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Vorticity Balance of Outcropping Isopycnals

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ABSTRACT

The authors extend Marshall and Nurser's analysis of potential vorticity (PV) flux into outcropping isopycnic layers of the oceanic thermocline to the nonstationary case, allowing for the seasonal migration of isopycnal surfaces under surface heating and cooling. The most important new result is that the bulk of the surface PV flux arising from seasonal heating is used up in creating stratification as an isopycnal outcrop moves northward, extending the stratified layers of the thermocline. Residual PV *transport* (flux times the separation distance between adjacent isopycnals) reaching the interior thermocline is small in quiescent regions where only mean advection (connecting to subduction or upwelling at the outcrop) operates, and is given by Marshall and Nurser's formula, unaffected by the migration of the isopycnals. Where geostrophic turbulence is vigorous, it supports another pathway of PV transport, via Reynolds flux of vorticity. Larger PV transports are then possible within the range of action of the geostrophic turbulence in locations where Ekman transport only partly balances wind stress.

1. Introduction

Isopycnic layers of the main oceanic thermocline terminate in surface "outcrops" at middle to high latitudes. As Iselin (1939) pointed out, surface heating, freshening, and mixing determine the temperature and the salinity of outcropping isopycnic layers in the North Atlantic. These properties then remain more or less unchanged over very great distances from the outcrops. Potential vorticity (PV) is also a conserved scalar property in the ocean, while at the surface wind stress and heating or cooling act as PV "sources" (or sinks). If the sources give rise to significant PV transport into the interior thermocline, one may expect a pattern of circulation to result because PV transport (flux times distance between adjacent isopycnal surfaces) acts as lateral friction (Csanady and Pelegri 1995).

Our present ideas on the behavior of PV originate from Ertel's (1942) celebrated theorem, followed later by several extensions and applications mainly to meteorology, collected in a recent anthology by Schröder and Treder (1993). Haynes and McIntyre (1987, 1990) have crystallized key ideas involved in an "impermeability theorem" for isopycnal surfaces, and in what might be called an "indestructibility theorem" for isopycnic PV. According to these theorems, the total stock of PV within an isopycnic layer of the ocean remains constant, except for inward or outward directed flux at the intersection of the layer with the free surface or the seafloor. This makes it possible to treat the vorticity balance of a single isopycnic layer independently of other layers. Boundary flux of PV into or out of a layer arises from friction force or from density change due to heating, freshening, or mixing.

Exploiting Haynes and McIntyre's analysis, Marshall and Nurser (1992) have recently discussed surface mixed layer processes responsible for PV fluxes into outcropping isopycnic layers of the main oceanic thermocline. Their approach supposes stationary isopycnal surfaces and surface outcrops. Observations show, however, very large seasonal excursions of isopycnal outcrops spanning distances of order 1000 km in periods of order 10^7 s, that is, at speeds of order 0.1 m s^{-1}. Because these excursions are plainly due to surface heating and cooling, they must have some effect on PV fluxes. On a shorter than seasonal timescale, where outcrops of adjacent isopycnals come close together, hydrodynamic instability of a frontal current (such as the Azores Current) causes chaotic displacements of isopycnal outcrops and associated vigorous geostrophic turbulence. Our primary aim here is to extend Marshall and Nurser's analysis to isopycnals migrating with the seasons. It is a small further step to explore effects of geostrophic turbulence on surface PV fluxes. Our approach employs idealizations similar to an earlier paper dealing with vorticity input at the intersection of isopycnal surfaces with the continental slope (Csanady and Pelegri 1995). We do not expect those idealizations to be realistic under all possible conditions, but they define what we believe is a minimally complex model of the phenomena to be investigated.

In a series of papers, Marshall and Nurser (1991, 1992, 1993; see also Nurser and Marshall 1991) also treated in some detail the important problem of ther-

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mocline circulation forced by surface PV input. Our goal is more limited: we confine our attention to PV transport in outcropping isopycnic layers, at the surface and within the range of any geostrophic turbulence without enquiring into the effect of such fluxes on thermocline circulation.

2. Seasonal migration of surface isopycnals

We will suppose that the surface layer is well mixed and the outcropping isopycnals are vertical near the surface. This is a realistic idealization because the surface layer is always convective to some depth on account of evaporation, as Simpson and Dickey (1981) have emphasized. At the bottom of the mixed layer the outcropping isopycnals bend over sharply and smoothly join their nearly horizontal configuration in the thermocline. On the sea surface, the outcrops are loci of constant density, which we represent as θ = const lines where θ is negative density anomaly, $\theta = 1 - \rho/\rho_0$, (ρ_0 a reference density). A convenient curvilinear coordinate system is along and across isopycnal outcrops, s and n, as illustrated in Fig. 1. In the eastern North Atlantic, which we take to be the prototype region to which our analysis applies, the outcrops generally include a smaller angle with the zonal than the meridional direction so that n is nearly northward and s nearly eastward. Under the influence of surface mechanical and thermal forcing, the isopycnal outcrops move northward or southward.

In analogy with isopycnic coordinates in the oceanic interior replacing the vertical coordinate, we here replace the normal coordinate n by the negative density anomaly, $\theta(s, n, t) = \theta[s, N(s, \theta, t), t]$, where $n = N(s, \theta, t)$ is the equation of an outcrop in the (s, n) coordinate system. The Jacobian of the coordinate transformation is $\partial\theta/\partial n$, and the derivatives are connected by

$$\frac{\partial\theta}{\partial t} + \frac{\partial\theta}{\partial n}\cdot\frac{\partial N}{\partial t} = 0, \quad \frac{\partial\theta}{\partial n}\cdot\frac{\partial N}{\partial\theta} = 1.$$

The total derivative of the negative density anomaly thus becomes

$$\dot{\theta} \equiv \frac{d\theta}{dt} = \frac{\partial\theta}{\partial t} + v_n\frac{\partial\theta}{\partial n} = \frac{\partial\theta}{\partial n}\left(v_n - \frac{\partial N}{\partial t}\right), \quad (1)$$

where v_n is the normal-to-outcrop surface velocity component. The term within parentheses is the surface diapycnal velocity, or velocity relative to a (possibly moving) isopycnal outcrop, analogous to entrainment velocity across a horizontal isopycnal surface. The negative density anomaly tendency $\dot{\theta}$ arises from the divergence of radiant heat flux and of Reynolds fluxes of heat and salt. Here we consider this to be an externally impressed quantity, and refer to it as seasonal heating or cooling, to the associated displacement of the isopycnal outcrops as seasonal migration. It is helpful to keep in mind that in the coordinates chosen $\partial\theta/\partial n$ is negative.

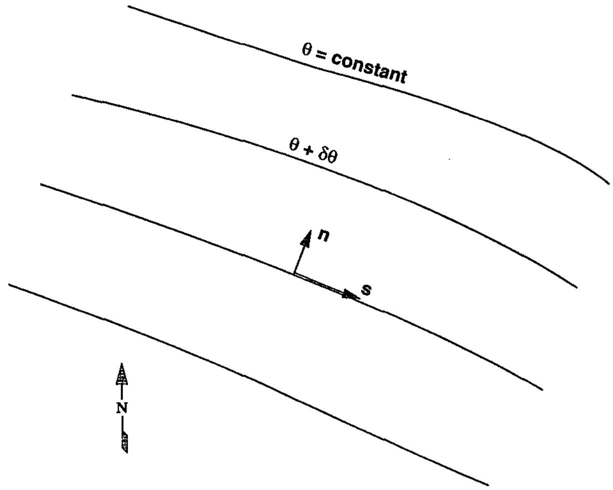

FIG. 1. Schematic illustration of isopycnal outcrops in the northeast Atlantic, showing unit vectors **s**, **n** of curvilinear (s, n coordinate system.

3. Surface flux of potential vorticity

Conservation of potential vorticity may be expressed in the following "flux" form (Haynes and McIntyre 1987, 1990; Marshall and Nurser 1992):

$$\frac{\partial q}{\partial t} + \nabla\cdot\mathbf{J} = 0, \quad (2)$$

with the definitions

$$q = \nabla\theta\cdot(f\mathbf{k} + \omega)$$
$$\omega = \nabla\times\mathbf{u}$$
$$J = \mathbf{u}q + \nabla\theta\times F - \dot{\theta}(f\mathbf{k} + \omega),$$

where q is potential vorticity, f the Coriolis parameter, **u** the velocity vector, **k** the vertical unit vector; **J** is PV flux vector, containing advective flux $\mathbf{u}q$ and contributions from the nonconservative force vector **F** (in kinematic units, i.e., divided by the reference density) and from the seasonal heating $\dot{\theta}$.

At the sea surface, we take **F** to be the wind stress force

$$\mathbf{F} = \mathbf{s}\frac{\partial\tau_s}{\partial Z} + \mathbf{n}\frac{\partial\tau_n}{\partial Z},$$

where τ_s, τ_n are Reynolds stress components along unit vectors **s** and **n**, along and normal to isopycnal outcrops. The vertical component of the PV flux at the surface is now, from Eq. (2), substituting for $\dot{\theta}$ from Eq. (1):

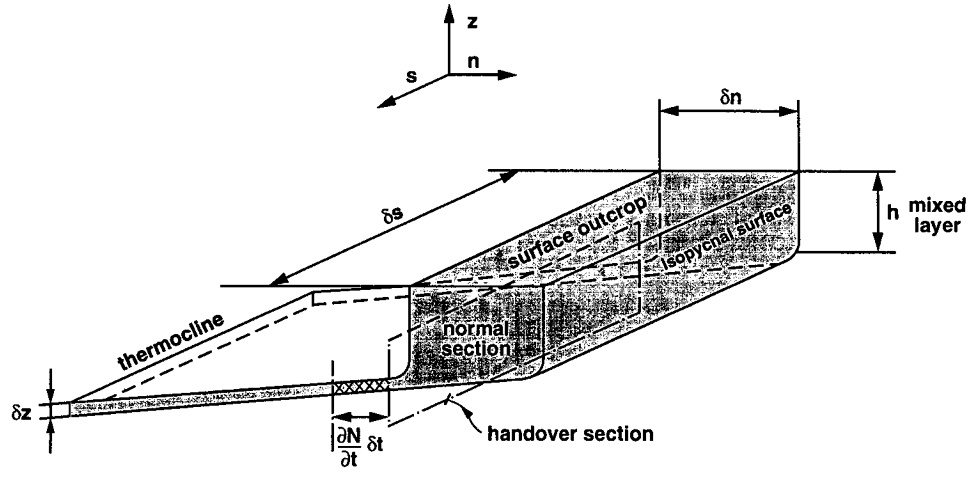

FIG. 2. Control volume used in the PV balance of a mixed layer element enclosed by two adjacent outcropping isopycnal surfaces, two normal sections, and a handover section.

$$\mathbf{J}\cdot\mathbf{k} = -\frac{\partial\theta}{\partial n}\left(v_n - \frac{\partial N}{\partial t}\right)(f+\zeta) - \frac{\partial\theta}{\partial n}\cdot\frac{\partial\tau_s}{\partial Z}, \quad (3)$$

with $\zeta = \partial v_n/\partial n - \partial v_n/\partial S$. The friction force in this result is subject to the along-outcrop momentum balance:

$$\frac{\partial\tau_s}{\partial Z} + v_n(f+\zeta) = \frac{\partial\Pi}{\partial S} + \frac{\partial v_s}{\partial t}, \quad (4)$$

where $\Pi = p + (v_s^2 + v_n^2)/2$ is the "total" pressure. A standard assumption in oceanography is that the left-hand side of this equation vanishes at the sea surface, as the friction force is balanced by Ekman drift. Where the balance is incomplete, the unbalanced (by Ekman drift) friction force is resisted by pressure gradient, or else it accelerates the fluid.

Combining the last two equations, we find for the PV flux across the free surface

$$\mathbf{J}\cdot\mathbf{k} = \frac{\partial\theta}{\partial n}\left[(f+\zeta)\frac{\partial N}{\partial t} - \left(\frac{\partial\Pi}{\partial s} + \frac{\partial v_s}{\partial t}\right)\right]. \quad (5)$$

The first term on the right,

$$\left(\frac{\partial\theta}{\partial n}\frac{\partial N}{\partial t}\right)(f+\zeta),$$

is the heating rate associated with the migration of isopycnal outcrops times absolute vorticity. The second term can also be put into a similar form by defining a generalized geostrophic velocity as

$$v_{ng}(f+\zeta) = \frac{\partial\Pi}{\partial s},$$

and putting $\partial v_s/\partial t = 0$. Equation (5) then reads

$$\mathbf{J}\cdot\mathbf{k} = \frac{\partial\theta}{\partial n}(f+\zeta)\left(\frac{\partial N}{\partial t} - v_{ng}\right),$$

in which form $(\partial\theta/\partial n)v_{ng}$ has the physical meaning of a heating rate, called "net heating" by Marshall and Nurser (1992). For stationary isopycnals, surface PV flux equals this net heating times absolute vorticity.

4. Handover of PV transport

What happens to the surface input of PV? In the idealized case of a mixed layer in which the density gradient is horizontal and absolute vorticity is vertical, PV is zero by definition [in a real mixed layer just small]. Equation (2) then implies that the surface input of PV to the mixed layer is balanced by outputs from the mixed layer somewhere. To examine exactly how and where we calculate the PV balance of a control volume illustrated in Fig. 2: its boundaries are the sea surface, two adjacent isopycnal surfaces θ and $(\theta + \delta\theta)$; two vertical planes normal to the outcrop δs apart, and a vertical plane parallel to the outcrop cutting off the quasi-horizontal interior portion of the isopycnal surfaces at a "handover section." In this section, the vertical separation of the isopycnals, $\delta z = \delta\theta/(\partial\theta/\partial z)$, is much smaller than their horizontal distance on the surface, $\delta n = -\delta\theta/(\partial\theta/\partial n)$, by a factor of some 10^3. When the isopycnal outcrops move northward, they extend the interior surfaces in a period δt by the distance $\delta t(\partial N/\partial t)$.

Potential vorticity fluxes out of the control volume are now $\mathbf{J}\cdot\mathbf{k}$ across the free surface, written down in Eq. (5). At the handover section density tendency and nonconservative force are negligible; hence the outward PV flux is $-\mathbf{J}\cdot\mathbf{n} = -v_n(f+\zeta)\partial\theta/\partial Z$, an advective contribution only. Across the normal planes there can similarly be only advective fluxes, $\mathbf{J}\cdot\mathbf{s} = v_s q$, but in the mixed layer they (nearly) vanish with q. In virtue of the impermeability theorem there is no flux across the isopycnal surfaces.

Gauss's theorem applied to Eq. (2) shows that the integral of $\partial q/\partial t$ over the control volume and the surface integral of the outward PV fluxes add up to zero. As already remarked, q (nearly) vanishes at the surface and over the mixed layer, and so does its time derivative. This holds over most of the control volume, with the exception only of a thin portion at the base where the isopycnal surfaces turn quasi-horizontal. That portion only contributes a quantity of higher order to the integral balances. Of first order, however, is the change of the control volume through the extension or shortening of the quasi-horizontal isopycnal surfaces, implying PV gain or loss. A similar volume change contribution would have to be taken into account if mixed layer base moved rapidly upward or downward.

Integrated outward PV fluxes from the control volume plus PV gain through volume change are therefore at the surface:

$$\mathbf{J}\cdot\mathbf{k}\left(-\frac{\partial n}{\partial\theta}\right)\delta\theta\delta s = -\left[(f+\zeta)\frac{\partial N}{\partial t} - \left(\frac{\partial\Pi}{\partial s} + \frac{\partial v_s}{\partial t}\right)\right]\delta\theta\delta s$$

handover section:

$$\mathbf{J}\cdot\mathbf{n}\frac{\partial Z}{\partial\theta}\delta\theta\delta s = -v_n(f+\zeta)\delta\theta\delta s$$

volume change:

$$(f+\zeta)\frac{\partial\theta}{\partial z}\frac{\partial N}{\partial t}\delta z\delta s = (f+\zeta)\frac{\partial N}{\partial t}\delta\theta\delta s.$$

The sum of these must add up to zero. After canceling the common factor $\delta\theta\delta s$, outward PV *transports* (flux times distance between isopycnals) across the surface and the handover section balance the volume change contribution. The absolute vorticity $(f + \zeta)$ at the control section is, to first order, the same as at the mixed layer base, that is, as at the surface, so that the volume change contribution cancels that part of surface PV transport proportional to $\partial N/\partial t$. In physical terms, surface PV transport arising from that part of seasonal heating that causes the northward migration of the isopycnals, creates the extra volume of stratified, that is, high PV, fluid at the base of the mixed layer. The remaining terms in the balance of PV inputs and outputs yield the relationship:

$$\frac{\partial\Pi}{\partial s} + \frac{\partial v_s}{\partial t} = (f+\zeta)v_n, \qquad (6)$$

where v_n is the normal to the outcrop velocity component *at the handover section,* a quasi-horizontal version of "subduction" velocity. The left-hand side is the residual surface PV transport, which remains after taking care of volume change. In terms of the generalized geostrophic velocity this may be written as $v_{ng}(f + \zeta)$, so that the normal-to-outcrop velocity at the handover section equals v_{ng}, the latter a surface quantity. The right-hand side is advective PV transport from the interior through the handover section.

With the aid of Eq. (4), the residual surface PV transport may also be expressed as shear force less Ekman drift, yielding in place of Eq. (6)

$$\frac{\partial\tau_s}{\partial Z} + (f+\zeta)v_{n|m} = (f+\zeta)v_{n|h}, \qquad (7)$$

where index m designates mixed layer and index h designates handover section quantities. The physical interpretation of the left-hand side is now a residual shear force acting as PV source or sink for the thermocline.

Multiplication of Eq. (6) by $\partial\theta/\partial n$ converts it into Marshall and Nurser's relationship between subduction velocity and net heating, the latter being the heating required to support geostrophic flow across stationary isopycnal outcrops as pointed out above following Eq. (5). North–south migration of the isopycnals reveals a new perspective: the associated heating rate, $(\partial N/\partial t)(\partial\theta/\partial n)$, is the primary source of "new" PV in the thermocline, created through the extension of isopycnic layers northward or destroyed by cooling and contraction of the same southward. One also notes that the total heating rate $\dot\theta$ may vanish at whatever value of "net heating" $v_{ng}(\partial\theta/\partial n)$, in which case Eq. (1) yields $v_{n|m} = \partial N/\partial t$, the flow simply advecting the isopycnal outcrops.

5. Pathways of PV transport

If the standard assumption in oceanography holds and surface shear stress is balanced by Ekman transport, then there is no PV transport into the interior thermocline and all of the surface PV flux is absorbed by extending or reducing the range of stratification. Let us call this case 1.

Suppose next that the shear force–Ekman drift balance is incomplete, leaving a residual shear force. With the left-hand side of Eq. (7) positive and ζ of smaller magnitude than f, $v_{n|h}$ has to be positive, connecting to upwelling in the mixed layer at a velocity some 10^3 times smaller. Excluding regions of vigorous upwelling such as found at the equator or along eastern ocean boundaries, $v_{n|h}$ can then be no higher than about 10^{-3} m s^{-1}. This implies a PV source alias residual shear force no higher than some 10^{-7} m s^{-2}. Given a surface wind stress of order $\tau_s = 10^{-4}$ m^2 s^{-2} and a mixing depth of order 30 m, the ratio of the implied residual shear force to the total is 1/30. If we think of the PV source as opposing pressure gradient instead of residual shear force and express it as a geostrophic velocity $v_{ng|m}$, equal to $v_{n|h}$, multiplication by the north–south temperature gradient yields the net heating rate, in the sense of Marshall and Nurser. With a temperature gradient of 1 K per 100 km this works out to be 1.2 W m^{-2}, a trivial amount compared to the typical seasonal heating rate of some 100 W m^{-2}. The Ekman drift velocity for the same wind stress and mixing depth as supposed above, and $f = 10^{-4}\ s^{-1}$, is $v_n = -0.029$ m s^{-1}, the migration velocity for the typical seasonal heating rate quoted $\partial N/\partial t = 0.051$ m s^{-1}, the surface PV transport absorbed by the extension of the isopycnal surfaces (with negligible ζ) 5.1×10^{-6} m s^{-2}, fifty times the PV transport at the handover section. In this case 2, the case of PV transport into the thermocline by subduction, only a small fraction of the surface PV transport makes it through the handover section.

Consider now a third possibility; the case of outcropping isopycnic layers under the influence of vigorous geostrophic turbulence. We suppose statistically steady conditions over an averaging period much shorter than the seasonal timescale of 10^7 s but longer than the lifetime of any eddies. In this case, the mean value of either Eq. (6) or (7) holds, and mean PV transport into an isopycnic layer equals, neglecting velocity change on the seasonal timescale

$$\overline{\frac{\partial \Pi}{\partial s}} = (f + \bar{\zeta})\bar{v}_n + \overline{v_n' \zeta'}. \tag{8}$$

The new term is the Reynolds flux of vorticity carried by geostrophic turbulence at the handover section. The mean pressure gradient along the isopycnal outcrop in a dynamically active region may well be of the same order as the shear force, say 10^{-6} m s^{-2}. Advection by mean subduction (the first term on the right) remains, however, much smaller. The Reynolds flux of vorticity could then be the means of conveying PV into the interior thermocline, if v_n' is of order 0.1 m s^{-1}, and ζ' of order $10^{-5}\ s^{-1}$, say, both conceivable in a vigorous unstable current. In isopycnal layers of a western boundary current in contact with the seafloor this appears to be the case, Reynolds flux of vorticity distributes PV transport originating as a shear force at the seafloor over the width of the boundary current (Csanady and Pelegri 1995).

A situation such as envisaged here may prevail in the Azores Current. Geostrophic turbulence is known to be present in this region, total pressure generally rises toward the east, and mean cross-isobath velocity $\bar{v}_n$ is small and in any case negative, on account of subduction. Thus, $\overline{v'\zeta'}$ should be positive, transporting negative vorticity downward from the mixed layer. One expects the Azores Current to be enhanced by this scenario, its surface eastward flow accelerated, the effect diminishing with distance from the isopycnal outcrops.

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