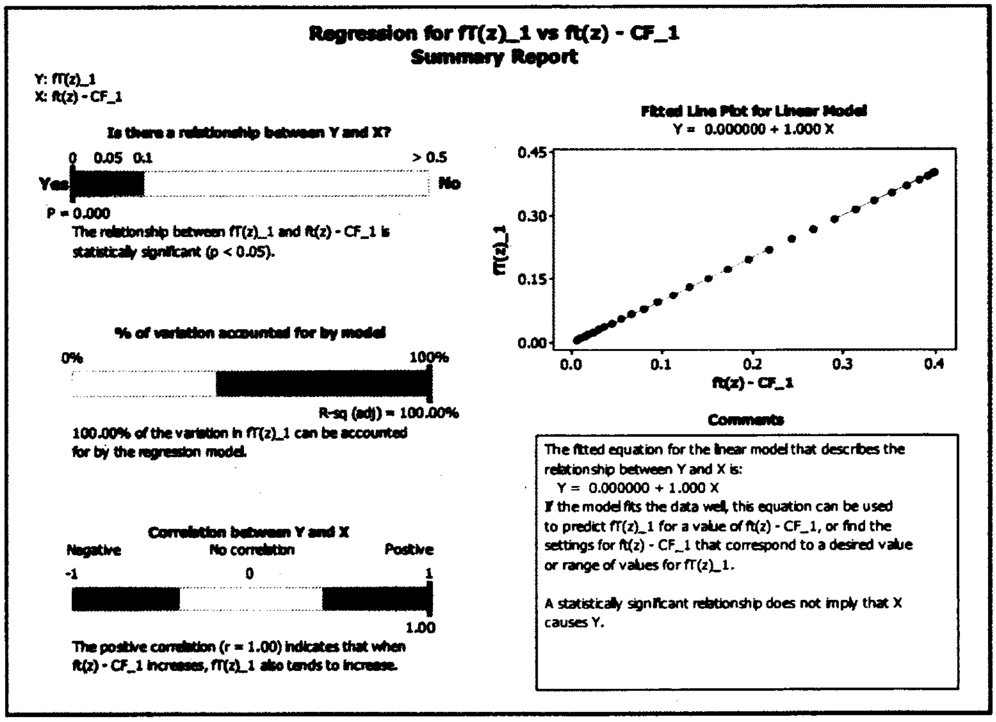


Figure H.4 - TSND Regression (-3 to 3)



P = 0.000

The relationship between fT(z)_1 and ft(z) - CF_1 is statistically significant ($p < 0.05$).

R-sq (adj) = 100.00%

100.00% of the variation in fT(z)_1 can be accounted for by the regression model.

The positive correlation (r = 1.00) indicates that when ft(z) - CF_1 increases, fT(z)_1 also tends to increase.

Y = 0.000000 + 1.000 X

The fitted equation for the linear model that describes the relationship between Y and X is:

Y = 0.000000 + 1.000 X

If the model fits the data well, this equation can be used to predict fT(z)_1 for a value of ft(z) - CF_1, or find the settings for ft(z) - CF_1 that correspond to a desired value or range of values for fT(z)_1.

A statistically significant relationship does not imply that X causes Y.

Figure H.5 - TSND Range (-2 to 2)

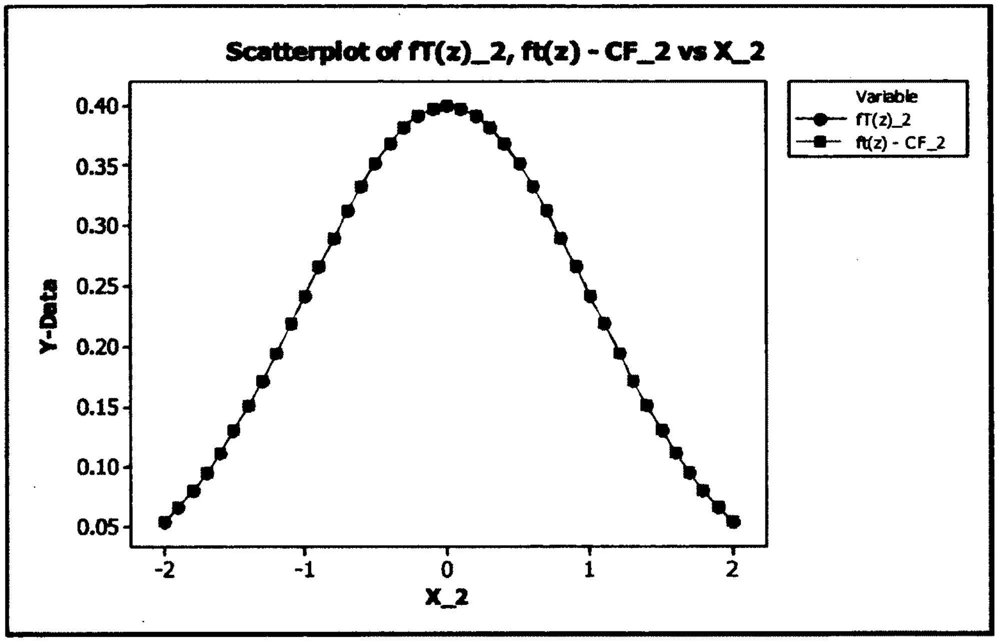
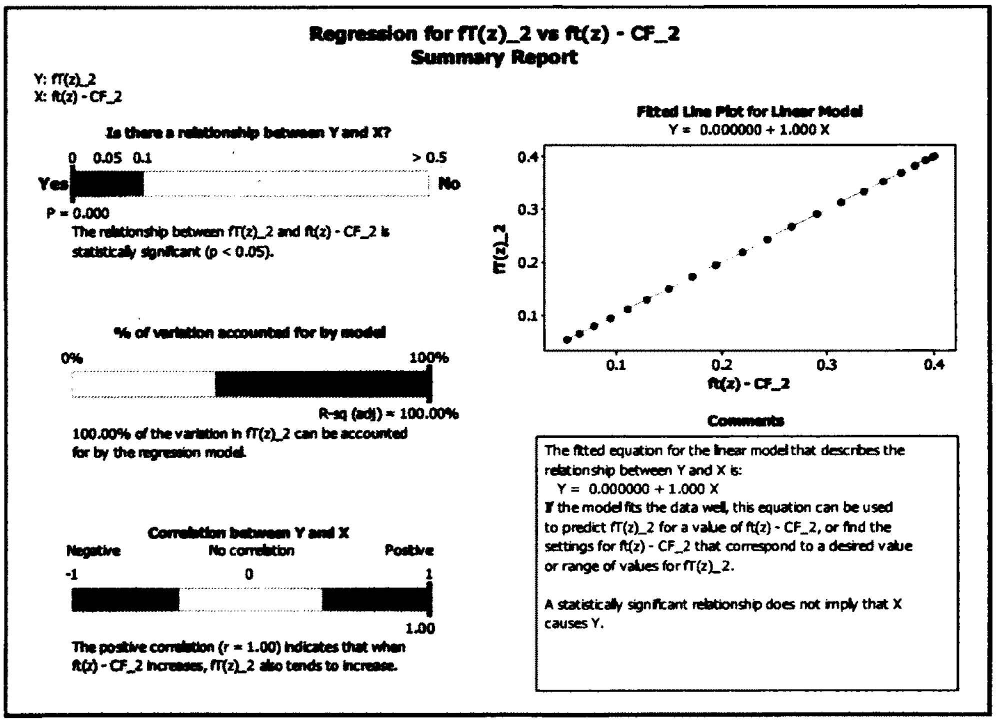


Figure H.6 - TSND Regression (-2 to 2)


Regression for fT(z)_2 vs ft(z) - CF_2
Summary Report

Y: fT(z)_2
X: ft(z) - CF_2

Is there a relationship between Y and X?

P = 0.000

The relationship between fT(z)_2 and ft(z) - CF_2 is statistically significant ($p < 0.05$).

% of variation accounted for by model

R-sq (adj) = 100.00%

100.00% of the variation in fT(z)_2 can be accounted for by the regression model.

Correlation between Y and X

The positive correlation (r = 1.00) indicates that when ft(z) - CF_2 increases, fT(z)_2 also tends to increase.

Fitted Line Plot for Linear Model

Y = 0.000000 + 1.000 X

Comments

The fitted equation for the linear model that describes the relationship between Y and X is:

Y = 0.000000 + 1.000 X

If the model fits the data well, this equation can be used to predict fT(z)_2 for a value of ft(z) - CF_2, or find the settings for ft(z) - CF_2 that correspond to a desired value or range of values for fT(z)_2.

A statistically significant relationship does not imply that X causes Y.

Figure H.7 - TSND Range (-1 to 1)

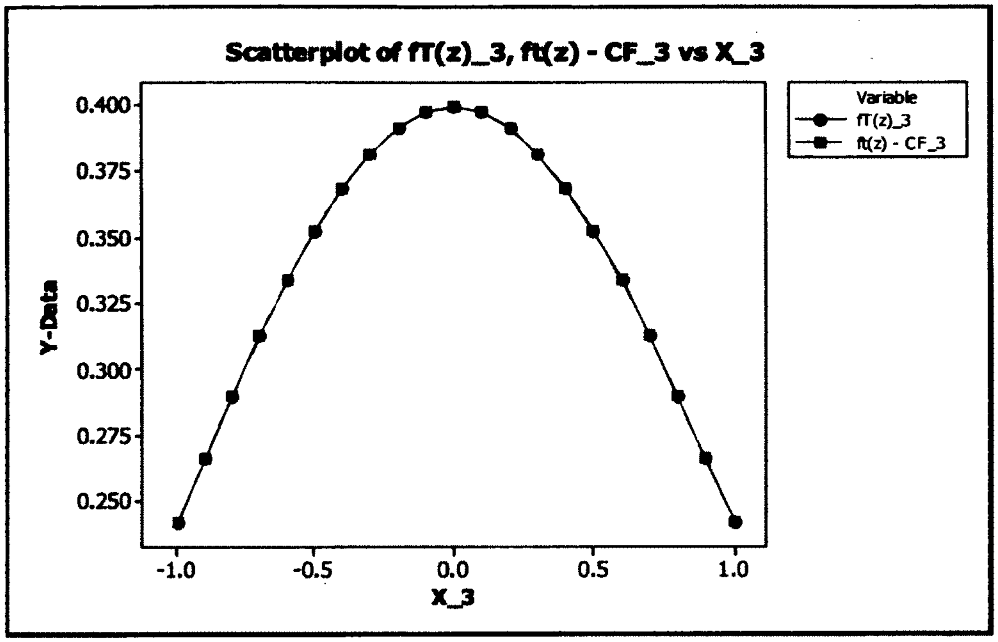
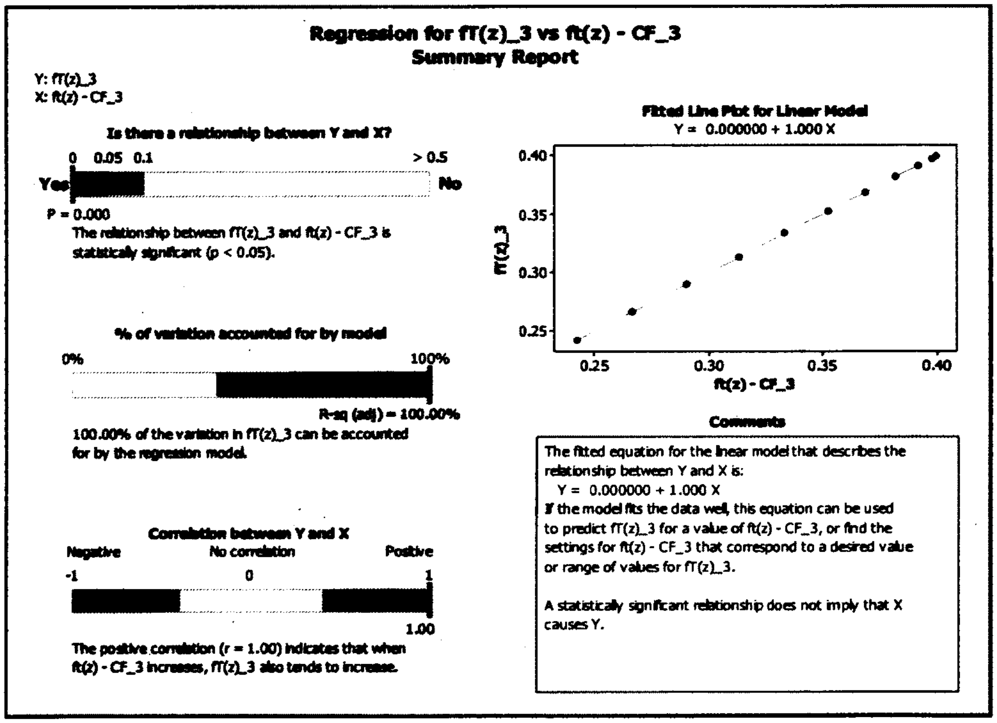


Figure H.8 - TSND Regression (-1 to 1) 1



The relationship between fT(z)_3 and ft(z) - CF_3 is statistically significant (p < 0.05).

100.00% of the variation in fT(z)_3 can be accounted for by the regression model.

The positive correlation (r = 1.00) indicates that when ft(z) - CF_3 increases, fT(z)_3 also tends to increase.

The fitted equation for the linear model that describes the relationship between Y and X is:

Y = 0.000000 + 1.000 X

If the model fits the data well, this equation can be used to predict fT(z)_3 for a value of ft(z) - CF_3, or find the settings for ft(z) - CF_3 that correspond to a desired value or range of values for fT(z)_3.

A statistically significant relationship does not imply that X causes Y.

APPENDIX I: HEURISTIC – TSND BASELINE USING CF INVERSION

This appendix documents the heuristic procedure developed for the baseline inversion of a characteristic function to a truncated standard normal distribution. The general equations are presented in Appendix A. A high-level graphical summary of this heuristic is found in Figure 8. The details for this heuristic are as follows:

Begin Heuristic:

Step 1: Initiate the General Parameters for the Truncated Standard Normal Distribution

- *I. Define Parameters $\sigma = 1$, $\mu = 0$, USL, LSL, x, n*
- *II. Define x as a variable between the USL and LSL*
  - *a. For a doubly truncated normal distribution (with CF inversion) per Appendix A, Equations 1-5.*
  - *b. For a probability density function (PDF) refer to Appendix A, Equation (6).*
  - *c. Calculate Z using Appendix A, Equation (4).*

Step 2: Calculate the probability density function (PDF) – (for information)

- *I. Using Appendix A, Equation (6) from Billingsley (1995), adapted to notation herein:*

$$f(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\left(\frac{(x-\mu)^2}{2\sigma^2}\right)}$$

Step 3: Calculate the Truncated Standard Normal Distribution

- *I. Using the defined parameters from step 1 and Appendix A, Equations (1), (2), (3), (4) and (5) from Khasawneh et al. (2005), calculate $f_T(z)$ as follows:*

$$f_T(z) = \int_{z_L}^{z} \frac{f(z)}{\left(\int_{z_L}^{z_U} f(z)dz\right)} dz \qquad z_L \leq z \leq z_u \qquad \text{(APPENDIX A, EQUATION 1)}$$

Where $$f(z) = \frac{1}{\sqrt{2\pi}} e^{\left(-\frac{1}{2} * z^2\right)} \qquad \text{(APPENDIX A, EQUATION 2)}$$

and

$$f(z)dz = \int_{Z_L}^{Z_U} \frac{1}{\sqrt{2\pi}} e^{\left(-\frac{1}{2} * z^2\right)} dz \tag*{(APPENDIX A, EQUATION 3)}$$

Given that:

a. $$z = \frac{x - \mu}{\sigma} \tag*{(APPENDIX A, EQUATION 4)}$$

b. $$\mu_{T_1}(z) = \int_{z_{L_1}}^{z_{U1}} z f_{T1}(z) dz \tag*{(APPENDIX A, EQUATION 5)}$$

This establishes the baseline for the CF inversion. Khasawneh et al. (2005) provides further insight into the calculation of a truncated standard normal distribution using Appendix A, Equations (1) through (5).

II. *Calculate $F_x(b)$ and $F_x(a)$ using Appendix A, Equation (6).*
   a. *For $F_x(b)$ the value of X = USL*
   b. *For $F_x(a)$ the value of X = LSL*

Step 4: Calculate the CF φ for the given distribution (Appendix A, Equations 2 and 11)

I. *Since a normal distribution* $= \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ *has a* $\varphi(t) = e^{iu\tau - \frac{\sigma^2 t^2}{2}}$ *then*

$$\varphi(t) = \int_a^b f_x(u) e^{iu\tau} du = e^{iu\tau - \frac{\sigma^2 t^2}{2}}$$

(Note: for a continuous distribution $b = +\infty$ and $a = -\infty$)

II. *Therefore for a truncated standard normal distribution (use Appendix A, Equations 12 and 13)*

$$\varphi(\tau) = \frac{1}{F_x(b) - F_x(a)} \int_a^b f_x(u) e^{iu\tau} du = \frac{1}{F_x(b) - F_x(a)} \left(e^{iu\tau - \frac{\sigma^2 t^2}{2}} \right) = \frac{e^{iu\tau - \frac{\sigma^2 t^2}{2}}}{F_x(b) - F_x(a)}$$

III. $$f_t(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-itx} \varphi(\tau) dt = f_t(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-itx} \left(\frac{e^{iu\tau - \frac{\sigma^2 t^2}{2}}}{F_x(b) - F_x(a)} \right) dt$$

   a. *Using Appendix A, Equations 11 and 15*

Step 5: Calculate the truncated standard normal distribution by inversion of the characteristic function using the inversion factor.

I. *Set the results of Step 3.I (for a given parameter set) equal to step 4.III. The difference equates to the equation and inversion factor (C_{TC}) in Step 5.II.*

II. $f_t(x) \approx \frac{1}{2\pi}\left(C_{Tc}\right)\left(\frac{e^{\frac{\sigma^2 x^2}{2}}}{F_x(b)-F_x(a)}\right)$, *where* $C_{TC} = \frac{1}{\sqrt{2\pi}}$

- *a. Noted as Appendix A, Equation 16*
- *b. Where* $C_{TC} =$ *is a constant for USL and LSL.*

Step 6: Baseline the results against a known truncated standard normal distribution

- *I. Generate a given distribution for a range of x value for a given sample size. For the purpose of this dissertation increments of 0.1 were used for a given TSND (e.g. USL/LSL from 4 to -4)*
- *II. Perform mathematical formulation in addition to correlation and regression analysis. An example is identified in Appendix H.*

End Heuristic

Note - Refer to Appendix A for additional information on equations, applications, and references.

APPENDIX J: HEURISTIC – TSND ASSEMBLY USING CF INVERSION

This appendix documents the heuristic procedure developed from the baseline inversion heuristic developed in Appendix B. The general equations utilized by this heuristic are presented in Appendix A. A high-level graphical summary of this heuristic is found in Figure 10. The details for this heuristic are as follows:

Begin Heuristic:

Step 1: Define the general parameters for the Truncated Standard Normal Distribution

- *I. Define parameters $\sigma = 1$, $\mu = 0$, USL, LSL, x, n*
- *II. Define x as a variable between the USL and LSL*
    - *d. For a doubly truncated normal distribution (with CF inversion) per Appendix A, Equations 1-5.*
    - *e. For a probability density function (PDF) refer to Appendix A, Equation (6).*
    - *f. Calculate Z using Appendix A, Equation (4).*

Step 2: Calculate the probability density function (PDF) – (for information)

- *I. Using Appendix A, Equation (6) from Billingsley (1995), adapted to notation herein:*

$$: f(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\left(\frac{(x-\mu)^2}{2\sigma^2}\right)}$$

Step 3: Calculate the Truncated Standard Normal Distribution

- *I. Using the defined parameters from step 1 and Appendix A, Equations (1), (2), (3), (4) and (5) from Khasawneh et al. (2005), calculate $f_T(z)$ as follows:*

$$f_T(z) = \int_{z_L}^{z} \frac{f(z)}{\left(\int_{z_L}^{z_U} f(z)dz\right)} dz \qquad z_L \le z \le z_u \qquad \text{(APPENDIX A, EQUATION 1)}$$

$$\text{Where } f(z) = \frac{1}{\sqrt{2\pi}} e^{\left(-\frac{1}{2} * z^2\right)} \qquad \text{(APPENDIX A, EQUATION 2)}$$

and

$$f(z)dz = \int_{z_L}^{z_U} \frac{1}{\sqrt{2\pi}} e^{\left(-\frac{1}{2}*z^2\right)} dz$$ *(APPENDIX A, EQUATION 3)*

Given that:

c. $$z = \frac{x-\mu}{\sigma}$$ *(APPENDIX A, EQUATION 4)*

d. $$\mu_{T_1}(z) = \int_{z_{L_1}}^{z_{U_1}} z f_{T_1}(z) dz$$ *(APPENDIX A, EQUATION 5)*

This establishes the baseline for the CF inversion. Khasawneh et al. (2005) provides further insight into the calculation of a truncated standard normal distribution using Appendix A, Equations (1) through (5).

II. Calculate $F_x(b)$ and $F_x(a)$ using Appendix A, Equation (6).
  - *e. For $F_x(b)$ the value of X = USL*
  - *f. For $F_x(a)$ the value of X = LSL*

Step 4: Calculate the CF φ for the given distribution (using Appendix A, Equations 2 and 11):

I. Since a normal distribution $= \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ *has a* $\varphi(t) = e^{iu\tau - \frac{\sigma^2 t^2}{2}}$ *then*

$$\varphi(t) = \int_a^b f_x(u) e^{iu\tau} du = e^{iu\tau - \frac{\sigma^2 t^2}{2}}$$

(Note: for a continuous distribution b = +∞ and a = -∞)

II. Therefore for a truncated standard normal distribution (Appendix 12 and 13):

$$\varphi(\tau) = \frac{1}{F_x(b) - F_x(a)} \int_a^b f_x(u) e^{iu\tau} du = \frac{1}{F_x(b) - F_x(a)} \left(e^{iu\tau - \frac{\sigma^2 t^2}{2}} \right) = \frac{e^{iu\tau - \frac{\sigma^2 t^2}{2}}}{F_x(b) - F_x(a)}$$

III. Given that $f_t(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-itx} \varphi(\tau) dt$, *then* = $f_t(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-itx} \left(\frac{e^{iu\tau - \frac{\sigma^2 t^2}{2}}}{F_x(b) - F_x(a)} \right) dt$,

(Appendix A, Equations 11 and 12)

Step 5: Define the characteristic function for x as: The characteristic function of a probability measure μ is defined for real t by (repeat for two identical distributions):

I. $$\varphi(t) = \varphi_x(t) := E\left[e^{itX}\right] = \int_{-\infty}^{\infty} e^{itx} \mu(dx)$$ *(Appendix A, Equation 7)*

a. where $\left[e^{itX}\right] = \cos(t) + i\sin(t)$ *(Appendix A, Equation 8)*

b. *where* $\varphi_{X+Y} = \varphi_x \varphi_y$ *(Appendix A, Equation 10)*

II. *Billingsley (1995) identifies that a Characteristic Function has 3 fundamental properties as follows:*

    i. *"If μ_1 and μ_2 have respective characteristic functions $\varphi_1(t)$ and $\varphi_2(t)$ then $\mu_1 * \mu_2$ has characteristic function $\varphi_1(t) * \varphi_2(t)$. Billingsley (1995) notes that "although convolution is essential to the study of sums of independent random variables, it is a complicated operation, and its often simpler to study the products of the corresponding characteristic functions.*
    ii. *The characteristic function uniquely determines the distribution. This shows that in studying the products in (i), no information is lost.*
    iii. *From the pointwise convergence of characteristic functions follows the weak convergence of the corresponding distributions. This makes it possible, for example, to investigate the asymptotic distributions of sums of independent random variables by means of their characteristic functions."*

Step 6: Calculate the truncated standard normal distribution by inversion of the characteristic function using the inversion factor.

I. *Set the results of Step 3.I (for a given parameter set) equal to step 4.III. The difference equates to the equation and inversion factor (C_{TC}) in Step 5.II.*

II. $$f_t(x) \approx \frac{1}{2\pi}\left(C_{Tc}\right)\left(\frac{e^{-\frac{\sigma^2 x^2}{2}}}{F_x(b)-F_x(a)}\right), \text{ where } C_{TC} = \frac{1}{\sqrt{2\pi}}$$ *(Appendix A, Equation 16)*

a. *Where C_{TC} = is a constant for USL and LSL*

Step 7: Abadir, K., & Magdalinos., T. (2002) d*efine the characteristic function for a doubly truncated normal distribution as: "the variate where x is doubly truncated to $y \in$ (a,b), where b>a, and its characteristic function is given by the integral (repeat for two identical distributions :*

I. $$\varphi_y(\tau) = \frac{1}{F_x(b)-F_x(a)}\int_a^b f_x(u)e^{iu\tau}du \text{ "}$$ *(Appendix A, Equation 12)*

II. *Then logically the sum of the characteristic functions for two doubly truncated normal distributions is given Equation 10 and Step 7.II.a (Appendix A, Equation 18)::*

a. $$\varphi_z(\tau) := \left(\frac{1}{F_x(b)-F_x(a)}\int_a^b f_x(u)e^{iu\tau}du\right)\left(\frac{1}{F_y(b)-F_y(a)}\int_a^b f_y(u)e^{iu\tau}du\right)$$

Step 8: Using an inversion formula the sum of two doubly truncated normal distributions can be used to determine the resulting probability density function for the combined distribution. Given the following:

I. Since $f(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-itx}\varphi(t)dt$ (Appendix A, Equation 14)

II. and Appendix A, Equation 10.

Step 9: Solve for $f_t(x)_{assy}=\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-itx}\left[\left(\frac{e^{iu\tau-\frac{\sigma^2 t^2}{2}}}{F_x(b)-F_x(a)}\right)_{d1}\left(\frac{e^{iu\tau-\frac{\sigma^2 t^2}{2}}}{F_x(b)-F_x(a)}\right)_{d2}\right]dt$

(Appendix A, Equation 19)

I. $f_t(x)_{assy}=\frac{1}{2\pi}\left(\frac{1}{(F_x(b)-F_x(a))_{d1}*(F_x(b)-F_x(a))_{d2}}\right)\int_{-\infty}^{\infty} e^{-itx}\left[\left(e^{iu\tau-\frac{\sigma^2 t^2}{2}}\right)_{d1}\left(e^{iu\tau-\frac{\sigma^2 t^2}{2}}\right)_{d2}\right]dt$

(Appendix A, Equation 20)

i. Where $\int_{-\infty}^{\infty} e^{-itx}\left[e^{iu\tau-\frac{\sigma^2 t^2}{2}}\right]dt = e^{-\frac{\sigma^2 t^2}{2}}$ (Appendix A, Equation 21)

II. $f_t(x)_{assy}\approx\frac{1}{2\pi}(C_{Tc})\left(\frac{\left(e^{-\frac{\sigma^2 t^2}{2}}\right)_{d1}\left(e^{-\frac{\sigma^2 t^2}{2}}\right)_{d2}}{(F_x(b)-F_x(a))_{d1}*(F_x(b)-F_x(a))_{d2}}\right)$, where C_{TC} $\frac{1}{\sqrt{2\pi}}$

(Appendix A, Equation 22)

Step 10: Baseline the results against a known truncated standard normal distribution (final state)

I. Generate a given distribution for a range of X value for a given sample size. For the purpose of this dissertation increments of 0.2 were used for a given TSND (e.g. USL/LSL from 8 to -8. Two identical distributions with an USL (4) and LSL (-4) were assembled. See Figure 9.

II. Perform mathematical formulation in addition to correlation and regression analysis. Assembly results are identified in Appendix D, E, and H.

End Heuristic

Note - Refer to Appendix A for additional information on Equations, Applications and References.

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Mr. John Walter Ralls has over 10 years of experience in engineering and management. He has significant project management experience in the area of manufacturing, design, system development, and front line engineering management. As a licensed professional engineer in the Commonwealth of Virginia, Mr. Ralls also holds certifications in project management, refrigeration, and holds a USCG 3rd Assistant Engineers License for Steam, Diesel, and Gas Turbines of Any Horsepower.