

Daniel Tjie  
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### Explorations of Finger Games

Finger Games, a topic in number theory, study certain 1-1 operations on sequences of 0's and 1's of length  $2f$  - viewing the left and right half as fingers of adjacent hands interpreted using what is called Gray Code. In particular, Finger Games involve the operation of counting in Gray Code by an even number  $C$ , alternating hands. As the operations are 1-1, they divide the sequences into orbits. A previous student, Brittany Rose, computed the orbits for  $C = 4$  and  $8$  and had most of the ideas for  $2^n$  (under the guidance of Professor Rosenthal). In Fall 2015, I was introduced to this research project in Junior Seminar (MATH 39700), a 1-credit research course where I used some results of Brittany to compute the even parts of orbits for  $C = 6$ . In Spring 2016, I continued my work in Research Experience in Mathematics (MATH 39810), a 3-credit course where using preliminary results of mine, I computed the even parts of orbits for  $C = 10$ . In Summer 2016, I was selected as a Summer Scholar to work full-time on my research project where I computed the even parts of orbits for  $C = 14$ . In August 2016 I attended MathFest, the national mathematics conference. In Spring 2017, I will be attending the National Conference of Undergraduate Research (NCUR) at the University of Memphis in Tennessee and compiling a manuscript hopefully to get published in the Mathematics journal, Pi Mu Epsilon (PME). Currently, I am computing even parts of orbits for  $C = 18$ . Based on my work with  $C = 6, 10, \text{ and } 14$ , I expect the orbits for  $C = 18$  to be discovered by a long sequence of results that build on previous results – both of previous students and mine. Such a long chain of results is typical of how lots of math is discovered.

Number theory, is a fascinating branch of mathematics focusing on the behavior of integers, developed by famous mathematicians including Euclid, Euler, and Gauss. Finger Games is a topic in number theory and my focus is finding even parts of orbits for counting by some even values other than powers of 2. Even orbits have special characteristics that include lead changes, of exchanges of leading and nonleading hands, cycles, short and long even orbits, bottom and top overflows and underflows, and other features.

Finger Games has plenty of notations and definitions. Doing each calculation by hand is tedious and so finding patterns (that can be explained) are useful in simplifying the work and reducing error. Finger Games is not just a research project of calculations but a way of connecting orbits since beginning positions will eventually return to themselves. Previous orbits have features that may be useful in discovering future ones. There needs to be some thought between the patterns and articulating those connections. Even the simplest results are sometimes difficult to prove and explain but my work often achieves this.

Counting by small numbers is easy but as those numbers get larger, there are more positions and longer orbits to investigate. There are techniques that can be used to check that orbits begin and end as we would expect. Knowing certain results obtained previously can be useful as orbits can be very closely related. Having devoted over sixteen months to my research project, I am at the stage of writing arguments for the results I obtained. As I graduate this May 2017, future students can continue from where I leave off most likely in the middle of  $C = 18$ . Other ideas include counting by counting by odd numbers, discovering odd parts of orbits, and creating a computer program to compute the orbits.

The main goal of Finger Games is to conjecture interesting and unexpected relationships between numbers and orbits and to prove (or at least explain) these relationships as true or not.