Winter 2003

Optimal Feedback Control for Ship Roll Motion Under Sea States

Anusit Anmanatarkul
Old Dominion University

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by

Anusit Anmanatarkul
M.E. August 1999, Old Dominion University
B.S. March 1995, King's Mongkut Institute of Technology Thonburi, Thailand

A Dissertation Submitted to the Faculty of
Old Dominion University in Partial Fulfillment of the
Requirement for the Degree of

DOCTOR OF PHILOSOPHY

MECHANICAL ENGINEERING

OLD DOMINION UNIVERSITY
December 2003

Approved by:

Jen-Kuang Huang (Director)

Sebastian Bawab (Member)

Gene J.-W. Hou (Member)

Duc T. Nguyen (Member)

ABSTRACT

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Anusit Anmanatarkul
Old Dominion University, 2003
Director: Dr. Jen-Kuang Huang

The primary influences of ship motion are roll motion. The purpose of this dissertation is to discuss a means to reduce the roll amplitude of ship motion in the case of zero forward speed using the roll mitigation device known as the flume tank, or U-tube, tank. Passive control and active control are studied. Optimal feedback control is the designated algorithm for activated roll mitigation device. The assumed model in this study is a submarine chaser. The linear coupled equation of swaying, rolling, and yawing motion of this assumed model is studied. The roll motion of this vessel is investigated under Sea State 3. The irregular wave, which attacked the ship hull, is studied with different encounter angles. The large roll amplitude is found when the wave encounters ship hull at Beam Sea ($\beta = 90^{\circ}$). The simulation results of passive control demonstrate that the flume tank creates high damping system under various wave angle's attacks. This mitigation device can cancel the roll amplitudes over 50%. For an activated anti-rolling tank, the optimal feedback control (LQR) shows that the full-state-variables method results in a high-damped system as well as the suboptimal feedback control. Technically, the linear coupled equation of motion should improve to the nonlinear range in order to obtain higher accuracy of the ship's motion and online control algorithms should be developed.

ACKNOWLEDGMENTS

I would like to thank my advisor, Dr. Jen-Kuang Huang, for his kindness, guidance, care, and motivation. Without his encouragement and support, my dissertation could not have been completed. I especially appreciate the member of my doctoral committee, Dr. Sebastian Bawab, Dr. Gene Hou, Dr. Duc T. Nguyen, for their constructive suggestions in revising the dissertation.

This dissertation would not have been possible without my mother and my brother in Thailand for their encouragement, and especially my wife Suwimol Anmanatarkul, for her patience, understanding and substantial support.

Nomenclature

ρ	Density
ω	The frequency
y	Sway displacement
ϕ	Roll angle
θ	Pitch angle
ψ	Yaw angle
ζ	Wave elevation
F_{sway_wave}	Swaying force generated by sea wave
M_{roll_wave}	Rolling moment generated by sea wave
M_{yaw_wave}	Yawing moment generated by sea wave
M_{pump}	Generated moment due to the pressure difference between two sides of the pump
M_{grav}	Generated moment due to the gravitational force acting on the fluid
M_{acc}	Generated moment due to the acceleration of fluid in the flume tank
$S(\omega)$	Wave spectrum
A_{tank}	Area of tank
A_{pipe}	Area of pipe
H_{tank}	Nominal height of the water in each tank from the center of the cross pipe
L_{stimu}	Distance between the centerlines of the tanks
$L_{X\_CM\_tank}$	Distance of the center of the cross pipe to the center of gravity of the ship in t X_0-direction
$L_{Z\_CM\_tank}$	Distance of the center of the cross pipe to the center of gravity of the ship in t Z_0-direction

$h_{\tan k}$	Change of water head from its nominal height of the water in the tank
ΔP	Pressure difference between two sides of the pump
$\dot{( )}$	First derivative with respect to time
$\ddot{( )}$	Second derivative with respect to time
$(\bullet)_{\tan k}$	Variable of tank
$(\bullet)_{pipe}$	Variable of pipe
$(\bullet)_{stimu}$	Variable of stimulator

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CHAPTER I

INTRODUCTION

Marine vehicles have been dramatically researched for over a decade. Currently, many types of ships have been operating on the seas or rivers such as cargo ships, container ships, fishing ships, and tanker ships. Most marine vehicles not only support business issues, but military issues are also concerned in ship stability. Motion stability and stationary stability are to be considered[1]. The former stabilizes the ship's direction. The latter is to stabilize the ship while being anchored for any matter's operation. For instance, we anchor merchant ships during the cargo loading. Stationary stability is the topic discussed in this dissertation.

The motivation of research is to control the stability of the ship and prevent the ship's capsizing. Undesirable motion could cause the ship's capsizing, as well as an uncomfortable sensation while onboard. M.A.S. Neves, et.al.[2] investigated the dynamic stability of fishing vessels in longitudinal regular wave. The shape of the ship has influence on the amplification of the motions. They studied both analytically and experimentally when specific parameters, which are wave amplitude, frequency, metacentric height and roll damping moment, are changed. Additionally, M. Taylan[3] studied both static and dynamic aspects of a capsize phenomenon. Stability margin is considered to analyze ship hydrodynamics.

The journal mode adapted for this Dissertation is JGCD

In order to reduce the undesirable motions of marine vehicles under various Sea States, the reducing forces and moments are significant issues to be considered. An engineer can reduce the forces and moments by increasing the damping coefficient, reducing natural frequency, or even directly reducing the excitation forces and moments[4].

In order to counteract undesirable motions, several roll stabilization devices have recently become available. These include bilge keels, gyroscopic stabilizers, movement of weight, rudder action, jet flaps, fins, or passive and active roll tanks[4]. To achieve roll cancellation, the control algorithm is considered. Ching-Yaw Tzeng, et.al,[5] proposed a control algorithm for rudder roll stabilization, which is called a sensitivity function. Ching-Yaw Tzeng, et.al, used this method to achieve good disturbance rejection and considered the wave disturbance as output noise. Satoru Yamaguchi[6] proposed an adaptive control algorithm for activated anti-rolling tank. The results showed that the activated anti-rolling tank could reduce the roll motion of the ship in the regular sea.

The anti-rolling tank is represented in this dissertation. In order to reduce the rolling moment amplitude, the water transfer in the tank will generate the anti-rolling moment. The anti-rolling tanks can be categorized by using passive and active controllability. This anti-rolling tank is known as the U-tube or the flume tank. The anti-rolling tanks are called passive tank stabilizers when fluid in the flume tank is allowed to move freely from one tank to another tank. The primary disadvantage of using a passive tank is that the response of the system could be too slow.

The controlled active tank has resolved the problem of using a pump to transfer the water in the U-tube tank, as well as to accelerate the system response. The natural frequency of U-tube tank should be very close to the ship's natural frequency for proper damping of the ship motion. The following parameters could have an effect on the amplitude as well as the phase of the U-tube tank: frequency encountered, amplitude of ship rolling motion, length of the tank, breadth of the tank, water height in the tank, and position of the tank[7-8]. In this dissertation, the roll stabilization using controlled active anti-roll tanks has been presented. In order to stabilize the ship roll motion under the Sea States, the pump controller must be effective. The control algorithm used for a controlled water pump is called the optimal feedback control algorithm.

The optimal control theory has been established since the 1960s. The objective of optimal control theory is to determine the control input signals that will drive the system (plant) to properly physical constraint with minimum (or maximum) performance criterion. The optimal control theory has been developed for over a decade while most researchers apply the algorithms to various applications. Y.M. Ram[9] studied the optimal control problem for structural vibration within the second-order differential equation. Transforming the system into the state-space form is not required. Likewise, the Ricatti equation is not required to be solved. R. S. Burns[10] proposed the optimal control algorithm for pitch, heave and roll stabilization of surface vessels. The controller device is a fin. He tested the effective controller at forward speed.

1.1 Objective

In general, a seaway, ship motion is the name given to the oscillations performed by a vessel, treated as a perfectly rigid body, as it floats on the surface of still water or disturbed water. Ship motions, generally, are surging, swaying, heaving, rolling, pitching, and yawing motions. We can treat ship motion as uncoupled motions or coupled motions. In this dissertation, the coupled swaying, rolling, and yawing motions are investigated.

As we know, unfavorable ship motions influence the seaworthiness of a vessel, and may occasionally lead to the following consequences: capsizing of the ship, damage to the hull or to the individual structure on the vessel, flooding of the deck, disturbing the operation on a vessel, decreasing the speed of a vessel, seasickness, and a decrease in the accuracy of gunnery in war-ships. Faltinsen O.M.[11] expressed examples of important seakeeping and wave load problems for ships. This is illustrated in Figure 1.1. A stabilization device, which is the flume tank in this dissertation, is considered in order to eliminate or minimize roll amplitude as much as possible.

The objective of the dissertation is to apply the optimal feedback control theory to control the flume tank. The control algorithm will be designed for the proper water head transferred in the tank to generate the counteractive excitation. As expected, the roll angle will be reduced.

1.2 Dissertation Outline

Chapter II introduces the general equations of ship motion. The coupling of swaying, rolling, and yawing motion is represented and the state-space model is stated.

Chapter III briefly represents the theory of wave disturbance caused by seaways. The kinds of wave spectrum are expressed. Force and the moments caused by waves are discussed.

Chapter IV designs the flume tank. The details of the flume tank such as the dynamic equations of the fluid motion in the flume tank, and the limitations of flume tank are discussed in this chapter.

As explanation of the optimal control theory is presented in chapter V. The linear quadratic regulator (LQR) is illustrated. Then, theory is applied to the controlled water's motion in the flume tank.

In Chapter VI, the numerical simulation, the effective passive and active control is presented.

In Chapter VII, the discussions of the conclusion as well as the suggestions for future works are explained.

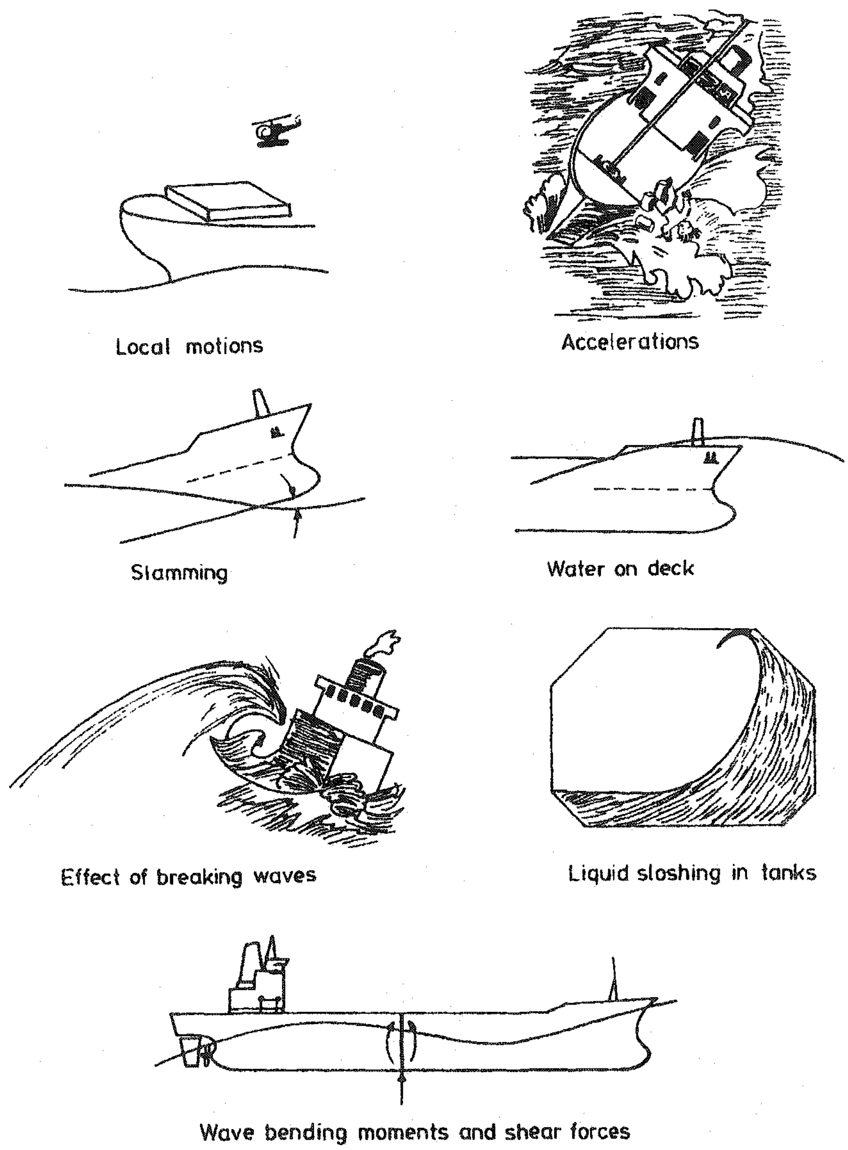

Figure 1.1 Example problems for the marine vehicles

CHAPTER II

DYNAMICS OF SHIP MOTION UNDER SEAWAY

The significance of control design is to realize the dynamic characteristics of the physical system to be controlled. Therefore, the equation of marine vehicle motion is the initiation of controller design. Mathematical models of the dynamic equation imply the modeling stability, control and motion response to the environmental disturbances[12].

Recently, many researchers have been approached as for the prediction of ship motion under Sea States. Analytical and experimental methods have been explored for over a decade. For the most important, the analysis of ship motions can be separated into the horizontal plane and vertical plane[13]. The motions in the horizontal plane include: surge, sway, roll, and yaw. The heaving, and pitching motions are in the vertical plane. Fukuzo Tasai[14] presents the equations of ship motion in the Beam sea that are derived from the Strip Theory as well as the experimental results that verified the theory. The ship's responses with coupling motions are represented in his research. Part of his research is the coupled equation of heaving and pitching motions. In the horizontal plane, the coupling motions of swaying and yawing motions as well as the coupling motions of swaying, rolling, and yawing motions are his derivatives[14]. In this dissertation, the most interesting is the linear coupling motions of swaying, rolling, and yawing motions. In recent years, the coupled equations of motion have been concerned because it might affect performance of ship stability control[14-16].

Generally, most engineers analyze ship's motion using the classical theory called the "Strip Theory". The Strip Theory is briefly stated in this dissertation. Recently, the methodology used to determine ship's motion has been continuously developed. The neural network is a modern technique to identify the ship's motion[17]. Haddra and Jinsong[17] proposed this method to define coupled heaving, and pitching motion in the random seas. Aryanpour M. and Ghorashi M.[18] investigate various ship's motion in the sea such as heaving and pitching motion. The analytical coupled equations of motion have predicted the heaving and pitching motion by applying the Strip Theory. A ship's behavior is subject to various moving forces and moments and is of great technical importance in the realistic sea because this circumstance is possible at any moment during a ship's operation. Not only is the prediction of ship motion determined in a frequency domain, but also in a time domain prediction[19-20]. Green's function and Bernoulli's equations have been applied for this circumstance. The more degrees of freedom there are, the more accurately the ship's predicted motion can be represented.

2.1 Ship Dynamic Modeling

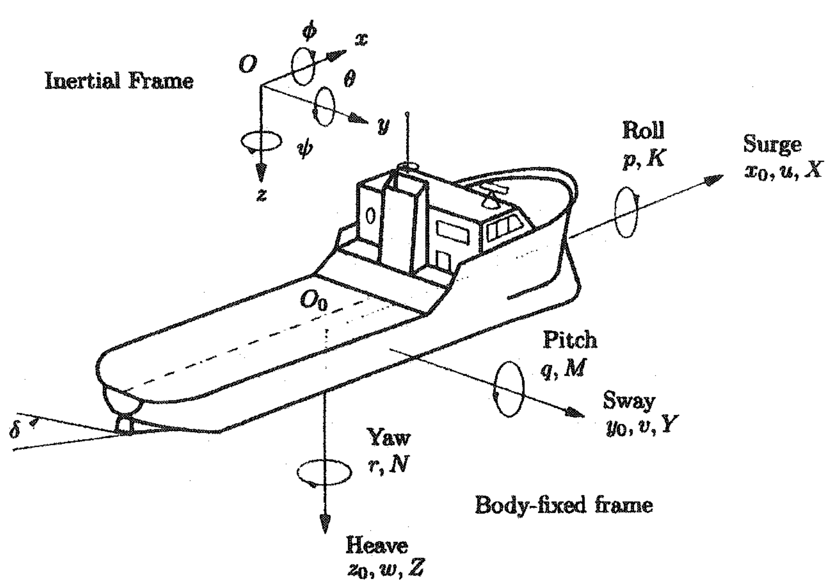

Figure 2.1 The Standard Ship Motions Coordinate System

The six degrees of freedom coordinates are necessary to determine the position and orientation of a rigid body

- the position and translational motions along the x-, y- and z- axes, and
- the orientation and rotational motions about the x-, y-, and z- axes,

The following are definitions of all motions for marine vehicle: surge, sway, heave, roll, pitch, and yaw. The notations used for marine vehicles are shown in Table 2.1.

Table 2.1: The notations used for marine vehicles.

DOF		Forces and Moments	Linear and Angular vel.	Positions and Euler angles
1	Motions in the x-direction (surge)	X	u	x
2	Motions in the y-direction (sway)	Y	v	y
3	Motions in the z-direction (heave)	Z	w	z
4	Rotation about the x-axis (roll)	K	p	ϕ
5	Rotation about the y-axis(pitch)	M	q	θ
6	Rotation about the z-axis (yaw)	N	r	ψ

Generally, the six degrees of freedom must be investigated in order to understand all responses of marine vehicles in regular waves[21]. The general form of the basic linearized equations in six degrees of freedom is based on Newton's second law.

$$\sum_{k=1}^{6} \Delta_{jk} \ddot{\eta}_k(t) = F_j(t) \tag{2.1}$$

When j is equal to the mode of motion (j = 1,2…6), Δ_{jk} are the generalized inertia matrix for the ship, $\ddot{\eta}_k$ are the accelerations in mode of motion (k), and F_j are the total forces or moments acting on the ship in j-direction.

For a ship with lateral symmetry, Equation (2.1) can be written as the following six degrees of freedom:

$$\Delta\left(\ddot{\eta}_1 + \bar{z}_c \ddot{\eta}_5\right) = \Gamma_1 \quad \text{(Surge)},$$

$$\Delta\left(\ddot{\eta}_2 - \bar{z}_c \ddot{\eta}_4 + \bar{x}_c \ddot{\eta}_6\right) = \Gamma_2 \quad \text{(Sway)},$$

$$\Delta\left(\ddot{\eta}_3 - \bar{x}_c \ddot{\eta}_5\right) = \Gamma_3 \quad \text{(Heave)}, \tag{2.2}$$

$$I_{44}\ddot{\eta}_4 - I_{46}\ddot{\eta}_6 - \Delta\bar{z}_c \ddot{\eta}_2 = \Gamma_4 \quad \text{(Roll)},$$

$$I_{55}\ddot{\eta}_5 + \Delta\left\{\bar{z}_c \ddot{\eta}_1 - \bar{x}_c \ddot{\eta}_3\right\} = \Gamma_5 \quad \text{(Pitch)},$$

$$I_{66}\ddot{\eta}_6 - I_{64}\ddot{\eta}_4 + \Delta\bar{x}_c \ddot{\eta}_2 = \Gamma_6 \quad \text{(Yaw)},$$

where Γ_j are the forces, which consist of two components. One is the component of the gravitational force acting on the ship in the j^{th} – direction. Another is the component of the fluid force acting in the ship in the j^{th} – direction. Once the hydrodynamic restoring forces, the added mass, and damping terms are determined, they are all brought to the left-hand side. Hence, Equation (2.1) yields the following governing equation.

$$\sum_{k=1}^{6}\left[\omega^2\left(\Delta_{jk} + A_{jk}\right) + i\omega B_{jk} + C_{jk}\right]x_k = F_j, \quad j = 1,2,\ldots,6 \tag{2.3}$$

where ω = wave frequency,

Δ_{jk} = the mass/inertia matrix for the ship,

A_{jk} = the added mass matrix,

B_{jk} = the damping matrix,

C_{jk} = the stiffness matrix,

F_j = the exciting force due to waves,

$\bar{x}_k$ = the unknown complex motion amplitude.

2.2 Strip Theory

The purpose of using the Strip Theory is to determine the coefficients of added mass, damping, restoring force, and restoring moment. The three dimensional hydrodynamic problem can then be reduced into a series of the two-dimensional hydrodynamic problem.

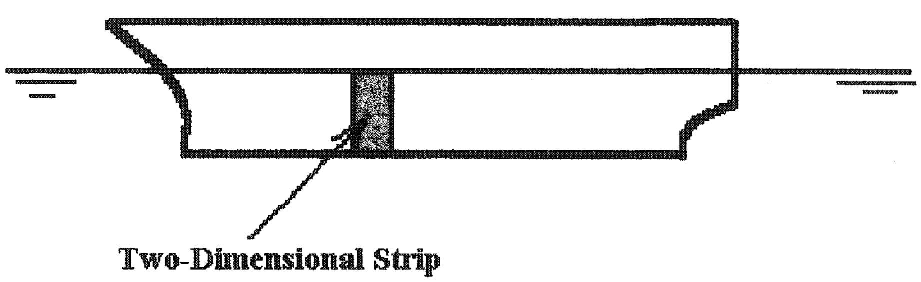

Figure 2.2 Two-dimensional diagram

There are some limiting assumptions associated with Strip Theory. They are briefly stated as follows:

- The vessel model is a slender body,
- Zero forward speed and high frequency,
- The ship's hull is the rigid body,
- The draft is much greater than the wavelength,
- The motions are small (i.e. sin $(\phi) \cong \phi$).

To apply Strip Theory to the ship motion problem, the ship is divided into several even sections of the ship's submerged hull. For a two-dimensional problem, each section has been calculated for the hydrodynamic properties such as added mass, damping, and stiffness. Once the two-dimensional problem has been solved for each section, the summation of the coefficients for each section are summed up over the length of the ship. The coefficients in the equation of motion using Strip Theory are shown in Appendix A. The following section states the derivatives of coupled equations of motions, which are swaying, rolling and yawing.

2.3 Coupled Equations of Swaying, Yawing, and Rolling Motion

Rolling moment and swaying forces are generated when the ship moves in sway's direction. In Figure 2.3, the explanation of the generated force and moment has been illustrated.

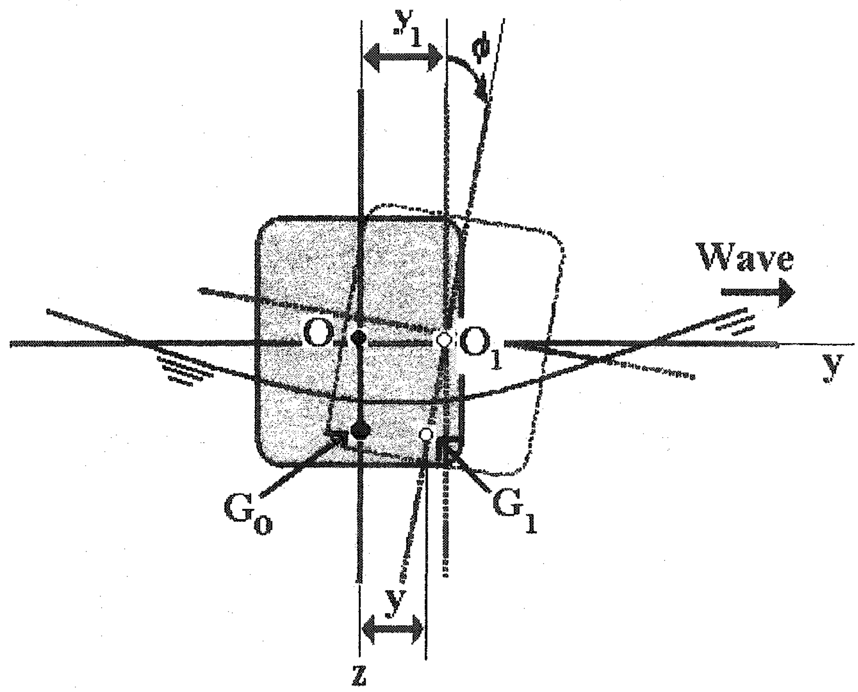

Figure 2.3 Free body of sway and roll motion

Hydrodynamic force and moment generate the swaying displacement (y) and the rolling moment (ϕ) about G_o. Letting F'_y and F'_ϕ be forces due to swaying motion (y_1) and rolling motion (ϕ) about O, respectively,

we obtain

$$F'_{y_1} = -m''\ddot{y}_1 - N_y\dot{y}_1 \tag{2.4}$$

$$F'_{\phi} = \frac{I'_x}{l_{\phi}}\ddot{\phi} + \frac{N_{\phi}}{l_w}\dot{\phi} \tag{2.5}$$

where m'' = added mass of swaying motion for the two-dimensional body

I'_x = added mass moment of inertia of rolling about O

N_y = coefficient of damping force of swaying motion

N_{ϕ} = coefficient of damping force of rolling motion

Letting m, and F'_{ye} be the mass and the external force for the two-dimensional body, the equation of sway motion, according to Equation (2.4), can be represented as the following equation:

$$m\ddot{y} + m''\ddot{y}_1 + N_y\dot{y}_1 - \frac{I'_x\ddot{\phi}}{l\phi} - \frac{N_{\phi}\dot{\phi}}{l_w} = F'_{ye} \tag{2.6}$$

The following steps can determine the equation of roll motion. Let the M_{y1}, and M_{ϕ} is the rolling moment about G_o, due to the swaying displacement (y_1) and the rolling motion (ϕ) about O, respectively. M_k is the linear restoring moment and $M'_{\phi e}$ is the exciting moment. All moments (M_{y1}, M_{ϕ}, M_k, and $M'_{\phi e}$) are shown as the following equations.

$$M_{y_1} = -m''\ddot{y}_1\left(\overline{OG_o} - l_y\right) - N_y\dot{y}_1\left(\overline{OG_o} - l_w\right) \tag{2.7}$$

$$M_{\phi} = \left(\frac{I'_x\overline{OG_o}}{l_{\phi}} - I'_x\right)\ddot{\phi} + \left(\frac{N_{\phi}\overline{OG_o}}{l_w} - N_{\phi}\right)\dot{\phi} \tag{2.8}$$

$$M_k = -W\overline{G_o M}\phi \tag{2.9}$$

$$M'_{\phi e} = F'_{ye}\left(\overline{OG_o} - l_w\right) \tag{2.10}$$

Finally, the equation of roll motion can be written by substituting the mass moment of inertia of rolling motion about G_0. We obtain

$$I_x\ddot{\phi} + W\overline{G_o M}\phi + \left(I'_x - \frac{I'_x}{l_\phi}\overline{OG_o}\right)\ddot{\phi} + \left(N_\phi - \frac{N_\phi}{l_w}\overline{OG_o}\right)\dot{\phi}\,\ddot{y}_1$$

$$+ m''\left(\overline{OG_o} - l_y\right) + N_\phi\left(\overline{OG_o} - l_w\right)\dot{y}_1 \quad = F'_{ye}\left(\overline{OG_o} - l_w\right) \tag{2.11}$$

According to a two-dimensional body, the relationship between the hydrodynamic moments generated by swaying motion and the forces generated by rolling motion are

$$\frac{N_\phi}{l_w} = \frac{N_y}{l_w}, \text{ and } \frac{I'_x}{l_\phi} = m''l_y. \tag{2.12}$$

Using the relation of Equation (2.12), Equations (2.6) and (2.11) becomes

$$m\ddot{y} + m''\ddot{y}_1 + N_y\dot{y}_1 - m''l_y\ddot{\phi} - N_y l_w\dot{\phi} \quad = F'_{ye} \tag{2.13 a}$$

$$\left(I_x + I'_x - m''l_y\overline{OG_o}\right)\ddot{\phi} + N_y l_w\left(l_w - \overline{OG_o}\right)\dot{\phi} + W\overline{G_o M}\phi$$

$$+ m''\left(\overline{OG_o} - l_y\right)\ddot{y}_1 + N_y\left(\overline{OG_o} - l_w\right)\dot{y}_1 \quad = F'_{ye}\left(\overline{OG_o} - l_w\right) \tag{2.13 b}$$

Simply, eliminate y_1 in Equations (2.13 a) and (2.13 b) using $y_1 = y + \overline{OG_o}.\phi$, then Equations (2.13 a) and (2.13 b) become

$$\left(m+m''\right)\ddot{y}+N_y\dot{y}+m''\left(\overline{OG_o}-l_y\right)\ddot{\phi}+N_y\left(\overline{OG_o}-l_w\right)\dot{\phi} \quad = F'_{ye} \tag{2.14}$$

$$\left(I_x+I_{add}\right)\ddot{\phi}+N_y\left(l_w-\overline{OG_o}\right)^2\dot{\phi}+W\overline{G_oM}\phi+m''\left(\overline{OG_o}-l_w\right)\ddot{y}$$

$$+N_y\left(\overline{OG_o}-l_w\right)\dot{y} \quad = F'_{ye}\left(\overline{OG_o}-l_w\right) \tag{2.15}$$

where $I_{add} = I'_x - 2m''l_y\overline{OG_o} + m''\overline{OG_o}^2$

The Equations (2.14) and (2.15) are only the coupled equations of sway and roll motion, respectively. Finally, the derivation of coupled swaying, rolling and yawing motion can be determined. The swaying oscillation (y), yawing (ψ), and rolling (ϕ) are about G_o-x_1 axis.

$$y_1 = y + \overline{OG_o}\phi + x\psi\,. \tag{2.16}$$

Similarly, substituting Equation (2.16) into Equations (2.13 a) and (2.13 b) in order to eliminate y_1, the hydrodynamic swaying force acting on a section distance x from G_o becomes

$$\frac{dF_y}{dx} = -m''\ddot{y}-N_y\dot{y}-m''x\ddot{\psi}-N_yx\dot{\psi}-m''\left(\overline{OG_o}-l_y\right)\ddot{\phi}-N_y\left(\overline{OG_o}-l_w\right)\dot{\phi}+F'_{ye}\,. \tag{2.17}$$

The hydrodynamic force due to the generated force as Equation (2.17) will lead to is

$$\frac{dM_\psi}{dx} = \left(\frac{dF_y}{dx}\right).x\,. \tag{2.18}$$

According to Equations (2.13 b) and equation (2.16), the hydrodynamic rolling moment about G_o-x_1 axis will become

$$\frac{dM_\phi}{dx} = \left\{-m''\overline{OG_o}\left(\overline{OG_o}-l_y\right)+m''l_y\left(\overline{OG_o}-l_\phi\right)\right\}\ddot{\phi}+N_y\left(l_w-\overline{OG_o}\right)\left(\overline{OG_o}-l_w\right)\dot{\phi}$$

$$-m''\left(\overline{OG_o}-l_y\right)\ddot{y}-N_y\left(\overline{OG_o}-l_w\right)\dot{y}-m''\left(\overline{OG_o}-l_y\right)x\ddot{\psi}$$

$$-N_y\left(\overline{OG_o}-l_w\right)x\dot{\psi}+F'_{ye}\left(\overline{OG_o}-l_w\right). \tag{2.19}$$

Using the Strip Theory method, we obtain the following equation of motions

$$m_o\ddot{y} = \int_{-l_1}^{l_2}\left(\frac{dF_y}{dx}\right)dx$$

$$I_\psi\ddot{\psi} = \int_{-l_1}^{l_2}\left(\frac{dF_y}{dx}\right).(x).dx \tag{2.20}$$

$$I_x\ddot{\phi}+W\overline{G_oM}\phi = \int_{-l_1}^{l_2}\left(\frac{dM_\phi}{dx}\right).dx\,.$$

After Equation (2.20) is integrated along the ship's length, the coupled equations of swaying, rolling, and yawing motion become

$$m_o\left(1+\overline{K}_y\right)\ddot{y}+m_o\overline{K}_y\overline{x}_4\ddot{\phi}+m_o\overline{K}_y\overline{x}_1\ddot{\psi}+\overline{N}_y\dot{y}+\overline{N}_y\overline{x}_5\dot{\phi}+\overline{N}_y\overline{x}_2\dot{\psi}=F_{ye}, \tag{2.21}$$

$$m_o\overline{K}_y\overline{x}_4\ddot{y}+\left(I_x+I'_x\right)\ddot{\phi}+m_o\overline{K}_y\overline{x}_6^2\ddot{\psi}+\overline{N}_y\overline{x}_5\dot{y}+\overline{N}_\phi\dot{\phi}+\overline{N}_y\overline{x}_7^2\dot{\psi}+W\overline{G_oM}.\phi=M_{\phi e} \tag{2.22}$$

$$m_o\overline{K}_y\overline{x}_1\ddot{y}+m_o\overline{K}_y\overline{x}_6^2\ddot{\phi}+\left(I_z+I'_z\right)\ddot{\psi}+\overline{N}_y\overline{x}_2\dot{y}+\overline{N}_y\overline{x}_7^2\dot{\phi}+\overline{N}_\psi\dot{\psi}=M_{\psi e}\,. \tag{2.23}$$

For more details of the derivation, see Reference 14.

2.4 State-Space Models

Equations (2.21), (2.22), and (2.23) are second-order differential equations; thus, those equations can be expressed in the following matrix form:

$$[M]\{\ddot{w}\}+[\zeta]\{\dot{w}\}+[K]\{w\}=[f(w,t)] \tag{2.24}$$

where $\ddot{w}$, $\dot{w}$, and w are vectors of generalized acceleration, velocity, and displacement, respectively. $f(w,t)$ is the force or input function over the period at the specific location.

Generally, the first-order matrix differential equation, which is known as the state-space model, has been applied in the modern control theory. In this dissertation, the discrete-time state-space model is represented. Recently, digital computer technology has had a profound effect; therefore, with digital control, an engineer is able to change a control strategy by writing a different program rather than constructing the analog control system. In this section, the continuous-time state-space model and discrete-time state-space model are briefly presented.

2.4.1 Continuous-Time State-Space Model

According to the second-order equation of motion in the matrix form [Equation (2.24)], we assume that the matrix M is invertible. Then, we can solve for $\ddot{w}$ as follows:

$$\ddot{w}=-[M]^{-1}[\zeta]\{\dot{w}\}-[M]^{-1}[K]\{w\}+[M]^{-1}f(w,t). \tag{2.25}$$

Hence, we can express the original second-order equations into the first-order form as

$$\frac{d}{dt}\begin{bmatrix} w(t) \\ \dot{w}(t) \end{bmatrix} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}\zeta \end{bmatrix}\begin{bmatrix} w(t) \\ \dot{w}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1}B \end{bmatrix} u(t) \tag{2.26}$$

where $f(w,t) = [B]u(t)$.

Rewriting Equation (2.26), we can obtain the compact form as

$$\dot{x} = A_c x + B_c u \tag{2.27}$$

where

$$A_c = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}\zeta \end{bmatrix}, \quad x = \begin{bmatrix} w(t) \\ \dot{w}(t) \end{bmatrix}, \quad B_c = \begin{bmatrix} 0 \\ M^{-1}B \end{bmatrix},$$

where x is represents the state variables. Additionally, The vector denotes another equation describing the output or measured quantities in a function of state variables as y.

$$y = C.x \tag{2.28}$$

Generally, matrix C is called the output influence matrix. Therefore, Equations (2.27) and (2.28) are called the continuous-time state-space model of the system.

2.4.2 Discrete-Time State-Space Model

In structural testing, the analog and digital signals are related via digital to analog (D/A) or analog to digital (A/D) converters. Most of the sensors continuously generate an analog signal within a period of time. Thus, the analog signal must be sampled for a digital computer to interpret it. This section will explain how a continuous-time state-space model can be converted into a discrete-time state-space model for digital control. In order to convert from continuous-time to discrete-time, a zero-order hold (or sample and hold) is required. A zero-order hold takes a continuous signal and turns it into a stepwise signal in which the signal is sampled and held for a certain interval of time (Δt).

Given the initial condition $x(t_0)$at some $t = t_0$, Equation (2.27) can be solved for $x(t)$.

$$x(t) = e^{A_c(t-t_0)}x(t_0) + \int_{t_0}^{t} e^{A_c(t-\tau)}B_c u(\tau)d\tau. \tag{2.29}$$

Then, consider the discrete sampling interval 0, Δt, $2\Delta t,\ldots,(k+1)\Delta t$. The $x(t)$ is changed from one time step to the next step. Substitution of $t = (k+1)\Delta t$ and $t_0 = k\Delta t$ into Equation 2.29, as well as, the input $u(k\Delta t)$ is held constant over the period of time with a zero-order hold yield

$$x[(k+1)\Delta t] = e^{A_c\Delta t}x(k\Delta t) + \left[\int_0^{\Delta t} e^{A_c\tau'}d\tau' B_c\right]u(k\Delta t), \tag{2.30}$$

where $\tau' = (k+1)\Delta t - \tau$.

Using a simplified notification k for the time argument $(k\Delta t)$, a discrete-time state-space model can be represented in following compact form.

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k), \\ y(k) &= Cx(k) + Du(k), \end{aligned} \tag{2.31}$$

where $A = e^{A_c\Delta t}$, $B = \int_0^{\Delta t} e^{A_c\tau'}d\tau' B_c$.

For more details of state-space model, see reference 26-27.

CHAPTER III

WAVE DISTURBANCE

Waves can be defined on transition with their energy being carried away from its origin[22]. Wind is the most common wave system energy source. Moreover, geological events cause the tidal waves due to seismic action. Current is one example of a wave energy source. The interaction of ocean currents can create a very large wave system as well.

Marine vehicles usually encounter waves in the sea. Therefore, the motion of a marine vehicle can be investigated due to wave action. However, the sea wave excites the ship, and thus, the ship's response can be measured with the surging, swaying, heaving, rolling, pitching, and yawing amplitudes.

3.1 Regular Wave Motion

The regular wave motion must be introduced in this section. Generally, the water wave is the phenomenon of a moving shape distortion of the water surface[22]. The reader may find more details for characteristics of waves in the reference sections. Usually, the sinusoidal wave is a wave that has been used to investigate a ship's behaviors under way. The shape of the sinusoidal wave is symmetric to both the crest and trough. The definitions of crest and trough are illustrated in Figure 3.1.

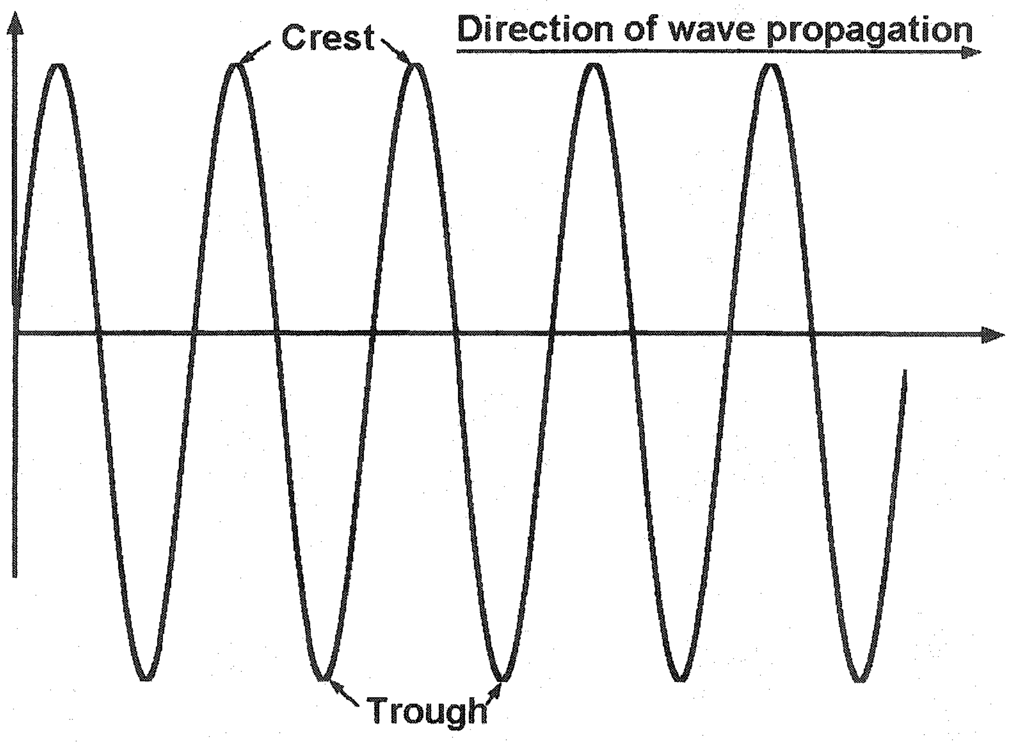

Figure 3.1 A regular wave diagram

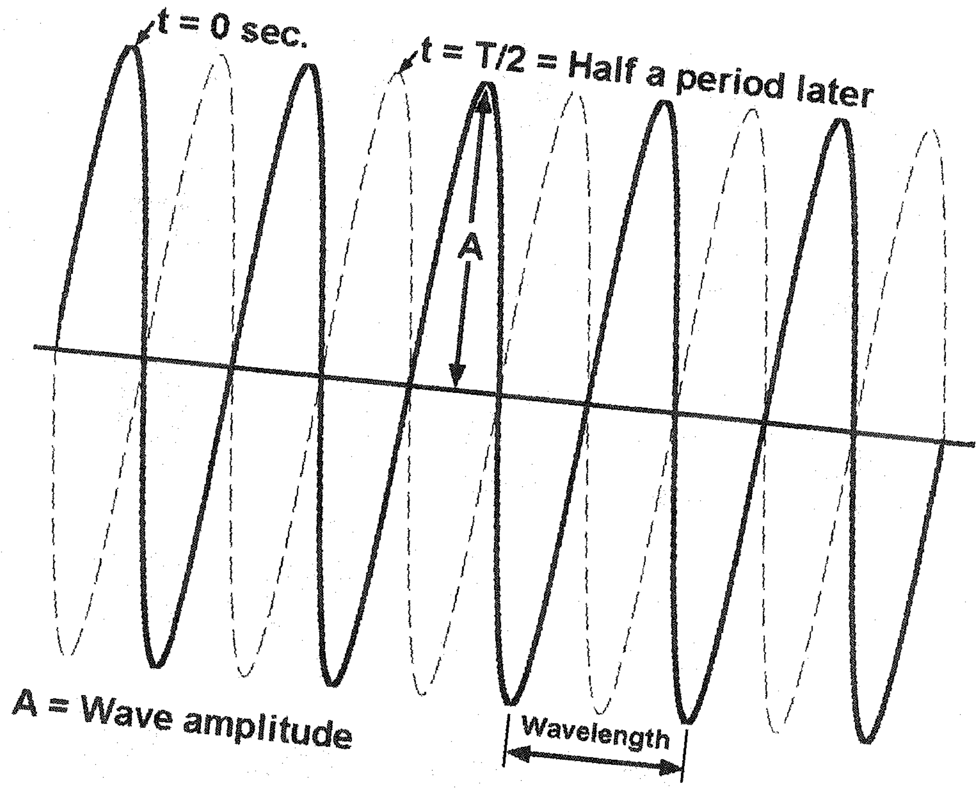

Figure 3.2 Regular sine wave

According to Figure 3.2, the sine (or cosine) function defines the regular wave as the following wave function $[\eta(x,t)]$.

$$\eta(x,t) = A.\sin\left\{\left(\frac{2\pi}{T}\right).(t) - \left(\frac{2\pi}{\lambda}\right).(x)\right\}. \tag{3.1}$$

The angular frequency (ω) equals $2\pi/T$ and is expressed in radians/second. The wave number (k) (rad/m) is the quantity of $2\pi/\lambda$. The constant "A" is called the wave amplitude. λ (Greek letter lambda) represents the wavelength. Equation (3.1) has the following properties:

- A sine (or cosine) function of (x) can be represented by a fixed period.

- Similarly, a sine function of (t) can be represented by a fixed displacement.
- The function can repeat itself each time but the x is increased with the wavelength (λ). It can be represented by the equality equations as $\eta(x,t)=\eta(x+\lambda,t)=\eta(x+n\lambda,t)$, where $n=\ldots-1,0,1,\ldots$.
- Similarly, the wave function can repeat itself each x but the time (t) is increased with the time period (T). $\eta(x,t)=\eta(x,t+T)=\eta(x,t+nT)$, where $n=\ldots-1,0,1,\ldots$.

3.2 Irregular Wave Motion

A ship motion can be disturbed by the wave under various Sea States. The sea state is the condition of the surface of the seas generated by prevailing wind. The simple pattern of the wave acting on the ship can be considered as the simple harmonic motion such as sinusoidal wave. Usually, it is called a regular wave. It does mean that the properties of wave, such as amplitude and period, remain the same. However, in the realistic sea, the properties of wave constantly change from time to time and place to place. This pattern is called the *irregular wave*.

Recently, researchers have paid attention to the realistic sea or irregular wave. The wave, which acts to the ship, in fact generates the forces and moments in the realistic sea. The ship's behaviors such as rolling, swaying, and yawing motions are more accurate if the irregular wave can be determined. Jianbo Hua and Wei-Hui Wang[23] studied roll motion under the irregular sea wave using a RoRo-ship for model testing. The JONSWAP spectrum was used as the irregular wave in their cases in order to investigate the roll behavior with various meta-centric height (GM). For this dissertation, the Pierson-Moskowitz spectrum has been proposed. The following section explains the determination of the wave function for irregular waves.

3.3 Wave Spectrum

As mentioned previously, wind is one source to generate waves. Generally, the process of wave generation is due to wind starts with small wavelets appearing over the water surface. The best representation of the wave's behavior under the irregular wave is the wave spectrum. Different sea states have different spectrums as shown in Figure 3.3.

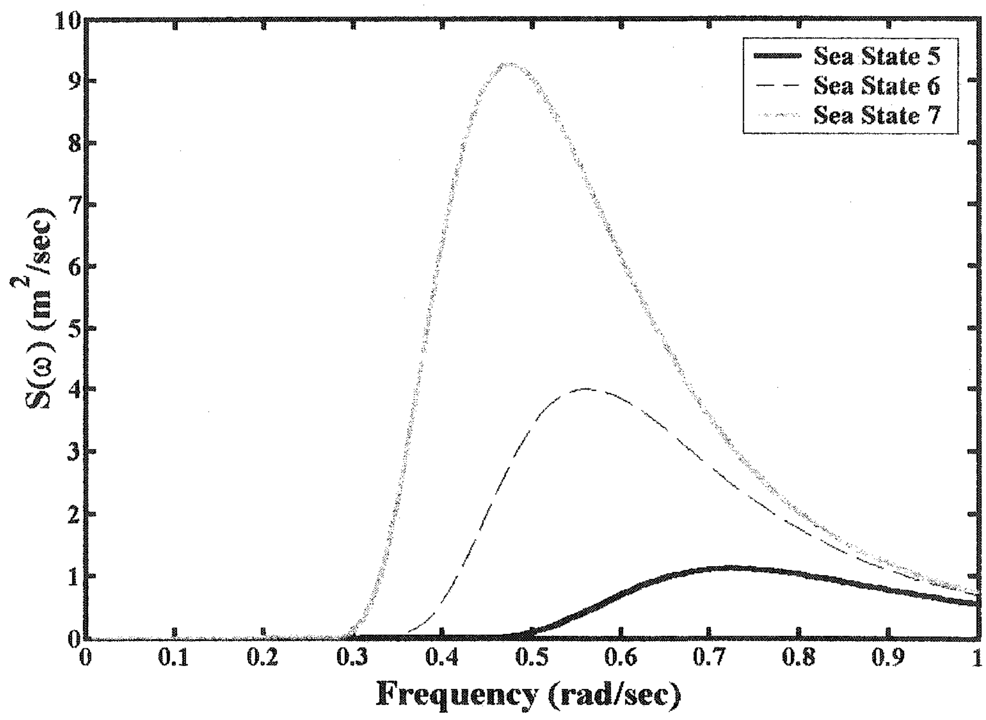

Figure 3.3 Wave spectrum under various sea states

The determination of wave spectrum amplitude is based on the standard wave spectra. The following section shows the current standard wave spectra.

3.4 Standard Wave Spectra

The primary wave spectrum used in this dissertation is based on the Pierson-Moskowitz spectrum. Other mathematical models briefly explain the wave spectrum, and are listed as follows.

3.4.1 Neumann Spectrum

This wave spectrum is the earliest formulation. Neumann had created in 1950. Neumann proposed the spectrum as follows:

$$S(\omega) = C\omega^{-6}\exp\left(-2g^2\omega^{-2}U^{-2}\right) \quad \left(m^2 - s\right), \tag{3.2}$$

where C is an empirical constant, U represents the wind speed, and g is the acceleration of gravity.

3.4.2 Bretschneider Spectrum

Bretschneider proposed this spectrum in 1959. He expressed the wave spectrum in term of significant wave height. The spectrum is written as:

$$S(\omega) = \frac{1.25}{4}\frac{\omega_0^4}{\omega^5}H_s^2\exp\left(-1.25\left(\omega_0/\omega\right)^4\right) \quad \left(m^2 - s\right), \tag{3.3}$$

where ω_0 is the modal frequency and H_s is the significant wave height.

3.4.3 Pierson-Moskowitz Spectrum

We concentrate on this particular spectrum in dissertation. This spectrum represents the Sea State conditions so called the Pierson and Moskowitz (PM – spectrum). The spectrum was proposed in 1963. The Pierson-Moskowitz Spectrum is written as:

$$S(\omega) = A\omega^{-5}\exp(-B\omega^{-4}) \quad (m^2 - s). \tag{3.4}$$

where

$$A = 8.1\times10^{-3}.g^2,$$

$$B = 0.74\left(\frac{g}{U}\right)^4.$$

Also, the Pierson-Moskowitz spectrum can be written as the significant wave height. Hence, only the parameter B has been changed, and the equation thus becomes:

$$B = 0.0323\left(\frac{g}{H_s}\right)^2 = \frac{3.11}{H_s^2}.$$

3.4.4 JONSWAP Spectrum

The Joint North Sea Wave Project (JONSWAP), known as the measurement program, was created in 1968-1969. The wave spectrum function is written:

$$S(\omega) = 155\frac{H_s^2}{T_1^4\omega^5}\exp\left(\frac{-944}{T_1^4\omega^4}\right)(\gamma)^Y. \tag{3.5}$$

Note that all standard wave spectra are obtained from different experiments as well as different locations in the sea.

3.5 Determination of Wave Function and Wave Slope

An irregular wave is assumed to be a combination of regular wave components. All components of regular waves can be identified by the wave spectrum shown in Figure 3.4. The determination of wave function and wave slope can be explained in the following steps.

- Divide the wave spectrum in Figure 3.4 into N intervals with equal lengths dω.
- Select a random frequency ω_i in each of the frequency intervals and determine $S(\omega_i)$.
- Determine the wave amplitude (A_i) and the wave number (k_i) as the following equation (i = 1…N).

$$A_i = \sqrt{2S(\omega_i).d\omega} \quad ,$$

$$k_i = \omega_i^2 / g \quad .$$

- Calculate the wave elevation (ς_i) and the wave slope (s_i) by the following equations.

$$\varsigma_i(0,t) = A_i \cos(\omega_i t + \phi_i)$$

$$s_i(0,t) = A_i k_i \sin(\omega_i t + \phi_i)$$

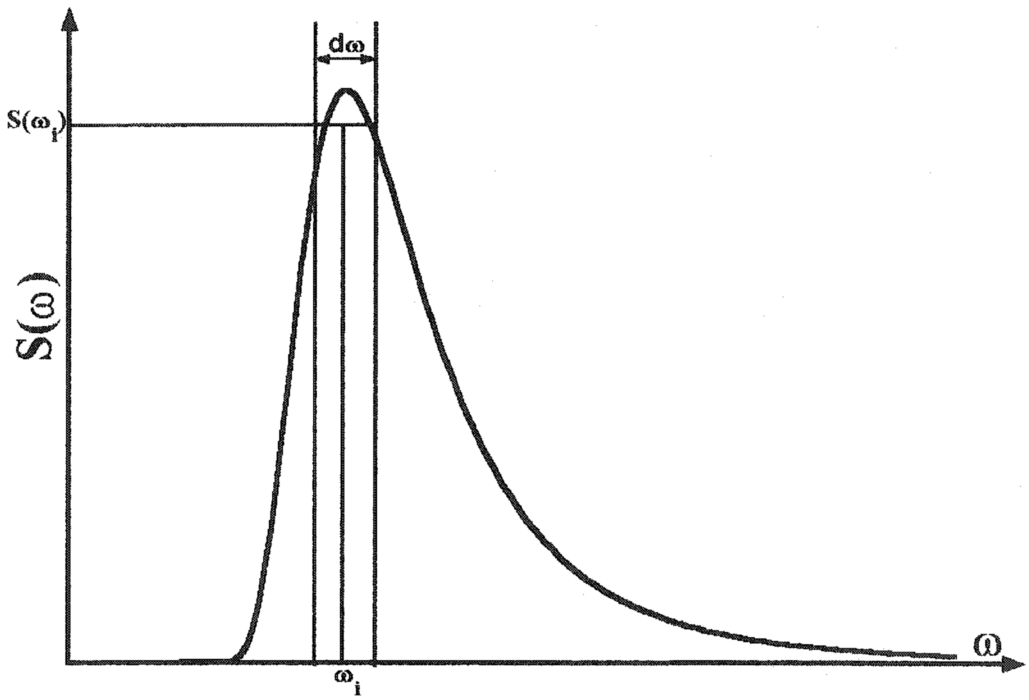

Figure 3.4 A Spectral Density of Sea Wave.

3.6 Connection between the Frequency Domain and Time Domain

In irregular waves, the configuration of the sea states is quite complicated due to the interaction of different wave systems. Therefore, a realistic sea cannot be represented by one wave pattern. The overall complicated wave system can be determined by considering many sinusoidal wave components as shown in Figure 3.5. Each wave component has its own spectral density, S (ω_i). The relationship between the time and frequency domain can also be illustrated in Figure 3.6.

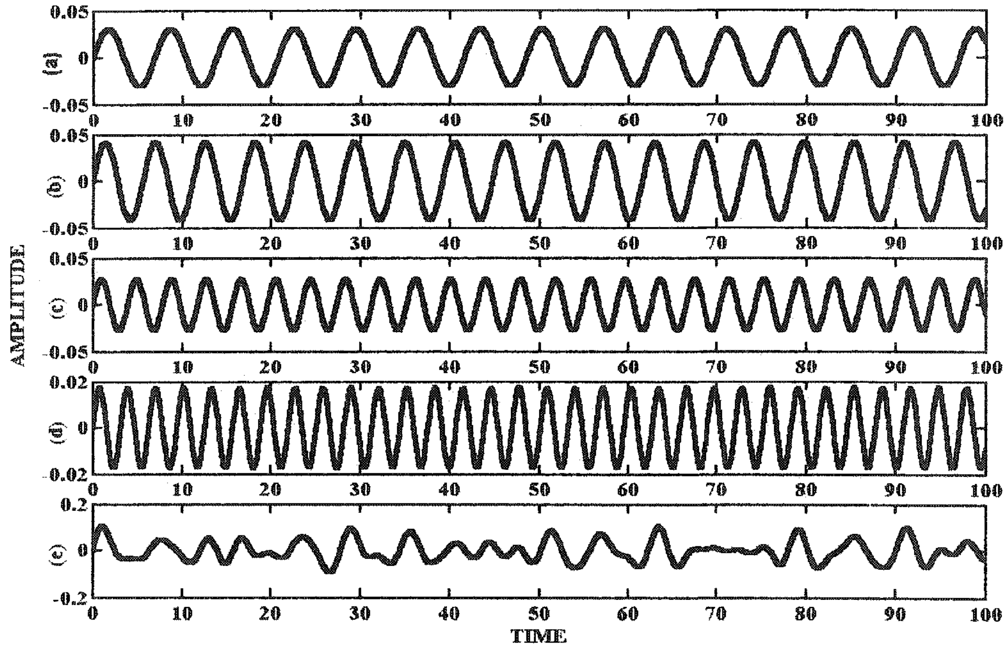

Figure 3.5 A time domain with various frequencies (a)-(d) and combination of all frequencies (e).

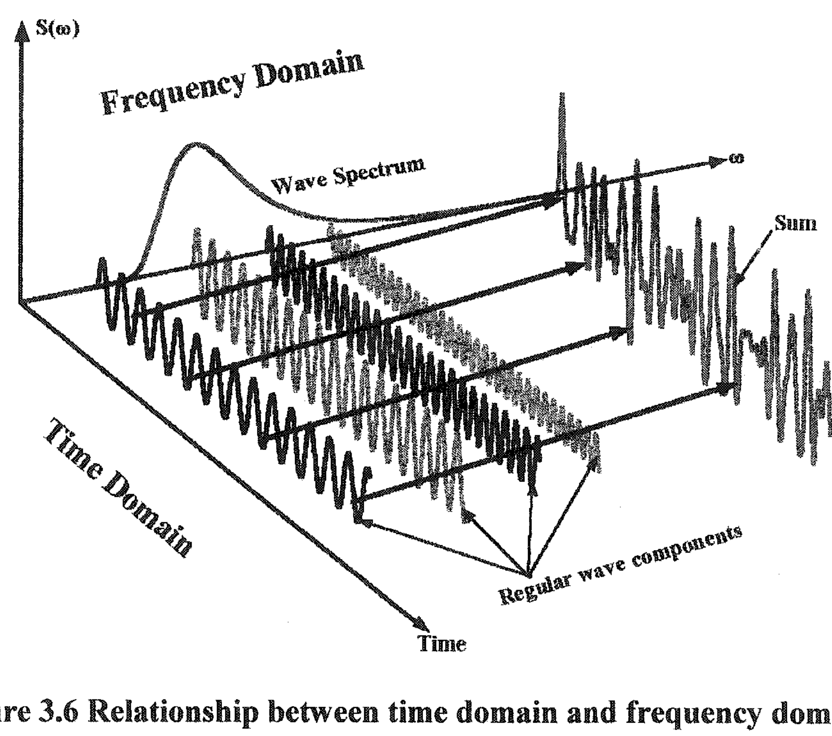

Figure 3.6 Relationship between time domain and frequency domain

3.7 Wave Induced Force and Moments

The 1st order wave disturbances generate the swaying force (Y_{wave}), rolling moment (K_{wave}), and yawing moment (N_{wave}) as the following formulas[24]:

$$F_{sway\_wave}(t) = \sum_{i=1}^{N} -\rho g B L T . \sin(\beta) s_i(t) \tag{3.6}$$

$$M_{roll\_wave}(t) = \sum_{i=1}^{N} \frac{1}{12} \rho g B^3 \sin(\beta) s_i(t) \tag{3.7}$$

$$M_{yaw\_wave}(t) = \sum_{i=1}^{N} \frac{1}{24} \rho g B L (L^2 - B^2) \sin(2\beta) . s_i(t) \tag{3.8}$$

where B is the vessel's breadth, L is the length of the ship, and T is the draft. The wave direction (β) can be described as following Figure 3.7.

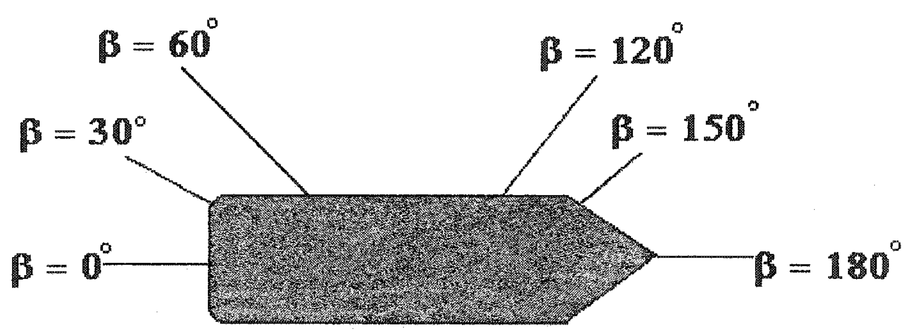

Figure 3.7 The Wave Direction Acting on the Ship

The wave direction between $\beta = 0^\circ$ and $\beta = 30^\circ$ is called "the Following Sea"; between $\beta = 30^\circ$ and $\beta = 60^\circ$ "the Quartering Sea"; between $\beta = 60^\circ$ and $\beta = 120^\circ$ "the Beam Sea"; between $\beta = 120^\circ$ and $\beta = 150^\circ$ "the Bow Sea"; and between $\beta = 150^\circ$ and $\beta = 180^\circ$ "the Head Sea".

CHAPTER IV

THE FLUME TANK DESIGN

Canceling of ship roll motion is done to comfort people onboard and prevent the ship from capsizing. Without a stabilizer device, the large roll amplitude may hamper the crew's ability to do their jobs. Motion stabilization devices are required in this circumstance. Recently, there have been many ship roll cancellation technologies introduced; most have successfully increased the stability of various marine vehicles. The following are some examples.

4.1 The Ship Roll Motion Stabilization's Technology

4.1.1 Bilge keels

Bilge Keels are the fixed fins attached to the exterior of the ship's hull almost perpendicular to the hull's surface. See Figure 4.1.

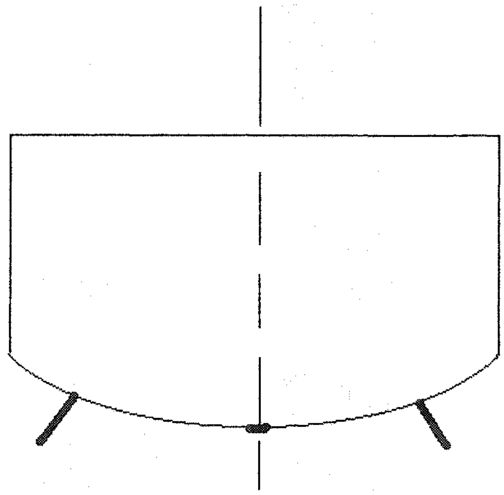

Figure 4.1 Mid-ship section with the bilge keels.

The lengths of bilge keels are usually from 25 to 50% of the ship's length. Also, the width of the bilge keels varies from 1 to 3 feet. With the bilge keels, the damping moment is generated by a component supplied by the pressure resistance of the bilge keel itself due to the change in the pressure distribution on the hull.

4.1.2 Gyroscopic Stabilizer

The Gyroscopic stabilizer consists of a heavy wheel free to rotate about an axis, which itself is confined within a framework and free to rotate about a perpendicular axis (see Figure 4.2).

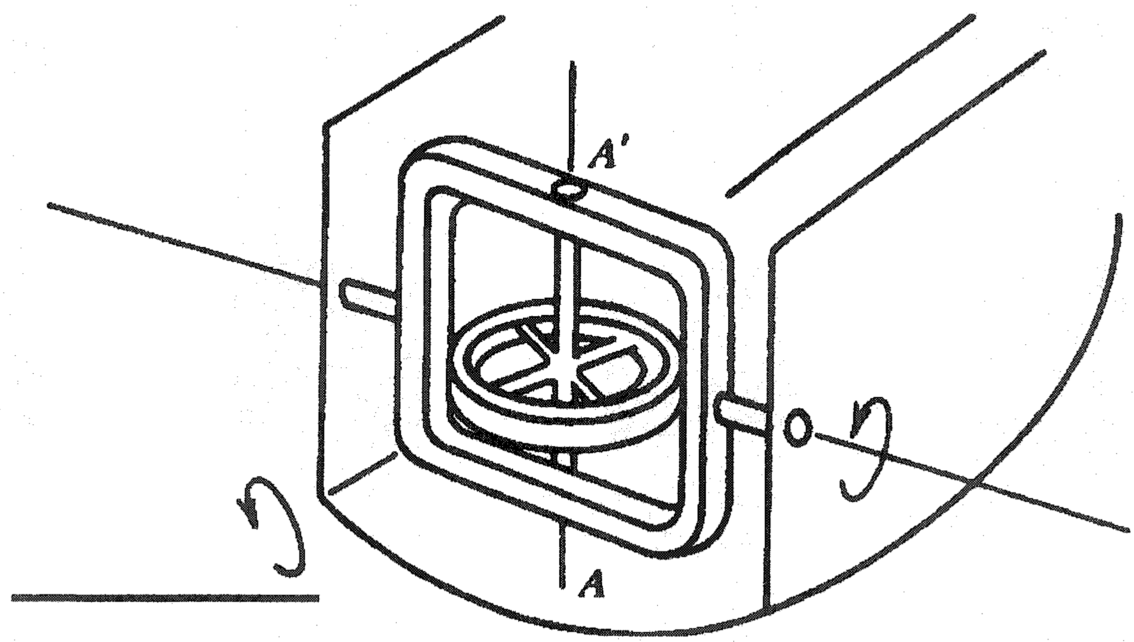

Figure 4.2 A gyroscopic stabilizer system

A gyroscopic stabilizer is very effective. It helps reduce roll from 60 to 80%. However, the gyroscopic device is very expensive, very heavy, and requires a large area to be properly installed.

4.1.3 Movement of Weight

This concept involves moving a large weight transversely across a ship to help balance the ship during excessive roll moment. For effective roll damping, the movement of the weight should be 90° behind the rolling motion of the ship in order to provide a moment to counteract the rolling moment. The more weights moving simultaneously in the proper direction there are, the greater the ship's stability.

4.1.4 Jet Flaps

Another development of stabilizer technology is jet flap (see Figure 4.3), which is similar to Bilge Keels. Jet flaps are suited for either low-speed or high-speed operation. In the jet flap system, a jet of fluid is ejected from or near the trailing edge of an aerofoil at an angle to the mainstream. The presence of the jet produces a much greater lift force than the vertical component of the resultant thrust.

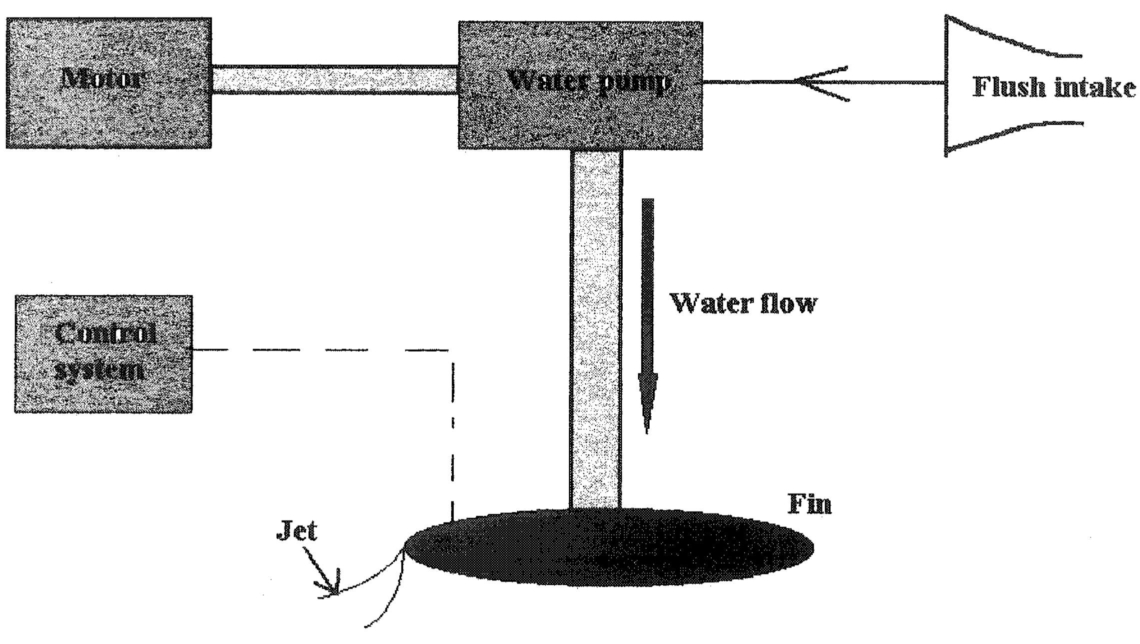

Figure 4.3 A Jet Flap Steam System

4.1.5 Rudder Stabilization

The roll moment usually generated by a rudder is small in magnitude unless its frequency is near the natural roll frequency of the ship. The rolling motion stabilized by rudder and a passive tank is more effective than by rudder alone.

4.1.6 Tank stabilizers

A tank stabilizer system consists of two tanks filled with water, which flows from one tank to another tank. The water's motion from one side to the other side is able to generate the required momentum to counteract the ship's roll motion. Hence, motion damping due to the U-tube tank is possible because there is a phase difference between the motion of the ship and the water in the tank. There are passive tank and active tank systems. The passive tank has the disadvantage of having a slower response than the active tank.

The active flume tank is the mitigation device represented in this dissertation. The dynamics mathematical model of the flume tank has been introduced prior to the control algorithm.

4.2 Dynamics Model of Flume Tank

The water's motion in the flume tank is the most influential component in the dynamic model of the flume tank. First, the water motion in the flume tank is analyzed. Either the ship's motion or the water pump usually forces the water motion in the flume tank. The forces and moments generated by the water motion in the tank certainly act

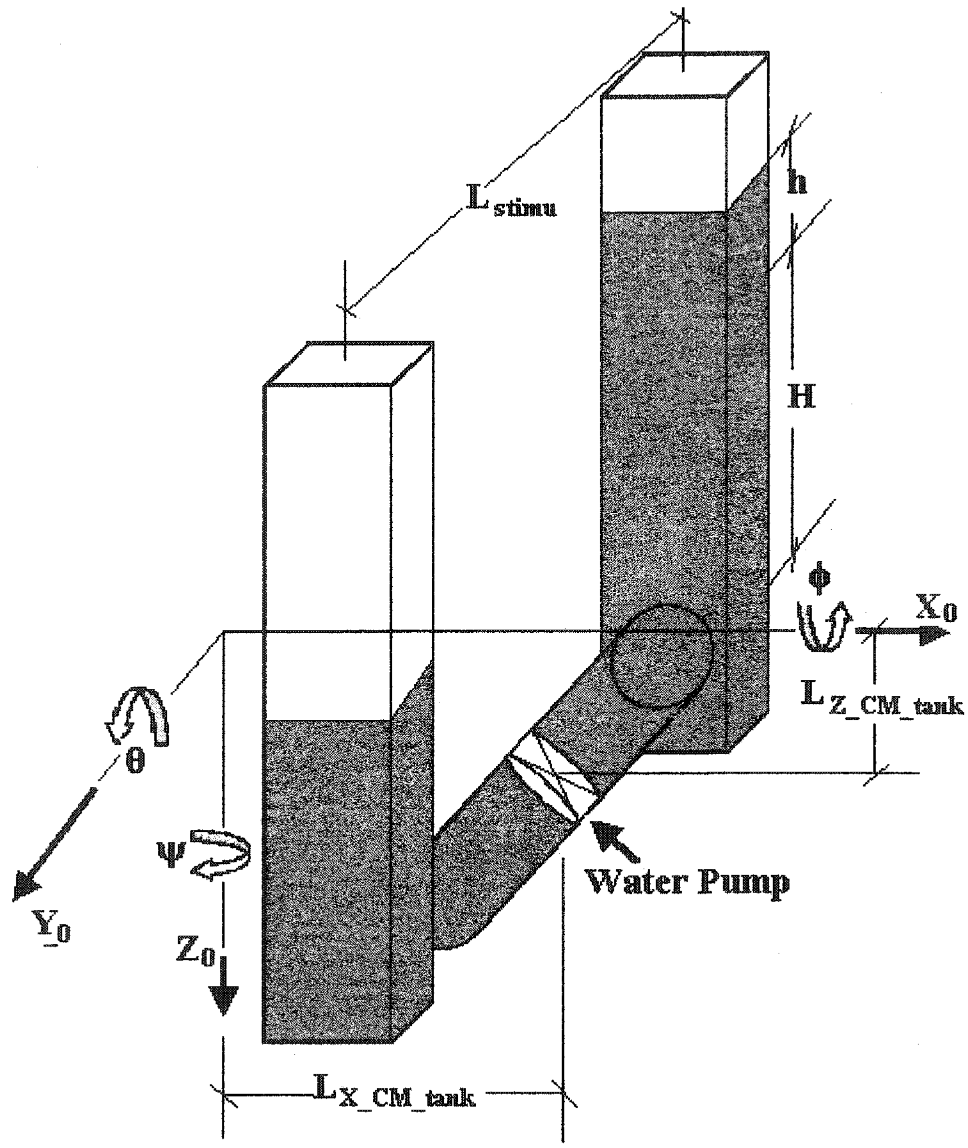

Figure 4.4 Diagram of Flume Tank

on the ship as well. The fluid motion can be determined by the Navier-Stokes equation and continuity equation.

$$\frac{\partial u}{\partial t} + (u.\nabla).u = -\frac{1}{\rho}\nabla p + \upsilon \nabla^2 u + f, \qquad (4.1)$$

and

$$\nabla \rho u = 0. \qquad (4.2)$$

Equation (4.1) and (4.2) are considered to represent three-dimensional motion. After these equations have been simplified to represent one-dimensional motion, the Navier-Stokes equation can be rewritten as the following equation:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x_{\sec}} = -\frac{1}{\rho}\nabla p + f. \qquad (4.3)$$

Equation (4.3) can be simplified by using two assumptions. The velocity either in the tank or in the pipe is equal, and the effect of fluid flow between the tank and the pipe can be ignored. Then, the simplified equation can be

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{dp}{dx_{\sec}} + f. \qquad (4.4)$$

The force (f) acting on the fluid element consists of the gravitation force, the acceleration force, and the frictional force.

$$f = f_{grav} + f_{acc} + f_{fric}.$$

Each force can be expressed by the following equations.

$$f_{grav} = g.\vec{V}_s. \qquad (4.5)$$

$$f_{acc} = -\left[\ddot{R} + 2\omega \times \dot{r} + \omega \times (\omega \times r) + \dot{\omega} \times r\right],$$

$$f_{fric} = -b_f u,$$

where $\vec{V_s}$ is the unit vector in the fixed ship axis,

$\ddot{R}$ is the linear acceleration,

$2\omega \times \dot{r}$ is the colioris acceleration,

$\omega \times (\omega \times r)$ is the centrifugal force,

$\dot{\omega} \times r$ is the tangential force,

and b_f is the linear damping coefficient.

Substituting all forces into Equation (4.4), results in

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{dp}{dx} + g.\vec{V_s} - \ddot{R} + 2\omega \times \dot{r} + \omega \times (\omega \times r) + \dot{\omega} \times r - b_f u \tag{4.6}$$

The details of derivation of Equation (4-6) are in Reference 25. Finally, the general dynamic of the flume tank can be written as follows:

$$\begin{aligned}
\left(2H + \frac{A_{\tan k}}{A_{pipe}} L_{stimu}\right)\ddot{h} = {} & -\frac{1}{\rho}(P_3 - P_4) - L_{stimu} g \sin\phi - 2gh\cos\phi \\
& + L_{stimu}\ddot{y} + 2h\ddot{z} - 2\left(\dot{\phi}^2 - \dot{\theta}^2\right)Hh \\
& + 2\dot{\phi}\dot{\psi} L_{X_CM_\tan k} H + \dot{\theta}\dot{\psi} L_{stimu} h \\
& + \dot{\phi}\dot{\psi} L_{X_CM_\tan k} L_{stimu} + \dot{\theta}\dot{\psi} L_{Z_CM_\tan k} L_{stimu} \\
& - \ddot{\phi} L_{stimu} H + 2\ddot{\theta} L_{X_CM_\tan k} H - \ddot{\phi} L_{Z_CM_\tan k} L_{stimu}
\end{aligned}$$

$$+\ddot{\psi} L_{X\_CM\_\tan k} L_{stimu} - b\left[2H + \frac{A_{\tan k}}{A_{pipe}} L_{stimu}\right]\dot{h}. \qquad (4.7)$$

After linearization, Equation (4.7) becomes

$$\left(2H + \frac{A_{\tan k}}{A_{pipe}} L_{stimu}\right)\ddot{h} = \frac{1}{\rho}\Delta P - L_{stimu} g\phi - 2gh + L_{stimu}\ddot{y}$$

$$-\ddot{\phi} L_{stimu} H - \ddot{\phi} L_{Z\_CM\_\tan k} L_{stimu}$$

$$+\ddot{\psi} L_{X\_CM\_\tan k} L_{stimu} - b\left[2H + \frac{A_{\tan k}}{A_{pipe}} L_{stimu}\right]\dot{h}. \qquad (4.8)$$

For further simplification, multiply Equation (4.8) by ρ on both sides and define some terms using the following variables.

$$m_s = \left(2H + \frac{A_{\tan k}}{A_{pipe}} L_{stimu}\right)$$

$$m_{sH} = \left(H + L_{Z\_CM\_\tan k}\right) L_{stimu}$$

$$\Delta P = P_4 - P_3.$$

The equation of motion of fluid in the flume tank becomes

$$\rho m_s \ddot{h} - \rho L_{stimu}\ddot{y} + \rho m_{sH}\ddot{\phi} - \rho L_{X\_CM\_\tan k} L_{stimu}\ddot{\psi} + \rho b m_s \dot{h} + 2\rho g h + \rho g L_{stimu}\phi = \Delta P. \qquad (4.9)$$

The combination of ship motion and fluid motion in the flume tank can be expressed in the following matrix form:

$$\begin{bmatrix} m_0\left(1+\overline{K}_y\right) & m_0\overline{K}_y\overline{x}_4 & m_0\overline{K}_y\overline{x}_1 & 0 \\ \left(m_0\overline{K}_y\overline{x}_4\right) & \left(J_x+I_x\right) & m_0\overline{K}_y\overline{x}_6^2 & 0 \\ m_0\overline{K}_y\overline{x}_1 & m_0\overline{K}_y\overline{x}_6^2 & \left(I_z+J_z\right) & 0 \\ -\rho L_{stimu} & \rho m_{sH} & -\rho L_{X\_CM\_\tan k}L_{stimu} & \rho m_s \end{bmatrix}\begin{Bmatrix} \ddot{y} \\ \ddot{\phi} \\ \ddot{\psi} \\ \ddot{h} \end{Bmatrix}+$$

$$\begin{bmatrix} \overline{N}_y & \overline{N}_y\overline{x}_5 & \overline{N}_y\overline{x}_4 & 0 \\ \overline{N}_y\overline{x}_5 & \overline{N}_{\phi e} & \overline{N}_y\overline{x}_7^2 & 0 \\ \overline{N}_y\overline{x}_2 & \overline{N}_y\overline{x}_7^2 & \overline{N}_\psi & 0 \\ 0 & 0 & 0 & \rho b m_s \end{bmatrix}\begin{Bmatrix} \dot{y} \\ \dot{\phi} \\ \dot{\psi} \\ \dot{h} \end{Bmatrix}+$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & W.\overline{GM} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \rho g L_{stimu} & 0 & 2\rho g \end{bmatrix}\begin{Bmatrix} y \\ \phi \\ \psi \\ h \end{Bmatrix} = \begin{bmatrix} F_{sway\_wave} \\ M_{roll\_wave} \\ M_{yaw\_wave} \\ \Delta P \end{bmatrix} \tag{4.10}$$

where F_{sway_wave}, M_{roll_wave}, and M_{yaw_wave} are the sea wave disturbances.

The roll moment, generated by the fluid motion in the flume tank, acts on the ship. Therefore, the excitation of ship roll motion in Equation (4.10) is generated not only from the sea (M_{roll_wave}) but also the flume tank (M_{stimu}). The general moments, which are generated by the fluid motion in the flume tank, consist of moments due to acceleration, gravitation, and the water pump.

$$M_{stimu} = M_{acc} + M_{grav} + M_{pump} \tag{4.11}$$

where

$$M_{acc} = -\rho\left(HA_{\tan k}L_{stimu} + L_{stimu}A_{\tan k}L_{Z\_CM\_\tan k}\right)\ddot{h}$$

$$+\frac{1}{2}\rho L_{stimu}A_{\tan k}\left(2\left(\dot{\phi}^2-\dot{\theta}^2\right)Hh - 2\dot{\phi}\dot{\psi}L_{X\_CM\_\tan k}H - \dot{\theta}\dot{\psi}L_{stimu}h\right)$$

$$+\rho L_{Z\_CM\_\tan k} A_{pipe}\left(\dot{\phi}\dot{\psi} L_{X\_CM\_\tan k} L_{stimu} + \dot{\theta}\dot{\psi} L_{Z\_CM\_\tan k} L_{stimu}\right)$$

$$-\frac{1}{2}\rho A_{\tan k} L_{stimu}\left(\ddot{\phi} L_{stimu} H - 2\ddot{\theta} L_{X\_CM\_\tan k} H\right)$$

$$-\rho A_{pipe} L_{Z\_CM\_\tan k}\left(\ddot{\phi} L_{Z\_CM\_\tan k} L_{stimu} - \ddot{\psi} L_{X\_CM\_\tan k} L_{stimu}\right)$$

$$+\rho L_{stimu} A_{pipe} L_{Z\_CM\_\tan k}\ddot{y} + \rho h A_{pipe} L_{stimu}\ddot{z}$$

$$M_{grav} = -\rho L_{stimu} g \sin(\phi) L_{Z\_CM\_\tan k} A_{pipe} - \rho g h \cos(\phi) L_{stimu} A_{\tan k}\rho$$

$$M_{pump} = -A_{pipe} L_{Z\_CM\_\tan k}\Delta P$$

Then, linearize them. First, each term of M_{stimu} becomes

$$M_{acc} = -\rho\left(H A_{\tan k} L_{stimu} + L_{stimu} A_{\tan k} L_{Z\_CM\_\tan k}\right)\ddot{h}$$

$$-\frac{1}{2}\rho A_{\tan k} L_{stimu}^2 H\ddot{\phi} - \rho A_{pipe} L_{Z\_CM\_\tan k}^2 L_{stimu}\ddot{\phi}$$

$$+\rho A_{pipe} L_{Z\_CM\_\tan k} L_{X\_CM\_\tan k} L_{stimu}\ddot{\psi}$$

$$+\rho L_{Z\_CM\_\tan k} L_{stimu} A_{pipe}\ddot{y}$$

$$M_{grav} = -\rho L_{stimu} g A_{pipe} L_{Z\_CM\_\tan k}\phi - \rho g h L_{stimu} A_{\tan k}$$

$$M_{pump} = -A_{pipe} L_{Z\_CM\_\tan k}\Delta P.$$

Then, define some parameters for simplification,

$$S_1 = H A_{\tan k} L_{stimu} + L_{stimu} A_{\tan k} L_{Z\_Cm\_\tan k}$$

$$S_2 = A_{\tan k} L_{stimu}^2 H$$

$$S_3 = A_{pipe} L_{Z\_CM\_\tan k}^2 L_{stimu}$$

$$S_4 = A_{pipe} L_{Z\_CM\_\tan k} L_{X\_CM\_\tan k} L_{stimu}$$

$$S_5 = L_{Z\_CM\_\tan k} L_{stimu} A_{pipe}$$

$$S_6 = L_{stimu} A_{pipe} L_{Z\_CM\_\tan k}$$

$$S_7 = A_{\tan k} L_{stimu}$$

$$S_8 = A_{pipe} L_{Z\_CM\_\tan k}$$

Then, substitute M_{acc}, M_{grav}, and M_{pump} into Equation (4.11). The Equation (4.11) becomes

$$M_{stimu} = -\rho S_1 \ddot{h} - \frac{1}{2}\rho S_2 \ddot{\phi} - \rho S_3 \ddot{\phi} + \rho S_4 \ddot{\psi} + \rho S_5 \ddot{y} - \rho g S_6 \phi - \rho g S_7 h - S_8 \Delta P. \quad (4.12)$$

The equation of motion, which is combined between the ship motion and the fluid motion from Equation (4.10), can be rewritten in the following matrix form:

$$\begin{bmatrix} m_0(1+\overline{K}_y) & m_0\overline{K}_y\overline{x}_4 & m_0\overline{K}_y\overline{x}_1 & 0 \\ (m_0\overline{K}_y\overline{x}_4)-\rho S_5 & M_{22} & m_0\overline{K}_y\overline{x}_6^{-2} - \rho S_4 & \rho S_1 \\ m_0\overline{K}_y\overline{x}_1 & m_0\overline{K}_y\overline{x}_6^{-2} & (I_z+J_z) & 0 \\ -\rho L_{stimu} & \rho m_{sH} & -\rho L_{X\_CM\_\tan k} L_{stimu} & \rho m_s \end{bmatrix} \begin{Bmatrix} \ddot{y} \\ \ddot{\phi} \\ \ddot{\psi} \\ \ddot{h} \end{Bmatrix}$$

$$\begin{bmatrix} \overline{N}_y & \overline{N}_y\overline{x}_5 & \overline{N}_y\overline{x}_4 & 0 \\ \overline{N}_y\overline{x}_5 & \overline{N}_{\phi e} & \overline{N}_y\overline{x}_7^{-2} & 0 \\ \overline{N}_y\overline{x}_2 & \overline{N}_y\overline{x}_7^{-2} & \overline{N}_\psi & 0 \\ 0 & 0 & 0 & \rho b m_s \end{bmatrix} \begin{Bmatrix} \dot{y} \\ \dot{\phi} \\ \dot{\psi} \\ \dot{h} \end{Bmatrix} +$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & W.\overline{GM} + \rho g S_6 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \rho g L_{stimu} & 0 & 2\rho g \end{bmatrix} \begin{Bmatrix} y \\ \phi \\ \psi \\ h \end{Bmatrix} = \begin{bmatrix} F_{sway\_wave} \\ M_{roll\_wave} - S_8\Delta P \\ M_{yaw\_wave} \\ \Delta P \end{bmatrix} \quad (4.13)$$

where M_{22} is equal to $\left\{(I_x + J_x) + \left[\frac{1}{2}\rho(S_2 + 2S_3)\right]\right\}$.

4.3 Designing the Anti-Roll U-Tube Tank

Mass is still a vital factor in designing tank stabilizers. For more details, see Reference 8. If we choose a light tank, it may not affect the roll's amplitude. However, the hull space may be restricted if we select a very heavy tank. Likewise, a heavy tank reduces the ship's stability. The recommended ratio of tank mass to ship mass should range from 2.0 to 6.0 %. If hull space is provided, we should design a tank mass ratio of 3.5 %. Another important consideration is the location of the pipe, which is connected to both tanks, with respect to the ship's center of gravity (C.G). In order to obtain an optimum roll response, the pipe should be placed below the center of gravity.

The important feature of flume tank design is the natural frequency of the tank. The natural frequency of the tank should be equal to the ship's roll natural frequency. Thus, water height is important for the "tuning" of the tank because the natural frequency is dependent on the water depth inside the tank.

Simply stated, the natural frequency of the flume tank is dependent on the natural frequency of the ship. The relationship of the natural frequency of the ship and flume tank can be expressed in the following equation[13].

$$\omega_{\tan k} \cong \omega_{ship} \equiv \sqrt{\frac{2g}{2.H_{\tan k} + \frac{A_{\tan k}}{A_{pipe}}.L_{\tan k}}} \tag{4.14}$$

In conclusion, there are several factors that may affect roll's amplitude using this mitigation device. Engineers should be aware of the following when they will design a flume tank.

- Frequency of encounter
- Amplitude of rolling motion
- Length of tank
- Breadth of tank
- Water height in the tank
- Vertical position of tank

CHAPTER V

OPTIMAL FEEDBACK CONTROL ALGORITHM

Optimal Control is a type of Modern Control Theory having been developed over the last decade. The result of optimal design is not only to stabilize the system but also to provide the best performance. Generally, the control problems involve a system, which can be categorized into the regulator problem and the tracking problem. The regulator problem is to apply a control to drive the system from a nonzero state to the zero state. This problem typically occurs when unwanted disturbances perturb the system. In addition, the tracking problem is how to provide a control in order to track the output of system. Only the regulator problem is discussed in this dissertation.

Linear optimal control is the special case of this dissertation. The concept of linear optimal control involves finding the constant gains subject to minimizing the performance index. This control methodology is also called Linear Quadratic[28] (LQ). Generally, the characteristic of an optimal control law based upon a quadratic performance index is a linear equation of state variables. Therefore, all state variables are required in this circumstance.

5.1 The Linear Quadratic Regulator Problem

Consider the linear system and the quadratic objective function (cost function, or performance index) as follows. Plant, or system, is given as the state-space model

$$\dot{x} = Ax + Bu$$
$$y = Cx. \tag{5.1}$$

A quadratic objective function or the performance index is

$$J = \frac{1}{2}\int_0^T \left(x^T Qx + u^T Ru\right)dt. \tag{5.2}$$

The problem is to minimize the performance index (J) with respect to the control input u (t). A simple interpretation of the cost function can be written in the scalar system as follows.

$$J = \frac{1}{2}\int_0^T \left(qx^2 + ru^2\right)dt. \tag{5.3}$$

The cost function (J) as Equation (5.3) represents the weighted sum of energy of the state and control. If r is very large relative to q, it means that the control energy is penalized very heavily. On the other hand, the state is penalized heavily if q is much larger than r. In the general case, Q and R represent respective weights on different states and control channels. The Q must be symmetric positive semi definite $(Q \geq 0)$ and R is symmetric positive definite $(R > 0)$ in order to optimize the problem.

5.2 LQR Solution Using the Minimum Principle

An optimal control problem can be solved using a variety of techniques. There are Euler-Lagrange equations, Hamilton-Jacobi-Bellman theory, and Pontragin's minimum principle. First of all, the form named as the Hamiltonian form is represented as:

$$H(x,\lambda,t)=\frac{1}{2}\left(x^T Qx+u^T Ru\right)+\lambda^T(Ax+Bu). \qquad (5.4)$$

The rules of the minimum principle must satisfy the following three equations.

$$\dot{x}=\frac{\partial H}{\partial \lambda} \quad x(0)=x_0 \quad \text{state equations} \qquad (5.5)$$

$$-\dot{\lambda}=\frac{\partial H}{\partial x} \quad \lambda(T)=0 \quad \text{costate or adjoint equations} \qquad (5.6)$$

$$\frac{\partial H}{\partial u}=0 \qquad (5.7)$$

Using the rules for differentiation of matrices and vectors, the preceding equations for the LQR case become:

$$\dot{x}=Ax+Bu, \qquad x(0)=x_0$$

$$-\dot{\lambda}=Qx+A^T\lambda, \qquad \lambda(T)=0$$

$$u^*=-R^{-1}B^T\lambda, \qquad u^* \text{ is the optimal control}$$

Note that R must be positive definite for its inverse to exist.

Substituting optimal control into the state equation, then, one obtains

$$\begin{bmatrix}\dot{x}\\ \dot{\lambda}\end{bmatrix}=\begin{bmatrix}A & -BR^{-1}B^T\\ -Q & -A^T\end{bmatrix}\begin{bmatrix}x\\ \lambda\end{bmatrix}\equiv H\begin{bmatrix}x\\ \lambda\end{bmatrix}, \qquad (5.8)$$

with H being the so-called Hamiltonian matrix.

Letting $\lambda = Sx$ and differentiating both sides with respect to the time, we then obtain

$$\begin{aligned}\frac{d\lambda}{dt} &= \frac{dS}{dt}x + S\frac{dx}{dt} \\ &= \frac{dS}{dt}x + SAx - SBR^{-1}B^{T}Sx \\ &= -Qx - A^{T}Sx. \qquad (5.9)\end{aligned}$$

Equation (5.9) must hold for any *x*, therefore, a sufficient condition for optimal control is that S must be satisfied.

$$-\frac{dS}{dt} = A^{T}S + SA + Q - SBR^{-1}B^{T}S, \qquad (5.10)$$

Note that $S(T) = 0$.

The Equation (5.10) is usually called the Riccati differential equation. It is a nonlinear first order differential equation that has to be solved backward in time. The optimal control problem usually is solved from the Riccati equation. According to the control law,

$$u^{*}(t) = -K(t)x(t),$$

where $K(t) = R^{-1}B^{T}S(t)$. It is found that the optimal control can be determined once $S(t)$ has been solved.

Even if the optimal control exists[29], it does not necessarily result in a stable closed loop system. $S(t)$ approaches a constant matrix, S, once $\frac{dS}{dt} \to 0$. Moreover, the positive definite solution of the algebraic Riccati equation (ARE) turns the result in an asymptotically stable closed loop system. The algebraic Riccati equation (ARE) is written in the following form:

$$A^T S + SA + Q - SBR^{-1}B^T S = 0, \qquad (5.11)$$

where the control law is

$$u^* = -Kx,$$

and

$$K = R^{-1}B^T S$$

Equation (5.11) holds for the following conditions:

- The pair (A, B) are stable,
- The matrix R must be positive definite ($R > 0$), and
- The matrix Q can be factored as $Q = C^T C$, where C is any matrix such that (C, A) is detectable.

These conditions are necessary and sufficient for existence and uniqueness of an optimal controller that will asymptotically stabilize the system.

5.3 Discrete-Time Linear Quadratic Regulator

Similarly, the solution of optimal control for discrete time domain follows the steps as same as we discussed as previous section. However, the optimal control will be solved as a sequence. The State and Costate equations are also derived[30]. Letting the linear plant can be described as follows:

$$x_{k+1} = A_k x_k + B_k u_k \qquad (5.12)$$

The performance index is the following quadratic function

$$J_i = \frac{1}{2} x_N^T S_N x_N + \frac{1}{2} \sum_{k=i}^{N-1} \left(x_k^T Q_k x_k + u_k^T R_k u_k \right) \tag{5.13}$$

The time interval is [*i*, *N*]. Letting matrix Q_k, R_k and S_N are then symmetric positives as well as the absolute R_k must be not equal to zeros for all *k*. Equation (5.13) intends to determine the control input u_k in order to minimize the performance index J_i. Solving the linear quadratic regulator problem can be described as following steps.

The Hamiltonian function is given by

$$H_k = \frac{1}{2} \left(x_k^T Q_k x_k + u_k^T R_k u_k \right) + \lambda_{k+1}^T \left(A_k x_k + B_k u_k \right). \tag{5.14}$$

Letting the state and costate equation are expressed as follows:

$$\begin{aligned} x_{k+1} &= \frac{\partial H_k}{\partial \lambda_{k+1}} = A_k x_k + B_k u_k, \\ \lambda_k &= \frac{\partial H_k}{\partial x_k} = Q_k x_k + A_k^T \lambda_{k+1} \end{aligned} \tag{5.15}$$

Set the stationary condition to be

$$0 = \frac{\partial H_k}{\partial u_k} = R_k u_k + B_k^T \lambda_{k+1}. \tag{5.16}$$

Finally, the control sequence can be determined as:

$$u_k^* = -R_k^{-1} B_k^T \lambda_{k+1}. \tag{5.17}$$

5.4 Determination of Closed-Loop Control

The restriction on the final state value (x_N) has not been given in the time. To derive the optimal control (u_k), the state and costate equations with the input u_k has been applied at the first time.

$$x_{k+1} = A_k x_k - B_k R_k^{-1} B_k^T \lambda_{k+1},$$
$$\lambda_k = Q_k x_k + A_k^T \lambda_{k+1}. \tag{5.18}$$

Given the initial state value is x_i and the final state value x_N is free., the x_N can be varied upon the constrained minimum even though it is given as free value. The linear relation of the Costate can be holds for all time $k \leq N$ as shown as

$$\lambda_k = S_k x_k. \tag{5.19}$$

Thus, if S_k can be determined, the proof of linear relation can be existed. Finally, the determination of S_k can be determined by Equation (5.20). The equation is called the *Riccati equation.*

$$S_k = A_k^T \left[S_{k+1} - S_{k+1} B_k \left(B_k^T S_{k+1} B_k + R_k \right)^{-1} B_k^T S_{k+1} \right] A_k + Q_k. \tag{5.20}$$

This Riccati equation can be determined as off-line. Note that if $|S_k|$ is not equal to zeros for all time k, the S_k in Equation (5.20) can be rewritten as shown below:

$$S_k = A_k^T \left(S_{k+1}^{-1} + B_k R_k^{-1} B_k^T \right)^{-1} A_k + Q_k. \tag{5.21}$$

The optimal control (u_k^*) is

$$u_k^* = -K_k x_k . \tag{5.22}$$

Note that K_k is called the *Kalman gain sequence*. It is given as

$$K_k = \left(B_k^T S_{k+1} B_k + R_k\right)^{-1} B_k^T S_{k+1} A_k . \tag{5.23}$$

In the free-final state linear quadratic regulator called LQR, it obviously shows that the closed-loop control law gives the optimal control. The conclusion of computational feedback gain can be shown in Table 5.1 and Figure 5.1.

Table 5.1 Discrete Linear Quadratic Regulator

Plant:

$$x_{k+1} = A_k x_k + B_k u_k , \qquad k > i$$

Performance index (PI):

$$J_i = \frac{1}{2} x_N^T S_N x_N + \frac{1}{2} \sum \left(x_k^T Q_k x_k + u_k^T R_k u_k\right)$$

Assumptions:

$S_N \geq 0, \quad Q_k \geq 0, \quad R_k > 0,$ and all three are symmetric

Optimal feedback control:

$$S_k = A_k^T \left[S_{k+1} - S_{k+1} B_k \left(B_k^T S_{k+1} B_k + R_k\right)^{-1} B_k^T S_{k+1}\right] A_k + Q_k , \quad k < N, \quad S_N \text{ given}$$

$$K_k = \left(B_k^T S_{k+1} B_k + R_k\right)^{-1} B_k^T S_{k+1} A_k , \quad k < N$$

$$u_k = -K_k x_k , \quad k < N$$

$$J_i^* = \frac{1}{2} x_i^T S_i x_i$$

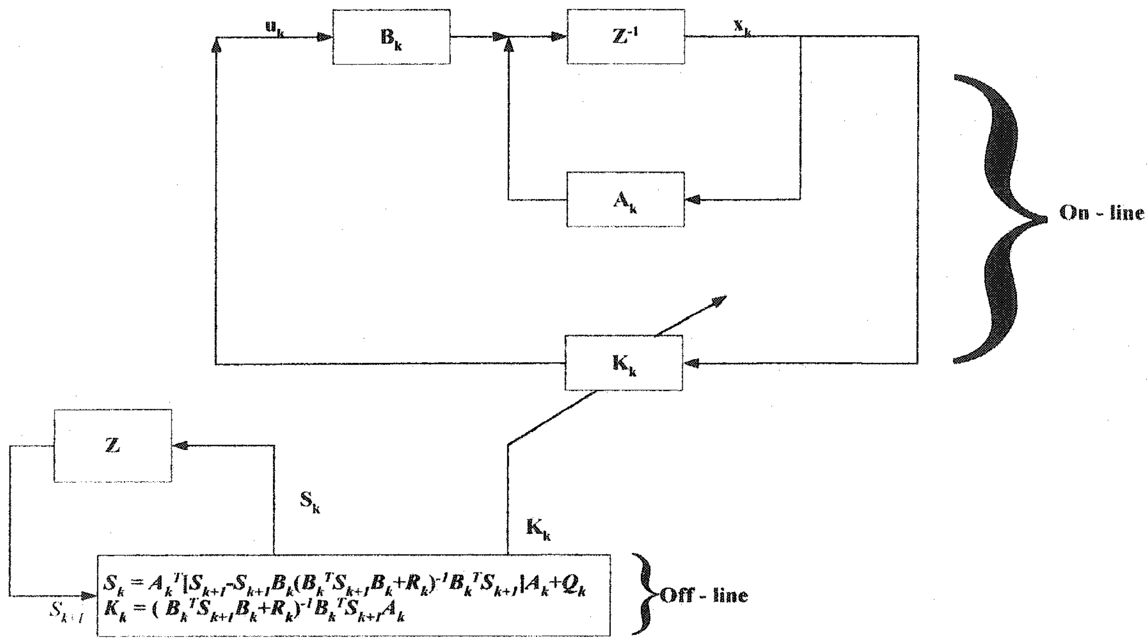

Figure 5.1 Diagram of LQR optimal control algorithm.

In order to determine the control action using LQR, many component devices are required to meet the performance index. Unfortunately, the availability of all state vectors is sometimes not possible. In this circumstance, the suboptimal feedback control can be selected rather than using the full state feedback control. We will discuss the LQ algorithm using suboptimal feedback in the following section.

5.5 Suboptimal Feedback Control

A suboptimal feedback controller is easily implemented. The configuration and parameters have been pre-calculated with an off-line computation. The optimal feedback gains K_k are time varying from the previous algorithm. However, suboptimal feedback gains can be treated as a constant (K).

According to the LQ optimal control problem, the feedback control is given as form:

$$u_k = -K_k . x_k \,, \tag{5.24}$$

with gain K_k given in the solution of S_k to the Riccati equation as Equation (5.20). In the case of fixed-gain suboptimal feedback control, the Riccati equation reduces to the well-known form, called the algebraic Riccati equation (ARE):

$$S = A^T \left[S - SB\left(B^T SB + R\right)^{-1} B^T S \right] A + Q \,. \tag{5.25}$$

Thus, all solutions to this algebraic Riccati equation (ARE) are not time-varying. If the solution to ARE exits, then the corresponding steady-state Kalman gain is

$$K = \left(B^T SB + R\right)^{-1} B^T SA \,. \tag{5.26}$$

For the feedback control law, the time-invariant is used in this circumstance instead of Equation (5.24). The feedback control law in the case of suboptimal control is

$$u_k = -K . x_k \,. \tag{5.27}$$

CHAPTER VI

NUMERICAL SIMULATIONS

This chapter will present numerical results that show the performance of the flume tank, particularly the output response, roll angle. The assumed model, which is a submarine chaser, is based on the Fukuzo Tasai model. The specification of this vessel is shown in Table 6.1.

Table 6.1: Specification of submarine chaser

Length of submarine chaser	59.0 m.
Beam	7.1 m.
Draft	2.33 m.
Volume	480,000 m^3
$\overline{KG_o}$	2.707 m.
$\overline{OG_o}$	-0.377 m.
$\overline{G_oM}$	0.736 m.

According to the equation of motion [Equation (4.10)], the coefficients mass, mass of inertia, and damping, are investigated from the experimental data. These coefficients are also based on the Fukuzo Tasai model and they are shown in the following Table 6.2.

Table 6.2: Coefficients on the left side of the equation of motion

$m_0(1+\overline{K}_y) = 99\ ton-\sec^2/m$	$\overline{N}_y = 8\ ton-\sec/m$
$m_0\overline{K}_y\overline{x}_1 = 254\ ton-\sec^2$	$\overline{N}_y\overline{x}_2 = 19\ ton-\sec$
$m_0\overline{K}_y\overline{x}_4 = 21\ ton-\sec^2$	$\overline{N}_y\overline{x}_5 = 4.7\ ton-\sec$
$(I_z+J_z) = 21{,}593\ ton-m-\sec^2$	$\overline{N}_\psi = 1209\ ton-m-\sec$
$m_0\overline{K}_y\overline{x}_6^2 = -433\ ton-m-\sec^2$	$\overline{N}_y\overline{x}_7^2 = -43\ ton-m-\sec$
$(J_x+I_x) = 437\ ton-m-\sec^2$	$\overline{N}_{\phi e} = 80\ ton-m-\sec$

For the specifications of the flume tank, with general assumptions, the motion of fluid from one tank to another tank will generate the moment in order to counteract the ship roll motion. Therefore, these two tanks should be located as far out on the beam of the vessel as possible to give the most effective control. For a submarine chaser, the specification of flume tank, which is given as following Table 6.3, is designed to be the best counteraction when the passive tank is trial simulation

Table 6.3: Specification of Flume-Tank

H_{tank} (m)	A_{pipe} (m^2)	ϕ_{pipe} (m)	A_{tank} (m^2)	L_{tank} (m)	$L_{z\_cm\_tank}$ (m)
2.3070	1.2272	1.2500	0.6173	6.0350	2.2070

The discrete-time state-space model for dynamic of flume tank with coupled swaying, rolling, and yawing motion of sub-marine chaser is derived in following form:

$$x(t+1) = [A].x(t) + [B].u(t)$$
$$y(t) = [C].x(t) + [D].u(t)$$

where

$$A=\begin{bmatrix} 1.0000 & 0.0045 & 0 & 0.0004 & 0.1984 & 0.0002 & -0.0012 & -0.0014 \\ 0 & 0.9837 & 0 & -0.0015 & -0.0001 & 0.1954 & 0.0009 & 0.0050 \\ 0 & -0.0004 & 1.0000 & -0.0000 & -0.0000 & -0.0001 & 0.1989 & 0.0001 \\ 0 & -0.0110 & 0 & 0.9938 & -0.0001 & 0.0003 & -0.0004 & 0.523 \\ 0 & 0.0448 & 0 & 0.0043 & 0.9843 & 0.0032 & -0.00123 & -0.0088 \\ 0 & -0.01627 & 0 & -0.0156 & -0.0014 & 0.9491 & 0.0088 & 0.0391 \\ 0 & -0.0038 & 0 & -0.0004 & -0.0000 & -0.0007 & 0.9892 & 0.0007 \\ 0 & -0.0728 & 0 & -0.0411 & -0.0006 & -0.0032 & -0.0027 & 0.0223 \end{bmatrix}$$

$$B= 1\times10^{-5}\begin{bmatrix} 0.0210 & -0.0012 & -0.0003 & 0.0009 \\ -0.0012 & 0.0045 & 0.0001 & -0.0032 \\ -0.0003 & 0.0001 & 0.0001 & -0.0001 \\ 0.0022 & -0.0016 & -0.0001 & 0.0357 \\ 0.2098 & -0.0122 & -0.0027 & 0.0075 \\ -0.0116 & 0.0445 & 0.0010 & -0.0272 \\ -0.0027 & 0.0010 & 0.0010 & -0.0006 \\ 0.0146 & -0.0108 & -0.0004 & 0.2355 \end{bmatrix}$$

$$C=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x = \left[y, \phi, \psi, h, \dot{y}, \dot{\phi}, \dot{\psi}, \dot{h} \right]^T$$

From Figure 6.1 to Figure 6.3, they show the ship roll response with single input, swaying force generated by random wave. Various encounter angles of wave: 45° (Quartering Sea), 90° (Beam Sea), and 150° (Bow Sea), are illustrated.

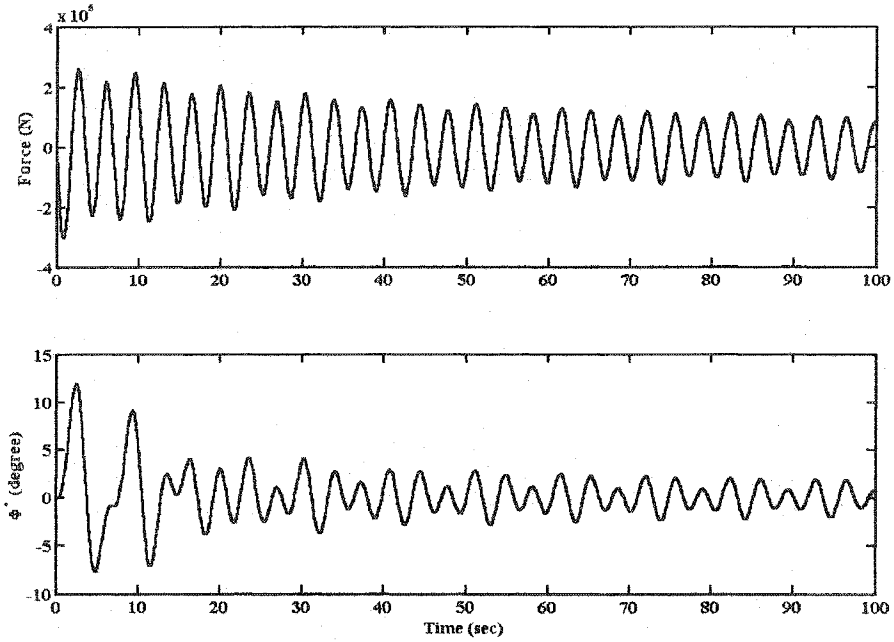

Figure 6.1 Roll amplitude (ϕ°), swaying force generated by random wave at Quartering Sea $(\beta = 45^\circ)$.

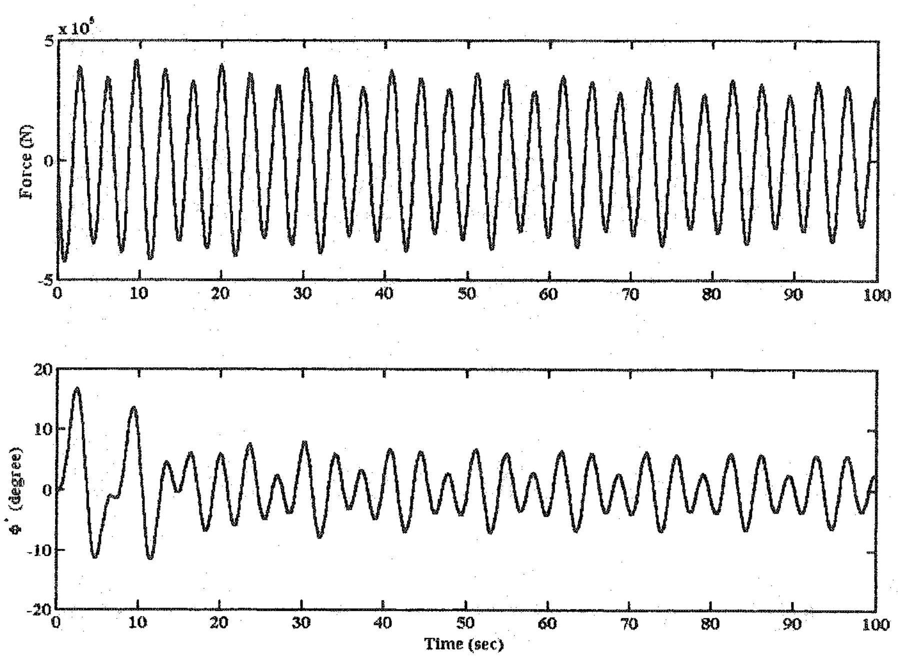

Figure 6.2 Roll amplitude (ϕ°), swaying force generated by random wave encounters at Beam Sea $(\beta = 90^\circ)$.

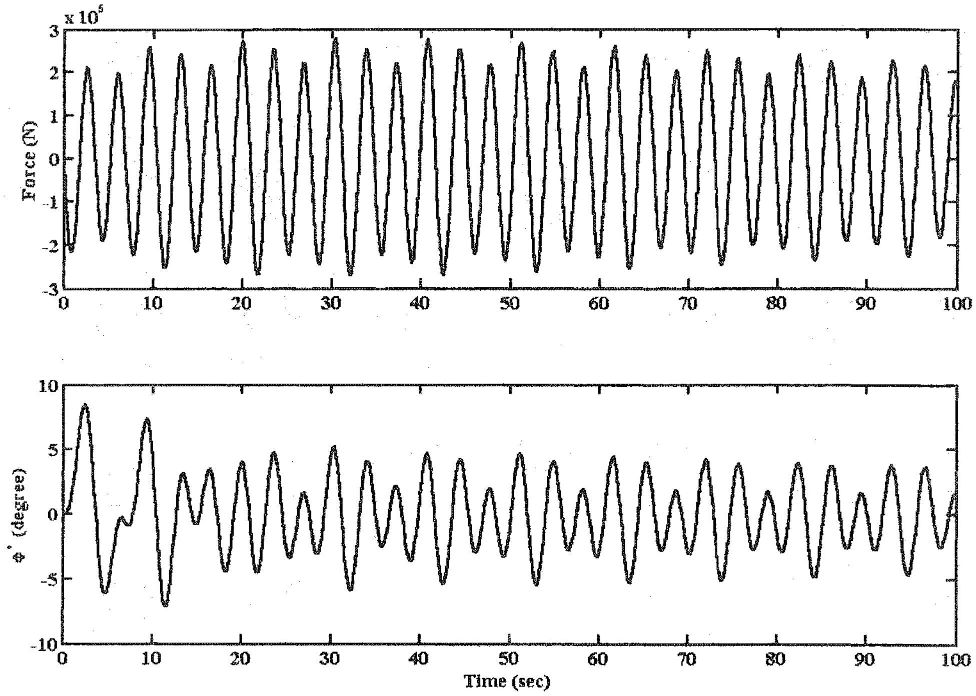

Figure 6.3 Roll amplitude (ϕ°), swaying force generated by random wave encounters at Bow Sea $(\beta = 150^\circ)$.

Figures 6.4 through Figures 6.6 illustrate the ship roll response with two inputs, swaying force, and rolling moment generated by random wave. Various encounter angles of wave: 45° (Quartering Sea), 90° (Beam Sea), and 150° (Bow Sea), are illustrated.

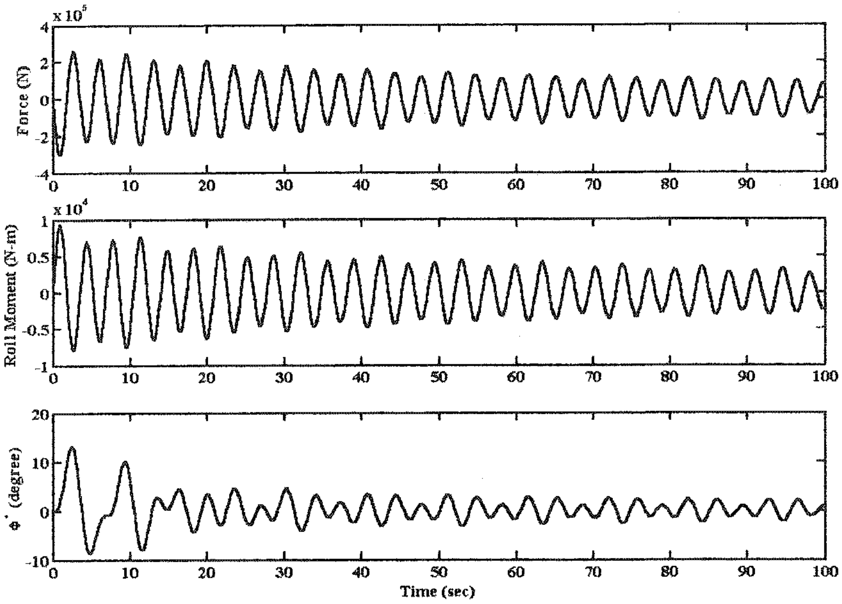

Figure 6.4 Roll amplitude (ϕ°), swaying force and rolling moment generated by random wave encounters at Quartering Sea $(\beta = 45^\circ)$.

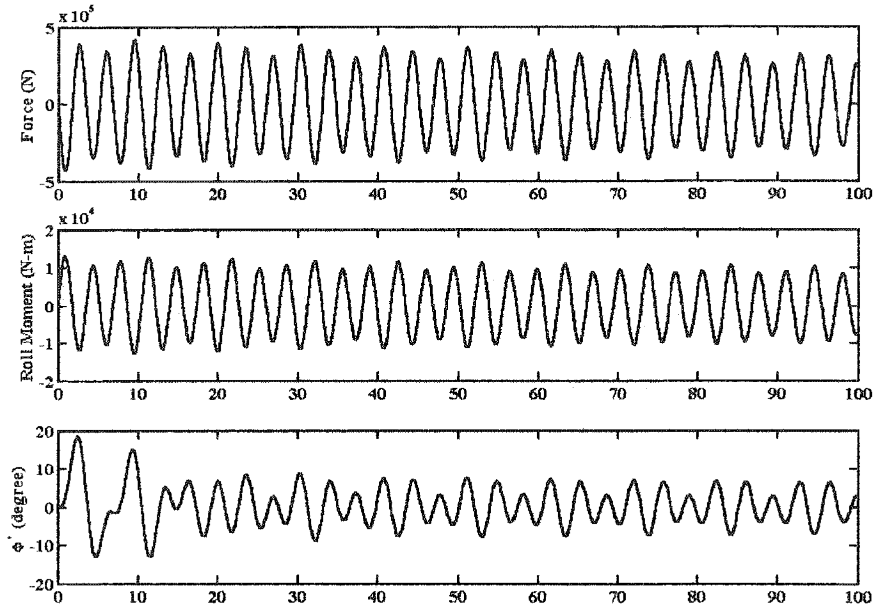

Figure 6.5 Roll amplitude (ϕ°), swaying force and rolling moment generated by random wave encounters at Beam Sea $(\beta = 90^\circ)$.

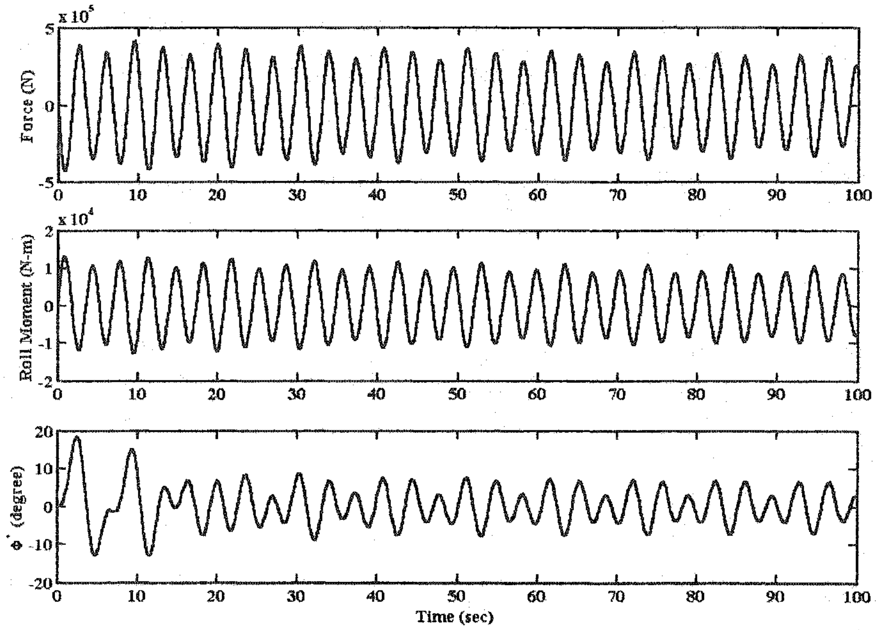

Figure 6.6 Roll amplitude (ϕ°), swaying force and rolling moment generated by random wave encounters at Bow Sea $(\beta = 150^{\circ})$.

Figures 6.7 through Figures 6.9 illustrate the ship roll response with three inputs, swaying force, rolling moment, and yawing moment generated by irregular wave under Sea State 3. Various encounter angles of wave: 45° (Quartering Sea), 90° (Beam Sea), and 150° (Bow Sea), are illustrated.

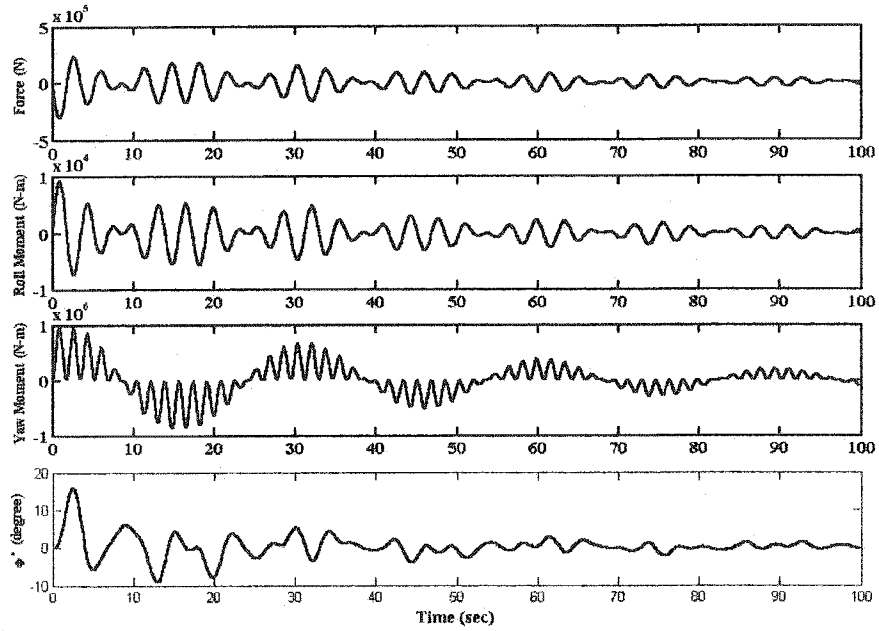

Figure 6.7 Roll amplitude (ϕ°), swaying force, rolling moment, yawing moment generated by irregular wave under Sea State 3 encounters at quartering sea $(\beta = 45^\circ)$.

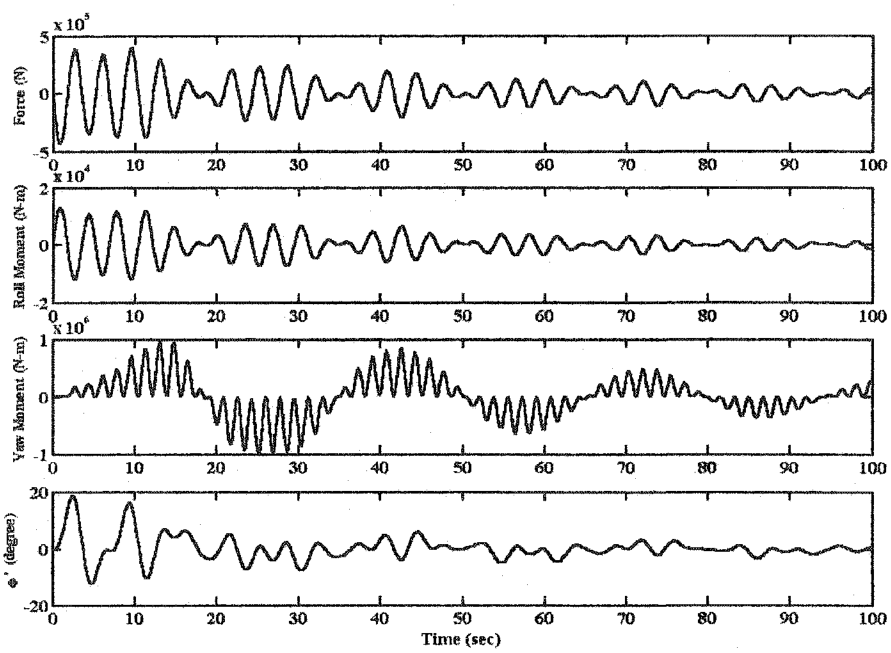

Figure 6.8 Roll amplitude (ϕ°), swaying force, rolling moment, yawing moment generated by irregular wave under Sea State 3 encounters at beam sea $(\beta = 90^\circ)$.

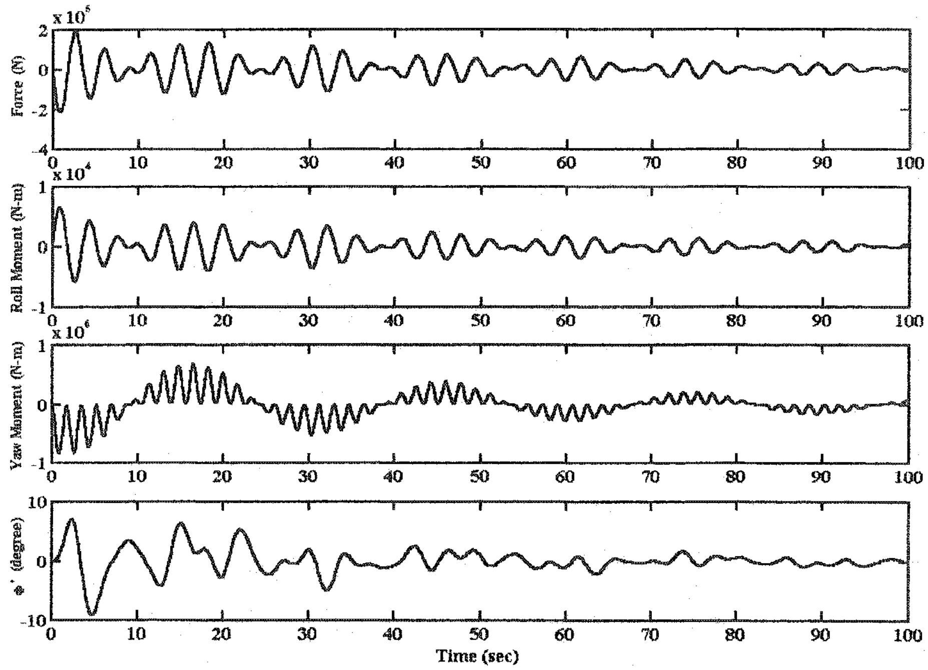

Figure 6.9 Roll amplitude (ϕ°), swaying force, rolling moment, yawing moment generated by irregular wave under Sea State 3 encounters at bow sea $(\beta = 150^\circ)$.

6.1 Results for The Passive Controller

The water in the flume tank moves to generate the moment without pump, and it also creates a high damped system. The roll amplitude of the vessel should be reduced. Figures 6.10 through Figures 6.12 illustrate the output response, which is the ship's roll angle. Passive controller is simulated when wave attacks to the ship hull at Beam Sea $(\beta = 90^\circ)$.

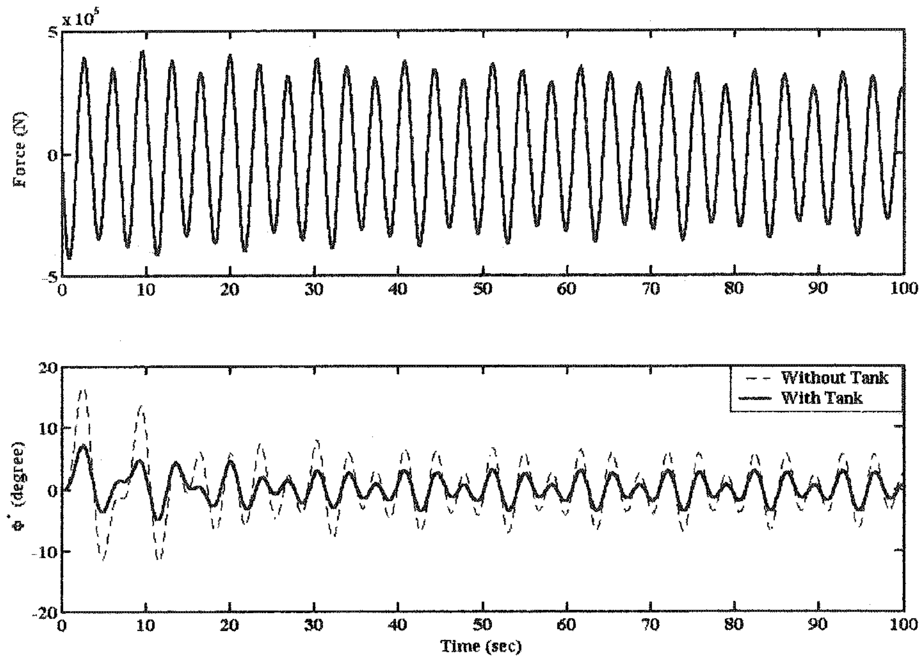

Figure 6.10 Swaying force generated by random wave encounters at Beam Sea $(\beta = 90^\circ)$. Without control (dash line) and passive control (solid line)

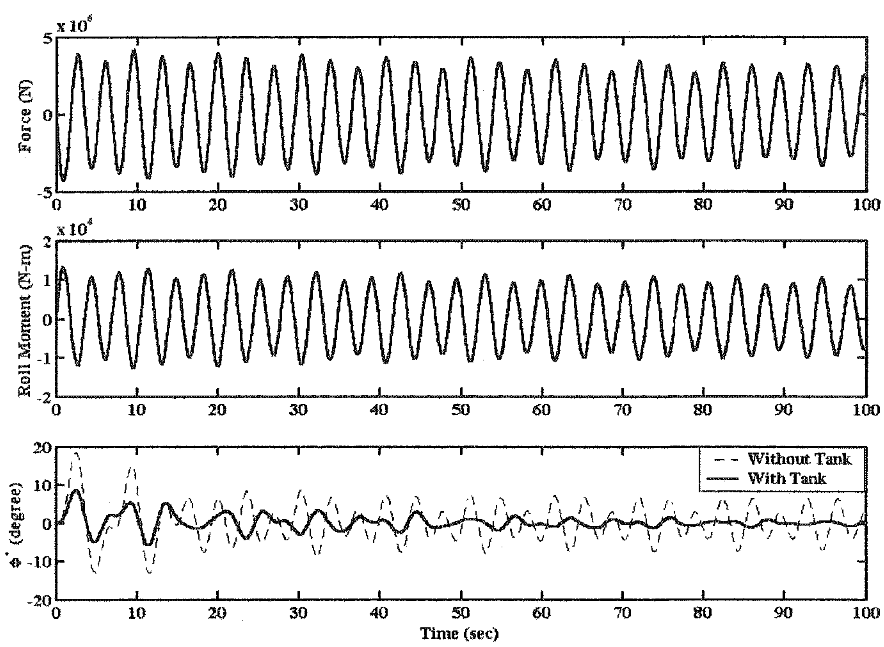

Figure 6.11 Swaying force, rolling moment generated by random wave at Beam Sea $(\beta = 90^\circ)$. Without control (dash line) and passive control (solid line).

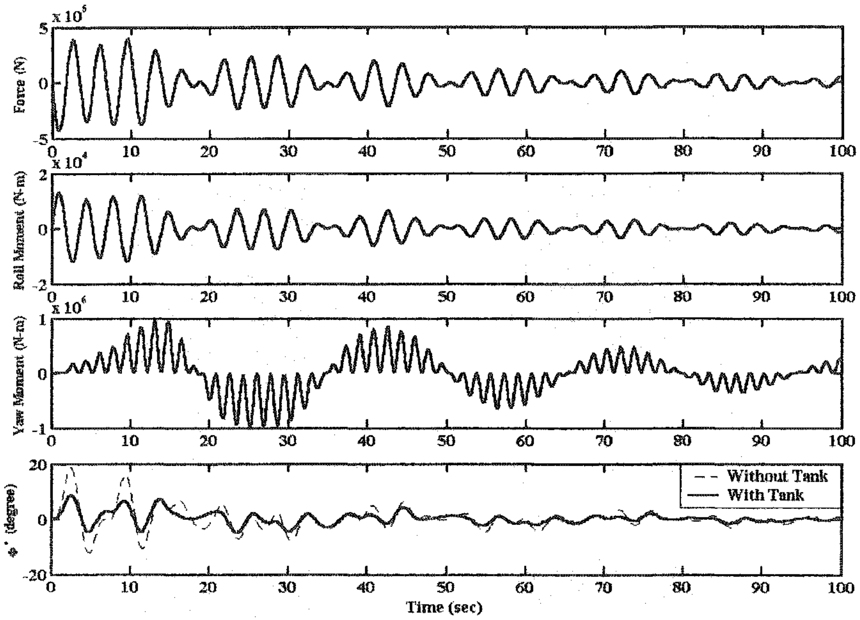

Figure 6.12 Swaying force, rolling moment, yawing moment generated by irregular wave encounters at Beam Sea $(\beta = 90^\circ)$. Without control (dash line) and passive control (solid line).

6.2 Results for Linear Quadratic Regulator (LQR)

The linear quadratic regulator (LQR) is a kind of full-state feedback controller that will minimize the cost function. The theory of this linear quadratic control is described in Chapter V. If we consider the minimum energy control case, the varying weighing matrix [R] should be considered. Likewise, the least expensive control case is considered when we vary weighing matrix [Q]. In this dissertation, we want to drive the system state x(t) to zero in a short time. Therefore, a larger weighing matrix [Q] should be used.

Figures 6.13 through Figures 6.15 compare roll angle with passive controller to roll angle with active controller. Various encountering waves are shown. The weighing matrix [Q] is used as Q = *diag* ([1, 1E+10, 1, 1, 1, 1, 1, 1]).

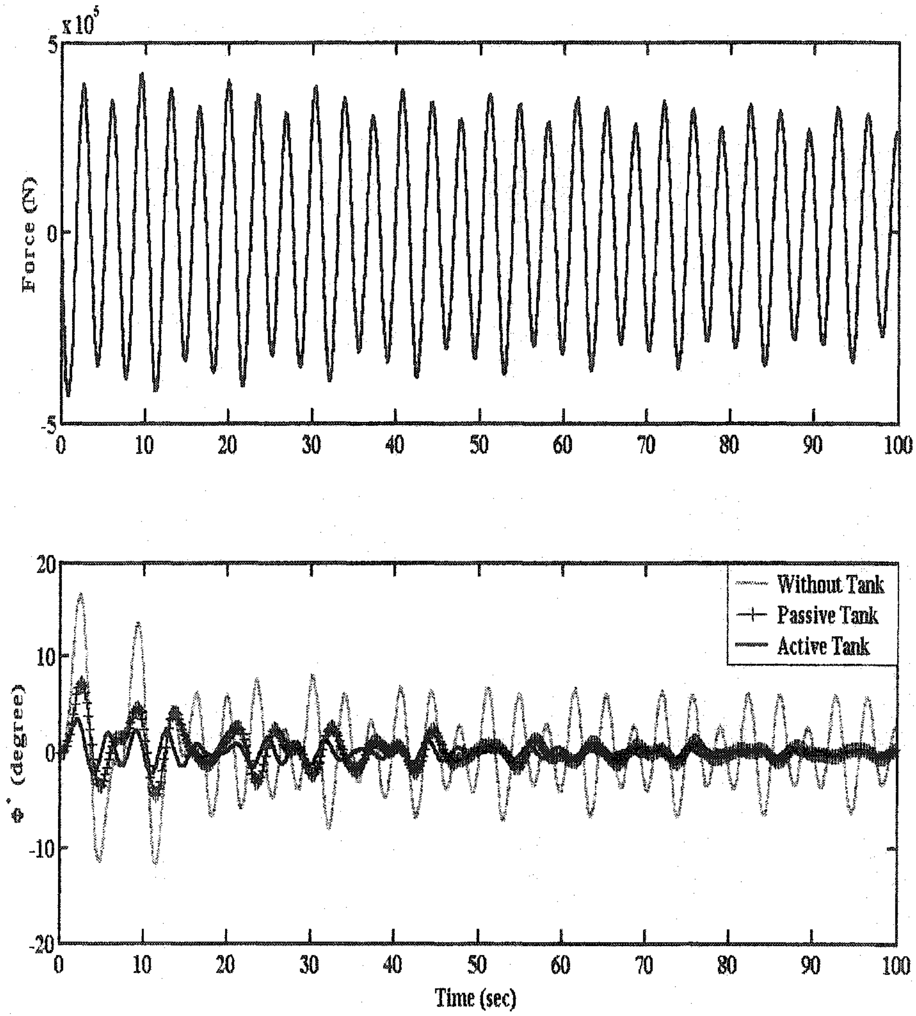

Figure 6.13 Swaying force generated by random wave encounters at Beam Sea $(\beta = 90^\circ)$. Without control (gray line), passive control (plus line), and control (solid line).

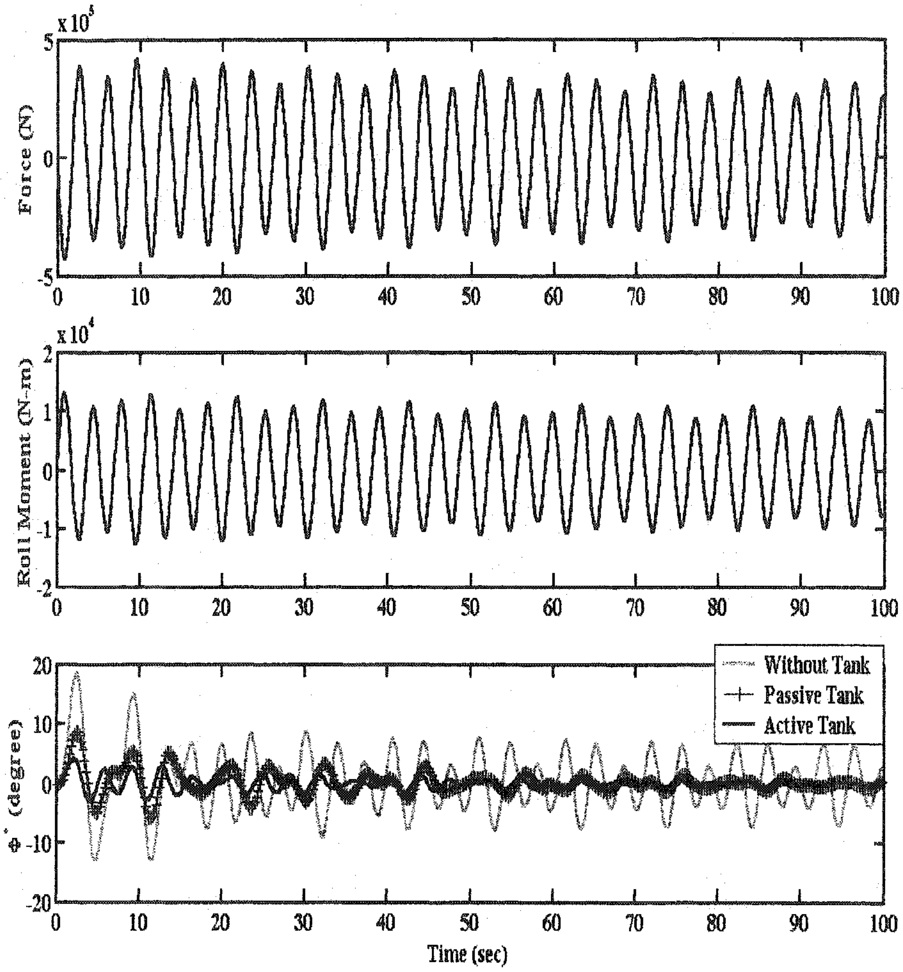

Figure 6.14 Swaying force, rolling moment generated by random wave encounters at Beam Sea $(\beta = 90^\circ)$. Without control (gray line), passive control (puls line), and active control (solid line).

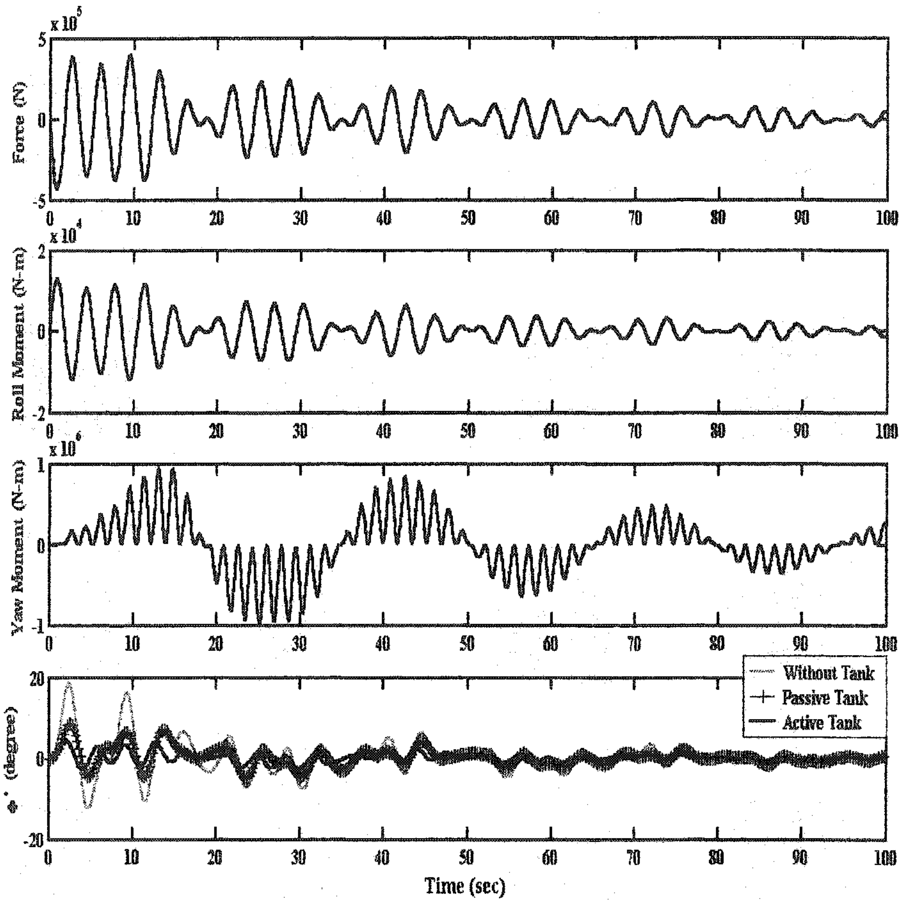

Figure 6.15 Swaying force, rolling moment, yawing moment generated by irregular wave under Sea State 3 encounters at Beam Sea $(\beta = 90^\circ)$. Without control (gray line), passive control (plus line), and active control (solid--line).

6.3 Results for Suboptimal Feedback Control

In practice, full state feedback control may not be implemented because of availability of sensors. In this dissertation, the sub-optimal feedback control will be accomplished because a few sensors are required. The state variables, which are significant variables, are roll angle(ϕ), water head(h), and roll rate$\left(\dot{\phi}\right)$. Figures 6.16

through Figures 6.18 illustrate that the sub-optimal feedback control is also effective. Comparing full state feedback system to suboptimal feedback system is shown. By examining Figure 6.19, we see that the extra control energy is required for using suboptimal feedback control.

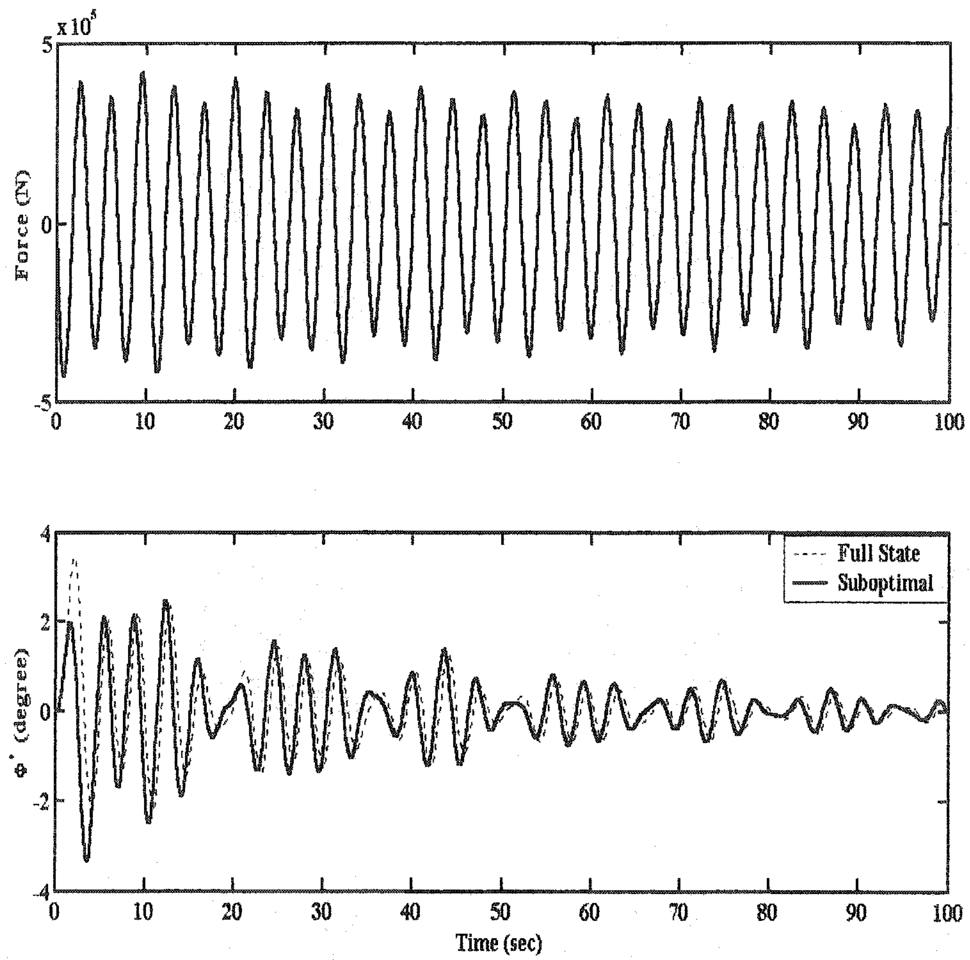

Figure 6.16 Swaying force generated by random wave under encounters at Beam Sea $(\beta = 90^\circ)$. Full state feedback control (dot line) and suboptimal feedback control (solid line).

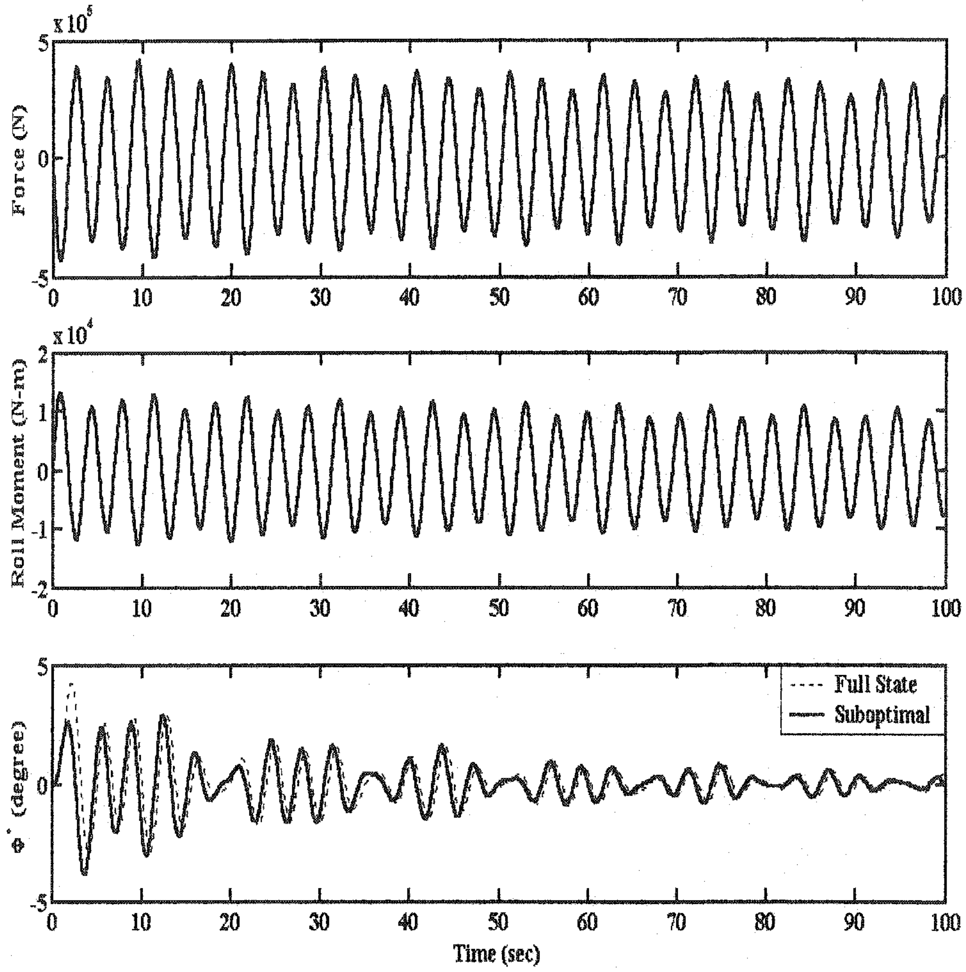

Figure 6.17 Swaying force, rolling moment generated by random wave encounters at Beam Sea $(\beta = 90^\circ)$. Full state feedback control (dot line) and suboptimal feedback control (solid line).

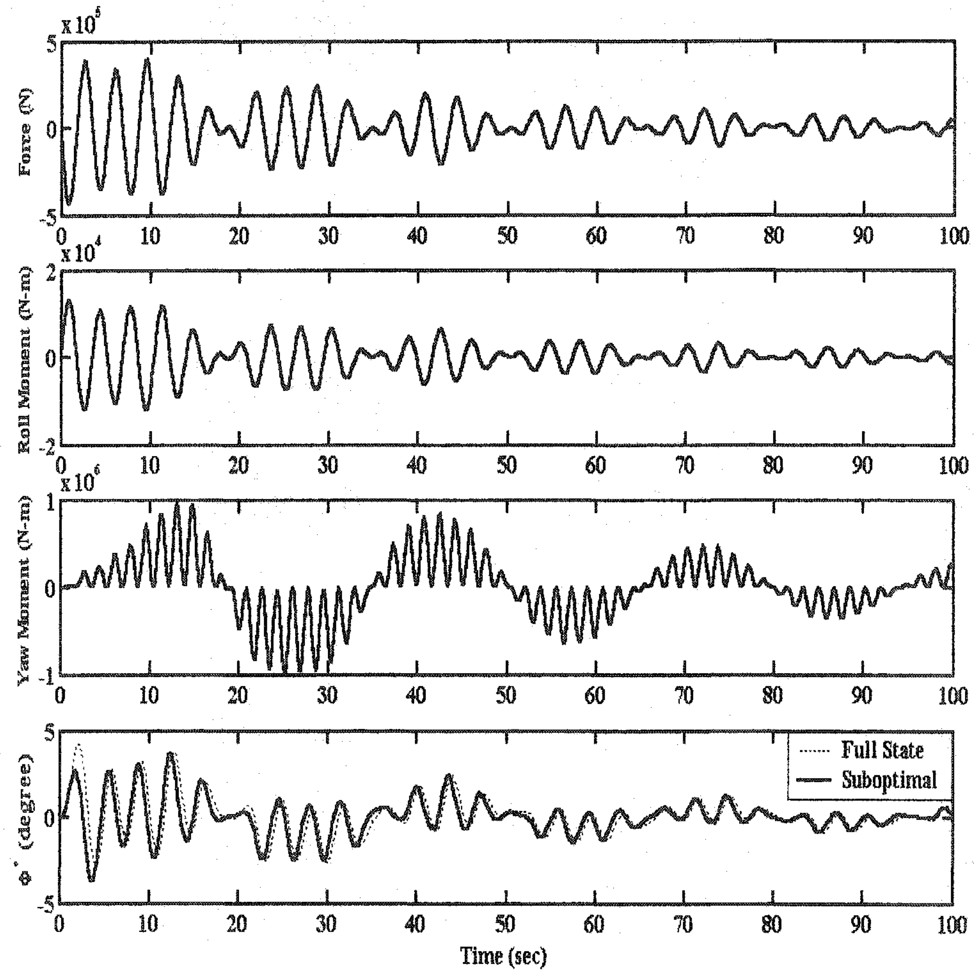

Figure 6.18 Swaying force, rolling moment, yawing moment generated by irregular wave under Sea State 3 encounters at Beam Sea $(\beta = 90^\circ)$. Full state feedback control (dot line) and suboptimal feedback control (solid line).

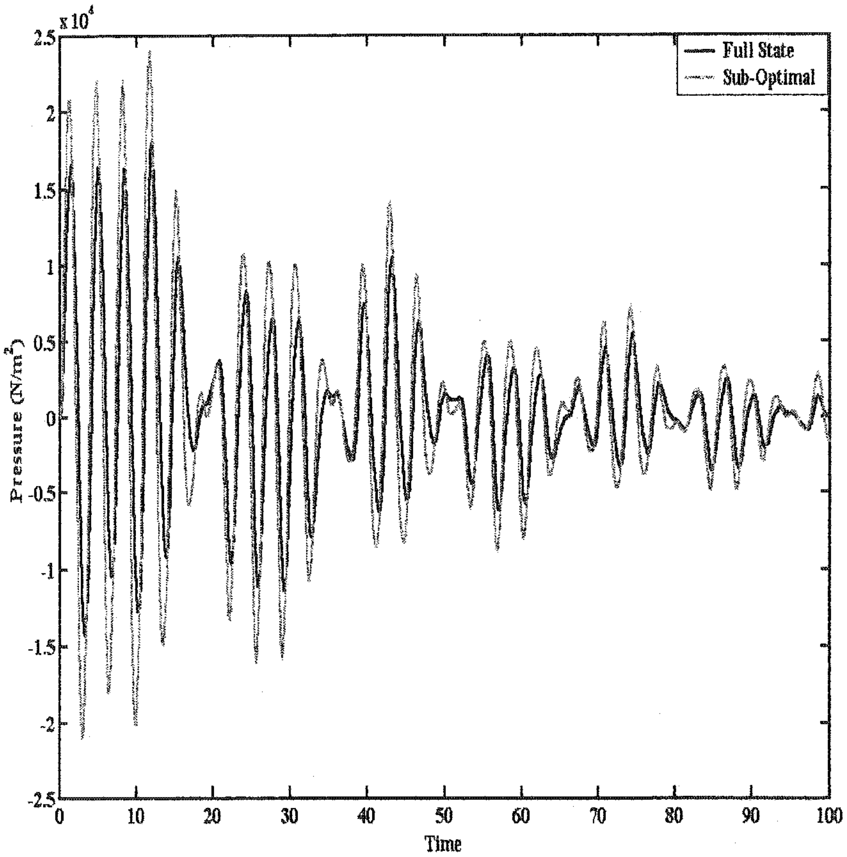

Figure 6.19: Comparing pressure input for full state feedback control (solid line) to pressure input for suboptimal feedback control (gray line).

6.4 Conclusion

This section illustrates the results of ship motion with the various excitations: swaying force, rolling moment, and yawing moment, which are generated by the irregular wave under sea. The large roll amplitude obviously appears when the wave encounters the ship at Beam Sea $(\beta = 90^\circ)$. The results imply that the swaying force has a great influence upon the roll amplitude. For the passive system, the results show that the flume tank is very effective. The roll amplitude can be reduced for over 50% with this mitigation device. A water pump is needed to move the fluid from one tank to another tank much faster. For the faster response, the activated-anti rolling tank has been used. According to the large weighing matrix [Q], the control input, in general, is larger; however, the active system results is more effective than the passive system.

The alternative active control, which represent in this dissertation, is suboptimal feedback control. It helps engineers to decide the economical controller because a few sensors are utilized. In practice, some sensor devices: the sway displacement, yaw angle, sway rate, and even the yaw rate, are not implemented. However, with suboptimal feedback control, the excessive control input is required.

CHAPTER VII

CONCLUSION AND FUTURE WORK

7.1 Conclusion

A ship in the sea will rotate and displace under the influence of waves. In the sea, waves can be categorized by Sea State level. Each level has its own wave strength. In order to counteract roll motion, stabilization is important. There are several ways to reduce roll motion such as using fins, and moving weight. However, the fins will not be effective when the ship is at zero forward speed. Stabilization via moving weight also requires more space. Therefore, the flume tank is presented as stabilizer in this dissertation. The motion of fluid in the tank will generate the counteractive moment in order to reduce roll amplitude. In reality, ship motion can be considered as a rigid body with coupled sway, roll, and yaw motion. In this dissertation, the interaction of ship motion due to the waves, and motion of fluid in the flume tank has been demonstrated. In a passive system, the flume tank is very effective. In order to have a quick response, the optimal feedback control is proposed for active systems. According to the results, roll amplitude using an active system is decreased more than when using only a passive system. In practice, the full state feedback control algorithm may not be implemented. Some sensors may not be available. In this circumstance, the suboptimal feedback control algorithm is feasible.

7.2 Further Extension of the Research

In this dissertation, the dynamics of a flume tank with the coupled sway, roll, and yaw motion are studied. However, in a realistic sea, the ship can possibly encounter six degrees of freedom rather than three. This issue would be an interesting subject for future research. Furthermore, less is better: using as few as possible sensors is one goal for practical reasons so that the results of using output feedback will be studied in order to increase controller performance. The experimental results should also be investigated to prove the simulations' results for extension of this work.

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Appendix A

The coefficients in equation of motion are divided into two modes, vertical mode and horizontal mode. The motions in the vertical mode are surging, heaving, and pitching motion. Likewise, the motions in the horizontal mode are swaying, rolling, and yawing motion.

Vertical Mode

$$A_{11} = \int a_{11} dx$$

$$A_{13} = \int a_{13} dx$$

$$A_{31} = A_{13}$$

$$B_{11} = \int b_{11} dx$$

$$B_{13} = \int b_{13} dx$$

$$B_{31} = B_{13}$$

$$A_{15} = -\int x.a_{13} dx - \frac{U_0}{\omega_e^2} B_{13}$$

$$B_{15} = -\int x.b_{13} dx - U_0 A_{13}$$

$$A_{51} = -\int x.a_{13} dx - \frac{U_0}{\omega_e^2} B_{31}$$

$$B_{51} = -\int x.b_{31} dx - U_0 A_{13}$$

$$A_{33} = \int a_{33} dx$$

$$B_{33} = \int b_{33}dx$$

$$A_{35} = -\int x.a_{33}dx - \frac{U_0}{\omega_e^2}B_{33}$$

$$B_{35} = -\int x.b_{33}dx + U_0 A_{33}$$

$$A_{53} = -\int x.a_{33}dx + \frac{U_0}{\omega_e^2}B_{33}$$

$$B_{53} = -\int x.b_{33}dx - U_0 A_{33}$$

$$A_{55} = \int x^2.a_{33}dx + \frac{U_0^2}{\omega_e^2}A_{33}$$

$$B_{55} = \int x^2.a_{33}dx + \frac{U_0^2}{\omega_e^2}B_{33}$$

$$C_{33} = \int c_{33}dx$$

$$= \rho g \int B(x)dx$$

$$C_{35} = -\int x.c_{33}dx$$

$$= \rho g \int x.B(x)dx$$

$$C_{53} = C_{35}$$

$$C_{55} = \rho g.\nabla.\overline{GM}_L + LCF^2 C_{33}$$

$$\approx \int x^2.c_{33}dx$$

$$= \rho g \int x^2.B(x)dx$$

Horizontal Mode

$$A_{22} = \int a_{22}dx$$

$$A_{24} = \int a_{24}dx$$

$$A_{42} = A_{24}$$

$$A_{26} = \int x.a_{22}dx + \frac{U_0}{\omega_e^2} B_{22}$$

$$A_{44} = \int a_{44}dx$$

$$A_{46} = \int x.a_{24}dx + \frac{U_0}{\omega_e^2} B_{24}$$

$$B_{22} = \int b_{22}dx$$

$$B_{24} = \int b_{24}dx$$

$$B_{42} = B_{24}$$

$$B_{26} = \int x.b_{22}dx - U_0 A_{22}$$

$$B_{44} = \int b_{44}dx + B_e$$

$$B_{46} = \int x.b_{24}dx - U_0 A_{24}$$

$$A_{62} = \int x.a_{22}dx - \frac{U_0}{\omega_e^2} B_{22}$$

$$B_{62} = \int x.b_{22}dx + U_0 A_{22}$$

$$A_{64} = \int x.a_{24}dx - \frac{U_0}{\omega_e^2} B_{24}$$

$$B_{64} = \int x.b_{24}dx + U_0 A_{24}$$

$$A_{66} = \int x^2.a_{22}dx + \frac{U_0^2}{\omega_e^2} A_{22}$$

$$B_{66} = \int x^2.b_{22}dx + \frac{U_0^2}{\omega_e^2} B_{22}$$

$$C_{44} \approx \rho g \nabla \overline{GM}_T$$

Note that all integrals as above are taken over the length of ship.

VITA

Anusit Anmanatarkul completed his Doctoral degree in Mechanical Engineering at Old Dominion University in December 2003. Also, he received his Master of Engineering degree from the Department of Mechanical Engineering at Old Dominion University in August 1998, and Bachelor of Science degree in Mechanical Technology Education from King's Mongkut Institute of Technology Thonburi (KMUTT), Thailand in 1995. After receiving B.S. degree, Mr. Anmanatarkul worked as a lecturer in Department of Mechanical Technology Education at KMUTT. He received a scholarship award from Thai government to study in the United States.

The address for the Department of Mechanical Engineering is Old Dominion University, Kaufman Hall Room 238, Norfolk, VA 23529-0247.